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Analysis and Simulation of MRC Diversity Reception in Correlated Composite Nakagami-Lognormal Fading Channels

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Abstract– The physical meaning of the composite Nakagami-lognormal fading model is not well understood by many researchers using the model. The signal power transfer and transform at the interface between the global lognormal shadowing sub-channels and the local Nakagami multipath sub-channels in the presence of correlation between these diversity sub-channels is rather complex. This is the main reason why a thorough analysis or a simulation model is absent to date for the case of *correlated* composite Nakagami-lognormal diversity channels. This paper presents a novel technique for the estimation of the probability density function (PDF) of the signal-to-noise (SNR) at the output of a maximum ratio combining (MRC) receiver operating in correlated composite diversity fading channels. The PDF is estimated using the recently proposed two-point *lossless* moment generating function (MGF) matching technique and a closed-form expression for the bit-error rate (BER) for QPSK signal is consequently presented using the Gauss-Hermite polynomial approximation. The paper also presents the complex Monte-Carlo simulation model for the MRC reception and BER counting in correlated composite Nakagami-lognormal fading channels.

Keywords– Nakagami-lognormal, maximum ratio combining (MRC), two point matching, moment generating function (MGF).

1 INTRODUCTION

It has long been established that diversity reception is a very effective technique for combating the detrimental effects of channel fading in wireless propagation, and for coherent reception, maximum ratio combining (MRC) is known to be the optimal linear combining technique. The principle of MRC is to sum up the powers, hence the signal-to-noise ratios (SNRs), of the diversity receiving signals. In a general radio propagation scenario over a large urban area, the signal in each diversity path encounters a series/cascade of scattering objects. This leads to what is commonly known as shadow fading referring to the fluctuation in the local mean of the received power as the mobile receiver moves through the shadow of an obstruction object.

The multiplicative nature of the cascading obstructions is widely accepted as having a lognormal distribution and the sum of lognormal powers underlying the analysis of MRC diversity reception has therefore become a challenging topic for research, particularly when the shadowing effects in the diversity channels are correlated. An exact closed-form expression for the probability density function (PDF) of the sum of lognormal RVs is not available to date but a number of analytical approximations do exist.

Most approximation techniques assume that the sum of lognormal RVs can be modelled as another single lognormal RV whose parameters are found by matching of the moments as in the Fenton-Wilkinson (F-W)

method [1] or by matching of the log-moments as in the Schwartz-Yeh (S-Y) technique [2] or by matching of the moment generating functions (MGF) at two points as in [3]. A relatively more recent paper in [4] proposes a two-step algorithm, via the Pearson-Type IV distribution, for approximating the lognormal sum and the linear combination with arbitrary weights of the lognormal RVs and shows that the proposed technique is superior to many other techniques currently available. The authors in [3] point out that the popular F-W approximation method in [1] puts more weight on larger SNRs resulting in a poor approximation in the tail portion of the PDF and a better tracking of the head portion.

On the other hand, the S-Y approximation method in [2] suffers from severe error in the head portion but achieves better tracking in the tail portion of the PDF. In [3] by adjusting the position of the two MGF matching points, the PDF tracking error can be controlled and distributed according to applications. In [5] a novel I two-point MGF matching technique is introduced to accurately locate the optimal MGF matching points.

However, in more realistic scenarios in mobile radio in urban areas, the multipath effect at the mobile receiver due to scattering from local scatterers such as buildings in the neighbourhood of the receiver causes a fast fading while the variation in the terrain configuration between the base-station and the mobile receiver causes a slow shadowing. Therefore, the mobile radio signal envelope is usually composed of a *small scale*

multipath fading component superimposed on a much *larger scale* or *slower shadowing* component [6]. It is well known that the signal envelope of the multipath component can be modelled as a Nakagami-*m* distributed random variable (RV) [7] or equivalently, its power can be modelled as a gamma distributed RV. Thus the path power gain, hence the SNR, has a mixed gamma-lognormal distribution which was first proposed in [8] as a generalization of the Rayleigh-lognormal, also known as the Suzuki fading distribution model in [9].

The sum-of-products model in Figure 1 is useful in studying this statistical mix in most physical mobile radio channels. The problem of approximating the power sum of correlated composite gamma-lognormal random variables is again more challenging than that of the power sum of correlated lognormal RVs. While [10] only presented the sum of the independent Nakagami-Lognormal fading channel, the aim of this paper is to present closed-form expressions for the approximated bit-error rate (BER) of the output from an MRC diversity receiver under the correlated composite Nakagami-lognormal fading channels. This is the main contribution of this paper.

The rest of the paper is organized as follows. Section 2 covers Composite Nakagami-lognormal fading channel model. The MRC diversity reception in correlated composite Nakagami-lognormal fading channels algorithm is described in Section 3. Monte Carlo simulation of diversity MRC repetition in correlated composite Nakagami-lognormal fading channels is described in Section 4. Finally, Section 5 gives the conclusions.

2 COMPOSITE NAKAGAMI-LOGNORMAL FADING CHANNEL MODEL

It is well known that, if *h* is the amplitude channel gain and *s(t)* and *n(t)* are transmit signal and channel noise then the receive signal can be defined

$$r(t) = hs(t) + n(t). \qquad (1)$$

The amplitude channel gain assumed to be constant over the transmission time of a symbol block. *n(t)* is modeled as a zero-mean additive white Gaussian noise (AWGN) process with one-sided power spectral density N_0. The received SNR is then

$$\gamma = \frac{|h|^2 E_s}{N_0}, \qquad (2)$$

where the signal energy is $E_s = E[s^2(t)]$.

In this paper we use the term power gain, $p = h^2$, and signal-to-noise ratio, γ, interchangeably where it is appropriate. If the energy is that of one bit, then we denote γ_b as the SNR per bit of transmitted information, then we express

$$\gamma_b(p) = pE_b/N_0. \qquad (3)$$

To avoid dealing with distance dependency, we normalize the average channel power gain $E|h^2| = 1$, thus

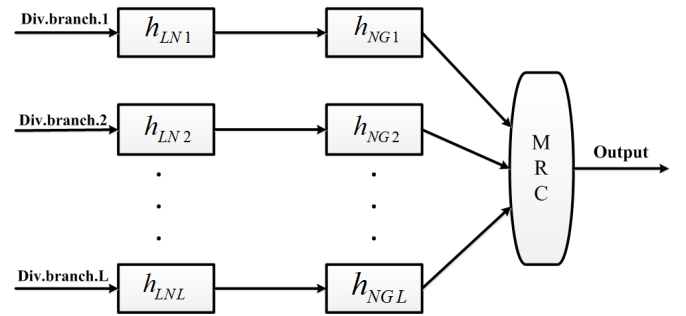
Figure 1. Modeling of diversity branches in a composite Nakagami-lognormal fading environment.

making the average received SNR per bit per channel $\overline{\gamma}_b = E_b/N_0$

At high SNR, the symbol-error-rate for rectangular M-QAM in AWGN with $M = 2^k$ when k is even, is approximated as [9].

$$\text{SER}_{\text{AWGN,MQAM}} \approx 4\left(1 - \sqrt{\frac{1}{M}}\right)Q\left(\sqrt{\frac{3}{M-1}\gamma_s}\right), \qquad (4)$$

in which γ_s is the average SNR per symbol per channel (without combining), and for equiprobable orthogonal signals the corresponding bit-error-rate is [11].

$$\text{BER}_{\text{AWGN,MQAM}} = \frac{M}{2(M-1)}\text{SER}_{\text{AWGN,MQAM}}. \qquad (5)$$

For QPSK signaling, $M = 4$ and without using Gray code, we have

$$\text{BER}_{\text{AWGN,QPSK}}(\gamma_b) = \frac{4}{3}Q(\sqrt{2\gamma_b}). \qquad (6)$$

From the model in Figure 1 for mobile radio propagation in urban areas, the path fading from the transmitter to the cluster of buildings in the neighbourhood of the receiver is predominantly large-scale or slow fading with lognormal distribution after the main wave encounters *multiplicative* reflections and or refractions by obstructions. In most situations the mobile receiver is surrounded by a local cluster of scatterers such as buildings in urban areas, the arriving main wave is scattered into multiple local paths and combined at the receiver causing multipath fading. These local paths arrive at the receiver roughly with the same strength but with different phases, giving rise to small-scale or fast fading with Nakagami-*m* distributed envelope, i.e. fading power gain is gamma-distributed.

The probability density function (PDF) of the SNR in the composite Nakagami-lognormal fading channel may be derived by equating the local average power of the much faster Nakagami-*m* fading signal, to the instantaneous power of the much slower arriving lognormal signal. This implies first, a complete transfer of signal power from the arriving global lognormal signal to the local multipath sub-channel, and second, no loss of power in the local multipath channel, i.e. the average power gain of the local Nakagami-*m* channel can be assumed to be unity. In the diversity composite fading model in Figure 1, in each branch the received channel power gain is the product of the main shadowing lognormal power gain $P_{\text{LN}} = |h_{\text{LN}}|^2$ and

its local multipath Nakagami-m power gain (gamma distributed) $p_{NG} = |h_{NG}|^2$. Since in this paper we are mainly interested in the gamma-distributed power of the local multipath Nakagami-m fading channels, we will denote this as p_{GM} instead of p_{NG} in what follows, i.e.

$$p_{\text{GM-LN}}(i) = p_{GM}(i)p_{LN}(i), \quad i = 1, 2, ..., L \quad (7)$$

and the average power gain of the local Nakagami-m channel is

$$\overline{p}_{GM} = 1, \quad \text{for all} \quad i. \quad (8)$$

The PDF of the power gain of the *Nakagami*(m, Ω) channel, i.e. a gamma(m, θ) random variable (RV), is

$$f_{GM}(GM) = \frac{1}{\Gamma(m)} \left(\frac{1}{\theta}\right)^m p_{GM}^{m-1} \exp\left(-\frac{p_{GM}}{\theta}\right), \quad m \geqslant 0.5 \quad (9)$$

where

$$\theta = \frac{\Omega}{m}, \quad (10a)$$

$$\Omega = E[p_{GM}], \quad (10b)$$

$$m = \frac{(E[p_{GM}])^2}{Var[p_{GM}]}, \quad (10c)$$

and the PDF of the power gain of the lognormal channel is given by

$$f_{LN}(P_{LN}) = \frac{1}{p_{LN}} \frac{\zeta}{\sigma_z \sqrt{2\pi}} \exp\left(-\frac{(10\log_{10} x - \mu_z)^2}{2\sigma_z^2}\right), \quad (11)$$

where the lognormal power fading is modeled as $p_{LN} = 10^{0.1Z} = \exp(z/\zeta)$ with $Z \sim \aleph(\mu_z, \sigma_z^2)$ in dB unit and $\zeta = 10/\log 10$ is the conversion constant between *dB* and *net*, and p_{LN} is in linear unit.

The PDF for the product power gain $p_{\text{GM-LN}}$ in (7) of each composite Nakagami-lognormal diversity channel, using the Jacobian transformation technique, can be found from (9) and (11) to be

$$f_{\text{NG-LN}}(p) = \int_0^\infty \frac{1}{\Gamma(m)} \left(\frac{m}{x}\right)^m p^{m-1} \exp\left(-\frac{mp}{x}\right)$$

$$\frac{1}{x} \frac{\zeta}{\sigma_z \sqrt{2\pi}} \exp\left(-\frac{(10\log_{10} x - \mu_z)^2}{2\sigma_z^2}\right) dx, \quad m \geqslant 0.5. \quad (12)$$

The composite PDF in (12) was first introduced in [7]. If $m = 1$, it reduces to the composite Rayleigh-lognormal, also known as Suzuki distribution [8].

3 MRC Diversity Reception in Correlated Composite Nakagami-lognormal Fading Channels

3.1 MGF of the Sum of Correlated Composite Gamma-lognormal Powers

In this section, the main contribution of our paper is presented. The mathematical derivation requires the following definitions and notations:

$$p = \sum_{i=1}^L p_{\text{GM-LN}}(i) = \sum_{i=1}^L p_i$$

is the resultant power gain at the output of the MRC combiner. $f_{\text{GM-LN}}(p_1, p_2, ..., p_L)$ is the joint distribution of the correlated composite gamma-lognormal input powers, and $f_{LN}(x_1, x_2, ..., x_L)$ is the joint distribution of the lognormal fading component with its associated generating Gaussian vector $\mathbf{z} = (z_1, z_2, ..., z_L)$ in which the elements z_i are identically distributed with mean μ_z and variance σ_z^2 and are correlated with covariance matrix C_z, i.e.

$$f_Z(\mathbf{z}) = \frac{1}{(2\pi)^{L/2}|C_z|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{z} - \mu_z)^T C_z^{-1}(\mathbf{z} - \mu_z)\right). \quad (13)$$

Then the MGF of p is

$$M_{p,\text{MRC}}(s) =$$
$$\int_0^\infty ... \int_0^\infty \prod_{i=1}^L \exp(-sp_i) f_{\text{GM-LN}}(p_1, ..., p_L) dp_1...dp_L$$

where s is the transform variable in the Laplace domain.

We can immediately recognize that the integral with respect to p_1 inside the square brackets gives the MGF of the gamma(m, θ_i)-distributed RV [12] with $\theta_i = x_i/m$, or by directly calculating the integral using [13], we have

$$M_{p,\text{MRC}}(s) = \int_{P_L}^\infty .... \int_{p_2}^\infty$$
$$\left\{ \int_0^\infty \left(\frac{1}{1 + sx_1/m}\right)^m f_{LN}(x_1, p_2, ..., p_L) dx_1 \right\} dp_2...dp_L.$$

We proceed in the same way for $p_2, p_3, ..., p_L$

$$M_{p,\text{MRC}}(s) =$$
$$\int_{x_L}^\infty ... \int_{x_1}^\infty \prod_{i=1}^L M_{GM}\left(s, \frac{x_i}{m}\right) f_{LN}(x_1, ..., x_L) dx_1...dx_L \quad (14)$$

where

$$M_{GM}\left(s, \frac{x_i}{m}\right) = \left(\frac{1}{1 + s\frac{x_i}{m}}\right)^m \quad (15)$$

is the MGF of the gamma(m, θ_i)-distributed RV [12] with $\theta_i = x_i/m$ and x_i is the local power of the corresponding lognormal diversity branch.

The expression in (14) is rather expected because, in the microdiversity model in Figure 1, the correlation is initially between the global lognormal arriving waves and the MRC detection acts on the local Nakagami diversity multipath sub-channels. The correlation between the latter is through their gamma-distributed powers being conditioned on the corresponding arriving local average lognormal power.

In the vector form for $\mathbf{x} = (x_1, x_2, ..., x_L)$, (14) can be rewritten as

$$M_{p,\text{MRC}}(s) = \int_0^\infty \prod_{i=1}^L M_{GM}(s, x_i/m) f_{LN}(\mathbf{x}) d\mathbf{x}.$$

By equating $f_{LN}(\mathbf{x})d\mathbf{x} = f_z(\mathbf{z})d\mathbf{z}$ and using (13) we have equation (16) (see next page).

To de-correlate (16), we make a change of variable $\mathbf{z} = \sqrt{2}\mathbf{C}_z^{1/2}\mathbf{u} + \mu_z$ and (10) becomes (17).

where c_{ij} is the (i, j) element of $\mathbf{C}_z^{1/2}$ which is obtained from $\mathbf{C}_z = \mathbf{C}_z^{1/2}(\mathbf{C}_z^{1/2})^T$ using Cholesky decomposition. Since $\mathbf{C}_z$ is symmetric and positive definite, $\mathbf{C}_z^{1/2}$ is lower triangular.

$$M_{p,\text{MRC}}(s) = \int\limits_{-\infty}^{\infty} \frac{1}{(2\pi)^{L/2}|\mathbf{C_z}|^{1/2}} \prod_{i=1}^{L} \left(\frac{1}{1 + \frac{s}{m}\exp(z_i/\zeta)} \right)^m \exp\left(-\frac{1}{2}(\mathbf{z} - \mu_\mathbf{z})^T \mathbf{C_z}^{-1}(\mathbf{z} - \mu_\mathbf{z}) \right). \tag{16}$$

$$M_{p,\text{MRC}}(s) = \int\limits_{-\infty}^{\infty} \frac{1}{\pi^{L/2}} \prod_{i=1}^{L} \left[1 + \frac{s}{m}\exp\left(\frac{\sqrt{2}}{\zeta}\sum_{j=1}^{L}c_{ij}u_j + \frac{\mu_j}{\zeta} \right) \right]^{-m} \exp\left(-\mathbf{u}^T\mathbf{u} \right) d\mathbf{u}. \tag{17}$$

$$M_{p,\text{MRC}}(s, \mu_\mathbf{z}, \mathbf{C_z}) \approx \sum_{n_l=1}^{N_p} \cdots \sum_{n_1=1}^{N_p} \frac{w_{n_1}...w_{n_L}}{\pi^{L/2}} \prod_{i=1}^{L} \left[1 + \frac{1}{m}s\exp\left(\frac{\sqrt{2}}{\zeta}\sum_{j=1}^{L}c_{ij}a_{n_j} + \frac{\mu_j}{\zeta} \right) \right]^{-m}. \tag{18}$$

The integral in (17) can be accurately approximated using Gauss-Hermite expansion [14]. The MGF of the sum of L correlated composite Rayleigh-lognormal power gains finally is given in (18). in which w_n, a_n and N_p are, respectively, the weights, the abscissas and the order of the Gauss-Hermite polynomial. As in [5] for the case of MRC diversity reception when only lognormal shadowing exists, in this paper we have found that (18) gives a very accurate result when $N_p = 12$.

3.2 Calculation of $\mathbf{C_z}$ from the Lognormal Channel Covariance Matrix $\mathbf{C_{LN}}$

The same as in [5], we use the popular decreasing correlation model proposed in [15] for shadow fading channels. The covariance matrix of the channel power gains, assuming micro-diversity channels being identically distributed, is

$$\mathbf{C}_{LN}(i,j) = \text{Cov}(p_{LN_i}, p_{LN_j}) = \text{Var}(p_{LN})\rho^{|i-j|}, \tag{19}$$

in which ρ is the correlation coefficient of two adjacent channel power gains.

The following statistical relationships are well known between the lognormal random variable p_{LN} and its associated Gaussian random variable Z:

$$E(p_{LN}) = \overline{p}_{LN} = e^{\mu_z/\zeta + \sigma_z^2/2\zeta^2},$$

$$\frac{\text{Var}(p_{LN})}{\overline{p}_{LN}^2} = \left(e^{\sigma_z^2/\zeta^2} - 1 \right).$$

Hence

$$\sigma_z^2 = \zeta^2 \ln\left(1 + \frac{\text{Var}(p_{LN})}{\overline{p}_{LN}^2} \right), \tag{20}$$

$$\mu_z = \zeta \ln(\overline{p}_{LN}) - 0.5\zeta \ln\left(1 + \frac{\text{Var}(p_{LN})}{\overline{p}_{LN}^2} \right), \tag{21}$$

and

$$\mathbf{C}_z(i,j) = Cov(Z_i, Z_j) = \zeta^2 \ln\left(1 + \frac{\text{Var}(p_{Ln})\rho^{|i-j|}}{\overline{p}_{LN}^2} \right). \tag{22}$$

In this paper, we normalize the lognormal channel's mean power gain $\overline{p}_{LN} = 1$ to avoid dependency on propagation distance and adopt a fixed Gaussian standard deviation $\sigma_z = 8$ dB. Therefore the MGF of

the MRC output in (18) can be completely known in terms of the correlation coefficient ρ. Note that this normalization automatically gives $\overline{p}_{\text{GM-LN}} = 1$ since from (16), $\overline{p}_{\text{GM}} = \Omega = 1$ which is inherent in the composite fading model in (17) and (18).

3.3 Estimation of Sum of Composite Gamma-lognormal RVs as a Single Lognormal RV

In this section, we approximate the sum of L correlated composite gamma-lognormal RVs by a *single* lognormal RV, $\hat{p}_{\text{Ln}} = 10^{0.1\hat{Z}}$ where $\hat{Z} \sim N(\hat{\mu}_z, \hat{\sigma}_z^2)$. Similar approximation has been investigated for the case of independent Suzuki RVs, i.e. Nakagami with $m = 1$, as a single lognormal RV in [2] using a two-point MGF matching technique, and as a single Suzuki RV in [16] by using the mean and variance matching technique. In a recent paper [5] we introduced a 'lossless'. two-point MGF matching technique which guarantees conservation of average signal power across the MRC combiner and which can locate the two matching points accurately, and therefore is more accurate than that proposed in [2]. By matching the MGF of the lognormal approximation $\hat{p}_{\text{LN}}$ with the MGF of the sum of the composite gamma-lognormal input powers p in (18) at two different positive real values s_1 and s_2 , a system of two simultaneous equations as in (17) is obtained which can then be used to solve for $\hat{\mu}_z$ and $\hat{\sigma}_z^2$.

$$\sum_{n=1}^{N_p} w_n \exp\left[-s_i \exp\{(a_n\hat{\sigma}_z\sqrt{2} + \hat{\mu}_z)/\zeta\} \right]$$
$$= \sqrt{\pi}M_{p,\text{MRC}}(s_i, \mu_z, \mathbf{C_z}), \quad i = 1, 2. \tag{23}$$

The principle of a lossless MRC thus gives the approximated average power gain at the output of the receiver as

$$\hat{\overline{p}}_{LN} = \exp\left[\frac{\hat{\mu}_z}{\zeta} + \frac{1}{2}\left(\frac{\hat{\sigma}_z}{\zeta} \right)^2 \right]. \tag{24}$$

Equation (25) provides a valid and reliable equation for iteratively improving the accuracy of the locations of the two MGF matching points s_1 and s_2. The percentage error of power loss is defined as follows [5].

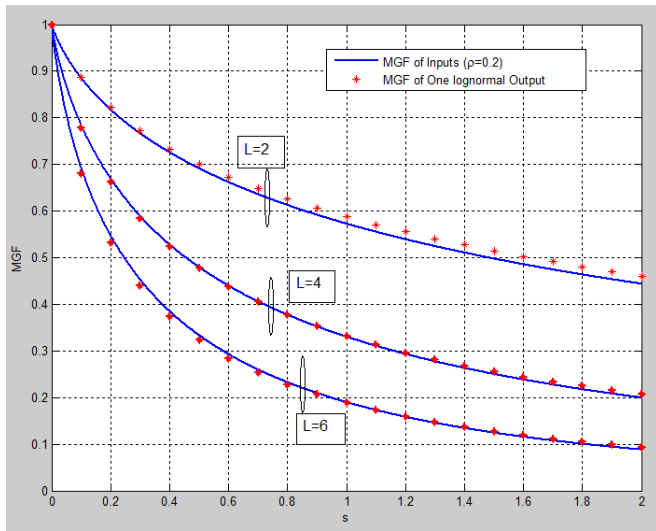$$\text{error}(\%) = 100\frac{L - \hat{\overline{p}}_{LN}}{L}. \tag{25}$$

Figure 2. MGF matching of the power sum of L correlated gamma-lognormal RVs to a single lognormal RV.

Figure 3. PDF of the single lognormal RV approximating the MRC output for $\rho = 0.2$ and $N = 2$.

A simple iterative search algorithm for the two matching locations, hence the values of $\hat{\mu}_z$ and $\hat{\sigma}_z$, in (22) is carried out until the power loss decreases to a specified error threshold which is set at 0.1% in this paper.

The PDF of the channel power gain at the output of the MRC receiver, approximated as a single lognormal RV, finally is

$$\hat{f}_{\text{LN,MRC}}(p) = \frac{1}{p}\frac{\zeta}{\hat{\sigma}\sqrt{2\pi}}\exp\left(-\frac{(10\log_{10}p - \hat{\mu}_z)^2}{2\hat{\sigma}_z^2}\right). \quad (26)$$

Table 1 presents the MGF matching results for matching points (s_1, s_2) and $(\hat{\mu}_z, \hat{\sigma}^2)$ for different shadowing channel correlations ρ and different multipath fading parameters m when the MRC output is approximated as a single lognormal RV. The error is limited to below 0.1%. As discussed in the introduction, the performance parameter in this paper is the agreement in the theoretical *BER* and the Monte-Carlo simulated BER. Therefore we want to give an equal weight, i.e. $\exp(-sp)$, as much as possible to the entire range of SNR in the MGF calculation, i.e. keeping the MGF matching points (s_1, s_2) as small as possible but still satisfying the specified matching error ceiling.

Figure 2 shows the MGF matching of the corresponding solutions in Table 1 for $m = 1$. It can be observed that when there are only two microdiversitybranches to add together, the sum of two exponential-distributed multipath components, i.e. a chi-square of only four degrees of freedom, is not sufficient to be approximated as a lognormal RV. As a result, the MGF matching of the sum of two gamma-lognormal RVs to a single lognormal RV cannot be expected to be as good as the matching when there are many moregamma-lognormal RVs, i.e. a chi square of many more degrees of freedom, to sum up.

The consequence of assuming a single lognormal RV for the output is to skew the distribution of SNR to the lower end as demonstrated in Figure 3, giving rise to higher BER as shown in Figure 4 for $L = 2$. We notice that this poor matching at low values of
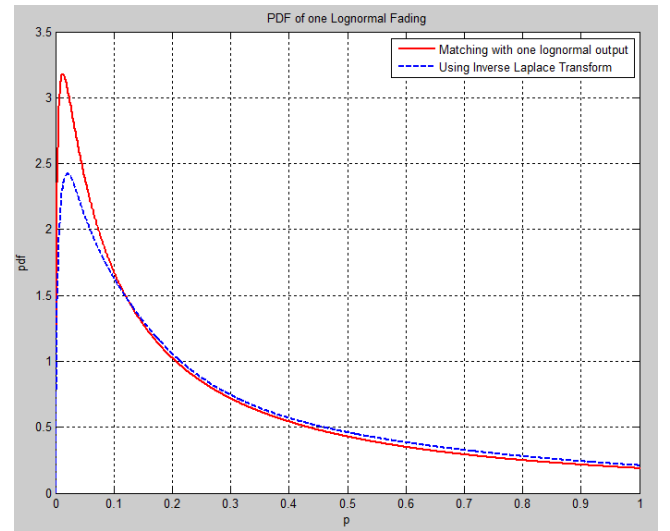
L is quite obvious but without authors'comments in Figure 7 of [3] for the sum of uncorrelated Suzuki RVs.

3.4 BER of QPSK in Diversity Reception in Correlated Composite Nakagami-lognormal Fading Channels

In this paper we use QPSK signaling for our analysis and simulation of mobile radio transmission using MRC diversity reception in a correlated composite Nakagami-lognormal fading environment.

Using PDF of the estimated single lognormal RV: The corresponding *BER* using QPSK signaling, $M = 4$ and without using Gray code:

$$\text{BER}_{\text{AWGN,QPSK}}(\gamma_b) = \frac{4}{3}Q(\sqrt{2\gamma_b}),$$

where γ_b is the SNR per bit and (26), is given by (27). In (27), $\gamma_b(p) = \frac{E_b}{N_0}p$ and term power gain $p = |h|^2$.

By setting

$$u = \frac{10\log_{10}p - \hat{\mu}_z}{\hat{\sigma}_z\sqrt{2}} \iff p = \exp\left(\frac{\hat{\mu}_z}{\zeta} + \frac{\sqrt{2}\hat{\sigma}_z}{\zeta}u\right) \quad (28)$$

Equation (27) can be reduced to

$$\text{BER}_{\text{NG-LN,QPSK,MRC}} =$$
$$\frac{1}{\sqrt{\pi}}\int\limits_0^\infty \text{BER}_{\text{AWGN,QPSK}}\left(\gamma_b(u)\right)e^{-u^2}du,$$

where $\gamma_b(u) = \frac{E_b}{N_0}\exp(\hat{\mu}_z/\zeta + u\hat{\sigma}_z\sqrt{2}/\zeta)$ is the argument of $\text{BER}_{\text{AWGN,QPSK}}(\gamma_b) = \frac{4}{3}Q(\sqrt{2\gamma_b})$.

The above expression for BER can then be accurately approximated by an N_p-order Gauss-Hermite polynomial expansion as given below

$$\text{BER}_{\text{NG-LN,QPSK,MRC}} =$$
$$\frac{1}{\sqrt{\pi}}\sum_{n=1}^{N_p}w_n\text{BER}_{\text{AWGN,QPSK}}\left(\gamma_b(a_n)\right). \quad (29)$$

Table I
RESULTS OF THE 2-POINT MGF MATCHING OF THE MRC OUTPUT TO A SINGLE LOGNORMAL R

Matching parameters		Shadowing correlation $\rho = 0.2$		Shadowing correlation $\rho = 0.6$	
		m=1.0	m=2.0	m=1.0	m=2.0
(s_1, s_2)	N=2	(0.0065, 0.0111)	(0.001,1.500)	(0.0178,0.0196)	(0.0005,1.5000)
	N=4	(0.001, 0.800)	(0.001,1.700)	(0.025, 0.800)	(0.0140,2.3000)
	N=6	(0.001, 1.100)	(0.0015,1.4500)	(0.042, 1.100)	(0.0350, 1.5000)
$(\hat{\mu}_z, \hat{\sigma}_z)$ dB	N=2	(-4.3096,0.9917)	(-3.1285,0.2993)	(-4.5674, 8.1148)	(-3.9369,0.7700)
	N=4	(1.341, 6.374)	(2.0638, 5.8660)	(0.390, 6.994)	(0.9599, 6.6376)
	N=6	(4.310, 5.491)	(4.8126, 5.0742)	(3.395, 6.174)	(3.8155, 5.8852)

$$BER_{\text{NG-LN,QPSK,MRC}} = \int_0^\infty BER_{\text{AWGN,QPSK}}\left(\gamma_b(p)\right) \frac{1}{p}\frac{\zeta}{\hat{\sigma}_z\sqrt{2\pi}} \exp\left(-\frac{(10\log_{10}p - \hat{\mu}_z)^2}{2\hat{\sigma}_z}^2\right) \tag{27}$$

4 Monte Carlo Simulation of Diversity MRC Reception in Correlated Composite Nakagami-lognormal Fading Channels

4.1 Simulation of correlated L-variates lognormal vector h_{LN}

In this section, we simulate the correlated L-variates lognormal vector h_{LN} by first generating the L-variates normal vector $z = N(\mu_{\mathbf{Z}}, \mathbf{C_Z})$ from Section 4.2.

- *Step 1:* Generate i.i.d **unit** L-variates $U_1, U_2, ..., U_L$.
- *Step 2:* Generate correlated normal variates $Z_1, Z_2, ..., Z_L$. For each $i = 1, 2, ..., L$

$$Z_i = \mu_{Zi} + \sum_{j=1}^{i} c_{ij}U_j, \tag{30}$$

where c_{ij} is the (i,j) element of the lower triangular $\mathbf{C}^{1/2}$ as in (17) obtained from matrix $\mathbf{C}_Z = \mathbf{C}_Z^{1/2}\left(\mathbf{C}_Z^{1/2}\right)^T$ using Cholesky decomposition.
- *Step 3:* Calculate the correlated lognormal variates $X_i = e^{Zi}$, for i=1,2,...,L. In which X is the channel power gain, i.e.$|h_{\text{LN}}|^2$.
- *Step 4:* Then calculate the fading transfer function of each lognormal diversity channels.

$$h_{\text{LN}} = e^{Zi/2}. \tag{31}$$

4.2 Simulation of Correlated L-variates Nakagami-m Vector r

To date there is no known technique to directly generate correlated Nakagami-m vector $\mathbf{r} \sim \mathbf{NG}(m, \mathbf{C_{NG}})$ except first to generate the corresponding correlated gamma vector $\mathbf{y} = \mathbf{p_{GM}} \sim \mathbf{GM}(m, \mathbf{C_{GM}})$ then using the fact that a *scalar* Nakagami RV is equal to the square root of the corresponding gamma RV [17]. In [17], it is shown that a gamma-distributed vector has a simple direct-sum decomposition of squares of *independent* Gaussian vectors $\mathbf{z_k}$, i.e

$$\mathbf{y} = \sum_{k=1}^{2m} \mathbf{z}_k^{o2}, \tag{32}$$

where the notation 'o' is to mean element–to–element operation, i.e. the element of the L-variate vector $\mathbf{y}$ is

$$y(i) = \sum_{k=1}^{2m} z_k^2(i), \quad i = 1, 2, ..., L. \tag{33}$$

The element of the corresponding L-variate Nagakami-m vector therefore can be generated as

$$r(i) = \sqrt{\sum_{k=1}^{2m} z_k^2(i)}. \tag{34}$$

The relationship between the covariance matrix $\mathbf{C_{GM}}$ of the gamma vector and the covariance matrix $\mathbf{C_z}$ of the corresponding generating Gaussian vector is [17]

$$\mathbf{C_z} = \frac{1}{2\sqrt{m}}\mathbf{C_{GM}^{o1/2}}. \tag{35}$$

From (4) with $E[p_{GM}] = 1$, we have $\text{Var}[p_{GM}] = 1/m$, and thus

$$\mathbf{C_{GM}} = \frac{1}{m}\begin{pmatrix} 1 & \rho & \rho^2 & ... & \rho^{L-1} \\ \rho & 1 & \rho^2 & ... & \rho^{L-2} \\ \rho & \rho^2 & 1 & ... & \rho^{L-3} \\ & & ... & & \\ \rho^{L-1} & ... & \rho^2 & \rho & 1 \end{pmatrix}, \tag{36}$$

and

$$\mathbf{C_Z} = \frac{1}{2m}\begin{pmatrix} 1 & \sqrt{\rho} & \sqrt{\rho^2} & ... & \sqrt{\rho^{L-1}} \\ \sqrt{\rho} & 1 & \sqrt{\rho^2} & ... & \sqrt{\rho^{L-2}} \\ \sqrt{\rho} & \sqrt{\rho^2} & 1 & ... & \sqrt{\rho^{L-3}} \\ & & ... & & \\ \sqrt{\rho^{L-1}} & ... & \sqrt{\rho^2} & \sqrt{\rho} & 1 \end{pmatrix}. \tag{37}$$

We are now ready to generate the independent correlated Gaussian vector $\mathbf{z}_k$, $k = 1, 2, ..., 2m$ required for (31), where

$$z_k(i) = \sum_{j=1}^{i} c_{ij}U_k(j), \tag{38}$$

where c_{ij} is the (i,j) element of $\mathbf{C_Z^{1/2}}$ obtained from matrix $\mathbf{C_Z}$ in (36) using Cholesky decomposition. Note that $\mathbf{C_Z^{1/2}}$ is a lower triangular matrix.

4.3 Generation of the Correlated Nakagami-m Channels h_{NG}

The correlated L-variate vector r in (33) only gives us the distribution of the *scalar* envelope of the signal in a correlated Nakagami-m fading channel.

Case $m = 1$:

For the case $m = 1$, the Nakagam distribution reduces to the Rayleigh distribution and the implementation of a physical Rayleigh channel $\mathbf{h_R}$ is well understood. The gamma-distributed power gain in (33) becomes exponentially distributed $p_R(i) = z_i^2(i) + z_2^2(i)$ and the Rayleigh channel is implemented as

$$h_R(i) = z_i(i) + jz_2(i), \qquad (39)$$

giving the composite (complex) Rayleigh-lognormal channel as

$$h_{\text{R-LN}}(i) = h_R(i)h_{\text{LN}}(i), \quad i = 1, 2, ..., L.$$

Now we are ready to simulate MRC reception using QPSK signalling in a composite Rayleigh-lognormal fading environment.

Case $m = 1.5, 2, 2.5,$:

For $m > 1$, the implementation of a physical Nakagami-m channel h_{NG} requires a $2m$-dimension signal space. Therefore in the conventional 2-D space of an M-QAM constellation, we have to be contented with the scalar, i.e. envelope magnitude, channel gain given in (34),

$$h_{\text{NG}}(i) = r(i), \qquad (40)$$

giving the composite (scalar) Nakagami-lognormal channel as

$$h_{\text{NG-LN}}(i) = h_{\text{NG}}(i)h_{\text{LN}}(i), \quad i = 1, 2, ..., L. \qquad (41)$$

Note this means both in-phase and quad-phase components of the QPSK signal has this same channel gain.

5 Comparison of Simulated and Theoretical BER

In Figure 4, we compare the simulated BER result using channel model in (32) with the theoretical BER calculated from (28) for the case of correlated Rayleigh-lognormal diversity channels when the output of the MRC receiver is approximated by a single lognormal RV. In both theory and simulation, we normalize the power of each channel to 1. By comparing BER performance QPSK modulation in correlated Nakagami-lognormal fading channels with $m = 1, 2$ and $L = 2, 4, 6$ obtained by theoretical analysis and Monte-Calor Simulation shown in Figures 4 and 5, the precision of mathematical analysis has been shown.

6 Conclusion

We have successfully presented a MGF two-point matching technique used to estimate the sum of several correlated composite Nakagami-m-lognormal RVs at the output of MRC to a single lognormal. The accurate estimation has been demonstrated by choosing two
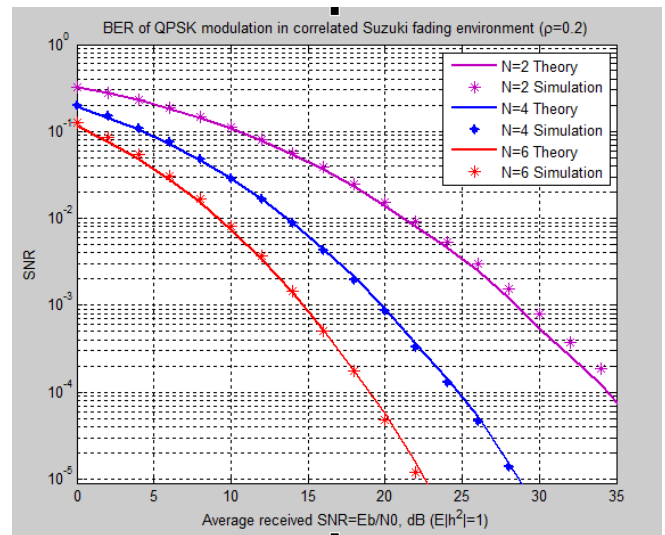

Figure 4.   Comparison of theoretical BER in (28) against simulated BER for the case of correlated Suzuki-fading environment
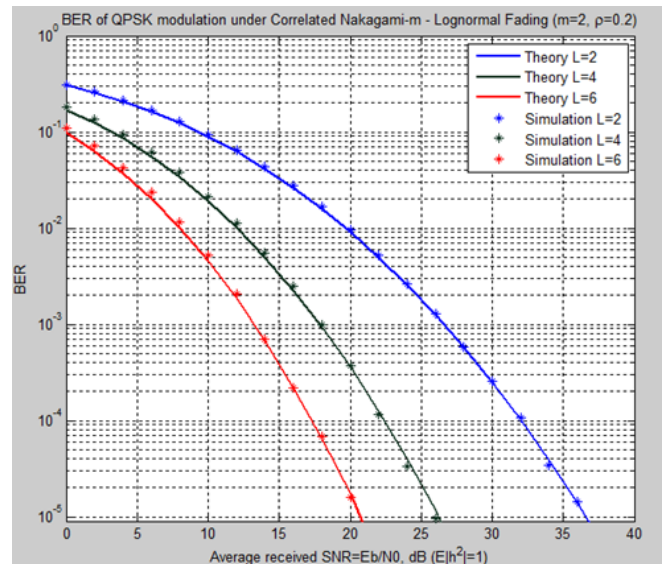

Figure 5.   Comparison of theoretical BER in (28) against simulated BER for the case of correlated Rayleigh-lognormal diversity channels ($m = 1$ and $m = 2$)

matching points, and the value of the percentage error power loss between the average power gain at the input and output of the receiver. The higher m value means the higher number of Rayleigh paths providing the better SNR. As the result, it is found that the BER is inversely proportional to the m. The accuracy of the mathematical analysis is also backed by Monte Carlo simulation of the BER of QPSK signal in Nakagami-lognormal fading environment using MRC diversity reception.

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