

Citation for published version:

Iglesias-Sarmiento, V., & Deaño, M. (2011). Cognitive Processing and Mathematical Achievement: A Study With Schoolchildren Between Fourth and Sixth Grade of Primary Education. *Journal of Learning Disabilities*, 44(6), 570-583.
<https://doi.org/10.1177/0022219411400749>

Peer reviewed version

Link to published version: <https://doi.org/10.1177/00315125221098635>

General rights:

© Hammill Institute on Disabilities 2011, [Article Reuse Guidelines](#).
Users who receive access to an article through a repository are reminded that the article is protected by copyright and reuse is restricted to non-commercial and no derivative uses. Users may also download and save a local copy of an article accessed in an institutional repository for the user's personal reference. For permission to reuse an article, please follow our [Process for Requesting Permission](#).

Cognitive processing and mathematical achievement: A study with schoolchildren between 4th
and 6th grade of primary education

Valentín Iglesias-Sarmiento¹ and Manuel Deaño-Deaño²

University of Vigo

¹ Valentín Iglesias-Sarmiento. Departamento de Psicología Evolutiva y Comunicación. University of Vigo. Campus Ourense. Spain. Telephone: (+34) 981956118. E-mail: visarmiento@uvigo.es

² Manuel Deaño-Deaño. Departamento de Psicología Evolutiva y Comunicación. University of Vigo. Campus Ourense. Spain. Telephone: (+34) 988387179. E-mail: deano@uvigo.es

*Correspondence and tests concerning this article should be sent to Manuel Deaño Deaño. Departamento de Psicología Evolutiva y Comunicación. Facultad de Ciencias de la Educación. Universidad de Vigo. Campus Ourense. As Lagoas S/N. 32004 Ourense. Spain.

Abstract

This investigation analyzed the relation between cognitive functioning and mathematical achievement in 114 students from 4th, 5th, and 6th grade. Differences in cognitive performance were studied concurrently in three selected achievement groups: Mathematical Learning Disability Group (MLD), Low Achieving Group (LA), and Typical Achieving Group (TA). For this purpose, performance in verbal memory and in the PASS cognitive processes of planning, attention, simultaneous and successive processing was assessed at the end of the academic course. Correlational analyses showed that the phonological loop and successive and simultaneous processing were related to mathematical achievement at all three educational grades. Regression analysis revealed simultaneous processing as a cognitive predictor of mathematical performance, although the phonological loop was also associated with higher achievement. Simultaneous and successive processing were the elements that differentiated the MLD from the LA groups. These results show that, of all the variables analyzed in this study, simultaneous processing was the best predictor of mathematical performance.

Cognitive Processing and Mathematical Achievement:

A Study with Schoolchildren between 4th and 6th Grade of Primary Education

Although the first scientific approximations to the study of people with disabilities in the mathematical domain are from almost a century ago, only recently has a series of investigations emerged that focuses specifically on mathematical learning disabilities (MLD) in childhood and adolescence. This reality in the field of research contrasts with the social interest aroused by the topic. The curricular area of mathematics includes a high percentage of academic failure, and some publications with high impact in the international sphere (e.g., Badian, 1983; Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005; Gross-Tsur, Manor, & Shalev, 1996; Hein, Bzufka, & Neumarker, 2004; Lewis, Hitch, & Walker, 1994) have situated the prevalence of the difficulty between 3.6 and 9.8%. These percentages usually increase by country. For example, the Programme for International Student Assessment (PISA) 2006 (OECD, 2007a, b) has revealed that, in the case of Spain, 8.6% of the participants did not even reach the minimum level.

Standardized tests have traditionally been promoted to select children with MLD. The assignation criterion has been discussed in recent years, suggesting the percentiles 25 (e.g., Koontz & Berch, 1996; Siegel, 1999), 30 (e.g., Geary, Hoard, & Hamson, 1999; Jordan & Montani, 1997), and 35 (e.g., Geary, 2005; Jordan, Hanich, & Kaplan, 2003) in order to put an end to all possible casuistry. Currently, various investigators (e.g., Chong & Siegel, 2008; Geary, Hoard, Nugent, & Byrd-Craven, 2008) have followed the proposal of Mazzocco and colleagues (e.g., Mazzocco, 2007; Murphy, Mazzocco, Hanich, & Early, 2007), based on the use of a criterion that is more consistent with the prevalence studies, placing the cut-off criterion for

MLD around percentile 10. However, these authors specify a low achieving group (LA), quantitatively located at scores that are equal to or lower than percentile 25.

Recent cognitive research has focused on the cognitive mechanisms underlying the mathematical deficit. There are two contrary positions although currently, various authors (e.g., Geary et al., 2009; Passolunghi, Vercelloni, & Schadee, 2007) have taken a comprehensive stance. The first position has indicated that MLDs are the result of a specific deficit in numerical representation (e.g., Dehaene, 1997; Landerl, Bevan, & Butterworth, 2004). The second theoretical perspective, which we have assumed in this study, focuses on the analysis of the relations between mathematics and diverse basic cognitive mechanisms. From this position, authors usually have focused their research on the mediator capacity of the different components of the working memory (e.g., Adams & Hitch, 1997; Geary, Brown, & Samaranayake, 1991; Passolunghi & Siegel, 2004; Swanson & Beebe-Frankenberger, 2004), although in recent years, diverse investigations have appeared (e.g., Fuchs et al., 2006; Krajewski & Schneider, 2009; Kroesbergen, van Luit, & Naglieri, 2003) that have revived interest in understanding the functioning of the underlying cognitive processes and their mediating role in mathematical achievement.

From a cognitive processing approach, Das, Naglieri, and Kirby (1994) expanded the conceptualization of intelligence based on the IQ, given its limitation to deal with the identification of learning disabilities and intervention (Das & Abbot, 1995; Mercer, 1997; Siegel, 1999; Stanovich, 1999). They redefined intelligence as a function of four basic psychological processes: planning, attention, and simultaneous and successive processing (PASS). These mental functioning units are based on the work of Luria (1966, 1980), who established that human cognitive processing requires the cooperation of three functional systems that work

together and whose participation is necessary for any type of mental activity (Luria, 1973). The first functional unit is responsible for regulating cortical tone and maintaining attention; the second unit receives, processes, and stores information, encoding it successively and simultaneously; and the third unit programs, regulates, and directs mental activity.

With these origins in neuropsychology, Das and colleagues elaborated a theory of cognitive processing (Das et al., 1994). According to these authors, planning is the process by which individuals determine, select, apply, supervise, and assess the possible solutions to problems, self-regulating their performance to achieve the desired goal. The process of attention, supported by Luria's first functional unit, allows individuals to perform a focalized cognitive activity, selective and sustained over time, focusing on some stimuli and inhibiting others depending on the goals pursued (Das et al., 1994). In order to deal with incoming information, individuals use two cognitive processes: simultaneous processing, by which they integrate stimuli into a perceptive or conceptual whole, and successive processing, by which they integrate stimuli into a specific serial order, forming a chain-like progression (Naglieri & Das 1997a). In simultaneous processing, the relations between the elements of incoming information are used to produce a single or integrated code (Kirby & Das, 1990). Successive processing is required to produce and store a set of sequentially ordered data, although the information may not have been presented sequentially. The only apparent relation in information seems to be sequential or temporal (Kirby & Das, 1990). Successive coding takes up as much space in the active memory as there are units within the code (Das et al., 1994).

The neuropsychological view of intelligence of the PASS model is different from the psychometric view in that it attempts to resolve how the mind works, anchoring its functions in the brain and discriminating dysfunctions (Das, 1988) and it has been operationalized through the Das-

Naglieri: Cognitive Assessment System (D.N:CAS, Naglieri & Das, 1997b), based on the PASS theory. Its authors have operationalized the subtests that measure the processing units (Das & Naglieri, 2001; Naglieri & Das, 1987; Naglieri, Das, Stevens, & Ledbetter, 1991) as a potential system to discover individual differences, dysfunctions, and disabilities (Das & Naglieri, 2001; Naglieri & Das, 1997b; Naglieri et al., 1991) and they also provide the basis for an intervention system (Das, 1999, 2000).

The PASS measures have been empirically related in diverse populations to measures of academic achievement such as reading (e.g., Joseph, McCachran, & Naglieri, 2003; Parrila, Kendrick, Papadopoulos, & Kirby, 1999; Solan, Shelley-Tremblay, Ficarra, Silverman, & Larson, 2003) and writing (e.g., Naglieri & Das, 1997b; Johnson, Bardos, & Tayebi, 2003; Naglieri & Rojahn, 2004). In mathematics, the literature has reported relations between measures of mathematical achievement and the PASS cognitive processes (Das et al., 1994) simultaneous and successive processing (e.g., Garofalo, 1986; Kroesbergen et al., 2003; Kroesbergen, Van Luit, Naglieri, Taddei, & Franchi, 2010; Naglieri & Das, 1987), planning (e.g., Ashman & Das, 1980; Das & Heemsbergen, 1983; Garofalo, 1986; Joseph & Hunter, 2001; Kirby & Ashman, 1984), and attention (Kroesbergen et al., 2003; Warrick, 1989) and has concluded that successive processing correlates with mathematical performance, but generally at a lower level than simultaneous processing (Das, 1988; Garofalo, 1986; Leong, Cheng, & Das, 1985).

Summing up, this is a multidimensional approach whose processes may explain the components of human performance in specific learning domains and cognitive learning strengths and weaknesses from an approach to *how* people encode information, rather than *how much* information they have (Das & Abott, 1995).

In the mathematical domain, various authors (e.g., Battista, 1994; Bishop, 1989; Hermelin & O'Connor, 1986) have held that mathematical reasoning is facilitated by the individual's capacity to interrelate spatial images and verbal propositions. Various studies have shown that students with high ability to solve spatial problems achieve good results in sciences and mathematics (e.g., Baker & Talley, 1972, 1974; Bodner & McMillen, 1986; Diezmann & Watters, 2000, Wai, Lubinsky & Benbow, 2009). In psychometric research, spatial ability is a construct usually identified as the result of the factor analysis of a battery of tasks, and not deduced from an established theory (Watters & English, 1995). The relations between spatial ability and individual characteristics may be reconsidered from Luria's (1973) neurophysiological theory and the operationalization carried out by Das et al. (1994) using the PASS model. From this theory, spatial ability can be understood in simultaneous and quasi-spatial format (Das & Varnhagen, 1986). Thus, tasks associated with spatial ability such as mathematics (Diezmann & Watters, 2000) seem to be easier for students who process information simultaneously rather than sequentially (Das & Varnhagen, 1986; Watters & English, 1995). Likewise, performance in other tasks such as conservation, transitive inference, or class inclusion is higher in children who use simultaneous processing rather than successive processing (Das & Verhagen, 1986).

This investigation focuses on the study of students' MLDs and their underlying cognitive skills; therefore, a test based on a theory like the D.N: CAS seems particularly useful, not only for diagnosis (Naglieri, 1999) and instruction (Naglieri & Gottling, 1997), but also to provide information about the cognitive strengths and weaknesses of MLD students, which are especially relevant to design instructional and specific intervention programs (Kroesbergen et al., 2003), and because of its reassessment of spatial ability.

Outside of the sphere of the PASS theory, some recent studies (Fuchs et al., 2006; Hecht, Torgesen, Wagner, & Rashotte, 2001; Krajewski & Schneider, 2009; Passolungi et al., 2007) have analyzed the relation between the level of phonological ability and performance in diverse mathematical and arithmetical tasks. The significant relation between simultaneous and successive processing and phonological processes was hypothesized by Kirby and Williams (1991) and recently confirmed by Joseph et al. (2003), who particularly identified successive processing as the best predictor of phonological processing. The results of studies of the relation of phonological processing and mathematical performance are still inconclusive because, although in various studies, the importance of phonological processing—and specifically of phonological awareness—in early arithmetic performance has been pointed out (Hecht et al., 2001; Koponen, Aunola, Ahonen, & Nurmi, 2007; Simmons, Singleton, & Horne, 2008), other investigations (e.g., de Jong & van der Leij, 1999; Passolungi et al., 2007) have found no predictive relations.

Following a developmental perspective, Naglieri and Das (1987) showed how planning and simultaneous and successive processing are related to mathematical achievement in 2nd grade. In 6th grade, the three processes also showed strong relations with mathematical achievement. However, in 10th grade, only successive processing remained at a similar level as simultaneous processing in the relation to mathematics. Kroesbergen et al. (2010), in their study with Italian and German kindergarten children, showed that simultaneous processing at early ages is more closely related to Piagetian-type tasks, whereas planning is more related to counting tasks.

A large part of the cognitive literature has analyzed the capacity of Baddeley and Hitch's (1974; see also Baddeley, 1986, 2000) model to predict mathematical achievement (e.g., Adams

& Hitch, 1998; De Smedt et al., 2009; Geary, Hoard, Nugent, & Byrd-Craven, 2007; Hecht et al., 2001; Swanson & Kim, 2007). From this viewpoint, although mathematical disability has been related to low performance in verbal memory tasks (e.g., Bull, Andrews-Espy, & Wiebe, 2008; Geary, et al., 1999; Passolunghi, Mammarella, & Altoè, 2008; van der Sluis, van der Leij, & de Jong, 2005), in some investigations, the link between an arithmetic deficit and a disorder of the working memory--or of a large part of it--as not been conclusively established (e.g., Landerl et al., 2004; Temple & Sherwood, 2002). Nonetheless, the working memory has been recurrently pointed out as the underlying mechanism in the deficits displayed by children with MLDs (e.g., Geary et al., 2007; Hitch, 1978; Swanson & Sachse-Lee, 2001).

Each component of Baddeley and Hitch's (1974) model seems to be related to specific aspects of mathematical performance. The phonological loop seems to participate in counting (Camos & Barrouillet, 2004; Imbo & Vandierendonck, 2006; Logie & Baddeley, 1987) and in calculation (DeSmedt et al., 2009; Fürst & Hitch, 2000; Logie, Gilhooly, & Wynn, 1994). The visio-spatial sketchpad seems to be involved in multidigit problems that require visual and spatial knowledge (Heathcote, 1994; Trbovich & Lefevre, 2003) and in estimation tasks (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007), although the results are not completely consistent (DeStefano & LeFevre, 2004; Krajewski & Schneider, 2009; Swanson, 2006). According to Baddeley (1996, 2007), the central executive is a set of processes aimed at the assignation of attentional resources (i.e., focalization and inhibition of distracters), the retrieval or change of plans, dealing with the input and the retention, manipulation, and strategic retrieval of information in the long-term memory. The central executive was studied with regard to arithmetic by McLean and Hitch (1999). The results of their study support the participation of this component in the coordination of diverse activities involved in counting and in arithmetic

problem solving (McLean & Hitch, 1999). Many recent investigations (e.g., Bull & Scerif, 2001; Gathercole & Pickering, 2000; Swanson, Jerman & Zheng, 2008; Wu et al., 2008) have confirmed the substantial involvement of the central executive in solving arithmetic word problems.

Currently, some authors have adopted a more developmental view when relating the components of the working memory to mathematical achievement. Specifically, DeSmedt et al. (2009) indicated the visio-spatial sketchpad as a unique predictor of mathematical performance in 1st grade, whereas the phonological loop emerged as a unique predictor in 2nd grade. Passolunghi et al. (2007) revealed the relevance of the central executive as a unique predictor of mathematical achievement at the end of 1st grade. Meyer, Salimpoor, Wu, Geary, and Menon (2010) confirmed the relevance of the phonological loop and the central executive in mathematical performance in 2nd grade, whereas the visio-spatial sketchpad predicted it in 3rd grade. Hecht et al. (2001) pointed out the phonological loop as a unique predictor of arithmetic performance in 2nd and 3rd grade.

Thus, with regard to Baddeley and Hitch's (1974) multicomponent model, MLDs seem to be linked to a deficit in the working memory in the phonological and executive processes, following a specific developmental pattern. As the role of the visio-spatial sketchpad in mathematical achievement is still not completely clear (DeStefano & LeFevre, 2004), we decided not to include this component in the study.

From an alternative viewpoint, Bull and collaborators (Bull & Johnston, 1997; Bull & Scherif, 2001; McKenzie, Bull, & Gray, 2003), following the postulates of Case (1985), have emphasized processing speed (an aspect normally assessed in working memory tasks) as the best

predictor of mathematical capacity, in terms of efficiency. Other recent investigations (e.g., Fuchs et al., 2006; Hecht et al., 2001) have confirmed processing speed as a correlate of mathematical skills, facilitating counting speed in young children and, thus, subsequent development of representations in the long-term memory (Geary et al., 1991). These results have been reported in 2nd- graders (e.g., Bull & Johnston, 1997; Hecht et al., 2001) and in 3rd-graders (Fuchs et al., 2006).

The present study investigates the relations between cognitive processes and mathematical achievement. We proposed three research questions in order to expand the results of previous investigations. The first question refers to the relations between cognitive processes and learning disabilities. Do cognitive processes differentiate achievement groups? And, specifically do they differentiate the LA group from the MLD group? To answer this question, in contrast to other studies, we distinguished TA students, LA students, and MLD students. The second question concerns the relation between cognitive processes and the educational grade: Do the relations between cognition and achievement vary as a function of the educational grade? We wished to determine whether each and every one of the diverse processes is related to achievement in all the grades and whether such relations are of the same intensity. The third question attempts to address the issue of whether mathematical achievement can be explained by underlying cognitive processes, specifically: can mathematical achievement be predicted from cognitive functioning?

Method

Participants

The sample comprised of a total of 114 students (59 boys and 55 girls) distributed equally among the 4th and 6th grades of Primary Education in eight schools from the urban and semiurban setting of the Galician community in Spain. The students were selected according to their performance in a standardized test, following the quantitative criteria recently proposed in the literature (e.g., Mazzocco & Myers, 2003; Geary et al., 2007; Murphy et al., 2007). The mathematical learning disability (MLD) group comprised 21 participants (7 each from 4th, 5th, and 6th grade), and included participants whose standard score was equal to or less than 80 in the test, which represents a score below percentile 11. The age of these children ranged between 8 years and 9 months and 13 years and 2 months, with a mean age of 11 years and 3 months. The low achieving (LA) group comprised 33 children (11 from each educational grade) and included participants whose scores were between percentiles 11 and 25. The age range of these children was between 8 years and 10 months and 13 years and 2 months, mean age 10 years and 8 months. Lastly, the typical achieving (TA) group comprised 60 children (20 from each educational grade), including children whose mathematical performance was equal to or higher than percentile 26 in the standardized test. The ages of this group were between 8 years and 5 months and 13 years, mean age 10 years and 6 months.

The diverse analyses carried out showed that the number of boys and girls did not differ significantly among the groups of mathematical competence at any educational grade, $\chi^2(1, N = 114) < 1$. However, significant differences in mean age were found, as indicated by the factorial ANOVA carried out, when the three groups of mathematical competence were considered conjointly ($p > .05$). As expected, significant differences were found between the mean ages of the different educational grades, $F(2, 105) = 125.514, p < .0005$. The factorial ANOVA did not

reveal any significant interaction among the groups of mathematical competence and the educational grade with regard to mean age ($p > .05$).

Measures

Mathematical achievement. To assess the children's mathematical competence, we used the Neuropsychological Battery of Assessment of Mathematical Skills (abbreviated to BANEVHAR; Iglesias-Sarmiento, 2009). This battery was designed as a comprehensive instrument that provides detailed information about the child's mathematical competence with regard to the child and to the group in 4th, 5th, and 6th grade of Primary Education. It is made up of 37 items, grouped into 4 scales. The battery provides different types of standardized scores associated with the scales and the tasks that comprise it. In this study, we administered all four scales in the established order and we used the standardized score (100, 15) of the Global scale as a descriptor of the child's global mathematical performance.

The Counting scale is made up of 8 subtests with which conceptual comprehension of counting and of seriation and counting speed are analyzed. The Arithmetical Conceptual Comprehension scale analyzes, through six tasks, comprehension of arithmetical operations, of the basic mathematical principles, semantic knowledge of the number, and numerical estimation. The Number Processing scale studies comprehension of the number and number production in its various annotations through 13 tasks. Lastly, in the Calculation scale, operational processing, retrieval of facts and mathematical rules, and the procedures of addition, subtraction, multiplication, and division are studied by means of 10 tasks.

The reliability of the battery was calculated with the split-half method (Spearman-Brown formula) for the Arithmetical Conceptual Comprehension, Number Processing, and Calculation

scales. For the Counting scale, we used retest. The reliability of the Global scale was calculated with Nunnally and Bernstein's (1994) formula of linear combinations. The indexes were .87 for the Counting scale, .75 for the Arithmetical Conceptual Comprehension scale, .82 for the Number Processing scale, .84 for the Calculation scale, and .93 for the Global scale. Construct validity was calculated with robust maximum likelihood estimation and the analysis of principal components with Kaiser's orthogonal Varimax rotation. The final solution extracted six factors (Counting, Counting speed, Arithmetical Conceptual comprehension, Number processing, Calculation, and Operational processing), which explained 49.8% of the total variance.

Cognitive processing. We used the Das-Naglieri Cognitive Assessment System (D.N:CAS; Naglieri & Das, 1997b) battery to measure planning, attention, and encoding of information. To assess memory span, we used the digit tests of the Wechsler Intelligence Scale for Children-revised (WISC-R; Wechsler, 1974; Spanish version, 1993), and processing speed was assessed from the results achieved in the Number Detection subtest of the Attention scale of the D.N:CAS.

Reliability of the D.N:CAS for the Spanish sample (Deaño, Alfonso, & Fernández, 2006) was calculated with the split-half procedure for all the simultaneous and successive subtests (except for speech rate), corrected with the Spearman-Brown formula. For the subtests planning, attention and speech rate, we used retest. The mean reliability of the sample of 1222 cases in each one of the scales was .90 (Planning), .89 (Attention), .92 (Simultaneous processing), and .91 (Successive processing). Construct validity was calculated with confirmatory factor analysis carried out separately in four age groups (5-7, 8-10, 11-13, and 14-17 years). The model was assessed through various goodness-of-fit and incremental indexes. The results (the goodness-of-fit [GFI] and adjusted goodness-of-fit [AGFI] indexes were all higher than .90 and

the root-mean-square-residual [RMSR] values were below .10) indicated a good correspondence between the PASS model and the data for each one of the four age groups.

Planning. The Planning subtests present tasks that require the children to make decisions in order to solve them. Solving the tasks requires the children to create an action plan, apply it, verify it according to the original goal, and modify it, if necessary. In turn, it provides the opportunity to observe children's strategies, which can help interpret their performance. Success in the Planning subtests requires the children to elaborate an action plan, assess its utility, control its effectiveness, correct or reject an old plan when the task requires a change, and to control impulsive performance. The tasks included in Matching Numbers require the children to locate and underline the two numbers that are the same in the different rows presented. In the next subtest, Planned Codes, a caption is presented that shows correspondence between letters and codes. The children's task is to fill in the empty boxes under each letter with the corresponding codes and discover their internal organization to solve the task. The last subtest, Planned Connections, requires the children to join a series of numbers that are randomly distributed in space in a sequential order and to alternately connect numbers and letters serially. The items are designed so that the children never complete a sequence by crossing one line over the other. This way, there are fewer areas to search when looking at the next number or letter. The use of problem-solving strategies in this task is obvious; it requires connecting the ordered stimuli to complete a sequential pattern.

Attention. In the Attention subtests, the children must use focal attention to detect a particular stimulus and avoid responding to irrelevant stimuli. The first one, Expressive attention, measures selective attention and is made up of the Stroop task. In the subtest, an interference condition is established that is applied after solving the items without this condition.

In this condition, the children are required to identify the color of a word that is printed in a different color ink than the color the word names. The notion of interference reflects the quality of selective attention. The children are requested to name the color ink the word is printed in, rather than read the word. This last item is applied to measure selective attention. In the second subtest, Number Detection, the children's selectiveness and capacity to resist distraction are measured. The children must underline the correct numbers among a large quantity of distracters. This task is carried out under time pressure, a measure we have used as a base score of processing speed. In the third subtest, Receptive Attention, letters are presented for the children to point out the physically identical pairs and then, the lexically similar pairs, underling row by row all the pairs of letters that are physically the same (e.e., TT, or tt) or the pairs of letters that have the same name (e.g., Aa).

Memory. To assess memory span, we used the Digit Span tests of the Wechsler Intelligence Scale for Children-Revised (WISC-R; Wechsler, 1974; Spanish version, 1993).

In the Digit Span Forward test, a series of numbers are read out loud, at a rate of one per second. Two series are presented for each element. The length of each element varies between 3 and 9 digits. This test was used as a measure of the storage capacity of the phonological loop.

The Digit Span Backward test presents a similar system, although the length of each element varies between 2 and 8 items. In this case, the children are asked to begin with the last number provided and to follow the sequence backwards. This task has been specifically involved in the assessment of the central executive of the working memory.

Processing speed. Processing speed was assessed from the results achieved in the Number Detection subtest of the Attention scale of the Spanish adaptation of the D.N:CAS (Naglieri & Das, 1997b). This individual test is carried out under time pressure and involves

seeking and underlining the digits with an identical format presented on a page divided into 15 rows with 14 numbers in each row. Response times were registered in seconds, analyzing it according to an inverse criterion (i.e., lower scores mean better performance).

Simultaneous Processing. This scale includes tasks that require the perception of the parts of a gestalt, the comprehension of logical-grammatical relations, and the synthesis of the parts into integrated groups, using both verbal and nonverbal content. This takes place through the examination of stimuli during the activity or the recall of the stimuli. To measure this kind of processing, firstly, we used the Nonverbal Matrixes subtest, in which the individual has to discover the relations among the parts of an element. Participants must choose one of the six options that complete the nonverbal analogy presented in the form of a matrix. In the second subtest, Verbal-Spatial Relations, illustrations are shown with a specific configuration of the elements; then, a question is asked and the children must select the option that corresponds to the verbal description. The logical-grammatical descriptions and spatial relations must be understood to perform this test. In the subtest, Figure Memory, a figure is shown for some seconds, then removed and another, more complex drawing is shown. The task consists of identifying the original figure that is embedded within the larger figure.

Successive Processing. The subtests of this scale require the individual to use the information presented in a specific order that is necessary to understand its meaning, the perception and reproduction of the natural sequence of stimuli, the comprehension of sentences based on syntactic relations, and the articulation of isolated sounds in a consecutive sequence. In the tasks of the successive processing subtests, the individual reproduces a particular sequence of questions about events or responses, which require the correct interpretation from the linearity of the events (Das et al., 1994). The first subtest corresponds to the so-called Word Series, in

which the children must repeat a series of monosyllabic, frequently used words in the same order as the examiner. Each series varies in length, from two to nine words. The second subtest, Sentence Repetition, requires the children to repeat sentences that are read out loud, which present semantic conflict. In the last subtest, Sentence Questions, phrases that were presented in the Sentence Repetition subtest are read, and the children are asked a series of questions about them. In order to respond to the questions, the children must have understood the implicit meaning of the sentence.

Procedure

The data of the study were collected during the final months of the school year (April-May). Eight investigators were chosen as the assessment team, as a function of their experience in the administration of psychological tests. The assessment team participated in various theoretical and practical training sessions with the two authors of study in order to unify the administrations of the tests. Each child was assessed individually in a specially prepared room in the child's school. Each child was assessed in two sessions. In the first session, the WISC-R was administered and, after a 5-minute rest, the BANEVHAR was administered. Because of the duration of the latter test, the children were allowed to rest for about 5 minutes after completing the first two scales of the battery. Each assessment with the BANEVHAR specifically followed the instructions that accompany the battery. The mean duration of this session was about 2 hours. In the second session, the DN:CAS was administered, according to the standard procedure established in the battery. The mean duration of the sessions was about 1½ hours.

Results

Table 1 presents the means and standard deviations of the main results obtained by the three mathematical competence groups in the proposed tests.

<INSERT TABLE 1 ABOUT HERE>

Group analyses

The results were analyzed with factorial ANOVAs in which the main factors were mathematical competence group (three groups: MLD, LA, and TA) and the educational grade (three groups: 4th, 5th, and 6th grade). The individual results obtained in each experimental task proposed were the dependent variables.

Planning. The factorial ANOVA yielded main effects of the mathematical competence group, $F(2, 114) = 2.90, p = .051$. No significant effects of educational grade were found ($p > .10$). The multiple comparison analyses showed that the MLD group obtained lower scores than the TA group ($p = .05$). No significant differences were observed in planning between the MLD and the LA groups or between the LA and the TA groups ($p > .10$).

Attention. The factorial ANOVA revealed significant effects of the mathematical competence group, $F(2, 114) = 3.29, p < .05$. No significant effects of educational grade were found ($p > .10$). The a posteriori contrasts showed that the performance of the MLD group was significantly poorer than that of the TA group ($p < .05$). No significant differences were found in the performance of the children from the MLD and the LA groups, or between the LA and the TA groups ($p > .10$).

Memory. The factorial ANOVAs revealed main effects of the mathematical competence group when solving the two memory span tasks: Digit Span Forward, $F(2, 114) = 8.64, p$

< .0005; and Digit Span Backward, $F(2, 114) = 5.10, p < .01$. No significant effects of educational grade were found ($p > .10$).

The a posteriori contrast showed a significantly better performance of the TA group in comparison to the MLD ($p < .0005$) and the LA groups ($p < .05$), both in the Digit Span Forward and in the Digit Span Backward ($p < .05$). No significant differences in memory span were observed between the two low achieving groups.

The factorial ANOVA also revealed no significant effects of the mathematical competence group ($p > .10$) in processing speed.

Coding. The factorial ANOVA yielded significant effects of the mathematical competence group in simultaneous processing, $F(2, 114) = 38.54, p < .0005$. No significant effects of educational grade were found ($p > .10$). The multiple comparisons of means revealed a significantly poorer performance of the MLD and LA groups compared to the TA group ($p < .0005$). The follow-up analyses also revealed poorer performance of the MLD group compared to the LA group ($p < .10$).

Likewise, the factorial ANOVA yielded main effects of the mathematical competence group in successive processing, $F(2, 114) = 8.94, p < .0005$. No significant effects of educational grade were found ($p > .10$). The a posteriori contrasts showed a significantly poorer performance of the MLD group compared to the TA ($p < .0005$) and the LA groups ($p < .10$). No significant differences were observed in the performance of the LA and the TA group ($p > .10$).

Correlational analyses

Correlational analyses were used to examine the pattern of relations between the predictor variables and the scores obtained in the Global scale of the BANEVHAR, conjointly and in each of the educational grades.

The review of the correlations for the entire sample showed significant relations among the predictor variables (more pronounced in memory span and coding tasks) and mathematical performance, except for the case of processing speed (see Table 2).

<INSERT TABLE 2 ABOUT HERE>

The analyses of the diverse educational grades showed that the relations among the cognitive measures and mathematical performance are mostly significant in 4th grade (except for executive skills). In 5th grade, these significant relations with mathematical achievement are limited to planning, the storage capacity of the phonological loop, and to both types of processing, simultaneous and successive. At the end of 6th grade, only the phonological loop and successive processing and, to a greater extent, simultaneous processing are significantly related to mathematical performance. Planning, attention, the central executive, or processing speed are not significantly associated with mathematical performance at this educational grade.

Regression analysis

The multiple regression analysis carried out conjointly indicates that simultaneous coding predicts mathematical achievement, $\beta = .55$, $t(111) = 7.50$, $p < .0005$. Likewise, the individual analyses carried out for each educational grade established the predictive capacity of simultaneous processing for mathematical performance in the 4th grade, $\beta = .79$, $t(36) = 7.4$, p

< .0005; 5th grade, $\beta = .40$, $t(35) = 2.89$, $p < .01$; and 6th grade of Primary Education, $\beta = .57$, $t(36) = 4.21$, $p < .0005$. Thus, as performance in simultaneous processing increases, so does mathematical performance in the test proposed.

Moreover, the highest scores in the Digit Span Forward were associated with a better mathematical performance, both in the group analysis $\beta = .25$, $t(111) = 3.32$, $p < .001$, and in the analysis carried out in 5th grade, $\beta = .39$, $t(35) = 2.85$, $p < .01$.

In contrast, no significant relations involving other cognitive variables were found in any of the educational grades analyzed.

Discussion

In this study, we analyzed the relations between mathematical achievement, disability, and the underlying cognitive functioning in a sample of ages that has traditionally been of little interest to researchers. From the operationalization of the deficit established, we studied the differences in the cognitive performance of the groups with mathematical learning disabilities; the relations between cognitive processing and achievement as a function of educational grade, and lastly, we analyzed the prediction of mathematical achievement from the variables planning, attention, information coding, and verbal memory.

The first issue refers to the differentiation among achievement groups and, specifically, the differentiation between the LA and MLD groups. At first glance, the results seem to indicate that the performance of the LA group is the same as that of the TA group in planning, attention, successive processing, and processing speed, but not in simultaneous processing and memory. The performance of the MLD group is as good as that of the TA and LA groups in processing

speed, the same as that of the LA group in planning, attention, and memory, but it differs significantly from the LA group in simultaneous and successive processing.

The three competence groups show a similar performance in processing speed. The studies that have explained the deficits of children with mathematical learning disabilities as a function of calculation speed (Garnett & Fleischner, 1983; Geary et al., 1991; Geary, Widaman, Little, & Cormier, 1987) have indicated that children with MLD are slower than their peers when solving arithmetic problems (Garnett & Fleischner, 1983; Geary & Brown, 1991) and that the speed of fact retrieval is related to performance in mathematical tests (Geary & Burlingham-Dubree, 1989). However, in this study, there was no clear evidence to suggest that the MLD group's processing was slower than that of their peers. The work of McLean and Hitch (1999) points in a similar direction. Thus, these data do not seem to provide consistency to the studies that indicate that processing speed is a specific predictor of mathematical achievement (e.g., Bull & Johnston, 1997; McKenzie et al., 2003). However, at lower educational grades, processing speed may be directly related to adequate mathematical performance, as was observed in the first grade of our study.

Cognitive processes moderately differentiate the LA and MLD groups. The difference is noted in the drop of the scores of simultaneous and successive processing in the MLD group compared to the LA group, and not in specific aspects of the working memory, and this latter result supports the findings of Geary et al. (2007). This difference in scores seems to point to a cognitive weakness in simultaneous processing in the MLD group. This is coherent with the studies of relations between simultaneous processing and mathematical achievement (Garofalo, 1986; Leong et al., 1985; Naglieri & Das, 1987) and the interpretation of specific learning disabilities using theory-based multidimensional tests (Naglieri, 1999; Naglieri & Kaufman,

2001). A more rigorous possibility is to consider the mean score in successive processing as a cognitive weakness, so the difference between the MLD and LA groups could be considered a difference in coding, along the lines of research that establish a direct link between simultaneous and successive processing and mathematical achievement (e.g., Deaño, 2000; Kroesbergen et al., 2010; Naglieri & Das, 1997b). These issues need to be reconsidered in the light of the response to the intervention from cognitive training based on the PASS processes (e.g., Das, 2000).

The second issue focused on the relations between cognition and educational grade. The correlational analyses showed that practically all the cognitive skills assessed correlated significantly with mathematical achievement. The pattern of relations changes across the grades studied. In the 4th grade of Primary Education, we observed a high relation between simultaneous processing and mathematical achievement, and a more moderate one between mathematical achievement and the tests that analyze memory span, successive processing, and attention. At this educational grade, the inverse relation between processing speed and mathematical achievement was also significant. At 5th grade, the pattern of correlational change favors the relation between mathematical achievement and performance in the tests that assess planning, the phonological loop, and coding. In contrast, in 6th grade of Primary Education, only the phonological loop and successive processing, and, to a greater extent, simultaneous processing are significantly related to mathematical achievement. These results could be interpreted inasmuch as 4th grade of Primary Education seems to require students to use attentional resources to deal with information. In 5th grade, the strategies of a plan or the parts of the strategies are placed in sequence. Sixth grade seems to require less of this type of sequences and more recognition of strategies and previously elaborated action plans, adequately coded, and

that can be more automatically applied to new situations without significant changes. Such recognition would mean more activity of simultaneous processing than of successive processing.

The results of the correlational analyses seem to indicate that the phonological loop (as measured by the Digit Span Forward) is related to mathematical performance at all educational grades, and these findings extend the results of previous investigations to these educational grades (e.g., DeSmedt et al., 2009; Hecht et al., 2001). Especially noteworthy is the robust relation of simultaneous and successive processing with mathematical achievement, as revealed by the correlational analyses. In this sense, significant correlations were found between the scores on the scales of simultaneous and successive processing and general mathematical performance in functions that may be more voluntary (4th grade), more strategic (5th grade), or more automatic (6th grade), and in which the successive function becomes less important than the simultaneous one. These results confirm the results of Naglieri and Das (1987), and extend them to other educational grades.

Traditionally, the importance of the central executive of the working memory (e.g., Bull et al., 2008; Geary, Hoard, Byrd-Craven, & De Soto, 2004; Siegel & Ryan, 1989; Swanson & Sachse-Lee, 2001) and of other closely linked processes such as planning (e.g., Kirby & Williams, 1991; Kroesbergen et al., 2003; Naglieri, 2000) or attentional skills (e.g. Lindsay, 2001; Kroesbergen et al., 2003) has been underlined in diverse aspects of academic performance and, specifically, in calculation. Although when analyzed conjointly, the present results provide some evidence that relate planning, attention, and memory span to mathematical achievement, they are more coherent with other investigations that have not been able to conclusively connect weaknesses in the central executive to MLDs (e.g., Landerl et al., 2004; Temple & Sherwood, 2002).

The final question refers to whether mathematical achievement can be explained by the underlying cognitive processes or, more specifically, whether it can be predicted by cognitive functioning. The regression analysis identified the phonological loop as a significant cognitive predictor of mathematical performance in the global analysis and in 5th grade. These results are in line with diverse investigations carried out with children who were younger than those of this study, which relate short-term memory to mathematical performance (Bull et al., 2008; Geary et al., 1991; Hecht et al., 2001; Passolunghi et al., 2008; Swanson, 1993). These data could be interpreted, along the lines of DeSmedt et al. (2009), as a reflection of the importance of verbally or phonologically coded information when solving arithmetical tasks, even at these educational grades.

The multiple regression analysis reveals simultaneous processing as a cognitive predictor of mathematical achievement at all educational grades. In this sense, along the lines of recent literature, the results of this study seem to support the proposals that emerged from the PASS theory of intelligence about the relevance of simultaneous coding for mathematical performance (e.g., Deaño, 2000; Naglieri & Das, 1997b; Kroesbergen, et al., 2003) and they extend the results to new educational grades.

One advantage of using the perspective of cognitive processing to examine disorders of the basic psychological processes that are related to academic disabilities is the explanatory power of this perspective (Naglieri & Das, 2002). In contrast to a discrepancy approach and a global consideration of MLDs, the results obtained herein refer to a basic psychological processing disorder and its relation to academic failures in mathematics. This disorder is produced in simultaneous processing. A disorder in a basic psychological process such as simultaneous processing can impair learning related to the comprehension of logical-

grammatical and geometric relations, number and number pattern recognition, and the representation and recognition of the outline of the problem. There is a difficulty to establish relations among the parts in the diverse learning domains, to integrate them into a whole and to understand them and lend meaning to the whole. The parts that make up the task must be interrelated (Das et al, 1994).

Summing up, the findings of this study relate mathematical achievement to underlying cognitive functioning. Moreover, it expands the data found in smaller children to a later developmental stage. New data that differentiate the groups with MLD and LA were also found. Lastly, along the lines of other works (e.g., Deaño & Tellado, 2009; Kroesbergen, et al., 2003; Naglieri & Johnson, 2000), the data obtained provide immediate educational implications related to the need to promote the improvement of the cognitive functioning of children with disabilities as an integrating part of the global intervention process. In this sense, training in mathematical tasks using the PASS processes with students of diverse characteristics such as those indicated in this study has led to an increase in simultaneous processing and in calculation (Deaño, 2005, 2006).

To conclude, an apparent limitation of this study is the cross-sectional treatment of the data. Along the lines of some recent works, future works should implement studies that allow longitudinal analysis of the data as extensively as possible. Likewise, in this study, we did not specifically analyze the viso-spatial aspects of the working memory, an aspect that has recently been considered by some authors and that may be potentially important when studying some subtypes of disabilities, such as procedural disabilities or disabilities related to number processing. However, it should be noted that, in this work, we analyzed verbal and spatial

performance as aspects linked to simultaneous processing. These aspects should be taken into account for future investigations.

References

- Adams, J.W., & Hitch, G. (1997). Working memory and children's mental addition. *Journal of Experimental Child Psychology*, 67(1), 21-38.
- Adams, J.W., & Hitch, G. (1998). Children's mental arithmetic and working memory. In C. Donlan (Ed.), *The development of mathematical skills* (pp. 201-205). Hove, UK: Psychology Press.
- Ashman, A.F., & Das, J.P. (1980). Relation between planning and simultaneous-successive processing. *Perceptual and Motor Skills*, 51, 371-382.
- Baddeley, A.D. (1986). *Working memory*. Oxford: Clarendon Press.
- Baddeley, A.D. (1996). Exploring the central executive. *Quarterly Journal of Experimental Psychology*, 49A, 5-28.
- Baddeley, A.D. (2000). The episodic buffer: a new component of working memory? *Trends in Cognitive Sciences*, 4, 417-423.
- Baddeley, A. D. (2007). *Working memory, thought and action*. Oxford: Oxford University Press.
- Baddeley, A.D., & Hitch, G. (1974). Working memory. In G.H. Bower (Ed.), *The psychology of learning and motivation* (Vol. 8, pp. 47-89). San Diego, CA: Academic Press.
- Baker, S.R., & Talley, L.H. (1972). Relationship of visualization skills to achievement in freshman chemistry. *Journal of Chemical Education*, 49, 775-776.
- Baker, S.R., & Talley, L.H. (1974). Visualization skills as a component of aptitude for chemistry - A construct validation study. *Journal of Research in Science Teaching*, 11, 95-97.
- Badian, N. (1983). Dyscalculia and nonverbal disorders of learning. In H. R. Mycklebust (Ed.), *Progress in Learning Disabilities* (Vol. 5, pp. 235-264). Nueva York: Grune & Stratton.

- Barbaresi, W.J., Katusic, S.K., Colligan, R.C., Weaver, A.L., & Jacobsen, S.J. (2005). Math learning disorder: Incidence in a population based birth cohort, 1976-1982, Rochester, Minn. *Ambulatory Pediatrics*, 5, 281-289.
- Battista, M. T. (1994). On Greeno's environmental/model view of conceptual domains: A spatial/geometric perspective. *Journal for Research in Mathematics Education*, 25, 86-94.
- Bishop, A. J. (1989). Review of research on visualisation in mathematics education. Focus on *Learning Problems in Mathematics*, 11(1), 7-16.
- Bodner, G. M. & McMillen, T. L. B. (1986). Cognitive restructuring as an early stage in problem solving. *Journal of Research in Science Teaching*, 23, 8, 727-737.
- Bull, R., Andrews-Espy, K., & Wiebe, S.A. (2008). Short-term memory, working memory, and executive functioning in preschoolers: longitudinal predictors of mathematical achievement at age 7 years. *Developmental Neuropsychology*, 33(3), 205-228.
- Bull, R., & Johnston, R.S. (1997). Children's arithmetical difficulties: contributions from processing speed, item identification and short-term memory. *Journal of Experimental Child Psychology*, 65, 1-24.
- Bull, R., & Scherif, G. (2001). Executive functioning as a predictor of children's mathematics ability: Inhibition, switching, and working memory. *Developmental Neuropsychology*, 19, 273-293.
- Camos, V., & Barrouillet, P. (2004). Adult counting is resource demanding. *British Journal of Psychology*, 9, 19-30.
- Case, R. (1985). *Intellectual development: Birth to adulthood*. San Diego, CA: Academic Press

- Chong, S.L., & Siegel, L.S. (2008). Stability of computational deficits in math learning disability from second through fifth grades. *Developmental Neuropsychology*, 33(3), 300-317.
- Das, J.P. (1988). Simultaneous-successive processing and planning. In R. Schmeck (Ed.), *Learning Styles and learning strategies* (pp. 101-129). New York: Plenum.
- Das, J.P. (1999). *PASS Reading Enhancement Program(PREP)*. Deal, NJ: Sarka Educational Resources.
- Das, J.P. (2000). PREP: A Cognitive Remediation Program in Theory and Practice. *Developmental Disabilities Bulletin*, 28 (2), 83-95
- Das, J. P. & Abbott, J. (1995). PASS: An alternative approach to intelligence. *Psychology and Developing Societies*, 7(2), 155-184.
- Das, J.P., & Heemsbergen, D.B. (1983). Planning as a factor in the assessment of cognitive processes. *Journal of Psychoeducational Assessment*, 1, 1-16.
- Das, J.P., & Naglieri, J.A. (2001). The Das-Naglieri cognitive assessment system in theory and practice. In J.J.W. Andrews, D.H.Sakolfske, & H.L. Janzen, (Eds.), *Handbook of Psychoeducational Assessment: Ability, achievement, and behavior in children* (pp. 34-64). San Diego, CA: Academic Press.
- Das, J.P., Naglieri, J.A., & Kirby, J.R. (1994). *Assessment of cognitive processes: The PASS theory of intelligence*. Boston: Allyn & Bacon.
- Das, J.P., & Varnhagen, C.K. (1986). Neuropsychological functioning and cognitive processing. In J.E. Obrzut and G.W. Hynd (Eds.), *Child neuropsychology, Vol. 1: Theory and research* (pp. 117-140). New York: Academic Press.
- Deaño M. (2000). *Cómo prevenir las dificultades de cálculo*. Málaga: Aljibe.

- Deaño, M. (2005). Mejora en el aprendizaje del cálculo en alumnos con debilidad cognitiva. *Revista Portuguesa de Pedagogía*, 39 (2), 365-389.
- Deaño, M. (2006). Programa de Recuperación y Enriquecimiento PASS-Matemático (PREP-M). Un programa cognitivo para alumnos con necesidades educativas. *Educación, Desarrollo y Diversidad*, 9(3), 37-64.
- Deaño, M., Alfonso, S, & Fernández, M.J. (2006). El D.N:CAS como sistema de evaluación cognitiva para el aprendizaje. In M. Deaño (Ed.), *Formación del profesorado para atender a las necesidades específicas de apoyo educativo. XXXII Reunión Científica Anual*. (pp.159-182). Ourense: AEDES.
- Deaño, M., & Tellado, F. (2009). Instrucción basada en los procesos PASS. In A. Barca (Ed.), *Motivación y Aprendizaje en Contextos Educativos* (pp. 369- 398). Granada: Grupo Editorial Universitario.
- Dehaene, S. (1997). *The number sense: How the mind creates mathematics*. New York: Oxford University Press.
- de Jong, P.F., & van der Leij, A. (1999). Specific contributions of phonological abilities to early reading acquisition: Results from a Dutch latent variable longitudinal study. *Journal of Educational Psychology*, 91, 450–476.
- De Smedt, B., Janssen, R., Bouwens, K., Verschaffel, L., Boets, B., & Ghesquière, P. (2009). Working memory and individual differences in mathematics achievement: A longitudinal study from first to second grade. *Journal of Experimental Child Psychology*, 103, 186-201.
- DeStefano, D., & Lefevre, J. A. (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology*, 16, 353–386.

- Diezmann, C.M., & Watters, J.J. (2000). Identifying and supporting spatial intelligence in young children. *Contemporary Issues in Early Childhood, 1*(3), 299-313.
- Fuchs, L., Fuchs, D., Compton, D., Powell, S., Seethaler, P., Capizzi, A., & Schatschneider, C., (2006). The cognitive correlates of third-grade skills in arithmetic, algorithmic computation and arithmetic word problems. *Journal of Educational Psychology, 98*(1), 29-43.
- Fürst, A.J., & Hitch, G.J. (2000). Different roles for executive and phonological components of working memory and mental arithmetic. *Memory and Cognition, 28*(5), 774-782.
- Garnett, K., & Fleischner, J.E. (1983). Automatization and basic fact performance of normal and learning disabled children. *Learning Disability Quarterly, 6*, 223-230.
- Garofalo, J. (1986). Simultaneous synthesis, regulation, and arithmetical performance. *Journal of Psychoeducational Assessment, 4*(3), 229-238.
- Gathercole, S.E., & Pickering, S.J. (2000). Working memory deficits in children with low achievement in the national curriculum at 7 years of age. *British Journal of Educational Psychology, 70*, 177-194.
- Geary, D. C. (2005). Role of cognitive theory in the study of learning disability in mathematics. *Journal of Learning Disabilities, 38*(4), 305-307.
- Geary, D. C., Bailey, D. H., Littlefield, A., Wood, P., Hoard, M. K., & Nugent, L. (2009). First-grade predictors of mathematical learning disability: A latent class trajectory analysis. *Cognitive Development, 34*, 411-429.
- Geary, D.C., & Brown, S.C. (1991). Cognitive addition: Strategy choice and speed-of processing differences in gifted, normal and mathematically disabled children. *Developmental Psychology, 27*, 398-406.

- Geary, D.C., Brown, S.C., & Samaranayake, Y.A. (1991). Cognitive addition: A short longitudinal study of strategy choice and speed-of-processing differences in normal and mathematically disabled children. *Developmental Psychology, 27*, 787-797.
- Geary, D.C., & Burlingham-Dubree, M. (1989). External validation of the strategy choice model for addition. *Journal of Experimental Child Psychology, 47*, 175-192.
- Geary, D.C., Hoard, M.K., Byrd-Craven, J., & DeSoto, M.C. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability. *Journal of Experimental Child Psychology, 88*, 121-151.
- Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development, 78*, 1343-1359.
- Geary, D.C., Hoard, M.K., & Hamson, C.O. (1999). Numerical and arithmetical cognition: patterns of functions and deficits in children at risk for a mathematical disability. *Journal of Experimental Child Psychology, 74*, 213-263.
- Geary, D.C., Hoard, M.K., Nugent, L., & Byrd-Craven, J. (2007). Strategy use, long-term memory and working memory capacity. In D. Berch & M.M.M. Mazzocco (Eds.), *Why is math so hard for some children. The nature and origins of mathematical learning difficulties and disabilities* (pp. 83-105). Baltimore: Paul H. Brookes.
- Geary, D.C., Hoard, M.K., Nugent, L., & Byrd-Craven, J. (2008). Development of number line representations in children with mathematical learning disability. *Developmental Neuropsychology, 33*(3), 277-299.

- Geary, D.C., Widaman, K.F., Little, T.D., & Cormier, P. (1987). Cognitive addition: Comparison of learning disabled and academically normal elementary school children. *Cognitive Development, 2*, 149-169.
- Gross-Tsur, V., Manor, O., & Shalev, R.S. (1996). Developmental dyscalculia: Prevalence and demographic features. *Developmental Medicine and Child Neurology, 38*, 25-33.
- Heathcote, D. (1994). The role of visuo-spatial working memory in the mental addition of multi-digit addends. *Cahiers de Psychologie/Current Psychology of Cognition, 13*, 207-245.
- Hecht, S.A., Torgesen, J. K., Wagner, R.K., & Rashotte, C.A. (2001). The relations between phonological processing abilities and emerging individual differences in mathematical computation skills: A longitudinal study from second to fifth grades. *Journal of Experimental Child Psychology, 79*, 192-227.
- Hein, J., Bzufka, M.W., & Neumarker, K.J. (2000). The specific disorder of arithmetic skills. Prevalence studies in a rural and an urban population and their clinico-neuropsychological validation. *European Child and Adolescent Psychiatry, 9*, II87-II/101.
- Hermelin, B., & O'Connor, N. (1986). Spatial representations in mathematically and in artistically gifted children. *British Journal of Educational Psychology, 56*, 150-157.
- Hitch, G. (1978). The numerical abilities of industrial apprentices. *Journal of Occupational Psychology, 51*, 163-176.
- Iglesias-Sarmiento, V. (2009). *Dificultades de aprendizaje en el dominio aritmético y en el procesamiento cognitivo subyacente*. (Doctoral dissertation, University of Vigo, 2008). Ann Arbor, MI: ProQuest LLC/ UMI Dissertation Publishing.

- Imbo, I., & Vandierendonck, A. (2006). The role of phonological and executive working memory resources in simple arithmetic strategies. *European Journal of Cognitive Psychology, 19*(6), 910-933.
- Johnson, J. A., Bardos, A. N., & Tayebi, K. A. (2003). Discriminant validity of the Cognitive Assessment System for students with written expression disabilities. *Journal of Psychoeducational Assessment, 21*, 180-195.
- Jordan, N.C., Hanich, L.B., & Kaplan, D. (2003). Arithmetic fact mastery in young children: a longitudinal investigation. *Journal of Experimental Child Psychology, 85*(2), 103-119.
- Jordan, N.C., & Montani, T. O. (1997). Cognitive arithmetic and problem solving: a comparison of children with specific and general mathematics difficulties. *Journal of Learning Disabilities, 30*(6), 624-634.
- Joseph, L.M., & Hunter, A.D. (2001). Differential application of a cue card strategy for solving fraction problems: exploring instructional utility of the cognitive assessment system. *Child Study Journal, 31*(2), 123-136.
- Joseph, L.M., McCachran, M. E., & Naglieri, J. A. (2003). PASS cognitive processes, phonological processes, and basic reading performance for a sample of referred primary-grade children. *Journal of Research in Reading, 26*(3), 304-314.
- Kirby, J.R., & Ashman, A.F. (1984). Planning skills and mathematics achievement: Implications regarding learning disability. *Journal of Psychoeducational Assessment, 2*, 9-22.
- Kirby, J. R., & Das, J. P. (1990). A cognitive approach to intelligence: Attention, coding and planning. *Canadian Psychology, 31*, 320-331.
- Kirby, J., & Williams, N.H. (1991). *Learning problems: A cognitive approach*. Toronto, Canada: Kagan and Woo.

- Koontz, K.L., & Berch, D.B. (1996). Identifying simple numerical stimuli: processing inefficiencies exhibited by arithmetic learning disabled children. *Mathematical Cognition*, 2(1), 1-23.
- Koponen, T., Aunola, K., Ahonen, T., & Nurmi, J.-E. (2007). Cognitive predictors of single-digit and procedural calculation skills and their covariation with reading skill. *Journal of Experimental Child Psychology*, 97, 220–241.
- Krajewski, K., & Schneider, W. (2009). Exploring the impact of phonological awareness, visual-spatial working memory, and preschool quantity-number competencies on mathematics achievement in elementary school: Findings from a 3-year longitudinal study. *Journal of Experimental Child Psychology*, 103, 516-531.
- Kroesbergen, E.H., Van Luit, J.E.H., & Naglieri, J.A. (2003). Mathematics learning difficulties and PASS cognitive processes. *Journal of Learning Disabilities*, 36(6), 574-582.
- Kroesbergen, E. H, Van. Luit, J. E.H., Naglieri, J.A., Taddei, S., Franchi, E. (2010, January). PASS processes and early mathematics skills in dutch and italian kindergartners. *Journal of Psychoeducational Assessment*. Retrieved from <http://jpa.sagepub.com/content/early/2010/01/04/0734282909356054.fullpdf>.
- Landerl, K., Bevan, A., & Butterworth (2004). Developmental dyscalculia and basic numerical capacities: a study of 8-9-year-old students. *Cognition*, 93, 99-215.
- Leong, C.K., Cheng, S.C., & Das, J.P. (1985). Simultaneous-successive syntheses and planning in Chinese readers. *International Journal of Psychology*, 20, 19-31.
- Lewis, C., Hitch, G.J., & Walker, P. (1994). The prevalence of specific arithmetic difficulties and specific reading difficulties in 9- to 10- year-old boys and girls. *Journal of Child Psychology and Psychiatry*, 35(2), 283-292.

- Lindsay, R.L. (2001). Attentional function as measured by a continuous performance task in children with dyscalculia. *Journal of Developmental and Behavioural Pediatrics, 22*, 287-292.
- Logie, R.H. & Baddeley, A.D. (1987). Cognitive processes in counting. *Journal of Experimental Psychology, 13*, 310-326.
- Logie, R.H., Gilhooly, K.J., & Wynn, V. (1994). Counting on working memory in arithmetic problem solving. *Memory and Cognition, 22* (4), 395-410.
- Luria, A.R. (1966). *Human Brain and Psychological Processes*. New York: Harper & Row.
- Luria, A.R. (1973). *The Working Brain: An Introduction to Neuropsychology*. New York: Basic Books.
- Luria, A. R. (1980). *Higher cortical functions in man*. Nueva York: Basic Books.
- Mazzocco, M.M.M. (2007). Defining and differentiating mathematical learning disabilities and difficulties. In D. Berch & M.M.M. Mazzocco (Eds.), *Why is math so hard for some children. The nature and origins of mathematical learning difficulties and disabilities* (pp. 29-47). Baltimore: Paul H. Brookes.
- Mazzocco, M.M.M., & Myers, G.F. (2003). Complexities in identifying and defining mathematics learning disability in the primary school age years. *Annal of Dyslexia, 53*, 218-253.
- McKenzie, B., Bull, R., & Gray, C. (2003). The effects of phonological and visual-spatial interference on children's arithmetical performance. *Educational and Child Psychology, 20*(3), 93-107.
- McLean, J.F., & Hitch, G.J. (1999). Working memory impairments in children with specific arithmetic learning difficulties. *Journal of experimental Child Psychology, 67*, 345-357.

- Mercer, C. (1997). *Students with learning disabilities*. Prentice Hall.
- Meyer, M.L., Salimpoor, V.N., Wu, S.S., Geary, D.C., & Menon V. (2010) Differential contribution of specific working memory components to mathematical skills in 2nd and 3rd graders. *Learning and Individual Differences* 20(2), 101-109.
- Murphy, M.M., Mazzocco, M.M.M., Hanich, L., & Early, M.C. (2007) Cognitive characteristics of children with mathematics learning disability (MLD) varies as a function of the cut-off criterion used to define MLD. *Journal of Learning Disabilities*, 40, 458-478.
- Naglieri, J. A. (1999). *Essentials of CAS assessment*. New York: Wiley.
- Naglieri, J.A., (2000). Can profile analysis of ability test scores work? An illustration using the PASS theory and CAS with an unselected cohort. *School Psychology Quarterly*, 15, 419-433.
- Naglieri, J.A., & Das, J.P. (1987). Construct and criterion related validity of planning, simultaneous and successive cognitive processing tasks. *Journal of Psychoeducational Assessment*, 4, 353-363.
- Naglieri, J.A., & Das, J.P. (1997a). *Cognitive Assessment System Interpretive Handbook*. Itasca, IL: Riverside Publishing.
- Naglieri, J.A., & Das, J.P. (1997b). *Cognitive Assessment System*. Itasca, IL: Riverside Publishing.
- Naglieri, J. A., & Das, J.P. (2002). Practical implications of general intelligence and PASS cognitive processes. In R. J. Sternberg & E. L. Grigorenko (Eds.), *The general factor of intelligence: How general is it?* (pp.855-884). New York: Erlbaum.

- Naglieri, J. A., Das, J. P., Stevens, J. J., & Ledbetter, M. F. (1991). Confirmatory factor analysis of planning, attention, simultaneous, and successive cognitive processing tasks. *Journal of School Psychology, 29*, 1-17.
- Naglieri, J.A., & Gottling, S. (1997). Mathematics instruction and PASS cognitive processes: an intervention study. *Journal of Learning Disabilities, 30*(5), 513-520.
- Naglieri, J.A., & Kaufman, J.C. (2001). Understanding intelligence, giftedness and creativity using the PASS theory. *Roeper Review, 23*(3), 151-156.
- Naglieri, J.A., & Johnson, D. (2000). Effectiveness of a cognitive strategy intervention in improving arithmetic computation based on the PASS theory. *Journal of Learning Disabilities, 33*(6), 591-597.
- Naglieri, J.A., & Rojahn, J. (2004). Construct validity of the PASS theory and CAS: Correlations with achievement. *Journal of Educational Psychology, 96*(1), 174-181.
- Nunnally, J. C., & Bernstein, I. H. (1994). *Psychometric theory*. Nueva York: McGraw-Hill.
- OECD (2007a). *Pisa 2006. Science competencies for tomorrow's world. Volume 1: Analysis*. Paris: Author.
- OECD (2007b). *Pisa 2006. Volume 2: Data*. Paris: Author.
- Parrila, R. K., Kendrick, M. E., Papadopoulos, T. C. & Kirby, J. R. (1999). Efficacy of cognitive reading remediation program for at-risk children in grade 1. *Developmental Disabilities Bulletin, 27*(2), 1-31.
- Passolunghi, M.C., Mammarella, I.C., & Altoè, G (2008). Cognitive abilities as precursors of the early acquisition of mathematical skills. *Developmental Neuropsychology, 33*(3), 229-250.

- Passolunghi, M. C., & Siegel, L. S. (2004). Working memory and access to numerical information in children with disability in mathematics. *Journal of Experimental Child Psychology, 88*, 348–367.
- Passolunghi, M. C., Vercelloni, B., & Schadee, H. (2007). The precursors of mathematics learning: Working memory, phonological ability, and numerical competence. *Cognitive Development, 22*, 165–184.
- Siegel, L.S. (1999). Issues in the definition and diagnosis of learning disabilities: A perspective on Guckenberger v. Boston University. *Journal of Learning Disabilities, 32*(4), 304–319.
- Siegel, L.S., & Ryan, E.B. (1989). The development of working memory in normally achieving and subtypes of learning disabled children. *Child Development, 60*, 973-980.
- Simmons, F., Singleton, C., & Horne, J. K. (2008). Phonological awareness and visual–spatial sketchpad functioning predict early arithmetic attainment: Evidence from a longitudinal study. *European Journal of Cognitive Psychology, 20*, 711–722.
- Solan, H.A., Shelley-Tremblay, J., Ficarra, A., Silverman, M., & Larson, S. (2003). Effect of attention therapy on reading comprehension. *Journal of Learning Disabilities, 36*(3), 556-563.
- Stanovich, K. E. (1999). The sociopsychometrics of learning disabilities. *Journal of Learning Disabilities, 32*(4), 350-361.
- Swanson, H.L. (1993). Working memory in learning disability subgroups. *Journal of Experimental Child Psychology, 56*, 87–114.
- Swanson, H.L. (2006). Cognitive processes that underlie mathematical precociousness in young children. *Journal of Experimental Child Psychology, 93*, 239–264.

- Swanson, H.L., & Beebe-Frankenberger, M. (2004). The relationship between working memory and mathematical problem solving in children at risk and not at risk for serious math difficulties. *Journal of Educational Psychology, 96*, 471–491.
- Swanson, H.L., Jerman, O., & Zheng, X. (2008). Growth in working memory and mathematical problem solving in children at risk and not at risk for serious math difficulties. *Journal of Educational Psychology, 100*(2), 343-379.
- Swanson, L., & Kim, K. (2007). Working memory, short-term memory, and naming speed as predictors of children's mathematical performance. *Intelligence, 35*, 151–168.
- Swanson, H. L., & Sachse-Lee, C. (2001). Mathematical problem solving and working memory in children with learning disabilities: Both executive and phonological processes are important. *Journal of Experimental Child Psychology, 79*, 294–321.
- Temple, C.M., & Sherwood, S. (2002). Representation and retrieval of arithmetical facts: developmental difficulties. *Quarterly Journal of Experimental Psychology, 55*(1), 733-752.
- Trbovich, P.L., & LeFevre, J. (2003). Phonological and visual working memory in mental addition. *Memory & Cognition, 31*, 738 – 745.
- van der Sluis, S., van der Leij, A., & de Jong, P. F. (2005). Working memory in Dutch children with reading- and arithmetic-related LD. *Journal of Learning Disabilities, 38*, 207–221.
- Wai, J., Lubinski, D., & Benbow, C.P. (2009). Spatial ability for stem domains: aligning over 50 years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology, 101*(4) 817–835

- Warrick, P.D. (1989). *Investigation of the PASS model (planning, attention, simultaneous, successive) of cognitive processing and mathematics achievement*. Unpublished doctoral dissertation, Ohio State University, Columbus.
- Watters, J.J. & English, L.S. (1995). Children's application of simultaneous and successive processing in inductive and deductive reasoning problems: Implications for developing scientific reasoning. *Journal of Research in Science Teaching*, 32(7), 699-714.
- Wechsler, D. (1974). *Manual for the Wechsler intelligence scale for children (revised)*. New York: Psychological Corporation [Escala de Inteligencia de Wechsler para niños-Revisada (WISC-R), 1993. Madrid: TEA].
- Wu, S.S., Meyer, M.L., Maeda, U., Salimpoor, V., Tomiyama, S., Geary, D.C., et al. (2008). Standardized assessment of strategy use and working memory in early mental arithmetic performance. *Developmental Neuropsychology*, 33(3), 365-393.