Bars overlapping in tensegrity structures belonging to the Octahedron family

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ABSTRACT

When performing the form-finding process of full-forms of tensegrity structures it is observed that beyond the double-expanded octahedron, struts overlapping occurs in space, making practicably impossible to materialize the tensegrity to employ it as a real structure. As the spatial coordinates of nodes that determine the geometry of the structure are provided as a linear combination of vectors composing a base of null space of the tensegrity force density matrix, an analytical study is carried out to check the existence of any combination of those vectors that avoids the overlapping of bars.

Keywords: tensegrity, octahedron family, form-finding, force density method.

1. INTRODUCTION

Tensegrity structures are, in general, spatial pin-jointed structures composed by compression (struts or bars) and tension (cables) elements which are pre-stressed so that the whole structure results free-standing and self-equilibrated [1]. The application of this kind of structures is becoming more and more widespread [2][3].

One of the key steps in the design of tensegrity structures is the form finding process. Although there are different approaches, as reviewed in [4], one of the most employed is the Force Density Method (FDM) proposed by Schek [5], based on the concept of force:length or force density ratio q to linearize the equations of equilibrium of the structure. The main equations in the FDM for a tensegrity structure are given by Eq. (1):

$$\begin{aligned} \mathbf{D}\cdot\mathbf{x} &= \mathbf{0} \\ \mathbf{D}\cdot\mathbf{y} &= \mathbf{0} \\ \mathbf{D}\cdot\mathbf{z} &= \mathbf{0} \end{aligned} \tag{1}$$

where $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ are n-dimension vectors that contains the x, y and z coordinates of the n nodes composing the tensegrity, and $\mathbf{D} \in \mathbb{R}^{n\times n}$ is the so-called force density matrix which is constructed according to Eq. (2):

$$D_{ij} = \begin{cases} \sum\limits_{k\in\Gamma} q_k & \text{for } i = j \\ -q_k & \text{if nodes } i \text{ and } j \text{ connected by member } k \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

being Γ the set of members (cables and struts) connected to node i. The total number of members in the structure is denominated by m. Given that the equilibrium equations to be solved (Eq. (1)) are a system of homogeneous equations, the solution implies $\mathbf{D}$ not to be full rank. Therefore, a set of proper

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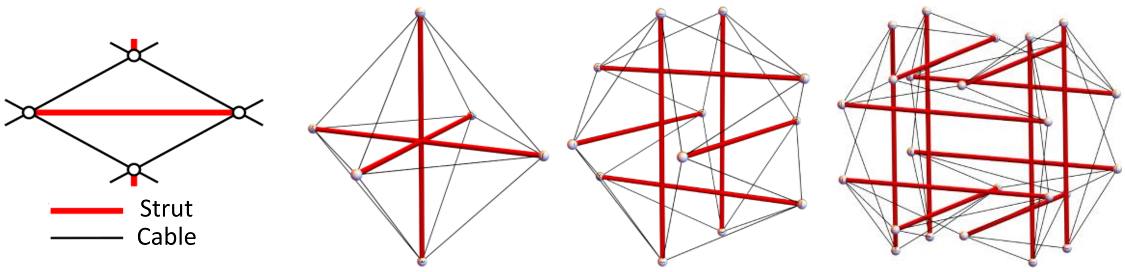
force densities q need to be sought in order to accomplish this fact. As proved by Zhang and Ohsaki [6], if a d-dimension tensegrity is to be obtained, the rank deficiency of $\mathbf{D}$ needs to be greater or equal to $d+1$.

2. THE OCTAHEDRON FAMILY

One of the main inputs to the FDM are the nodal connectivity and the force density assignation for the members composing the connectivity of the structure. In this work, the connectivity or topology of the Octahedron family [1] is employed to build up the structure. The Octahedron family is based on 4-nodes rhombical cells which are composed by four tensioned cables and a compressed strut. The construction of the structure is made so that each node only receives a compressed strut. All the members in the family are built by duplicating nodes, cables and struts from the first member, the octahedron, that consists of three rhombical cells, that is, six nodes linked by three compressed bars and twelve tensioned elements. Figure 1 shows the basic rhombical cells and the three first members of the family. It is observed that the bars in the structures in the Octahedron family are grouped in 3 three groups of equal length and parallel bars.



(a) (b) (c) (d)

Figure 1. *The Octahedron family: (a) basic rhombical cell, (b) octahedron ($p=1$), (b) expanded octahedron ($p=2$), (c) double-expanded octahedron ($p=3$).*

An important aspect in the form-finding process of the different members of the Octahedron family is fullness. This is the property of the structure according to which there is no node with the same spatial coordinates as another in the structure. If this is accomplished, the structure is full. If not, then the structure is said to be folded. Fernández-Ruiz et al. [7] proposed that assigning all the same value of force density ratio, q_c, to the cables in the structure, and doing the same for the struts, q_s, then the following force density assignation leads to a 3D full form of the p-th member in the Octahedron family:

$$\frac{q_s}{q_c} = -\frac{p+1}{p} \tag{2}$$

The observed problem in the structures belonging to the Octahedron family appears when $p \geq 4$, that is, the triple-expanded octahedron. In those structures there are bars that overlap each other, as shown in Figure 2.

3. NODAL COORDINATES AS LINEAR COMBINATION OF BASE OF NULL SPACE OF $\mathbf{D}$ MATRIX

The force:length assignation performed by the relation given by Eq. (2) assures the matrix $\mathbf{D}$ rank deficiency of 4 to get a 3D tensegrity structure. By observing Eq. (1), the solutions $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ are vectors which belong to the null space of matrix $\mathbf{D}$. Therefore, any linear combination of the vectors composing a basis of the null space of $\mathbf{D}$ will provide a possible spatial realization of the tensegrity structure. If $\mathbf{e}^i \in \mathbb{R}^n$, with $i = \mathrm{I}, \ldots, \mathrm{IV}$, compose a basis of $\ker(\mathbf{D})$, then the nodal coordinates of the structure are given by:

$$\mathbf{j} = \sum_{i=\mathrm{I}}^{\mathrm{IV}} \alpha_j^i \cdot \mathbf{e}^i \tag{2}$$

with $j = x, y, z$, and α_j^i are twelve arbitrary real values chosen so that the vectors $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are linearly independent. Different values of α_j^i will provide different geometries of the tensegrity structure.

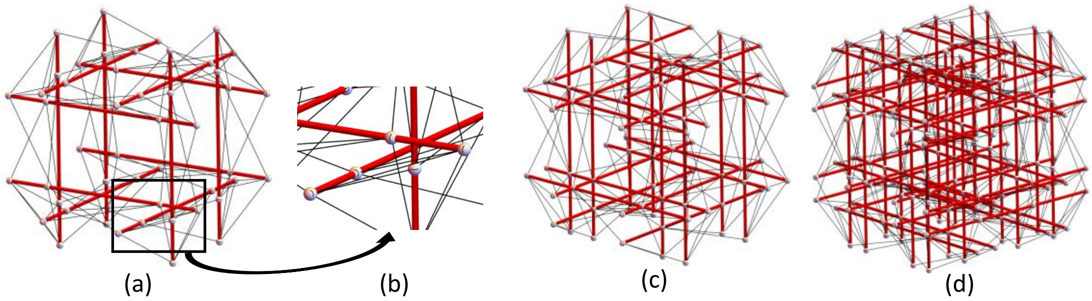


Figure 2. *Bars overlapping in the octahedron family when $p \geq 4$: (a) 3-expanded octahedron ($p = 4$), (b) detail of overlapped bars, (c) 4-expanded octahedron ($p = 5$), (d) 5-expanded octahedron ($p = 6$)..*

4. BARS OVERLAPPING AND LINEAR COMBINATION OF BASE OF NULL SPACE OF $\mathbf{D}$ MATRIX

By observation of the resulting structures when $p \geq 4$ and $\mathbf{X} = (\mathbf{x}, \mathbf{y}, \mathbf{z}) = (\mathbf{e}^i, \mathbf{e}^k, \mathbf{e}^l)$**,** with $i = \mathrm{I}, \ldots, \mathrm{IV}$, $k = \mathrm{I}, \ldots, \mathrm{IV}$, $l = \mathrm{I}, \ldots, \mathrm{IV}$, and $i \neq k \neq l$ it can be concluded that some bars in the structure are collinear and they overlap at some extent (Figure 2). Let us pay attention at to two of those overlapped bars, joining, say, nodes A and B and C and D. The parametric expression of the line defined by nodes A and B is:

$$\begin{aligned} x &= x_A + \gamma(x_B - x_A) = e_A^i + \gamma(e_B^i - e_A^i) = e_A^i + \gamma \Delta e_{AB}^i \\ y &= y_A + \gamma(y_B - y_A) = e_A^k + \gamma(e_B^k - e_A^k) = e_A^k + \gamma \Delta e_{AB}^k \\ z &= z_A + \gamma(z_B - z_A) = e_A^l + \gamma(e_B^l - e_A^l) = e_A^l + \gamma \Delta e_{AB}^l \end{aligned} \tag{3}$$

with $\gamma \in \mathbb{R}$. As the extremal node C of the other overlapped bar is on that line, then:

$$\left.\begin{aligned} x_C &= e_C^i = e_A^i + \gamma_C \Delta e_{AB}^i \\ y_C &= e_C^k = e_A^k + \gamma_C \Delta e_{AB}^k \\ z_C &= e_C^l = e_A^l + \gamma_C \Delta e_{AB}^l \end{aligned}\right. \Leftrightarrow \gamma_C = \frac{\Delta e_{AC}^i}{\Delta e_{AB}^i} = \frac{\Delta e_{AC}^k}{\Delta e_{AB}^k} = \frac{\Delta e_{AC}^l}{\Delta e_{AB}^l} \tag{4}$$

The question now is: is there any combination of α_j^i so that $(\mathbf{x}, \mathbf{y}, \mathbf{z}) \neq (\mathbf{e}^i, \mathbf{e}^k, \mathbf{e}^l)$ and node C of the overlapped bar changes its relative position to A and B and the overlapping disappears? If the nodal coordinates are given as a linear combination of a base of $\ker(\mathbf{D})$, Eq. (2), now the line AB remains:

$$j = \sum_{i=\mathrm{I}}^{\mathrm{IV}} \alpha_j^i \cdot e_A^i + \gamma \sum_{i=\mathrm{I}}^{\mathrm{IV}} \alpha_j^i \cdot \Delta e_{AB}^i \tag{5}$$

with $j = x, y, z$. Let us assume that the coordinates of the extremal node C of the other bar can be written as:

$$j_C = \sum_{i=\mathrm{I}}^{\mathrm{IV}} \alpha_j^i \cdot e_C^i = \sum_{i=\mathrm{I}}^{\mathrm{IV}} \alpha_j^i \cdot e_A^i + \gamma_j \sum_{i=\mathrm{I}}^{\mathrm{IV}} \alpha_j^i \cdot \Delta e_{AB}^i \tag{6}$$

with $j = x, y, z$. Therefore, if the extremal node C of the other bar is not on line AB, then $\gamma_x \neq \gamma_y \neq \gamma_z$. Rearranging Eq. (6) and considering that $\gamma_C = \frac{\Delta e_{AC}^i}{\Delta e_{AB}^i}$ (Eq. (4)):

$$\sum_{i=\mathrm{I}}^{\mathrm{IV}} \alpha_j^i \cdot \Delta e_{AC}^i = \gamma_j \sum_{i=\mathrm{I}}^{\mathrm{IV}} \alpha_j^i \cdot \Delta e_{AB}^i \Leftrightarrow \gamma_C \sum_{i=\mathrm{I}}^{\mathrm{IV}} \alpha_j^i \cdot \Delta e_{AB}^i = \gamma_j \sum_{i=\mathrm{I}}^{\mathrm{IV}} \alpha_j^i \cdot \Delta e_{AB}^i \tag{6}$$

Therefore $\gamma_C = \gamma_x = \gamma_y = \gamma_z$ and it can be concluded that for any linear combination of vectors composing a basis of $\ker(\mathbf{D})$, node C is always on line AB and retains its relative position with respect to the AB nodes. The same fact can be stated for the other extremal node D. So, the answer to the previously stated question is clear: no, bars AB and CD will always overlap no matter the chosen linear combination of basis of $\ker(\mathbf{D})$.

5. CONCLUSIONS

The form-finding process of tensegrity structures by means of FDM implies nodal coordinates of the structure to be calculated as a linear combination of vectors composing a basis of null space of the force density matrix $\mathbf{D}$. It has been shown that, although the Octahedron family is a very interesting source of tensegrity structures, a problem arises when dealing with high order members of the family: some bars overlap each other. It has been proven that no matter the chosen real coefficients to calculate the referred linear combination, the overlapping cannot be avoided, hindering the real materialization of the structure beyond the double-expanded octahedron.

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