Vol. 62, p. 1528-1547, jul./dez. 2023. DOI: 10.5380/dma.v62i0.87977 e-ISSN 2176-9109

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Reassessing the neoclassical substitution model: the increasing flows evidence

Reavaliando o modelo de substituição neoclássico: a evidência dos fluxos crescentes

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Article received on October 14, 2022, final version accepted on April 5, 2023, published on December 21, 2023.

ABSTRACT: In this article we assess the validity of the elasticity of substitution indicator to evaluate natural resource substitution. After clarifying the importance of the static equilibrium hypothesis for the development of such an indicator, we demonstrate that its application to growth conditions imposes the observation of decreasing inflows of natural resource as a necessary consequence. We derive this result without making any assumptions not recognized by advocates of the neoclassical approach. In addition to refuting the neoclassical substitution model, this result helps clarify the relationship observed since the Industrial Revolution between the physical and economic substitution of natural resources.

Keywords: elasticity of substitution; strong sustainability; weak sustainability; degrowth; economic growth

RESUMO: Neste artigo investigamos a validade do indicador elasticidade de substituição para avaliar a substituição de recursos naturais. Depois de esclarecer a importância da hipótese de equilíbrio estático para o desenvolvimento de tal indicador, demonstramos que a sua aplicação a condições de crescimento impõe a observação de fluxos decrescentes de recursos naturais como uma consequência necessária. Derivamos tal resultado sem fazer nenhuma suposição não reconhecida pelos defensores da abordagem neoclássica. Além de refutar o modelo de substituição neoclássico, esse resultado ajuda a esclarecer a relação observada desde a Revolução Industrial entre as substituições física e econômica dos recursos naturais.

Palavras-chave: elasticidade de substituição; sustentabilidade forte; sustentabilidade fraca; decrescimento; crescimento econômico

1. Introduction

Natural resource substitution is central to the weak versus strong sustainability debate (Neumayer, 2013). Weak sustainability considers natural resources as a form of capital like any other. In such a perspective, it is possible describe the behavior of the economic system using only indicators that are endogenous to market operations (such as prices and elasticies of substitution). In particular, it is possible to assess the dependence of economic system on natural resources by observing the elasticity of substitution between natural and manufactured capital.

In contrast, strong sustainability considers natural and manufactured capital as complements and only marginally as substitutes (Daly, 1990). The assessment of strong sustainability advocates stems from its interpretation of the relevance of the laws of thermodynamics to productive activities: the economic system cannot reproduce itself over time without continually appropriating low-entropy that are extracted by the economic system, not manufactured. Based on of this interpretation, strong sustainability combines endogenous indicators with indicators that can be considered exogenous to market operations, such as per capita consumption of energy and materials (Fischer-Kowalski 1998; Fischer-Kowalski & Huttler 1998).

Neumayer's (2013) interpretation is quite influential in appreciating the weak versus strong sustainability dissent:

> Chapter 3 has tried to assess the validity of the opposing claims of WS [Weak Sustainability] and SS [Strong Sustainability] with respect to the substitutability of natural capital. The conclusion that arises from the analysis is that both paradigms rest on certain assumptions as well as hypotheses and claims about the (distant) future that are non-falsifiable. That does not mean, of course, that either paradigm is nonsensical. Both of them have some theoretical plausibility as well as some empirical evidence in their support (Neumayer, 2013, p. 96).

Although we recognize Neumayer's (2013) contribution as a helpful synthesis of the debate, this article diverges from his assessment. The assumption that refutation of disputed paradigms can only be accomplished by evaluating evidence in the distant future ignores the possibility that disputing parties may ignore refuting evidence that is currently available. However, one should not rule out the possibility that refuting evidence requires some theoretical elaboration for its meaning determination. Furthermore, such meaning arises only from specific theoretical perspectives; it is not merely an empirical question. More specifically, it is not yet sufficiently clear how natural and manufactured capital are generally complements and only marginally substitutes. How should we interpret this point to explain substitutability and complementarity in a growth economy?

Daly (1990)'s emphatic statement has been made more than once (Daly 1990, 1994, 1997; Daly & Farley 2004). It is shared by ecological economists (see for instance Cleveland & Ruth 1997). Nevertheless, as it stands, it can be either seen as a plausible assertion[1] or plainly nonsense[2], depending on the position taken in the debate. After all, are

[1] Daly (1990, pp. 2-3): "A house is no doubt a superior substitute for a cave or a tree as a place to live, but that is not the issue. The issue is the nature of the roles played by resources and capital in the construction of a house. Are they complements or substitutes? It should be obvious that they are basically complementary and only very marginally substitutable."

[2] In the debate, Solow (1997, p. 267) considered that Daly (1997) did not even understand the economist substitution concept: "[…] Contrary

natural and manufactured capital complementary or substitutable? Can they be both at the same time? A redescription may help to specify more rigorously the meaning of Daly's (1990) proposition.

This article reintroduces the elasticity of substitution indicator, emphasizing the boundary conditions that guided its construction. In this redescription, we indicate the importance of the premise that the marginal rate of substitution depends exclusively on the quotient between factors of production (Section 2). Although economists present such a premise considering static equilibrium conditions, they have used it to evaluate contexts characterized by economic growth trajectories. Section 3 investigates the consistency of such a procedure. We conduct the investigation without introducing any premises that are not recognized by neoclassical economists. More specifically, in Section 3 we show that consistent use of the elasticity of substitution indicator under growth conditions implies as a necessary implication, a declining rate of natural resource use $\frac{dn}{dt}<0$. In Section 4, we retake (Daly 1990)'s strong statement to emphasize the need to explicitly recognize the different hierarchical levels at which production operations take place when using indicators to describe the economic system behavior. In the conclusion, we argue that the available evidence contradicts the weak model of sustainability substitution.

2. Boundary conditions of the neoclassical model

In this Section, we present a restatement of the elasticity of substitution indicator in which we emphasize the boundary conditions that guide its construction. Although built to assess static equilibrium situations, the indicator has been used in situations involving dynamic boundary conditions and development processes. After introducing the elasticity of substitution indicator (Section 2.1)[3], we contrast the static conditions underlying the neoclassical construction with what scientists typically understand to be dynamic (Section 2.2) and developmental situations (Section 2.3). Section 2.4 emphasizes that neoclassical economists have used the elasticity of substitution indicator to make inferences beyond the static boundary conditions that guided its construction. Only in Section 3 will we develop the implication of using the neoclassical static formulation in dynamic contexts.

2.1. The indicator of elasticity of substitution

The boundary conditions that guide the neoclassical model are that of a firm operating in a market with many other firms, responds in the short term to variations in the prices of production factors that it acquires in the market, and practicing selling prices over which it has no control. Also, the firm

to what Daly says, the substitution between renewable and nonrenewable resources is the essence of the matter. (One of his lesser problems is that he does not understand what economists mean when they speak of complements and substitutes) […]".

[3] In our description of the elasticity of substitution indicator, we follow the valuable and concise presentations made by Allen (1962) and Stiglitz (1979). As our objective is mainly to clarify the conceptual elements involved in the neoclassical formulation, we will not refer to the vast empirical literature involving the use of the elasticity of substitution.

has already determined at what level of production it should operate. The firm's decisions cannot change either the optimal level of production or the selling prices. Under such boundary conditions, the profit maximization by the firm is equivalent to the minimization of its costs. The firm is then left with the problem of deciding what combination of factors of production combination to acquire in the market to produce product q at the lowest possible cost. Analytically, the possible operations of this hypothetical firm reduce to determining how to substitute factors of production as it observes price changes.

Figure 1 below illustrates substitutability in neoclassical models. The case allows for an intuitive representation since it involves only two factors of production. The constant output curve (isoquant) represents the behavior of the production function under static equilibrium conditions. In the simplest analytical case, infinite combinations of production factors can keep the product q constant; all possible combinations of production factors fall on the red curve in Figure 1. When the restriction that production processes cannot occur without using non-zero amounts of a production factor is incorporated, the factor in question is called essential (Stiglitz 1979).

Thus, the problem conditions are such that $q = f(k,n) = constant$, $k > 0$, $n > 0$. Since q is constant, its differentiation gives:

$$q = f(k,n) = constant \Rightarrow f_k * dk + f_n * dn$$

dk and dn are the differential increments along the curve (k,n); $f_k = \frac{\partial f}{\partial k}$ and $f_n = \frac{\partial f}{\partial n}$ denote k and n marginal products, respectively. Since $q = f(k,n)$ for all points on the curve (k, n), this relationship holds for any selected point in the above red curve. Rewriting the above equation:

$$\frac{dk}{dn} = \frac{-f_n}{f_k}$$

By definition, the marginal rate of substitution of natural capital with manufactured capital in the making of q units of a good produced by combining manufactured and natural capital is:

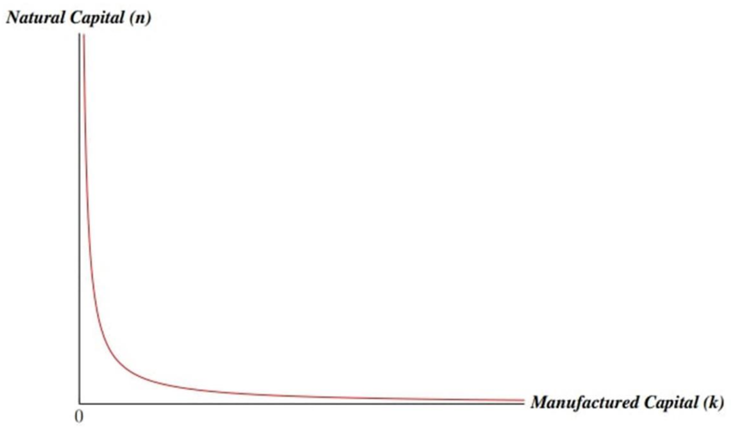

FIGURE 1 – Neoclassical substitution between natural and manufactured capital
SOURCE: adapted from Stiglitz (1979).

$$r_{kn} = \frac{-dk}{dn} = \frac{f_n}{f_k} \tag{2.1}$$

r_{kn} represents the additional amount of manufactured capital in a given combination of factors required to keep output unchanged when there is a small reduction in natural capital takes place[4]. In such a definition, r_{kn} is the negative of the quotient between a tiny increase in manufactured capital use to a tiny decrease in natural capital use, implying $r_{kn}>0$.

It usually becomes increasingly difficult to substitute one factor of production for another when substitution occurs. Note that the value r_{kn} indicates the marginal substitution rate at a point on the isoquant. In practice, there is interest to evaluate the substitution possibilities in a range of values rather than at a point. Another difficulty in using Equation 2.1 is that the value obtained depends on the specific units of manufactured and natural capital. Regardless of the units used to measure natural n and manufactured k capital, to ascertain what happens to the marginal rate of substitution in the isoquant region in which the quantities of manufactured k and natural n factors are located?[5] The elasticity of the substitution indicator overcomes such limitations.

For each change along the constant curve (k, n), $d\left(\frac{k}{n}\right)$ represents the increase or (decrease) in the use of manufactured capital compared to that of natural capital, and $dr = d\left(\frac{f_n}{f_k}\right)$ is the corresponding increase (or decrease) in the marginal rate of substitution. The ratio of these differentials, expressed proportionally to make them independent of the units of measure, is defined as the elasticity of substitution between the factors at the combination of factors considered (Allen, 1962, p. 341):

$$\sigma_{kn} \equiv \frac{\frac{d\left(\frac{k}{n}\right)}{\left(\frac{k}{n}\right)}}{\frac{d(r)}{(r)}} \tag{2.2}$$

The condition of cost minimization imposes that the factors of production must be employed in amounts such that their marginal products are proportional to their respective prices (Allen, 1962, p. 370):

$$\frac{p_k}{f_k} = \frac{p_n}{f_n} \Rightarrow \frac{f_n}{f_k} = \frac{p_n}{p_k} \tag{2.3}$$

From Equations 2.1, 2.2 and 2.3:

$$\sigma_{kn} \equiv \frac{\frac{d\left(\frac{k}{n}\right)}{\left(\frac{k}{n}\right)}}{\frac{d(r)}{(r)}} \Rightarrow \sigma_{kn} = \frac{\text{relative change in } \left(\frac{k}{n}\right)}{\text{relative change in } \left(\frac{p_n}{p_k}\right)} \tag{2.4}$$

[4] Our description is analogous to that given by Allen (1962, pp. 340-341). The formal approach is the same, only applied to the specific problem of replacing natural capital with manufactured capital.

[5] The uppercase letters K and N indicate the specific qualities required by the production process, while the lowercase letters k and n respectively indicate the required amounts of such qualities. In classical economics, the term use value was used to indicate the concrete qualities associated with commodities, in distinction to quantitative monetary aspects, such as price (exchange value). Neoclassical economics tends to consider the distinction between monetary and other dimensions of production processes secondary or irrelevant. In part, the debate between ecological and neoclassical economists is a divergence as to the relevance of the concept of use value in the description of economic facts, particularly concerning the relationship between thermodynamic use values and monetary economic values.

Equation 2.4 allows the firm to change the proportion of factors of production used while observing price changes in those factors. It is common in Economic textbooks to present Equation 2.4 directly as an identity definition (see for instance Chiang & Wainwright, 2005, p. 396). Such presentation obscures that elasticity of substitution is not a primary variable but an indicator constructed from the marginal rate of substitution under the following conditions: firm level, isolated system (there is no other system beyond the market), perfect competition, and static equilibrium.

In the short term, there is no room for significant changes in the technologies adopted (new investments). In general, there is also little room for changes in strategy. These conditions are reasonably well represented by assuming that the market is in static equilibrium, with minor changes around a certain level of production, corresponding to the balance between supply and demand at a given moment. In this context, the firm can respond to small changes in the market state by modifying the proportion with which it uses factors K and N. If the relative price of factor K increases, it must reduce its use and increase the use of factor N, and vice versa.

In the neoclassical formulation, it does not even make sense to refer to the extraction of factors of production, since the firm necessarily acquires the factors on the market. The question of how the factors of production come to the market does not appear in the neoclassical interpretation. For the same reason, the neoclassical model does not represent the relationship of the economic system to other systems (for example, the biophysical system that makes up planet Earth). We will return to this point later in Section 4.

The most crucial point of the above exposition is to clarify that the elasticity of substitution is an indicator derived from the marginal rate of substitution under static equilibrium conditions.

If the time variable were irrelevant, models focusing on static equilibrium conditions would suffice. But both the states of the system and the relationships between those states can change over time. Nevertheless, the neoclassical formulation abstracts from both dynamic and development possibilities. Therefore, some brief comments on such possibilities follow to clarify the meaning of the neoclassical procedure.

2.2. Dynamic conditions

The neoclassical firm responds only to market conditions; it does not have the means to keep internal variables such as production level and profit margin within a specific range of values to represent a genuinely dynamic equilibrium. Although real systems can exhibit both static and dynamic equilibrium, modeling more dynamic behavior can be difficult because various feedback mechanisms are required to recognize it.

Dynamic equilibriums can refer to an isolated feedback structure, indicating specific negative feedback, or to the entire system. In the latter case, the expression homeostasis is often used to refer to the observed dynamical equilibrium (*e.g.*, Damasio & Damasio 2016), which may include numerous negative and positive feedback structures. To model dynamic equilibrium, it is necessary to describe which structures of an organization respond to environmental changes to maintain the organizational identity. For example, the body is constantly working to ensure that the internal temperature does not deviate from 36 degrees Celsius, regardless of variations in the ambient temperature. Unlike neoclassical static equilibrium, the system does not passively adapt to external conditions. Nevertheless, it has structures to maintain specific variables in specific ranges of values, with both specificities being the product of peculiar historical development. Such a perspective differs from the neoclassical approach. In the static equilibrium model of the neoclassical firm, it is assumed that the quantity the firm must produce is given, and there is no decision to be taken in this sense. Eventually, a dynamic model would make it possible to determine both quantity and prices as a function of changes in market prices and other environmental changes (e.g., an agricultural firm responding to unforeseen levels of rainfall).

2.3. Development

In the neoclassical model, there is no firm development since the production function is the same throughout the analysis. Such an assumption is very useful for short-term situations. However, considering alternative functional forms may be relevant when evaluating long-term trajectories. Such consideration is essential not only for the processes of substituting production factors but for production processes in general (Georgescu-Roegen 1971, p. 236)[6]:

> A catalog of all *feasible* and *non wasteful* recipes then consists of a set of points in an abstract space, as opposed to Euclidean space. The set may be regarded as a variety within the abstract space and, hence, represented by a relation of the form.
>
> $$(11) \underset{0}{\overset{T}{Q}}(t) = \mathcal{F}\left[\underset{0}{\overset{T}{R}}(t)\underset{0}{\overset{T}{I}}(t)\underset{0}{\overset{T}{M}}(t)\underset{0}{\overset{T}{W}}(t);\ \underset{0}{\overset{T}{L}}(t)\underset{0}{\overset{T}{K}}(t)\underset{0}{\overset{T}{H}}(t)\right]$$
>
> which in mathematical jargon is called a *functional*. This is a relation from *a set of functions to one function*. Consequently, (11) is a far cry from the Neoclassical production function [...], which is a point function, i.e., *a relation from a set of numbers to one number*.

The functional approach proposed by (Georgescu-Roegen, 1971) has the merit of indicating more general situations than those captured by neoclassical models. However, such an approach does not exhaust the problem of representing development. Since development involves qualitative changes, it is impossible to understand it in the strictly arithmetical terms that prevail in the construction of mathematical models (by the way, a fact amply emphasized by Georgescu-Roegen,1971). In general, the qualitative changes that characterize modifications in the development pattern cannot be predicted. Nevertheless, one can attempt to build a new model that captures such changes once a qualitative characteristic or change has been identified as relevant.

2.4. The scope of the indicator for elasticity of substitution

It seems plausible that static equilibrium models can capture a relevant part of the short-term behavior of individual firms operating in markets

[6] All italics in the citation are from the original.

with several other firms with similar technical and economic capacity. However, while there are boundary conditions under which the neoclassical approach is appropriate, there are also numerous economically relevant situations in which the boundary conditions strongly suggest that it is at least inadequate.

The problem is not just knowing if the assertions produced by neoclassical models are false or true. There are relevant situations that have no clear meaning in the neoclassical model. Since the neoclassical model considers that all factors of production come from the market through buying and selling operations, it is not clear how to interpret the extraction of primary sources. After all, primary resources need to be mined somewhere before they exist on the market in purchase and sale operations. The neoclassical approach completely abstracts from non-market material (for example, the planet's biosphere) and institutional conditions allowing numerous agents to carry out purchase and sale operations inside the market. It is not evident that primary resource extraction operations can be interpreted in the same sense as the purchase and sale operations that occur within the market.

It must be clear that the limitations of neoclassical are a methodological problem only if one does not consider the boundary conditions that support the use of the model. To use models is to use abstractions; the question is whether the abstractions used are appropriate for dealing with the problems under study.

In constructing indicators of elasticity of substitution, neoclassical economists consider that these indicators describe the behavior of individual firms buying and selling in perfect competition under static equilibrium conditions. However, this model has been used to assess situations in which the time scale and range of behaviors that firms can exhibit are much larger. In fact, elasticity of substitution is used to support the assertion that the economic system can replace natural resources along long paths with dynamic and developmental situations, regardless of the size that the economic system can assume. The assertion by Solow (1974a, 1974b) that if the elasticity of substitution is greater than one, continuous growth in consumption is possible is well known and influential. His perspective considers it indisputable that elasticities of substitution are sufficient to assess long-term trajectories. Since such use of the indicator is not inherent to the boundary conditions used in its construction, it is worth checking the consistency of such a procedure. The following section reintroduces the neoclassical substitution model by emphasizing the implications of different boundary conditions.

3. How to test the neoclassical substitution model

The substitution of production factors in static situations behaves differently from substitution in dynamic situations. Based on this understanding and having recognized the static philosophy of the neoclassical formulation of the problem of substitutability of factors of production, it may be helpful to assess the implications of such a static formulation in known dynamic contexts. Section 3.1 indicates how the neoclassical methodology encodes factor substitution in the production function. Section 3.2 evaluates the consistency of such a procedure in static boundary conditions; in Section 3.3, we evaluate the neoclassical procedure in dynamic boundary conditions. A relevant result of the evaluation carried out in Section 3.3 is the demonstration that the application of neoclassical modeling under dynamic conditions (more specifically, under conditions in which the output increases continuously over time) imposes a necessary implication, starting solely from neoclassical premises alone and without using assumptions not recognized by the neoclassical paradigm, $\frac{dn}{dt}<0$. Such an implication can be used to assess the empirical consistency of the neoclassical model of natural resource substitution.

3.1. Factor substitution in the neoclassical production function

The homogeneous production function is central to the neoclassical approach. For any scalar h, a real-valued function $f(x_1,\ldots,x_n)$ is homogeneous of degree h if (Simon & Blume 1994):

$f(tx_1, \ldots, tx_n) = t^h f(x_1, \ldots, x_n)$ for all $x_1, \ldots, x_n$ and all $t > 0$.

Depending on the value of h, the factor t^h will be larger or smaller than t and output increases more or less than the input vector (Léonard & Long 1992):

$h < 1 \Rightarrow t^h < t$: $f(\boldsymbol{x})$ exhibits decreasing return to scale

$h = 1 \Rightarrow t^h = t$: $f(\boldsymbol{x})$ exhibits constants returns to scale

$h > 1 \Rightarrow t^h > t$: $f(\boldsymbol{x})$ exhibits increasing returns to scale

The bold format (x) is just a synthetic way of writing $(x_1, \ldots, x_n)$. Our analysis considers a production function whose input vector elements are only natural n and manufactured k capital. That is, $\boldsymbol{x} = (k, n)$.

Note that the homogeneous function parameterizes scale effects: multiplying all inputs by t gives the same effect as multiplying the function f by the parameter t^h. Simon & Blume (1994) consider the use of such a procedure natural. Nevertheless, verifying whether such strategy applies in each concrete situation is always advisable. The typical neoclassical models consider decreasing or constant returns to scale in a homogeneous production function:

$$0 \leq h \leq 1 \tag{3.1}$$

$$f(tk, tn) = t^h * f(k, n) \tag{3.2a}$$

It is possible to rewrite Equation 3.2a so that the only independent variables are the quotient *(k /n)* and the quantity of natural resources n. Choosing the parameter t so that $t=(1/n)$ in Equation 3.2a:

$$\left.\begin{array}{c} f(tk,tn) = t^h * f(k,n) \\ t = \left(\frac{1}{n}\right) \end{array}\right\} \Rightarrow f\left(\frac{1}{n} * k, \frac{1}{n} * n\right) = f\left(\frac{k}{n}, 1\right) = \left(\frac{1}{n}\right)^h f(k,n)$$

Writing $\phi\left(\frac{\mathrm{k}}{\mathrm{n}}\right) = f\left(\frac{\mathrm{k}}{\mathrm{n}}, 1\right)$:

$$q = f(k,n) = n^h \phi\left(\frac{\mathrm{k}}{\mathrm{n}}\right) \tag{3.2b}$$

Why is Equation 3.2b important? The importance of this equation is best illustrated by the debate over weak versus strong sustainability debate. Critics of neoclassical economics (Georgescu-Roegen 1979; Daly 1997) have pointed out that the validity of the neoclassical process implies an indefinitely long trajectory of economic growth with natural resource quantities small as we wish, provided we have sufficiently large amounts of manufactured capital. The neoclassical procedure is mathematically equivalent to claiming that the production function f does not depend directly on the absolute quantities of manufactured and natural capital. Invariably, neoclassical models codify such a relation by making f depend on the quotient of the factors of production.

To operationalize such a codification, we can work with $\left(\frac{k}{n}\right)$ or with $\left(\frac{n}{k}\right)$:

(a) In demonstrating Equation 3.2b, we used a variable transformation that produces the quotient $\left(\frac{k}{n}\right)$ and (n) as independent variables.

(b) If we set $t=\left(\frac{1}{k}\right)$, we obtain the ratio $\left(\frac{n}{k}\right)$ and (k) as independent variables.

In the debate over strong versus weak sustainability, the central question is what happens to the trajectory of natural resources. Letting n as an independent variable opens the possibility of comparing the model results with observed patterns of natural resource use. Therefore, our option for (a) is justified (Equation 3.2b).

To assess the impact of changes in the quantities of n and k used, we can evaluate the partial derivatives of the production function f with respect to n and k. Using Equation 3.2b in this evaluation:

$$f_n = n^{h-1} \cdot [h \cdot \phi\left(\frac{k}{n}\right) - \left(\frac{k}{n}\right) \cdot \phi'\left(\frac{k}{n}\right)]^7 \quad \textbf{(3.3)}$$

$$f_k = n^{h-1} \cdot [\phi'\left(\frac{k}{n}\right)] \quad \textbf{(3.4)}$$

[7] $\phi'\left(\frac{k}{n}\right)$ is $\phi\left(\frac{k}{n}\right)$ derivative with respect to $\left(\frac{k}{n}\right)$.

In addition to the equations describing the behavior of the homogeneous production function (Equations 3.2b, 3.3 and 3.4), it is also necessary to consider the equation defining the substitution of natural capital with manufactured capital (Equation 2.1):

$$r_{kn} = \frac{-dk}{dn} = \frac{f_n}{f_k} \quad \textbf{(2.1)}$$

To verify the neoclassical procedure, we need to investigate the consistency of the implications of the models when a homogeneous production function is used to describe substitution processes under different boundary conditions. More specifically, the objective implications generated by the neoclassical methodology must be consistent both in static conditions ($dq = 0$) and dynamic conditions ($dq \neq 0$).

A relationship between the marginal productivities of the factors of production (f_k *and* f_n) and the marginal rate of substitution can be obtained using the total derivative of the production function q with respect to natural resources n:

$$q = f(k,n)$$

$$\frac{dq}{dn} = f_k * \frac{dk}{dn} + f_n$$

Combining equation 2.1 with the total derivative equation:

$$r_{kn} = \frac{-dk}{dn} = \frac{f_n}{f_k} - \left(\frac{1}{f_k}\right) * \left(\frac{dq}{dn}\right) \quad \textbf{(3.5)}$$

There are two main situations to investigate: static ($dq = 0$) and dynamic ($dq \neq 0$) equilibrium conditions.

3.2. Static conditions

Using the constraint $dq = 0$ referring to static equilibrium conditions in Equation 3.5, we arrive at the case analogous to what is traditionally presented in economics textbooks, $r_{kn} = \frac{f_n}{f_k}$. Putting Equations 3.3 and 3.4 into Equation 3.5:

$$r_{kn} = h \cdot \left[\left(\frac{\phi\left(\frac{k}{n}\right)}{\phi'\left(\frac{k}{n}\right)} \right) - \left(\frac{k}{n}\right) \right] \tag{3.6}$$

Equation 3.6 expresses the marginal rate of substitution as a function of the independent variables $\left(\frac{k}{n}\right)$. Considering that neoclassical economics always works with differentiable functions, there is a practical way to assess whether the marginal rate of substitution depends only on the quotient $\left(\frac{k}{n}\right)$. The practical approach is to use the fact that if r'_{kn}[8] does not change sign over the entire domain of observations, then $r_{kn}\left(\frac{k}{n}\right)$ is a one-to-one function (see for instance Simon and Blume 1994, p. 78; Chiang 2005). In this case, different values of r_{kn} correspond to different values of $\left(\frac{k}{n}\right)$, and only changes in $\left(\frac{k}{n}\right)$ modify r_{kn}. Under such conditions, the absolute quantities k and n do not matter directly but only as they impact the quotient $\left(\frac{k}{n}\right)$. Deriving Equation 3.6 with relation to $\left(\frac{k}{n}\right)$ and using the abbreviations

$$\phi = \phi\left(\frac{k}{n}\right), \phi' = \phi'\left(\frac{k}{n}\right), \phi'' = \phi''\left(\frac{k}{n}\right):$$

$$r'_{kn} = -h \cdot \left[\frac{\phi \cdot \phi''}{\left(\phi'\right)^2} \right] \tag{3.7}$$

Regarding Equation 3.7, the following points should be noted:

(i) $0 \leq h \leq 1$ (from Equation 3.1) ;

(ii) Φ is a positive number (see Equation 3.2b);

(iii) Φ′ is a real number$\Rightarrow \left(\phi'\right)^2 > 0$;

Since natural resources are essential, no combination exists in which $n = 0$. It then follows that $k<q$ (k,n):

$$k < q$$

$$\ln\left(\frac{k}{n}\right) < \ln\left(\frac{q}{n}\right)$$

$$dln\left(\frac{k}{n}\right) = dln\left(\frac{q}{n}\right) \tag{3.8}$$

Observing that:

$$\frac{q}{n} \leq \frac{q}{n^{h-1}} = \phi \cdot \mathrm{d}\phi \tag{3.9}$$

[8] r'_{kn} is the derivative of r_{kn} with respect to $\left(\frac{k}{n}\right)$.

Using equations 3.8, 3.9 and $d\ln(x) = \left(\frac{1}{x}\right) dx$:

$$\left(\frac{1}{\left(\frac{k}{n}\right)}\right) d\left(\frac{k}{n}\right) \leq \left(\frac{1}{\phi}\right) d\phi$$

$$\left(\frac{n}{k}\right) \leq \left(\frac{1}{\phi}\right) \cdot \frac{\mathrm{d}\phi}{d\left(\frac{k}{n}\right)}$$

Since $\phi' = \frac{\mathrm{d}\phi}{d\left(\frac{k}{n}\right)}$:

$$\left(\frac{n}{k}\right) \leq \left(\frac{\phi'}{\phi}\right)$$

Then:

$$\phi - \phi' \cdot \left(\frac{k}{n}\right) \leq 0 \tag{3.10}$$

Deriving Equation 3.10 with respect to $\left(\frac{k}{n}\right)$:

$$\phi' - \phi'' \cdot \left(\frac{k}{n}\right) - \phi' \cdot (1) \leq 0$$

$$\phi'' \cdot \left(\frac{k}{n}\right) \geq 0$$

Since $\left(\frac{k}{n}\right)$ is always positive:

$$\phi'' \geq 0 \tag{3.11}$$

Therefore, facts (i), (ii) and (iii) together with Equations 3.11 and 3.7 guarantee that $r'_{kn} < 0$, for all observable *k, n*. Therefore, it is always true that the marginal rate of substitution is completely controlled by the quotient $\left(\frac{k}{n}\right)$ under static equilibrium conditions.

3.3. Dynamic conditions

What happens under dynamic conditions, that is, when $dq \neq 0$? Substituting Equations 3.3 and 3.4 into Equation 3.5, taking into account this new situation:

$$r_{kn} = \left(h \cdot \frac{\phi}{\phi'}\right) - \left(\frac{\mathrm{k}}{\mathrm{n}}\right) - \left(\frac{1}{\phi'}\right) \cdot \left(\frac{1}{n^{h-1}}\right) \cdot \left(\frac{dq}{dn}\right) \tag{3.12}$$

By definition, $r_{kn} > 0$:

$$r_{kn} = \left(h \cdot \frac{\phi}{\phi'}\right) - \left(\frac{\mathrm{k}}{\mathrm{n}}\right) - \left(\frac{1}{\phi'}\right) \cdot \left(\frac{1}{n^{h-1}}\right) \cdot \left(\frac{dq}{dn}\right) > 0 \tag{3.13}$$

Using Equation 3.10:

$$\phi - \phi' \cdot \left(\frac{k}{n}\right) < 0$$

$$\phi < \phi' \cdot \left(\frac{k}{n}\right)$$

$$\phi' > \frac{\phi}{\left(\frac{k}{n}\right)} \Rightarrow \phi' > 0$$

As ϕ, k, n are all positive, $\phi' > 0$. Therefore, we can multiply both sides of Equation 3.13 by ϕ' to get:

$$(h \cdot \phi) - \left(\frac{k}{n}\right) \cdot \phi' - \left(\frac{1}{n^{h-1}}\right) \cdot \left(\frac{dq}{dn}\right) < 0$$

$$(h \cdot \phi) - \left(\frac{k}{n}\right) \cdot \phi' > \left(\frac{1}{n^{h-1}}\right) \cdot \left(\frac{dq}{dn}\right)$$

Recognizing that $0 \leq h \leq 1$ we can modify the above equation:

$$\phi - \left(\frac{k}{n}\right) \cdot \phi' > \left(\frac{1}{n^{h-1}}\right) \cdot \left(\frac{dq}{dn}\right) \tag{3.14}$$

Combining Equations 3.10 and 3.14:

$$\left(\frac{1}{n^{h-1}}\right) \cdot \left(\frac{dq}{dn}\right) < 0$$

Since $\left(\frac{1}{n^{h-1}}\right) > 0$:

$$\left(\frac{dq}{dn}\right) < 0 \tag{3.15}$$

The main point is to assess what happens in a growth economy ($\frac{dq}{dt}$>0). Using the time derivative:

$$\frac{(\frac{dq}{dt})}{(\frac{dn}{dt})} < 0$$

By definition, in a growth economy $\frac{dq}{dt}$>0. Therefore:

$$\left(\frac{dn}{dt}\right) < 0 \tag{3.16}$$

Equation 3.16 is our test guide. It indicates that the flow of primary inputs through the economic system decreases over time s in a growth economy where the marginal rate of substitution depends only on the quotient between the factors. The relationship $\frac{dn}{dt}$<0 is a necessary condition both to validate the assumption that factor substitution depends only on the quotient between factors and to use the elasticity of substitution as a measure of the marginal rate of substitution under dynamic conditions ($\frac{dq}{dt}$>0). Correspondently, the empirical observation of long-term trajectories where $\frac{dq}{dt}$>0 is a refutation of both the assumption that substitution depends only on the quotient factors and the conditions that make it possible for using elasticity of substitution to consistently measure substitutability in a growing system.

It is important to emphasize that Equation 3.16 is a result obtained without imposing any extraneous assumptions to the neoclassical framework. It allows the debate on weak versus strong sustainability to proceed on terms accepted by both participants in the divergence.

It is also important to note that models around the elasticity of substitution are not the only neoclassical models possible in the strong versus weak sustainability debate. Baumol (1986) proposed an alternative neoclassical model that explains how an indefinitely long economic growth trajectory would be possible, even amid the entropic degradation of a finite stock of natural resources. Investigating Baumol (1986)'s model, Amado *et al.* (2017) have demonstrated that a necessary implication of his model is $\frac{dn}{dt}$<0. Amado *et al.* (2017) also have performed an empirical test of $\frac{dn}{dt}$<0. They investigated 153 countries covering the 1991-2011 period, using the per capita energy consumption as a proxy for natural resource consumption. The results reject the hypothesis of an overall downward trend in total primary energy consumption per capita ($\frac{dn}{dt}<0$) for 124 out of the 153 countries. Out of the 29 countries had downward trends in total primary energy consumption per capita, among which 20 did not maintain economic growth trends. Thus, only 9 countries out of 153 (of which only Germany belongs to the G-20) have shown energy consumption per capita downward trends and, simultaneously, economic growth trajectories (Amado *et al.* 2017).

Although the empirical analysis of *t* by Amado *et al.* (2017) is relevant, it should be noted that from a strong sustainability perspective it is not enough that isolated countries present $\frac{dn}{dt}<0$: the world economy as a whole must present such a trend.

Why do both Baumol (1986)'s and elasticity of substitution models impose $\frac{dn}{dt}<0$ as necessary? On an indefinitely long economic growth trajectory from a finite resource base, $\frac{dn}{dt}\geq 0$ (that is, increasing or constant flows over time) is necessarily impossible: at some point, the amount of resources extracted will exceed the size of the resource stock. Therefore, any well-built neoclassical model implies $\frac{dn}{dt}<0$. There is no other alternative, or the technological and institutional arrangements can avoid $\frac{dn}{dt}\geq 0$ and produce $\frac{dn}{dt}<0$ or else economic growth trajectories will need to be limited at some point. Therefore, to decide whether the neoclassical models are mere paper-and-pencil constructions (Georgescu-Roegen 1971, p. 397) or whether they are confirmed, we can confront Equation 3.16 with the empirical evidence.

Until now, the empirical evaluation of Equation 3.16 corroborates the importance of considerations made by strong sustainability supporters, including arguments in defense of degrowth (Khmara & Kronenberg 2020; Targa 2020; Fitzpatrick *et al.* 2022). Given the binomial role of technology and the market described by neoclassical economics[9], why does economic growth seem to imply increasing demand for very specific natural resources such as fossil fuels? Should not the homogenization of the resource base make the economic system indifferent to the available physical resource base? Why does there seem to be increasing planetary boundaries despite the enormous technological development in recent centuries (Rockstrom *et al.* 2009a; Rockström *et al.* 2009b)?

Indeed, it is difficult to see how a global environmental crisis could emerge if we observed that Equation 3.16 is actual. An economic system in which the rate of consumption of natural resources decreases over time tends to be more environmentally sustainable, not less. The political relevance of the sustainability agenda alone suggests that Equation 3.16 is false.

Actually, even reducing the flow of a subset of natural resources (replacing fossil fuels with renewables), which is undoubtedly easier than

[9] (Barnett & Morse 1963, p. 11): "Advances in fundamental science have made it possible to take advantage of the uniformity of energy/matter- a uniformity that makes it feasible without preassignable limit, to escape the quantitative constraints imposed by the character of the earth's crust [...]. Nature imposes particular scarcities, not an inescapable general scarcity. Man is therefore able, and free, to choose among an indefinitely large number of alternatives. There is no reason to believe that these alternatives will eventually reduce to one that entails increasing cost-that it must sometime prove impossible to escape diminishing quantitative returns. *Science, by making the resource base more homogeneous, erases the restrictions once thought to reside in the lack of homogeneity. In a neo-Ricardian world, it seems, the particular resources with which one starts increasingly become a matter of indifference*". The italics are ours.

reducing the consumption of natural resources as a whole, has been proven to be a tough challenge. Today, 50 years after the Stockholm Conference (1972), the so-called modern renewables (which exclude hydroelectric plants) account for 6.7% of the world's primary energy consumption(BP 2022)[10]. Despite all the political consensus around the energy transition, changes are still evidently timid.

The recurring refutation of Equation 3.16 indicates that physical and economic substitution of natural resources are not synonymous. Common sense tends to think of substitution in the physical sense (replace A with B implies increasing the use of B while decreasing the use of A). Nevertheless, economic substitution at the system level normally occurs in opposition to physical substitution at the firm level because it depends on increases in the scale of production[11]. Therefore, it is necessary to detail further the relationship between physical and economical substitution, which we do in the next Section.

4. Marginally substitutes at one level and complementary at another

If the economic system were to operate in such a way that we observe Equation 3.16, Daly (1990)'s assertion that natural and manufactured capital are marginally substitutes and basically complements would be contradictory and meaningless. On the other hand, a reality in which Equation 3.16 is repeatedly refuted indicates that what is meaningless is the use of the elasticity of substitution indicators to assess economic growth trajectories. It is important to note that the apparently contradictory character of Daly (1990)'s bold assertion breaks down only when we recognize that substitution and complementarity in Daly (1990) refer to different hierarchical levels (Giampietro & Mayumi 2018), individual firms versus the economy as a system. The substitution that occurs at the level of the individual firm at the margin is not evidence of substitution at the level of the economic system over time. Actually, all indicate complementarity happens at the system level, increasing substitution possibilities at the firm level.

At the level of individual firms, it is possible to use the indicator of elasticity of substitution to selectively describe the optimal behavior of the firm (constant product q, therefore, $\frac{dq}{dt}=0$), to determine the allocation of natural and manufactured capital at the margin. However, this indicator does not make it possible to consistently describe the allocation of the same production factors at the system level along a trajectory ($\frac{dq}{dt}\neq 0$, in a growth economy $\frac{dq}{dt}>0$), as evidenced by the persistent refutation of Equation 3.16 since the Industrial Revolution.

Thus, everything we know indicates that at the systems level, strictly economic operations must always be complemented by new flows of materials and energy, in a process that can be called extensive complementation. At the systems level, there cannot even be static equilibrium. The very possibility

[10] World primary consumption in 2021 was 595.15 exajoules (EJ), with non-hydro renewables amounting to 39.91 EJ. The hydroelectric plants totaled 40.26 EJ. If we count renewables including hydropower, then renewables accounted for 13.47 % of the world's primary consumption (BP 2022, p. 9).

[11] Georgescu-Roegen (1984, p. 29): “The special stumbling block thus comes to the surface: from all we know, to tap nature for her treasures (fossil fuels and even waterfalls) ‘tools’ of greater and greater dimensions had to be used. More efficient machines need a greater amount of matter and energy to go through the whole economic process. A thermonuclear reactor may very well be as great as the whole Manhattan.”

of carrying out work is based on the exploitation of a pre-existing energy imbalance, which allows redirecting a high concentration of low-entropy resources in some regions of the biosphere into the economic system. Neither physical theory nor empirical observation indicate that this can be a generalized substitution of primary inputs from the point of view of the system. On the contrary, evidence accumulated in centuries of economic growth corroborates the centrality of recurrent access to regions external to the economic system to control material objects made available by the biosphere, both to be used directly as natural capital and to manufacture manufactured capital. And in addition to providing the primary inputs, the biosphere also processes the results of entropic degradation associated with the socioeconomic processes of production and consumption.

It has not been emphasized enough that the transformation of the biosphere into capital is simultaneously biophysical and social. On the one hand, the biosphere must be thermodynamically transformed during production and consumption. On the other hand, resources must be socially transformed to present themselves as capital: at first, natural resources exist only as material objects made available by the planet's biosphere. For them to exist as capital, they need to be inserted into production processes through socially constructed rules. Only through specific institutional rules built throughout history can matter and energy present in the biosphere be socially transformed and to be systematically recognized as capital.

One of the functions of the socio-physical transformation of the biosphere into capital is to allow individual economic agents to abstract from non-economic considerations in the allocation of factors of production, and focus on increasing the social product. Thus, while the technological and institutional apparatus is successful, at the level of the individual firm manufactured and natural capital are usually treated in absolutely identical terms and are evaluated solely in terms of their contribution to the increasing of financial wealth in the hands of the capitalist. At this level of operations, the natural resource always appears as a commodity (a useful object acquired in the market involving only intra-market relationships), so that its acquisition and use are based on typical microeconomic instruments (*e.g.*, the elasticity of substitution). In this context, the decision on the intensity and form of use of natural or manufactured capital depends exclusively on their respective relative contributions to the product of individual agents. Therefore, in normal reproduction processes[12] of the economic system, for agents that operate on the margin, manufactured and natural capital will often present themselves as substitutes.

But the abstraction that holds at the hierarchical level of the individual firm does not hold at the level of the economic system as a whole. As using natural resources involves different hierarchical levels, it is necessary to integrate non-equivalent representations of the metabolic process. In other words, the analysis must combine assessments based on both extensive (size) and intensive variables (flow per unit of size) to establish bridges across different hierarchical levels (Giampietro & Mayumi 2018, p. 11). The elasticity of substitution measures the relative substitution of factors of production,

[12] We use the expression 'normal' in strictly empirical terms to indicate the happenings in which reproduction occurs without causing a generalized breach of expectations among the agents.

parameterized at a given level of output (constant $q \Rightarrow \frac{dq}{dt} = 0$), referring to the natural resources as an intensive variable. At another hierarchical level, the problem of sustainability as formulated by strong sustainability or evaluated by $\frac{dn}{dt}$ refers to trends in the total amounts of primary inputs of the biosphere used by the economic system as a whole, and thus refers to the natural resources as an extensive variable. Weak sustainability can be seen as a defense that the behavior of the economic system can be entirely described by using only monetary and intensive variables.

While weak sustainability tends to neglect on extensive variables, strong sustainability insists on the need to combine intensive and extensive variables to consistently describe the behavior of the economic system. For example, when changing variables from the production function $q = f(k,n)$ to the homogeneous function $q = n^h \phi\left(\frac{k}{n}\right)$, the focus of the investigation will be on the quantities of manufactured capital per unit of natural capital, not the total quantities of factors of production. We then move from a model based on extensive variables (total amounts of natural and manufactured capital, indicated as independent variables in the original production function) to another whose focus is on the intensive variable $\frac{k}{n}$, which in practice starts to function as an independent variable in neoclassical modeling. Thus, neoclassical analysis tends to investigate intensive substitution and abstract from the extensive complementation of resources provides by the biosphere. Ecological economic does not deny the market laws but emphasizes the importance of extensive use of resources. In line with the methodological perspective of ecological economics, the observations of Daly (1990) indicate that natural and manufactured capital are extensively complements at the system level, though intensively substitutable at the margin.

It should be emphasized that while elasticity of substitution is meaningless in assessing what happens at the system level over time, the global availability of natural resources has implications for the elasticity of substitution of natural capital. From a thermodynamic point of view, extensive complementation tends to increase intensive substitution. For example, it is well known that the efficiency of heat engines operating at higher temperatures (hence consuming greater amounts of fuel) tends to be greater than that of heat engines operating at lower temperatures. In fact, a relevant part of the innovations that increase the efficiency of heat engines is related to the use of materials and strategies that allow them to operate at higher levels of temperature and pressure (Masuyama, 2001). Therefore, with greater availability of primary inputs in the form of fuels and specific materials, it is possible to operate with higher levels of temperature and pressure and, consequently, higher levels of efficiency, reducing the amount of primary inputs per unit of product. Observing technological innovation from this perspective, the extensive complementation of primary inputs is one of the causes of the intensive substitution of these same inputs. It does not seem absurd to consider the extensive complementation of resources a cause possibly more critical than human ingenuity. Once one recognizes such a reality, it is clear that the role of science has not been to homogenize factors of production but to allow for an increase in the scale of use of very specific production factors.

The dynamic relation between intensive substitution and extensive complementation produces important methodological implications. The facts indicate that the economical use of natural resources from the perspective of individual actors does not allow conclusions about the economic use of resources from the system's perspective as a whole. Therefore, the natural resources use trajectory by the economic system calls into question the methodological individualism (Basu, 2018) that guides the neoclassical approach and corroborates the need to incorporate methodologies compatible with complexity theory (Leff, 2007; Foxon *et al.*, 2013) and post-normal science (Ravetz, 1999; Lauda-Rodriguez & Ribeiro, 2019; Lampis *et al.*, 2021).

5. Conclusion

The elasticity of substitution is a consistent indicator for evaluating the substitution of natural by manufactured capital at the level of individual firms operating close to the condition of static equilibrium under perfect competition. However, to use the elasticity of substitution to assess the substitution of natural capital by manufactured capital at system-wide level during economic growth paths, it would be necessary to observe $\frac{dn}{dt}<0$ along natural resource utilization paths. The environmental crisis itself indicates that such usage trajectories are not occurring. Therefore, the neoclassical model is at odds with the evidence available today. Thus, contrary to what Neumayer (2013) argues, the claims of Weak Sustainability are falsifiable, and it has been rejected. For this reason, it is not necessary to wait for the distant future to submit weak sustainability to critical tests.

Acknowledgments

The first author gratefully recognizes the discussions with João Marcos Mott Pavanelli about institutional aspects of energy transition, which stimulated the approach adopted in the paper. The authors also are grateful for the contribution of Alcântaro Lemes Rodrigues in writing the equations in Word.

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