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IMPORTANCE OF THE GLOBAL SENSITIVITY ANALYSIS IN DEVELOPMENT OF METAMODELS FOR GEOTECHNICAL BACK ANALYSIS

Summary: *This paper investigates the use of global sensitivity analysis during the development of metamodels for back-analysis of geotechnical problems. Variance-based global sensitivity analysis was combined with Particle swarm optimization algorithm and POD-ERBF metamodel to back-calculate the hypothetic (synthetic) problem of anchor-supported excavation. PLAXIS 2D FE code was used for numerical simulations. The results of the back analysis for different sets of model parameters were compared to emphasize the importance of global sensitivity analysis before performing the back analysis and the recommendations for the robust modeling of presented geotechnical problem were given.*
Keywords: *metamodel, PLAXIS 2D, variance-based sensitivity analysis, back analysis, particle swarm optimization*

ZNAČAJ GLOBALNE ANALIZE OSETLJIVOSTI U RAZVOJU METAMODELA ZA GEOTEHNIČKU POVRATNU ANALIZU

Rezime: *U ovom radu se istražuje primena globalne analize osetljivosti prilikom razvoja metamodela za povratnu analizu geotehničkih problema. Variance-based globalna analiza kombinovana je sa PSO optimizacionim algoritmom i POD-ERBF metamodelom i izvršena je povratna analiza hipotetičkog geotehničkog problema iskopa zaštićenog potpornom sidrenom konstrukcijom. PLAXIS 2D MKE kod je korišćen za numeričke simulacije. Rezultati povratne analize za različite setove parametara modela su upoređeni kako bi se istakao značaj sprovođenja globalne analize osetljivosti pre izvođenja povratne analize. Date su preporuke za robusne modele prikazanog geotehničkog problema.*
Ključne reči: *metamodel, PLAXIS 2D, variance-based analiza osetljivosti, povratna analiza, PSO algoritam*

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1. INTRODUCTION

Optimization methods are indispensable tools in civil engineering that enable engineers to find optimal solutions to complex problems while considering various constraints and objectives. Whether in structural design, transportation planning, water resources management or construction management, these methods play a crucial role in ensuring the efficiency, safety and sustainability of civil engineering projects. As technology advances, the integration of optimization methods into civil engineering practices will continue to drive innovation and improve the quality of infrastructure worldwide.

Important engineering activities such as design optimization, probabilistic investigation of uncertainties, parameter determination, inverse problems or sensitivity analyses become impossible because they require thousands or even millions of computationally expensive simulations. Engineers commonly address this challenge by adopting simplified models to closely approximate the original model's behavior with high precision [3]. Such approximated models, capable of simulating the original model, are known as metamodels or surrogate models. While metamodels offer numerous advantages, they are not without challenges. The accuracy of metamodels heavily depends on the quality and quantity of data used for model training. Ensuring the reliability of metamodels requires rigorous validation against field measurements or laboratory tests. Calibration is often necessary to fine-tune model parameters and improve accuracy. Advances in machine learning and artificial intelligence are expected to enhance the predictive capabilities of metamodels. These technologies can handle complex relationships and adapt to varying data inputs, making them valuable tools for geotechnical analysis.

In geotechnical problems, obtaining representative and diverse data sets can be challenging, especially in regions with limited geotechnical information. Soil behavior is influenced by numerous factors, including soil type, moisture content, loading conditions and structural geometry. The availability of big data and remote sensing technologies can provide geotechnical engineers with more extensive and real-time information about soil properties and behavior, enabling the development of more accurate and responsive metamodels. Metamodels can be integrated with multi-objective optimization techniques to consider multiple design objectives simultaneously, such as safety, cost, and environmental impact, leading to more sustainable and efficient designs.

2. METAMODELING CONCEPT

The main idea behind metamodeling is to approximate (replace) an unknown function u that describe the behaviour of an considered engineering problem. In order to construct a metamodel, two main components are necessary: (1) the input parameter matrix (**P**) which includes the s parameters of n_p sample points; (2) the matrix of system responses or snapshot matrix (**U**), in which the n_p function values of m observation points are stored. Therefore, **P** and **U** matrices are of size $s \times n_p$ and $m \times n_p$, respectively. Khaledi et al. [3] performed a comparative study and evaluated the performance of various metamodels: *Polynomial Regression (PR), Moving Least Squares (MLS), Proper Orthogonal Decomposition with Radial Basis Function (POD-RBF) and Proper Orthogonal Decomposition with Extended Radial Basis Function (POD-ERBF)*, using different types of mathematical functions for calculating the systems response. All the evaluation results obtained from this comparative study are summarized and they showed POD-ERBF (along with POD-RBF) excels in most categories. In this paper, Proper Orthogonal Decomposition (POD) combined with Extended Radial Basis Functions (ERBF), proposed by [7], is used to construct a reliable metamodel. The algorithm consists of two main parts: (1) proper orthogonal decomposition of the snapshot matrix and (2) interpolation using a combination of radial and non-radial basis functions.

The performance of each metamodeling technique can be evaluated considering the following criteria [3]: (1) accuracy (an accurate metamodel should be capable of making predictions across the entire design space with minimal error); (2) problem dependency (a metamodel that is independent of the specific problem should achieve high accuracy across different problems) and (3) efficiency (an efficient metamodeling approach should require minimal computational effort during the metamodel construction process). The overall performance of the metamodels can be evaluated using standard accuracy measure: Normalized Root Mean Squared Error (NRMSE), which provides a global error measure over the entire design domain.

2.1. Sampling technique and sample size

The precision of a metamodel relies heavily on how samples (training points) are distributed within the design space. Consequently, sampling techniques are employed to identify the optimal sample points within the design domain. The selection of an appropriate sampling technique is generally considered a critical factor influencing the effectiveness of any metamodeling approach [3]. The general concept behind a sampling strategy is to generate a series of points that are uniformly dispersed throughout the input parameter space. Latin Hypercube Sampling (LHS) sampling technique has been applied in this paper. It is a statistical sampling technique widely used in fields such as engineering design, computer experiments (simulation studies), optimization, sensitivity analysis and uncertainty quantification. It is particularly useful when you want to generate a representative and evenly distributed set of samples across multiple dimensions or variables. It helps ensure that the entire parameter space is adequately explored while avoiding over-sampling in any particular region. Therefore, researchers and engineers can efficiently explore and analyze complex systems by providing a balanced and representative set of input values for their models or experiments.

There is a strong connection between the quantity of sample (training) points and the accuracy of the metamodel. The number of sample points needed for creating a metamodel relies on two primary factors: the dimensionality of the unknown function and the level of nonlinearity. Consequently, quantifying the optimal sample size can be a challenge. Zhao and Xue [10] suggested the following equations to determine the suitable size of samples: $l(s+1)(s+2)$ for high dimensional and $3l(s+1)(s+2)$ for low dimensional problems, where $l=0.5$-2 is a scaling parameter and s is the number of the input parameters.

3. GLOBAL SENSITIVITY ANALYSIS

Sensitivity analysis is a valuable technique in numerical modeling. The main goal of sensitivity analysis is to help engineers to understand how the input parameters of a numerical model influence the model's output and provides insights into model's behavior, reliability and robustness. Global sensitivity analysis (GSA) is widely applied in numerical modeling in civil engineering for various problems, such as uncertainty assessment, model calibration, design optimization, risk assesment, decision support etc. In this paper, GSA is used for model parameter ranking and dimensionality reduction of initial, more complex numerical model.

Among different global sensitivity analysis methods, Variance-based GSA metod is widely recognized. The output variance is decomposed to the sum of contributions of each individual input parameter and the interactions between different parameters [6].

$$1 = \sum_i S_i + \sum_i \sum_{j>i} S_{ij} +, \ldots, + S_{12,\ldots k} \tag{1}$$

The S_i represents the first order sensitivity measure, evaluating the impact of input parameter x_i on the model Y without considering interactions between input parameters. On the other hand, the total effect sensitivity index S_{Ti} is a more comprehensive metric that accounts for interactions between parameters [2].

The procedure for calculating S_i and S_{Ti} begins with the generation of two matrices (n_p x s), A and B, each containing random parameters sets. Here, n_p represents the number of samples and s represents the number of input parameters. A third matrix C_i is defined, where all its columns are copied from matrix B except ith column copied from its corresponding column in A. The next step involves calculating model outputs for all input values present in the sample matrices A, B and C_i. Finally, sensitivity indices for each parameter are obtained with following equations:

$$S_i = \frac{y_A^T y_{C_i} - n_p(\bar{y}_A)^2}{y_A^T y_A - n_p(\bar{y}_A)^2} \tag{2}$$

$$S_{T_i} = \frac{\left(y_B - y_{C_i}\right)^T \left(y_B - y_{C_i}\right)}{2y_B^T y_B - 2n_p(\bar{y}_B)^2} \tag{3}$$

Where $\mathbf{y}_A$, $\mathbf{y}_B$ and $\mathbf{y}_{Ci}$ are vectors containing model evaluations for matrices A, B and C_i respectively. $\bar{y}_A$ and $\bar{y}_B$ are the mean value estimates for the components of $\mathbf{y}_A$ and $\mathbf{y}_B$.

4. BACK ANALYSIS

The material parameters of soil around the structure can be determined through inverse analysis of experimental measurements. In order to obtain the optimized values of related model parameters which can give a good match between predicted and measured values, back analysis is applied to conduct the parameter optimization.

Due to the complexity of the optimization problems in geotechnical engineering, a robust optimization algorithm is required to obtain the global minimum of the objective function. In this paper, the particle swarm optimization (PSO) algorithm is used to perform the optimization process. It is one of the evolutionary computational techniques, originally introduced by Kennedy & Eberhart [1], inspired by the social behaviour of birds flocking or fish schooling.

If the objective function is highly non-linear with a large number of input variables, a large number of evaluations of the objective function are needed before the best set of parameters is identified. This high computation cost makes the algorithm of inverse analysis inefficient. Therefore, a practical solution is to replace the original FE model by the metamodel (POD-ERBF). The flow chart shown in Figure 1 describes the parameter identification procedure using the particle swarm optimization algorithm with the proposed metamodel.

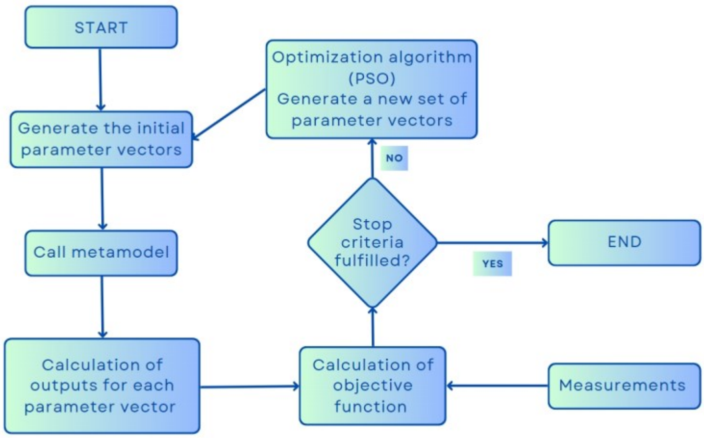

Figure 1. Flowchart of back analysis procedure

5. NUMERICAL EXAMPLE

The primary objective of this section is to demonstrate effectiveness and accuracy of the proposed metamodeling technique in a real geotechnical problem. A 2D FEM model of the construction of an excavation is replaced with POD-ERBF metamodel in a computationally expensive parameter back calculation problem. The original model has been simulated by the finite element method using the commercial code PLAXIS 2D (Figure 2) and user python scripts for model pre-/post-processing [5].

The excavation is 30 *m* wide and the final depth is 20 *m*. It extends in longitudinal direction for a large distance, so that a plane strain model is applicable. The sides of the excavation are supported by 30 *m* long diaphragm walls, which are braced by horizontal struts at an interval of 5 *m*. Since the geometry is symmetric, only one half (the left side) is considered in the analysis. The excavation process is simulated in three separate excavation stages. The diaphragm wall is modelled using plate finite elements. The interaction between the wall and the soil is modelled at both sides by means of interfaces. The interfaces allow for the specification of a reduced wall friction compared to the friction in the soil. The strut is modelled as a spring element for which the normal stiffness is a required input parameter. Table 1 provides the description of material parameters and their value ranges. The excavation example presented in this paper concerns only with the homogeneous soil conditions. In reality, the material properties of soil can change along the excavation. However, uncertainties and spatial variability of soil properties were not in the framework of this research.

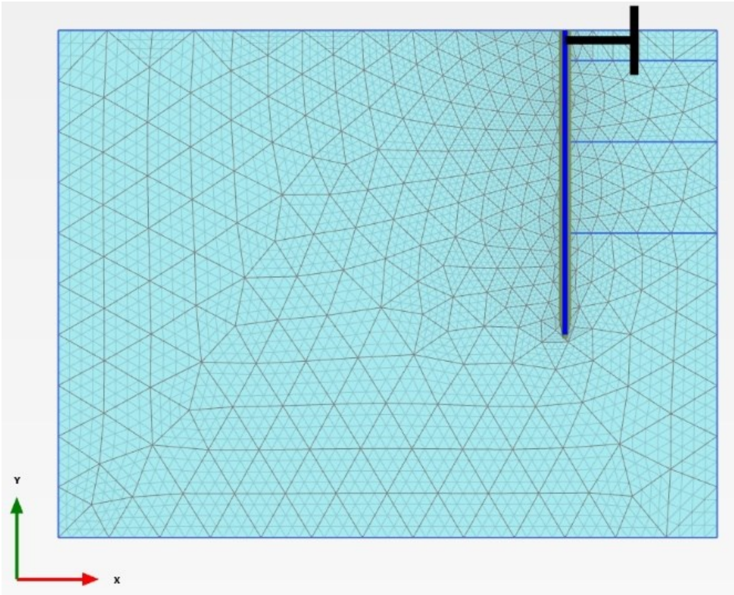

Figure 2. Model of an excavation in PLAXIS 2D

Parameters		*Parameter Ranges*
Soil (HS model)		
c	Effective cohesion at failure	[0, 7] kPa
φ	Effective friction angle at failure	[33, 37] deg
υ	Poisson's ratio	[0.1, 0.3]
m	Power for stress level dependency of stiffness	[0.4, 0.7]
γ_{unsat}	Volumetric weight of unsaturated soil	[16, 19] kN/m^3
E_{ur}	Stiffness modulus for unloading/reloading in drained triaxial test	[125, 175] MPa
E_{oed}	Stiffness modulus for primary loading in oedometer test	[25, 35] MPa
ψ	Dilatancy angle at failure	$=\varphi-30$
$K_{0,NC}$	Earth pressure coefficient at rest	$=1-\sin\varphi$
E_{50}	Stiffness modulus for primary loading in drained triaxial test	$=E_{oed}$
Diaphragm wall		
EA_1	Axial stiffness	7.5x10^6 kN/m
EI	Bending stiffness	1.0x10^6 kNm^2/m
Anchor		
EA	Axial stiffness	2.0x10^6 kN

Table 1. Description of material parameters and their predefined value and ranges

The Hardening soil (HS) model [8] has been used as the constitutive model for describing the elasto-plastic behavior of soil elements. The Hardening Soil model is an advanced model for the simulation of soil behavior. As for the Mohr-Coulomb model, limiting states of stress are described by means of the key parameters, including the friction angle φ, the cohesion c and the dilatancy angle ψ. One distinctive feature of the HS model is its use of multiple input stiffness parameters to describe soil stiffness more accurately. These parameters include: the triaxial stiffness E_{50}, the triaxial unloading/reloading stiffness E_{ur} and the oedometer loading stiffness E_{oed}. Unlike simpler models like Mohr-Coulomb, the HS model takes into account the stress-dependency of stiffness moduli. This means that as the stress on the soil changes, the stiffness of the soil also changes, and this relationship is described using a power law. Also, the HS model incorporates hardening mechanisms to simulate irreversible plastic deformation. It includes: (1) shear hardening (this accounts for the accumulation of irreversible plastic strains during primary deviatoric loading - shear loading), and (2) compression hardening (this models irreversible plastic strains due to primary compression during oedometer loading tests). More details about HS model can be found in [8, 9]. Replacing the FE model with POD-ERBF metamodel

The data obtained at various observation points (snapshots) can be any physical properties such as pressure, strain, force, displacement and temperature. In this numerical example, we focused on the maximum horizontal displacement of the diaphragm wall and the force in the strut. These two values are read and recorded as output.

Modelling the elasto-plastic behaviour of soil according to the HS model requires ten parameters: φ, c, ψ, E_{ur}, E_{50}, E_{oed}, ν, m, γ_{unsat} and $K_{0,NC}$ where, as given in Table 1, ψ and $K_{0,NC}$ are functions of the friction angle, and E_{50}=E_{oed}. Now, the number of independent material parameters reduces to seven. The constructed metamodel for this numerical example will have 7 inputs and 2 outputs.

We generated n_p sample points within the range of input parameters values as specifies in Table 1. The size of the input parameter matrix is 7 x n_p. Samples were generated using the LHS method. It is very important to determine the most appropriate sample size for this numerical example. To do so, the metamodel's accuracy need to be assessed across different sample sizes. Since we were not able to perform such an analysis in this work, the sample size was determined based on the recommendations given in [3]. The adopted sample size is 100. The adopted number of additional test points is 20. For relatively simple 2D problems like this, the sample size could be even smaller. But when we talk about numerical models of high complexity, such as 3D models of tunnel excavations or some other complex structures, this kind of analysis plays a very important role. Once the metamodel is created, computation time drastically decreases.

In order to establish a metamodel, it is necessary to go through the following steps:

1. The 2D finite element model is run for each parameters set, and the resulting maximal horizontal displacement of the diaphragm wall and force in the strut were saved in the snapshot matrix. This matrix has 2 rows and n_p columns.
2. The input matrix is normalized between 0 and 1 in order to prevent the potential scaling errors caused by varying input parameter magnitudes.
3. With the input parameter and snapshot matrices ready, the construction of POD-ERBF metamodel can be completed
4. Constructed metamodel is tested (validated) against additional training points, by calculating the total NRMSE

5.1. Global sensitivity analysis

In order to estimate the parameters of the model using back analysis, ranking input parameters by their importance and selecting the most sensitive ones is really important. In order to evaluate the importance of input parameters to the model responses global sensitivity analysis has been conducted. The results are shown below.

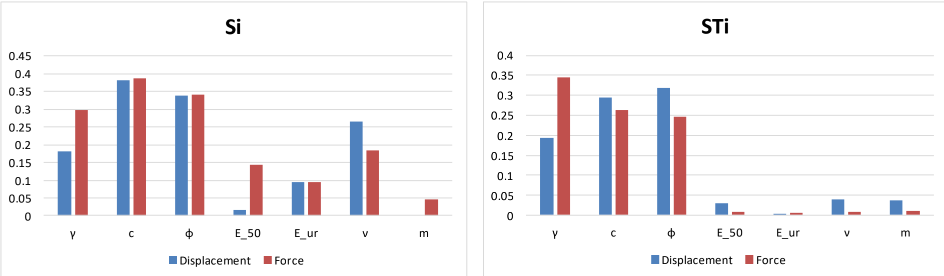

Figure 3. Results of Variance-based GSA

As can be seen, the sensitivity indices show that the maximal horizontal displacement of the diaphragm wall is sensitive to the change of cohesion, Poisson's ratio and friction angle, while the force in the strut is highly sensitive to parameters such as volumetric weight, cohesion and friction angle. According to the sensitivity analysis, the parameters E_{ur} and m do not significantly affect the result, so they can be excluded from the analysis in the next step.

5.2. Back analysis

The primary goal of this section is to obtain the material parameters by inverse analysis of synthetic data generated for a simulated excavation problem. The term "synthetic" is used here because the data used for analysis come from numerical simulations and not from actual field measurements. To achieve this, we calculate the maximum horizontal displacement of the diaphragm wall and force in the strut using a finite element solver and a predefined set of model parameters. These calculated results correspond to in-situ measurements and can be used in the parameter identification algorithm shown in Figure 1. In order to consider the effects of the probable errors, the measurements errors are assumed to be ±3%. The original FEM model is replaced by POD-ERBF metamodel constructed in previous section and parameter back calculation is performed according to the flowchart given in Figure 1. The particle swarm optimization algorithm was used to perform the optimization process, using optimum algorithm parameters [4].

The goal is to examine the ability of metamodel to find same set of pre-defined model parameters. First, a test was conducted to identify seven parameters of HS model (designated MM7x2). To assess the accuracy of identified parameters, they have been inserted into the FE model and responses at the observation points have been compared with the original responses (measurements). The obtained results for metamodel, PLAXIS 2D and measurements are given in the Table 2.

As already mentioned, the results of the sensitivity analysis showed that the importance level of each parameter is not the same. Therefore, in the second step, the results of the sensitivity analysis were taken into account and less important input parameters (E_{ur} and m) were excluded from the new metamodel (designated MM5x2). The same procedure was repeated again, now excluding 3 initial input parameters that showed minimum sensitivity (E_{ur}, m and E_{oed}), and another metamodel (designated MM4x2) was created.

Back analysis was repeated for both new metamodels, and results are shown in Table 2. In order to create a robust and reliable metamodel, this analysis can be continued until an even smaller set of the most important parameters is found. The parameter sets identified by the metamodel for all three cases are given in Table 3 and compared with pre-defined set used to obtain synthetic measurements. As shown in Tables 2 and 3, all three metamodels show great effectiveness in replacing the initial Plaxis 2D numerical model, as well as in model parameter identification via back analysis.

Outputs	**Measurements**	**MM (7x2)**		**MM (5x2)**		**MM (4x2)**	
		MM	**P2D**	**MM**	**P2D**	**MM**	**P2D**
U_x [mm]	0.120	0.120	0.117	0.120	0.119	0.120	0.120
F [kN]	-2535	-2535	-2522	-2535	-2528	-2535	-2558

Table 2. The results of back analysis, for all metamodels (designated MM7x2, MM5x2, MM4x2), compared with synthetic measurements and Plaxis 2D (P2D) results

	γ [kN/m^3]	**c [kPa]**	**ϕ [deg]**	**E_{50} [MPa]**	**E_{ur} [Mpa]**	**ν**	**m**
Measurements	18.66	2.201	33.32	31480	152267	0.254	0.479
MM 7x2	18.97	3.563	33.00	31756	129540	0.201	0.7
MM 5x2	18.57	2.861	33.12	27715	/	0.278	/
MM 4x2	18.13	2.331	33.00	/	/	0.274	/

Table 3. The parameter sets identified by back analysis, for all three cases of analysis, compared with synthetic measurements

6. CONCLUSIONS

The main objective of this paper was to demonstrate how metamodels can reliably replace the finite element simulation model and drastically reduce the expensive computation time of the back analysis. Inverse analysis of measurements has been performed for excavation problem to identify the material parameters for HS constitutive model. In order to do this, the error between the synthetic measurements and obtained results has been minimized by particle swarm optimization algorithm combined with metamodeling technique. The obtained results show that with the aid of accurate and efficient metamodeling method such as POD-ERBF, it's possible to obtain the solution of the optimization problem with a small error in a significantly shorter time. In this way, solving computationally expensive problems such as parameter identification and sensitivity analysis becomes possible. The shown example is relatively simple due to limited resources and may not clearly show the time efficiency of this approach, but for some more complex examples such as those shown in [3] and [2], it is quite clear. However, the principle is exactly the same.

One of the most important parts of this work is the global sensitivity analysis, which provides clear information on the level of importance of the considered parameters. GSA can greatly help to construct a robust and reliable metamodel, with gradual reduction of number of input parameters. Also, the numerical example presented in this paper concerns only with homogeneous soil conditions, and in reality soil parameters are quite variable in space. However, uncertainties and spatial variability of soil parameters were not considered in this paper. Further work is foreseen to be done for investigating this type of problem.

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