A new approach of accounting for impedance effects in Gardner's method of determining the hydraulic conductivity of unsaturated soils

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Abstract. Based on tests carried out on a specific device allowing to determine the water retention and transport properties of granular media at low suctions, an alternative approach to Kunze and Kirkham's method of accounting for the impedance effects due to the high air entry value ceramic disk when using Gardner's method is proposed. Impedance effects are accounted for by proposing analytical solutions to the equations governing water transfers occurring within the specimen and the ceramic disk. By using some experimental data obtained on a volcanic granular substrate used for urban green roofs, the method is successfully compared to Kunze and Kikham's graphical method. Its advantages are to be simpler of use and not operator dependent. A detailed examination of the performance of our method compared to those of Gardner and Kunze and Kirkham is carried out based on experimental data, that confirm its validity.

1. Introduction

Gardner's method [1] was the first analytical method of calculating the hydraulic conductivity of unsaturated porous media based on the measurement of transient outflow under suction step in the pressure plate apparatus. Gardner's method assumes both the linearity of the water retention curve (WRC) and a constant diffusivity over the suction step. However, Gardner's method doesn't account for the impedance effects of the plate (made up of a saturated ceramic porous disk with high air entry value) that may have a significantly lower hydraulic conductivity than that of the saturated specimen. Miller & Elrick [2] were the first to consider the impedance effect, while based on their analytical solution Kunze & Kirkham [3] developed a well-known graphical method. This method is nowadays rarely used, since it has been replaced by numerical back analysis methods (e.g. [4, 5]) that deal with impedance issue through the simulation of the water flow in two-layered media (specimen - disk), by numerically solving Richards equation [6].

In order to avoid some level of subjectivity in Kunze & Kirkham's method, or tedious numerical computation related to the selection of the hydraulic properties model in numerical back analysis method, a new and simpler approach of accounting for the impedance effects in Gardner's method has been developed. The advantage of the new approach compared to Gardner's and Kunze & Kirkham's method is validated based on the comparison with experimental data for three different materials.

2. Theory

The method presented in this work originates from an experimental investigation carried out by [7] based on the device represented in Fig. 1, that schematically illustrates the hanging column apparatus used for simultaneous determining the WRC and the hydraulic conductivity function (HCF). It consists of a metal cell in which the specimen is placed on a saturated HAEV ceramic disk. A suction step is applied by moving down a mobile system in which the constant suction is controlled by the level of the top of the inner tube. The water extracted due to the suction step overflows in the outer tube where the change in water level is monitored by means of a high precision differential pressure gauge ([7]).

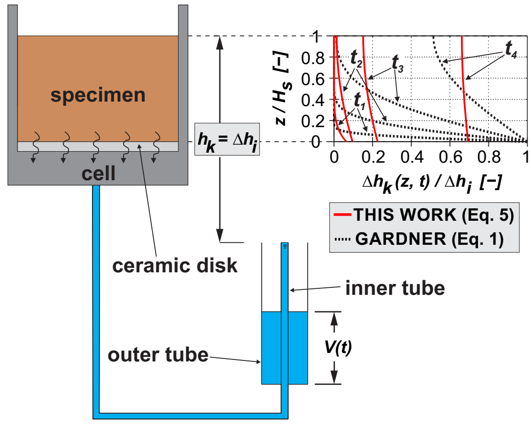

Fig. 1. Scheme of the Hanging column device used in [7].

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2.1 Gardner's method

If the hydraulic conductivity of the ceramic disk is enough higher than that of the specimen, there is no impedance effects and the constant imposed suction increment Δh_i is immediately transferred through the disk to the specimen bottom ($\Delta h_k\,(z=0,\,t) = \Delta h_i = const.$) – see dotted lines in Fig. 1.

Since the suction step at bottom $(\partial \Delta h_k(z,t)/\partial z)|_{z=0}$ governs the outflow from the specimen through Richards equation, Gardner proposed to express $\Delta h_k\,(z,\,t)$ based on an analogy with Terzaghi-Fröhlich consolidation equation [8]:

$$\Delta h_k\left(z,t\right)=\Delta h_i\left(1-\frac{4}{\pi}\sum_{n=1,3,5...}^{\infty}\frac{1}{n}e^{-\left(n/2\right)^2\pi^2\frac{tD\left(h_k\right)}{H_s^2}}sin\frac{\frac{n}{2}\pi z}{H_s}\right) \quad (1)$$

where the drainage length is equal to the thickness of the specimen H_s [L], while $D(h_k)$ is diffusivity [L^2/T], considered as constant over the suction increment Δh_i. By integrating the outflow over time, the standard form of Gardner's solution, describing the evolution of the water volume extracted from the specimen, is obtained:

$$V\left(t\right)=V_\infty\left(1-\frac{8}{\pi^2}\sum_{n=1,3,5...}^{\infty}\frac{1}{n^2}e^{-\left(n\pi/2\right)^2 tD\left(h_k\right)/H_s^2}\right) \quad (2)$$

Where V_∞ is the total volume of water [L^3] extracted from the specimen during the suction step. Finally, the hydraulic conductivity $K(h_k)$ is calculated as $D(h_k)C(h_k)$, where $D(h_k)$ is adjusted to obtain the best possible agreement between Equation (2) and experimental data, while $C(h_k)$ is computed based on the measured values as $\Delta\theta/\Delta h_i = V_\infty/(AH_s\Delta h_i)$.

2.2 Kunze & Kirkham's method

The governing equation in this case is modified equation presented in [2]:

$$\frac{V\left(t\right)}{V_\infty}=1-\sum_{n=1}^{\infty}\frac{2e^{-\lambda_n^2 Dt/H_s^2}}{\lambda_n^2\left(a+csc^2\,\lambda_n\right)} \quad (3)$$

where λ_n is the nth solution of the equation $a\lambda_n = \cot\lambda_n$, while a is the ratio between the impedance of the ceramic disk and that of the specimen. Kunze & Kirkham's solution is graphically presented through various curves showing the changes in $V(t)/V_\infty$ with respect to the dimensionless variable $\lambda_1^2 D(h_k)t/H_s^2$. The various curves correspond to various values of parameter a. Experimental data are presented in the form $V(t)\ /\ V_\infty$ versus t, and they are shifted along the axis $\lambda_1^2 D(h_k)t/H_s^2$ in order to find the best fitting theoretical curve that defines the value of a. Based on the chosen value of a, the corresponding value of λ_1^2 is adopted from the table presented in [3], while a reference time t_{RP} is graphically determined for $\lambda_1^2 Dt\,/\,H_{soil}^2 = 1$. Finally, the diffusion coefficient is calculated as $D(h_k) = H_s^2\ /\ \lambda_1^2 t_{RP}$ and the hydraulic conductivity as $K(h_k) = D(h_k)\Delta\theta/\Delta h_i$.

3. A new method accounting for impedance effects

As an alternative to existing methods of accounting for impedance effects, it is proposed to first apply Darcy's law to the saturated porous disk of thickness Δz_d, of saturated hydraulic conductivity K_d and of cross sectional area A, like in [4]. One obtains the following expression of the changes in the increment of suction at the specimen bottom:

$$\Delta h_k\left(z=0,t\right)=\Delta h_i-\Delta z_d\,\frac{\Delta V\left(t\right)}{AK_d\Delta t} \quad (4)$$

where ΔV [L^3] is the extracted water volume during the time interval Δt. The sooner Δh_k (z = 0, t) reaches Δh_i, the less significant the impedance effect is, and vice versa.

Based on the time superposition principle ([9, 10], among others), it is hence proposed:

- to decompose a suction increment at the specimen bottom Δh_k (z = 0, t) as the sum of N_s very small successive suction increments $\Delta h_m = \Delta h_i/N_s$, occurring at time t_m,
- to apply the analytical solution (Equation 1) to each suction increment and
- to superpose in time all suction increments, giving the following expression of the suction changes:

$$\Delta h_k\left(z,t\right)=\sum_{m=1}^{N_s}\Delta h_m\left(1-\frac{4}{\pi}\sum_{n=1,3,5...}^{\infty}\frac{1}{n}e^{-\frac{(n\pi/2)^2(t-t_m)D(h_k)}{H_s^2}}sin\frac{n\pi z}{2H_s}\right) \quad (5)$$

resulting in the following expression of extracted volume:

$$V\left(t\right)=\frac{V_\infty}{N_s}\sum_{m=1}^{N_s}\left(1-\frac{8}{\pi^2}\sum_{n=1,3,5...}^{\infty}\frac{1}{n^2}e^{-\left(n\pi/2\right)^2\left(t-t_m\right)D\left(h_k\right)/H_s^2}\right) \quad (6)$$

Suction profiles when accounting for impedance effects are calculated using Equation (5) (solid lines in Fig. 1). Compared to Gardner's method (Equation 1) that does not account for impedance effects (dotted lines in Fig. 1), in this case $\Delta h_k\,(z=0,\,t)$ gradually approaches Δh_i - the longer the delay, the stronger the resistance of the ceramic disk. Note that larger N_s secures smoother curves obtained using Equations (5) and (6), where the adequate value of this parameter can be obtained based on the sensitivity analysis. Since the computation is not time consuming, N_s = 1000 was adopted as the value large enough for all cases presented.

$K(h_k)$ is calculated in the same way as in Gardner's method (Equation 2), by adjusting value of $D(h_k)$ and by computing $C(h_k) = \Delta\theta/\Delta h_i$ based on the measured WRC.

https://doi.org/10.1051/e3sconf/202019503012

4. Experimental validation

The validity of the method was established by considering the experimental data obtained on three quite different materials, provided by [7] on a coarse granular material, and [5] on both a poorly graded sand and an undisturbed silty clay.

4.1 Experimental data

4.1.1 Data of [7]

In the apparatus presented in [7] (Fig. 1), a 70 mm diameter and 24 mm height specimen is placed on a Δz_d = 5 mm thick ceramic porous disk with an air entry value of 50 kPa, and a saturated hydraulic conductivity K_d = 4.02 x 10^{-8} m/s. Water exchanges are monitored by using an outer tube (15 mm diameter) that surrounds the thin inner tube (inner diameter 5 mm, outer diameter 8 mm).

4.1.2 Data of [5]

Wayllace and Lu in [5] developed a transient water release and imbibition (TWRI) method for determining the WRC and HCF of two materials along both the drying and wetting paths. In this device, they imposed, through the axis translation method, two suction increments to drain water from the soil specimen, followed by a suction decrease, allowing for subsequent water imbibition. The TWRI apparatus consisted of:

- a flow cell accommodating a soil specimen of 60.7 mm diameter placed on 300 kPa HAEV ceramic disk (saturated hydraulic conductivity K_d = 2.5x10^{-9} m/s, thickness Δz_d = 3.2 mm),
- a pressure regulator connected to cell top,
- a water jar placed on a weight scale connected to the cell bottom to collect the drained outflow (more details in [5]).

4.2 Validation of the method

Gardner's, Kunze & Kirkham's and our method are now compared with the transient outflow data of the three materials presented. The data sets for each of them are related to the first suction step, during which the impedance effects are the most significant.

4.2.1 Coarse volcanic substrate ([7])

By imposing Δh_i = 0.185 m at the saturated specimen (θ_s = 0.395), a total volume V_∞ = 14.41 cm^3 of water was extracted ($\Delta\theta$ = 0.16). Thus, $C(h_k$ = 18.5 cm) = 0.16 / 0.185 = 0.86 m^{-1}.

In Fig. 2*a* is illustrated the evolution of Δh_k (z = 0, t) for step 1 (connected filled dots), calculated using measured volumes and Darcy's law (Equation 4), that gradually reaches Δh_i imposed at the disk bottom.

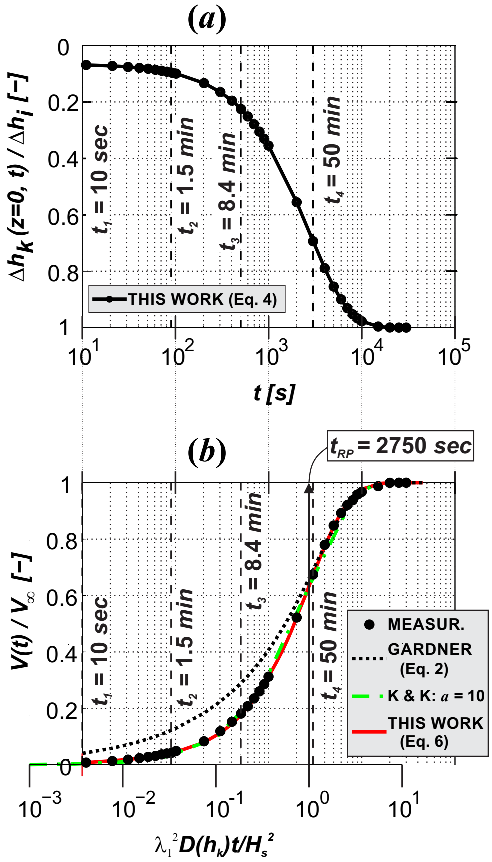

Fig. 2. Suction step 1 for coarse material: (a) evolution of Δh_k (z = 0, t); (b) comparison between measured (dots) and calculated V(t) using three different methods.

Fig. 2*b* presents a comparison of the calculated values of $V(t)/V_\infty$ (in a time logarithmic scale) using Gardner's method, Kunze and Kirkham's method and our method (Kunze and Kirkham's non-dimensional time variable $\lambda_1^2 D(h_k)t/H_s^2$ is also reported on the bottom *x*-axis).

Fitting our experimental data following Kunze and Kirkham's method provided λ_1^2 = 0.097 and t_{RP} = 2750 s, with a = 10, giving $D(h_k)$ = 2.17x10^{-6} m^2/s and finally $K(h_k)$ = 1.82x10^{-6} m/s. However, the choice of the adequate theoretical curve (value of a) is somewhat operator-dependent, especially for higher values of a (> 0.5), where some of the curves are almost overlapping. A wrong choice of parameter a can lead to significantly different values of $K(h_k)$, up to one or two orders of magnitude.

The best fit between Equation (6) and experimental data is obtained for $D(h_k)$ = 1.2x10^{-6} m^2/s, which finally gives $K(h_k)$ = 1.04 x 10^{-6} m/s. The Figure shows excellent agreement between experimental data and both Kunze & Kirkham and our method. Also, both methods confirm the occurrence of impedance effects, since $K(h_k)$ > K_d in both cases. Unsurprisingly, the extracted volume estimated by Gardner's method ($D(h_k)$ = 7.5x10^{-8} m^2/s) for times smaller than 1 *h* is higher than those measured and calculated with the two other methods.

4.2.2 Poorly graded sand and Undisturbed silty clay ([5])

In Fig. 3 are presented the same kind of data as in Fig. 2, for the poorly graded sand. In this case H_s = 2.67 cm, Δh_i = 0.2 m and V_∞ = 4.66 x 10^{-6} m^3, thus giving $C(h_k)$ = 0.302 m^{-1}. The best agreement between measurements and Equations (2) and (6) is obtained for $D(h_k)$ = 1.0x10^{-8} m^2/s ($K(h_k)$ = 3.0x10^{-9} m/s) and $D(h_k)$ = 1.8x10^{-8} m^2/s ($K(h_k)$ = 5.4x10^{-9} m/s), respectively. In case of Kunze & Kirkham's method, the best fitting theoretical curve is a = 0.389 (λ_1^2 = 1.323) with t_{RP} = 23700 s, which finally gives $K(h_k)$ = 6.9x10^{-9} m/s.

For the undisturbed silty clay (Fig. 4), H_s = 2.41 cm, Δh_i = 0.2 m and V_∞ = 1 x 10^{-6} m^3, thus giving $C(h_k)$ = 0.07 m^{-1}. The value of $D(h_k)$ is adjusted to 8x10^{-8} and 5.0x10^{-7} m^2/s for Equations (2) and (6), respectively. Also, the theoretical curve at a = 0.5 (λ_1^2 = 1.16) shows the best agreement with experimental data (not in small times) for t_{RP} = 2300 s, leading to $K(h_k)$ = 1.6x10^{-9} m/s.

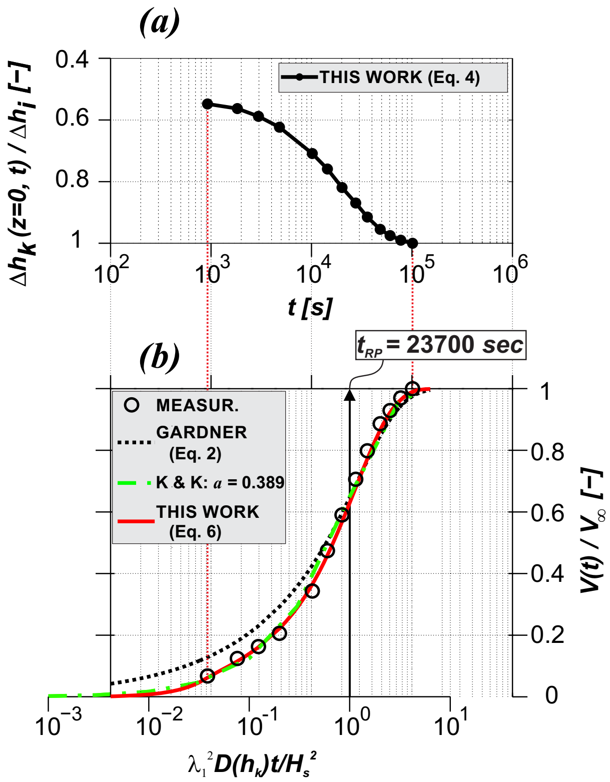

Fig. 3. Same data as in Fig. 2, for the poorly graded sand ([5])

Based on the data from both our method and Kunze & Kirkham's one, it can be concluded that the impedance effect does occur (with $K(h_k) > K_d$ for both soils), especially in case of the silty clay where the calculated value of $K(h_k)$ is about an order of magnitude larger than K_d for both methods. Unsurprisingly, Gardner's method shows significantly lower $K(h_k)$ values, because of the perturbation caused by the low hydraulic conductivity ceramic disk.

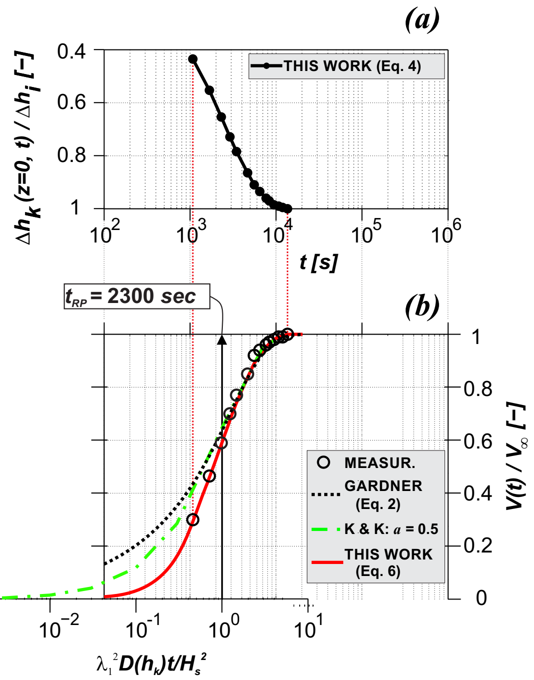

Fig. 4. Same as in Fig. 2 and Fig. 3, undisturbed silty clay ([5])

5. Conclusion

The experimental data from various materials analyzed (a coarse green roof substrate, a poorly graded sand and an undisturbed silty clay) showed that the proposed simple analytical method fairly well accounts for the impedance effects of the ceramic disk. This method is believed to be more reliable than Kunze & Kirkham's graphical method, especially in the case of significant impedance effect, because it is not dependent of the difficulty in choosing the best fitting theoretical curve among the family of curves provided by Kunze & Kirkham. The proposed method, based on the analytical resolution of the water transfer equations in the different parts of the system, only requires the accurate monitoring of outflow measurements, a requirement that is typical of any method of determining the hydraulic conductivity of multiphase porous material.

Compared to numerical back analyses method, our method provides the values of hydraulic conductivity without the need to assume a parametric expression for the hydraulic conductivity function. Also, this analytical method is considered simpler in the sense that it does not require the use of any numerical simulations with optimization algorithms, since the analysis of outflow data and the derivation of hydraulic conductivity value is much more straightforward.

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