DYNAMIC MODEL OF A NON-LINEAR SERVO CONTROL SYSTEM USING TRANSMISSION LINE MODELLING TECHNIQUE

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ABSTRACT

The modelling and simulation of a test rig based on the transmission line modelling technique is presented in this paper. The studied test rig is a non-linear servo control system and represents a CNC machine tool feed drive where the controller commands the movement of worktable linked to a motor through a ball-screw.

The transmission line modelling technique has been successfully applied until now for hydraulic systems, electromagnetic, acoustics and several mechanical applications (not much emphasis on digital controllers).

The paper presents a novel application of this method for modelling the interaction between the position loop controller, servo system and the worktable mechanism. It addresses the modelling of the non-linearities (Coulomb and viscous friction, backlash) and axial and torsional forces applied on the ball-screw nut during its linear movement.

The simulation results generated by the MATLAB program compare well with the measured data when same stimuli are applied to the test rig. So this numerical method is successfully applied for accurate modelling of non-linear and dynamic behaviour of CNC machine tools feed drives.

1 INTRODUCTION

An optimal operation of feed drives within CNC machine tools requires a fine co-ordination between controls, electrical drives, linear guides, pneumatics and hydraulics. Its main advantages are: increased high-speed performance, reduced costs, improved robustness, etc. This requires the development of precise models reflecting the dynamic behaviour of CNC machine tools for various running conditions.

The lumped parameter models (1) with the load inertia referred to the motor shaft did not reflect the real dynamic behaviour of the machine tools although they were frequently used in the design process.

Then basic software modules (2) were developed for the machine's mechanical and electrical components. A perturbing load torque represented the effect of mechanical load upon the electrical drive. However, the dynamic performance of the simulated results did not compare well to the machine measured data.

Therefore Pislaru et al (3) have developed a hybrid model with distributed and lumped elements for an analogue feed drive. The simulated dynamic responses when the nut oscillated at the middle of ball-screw were similar with the response of the feed drive system. Still further research should be performed in order to reflect the complex interactions within the versatile hybrid multi-body systems such as CNC machine tools.

Christopolous (4) showed the analogy between electrical circuits and physical systems based on the transmission line modelling (TLM) method. The elements of various systems could be represented by capacitors, inductors and resistances considering the wave propagation through a variety of mediums.

Bartlett and Whalley (5) validated this method of approach by developing the admittance and impedance modules for two hybrid systems - ventilation systems used in long tunnels and rotor shells. The torsional oscillations of a wide-faced rotor were simulated using an analogue solution of TLM method. The system response reflected the effect of distributed inertia and stiffness, but a feed drive contains a controller commanding the worktable movement through a motor and a ball-screw system.

Moreno-Castaneda et al. (6) have built a TLM model for a digital drive fitted on the test rig which is the subject of this investigation. The dynamic behaviour of the elements has been described by differential equations and corresponding TLM models have been derived. The development for the first time of a TLM model for a digital controller represents an important contribution.

The main advantages of using TLM to model the feed drive system behaviour are the relatively simple procedures (enabling both continuous and discrete models to be accommodated) and the high speed of processing. The developed model had only linear elements even though the real feed drive system has non-linearities (friction, backlash, quantisation, etc.).

Eun-Chan et al. (7) underlined that the main disturbances which deteriorate the tracking performance of machine tool (especially at velocity reversal) are friction, the backlash and the torsional displacement between motor and the worktable.

Berger (8) mentioned that "the success of models in predicting experimental results remains strongly sensitive to the friction model". He underlined that friction depends on a large number of parameters such as: normal load, sliding speed, acceleration, temperature, humidity, surface quality, material combination. Therefore the research into computational

tools and approaches capable of assessing the multi-scale effects of friction contacts is an ongoing process.
Burton et al (9) have developed accurate models for simulating distributed laminar friction and frequency dependent friction within hydraulic systems. Their approach generated computationally efficient models. Hence it will be used in future as a guide during the implementation of TLM model for CNC machine tool feed drive into on-line condition-monitoring systems. The analysis will be performed especially in the frequency domain, therefore the method of modal approximation to the distributed friction TLM functions developed by Watton (10) will be applied.
The authors of this paper model Coulomb and viscous friction for worktable-slides and screw-nut contacts using the approach suggested by Karnopp (11). The backlash is modelled as a hysteresis loop. Also the axial and torsional forces applied on the ball-screw nut during its linear movement and the structural stiffness of various elements are analysed and included in the TLM model of the studied hybrid system.
The paper contains a brief representation of the TLM method and the differential equations describing their dynamic behaviour of the feed drive components. Also their corresponding TLM models are developed and analysed. Then friction, backlash and axial and torsional forces applied on the ball-screw nut during its movement are included into the model.
The simulation results obtained by implementing the TLM model in MATLAB reflect the hybrid nature of the feed drive and are the same as those measured from the test rig when the same stimuli are applied.

2 GENERALITIES REGARDING TLM METHOD

TLM is a numeric differential method usually used to solve wave propagation problems through a medium. The system equations are made equivalent with the equations for voltages and currents for a mesh of transmission lines. TLM technique uses two circuits:

- the stub - one-port unit used for solving circuits and equations;
- the link - two-port block that can be used for one-, two- or three-dimensional modelling.

Christopolous (4) demonstrated that the reactive components of any electrical circuit could be replaced with their corresponding TLM stubs. The voltage and current are considered as discrete pulses bouncing to and from the nodes of these stubs at each time step.
The incident *(e^i)* and reflected (e^r) pulses in a port determine the voltage and current in each component (Fig. 1). The incident pulses are injected into the TLM network and a time (Δt) is necessary for their travel between ports. The reflected pulses are generated according to boundary conditions are generated when incident pulses reach a port (node). The reflected pulses will become the incident pulses in the next time step.

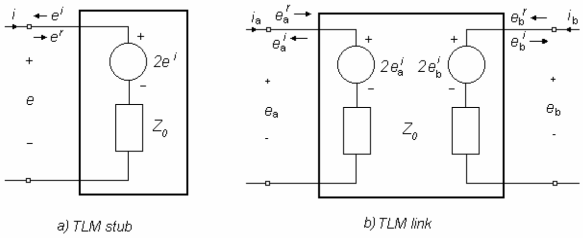

Figure 1: TLM Units

The voltages and current can be calculated knowing the incident pulse $_k e^i$ at time step k. The current for $_k e$ as the discrete stimulus applied to the TLM yields:

$$_k i = \left(_k e - 2\, _k e^i \right) / Z_0 \tag{1}$$

The result of applying Ohm's law to the TLM stub is:

$$_k e = 2\, _k e^i + {}_k i Z_o \tag{2}$$

The reflected pulse is $\quad _k e^r = {}_k e - {}_k e^i \quad (3)$

Like it was mentioned, the reflected pulse becomes the next incident pulse, hence $\quad _{k+1} e^i = \Gamma\, _k e^r \quad (4)$

The current for the next time step $_{k+1} i$ may be obtained from the next incident pulse $_{k+1} e$. This scattering algorithm is repeated as many times as necessary.
The variables Z_0 and Γ depend on the represented element.

3 TLM MODEL OF SERVO CONTROL SYSTEM

A typical digital drive from a CNC machine tool consists of the basic elements shown in Fig. 2.

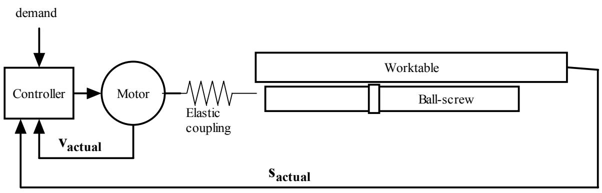

Figure 2: Experimental test rig configuration

Moreno-Castaneda et al. (6) have developed the TLM models for the test rig components:

Digital axis-controller model - based on the principle of cascade control and contains an inverter which commands the motor by pulse width modulated (PWM) signals. The set point generator calculates the velocity profile from the feed rate, maximum acceleration, and jerk. The position controller uses a linear interpolation algorithm to determine the nominal velocity value V_{nom} for the velocity controller:

$$V_{nom} = K_v (s_{x0} + V\, \Delta t - s_{actual}) + V_{ff} \tag{5}$$

It depends on velocity constant (K_V), previous nominal position (s_{x0}), calculated velocity (V), controller cycle time (Δt=3 ms), measured position (s_{actual}) and the value calculated by the velocity feed forward loop (V_{ff}).
The equation for the equivalent TLM stub yields:

$$V_{nom} = K_v (s_{x0} + V\,\Delta t - s_{actual}) + 2e^i_{ff} + \\ + (s_{x0} + V\,\Delta t - s_{actual}) Z_{ff} \qquad (6)$$

The equivalent impedance is $Z_{ff} = 1/(\Delta t_s/2)$ for simulation time Δt_s.

The velocity controller generates the nominal current every 0.6 ms using a PID strategy:

$$I_{nom} = D_1 \frac{d}{dt} V_{ff} + PV_e + I \int V_e dt + D \frac{d}{dt} V_e \qquad (7)$$

The current depends on proportional (P), integral (I), differential (D) factors and differential factor for acceleration feed forward (D_1). The spikes caused by changes in velocity are minimised by the acceleration feed forward used in parallel with the speed controller.

The equivalent TLM representation is

$$I_{nom} = Z_{Aff} V_{ff} + 2e^i_{Aff} + PV_e + Z_I V_e + 2e^i_I + Z_D V_e + 2e^i_D \qquad (8)$$

where $V_e = V_{nom} - V_{actual}$, $Z_{Aff} = D_1/(\Delta t_s/2)$

$Z_I = I(\Delta t_s/2)$ $\quad Z_{Aff} = D/(\Delta t_s/2)$

The current controller voltage uses a voltage source inverter (VSI) to ensure that the rotor flux vector is always kept in quadrature with the stator current vector.

Permanent magnet synchronous motor model – contains the equivalent TLM stub when the machine performance equations are simplified to its reference frame (11). Also the TLM model of the mechanical dynamics of the motor considering the moment of inertia, friction in the motor bearings and the load torque is included.

Dynamic model of the ball-screw – takes into consideration the effect of the table and nut on the screw shaft (Fig. 3).

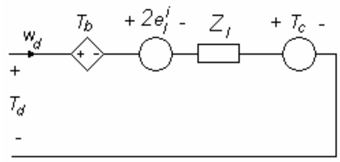

Figure 3: TLM model of the effect of the table and nut on the screw shaft (6)

The TLM stub generating this effect is:

$$T_d = T_c + T_b + Z_l w_d + 2e^i_l \qquad (9)$$

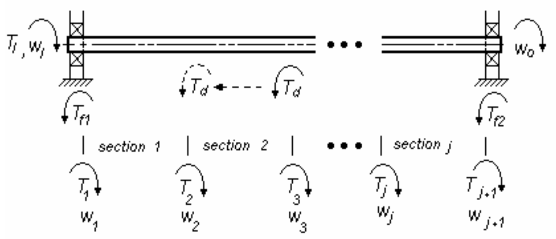

Figure 4: Screw shaft divided into j sections (6)

The torque generated by the motor (T_d) has to overcome the cutting torque (T_c), Coulomb friction torque and pre-load torque (T_b) and torque due to load inertia because $Z_l = J_{load} / (\Delta t_s/2)$.

The effect of moving nut is modelled by dividing the length of the shaft into j equal sections (Fig. 4). The point where the torque T_d is applied on the screw shaft changes as the nut moves.

Each section was modelled as a TLM link and the equations and equivalent diagram were presented in (6).

4 IMPROVEMENTS OF TLM MODEL FOR THE BALL-SCREW WITH MOVING NUT

4.1. TLM model for stick-slip friction

The model of stick-slip friction proposed by Karnopp (12) is implemented in TLM model of the test rig. The friction force F_f is always a function of v and the author proposes to consider the velocity v to be zero in a small region surrounding v=0 (DV from Fig. 5). This region is necessary for digital computation time since an exact value of zero cannot be calculated.

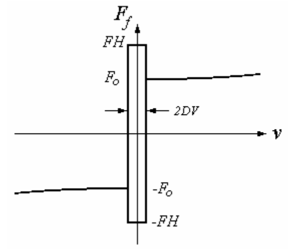

Figure 5. Stick – slip friction law proposed by Karnopp

The force F necessary to move the mass m from rest with velocity v and momentum P should overcome the friction force F_f. The block diagram from Figure 6 shows the algorithm implementing the friction approach described above.

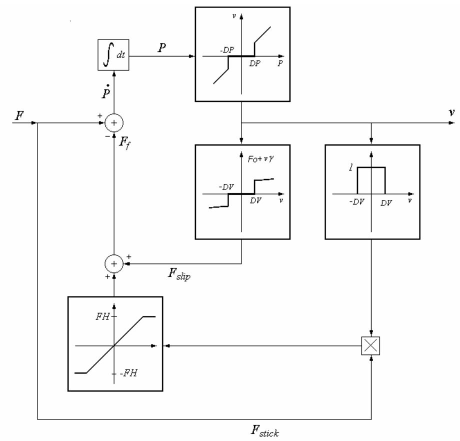

Figure 6: Block diagram of stick-slip friction (12)

The values included in the MATLAB program are:

DV=0 ; DP= μ * DV=0

FH= 1.5 F_C=1.5*9.81 * μ * m=1.5 * 9.81 * 0.003* 850

where μ - viscous friction coefficient for linear guides

m - mass of the worktable

4.2. TLM model for the angular and torsional displacement of the nut

The general form of wave equation (6) is:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{u^2}\frac{\partial^2 y(x,t)}{\partial t^2} \tag{10}$$

This relation can be written under the following form:

$$DP\frac{\partial^2 y(x,t)}{\partial x^2} = MP\frac{\partial^2 y(x,t)}{\partial t^2} \tag{11}$$

The equivalent TLM link is displayed in Fig. 7 and the parameters are:

$u = \sqrt{D/M}$ velocity of propagation (12)

$Zo = MPu$ equivalent impedance (13)

The function y(x, t) can represent either voltage or current on a transmission line.

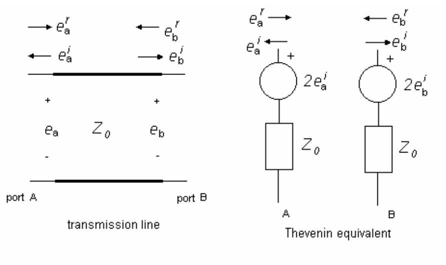

Figure 7: TLM representation for wave propagation equation

The same method is applied for the equation of the longitudinal vibration of a bar (13):

$$EA\frac{\partial^2 y(x,t)}{\partial x^2} = \rho A\frac{\partial^2 y(x,t)}{\partial t^2} \tag{14}$$

The parameters of the equivalent TLM link are:

$$u = \sqrt{E/\rho} \tag{15}$$

$$Zo = \rho A u \tag{16}$$

where E- Young's modulus ρ - density of material
G- rigidity modulus A- cross sectional area
J- polar moment of inertia

The function y (x, t) represents either axial force or longitudinal displacement.

The equation for torsional vibration of a shaft (4) yields:

$$GJ\frac{\partial^2 y(x,t)}{\partial x^2} = \rho J\frac{\partial^2 y(x,t)}{\partial t^2} \tag{17}$$

The equivalent TLM link has the following parameters:

$$u = \sqrt{G/\rho} \tag{18}$$

$$Zo = \rho G u \tag{19}$$

The function y (x, t) signifies either torque or angle of twist.

4.3. TLM model for backlash

The TLM model is derived from the mathematical model of backlash suggested by Kao et al. (14). The backlash has the distance *D* between the position feedback X_{act} and the worktable position X_{tab}.

The relative distance x_i between the position feedback X_{act} and the worktable position X_{tab} must be known first in order to determine correctly the worktable position X_{tab}. The relative distance x_i at the i^{th} time step could be determined by the following relation:

$$x_i = x_{i-1} + dX_{act} \tag{20}$$

where dX_{act} - the incremental position feedback at the i^{th} time step,

x_{i-1} - the relative distance at the *(i-1)th* time step.

The four possible situations are depicted in TABLE 1:

TABLE 1 – Simplified backlash model for various input-output characteristics

Input conditions	Output function
$0 < x_i < D$ $dX_{act} > 0$	X_{tab} is stationary
$0 < x_i < D$ $dX_{act} < 0$	X_{tab} is stationary
$x_i > D$	$X_{tab}^i = X_{tab}^{i-1} + x_i - D$ $x_i = D$
$x_i < 0$	$X_{tab}^i = X_{tab}^{i-1} + x_i$ $x_i = 0$

5 COMPARISON BETWEEN SIMULATION AND MEASURED RESULTS

A trapezoidal velocity signal is used as an input stimulus to check out the performance of the test rig during acceleration, deceleration and functioning at constant rate (6000 mm/min in this case). The same signal is applied to the TLM model implemented in MATLAB and the error between simulated and measured velocity of the nut is shown in Fig. 8.

The error has great values during the acceleration and deceleration time (as expected), and its approximate mean value of 10 mm/min indicates that further research should be performed in order to optimise the coefficients of the filters included into the TLM model of the controller.

The difference between the worktable position measured by a linear encoder and that generated by the TLM model is displayed in Fig. 9. Its mean value of 20 μm points out that other factors (such as stiffness, presliding friction etc.) should be analysed and included into the TLM model for the feed drive.

Also the TLM model should include the effect of system uncertainties (parameter variation, modelling error, external load, electronic noises) and dynamic constrains (system bandwidth, mechanical response).

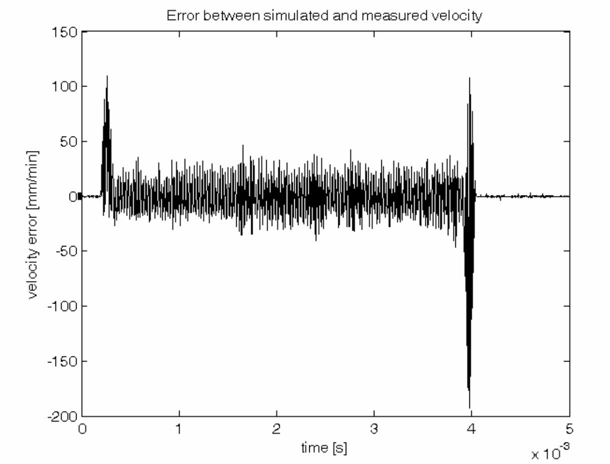

Figure 8: Error between simulated and measured linear velocity of the nut

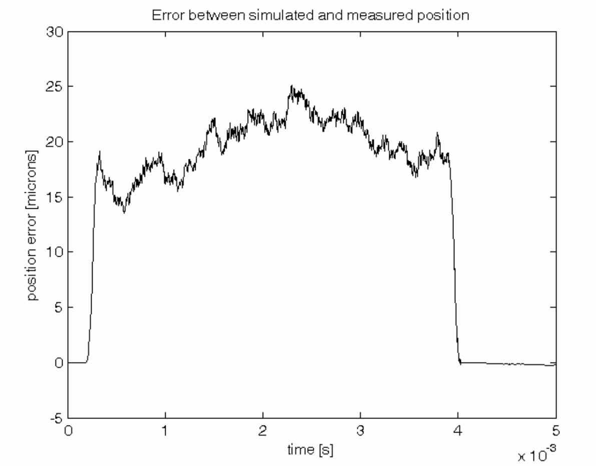

Figure 9: Error between simulated and measured position of the worktable

6 CONCLUSIONS

The paper presents the comparison between measured and simulation results when a trapezoidal velocity signal is applied to the test rig and its TLM model. The TLM model is validated by good comparison of very fast and accurate simulated results to the direct responses from the actual digital axis-controller.

The paper describes the TLM modelling of the non-linearities (Coulomb and viscous friction, backlash) and axial and torsional forces applied on the ball-screw nut during its linear movement. The simulation results are encouraging and improvements in the modelling of other factors (such as stiffness, presliding friction, system uncertainties, dynamic constraints etc.) are needed in order to tune the TLM model to the measured dynamic behaviour of modern CNC machines.

This numerical method could be the base of a powerful, flexible and effective approach to improve the accuracy of the complete servo control system.

The TLM model accurately representing the non-linear and dynamic behaviour of the feed drives is intended to become a part of on-line condition-monitoring methods.

The future work will include TLM implementation in the frequency domain allowing the easy construction of an accurate universal mathematical model for modern CNC machine tools.

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