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Running head: MATHEMATICAL DIFFICULTIES vs. HIGH ACHIEVEMENT

Mathematical difficulties vs. high achievement: an analysis of arithmetical cognition in
elementary school

Abstract

This study analyzed the contribution of cognitive processes (planning, attention, simultaneous and successive processing) and domain-specific skills (counting, number processing and conceptual comprehension) to the arithmetic performance achieved in the last three grades (4th, 5th, and 6th) of elementary school. Three groups of students with a different arithmetic achievement level were characterized. The predictive value of the cognitive processes and the math specific skills are explored through diverse covariance and discriminant analyses. Participants were 110 students ($M = 10.5$ years, $SD = 1.17$) classified in three groups: mathematical difficulties (MD; $n = 26$), high achieving (HA; $n = 26$), and typical achieving (TA; $n = 58$). Cognitive processes and domain-specific skills were evaluated in two individual sessions at the end of the school year. Nonverbal intelligence was assessed in a final collective session with each class. The mathematical difficulties group's achievement was deficient in simultaneous and successive processing, number processing, and conceptual comprehension compared to the typical achievement group. High achievement children obtained significantly better results than the typical achievement children in simultaneous processing, counting, number processing, and conceptual comprehension. Number processing and conceptual comprehension were the most consistent classifiers, although successive and simultaneous processing, respectively, also contributed to identifying students with mathematical difficulties and high achievement. These findings have practical implications for preventive and intervention proposals linked to the observed profiles. *Keywords:* arithmetic, cognitive processes, mathematical difficulties, high achievement, elementary school

Mathematical difficulties vs. high achievement: an analysis of arithmetical cognition in elementary school

The number of studies of mathematical achievement in the first three grades of elementary school, when children are acquiring single-digit and multi-digit arithmetic skills, has increased exponentially over the past three decades (e.g., Passolunghi, Cargnelutti, & Pastore, 2014; Träff, Olsson, Skagerlund, & Östergren, 2018). As of 4th grade, when arithmetic abilities should be consolidated, the number of studies is considerably lower, although some educational reports such as the Trends in International Mathematics and Science Study (TIMSS; IEA, 2016) within the OECD context alert us about the great heterogeneity in the classrooms, with 6% of the 4th-grade students at a very low level (7% in the case of Spain) and, at the other extreme, 10% at an advanced level (3% in the case of Spain). As arithmetic conditions performance in other mathematical domains—such as algebra, geometry, or trigonometry, which are introduced in the final grades of elementary school—and even access to higher studies or adult employability (Geary, 2011), it seems important to understand the factors underlying the differential arithmetic achievement of children in the final elementary school grades and to establish their specific contribution (Träff, 2013). Arithmetic has been defined operationally as the ability to solve additions, subtractions, multiplications, and divisions, recovering facts and rules and correctly applying the corresponding algorithms (Iglesias-Sarmiento & Deaño, 2016). Arithmetic skills are acquired gradually throughout the first educational grades and they require other domain-specific skills such as counting, number processing, or the understanding of arithmetic principles that are taught and learned hierarchically and cumulatively at school (Anderson, 2008). Current literature has provided evidence suggesting that these skills are necessary but do not guarantee per se adequate arithmetic performance, because different general skills such as planning, working memory, or executive functions, among others, may also be necessary (e.g., Sowinski, LeFevre,

Skwarchuk, Kamawar, Bisanz, & Smith-Chant, 2015; Träff et al., 2018), and these relationships tend to change during development (Raghubar, Barnes, & Hecht, 2010). Butterworth (2010) claimed that counting, number magnitude processing, or estimation are precursors of arithmetic skills. However, throughout elementary education, this relationship is not linear, nor are the skills involved always the same ones (Geary, 2011; Sowinski et al., 2015). As indicated by Noël and Rouselle (2011), counting can be understood as the ability by which children are able to apprehend the cardinality and ordinality of the number based on the principles of Gelman and Gallistel (1978). There is longitudinal evidence of the predictive relationships between counting assessed in kindergarten and subsequent arithmetic achievement in 5th grade of elementary school (Hannula-Sormunen, Lehtinen, & Räsänen, 2015). Regarding number processing skills, Arabic magnitude processing has been related to tasks of subtraction with multi-digit numbers in 3rd and 4th grade (Linsen, Verschaffel, Reynvoet, & De Smedt, 2014, 2015). There is also evidence that relates comprehension of the numerical system, analyzed through tests of place value, with arithmetic achievement in 3rd grade (Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011). In addition, it has been claimed that transcoding between verbal and written notations with multi-digit numbers seems to evolve throughout elementary school (Skwarchuk & Anglin, 2002), from the initial moments when school children display difficulties in all cases (Dowker, 2005) until the final grades in which 5th-grade children still show significant differences in verbal production when using thousands (Skwarchuk & Betts, 2006). Although arithmetic conceptual comprehension has received less attention in the literature, and its definition is not sufficiently consensual (Crooks & Alibali, 2014), this construct has been analyzed through the knowledge of arithmetic symbols (Ploger & Hetch, 2009) and, more concretely, drawing on the derivation of exact and approximate responses, respectively, related to managing the basic arithmetical principles or to decomposing numbers into parts (Baroody, 2006) and to arithmetic estimation

skills (Dowker, 2005). Research of conceptual comprehension indicated that arithmetic achievement can be predicted in 2nd and 3rd grade from measures linked to knowledge of the basic arithmetical principles, after controlling for cognitive skills (Cowan et al., 2011), and that arithmetic or computational estimation continues to be acquired throughout late elementary school (LeFevre, Greenham, & Waheed, 1993). Different meta-analyses and reviews of heterogeneous studies have situated the deficits of elementary school children with mathematical difficulties (MD) in counting (Geary, 2004), number processing (Noël & Rouselle, 2011), and strategies related to the comprehension of arithmetic concepts (Shin & Bryant, 2015). When sample selection is based on arithmetic achievement, the results vary according to the selection criteria used. The use of a stricter empirical criterion places the counting difficulties of children with MD throughout elementary school (Moll, Göbel, & Snowling, 2015) in comprehension of the decimal number system between 3rd and 6th grade (Anderson, 2010), transcoding between notations between 4th and 6th grade (Iglesias-Sarmiento & Deaño, 2016), and the use of the arithmetical principles and arithmetical estimation between 3rd and 6th grade of elementary school (Anderson, 2010). There is also evidence pointing to numerical processing skills linked to the processing of symbolic and non-symbolic magnitudes as defining characteristics of the group with high achievement (HA) in a sample of children (5% sample) between kindergarten and 3rd grade elementary school (Geary et al., 2009). In addition, it has been noted that students who showed the greatest success in arithmetic tasks during elementary school employed more sophisticated strategies (Dowker, 2005; Star, Rittle-Johnson, Lynch, & Perova, 2009).

Like the indicated domain-specific skills, domain-general skills also help explain arithmetic performance (Geary, 2011). Traditionally, variables that have produced mixed results linked to working memory and executive function constructs seem to point to visuo-spatial skills as the differentiating variables between groups with MD (e.g., Berg, 2008; Meyer, Salimpoor,

Wu, Geary, & Menon, 2010), HA groups (Geary et al., 2009; Leikin, Paz-Baruch, & Leikin, 2013), and their peers. Among the executive variables, inhibitory control (e.g., Geary, 2011; Murphy, Mazzocco, Hanich, & Early, 2007) has been pointed out as one of the main sources of alteration in children with MD in the cognitive area, and the central executive, unitarily, has been indicated as the differentiating element of the HA group compared to their age-matched peers (Swanson, 2006).

This multiplicity of general cognitive and domain-specific skills that contribute to differential arithmetic competence requires multidimensional and comprehensive tools or measures to predict the future development of mathematical competence and its deviation, to explain how different general skills work and how they relate to specific skills to achieve competent performance.

In this context, the theoretical explanation of cognitive functioning based on the PASS theory (Das et al., 1994; Naglieri, 2015), operationalized through the Cognitive Assessment System (CAS; Naglieri & Das, 1997) test, is an appropriate approach to explain of how students perform academic tasks differentially (Das & Naglieri, 2001) and to obtain cognitive profiles for the development of concrete action proposals (e.g., Deaño, Alfonso, & Das, 2015; Iseman & Naglieri, 2011).

Another strength of the theory is its permeability to incorporate explanations based on other approaches (Iglesias-Sarmiento, Deaño, Alfonso, & Conde, 2017). Executive functioning is explained in PASS theory through the processes of planning and attention (Das & Misra, 2015). Planning is defined as the executive function that provides cognitive control, selecting, supervising, and self-regulating solutions to problems to achieve the goal, whereas attention refers to focused, selective, and sustained action, concentrating on a few stimuli and inhibiting others depending on the goals (Das et al., 1994). The characteristics of attention link it intuitively

to any arithmetic task, as they involve selectively attending to the task's relevant aspects (Naglieri & Das, 1997), while inhibiting the aspects that are irrelevant to its resolution (Wang, Georgiou, Li, & Tavouktsoglou, 2018). The functions of generation of plans, and of regulation and alternation linked to the planning process, are necessary for the comprehension of all kinds of mathematical tasks (Cai, Georgiou, Wen, & Das, 2016), specifically, for the recovery and use of arithmetic facts (Das et al., 1994), the alternation between numerical notations and arithmetic operations (Wang et al., 2018), and the development of potential strategies to resolve non-automated tasks (Das & Janzen, 2004). Various correlational studies have linked planning to mathematical achievement in 3rd grade (Manamaa, Kikas, Peets, & Palu, 2012) and as of 4th and 6th grade of elementary school (Iglesias-Sarmiento & Deaño, 2011), as well as to arithmetic achievement in 5th grade (Garofalo, 1986). With regard to attention, although it has been established as an important process in academic tasks (Naglieri & Das, 1997), few studies with school samples have reported direct relationships with math achievement (Iglesias-Sarmiento & Deaño, 2011; Warrick, 1989).

The theory uses two cognitive processes to operate with the incoming information: simultaneous processing, whereby the subject integrates stimuli in a perceptual or conceptual entirety, and successive processing, through which stimuli are integrated in a specific order (Naglieri & Das, 1997). At a deeper level of semantic information analysis, simultaneous processing helps to see the interrelationship between separate units of information and to integrate them into more global information units, thus playing an important role in the comprehension of the meaning of the tasks. Successive processing involves processing serially organized information by encoding it in the order provided so that, when it is recovered from memory, it can be executed in the same sequence. The simultaneous and successive processing

tasks of the CAS have been associated, respectively, with tasks that assess the visuo-spatial sketchpad and the phonological component of the working memory (Cai, Li, & Ping, 2013).

Das, Kirby, and Jarman (1979) implicated simultaneous processing in the coding of the decimal numerical system and in the recovery of the arithmetic facts, although, due to its visuo-spatial nature, its implication can be extended to the resolution of multidigit tasks regarding numerical decomposition, the alignment of numbers when counting, and the integration of the different arithmetic procedures for their resolution when the tasks require this, as in the case of multiplication or division (Cowan & Powell, 2014; Raghubar et al., 2009). The implication of successive processing in arithmetic is more closely related to the maintenance of the information during the task and to the recovery of the items and the implementation of the procedures in the correct sequence (Das & Janzen, 2004). Two recent studies have pointed to simultaneous processing as the best predictor of mathematical (Iglesias-Sarmiento & Deaño, 2011) and arithmetic achievement (Iglesias-Sarmiento & Deaño, 2016) in general samples of 4th to 6th grade. In addition, significant relationships have been found between successive processing and math achievement in these grades (Iglesias-Sarmiento & Deaño, 2011).

Results from studies of groups of children with MD linked to their overall mathematical performance show significantly lower performance than controls in the four PASS processes in the last elementary school grades (Iglesias-Sarmiento & Deaño, 2011; Kroesbergen, Van Luit, & Naglieri, 2003). In a recent study with children with severe MD (percentile ≤ 10), selected for their arithmetic performance, Iglesias-Sarmiento and Deaño (2016) reported a different cognitive profile linked to deficits in the processes of attention and simultaneous and successive processing. In addition, within the group of children with difficulties, the data showed that the simultaneous and successive processes differentiated children with MD (percentile ≤ 10) from those with low achievement (performance between the 11th and 25th percentiles).

This study

In this study, we explored the joint contribution of the cognitive PASS processes (planning, attention, simultaneous and successive processing) and domain-specific skills (counting, number processing, and conceptual comprehension) to the differential arithmetic performance achieved by MD and HA pupils compared to their typical achieving (TA) peers in the last three grades of the Spanish elementary school (4th, 5th, and 6th grade). As the ultimate purpose of such a work is to establish concrete profiles that facilitate proper classification and subsequent intervention (Szűcs & Goswami, 2013), the study focused on these three grades in order to link arithmetic achievement to earlier educational grades. Learning arithmetic is completed in the 3rd grade of the Spanish elementary school, so that, in the next grades, the influence of the cognitive and domain-specific variables on the different established levels of arithmetic achievement should be observable. Specifically, in this study, we proposed the following main objectives: (a) to characterize the groups selected according to their arithmetic achievement from a cognitive and mathematical point of view and (2) to explore the ability of cognitive and domain-specific skills to predict membership in the arithmetic achievement groups. The study expands the results of previous research, drawing on a multidimensional approach that provides alternative information about the relationship between general cognitive processes and domain-specific skills when analyzing arithmetic achievement in the final grades of elementary school. Although some studies relate PASS cognitive processes to mathematics, they have not been consistently conducted in the arithmetic domain or with ungifted HA children. This approach is also novel to describe students with MD and HA. In this sense, the joint use of PASS processes and the domain-specific skills that literature has considered as antecedents in earlier educational grades provides differential profiles that can contribute to developing preventive and intervention programs. Finally, this study interprets the results from a

comprehensive approach that reconciles the results obtained from different theoretical positions linked to domain-general skills such as working memory and executive functioning with positions linked to domain-specific skills.

Methodology

Participants

The sample of the study was selected from a cohort made up of all the students between 4th and 6th grade in 7 public schools of the urban and semi-urban areas of the provinces of Ourense and Pontevedra (northwestern Spain), from families of medium sociocultural level and, mostly, with secondary studies (Instituto Galego de Estatística [Galician Institute of Statistics], 2017). In these schools, pupils are taught in the two co-official languages (Spanish and Galician). Students with developmental disorders and children identified by the educational administration as gifted or with special educational needs due to sociocultural aspects (immigrant students who do not know the co-official languages or who are in situations of extreme poverty) were explicitly excluded. Students whose performance was between the 11th and 25th percentiles were also excluded from the study because several recent investigations (e.g., Geary, 2011; Iglesias-Sarmiento & Deaño, 2016) have located a differential cognitive and mathematical profile for this group. Finally, children who did not complete all the tests were also excluded from the study, following the established procedure. The final sample of the study included 110 students (53 boys: $M = 10.5$ years, $SD = 1.17$) from 4th grade ($n = 37$), 5th grade ($n = 36$), and 6th grade ($n = 37$). Student assignment to the different experimental conditions followed empirical criteria based on the performance achieved on the Calculation Scale of the "*Batería Neuropsicológica de Evaluación de las Habilidades Aritméticas*" (BANEVHAR; [Neuropsychological Battery of Assessment of Arithmetic Abilities] Iglesias-Sarmiento, 2009). The inclusion criterion for students in the MD group was having obtained a score equal to or lower than the 10th percentile.

Twenty-six students (16 boys: $M = 10.8$ years, $SD = 1.46$) were selected from 4th ($n = 9$), 5th ($n = 8$), and 6th ($n = 9$) grade. This criterion was adapted to the recent criteria established in the specialized literature for students with severe difficulties (e.g., Geary, 2011; Mazzocco, 2007). The results obtained were contrasted with the teachers' observations. The inclusion criterion for the HA group was having obtained a score equal to or greater than the 90th percentile. Twenty-six students (13 boys: $M = 10.4$ years, $SD = .99$) were selected from 4th ($n = 9$), 5th ($n = 9$), and 6th ($n = 8$) grade. This criterion has a ratio of 1:10, matching the first level established in Gagné's Metric-Based (MB) System (2005) for gifted and talented population. This criterion also coincides with the cut-points established in the two batteries used in this study to identify high general or specific achievement. None of the children attended special units or was identified as gifted by the educational administration. As a control group (26 boys: $M = 10.1$ years, $SD = 1.10$), we selected 58 students: 19 from 4th, 19 from 5th, and 20 from 6th grade, with arithmetic achievement between the 26th and 89th percentiles.

Measures

Cognitive skills. We used the CAS (Naglieri & Das, 1997) battery for the assessment of cognitive functioning. The Spanish adaptation provides standardized scores (100, 15) of the processes of planning, attention, and simultaneous and successive processing, calculated by age groups at intervals of six months for subjects between 5 and 18 years of age. The reliability for the Spanish sample (Deaño, Alfonso, & Fernández, 2006) was .90 (Planning), .89 (Attention), .92 (Simultaneous Processing), and .91 (Successive Processing).

The *Planning* tests, of an executive nature, require the child to develop a plan, evaluate its usefulness, control its effectiveness and, if necessary, correct it or reject it and create a new one according to task demands. The tasks are pencil-and-paper tasks and require the child to locate equal numbers of different complexity in sequences of rows (*matching numbers*), to convert

letters according to a specific code (*planned codes*), and to alternately connect letters and numbers distributed randomly in a sequential order (*planned connections*). The *Attention* scale contains three tasks that analyze focused, sustained, and selective attention (*expressive attention*) through Stroop tests, selecting numbers from a large amount of distractors (*number detection*), and recognizing physically identical pairs of numbers or letters that are alike from the lexical viewpoint (*receptive attention*). The *Simultaneous Processing* scale, which is visuo-spatial, includes verbal and non-verbal contents and requires the integration of the parts in a Gestalt, the comprehension of logical-grammatical relations, and the synthesis of the parts in a group. The child must discover the relationships between the parts of an element (*nonverbal matrices*), choose the correct answer from among six options based on the spatial information presented in writing and orally (*verbal spatial relations*), and identify previously presented figures within complex figures (*figure memory*). In the *Successive Processing* subtests, the child should remember information presented orally. The first two tests require the repetition of monosyllabic words in a particular order (*word series*) and of sentences presenting semantic conflict (*sentence repetition*). The last test (*sentence questions*) requires comprehension of the implicit meaning of the sentences.

Domain-specific skills. We used the tasks of the BANEVHAR (Iglesias-Sarmiento, 2009) for the evaluation of domain-specific skills. This battery, validated in the Spanish context, provides standardized scores (100, 15), calculated by educational level, of the person's individual competence in counting, arithmetical conceptual comprehension, number processing, and calculation. The reliability indices obtained with the standardization sample were .87 (Counting), .82 (Number Processing), .75 (Conceptual Comprehension) and .84 (Calculation). The *Counting* scale includes three groups of different tasks that evaluate the oral knowledge of the numeric string with non-consecutive numbers forwards and backwards (*counting comprehension*), the

specification of the numbers before and after the number provided in written form (*seriation*), and a group of five tasks involving verbal counting and counting of images under timed conditions (*counting speed*).

The *Number Processing* scale includes a group of three classical tasks (*magnitude comparison*) that assess the processing of numerical magnitude in Arabic, verbal, and written notation. Another group of 6 tasks (*transcoding*) analyzes all possible conversions between the three notations. The last three tasks of the scale analyze the processing of the complex structure of the number using bills of fixed values (*numerical value comprehension*), the comprehension of the number as a stable sequence, requesting the child to order a set of 10 numbers with several digits (*ordering multi-digit numbers*), and the place value of the digits in the decimal number system (*place value*).

In the first two tasks of the scale of *Conceptual Comprehension*, the child is requested to write out what each of the arithmetic operations means (*operational definitions*) and to complete five problems with their arithmetical symbol (*verification of arithmetical operations*). The *arithmetical principles* test evaluates the application of the commutative property of addition and multiplication, the principle of $n+1$, the principle of $n-1$, the principle of $n \times 10$, the inverse principle of subtraction and addition, and the inverse principle of multiplication and division. The fourth task (*understanding of quantity*) evaluates numerical semantic information. The fifth task (*estimation*) evaluates the ability to quickly reach the correct result without counting. The final test (*arithmetical routines*) assesses, through an implicit task, the capacity to derive exact answers to non-routine items.

The arithmetic *Calculation* scale includes two tasks that assess the comprehension of arithmetic signs in oral and written form (*operational processing*), four tasks in which the recovery of facts of addition, subtraction, multiplication, and division are evaluated separately

(*mental arithmetic*), and four tasks that evaluate the recovery of facts, rules, and procedures of the four arithmetic operations (*written arithmetic*).

Nonverbal intelligence. In order to control for nonverbal intelligence, the Raven's Progressive Matrixes SPM (General) of Raven, Court, and Raven (1996) were used. The general scale has 60 items organized in five sets (A to E) with 12 items in each set. In each item, the child must complete a series of complex spatial figures by means of analogical reasoning. The test provides centile scores for each educational level. The reliability of the overall scale is between .83 and .90.

Procedure

The study was carried out respecting the ethical standards applicable to this type of research established in general by the Ethical Committee of Clinical Research of Galicia (CEIC). For the initial selection of the sample, we requested the collaboration of the professional specialists of the schools in order to confirm that none of the children showed developmental disorders or special educational needs resulting from sociocultural aspects. Family consent was subsequently requested. The evaluation was carried out in three different sessions in the schools of origin at the end of the school year. The first two sessions were individual, and the third was collective. In the first session, held during the month of May, the CAS battery was applied. During the month of June, the individual evaluation was completed with the BANEVHAR. The average duration of the individual sessions was around 1.5 hours for the first session and 2 hours for the second one. Raven's test was administered in a final collective session of about half an hour with each group. As they are regulated and standardized tests, we explicitly followed the application and scoring instructions contained in them.

Data analysis

In order to characterize the groups, several covariance analyses (ANCOVA) were carried out on the standardized scores of cognitive (planning, attention, simultaneous and successive processing) and domain-specific skills (counting, number processing, and conceptual comprehension) as a function of the arithmetic achievement of the groups (3 groups: MD, TA, HA) and the educational grade (3 grades: 4th, 5th, and 6th grade). Nonverbal intelligence was included as a covariate in order to control for its effect. In the case of domain-specific skills, several analyses were carried out in which the dependent variables were the raw scores of the tasks/groups of tasks included in the Counting (counting comprehension, seriation, and counting speed), Number Processing (magnitude comparison in the Arabic, oral, and verbal written notations, place value, and transcoding), and Conceptual Comprehension scales (operational comprehension, arithmetical principles, and estimation). In this case, the participants' age group (six-month intervals) and nonverbal intelligence were controlled. Bonferroni correction was used for inter- and intra-group comparisons.

Secondly, several discriminant analyses were conducted to examine the extent to which cognitive and domain-specific skills could individually or jointly predict membership in the arithmetic achievement groups. Variables in which significant group differences were found in the previous analyses were selected as predictors.

The *Statistical Package for Social Sciences* (SPSS) program, version 18.0, was used to perform the diverse statistical analyses.

Results

Cognitive achievement by groups and grades

The cognitive achievement of the groups differed in Planning, $F(2,100) = 4.232, p = .05$, $\eta^2_{partial} = .077$, Simultaneous Processing, $F(2,100) = 18.013, p < .001$, $\eta^2_{partial} = .265$, and

Successive Processing, $F(2,100) = 7.961, p = .001, \eta^2_{partial} = .137$ (Table 1). Students with MD obtained significantly lower mean scores than those of the HA group in Planning ($\Delta M = -11.227, p < .05$), Simultaneous Processing ($\Delta M = -21.461, p < .001$), and Successive Processing ($\Delta M = -12.813, p < .01$). The MD group also obtained significantly lower mean scores than the TA group in Simultaneous Processing ($\Delta M = -9.427, p < .01$) and Successive Processing ($\Delta M = -11.570, p = .001$). The TA Group obtained significantly lower means than the HA group in Simultaneous Processing ($\Delta M = -12.034, p = .001$).

<Table 1>

Math achievement by groups and grades

Standard scores. Significant variations in the mean scores by groups were found for counting, $F(2,100) = 4.915, p < .01, \eta^2_{partial} = .090$, number processing, $F(2,100) = 18.886, p < .001, \eta^2_{partial} = .274$, and conceptual comprehension, $F(2,100) = 20.337, p < .001, \eta^2_{partial} = .289$ (Table 2).

In counting, the MD group obtained significantly lower mean scores than the HA group ($\Delta M = -13.027, p < .01$). In number processing and conceptual comprehension skills (Table 2), the MD group obtained significantly lower mean scores than the HA group ($\Delta M_{number\ processing} = -21.214, p < .001; \Delta M_{conceptual\ comprehension} = -25.461, p < .001$) and the TA group ($\Delta M_{number\ processing} = -10.023, p < .01; \Delta M_{conceptual\ comprehension} = -12.534, p = .001$). In turn, the TA group obtained significantly lower mean scores than the HA group ($\Delta M_{number\ processing} = -11.010, p = .001; \Delta M_{conceptual\ comprehension} = -12.927, p = .001$).

<Table 2>

Counting. The results showed significant variations according to the educational grade in the scores of seriation, $F(2,100) = 4.776, p = .01, \eta^2_{partial} = .088$, and counting speed, $F(2,100) =$

3.750, $p < .05$, $\eta^2_{partial} = .070$ (Table 3). In Seriation, 4th graders obtained significantly lower raw scores than 5th graders ($\Delta M = -1.014$, $p < .01$) and 6th graders ($\Delta M = -1.273$, $p < .05$). In Counting speed, 6th-grade students obtained better achievement than 4th ($\Delta M = -46.942$, $p < .05$) and 5th graders ($\Delta M = -33.233$, $p < .05$).

Moreover, a significant Arithmetic Achievement Group x Grade interaction was obtained for the raw scores of seriation, $F(4, 110) = 3.208$, $p < .05$, $\eta^2_{partial} = .115$. Within the group of students with MD, 4th graders obtained significantly lower raw scores than 5th ($\Delta M = -2.083$, $p < .05$) and 6th graders ($\Delta M = -2.222$, $p < .005$). By grade, in 4th grade, the MD group obtained significantly lower mean scores than the HA Group ($\Delta M = -1.778$, $p < .05$).

Number processing. There were significant group differences in the mean scores of verbal-written magnitude comparison, $F(2,100) = 8.736$, $p < .001$, $\eta^2_{partial} = .150$, oral magnitude comparison, $F(2,100) = 4.488$, $p < .05$, $\eta^2_{partial} = .083$, transcoding, $F(2,100) = 6.453$, $p < .01$, $\eta^2_{partial} = .115$, and place value, $F(2,100) = 4.456$, $p < .01$, $\eta^2_{partial} = .083$ (Table 3). Comparing the verbal-written magnitude, the MD group obtained significantly lower raw scores than the students of the HA ($\Delta M = -1.294$, $p < .001$) and TA groups ($\Delta M = -.644$, $p < .05$). The TA students also obtained significantly lower raw scores than the HA students ($\Delta M = -.650$, $p < .05$). Comparing oral magnitude, the MD group obtained significantly lower raw scores than the HA group ($\Delta M = -.950$, $p < .05$). In the transcoding task, the MD group obtained significantly lower raw scores than the HA ($\Delta M = -4.231$, $p < .01$) and TA groups ($\Delta M = -2.684$, $p < .05$). In the task of place value, the MD group obtained significantly lower raw scores than the HA group ($\Delta M = -1.696$, $p < .05$).

By educational grade, the mean scores varied significantly for the tasks of verbal-written magnitude comparison, $F(2,100) = 3.605$, $p < .05$, $\eta^2_{partial} = .068$, place value, $F(2,100) = 3.818$, $p < .05$, $\eta^2_{partial} = .072$, and transcoding, $F(2,100) = 5.330$, $p < .01$, $\eta^2_{partial} = .097$ (Table 3). The

4th-grade students obtained significantly lower raw scores than the 5th and 6th graders in transcoding ($\Delta M_{fifth} = -3.439, p < .05$; $\Delta M_{sixth} = -4.023, p < .01$), and place value ($\Delta M_{fifth} = -1.747, p < .05$; $\Delta M_{sixth} = -2.746, p < .05$), and than the 6th graders in verbal-written magnitude comparison ($\Delta M = -1.532, p < .05$). Fifth grade students also obtained a significantly lower mean score than 6th grade students in transcoding ($\Delta M = -4.340, p < .05$).

A significant Arithmetic Achievement Group x Grade interaction was obtained for the task of transcoding, $F(4,100) = 3.420, p < .05, \eta^2_{partial} = .121$. Within the TA group, the 4th and 5th graders obtained significantly lower raw scores than the 6th graders ($\Delta M_{fourth} = -3.131, p = .01$; $\Delta M_{fifth} = -2.552, p < .05$). By educational grade, in 4th grade, the MD group obtained significantly lower mean scores than the HA ($\Delta M = -10.000, p < .01$) and the TA group ($\Delta M = -6.923, p < .001$).

Conceptual comprehension. In this mathematical skill, the mean scores of the tasks by groups varied significantly in operational comprehension, $F(2,100) = 6.855, p < .01, \eta^2_{partial} = .122$, arithmetical principles, $F(2,100) = 12.648, p < .001, \eta^2_{partial} = .204$, and arithmetic estimation, $F(2,100) = 12.370, p < .001, \eta^2_{partial} = .150$ (Table 3). In operational comprehension, the MD group obtained significantly lower raw scores than the HA ($\Delta M = -2.109, p < .01$) and TA groups ($\Delta M = -1.462, p < .05$). In arithmetical principles, the MD group obtained significantly lower raw scores than the HA ($\Delta M = -1.870, p < .001$) and TA groups ($\Delta M = -.985, p < .01$). The TA group obtained significantly lower raw scores than the HA students ($\Delta M = -.885, p < .05$). In arithmetical estimation, the MD group obtained significantly lower raw scores than the HA ($\Delta M = -1.382, p < .001$) and TA groups ($\Delta M = -.953, p < .01$).

There were also significant variations in the mean scores by grade in the task of arithmetical principles, $F(2,100) = 4.436, p < .05, \eta^2_{partial} = .082$, (Table 3). The 4th-grade

students obtained significantly lower raw scores than the 5th ($\Delta M = -1.238, p < .05$) and 6th graders ($\Delta M = -1.736, p < .05$).

There was a significant interaction of Arithmetic Achievement Group x Grade for the task of arithmetical principles, $F(4,100) = 3.230, p < .05, \eta^2_{partial} = .115$ (Table 3). In the group of students with MD, the 4th graders obtained significantly lower raw scores than the 5th ($\Delta M = -1.847, p < .05$) and 6th graders ($\Delta M = -3.556, p < .001$). In the TA group, the 4th graders obtained significantly lower raw scores than the 5th ($\Delta M = -1.368, p < .05$) and 6th graders ($\Delta M = -1.463, p < .01$). Depending on the grade, the 4th graders of the MD group obtained significantly lower raw scores than the HA ($\Delta M = -3.556, p < .001$) and the TA group ($\Delta M = -1.959, p < .05$). In the 5th grade, students with MD obtained significantly lower mean scores than HA ($\Delta M = -2.264, p < .01$) and TA students ($\Delta M = -1.480, p < .05$).

<Table 3>

Prediction of arithmetic achievement by groups

With the first group of discriminant analyses, we determined the extent to which cognitive (C1: planning, simultaneous and successive processing) and domain-specific skills (M1: counting, number processing, and conceptual comprehension) can predict individual (C1; M1) and conjoint (C1+M1) membership in the MD, TA, and HA groups (Table 4). Wilks' Lambda was significant for cognitive variables in the first analysis, $\lambda = .65, \chi^2(6) = 45.12, p < .001$, with 51.8% of the students correctly classified (57.7% of the students with MD; 65.4% of the HA students). Wilks' Lambda was also significant in the second analysis performed with the domain-specific variables individually, $\lambda = .63, \chi^2(6) = 48.80, p < .001$, with 59.1% of the students correctly classified (80.8% MD; 69.2% HA). The conjoint introduction of cognitive and domain-specific variables in the third analysis provided the best prediction, correctly classifying 63.6% of

the original cases (84.6% of the students with MD; 80.8% of the HA students), $\lambda = .55$, $\chi^2(12) = 61.85$, $p < .001$. Based on the standardized coefficients of canonical discriminant functions and the structure coefficients, the variables that contributed the most to the discriminant function were conceptual comprehension, number processing, and simultaneous processing (Table 5).

<Table 4>

In addition, three discriminant analyses were carried out in order to specify the individual and conjoint contribution of cognitive (C2: simultaneous processing and successive processing) and domain-specific skills (M2: number processing and conceptual comprehension) when predicting membership in the group with MD versus their TA peers. Wilks' Lambda was significant for the cognitive variables, $\lambda = .81$, $\chi^2(2) = 16.94$, $p < .001$, with 63.1% of the students correctly classified (61.5% of the students with MD), and for domain-specific variables, $\lambda = .77$, $\chi^2(2) = 20.90$, $p < .001$, with 75% of the students correctly classified (76.9% of the MD students). Although the conjoint introduction of cognitive and domain-specific variables improved the prediction, correctly classifying 76.2% of the original cases, in the case of MD, the percentage of classified students did not increase (Table 4), $\lambda = .72$, $\chi^2(4) = 26.51$, $p < .001$. The variables that contributed the most to the conjoint discriminant function were number processing, conceptual comprehension, and successive processing (Table 5).

<Table 5>

Similarly, with the aim of investigating the predictive value of cognitive (C3: simultaneous processing) and domain-specific skills (M3: number processing and conceptual comprehension) in the differentiation between the HA and TA groups, three analyses were performed (Table 4). Wilks' Lambda was significant for simultaneous processing, $\lambda = .81$, $\chi^2(1) = 16.77$, $p < .001$, with 71.4% of the students correctly classified (65.4% of the HA students) and for domain-specific variables, $\lambda = .81$, $\chi^2(2) = 17.23$, $p < .001$, also with 71.4% of the students

correctly classified (73.1% of the students with HA). The conjoint introduction of cognitive and domain-specific variables improved the prediction, correctly classifying 77.4% of the original cases, although in the case of students with HA, it did not improve the prediction based on the domain-specific variables, $\lambda = .78$, $\chi^2(3) = 20.5$, $p < .001$. The standardized coefficients of the canonical discriminant and structural functions, respectively, indicated that simultaneous processing, number processing, and conceptual comprehension were the variables that contributed the most to the conjoint discriminant function (Table 5).

Discussion

The central goals of the current study were to characterize the groups selected according to their arithmetic achievement and to explore the ability of cognitive and domain-specific skills to predict membership in the achievement groups.

Regarding the first objective, the results obtained by the children with MD place their cognitive achievement below that of the groups with TA and HA in simultaneous and successive processing, coinciding with recent research pointing to cognitive weaknesses in these processes in children with MD (Iglesias-Sarmiento & Deaño, 2016; Kroesbergen et al., 2003), and with the literature emerging from the working memory construct, which places the children's deficits in the visuo-spatial sketchpad and in verbal memory span (Raghubar et al., 2010). Regarding the executive variables analyzed from the PASS model, no differences were found in the MD group compared with the TA group. These results contrast with those obtained in other studies with samples selected for their overall math achievement (Cai et al., 2013; Kroesbergen et al., 2003). In the line established by Murphy et al. (2007), the use of stricter cut-off criteria in the case of the MD and TA groups and focusing on the arithmetic domain, including the four arithmetic operations, may explain the differences with other investigations that have used broader selection

criteria and/or curricular mathematical tests with higher executive demands (Cai et al., 2013; Kroesbergen et al., 2003). Another possible interpretation of these results involves lower planning in the MD group's performance compared to that of the HA group. The inclusion of HA students in the control groups in some research (e.g., Iglesias-Sarmiento & Deaño, 2016; Kroesbergen et al., 2003) could condition the results obtained, depending on the weight of this sample. Regarding the domain-specific skills of the MD group, the global data support and extend the findings of the literature related to the deficient achievement in number processing and conceptual comprehension in these educational grades, both compared with the TA group (e.g., Noël & Rouselle, 2011; Shyn & Bryant, 2015), and, in a novel way, with the HA group. Specifically, in the lines of previous research, the study data place the difficulties of children with severe MD compared to their TA peers in transcoding between notations (Moll et al., 2015), number processing in the verbal magnitude (Iglesias-Sarmiento & Deaño, 2016), comprehension of operations and arithmetic principles, and the use of arithmetic estimation (Anderson, 2010). In the case of the Arabic notation, the results of the study contradict those of other studies (e.g., Landerl et al., 2009; Moll et al., 2015) that consider deficits in Arabic notation an intrinsic characteristic of children with severe MD in these educational grades. In this study, the selection of the sample was unusual, as we used the correct answers in the task as a criterion of success instead of its solving times. The use of both criteria may be of interest for future research, given the relevance of the processing of the Arabic magnitude as a predictor of arithmetic achievement. However, the literature has not managed to specify the nature of this relationship (De Smedt, Noël, Gilmore, & Ansari, 2013). Regarding the PASS cognitive functioning of the HA group, the results indicate the simultaneous processing as the only discriminating variable with respect to the TA control group. These novel results in this theoretical context appear to converge with the studies of 3rd grade HA children selected for their mathematical achievement, which indicate the

visuo-spatial sketchpad as one of the differentiating variables with regard to the control group (Geary et al., 2009; Leikin et al., 2013). On another hand, no significant differences were found with the TA group in the executive variables, in contrast to the results of Swanson (2006), who, from the construct of the working memory, indicated the central executive as the differentiating element. In addition to the theoretical construct and the tests employed, in that study, the educational grades and the selection of the samples with HA and TA were different, and age was not specifically controlled, nor were standardized scores used. It might be interesting to repeat this study with children in earlier grades to determine whether the PASS variables that differentiate the groups are the same and to introduce variables related to the executive function and working memory to control for this effect beyond the differences in planning with the MD group. Regarding the domain-specific skills analyzed, the performance achieved by the HA group was significantly higher than that of the TA group in two of the three standard measures: number processing and conceptual comprehension. These data are novel because no comparable studies that use factorially formed variables have been located. The individual analysis of the domain-specific tasks focuses on differences in processing the written verbal magnitude and in the mastery of the basic arithmetical principles. In contrast to the study carried out by Geary et al. (2009), which established differences in 1st grade associated with the task of number sets, in this study, no differences were found in the processing of the symbolic Arabic magnitude. This may be due to the fact that we used the final grades of elementary school, we compared groups with average or high arithmetic competence, and their selection was associated with oral and written arithmetic tasks with Arabic numbers. An interesting finding is that, in HA students, the domain-specific skills appear to be consolidated when they enter the 4th grade, whereas the other groups continue to progress throughout the analyzed grades in aspects like counting speed, seriation, transcoding, place value, and the use of arithmetical principles. In this line, it could be interesting

to study whether the greater mastery of these mathematical variables could represent an advantage for HA children when they face new, more complex, or formulated tasks, as in the case of solving arithmetic problems, in written oral or verbal format. It would also be important to establish longitudinally at which moment the different skills are consolidated. One of the key findings of this study, related to the second objective of the study, is that domain-specific skills seem to classify children better than cognitive skills, although the predictive value of both skills is confirmed when identifying students with MD and HA. Number processing and conceptual comprehension were shown to be the most consistent classification variables throughout the discriminant analyses performed, as they both contributed to the identification of students with MD and HA. These results seem to provide new evidence about the link between low arithmetic performance and deficits in number processing (Noël & Rouselle, 2011) and conceptual tasks that evaluate estimation or arithmetic principles (Shin & Bryant, 2015). In addition, the study data point to conceptual comprehension as a concrete predictor of high arithmetic achievement and extend to these educational grades the results of Geary et al. (2009), who considered numerical competence as the defining variable of the HA group. In the case of cognitive skills, in line with the results obtained in studies that have used regression analysis with undifferentiated samples (Iglesias-Sarmiento & Deaño, 2011, 2016), simultaneous processing was the cognitive variable that best classified the three groups. In any case, the paired discriminant analyses performed delimit the cognitive functioning of the groups, showing that simultaneous and successive processing, respectively, are the cognitive variables that best predict membership in the groups with HA and MD. The results obtained with the HA group seem to converge with the few existing comparative studies of adolescent students that consider visuo-spatial performance as the general differentiating variable (Dark & Benbow, 1991; Leikin et al., 2013). Although there are no studies so far that have considered successive processing as a predictor of severe

MD, the findings of this study may be supported by research that has linked elementary school children's arithmetic achievement to successive processing (e.g., Iglesias-Sarmiento & Deaño, 2016; Manamaa et al., 2012) and also with other studies that, using related constructs, have pointed to updating/working memory as predictors of MD (e.g., Mabbott & Bisanz, 2008; Toll et al., 2011). Along the lines of Das and Janzen's (2004) argument, these findings could be indicative of the problems of students with MD to keep information in the working memory active and to retrieve arithmetic facts and implement the specific procedures of each operation in the correct sequence. Summing up, the results of the study seem to differentiate the MD, HA, and TA groups on a continuum of cognitive and domain-specific variables. In addition, the characterization of a non-gifted HA group contributes new findings to be considered in future research. In any case, it would be interesting to incorporate students with poor achievement, intellectually gifted students, or students with disorders such as ADHD to address the exact reality of the classroom. Secondly, the data found suggest the relevance of cognitive and domain-specific skills in predicting the arithmetic achievement in final grades of elementary school (Sowinski et al, 2015; Träff, 2013). In this case, the main contribution of the study is that a different combination of cognitive and domain-specific skills seems to underlie the arithmetical achievement shown by the groups with MD and HA compared to their peers. These findings have practical educational implications when establishing preventive and intervention proposals linked to the profiles observed. From a mathematical point of view, it seems necessary to reinforce in early educational programs the development of the comprehension and production of number in its different notations and to promote the strategic skills linked to numerical estimation and the basic arithmetic principles. In addition, the results indicate the importance of using multidimensional proposals to promote cognitive skills such as spatial or executive skills, which, in this study, seem to be linked to students who show higher arithmetic performance. These

cognitive skills may have an important facilitating effect on math skills at early ages, when such math skills are not yet automated. The use of empirical criteria, as in this study, limits the generalization of results to contexts where similar criteria are followed. Another limitation of the study is related to the size of the sample of the MD and HA groups. In any event, the use of restricted criteria like those used necessarily implies a readjustment in the size of the groups. It would also be desirable for the sample to be more representative of the educational system, including private schools and introducing other variables that the literature has taken into account, such as reading achievement, language, motivation, anxiety, or socio-educational variables such as parental education, and, especially, to extend the study to earlier grades in order to longitudinally deepen the relationship among the variables and thereby to design early interventions, adjusted by age and group achievement.

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