GREEN'S FUNCTIONS IN LAYERED MEDIA: IMAGINARY AXIS INTEGRATION AND ASYMPTOTIC BEHAVIOR

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ABSTRACT

This paper presents an efficient technique for evaluating Green's functions associated to layered media, when cast in the space domain as Sommerfeld integrals. The theoretical developments needed to set up the numerical algorithm throw a new light on the asymptotic behavior of these Green's functions for large transverse source-observer distances

1. INTRODUCTION

Printed antennas and associated circuits have been extensively investigated in the last decades. Among the currently used models, methods based on integral equations techniques, both in the spectral and in the space domain, have become specially attractive since they provide excellent accuracy and good computational speed. Details of space domain integral equations techniques applied to the analysis of infinite multilayered printed structures can be found for instance in [1]. Of paramount relevance in this formulation is the concept of Green's function, defined as the fields or potentials created by a point unit source embedded in a layered medium (fig.1).

The developments in this paper are based on previous work by one of the authors [2,3,4]. In this work, the relevant spatial domain Green's functions were formulated as Sommerfeld integrals of the corresponding spectral domain counterparts and the accurate numerical evaluation of these Sommerfeld integrals was shown to be a critical point.

Traditionally, integration through the real axis combined with pole extraction techniques and averaging methods has been employed, leading to very efficient algorithms [3]. Fig.2 shows the modulus of the function to be integrated on the real axis for a typical Green's function evaluated at a transverse source-observer distance $k_0\rho=5$.

It is clear that abrupt variations and oscillating behavior make the numerical evaluation very time consuming. Moreover, the situation worsens for higher values of the transverse distance. However, many current practical problems involve distances of several tens or even hundreds of wavelengths. This is the case when computing mutual coupling inside large arrays or when modeling cavity backed antennas by using space images respect to the cavity's lateral walls.

In this paper a new, accurate and efficient technique is developed for the computation of Sommerfeld integrals for large values of $k_0\rho$. The technique is based on a new choice of the integration contour which is closed through the imaginary axis of the spectral plane. This idea was first proposed by one of the present authors in [2], and its potentialities were already outlined. However, the imaginary axis integration algorithm developed in [2] is only valid for lossless

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layers and the mathematical procedure must be modified and generalized when the layers are lossy.

2. THEORY

A general form for a Green's function (Sommerfeld integral) associated to the problem of figure 1 is:

$$G_n(\rho) = \int_0^{\infty} J_n(k_\rho \rho) g(k_\rho) k_\rho^{n+1} \, dk_\rho \qquad (1)$$

where J_n is a Bessel function and $g(k_\rho)$ the spectral Green's function, which can be obtained analytically for a layered medium [3].

Multilayered spectral domain Green's functions are functions of the complex spectral variable k_ρ. For a lossless configuration, they exhibit poles laying on the real axis Re[k_ρ], in the interval $k_0 < k_\rho < k_0\sqrt{\max(\mu_r \varepsilon_r)}$. When a real lossy geometry is considered, the poles, originally in the real axis, migrate to the lower-half complex plane as shown in fig.3. In addition to the complex poles, multilayered spectral domain Green's functions also exhibit branch cuts in the complex k_ρ plane due to the appearance of the multivalued function $u_0 = \sqrt{k_\rho^2 - k_0^2}$

Care must be exerted when selecting the proper branch of this function, and the final choice must be both mathematically convenient and physically sound. Here, we propose to introduce a branch cut as (fig. 3) : $-1<\mathrm{Re}[k_\rho/k_0]<+1$.

Then by integrating around the contours of fig.3, we can generalize the procedure described in [2] and obtain the result:

$$G_n(\rho) = \int_0^{k_0} H_n^{(2)}(x\rho)[g_1(x) - g_2(x)]x^{n+1} \, dx$$
$$+\frac{j}{\pi}\int_0^{\infty} K_n(y\rho)[g(jy) - g(-jy)]y^{n+1} \, dy + \frac{\pi}{j}\sum_i R_i H_n^{(2)}(k_{\rho i}\rho) = T_1 + T_2 + T_3 \qquad (2)$$

with $k_\rho = x + jy$. The first term T_1 is an integral over the bounded interval $[0,k_0]$ of a function without singularities in the integration interval. The term T_2 corresponds to an improper integral, extended to the unbounded interval $[0,\infty]$ of a function involving the fast decaying modified Bessel function K_n. We can clearly see that the absolute value of the integrand decreases exponentially and the decreasing rate is proportional to ρ. Indeed, the main impact of the new technique from a numerical point of view resides in this term T_2. Finally, the T_3 term corresponds clearly to a sum of surface wave contributions directly linked with the poles of the spectral Green's function.

It is clear that numerical evaluation of integrals in T_1, T_2 will be competitive for large values of source-observer distances.

The point that remains to be discussed, before ascertaining the potential validity of this technique as an useful numerical tool, is the relative difficulty to exactly locate the poles of the spectral Green's function. Fortunately, this is not a problem for real life substrates with small losses, since a numerical search is easily performed in the lossless case, where the number and bound values of the poles are known. Then, for a lossy situation, a tracking procedure will converge in a few iterations to the locations of the poles in the complex plane.

3. ASYMPTOTIC BEHAVIOR

In addition to provide an interesting way for numerically computing the Green's functions, the decomposition of any Sommerfeld integral into the three terms of equation (3) gives us some useful theoretical insights concerning the behavior of Green's functions in the near and far field regions.

As it has been pointed out in reference [2], the term T_1, given by the integral of the modified Bessel function, represents the quasistatic term. Therefore, this term is dominant in the near field and the behavior of Green's functions for vanishing source-observer distances can be inferred from the study of this term alone. Moreover, for a lossless case T_1 becomes a pure real quantity decreasing exponentially as the source-observer distance decreases.

The term T_2 corresponds essentially to the branch point contribution and can be viewed as the space wave term. Alone (lossless case) or combined with T_1 (lossy case) it provides an asymptotic behavior of free space type. Thus a scalar potential will behave as $1/\rho^2$ for high values of ρ.

Finally, the sum referred as T_3 corresponds to the set of surface waves generated by the multilayered medium. Their propagation constants are directly given by the values of the poles in the spectral plane and they can interfere given rise to sharp oscillations in the value of the Green's functions as a function of the distance. It is currently said that surface waves are the dominant contribution in the far field region. This is only true for strictly lossless structures. A non zero loss tangent, will push the poles away from the real axis and will introduce a small exponentially decreasing behavior in the Hankel functions appearing in T_3.

As a result, the surface wave behavior dominates only till a given distance. Then the terms devoid of exponential attenuation take over and the overall Green's function behaves as a Zenneck wave, for instance decreasing like in the case of a scalar potential.

All these phenomena are clearly seen in fig. 4 where the modulus of a typical multilayered Green's function for the scalar potential is depicted as a function of the radial distance, together with the partial contributions T_1+T_2 and T_3.

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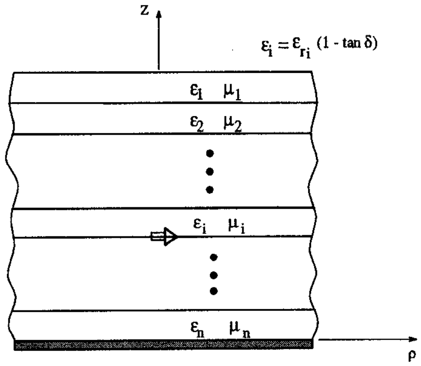

Figure 1: General multilayered structure analyzed in this paper.

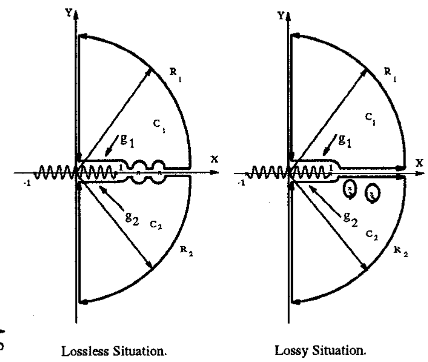

Figure 3: Complex spectral plane showing the alternative integration contours proposed in this paper, leading to the new formulation.

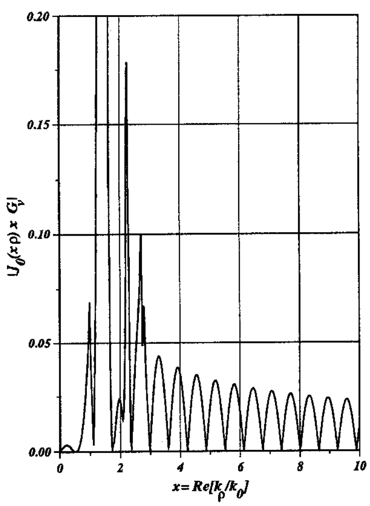

Figure 2: Typical function to be integrated if the real axis is chosen to evaluate Sommerfeld integral.

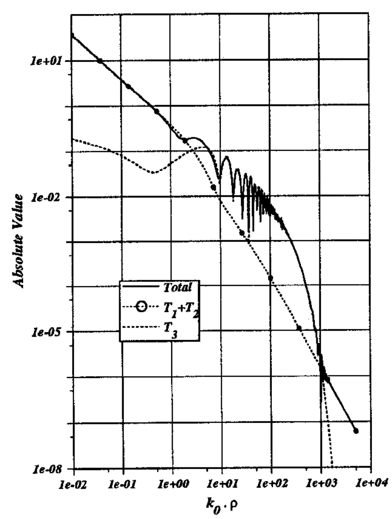

Figure 4: Example of a spatial domain Green's function obtained using the new approach, showing surface and Zenneck wave interactions.