Decomposition & synergy: a study of the interactions and dependence among the 5 Brazilian macro regions*

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RESUMO

A metodologia originalmente desenvolvida por Sonis, Hewings e Miyazawa (1997) neste artigo é expandida e discutida mais intensamente quando aplicada a um sistema inter-regional de insumo-produto no nível das 5 macrorregiões da economia brasileira para o ano de 1995. A metodologia utilizada neste trabalho é baseada num sistema particionado de insumo-produto e explora técnicas da matriz inversa de Leontief via natureza das interdependências internas e externas fornecidas pelas ligações, o que permite classificar os tipos de interações sinergéticas dentro de uma combinação de hierarquias de sub-sistemas econômicos interligados. Os resultados mostram que: a) a região Norte praticamente não possui relações com a região Nordeste e vice-versa; b) enquanto a região Sul produz algum impacto na produção da região Norte, o inverso não é verdade; c) apesar do fato das demandas da região Centro-Oeste possuírem algum impacto na produção das outras regiões, a produção da região Centro-Oeste possui as suas relações concentradas nas regiões Sudeste e Sul; e d) as regiões Sul e Sudeste se apresentam como as regiões mais importantes no sistema.

Palavras-chave: economia brasileira, estrutura produtiva, economia regional, insumo-produto.

ABSTRACT

The methodology originally developed by Sonis, Hewings, and Miyazawa (1997) is now expanded and discussed more thoroughly when applied to an interregional table at the level of the 5 macro regions of the Brazilian economy for the year of 1995. The methodology used in this work is based on a partitioned input-output system and exploits techniques of the Leontief inverse through the nature of the internal and external interdependencies giving by the linkages, which allows to classify the types of synergetic interactions within a preset pair-wise hierarchy of economic linkages sub-systems. The results show that: a) the North region has practically no relation with the Northeast region and vice-versa; b) while the South region has some impact on the production of the North region, the inverse is not true; c) despite the fact that the demands from the Central West region have some impact on the production of the other regions, the production in the Central West region has its relations concentrated with the Southeast and South regions; and d) the South and Southeast regions show to be the most important regions in the system.

Key words: Brazilian economy, productive structure, regional economics, input-output.

JEL classification: C67, R15, O18.

* We would like to thank the comments made by Daniel Czamanski, Maria Cristina Ortiz Furtuoso, and two anonymous referees.

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This author would like to thank FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo) for the financial support that made possible to attend and to present this paper at the 39th European Congress of the European Regional Science Association in Dublin, Ireland and at the International Input-Output Seminar in Guadalajara, Mexico.

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Recebido em setembro de 1999. Aceito em fevereiro de 2001.

I Introduction

The methodology originally developed by Sonis, Hewings, and Miyazawa (1997), which classifies the types of synergetic interactions and allows to examine the structure of the trading relations among the regions, and in a exploratory way applied by Guilhoto, Hewings, and Sonis (1999) to an interregional input-output table at the level of 2 regions for the year of 1992 for the Brazilian economy is now expanded and discussed more thoroughly when applied to an interregional table at the level of the 5 macro regions (North, Northeast, Central West, Southeast, and South) of the Brazilian economy. (Guilhoto, 1999)

This work is organized in the following way: a) the theoretical background will be presented in the next section; b) the third section will present the results for the Brazilian economy; and c) some final remarks will be made in the last section.

II Theoretical background

This methodological section will be divided into two parts: a) in the first one it is made reference to the theory originally developed for the two regions case; and b) in the second it is showed how this theory can be extended to the n regions case.

II.1 The two regions case

A complete description for the 2 regions case is presented in Sonis, Hewings, and Miyazawa (1997), which is the basis for this section.

Consider an input-output system represented by the following block matrix, A, of direct inputs:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{1}$$

where A_{11} and A_{22} are the quadrat matrices of direct inputs within the first and second regions, respectively, and A_{12} and A_{21} are the rectangular matrices showing the direct inputs purchased by the second region and vice versa.

The building blocks of the pair-wise hierarchies of sub-systems of intra/interregional linkages of the block-matrix Input-Output system are the four matrices $A_{11,} A_{12,} A_{21}$ and A_{22} corresponding to four basic block-matrices:

$$A_{11} = \begin{bmatrix} A_{11} & 0 \\ 0 & 0 \end{bmatrix}; \quad A_{12} = \begin{bmatrix} 0 & A_{12} \\ 0 & 0 \end{bmatrix}; \quad A_{21} = \begin{bmatrix} 0 & 0 \\ A_{21} & 0 \end{bmatrix}; \quad A_{22} = \begin{bmatrix} 0 & 0 \\ 0 & A_{22} \end{bmatrix} \tag{2}$$

This section will usually consider the decomposition of the block-matrix (1) into the sum of two block-matrices, such that each of them is the sum of the block-matrices (2) $A_{11,}A_{12,}A_{21}$ and A_{22}. From (1) 14 types of pair-wise hierarchies of economic sub-systems can be identified by the decompositions of the matrix of the block-matrix A (see Figure 1 and Table 1).

Consider the hierarchy of Input-Output sub-systems represented by the decomposition $A = A_1 + A_2$ Introducing the Leontief block-inverse $L(A) = L = (I - A)^{-1}$ and the Leontief block-inverse $L(A_1) = L_1 = (I - A_1)^{-1}$ corresponding to the first sub-system, the outer left and right block-matrix multipliers M_L and M_R are defined by equalities:

$$L = L_1 M_R = M_L L_1 \tag{3}$$

The definition (3) implies that:

$$M_L = L(I - A_1) = (I - L_1 A_2)^{-1} \tag{4}$$

$$M_R = L(I - A_1)L = (I - A_2 L_1)^{-1} \tag{5}$$

The calculation of the outer block-multiplier M_L and M_R is based on the particular form of the Leontief block-inverse $L(A) = L$. This work will presented the application of formulas (3), (4) and (5) to the derivation of a taxonomy of synergetic interactions between regions. The possibilities for the A_1 matrix are presented in Table 1. Also, Figure 1 shows the schematic representation of the possible forms of the A_l matrices.

Based on hierarchy of input-output sub-systems represented by the decomposition $A = A_1 + A_2$, their Leontief block-inverse $L(A) = L = (I - A)^{-1}$ and the Leontief block-inverse $L(A_1) = L_1 = (I - A_1)^{-1}$ corresponding to the first sub-system, the multiplicative decomposition of the Leontief inverse $L = L_1 M_R = M_L L_1$ can be converted to the sum:

$$L = L_1 + (M_L - I)L_1 = L_1 + L_1(M_R - I) \tag{6}$$

If f is the vector of final demand and x is the vector of gross output, then from the decomposition (6) is possible to divide the gross output into two parts: $x_1 = L_1 f$ and the increment $D_x = x - x_1$. Such decomposition is important for the empirical analysis of the structure of actual gross output and for the contribution that the relations among the regions have to the total gross output.

While 14 types of pair-wise hierarchies of economic linkages have been developed (Figure 1 and Table 1), it is possible to suggest a typology of categories into which these types may be placed. The following characterization is suggested:

1. backward linkage type (VI, IX): power of dispersion

2. forward linkage type (V, X): sensitivity of dispersion

3. intra- and inter- linkages type (VII, VIII): internal and external dispersion

4. isolated region versus the rest of the economy interactions style (I, XIV, IV, XI)

5. triangular sub-system versus the interregional interactions style (II, XIII, III, XII).

Figure 1
Schematic Representation of the Possible Forms of the A_1 Matrix - 2 Regions Case

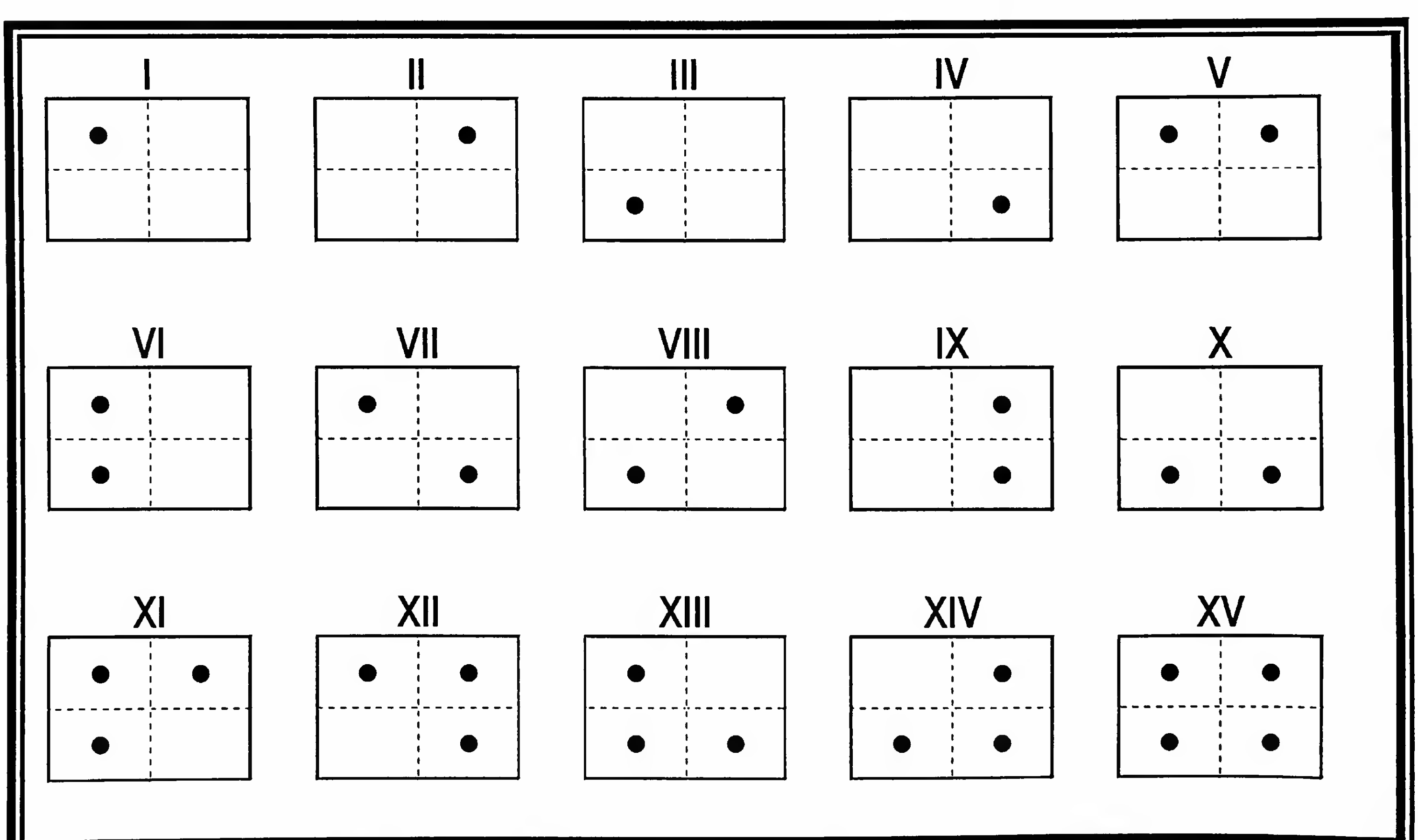


Table 1

Taxonomy of Synergetic Interactions Between Economic Sub-Systems

[Each entry presents a description of the structure and the corresponding form of the A_1 matrix]

I. *Hierarchy of isolated region versus the rest of economy*	$A_1 = \begin{bmatrix} A_{11} & 0 \\ 0 & 0 \end{bmatrix}$
II. *The order replaced hierarchy of interregional linkages of second region versus lower triangular sub system*	$A_1 = \begin{bmatrix} 0 & A_{12} \\ 0 & 0 \end{bmatrix}$
III. *The order replaced hierarchy of interregional linkages of first region versus upper triangular sub system*	$A_1 = \begin{bmatrix} 0 & 0 \\ A_{21} & 0 \end{bmatrix}$
IV. *The order replaced hierarchy of backward and forward linkages of the first region versus rest of economy*	$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & A_{22} \end{bmatrix}$
V. *Hierarchy of forward linkages of first and second regions*	$A_1 = \begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \end{bmatrix}$
VI. *Hierarchy of backward linkages of first and second regions*	$A_1 = \begin{bmatrix} A_{11} & 0 \\ A_{21} & 0 \end{bmatrix}$
VII. *The hierarchy of intra-versus inter-regional relationships*	$A_1 = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix}$
VIII. *The hierarchy of inter versus intra regional relationships*	$A_1 = \begin{bmatrix} 0 & A_{12} \\ A_{21} & 0 \end{bmatrix}$
IX. *Order replaced hierarchy of backward linkages*	$A_1 = \begin{bmatrix} 0 & A_{12} \\ 0 & A_{22} \end{bmatrix}$
X. *Order replaced hierarchy of forward linkages*	$A_1 = \begin{bmatrix} 0 & 0 \\ A_{21} & A_{22} \end{bmatrix}$
XI. *The hierarchy of backward and forward linkages of the first region versus rest of economy*	$A_1 = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & 0 \end{bmatrix}$
XII. *The hierarchy of upper triangular sub system versus interregional linkages of first region*	$A_1 = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$
XIII. *The hierarchy of lower triangular sub system versus interregional linkages of second region*	$A_1 = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}$
XIV. *Hierarchy of the rest of economy versus second isolated region*	$A_1 = \begin{bmatrix} 0 & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

By viewing the system of hierarchies of linkages in this fashion, it will be possible to provide new insights into the properties of the structures that are revealed. For example, the types allocated to category 5 reflect structures that are based on order and circulation. Furthermore, these partitioned input-output systems can distinguish among the various types of dispersion (such as 1, 2 and 3) and among the various patterns of interregional interactions (such as 4 and 5). Essentially, the 5 categories and 14 types of pair-wise hierarchies of economic linkages provide the opportunity to select according to the special qualities of each region's activities and for the type of problem at hand; in essence, the option exists for the basis of a typology of economy types based on hierarchical structure. The use of different synergetic interactions allows one to analyze and to measure how the transactions do occur among the regions, being possible to verify how much the relation of production on a given region do affect the production in another region.

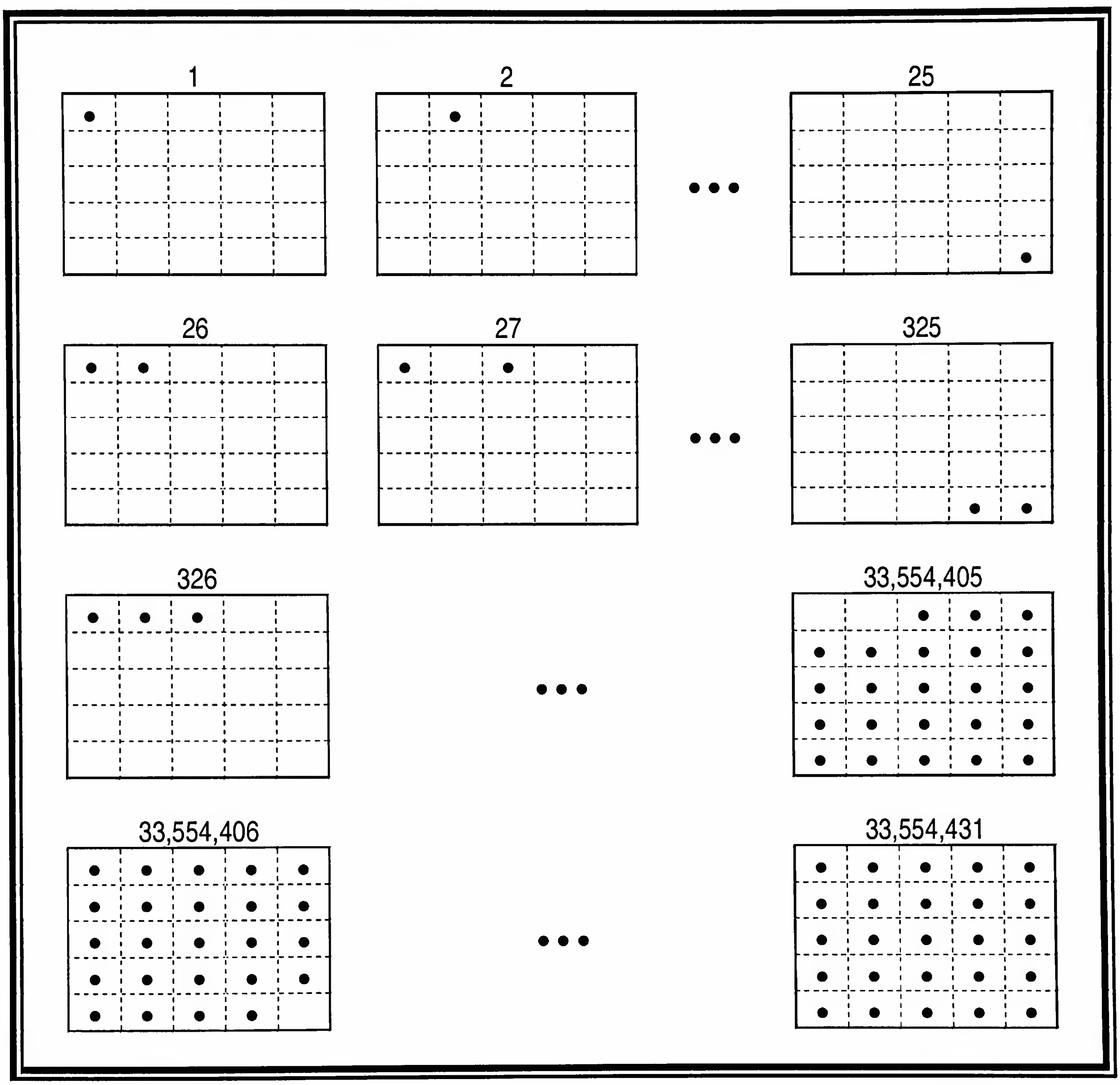
II.2 The n regions case

For the *n* regions case the number of decompositions increases dramatically as one increases the number of regions, such that from the 15 decompositions (including the whole system) for the 2 regions case, one goes to: a) 511 decompositions for the three regions case; b) 65,535 decompositions for the 4 regions; c) 33,554,431 decompositions for the 5 regions; and so on. In this way, the equation representation of the system for the *n* regions case becomes very complex, so what is presented here is a general idea of how the system works, as can be seen in a schematic way for the 5 regions case, as it is presented in Figure 2. From this figure one can see that in the 5 regions case one has 25 matrices. At first, one has to consider each matrix isolated, the next step is to consider the 25 matrices combined 2 at time, then 3 at time, and so forth, until one gets to the whole system. To measure the net contribution of each combination for the production in the productive process one has to subtract from the result of the combination of *k* matrices all the possible lower level combinations of these matrices, e.g., the result of a set of 5 matrices must be subtracted from the results of all the possible combination of these five matrices at the level of 4, 3, 2, and 1 matrices.

Some works have already being developed for Brazil using the methodology proposed by por Sonis, Hewings, and Miyazawa (1997). For the two regions case one has the work of Guilhoto, Hewings and Sonis (1999), while Moretto (2000) and Silveira (2000) explore the methodology for the 4 regions case. The two regions used in Guilhoto, Hewings and Sonis (1999) are the Northeast and the Rest of Brazil regions. Moretto (2000) works with a four regions interregional input-output output system construct for the state of Paraná. The work of Silveira (2000) uses an interregional system that includes the Brazilian states of Minas Gerais, Bahia, Pernambuco, and the Rest of Brazil economy.

The next section will present the results when the above methodology is applied to the interregional system of the 5 Brazilian macro regions.

Figure 2
Schematic Representation of the Possible Forms of the A1 Matrix - 5 Regions Case



III An application to the Brazilian economy

In this section it is made first a general presentation of the main aspects of the five Brazilian macro regions and then it is made an analysis of the results derived from the application of the theory presented in section II.

III.1 The Brazilian macro regions

According to the classification of the Brazilian Institute of Geography and Statistics (IBGE) the Brazilian Economy is divided into 5 macro regions, see Figure 3· a) North (7 States); b) Northeast (9 States); c) Central West (3 States and the Federal District); d) Southeast (4 States); and e) South (3 States).

Figure 3
Map of Brazil and Its 5 Macro Regions

The overall size of the Brazilian territory is 8,511,996 Km^2 of which 45.25% belongs to the North region, 18.25% to the Northeast, 18.85% to the Central West, 10.85% to the Southeast, and 6.76% to the South. However the economic and population distribution do not follow the geographical distribution, as can be seen in Table 2.

Table 2
Main Economical and Geographical Characteristics of the Brazilian Macro Regions.

Macro Regions	Size		Population (1996)		Urban Population	GDP 1995
	km2	Share (%)	Number (1,000)	Share	%	Share (%)
North	3,851,560	45.25	11,288	7.19	62.36	5.27
Northeast	1,556,001	18.28	44,767	28.50	65.21	13.62
Central West	1,604,852	18.85	10,501	6.69	84.42	7.25
Southeast	924,266	10.86	67,001	42.66	89.29	56.97
South	575,316	6.76	23,514	14.97	77.22	16.89
Brazil	8,511,996	100.00	157,070	100.00	78.36	100.00

Source: IBGE (1997a and 1997b), Considera and Medina (1998).

Having 45.25% of the Brazilian territory the North region has only 7.19% of the Brazilian population and the smallest number peoples living per km^2, it also has the smallest share of population living in the cities (62.36%) and the smallest share in the Brazilian GDP (5.27%). The most developed regions in Brazil are the Southeast and the South region. The Southeast region has a share of 56.97% of the Brazilian GDP with 42.66% of its population and 10.86% of the territory, while the South region has a share of 16.89% in the Brazilian GDP with 6.76% of the territory and 14.97% of the population. The Southeast region is the most industrialized region in Brazil, while the South region is the one more closed to the Mercosur countries which is the region that due to the continental size of Brazil could be the one to get the most benefits from the Mercosur integration. The Central West region has been an important region for Brazil in terms of agriculture, mainly because of the favorable type of land that this region has, and it has a reflex in its share in the population (6.69%) and GDP (7.25%) of Brazil. The Northeast region has serious problems of draught and in the beginning of the formation of the Brazilian State it used to be it most important region. This region has 18.28% of the Brazilian territory, 28.50% of its population and 13.62% of its GDP. Recently oil extraction and processing has been one of the most growing business in the region and with the openness of the Brazilian economy a lot of industries have been installing they production units in the region (in part due to the fiscal incentives giving by the various levels of the state).

III.2 The productive relation among the regions

Using a set of interregional input-output tables built by Guilhoto (1999) at the level of 22 sectors for the year of 1995 for the 5 Brazilian macro regions (North (N), Northeast (NE), Central West (CW), Southeast (SE), and South (S)), the methodology presented in section II is applied, and the results are presented in this section.[1]

1 Attention should be called here about the number of sectors used in the analysis, i.e., the relatively small number of sectors used may not completely reflect the Brazilian economy and as so one should expect that as the number of sectors increase better results might be achieved.

Due to computational problems, i.e., the computer resources available to the author were not enough to carry out the estimations directly at the 5 regions level, the estimations were carried in the following way· a) first, it was considered each region against all the others aggregated; and b) then, the results for the five regions where derived from the results obtained from five four regions cases where two regions were aggregated.

It was necessary to derive the five regions case from the four regions case due to computer time requirements. In the 4 regions case the computer resources required are considerable, the time to estimated all the 65,535 combinations on a 120 MHz Pentium computer (used by the authors) would be more than one week. Fortunately, in practical terms, the combinations of 1, 2, 3, 4, and 5 matrices generates more than 99.90% of production explanation for a given region, which allows to take the remaining explanation as a residual of all the other combinations (even in this case the computer takes more than 6 hours to generate the results for each interregional system of 4 regions).

To aggregate the 5 regions into 4 it was taken into consideration the geographic localization of the regions as well as their economic relations, resulting into 5 combinations: a) N+NE, CW, SE, S; b) N+CW, NE, SE, S; c) NE+CW, N, SE, S; d) N, NE, CW+SE, S; and e) N, NE, CW, SE+S.

Below it is made an analysis of the results for the 2 regions and 5 regions cases. The results for the 2 regions case allow on the one hand a first view of how each region interacts with the rest of the economy and on the other hand permits to see the importance of each interaction to generated the production in each region. The 5 regions case will give more emphasis on the analysis of the importance of the links among the regions to the production generated into each region.

III.2.1 The 2 regions case (one region against all the others)

Starting from the isolated regions (block matrices) and then adding the interactions among them it is possible to measure how each interaction adds to the total production. These results are presented in Table 3 and in Figure 4 for each of the 2 regions case, i.e., one region against the rest of Brazil.

The results show that decomposition **I**, that measures the contribution of the production inside the region to the total production in the productive process, is the most important element in all of the 5 Brazilian regions, however it presents the highest values in the most developed regions, Southeast (84.52%) and South (76.86%). For the Northeast region it represents

73.12%, it also shows that the Central West (68.44%) and the North (64.33%) are the regions more dependents on the other regions for their productive process.

The most important decompositions for the region 1 (isolated Brazilian region), in the 2 regions case, are decompositions **I, II, V, IX,** and **XII**, which are related with the matrices A_{11}, A_{12}, and A_{22} (Table 3 and Figure 4). This meaning that the inputs that each Brazilian region buys from the rest of the economy has practically no impact over its production. From the data one has that the inputs that the rest of the economy buys from a given region (A_{12}) represents from 12.15% (Southeast) to 27.32% (North) of the production in this region, while the production relations inside the rest of Brazil (A_{22}) represents from 2.72% (Southeast) to 8.12% (North) of the production in this region.

Table 3

Contribution (%) of Each Pair-Wise and Block Matrix to the Total Share of $(x_1\text{-}f)$ in x

	North and Rest of Brazil									
	North					Rest of Brazil				
Decomp.	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22
I	60.24	60.24								
II	16.34		16.34							
III						0.80			0.80	
IV						97.88				97.88
V	5.40	2.70	2.70							
VI						0.20	0.10		0.10	
VII										
VIII	0.25		0.12	0.12		0.05		0.03	0.03	
IX	13.44		6.72		6.72					
X						0.73			0.37	0.37
XI	0.11	0.04	0.04	0.04		0.02	0.01	0.01	0.01	
XII	4.00	1.33	1.33		1.33					
XIII						0.17	0.06		0.06	0.06
XIV	0.14		0.05	0.05	0.05	0.11		0.04	0.04	0.04
XV	0.08	0.02	0.02	0.02	0.02	0.04	0.01	0.01	0.01	0.01
Total	100.00	64.33	27.32	0.23	8.12	100.00	0.17	0.08	1.40	98.35

(continue)

Table 3
Contribution (%) of Each Pair-Wise and Block Matrix to the Total Share of $(x_1\text{-}f)$ in x (Continued)

Northeast and Rest of Brazil

Decomp.	Northeast					Rest of Brazil				
	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22
I	68.24	68.24								
II	8.82		8.82							
III						1.20			1.20	
IV						96.28				96.28
V	4.84	2.42	2.42							
VI						0.49	0.25		0.25	
VII										
VIII	0.22		0.11	0.11		0.08		0.04	0.04	
IX	10.23		5.12		5.12					
X						1.10			0.55	0.55
XI	0.34	0.11	0.11	0.11		0.04	0.01	0.01	0.01	
XII	6.85	2.28	2.28		2.28					
XIII						0.42	0.14		0.14	0.14
XIV	0.19		0.06	0.06	0.06	0.24		0.08	0.08	0.08
XV	0.28	0.07	0.07	0.07	0.07	0.15	0.04	0.04	0.04	0.04
Total	100.00	73.12	18.99	0.35	7.53	100.00	0.44	0.17	2.30	97.09

Central West and Rest of Brazil

Decomp.	Central West					Rest of Brazil				
	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22
I	63.53	63.53								
II	15.29		15.29							
III						0.85			0.85	
IV						97.10				97.10
V	6.82	3.41	3.41							
VI						0.40	0.20		0.20	
VII										
VIII	0.08		0.04	0.04		0.10		0.05	0.05	
IX	9.70		4.85		4.85					
X						0.83			0.41	0.41
XI	0.08	0.03	0.03	0.03		0.05	0.02	0.02	0.02	
XII	4.33	1.44	1.44		1.44					
XIII						0.37	0.12		0.12	0.12
XIV	0.08		0.03	0.03	0.03	0.21		0.07	0.07	0.07
XV	0.08	0.02	0.02	0.02	0.02	0.10	0.02	0.02	0.02	0.02
Total	100.00	68.44	25.11	0.11	6.34	100.00	0.36	0.16	1.74	97.73

(continue)

Table 3

Contribution (%) of Each Pair-Wise and Block Matrix to the Total Share of $(x_1\text{-}f)$ in x

(Continued)

	Southeast and Rest of Brazil									
	Southeast					Rest of Brazil				
Decomp.	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22
I	80.68	80.68								
II	6.41		6.41							
III						8.43			8.43	
IV						76.05				76.05
V	5.22	2.61	2.61							
VI						5.58	2.79		2.79	
VII										
VIII	0.34		0.17	0.17		0.47		0.23	0.23	
IX	3.30		1.65		1.65					
X						4.87			2.44	2.44
XI	0.70	0.23	0.23	0.23		0.37	0.12	0.12	0.12	
XII	2.64	0.88	0.88		0.88					
XIII						3.10	1.03		1.03	1.03
XIV	0.24		0.08	0.08	0.08	0.63		0.21	0.21	0.21
XV	0.47	0.12	0.12	0.12	0.12	0.50	0.13	0.13	0.13	0.13
Total	100.00	84.52	12.15	0.60	2.72	100.00	4.07	0.69	15.38	79.85

	South and Rest of Brazil									
	South					Rest of Brazil				
Decomp.	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22	Pair-Wise	Matrix A11	Matrix A12	Matrix A21	Matrix A22
I	72.04	72.04								
II	10.57		10.57							
III						2.96			2.96	
IV						90.52				90.52
V	6.96	3.48	3.48							
VI						1.69	0.85		0.85	
VII										
VIII	0.18		0.09	0.09		0.21		0.11	0.11	
IX	6.02		3.01		3.01					
X						2.36			1.18	1.18
XI	0.27	0.09	0.09	0.09		0.16	0.05	0.05	0.05	
XII	3.58	1.19	1.19		1.19					
XIII						1.43	0.48		0.48	0.48
XIV	0.15		0.05	0.05	0.05	0.39		0.13	0.13	0.13
XV	0.23	0.06	0.06	0.06	0.06	0.28	0.07	0.07	0.07	0.07
Total	100.00	76.86	18.54	0.29	4.31	100.00	1.44	0.36	5.82	92.38

Source: Estimated by the authors.

Figure 4
Schematic Representation of the Results for the 2 Regions Case

North

	N	RB
N	64.33	27.32
RB	0.23	8.12

Rest of Brazil

	N	RB
N	0.17	0.08
RB	1.40	98.35

Northeast

	NE	RB
NE	73.12	18.99
RB	0.35	7.53

Rest of Brazil

	NE	RB
NE	0.44	0.17
RB	2.30	97.09

Central West

	CW	RB
CW	68.44	25.11
RB	0.11	6.34

Rest of Brazil

	CW	RB
CW	0.36	0.16
RB	1.74	97.73

Southeast

	SE	RB
SE	84.52	12.15
RB	0.60	2.72

Rest of Brazil

	SE	RB
SE	4.07	0.69
RB	15.38	79.85

South

	S	RB
S	76.86	18.54
RB	0.29	4.31

Rest of Brazil

	S	RB
S	1.44	0.36
RB	5.82	92.38

Source: Table 3.

Giving the size of the Brazilian economy and the importance of the Southeast and South regions economy, for region 2 (the Rest of Brazil), in the 2 regions case, one has that the most important decompositions are the decompositions **III, IV, VI, X,** and **XIII**, which are related with the matrices A_{22}, A_{21}, and A_{11} (Table 3 and Figure 4). A closer look at the data also shows that with the exceptions of the cases where the Southeast and the South regions are taken isolated the relations inside the rest of Brazil economy (A_{22}) responds for around 97% of the production in the productive process.

In general, for the Brazilian case one has that the size of the regional economy really has an impact on the results, the North and the Central West regions being the more open economies, the South and the Southeast regions being the more closed ones and the Northeast region being in a middle condition among the other regions. In the next section when it will be taking into consideration the relation among the five regions it will be possible to see how each region has its production in the productive process related with the production on the other regions.

III.2.2 The 5 regions case

The results for the 5 regions case are presented in Figure 5 which are derived from combinations using the 4 regions case as described in III.2.

When comparing the results presented in this section with the results of the previous section one has that with minor differences (probably due to rounding problems) the sum of the partial results are the same as the aggregated result, which give us confidence in the results obtained in this section and at the same time validate the analysis in the previous section.

Taking a closer look at the relations among the 5 Brazilian macro regions it is clear the importance of the Southeast and the South region for the Brazilian economy. Also, it is possible to identify a set of at most 6 relations that responds for more than 97% of the production in the productive process in a given region.

Starting with the North region, one can see that the internal relations in the productive process were responsible for 64.27% of the total production in the productive process of this region. Furthermore, 17.60% o this production is due to the sales of inputs used in the production process of the Southeast region. The South region has influence on the production of the North region, given that the relation between then generates 7.01% of the North region production. It is observed a low relation of the North region with the Northeast and the Central West regions. The production relations inside the Southeast and the South regions have an impact of respectively, 4.97% and 1.64%, on the North region production.

For the Northeast region it is verified that 73.03% of its production in the productive process are due to the sales for production inside the region. It is possible to observe a strong relation with the Southeast region, giving that 12.76% of the production in the Northeast region is due to sales to the Southeast region. The sales to the South and Central West regions generate respectively, 4.03% and 0.98% of the Northeast production. Concerning the internal relation of production, one observe that the productive process inside the Southeast and South regions is responsible for respectively, 4.91% and 1.41%, of the Northeast region production.

The results for the Central West region show a productive structure in which the internal relations in the productive process are responsible for 68.41% of the total production, which shows that this region is the second most opened regional economy of Brazil. This region also shows a dependence with the Southeast and the South regions, giving that the sales to the Southeast region were responsible for 20.42% of its production, while the value for the South region is 3.46%. Also, the internal relations of production in the Southeast region were responsible for 4.65% of the production in the Central West region.

The Southeast region shows the productive structure less dependable on the other regions, given that the internal production relations are responsible for 84.49% of the total production in the productive process. The sales to the other regions are responsible for 12.11% of its production, with the South region having the biggest share, 6.02%. Considering all the regions, the only internal production relation that affects the Southeast region is the one of the South region, 1.49%.

Figure 5
Contribution (%) of Each Block Matrix to the Total Share of $(x_1\text{-}f)$ in x to the Regions North, Northeast, Central West, Southeast, and South.

North

	N	NE	CW	SE	S	
N	64.27	0.49	1.68	17.60	7.01	91.05
NE	0.01	0.18	0.00	0.04	0.01	0.24
CW	0.00	0.01	0.34	0.12	0.02	0.49
SE	0.19	0.21	0.15	4.97	0.47	5.99
S	0.03	0.06	0.03	0.44	1.64	2.20
	64.50	0.95	2.20	23.17	9.15	99.97

Northeast

	N	NE	CW	SE	S	
N	0.13	0.00	0.00	0.05	0.01	0.19
NE	0.81	73.03	0.98	12.76	4.03	91.61
CW	0.00	0.01	0.29	0.08	0.02	0.40
SE	0.12	0.28	0.19	4.91	0.48	5.98
S	0.02	0.06	0.03	0.24	1.41	1.76
	1.08	73.38	1.49	18.04	5.95	99.94

Central West

	N	NE	CW	SE	S	
N	0.06	0.00	0.00	0.01	0.01	0.08
NE	0.00	0.17	0.00	0.04	0.01	0.22
CW	0.32	0.83	68.41	20.42	3.46	93.44
SE	0.06	0.18	0.09	4.65	0.28	5.26
S	0.02	0.02	0.01	0.13	0.79	0.97
	0.46	1.20	68.51	25.25	4.55	99.97

Southeast

	N	NE	CW	SE	S	
N	0.20	0.01	0.00	0.06	0.01	0.28
NE	0.01	0.51	0.01	0.14	0.04	0.71
CW	0.01	0.00	0.40	0.13	0.02	0.56
SE	1.67	2.53	1.89	84.49	6.02	96.60
S	0.02	0.05	0.04	0.24	1.49	1.84
	1.91	3.10	2.34	85.06	7.58	99.99

South

	N	NE	CW	SE	S	
N	0.12	0.00	0.00	0.03	0.01	0.16
NE	0.01	0.32	0.00	0.07	0.02	0.42
CW	0.00	0.01	0.25	0.11	0.01	0.38
SE	0.05	0.10	0.07	3.39	0.22	3.83
S	0.86	1.95	1.16	14.41	76.82	95.20
	1.04	2.38	1.48	18.01	77.08	99.99

Shares of Main Relations

	N	NE	CW	SE	S
N. of Matrices	6	6	4	6	5
% Prod.	97.17	97.12	96.94	98.09	97.73

Source: Estimated by the authors.

The South region is the second less dependable region of the Brazilian regions presented here, giving that 76.82% of its total production in the productive process are due to internal production relations. This region shows a strong link with the Southeast region, as 14.41% of its production is giving to sales to the Southeast region. The sales to the Northeast and Central West regions are responsible, respectively, for 1.95% and 1.16% of its production. The production relations inside the Southeast region are responsible for 3.39% of the production in the South region.

In the next section some final remarks will be made.

IV Conclusions

In this paper the methodology originally developed by Sonis, Hewings, and Miyazawa (1997) to a 2 regions case is extended to a *n* regions case and given a new dimension, such that it is possible to measure the contribution of each block matrix, that represents the relations among the regions, to the production in the productive process of a given region.

This methodology was applied to a set of interregional tables constructed by Guilhoto (1999) for 1995 for the 5 Brazilian macro regions. The results were derived for the 2 regions case, one region against the rest of the economy, as well as for the 5 regions case.

An overview of the relations among the regions, in the productive process, shows that: a) the North region has practically no relation with the Northeast region and vice-versa; b) while the South region has some impact on the production of the North region, the inverse is not true; c) despite the fact that the demands from the Central West region have some impact on the production of the other regions, the production in the Central West region has its relations concentrated with the Southeast and South regions; d) the Southeast and the South regions show a productive structure more closed and less integrated to the Brazilian economy as a whole, while the North and the Central West economies are the more open and dependent economies of the system, the Northeast region, in terms of openness and dependence, is in the middle way; e) the South and Southeast regions show to be the most important regions in the system.

Despite the progress achieved in this paper, there are still some points left out that need further investigation, i.e.. a) applying the above methodology to a large set of data shows to be very demanding in terms of computer time, so there is a need for the construction of better algorithms of solution; b) how would the results change with an increase in the number of sectors; c) when measuring the contribution of the synergy among a set of matrices, that represent the relations among the regions, it was given an equal importance to each matrix, if this is not the case what it is the right way to weight the contribution of each matrix to the final result of the synergy?; and d) what would be the right way to apply this methodology to measure how the relations among the regions have evolved through time and how this change has contributed to the growth of the regions.

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