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# Solution of Integral Equation using Second and Third Order B-Spline Wavelets

Kanchan Lata Gupta <sup>α</sup>, Gopal Gupta <sup>σ</sup>, V. K Singh <sup>ρ</sup> & B. Kunwar <sup>ω</sup>

**Abstract-** It was proven that semi-orthogonal wavelets approximate the solution of integral equation very finely over the orthogonal wavelets. Here we used the compactly supported semi-orthogonal B-spline wavelets generated in our paper "Compactly Supported B-spline Wavelets with Orthonormal Scaling Functions" satisfying the Daubechies conditions, to solve the Fredholm integral equation. The generated wavelets satisfies all the properties on the bounded interval. The method is computationally easy, which is illustrated with two examples whose solution closely resembles the exact solution as the order of wavelet increases.

**Keywords:** B-spline wavelets, dual wavelets, integral equation.

## I. INTRODUCTION

Integral equations are find very vast usage in many areas of engineering, physics, applied mathematics and many more. Here we seek to resolve a class of integral equation called Fredholm integral equation. There are various methods like variational method, collocation type method and integrated collocation method are known to estimate the solution of integral [1]. Some of the methods convert the integral equation into non linear equation while in some others method it transform to a set of algebraic equations.

## II. SECOND AND THIRD ORDER B-SPLINE WAVELETS ON [0,1]

The wavelets are generally defined as

$$\psi_{j,k}(x) = \psi(2^j x - k) \quad 0 \leq k \leq 2^j - 1$$

that is, the translation and dilation of mother wavelets. Here  $j$  is called the octave level and  $j = j_0$  is lowest octave level, first obtained in [2] for B-spline semi-orthogonal wavelets of order  $m$  as

$$2^{j_0} \geq 2m - 1$$

to give a complete wavelet in interval [0,1]. Here the actual coordinate position  $x$  is related to  $x_j$  as  $x = 2^j x_j$ .

The second order B-spline scaling function are given by

$$B_{j,k}(x) = \begin{cases} x_j - k & k \leq x_j \leq k + 1 \\ 2 - (x_j - k) & k + 1 \leq x_j \leq k + 2 \\ 0 & \text{otherwise} \end{cases} \quad k = 0, \dots, x_j - 2 \quad (2.1)$$

For  $j=2$  the inner scaling functions are obtained by

Wavelets due to its outstanding properties like vanishing moment, compact support, are good candidates for providing fast algorithm in numerical aspects in approximating [3,4,5,6]. In the present paper, we apply compactly supported semi orthogonal B-Spline wavelet generated in our paper[7] for bounded interval to solve the linear Fredholm integral equation of form

$$y(x) = f(x) + \int_0^1 k(x,t)y(t)dt \quad 0 \leq x \leq 1$$

where  $f$  and  $k$  are given continuous function. Due to the interesting features like smoothness which increases with order of vanishing moment and closed form expression of compactly supported spline wavelets, it was widely used in solving numerical problems. The wavelet formed satisfy all the properties on a bounded interval. In [8] it was shown that semi-orthogonal wavelets are better than orthogonal for integral equation application.

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$$B_{2,0}(x) = \begin{cases} 4x & 0 \leq x \leq \frac{1}{4} \\ 2 - 4x & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$B_{2,1}(x) = \begin{cases} 4x - 1 & 0 \leq x \leq \frac{1}{2} \\ 3 - 4x & \frac{1}{2} \leq x \leq \frac{3}{4} \\ 0 & \text{otherwise} \end{cases}$$

$$B_{2,2}(x) = \begin{cases} 4x - 2 & \frac{1}{2} \leq x \leq \frac{3}{4} \\ 4 - 4x & \frac{3}{4} \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

The L.H.S and R.H.S boundary scaling functions are

$$B_{2,-1}(x) = \begin{cases} 1 - 4x & 0 \leq x \leq \frac{1}{4} \\ 0 & \text{otherwise} \end{cases}$$

$$B_{2,3}(x) = \begin{cases} 4x - 3 & \frac{3}{4} \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

The second order B-spline wavelet obtained in [7] are given by

$$\psi_{j,k}(x) = \frac{1}{2} \begin{cases} \frac{-2}{\sqrt{2}}(x_j - k) & k \leq x_j \leq k + \frac{1}{2} \\ 2\left(\frac{3}{\sqrt{2}} - 1\right)(x_j - k) - \frac{4}{\sqrt{2}} + 1 & k + \frac{1}{2} \leq x_j \leq k + 1 \\ 2\left(3 - \frac{2}{\sqrt{2}}\right)(x_j - k) + \frac{6}{\sqrt{2}} - 7 & k + 1 \leq x_j \leq k + \frac{3}{2} \\ -2\left(3 + \frac{2}{\sqrt{2}}\right)(x_j - k) + \frac{6}{\sqrt{2}} + 11 & k + \frac{3}{2} \leq x_j \leq k + 2 \\ 2\left(\frac{3}{\sqrt{2}} + 1\right)(x_j - k) - \frac{14}{\sqrt{2}} - 5 & k + 2 \leq x_j \leq k + \frac{5}{2} \\ -\frac{2}{\sqrt{2}}(x_j - k) + \frac{6}{\sqrt{2}} & k + \frac{5}{2} \leq x_j \leq k + 3 \\ 0 & \text{otherwise} \end{cases} \tag{2.2}$$

The inner wavelet functions are obtained as

$$\psi_{2,0}(x) = \frac{1}{2} \begin{cases} \frac{-2}{\sqrt{2}}(4x) & 0 \leq x_j \leq \frac{1}{8} \\ 2\left(\frac{3}{\sqrt{2}} - 1\right)(4x) - \frac{4}{\sqrt{2}} + 1 & \frac{1}{8} \leq x_j \leq \frac{1}{4} \\ 2\left(3 - \frac{2}{\sqrt{2}}\right)(4x) + \frac{6}{\sqrt{2}} - 7 & \frac{1}{4} \leq x_j \leq \frac{3}{8} \\ -2\left(3 + \frac{2}{\sqrt{2}}\right)(4x) + \frac{6}{\sqrt{2}} + 11 & \frac{3}{8} \leq x_j \leq \frac{1}{2} \\ 2\left(\frac{3}{\sqrt{2}} + 1\right)(4x) - \frac{14}{\sqrt{2}} - 5 & \frac{1}{2} \leq x_j \leq \frac{5}{8} \\ -\frac{2}{\sqrt{2}}(4x) + \frac{6}{\sqrt{2}} & \frac{5}{8} \leq x_j \leq \frac{3}{4} \\ 0 & \text{otherwise} \end{cases}$$

$$\psi_{2,1}(x) = \frac{1}{2} \begin{cases} \frac{-2}{\sqrt{2}}(4x - 1) & \frac{1}{4} \leq x_j \leq \frac{3}{8} \\ 2\left(\frac{3}{\sqrt{2}} - 1\right)(4x - 1) - \frac{4}{\sqrt{2}} + 1 & \frac{3}{8} \leq x_j \leq \frac{1}{2} \\ 2\left(3 - \frac{2}{\sqrt{2}}\right)(4x - 1) + \frac{6}{\sqrt{2}} - 7 & \frac{1}{2} \leq x_j \leq \frac{5}{8} \\ -2\left(3 + \frac{2}{\sqrt{2}}\right)(4x - 1) + \frac{6}{\sqrt{2}} + 11 & \frac{5}{8} \leq x_j \leq \frac{3}{4} \\ 2\left(\frac{3}{\sqrt{2}} + 1\right)(4x - 1) - \frac{14}{\sqrt{2}} - 5 & \frac{3}{4} \leq x_j \leq \frac{7}{8} \\ -\frac{2}{\sqrt{2}}(4x - 1) + \frac{6}{\sqrt{2}} & \frac{7}{8} \leq x_j \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

The L.H.S and R.H.S boundary wavelet function are given as

$$\psi_{2,-1}(x) = \frac{1}{2} \begin{cases} 2\left(4 - \frac{5}{\sqrt{x}}\right)(4x + 1) + \frac{14}{\sqrt{2}} - 10 & 0 \leq x \leq \frac{1}{8} \\ -2\left(3 + \frac{1}{\sqrt{2}}\right)(4x + 1) + \frac{2}{\sqrt{x}} + 11 & \frac{1}{8} \leq x \leq \frac{1}{4} \\ 2\left(\frac{3}{\sqrt{2}} + 1\right)(4x + 1) - \frac{14}{\sqrt{2}} - 5 & \frac{1}{4} \leq x \leq \frac{3}{8} \\ -\frac{2}{\sqrt{2}}(4x + 1) + \frac{6}{\sqrt{2}} & \frac{3}{8} \leq x \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$\psi_{2,2}(x) = \frac{1}{2} \begin{cases} \frac{-2}{\sqrt{2}}(4x - 2) & 0 \leq x \leq \frac{1}{8} \\ 2\left(\frac{3}{\sqrt{2}} - 1\right)(4x - 2) - \frac{4}{\sqrt{2}} + 1 & \frac{1}{8} \leq x \leq \frac{1}{4} \\ 2\left(3 - \frac{1}{\sqrt{2}}\right)(4x - 2) + \frac{4}{\sqrt{2}} - 7 & \frac{1}{4} \leq x \leq \frac{3}{8} \\ -2\left(4 + \frac{5}{\sqrt{2}}\right)(4x - 2) + \frac{16}{\sqrt{2}} + 14 & \frac{3}{8} \leq x \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

The third order B-spline scaling function and B-spline wavelet function are given by

$$B_{j,k}(x) = \begin{cases} \frac{1}{2}(x_j - k)^2 & k \leq x_j \leq k + 1 \\ -(x_j - k)^2 + (x_j - k) - \frac{3}{2} & k + 1 \leq x_j \leq k + 2 \\ \frac{1}{2}(x_j - k)^2 - 3(x_j - k) + \frac{9}{2} & k + 2 \leq x_j \leq k + 3 \\ 0 & \text{otherwise} \end{cases} \quad (2.3)$$

$$\text{and } \psi_{j,k}(x) = \frac{1}{2} \begin{cases} I & k + \frac{1}{2} \leq x_j \leq k + \frac{3}{2} \\ II & k + \frac{1}{2} \leq x_j \leq k + 1 \\ III & k + 1 \leq x_j \leq k + \frac{3}{2} \\ IV & k + \frac{3}{2} \leq x_j \leq k + 2 \\ V & k + 2 \leq x_j \leq k + \frac{5}{2} \\ VI & k + \frac{5}{2} \leq x_j \leq k + 3 \\ VII & k + 3 \leq x_j \leq k + \frac{7}{2} \\ VIII & k + \frac{7}{2} \leq x_j \leq k + 4 \\ 0 & \text{otherwise} \end{cases} \quad k = 0, \dots, 2^j - 4 \quad (2.4)$$

where

$$I = -3(x_j - k)^2$$

$$II = -\frac{3}{2} \left[ -(2x_j - 2k)^2 + 3(2x_j - 2k) - \frac{3}{2} \right] + \frac{7}{4}(2x_j - 2k - 1)^2$$

$$III = \frac{7}{2} \left[ -(2x_j - 2k - 1)^2 + 3(2x_j - 2k - 1) - \frac{3}{2} \right] - \frac{3}{2} \left[ \frac{1}{2}(2x_j - 2k)^2 - 3(2x_j - 2k) + \frac{9}{2} \right]$$

$$IV = \frac{7}{2} \left[ \frac{1}{2}(2x_j - 2k - 1)^2 - 3(2x_j - 2k - 1) + \frac{9}{2} \right] - 3(2x_j - 2k - 3)^2$$

$$V = -6 \left[ -(2x_j - 2k - 3)^2 + 3(2x_j - 2k - 3) - \frac{3}{2} \right] + \frac{11}{4}(2x_j - 2k - 4)^2$$

$$VI = -6 \left[ \frac{1}{2}(2x_j - 2k - 3)^2 - 3(2x_j - 2k - 3) + \frac{9}{2} \right] + \frac{11}{2} \left[ -(2x_j - 2k - 4)^2 + 3(2x_j - 2k - 4) - \frac{3}{2} \right] - \frac{3}{4}(2x_j - 2k - 5)^2$$

$$VII = \frac{11}{2} \left[ \frac{1}{2}(2x_j - 2k - 4)^2 - 3(2x_j - 2k - 4) + \frac{9}{2} \right] - \frac{3}{2} \left[ -(2x_j - 2k - 4)^2 + 3(2x_j - 2k - 4) - \frac{3}{2} \right]$$

$$VIII = -\frac{3}{2} \left[ \frac{1}{2}(2x_j - 2k - 5)^2 - 3(2x_j - 2k - 5) + \frac{9}{2} \right]$$

The L.H.S and R.H.S boundary scaling function and wavelet function are given by

$$B_{3,-1}(x) = \begin{cases} -(8x+1)^2 + (1-8x)^2 + 3(8x+1) - \frac{3}{2} & 0 \leq x \leq \frac{1}{8} \\ \frac{1}{2}(8x+1)^2 - 3(8x+1) + \frac{9}{2} & \frac{1}{8} \leq x \leq \frac{1}{4} \\ 0 & \text{otherwise} \end{cases}$$

$$B_{3,6}(x) = \begin{cases} \frac{1}{2}(8x-6)^2 & \frac{3}{4} \leq x \leq \frac{7}{8} \\ -(8x-6)^2 + 3(16x-16) + \frac{1}{2}(10-8x)^2 + \frac{6}{2} & \frac{7}{8} \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\psi_{3,-1}(x) = \frac{1}{2} \begin{cases} I' & 0 \leq x \leq \frac{1}{16} \\ II' & \frac{1}{16} \leq x \leq \frac{1}{8} \\ V & \frac{1}{8} \leq x \leq \frac{3}{16} \\ VI & \frac{3}{16} \leq x \leq \frac{1}{4} \\ VII & \frac{1}{4} \leq x \leq \frac{5}{16} \\ VIII & \frac{5}{16} \leq x \leq \frac{3}{8} \\ 0 & \text{otherwise} \end{cases}$$

and

$$\psi_{3,5}(x) = \frac{1}{2} \begin{cases} I & \frac{5}{8} \leq x \leq \frac{11}{16} \\ II & \frac{11}{16} \leq x \leq \frac{3}{4} \\ III & \frac{3}{4} \leq x \leq \frac{13}{16} \\ IV & \frac{13}{16} \leq x \leq \frac{7}{8} \\ V' & \frac{7}{8} \leq x \leq \frac{15}{16} \\ VI' & \frac{15}{16} \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where  $I' = \frac{7}{2} \left\{ -(16x+1)^2 + 3(16x+1) - \frac{3}{2} \right\} - \frac{3}{2} \left\{ \frac{1}{2}(16x+2)^2 - 3(16x+2) - (2-16x)^2 + 3(2-16x) + \frac{6}{2} \right\} + \frac{7}{4}(-16x+1)$

$$II' = \frac{7}{2} \left\{ \frac{1}{2}(16x+1)^2 - 3(16x+1) + \frac{9}{2} \right\} - 3(16x-1)^2 - 3(-8x+1)^2$$

$$V' = -6 \left\{ -(16x-13)^2 + 3(16x-13) - \frac{3}{2} \right\} + \frac{11}{4}(16x-14)^2 - \frac{3}{2} \left\{ \frac{1}{2}(16x-17)^2 + 3(16x-17) + \frac{9}{2} \right\}$$

$$VI' = -6 \left\{ \frac{1}{2}(16x-13)^2 - 3(16x-13) + \frac{9}{2} \right\} - \frac{3}{4}(16x-15)^2 - \frac{3}{2} \left\{ -(16x-17)^2 - 3(16x-17) - \frac{3}{2} \right\} + \frac{11}{2} \left\{ -(16x-14)^2 + 3(16x-14) + \frac{1}{2}(16x-18)^2 + 3(16x-18) + 3 \right\}$$

The inner third order B-spline scaling function are obtained by substituting  $j = 3$  and  $k = 0,1,2,3,4,5$  in eqn(2.3). And the inner wavelet functions are obtained by putting  $j = 3$  and  $k = 0,1,2,3,4$  in equation (2.4). Fig (2.1) shows the B-spline wavelets for  $m = 2$  &  $m = 3$ .

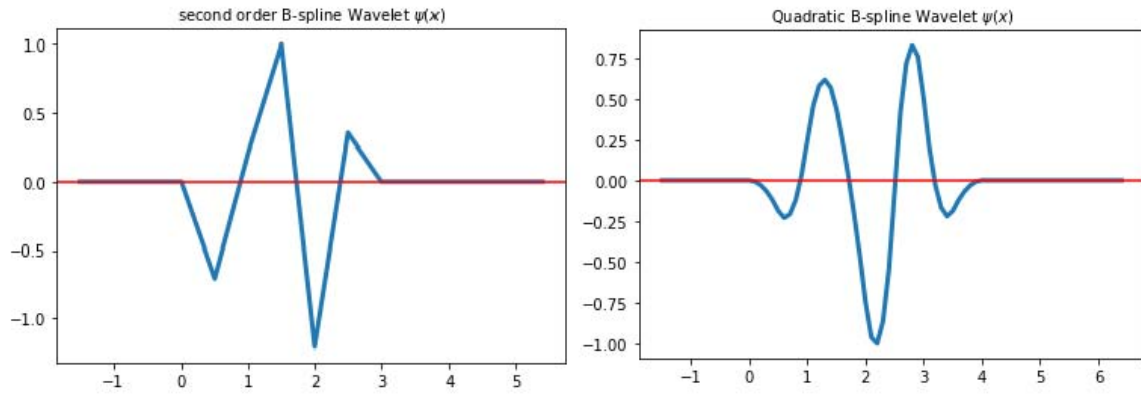


Fig. (2.1): Second and third B-spline wavelet i,e for m=2 and m=3 resp

### III. FUNCTION APPROXIMATION

A function  $g(x)$  defined over  $[0, 1]$  may be represented by B-spline wavelets as

$$g(x) = \sum_{k=-1}^{2^j-(m-1)} \alpha_{j,k} B_{j,k}(x) + \sum_{k=-1}^{2^j-m} \alpha_{j,k} \psi_{j,k}(x), \quad j = 2, 3, \dots, M$$

where  $B_{j,k}$  and  $\psi_{j,k}$  are the scaling and wavelet functions and  $m$  is the order of B-spline wavelets. Also,

$$g(x) = A_g^T \Psi(x)$$

where

$$A_g = [\alpha_{j,-1}, \alpha_{j,0}, \dots, \alpha_{j,2^j-(m-1)}]^T$$

$$\Psi = [B_{j,-1}, B_{j,0}, \dots, B_{j,2^j-(j-1)}, \psi_{j,-1}, \psi_{j,2}, \dots, \psi_{j,2^j-m}]^T$$

where  $\alpha_{j,k} = \int_0^1 g(x) \tilde{B}_{j,k}(x) dx$  and  $\alpha_{j,k} = \int_0^1 g(x) \tilde{\psi}_{j,k}(x) dx$ .

$\tilde{B}_{j,k}$  and  $\tilde{\psi}_{j,k}$  are duals of  $B_{j,k}$  and  $\psi_{j,k}$  resp. The duals can be obtained as follows:

Let  $\phi = [B_{j,-1}, B_{j,0}, \dots, B_{j,2^j-(m-1)}]^T$

$$\psi = [\psi_{j,-1}, \psi_{j,0}, \dots, \psi_{j,2^j-m}]^T$$

$$\text{then, } \int_0^1 \phi \phi^T dx = P_1 \quad \text{and} \quad \int_0^1 \psi \psi^T dx = P_2 \tag{3.1}$$

Let  $\tilde{\phi}$  and  $\tilde{\psi}$  are the dual function of  $\phi$  and  $\psi$  resp. given by

$$\tilde{\phi} = [\tilde{B}_{j,-1}, \tilde{B}_{j,0}, \dots, \tilde{B}_{j,2^j-(m-1)}]^T$$

$$\tilde{\psi} = [\tilde{\psi}_{j,-1}, \tilde{\psi}_{j,0}, \dots, \tilde{\psi}_{j,2^j-m}]^T \tag{3.2}$$

$$\text{such that, } \int_0^1 \tilde{\phi} \phi^T dx = I_1 \quad \text{and} \quad \int_0^1 \tilde{\psi} \psi^T dx = I_2 \tag{3.3}$$

where  $I_1$  and  $I_2$  are identity matrices.

From (3.1) and (3.2),

$$\tilde{\phi} = P_1^{-1} \phi \quad \text{and} \quad \tilde{\psi} = P_2^{-1} \psi$$

#### IV. FREDHOLM INTEGRAL EQUATIONS

Here we consider Fredholm Integral Equations of type

$$y(x) = f(x) + \int_0^1 k(x, t) y(t) dt \quad 0 \leq x \leq 1 \tag{4.1}$$

and solve this equation by second order B-spline wavelets for  $j = 2$ . Let first approximate  $y(x)$  as

$$y(x) = A_y^T \Psi(x) \tag{4.2}$$

where  $\Psi(x) = [B_{2,-1}, B_{2,0}, \dots, B_{2,3}, \psi_{2,-1}, \dots, \psi_{2,2}]^T$

Also denoted as  $\Psi(x) = [\psi_1, \psi_2, \dots, \psi_6, \dots, \psi_9]^T$  (4.3)

we expand  $f(x)$  and  $k(x, t)$  by B-spline dual wavelets defined as in (3.2) as

$$f(x) = A_f^T \tilde{\Psi}(x) \tag{4.4}$$

and  $k(x, t) = \tilde{\Psi}^T(t) \Theta \tilde{\Psi}(x)$  (4.5)

where  $\Theta$  is a  $9 \times 9$  matrix for second order B-spline wavelet with  $j = 2$  with its elements

$$\Theta_{ij} = \int_0^1 [\int_0^1 k(x, t) \psi_i(t) dt] \psi_j(x) dx$$

From (4.2) and (4.5),

$$\int_0^1 k(x, t) y(t) dt = \int_0^1 \tilde{\Psi}^T(t) \Theta \tilde{\Psi}(x) A_y^T \Psi(t) dt = \Theta \tilde{\Psi}(x) A_y^T = \Theta A_y^T \tilde{\Psi}(x) \tag{4.6}$$

Using (4.2), (4.4) and (4.6) in (4.1), we get

$$A_y^T \Psi(x) = A_f^T \tilde{\Psi}(x) + \Theta A_y^T \tilde{\Psi}(x)$$

Multiplying the above equation by  $\tilde{\Psi}^T(x)$  and integrating from 0 to 1, we get

$$A_y^T \int_0^1 \tilde{\Psi}^T(x) \Psi(x) dx = A_f^T + \Theta A_y^T \tag{4.7}$$

Putting  $\int_0^1 \tilde{\Psi}^T(x) \Psi(x) dx = P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$ . For 2nd order B-spline scaling function and wavelet function, for  $j=2$ ,

$$P_1 = \begin{bmatrix} 0.0833 & 0.0417 & 0.0000 & 0.0000 & 0.0000 \\ 0.0417 & 0.1667 & 0.0417 & 0.0000 & 0.0000 \\ 0.0000 & 0.0417 & 0.1667 & 0.0417 & 0.0000 \\ 0.0000 & 0.0000 & 0.0417 & 0.1667 & 0.0417 \\ 0.0000 & 0.0000 & 0.0000 & 0.0417 & 0.0833 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0.1347 & -0.034 & 0.0000 & 0.0000 \\ -0.034 & 0.1667 & -0.0417 & 0.000 \\ 0.0000 & -0.0417 & 0.1667 & -0.049 \\ 0.0000 & 0.0000 & -0.049 & 0.2819 \end{bmatrix}$$

thus,

$$A_y^T P = A_f^T + \Theta A_y^T$$

$$A_y^T (P - \Theta) = A_f^T$$

$$\text{or, } A_y^T = A_f^T(P - \Theta)^{-1}$$

In this way, from equation(4.2) , we get the numerical solution of the integral equation (4.1).Similar process is applied for higher order wavelets and higher octave levels.

### V. ILLUSTRATIVE EXAMPLES

1. Consider the equation

$$y(x) = e^x - \frac{e^{x+1} - 1}{x + 1} + \int_0^1 e^{xt} y(t)dt, \quad 0 \leq x \leq 1$$

with exact solution  $y(x) = e^x$ .

The solution for  $y(x)$  is obtained by the method explain in section 4 with second ( $m=2$ ) and third ( $m=3$ ) order B-spline wavelet for different values of  $j$ . The numerical solution obtained with exact solution  $y(x) = e^x$  are given in Table 5.1

Table 5.1: Exact and Obtained Solution

x	m = 2		m = 3	Exact Sol.
	j = 2	j = 3	j = 3	
0	0.954713	0.981062	1	1
0.1	1.06087	1.12446	1.09688	1.10517
0.2	1.26066	1.20488	1.21838	1.2214
0.3	1.38671	1.34623	1.35443	1.34986
0.4	1.45637	1.49318	1.49702	1.49182
0.5	1.67171	1.6511	1.6467	1.64872
0.6	1.80969	1.82285	1.82085	1.82212
0.7	2.01761	2.01214	2.01787	2.01375
0.8	2.20405	2.22317	2.21904	2.22554
0.9	2.41732	2.44861	2.45811	2.4596

2. Consider the equation

$$y(x) = e^{2x+\frac{1}{3}} - \frac{1}{3} \int_0^1 e^{2x-\frac{5t}{3}} y(t)dt, \quad 0 \leq x \leq 1$$

with exact solution  $y(x) = e^{2x}$ .

The numerical solution obtained for  $m=2$  and  $m=3$  for different values of  $j$  together with exact solution  $y(x) = e^{2x}$  are given in Table 5.2

Table 5.2: Exact and Approximate Solution

x	m = 2		m = 3	Exact sol <sup>n</sup>
	j = 2	j = 3	j = 3	
0	0.892668	0.957694	1	1
0.1	1.14382	1.26296	1.20363	1.2214
0.2	1.58615	1.45818	1.48601	1.49182
0.3	1.90841	1.81305	1.82929	1.82212
0.4	2.16427	2.2306	2.23898	2.22554
0.5	2.77233	2.72857	2.7078	2.71828
0.6	3.2909	3.32686	3.32117	3.32012
0.7	4.0913	4.04803	4.07837	4.0552
0.8	4.84134	4.93798	4.91508	4.95303
0.9	5.85308	5.99318	6.03669	6.04965



## VI. CONCLUSION

In this paper, we present a method to solve linear Fredholm integral equations. Our approximation is based on compactly supported semi-orthogonal B-spline wavelet generated in our previous paper. Two examples are illustrated to check the validity and significance of the proposed technique. The solution of the examples reveals that the exactness of solution increases as the order of B-spline wavelet and octave level increases.

## REFERENCES RÉFÉRENCES REFERENCIAS

1. Delves LM, Mohamed JL. Computational methods for integral equations. CUP Archive; 1988 Mar 31.
2. Goswami JC, Chan AK, Chui CK. On solving first-kind integral equations using wavelets on a bounded interval. IEEE Transactions on antennas and propagation. 1995 Jun; 43 (6): 614-22.
3. Lakestani M, Razzaghi M, Dehghan M. Solution of nonlinear Fredholm-Hammerstein integral equations by using semiorthogonal spline wavelets. Mathematical problems in engineering. 2005 Jan 26; 2005 (1): 113-21.
4. Askari-Hemmat A, Rahbani Z. Pantic B-spline wavelets and their application for solving linear integral equations. Iranian Journal of Science and Technology (Sciences). 2012 Feb 6; 36 (1): 47-50.
5. Maleknejad K, Rostami Y. Numerical Solution for Integro-Differential Equations by Using Quartic B-Spline Wavelet and Operational Matrices. International Journal of Mathematical and Computational Sciences. 2014 Jun 1; 8 (7):1031-9.
6. Bahmanpour M, Fariborzi Araghi MA. Numerical solution of Fredholm and Volterra integral equations of the first kind using wavelets bases. J Math Comput Sci. 2012; 5 (4): 337-45.
7. Gupta KL, Kunwar B, Singh VK. Compactly Supported B-spline Wavelets with Orthonormal Scaling Functions. Asian Research Journal of Mathematics. 2017 Mar 9:1-7.
8. Nevels RD, Goswami JC, Tehrani H. Semi-orthogonal versus orthogonal wavelet basis sets for solving integral equations. IEEE Transactions on Antennas and Propagation. 1997 Sep; 45 (9):1332-9.