Keynote Speakers

“From symmetry breaking to functionality: Examples from nonlinear mechanics of beams, plates and shells”

Dr Miha Brojan

is an Associate Professor of Mechanics at the Faculty of Mechanical Engineering, University of Ljubljana. He graduated from the same institution in 2003 and received his Ph.D. degree in 2010. He completed his postdoctoral training at the Massachusetts Institute of Technology – MIT between 2012 and 2013 and in 2016 at Princeton University. For the last two years, he has been working closely with researchers from Princeton University on nonlinear deformations and researchers from UCLA on activation of slender elastic structures based on the loss of stability, transformative structures and the inclusion of artificial intelligence in solving highly nonlinear problems.

Since 2015, he has been principal investigator at the Laboratory for Nonlinear Mechanics (LANEM) at the Faculty of Mechanical Engineering, University of Ljubljana. In recent years, he has been a lecturer on seven subjects, including: Strength of Materials, Thermomechanics, Mechanical Engineering Reporting, Advanced Strength of Materials, Engineering Mechanics 3, Mechanics of Light-weight Structures and Aircraft Flight Mechanics. He is currently mentoring seven undergraduate students, five master’s students and nine doctoral students.

The research he conducts together with his colleagues focuses on the mechanical responses and supercritical stability of slender beams, plates and shells, shape memory materials used in elasto-calorics, viscoelasticity and phase transformations in liquid polymers. They pay special attention to the formation of periodic structures, which are created in e.g. wrinkling of plates and shells, torsion of beams, etc. In doing so, the research group develops (ab initio) theoretical and numerical predictive models and, more recently, precise desktop experiments, which serve to validate the predictive models or to discover new mechanisms of operation of such structures.

Abstract

Symmetry breaking is frequently observed in mechanics of structural elements when an equilibrium system transitions from a symmetric phase to a phase with lower energy where the system is at least meta-stable. However, predicting this phenomenon for general problems using current theoretical models and even numerical tools can be rather challenging. A correct description of this process usually requires taking into account significant geometric nonlinearities, a large number of available equilibrium meta-stable states, the possibility of multiple phase transitions (switching between deformation modes), complex equilibrium paths, etc. It should be noted that this is not just of interest to theoretical research; it also has practical implications in engineering. Structural elements such as beams, plates, and shells must be designed according to specific standards to prevent reaching and exceeding critical stresses and

strains during operation. Additionally, for slender structures, it is crucial that all parts of the structure remain stable. However, recent studies have been exploring the vast array of deformation patterns and mechanical phenomena that can arise after the loss of stability, particularly in systems that exhibit periodic topography in a deformed configuration. These phenomena can be harnessed to provide advanced functionality using slender structures.
The lecture will cover three examples of symmetry breaking in a highly symmetric structure and transitioning to the supercritical phase by forming periodic patterns. These include:

1. a slender beam subjected to torsional deformation,
2. a thin two-layer plate with active deformation in one layer,
3. a thin hemispherical shell on an elastic substrate subjected to hydrostatic pressure.

In the first example, the critical torsional stress leads to the formation of a helix, the only periodic spatial curve with constant torsion and bending curvature. A theoretical and experimental model of a deformed beam made of extruded polyamide will be presented. This beam has an effectively high negative thermal expansion coefficient due to its geometry and production technology. The second example includes a theoretical and experimental model of a homogeneous deformation of a round composite plate with active deformation in one layer. It will be shown that in the supercritical region the characteristic periodic deformation forms which emanate from the outer edge of the plate are load amplitude dependent. The results are useful in applications of morphing planar structures into preprogrammed three-dimensional objects. The third example is a hemispherical bilayer structure consisting of a thin, stiffer shell and a softer, thicker substrate. Under the influence of hydrostatic pressure, this system loses its stability and forms a surface periodic structure that minimizes mechanical energy by bending and arranging the surface crystal structure. The application of this model has been demonstrated in realizations of smart wetting (to induce super-hydrophobicity/-hydrophilicity), smart bonding, dynamic tuning of aero/hydrodynamic drag, etc.

“Elements of the Theory of Constitutive Relations and Formulations of the Linearized Problems on Stability”

Dr Dimitri V. Georgievskii

is a Professor and Head of the Chair of Elasticity Theory at Mechanical and Mathematical Department of the Moscow State University. He is Director of the Institute of Mechanics (MSU), Director of the world-class scientific center “Supersonic-MSU” and professor and expert of the Russian Academy of Sciences. In the period 2014-2015 was a Vice-Rector of the MSU.

The field of the main scientific interests of Prof. Georgievskii includes the following problems:

- Theory of constitutive relations in continuum mechanics. Instrument of tensor functions, Integral representations. Theory of adjusting experiment for obtaining of material functions.
- Phenomenological description of stress-strain state by multiscale simulation.
- Asymptotic methods in theory of thin solids. Deformation of solids with strongly distinguishing sizes.
- Stability of deformation processes in solid mechanics and composite mechanics. The Liapunov – Movchan’ method and its development.
- Hydrodynamic stability of flows with complex rheology. Spectral problems on stability. The generalized Orr – Sommerfeld’ problems. The integral relations methods and energetic (variational) estimates of stability.
- Non-Newtonian and viscoplastic flows. Analytical and numerical solutions. Self-similarity and the Stefan’ problem. Applications in oil industry, biomechanics, geotectonics, glaciology.
- Weak nonhomogeneous flows. Description of transitions to mixing.
- Heavy gravitational non-stable multilayer systems. Dominating waves. Salt diapirism.
- Mechanics of n-dimensional solid and n-dimensional continuum.

He is executive secretary of the Editorial Board of Journal “Vestnik Moskovskogo Universiteta. Ser. 1. Matematika, mekhanika”, Scientific Editor (2006-2014) and the Member of the Editorial Board of Journal “Izvestiya Akademii Nauk. Ser. Mekhanika Tverdogo Tela”. The Member of Editorial Board of Journals “Applied Mathematics and Mathematical Physics”, “Composites and Nanostructures”, “Vestnik MGTU. Ser. Estestvennye nauki”, “Mathematical Modeling and Computational Methods”, “Vestnik ChGPU. Ser. Mekhanika Predelnogo Sostoyaniya”, “Vestnik Udmurtskogo Universiteta. Ser. Matematika. Mekhanika. Komp’yuternye nauki”.

Prof. Georgievskii is author or coauthor of four monographs and textbooks, as well as in more than 250 published papers.

Professor Georgievskii is the member of International Higher Education Academy of Sciences, the

Member of the Expert Council in mathematics and mechanics of the Russian High Attestation Committee and the Member of the Expert Council in the Russian Fund for Basic Research. He is Member of the Gesellschaft fur Angewandte Mathematik und Mechanik (GAMM), American Mathematical Society (AMS), International Society for Analysis, its Applications, and Computation (ISAAC).

He is decorated with five medals and prizes for his scientific work.

Abstract

The report consists of three interconnected parts, one way or another concerning the various aspects of continuum mechanics and the theory of constitutive relations. In the first of them, the basic properties of the operators included in the constitutive relations are described and on this basis a certain classification of media is given. In the second part, the apparatus of tensor nonlinear isotropic functions in the theory of constitutive relations in relation to tensor (vector) nonlinear, or quasilinear media, is developed. In the third part, for the environments described in the previous two parts, the formulations of linearized stability boundary-value problems are given in terms of small perturbations of both initial data and material functions.

Elements of the Theory of Constitutive Relations (CR)

Tangent module and tangent pliability as the inverse tensors of the fourth rank. Bounded above, nonnegative and positive definite tangent modulus. Soft and hard characteristics of material.

Material functions as the quantities that involve in definitions of the CR-operators and may be determined only in setting experiments.

Rheonomic and scleronomous media. Aging and ageless materials.

Homogeneous and nonhomogeneous media. Composites as the media where the CR-operators depends on coordinates by discontinuous way. Layered (laminated, stratified), fibrous and granular composites. Micro- and nanocomposites.

Elastic solid as the medium where at any time moment the strain tensor and temperature in this time moment are the only independent state parameters.

Viscous fluid as the medium where at any time moment the strain rate tensor and temperature in this time moment are the only independent state parameters.

Media with memory. Viscoelastic solid.

Non-local media. Strong and weak nonlocality.

Isotropic Tensor Functions and Their Invariants in the Theory of Constitutive Relations

Most general form of the isotropic tensor function in three-dimensional space.

Media possessing a scalar potential. The conditions of potentiality.

Incompressible media (fluids). Most general form of the isotropic tensor function in case of incompressibility. Algebraic connection of invariants.

Nonlinear elastoviscoplastic media in mechanics of continuum and their possible classification. Tensor (vector) CR and scalar CR. Tensor linear (quasilinear) and tensor nonlinear CR.

Formulation of the Boundary-Value Problem in Terms of Perturbations

The closed systems of linearized equations in terms of perturbations.

Linearization of constitutive relations.

Linearization of boundary conditions and their transfer from perturbed boundaries to non-perturbed ones.

Integral measures evaluating initial and current perturbations. Stability by two measures.

Stability with respect to perturbations of material functions.

“Nonlinear Wave Propagation in Cables and Beams Resting on a Bilinear Foundation”

Dr Stefano Lenci

researched and taught at the Universities of Ancona (now Polytechnic University of Marche), Camerino, Pisa, Rome “La Sapienza” and Paris 6, where he stayed for two years and half. He was the responsible of various national and international (with UK, Poland, France, Spain, Brazil, Serbia, Germany, China) scientific projects. He was visiting at the Universities of Paris 6, Wien, Montpellier II, London, Aberdeen, Lublin, Extremadura, Sao Paulo, Rio de Janeiro, Hiroshima, Jeddah, Harbin, Nanjing, Lodz, Novi Sad, Hunan University, Sichuan University of Science and Engineering. He was invited to deliver 40+ seminars in different Universities. He supervised more than 200 undergraduated students and 25 PhD students (4 ongoing). He authored about 400 scientific publications, among which 12 books (2 international) and 204 papers on international, peer reviewed, scientific journals with high impact factor. He has been Guest Editor of Special Issues of “Phil. Trans. Royal Soc. London,” “Nonlinear Dynamics,” “Meccanica,” “Int. J. Non-Linear Mech”, “Math. Prob. Eng.”, “Nonlinear Theory and Its Applications (NOLTA)”, “Actuators”, “Chaos”, “Theoretical and Applied Mechanics Letters”. He is member of various scientific organization, national and international, and he is reviewer for 187 international scientific journals and for research projects of various scientific Institutions (Austria, Belgium, Canada, Israel, Japan, Kazakhstan, Poland, Romania, UK, The Netherlands, Horizon 2020). He delivered invited/keynote/plenary lectures in various Conferences, and chair several sessions in international Congresses. He was the Chairman of the XIX Italian Congress of Mechanics, Ancona, Italy, 14-17 September 2009, of the Euromech 541, Senigallia, Italy, 3-6 June 2013, of the IUTAM Symposium “Enolides,” Novi Sad, Serbia, 15-19 July 2018, of the Euromech Symposium n. 603,” Porto, Portugal, 5-7 September 2018. He was the co-organizer of the CISM course “Global Nonlinear Dynamics for Engineering Design and System Safety,” Udine, Italy, 13-17 June 2016.

His research is focused on the investigation of several aspects of the nonlinear dynamics of various mechanical systems and models, including buckled beams, shallow arches, rolling ships, inverted pendulum between lateral walls, mathematical pendulum, rigid block, infinite beams on unilateral soil, frictional impact oscillator, bilayer beams. An original method for controlling the nonlinear dynamics and chaos has been developed and applied to various mechanical systems. The dynamical integrity of mechanical systems has been investigated, too. Other specific research issues include laying of marine pipelines in deep and ultra-deep waters (the J-lay problem), the dynamic of windscreen wiper, various aspects of the mechanical behaviour of interfaces, and mechanical models for detecting elastic and damaged behaviour of composites.

Recently, he has investigated the nonlinear vibrations of non-uniform beams, and the effects of the boundary conditions of the nonlinear oscillations of beams, the isochronous oscillations of nonlinear beams, wave propagation in beams on unilateral soil, and exact solutions for nonlinear oscillators.

Abstract

Wave propagation on cable and beams is an old mechanical problem, which is relevant both from theoretical and practical points of view. In the past much has been done, but mainly in the linear realm, even in presence of a (linear) foundation. Much less instead has been developed when there are nonlinearities, and there is still a gap of knowledge in this respect. Various studies are based on numerical simulations that solve specific problems but do not give an overall understanding of the mechanical behaviour.

In this work I focus on exact mathematical solutions of the nonlinear problem that are an important to have a full perception of the system characteristic. In the nonlinear realm exact solutions are very few and restricted to particular cases. Fortunately, I found that when the nonlinearity comes from a bilinear foundation, i.e. a substrate that has different stiffnesses in tension and compression, the solution can be obtained analytically.

The obtained solutions are illustrated for the case of cables, beams and nonlocal beams. They have increasing difficulty from a mathematical point of view, and in parallel (and as a consequence, indeed) have more complex and interesting mechanical behaviours.

“Large Eddy Simulation at Affordable Cost: Application to a Full Aircraft Configuration”

Dr Parviz Moin

is Franklin and Caroline Johnson Professor in the School of Engineering at Stanford University. He is the founding director of the Center for Turbulence Research, CTR, at Stanford University. CTR is widely recognized as the international focal point for turbulence research.

Professor Moin pioneered the use of direct and Large Eddy Simulation techniques for the study of turbulence physics, control and modelling concepts and has written widely on the structure of turbulent shear flows. His current interests include: large eddy simulation of complex flows, multi-phase flows with application to aircraft icing, hypersonic flows, propulsion, computational science, and flow control. He is a co- Editor of the Annual Review of Fluid Mechanics and Associate Editor of Journal of Computational Physics. Professor Moin is a member of the U.S. National Academy of Sciences, and the National Academy of Engineering. He is a Fellow of the American Academy of Arts and Sciences, AIAA and APS.

Abstract

The use of computational fluid dynamics (CFD) for external aerodynamic applications has been a key tool for aircraft design in the modern aerospace industry. In takeoff and landing configurations, predicting the maximum lift an aircraft can produce, and the associated onset of boundary layer separation encountered at high angles of attack is critically important. Flow solutions from state-of-the-art solvers are unable to routinely comply with the stringent accuracy and computational efficiency requirements demanded by industry. In this lecture, I will demonstrate that leveraging large eddy simulation with appropriate wall/subgrid-scale models and low dissipation numerical methods suitable for complex geometries on modern compute architectures offers a tractable path towards meeting these accuracy and affordability requirements.

“Bio-electro-mechanical System of the Human Middle Ear”

Dr Rafal Rusinek

educated in Poland, is a head of Biomechanics Department at Lublin University of Technology. His area of research is theoretical and experimental nonlinear dynamics, which he applies to various engineering and biomechanical problems, especially of the middle ear and cutting processes. He has published over 100 journal and conference papers. Rafal Rusinek is the inventor of new patented shape memory prosthesis dedicated for the middle ear and active implant elements. He has established unique experimental laboratory allowing to investigate complex nonlinear dynamic of the middle ear and cutting processes of modern materials.

Abstract

The middle ear is the smallest biomechanical structure in the human body which is responsible for sound transmission from the outer ear canal to the cochlea. Sound approaching the outer ear as an acoustic pressure is transformed first into mechanical vibrations in the middle ear and then into electrical signal in the inner ear. This lecture focuses on sound transfer process through the middle ear in case of healthy and damaged (unhealthy) ear. Analyses are performed using mathematical models and verified experimentally by means of the Laser Doppler Vibrometer on the human temporal bones. The damaged ossicular chain is reconstructed with a new designed shape memory prosthesis or alternatively the ear is provided with an active implant. Both methods of treatments are solved numerically to find better conditions to improve hearing loss and to describe ear dynamics with possible regular and irregular vibrations.

“Numerical Investigation of Flows Around Small-Scale Propellers: Possibilities and Challenges”

Dr Jelena M. Svorcan

is currently an Associate Professor at the Department of Aerospace Engineering of the University of Belgrade, Faculty of Mechanical Engineering in Belgrade, Serbia where she had also previously obtained her PhD in 2014. She was a Fulbright Visiting Scholar at Stanford University, Center for Turbulence Research from November 2021 till July 2022. Throughout an over a decade-long scientific career, her research has mainly been focused on computational aerodynamics, rotating lifting surfaces, wind energy, turbulence, efficiency improvement, numerical simulations, aircraft design and optimization. So far, Svorcan has participated in several national research projects, mainly focused on fluid flow analysis, composite rotors, and drones, including various collaborations with partners from the industry and has authored or co-authored over 100 scientific publications and technical solutions. Svorcan also continuously performs reviews for international scientific journals. She has prepared and given lectures in 10 different subjects at all three levels of study (BSc, MSc, PhD). Svorcan continuously works with students and younger colleagues-researchers, and so far has guided and taken part in several MSc and PhD theses committees.

Abstract

This talk focuses on flows induced by small-scale propeller blades and the wakes shedding from their tips. Flows around propellers for unmanned air vehicles (approximately 25-75 cm in diameter) in hover are simulated by different approaches to considering turbulence. The challenges to simulating these kinds of flows mainly arise from the relatively low values of Reynolds numbers (several tens to several hundreds of thousands) when transition and other flow phenomena may be expected. The adopted numerical set-ups are validated though comparisons with available experimental data. It can be concluded that global aerodynamic performance can be determined with satisfactory accuracy (the discrepancies between computed and measured values of thrust and torque remain below several percents). However, discerning the actual flow characteristics remains challenging. Here, some distinguishing features of small Re rotational flows are accentuated and discussed. Vortex wakes shedding from the blades are visualized and analyzed. These two benchmark examples provide useful guidelines for further numerical and experimental studies of small-scale propellers.

Technical Program

WED 5 July 2023

09:00 - 09:30	Opening Ceremony - Day I: **Prof. Jelena Begović,** Minister of Science, Technological Development and Innovation **Boban Đurović**, Mayor of Vrnjačka Banja **Prof. Nenad Filipović**, Rector of the University of Kragujevac
09:30 - 10:00	Keynote speaker: **Miha Brojan: From symmetry breaking to functionality: Examples from nonlinear mechanics of beams, plates and shells** Chair: Milan Cajić

Session W.1: 10:00-11:30

MS I – Mechanical Metamaterials

Chair: Milan Cajić, Danilo Karličić

W.1.1 – A Brief Review of the Results of Forced Vibrations of Elastically Coupled Nano Structures – *Marija Stamenković Atanasov, Ivan Pavlović*

W.1.2 – Wave Propagation in Periodic Timoshenko Beams on Different Elastic Foundation Types – *Nevena Rosić, Danilo Karličić, Milan Cajić, Mihailo Lazarević*

W.1.3 – Frequency Band Structure Analysis of a Periodic Beam-Mass System for Piezoelectric Energy Harvesting– *Stepa Paunović*

W.1.4 – Wave Propagation Characteristics of Curved Hexagonal Lattice with Resonators – *Shuvajit Mukherjee, Milan Cajić, Sondipon Adhikari, Steffen Marburg*

W.1.5 – Experimental and Numerical Approach to Natural Frequency of Tapered 3D Printed Cantilever Beam a Tip Body – *Marko Veg, Aleksandar Tomović, Goran Šiniković, Stefan Dikić, Nemanja Zorić, Slaviša Šalinić, Aleksandar Obradović, Zoran Mitrović*

W.1.6 – Parametric Amplification in Periodic Chain System – *Milan Cajić, Nikola Nešić, Danilo Karličić*

11:30 - 12:00	Coffee Break

Session W.2: 12:00-14:00

Mechanics of Solid Bodies *(part I)*

Chair: Dragan Rakić

W.2.1 – A Note on the Effect of Statistical Sample Size on Fracture Toughness Characterization in the DTB Transition Region – *Sreten Mastilović, Branislav Đorđević, Aleksandar Sedmak*

W.2.2 – Upgraded Two-step Scaling Approach to the DTB Characterization of Ferritic Steels – *Sreten Mastilović, Branislav Đorđević, Aleksandar Sedmak*

W.2.3 – Plane Motion of a Body Resting on One Cylindrical Hinge and One Sliding Elastic Support Resting on a Rough Plane – *Marat Dosaev, Vitaly Samsonov*

W.2.4 – Phase-field Modeling of High Cyclic Fatigue in Brittle Materials – *Vladimir Dunić, Miroslav Živković, Vladimir Milovanović, Jelena Živković*

W.2.5 – Flexible Deployables Made from Soft Kirigami Composites – *Jan Zavodnik, Mohammad Khalid Jawed, Miha Brojan*

W.2.6 – Determination of the Overall Material Parameters in the Series-parallel Rock-mass Mixture – *Dragan Rakić, Miroslav Živković, Milan Bojović*

W.2.7 – Stochastic Stability of the Timoshenko Beam Resting on the Modified Elastic Foundation – *Dunja Milić, Jian Deng, Vladimir Stojanović, Marko Petković*

W.2.8 – Stability of Parametric Vibrations of the Coupled Rayleigh Beams – *Dunja Milić, Jian Deng, Vladimir Stojanović, Marko Petković*

14:00 - 15:00	Buffet Lunch
15:00 - 15:30	Keynote speaker: **Dimitri V. Georgievskii: Elements of the Theory of Constitutive Relations and Formulations of the Linearized Problems on Stability** Chair: Vladimir Simić

Session W.3: 15:30-17:00

Mechanics of Solid Bodies *(part II)*

Chair: Vladimir Dunić

W.3.1 – Mechanical Response of V-shaped Protective Plates with Different Angles under Blast Loading – *Miloš Pešić, Aleksandar Bodić, Živana Jovanović Pešić, Nikola Jović, Miroslav Živković*

W.3.2 – Phase-field Modeling of Low Cyclic Fatigue in Ductile Materials – *Vladimir Dunić, Miroslav Živković, Vladimir Milovanović, Jelena Živković*

W.3.3 – Experimental and Numerical Analysis of the Strength of a Drone Arm Made of Composite Material – *Petar Ćosić, Miloš Petrašinović, Aleksandar Grbović, Danilo Petrašinović, Mihailo Petrović, Veljko Petrović, Nikola Raičević, Boško Rašuo*

W.3.4 – Parameter Identification of Viscoelastic Materials using Different Deformation Velocities –*Iva Jankovic, Nenad Grahovac, Miodrag Žigić*

W.3.5 – Application of Composite Smeared Finite Element for Mechanics (CSFEM) on Tumor Growth Model – *Vladimir Simić, Miljan Milošević, Miloš Kojić*

W.3.6 – Vibrations of Fluid-conveying Functionally Graded Nanotubes – *Nikola Despenić, Goran Janevski*

17:00 - 17:30	Coffee Break

Session W.4: 17:30-19:45

MS III - Biomechanics and Mathematical Biology

Chair: Anđelka Hedrih, Marat Dosaev

W.4.1 – An Exergame-integrated IoT-Based Ergometer System for Personalized Training of the Elderly – *Su Fong Chin, Lin Chih-Chun, Kuo Li-Chieh, Lin Yu-Sheng, Chang Chia-Ming, Hu Fang Wen, Chen Yi-Jing, Lin Chun-Tse* - ***Invited talk***

W.4.2 – Effect of Smooth-muscle Activation in the Static and Dynamic Mechanical Characterization of Human Aortas – *Marco Amabili, Ivan Breslavsky, Francesco Giovanniello, Giulio Franchini, Ali Kassab, Gerhard Holzapfel*

W.4.3 – A SIS Model with a Saturated Incidence Rate – *Marcin Choiński*

W.4.4 – Memristive Neural Networks for Predicting Epileptiform Activity – *Svetlana Gerasimova, Nikolay Gromov, Albina Lebedeva, Tatiana Levanova*

W.4.5 – Three-link Snake Robot Controlled by An Internal Flywheel – *Yury Selyutskiy, Liubov Klimina, Anna Masterova*

W.4.6 – Modeling an Indentation of a Head of Video-tactile Sensor into a Linear Elastic Tissue – *Marat Dosaev, Anfisa Rezanova*

W.4.7 – Left Ventricle Cardiac Hypertrophy Simulations using Shell Finite Elements – *Bogdan Milićević, Miljan Milošević, Vladimir Simić, Danijela Trifunović; Goran Stanković; Nenad Filipović; Miloš Kojić*

W.4.8 – An Overview: About Three Models of Mitotic Spindle Oscillations and their Mods – *Anđelka Hedrih, Katica Stevanović Hedrih*

W.4.9 – L-Tyrosine Influence on the Reaction Kinetics of Iodate-Hydrogenperoxide Oscillatory Reaction –*Jelena Maksimović, Ana Ivanović-Šašić, Stevan Maćešić, Željko Čupić, Ljiljana Kolar-Anić*

20:30 - 24:00	Party for Young Scientists

THU 6 July 2023

Session T.1: 8:30-10:30
MS II - Turbulence *(part I)*
Chair: Đorđe Čantrak, Mihailo Jovanović

T.1.1 – MS II - Keynote Speaker - Probing Turbulence Physics sing Numerical Simulation Databases – A Case Study in Predictive Science – *Parviz Moin*

T.1.2 – MS II - Keynote Speaker - Secondary Flows of Prandtl's Second Kind. Mechanism of Formation and Method of Prediction – *Nikolay Nikitin*

T.1.3 – Design and Optimization of Splitter Blade of Return Channel for Improvement of Pump Turbine Performance – *Geyuan Tai, Wenjie Wan, Ji Pei, Giorgio Pavesi, Shouqi Yuan*

T.1.4 – Leading Edge Shape Optimization of a Novel Family of Hybrid Dolphin Airfoils – *Zorana Dančuo, Ivan Kostić, Olivera Kostić, Aleksandar Bengin, Goran Vorotović*

T.1.5 – The Influence of Magnus Force on Turbulent Particle-Laden Flows in Horizontal Narrow Channel – *Darko Radenković, Milan Lečić*

T.1.6 – Enstrophy Study of the Turbulent Swirling Flow in Pipe – *Đorđe Čantrak, Novica Janković, Dejan Ilić, Lazar Lečić*

10:30 - 11:00	Coffee Break
11:00 - 11:30	Keynote speaker: **Stefano Lenci: Nonlinear Wave Propagation in Cables and Beams Resting on a Bilinear Foundation** Chair: Katica (Stevanović) Hedrih

Session T.2: 11:30-13:15
MS II - Turbulence *(part II)*
Chair: Mirjana Stamenić, Jelena Svorcan

T.2.1 – MS II - Keynote Speaker - Research on High Efficiency and High Reliability Pumps in Jiangsu University – *Ji Pei, Wenjie Wang*

T.2.2 – Oblique Transition in High-speed Separated Boundary Layers–*Anubhav Dwivedi, G.S. Sidharth, Mihailo Jovanović*

T.2.3 – Improving Airfoil Performance by Designed Blowing – *Jelena Svorcan, Toni Ivanov, Aleksandar Simonović*

T.2.4 – Deep Learning in PIV Applications – *Jelena Ilić, Ivana Medojević, Novica Janković*

T.2.5 – Numerical Simulations in the Design and Optimization of a Fluid-dynamical Valve in Regenerative Burners Installation – *Mirjana Stamenić*

T.2.6 – Experimental Investigation and Mathematical Modelling of Vortex Structures Found at Impinging Turbulent Axisymmetric Air Jet Modified by Low-Amplitude Sound Modulation – *Dejan Cvetinović, Aleksandar Erić, Nikola Ćetenović, Đorđe Čantrak, Jaroslav Tihon, Kazuyoshi Nakabe*

Session T.3: 13:15-14:30

MS IV – Nonlinear Dynamics

Chair: Julijana Simonović

T.3.1 – An Overview: With the Anđelić and Rašković Tensor into Transformations of the Base Vectors in the Tangent Space of the Position Vector of the Kinetic Point – *Katica (Stevanović) Hedrih*

T.3.2 – Steady State Solution for Dynamics of a Nonideal Crank-Slider Mechanism with an Active Mass Damper (AMD) – *Julijana Simonović, Nikola Nešić, José Manoel Balthazar, Maurício Aparecido Ribeiro, Jorge Luis Palacios Felix*

T.3.3 – Dynamic Behavior of Nano-system under the Influence of Moving External Nanoparticle – *Marija Stamenković Atanasov, Danilo Karličić, Ivan Pavlović*

T.3.4 – On Deviations in Nonlinear Time Domain Regime of Vibrations of the Partly Coupled Structures with The Curvatures – *Vladimir Stojanović, Jian Deng, Dunja Milić, Marko Petković*

T.3.5 – Nonlinear Characterization of a Vibration System Model– *Tamara Nestorović, Umaaran Gogilan, Atta Oveisi*

14:30 - 15:30	Buffet Lunch
15:30 - 16:00	Keynote speaker: **Parviz Moin: Large Eddy Simulation at Affordable Cost: Application to a Full Aircraft Configuration** Chair: Miloš Kojić

Session T.4: 16:00-18:15

General Mechanics *(part I)*

Chair: Borislav Gajić

T.4.1 – An Overview: About Two Doctorates in Serbian Science on Ball Rolling and New Modern Results – *Katica (Stevanović) Hedrih*

T.4.2 – An Overview: On Nonlinear Differential Equations and Integrals of the Dynamics of Ball Rolling Along Curved Lines and Surfaces – *Katica (Stevanović) Hedrih*

T.4.3 – The Brachistochronic Motion of Chaplygin Sleigh in a Vertical Plane with Unilateral Nonholonomic Constraint –*Aleksandar Obradović, Oleg Cherkasov, Luka Miličić*

T.4.4 – Thin Composite Plates with Stress Concentrators Analyzed by Theory of Critical

Distances – *Ivana Atanasovska; Dejan Momčilović*

T.4.5 – Free Vibration Analysis of FGM Plates by using Layer Wise Displacement Model – *Marina Ćetković*

T.4.6 – Stability Analysis of FGM Plates by using Layer Wise Displacement Model – *Marina Ćetković*

T.4.7 – Examples of Integrable Nonholonomic Systems with an Invariant Measure – *Vladimir Dragović*, *Borislav Gajić, Božidar Jovanović*

T.4.8 – Chaplygin Systems with Gyrosopic Forces –*Vladimir Dragović, Borislav Gajić, Božidar Jovanović*

T.4.9 – Vibrations of a Viscoelastic Rod Modeled by Fractional Burgers Constitutive Equations – *Slađan Jelić, Dušan Zorica*

Session T.5: 18:15-19:00

Robotics

Chair: Mihailo Lazarević

T.5.1 – Analytical Design of Resonant Controller Applied for Solving Robot Arm Tracking Problem –*Petar Mandić, Tomislav Šekara, Mihailo Lazarević*

T.5.2 – Systematic Design of a Desktop Robot Arm in Solidworks and Matlab Simulink – *Andrija Dević, Jelena Vidaković, Nikola Živković, Mihailo Lazarević*

T.5.3 – PSO-optimized Fractional Order Iterative Learning Controller For 3DOF Uncertain Exoskeleton System – *Nikola Živković, Mihailo Lazarević*

19:00 - 20:30	General Assembly of SSM
21:00 - 24:00	Gala Dinner

FRI 7 July 2023

Session F.1: 9:00-11:15 **General Mechanics *(part II)*** Chair: Nenad Grahovac

F.1.1 – Further Results on Robust Finite-time Stability Nonstationary Two-term Neutral Nonlinear Perturbed Fractional – Order Time Delay Systems – *Mihailo Lazarević, Darko Radojević, Petar Mandić, Stjepko Pišl*

F.1.2 – Closed-form Solutions and Stability of Shellsunder the White Noise Excitation – *Vladimir Stojanović, Jian Deng, Dunja Milić, Marko Petković*

F.1.3 – Dynamics of a Multilink Aerodynamic Pendulum – *Yury Selyutskiy, Andrei Holub*

F.1.4 – Dynamics of Asymmetric Mechanical Oscillator Moving Along an Infinite Beam-Type Complex Rail System – *Vladimir Stojanović, Jian Deng, Dunja Milić, Marko Petković*

F.1.5 – Thermodinamical Restrictions for Moving Point Load Model Involving Generalized Viscoleastic Foundation – *Lidija Rehlicki Lukešević, Marko Janev, Branislava Novaković, Teodor Atanacković*

F.1.6 – Analysis of Planar Complex Motion of a Homogeneous Disk and a Material Point with Euler-Lagrange Equations in Quasi-velocities – *Marko Gavrilović*

F.1.7 – An Alternative for the Grünwald-Letnikov-Turner Method for Solving Set-valued Fractional Differential Equations of Motion –*Filip Jakovljević, Miodrag Žigić, Nenad Grahovac, Dragan Spasić*

F.1.8 – Forced Vibrations with Burgers Type of Damping – *Dragan Spasić*

F.1.9 – On The Trail of Vujičić's Coordinates-Independent Position Vector Form: Rotationally Invariant/(Classically)-covariant Trajectorial Coordinates System Formulation and Other Repercusios for the Mechanics / Dynamics Modeling – *Slobodan Nedić*

11:15 - 11:45	Coffee Break
11:45 - 12:15	Keynote speaker: **Rafal Rusinek: Bio-electro-mechanical System of the Human Middle Ear** Chair: Anđelka Hedrih

Session F.2: 12:45-14:00 **Interdisciplinary and Multidisciplinary Problems *(part I)*** Chair: Migleva Doneva

F.2.1 – Modeling of Penetration Depth of a Shaped Charge Jet – *Predrag Elek, Miloš*

Marković, Dejan Jevtić, Radovan Đurović

F.2.2 – Age-related Problems of Polypropylene Hernia Meshes – *Miglena Kirilova-Doneva, Dessislava Pashkouleva*

F.2.3 – Modelling of Landslide Dynamics: Role of Displacement Delay and Natural Background Noise – *Srđan Kostić*

F.2.4 – Comparative Structural Analysis of Aluminum and Composite Wing of Passenger Aircraft – *Radoslav Radulović, Milica Milić, Fei Xun*

F.2.5 – Development of a Hybrid Fixed-wing VTOL Unmanned Aerial Vehicle – *Radoslav Radulović, Milica Milić*

14:00 - 15:00	Buffet Lunch
15:00 - 15:30	Keynote speaker: **Jelena Svorcan: Numerical Investigation of Flows around Small-Scale Propellers: Possibilities and Challenges** Chair: Tijana Geroski

Session F.3: 15:30-16:45

Interdisciplinary and Multidisciplinary Problems *(part II)*

Chair: Smiljana Tomašević

F.3.1 – 3D Printing Technology in Cranioplasty: Case Study – *Tijana Geroski, Dalibor Nikolić, Vojin Kovačević, Nenad Filipović*

F.3.2 – Development and Mould Technology for Testing of Biocomposite Structures (Application for Thermoinsulated Bio Plates) – *Marija Baltić, Milica Ivanović, Dragoljub Tanović, Miloš Vorkapić*

F.3.3 – Inhibitory Effect of 4-Hydroxycoumarin Derivative on Kras Protein – *Marko Antonijević, Žiko Milanović, Edina Avdović, Dušan Dimić, Zoran Marković*

F.3.4 – Gallic Acid Derivatives as Inhibitors of Carboxy Anhydrases – *Marko Antonijević, Dušica Simijonović, Jelena Đorović-Jovanović, Zoran Marković*

F.3.5 – Computational Analysis of Drug Effects on Hypertrophic Cardiomyopathy – *Smiljana Tomašević, Miljan Milošević, Bogdan Milićević, Vladimir Simić, Momčilo Prodanović, Srboljub Mijailović; Nenad Filipović*

16:45 - 17:15	Coffee Break

Session F.4: 17:15-19:15

Fluid Mechanics

Chair: Srboljub Simić

F.4.1 – Nonlinear Sea Surface Waves – *Teodor Vrećica*

F.4.2 – Development of a Method for the Calculation of Multistage Gas Turbines and Estimation of the Required Amount of Cooling Air – *Nikola Marković*

F.4.3 – Analytical and Numerical Analysis of Compressible Isothermal Flow between Parallel Plates – *Petar Vulićević, Snežana Milićev, Nevena Stevanović*

F.4.4 – Comparative Analysis of SPH and FVM Numerical Simulations of Bloodflow through Left Ventricle – *Aleksandar Bodić, Marko Topalović, Miljan Milošević, Miroslav Živković, Miloš Pešić*

F.4.5 – Influence of Reflected Shockwaves on Normal Force Coefficient of Grid Fins in Supersonic Flight Regime – *Ognjen Ristić*

F.4.6 – Designing, Optimising and Fabricating of Microfluidic Devices, Based on Topology Optimisation and 3D Printing – *Dalibor Nikolić, Nevena Milivojević, Ana Mirić, Marko Živanović, Nenad Filipović*

F.4.7 – Viscous Generalized Maxwell-Stefan Model of Diffusion –*Damir Mađarević, Srboljub Simić*

F.4.8 – A Simplified Nonlinear Dynamic Mathematical Model of a Controlled Real Turbojet Engine – *Miloš Živanović*

Session F.5: 19:15-20:15

Biomechanics

Chair: Igor Saveljić

F.5.1 – Parallelized Software for Fast Virtual Stenting Simulation of Patient-specific Coronary Artery – *Tijana Đukić, Igor Saveljić, Nenad Filipović*

F.5.2 – Biomimetic Materials for Military Applications – *Marina Simović Pavlović, Katarina Nestorović, Darko Janković, Aleksandra Radulović, Maja Pagnacco*

F.5.3 – Corrugation Elasticity as a New Property of Nanostructured Material: Holographic Analysis of Apatura Butterfly Wings – *Marina Simović Pavlović, Katarina Nestorović, Aleksandra Radulović, Maja Pagnacco*

F.5.4 – Comparative Numerical Analyses of Tooth Restored with Hydroxyapatite Ceramic Insert Versus Traditional Composite Restoration – *Aleksandar Bodić, Maja Ležaja-Zebić, Milan Bojović, Đorđe Veljović, Vladimir Milovanović*

20:30 - 21:00	Closing Ceremony

9th International Congress of the Serbian Society of Mechanics
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FREE VIBRATION ANALYSIS OF FGM PLATES BY USING LAYER WISE DISPLACEMENT MODEL

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Abstract

In this paper, the free vibration analysis of simply supported functionally graded material (FGM) plate is analyzed. The displacement model based on Generalized Laminate Plate Theory (GLPT) assumes layer wise (LW) linear variation of in–plane displacements, constant transverse displacement, linear strain–displacement relations and linear material properties. The effective material properties of FGMs are assumed to be given by the Voigt's rule of mixture (ROM). The Power law distribution of volume fraction is assumed through the plate thickness. The mathematical model includes the quadratic variation of transverse shear stresses within each mathematical layer of the plate. The principle of virtual displacements (PVD) is used to derive Euler–Lagrange differential equations of motions for free vibration problem. The Closed form solution is derived following the Navier's technique and solving the eigenvalue problem. An original MATLAB computer program is coded for the numerical solution. The results reveal that the effects of side–to–thickness ratio, power–low index and material properties have significant effect on free vibration frequencies of FGM plates.

Key words: Free vibration, FGM plate, LW Theory, Navier solution, MATLAB program

1. Introduction

Functionally graded materials (FGM) were initially developed in Japan in the 1980s for thermal insulation purposes during the space plane project [1]. After that, the improvement of the material properties of FGM have extended their applications towards the biomedical, aerospace and automotive, bullet–proof and defence industries [2]. The initially assumed concept of FGM was the mixing of two materials, usually metal (ductile phase) and ceramic (brittle phase), in order to obtain the gradual variation of material properties. These material properties, often called the effective material properties, despite of their complex heterogeneous microstructure, are mathematically defined by specific homogenization scheme. The homogenization scheme includes different material models, such as Power law model (P–FGM), Exponential model (E–FGM), Sigmoid law model (S–FGM), Mori–Tanaka scheme etc. When the appropriate homogenization scheme is adopted, the mathematical formulation, numerical solution, and finally the behaviour of FGM structural components under various loading conditions may be found.

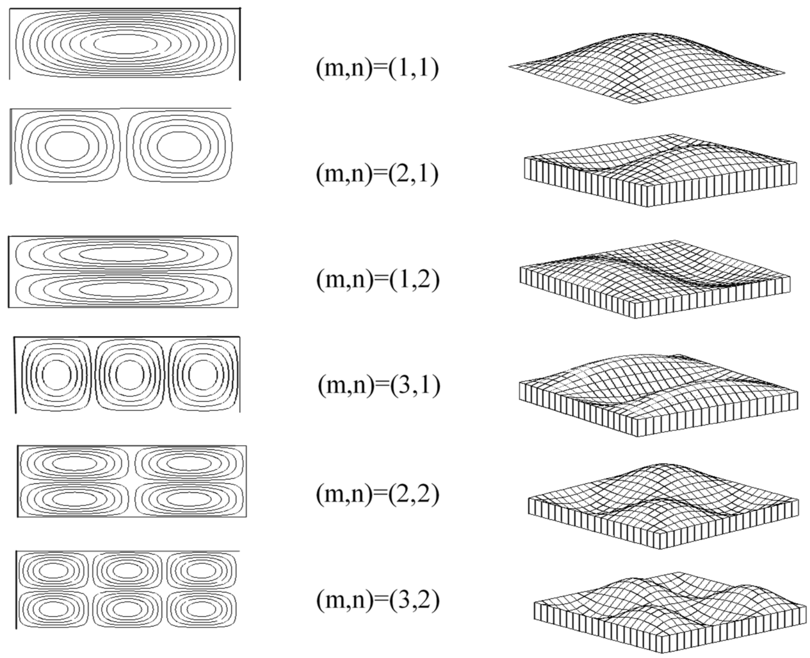

Fig. 1. The contour lines and mode shapes for simply supported rectangular plate

Among the critical loading conditions, important for the safe design of FGM structures is the failure due to vibrations. The vibrations may be initiated from the external source, or by initial disturbance, when no external force is applied. The first one are the forced vibrations, and the second one are the free vibrations. Both vibrations may be damped or undamped, linear or nonlinear [3]. In this paper linear free vibrations of P─FGM plates are analysed.

Mathematical models for free vibration analysis of FGM plates are formulated using either 3D theory of elasticity, Equivalent Single Layer Theories (ESL), and more recently Carrera's Unified Formulation (CUF). Since there are limited number of 3D elasticity solutions, the most literature regarding free vibrations of FGM plates are based on ESL theories, which are Classical Plate Theory (CPT), First–order Shear Deformation Theory (FSDT) and Higher–order Shear Deformation Theory (HSDT).

Batra and Vel presented 3D elasticity solution for free and forced vibration analysis of simply supported P─FGM sandwich plates [4]. The three displacement componenets are expanded by a series of Chebyshev polynomials multiplied by appropriate functions to satisfy the essential boundary conditions. The governing steady state coupled ordinary differential equations are solved by employing the power series method. The parametric study including the volume faction index, layer thickness ratio and aspect ratio is performed. It is observed that for thick FGM plates, there are significant differences between the exact solution and results obtained from CPT theory, even when the transverse shear and the transverse normal stresses are computed by integrating the 3D elasticity equations. However, the results from FSDT and HSDT are shown the good agreement with exact solution.

Since there are limited number of 3D elasticity solutions for FGM plates, and considerable computational cost is needed, an ESL theories are used. Abrate [6] used CPT to analyze free vibrations of P─FGM plates. He showed that by selecting an appropriate reference plane, the uncoupled governing equations may be obtained. As a consequence, the FGM plate will behave like homogeneous isotropic plate. Chen et al. [7] studied linear and nonlinear free vibrations frequencies of an initially stressed P─FGM plate with initial stresses and initial geometrical imperfections. The perturbation technique is used to derive the nonlinear governing equations using the Galerkin method. The governing equations are solved by the Runge–Kutta method. It is shown that linear and nonlinear vibration frequency of the FGM plate increases with the volume fraction of ceramic. Since, CLPT neglects transverse shear deformation it becomes inadequate for the analysis of moderately thick to thick plates. In these cases, FSDT which assumes constant transverse shear strains, should be adopted. Zhao et al. [8] presented free vibration analysis of P─FGM plates using FSDT and the element–free kp–Ritz method. It is found that significant influence on the frequency has volume fraction exponent that ranges between 0 and 5. Natarajan et al. [9] used FSDT to study static bending, free vibration, and mechanical and thermal buckling behaviour of FGM plates. The effective properties of the FGM plates are computed using the rule of mixtures or employing the Mori–Tanaka homogenization scheme. The cell–based smoothed finite element method with discrete shear gap technique is employed to find the numerical solution of governing equations. The effects of material gradient index n, skewness of the plate ψ, the plate aspect ratio a/b, the plate thickness a/h, and boundary conditions on the global response, are numerically presented. In FSDT the shear correction factor is not easy to accurately define and HSDT should be adopted. Matsunaga [10] studied natural frequencies and buckling stresses of P─FGM plates using HSDT with power series expansions of displacement components, subjected to in–plane stresses. The fundamental set of equations are derived through Hamilton's principle. The governing equations are solved using closed form Navier solution. It is noticed that the critical buckling stresses for thick plates occur at higher displacement modes as in the isotropic case. Jha et al. [11] presented free vibration response of P─FGM and E─FGM rectangular simply supported plates based on HSDT. The equations of motion are derived using Hamilton's principle, while numerical solutions are obtained using Navier technique and are incorporated into a MATLAB computer program. It is shown that natural frequency decreases with increase of the metal percentage of FGM. This is due to the fact that the Young's modulus of ceramic is higher than metal. For the same value of power low index, the natural frequency increases for the higher modes. Also, as a/h is increased, the plates become thin, more flexible thus giving the decrease of the natural frequencies. Gupta et al. [12] used HSDT for free vibration of geometrically imperfect P─FGM plates. Mori–Tanaka and Voigt micromechanics models are employed to determine the effective material properties of the plate. It is found that the proposed theory can accurately predict the free vibration responses of functionally graded ceramic–metal plates. In wish to obtain generalized expression, Carrera proposed so called Carrera's Unified Formulation (CUF). The CUF enables the formulation of a large variety of 2D and quasi–3D hierarchical models using unified approach. The unified approach assumes that the variational statement and governing differential equations are given in terms of fundamental nuclei, which are mathematically and formally independent from the expansion orders used in the displacement field and from the kinematic description used such as ESL or LW theories. Cinefra et al.[13] used Carrera's Unified Formulation to study free vibration analysis of P─FGM plate. PVD is used to derive governing differential equation and are solved using the Navier's closed form technique. The obtained results are in good agreement with the reference solutions. Also, the analysis is showen that HSDT are necessary to provide an adequate accuracy for thick plates, while CPT gives erroneous results, especially for high wave numbers and high–order frequencies.

In order to fulfill the lack of free vibration solutions based on LW plate theories in the literature, in this paper an analytical solution for free vibrations of P─FGM plates using Reddy's LW theory [14] is formulated. After establishing the accuracy of the present LW model for linear and geometrically nonlinear bending, vibration and buckling analysis of perfect and imperfect laminated composite and sandwich plates subjected to thermo–mechanical load in the authors previous papers [15–20], in this paper free vibration analysis of FGM plates is further investigated. The mathematical model assumes layer wise variation of in–plane displacements and constant transverse displacement through the plate thickness, linear strain–displacement relations and linear mechanical material properties. The material properties of FGM plates are assumed to be constant in xy–plane and vary through the thickness by a power law function in terms of the volume fraction of the constituents. The effective materials properties are given by the rule of mixture. The governing Euler–Lagrange differential equations of free vibration problem are derived using Principle of virtual displacements (PVD). Euler–Lagrange differential equations of motions are solved using Navier's technique. The original MATLAB program is coded and used to study the effects of side to thickness ratio, power law index and material properties on free vibration frequencies. The accuracy of the numerical model is verified by comparison with the available results from the literature.

2. Theoretical Formulation

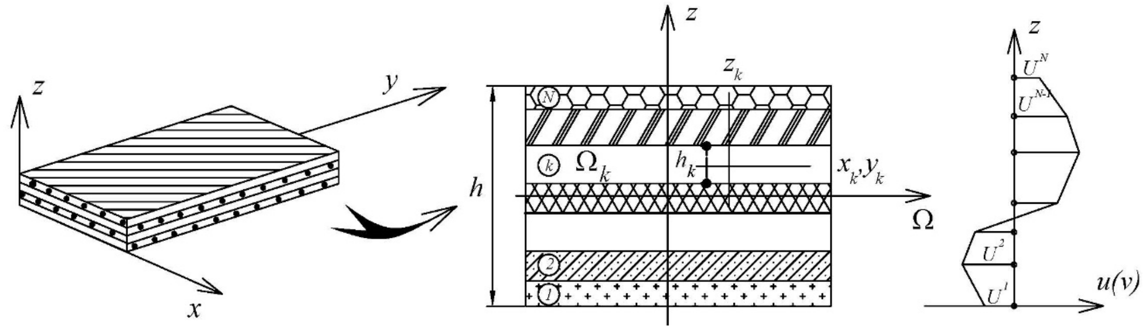

Fig. 2. Plate geometry and LW in–plane displacement field

A LW plate model is composed of n layers. It is assumed that 1) layers are perfectly bonded together, 2) material of each layer is linearly elastic, 3) strains are small, 4) each layer is of uniform thickness, 5) inextensibility of normal is imposed, Figure 2.

2.1 Displacement field

The displacements components (u_1, u_2, u_3) at a point (x, y, z) of plate are expressed as [14]:

$$
\begin{aligned}
u_1(x,y,z) &= u(x,y) + \sum_{I=1}^{N} U^I(x,y)\cdot\Phi^I(z), \\
u_2(x,y,z) &= v(x,y) + \sum_{I=1}^{N} V^I(x,y)\cdot\Phi^I(z), \\
u_3(x,y,z) &= w(x,y).
\end{aligned}
\tag{1}
$$

where (u, v, w) are displacements of a point $(x, y, 0)$ on the reference plane, functions $\Phi^I(z)$ are 1D linear Lagrange interpolation functions of thickness coordinates, while (U^I, V^I) are the values of (u_1, u_2) at the I–th plane, Fig. 2.

2.2 Strain–displacement relations

The strains associated with the displacement field (1) are given using linear strain–displacement relations:

$$
\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} + \sum_{I=1}^{N} \frac{\partial U^I}{\partial x}\Phi^I, \\
\varepsilon_{yy} &= \frac{\partial v}{\partial y} + \sum_{I=1}^{N} \frac{\partial V^I}{\partial y}\Phi^I, \\
\gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \sum_{I=1}^{N}\left(\frac{\partial U^I}{\partial y} + \frac{\partial V^I}{\partial x}\right)\Phi^I, \\
\gamma_{xz} &= \sum_{I=1}^{N} U^I \frac{d\Phi^I}{dz} + \frac{\partial w}{\partial x}, \\
\gamma_{yz} &= \sum_{I=1}^{N} V^I \frac{d\Phi^I}{dz} + \frac{\partial w}{\partial y}.
\end{aligned}
\tag{2}
$$

2.3 Constitutive equations

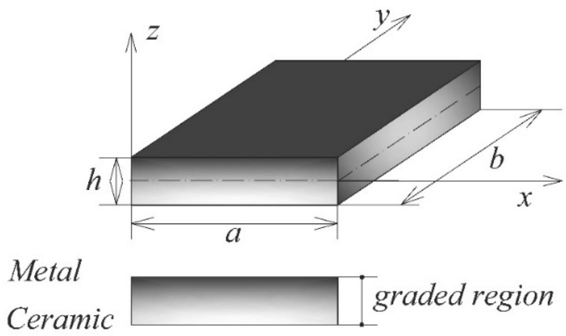

Fig. 3. Geometry and material gladiation of FGM plate

The plate is made from a mixture of ceramic and metal Figure 3, where the rule of mixture is defined as:

$$P_e = V_m + \left(P_c - P_m\right) V_c\left(z\right) \tag{3}$$

The P_e denotes the effective material properties of FGM plate, such as Young's modulus E and mass density ρ , while Poisson's coefficient ν is assumed to be constant. The subscripts c and m denote the ceramic and metal, corresponding the material property of the lower and upper surface of the plate, respectively. The V_c is volume fraction of ceramic. The volume fraction is given by the power low distribution, Figure 4 in the thickness direction as:

$$V_c\left(z\right) = \left(\frac{z}{h} + \frac{1}{2}\right)^n \tag{4}$$

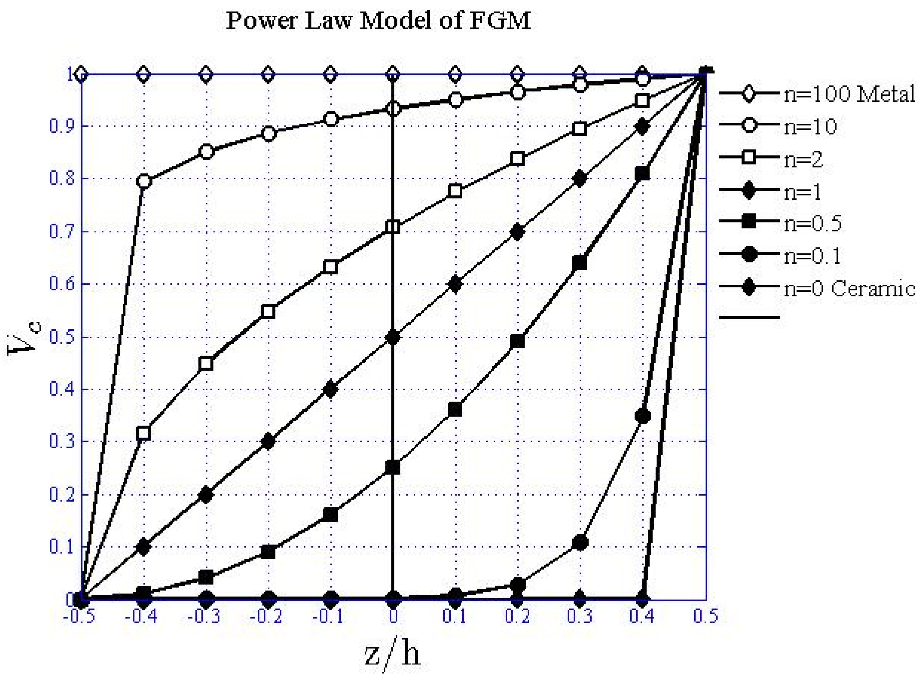

Fig. 4. Volume fraction V_c distribution along the plate thickness for different values of the volume fraction index n

Where n denotes the power low index by which the gradation of the constituents are controlled and may take the values $[0, \infty]$. When the volume fraction exponent is 0 plate is fully made of ceramic, and when the volume fraction exponent is 1 the variation of the volume fraction is linear.

A linear elastic material behavior is considered. The stress–strain relations are given by the generalized Hook's law as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}^{(k)} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix}^{(k)} \times \left(\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}^{(k)} \right), \tag{5}$$

where Q_{ij} are elastic stiffness of FGM plate, given as:

$$Q_{11} = Q_{11} = \frac{(1-\nu)}{(1-2\nu)(1+\nu)} E(z), Q_{12} = Q_{13} = Q_{23} = \frac{1}{(1-2\nu)(1+\nu)} E(z),$$

$$Q_{33} = Q_{44} = Q_{55} = \frac{1}{2(1+\nu)} E(z). \tag{6}$$

2.4 Governing equations and boundary conditions

The governing Euler–Lagrange equations of motion, defining free vibration of FGM plates are given in the following form:

$$\begin{aligned} & N_{xx,x} + N_{xy,y} = I_0 \ddot{u} + \sum_{J=1}^{N} I^J \ddot{U}^J \\ & N_{xy,x} + N_{yy,y} = I_0 \ddot{v} + \sum_{J=1}^{N} I^J \ddot{V}^J \\ & Q_{x,x} + Q_{y,y} = I_0 \ddot{w} \\ & N_{xx,x}^I + N_{xy,y}^I - Q_{xz}^I = I^I \ddot{u} + \sum_{J=1}^{N} I^{IJ} \ddot{U}^J \\ & N_{xy,x}^I + N_{yy,y}^I - Q_{yz}^I = I^I \ddot{v} + \sum_{J=1}^{N} I^{IJ} \ddot{V}^J \end{aligned} \tag{7}$$

Where $I_0 = \int_{-h/2}^{h/2} \rho(z)\, dz$, $I^I = \int_{-h/2}^{h/2} \rho(z)\, \Phi^I dz$, $I^{IJ} = \int_{-h/2}^{h/2} \rho(z)\, \Phi^I \Phi^J dz$, and ρ is mass density, while appropriate force and displacement boundary conditions are:

$$\{N_{nn}\ N_{ns}\ Q_n\ N_{nn}^I\ N_{ns}^I\} = \{N^*{}_{nn}\ N^*{}_{ns}\ Q^*{}_n\ N_{nn}^{*I}\ N_{ns}^{*I}\}, \{u_n\ u_s\ w\, U_n^I\ V_n^I\} = \{u^*{}_n\ u^*{}_s\ w^*\ U_n^{*I}\ V_n^{*I}\} \tag{8}$$

3. Analytical solution

Navier's solution of Euler–Lagrange equilibrium equations is derived for rectangular plates axb with the following simply supported boundary conditions:

$$v = w = V^I = N_{xx} = N_{xx}^I = 0 \quad \text{at} \quad x = 0, a$$
$$u = w = U^I = N_{yy} = N_{yy}^I = 0 \quad \text{at} \quad y = 0, b \tag{9}$$

The displacement field which satisfies the boundary conditions (9) and Euler–Lagrange equations of motions (7), is assumed in the form:

$$\left(u(x,y); U^I(x,y)\right) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(u_{mn}; U^I{}_{mn}\right)\cdot e^{i\omega_{mn}t}\cdot\cos\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y$$
$$\left(v(x,y); V^I(x,y)\right) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(v_{mn}; V^I{}_{mn}\right)\cdot e^{i\omega_{mn}t}\cdot\sin\frac{m\pi}{a}x\cdot\cos\frac{n\pi}{b}y \tag{10}$$
$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}\cdot e^{i\omega_{mn}t}\cdot\sin\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y$$

Substituting displacement field (10) into the governing equations of motions (7) the governing free vibration equations of FGM plates are obtained:

$$\left(\left[\mathbf{K}\right] - \omega_{mn}^2\left[\mathbf{M}\right]\right)\left\{\boldsymbol{\Delta}\right\} = \left\{\mathbf{0}\right\} \tag{11}$$

For each choice of (m,n) the free vibration frequency or eigenvalues ω_{mn} is obtained. The smallest of all ω_{mn} not equal to zero is the fundamental frequency ω_{mn}^{cr}. The vector of free vibration mode shapes is then $\{\Delta\}^T = \{X_{mn}, Y_{mn}, W_{mn}, U_{mn}^I, V_{mn}^I\}$, while $[\mathbf{K}]$ and $[\mathbf{M}]$ are given in [16].

4. Numerical results and discussion

Using previously derived analytical solutions, an original computer program was coded using MATLAB programming language, for free vibration of simply supported FGM plates. The parametric effect of side to thickness ratio b/h, power law index n and material properties on fundamental frequency are analyzed.

Example 1: The first example is the FGM ($Ti-6Al-4V/Al_2O_3$) simply supported plate made of following material properties:

Metal ($Ti-6Al-4V$): $E_m = 70\,GPa,\quad \nu = 0.3,\quad \rho_m = 4429\,kg/m^3$

Ceramic (Al_2O_3): $E_c = 380\,GPa,\quad \nu = 0.3\quad \rho_c = 2370\,kg/m$

The fundamental frequency is presented in nondimensional form: $\bar{\omega} = \omega_{cr}\sqrt{\dfrac{12(1-\nu^2)\rho_c a^2 b^2}{\pi^4 E_c h^2}}$

Theory	n	b/h 5	10	20	30	50	100
HSDT [12]	0	1.6999	1.9391	2.0053	2.0202	2.0295	2.0343
Present LW		1.6230	2.2801	2.3342	2.3458	2.3528	2.3565
HSDT [12]	1	1.2592	1.4248	1.4706	1.4809	1.4873	1.4906
Present LW		1.3524	1.4801	1.5146	1.5221	1.5266	1.5289
HSDT [12]	2	1.1370	1.2935	1.3375	1.3474	1.3536	1.3567
Present LW		1.2104	1.3131	1.3446	1.3514	1.3555	1.3576
HSDT [12]	5	1.0312	1.1922	1.2387	1.2491	1.2557	1.2591
Present LW		1.0919	1.1897	1.2200	1.2266	1.2305	1.2326
HSDT [12]	10	0.9760	1.1306	1.1752	1.1852	1.1916	1.1949
Present LW		1.0276	1.1207	1.1497	1.1559	1.1597	1.1617
HSDT [12]	100	0.8577	0.9708	1.0151	1.0229	1.0277	1.0302
Present LW		0.9117	0.9869	1.0101	1.0151	1.0182	1.0197

Table 1. Comparison of nondimensional fundamental frequency $\bar{\omega}$ of simply supported $Ti-6Al-4V/Al_2O_3$ plate for various power low index n and side to thickness ratio b/h

Table 1 shows that the present LW solution is in close agreement with HSDT [12] solution, for all thickness ratios. It is shown that frequency parameter decreases with the increase of volume fraction index, or power low index. This is due to the fact that the larger power low index means the smaller the ceramic phase and the reduced stiffness. Also, as noticed by Gupta et al. [12] the considerable rise in the frequency parameter is up to b/h=30, after that no significant change may be observed.

Example 2: The second example is the FGM (Al/Al_2O_3) simply supported plate made of following material properties:

The FGM (Al/Al_2O_3) plate is made of following material properties:

Metal (Al): $E_m = 70\,GPa, \quad \nu = 0.3, \quad \rho_m = 2707\,kg/m^3$

Ceramic (Al_2O_3): $E_c = 327.27\,GPa, \quad \nu = 0.3, \quad \rho_c = 2370\,kg/m^3$

In Figure 5 the close agreement of present LW solution with HSDT [12] solution for all power low indexes and side to thickness ratios a/h. As observed from the previous example, the fundamental frequency parameter decreases with the increase of power low index and side to thickness ratio a/h.

The fundamental frequency is presented in nondimensional form: $\bar{\bar{\omega}} = \omega_{cr} h \sqrt{\frac{\rho_c}{E_c}}$

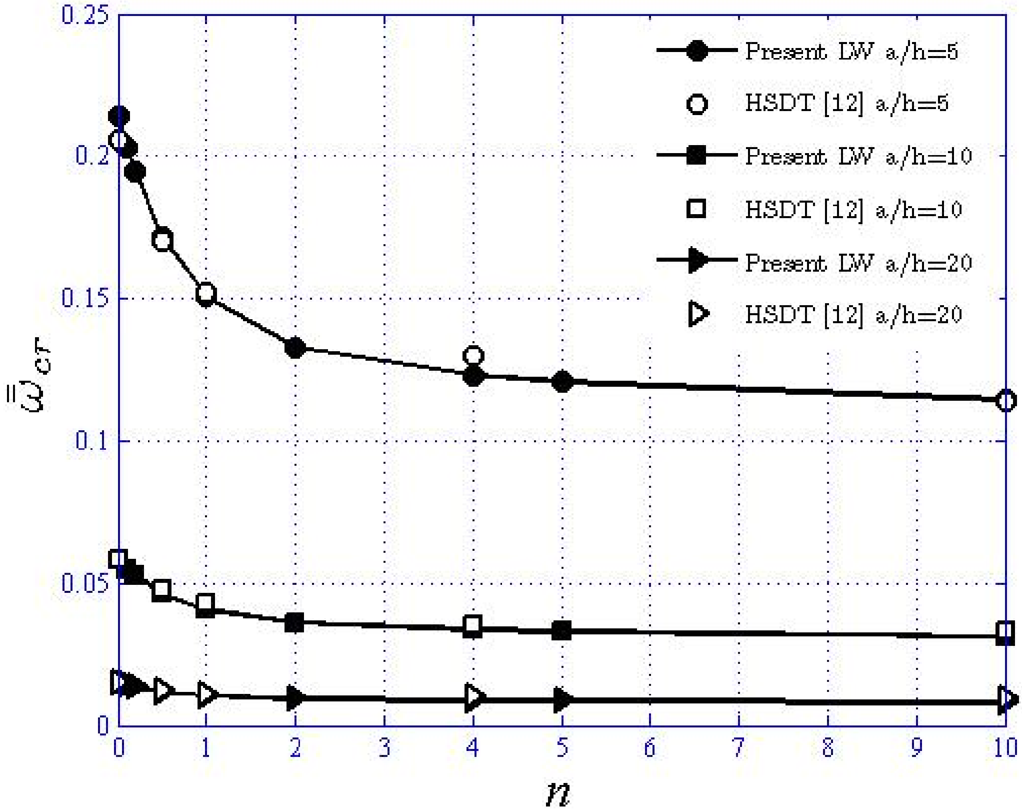

Fig. 5. Comparison the effects of power low index n and side to thickness ratio a/h on fundamental frequency $\bar{\bar{\omega}}$ for simply supported Al / Al_2O_3 plate

5. Conclusion

In this paper, the free vibration analysis of FGM plates is formulated using LW displacement model [14]. The principle of virtual displacement (PVD) is used to derive Euler–Lagrange differential equation of motion. The governing differential equations are solved following the Navier's technique and solving the eigen value problem. An original MATLAB program is coded for numerical solution and used to analyze the effects of side to thickness ratio, power low index and material properties on free vibration frequency of simply supported FGM plate. The results have shown that the proposed model gives acceptable results when compared to HSDT [12] solutions from the literature and may serve to verify other numerical solutions, such as finite element one.

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