

## Article

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"Beyond Infinity: Augustine and Cantor"

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## BEYOND INFINITY : AUGUSTINE AND CANTOR

Adam DROZDEK

*RÉSUMÉ* : Cet article entend montrer que pour Augustin : 1) l'infini est un concept inné qui est prérequis à toute connaissance ; 2) les mathématiques – parce qu'elles reposent sur le concept d'infini – constituent le meilleur outil pour parvenir à la connaissance de Dieu ; et 3) Dieu n'est ni fini ni infini : sa grandeur est au-delà de l'infini. Augustin est original en ce qu'il allie ces trois aspects dans sa philosophie ; or ces mêmes trois aspects se trouvent à nouveau chez Cantor.

*SUMMARY* : It is argued in this paper that for Augustine 1. infinity is an inborn concept which is a prerequisite of any knowledge ; 2. mathematics – because it relies on the concept of infinity – is the best tool of acquiring knowledge about God, and 3. God is neither finite nor infinite and his greatness is beyond infinity. Augustine is original in combining these three aspects in his philosophy, and all three aspects can again be found in Cantor.

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An eminent theologian and philosopher, Étienne Gilson, wrote that although “there is no other attribute of God more known to theologians than infinity,” Augustine in listing God’s attributes curiously “does not remember infinity.” Notwithstanding this omission, Gilson suggests that Augustine “is sure that God is infinite”.<sup>1</sup> It seems, however, that this conclusion is unjustified and according to Augustine, infinity is not one of God’s attributes, since God transcends even the infinite.

### I

In his Manichean period, Augustine saw God as an infinite corporeal reality surrounding the world like “an infinite sea” (*Conf.* 7.5), as extended infinitely in the space (7.14), but he abandoned this view after his conversion. Then he saw that God

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1. Étienne GILSON, *L'infinité divine chez saint Augustin. Études augustiniennes*, Paris, Vrin, vol. I, 1954, p. 569-570. The assumption that Augustine considers God infinite is, in fact, rather common, especially in cursory accounts of Augustine’s views, e.g., Rudolf RUCKER, *Infinity and the mind: The science and philosophy of the infinite*, New York, Bantam Books, 1983, p. 4, and A.W. MOORE, *The infinite*, London, Routledge, 1990, p. 46.

is infinite in a different way (*vidi te infinitum aliter*) than he had imagined before. One thing was certain, namely that this knowledge was not derived from sensory perception (7.14). Where did it come from? And in what “different way” can God’s infinity be understood (if in any)?

Augustine sets knowing God and souls as the goal of his philosophical quest – and nothing else (*Solil.* 1.2.7), or rather, knowing anything else is just a means to reach this goal. It is obvious that sensory perception is the least reliable and the least useful in this quest, since not only perceptual knowledge is subject to change and delusion, but also the objects of perception are of temporary, mutable nature. Knowledge, however, has to bring true cognition, and truth cannot change with time; truth is eternal, immutable, truth stands the test of time and cannot be abolished, “truth will exist even after the world perishes” (2.2.2). Is such a truth possible? Is the goal of reaching it not a chimera? The existence of this goal has to be assumed before knowledge can be gained; therefore, belief precedes the process of cognition, and faith enables us to launch this cognitive quest. “It is not enough to have eyes [of mind], it is not enough to look to see”; our mental sight has to be freed from sensory impurities, and “initially, only faith can give soul this purity” (1.6.12). Also, understanding is the reward of faith; therefore, belief enables understanding, not vice versa.

Knowing God is the ultimate goal. However, the cognitive apparatus can be exercised with respect to other objects, to geometrical objects and numbers, for example, since the difference between knowing God and knowing numbers lies not in the cognition itself but in the nature of cognitive objects (*Solil.* 1.5.11). Hence, analysis of how and what we know about objects of mathematics is a stepping stone to analysis of our knowledge of God. Mathematical sciences make us more sensitive to more subtle kinds of knowledge, and without the preparatory exercise of mathematics, “our mind could not bear great light” of this knowledge (*De quant. animae* 15.25).

Augustine ascribes very high status to the knowledge of mathematical objects, so high, in fact, that this knowledge is used as a distinctive mark of humanity: men are better than animals since they can know numbers (*De ordine* 19.49) and also geometrical objects since the truths of geometry are accessible to the reasoning mind alone, and man is different from animal because of this reasoning faculty. Although animals can craft things more harmoniously than we can – e.g., birds building their nests or bees constructing their honeycombs – we are better than animals because we are able to know numbers (2.19). Numbers have a divine status since God himself used them (and laws) when creating the universe (Augustine refers here to Is. 40:26 – LXX; Wisdom 11:20).

Truth of numbers and their relations is accessible to anyone who reasons (*De lib. arb.* 2.8.20). But knowledge about numbers does not come from the senses, and senses cannot give any knowledge concerning operations on numbers. The laws of numbers are eternal; therefore, how could a mutable body imprint in us eternal laws (2.8.21)? Only “the light of mind” enables one to say that the result of an operation is wrong (2.8.21). Moreover, the concept of number does not come from senses. Each number can be considered a sum of ones, but the senses impose on us the

concept of multitude, not unity, and hence they render impossible creating the concept of the unit (2.8.22). Also, we perceive only some numbers, how then could we arrive at laws valid for all of them? And finally, "no one is able to grasp with the senses all numbers, since they are uncountable," but with the inner light it is possible (2.8.23). There is a hierarchy of numbers described in *De musica* 6.2-9: corporal numbers, remembered numbers, reacting numbers, active numbers, sensible numbers, and finally rational numbers. The latter are the constitutive numbers of our reason; they, however, are only reflections of the eternal, divine numbers which are in God, whereby the absolute is present in us. The rational numbers are imprinted, impressed in our mind (*De musica* 12.35); that is, they are created as the mind itself was created, and they communicate the eternal numbers.

This interest in numbers is not accidental. Numbers are so important because reality can be comprehended only through numbers. First, physical objects can be counted and a number is always implicitly present in perception: I see *a* man, *i.e.*, *one* man. Also, all things have shapes and proportions which can be measured. Therefore, if numerical dimensions are removed from objects, the objects would disappear. Furthermore, numerical laws are present in manufacturing, in dance, art, architecture, music, etc. (*De lib. arb.* 2.16.42). For example, an architect does not put windows in a building randomly, but led by numerical laws he creates a harmony, making windows pleasing to the eye as well as usable. Thus, not surprisingly, Augustine sees numbers as belonging to the world of intelligibles. They exist since they are known. One cannot know what does not exist. Numbers and their laws are immutable and independent of changes in the world. After all, "who would dare say that the concept of number is mutable?" (*De imm. animae* 4.5). And these eternal and immutable numbers have to be known in order to have any knowledge about the changing world, not vice versa.

Equally high status is ascribed to the knowledge of geometrical objects, since, for example, "what can be more eternal than the concept of the circle?" (*De imm. animae* 4.6). The truths about these objects cannot be extracted from sensory perception. This perception can give us certain ideas about these truths, but it can never be an ultimate basis for their derivation. For example, each line we are able to draw has a certain width; how can we see, then, a line which touches the circumference of a circle at only one point? Also, if we draw, in a circle, two radii very close to each other, then there will be no room for drawing a radius between them. And yet "our reason says that we may draw an infinite number of such lines and in this improbably small space they should touch each other only in the center" (*Solil.* 2.20.35). To be sure, reason can know this fact only if it possesses an idea of infinity; this idea precedes and is independent of sensory knowledge. This idea of infinity allows us to extend the line indefinitely in both directions, which is a possibility that the sight of the mind sees "perfectly and without any trouble" (*De quant. animae* 1.6.11).

Knowledge of numbers and geometrical objects indicates that "there exists some eternal and immutable perfection which does not let mutable things to perish, but in a sense it allows them to traverse the paths of time in all the variety of decided

shapes and calculated movements” (*De lib. arb.* 2.16.44). These truths are not created, they are discovered in the soul. Their existence is independent of the existence of the soul ; they only empower the soul to do what it can and should do : know God and itself, the soul. Knowledge of numbers is a path leading to knowledge of God, and if we have in the soul an idea of eternal numbers, then we ask ourselves about their source. But what other source can it be if not God ?

Reason itself could not be the author of these mathematical truths, since reason is mutable : sometimes it wants to know the truth, sometimes it does not ; yet reason is able on its own strength to know that there is an eternal and immutable being and it must admit that it, reason, is lower than this being and that this being is God (*De lib. arb.* 2.6.14). It is natural for reason to seek this truth, and it is also its primary task. It would be, however, unable to know the truth without divine help. This help manifests itself in the fact that some truths are already imprinted in our soul. Except for mathematical truths, we can mention the concept of happiness and wisdom impressed (*inpressa*) upon our mind ; therefore, we answer affirmatively when asked whether we want to be happy and wise (2.9.26). Also, all human justice is derived from an eternal an immutable law which is engraved in us (1.6.15).

Although the soul contains eternal truths, it is not always aware of them : “All true concepts are in its hidden recesses, although it seems they are not there” (*De imm. animae* 4.6), and a deep knowledge, God’s law, exists engraved in the soul and most people do not even suspect they exist (*De ordine* 2.8.24-25). Therefore, some direction or enlightenment is needed to reach these hidden recesses and to bring light to them. This direction comes from none other than God who through the divine light illuminates not only the truths, but also the mind. Consequently, not only the object of cognition can be seen through this light, but also the faculty of the mind is empowered through it. Without this illumination we would be powerless and unable to surpass the animal level of sensory perception. Our human existence hinges upon it to the extent that “illumination directly determines the nature itself of ideas and of intelligence”.<sup>2</sup> These ideas could not be what they are without containing, although in a dimmed form, the divine light. This light in us, however, is not – and cannot be – the ultimate source of knowledge. The soul is created and along with it its inner light. This light is also mutable, which can be seen in the fact that all people do not see eternal truths equally clearly. Thus, although the inner light enables knowledge, it is itself dependent upon the eternal and immutable source ; or rather, it is an intermediary of the eternal light in which our souls get kindled like lamps (*Enarr. in Ps.* 7.8 ; *Epist.* 19), although Augustine often does not make a distinction between these two lights.<sup>3</sup> After all, God is a true light illuminating everyone coming into the world (John 1 :9), thus the truth is in man only thanks to God’s immutability, and it is sufficient to see the presence of truth in thought to know that God exists.<sup>4</sup> This illumination is not reserved to the believers alone. It can be found, although in various degrees of clarity, in all humans, so that because of illumination “even the ungodly

2. Charles BOYER, *L'idée de vérité dans la philosophie de saint Augustin*, Paris, Beauchesne, 1941, p. 234.

3. *Ibid.*, p. 224.

4. Étienne GILSON, *Introduction à l'étude de saint Augustin*, Paris, Vrin, 1949, p. 28.

have a notion of eternity" (*De Trinitate* 14.21). This inner light is brought to us by God himself, and realization of this source is the greatest gain we can have in the process of cognition. Therefore, it is true that this dependence of human spirit on the supreme light is the very center of Augustine's philosophy.<sup>5</sup>

Most people acquire true knowledge using reason and its faculties ; some people, however, may have an immediate contact with God and his knowledge. In fact, this type of knowledge acquirement is an ideal accessible to everyone after death of the body (*De ordine* 2.19). In this life such immediate contact with the truth is very rare, and it happens to very few in their mystical experience (*Solil.* 1.6.13 ; *De quant. animae* 33.76). To be sure, this type of contact with the truth is not unprecedented in philosophy, to mention Parmenides' experience, Socrates' hope for out-of-body contemplation of true reality (*Phaedon* 10), or Philo's concept of reason ascending to the contemplation of the One (*Enneads* 1.6 ; 6.9). Augustine himself described his own mystical experiences of reaching the truth through immediate contact (*e.g.*, *Conf.* 10.27).

The entire cognitive process is possible only because the concept of infinity, in its spatial and temporal aspects, is given to us before the process even starts. Infinity is not developed through the cognitive process, it is this process which develops through the concept of infinity. Even gaining some knowledge about the finite and mutable world would not be possible without being endowed with the concept of infinity. In this sense, the infinite precedes the finite in the cognitive order. But the infinite comes first in the cognitive order only because it is first in the ontological order. The world is created by an infinite power, by eternal and immutable God without whom the existence of the world would not be possible.

The essence of God is founded on the biblical concept of "I am who I am" which says that God's being, or the fact that God is, constitutes supreme essence, *i.e.*, he is the supreme being. God, creator of all things, surpasses the limits of time and space, since the latter are also his creations. God is eternal, because "in eternity proper, nothing disappears in the past, nothing is to become, but everything in it simply is" (*De div. quaest.* 83.19). Divine life exists all at once (*Conf.* 11.13.16). Importantly, eternity is not just God's attribute but his substance that has nothing mutable (*Enarr. in Ps.* 101.24). Augustine identifies eternity with divine essence : "In the nature of God [...] there is only what *is*, and this is eternity itself" (*Enarr. in Ps.* 9.11 ; *De Trinitate* 4.1). Hence, God's eternity is not simply existence in time without beginning or end, but existence proper outside of time. Existence in time implies imperfect existence, since the past already does not exist and the future does not exist yet. Hence the exclamation : "If you want to be, elevate yourselves above the boundaries of time" (*In Joannis Ev. tract.* 38.10). To exist in time is to be a subject of time's flux, is to have no power over time, since time does not wait, does not flow faster according to our whims – it has its own course independent of things immersed in time. God, however, being a ruler of everything, rules also over time. He is eternal not because he can exist timelessly, but because he can exist outside

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5. *Ibid.*, p. 179.

the confines of time. Even if time did not have limits, and hence it were infinite, God's infinity would still be of a higher magnitude, an infinity of different kind. His infinity is above all possible temporal (and spatial) infinity; it is an infinity of infinities, whose magnitude can be dimly imagined by means of mathematical infinity.<sup>6</sup> It is an infinity of infinities also in that "all infinity is in some ineffable way made finite to God," since no infinity is incomprehensible to God (*De civ. Dei* 12.18), he can count numbers without succession of thought. God is even able to count without numbers (12.17), which assumes that there is no number equal to the quantity of all numbers, that is, no number, to use modern parlance, expressing cardinality of integers (which is aleph zero). This is no hindrance to God who is able to see the entire sequence of numbers without looking at these numbers one by one. Infinity of these numbers can be grasped in one act of comprehension.

God *is* – and that is where we can find God's essence. God is, and hence he does not change; he is always (meant atemporally) the same, immutable: "that truly is which abides immutably" (*Conf.* 7.11.17). Immutability is implied by His uncreatedness: "the nature, which is not made, is on that account the only immutable one" (*Contra Julianum* 5.44; 5.60; 6.5). Eternity, on the other hand, is a denial of temporality, that is, of temporal mutability. In this sense eternity "designates the way in which God is immutable; Augustinian 'eternity' is immutability considered in reference to time: God is that 'true eternity' whereby He is immutable, without beginning, without end" (*De Trinitate* 15.5.7).<sup>7</sup>

From the foregoing it is clear that for Augustine bestowal of existence is not from God's substance, since it would mean that a part of the divine substance can become finite and mutable; therefore, the world could only have been created out of nothing. Creating the cosmos from God's substance would also mean that the cosmos can become as immutable and eternal as God himself; however, only God is eternal (*Sermo* 7.7). The world exists in time and it is as mutable as time itself. Mutability is the nature of time, since only the present exists. Therefore, claiming that it is possible to have the world coeternal with God would mean that time is coeternal with God, but, as Augustine writes, "no time was coeternal with you, since you are permanent, and if time were permanent, it wouldn't be time" (*Conf.* 11.14; also, *De civ. Dei* 12.12.1; 12.16.2-3). Some neoplatonists defend coeternality by using this analogy: suppose that a foot was eternally put in sand, then a footprint would be in the sand eternally; the foot would be its cause, but cause neither precedes the effect nor vice versa. It may be admitted that God always existed and that he always was creating the world, so that the creation was a principle in the order of being but not

6. For this reason, it is difficult to agree with the statement that for Augustine "an eternal being must have [...] at least duration" (Christopher KIRWAN, *Augustine*, London, Routledge, 1991, p. 169); eternity surpasses duration, it is mentioned as an opposite of duration or of immersion in time; eternity is an extra-temporal immutability.

7. James F. ANDERSON, *St. Augustine and being: A metaphysical essay*, The Hague, Martinus Nijhoff, 1965, p. 15; however, because Augustine identifies eternity as God's substance, one cannot agree with Anderson's statement that it is just God's attribute. Also Boyer writes that "immutable and eternal are, in fact, synonymous" (*op. cit.*, p. 118), since eternity is immutability with respect to time, eternity is a mode of immutability.

in the order of time (*De civ. Dei* 10.31). But even assuming that the world always (perpetually) existed, it is obvious that time also perpetually existed. This does not imply that the world is eternal since perpetual time is not the same as eternal time. Time exists only partially, in the present moment (*Conf.* 11.20).

We can see that infinity in spatial and temporal dimensions cannot be found in God, he is above both of them. He is infinite in the sense of not being limited by anything, but this is a consequence of his being a spiritual being. Infinity is in us as part of our cognitive endowment with which even the finite can be understood, and this infinity in us comes from God who created us along with what is needed to live. But God himself is above all infinity, and when Augustine describes God as eternal, he wants to stress that reality of God is outside of time. Eternity of the world, if it were possible, would be incomparable with eternity of God, since the world exists in time and through time.

The concept of infinity has a negative ring in Augustine because, as quoted, all infinity is finite for God, since it can be comprehended by him. But because God can also comprehend himself, he is not, in this sense, infinite. Moreover, he is not finite, either, since he is not limited by anything, which rules out infinity as an attribute of God.<sup>8</sup> Infinity, at best, exists in God as an idea, and it can be used as the means to, for example, see all numbers at once. After all, numbers are very important because God used numbers and laws when creating the universe.<sup>9</sup> God can think infinite thoughts thereby encompassing infinity, but he himself is beyond infinity. This, therefore, adds to the transcendence of God: we are unable to comprehend the finite without having an infinite engrafted in our mind, but God is even higher than infinity. The concept of infinity in our mind is just the means of directing us to knowledge about God, at least, to some knowledge. Even the lofty concept of infinity is too weak to gain the fullness of such knowledge, one reason being that God is greater than infinity. But without the concept of infinity we would not be able to gain as much (or as little) as we can gain; hence, this concept should not be neglected just because it will not bring us the fullness of knowledge. Therefore, very much in the Platonic spirit, Augustine agrees that mathematics is the splendid means by which we can raise our spiritual eyes to the divine, and that the ultimate purpose of mathematical analyses is to approach God as much as powers available to humans empower us.

To summarize, there are three important aspects of Augustine's discussion of the problem of infinity. First, infinity is an inborn concept which enables any knowledge. Second, infinity can be found in the purest form in mathematics, and thus mathematics is the best tool of acquiring knowledge about God. Third, God is neither finite nor infinite and his greatness surpasses even the infinite. Augustine is original in combining these three aspects in his philosophy; some of them can be found in other philosophers and theologians, but also in mathematicians.

8. Therefore, Lévy is correct in saying that "infinity is not, for Augustine, a *positive* mark of God", but not in saying that "God is infinite, *because* he is immaterial" (Tony LÉVY, *Figures de l'infini: Les mathématiques au miroir des cultures*, Paris, Seuil, 1987, p. 129).

9. See GILSON, *Introduction*, p. 251-252.



## II

As discussed, Augustine ascribes a very important role to mathematics as the path toward the knowledge of God. It is worth noting that Augustine's treatment of mathematics is unlike Plato's approach. For Plato, the knowledge of geometry was a necessary passport to the Academy, and the knowledge of mathematics was a necessary prerequisite for gaining truths of higher magnitude. Mathematical truths were interesting primarily because of their non-utilitarian value, to acquire knowledge for knowledge's sake and to use them as the means to approach the world of ideas and the idea of ideas: the idea of Good (*Rep.* 525b-527b). Therefore, some comprehension of infinity inherent in mathematical objects enabled man to gain some knowledge about the infinity of the truth, about what surpasses the mutability of our world, *i.e.*, about what is really important.

A reflection of the Augustinian view on the position of mathematics may be seen in the role of quadrivium (geometry, arithmetics, astronomy, and music) which along with trivium was the basis of the Middle Ages academic curriculum. An Augustinian interpretation of quadrivium was especially strong in the school of Chartres. In the thirteenth century the perception in this school was that both trivium and quadrivium are the paths leading to God. Thierry of Chartres wrote that "there are four kinds of reasons leading man to the knowledge of the Creator. They are arithmetical proofs (*arithmeticae probationes*), musical proofs, geometrical proofs, and astronomical proofs." Quadrivium, also called simply mathematics, is thus a foundation upon which rational faith can be built, which – as Thierry remarks – is not unprecedented: "The custom of the ancients was to teach first of all mathematics in order to approach the knowledge of the divine." Mathematics mediates between theology and physics, and serves as the bridge between the physical world and the world of intelligibles, the latter being the proper object of both mathematics and theology.<sup>10</sup>

Among Christian theologians, mathematics acquired the highest status probably in the philosophy of Nicolas Cusanus in whose metaphysical discussions mathematics plays a central role. Mathematics is the foundation of the knowledge of God; it is the best tool enabling man to approach the truth. Analysis of infinity in mathematics is especially important since it can give us some knowledge concerning God's infinity, although in that respect Cusanus professed learned ignorance: learning the inadequacy of our knowledge especially in the face of infinity. Infinity will never be completely known to us and "the infinite as infinite is unknown" (*De docta ignorantia* 1.1). Although we cannot grasp the infinite in its entirety, we can approach it using mathematical reasoning since we "cannot do better" in approaching "knowledge of things divine" than through mathematics (1.11). Mathematics analyzes its objects in terms of finite attributes, and this finitude has to be transcended by extrapolating properties of finite figures onto infinite objects, thereby transcending the realm of mathematics and approaching the things divine. But afterwards, properties inferred about these infinite geometrical objects are used to transcend even this level, to be

10. Edouard JEAUNEAU, "Mathématiques et Trinité chez Thierry de Chartres", in Paul WILPERT (ed.), *Die Metaphysik im Mittelalter*, Berlin, Walter de Gruyter, 1963, p. 293-294.

as close to the things divine as the human reasoning allows us to approach. Hence, mathematical finitude leads to extramathematical infinity and the latter to the divine infinity. For example, mathematics can help us to approach the truth of the statement that God is *coincidentia oppositorum*. This *coincidentia* cannot be comprehended by *ratio*, but *intellectus* can gain some insight of it. In particular, the circumference of a circle whose radius grows infinitely more and more, resembles a straight line. If this circle became infinite then the circumference would merge with the straight line, whereby the opposition of the straight line and a curve disappears. Such reasoning can be repeated with other geometrical figures, which leads to the conclusion that although such infinite figures exist only potentially, the absolute being, whose infinity is actual, is truly a coincidence of the opposites and only this being can be in reality what these figures are potentially.

Mathematics allows us to approach infinity more than any other science, and without some approximate and imperfect comprehension of infinity, any knowledge of God would be seriously wanting. This insistence on the role of mathematics in Cusanus' speculations stems from the fact that for him the concept of infinity "is the point of departure and the goal of his thought, and also the aim and the means of knowledge".<sup>11</sup> This concept must be the point of departure, since without a concept or merely a premonition of the infinite, the entire process of abstracting and transcending would not lead Cusanus anywhere. The infinite is assumed before the reasoning in finite terms even starts; it is the light allowing Cusanus to bring his mathematical speculations to metaphysical conclusions. The learned ignorance is learned in knowing how little it knows about infinity, not in relying entirely on the finite. The learned ignorance is a realization that we are "guided by Infinite Truth" (1.12) in order to come closer to this truth – thanks to mathematics.

### III

Augustine's concept of God surpasses the vision of many other Christian theologians and most of them consider infinity to be one of the primary characteristics of God. For example, John Damascene uses a famous simile which compares God to "an infinite ocean of substance" (*De fide orthodoxa* 1.9); Bonaventura sees in infinity an indispensable component of divine essence (*De mysterio Trinitatis* 4.1); Roger Bacon writes, "God [...] is of infinite substance and essence, so that it follows that he is best, wisest and most powerful" (*Opus majus* ch. 4); Thomas Aquinas proves that "God is infinite and perfect" (*Summa th.* 1.7.1); Duns Scotus shows that the first, the most perfect being is infinite (*Opus oxoniense* 1.2.1); and Fénelon writes "when I say about an infinite being that it is God and I do not add anything to it – I said everything about it" (*Traité de l'existence de Dieu* 2.5). However, Augustine is an exception to this rule. To him, God encompasses infinity, himself not being infinite. But although his position is exceptional, Augustine is not the only

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11. Siegfried LORENZ, *Das Unendliche bei Nicolaus von Cues*, Fulda 1926, p. 10.

one who maintained this view. Augustinian legacy can be found in the philosophy of Jean de Ripa and Georg Cantor.

Jean de Ripa taught theology in the University of Paris in 14th century. In his time it was assumed, contra Aristotle, that there is a void outside the heavens and that this void was infinite. It was also created, and hence the creator has to exceed the infinity of the void to the extent that, in contradistinction to the created infinity, Ripa considers God to be immense, greater than infinite: "Immensity of the real presence of God exceeds immensely infinity of all the possible void".<sup>12</sup> So Ripa makes explicit what can be found implicit in Augustine: immensity exceeds spatial infinity as eternity exceeds temporal infinity. Ripa uses a proportion to explain this. If *c* is God's eternity, *d* is temporal eternity, *a* is God's being (*entitas Dei*), and *b* is the totality of all created things (*tota latitudo entis creabilis*) then *a* is to *b* what *c* is to *d*, or as *c* precedes *d* and reaches beyond, *i.e.*, *c* exceeds *d*, so *a* by its immensity exceeds *b*.<sup>13</sup> God necessarily has to exceed infinity; since infinity can be real, it is conceivable for it to be part of the physical world. For example, the world can be envisioned as a sphere, but this sphere would be limited and it would always be possible to conceive a larger sphere – even an infinite number of such spheres. Hence, God exceeds any such imaginable sphere, and since it is the very essence of God that nothing limits him (*divina essentia incircumscripta*), then – according to the famous saying – God is an infinite sphere with center everywhere and circumference nowhere.<sup>14</sup> Also, it is possible to have an infinite causal chain between the first cause and any secondary cause, which is the view defying a stance popular in Ripa's time, so that an infinite process is possible.<sup>15</sup> This seriously weakens the ontological proof which relies on the impossibility of an infinite causal chain. For Ripa the proof would have to rely on the nature of God rather than on infinity, and this nature has to surpass that of the cosmos. God is perfect since he surpasses limitations of infinity in all respects. Only an immense being can be the creator, and there is nothing in nature which meets that requirement. The vision Ripa offered is of rare imagination and it is a great deal of truth in the emphatic statement that "probably never a metaphysical doctrine separated with more force from the totality of real or possible being, the being absolutely transcending whose essence, and only this [essence], does not admit any kind of degree, real or imaginary".<sup>16</sup>

#### IV

It is interesting that the fullest continuation of the Augustinian view on the role of infinity in theology can be found in views of the founder of set theory, Georg Cantor, who on many occasions indicates that he was theologically motivated in his

12. André COMBÈS, Francis RUELLO, Paul VIGNAUX, "Jean de Ripa I Sent. dist. XXXVII: De modo inexistendi divine essentie in omnibus creaturis", *Traditio*, 23 (1967), p. 200, 235.

13. Jean DE RIPA, *Quaestio de gradu supremo*, Paris, Vrin, 1964, p. 166-167.

14. *Ibid.*, p. 177-178.

15. *Ibid.*, p. 207, 209.

16. André COMBÈS, "La métaphysique de Jean de Ripa", in Paul WILPERT, *op. cit.*, p. 555.

mathematical venture. He is generous in theological comments of his mathematical analyses and does not leave any doubt that his main interest lies in theology rather than in mathematics.

Cantor distinguishes an immanent reality from transcendent (*transiente*) reality. Mathematics is interested in the immanent reality, and hence it is not obligated to prove existence of such immanent infinity. Mathematics has to obey the rule of consistency which guarantees that its creations "can and must be treated in mathematics as existent and real".<sup>17</sup> This statement does not imply a complete arbitrariness, and although Cantor calls on the uselessness and fruitlessness of arbitrarily constructed mathematical entities, it seems that his reference to "the unity of the universe" is a more important argument for this understanding of freedom in mathematics: this freedom is limited by the nature of the universe and hence by the nature of our mind, thus there must be some necessary correspondence and similarity between immanent and transcendent realities. Fruitfulness of mathematics is founded on this unity of the universe so that creating new constructs turns out to be just a rational discovering of what the transcendent reality contained all along. But this unity of the cosmos is, in turn, founded on the world being created by God. Therefore, as Cantor writes to E. Illigenes, "if I know that a concept signifying an existing entity (*ein Sein*) is internally consistent, then the idea of Almighty God forces (*zwingt*) me to think about the entity signified by this concept to be in some way realizable in actuality," whereby the entity is possible, although the nature of its existence may not be now clear.<sup>18</sup> Therefore, mathematical statements are not divorced from reality, and, for instance, set theory makes certain pronouncements about things in themselves, about "true being," and "the general set theory [...] belongs entirely to metaphysics" and is its servant.<sup>19</sup>

This unity of the universe also has another meaning for Cantor, which brings him close to Augustine. Cantor is opposed to epistemology of sensualism, since this type of knowledge allows for no certainty. True knowledge is based on "concepts and ideas," which are "stimulated" by experience and are basically "built by the inner induction and deduction as what in us, as it were, already lay and was only awakened and brought to consciousness".<sup>20</sup> Although Cantor refers in that respect to Plato, he is not committed to the theory of anamnesis. He only wants to stress the fact that abstract knowledge is already in us, implanted and dormant, enlivened by our quest for it. In particular, infinity cannot be recognized unless it is inborn, since infinity "even inhabits our mind (*Geiste*)".<sup>21</sup> Therefore, mathematics has not only a purely theoretical interest, but it is also of philosophical and theological bearing.

17. Georg CANTOR, *Gesammelte Abhandlungen*, Hildesheim, Georg Olms, 1962, p. 181-182.

18. Letter from May 21, 1886, quoted in Walter PURKERT, *Georg Cantor: 1845-1918*, Basel, Birkhäuser Verlag, 1987, p. 107.

19. Letter to Th. Esser, Feb. 1, 1896, Herbert MESCHKOWSKI, *Probleme des Unendlichen: Werk und Leben Georg Cantors*, Braunschweig, Vieweg, 1967, p. 111, 123.

20. CANTOR, *op. cit.*, p. 207.

21. *Ibid.*, p. 375.

First, we have to realize that there is a great difference between an absolute infinity, or simply an Absolute which can only be realized in God, and the transfinite (*Transfinitum*), *i.e.*, the infinities in the world and in the mind.<sup>22</sup> The transfinite is a subject of change, of increase, and hence it is the augmentable (*Vermehrbares*), but the Absolute cannot be modified and thus it cannot be mathematically determined.<sup>23</sup> The former can be found in the material and in the spiritual worlds, whereas the absolute infinity is only God's attribute: "the transfinite with its profusion of transformations and forms by necessity points to an Absolute, to a 'true infinity,' whose magnitude can be neither augmented nor diminished and which, therefore, is considered quantitatively, is an *absolute maximum*".<sup>24</sup> But whereas different transfinite levels can be known (*erkannt*), the Absolute can only be recognized (*anerkannt*), not known, not even approximately; however, an "absolutely infinite sequence of numbers," *i.e.*, sequence of all infinities, can be considered "a suitable symbol of the Absolute".<sup>25</sup> Set theory shows that there is no set encompassing all sets, and yet God is able to comprehend all these infinities, hence he is above infinity, he is the Absolute. The transfinite, unlike the Absolute, "clearly appears to us as limited, capable of being augmented and thus related to the finite".<sup>26</sup> With this statement Cantor returns to Augustine's conviction that "all infinity is in some ineffable way made finite to God."

The transfinite numbers are not pure creations of our mind, they are only discovered in the mind and in the world. They cannot be our creations since they precede our very existence and the existence of the world. As Augustine, whom he quotes, Cantor believes that God utilized numbers to create the world. In particular, "the whole numbers [...] exist in the highest degree of reality *in intellectu Divino* as eternal ideas".<sup>27</sup> But we can repeat the same about all levels of infinity, since, "the transfinite kind of numbers are as much at the disposal of the intention of the Creator and of his absolutely unbound power of will as finite numbers".<sup>28</sup> But this means that "all these particular modi of the transfinite exist from eternity as ideas *in intellectu Divino*".<sup>29</sup> Hence, mathematicians and set theorists in particular can be certain about the validity of their pronouncements since their domain is divine ideas, if only existing in some imperfect reflection in their own minds. After all, in a Leibnizian way, it may be claimed that omnipotent God realized any consistent possibility, if only in his unbound mind.<sup>30</sup>

22. *Ibid.*, p. 378.

23. *Ibid.*, p. 375.

24. *Ibid.*, p. 405.

25. *Ibid.*, p. 205.

26. *Ibid.*, p. 378.

27. Letter to Hermite, Nov. 30, 1895, MESCHKOWSKI, *op. cit.*, p. 262.

28. CANTOR, *op. cit.*, p. 404.

29. Letter to Jeiler, Oct. 13, 1895, MESCHKOWSKI, *op. cit.* p. 258.

30. See Joseph W. DAUBEN, *Georg Cantor: His mathematics and philosophy of the infinite*, Princeton, Princeton University Press, 1990, p. 229.

If mathematics, and its foundation, set theory, is so certain, it should be brought in service of theology. Set theory is founded on the structure of the created universe, and it can be used to disclose this structure to direct people's attention to God. In this sense, Cantor saw his work as Christian service and as a means of bringing people to rational (*vernunftgemäßen*) theism.<sup>31</sup> In particular, the theory of transfinite numbers could help in proving that the world was created.<sup>32</sup> Also, in spite of the difference between the transfinite and the Absolute, the latter can be approached through the avenue of the transfinite. Not surprisingly then, different infinite cardinalities "were for Cantor somewhat sacred and in a sense steps leading to the throne of the infinity, to the throne of God".<sup>33</sup>

This discussion indicates that we can find in Cantor, more or less clearly verbalized, all three aspects of Augustinian philosophy of infinity. Interestingly, although Cantor enthusiastically quotes Augustine, he does not consider himself his intellectual heir. A comparison of their views of infinity indicates that he is a heir of Augustine more than any theologian and that Cantor is more a successor of Augustine than any other thinker. First, infinity is in us, we do not create it. Second, mathematics is most competent in dealing with infinity, and Cantor himself spent his life to actualize the part of mathematics which deals with infinity in the purest form, set theory. But this infinity is only an adumbration of what goes beyond infinity, the Absolute which dwells *in Deo*. And this is the third aspect – Absolute as surpassing the transfinite. In all these, Cantor joins Augustine in theological pronouncements and – as far as mathematics allows him – he also shows that it is a mathematically proven fact. There is no set of all sets, the number of infinities surpasses any number. This fact can be used by theologians that God simply must surpass all infinities and in this sense he is not infinite – he is the Absolute.

## V

The legacy of Augustine is the view that people have the idea of infinity and this idea is first of all a vehicle for directing us toward God. The primary task of science, especially mathematics, it to enable us better comprehension of the spiritual sphere. Its pragmatic use is only secondary and contingent. Mathematics, by using infinity, shows us our limitations and our finitude, thereby directing us to what surpasses us, our world, and the infinity which was used to create us and the world.

The concept of infinity directs our eyes toward God, and in a sense the presence of infinity in us can be considered a proof of God's existence: we, the finite and mutable beings, could not engender that concept; who else could do it if not God? Also, infinity in us can be appreciated and known best through mathematics, through analysis of numbers. In that sense the existence of numbers can be considered a more fundamental proof of God than cosmological argument – since the world would not

31. See letter to Hermite, Jan. 21, 1894, MESCHKOWSKI, *op. cit.*, p. 124-125.

32. Letter to A. Schmidt, Aug. 5, 1887, MESCHKOWSKI, *op. cit.*, p. 125.

33. Gerhard KOWALEWSKI, *Bestand und Wandel*, München, Oldenburg, 1950, p. 201.

exist without numbers – and teleological argument – since the design and order in the world can be recognized only through numbers, since order and design are due only to numbers. Therefore, although ontological proof, announced already in Psalm 19:1 that “the heavens declare the glory of God,” has always been considered most important, Augustine could consider mathematical proof as the most fundamental: God exists since the number and infinity exist in our mind.

A remarkable fact is that all these Augustinian ideas can be found in views of a mathematician rather than a theologian. And it can be claimed that without them Cantor’s set theory would not emerge. Cantor’s theory was truly groundbreaking and revolutionary, but it is difficult to agree that “as much as he was revolutionary in his mathematical ideas, Cantor appears to be conservative in questions of religion”.<sup>34</sup> It seems, however, that only because he was so conservative in his religious views, Cantor could be revolutionary in mathematics. Strong conviction in the veracity of his theory and in its divine provenance – he believed that it was revealed to him directly by God –<sup>35</sup> enabled him to persist in constructing it and later in defending it in the face of strong opposition on part of mathematicians and philosophers. It was this mixture of religious and mathematical views that brought Cantor’s theory to fruition, and although we divorce today all too easily the religious from the mathematical in Cantor, “the theological side of Cantor’s set theory [...] is [...] essential for the full understanding of his theory and the development he gave it,” and this side “should not be discounted as merely an aberration”.<sup>36</sup>

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34. MESCHKOWSKI, *op. cit.*, p. 126.

35. DAUBEN, *op. cit.*, p. 232.

36. *Ibid.*, p. 291.