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Title DESIGN OF AN ELECTRICAL ANALOG FOR SPHERICAL WAVE PROPAGATION IN SOLID ELASTIC MEDIA

Abstract approved [redacted]
(Major professor)

With the advent of underground testing, the study of the propagation of seismic signals has become of great importance. This thesis investigates the design of electrical analogs to represent the propagation of seismic waves.

The analogs developed allow for the study of the effects of variations in the characteristic constants of the medium through which the wave propagates and further allow for the characteristics of the source to be imposed upon the compressional displacement propagation. It is assumed that the source may be represented by a spherical radiator and by solving the wave equation in spherical coordinates, the transfer function for the wave is obtained. The design of the analog is based on this transfer function and through the use of network synthesis, a lumped element network is synthesized.

Variation of the elements in the synthesized network will permit the study of the effects of variations in velocity. Poisson's ratio, range, size of the radiator, and excitation functions on the compressional wave propagation.

To include the effects of multiple layering, a second analog was designed. This was accomplished by connecting, in tandem, analogs representing single layer of specified thickness. The individual stages of this analog were synthesized using a different method from the one used in the synthesis of the first analog in that they were terminated. The application of the second analog is restricted to the case for which we can assume, within reasonable error, that the spherical wave approaches a plane wave in character.

DESIGN OF AN ELECTRICAL ANALOG FOR SPHERICAL WAVE PROPAGATION IN SOLID ELASTIC MEDIA

by

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INTRODUCTION

The study of seismic wave propagation is of great interest in seismology. However, the complexity of the analytical expressions for the characteristic constants, (velocity, Poisson's ratio, density), of the medium in which the waves propagate as well as complex excitation functions make this study very difficult. Proposed mathematical models encounter obstacles because either the solutions are not in a feasible form, or they are arrived at only after tedious mathematical computations. In general a solution of the wave equation can be arrived at rather easily if certain simplifying restrictions about the problem are made. These restrictions will involve approximations such as infinite medium, homogeneity and isotropicity of the medium, character of the source, and shape of the excitation function.

Recently, with the data from underground testings, the different solutions of the wave equation can be checked with well-known detailed observations. The plot of these solutions will normally require the use of computers. An electrical analog circuit can, however, be built to simulate the displacement resulting from the solution of the wave equation, and at the same time make feasible the study of the effects of variations in the characteristic constants of the medium upon the displacement wave propagation. The proposed analogs will be built using the ratio of the output response to the input excitation of the system thus giving a representation of the medium in terms of its characteristic constants. Such a simulation will permit the study of variations in radiator size, effects of distance from the radiator and variations of the characteristic constants of the medium to be imposed upon the displacement wave propagation; but it will be limited to non-dispersive waves generated from any type of excitation applied to the walls of a spherical radiator.

Simulation of variations in radiator size and distance from radiator will be accomplished by changes in the elements in the circuit, while the simulation of variations in velocity, Poisson's ratio and density of the medium will be accomplished by cascading single networks corresponding to a lumped parameter representation of a layer of any specified thickness where all the above parameters are constant.

TRANSFER FUNCTION

One convenient method of describing a physical system is in terms of its transfer function. A transfer function is defined as the output response divided by the input excitation.

The above definition does not imply that a transfer function is a dimensionless quantity; a transfer function may or may not have dimensions depending on how the input and output are specified. Usually a transfer function is expressed as a function of the Laplace transform variable $s = \sigma + j\omega$, otherwise related to the frequency response of the system.

$$\text{Transfer function} = \frac{\text{Laplace transform of output}}{\text{Laplace transform of input}}$$

Moreover, for a lumped and finite system, the transfer function will be a rational function of the complex variable s and may be written as the ratio of two polynomials.

$$\text{Transfer function} = K \frac{a_0 s^n + a_1 s^{n-1} + \ldots + a_{1n}}{b_0 s^m + b_1 s^{m-1} + \ldots + b_m}$$

If the system is stable, that is, when the input is removed the output dies out with time, the polynomial of the denominator in the above expression must be of the class of Hurwitz. A polynomial is of the class of Hurwitz if all its zeros lie on the left half of the s-plane.

For every rational transfer function there corresponds an electrical network composed of linear elements. Such networks can be found using known methods of network synthesis.

The configuration, the number of elements and the element values of such networks will not be unique; but will depend on the method used for their synthesis. Generally, we can classify transfer functions as either immittance transfer functions or gain transfer functions. With existing methods these functions can be synthesized in the form of an RL, RC, or RLC network.

PHYSICAL CONSIDERATIONS

The methods used in constructing electrical analogs are characterized by the fact that the analog has to be described by equations similar to that of the physical system. Generally a mechanical analog is sought first, then the electrical analog is constructed from it in accordance with the basic system of electro-mechanical analogs.

In this work, no mechanical analog is necessary because the electrical analog is derived directly from the transfer function of the system.

If we assume an infinite, elastic, homogeneous, and isotopic medium, the equation governing the wave propagation is:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} \tag{1}$$

where ϕ = potential function,

c = compressional wave propagational velocity

With the existing methods for modeling a system described by equation (1), a three-dimensional analog has to be devised. B. D. Ivakin [7, p. 481] gives a two-dimensional analog of the two-dimensional wave equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} .$$

This analog is shown in Figure 1.

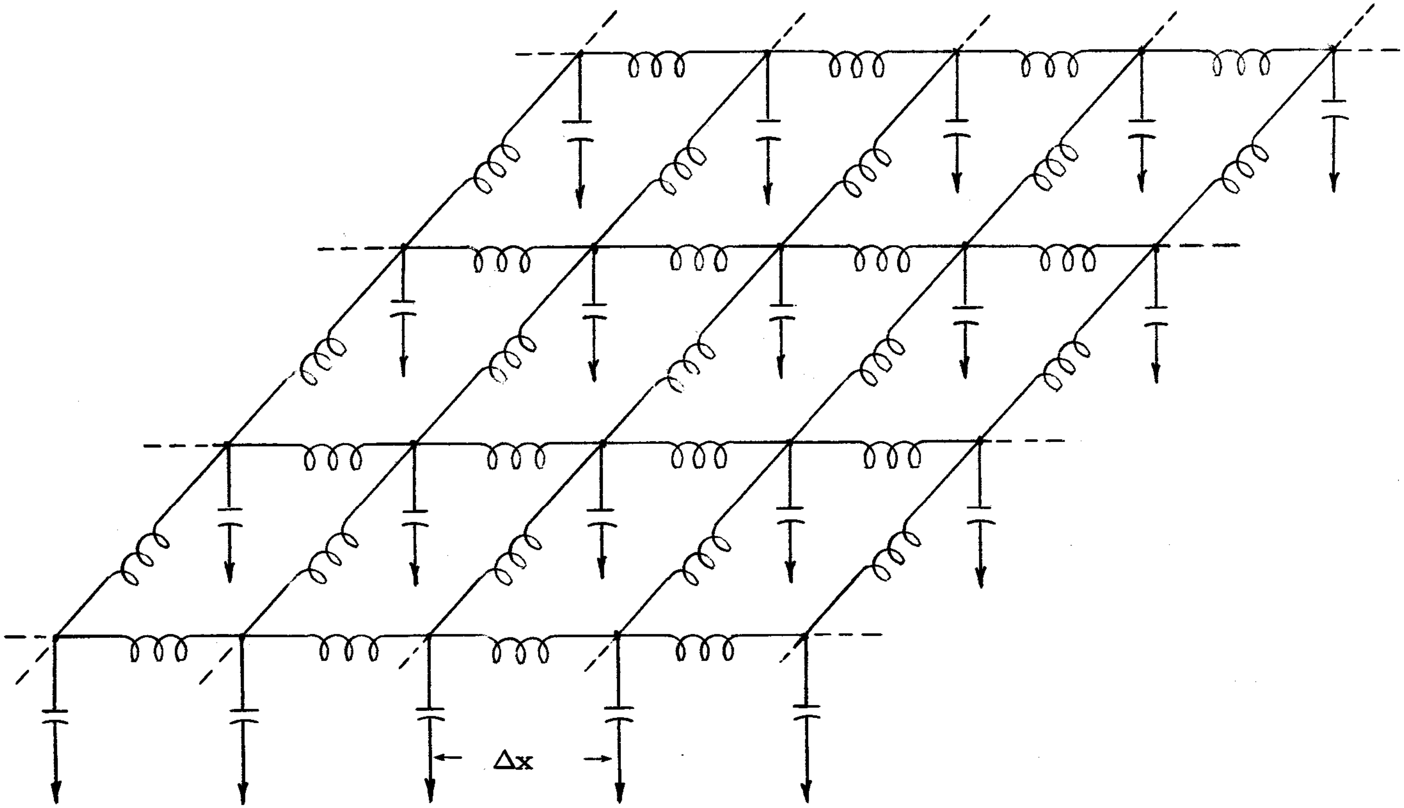


Figure 1

Two dimensional analog for an elastic medium

Ivakin used this analog for modeling some geophysical phenomena such as, transmission and propagation processes of seismic waves, modeling of earthquake focus, and modeling shotpoint areas. A three-dimensional analog similar to the one of Figure 1 can be constructed, In an analog of this type the problems associated with electric analogs will be more profound because the number of components will increase and the effect of inductive coupling and stray capacitance will be more difficult to handle. In addition, since the analog will have three dimensions its construction and measurements on it may constitute a problem.

In the proposed model we avoid some of these difficulties because of the simplicity of its construction.

The solution of equation (1) in spherical coordinates, for a pulsating cavity of radius a will be of the form:

$$\phi = \frac{A}{r} f(\tau)$$

where A is a complex constant, r is the range and τ is a parameter which is a function of time and the radius of cavity.

F. G. Blake [2, p. 212] has carried out the solution of this equation subjected to the boundary condition that the pressure in the cavity is equal to the radial stress within the cavity walls. This can be expressed mathematically as:

$$P(t) = -\rho c^2 \left[\frac{\partial u}{\partial r} + \left(\frac{2\sigma}{1-\sigma}\right)\frac{u}{r}\right]_{r=a}$$

where, u is the displacement, ρ in the density of the medium, and σ is the Poisson's ratio for the medium.

For a step pressure function of the form:

$$P(t) = P_0 \qquad t > 0$$

$$P(t) = 0 \qquad t < 0$$

the displacement potential is

$$\phi = \frac{P_0 a}{\rho r(\omega_0^2 + \alpha_0^2)}\left[-1 + e^{-\alpha_0 \tau}\left(1 + \frac{\alpha_0^2}{\omega_0^2}\right)^{\frac{1}{2}} \cos\left(\omega_0 \tau - \tan^{-1}\frac{\alpha_0}{\omega_0}\right)\right]$$

where

$$\tau = \frac{t - (r-a)}{c}, \quad \alpha_0 = \frac{c}{2a_0 K}$$

$$\omega_0 = \frac{c}{2a_0 K}(4K-1)^{\frac{1}{2}}, \quad K = \frac{1-\sigma}{2(1-2\sigma)} .$$

The displacement, u, can be obtained by direct differentiation of the displacement potential with respect to range (See Appendix I).

$$u = \frac{P_0 a}{\rho r(\omega_0^2 + \alpha_0^2)}\left[1 + e^{-\alpha_0 \tau}\left(-\cos\omega_0\tau + \frac{\omega_0^2 r + \alpha_0^2 r - c\,\alpha_0}{\omega_0 c}\sin\omega_0\tau\right)\right] .$$

The transfer function of the system can be obtained from the ratio of the displacement to the input pressure function. In terms of the Laplace variable s it will be of the form (See Appendix II),

$$\text{Transfer Function} = G_{12} = \frac{\dfrac{a}{r\rho c} s + \dfrac{a}{\rho r^2}}{s^2 + 2a_0 s + 3a_0^2} . \qquad (2)$$

SYNTHESIS OF THE ANALOG

The transfer function derived in the preceding section may be represented by an electrical circuit. The process of obtaining a particular electrical circuit from the transfer function is called synthesis and as mentioned before it will result in a network the form of which will depend upon the method used for its synthesis.

The method used below results in an open circuit symmetrical lattice network. This method was chosen primarily for two reasons; (1) it is straightforward in application and (2) it yields a simple network that may be decomposed into an unbalanced network.

An open-circuit synthesis of the transfer function is justified in this case because terminations have been taken into account in the solution of the wave equation. Since the equation is solved for an infinite medium, a lumped parameter representation of any section of specified thickness, when synthesized with any type of termination, will give a correct representation of the medium.

For a symmetrical lattice network as shown in Figure 2, we have:

$$V_2 = \frac{Z_b V_1}{Z_a + Z_b} - \frac{Z_a V_1}{Z_a + Z_b}$$

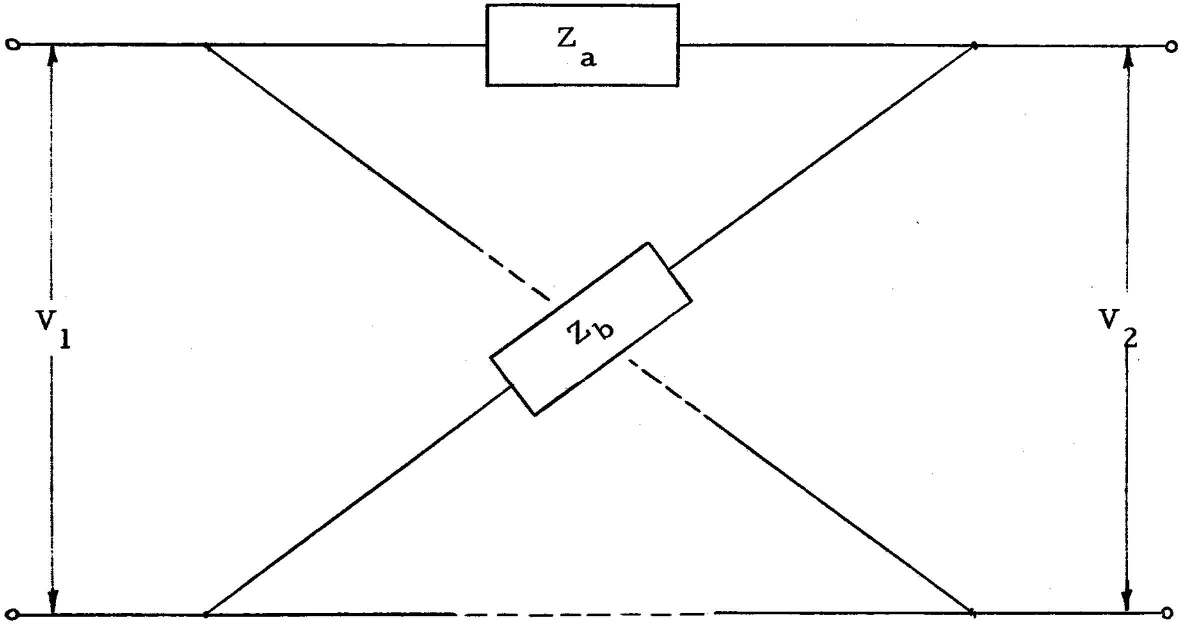


Figure 2

Symmetrical lattice network

$$\frac{V_2}{V_1} = \frac{Z_b - Z_a}{Z_b + Z_a}$$

If our transfer function represents the voltage ratio, we have:

$$G_{12} = \frac{Z_b - Z_a}{Z_b + Z_a}$$

Solving for the ratio $\frac{Z_a}{Z_b}$ we obtain:

$$\frac{Z_a}{Z_b} = \frac{1 - G_{12}}{1 + G_{12}} \quad . \tag{3}$$

If $p(s)$ and $q(s)$ represent the numerator and denominator polynomials of our transfer function, equation (2) can be re-written as:

$$\frac{Z_a}{Z_b} = \frac{q(s) - p(s)}{q(s) + p(s)} \tag{4}$$

from which the forms of Z_a and Z_b can be selected directly.

Lewis [9, p. 282] has shown an interesting synthesis procedure for the case where either the even or the odd parts of the polynomials $q(s) - p(s)$ and $q(s) + p(s)$ are proportional. Assume the odd parts are proportional and that m_1 and n_1 represent the even and odd part respectively of the polynomial $q(s) - p(s)$ and m_2 and n_2 represent the even and odd part of the polynomial $q(s) + p(s)$. Equation (4) will then take the form

$$\frac{Z_a}{Z_b} = \frac{m_1 + n_1}{m_2 + n_2} = \frac{1 + \frac{m_1}{n_1}}{k + \frac{m_2}{n_1}} \tag{5}$$

where k is the constant of proportionality of the odd parts of the above polynomials.

From equation (5) we can make direct assignments for the expressions of Z_a and Z_b as:

$$Z_a = 1 + \frac{m_1}{n_1}$$
$$Z_b = k + \frac{m_2}{n_1} \tag{6}$$

or

$$Z_a = \frac{1}{k + \frac{m_2}{n_1}}$$
$$Z_b = \frac{1}{1 + \frac{m_1}{n_1}} \; . \tag{7}$$

Functions of the form m_1/n_1 and m_2/n_1 have been shown to be reactance functions. Hence they can be synthesized as one terminal-pair LC network. Z_a and Z_b in equations (6) can be synthesized into impedances consisting of a resistor in series with an LC network, while the set of equations (7) can be synthesized into impedances consisting of a resistor in parallel with an LC network.

From our transfer function

$$q(s) - p(s) = s^2 + (2\alpha_0 - \frac{a}{rc\rho})\, s + 3\alpha_0^2 - \frac{a}{\rho r^2}$$

$$q(s) + p(s) = s^2 + (2\alpha_0 + \frac{a}{rc\rho}) s + 3\alpha_0^2 + \frac{a}{\rho r^2} \ .$$

The series and parallel branch impedances for the desired lattice network can be written directly from equations (6) as:

$$Z_a = 1 + \frac{1}{2\alpha_0 - \frac{a}{rc\rho}}\, s + \frac{3\alpha_0^2 - \frac{a}{\rho r^2}}{2\alpha_0 - \frac{a}{rc\rho}}\, \frac{1}{s}$$

$$Z_b = \frac{2\alpha_0 + \frac{a}{\rho cr}}{2\alpha_0 - \frac{a}{\rho cr}} + \frac{1}{2\alpha_0 - \frac{a}{rc\rho}}\, s + \frac{3\alpha_0^2 + \frac{a}{\rho r^2}}{2\alpha_0 - \frac{a}{rc\rho}}\, \frac{1}{s} \ .$$

These equations are of the form $Z(s) = R + Ls + \frac{1}{Cs}$. Therefore the branches of the network will consist of a resistor, a capacitor, and an inductor connected in series as shown in **Figure 3.** If R_s, C_s, L_s, and R_p, L_p, C_p correspond to the elements in the series and parallel branches respectively, their values will be given as:

$$R_s = 1 \qquad \text{ohm}$$

$$C_s = \frac{2\alpha_0 - \frac{a}{\rho cr}}{3\alpha_0^2 - \frac{a}{\rho r^2}} \qquad \text{farads}$$

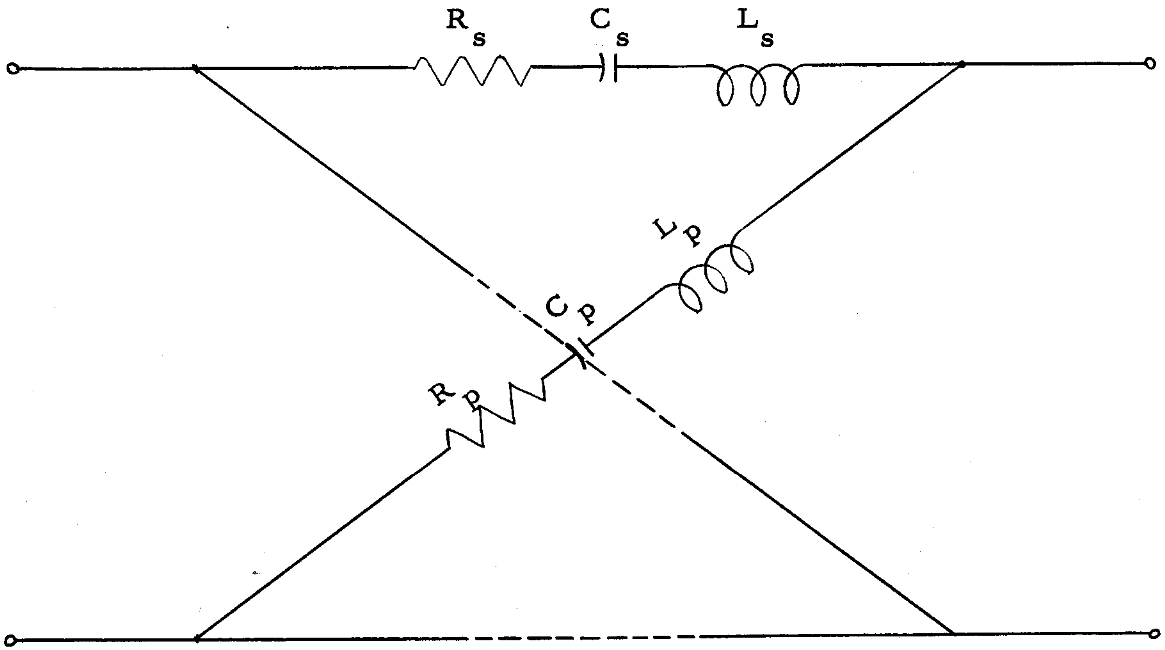


Figure 3

Element configuration in the analog circuit

$$L_s = \frac{1}{2a_0 - \frac{a}{\rho c r}} \qquad \text{henries}$$

$$R_p = \frac{2a_0 + \frac{a}{\rho c r}}{2a_0 - \frac{a}{\rho c r}} \qquad \text{ohms}$$

$$L_p = \frac{1}{2a_0 - \frac{a}{\rho c r}} \qquad \text{henries}$$

$$C_p = \frac{2a_0 - \frac{a}{\rho c r}}{3a_0^2 + \frac{a}{\rho r^2}} \qquad \text{farads}$$

LATTICE DECOMPOSITION

Because of the absence of a common ground, the above circuit will create certain instrumentation problems. In certain cases, however, a lattice network can be decomposed into a ladder network. In Figure 4a the impedance of the two branches in the symmetrical lattice network is so arranged that an equal impedance Z appears in both branches. This impedance Z can be pulled out of the two branches as indicated in Figure 4b. The two networks have the same open-circuit input and output impedance and the same transfer impedance; hence, they are equivalent networks.

Network (a)

$$Z_{in} = Z_{out} = \tfrac{1}{2}[Z_a + Z_b] = \tfrac{1}{2}[2Z + Z'_a + Z'_b] = Z + \tfrac{1}{2}[Z'_a + Z'_b]$$

$$Z_{trans} = \tfrac{1}{2}[Z_a - Z_b].$$

Network (b)

$$Z_{in} = Z_{out} = Z + \tfrac{1}{2}[Z'_a + Z'_b]$$

$$Z_{trans} = \tfrac{1}{2}[Z'_a + Z'_b].$$

For our network in Figure 3, the total impedance of the series branch can be pulled out, that is $Z'_a = 0$.

The resulted network is shown in Figure 5, and the values of the elements are:

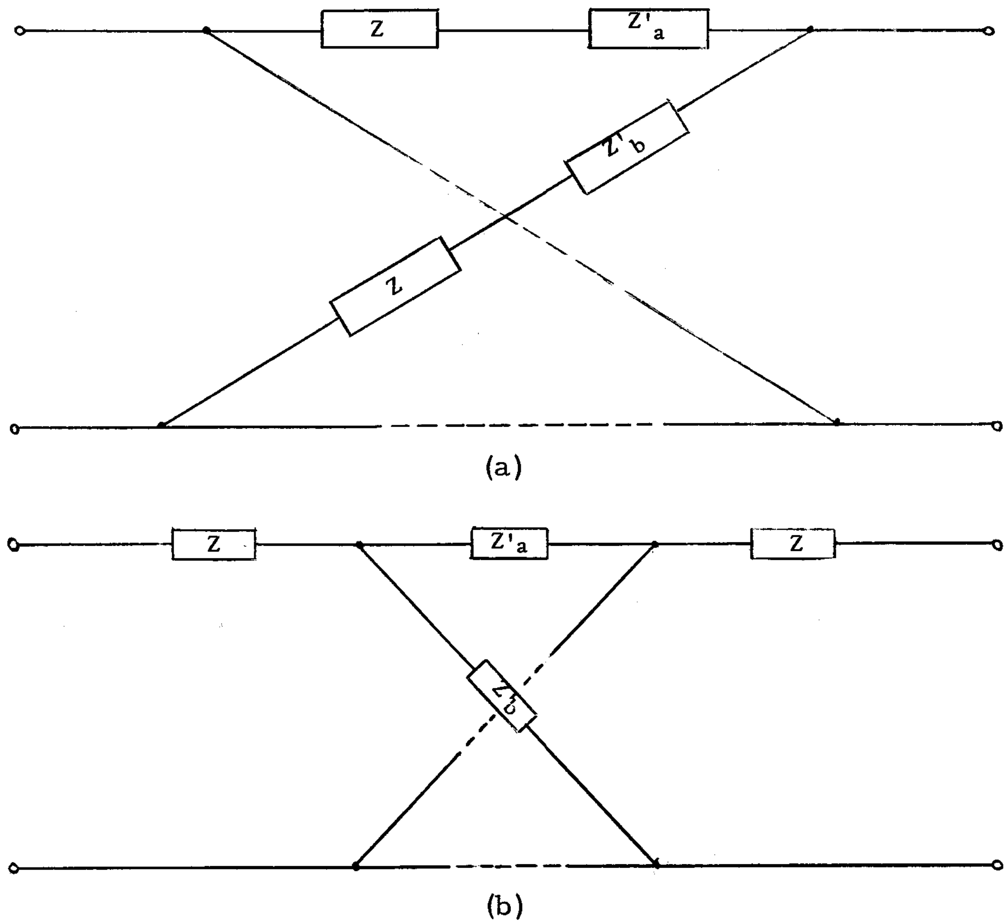


Figure 4a, 4b

Lattice decomposition

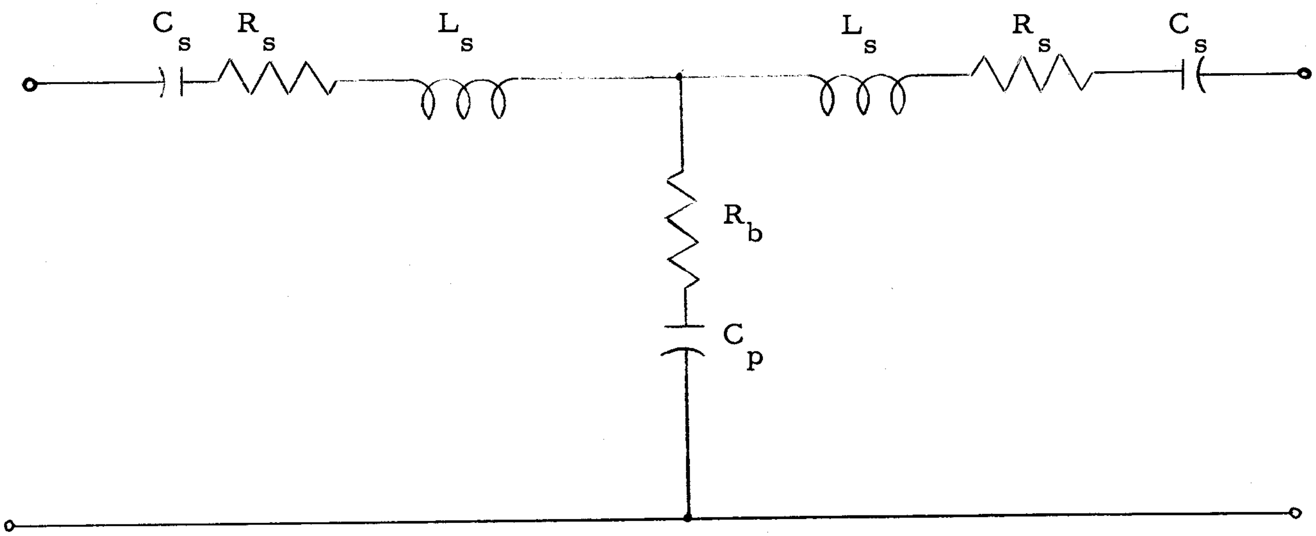


Figure 5

Decomposed analog network

$$R_s = 1 \qquad \text{ohm}$$

$$L_s = \frac{1}{2\alpha_0 - \frac{a}{\rho c r}} \qquad \text{henries}$$

$$C_s = \frac{2\alpha_0 - \frac{a}{\rho c r}}{3\alpha_0^2 - \frac{a}{\rho r^2}} \qquad \text{farads}$$

$$R_p = \frac{a}{\rho c r (2\alpha_0 - \frac{a}{\rho c r})} \qquad \text{ohms}$$

$$C_p = \frac{4a}{\rho r^2 (3\alpha_0^2 - \frac{a}{\rho r^2}} \qquad \text{farads}$$

ANALOG FOR A LAYERED MEDIUM

In the case where the variation of the characteristic constants of the medium with radial distance has to be considered a single-layer analog will not give an accurate representation of the physical system. If a stratified medium is assumed, such that different layers of finite thickness have discrete characteristic constants, an analog can be constructed, composed of lumped parameter networks connected in tandem. Each network will then represent a particular layer and its elements will be a function of the characteristic constants of that layer.

Networks similar to the one in Figure 5, based on open-circuit synthesis, cannot be used in this analog. Since the output of each stage is going to drive the following stage, the impedances at the input and output of each stage must be matched to avoid distortion of the waveform. On the other hand, since the transfer function derived is for a medium where no reflections are allowed, the networks must be terminated in the acoustic radiation impedance of the medium they represent. Hence each of the networks representing a layer will be synthesized with an impedance termination at both input and output corresponding to the acoustic radiation impedance of the previous layer and the layer the network represents.

The character of the impedance for the different layers, as seen by a spherical propagating wave, is complex in character. For values of $r \gg a$ the impedance will be that of a plane wave, which is resistive in character, and its value will be given by:

$$Z \doteq \rho c$$

For the synthesis of this analog the problems will be restricted to the case where such an approximation can be made.

The method chosen for the synthesis of the individual sections in this analog is given by E. C. Ho [6, p. 150]. This method has the advantage that it can synthesize transfer functions with complex poles without the requirement of ideal transformers. It is based on a new factorization of the ABCD matrix, see Appendix III, and if only the voltage transfer ratio $A = \dfrac{E_{in}}{E_{out}}$ is known, we can make the synthesis directly from the following matrix equation,

$$\begin{vmatrix} A & B' \\ C' & D' \end{vmatrix} = \begin{vmatrix} 1 & R_1 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ \frac{1}{R_1}\left(B_1A_1 - 2 + \frac{1}{B_2A_2}\right) & 1 \end{vmatrix} \cdots$$

$$\begin{vmatrix} 1 & R_1 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ \frac{1}{R_1}\left(B_{n-1}A_{n-1} - 2 + \frac{1}{B_nA_n}\right) & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & R_1 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ \frac{1}{R_1}(B_nA_n - 1) - \frac{1}{R_2} & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ \frac{1}{R_2} & 1 \end{vmatrix} . \quad (8)$$

The primes on the elements on the left-hand side of the equation indicate that these functions are not specified. $A_1, A_2 \ldots A_n$ represent the non-minimum resistive positive real function to which A can be decomposed, and $B_1, B_2, B_3 \ldots B_n$ are constants to be chosen later. The values of the different B's are chosen such that they will make each of the admittance functions synthesizable with real elements or, in other words, that they will make the admittance functions positive real. If the transfer function to be synthesized is not positive real, surplus factors can be added to make it positive real.

In the construction of an individual stage for this analog, assuming a definition of the transfer function as the ratio of the displacement at any distance r to the input pressure, the output of one stage must be differentiated before being fed into the following stage since the transfer function is defined as pressure over displacement.

The transfer function of equation (2) is positive real for values of $a > \frac{c}{K}$. Since we are only interested in values of "a" in this domain, no surplus factors are needed in the synthesis of the analog. Applying equation (8) we then get,

$$\begin{vmatrix} A & B' \\ C' & D' \end{vmatrix} = \begin{vmatrix} 1 & R_1 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ \frac{1}{R_1}\left(B_1 \frac{s^2 + 2a_0 s + 3a_0^2}{s + c/r} - 1\right) - \frac{1}{R_2} & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 \\ \frac{1}{R_2} & 1 \end{vmatrix}$$

which will result in the network of Figure 6.

As an example, consider an explosion of 1 kilo-ton of TNT at a depth of 16 km in a layered medium as shown in Figure 7a, with specified characteristic constants as given in the figure. The analog synthesized for this example using the above method will have the configuration and element values as shown in Figure 7b.

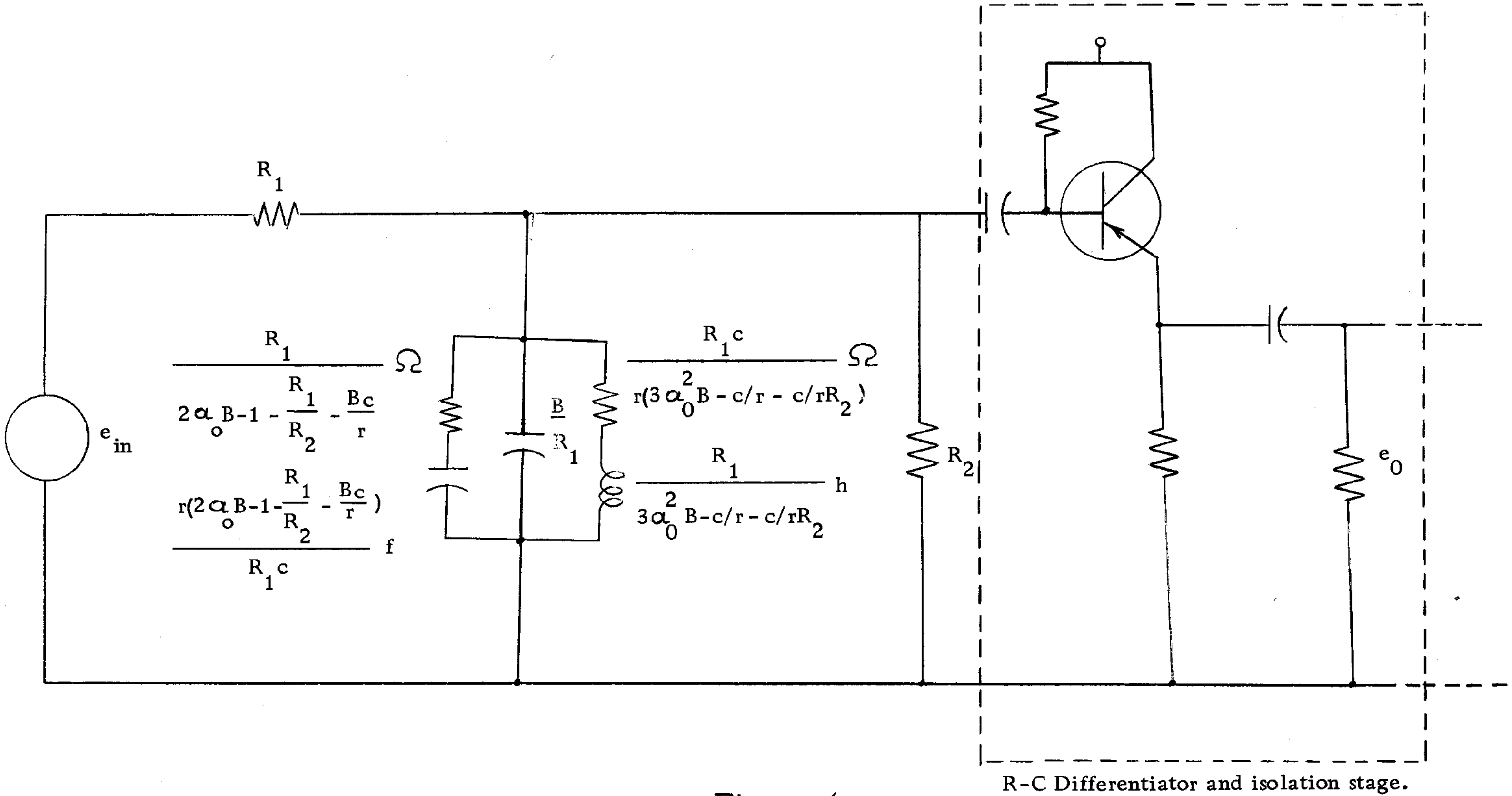
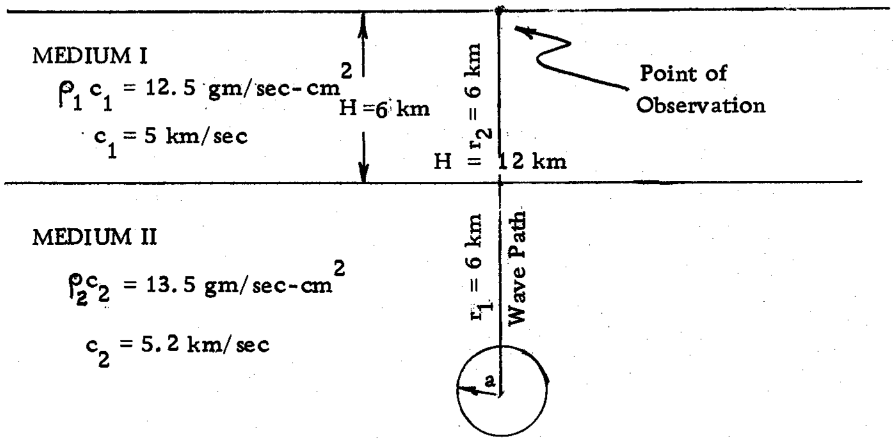
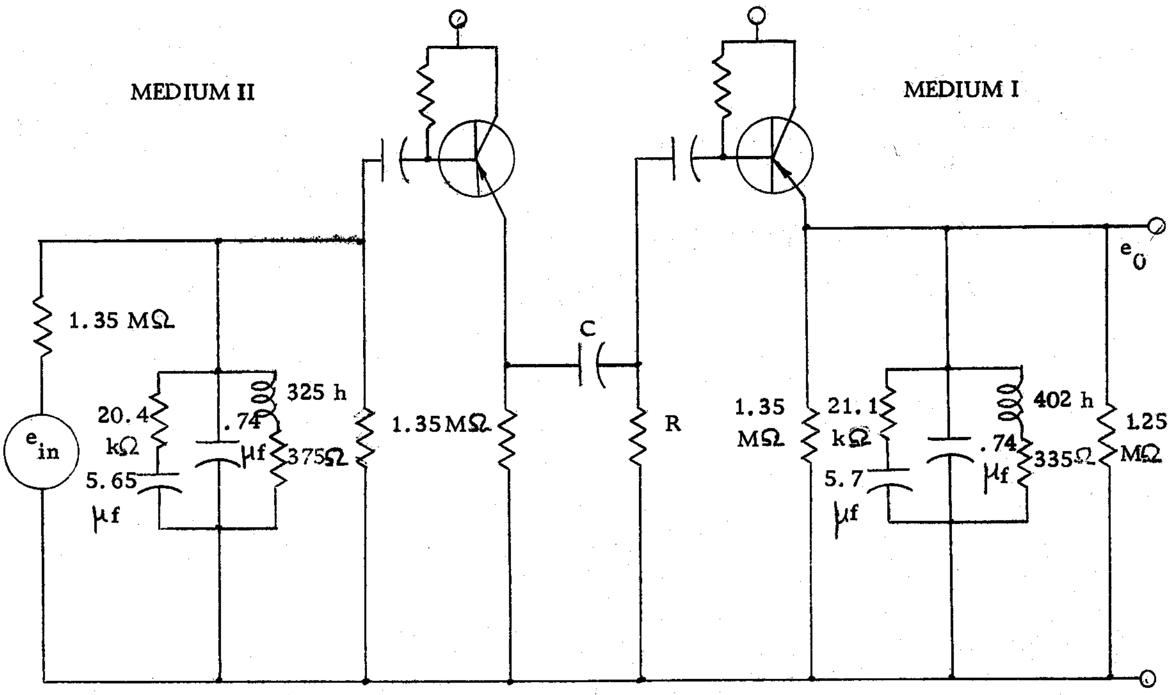


Figure 6

Representative network of the analog synthesized from the $\frac{\text{input pressure}}{\text{output displacement}}$ transfer function, plus isolation and differentiation stages.



(a)



(b)

Figure 7

a) Physical system; b) Analog for displacement compressional waves of the above physical system; value of B = 1

EXPERIMENTAL ANALYSIS

To illustrate a typical experimental analysis of the previous designed analogs, the analog representing a single medium was built in the laboratory.

The analog circuit constructed represents a section of 0.3 km of a single-layer medium for parameter values and characteristic constants as indicated in Figure (8). The circuit was synthesized within a constant multiplier $\left(\frac{a}{rc\rho} = 19.8 \times 10^{-8} \frac{\text{sec-cm}^2}{\text{gm}}\right)$ and it was frequency and amplitude scaled to achieve practical component values. Table I shows the value of the elements before scaling, after scaling, and actual values used.

Figure 9 shows the wave displacement curve obtained from computer plotting of the wave equation at the distance of 0.3 km, and the displacement curve obtained from the analog. The comparison is excellent.

To check the sensitivity of the output response of the analog to variation of element values of the circuit components, a computer solution of the circuit response was performed assuming ± 10 percent variation of the circuit element values. The results obtained are shown in Figures 10a and 10b.

From the above sensitivity analysis it can be concluded that the

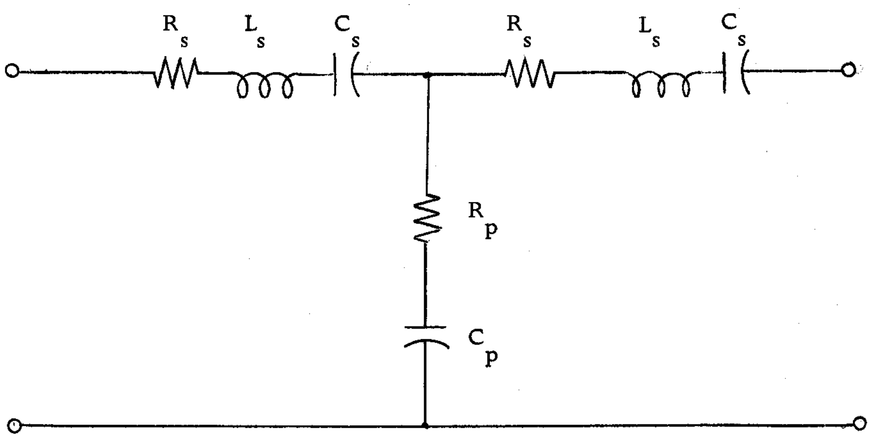


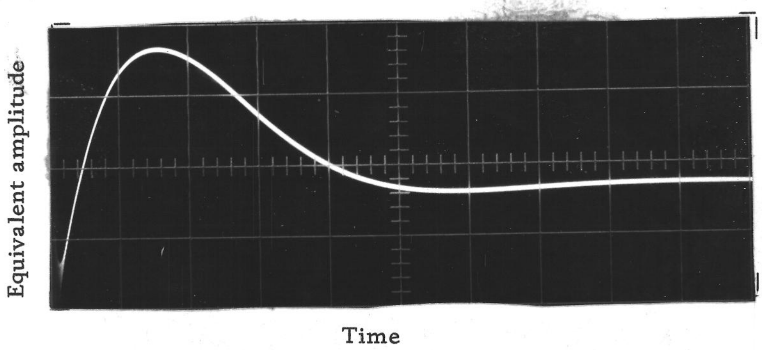
Figure 8

Network representing a .3 km range of a single layer medium, of velocity of 2.44 km/sec, Poisson's ratio of .25 and density of 1. gm/cm^3. Radiator size a = .145 km.

Table I

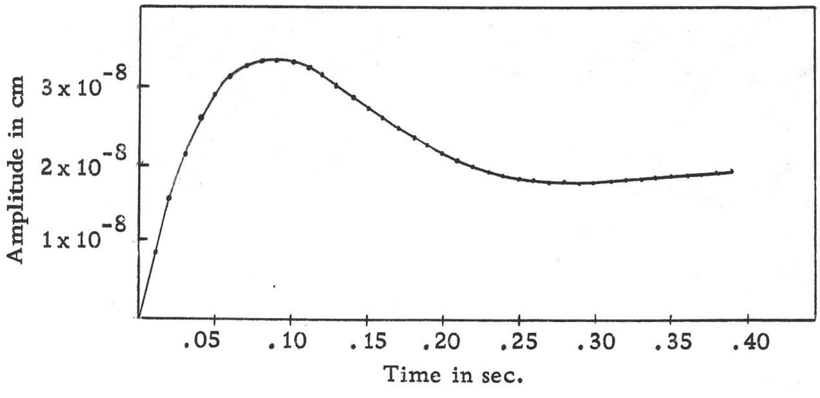
Elements	Calculated Values	Scaled Values $C^* = \frac{1}{ab} C$, $L^* = \frac{b}{a} L$, $R^* = bR$	Actual Values Used
R_s	1. Ω	100. Ω	98.
L_s	.04644 h	4.644 mh	4.68 mh
C_s	.0582 f	.582 μf	.58 μf
R_p	.04663 Ω	4.663 Ω	4.5 Ω
C_p	2.63576 f	26.3576 μf	27.3 μf

Where * corresponds to scaled values, a = 1000 is the frequency scaling factor and b = 100 is the amplitude scaling factor.



(a)

Analog output for single medium, for a = . 145 km, r = . 3 km, ρ = 1. gm/cm^3, Poisson's ratio of . 25 and c = 2. 44 km/sec. Input voltage one volt step. Time scale . 5 sec/cm



(b)

Theoretical waveform obtained from plotting the solution of the wave equation, for a = . 145 km, r = . 3 km, ρ = 1.0 gm/cm^3, Poisson's ratio of . 25, c = 2. 44 km/sec. P_0 = 1 gm/cm sec^2

Figure 9

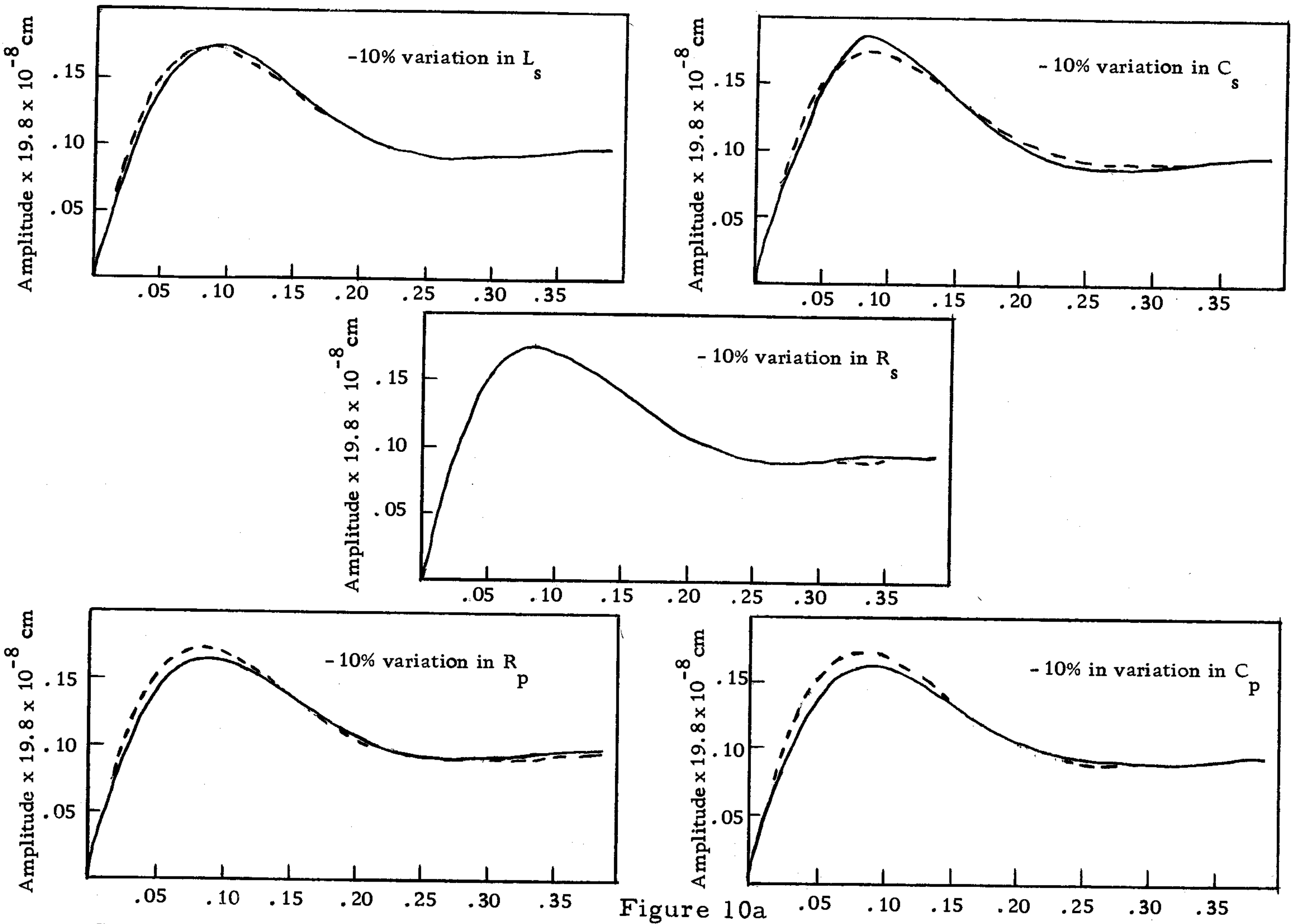


Figure 10a

Sensitivity analysis of analog showing effect of -10 percent of component values.
Dotted line indicates original curve. (Horizontal scale time in sec.)

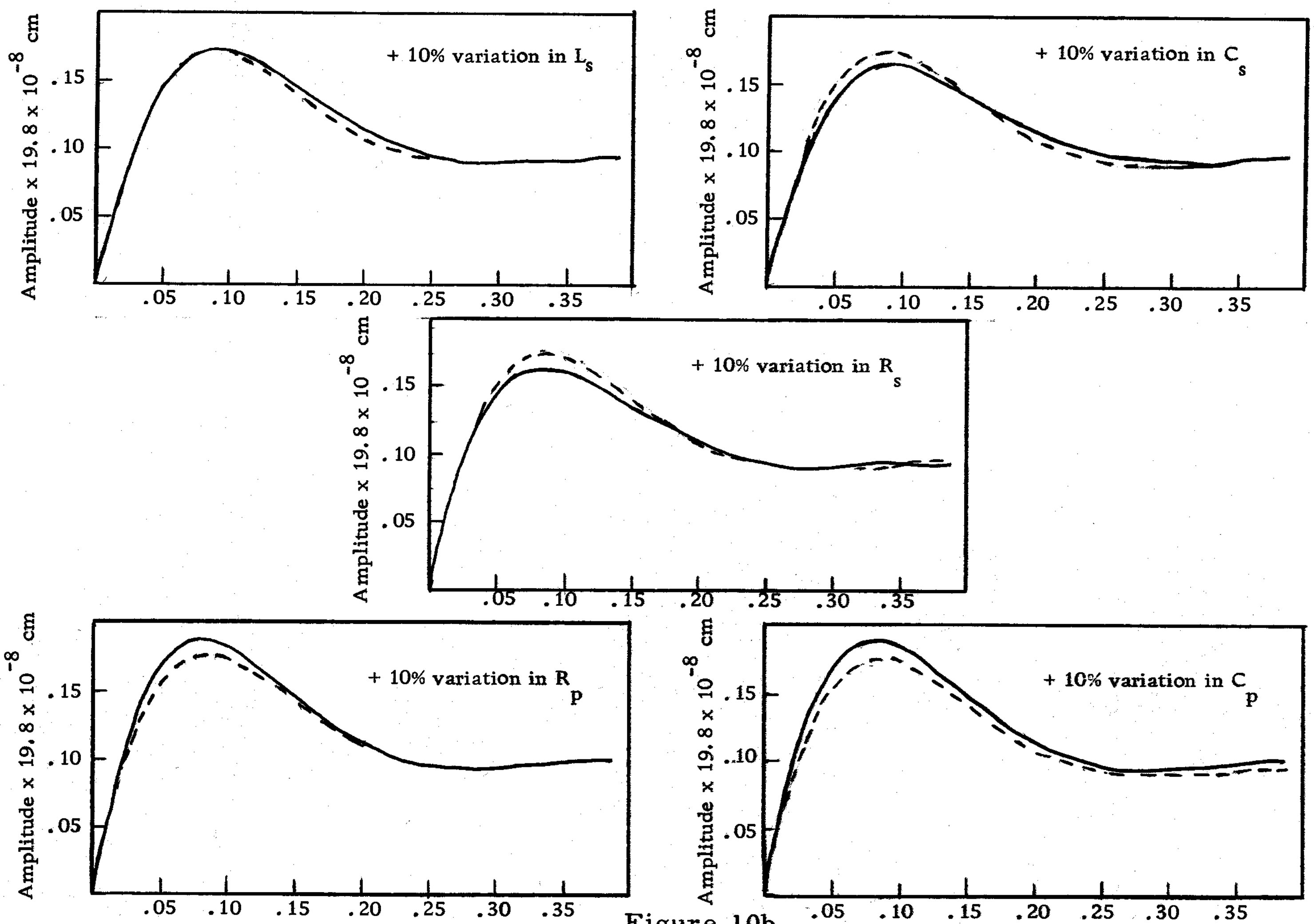


Figure 10b

Sensitivity analysis of analog showing effect of 10 percent of component values. Dotted line indicates original curve. (Horizontal scale time in sec.)

circuit response is not very sensitive to component variation and that within a good approximation components with 10 percent tolerances can be used in the construction of such a model.

SUMMARY

Each of the two types of analogs designed has its advantages and disadvantages in simulating propagation of displacement compressional waves generated from spherical radiators in solid media.

The analog representing a single-layer medium will simulate the propagation of displacement waves at any distance from a spherical radiator. It allows the displacement wave to be subjected to variations of the characteristic constants of the medium and to variations in the size of the radiator and form of the excitation function. A typical analog of this type was built and the results obtained compared extremely well with results obtained from the solution of the wave equation with the same boundary conditions.

One application of an analog of this type is that if it is built from two experimental displacement waveforms which are obtained at close-in recordings from an underground explosion, the pressure waveform at the walls of the elastic-inelastic boundary of the different mathematical models can be checked and the correct mathematical model can thus be obtained.

The second analog, taking into consideration the effect of layering, was designed, but the simulation of displacement wave-forms obtained are restricted to the case for which the point of observation is at a distance very large compared to the radius of the radiator.

Some problems that are of interest to the person pursuing this type of work might be considered from the viewpoint of adapting the transfer-function synthesis approach described in this paper to the particular problem. For example, several suggested lines of further research are the adoption of this method to:

1. Synthesis of an analog for a cylindrical, or a line, radiator instead of a spherical radiator. A typical use of such a radiator would be the simulation of waves generated from earthquakes.

2. Synthesis of an analog which will simulate spherical-wave propagation in a layered medium. This will require synthesis of the transfer function with complex impedance terminations.

3. Pursuit of synthesis of a transmission line, for spherical or cylindrical waves, which will allow the construction of theoretical seismograms.

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APPENDICES

APPENDIX I

To obtain the displacement function we have to take the first space derivative of the displacement potential.

For a step input the displacement potential ϕ is [2, p. 213]

$$\phi = \frac{P_0 a}{\rho r(\omega_0^2 + \alpha_0^2)} \left[-1 + e^{-\alpha_0 \tau} \left(1 + \frac{\alpha_0^2}{\omega_0^2}\right)^{\frac{1}{2}} \cos\left(\omega_0 \tau - \tan^{-1} \frac{\alpha_0}{\omega_0}\right) \right] .$$

Treating ϕ as a product, its partial derivative with respect to r will be:

$$u = \frac{\partial \phi}{\partial r}$$

$$= - \frac{P_0 a}{\rho r^2 (\omega_0^2 + \alpha_0^2)} \left[-1 + e^{-\alpha_0 \tau} \left(1 + \frac{\alpha_0^2}{\omega_0^2}\right)^{\frac{1}{2}} \cos\left(\omega_0 \tau - \tan^{-1} \frac{\alpha_0}{\omega_0}\right) \right]$$

$$+ \frac{P_0 a}{\rho r(\omega_0^2 + \alpha_0^2)} \left[\frac{\alpha_0}{c} e^{-\alpha_0 \tau} \left(1 + \frac{\alpha_0^2}{\omega_0^2}\right)^{\frac{1}{2}} \cos\left(\omega_0 \tau - \tan^{-1} \frac{\alpha_0}{\omega_0}\right) \right.$$

$$\left. + \frac{\omega_0}{c} e^{-\alpha_0 \tau} \left(1 + \frac{\alpha_0^2}{\omega_0^2}\right)^{\frac{1}{2}} \sin\left(\omega_0 \tau - \tan^{-1} \frac{\alpha_0}{\omega_0}\right) \right]$$

where $\tau = t - (r-a)/c$ and $\frac{\partial \tau}{\partial r} = -1/c$.

If the following substitutions are made in the above equation,

$$\cos\left(\omega_0 \tau - \tan^{-1} \frac{\alpha_0}{\omega_0}\right) = \cos(\omega_0 \tau)\cos\left(\tan^{-1} \frac{\alpha_0}{\omega_0}\right) + \sin(\omega_0 \tau)\sin\left(\tan^{-1} \frac{\alpha_0}{\omega_0}\right)$$

$$\sin(\omega_0 T - \tan^{-1}\frac{a_0}{\omega_0}) = \sin(\omega_0 T)\cos(\tan^{-1}\frac{a_0}{\omega_0}) - \cos(\omega_0 T)\sin(\tan^{-1}\frac{a_0}{\omega_0})$$

and

$$\cos\,(\tan^{-1}\frac{a_0}{\omega_0}) = \frac{1}{\sqrt{\left(\frac{a_0}{\omega_0}\right)^2 + 1}}$$

$$\sin\,(\tan^{-1}\frac{a_0}{\omega_0}) = \frac{a_0}{\sqrt{\left(\frac{a_0}{\omega_0}\right)^2 + 1}}$$

the displacement function u, after collecting terms, will be written as:

$$u = \frac{P_0 a}{\rho r^2 (a_0^2 + \omega_0^2)} \left[1 + e^{-a_0 T}\left(-\cos\omega_0 T + \frac{\omega_0^2 r + a_0^2 r - c\,a_0}{\omega_0 c}\sin\omega_0 T\right)\right] .$$

APPENDIX II

Let U_{out} be the output of our analog and U_{in} its input corresponding to the wave displacement at any distance r and to the pressure at the walls of a radiator of radius "a", respectively. From Reference 1 we have:

$$U_{out} = \frac{P_0 a}{\rho r^2 (\alpha_0^2 + \omega_0^2)} \left[1 + e^{-\alpha_0 \tau} \left(-\cos \omega_0 \tau + \frac{\omega_0^2 r + \alpha_0 r^2 - c\alpha_0}{\omega_0 c} \sin \omega_0 \tau \right) \right].$$

$$\text{Let } A = \frac{P_0 a}{\rho r^2 (\alpha_0^2 + \omega_0^2)}$$

$$B = \frac{\omega_0^2 r + \alpha_0^2 r - c\alpha_0}{\omega_0 c}$$

then

$$U_{out} = P_0 A \left[1 + e^{-\alpha_0 \tau} \left(-\cos \omega_0 \tau + B \sin \omega_0 \tau \right) \right].$$

The Laplace transform of the above function will be:

$$L\left[U_{out}\right] = P_0 A \left[\frac{1}{s} + \frac{B\omega_0}{(s+\alpha_0)^2 + \omega_0^2} - \frac{s+\alpha_0}{(s+\alpha_0)^2 + \omega_0^2} \right]$$

if

$$U_{in} = P_0$$

its Laplace transform is:

$$L\left[U_{in}\right] = \frac{P_0}{s}$$

The transfer ratio G_{12}, defined as the ratio of the Laplace transform of the output response to the Laplace transform of the input excitation, will take the form:

$$G_{12} = \frac{P_0 A \left[\frac{1}{s} \quad \frac{B\,\omega_0}{(s+a_0)^2 + \omega_0^2} - \frac{s + a_0}{(s+a_0)^2 + \omega_0^2}\right]}{\frac{P_0}{s}}$$

$$= \frac{a}{rc\rho} \quad \frac{s + \frac{c}{r}}{s^2 + 2a_0 s + a_0^2 + \omega_0^2}$$

If we assume a Poisson's ratio of .25

$$\omega_0 = \sqrt{2}\, a_0$$

and

$$G_{12} = \frac{a}{rc\rho} \quad \frac{s + \frac{c}{r}}{s^2 + 2a_0 s + 3a_0^2} .$$

APPENDIX III

A finite RLC two-terminal-pair network is completely specified by a two-by-two matrix. referred to as the ABCD matrix, which relates the voltages and the currents at the input and output terminals by the following relationship (see Figure 11a).

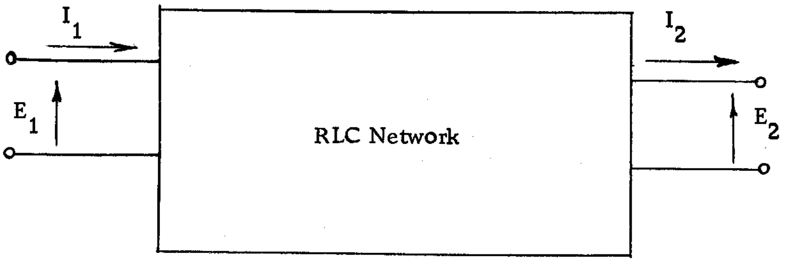
$$\begin{vmatrix} E_1 \\ I_1 \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix} \begin{vmatrix} E_2 \\ I_2 \end{vmatrix}$$

If the RLC network is a simple ladder network (see Figure 11b), terms of the ABCD matrix specification, the over-all network matrix may be obtained from the following matrix equation.

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \begin{vmatrix} 1 & Z_1 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ y_2 & 1 \end{vmatrix} \begin{vmatrix} 1 & Z_3 \\ 0 & 1 \end{vmatrix} \cdots$$

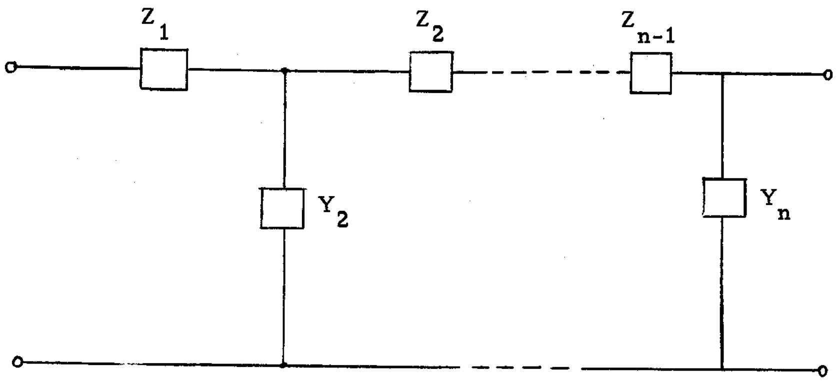
$$x \begin{vmatrix} 1 & Z_{n-1} \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ y_n & 1 \end{vmatrix}.$$

It may be noted that the right-hand side of the above equation is a simple product of n-matrices each specifying a two-terminal pair network containing either a series or a shunt impedance. Based on this analysis E. C. Ho performed a realization of a ladder network by a reverse process. For a more detailed explanation see reference 10, p. 134.



(a)

RLC network showing input and output voltages and currents



(b)

Simple ladder network

Figure 11

APPENDIX IV

TABLE OF SYMBOLS

ρ	density
c	velocity of propagation of a compressional elastic wave
a	radius of spherical cavity
t	time
P_o	pressure amplitude (real)
ϕ	Scalar displacement potential
r	distance from center of cavity
τ	t - (r - a)/c
σ	Poisson's ratio, ratio of resultant lateral contraction to the longitudinal extension
u	radial particle displacement
k	$1/2(1 - \sigma)(1 - 2\sigma)^{-1}$
α_0	radiation damping factor
ω_0	natural frequency of oscillating cavity
s	Laplace transform variable
Z	complex impedance
L	inductor
C	capacitor
R	resistor
Ls	impedance associated with inductor

$1/Cs$ impedance associated with capacitor

V voltage

I current

Z_{in} open-circuit driving-point impedance,

$$\left.\frac{V_{in}}{I_{in}}\right|_{I_{out} = 0}$$

Z_{trans} open-circuit transfer impedance,

$$\left.\frac{V_{out}}{I_{in}}\right|_{I_{out} = 0}$$

G_{12} open-circuit transfer voltage ratio,

$$\frac{Z_{trans}}{Z_{in}}$$