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Optimal competitive capacity strategies: Evidence from the container shipping market[☆]

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ABSTRACT

For nearly two decades, ocean carriers have been locked in an arms race for capacity, which has led to huge losses for many and even bankruptcy for some. We investigate the nature of this investment race by studying a long-term capacity investment problem in a duopoly under demand uncertainty. In our model, two firms make sequential capacity decisions, responding to each other's current and future capacity. We consider two types of strategies which differ in terms of how a firm considers the opponent's future capacity in its own strategy: a proactive strategy where the firm assumes that the opponent will respond using a certain strategy, or a reactive strategy where the firm assumes that the opponent's future capacity remains unchanged. In the proactive case, we allow the firm to have different assumptions on the opponent's strategy, representing different amounts of information the firm has on the opponent. For each type of strategies, we derive the firm's optimal decisions on both the timing and size of capacity adjustments, specified by an array of intervals for the optimal capacity in a given capacity space in each period. Using detailed data from the container shipping market (2000–2015), we illustrate how to plan competitive capacity investments, following our model. By comparing the optimal decisions specified by our model with the reality, we show that the realized capacity decisions of the leading carriers, which were often questioned as irrational, are close to optimal, assuming these carriers follow proactive strategies. By revealing the underlying structures of different strategies, that is, the stayput intervals, we show how a specific strategy brings value to firms under competition. Based on our results, we provide practical guidelines to carriers and firms which operate in a similar competitive market for implementing an effective competitive capacity strategy.

1. Introduction

Over the past two decades, we have observed a striking investment race among ocean carriers for capacity. Since the launch of the first *ultra large container vessel*[1] in 2006, by 2015 the size of the largest container vessel increased by 24%. Between 2010 and 2017, the world fleet capacity in fully cellular containerships increased by over 56.9%. However, in stark contrast to the enormous increase in fleet capacity, the shipping industry has had a difficult ride since the 2008 global recession [9]. The battle of survival for carriers can only be partially pinned on the recession or to buying too many ships before the recession started, in anticipation of continued demand growth. The situation was drastically aggravated by post-crisis investment cascades [33]. Except the first one, all *ultra large container vessels* were ordered after 2008. In 2011 alone, CMA-CGM increased the capacity option of its three on-order vessels (Marco Polo vessels) by 15.7% [18], followed by Maersk's $3.8 billion investment in 20 Triple-E-class vessels, which led the size of the largest containership to instantly rise by another 14.2% [34]. Instead of boosting profit, these investments caused high volatility in freight rates and profit losses for many carriers [58–60]. Immediately after CMA-CGM and Maersk had placed their orders, the spot rates in the Asia-Europe market hit rock bottom, dropping from an average value of $1789 per TEU in 2010 to $450 per TEU in December 2011 [41,59]. Consequently, in 2011 many carriers depleted their cash reserves. Due to the intensity of the investment race and the destructive outcome, industry experts have been calling these investments irrational and increased research efforts have

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[1] Container vessels are distinguished into seven major size categories and the category of *ultra large container vessels* includes container vessels with a capacity of 14,501 TEU and higher [66].

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been called for to explain excess capacity and persistent overinvestment in the container shipping market [31,33].

We propose an investment framework for explaining the nature of the capacity investment race in the container shipping market. At the initial stage of our research in 2016, we conducted interviews with multiple carriers, asking about why they invested in recent years. Managers of newbuilding explicitly stated that among demand growth, economies of scale, and other common investment drivers, competition was a key driver for the post-crisis investments:

"If the no.1 does it, no.15 also has to do likewise or he will be kicked out of the market, because they are no longer competitive." (shipping line A); "It is a must to survive! When you cannot beat them, join them." (shipping line B); "You need to stay relevant for your alliance, how fragile that might be. If you are very much behind on your investment, nobody would like to partner with you." (shipping line C); "Everybody who is doing investments now (2016) in new ships, is not based on growth of world market." (shipping line D).

Having known what the other firms have purchased, carriers responded to these investments through their own investments. Little research has explicitly focused on an investment race with such a competitive nature and on the optimal structure of a firm's strategy in the race. Our research fills the gap by studying a long-term capacity investment problem in which two firms make sequential capacity decisions, responding to each other's current and future capacity. Because capital assets like containerships have long lifetimes, present investment decisions influence decisions in the future. A long-term strategy should address the optimal timing and size of capacity adjustments. Capturing the optimal timing under competition requires firms to balance the financial risk of investing and the competitive risk of not investing. Once-in-a-cycle delays can create a lasting competitive disadvantage in a multi-round investment race. Similarly, the optimal size of competitive capacity requires a careful trade-off analysis. Investing too little certainly does not grant firms a competitive advantage, e.g., it is not enough to deter the opponent's future investments. However, competition does not necessarily drive firms to build the maximum possible capacity. Examples such as the lack of success of the Airbus 380 have demonstrated that more is not always better under competition [10].

We consider two types of strategies in the investment race: a proactive strategy where the firm plans its capacity, assuming that the opponent will respond using a certain strategy, or a reactive strategy where the firm plans its capacity, assuming that the opponent's future capacity will remain unchanged. Being proactive is not necessarily better than being reactive. In a proactive strategy, the firm needs to have information on the opponent's strategy and this information may be incorrect. In addition, proactive strategies likely lead to more capacity adjustments, potentially resulting in lower prices and higher capacity costs for the firm. In our model, we allow the proactive firm to have different assumptions on the opponent's strategy, representing different amounts of information the firm has on the opponent. We derive the firm's optimal strategy in terms of the timing and size of capacity adjustments, following the structure of an ISD (*Invest, Stayput, Disinvest*) policy. Such a policy is characterized by a set of stayput intervals in the solution space. If the firm's current capacity falls in a stayput interval, the optimal decision is to stay put; otherwise, it should adjust its capacity to the closest boundary of a close-by interval. We apply our model to the container shipping market using 16-year (2000–2015) data. By comparing the optimal decisions specified by our model with the reality, we show that the investments of the leading carriers, which were often questioned to be irrational, follow an optimally proactive structure. By revealing the underlying structure of different strategies, that is, the stayput intervals, we show how a specific strategy brings value to firms under competition. Based on our results, we provide practical guidelines to carriers and firms which operate in a similar competitive market for implementing an effective competitive capacity strategy.

We contribute to theory and practice in five ways: *First*, to theory by investigating the strategic implications of capacity in the investment race and by providing a theory that can explain the investment phenomena observed in practice. Current models do not fully explain these phenomena as they do not explicitly consider the competitive feedback nature of the race; *Second*, to theory by investigating the optimal structure of the long-term competitive strategy, answering the question of how firms should timely respond to the current competition, in anticipation of future changes. Current research typically focuses on myopic strategies which assume that the future is fixed forever or Cournot-Nash equilibrium strategies which neglect the direct feedback between firms' decisions; *Third*, to theory by studying proactive strategies with different amounts of information on the opponent's strategy. Current models mostly assume a symmetric scenario in which all firms know each other's strategy; *Fourth*, to theory by deriving the full optimal policy in terms of both the timing and size of capacity adjustments. Existing dynamic models often focus on timing only, given fixed-sized capacity options. In addition, different from the majority of the existing models which only focus on a single period and the ultimate decision, i.e., the optimal capacity in that period, our method derives the complete set of optimal decision intervals in the available capacity space, revealing the competitive value of capacity; *Fifth*, to practice by providing a framework for planning capacity investments in the long-term competition.

2. Literature review

Our research is related to capacity models that are concerned with strategically determining the timing and size of buying or selling additional capacity (see [62] and [15] for a detailed literature review). Two categories of capacity models can be distinguished: (1) *static* models and (2) *dynamic* models. Table 1 gives a comparison of some existing capacity models in each category and our model, in terms of whether the model considers multiple rounds of decisions, whether the optimal solution specifies timing or size of capacity adjustments, whether it considers competition between vertical firms such as manufacturers and retailers or horizontal firms who supply a common market, and whether it considers simultaneous or sequential decisions of horizontal firms.

Static models investigate the optimal capacity locations and sizes in a processing network for a single or for multiple decision makers in a stationary environment where there is no managerial flexibility to cope with market changes [11,63]. This collapses the problem to a single initial capacity investment where the optimal capacity remains constant over time. This category of capacity models adopts queuing [13,32] and newsvendor network formulations [30,39,61]. *Dynamic* models allow time-dependent investments to respond to the resolution of uncertainty. They emphasize the timing of capacity adjustments in a single-shot or a multi-round game [5,12,27,28,38,48]. Some noted approaches in this category are decision-tree analysis, dynamic programming, control theory, and real-options models [20,22,36,51,55]. Often, investment is viewed as an optimal stopping problem, focusing on finding the demand values at which capacity should be adjusted to maximize the expected reward. Eberly and Van Mieghem [21] derive the optimal investment dynamics as an ISD (invest, stayput and disinvest) policy which is characterized by a single capacity interval.

Based on Eberly and Van Mieghem [21]'s solution approach, we develop a method to analyze investment opportunities involving important competitive and strategic implications under uncer-

Table 1
Comparison of some existing capacity models and our model.

Capacity models	Examples	Multi-round	Investment decisions		Firm interaction		
			timing	*size*	*vertical*	*horizontal*	
						simultaneous	*sequential*
Static	[39]			✓			
	[30]			✓			
	[61]	✓		✓			
	[44]			✓	✓		
	[14]			✓	✓		
	[32]			✓		✓	
	[64]			✓		✓	
Dynamic	[21]	✓	✓	✓			
	[48]	✓	✓	✓			
	[28]	✓	✓	✓			
	[19]		✓				✓
	[26]		✓				✓
	[42]		✓				✓
	[54]		✓				✓
	[53]		✓			✓	
	[24]	✓	✓			✓	
	[1,2]	✓	✓			✓	
	Our model	✓	✓	✓			✓

tainty. Our model belongs to the group of *dynamic* capacity models in which firms condition their decisions not only on the resolution of exogenous uncertainty, but also on the (re)actions of competitors [6,15,51]. Numerical results of a multi-round investment problem can be derived using stochastic dynamic programming and Monte Carlo simulation [37]. For analytical results, control theory is used to derive capacity strategies in a Nash framework [24]. Research that examines dynamic competitive investments with an explicit focus on a firm's strategic thinking is still lacking. Such research has been applied to some extent in the financial domain, e.g., to research and development investments. A limitation of these studies is that they specify a fixed size for the action available to firms and focus only on the timing of taking this action. In addition, current research has been limited to Cournot competition (e.g., [45]) in which a firm sees the opponents' strategies as exogenous stochastic processes. The Cournot-Nash equilibrium strategy, although mathematically tractable, is considered as an open-loop strategy. It fails subgame perfection: if one firm deviates from the equilibrium strategy, driving the price down or up, other firms ignore this and continue to invest [7].

Our model considers a sequential competition between firms and a feedback strategy. In addition to timing, our model allows the size of an investment to be determined by the optimal policy and thus we study more complete features of a capacity strategy. Sarkar [51] considers both the timing and size, and the impact of competition on the investment. However, he captures competition as an exogenous factor, measuring the firm's market power by the price-sensitivity of the product, and thus viewing the firm in a competitive market as having less market power. His results show that the firm's investment is more sensitive to demand uncertainty if it operates in a competitive market. Rau and Spinler [45] used real-options models to study the capacity strategy in the container shipping market. Focusing on a Cournot-Nash equilibrium, they consider competition from two perspectives: the price elasticity of demand and the number of firms. Our research differs as we use a Stackelberg framework. We contribute to the extant literature by studying sequential feedback strategies in which all firms respond to the investment of any other firm like a Stackelberg follower throughout the investment game. Moreover, our feedback strategies incorporate a firm's assumption on the opponent's strategy, instead of the real strategy. Therefore, we study the value of information in the firm's strategic investments.

3. Our model

The main notations used in our model are summarized in Table 5 in the appendix. We consider two firms (l and f) selling a homogeneous product, e.g., shipping service, in an oligopolistic market within a finite time horizon $\Gamma = \{1, \cdots, T\}$, assuming capacity is instantaneously adjustable. We use the subscript i, $i \neq j$, to refer to the opponent of firm j, $i, j \in \{l, f\}$. For $t \in \Gamma$ and $j \in \{l, f\}$, let k_{tj} represent firm j's capacity level in period t and let the finite set $\mathscr{K}_{tj} \subseteq \mathbb{R}_{\geq 0}$ denote the set of available capacity choices, $k_{tj} \in \mathscr{K}_{tj}$. The origin and final values of $\mathscr{K}_{tj}$ are denoted as k^o_{tj} and k^e_{tj}, respectively: $k^o_{tj} = \inf \mathscr{K}_{tj}$ and $k^e_{tj} = \sup \mathscr{K}_{tj}$. At the beginning of period $t \in \Gamma$, firm l first changes its capacity from k_{t-1l} to k_{tl}, and then firm f observes k_{tl} and changes its capacity from k_{t-1f} to k_{tf}. Based on the decision sequence, we refer to firm l as the leader and firm f as the follower. The initial capacity of the two firms are denoted as k_{0l} and k_{0f}, respectively.

A firm's optimal capacity in each period is determined based on the demand, supply, and investment cost information then available to the firm and on its assessment of the uncertain future. At the beginning of period t, firms observe the current demand, denoted as, ω_t, and predict the demand growth of this year. $\omega_t \in \Theta$, where Θ is the set of demand values and $\Theta \subseteq \mathbb{R}_{\geq 0}$. We assume that exogenous uncertainty exists in ω_τ, $\forall \tau > t$, and it possesses a Markov property. Thus, demand information relevant to the capacity decision in period t includes only the current demand ω_t and the transition probability. We denote the transition probability function of demand as $Pr : \Theta \times \Theta \times \Gamma \to [0, 1]$, $\Pr\{\omega_{t+1} = x_{t+1} \mid \omega_t = x_t\} = Pr(x_t, x_{t+1}, t)$. Let the two firms' capacity, demand, and time define the state of the system. At the beginning of period t, the leader observes state $\mathbf{Y}_{tl} = (k_{t-1l}, k_{t-1f}, \omega_t, t) \in \mathscr{K}_{t-1l} \times \mathscr{K}_{t-1f} \times \Theta \times \Gamma$ and decides k_{tl}. The follower then observes state $\mathbf{Y}_{tf} = (k_{tl}, k_{t-1f}, \omega_t, t) \in \mathscr{K}_{tl} \times \mathscr{K}_{t-1f} \times \Theta \times \Gamma$ and decides k_{tf}. Hereinafter, we omit the time parameter t in state vectors.

After having made capacity decisions, the two firms face a single-period production problem. Production decisions (i.e., capacity usages) do not have the same strategic implications as capacity decisions [37,62]. Some capacity models study volume flexibility, but in a single-firm setting [47,49,50,67]. Since our focus is on strategic capacity decisions for a single product, we do not consider flexible capacity usage in the long-term problem. We assume that the two firms will produce up to their capacity. Given the total available capacity, the price of the homogeneous product is specified by an inverse demand function, $P_t(k_{ti}, k_{tj}, \omega_t)$. Let $H_{tj}(k_{tj})$ represent firm j's setup and operating cost function in period t, dependent on the firm's capacity k_{tj}. Equation (1) lists firm j's operating profit in period t.

$$\pi_{tj}(k_{ti}, k_{tj}, \omega_t) = P_t(k_{ti}, k_{tj}, \omega_t)k_{tj} - H_{tj}(k_{tj}), \qquad \forall t \in \Gamma \tag{1}$$

It is possible that $P_t(k_{ti}, k_{tj}, \omega_t)k_{tj} < H_{tj}(k_{tj})$, implying that firm j can have negative profits for a period. This is rational as the firm's objective considers the long-term profit, and thus a single-period loss will unlikely lead the firm to sell all its assets. It is also consistent with the common practice in the container shipping market, that is, in order to remain active in the market, carriers continue to operate even if they are facing a loss. For instance, in 2016 and 2017, none of the major carriers expected to be profitable, yet they still maintained their operations [65].

At the end of the time horizon Γ, the salvage value of firm j is determined by the function $F_j(k_{Ti}, k_{Tj}, \omega_{T+1})$. Since the marginal profit of an investment is usually non-increasing, in Assumption 1 we formalize our assumption on firm j's operating profit and salvage value being concave in its capacity decision, given a fixed capacity of the opponent. Examples of functions that satisfy this assumption include those associated with the market-clearing price or isoelastic prices.

Assumption 1. For any given and fixed capacity of the opponent $k_{ti} \in \mathscr{K}_{ti}$ and for each $\omega_t \in \Theta$, firm j's operating profit function $\pi_{tj}(k_{ti}, \cdot, \omega_t)$ and salvage value function $F_j(k_{Ti}, \cdot, \omega_t)$ are concave in its own decision k_{tj}.

We define the investment cost function of firm j in period t as a kinked piece-wise linear function: $C_{tj}(k_{tj}) = c_{tj} \times (k_{tj} - k_{t-1j})^+ - r_{tj} \times (k_{t-1j} - k_{tj})^+$, where $(x)^+$ denotes $\max\{0, x\}$. The parameters c_{tj} and r_{tj} are the marginal investment cost and marginal disinvestment revenue, respectively. As purchasing capital assets or technology is partially irreversible, we make the following assumption on the investment cost parameters c_{tj}, r_{tj} and the discount rate δ:

Assumption 2. Capacity investment is costly to reverse as $c_{tj} > r_{tj}$. In addition, the present value of a unit of used capacity cannot be higher than a new unit: $c_{tj} \geqslant \delta^{\tau-t} r_{\tau j}$, $\forall \tau \in \{t, \cdots, T\}$, where $\delta > 0$ is the single-period discount factor.

The optimal value function

The value of a firm's long-term strategy consists of the value of each capacity decision planned for the future. Let $\mathbf{K}_{tj} = (k_{tj}, k_{t+1j}, \cdots, k_{Tj})$ denote firm j's capacity strategy vector from period t to the end of the time horizon Γ and let $\mathcal{K}_{tj}$ denote the set of all capacity strategy vectors, $\mathbf{K}_{tj} \in \mathcal{K}_{tj}$. At the beginning of period t, the follower's expected net present value (NPV) depends on the state $\mathbf{Y}_{tf} = (k_{tl}, k_{t-1f}, \omega_t)$ that is observed, and on the two firms' future capacity, $\mathbf{K}_{t+1l}$ and $\mathbf{K}_{tf}$. The leader's expected NPV depends on the state $\mathbf{Y}_{tl} = (k_{t-1l}, k_{t-1f}, \omega_t)$, $\mathbf{K}_{tl}$, and $\mathbf{K}_{tf}$. Equation (2) gives firm j's NPV function:

$$V_{tj}(\mathbf{Y}_{tj}, \mathbf{K}_{t+1i}, \mathbf{K}_{tj}) = E\Bigg[\sum_{\tau=t}^{T} \delta^{\tau-t}(\pi_{\tau j}(k_{\tau i}, k_{\tau j}, \omega_\tau) - C_{\tau j}(k_{\tau j})) + \delta^{T+1-t} F_j(k_{Ti}, k_{Tj}, \omega_{T+1}) \mid \omega_t\Bigg], \tag{2}$$

In firm j's expected NPV function above, the opponent's future capacity $\mathbf{K}_{t+1i}$, is specified by firm j's assumption of the opponent's strategy (note that if $j = l$, then the opponent's future capacity is $\mathbf{K}_{ti}$; hereinafter, unless specified otherwise, we present solutions for $j = f$). In order to derive a structured optimal strategy, we assume that for each $k_{\tau j}$ in $\mathbf{K}_{tj}$, a value of $k_{\tau+1i}$ is projected according to a rule and this rule is consistent for all $\tau \in \{t, \cdots, T-1\}$.

All existing oligopoly capacity models implicitly assume that both firms know ex ante each other's exact response to the firm's own strategy (e.g., [24,40]). This assumption may hold true for the leader, as it can exert some control over the market and thus knows the follower's possible responses. However, the same assumption is not always applicable to the follower. We assume that when the leader can plan its investments proactively, considering the follower's responses, the leader also has full information on the follower's responses as part of the first mover advantage. Full information includes the follower's assumption on the leader's strategy as it is considered in the follower's long-term strategy. When the follower can adopt a proactive strategy, we consider two scenarios where the follower's proactive thinking has complete information and incomplete information, respectively. An example of the latter scenario is that while the leader is being self-interested, the follower sees the leader being adversarial, i.e., the follower thinks that the leader decides its capacity to minimize the follower's value. It is not unrealistic for a firm to speculate that the opponent is being adversarial in the investment race. The container shipping market is controlled by a small number of carriers who can influence the price of the common product through their capacity. Taking advantage of this market feature, leading carriers can squeeze the profit margin of other carriers by adding more capacity [31,45]. Once competitors have been squeezed out of the market, leading carriers can then change their objective to maximize their own profit.

After specifying the rule on the relation between $k_{\tau j}$ and $k_{\tau+1i}$, we can omit $\mathbf{K}_{t+1i}$ in firm j's value function, i.e., $V_{tj}(\mathbf{Y}_{tj}, \mathbf{K}_{tj})$. Firm j's optimal value function at the beginning of period t is: $V^*_{tj}(\mathbf{Y}_{tj}) = \sup_{\mathbf{K}_{tj} \in \mathcal{K}_{tj}} V_{tj}(\mathbf{Y}_{tj}, \mathbf{K}_{tj})$. In any proactive case, the optimal value function V^*_{tj} suffers the curse of dimensionality. We use recursive optimality equations to derive the optimal strategy $\mathbf{K}^*_{tj}$. At the end of the time horizon Γ (or at the beginning of period $T+1$), firm j's salvage value associated with the state $\mathbf{Y}_{T+1j} = (k_{Tl}, k_{Tf}, \omega_{T+1})$ is: $V^*_{T+1j}(\mathbf{Y}_{T+1j}) = F_j(k_{Ti}, k_{Tj}, \omega_{T+1})$. According to Bellman's principle of optimality, at the beginning of period $t \in \Gamma$, firm j's optimal value function associated with the state $\mathbf{Y}_{tf} = (k_{tl}, k_{t-1f}, \omega_t)$ or $\mathbf{Y}_{tl} = (k_{t-1l}, k_{t-1f}, \omega_t)$ equals the following:

$$V^*_{tj}(\mathbf{Y}_{tj}) = \sup_{k_{tj} \in \mathscr{K}_{tj}} \Big\{\pi_{tj}(k_{ti}, k_{tj}, \omega_t) - C_{tj}(k_{tj}) + \delta E[V^*_{t+1j}(\mathbf{Y}_{t+1j}) \mid \omega_t]\Big\}, \tag{3}$$

We define a function G_{tj} as firm j's expected NPV evaluated in period t, given that its capacity has been adjusted to k_{tj} and an optimal follow-up capacity strategy will be implemented: $G_{tj}(\mathbf{Y}_{tj}, k_{tj}) = \pi_{tj}(k_{ti}, k_{tj}, \omega_t) + \delta E[V^*_{t+1j}(\mathbf{Y}_{t+1j}) \mid \omega_t]$. Substituting G_{tj} into equation (3), the optimization problem of firm j in period t equals the following:

$$V^*_{tj}(\mathbf{Y}_{tj}) = \sup_{k_{tj} \in \mathscr{K}_{tj}} \Big\{G_{tj}(\mathbf{Y}_{tj}, k_{tj}) + r_{tj} \times (k_{t-1j} - k_{tj})^+ - c_{tj} \times (k_{tj} - k_{t-1j})^+\Big\} \tag{4}$$

The intuition behind Eq. (4) is that given an optimal follow-up strategy, the firm's optimal capacity in the current period can be determined by comparing the cost of increasing the current capacity with the revenue of reducing it. Eberly and Van Mieghem [21] solve the above optimization problem for a single-firm case. They show that if the optimal value function is strictly concave, the optimal policy can be represented in the form of a unique stayput interval, which is a continuum of optimal solutions to the investment problem. The boundaries of the stayput interval define the decision rule for investments: if the current capacity falls within the boundaries (i.e., inside the stayput interval), it is optimal not to adjust capacity; otherwise, capacity should be adjusted to an appropriate point on the interval' s boundary. The policy which takes such a form is referred to as an ISD (invest, stayput and disinvest) policy. In the next section, we focus on a multi-period sequential competition between the two firms. We adopt the solution approach from Eberly and Van Mieghem [21], while our contribution lies in the development of a method which incorporates the opponent's ISD policy in a firm's own ISD policy.

4. The competitive capacity investment policy: Reactive vs proactive

In the duopoly, if firm j's optimal value function V^*_{tj} is jointly concave in (k_{t-1j}, k_{tj}) for any given $k_{ti} \in \mathscr{K}_{ti}$ and for each $\omega_t \in \Theta$, its ISD policy can be derived following similar lines as in the proof of *Theorem 2* of [21]. In Proposition 1, we present such an ISD policy. All proofs are given in the appendix.

Proposition 1. *Given the current state $\mathbf{Y}_{tj}$, if firm j's optimal value function V^*_{tj} is jointly concave in (k_{t-1j}, k_{tj}), then its optimal capacity in period t is specified by an ISD policy that is characterized by the following functions:*

$$k^L_{tj} = \sup\left\{\{k^o_{tj}\} \cup \{k_{tj} : \frac{\nabla_- G_{tj}(\mathbf{Y}_{tj}, k_{tj})}{\nabla k_{tj}} \geq c_{tj}, \quad k_{tj} \in \mathscr{K}_{tj}\}\right\} \tag{5}$$

$$k^H_{tj} = \inf\left\{\{k^e_{tj}\} \cup \{k_{tj} : \frac{\nabla_+ G_{tj}(\mathbf{Y}_{tj}, k_{tj})}{\nabla k_{tj}} \leq r_{tj}, \quad k_{tj} \in \mathscr{K}_{tj}\}\right\}, \tag{6}$$

where $\frac{\nabla_- G_{tj}(\mathbf{Y}_{tj},k_{tj})}{\nabla k_{tj}}$ and $\frac{\nabla_+ G_{tj}(\mathbf{Y}_{tj},k_{tj})}{\nabla k_{tj}}$ are the infimum of all left-sided difference quotients and the supremum of all right-sided difference quotients of the function $G_{tj}(\mathbf{Y}_{tj}, k_{tj})$ at the point k_{tj}, respectively: $\frac{G(\mathbf{Y}_{tj},a)-G(\mathbf{Y}_{tj},k_{tj})}{a-k_{tj}} \geq \frac{\nabla_- G(\mathbf{Y}_{tj},k_{tj})}{\nabla k_{tj}}$, $\forall a < k_{tj}$, and $\frac{G(\mathbf{Y}_{tj},b)-G(\mathbf{Y}_{tj},k_{tj})}{b-k_{tj}} \leq \frac{\nabla_+ G(\mathbf{Y}_{tj},k_{tj})}{\nabla k_{tj}}$, $\forall b > k_{tj}$, where a, b and k_{tj} are in the domain of G. Set $S_{tj} = [k^L_{tj}, k^H_{tj}]$. Firm j's optimal capacity in period t is determined based on its current capacity k_{t-1j} and S_{tj}: if $k_{t-1j} \in S_{tj}$, no adjustment should be made, i.e., $k^*_{tj} = k_{t-1j}$; if $k_{t-1j} < k^L_{tj}$, an investment should be made such that the new capacity hits the lower boundary of S_{tj}, i.e., $k^*_{tj} = k^L_{tj}$; if $k_{t-1j} > k^H_{tj}$, a disinvestment should be made such that the new capacity hits the higher boundary of S_{tj}, i.e., $k^*_{tj} = k^H_{tj}$.

In Proposition 1, $\frac{\nabla_- G_{tj}(\mathbf{Y}_{tj},k_{tj})}{\nabla k_{tj}}$ and $\frac{\nabla_+ G_{tj}(\mathbf{Y}_{tj},k_{tj})}{\nabla k_{tj}}$ can be seen as firm j's (minimal) marginal value of investment and (maximal) marginal value of disinvestment at capacity k_{tj}, respectively. Thus, the ISD policy specifies an interval S_{tj} of which the (minimal) marginal value of investment equals the marginal investment cost c_{tj} at the lowerbound and the (maximal) marginal value of disinvestment equals the marginal disinvestment revenue r_{tj} at the upperbound. The concavity of V^*_{tj} indicates that capacity outside S_{tj} should be adjusted to the closest boundary of S_{tj}. In a single-firm case, Eberly and Van Mieghem [21]'s method directly calculates the boundaries of the stayput interval using difference quotients of the value function. However, in the duopoly, such boundaries depend on the other firm's future responses.

If firm j adopts the reactive strategy, it observes the latest decision of the opponent and assumes that it will remain unchanged for the rest of the timespan. In Proposition 2, we show that under Assumptions 1 and 2, the firm's optimal value function is concave in its capacity decision. Thus, the reactive ISD policy takes the same form as in Proposition 1 and the resulting interval S_{tj} is a function of the opponent's last observed decision k_{ti}.

Proposition 2. *Under Assumptions 1 and 2, if firm j adopts the reactive strategy, the optimal value function V^*_{tj} is jointly concave in (k_{t-1j}, k_{tj}) for any given current capacity of the opponent $k_{ti} \in \mathscr{K}_{ti}$, if $i = l$ (or $k_{t-1i} \in \mathscr{K}_{t-1i}$, if $i = f$) and for each $\omega_t \in \Theta$.*

If firm j adopts a proactive strategy, it perceives that the opponent responds to the firm's decision. Thus, changing any capacity in its strategy $\mathscr{K}_{tj}$ will potentially trigger a different response in the opponent's strategy $\mathscr{K}_{t+1i}$. Since the opponent's future capacity cannot be fixed at one value, firm j's optimal value function V^*_{tj}, as well as its expected NPV function G_{tj}, will unlikely be concave in its capacity k_{tj}. To efficiently derive firm j's proactive ISD policy S_{tj} in a discrete capacity space, we propose a three-step method. The principle of our method is to "divide and conquer", i.e., dividing the problem into a number of subproblems that are smaller instances of the same problem and conquering the subproblems by solving them recursively (see a similar method used by Groenevelt [25]). First, we divide the capacity space into mutually exclusive and complementary *ranges*, each associated with a competitive goal that the firm can define. After that, we determine the optimal decision *interval* in each competitive-goal *range*, separately, and compare the values of switching between different *ranges* to obtain the decision *intervals* that are optimal across the entire capacity space. Hereinafter, we use *interval* to refer to a stayput interval, of which the boundaries define the decision rule for investments, and use *range* to refer to a range of capacity values in the available capacity space.

We classify a firm's competitive goal based on the type of response of the opponent the firm aims at. Considering only the immediate response of the opponent, a firm can have one of the following three potential goals: (1) a *passive* goal which leads the opponent to invest, (2) a *neutral* goal which leads the opponent to stay put, and (3) a *progressive* goal which leads the opponent to disinvest. These names indicate the firm's different attitudes towards the opponent's immediate future capacity growth, knowing the impact of the firm's own decision on the opponent. For instance, with a *passive* goal, the firm passively accepts that its decision will lead the opponent to increase its capacity in the future. In our model, the firm determines its capacity strategy to maximize its long-term value (see Eq. (3)), which not only includes the present profit but also the profits in the follow-up periods. Therefore, the motivation behind the firm's capacity decision can be that it drives the opponent to take a specific next action, so that the firm profits in the future. Our algorithm follows this intuition.

In the single-period problem, there will be at most three competitive-goal ranges in the firm's capacity space, each associated with a potential competitive goal. In the long-term problem, considering a series of responses of the opponent and a number of possible goals associated with each response, the capacity space can be divided into many ranges. Since the number of resulting stayput intervals is directly determined by the number of competitive-goal ranges, an ISD policy with more intervals indicates that the firm's capacity decision will be more responsive to the opponent's decision. Depending on the number of stayput intervals contained in the capacity space, the same space can have different strategic implications. Our method derives the complete set of stayput intervals in the capacity space, revealing the competitive value of capacity. In addition, our method predicts the volatility of the firm's investments by displaying the size of each stayput interval. The shorter each interval, the more likely it is that the firm's current capacity falls outside the interval and it thus should be adjusted, indicating a higher level of volatility. Below, we elaborate on each step of our method. We first show in Section 4.1 how to apply our method to a two-period game, and then in Section 4.2, formulate our method as an algorithm for com-

Table 2
Three-step method and *Decomposition Algorithm*.

	Three-step method	***Decomposition Algorithm*** for firm j's long-term proactive ISD policy: $S_{tj} = \cup_n \mathbf{S}_n$
Step 1	Divide the capacity space into mutually exclusive and complementary ranges	Divide the capacity space $[k^L_{tj}, k^H_{tj}]$ into ranges $[I^o_n, I^e_n]$: $[k^L_{tj}, k^H_{tj}] = \bigcup_n [I^o_n, I^e_n]$
Step 2	Determine the optimal decision interval in each range, separately	In capacity interval $[I^o_n, I^e_n]$, $\forall n$, derive the set of stayput values, $\mathbf{N}_n$, following the process of elimination: $\mathbf{N}_n \subseteq [I^o_n, I^e_n]$
Step 3	Determine the decision interval that is optimal across the entire capacity space	Comparing $\mathbf{N}_n$, $\forall n$, with each other to eliminate non-stayput values: in $\mathbf{N}_n$, the remaining interval is $\mathbf{S}_n$, $\mathbf{S}_n \subseteq \mathbf{N}_n$

puting a firm's long-term proactive ISD policy. A summary of our method and algorithm is presented in Table 2.

4.1. The two-period proactive ISD policy

We solve a multi-round game backwards, starting with the last period T. In the last period, the leader is the only firm who can have a proactive strategy as the leader moves first and the follower knows that the leader's capacity will remain unchanged after the last decision k_{Tl}. Under Assumptions 1 and 2, the follower's value function V_{Tf} is concave in its own decision k_{Tf}, according to Proposition 2. Thus, following Proposition 1, the follower's ISD policy contains a single stayput interval, $S_{Tf} = [k^L_{Tf}, k^H_{Tf}]$, which responds to a possible value of the leader's capacity k_{Tl}. The follower's optimal decision is either to invest, stay put, or disinvest, i.e., $k^*_{Tf} = k^L_{Tf}$, k_{T-1f}, or k^H_{Tf}.

To derive the leader's proactive ISD policy in period T, the *first* step of our method is to divide the leader's capacity space, given that it is sufficiently large and connected, into three competitive-goal ranges. Each range corresponds to a response of the follower. The *second* step is to determine the optimal decision interval for each competitive goal separately. Within each competitive-goal range, the leader's value function is concave in the leader's capacity. This is because in the last period, the leader only needs to consider one response of the follower and this response is fixed, given any decision of the leader within the same competitive-goal range. Using Proposition 1, we derive the boundaries of the leader's stayput interval for each competitive-goal range. We denote the stayput interval within the nth competitive-goal range as $\mathbf{N}_n$, $n = 1, 2, 3$. If the leader's current capacity is within a stayput interval, according to Proposition 1, the leader has more value staying put than adjusting to any other point within the same interval. However, the value of adjusting to another competitive-goal range may be higher than the value of staying put, considering that the follower may respond differently. The *third* step is to obtain the decision intervals that are optimal across the entire capacity space by comparing the value of staying in one stayput interval and the value of adjusting to either the lowerbound or upperbound of the other interval. The minimal difference between the two values is concave in the leader's capacity within $\mathbf{N}_n$. Hence, there exists a single interval $\mathbf{S}_n \subset \mathbf{N}_n$ satisfying that $\forall k \in \mathbf{S}_n$, the value of staying at k is larger than the maximal value of adjusting. We denote the lowerbound and upperbound of $\mathbf{S}_n$ as s^L_n and s^H_n, respectively. Some competitive-goal ranges may not have a stayput interval, $\mathbf{S}_n = \emptyset$. This means, the value of staying in a range is smaller than the maximal value of adjusting elsewhere. Since the competitive-goal ranges are non-overlapping, the resulting stayput intervals are non-overlapping.

The leader's ISD policy in period T consists of all $\mathbf{S}_n$, i.e., $S_{Tl} = \cup_n \mathbf{S}_n$. Figure 1 gives an example of such an ISD policy in the capacity space $[k^o_{Tl}, k^e_{Tl}]$. The first, second, and third stayput intervals, $[s^L_n, s^H_n]$, $n = 1, 2, 3$, are colored in black, green, and blue, respectively. According to S_{Tl}, if at the beginning of period T the leader's capacity is inside a stayput interval, e.g., $k_{T-1L} \in [s^L_3, s^H_3]$, its best decision is to stay put, i.e., $k^*_{Tl} = k_{t-1l}$. If its current capacity falls outside all stayput intervals $\mathbf{S}_n$, $\forall n = 1, 2, 3$, the value of staying put is smaller than the value of adjusting to an interval boundary. For capacity values that are on one side of all stayput intervals, $k < s^L_1$ or $k > s^H_3$, the optimal decision is to either invest or disinvest to hit the closest interval boundary. For capacity values that fall between two adjacent intervals, $k \in (s^H_n, s^L_{n+1})$, the interval to which the current capacity should be adjusted depends on the values of the two adjacent competitive goals n and $n+1$, respectively. The difference between the two values is monotonously changing in the leader's capacity, $\forall k \in (s^H_n, s^L_{n+1})$. Thus, there exists a unique investment threshold $s_{n,n+1} \in [s^H_n, s^L_{n+1}]$ such that the optimal decision for all $k \in (s^H_n, s_{n,n+1})$ is to disinvest to hit s^H_n, and the optimal decision for all $k \in (s_{n,n+1}, s^L_{n+1})$ is to invest to hit s^L_{n+1}. If $k = s_{n,n+1}$, adjusting to either boundary yields the same value. In Fig. 1, the investment thresholds are presented by purple crosses and the black lines with an arrow on one side indicate the optimal decision for a non-stayput point.

As demonstrated above, a firm's proactive ISD policy in the duopoly can largely differ from a single firm's ISD policy which contains only one stayput interval. A direct implication of having multiple separate stayput intervals is that the optimal investments will show a high level of volatility and irregularity. With one interval, a firm's decision is monotonously changing from investing if the current capacity is low, then staying put, last, disinvesting if the current capacity is high. With multiple intervals, the firm's decision is continually changing in the cycle of investing, staying put, and disinvesting. As a result, there may be "irrational" investment phenomena. e.g., when the current capacity falls between two adjacent intervals. If the current capacity is higher than the investment threshold between the two intervals, the optimal decision is to invest more to hit the higher stayput interval; if it is lower, the optimal decision is to disinvest to hit the lower stayput interval. In other words, holding more assets may trigger investments, while having fewer assets may trigger disinvestment.

Given three stayput intervals in period T, the leader has 7 possible decisions for k_{Tl}: invest or disinvest to hit a stayput interval boundary in a competitive-goal range ($k_{Tl} = s^L_n$ or s^H_n, $n = 1, 2, 3$; see Fig. 1) or stay put at the current capacity ($k_{Tl} = k_{T-1l}$). Extending the game to a two-period setting, i.e., $T-1$ and T, a proactive follower decides its capacity in period $T-1$ taking into account the leader's response in the last period, k_{Tl}. Since there are 7 pos-

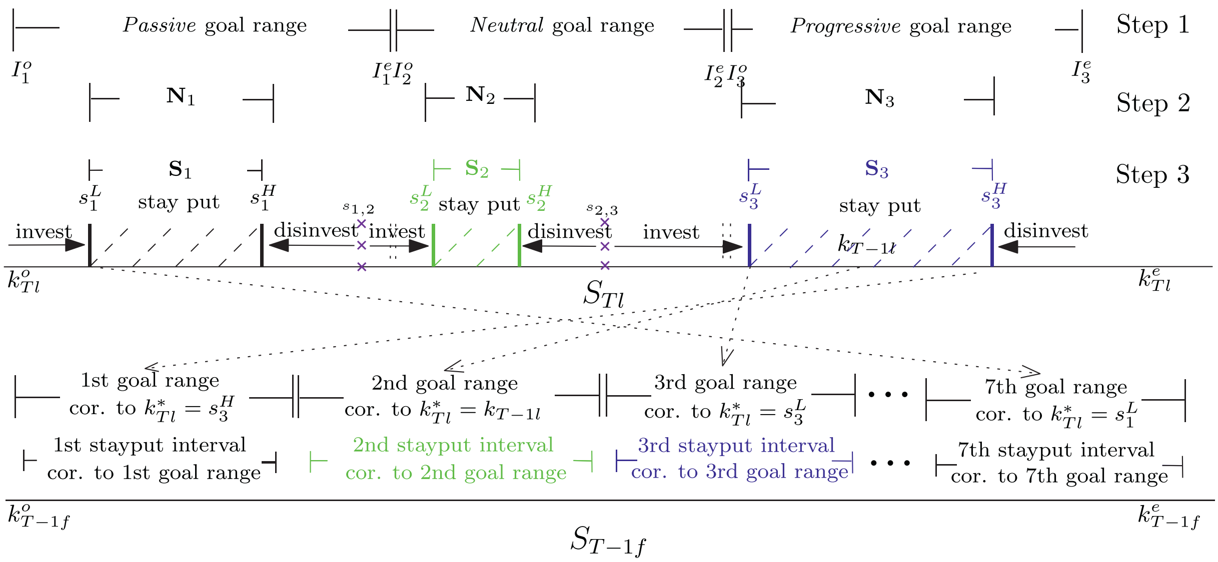


Fig. 1. Example of the leader's ISD policy in the last period T and the follower's ISD policy in period $T-1$.

Table 3
Example of the two firms' ISD policy in the two-period game.

t	j	$[I_n^o, I_n^e]$, $\forall n$ each corresponding to a value of k_{t+1i}^*	S_{tj}, dependent on $[I_n^o, I_n^e]$	k_{tj}^*, according to S_{tj}
$t=T$	Follower ($j=f$)	NA	$[s_1^L, s_1^H]$	s_1^L or s_1^H, or k_{t-1j}
	Leader ($j=l$)	$[I_1^o, I_1^e]$, $[I_2^o, I_2^e]$, $[I_3^o, I_3^e]$	$\cup\{[s_1^L, s_1^H]$, $[s_2^L, s_2^H]$, $[s_3^L, s_3^H]\}$	s_1^L or s_1^H, s_2^L or s_2^H, s_3^L or s_3^H, or k_{t-1j}
$t=T-1$	Follower ($j=f$)	$[I_n^o, I_n^e]$, $n=1,2,\ldots,7$	$\cup_n\{[s_n^L, s_n^H]$, $n=1,2,\ldots,7\}$	s_n^L or s_n^H, $n=1,2,\ldots,7$, or k_{t-1j}
	Leader ($j=l$)	$[I_n^o, I_n^e]$, $n=1,2,\ldots,15$	$\cup_n\{[s_n^L, s_n^H]$, $n=1,2,\ldots,15\}$	s_n^L or s_n^H, $n=1,2,\ldots,15$, or k_{t-1j}

sible values for k_{Tl}, the follower can have up to 7 competitive goals in period $T-1$, each corresponding to a value of k_{Tl}. In Fig. 1, we show how the leader's stayput intervals in period T lead to the follower's stayput intervals in period $T-1$. These competitive goals lead to potentially 7 stayput intervals. Consequently, the follower can have 15 possible decisions for k_{T-1f} (invest or disinvest to hit a stayput interval boundary, i.e., 2×7 decisions, and 1 decision that is to stay put). The leader's proactive ISD policy in period $T-1$ is based on k_{T-1f}. Following the same logic as above, the leader can have 15 (or more since the leader's value function may not be concave in k_{T-1l} within the same competitive-goal range, considering that there are now two responses of the follower, k_{T-1f} and k_{Tf}) stayput intervals in period $T-1$. In Table 3, we list an example of the two firms' respective ISD policy in the two-period game. Note that the exact number of competitive goals or stayput intervals directly depends on the size and form (whether it is connected) of the available capacity space. For instance, if the follower's capacity space in period $T-1$ is constrained such that all of its available choices can only trigger one type of response from the leader, although there are 7 possible responses (7 possible values for k_{Tl}), then only one competitive goal is available to the follower. In this scenario, the number of the follower's stayput intervals in period $T-1$ will be largely reduced. Given a sufficiently large capacity space, the number of stayput intervals can increase exponentially in the long-term problem. Next, we develop an exact algorithm (referred to as the *Decomposition Algorithm*) to derive the optimal long-term proactive ISD policy.

4.2. The long-term proactive ISD policy

In the long-term problem, we first identify the range of capacity which contains the final stayput interval S_{tj} and then apply our method using this range as the new capacity space, instead of considering the entire space $\mathscr{K}_{tj}$. We use the two boundary functions in Proposition 1 to identify this capacity range, $[k_{tj}^L, k_{tj}^H]$. Since the value function is not necessarily concave, the computed lowerbound k_{tj}^L is not guaranteed to be smaller than or equal to the computed upperbound k_{tj}^H. In Proposition 3, we show that if there exists a solution to the optimization problem, then $k_{tj}^L \le k_{tj}^H$ and the stayput interval S_{tj} is exclusively contained in the range $[k_{tj}^L, k_{tj}^H]$. Otherwise, $S_{tj}=\emptyset$. In addition, k_{tj}^L and k_{tj}^H are contained in S_{tj}.

Proposition 3. *$S_{tj} \subseteq [k_{tj}^L, k_{tj}^H]$, $k_{tj}^L = \inf S_{tj}$ and $k_{tj}^H = \sup S_{tj}$, where $k_{tj}^L = \sup\{\{k_{tj}^o\} \cup \{k_{tj} : \frac{\nabla_- G_{tj}(\mathbf{Y}_{tj}, k_{tj})}{\nabla k_{tj}} \ge c_{tj}, k_{tj} \in \mathscr{K}_{tj}\}\}$ and $k_{tj}^H = \inf\{\{k_{tj}^e\} \cup \{k_{tj} : \frac{\nabla_+ G_{tj}(\mathbf{Y}_{tj}, k_{tj})}{\nabla k_{tj}} \le r_{tj}, k_{tj} \in \mathscr{K}_{tj}\}\}$.*

We use two rolling procedures, *rolling up* and *rolling down*, to scrutinize a capacity range. Starting with an inspection range, these two rolling procedures iteratively extend the inspection range on one side, while keeping the other side fixed. Unless specified otherwise, a rolling procedure continues until the extending end reaches the end or the start of the capacity range to which the procedure is applied. In Definition 1, we define how these two rolling procedures extend the inspection range in an iteration.

Definition 1. Given $k \in \mathscr{K}$, $\triangle^+ k$ and $\triangle^- k$ denote the smallest possible capacity larger than k and the largest possible capacity smaller than k in $\mathscr{K}$, respectively: $\triangle^+ k = \inf\{k' : k' > k \text{ and } k' \in \mathscr{K}\}$ and $\triangle^- k = \sup\{k' : k' < k \text{ and } k' \in \mathscr{K}\}$. Applying the two rolling procedures to capacity range $[k^o, k^e]$ and given that the $n-1$th inspection range is $[k^o, k^{n-1}]$ or $[k^{n-1}, k^e]$, the nth iteration of the *rolling up* procedure extends the range $[k^o, k^{n-1}]$ to $[k^o, \triangle^+ k^{n-1}]$, whereas that of the *rolling down* procedure extends the range $[k^{n-1}, k^e]$ to $[\triangle^- k^{n-1}, k^e]$.

Similar rolling procedures have been widely used in forecasting and planning [8,29]. The most common approach, i.e., the *rolling window* method, is to divide the time horizon into equal overlapping windows and to use the observations in each window to construct an aggregated observation [43]. The main differences between the rolling window method and our rolling procedures are that: first, we divide the capacity space, instead of the time horizon, and second we iteratively extend the inspection capacity range, rather than maintaining equal-sized ranges.

To identify the stayput values in the capacity space $[k^L_{tj}, k^H_{tj}]$, our *Decomposition Algorithm* follows three steps. In the appendix, we provide the pseudocode and detailed explanation of the algorithm. The first step is to divide the capacity space $[k^L_{tj}, k^H_{tj}]$ into N, $N \subset \mathbb{Z}$, competitive-goal ranges, each corresponding to an immediate response of the opponent, assuming that the opponent's all future capacity stays the same afterwards. We use I^o_n and I^e_n (see Table 2 for the notations used in each step) to denote the origin and final values of the nth range, respectively, $\forall n \in N$.

The second step is to derive the set of stayput values, denoted as $\mathbf{N}_n$, in each competitive-goal range $[I^o_n, I^e_n]$, separately. Denote the lowerbound and upperbound of $\mathbf{N}_n$ as N^o_n and N^e_n, respectively: $N^o_n = \inf \mathbf{N}_n$ and $N^e_n = \sup \mathbf{N}_n$. Given any capacity within a competitive-goal range, the opponent's immediate response is fixed, however its later responses can vary. Therefore, within each competitive-goal range, the firm's value function is not necessarily concave in its capacity and the stayput interval in the competitive-goal range cannot be directly derived using Proposition 1. Our *Decomposition Algorithm* derives the stayput interval by elimination: it determines whether a capacity value is a non-stayput value by comparing it to a known stayput value and eliminates it from the solution set if a non-stayput value is identified. According to Proposition 3, we can use Eqs. (7) and (8) in Proposition 1 to derive two known stayput values in the competitive-goal range, i.e., the lowerbound and upperbound of the stayput interval. These two bounds can then be used in the process of elimination.

The third step is to eliminate capacity values from $\mathbf{N}_n$ that are non-stayput values across all competitive-goal ranges. Proposition 4 shows that whether a capacity value in N_n is a stayput value across another competitive-goal range N_i, $i \neq n$, can be determined by comparing this capacity value with the upperbound or lowerbound of N_i. In Proposition 5, we show that when using the rolling procedure to sequentially scrutinize $\mathbf{N}_n$, once a stayput value is confirmed, the procedure can stop at the current iteration. All remaining capacity values in $\mathbf{N}_n$ are denoted as $\mathbf{S}_n$.

Proposition 4. *Given the stayput set in the nth competitive-goal range,* $\mathbf{N}_n$, $N^o_n = \inf \mathbf{N}_n$ *and* $N^e_n = \sup \mathbf{N}_n$*:* $\forall k$, *if the expected NPV value of staying at k equals the value of adjusting to* N^e_n *or to* N^o_n, *then the value of staying at k equals the value of adjusting to any value in* $\mathbf{N}_n$.

Proposition 5. *Given the stayput set in the nth competitive-goal range,* $\mathbf{N}_n$ *and a capacity value* $\bar{k} \in \mathbf{N}_n$*: if there exists a capacity value* $L < \bar{k}$ *(or* $H > \bar{k}$*) such that* $\bar{k} \le K^H([L, \bar{k}])$ *(or* $\bar{k} \ge K^L([\bar{k}, H])$*), then* $k \le K^H([L, k])$, $\forall k \in \mathbf{N}_n$ *and* $k > \bar{k}$ *(or* $k \ge K^L([k, H])$, $\forall k < \bar{k}$*).*

After the three steps, the final stayput interval is $S_{tj} = \cup_{n=1,\cdots,N} \mathbf{S}_n$. In Theorem 1, we elaborate on the decision rule specified by a proactive ISD policy with multiple separate stayput intervals. For stayput capacity, the optimal decision is to do nothing, i.e., staying put. For non-stayput capacity that falls at one side of all stayput intervals, the optimal decision is to adjust to the closest stayput interval boundary. For non-stayput capacity that falls between two adjacent stayput intervals, the optimal decision is guided by an investment threshold between the two intervals: below the threshold, capacity should be adjusted to the upperbound of the lower stayput interval; otherwise, it should be adjusted to the lowerbound of the higher stayput interval. The investment threshold can be identified through a binary search.

Theorem 1. $\forall k \in S_{tj}$, *no adjustment should be made, i.e.,* $k^*_{tj} = k$. $\forall k < \inf S_{tj}$ *or* $k > \sup S_{tj}$, *the optimal decision is to adjust to the closest stayput interval boundary. Given two disjoint stayput intervals,* $[s^L_n, s^H_n]$ *and* $[s^L_{n+1}, s^H_{n+1}]$, $\forall n \ge 1$, *there exists an investment threshold* $s_{n,n+1} \in [s^H_n, s^L_{n+1}]$ *such that the optimal investment policy assigns all capacity in* $(s^H_n, s_{n,n+1}]$ *to be adjusted downwards to* s^H_1, *and assigns all capacity in* $[s_{n,n+1}, s^L_{n+1})$ *to be adjusted upwards to* s^L_{n+1}.

5. Case study on the container shipping market

In this section, we illustrate how to plan competitive capacity investment following our model, using data from the container shipping market. In our case study, Maersk and MSC, which are the biggest and the second biggest carriers based on their fleet capacity [4], are selected as the leader and the follower, respectively. The criterion for the selection of the competing firms is that whether these firms can directly influence the price of the shipping service through their capacity investments. Having large and relatively similar market shares is a good indicator for this. The criterion for the selection of the leader between the two firms is the market share. In practice, leader firms have superior access to supply, which leads them to move first in the capacity investment race, and firms which have more access to supply usually also have more capacity. Thus, having a larger market share can be an indicator of the leader.

Mixing the two firms' proactive or reactive strategies, as well as information on the opponent's strategy used in the follower's proactive strategy, we consider four cases (see a summary in Table 4). (1) Case *stayput*: the leader is proactive, while the follower responds to the competition by assuming that the leader will stay put in the next period. (2) Case *adversarial*: both firms are proactive, however, the follower has incorrect information on the leader's strategy and assumes that the leader is adversarial. In other words, in period t, the follower assumes $k_{\tau l} = \operatorname{argmin}_{k \in \mathscr{K}_{\tau l}} V^*_{\tau f}(\mathbf{Y}_{\tau f} = (k, k_{\tau-1f}, \omega_\tau))$, $\forall \tau \in \{t+1, \cdots, T\}$. (3) Case *optimal*: both firms are proactive, and the follower has full information on the leader's optimal strategy. (4) Case *reactive*: both firms are reactive by assuming that the other firm will stay put in the next period. A case in which the leader is reactive and the follower is proactive is not mentioned here, as it is identical to case *stayput* with a delayed starting point (i.e., the follower moves first). We refer to cases *optimal* and *reactive* as symmetric cases since both firms adopt the same strategy and have the same amount of information on each other's strategy, whereas we refer to cases *stayput* and *adversarial* as asymmetric cases as the situations are different for the two firms.

We use the demand and supply data of the container shipping market over a timespan of 16 years (2000 – 2015). We use

Table 4
Leader and follower firms' strategies in the four cases.

Type of cases	Name of cases	Leader			Follower		
		Proactive		Reactive	Proactive		Reactive
		Full information	Incorrect information		Full information	Incorrect information	
Asymmetric	*Stayput*	✓					✓
	Adversarial	✓				✓	
Symmetric	*Optimal*	✓			✓		
	Reactive			✓			✓

a fixed planning horizon and derive the series of optimal capacity decisions at the beginning of the horizon. This implies that in the later periods in the horizon, there may be a terminating effect: as it reaches the end of planning, firms weigh their salvage values more. In practice, both Maersk and MSC's planning horizons are likely more than 16 years, not planning to end business in 2015. In order to compare our model results, especially those in the later periods in the 16-year timespan, with the reality, we extend the time horizon to 2017, setting $\Gamma = \{1, \cdots, 18\}$. At the beginning of 2000, the first observed demand, ω_1, is set to be 1. At the beginning of each year, both firms observe the current demand and predict the demand growth of this year. In order to mitigate the risk, a demand forecast consists of several future scenarios with different probabilities, each taking into account the forecast error to a varying degree. Thus, we use the following categorical distribution to represent the demand scenario forecast. At the beginning of period $t \in \Gamma$, firms observe ω_t and expect that ω_{t+1} evolves as follows: $\omega_{t+1} = \omega_t \times (1 + \mu_t + x\sigma_t)$, $\forall x = \{-2, -1, 0, 1, 2\}$, with probability pr_x. The transition probabilities, pr_x, are taken from the Z-score table. μ_t is the forecast of the demand growth of period t and σ_t is the forecast error in period t, which is the average discrepancy between all previous forecasts μ_τ and the realized demand growths μ^r_τ, $\forall \tau \in \{1, \cdots, t-1\}$. We use a deviation of 5%, a standard forecast deviation in the container shipping market, as the first forecast error σ_1. The values of μ_t and μ^r_t are the average values based on the two half-yearly *Clarkson Shipping Review and Outlook* reports [16,17]. We use the same demand-growth forecast of 2015 for the artificial years (2016 and 2017) in our time horizon, setting $\mu_{18} = \mu_{17} = \mu_{16}$, and assume a zero demand growth after 2015, setting $\mu^r_{18} = \mu^r_{17} = 0$.

In the long-term investment problem, we adopt a standard discount rate of 0.89 [23]. We keep supply and investment cost parameters constant throughout the entire time horizon Γ. For simplicity, we assume that both firms' setup and operating costs are zero in each period: $H_{tj}(k_{tj}) = 0$, $\forall t \in \Gamma$. We use discrete market share values to present the capacity choices available to the two firms: $\mathscr{K}_{tl} = \mathscr{K}_{tf} = \{0, 1, \cdots, 19\}$, $\forall t \in \Gamma$. An upper limit of 19 is set to both firms' capacity spaces, due to the fact that a single operator's market share (based on its fleet capacity) has never exceeded 19% to date [4]. At the beginning of 2000, the market shares of Maersk and MSC were around 12% and 5%, thus we set $k_{0l} = 12$ and $k_{0f} = 5$ [56]. According to *Maersk's container market weekly report* [35], the average second-hand vessel price is \$4,837 per TEU and the average newbuilding vessel price is \$10,741 per TEU, thus we set both firms' investment cost parameters as $c_{tl} = c_{tf} = 10.7$ and $r_{tl} = r_{tf} = 4.8$, $\forall t \in \Gamma$. Assumption 2 is thus satisfied. Notice that these are not the prices for changing 1% market share, however, the difference between c and r still represents the investment irreversibility level in the container shipping market. In the appendix, we investigate the impact of different levels of investment irreversibility on capacity strategies, using Monte Carlo simulations (10,000 demand paths). We find that our results here persists.

Since the freight rate fluctuates heavily with capacity investments of dominant firms, we use a linear market-clearing price: $P_t = \alpha\omega_t - k_{tl} - k_{tf}$, $\forall t \in \Gamma$, where α is a positive marginal impact of demand on price, given the supply. We use the same form of the operating profit function for the salvage value function: $F_j = (\alpha\omega_{T+1} - k_{Tl} - k_{Tf}) \times k_{Tj}$. With this function, Assumption 1 is satisfied. We determine the value of α using the historical freight rates (dollars per TEU), demand and supply data (both in thousand TEUs) on the three major trade routes, transpacific, Europe-Asia, and transatlantic. Using the linear market-clearing price, the marginal demand impact on freight rate ranged from 1.2 to 2.1 on the three routes at the beginning of 2000 [57]. We choose the average value of 1.5 as the marginal demand effect and scale up this value by 10, setting $\alpha = 15$. This is because when applying our model, we have scaled the initial demand value to $\omega_1 = 1$ and thus all demand values are single-digit. However, the two carriers' market shares are mostly double-digit. Scaling α helps to achieve comparable demand and supply data.

5.1. Optimal ISD capacity strategies in the container shipping market

Using the *Decomposition Algorithm*, we determine a firm's ISD policy in each period. In Fig. 2, we present the complete set of stayput intervals of the two firms in the four cases. We also list the key parameter values in the figure title. We use abbreviations to refer to a firm in a specific case, e.g., the leader (L) in the *stayput* case (S) is abbreviated as *SL*. The black, green, and blue boxes indicate the first, second, and third stayput intervals, respectively. Note that in the figure, there are not as many stayput intervals as mentioned in Section 4.1 (see Fig. 1 and Table 3). This is mainly because here we have a relatively small capacity space, which directly influences the number of competitive-goal ranges and thus the number of stayput intervals. When an interval contains only one value, e.g., *SL* at $t = 15$, it shows as a single line piece. The purple crosses indicate the investment threshold between two adjacent intervals. In our examples, all investment thresholds are located at a boundary of a stayput interval. The red solid line depicts the optimal capacity, computed based on value function maximization.

As shown in Fig. 2, the two firms adjust their capacity based on their ISD policy. Taking subfigure (g) *OF* as an example, at the beginning of period 8, the follower's stayput interval in this period contains two pieces: [8,9] and 11, with a threshold at 9. Since the follower's current capacity, $k_{7f} = 9$, falls in one of the two intervals, it is optimal for the follower to stay put, i.e., $k_{8f} = 9$. By revealing the complete set of stayput intervals in a given capacity space, carriers can better evaluate different capacity options. Still using *OF* as an example, assume that at the beginning of period 5 and period 6, the offers of the current ship builder are equivalent to increasing the follower's market share to the range from 7% to 8% (the maximal building capacity is equivalent to 8% market share). According to the stayput intervals revealed in subfigure (g), in period 5 there is no competitive value of increasing the upper-

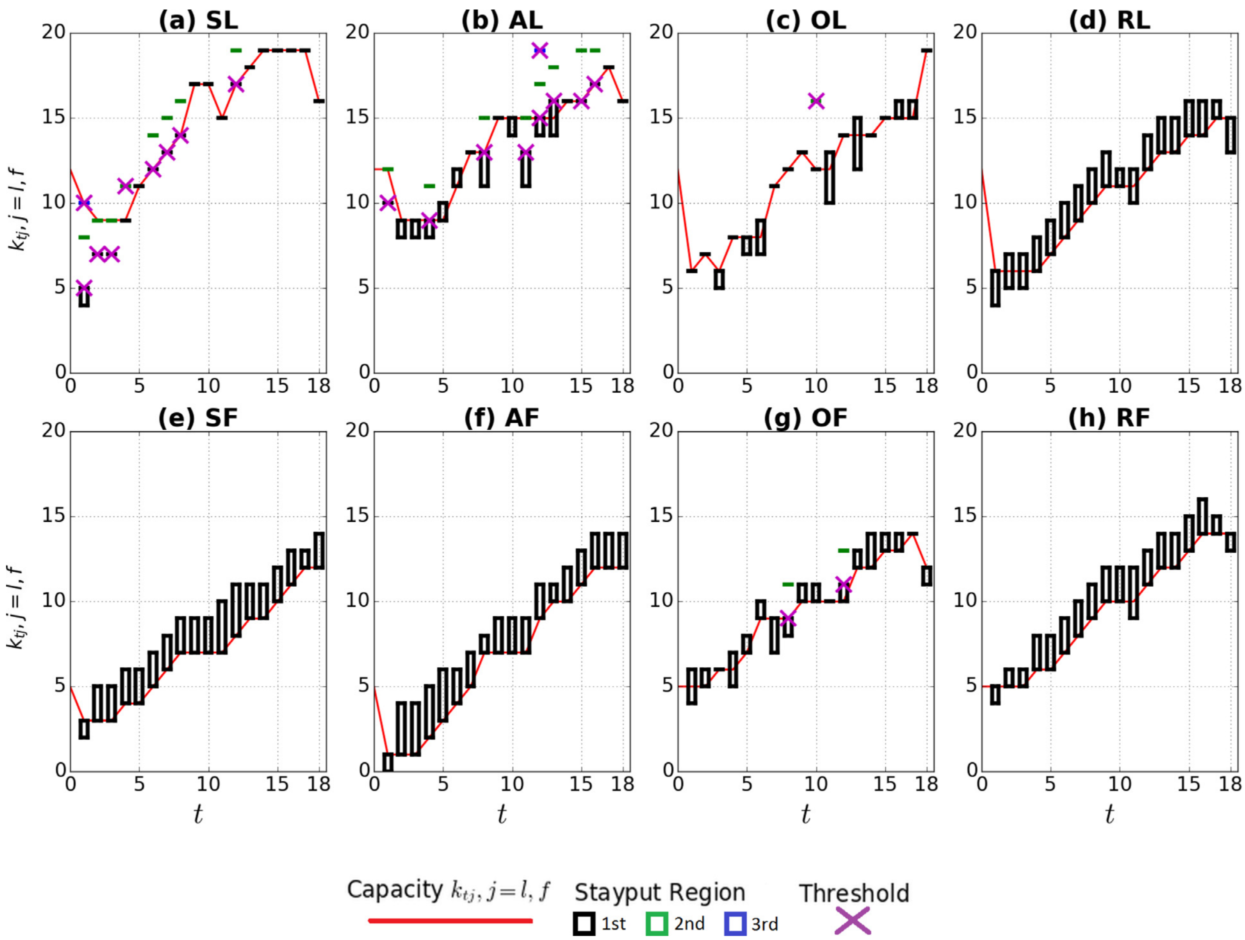


Fig. 2. Stayput intervals of in each strategy case ($\delta = 0.89$, $\alpha = 15$, $k_{0l} = 12$, $k_{0f} = 5$, $c_{tj} = 10.7$, $r_{tj} = 4.8$, $\forall t, j$).

bound of the current offers since there is only one stayput interval in this period and it is contained in the capacity space [7,8]. However in period 6, the follower needs to look for a larger capacity option than the current offers as the stayput interval in this period, i.e., [9,10] is outside the capacity space [7,8]. Our results also show that there is no need to consider capacity higher than 10% market share.

In our examples, a more (effective) proactive planning brings both the leader and the follower benefits. The difference between the leader's and the follower's capacity in each period is almost always bigger in the asymmetric cases than that in the symmetric cases. Figure 3 presents the profits of each firm in each strategy case, averaged over 10,000 simulated demand paths. Compared with Figs. 2 and 3 shows that higher capacity almost always leads to more profits. In the title of Fig. 3, we also list the cumulative profits of the leader and the follower in each strategy case, respectively. The difference between the leader's and the follower's cumulative profits is much bigger in the asymmetric cases than in the symmetric cases. By adopting a proactive strategy, the leader can gain up to 20.87% more cumulative profits in 18 years, compared to when both firms are reactive (comparing the leader's highest total profit in the proactive cases, i.e., *AL*, with *RL*). The follower also benefits from proactively responding to the leader's proactive strategy, compared to where it responds reactively. The follower can gain 36.89% more cumulative profits if it responds proactively to the leader's proactive strategy rather than being reactive (comparing *OF* with *SF*). Even in the case where the follower has inaccurate information about the leader's strategy, the follower can still gain approximately 12.43% more cumulative profits by acting proactively (comparing *AF* with *SF*).

The reason why proactive thinking brings benefits to firms can be traced back to the underlying structure of the proactive strategies. Proactive thinking leads a firm to have multiple stayput intervals and an ISD policy with such intervals indicates that the firm's capacity will be highly responsive to the opponent's decisions. Comparing subfigure (a)–(d) with (e)–(h) in Fig. 2, the leader has more and shorter stayput intervals in each period than the follower and the difference between the two firms' stayput intervals is especially obvious in the asymmetric cases. Comparing the capacity of *OF* and that of *AF* with the capacity of *SF*, respectively, *OF* acquires a higher capacity than *SF*, and the capacity of *AF* gradually surpasses the capacity of *SF* starting from $t = 8$. This is because *OF* has multiple stayput intervals compared to *SF*, attributed to the follower's proactive thinking in *OF*. Even when the follower has incomplete information on the leader's strategy, i.e., *AF*, the follower still benefits from proactive thinking as it constrains the leader's proactive thinking. Figure 2 shows that *AL* has larger intervals than *SL*, indicating that the strategic responsiveness of *AL* is restrained. Consequentially, *AL* has lower capacity than *SL*.

Figure 4 presents the two firms' optimal capacity (same as the optimal capacity of different strategies in Fig. 2), compared to the realized end-of-year capacity of Maersk and MSC in period $t \in \{1, \cdots, 16\}$ (2000–2015), which is extracted from the yearly *UNCTAD Review of Maritime Transport* report (for 2000–2014) and from Alphaliner [3] (for 2015). The realized capacity of the leader and the follower carriers are denoted as *ReL* and *ReF*, respectively. In each strategy case, we list the mean squared errors (MSE), aver-

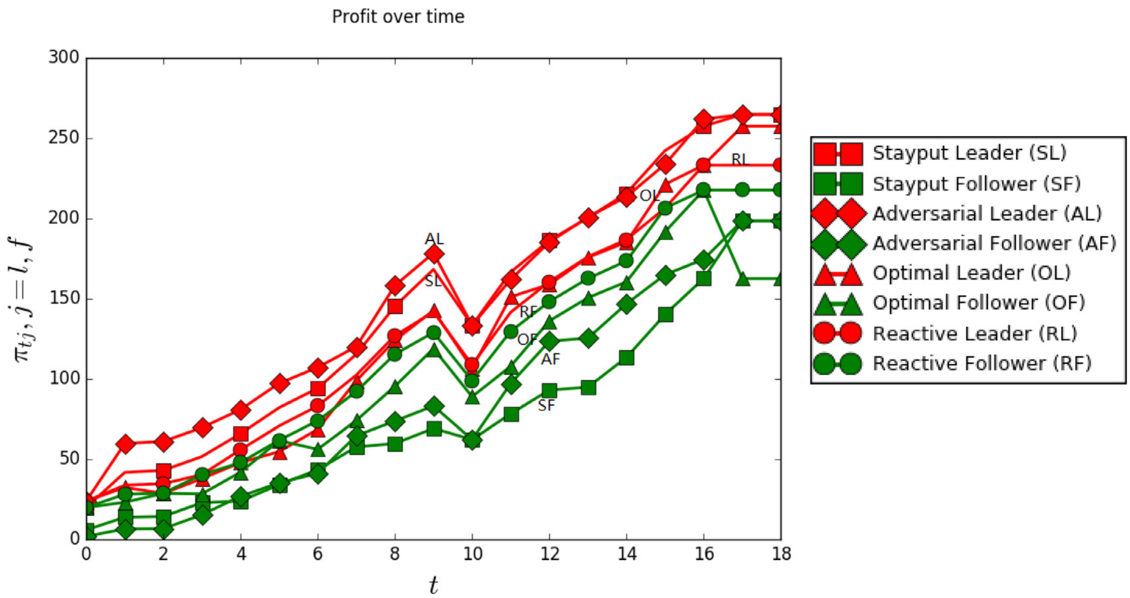
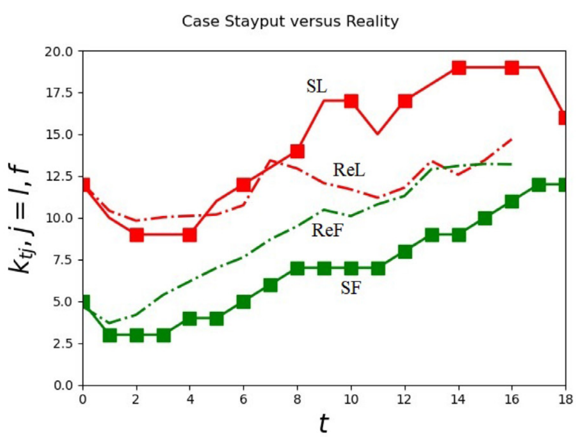
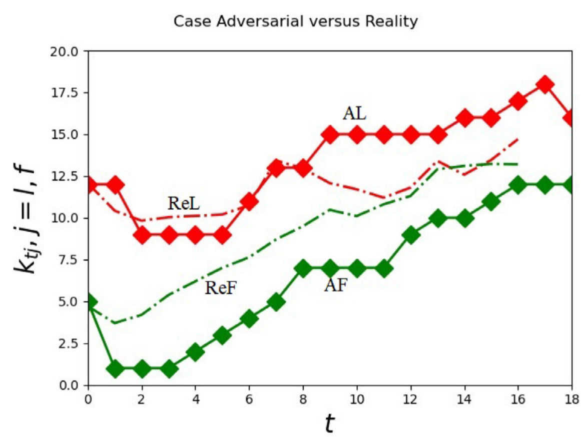


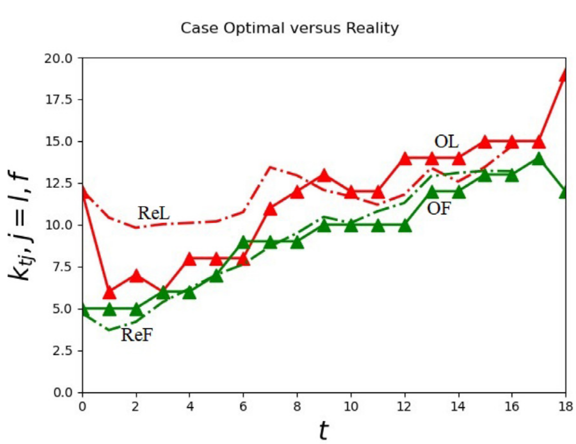
Fig. 3. Optimal profit in each strategy case ($\sum_t \pi_{tj}(j = SL) = 2495.4$, $\sum_t \pi_{tj}(j = AL) = 2613$, $\sum_t \pi_{tj}(j = OL) = 2150.65$, $\sum_t \pi_{tj}(j = RL) = 2161.8$, $\sum_t \pi_{tj}(j = SF) = 1289.75$, $\sum_t \pi_{tj}(j = AF) = 1450.05$, $\sum_t \pi_{tj}(j = OF) = 1765.6$, $\sum_t \pi_{tj}(j = RF) = 1993.5$).



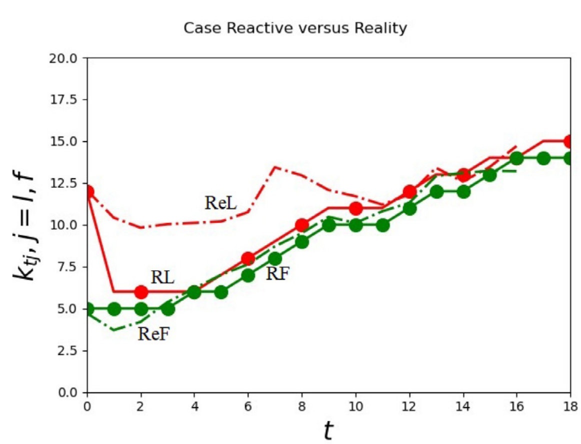
$1 \leq t \leq 8$ MSE: 3.023
(a) $9 \leq t \leq 16$ MSE: 18.735
$1 \leq t \leq 16$ MSE: 10.879



$1 \leq t \leq 8$ MSE: 6.895
(b) $9 \leq t \leq 16$ MSE: 8.498
$1 \leq t \leq 16$ MSE: 7.696



$1 \leq t \leq 8$ MSE: 4.521
(c) $9 \leq t \leq 16$ MSE: 0.998
$1 \leq t \leq 16$ MSE: 2.76



$1 \leq t \leq 8$ MSE: 7.373
(d) $9 \leq t \leq 16$ MSE: 0.407
$1 \leq t \leq 16$ MSE: 3.89

Fig. 4. Optimal capacity in each strategy case versus reality.

aged for the two firms, during the first 8 years, the second 8 years and the entire 16 years, respectively. The case which fits the realized capacity of the first 8 years the best, i.e., case *stayput*, fits the realized capacity of the second 8 years the worst, whereas the case which fits the second 8 years the best, i.e., case *reactive* or *optimal* fits the first 8 years poorly. During the 16-year timespan, the strategy case *optimal* fits the realized capacity the best. At the end of 2015 ($t = 16$), the realized capacity of the two carriers grew by 22.5% and 164%, respectively, reaching market shares of 14.7% and 13.2% [3], which are best matched by *OL* and *OF* (15% and 13%). While the dominant firms followed their respective competitive strategies, other firms were squeezed out of the market. According to the Reuters news, 70% of the carriers each own fewer than 51 vessels in 2015 [46] and this trend is continued. Assuming that the leader and the follower follow the trend to increase and decrease its capacity, respectively, beyond 2015 ($t > 16$), the *optimal* case shows a better prediction, compared to the other three cases.

According to the results in Fig. 4, it is plausible that the 2008 ($t = 8$) global recession is a turning point where carriers have changed their capacity strategies. Although it is difficult to identify which one of the four cases exactly matches the two firms' strategies before 2008 ($1 \leq t \leq 8$ the MSE in the four cases varies from 3.023 to 7.373), it is noticeable that after 2008 the two firms' strategies are close to *optimal*. In other words, since 2008, both firms adopt proactive strategies with full information on each other's future investments. An explanation could be that market downturns attract extra attention on the opponent's moves, making firms' strategies transparent to each other. The observation in the numerical experiment is also consistent with the results in our early interviews with the carriers. Our interviewees explicitly mentioned that since 2008 pricing downward to marginal costs has become necessary due to overcapacity and they have started to pay close attention to each other's orderbook since any new ship orders will further deteriorate the freight rate (also mentioned in [52]):

"We follow our competitors carefully and closely." (shipping line B); "The newsletter on newbuilding is the first thing we read every morning. You need to know what others have just bought and even better if you know their purchases in planning. But it is difficult, the hidden agenda is everywhere." (shipping line C); "There is no secret (regarding ship orders) in this industry after 2008. Suddenly, people know what each other is buying, and even if you do not want to know, they (shipping lines who have ordered new ships) will find a way to let you know and then you just (have to) follow." (shipping line E)

5.2. Managerial insights

Existing studies emphasize that in the capacity investment race, it is crucial to act first, as the first mover usually gains a competitive advantage through control of resources. Our case study shows that the key to success in the race is to plan investments proactively and collect as much information as possible on the competitor's strategy to be used in the firm's proactive strategy. Our recommendation for competing firms is to ***act proactively at the earliest opportunity and to make one's proactive thinking visible in the race***. The essence of a proactive strategy is for the firm to realize that its decisions can influence other firms' future plans. Regardless of how accurately the firm knows its impact, if it is able to convey the message that its strategy considers its competitor's potential responses, then the proactive strategy serves as a credible threat. This may alter the competitor's expectations of the firm's future decisions, and thereby induce them to make decisions that are favorable to the firm, or deter them from making harmful moves. Our method provides a useful tool for both leader and follower firms to plan their long-term capacity investments under competition. Based on our results, we formulate the following three steps for implementing an effective competitive capacity strategy.

1. ***Find the most relevant competitor.*** The relevance of a competitor is judged based on the impact of its capacity decisions on the product price. In the example of the container shipping market, a carrier's most relevant competitor is another carrier that operates on the same routes and has a similarly large fleet size that allows it to influence the price of the shipping service.
2. ***Set the competitive goal(s) and identify good investment options for each goal.*** A competitive goal is the firm's attitude towards the competitor's future capacity growth. In general, there are three types of competitive goals: *passive, neutral* and *progressive*. An appropriate competitive goal is set based on the current market condition, including demand (growth), the competitor's current capacity and investment costs. If the market is highly volatile, the firm should always make plans for several feasible goals, which helps the firm cope better with the rapidly changing market.
3. ***Find the best option by comparing options that are close to the current capacity.*** An optimal ISD policy directs the firm's current capacity to a close-by stayput interval. This implies that when considering the best investment plan, the firm should first evaluate the competitive goal and investment options that are easy to reach from the current position, and then decide how much further it can stretch its competitive strength based on the current investment costs. The approach of comparing close plans is sensible as there could still be sizable economies to be gained from adjusting the plan slightly, while overextending is highly risky.

In a competitive market, the firm should focus on evaluating the impact of its options on the competitor's future investments. This may lead to some non-obvious decisions in practice. For instance, ***holding more assets in a competitive market may trigger investments, while having fewer assets may trigger disinvestments.*** In some market positions, investing to reach a higher market share can put off the competitor's future investments and lead to a higher profit. In this situation, the optimal decision is to invest, even if the current capacity is high. With a lower market share, it may be better to stay at the current capacity or even fall back since the amount of investment required to hold back the competitor's investments may be so large that the plan is unrewarding.

6. Conclusion

We study a long-term capacity investment problem in a duopoly under demand uncertainty. Different from the majority of oligopoly capacity models in the literature which focus on simultaneous investments of firms, we investigate a problem where two firms sequentially adjust their capacity to respond to each other's decisions. In the capacity investment race, a firm can either proactively or reactively plan its investments. We derive the optimal strategy in the form of an ISD policy. We contribute to the literature by investigating the strategic implications of capacity in the investment race. In the race, a firm's proactive thinking is realized by first deciding the set of possible competitive goals towards the competitor's future capacity growth, then determining the optimal capacity for each goal, and last comparing profits to make the best decision. The multiple competitive goals and the optimal investment policy which specifies the current capacity to be adjusted to the closest goal range lead to non-obvious investment decisions under competition. Moreover, we contribute to the literature by revealing the optimal structure of the long-term competitive capacity strategy, shown as an array of separate intervals for the optimal capacity in each period. We discover the core of a competitive strategy by revealing the meanings behind different stayput intervals. In addition, by deriving the complete set of stayput intervals in the available capacity space, our method reveals the competitive value of capacity, helping firms evaluate different capacity options.

We illustrate the optimal ISD capacity strategies using detailed data from the container shipping market. Our results show that the realized investments, which are questioned to be irrational decisions, followed a competitive and optimal structure. In our case study, the leader generally performs better than the follower in terms of capacity and profit, and the leader enhances its first mover advantage by adopting a proactive rather than a reactive strategy. The follower's best response to the leader's proactive strategy is also to be proactive. For our case study which contains two firms, 18 time periods and 19 capacity options for each firm, our algorithm can compute the optimal strategy of each firm relatively fast: seconds for the reactive strategy and 1–3 h for a proactive strategy (with different amounts of information on the opponent's strategy) on a laptop with i5-7500 CPU and 8 GB RAM. Although theoretically, the capacity space can contain more than 19 options and the investment timespan can contain more than 18 periods, practical investment problems usually have a limited number of capacity options and are for a limited timespan. Therefore, we believe our algorithm is effective for solving most of the practical investment problems.

Investment in practice is complex. Firms can switch between multiple strategies in the race. Future research should address the optimal structure of such a competitive capacity strategy, answering when the firm should change its strategy and how. In the investment race, firms can also learn from previous experience and update their knowledge on the competitors' strategies. Future research can incorporate competition learning. In addition, it can incorporate a delay between the capacity decision and the production decisions and study the implication of this delay in the long-term investment race. Efficient algorithms can be developed to solve this combined capacity and production problem.

Acknowledgment

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Appendix: Model parameters and proofs

Proposition 1. *Given the current state $\mathbf{Y}_{tj}$, if firm j's optimal value function V^*_{tj} is jointly concave in (k_{t-1j}, k_{tj}), then its optimal capacity in period t is specified by an ISD policy that is characterized by the following functions:*

$$k^L_{tj} = \sup\left\{\{k^o_{tj}\} \cup \{k_{tj} : \frac{\nabla_- G_{tj}(\mathbf{Y}_{tj}, k_{tj})}{\nabla k_{tj}} \geq c_{tj}, \quad k_{tj} \in \mathscr{K}_{tj}\}\right\} \tag{7}$$

$$k^H_{tj} = \inf\left\{\{k^e_{tj}\} \cup \{k_{tj} : \frac{\nabla_+ G_{tj}(\mathbf{Y}_{tj}, k_{tj})}{\nabla k_{tj}} \leq r_{tj}, \quad k_{tj} \in \mathscr{K}_{tj}\}\right\}, \tag{8}$$

where $\frac{\nabla_- G_{tj}(\mathbf{Y}_{tj},k_{tj})}{\nabla k_{tj}}$ and $\frac{\nabla_+ G_{tj}(\mathbf{Y}_{tj},k_{tj})}{\nabla k_{tj}}$ are the infimum of all left-sided difference quotients and the supremum of all right-sided difference quotients of the function $G_{tj}(\mathbf{Y}_{tj}, k_{tj})$ at the point k_{tj}, respectively: $\frac{G(\mathbf{Y}_{tj},a)-G(\mathbf{Y}_{tj},k_{tj})}{a-k_{tj}} \geq \frac{\nabla_- G(\mathbf{Y}_{tj},k_{tj})}{\nabla k_{tj}}$, $\forall a < k_{tj}$, and $\frac{G(\mathbf{Y}_{tj},b)-G(\mathbf{Y}_{tj},k_{tj})}{b-k_{tj}} \leq \frac{\nabla_+ G(\mathbf{Y}_{tj},k_{tj})}{\nabla k_{tj}}$, $\forall b > k_{tj}$, where a, b and k_{tj} are in the domain of G. Set $S_{tj} = [k^L_{tj}, k^H_{tj}]$. Firm j's optimal capacity in period t is determined based on its current capacity k_{t-1j} and S_{tj}: if $k_{t-1j} \in S_{tj}$, no adjustment should be made, i.e., $k^*_{tj} = k_{t-1j}$; if $k_{t-1j} < k^L_{tj}$, an investment should be made such that the new capacity hits the lower boundary of S_{tj}, i.e., $k^*_{tj} = k^L_{tj}$; if $k_{t-1j} > k^H_{tj}$, a disinvestment should be made such that the new capacity hits the higher boundary of S_{tj}, i.e., $k^*_{tj} = k^H_{tj}$.

Proof. Assumption 1 of the model specifies that for any given and fixed capacity of the opponent $k_{ti} \in \mathscr{K}_{ti}$ and for each $\omega_t \in \Theta$, firm j's operating profit function $\pi_{tj}(k_{ti}, \cdot, \omega_t)$ and salvage value function $F_j(k_{Ti}, \cdot, \omega_t)$ are concave in its own decision k_{tj}. If firm j's optimization problem V^*_{tj} is also jointly concave in (k_{t-1j}, k_{tj}) for any given $k_{ti} \in \mathscr{K}_{ti}$ and for each $\omega_t \in \Theta$, then firm j's expected NPV function G_{tj}, $G_{tj}(\mathbf{Y}_{tj}, k_{tj}) = \pi_{tj}(k_{ti}, k_{tj}, \omega_t) + \delta E[V^*_{t+1j}(\mathbf{Y}_{t+1j}) \mid \omega_t]$, is concave in k_{tj}, as a sum of concave functions is concave. *Theorem 2* of Eberly and Van Mieghem [21] proves that in a single-firm setting, given $\omega_t \in \Theta$, the unique solution to the firm's concave optimization problem G_{tj} is an ISD policy. It is defined by an unique stayput interval $[k^L_{tj}, k^H_{tj}]$, where the two boundaries of the interval, i.e., k^L_{tj} and k^H_{tj}, can be computed using *Equation (18)* and *Equation (19)* of Eberly and Van Mieghem [21]. In the duopoly, these two boundaries then take the forms of Eqs. (7) and (8), respectively. Under Assumption 2, which specifies $c_{tj} > r_{tj}$, and because of the concavity of the optimization problem V^*_{tj} (see Eq. (4)), it guarantees that $k^L_{tj} \leq k^H_{tj}$. The theorem then follows. □

Proposition 2. *Under Assumptions 1 and 2, if firm j adopts the reactive strategy, the optimal value function V^*_{tj} is jointly concave in (k_{t-1j}, k_{tj}) for any given current capacity of the opponent $k_{ti} \in \mathscr{K}_{ti}$, if $i = l$ (or $k_{t-1i} \in \mathscr{K}_{t-1i}$, if $i = f$) and for each $\omega_t \in \Theta$.*

Proof. The salvage value function F_j is concave by assumption. We then use induction on t and assume V_{t+1j} is concave. Using a concavity preservation lemma (also used in *Theorem 1* of [21]), that is, $\{k_t \geq 0\} =_{\geq 0}$ is nonempty, $A =^2_{\geq 0}$ is a convex set, and $\pi_t(k_t, \omega) - C_t \times (k_t - k_{t-1})^+ + r_t \times (k_{t-1} - k_t)^+ + \delta E[V_{t+1}(k_t) \mid \omega]$ is jointly concave in $(k_{t-1}, k_t) \in A$ as a sum of jointly concave functions. In other words, a single firm's optimal value function $V_t(k_{t-1}, \omega_t)$ is jointly concave in (k_{t-1}, k_t) for each $\omega_t \in \Theta$. In the reactive case with a given and fixed capacity of the opponent, $k_{\tau i}$, which is invariant for all $\tau > t$, firm j's optimal value function $V^*_{tj}(k_{ti}, k_{t-1j}, \omega_t)$ is also jointly concave in (k_{t-1j}, k_{tj}) for each $\omega_t \in \Theta$. □

Proposition 3. $S_{tj} \subseteq [k^L_{tj}, k^H_{tj}]$, $k^L_{tj} = \inf S_{tj}$ *and* $k^H_{tj} = \sup S_{tj}$, *where* $k^L_{tj} = \sup\{\{k^o_{tj}\} \cup \{k_{tj} : \frac{\nabla_- G_{tj}(\mathbf{Y}_{tj},k_{tj})}{\nabla k_{tj}} \geq c_{tj}, k_{tj} \in \mathscr{K}_{tj}\}\}$ *and* $k^H_{tj} = \inf\{\{k^e_{tj}\} \cup \{k_{tj} : \frac{\nabla_+ G_{tj}(\mathbf{Y}_{tj},k_{tj})}{\nabla k_{tj}} \leq r_{tj}, k_{tj} \in \mathscr{K}_{tj}\}\}$.

Proof. We abbreviate firm j's expected NPV function $G_{tj}(\mathbf{Y}_{tj}, k_{tj} = k)$ as $G(k)$. According to Eqs. (7) and (8) in Proposition 1, $\frac{G(k)-G(k^L_{tj})}{k-k^L_{tj}} \geq c_{tj}$, $\forall k < k^L_{tj}$, and $\frac{G(k)-G(k^H_{tj})}{k-k^H_{tj}} \leq r_{tj}$, $\forall k > k^H_{tj}$. This means that $\forall k < k^L_{tj}$ or $k > k^H_{tj}$, $k \notin S_{tj}$. If $k^L_{tj} \leq k^H_{tj}$, $S_{tj} \subseteq [k^L_{tj}, k^H_{tj}] \neq \emptyset$. Otherwise, $S_{tj} = \emptyset$.

Assume that there exists $a = \inf\{k : \frac{G(k^L_{tj})-G(k)}{k^L_{tj}-k} \geq c_{tj}\}$ and $b = \sup\{k : \frac{G(k^H_{tj})-G(k)}{k^H_{tj}-k} \leq r_{tj}\}$ (by definition, $a \neq k^L_{tj}$ and $b \neq k^H_{tj}$). Thus, $\forall k < a$, $\frac{G(k)-G(a)}{k-a} > c_{tj}$; $\forall k > b$, $\frac{G(k)-G(b)}{k-b} \leq r_{tj}$. This corresponds to the definitions of k^L_{tj} and k^H_{tj}, indicating that $a = k^L_{tj}$ and $b = k^H_{tj}$. It means that such a and b do not exist. Therefore, k^L_{tj} and k^H_{tj} are the lowerbound and the upperbound of S_{tj}. □

Proposition 4. *Given the stayput set in the nth competitive-goal range, $\mathbf{N}_n$, $N^o_n = \inf \mathbf{N}_n$ and $N^e_n = \sup \mathbf{N}_n$: $\forall k$, if the expected NPV*

Table 5
Model parameters.

Parameter	Description
Γ	set of time periods
k_{tj}, $\mathscr{K}_{tj}$	firm j's capacity and the set of its available capacity choices in period t
k^o_{tj}, k^e_{tj}	origin and final values of the capacity space $\mathscr{K}_{tj}$
$\mathbf{K}_{tj}$	firm j's investment strategy vector from period t to the end of Γ
$\mathcal{K}_{tj}$	the set of all possible $\mathbf{K}_{tj}$
ω_t, Θ	demand indicator of period t and the set of all possible ω_t
Pr	transition probability function of the demand
$\mathbf{Y}_{tj}$	state vector observed by firm j at the beginning of period t
P_t	price function in period t
H_{tj}	firm j's setup and operating cost function in period t
π_{tj}, F_j	firm j's operating profit function in period t and salvage value function
c_{tj}, r_{tj}	firm j's marginal investment cost and marginal disinvestment revenue in period t
C_{tj}	firm j's investment cost function in period t
δ	single-period discount factor
V_{tj}, V^*_{tj}	firm j's value function and optimal value function at the beginning of period t
S_{tj}	firm j's stayput interval in period t
k^L_{tj}, K^L_{tj}	lowerbound and lowerbound function of firm j's stayput interval in period t
k^H_{tj}, K^H_{tj}	upperbound and upperbound function of firm j's stayput interval in period t
I^o_i, I^e_i	origin and final values of the ith competitive-goal range
N_i, N^o_i, N^e_i	stayput interval in the ith competitive-goal range, origin and final values of the interval
s_n, s^o_n, s^e_n	nth stayput interval in the capacity space, origin and final values of the interval

value of staying at k equals the value of adjusting to N^e_n or to N^o_n, then the value of staying at k equals the value of adjusting to any value in $\mathbf{N}_n$.

Proof. We abbreviate firm j's expected NPV function $G_{tj}(\mathbf{Y}_{tj}, k_{tj} = k)$ as $G(k)$. Given N_n, $N^o_n = \inf N_n$ and $N^e_n = \sup N_n$, $G(N^e_n) - G(k) > r \times (N^e_n - k)$, $\forall k \in N_n$. Assume that there exists a capacity value i that $i > N^n_e$ satisfying $i = K^H([N^e_n, i])$, i.e., $G(i) - G(N^e_n) > r \times (i - N^e_n)$. Adding the two inequality, we get: $G(i) - G(k) > r \times (i - k)$, $\forall k \in N_n$. Therefore, $i = K^L([k, i])$, $\forall k \in N_n$. The proof for the case where $i < N^o_n$ satisfying $i = K^L([i, N^o_n])$ follows the same argument. The proposition then follows. □

Proposition 5. *Given the stayput set in the nth competitive-goal range, $\mathbf{N}_n$ and a capacity value $\bar{k} \in \mathbf{N}_n$: if there exists a capacity value $L < \bar{k}$ (or $H > \bar{k}$) such that $\bar{k} \le K^H([L, \bar{k}])$ (or $\bar{k} \ge K^L([\bar{k}, H])$), then $k \le K^H([L, k])$, $\forall k \in \mathbf{N}_n$ and $k > \bar{k}$ (or $k \ge K^L([k, H])$, $\forall k < \bar{k}$).*

Proof. We abbreviate firm j's expected NPV function $G_{tj}(\mathbf{Y}_{tj}, k_{tj} = k)$ as $G(k)$. Given $\bar{k} \in N_n$, for all k that $\bar{k} < k \in N_n$, $G(k) - G(\bar{k}) > r \times (k - \bar{k})$. Assume that there exists $i < \bar{k}$ satisfying $\bar{k} = K^H([i, \bar{k}])$, i.e., $G(\bar{k}) - G(i) > r \times (\bar{k} - i)$. Adding the two inequalities, we get: $G(k) - G(i) > r \times (k - i)$. The proof for the case where $i > \bar{k}$ satisfying $\bar{k} = K^L([\bar{k}, i])$ follows the same argument. The proposition then follows. □

Theorem 1. *$\forall k \in S_{tj}$, no adjustment should be made, i.e., $k^*_{tj} = k$. $\forall k < \inf S_{tj}$ or $k > \sup S_{tj}$, the optimal decision is to adjust to the closest stayput interval boundary. Given two disjoint stayput intervals, $[s^L_n, s^H_n]$ and $[s^L_{n+1}, s^H_{n+1}]$, $\forall n \ge 1$, there exists an investment threshold $s_{n,n+1} \in [s^H_n, s^L_{n+1}]$ such that the optimal investment policy assigns all capacity in $(s^H_n, s_{n,n+1}]$ to be adjusted downwards to s^H_1, and assigns all capacity in $[s_{n,n+1}, s^L_{n+1})$ to be adjusted upwards to s^L_{n+1}.*

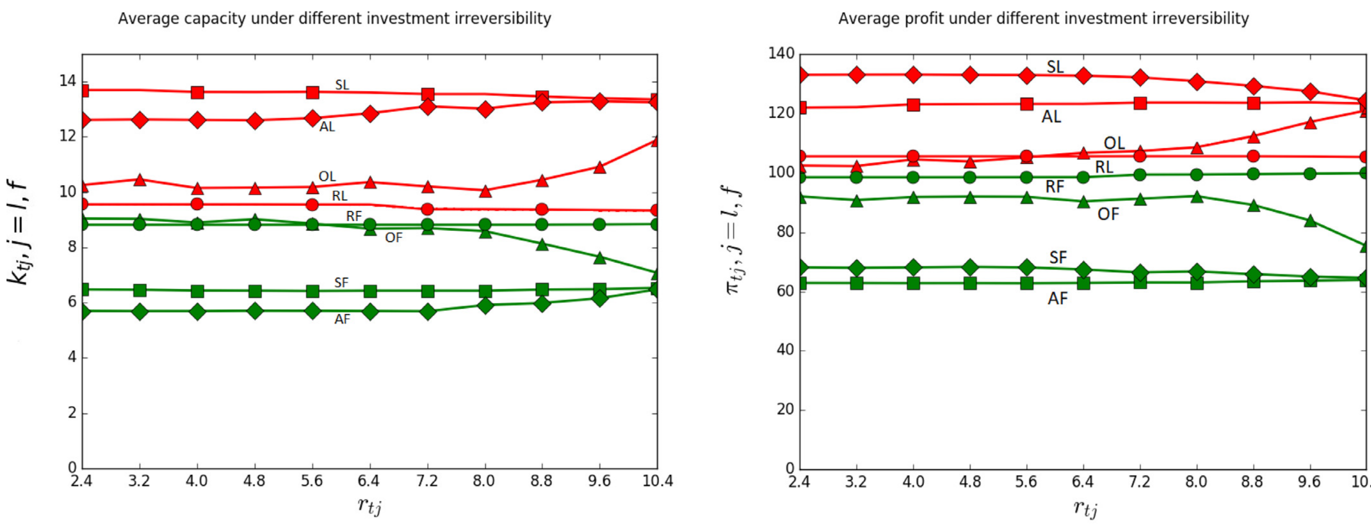
Proof. According to Propositions 3–5, the set of remaining capacity values after the three steps of the *Decomposition Algorithm* composes firm j's stayput interval in period t, S_{tj}. This interval is a set of capacity values, of which the associated value of Eq. (4) cannot be improved. According the procedure following which we derive the stayput values, for any capacity values outside S_{tj}, the optimal decision is to adjust to a stayput interval boundary since the value of adjusting to a stayput interval is larger than the value of staying or that of adjusting to any other non-stayput value. After the adjustment, the value of staying is larger than the value of adjusting to any other value. Considering all available stayput intervals and all values in each interval, the optimal decision for a non-stayput capacity is to adjust to the closest boundary of a close-by interval. This is because the value of adjusting incorporates the cost of adjusting and it depends on the distance between the current capacity and the target capacity.

For any non-stayput capacity value k, $s^H_1 < k < s^L_2$, that is between the two consecutive stayput intervals, $[s^L_1, s^H_1]$ and $[s^L_2, s^H_2]$, an adjustment should be made to either s^L_2 or s^H_2. By comparing the expected NPV of adjusting k to s^H_1 with the one of adjusting k to s^L_2, a decision whether to invest or disinvest can be made. We abbreviate firm j's expected NPV function $G_{tj}(\mathbf{Y}_{tj}, k_{tj} = k)$ as $G(k)$. Under Assumption 2, which specifies $r_{tj} < c_{tj}$, the function $Threshold(k, s^H_1, s^L_2) = G(s^L_2) - c_{tj} \times (s^L_2 - k) - G(s^H_1) - r_{tj} \times (k - s^H_1)$ is monotonously increasing in k. The theorem then follows. □

Appendix: Impact of investment irreversibility

Using Monte Carlo simulations, we investigate the impact of investment irreversibility on capacity strategies. Using the same remaining parameters as in Section 5, we change the value of disinvestment unit price, i.e., r_{tj}, to represent different levels of investment irreversibility. In each experiment, 10,000 demand paths that follow the transition rule in Section 5.1 are simulated and the same demand paths are applied to all four cases. In each simulation in a case, the optimal capacity and profit of each period are computed and then averaged over the entire time horizon Γ. These mean values are then averaged over the 10,000 simulations and the resulting final values serve as an indicator of a player's average performance in a case. Figs. 5a and 5b show the average capacity and profits of all four cases with different r_{tj} values.

We find that the results in Section 5 are robust with respect to different r_{tj} values. For instance, the leader generally performs better than the follower in terms of average capacity and profit and the leader performs better in asymmetric cases compared to that in symmetric cases, while the opposite holds for the follower. When the level of investment irreversibility is very low, i.e., r_{tj} is large compared to c_{tj}, the leader in case *optimal, OL*, has slightly more chances than the follower to exercise its competitive strategy, leading the follower's, *OF*'s, average capacity and profits to decrease. The two firms in other cases keep the average capacity and profit relatively constant.



(a) Average capacity (b) Average profit

Fig. 5. Impact of investment irreversibility on capacity strategies ($c_{tj} = 10.7$).

Appendix: Pseudocode

Table 6 presents the pseudocode of our *Decomposition Algorithm*. Below, we elaborate on each step of the algorithm.

Step 1. Starting with $[k_{tj}^L, k_{tj}^L]$ as the first inspection range, we apply the *rolling up* procedure to the capacity space $[k_{tj}^L, k_{tj}^H]$ to identify the lists of I_n^o and I_n^e, denoted as $(I_n^o)_{n\in N}$ and $(I_n^e)_{n\in N}$. The first value in $(I_n^o)_{n\in N}$ is set to be the start of the capacity space, i.e., $I_1^o = k_{tj}^L$. Given firm j's capacity k_{tj}, we denote the opponent's response function as follows: $K_{t+1i}(k_{tj}) = \text{argmax}_{k_{t+1i}} V_{t+1i}(\mathbf{Y}_{t+1i}, \mathbf{K}_{t+1i})$, where in $\mathbf{K}_{t+1i}$, $k_{\tau i} = k_{t+1i}$, $\forall \tau \in \{t+2, \cdots, T\}$. In each iteration of the rolling procedure, we check whether the right end of the inspection range, assumed to be I, satisfies the following: $K_{t+1i}(I) \neq K_{t+1i}(\triangle^+ I)$. If so, I is added to $(I_n^e)_{n\in N}$; if not, the rolling procedure proceeds to the next iteration $[k_{tj}^L, {}^+I]$ following Definition 1. For each value in $(I_n^e)_{n\in N}$, we set $I_{n+1}^o = \triangle^+ I_n^e$ and add it to $(I_n^o)_{n\in N}$. At the end of the rolling procedure, we add k_{tj}^H to the list $(I_n^e)_{n\in N}$ as the last value, i.e., $I_N^e = k_{tj}^H$.

Step 2. We define the following two boundary functions, dependent on the capacity space $\mathscr{K} = [k^o, k^e]$: $K^L([k^o, k^e]) = \sup\left\{\{k^o\} \cup \{k : \frac{\nabla_- G_{tj}(\mathbf{Y}_{tj}, k)}{\nabla k} \geq c_{tj}, k \in \mathscr{K}\}\right\}$ and $K^H([k^o, k^e]) = \inf\left\{\{k^e\} \cup \{k : \frac{\nabla_+ G_{tj}(\mathbf{Y}_{tj}, k)}{\nabla k} \leq r_{tj}, k \in \mathscr{K}\}\right\}$. In each competitive-goal range $[I_n^o, I_n^e]$, we first identify the lowerbound and upperbound of the stayput set $\mathbf{N}_n$, N_n^o and N_n^e, as follows: if $K^L([I_n^o, I_n^e]) \leq K^H([I_n^o, I_n^e])$, $N_n^o = K^L([I_n^o, I_n^e])$ and $N_n^e = K^H([I_n^o, I_n^e])$; if $K^L([I_n^o, I_n^e]) > K^H([I_n^o, I_n^e])$, we set $\mathbf{N}_n$ to be an empty set. Next, we initialize $\mathbf{N}_n = [N_n^o, N_n^e]$ and apply the *rolling up* procedure to the range $[N_n^o, N_n^e]$, starting with the inspection range $[N_n^o, N_n^o]$. At each iteration with the inspection range $[N_n^o, k]$, k is eliminated from $\mathbf{N}_n$ if $k > K^H([N_n^o, k])$. After the *rolling up* procedure, we apply the *rolling down* procedure in $\mathbf{N}_n$, starting with the inspection range $[N_n^e, N_n^e]$. At each iteration with the inspection range $[k, N_n^e]$, k is eliminated from $\mathbf{N}_n$ if $k < K^L([k, N_n^e])$.

Step 3. We first initialize $\mathbf{S}_n = \mathbf{N}_n$. Then $\forall i = 1, \cdots, n-1$, using the upperbound of N_i, N_i^e, as a benchmark, we apply the *rolling up* procedure to examine all capacity values in N_n. The first iteration starts with the inspection range $[N_i^e, N_n^o]$. At each iteration with the inspection range $[N_i^e, k]$, k is eliminated from $\mathbf{S}_n$ if $k > K^H([N_i^e, k])$; otherwise, we stop the rolling procedure. The *rolling up* procedure is applied $n-1$ times. Then we apply the *rolling down* procedure in remaining $\mathbf{S}_n$, using the lowerbound of N_i, $\forall i = n+1, \cdots, N$, N_i^o, as a benchmark. The first iteration starts with the inspection range $[N_n^e, N_i^o]$. At each iteration with the inspection range $[k, N_i^o]$, k is eliminated from $\mathbf{S}_n$ if $k < K^L([k, N_i^o])$; otherwise, we stop the rolling procedure. The *rolling down* procedure is applied $N-n$ times. Figure 6 shows an illustration of the cross-interval comparison. In the figure, there are four competitive-goal ranges, from the $n-1$th to the $n+2$th. In each range, the red lines indicate the stayput intervals after the second step, $\mathbf{N}_{n-1}, \cdots, N_{n+2}$. The black solid line with an arrow indicates the $n-1$th *rolling up*

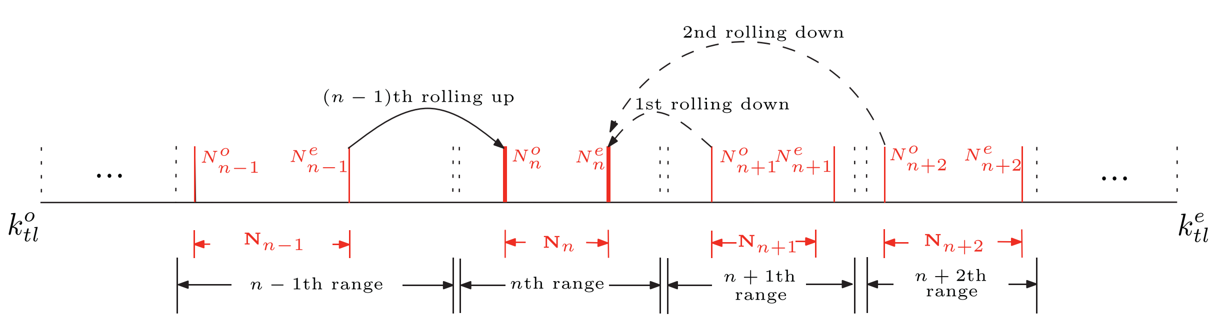


Fig. 6. Illustration of the cross-interval comparison (step 3 of the *Decomposition Algorithm*).

Table 6
Decomposition Algorithm for computing firm j's stayput interval in period t: S_{tj}.

Data: $[k_{tj}^L, k_{tj}^H]$

Result: $S_{tj} = \cup_n S_n$

Step 1: Initialize $I_n^o list \leftarrow \{\ \}$, $I_n^e list \leftarrow \{\ \}$

APPEND $(I_n^o list, k_{tj}^L)$

Initialize $I \leftarrow k_{tj}^L$

while $I = k_{tj}^H$ *is not met* **do**
  while $K_{ti}(I) \neq K_{ti}(\triangle^+ I)$ *is not met* **do**
    $I \leftarrow \triangle^+ I \in [k_{tj}^L, k_{tj}^H]$
  end
  APPEND $(I_n^e list, I)$, $(I_n^0 list, \triangle^+ I)$
  $I = \triangle^+ I \in [k_{tj}^L, k_{tj}^H]$
end

$N \leftarrow n(I_n^e list)$

Step 2: initialize $n \leftarrow 1$

while $n = N$ *is not met* **do**
  $L \leftarrow \sup\{\{I_n^o\} \cup \{k_{tj} : \frac{\nabla_- G_{tj}(\mathbf{Y}_{tj}, k_{tj})}{\nabla k_{tj}} \geq c_{tj}, \quad k_{tj} \in [I_n^o, I_n^e]\}\}$ and
  $H \leftarrow \inf\{\{I_n^e\} \cup \{k_{tj} : \frac{\nabla_+ G_{tj}(\mathbf{Y}_{tj}, k_{tj})}{\nabla k_{tj}} \leq r_{tj}, \quad k_{tj} \in [I_n^o, I_n^e]\}\}.$
  while $L > H$ **do**
    $\mathbf{N}_n \leftarrow \emptyset$
  end
  while $L \leq H$ **do**
    $N_n^o \leftarrow L$ and $N_n^e \leftarrow H$, $\mathbf{N}_n \leftarrow [N_n^o, N_n^e]$
    initialize $k \leftarrow \triangle^+ N_n^o$
    while $k \geq N_n^e$ *is not met* **do**
      while $k > K^H([N_n^o, k])$ **do**
        $\mathbf{N}_n - k$
      end
      $k \leftarrow \triangle^+ k \in \mathbf{N}_n$
    end
    initialize $k \leftarrow \triangle^- N_n^e$
    while $k \leq N_n^o$ *is not met* **do**
      while $k < K^L([k, N_n^e])$ **do**
        $\mathbf{N}_n - k$
      end
      $k \leftarrow \triangle^- k \in \mathbf{N}_n$
    end
  end
  $n \leftarrow n + 1$
end

(*continued on next page*)

Table 6 (*continued*)

$$
\begin{array}{l}
\textbf{while } n = N \textit{ is not met } \textbf{do} \\
\quad k \leftarrow N_n^o \\
\quad \text{initialize } i = 1 \\
\quad \textbf{while } i = n-1 \textit{ is not met } \textbf{do} \\
\quad\quad \textbf{while } k = N_n^e \textit{ is not met } \textbf{do} \\
\quad\quad\quad \textbf{while } k > K^H([N_i^e, k]) \textbf{ do} \\
\quad\quad\quad\quad \mathbf{S}_n - k \\
\quad\quad\quad \textbf{end} \\
\quad\quad\quad k \leftarrow \triangle^+ k \in \mathbf{S}_n \\
\quad\quad \textbf{end} \\
\quad\quad i \leftarrow i+1 \\
\quad \textbf{end} \\
\quad k \leftarrow N_n^e \\
\quad \text{initialize } i \leftarrow n+1 \\
\quad \textbf{while } i = N \textit{ is not met } \textbf{do} \\
\quad\quad \textbf{while } k = N_n^o \textit{ is not met } \textbf{do} \\
\quad\quad\quad \textbf{while } k < K^L([k, N_i^o]) \textbf{ do} \\
\quad\quad\quad\quad \mathbf{S}_n - k \\
\quad\quad\quad \textbf{end} \\
\quad\quad\quad k \leftarrow \triangle^- k \in \mathbf{S}_n \\
\quad\quad \textbf{end} \\
\quad\quad i \leftarrow i+1 \\
\quad \textbf{end} \\
\quad n \leftarrow n+1 \\
\textbf{end}
\end{array}
$$

procedure for $\mathbf{N}_n$, and the black dashed lines with an arrow indicate the first two *rolling down* procedures for $\mathbf{N}_n$.

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