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TOWARDS A MORE EFFICIENT EVACUATION OF CROWDS BY MEANS OF AN OPTIMAL LOCATION OF EXIT DOORS

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Summary. In this work we present a new strategy, employing optimal control techniques of partial differential equations, to automate the optimization of locations for a given number of exit doors at gathering places, so that the evacuation of crowds takes place in a safer and faster way. Once given a detailed mathematical formulation of the problem, in order to solve the constrained optimal control problem numerically, we propose its full discretization, with a space semi-discretization via the finite element method over a family of triangular meshes of the domain under study, and a time semi-discretization via the Euler algorithm. Finally, for the resulting discretized minimization problem, we try its optimization by means of a derivative-free algorithm. Numerical examples, corresponding to different scenarios for a real-world study case posed on "Plaza de la Liberación" (Guadalajara, Mexico), are presented and discussed to assess the effectiveness of our approach.

1 INTRODUCTION

Effective crowd evacuation in emergencies is a key public safety priority. Modelling and analysis of crowd dynamics have been a very active study area in traffic engineering in recent decades, both from a numerical and an analytical viewpoint [1, 2]. Nevertheless, optimal control and optimization of these evacuation processes have been much more sparsely addressed within the scientific literature.

In this work we introduce a method, based on optimal control techniques of partial differential equations, to automate the optimization of locations for a given number of exit doors at gathering places, so that the evacuation of crowd takes place in a safer and faster way.

For a rigorous mathematical setting of the problem, we consider a reformulation of the classical Hughes system with a suitable set of initial/boundary conditions -modelling the flow of pedestrians, characterized by their density and walking velocity- which constitutes the state system of the optimal control problem. The objective function to be optimized in our problem corresponds to minimizing the number of pedestrians left inside the place at the end of the evacuation process. Moreover, we also need to include some constraints on the control (the location of the exit doors), since not all possible door positions are admissible for geometric, organizational or security reasons.

In order to obtain the numerical solution of the constrained optimal control problem, we give a full discretization of the state system, where space is discretized by a finite element method (for a family of triangular meshes of the domain under study), and time is discretized by the Euler algorithm. Then, once rewritten the discretized minimization problem, we suggest its optimization by means of any derivative-free algorithm, due to the hard numerical difficulties involved in the possible computation of the cost functional gradients. In our case, we have chosen two of them: the classical Nelder-Mead algorithm, and a controlled random search procedure.

Some numerical examples, corresponding to two different scenarios for a real-world study case posed on "Plaza de la Liberación" in Guadalajara (Mexico), are presented and discussed in the congress, to assess the effectiveness of our approach. Interested readers can find all of them in our recent publication [3], since only one is shown here.

This full paper is organized as follows: First, the mathematical model proposed to simulate the evolution of pedestrian flow is given. Then, the full details of the formulation of our problem under the structure of an optimal control problem are presented, including some details of the numerical algorithm to solve this problem. Finally, a computational example and some concluding remarks are summarized in last section.

2 THE MATHEMATICAL MODEL

2.1 The State Equations

In the numerical resolution of our control problem we will use a two-dimensional mathematical model, where we denote by $\Omega \subset \mathbb{R}^2$ and $[0,T] \subset \mathbb{R}$ the meeting place and the time interval under study, respectively.

Our formulation is based on the original model introduced by Hughes [4] for the flow of pedestrians, which couples the eikonal system with the continuity equation:

$$\|\nabla\phi\| = \frac{1}{f(\rho)} \quad \text{in } \Omega\times(0,T), \tag{1}$$

$$u(\rho) = -f(\rho)\frac{\nabla\phi}{\|\nabla\phi\|} \quad \text{in } \Omega\times(0,T), \tag{2}$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho u(\rho)) = 0 \quad \text{in } \Omega\times(0,T), \tag{3}$$

that must be closed with a suitable set of boundary conditions on $\Gamma = \partial\Omega$ and initial conditions at $t = 0$. Equation (2) describes the walking velocity $u(x,t)$ for pedestrians, with the direction is prescribed by the normalized gradient of $\phi(x,t)$ and the speed computed by a fundamental diagram f. In our case, we will use the affine diagram: $f(\rho) = u_{max}(1 - \frac{\rho}{\rho_{max}})$. For this fundamental diagram, equation (1) represents walking difficulties for high density cases. At last, equation (3) guarantees the conservation of pedestrians' mass, where its density is represented by $\rho(x,t)$.

Rewriting equation (1) as $\|\nabla\phi\|^2 = \frac{1}{f^2(\rho)}$, and including a Laplacian term in order to increase the stability in its numerical resolution, we can reformulate equation (1) as the nonlinear second order partial differential equation:

$$\|\nabla\phi\|^2 - \epsilon_1^2\Delta\phi = \frac{1}{f^2(\rho)}. \tag{4}$$

In this way, for a small enough parameter $\epsilon_1 > 0$, the viscosity solution of (4) can be understood as a regularization of the original solution of (1).

Then, by introducing the standard direct transformation:

$$\psi = e^{-\frac{\phi}{\epsilon_1}} \tag{5}$$

(with inverse transformation given by $\phi = -\epsilon_1 \ln(\psi)$ [5]), we arrive to the following equivalent equation:

$$\frac{1}{f^2(\rho)}\psi - \epsilon_1^2\Delta\psi = 0. \tag{6}$$

With respect to boundary conditions, we will consider the Γ split into three parts: $\Gamma = \Gamma_w \cup \Gamma_{in} \cup \Gamma_{out}$, representing, respectively, the lateral walls, the entry doors, and the exit doors. Original Hughes model imposes that ϕ must be null on the exit doors, and that $u(\rho)\cdot n$ must vanish on the rest of the boundary (with n the outward unit normal vector to Γ). Thus, from above transformation (5), we need to impose on ψ the boundary conditions $\psi = 1$ on $\Gamma_{out}\times(0,T)$, and $\nabla\psi\cdot n = 0$ on $(\Gamma_w\cup\Gamma_{in})\times(0,T)$.

Finally, to prevent computational instabilities in (2) and (3), we replace those equations by the respective regularized ones:

$$u(\rho) = -f(\rho)\frac{\nabla\phi}{\sqrt{\|\nabla\phi\|^2 + \epsilon_2^2}}, \tag{7}$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho u(\rho)) - \epsilon_3^2\Delta\rho = 0, \tag{8}$$

where $\epsilon_2, \epsilon_3 > 0$ are small enough parameters.

Thus, the state system used in this work will be the following novel reformulation of the Hughes model:

$$\frac{1}{f^2(\rho)}\psi - \epsilon_1^2 \Delta\psi = 0 \quad \text{in } \Omega \times (0,T), \tag{9}$$

$$\nabla\psi \cdot n = 0 \quad \text{on } (\Gamma_w \cup \Gamma_{in}) \times (0,T), \tag{10}$$

$$\psi = 1 \quad \text{on } \Gamma_{out} \times (0,T), \tag{11}$$

$$\phi = -\epsilon_1 \ln(\psi) \quad \text{in } \Omega \times (0,T), \tag{12}$$

$$u(\rho) = -f(\rho)\frac{\nabla\phi}{\sqrt{\|\nabla\phi\|^2 + \epsilon_2^2}} \quad \text{in } \Omega \times (0,T), \tag{13}$$

$$\frac{\partial\rho}{\partial t} + \nabla \cdot (\rho u(\rho)) - \epsilon_3^2 \Delta\rho = 0 \quad \text{in } \Omega \times (0,T), \tag{14}$$

$$\nabla\rho \cdot n = 0 \quad \text{on } \Gamma_w \times (0,T), \tag{15}$$

$$\rho = \rho_{in} \quad \text{on } \Gamma_{in} \times (0,T), \tag{16}$$

$$\rho(0) = \rho^0 \quad \text{in } \Omega. \tag{17}$$

Then, arguing in the standard way, that is, multiplying equations (9) and (14) by test functions ω, integrating by parts in Ω, applying Green's formula and considering above Dirichlet and Neumann boundary conditions (10), (11), (15) and (16), we arrive to a classical variational formulation of the state system. However, to enhance the evacuation of pedestrians avoiding jams in the exit doors, we will replace in this formulation the boundary term $\int_{\Gamma_{out}} (\rho u(\rho) \cdot n)\, \omega\, d\Gamma$ by the reinforcement term $\int_{\Gamma_{out}} \gamma_{out}\, (\rho u(\rho) \cdot n)\, \omega\, d\Gamma$, where $\gamma_{out} \geq 1$ represents a strengthening parameter.

2.2 The Optimal Control Problem

This subsection is devoted to formulating, in a mathematically rigorous way, our control problem: the characterization of the optimal locations of the exit doors -that must remain inside an admissible part Γ_{ad} of the boundary Γ of Ω- in such a way that the evacuation of the mass of pedestrians gathered together in Ω can be carried out as quickly as possible.

With this objective in mind, we will choose as the cost function to be minimized, the number of pedestrians still remaining inside Ω at final time, that is,

$$J = \int_\Omega \rho(x,T)\, dx, \tag{18}$$

where ρ represents the pedestrians density, solution of above variational formulation of the state system (9)-(17). Other possible alternative expressions for the cost functional J can be found in [3].

Thus, the optimal control problem to be solved consists of finding the optimal locations of the exit doors, such that these locations minimize the cost function J, remaining in the admissible part of the boundary Γ_{ad}.

3 THE NUMERICAL MODEL

Above optimal control problem needs to be treated with a suitable computational approach. Firstly, in the discretization of the state system (9)-(17) we will propose a standard finite element method, in order to compute a numerical approximation of the nonlinear optimization problem (resulting from the full space-time discretization of the control problem) by any gradient-free algorithm.

3.1 Discretization in Space and Time

For the time semi-discretization, we consider a natural number $N \in \mathbb{N}$, and define the time step $\Delta t = \frac{T}{N}$. Then, we take the discretized times $t^n = n\,\Delta t$, for $n = 0, \ldots, N$. Then, we discretize the time derivative of ρ in (14) by the Euler explicit method, that is:

$$\frac{\partial \rho}{\partial t}(\cdot, t^n) \simeq \frac{\rho(\cdot, t^n) - \rho(\cdot, t^{n-1})}{\Delta t}, \qquad \text{for } n = 1, \ldots, N. \tag{19}$$

For the space semi-discretization of the domain Ω, we consider a family of triangular meshes τ_h for the polygonal approximation Ω_h of Ω, with characteristic size h, associated to the Lagrange finite element space P_1 (globally continuous, piecewise linear polynomials on Ω_h).

Thus, taking into account this space-time discretization, above variational formulation of the state system can be rewritten as a large system of nonlinear equations, whose solution $\rho_h^n(\cdot) \simeq \rho(\cdot, t^n),\ n = 0, \ldots, N$, will be used to compute the value of the discretized version of the cost function J defined by (18):

$$J_h = \sum_{\tau \in \tau_h} \int_\tau \rho_h^N(x)\, dx, \tag{20}$$

where the integral on each element τ of the mesh τ_h can be approximated by any quadrature formula.

It is worthwhile remarking here that the location of the exit doors (the control in our problem) enters the value of J_h via the definition of the exit boundary Γ_{out} in boundary condition (11).

3.2 Numerical Optimization

Once we know how to calculate the value of the discretized cost function J_h for any arbitrary location of the exit doors, we will now center our attention into the minimization of this function J_h.

In our particular case, due to the fact that we are managing a control-constrained optimal control problem, we previously have to rewrite our original constrained optimization problem as an unconstrained problem by means of the addition of a penalty term P_h to the discretized cost function J_h, where the penalty term P_h corresponds to the compliance with constraints $\Gamma_{out} \subset \Gamma_{ad}$ (that is, P_h takes a very high value if the constraints are not satisfied, and is zero in the other case).

Figure 1: Satellite photo of the real-world domain Ω: Plaza Liberación in Guadalajara (Mexico), a rectangular square of approximately 180 by 92 meters, including many green areas with access restricted to pedestrians.

Then, to minimize this new cost function $F_h = J_h + P_h$, we propose the use of a gradient-free algorithm. In particular, we will use here two alternative methods: the Nelder-Mead simplex algorithm [6], and a controlled random search procedure for global optimization [7].

4 COMPUTATIONAL EXAMPLES

We have developed many computational simulations for a real-world case posed in the main square (Plaza Liberación) of the city of Guadalajara (Mexico), whose satellite photo can be seen in Fig. 1. Several numerical results are presented at the congress, and can be found in our recent paper [3]. A particular example of one of the several finite element meshes of the square employed in our optimization process can be found in Fig. 2.

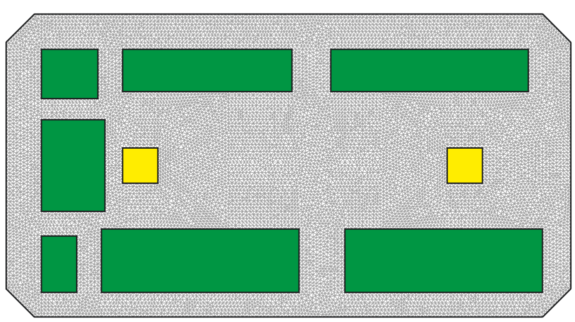

Figure 2: Example of one of the many finite element meshes for the computational domain Ω_h used in the optimization process.

In order to perform our numerical simulations, we have employed the open source scientific software FreeFem++ [8], interfaced with the optimization packages NelderMead (for the Nelder-Mead simplex algorithm) and CRS2 (for the controlled random search with local mutation method).

We have performed a very large number of numerical experiences for very different scenarios, in particular for the optimal location of two exit doors -with fixed width- in two different configurations of the admissible set Γ_{ad}: in a first case the exit doors will be located on the oblique sides at the bottom left and top left corners, and in a second case, in the longer left and right sides. In Fig. 3 we show a graphical description of these two possible admissible regions Γ_{ad} for the computational examples.

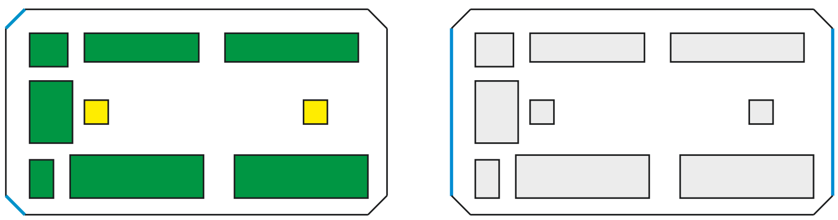

Figure 3: Admissible parts Γ_{ad} (marked with solid blue lines) of the boundary Γ where exit doors can be located for the first case (left) and for the second one (right).

For both scenarios we have solved the optimal control problem by two optimization algorithms (the Nelder-Mead simplex algorithm and the controlled random search with local mutation method), being the former the one that gives better results in these cases, with a lower value of the cost function F_h.

From the results obtained by our methodology, we can deduce that the choice of exit doors in the left corners is a better option (in the sense of an easier evacuation) than doors located in the lateral sides, since the number of pedestrians remaining inside the square in lower for the optimal locations obtained in the first case. In Fig. 4 we show the pedestrian density ρ at final time for the optimal configuration achieved in one of the numerical experiments. We can observe how the area is almost evacuated at final time, except in the neighbourhoods of the exit doors, where a small number of pedestrians remains still inside the square. This fact may show the need for a greater number of exit doors or the extension of their width, in order to ensure the complete evacuation of the enclosure in the given time.

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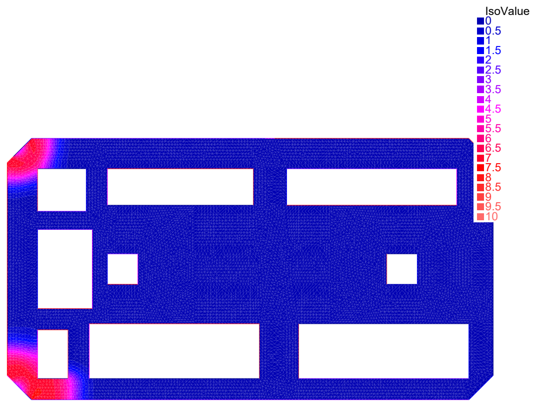

Figure 4: Pedestrian density ρ at final time $T = 350$ seconds for the optimal location of two exit doors, corresponding to the first case of scenario (left corners).

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