AN ABSTRACT OF THE THESIS OF

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A common problem in energy allocation problems is managing the trade-off between selling surplus energy to maximize short term revenue, versus holding surplus energy to hedge against future shortfalls. For energy allocation problems, this surplus represents resource *flexibility* and is quantified as the surplus energy after meeting the demand. The decision maker has an option to sell or hold the flexibility for future use. As a decision in the current period can affect future decisions significantly, future risk evaluation of negative shocks (or uncertainties) is recommended for the current decision in which a traditional robust optimization is not efficient. Therefore, an approach to Flexible-Robust Optimization has been formulated by integrating a Real Options Model with the Robust Optimization framework. Real options analysis is an efficient economic model for risk evaluation in investment problems. In the energy problem, the real options model evaluates the future risk, and provides the value of holding flexibility, whereas the robust optimization quantifies uncertainty and provide a robust solution (i.e. a solution which is generally insensitive to uncertainties) of net revenue by selling flexibility. This integration or models has introduced compatibility issues which have been discussed extensively in the literature. However, the limitations have been overcome successfully by implementing bi-level programming in this work. Therefore, a complete general mathematical formulation of *Bi-Level Flexible-Robust Optimization* model is presented and results shown to provide an efficient decision making process in energy sectors.

Bi-level Flexible-Robust Optimization for Energy Allocation Problems

by
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I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

Arpan Biswas, Author

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CONTRIBUTION OF AUTHORS

Dr Yong Chen has shared valuable knowledge in Real Options analysis and was involved in the formulation of Real Options Model. He has also contributed in providing figures 2.1, 2.2, 2.3 and writing of following sections in Chapter 2: a. *The Real Option Model to compute* V(h_t)

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DEDICATED
TO

My mother, Kabita Biswas

My father, Nepal Biswas

My wife, Ananya Sinha Chowdhury

CHAPTER 1
GENERAL INTRODUCTION

In the modern era, renewable energy is a boon in any developed countries. There are various sources of renewable energy like Solar, Hydro, Wind, etc. With the help of advanced technologies, we can convert these energies efficiently into electricity. Electricity plays one of the key roles in our daily life, from operation of fans, lights, vehicles to operations of large machines in industries. Without electricity, the rapid development of human advancement will come to a halt. This is why renewable sources of energy are very important. Day by day, the population is increasing significantly which leads to an increase in the use of electricity. Therefore, research has been conducted on how to increase the efficiency of generation of electricity from these renewable resources and to allocate it optimally. This will increase the revenue of the energy sectors, and also minimize power failures. In this thesis, we will focus on one particular renewable source of energy: Hydro energy. Like all the renewable sources, Hydro energy also has many sources of uncertainty, thereby making the energy allocation problem very complex. For example, we have low uncertainty in water level and demand in the initial time period as we will have a good idea of the inflows and demands on the present day; however, we are not as sure about that in the future dates, and therefore uncertainty in the system increases. This could make a significant impact in the decision making of optimal allocation of electricity generated from hydro energy. Thus, the Robust Optimization approach is mandatory in these problems to quantify uncertainty in the system. Though Robust Optimization quantifies uncertainty in the system, it does not have an efficient method to value resource *flexibility*. Resource flexibility is the defined as the surplus hydro energy after meeting the demand, and is expressed in energy units (MWh.). As we will focus only on resource flexibility in the thesis, we will simply call this flexibility. The value of flexibility refers to the economic value created by the ability to move this hydro energy generation from one time period to another. The Robust Optimization approach provides a robust solution of the optimal generation of electricity; however, in energy allocation problems considering all the uncertainties, we need to decide whether to allocate the flexibility in the current period or to hold it for the future to overcome any negative shocks (i.e. uncertainties) in the energy market. Therefore, the valuation of flexibility is required to realize such decisions of selling or holding the flexibility which standalone Robust Optimization is not capable of providing.

To summarize, the list of research challenges is provided below:

1. **Integration of the Real Options model with Robust Optimization** to resolve the trade-off between getting revenue now versus holding water to overcome future risks. Others work consider either standalone Robust Optimization or only Real Option Analysis.
2. **Implementation of the bi-level programing** in the model to enforce the complex operation constraints and to enable estimation of the outflows for the entire reservoir system to meet target flexibility allocations.
3. **Computation efficiency of solving the problem**. We have resolved partially by formulating constraints to manage search size space and also by choosing efficient algorithm after being compared by running the model with different algorithms like SQP, GA. However, a more efficient solution will be considered for future research.

The first two challenges are the main focuses in the next chapters and have been resolved successfully. For general allocation problems, economic valuation is done by various economic models for any optimal decision making in investment problems. This valuation helps in making choices on whether to invest now or later. We can relate this similar scenario to our problem, where during each period we are making choices whether to use the flexibility or hold it for future. Therefore, an approach to bi-level Flexible-Robust Optimization for optimal energy allocation problems has been presented by integrating an economic model to evaluate flexibility with Robust Optimization framework. Chapter 2 focuses on the mathematical formulation of a simplified Flexible-Robust Optimization model (Figure 1) to assess the feasibility of the Flexible-Robust Objective. Chapter 3 talks about the limitations of the simplified model presented in Chapter 2 and introduces a Bi-Level programming method as a way to resolve the issues in the solving the flexible-robust optimization problem. Chapter 3 presented a mathematical formulation of Bi-level Flexible-Robust Optimization (Figure 2) for any generalized energy allocation problem.

FIGURES OF CHAPTER 1

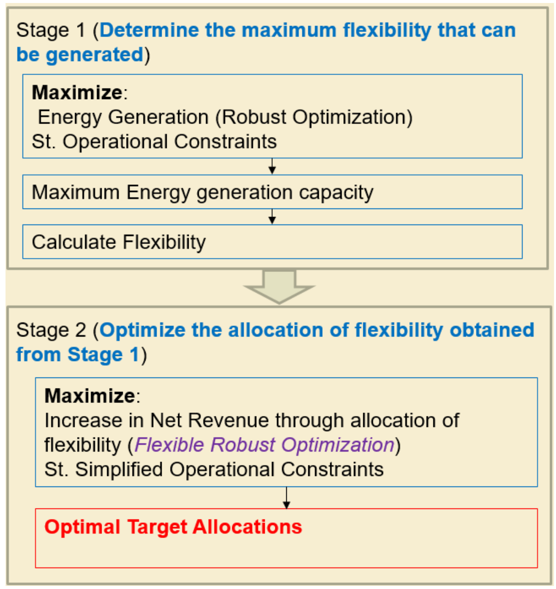

Figure 1. High-Level Structure of Flexible-Robust Optimization Framework

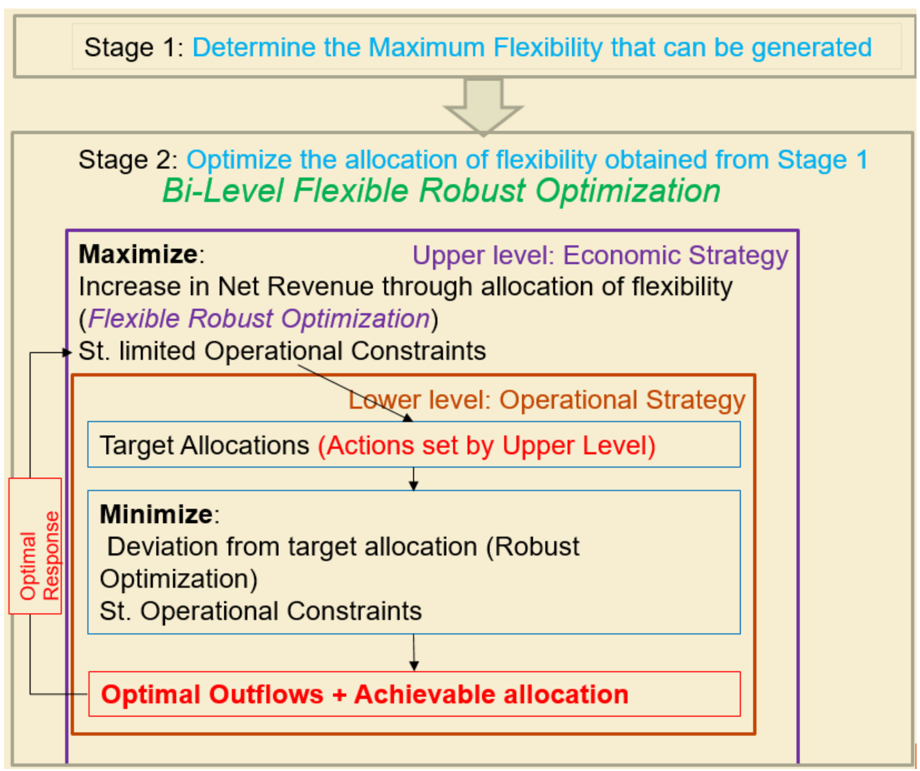

Figure 2. Implementation of Bi-level programming into Flexible-Robust Optimization Framework

CHAPTER 2
AN APPROACH TO FLEXIBLE-ROBUST OPTIMIZATION OF LARGE-SCALE SYSTEMS

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ABSTRACT

Though Robust Optimization has proven useful in solving many design problems with uncertainties, it is not suitable for certain problems which have sequential options in the decision making process. In this work, an integration of a Real Option model with the Robust Optimization technique is presented. This approach aims to eliminate the shortcomings of robust optimization for sequential decision making problems. We provide an example of applying this new integrated model to the operational control of a single reservoir of the Oregon-Washington Columbia River system by optimizing the flexibility of the system. Flexibility for an engineering system is the ease with which the system can respond to uncertainty in a manner to sustain or increase its value delivery through decision-making. In this paper, we define flexibility as the amount of water left in the storage reservoir to produce electricity after meeting demand. Real Option analysis is an economic tool which helps to value the multiple courses of actions in a decision: that is to either sell the flexibility or hold it for future use based upon the future value of flexibility. Selling flexibility causes one to lose some future value because one may be forced to repurchase that flexibility from the market at higher prices later due to shortages; Real Options analysis values future purchases to support decision-making. Robust optimization focuses on for selling the flexibility in a daily market and gives an optimal result by maximizing net revenue, considering all the physical and operational constraints of the reservoir to avoid floods or other environmental calamities. Net revenue is defined as cost of selling and cost of future purchase of the flexibility. We provide an optimization result of 27 random inflow scenarios which gives high, medium and low flexibility to allocate using the integrated model. We compare the optimal solutions given by the integrated model with that given by robust optimization. The integrated real option-robust optimization model improves the revenue from allocating flexibility as much as 40 percent over the robust optimization result.

1. INTRODUCTION

Renewable energy, such as hydropower, is one of the major sources for electricity generation for power sectors. Over many decades, there has been continuous development of water resource management for the economic benefit of electricity industries. Various studies are being conducted in wide range of domains, such as water allocation, infrastructure capacity expansion, water quality, drought control mitigation, flood control and conservation of aquatic ecosystems [1]. A key consideration for the power sector is identifying optimal strategies for buying and selling electricity, thereby using the water optimally to increase their revenue and also meet the demands. Fundamental ideas of engineering have been studied in the water allocation optimization problem; literature related to hydro-economic optimization models are available [1]–[3]. The problem of water allocation is complicated by uncertainty. Generally, several factors are uncertain in river system and energy markets, such as water inflows, weather, market demands and prices. Therefore, in an optimization model, these parameters cannot be expressed as deterministic values. Pulido-Velazquez [1] presented a scholastic hydro-economic optimization model where optimal allocation of water is provided based on the water values and the objective was addressing water scarcity and reducing water conflicts considering environmental constraints. The valuation of water is calculated based on its availability; it is generally assumed that the value of water increases in future time periods due to having greater uncertainty for a shortage. Under such an assumption, it is very likely that the future value of water will dominate the present value of water. Therefore, there will be a tendency to save water for future time periods due to the potential of a water shortage, even when the market price for the water in the present time period is much higher than the future value of water. However, it would be more profitable if the optimal decision is made to allocate water comparing the present market value and the future value of water considering all the uncertainties.

The main premise of this paper is to present an optimization model using a *flexible-robust objective function* and applying it to a single reservoir in Columbia River System as a case study. The model integrates the real options concept for the valuation of flexibility with the robust design optimization framework to find a robust solution. The objective is to maximize revenue considering the future value of flexibility to provide an optimal decision to allocate flexible water (i.e. water not needed to satisfy contracted demand). Robustness for an engineering design is the state when the design is minimally sensitive to factors causing uncertainties. Flexibility for an engineering system is the ease with which the system can respond to uncertainty in a manner to sustain or increase its value delivery. In this paper, we define flexibility as the amount of water left in the storage to produce electricity after meeting contracted demand. This flexibility allows us to take multiple courses of actions; for example, to sell water or to hold water for future use. If used wisely, flexibility also helps one to cope the unplanned shortages of electricity during natural calamities, sudden rises in demands, or sudden decrease of electricity generation due to turbine failure. We determine the future value of flexibility using Real Options Analysis. However, robustness in the design of our project will ensure insensitiveness to the course of action taken, considering the uncertainties in inflows, demand, forecast, market price of electricity.

2. BACKGROUND

To implement a method to optimize operations and formulate the flexible-robust objective, real options analysis and robust optimization are investigated.

Real Options in Valuing Flexibility in Large Scale Systems

The structure of energy markets, which face an increase in competition and an goal of improved economic efficiency, face various risks and uncertainties. As the level of risk and uncertainty increases, traditional deterministic discounted cash flow (DCF) modeling approaches used for capacity investment planning need to be complemented by other, more sophisticated methods to deal with the potential fluctuations in both demand and price, among others. The real options (RO) approach to investment decision planning provides an attractive opportunity to evaluate investment alternatives in power generation in a deregulated market environment [4]. Kumbaroğlu et al. [4] presented a policy planning model which can guide policy planning in the electricity supply sector, and is based on the real options approach to investment. Several other studies have been conducted using the real options approach to investment problems in power sectors for the valuation of flexible renewable energy where uncertainties are high [5], [6]. Marreco and Carpio present a valuation study of operational flexibility using Real Option Theory in order to determine the fair premium to be paid by the thermal capacity installed and applied in the complex Brazilian Power System, considering uncertainties in natural affluences [7].

In the case of renewable energy facilities (e.g. hydroelectric), the system is highly dependent on hydrological conditions; therefore, uncertainties such as inflows, weather forecast, market electricity demands and prices, chances of negative shocks are high. It has been a problem for decision makers in these facilities to determine how to allocate remaining water after meeting the contracted demands. This can lead to a wrong decision which can decrease revenue significantly and, in a worst scenario, can cause environmental damage. For example, if a facility empties storage due to high market prices and suddenly there are shortages in the next days, they have to forcefully buy electricity from the market to meet the demand, which will decrease revenue. Also, if they decide to allocate the flexibility later, and if inflows suddenly increase due to unpredicted rain, reservoirs may spill and cause flooding. Thus valuing the flexible water is necessary to address potential negative impacts. In other words, it is beneficial for these facilities to understand the future value of allocated flexibility to help them determine better scheduling plan: whether to generate and sell the electricity now with the flexible water or to hold the water for future use. Real Option Theory is an appropriate technique to determine the value of flexibility.

In this paper, a Real Option (RO) model is proposed for valuation of the flexible water, by adopting real option theory. This real option model is integrated with a stochastic optimization framework as part of the objective function in the proposed *Flexible-Robust Objective*. Details of flexible robust objective will be explained in section 4.

Robust Optimization

In this section, we provide a brief discussion about robust optimization and why it is necessary in hydropower generation projects. As noted in the previous section, there are multiple sources of uncertainty in hydropower generation, and the optimal operational decisions should provide consistent results, even when there are variations of the uncertain parameters from the expected value. Robust optimization is the best approach for these types of problems, where the decision can be made based on the risk attitude of the decision maker. Much literature is available which presents a stochastic model for solving problems in the energy market to deal with uncertainties, and to provide an optimal result with minimal risk [8]–[10]. Robustness has also been studied extensively in the design literature as a means to account for uncertainty. Robustness is defined as the ability of a given system configuration to perform well over a wide range of conditions over the product lifecycle, such as the occurrence of faults and resulting functional losses. Taguchi-based approaches [11]–[14] utilize an optimization framework in which the system design is optimized based on an objective which considers both the mean and variance of the system performance. Variance in the system performance can result from multiple sources of random noise, both external and internal to the system. Robustness has also been investigated in the biological network literature, with principles for achieving robustness defined as system control, redundancy, diversity, modularity, and decoupling [15]. McIntire et al. applied a Robust Design Optimization framework to the Columbia River System to provide an optimal outflow for maximizing expected revenue considering the inflow uncertainties. The probabilistic framework results in lower risk solution than the deterministic approach, when uncertainties are accounted for [16].

3. FLEXIBLE-ROBUST OPTIMIZATION FRAMEWORK

In the previous section, we discussed the application of the robust optimization framework in large-scale systems. In previous work by McIntire et al. [16], it was assumed that the market selling price is constant throughout the optimization period. Therefore, the optimal decision is to empty the reservoir each day for electricity generation and to sell the remaining electricity after meeting the demand. However, in reality the market prices are always variable. In such cases, a better result could be possible by holding the flexible water for future use when the market prices will be higher. In our model, we consider price changes every time period, which is one day in our model. Although robust optimization provides insensitiveness to uncertain factors, it does not provide the value of flexibility which is required to address potential water shortfalls due to *negative shocks*. For example, robust optimization in hydropower generation does not value flexible water, and therefore if the market price is substantially higher on Day 2 than in Day 4, but Day 4 has greater uncertainty, the robust optimization may still provide an optimal result to sell the water on

Day 2 based upon higher revenue for that day. However, the value of flexibility will be higher on Day 4 due to higher uncertainties. So, in the case of negative shocks on Day 4 (i.e. uncertainty leads to shortage of water), the facility will need to forcefully buy electricity to meet contracted demand, thus decreasing total revenue. Thus, valuing flexibility is necessary to better manage risks.

Several papers have been published based on the optimization of flexibility using real Option Model. Martínez-Cese˜na, and Mutale [17] and Yang and Blyth [18] provided an optimization model for investment planning in renewable energy generation projects using Real Options Theory. Hu and Solana [19] presented an optimization of operational options of a hybrid diesel-wind generation plant using real option analysis maximizing the value of flexibility. However, in the case of projects such as the optimal water allocation problem for hydropower generation, only maximizing the value of flexibility does not provide the best solution. As an example, if future days have higher uncertainty, the probability of a negative shock is higher on future days. Therefore, the value of flexibility of future days will be more compared to the present day in an optimization period. Thus, the optimizer will provide a solution to hold the flexible water for the future, irrespective of the higher market price at the present day. This result may be overly risk-averse as there might, in fact, be no realized negative shocks on future days, and the hydropower facilities will have to sell the flexible water on those days at a cheaper price to fulfill operational constraints such as storage capacity, etc. To overcome such limitations and to solve the optimum allocation of flexible water problem, it is necessary to have an objective to maximize the revenue based on the daily market prices *and* an objective to maximize the value of flexibility to address any negative shocks to lower the chances of losing revenue. The first objective can be quantified using the robust optimization technique, while the second objective quantification requires real options analysis. This integration of Real Options Theory into the Robust Optimization is the ***Flexible-Robust Optimization framework***. Detailed description of our proposed model incorporating this Flexible-Robust Optimization framework along with a case study is discussed in the remainder of this paper.

In order to illustrate the concept of the flexible-robust optimization framework and the importance of the integration of real option analysis and robust optimization in a framework to solve the optimal water allocation problem for hydropower generation facilities, we created a generalized model for any large-scale system. However, in this paper, we have limited our case study to single reservoir of Columbia River system: Grand Coulee which is managed by Bonneville Power Administration (BPA). The descriptions of Grand Coulee are provided by BPA.

Lower Columbia River: Grand Coulee Reservoir

An optimization model is proposed which takes into account the reservoirs of Columbia River system according to the problem description that follows. We propose a multi-stage optimization method to model the system. Similar approaches have been studied previously. van der Weijde and Hobbs proposed an optimization model that captures the multistage nature of transmission planning under uncertainty and use it to evaluate interregional grid reinforcements in Great Britain where transmission decisions are modelled as a two-stage, bi-level game. Transmission planners take the first step and commit to certain investment options, to which the generators react. Subsequently, a wide range of future realizations could occur. After a decision and response, transmission planners can again make decisions followed by a market response, but the set of alternatives available at this next time step is constrained by the first-period decisions [20].

In our approach, we utilize a two-stage optimization approach. In the **Stage 1** optimization, we identify the maximum generation available, given the current state and constraints on the system. In **Stage 2**, we optimally allocate available flexibility. The two-stage optimization is described in detail as follows:

<u>Stage 1 Optimization</u>

In the first stage of optimization, the goal is to maximize electricity generation capacity while maintaining the physical and operational constraints, such as Water Balance Constraints, Reservoir WSE Constraints, Turbine Flow Constraints, output constraints, and reservoir WSE Constraints on the end of the period or optimization. Detailed information about the objective function and constraints are detailed below. The Stage 1 optimization will provide a robust solution of the maximum power that can be generated each time step given the uncertainties in the inflows and satisfying the physical and operational constraints of reservoir. This is helpful for BPA to know the flexibility they will have each day during the optimization period after meeting the demands. This information will be treated as input in the Stage 2 optimization and also to calculate the total predicted flexibility throughout the optimization period.

Note:

In the below mathematical formulation,

- Variables having the overscript ~ are <u>uncertain variables.</u>
- Variables without the overscript ~ are <u>deterministic variables.</u>
- Variables having superscript * are <u>optimal solutions</u>.

Model Input:

Inflows, $\tilde{Q}_{in}$

Model Decision Variables:

The total outflows of GCL reservoir at each time step are defined as decision variables in the optimization. We have considered an optimization period of 14 days with daily a time-step; therefore, the number of the decision variables is 14 for the single reservoir and is given as an array of flow rate, $Q_{\ out,Stage1}$decision variables as follows:

$$[Q_{out,1}\ Q_{out,2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ Q_{out,14}]$$

Model Objective:

Maximize Energy generation capacity,

$$\max_{Q_{out,t}} \sum_{t=1}^{14} \tilde{\mathrm{e}}_{\mathrm{t}}(Q_{out,t}, \widetilde{hd}_t(\widetilde{FB}_t,\ \mathrm{TW_t}), \xi_t) \qquad (1), (2)$$

Where,

$$\tilde{\mathrm{e}}_{\mathrm{t}} = \ \eta * 9.81 * \widetilde{hd}_t(\widetilde{FB}_t,\ \mathrm{TW_t}) * Q_{out,t} * 8.6310 * 10^{-3} * \xi_t$$

In this equation, $\tilde{e}_t$ is the energy generation at each time step t in MWh, where η is the efficiency of the reservoir, taken as 0.75; ξ_t is considered as 1 hour; $\widetilde{hd}_t$ is the head in ft. and is calculated as below:

$$\widetilde{hd}_t = \widetilde{FB}_t(\tilde{S}_t\ (\tilde{Q}_{in,t}, \tilde{Q}_{in,t-1}, Q_{out,t}, Q_{out,t-1}, \mathrm{delt_t})\ ) - \mathrm{TW_t} \quad (3)$$

$\widetilde{FB}_t$ is the reservoir water level in ft at time *t* which is a function of is reservoir storage $\tilde{S}_t$; $\mathrm{TW_t}$ is the tailwater in ft which is considered as constant for single reservoir. For multiple reservoir system, the tailwater will be subject to be an uncertain variable where the uncertainty will propagate from Inflows.

Subject to:

Model Constraints:

a. *Water Balance Constraints*

$$0 \le \tilde{S}_t\ (\tilde{Q}_{in,t}, \tilde{Q}_{in,t-1}, Q_{out,t}, Q_{out,t-1}, \mathrm{delt_t}) \le S_{max} \quad (4), (5)$$

Where,

$$\tilde{S}_{t+1} = ((\tilde{Q}_{in,t} + \tilde{Q}_{in,t+1})/2 - (Q_{out,t} + Q_{out,t+1})/2). \mathrm{delt_t} + \tilde{S}_t$$

In these equations, $\tilde{S}_t$ is reservoir storage in kcfs-day. S_{max} is the maximum storage capacity, $\tilde{Q}_{in}$ and Q_{out} are inflow and outflow to reservoir in kcfs, respectively, and $\mathrm{delt_t}$ is time (day) between each time step. At this stage, the water leakage and natural water loss is not considered We consider Eq. (4) as a *Reliability Constraint*. Therefore, equation 4 becomes:

$$Pr\{0 \le \tilde{S}_t\left(\tilde{Q}_{in,t}, \tilde{Q}_{in,t-1}, Q_{out,t}, Q_{out,t-1}, \mathrm{delt_t}\right) \le S_{max}\ \} \ge R,$$
$$0 \le R \le 1$$

where *R* is the reliability factor.

b. *Reservoir Water Surface Elevation (WSE) Constraints*

$$FB_{min} \le \widetilde{FB}_t(\tilde{S}_t) \le FB_{max} \quad (6), (7)$$

Where,

$$\widetilde{FB}_t = c_1 * (\tilde{S}_t)^2 + c_2 * (\tilde{S}_t) + c_3$$

where $\widetilde{FB}_t$ is the reservoir water level in ft at time *t*; FB_{min} and FB_{max} are the allowable minimum and maximum reservoir water elevation respectively; c_1 = -3.63*10^{-6}, c_2 = 0.0406 and c_3 =1208. The constants are determined by fitting actual forebay elevation observations with a polynomial regression model.

c. *Turbine Flow Constraints*

$$Q_{tb-min} \leq Q_{tb,t} \leq Q_{tb-max} \quad (8)$$

In this constraint, $Q_{tb,t}$ is turbine flow for power generation in kcfs at each time step and Q_{tb-min} and Q_{tb-max} are allowed minimum and maximum discharge respectively. Since we are not ignoring spill flow, Turbine flow and Outflow will be same. Therefore, we can re-write equation 8 as below:

$$Q_{tb-min} \leq Q_{out,t} \leq Q_{tb-max} \quad (9)$$

d. *Power Output Constraints*

$$N_{d-min} \leq \widetilde{N}_{d,t}(Q_{out,t}, \widetilde{hd}_t) \leq N_{d-max} \qquad (10), (11)$$

Where,

$$\widetilde{N}_{d,t} = \eta * 9.81 * \widetilde{hd}_t\left(\widetilde{FB}_t, \mathrm{TW_t}\right) * Q_{out,t} * 8.6310 * 10^{-3}$$

In the output constraint, $\widetilde{N}_{d,t}$ is power output in MW at time *t*. N_{d-min} and N_{d-max} are the minimum and maximum output capacity respectively.

e. *Reservoir Water Surface Elevation (WSE) Constraints on the end-of-period*

The optimization is conducted for 14 days, which is a relatively short term for the reservoir operation. To be consistent with middle-term or long-term operation, the water surface elevation (WSE) in the reservoir at the end of optimization period is expected to stay within a target WSE to fulfill future requirements. In the example problem we have formulated, the historical data from the actual operation scheme is used as the target WSE for the optimization model. To avoid equality constraints, a small range on the target WSE is used to restrain the WSE on the end-of-period to be close to the target WSE:

$$FB_{tar,end} - \Delta \leq \widetilde{FB}_t(\tilde{V}_t) \leq FB_{tar,end} + \Delta \qquad (12)$$

where $FB_{tar,end}$ is the target WSE on the end-of-period and Δ is the deviation from the target WSE. The Δ is set as 1% in the model and $FB_{tar,end}$ is taken as 1280 ft.

Model Output:

- Maximum energy generation capacity $\tilde{E}^{max,1}$ = [$\tilde{e}_1^{max,1}, \tilde{e}_2^{max,1}, \ldots\ldots, \tilde{e}_{14}^{max,1}$]

Stage 2 Optimization

In the Stage 2 optimization, we need to know how much water is available after meeting demand and other obligations; this is the available *flexibility*. Thus the Stage 1 optimization will provide the maximum electricity generation capacity each day. Once we deduct the daily demand and obligations, we will quantify the flexibility for each day. The purpose of the Stage 2 optimization is to determine how to allocate the flexibility and provide an optimal decision on a particular day: whether to sell the flexibility (and how much) to maximize revenue of BPA, or to hold because the value of the flexibility (value of holding the water) to BPA is greater than the market value. Thus, holding it on that day and using that on future when the market value will be greater than value of flexibility will give more profit to BPA, thus increasing the revenue. The robustness in the objective will account for the uncertain parameters, thereby ensuring the insensitiveness of the optimal allocation of flexibility each day to the uncertainty.

The objective of Stage 2 optimization is to maximize the profit of selling flexible water. In this step, few constraints are specified such as the minimum and maximum storage constraints, flexibility at the end of period etc. The decision variable will be the allocated flexibility, $h_t$each time step.

Model Input:

- Demand, D = $[d_1, d_2, \ldots \ldots \ldots \ldots, d_{14}]$
- Flexibility each day $\tilde{F}$= $\tilde{E}^{max,1}$ - D = $[\, \tilde{e}_1^{max,1} - d_1, \tilde{e}_2^{max,1} - d_2, \ldots \ldots ..] = [\tilde{f}_1, \tilde{f}_2 \ldots \ldots \ldots \tilde{f}_{14}]$
- Total Flexibility in 14 days' period, $\tilde{F}_{total} \; = \; \sum_{t=1}^{14} \tilde{f}_t$
- Price, P

Model Decision Variables:

We propose the decision variable to be the allocated flexibility *H*. Since it is daily-based model, the number of the decision variables is 14 for a single reservoir, given as an array of decision variables, *H*:

$$[h_1, h_2, \ldots \ldots \ldots, h_{14}]$$

Model Objective:

The objective has two components: maximizing expected revenue and the minimizing the value of flexibility of holding the water:

(1) Maximize Expected Increase in Net Revenue through allocation:

$$\max_{h_t} \sum_{t=1}^{14} ((\mathrm{p}_t * (\mathrm{h}_t - \tilde{f}_t(\tilde{e}_t^{max,1}, d_t))) * 24\,) \qquad (13)$$

$\tilde{f}_t$ is the flexibility obtained in period *t* in MWh; p_t is the price/ MWh to sell electricity in each day over 14 days period; and h_t is the allocated flexibility in period *t* in MWh.

(2) Valuation of Flexibility

The above equation (13) does not consider the scenario that, after allocating flexibility on a certain day, if there are any unplanned shortages of power on *future days*, BPA will need to buy that power from the market at a potentially higher price to meet the demand. Since BPA controls a significant portion of the electricity market, when there is a shortage of power, the market prices will tend to rise, and thus BPA may potentially have to buy power with higher prices than the prices they have sold. Therefore, flexibility has value and we will treat this value of *allocated flexibility* $\mathrm{V}(h_t)$ as the dollar value to BPA that they have to spend to buy power h_t from the market in the future to meet demand, considering probabilities stemming from the uncertainties in inflows and other potential sources. Thus, along with the objective function of maximize profit (13), we propose to integrate the valuation of allocated flexibility, $\mathrm{V}(h_t)$, leading to the flexible-robust objective.

The Real Option Model to compute $\mathbf{V}(\boldsymbol{h_t})$

Given the future economic value of flexibility (one value for each future time point), we need to figure out the current value of flexibility so that it is directly comparable to the current sales revenue. This is accomplished using *option theory*. To facilitate illustration, we start with a simplified, discretized model.

As shown in Figure 1, the realization of past uncertainties leads us to the current state, denoted by the red dot. We can sell h amount of flexibility now and get current revenue. Or we can hold it on to the future. Depending on the realization of future uncertainties, we may evolve to different states in period $t+1$ along different future paths as denoted by the broken lines. The different realizations of uncertainties generate different future scenarios with different flexibilities. The probability distribution is given by the blue line in the figure. As discussed earlier, this probability distribution also stipulates the probability of energy shortage as denoted by the shaded area. If provided with the information about the market supply function, we can derive $V(h, t+1)$, the future value of flexibility for period $t+1$. Following the same procedure, we can derive $V(h, t+2)$, the future value of flexibility for period $t+2$ and so on.

As flexibility can only be used once, the opportunity cost of selling h in the current period can only take one value in $V(h, t+k)$. It must be the one that gives the highest current value, that is,

$$V(h) = \max_k r^{-k} V(h, t+k) \quad \text{for } k = 1, 2, \ldots, 14, \tag{14}$$

where r is the interest rate. Given that the time step in this project is daily, $r \approx 1$.

Next, we need to convert all the possible future values of flexibility into one single value, the value of holding h in the current period, or equivalently the foregone opportunity cost of selling h in the current period t. This can be accomplished using option theory. For the discrete case as shown in Figure 2, the current value of the foregone opportunity cost can be derived using the multi-period multinomial option price model as described in Madan et al. [21] and Lee and Lee [22].

The example shown in Figure 2 generates a classical multi-period binomial option model (Cox et al., 1979) as represented in Figure 3. Figure 2, 3 is a binomial decision tree structure for a three-period option. Given the flexibility in the current period t, there are two possible scenarios associated with different levels of flexibility in period t+1: *shortage* and *no shortage*. Due to the uncertainties in the system, the incidence of shortage is governed by the probabilistic event $f_{t+1} < 0$, which occurs with the probability given by the blue-shaded area in the Figure 2. Conditional on the realization of flexibility in period t+1, there are two possible scenarios for the period t+2. This structure can be extended to multiple periods. The cost of doing so is the increase in computing time. In the current period t, we can either sell the h amount of flexibility for sales revenue or hold on the flexibility as an option to use it in future periods. Depending on the realization of uncertainties, the option value of holding h amount of flexibility to period t+1 may equal either $V_1(h, t+1)$ or $V_2(h, t+1)$. Then conditional on the amount of flexibility in period $t+1$, we may have two additional possible scenarios corresponding to the incidence of shortage in period $t+2$. Depending on the uncertainties in period t+2, the value of holding h amount flexibility to period t+2 may have different values denoted as $V_1(h, t+2), V_2(h, t+2), V_3(h, t+2)$, and $V_4(h, t+2)$. The determination of the option value for each period $V_k(h, t+\tau)$ $(\tau = 1, 2, \ldots, T)$ uses the backward induction scheme. Starting from the last period T (*T=t+2* in case 3-day optimization period decision tree structure as shown in Figure 2, 3), if the flexibility h is held to the last period, its value equals the purchase cost saved if shortage occurs:

$$V(h, T) = -\min(0, \max(-h_t, f_T)) \times P_T = \begin{cases} -\max(-h_t, f_T) \times P_T, & if\ f_T < 0 \\ 0 & if\ f_T \geq 0 \end{cases} \quad (15)$$

Note: shortage occurs when $f_T < 0$.

Similarly, we can calculate the value of using h to avoid purchase cost if shortage occurs in period T-1, denote the value as: $\hat{V}(h, T-1)$:

$$\hat{V}(h, T-1) = -\min(0, \max(-h_t, f_{T-1})) \times P_{T-1} = \begin{cases} -\max(-h_t, f_{T-1}) \times P_{T-1}, & if\ f_{T-1} < 0 \\ 0 & if\ f_{T-1} \geq 0 \end{cases} \tag{16}$$

The discounted expected value of the last period option value $V(h, T)$ is then compared with $\hat{V}(h, T-1)$, the option value of using h in period T-1. The larger value is taken as the option value for period T-1. This reflects the defining feature of American Option that it can be exercised any time before the expiration date:

$$V(h, T-1) = \max(\hat{V}(h, T-1), \delta EV(h, T)) \tag{17}$$

This is done for each of the possible scenarios in period T-1. Then, taking the period T-1 as the final period and using this procedure iteratively. The option value at the current time $V(h, t)$ can be calculated. In our problem, the decision tree includes a 14- day period. That is T=t+13.

(3) The Flexible-Robust Objective:

As discussed in the previous section, integrating the Real Option Model with Robust Optimization technique, we modify Eq. (13) into **flexible-robust objective**:

$$\max_{h_t} \sum_{t=1}^{14} (((\mathrm{p}_t * (\mathrm{h}_t - \tilde{f}_t(\tilde{e}_t^{max,1}, d_t))) * 24) - \widetilde{\mathrm{V}}(\mathrm{h}_t * 24)) \tag{18}$$

$\tilde{f}_t$ is the flexibility obtained in period *t* in MWh; p_t is the price/Mwh to sell electricity in each day over 14 days period; and h_t is the allocated flexibility in period *t* in MWh. $\widetilde{\mathrm{V}}(\mathrm{h}_t * 24)$ is the value of the allocated flexibility $(\mathrm{h}_t * 24)$ in MW-day. Uncertainty of the Value of allocated flexibility is incorporated from actual availability of flexibility in future periods. Thus during optimization, there will be a comparison between the revenue generated by the allocated flexibility, and the expected loss of dollar value due to the allocated flexibility, i.e., the value of holding or the future value of allocated flexibility.

The constraints in Stage 2 are as follows:

Model Constraints:

1. Maximum Storage Constraint:

$$\tilde{A}_t(\tilde{A}_{t-1}, \mathrm{h}_{t-1}, \tilde{f}_t) - \mathrm{P}_{\max} \leq \delta \qquad (19, 20)$$

Where,

$$\tilde{A}_t = \tilde{A}_{t-1} - \mathrm{h}_{t-1} + \tilde{f}_t$$

$\tilde{A}_t$ is the actual availability of flexibility in MWh after allocating flexibility for t-1 days as per decision made and h_{t-1} is the allocated flexibility on Day (t -1) in MWh, $\tilde{f}_t$ is the flexibility obtained in period *t* in MWh, δ is the maximum tolerance of the violation and is set as 1 MWh. $\mathrm{P}_{\max}$ is the maximum flexibility generated in Mwh for the maximum amount of water a given reservoir can store. In the next chapter, we will modify the storage constraint in terms of units of volume. We consider Eq. (19) as a *Reliability Constraint*, thus becoming:

$$Pr\{\, \tilde{A}_t(\tilde{A}_{t-1}, \mathrm{h}_{t-1}, \tilde{f}_t) - \mathrm{P}_{\max} \leq \delta \,\} \geq R, \ \ 0 \leq R \leq 1$$

where *R* is the reliability factor.

2. *Maximum Allocation of Flexibility*

This constraint is applied to validate that allocated flexibility is less than the actual availability of flexibility each day. It is unrealistic to allocate more flexibility than actually will have on a day.

$$\mathrm{h}_t - \tilde{A}_t(\tilde{A}_{t-1}, \mathrm{h}_{t-1}, \tilde{f}_t) \leq \delta \qquad (21)$$

δ is the maximum tolerance of the violation and is set as 1 MWh. We consider Eq. (21) as a *Reliability Constraint*. Therefore, equation 21 becomes:

$$Pr\{\, \mathrm{h}_t - \tilde{A}_t(\tilde{A}_{t-1}, \mathrm{h}_{t-1}, \tilde{f}_t) \leq \delta \,\} \geq R, \ \ 0 \leq R \leq 1$$

where *R* is the reliability factor.

Model Output:

- Optimal allocated flexibility H*
- Maximum Net Revenue in selling the Flexibility, $\tilde{R}^{max}$

4. SOLUTION METHODOLOGY

Figure 4 shows a high level flowchart of the framework, along with the optimization technique and methodology applied to convert the problem described in previous section to investigate the efficiency of flexible-robust objective. For stage 1 optimization, a gradient based nonlinear constrained optimization algorithm (SQP) is used. For stage 2 optimization, stochastic optimization is used, more specifically a Genetic Algorithm (GA). *Utility theory* has been applied with a risk-averse attitude to find out the expected utility of the objective function to address the robust optimization. Uncertainty quantification was done using Full Tensor Numerical Integration (FTNI) and the validation of probabilistic constraints is handled using the Inverse Reliability method.

Uncertain Parameters

We considered inflows, $\tilde{Q}_{in}$, each day as uncertain parameters for Stage 1. Thus, we consider 14 uncertain parameters in Stage 1 for the two-week optimization period with daily time steps. The model will provide the results of maximum hydroenergy, $\tilde{e}_t$ from Stage 1, which has uncertainty due to uncertainty on inflows; the maximum energy generation each day, $\tilde{e}_t$ results in uncertainty in flexibility, $\tilde{f}_t$, each day after deducting estimated demand from $\tilde{e}_t$ In the Real Option analysis, the probabilities of shortages are calculated from the uncertainty of flexibility. In the future, we will also consider prices and demands as uncertain parameters. In this example, we consider 14 uncertain parameters, $\tilde{f}_t$ in the Stage 2 optimization.

Techniques used for improving the optimization

Though the model created with the previously described approach is successful in providing an optimum allocation of flexibility, it does not prove computationally efficient. The primary issue is in the numerical integration of uncertainty in each iteration. In the

FTNI method used for numerical integration, we used 3 nodes to represent uncertainty in each of the 14 decision variables, which resulted in 3^{14} (4782969) calculations for quantification of uncertainty of the objective function in each population of the Genetic Algorithm. Therefore, the method proves computationally too expensive. Therefore, in the calculation of expected value, we ignore all the calculations which add very little to the total value. Consequently, we ignore any set of data which has a weight of less than δ. We considered δ as 10^{-6}. This reduces the calculations to approximately 3000, which reduces ¾ of the previous computational time. The accuracy of the result was shown to be very good with this approximation

Convergence Criteria

For Stage 1, the convergence criteria are step size tolerance, constraint violation tolerance, function tolerance; all set as $\xi = 10^{-5}$ and maximum function evaluations set as 40000. For Stage 2, the convergence criteria are maximum number of generations set by the users.

5. RESULTS FOR THE SINGLE RESERVOIR MODEL

In this section, we show the decisions provided by the model **with** and **without** the consideration of Real Option Analysis. We use this approach to show the effect of including the real option model as part of the objective (i.e. the flexible-robust objective). We therefore have two model policies to evaluate:

a) **Policy 1** in which the Real Option valuation is **ignored** (equation 13).
b) **Policy 2** in which the Real Option valuation is **included** (equation 18).

We will demonstrate the results of two policies on two sample scenarios:

a) **Scenario 1** in which there is large amount of total flexibility in the 14 days optimization period, i.e., low chance of water shortage.
b) **Scenario 2** in which there is small amount of total flexibility, i.e, greater chance of a shortage.

The following figures show the results for the two policies and the two scenarios.

In the Figures 5 and 6, the blue lines are the flexibility after Stage 1 optimization, while the green lines are the optimal decisions for rescheduling the operations by allocating flexibility. The red dashed line shows the actual availability of flexibility each day. As we cannot allocate more flexibility more than available: the green line will always be under the red dashed line. Figures 7 and 8 are the graphs of price each day and the uncertainty each day as quantified by the variance in flexibility.

In Figures 5A and 6A (which represent Policy 1), we see that the optimal decision has been made to allocate more flexibility on day 8 than day 12, since the price on day 8 is higher. However, the uncertainty of available flexibility is higher on Day 12 than on Day 8. Therefore, when we considered the Real Option valuation in our optimization (Policy 2), we see that the optimal decision is to allocate less on Day 8 and hold the flexibility to allocate on Day 12 (Fig 5B) Also, we see a greater amount of flexibility to allocate on Day 14 despite that it has a lower selling price than day 8 (Fig 6B). Thus, it is seen that inclusion of the Real Option valuation in Policy 2 provides additional weightage to the uncertainties than just robust optimization in both Scenarios 1 and 2, resulting in decisions which hold onto the water until later in the time period. The effect is greater in Scenario 2, which has lower flexibility, and thus has a greater chance of having a water shortage. It is clear that there is a trade-off between the two objectives: whether to ***sell*** the flexibility or to ***hold*** it for future use. One of the objectives will dominate the decision process depending on different conditions such as market price, uncertainty, and the availability of total flexibility.

The revenue for the different policies and scenarios is shown in Table 1. The Net Revenue in Table 1 is defined as the cost of selling the allocated flexibility minus the cost of purchase. The cost of purchase is the expected power that needs to be purchased from markets due to a probability of shortages, as quantified using the Real Option Model. Also, we consider the revenue BPA will attain by selling the flexible water after meeting daily demand; that is, it does not consider the revenue attained by selling the electricity to meet daily demands, only that from selling flexibility.

Table 1 shows the increase in Net Revenue which results by integrating the Real Option model with the Robust Optimization framework (Policy 2) for both scenarios. We can see the percentage of increase by implementing Policy 2 is greater in Scenario 2 (25%) than that in Scenario 1 (13%). This is reasonable since there are higher probabilities of shortages due to low flexibility in Scenario 2, and the real option model will tend to provide more value in holding the flexibility. Thus real option analysis will have greater benefit when there is less flexibility, i.e., in dry seasons. However, it provides a significant profit increase even when there is large amount of flexibility available.

In Figure 9, we show the optimum allocation of flexibility in terms to optimal outflows, Q_{out}, for Scenario 1. The blue line is the optimal outflow after Stage 1 optimization, i.e., the optimal outflow to generate maximum electricity without violating any constraints. The red and green lines are the optimal outflow after Stage 2 optimization following Policy 1 and 2, respectively. We know from figure 5A that in Policy 1, the optimal decision is to allocate more flexibility on day 8 than on Day 12. Thus, we see a higher release of water on Day 8. Integrating the real option analysis in the robust optimization (Policy 2) changes the decision (Fig 5B). Thus, we see higher release of water on Day 12 and 14 respectively. Figure 10 represents the corresponding storage of water. The blue line is the storage each day at the optimal decision following Policy 1 and the green line is the same for Policy 2. We see in Fig 9, the optimal decisions in both Policy1 during the first 7 days are mostly to hold water, therefore, the storage increases on Day 7 as shown in Fig 10. However, there is a large amount of water released on Day 8 and Day 12 which causes a sharp decrease in storage on these days. The same is true for Policy 2, where we see the storage increases until Day 12 and then goes down sharply due to high release of water on future days. (Fig 9, Fig 10). We also validate the rescheduling of operation in Stage 2 optimization did not violate the physical storage constraint of the dam, Grand Coulee.

Figures 11 show the results of the 14 day period from the optimization with 27 different inflow scenarios. Fig. 11A shows the increase in Net Revenue BPA will attain by selling flexibility following Policy 2 instead of Policy 1 (i.e., considering the Real Option analysis). Fig. 11B shows the increase (percentage) for the respective 27 scenarios. We can see the percentage increases are generally in the range of **2% to 40%** which is roughly **10 to 80 thousand dollars**. However, when there are very high inflows and BPA needs to sell water to avoid floods, the real option model will not provide any valuation, since the future value of water is dominated by the constraints such as the storage constraint. Therefore, we can see in the graph a few scenarios where there is no increase in revenue, since the storage constraint is active.

6. CONCLUSION

We presented a new approach to large-scale system optimization, using a flexible-robust objective, by successfully integrating Real Option theory with Robust Optimization. Based on the results shown, we can conclude that valuation of flexible water is beneficial in managing operations of large system and can minimize the risk of shortages in electricity and maximize the profit of power sectors. While we have demonstrated the concept, we will be addressing the future work. The first issue is that this is a simplified version of the system model and we must scale our work to address the entire reservoir system. Our next task is to include more physical and operational constraints in Stage 2 to provide more realistic results. Currently, we assume that the inflows are independent and normally distributed. In the future, we will focus on executing our model with the actual distribution of the inflows which can be estimated from historic data provided by BPA. We will also quantify uncertainty more accurately by considering dependence among the inflows. Also, we will incorporate price and demand uncertainty; the real option valuation will provide better results with a better characterization of uncertainty. One of the drawbacks in the Stage 2 stochastic optimization is that the decision variables are discrete. Our future work will be to eliminate that restriction and the Stage 2 optimization can work with continuous random decision variable. Also, our future research interest will focus on including different Robust Optimization techniques which will provide fast convergence, better solutions and handle the decision variable and constraints more efficiently.

ACKNOWLEDGMENTS

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TABLES OF CHAPTER 2

Scenario 1		
Net Revenue ($)	***Policy 1***	$ 551450
	Policy 2	$ 622958
Increase in Net Revenue ($)		$ 71508
Percentage Increase		13 %
Scenario 2		
Net Revenue ($)	***Policy 1***	$ 48005
	Policy 2	$ 59980
Increase in Net Revenue ($)		$ 11975
Percentage Increase		25 %

Table 1: Summary of Results

FIGURES OF CHAPTER 2

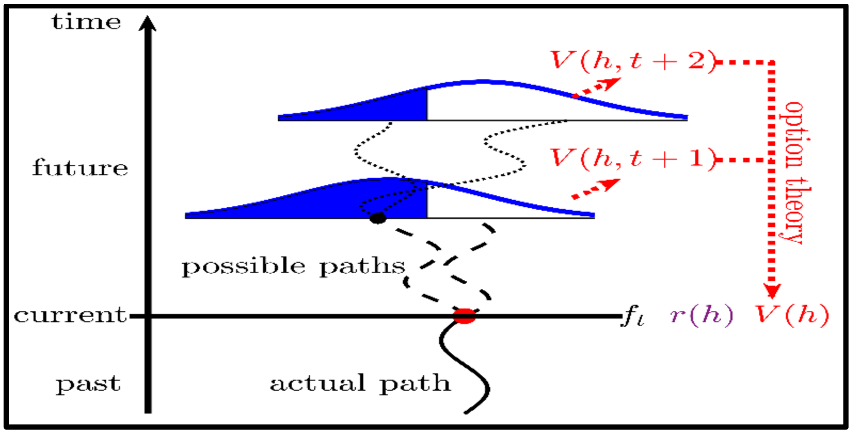

Figure 1. Evolution of States

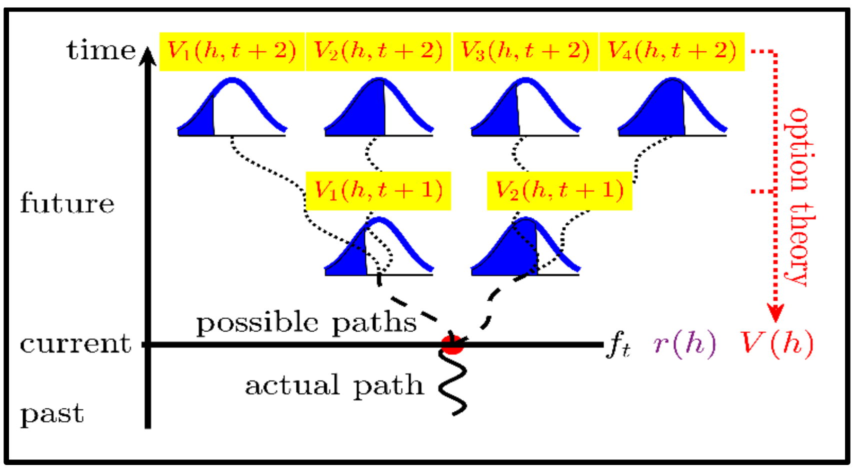

Figure 2. Possible Future States

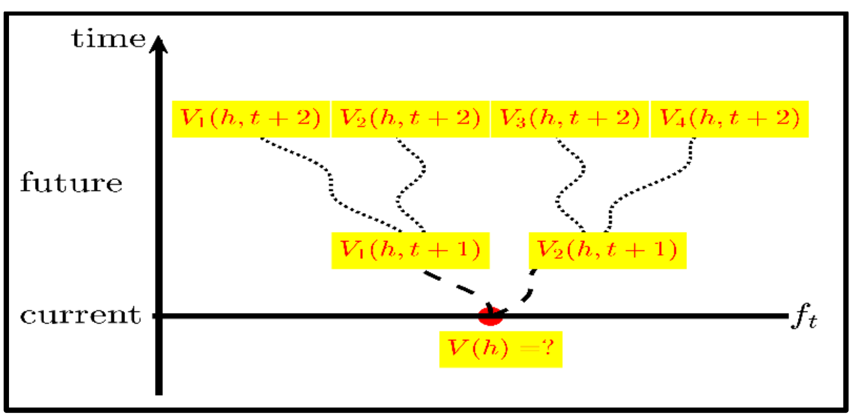

Figure 3. Binomial Option Model

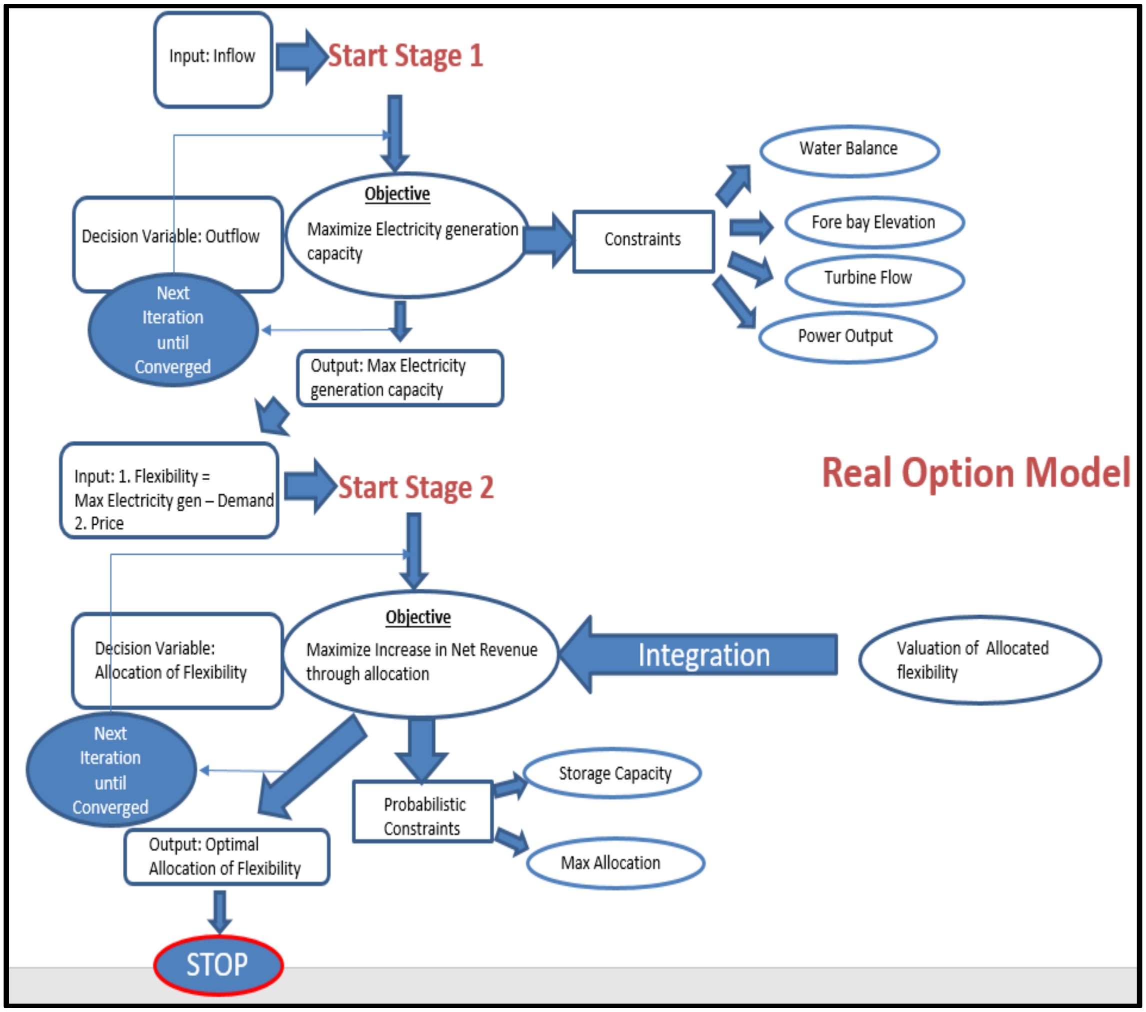

Figure 4. Overall Flexible-Robust Approach

Scenario 1

Policy 1

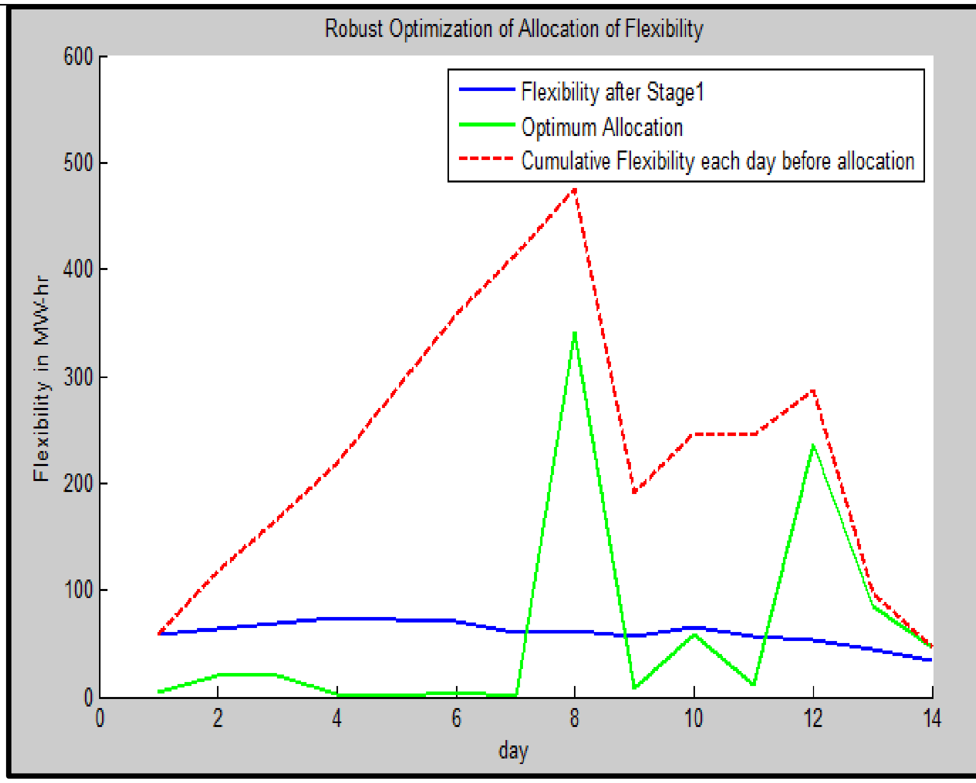

Figure 5A. Optimal Decision of Allocated Flexibility

Policy 2

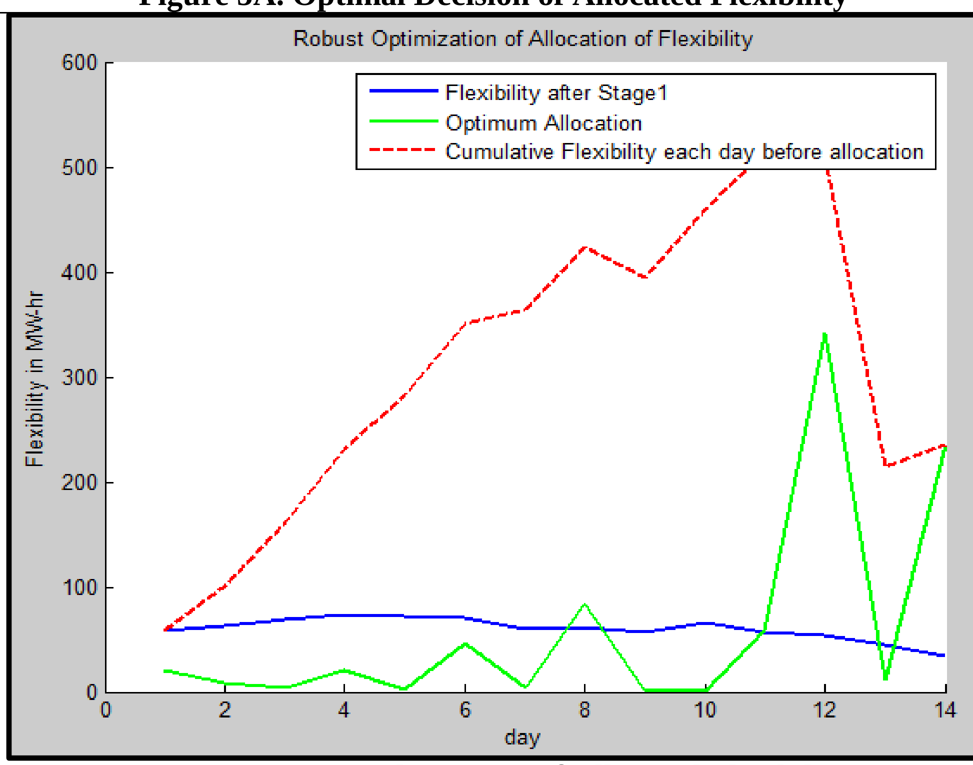

Figure 5B. Optimal Decision of Allocated Flexibility

Scenario 2

Policy 1

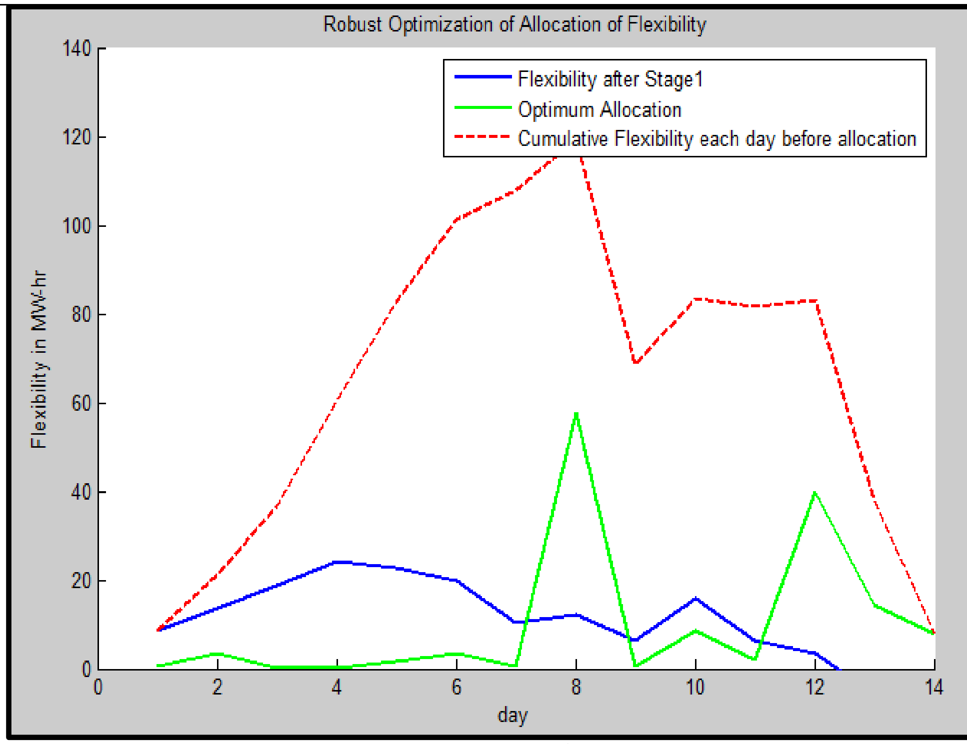

Figure 6A. Optimal Decision of Allocated Flexibility

Policy 2

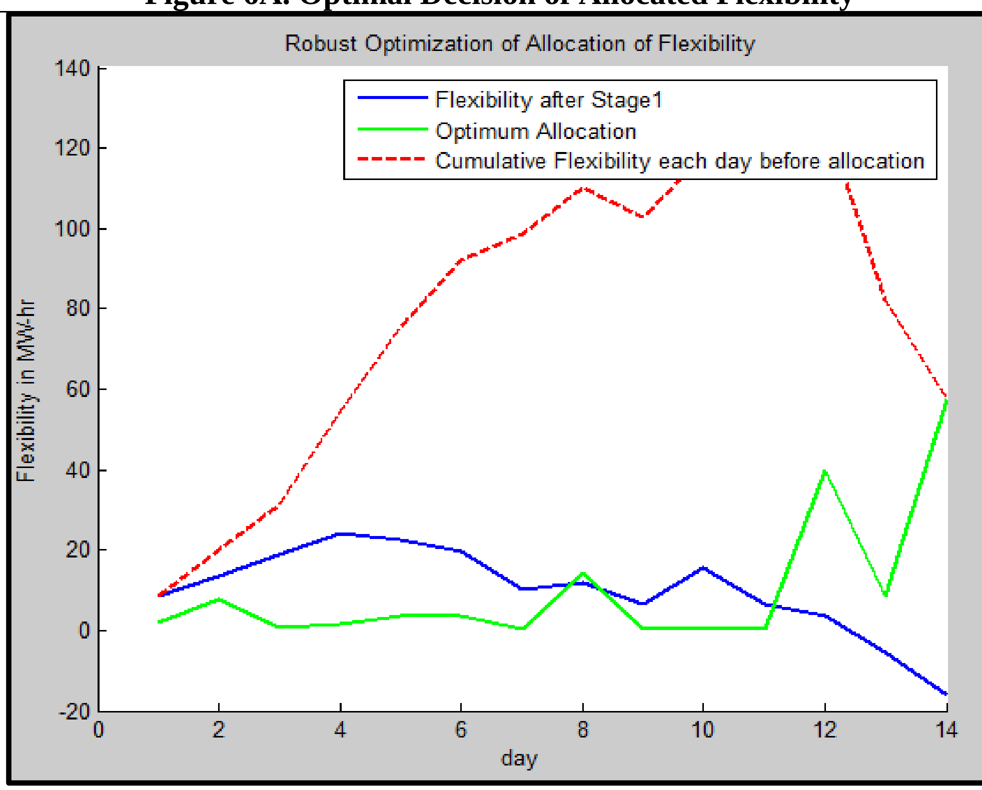

Figure 6B. Optimal Decision of Allocated Flexibility

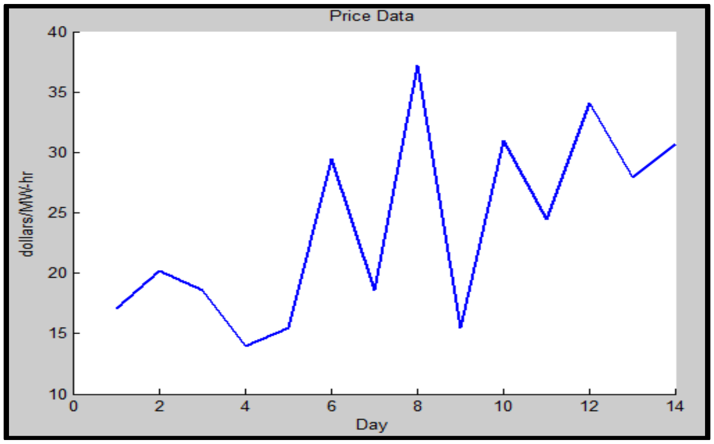

Figure 7. Price of Electricity Each Day

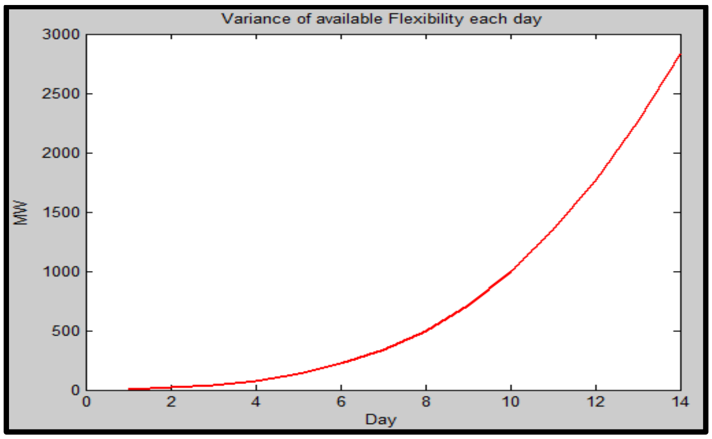

Figure 8. Uncertainty on Available Flexibility Each Day

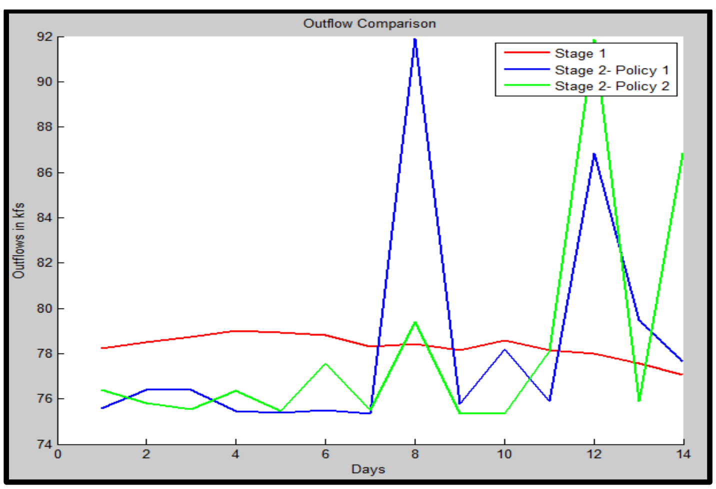

Figure 9. Optimal Outflows Comparison

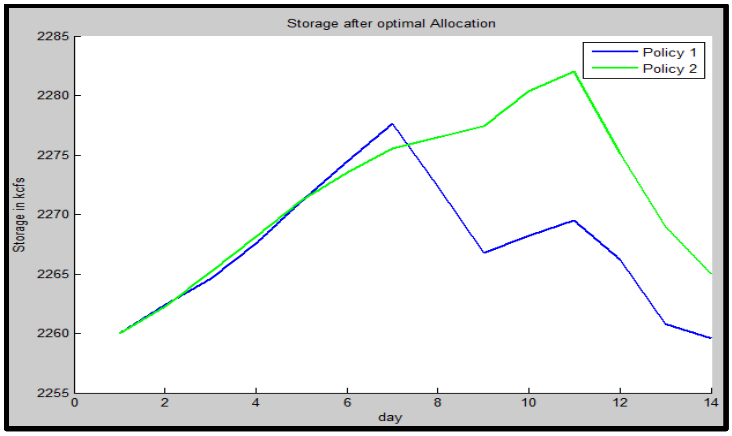

Figure 10. Storage Comparison at Optimal Outflows

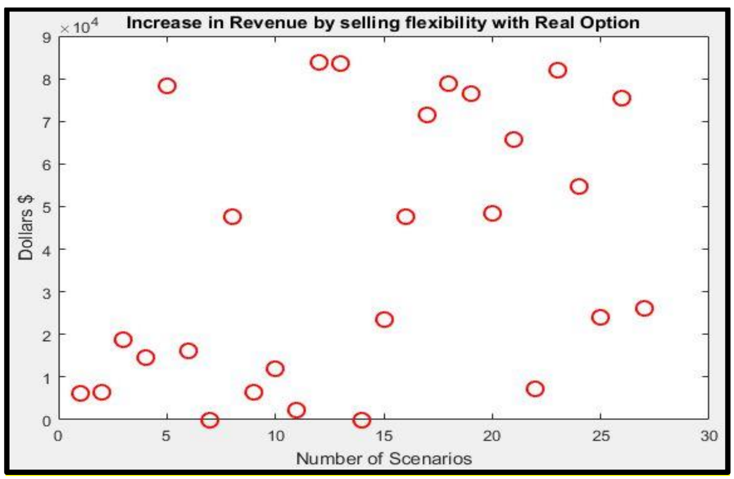

Figure 11A. Increase in Revenue ($) by selling flexibility with Real Option

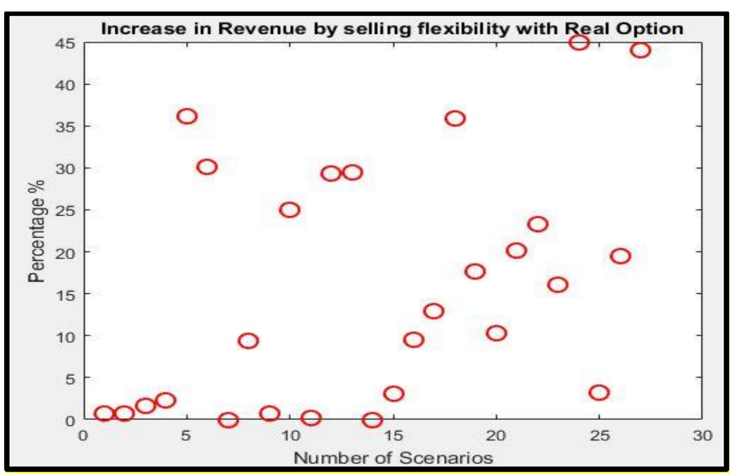

Figure 11B. Increase in Revenue (%) by selling flexibility with Real Option

CHAPTER 3

A BI-LEVEL OPTIMIZATION APPROACH FOR ENERGY ALLOCATION PROBLEMS

ABSTRACT

*In our previous chapter, we have integrated the Robust Optimization framework with the Real Options model to evaluate flexibility, introducing the Flexible-Robust Objective. Flexibility is defined as the energy left to allocate after meeting daily demands. This integration proved more efficient in risk evaluation in energy allocation problems. However, the integration has some limitations in applying operational and physical constraints of the reservoirs. In this paper, an in-depth analysis of all the limitations is discussed. To overcome those limitations and ensure a conceptually correct approach, a bi-level programming approach has been introduced in the second stage of the model in solve the energy allocation problem. We define the proposed model in this paper as **Two-Stage, Bi-Level Flexible-Robust Optimization.** Stage 1 provides the maximum total flexibility that can be allocated throughout the optimization period. Stage 2 uses bi-level optimization. The Stage 2 upper level sets the target allocation of flexibility in each iteration and maximizes net revenue along with the evaluation of allocated flexibility by the real options model. The Stage 2 lower level minimizes the deviation between the level 1 target and the achievable solution, ensuring no violation in physical and operational constraints of the reservoirs. Some compatibility issues have been identified in integrating the two levels, which have been discussed and solved successfully; the model provides an optimal achievable allocation of flexibility by maximizing net revenue and minimizing violation of constraints. Uncertainty in the objective function and constraints has been handled by converting into a robust objective and probabilistic constraints, respectively. Both classical methods (SQP) and evolutionary methods (GA) with continuous decision variables have been applied to solving the optimization problem, and the results are compared. It is shown that SQP provides much faster and better results. Also, the result has been compared with the simplified version in previous chapter, which was limited to randomly generated discrete decision variables. The new results provided an 8% improvement over the previous simplified model.*

1. INTRODUCTION

In real world complex design problems for large scale systems, multidisciplinary optimization has been utilized and plays a key role in design research. In large-scale complex systems, it is often difficult to optimize for the whole system in a single level consisting of large numbers of design variables, objective functions and constraints. Therefore, it is desirable to break down the system into several components or subsystems. It is easier to optimize each of the subsystems which guarantee an optimal solution to the main system. This idea arises in the multidisciplinary optimization framework, where the optimization of a large-scale system is done by optimizing each of the subsystems, which are coupled with each other. Analytical Target Cascading (ATC) and Collaborative Optimization (CO) are the two methods of MDO. Several studies have been done in recent years in MDO [23], [24]. McAllister and Simpson [25] presented a CO framework for an Internal Combustion Engine. Another emerging approach in the area of Multi-disciplinary optimization is the Bi-Level Optimization method. Details of Bi-Level Optimization with literature reviews will be described in the later sections.

In the field of river systems and hydropower energy sectors, uncertainties play a key role in terms of quantifying inflows, weather forecasts, market prices and demands for electricity. To incorporate these uncertainties in design, Robust Optimization is the best approach for managing such uncertainties. Much literature is available on Robust Optimization, where robustness has been studied extensively to handle uncertainties in several design areas, including hydropower energy sectors [16], [26]. Robustness is defined as the ability of a given system configuration to perform well over a wide range of conditions over the product lifecycle, such as the occurrence of faults and resulting functional losses. Taguchi-based approaches [11]–[14] utilize an optimization framework in which the system design is optimized based on an objective which considers both the mean and variance of the system performance. Variance in the system performance can result from multiple sources of random noise, both external and internal to the system. Robustness has also been investigated in the biological network literature, with principles for achieving robustness defined as system control, redundancy, diversity, modularity, and decoupling [15]. McIntire et al. applied a Robust Design Optimization framework to the Columbia River System to provide optimal outflows for maximizing expected revenue, considering the inflow uncertainties. The probabilistic framework results in lower risk solution than the deterministic approach, when uncertainties are accounted for [16].

This paper is premised upon a Two-Stage Robust Bi-level Optimization approach for Complex Large Scale Systems integrated with a Real Option model to evaluate flexibility in allocation of energy. For the simplicity of this paper, we will start with single reservoir of Lower Columbia river as an example. However, the mathematical model described in later sections is also applicable for multiple reservoirs. However, the result for multiple reservoirs is out of scope for this paper and will be considered in future. A detailed description of the problem will be presented in a separate section.

2. AN OVERVIEW ON BI-LEVEL OPTIMIZATION

Bi-level optimization is a certain type of optimization where one problem is embedded (nested) within another. The outer optimization task is commonly referred to as the upper-level optimization task, and the inner optimization task is commonly referred to as the lower-level optimization task. The lower level optimization acts as a constraint in the upper level. These problems involve two kinds of variables, referred to as the upper-level variables and the lower-level variables [27]. The lower level optimization, also referred to as follower's optimization problem, is solved first. The upper level optimization, also referred as leader's optimization problem, considers the optimal solution of the follower. Therefore, the optimal solution of the upper level optimization problem will guarantee optimality also in the lower level optimization problem. Bi-level optimization was first realized in the field of game theory by a German economist Heinrich Freiherr von Stackelberg in 1934 that described this hierarchical problem.

A simple formulation of Bi-Level Optimization can be written below as:

$\min_{\boldsymbol{x}} \boldsymbol{F}(\boldsymbol{x}, \boldsymbol{y})$ (*Upper Level*)

s.t (*Upper Level Constraints*)

$$G_i(x, y) \leq 0 \; for \; i = \{1,2 \ldots N\}$$

$$\min_{y} f(x, y) \; (\textit{Lower Level})$$

s.t (*Lower Level Constraints*)

$$g_j(x, y) \leq 0 \; for \; j = \{1,2 \ldots n\}$$

Where x and y are upper and lower level decision variables, respectively; $\boldsymbol{G_i}$ and $\boldsymbol{g_j}$ are the $\boldsymbol{i^{th}}$ and $\boldsymbol{j^{\,th}}$ inequality constraints in upper and lower level, respectively.

Papers have been published attempting Bi-Level programming in various design problems. This approach has been extensively applied in the field of transportation and defense strategy. Labbe, Marcotte and Savard in 1998 proposed a bilevel model of taxation and its application in toll-setting problem in highways. In this bi-level model the leader wants to maximize revenue from taxation schemes, while the follower rationally reacts to those tax levels [28]. Chen and Subprasom in 2006 [29] formulated a stochastic bi-level programming model for a Build-Operate-Transfer (BOT) road pricing problem under demand uncertainty. Braken and McGill in 1973 [30] proposed a bi-level optimization model in defense applications which includes strategic force planning problems and two general purpose force planning problems. In recent years, this approach has been accepted and is being widely used in strategic bomber force structure, and allocation of tactical aircraft to missions. Roghanian, Sadjadi and Aryanezhad in 2006 [31] presented a bi-level multi-objective programming model in enterprise wide supply chain planning problem considering uncertainties on market demand, production capacity and resource availability. The bi-level programming method has also been applied in water resource allocation planning problems [32], [33].

3. PROBLEM DESCRIPTION

In this section, we will provide an overview of our problem and introduce the bi-level optimization approach for optimal hydropower scheduling in energy sectors. In the hydropower scheduling problem, the main sources of uncertainties are the inflows, market price, and market demand of electricity. Huang in 2010 [32] and Xu in 2013 [33] attempted a bi-level optimization technique for a water allocation problem; however, the uncertainties were handled in a fuzzy random environment in both works. Also, the operational constraints of the reservoirs were not considered in the model, which leads to a complex design. In our paper, we will handle the uncertainties with an approach to Flexible-Robust Optimization framework as discussed in Chapter 2. Flexibility is defined in this case as the option to either sell the remaining electricity to market after meeting daily demand or to hold it for future use to address any negative shocks. Evaluation of flexibility is done using Real Option (RO) analysis. RO analysis has been used previously in bi-level programming in the FACTS investment problem [34]. However, the model was not integrated within a robust optimization framework. In Chapter 2, we have described why both robust optimization and real options analysis are simultaneously necessary in hydropower scheduling problems: the robust optimization framework focuses on maximizing revenue and minimizing future risks based on the evaluation of the RO model by allocating flexibility in the current period accounting inflow uncertainty. We have provided a formulation of the integrated model along with the details of RO analysis for future risk evaluation of flexibility. The benefit of integration of RO model with the Robust Optimization framework (Flexible-Robust Optimization) is demonstrated by applying it to a single reservoir in the Lower Columbia River System; the results demonstrate a better revenue by optimal allocation of flexibility by as much as 40% when compared with standalone Robust Optimization. However, there are some limitations in the integrated model. Also, the problem mentioned in our previous paper in Stage 2 did have limited constraints. In this paper, we target to address those limitations in the previously simplified model.

We will discuss in this section the necessity of Bi-level programming as an approach to overcoming the limitations in the mathematical model in Chapter 2, in a conceptually sound manner. In the scheduling of hydropower, the decision makers need to know optimal water releases at each period from the reservoir which will generate total optimal allocated energy in each period. The total optimal allocated energy is the sum of energy to meet demand and the optimal allocated flexibility in each period. It will not be useful to them to know only the total optimal allocation of energy, as they will ultimately need to understand how much water needs to be released from *each* reservoir to achieve the total allocation. This was the limitation in the model presented in our previous paper, where we assumed that it is sufficient to provide the total optimal allocation of flexibility as the output of the model, and the optimal outflows for each reservoir can be calculated separately. Also, the model in Stage 2 had limited constraints, the completion of which were part of the future work. A particular challenge for constraints is that in a real scenario, the storage of water for future use will affect the physical and operational constraints of the reservoirs. Therefore, the new version of the previous simplified model addresses this limitation by introducing *bi-level programming*.

In our approach, the Stage 2 optimization has a flexible-robust objective function where evaluation of allocation of flexibility is done by RO analysis. The RO analysis considers different decision scenarios with respect to allocation of flexibility. Therefore, in the RO analysis, allocated flexibility should be a deterministic quantity because it is unreasonable as a part of RO analysis to evaluate the value of a *decision* which has randomness. This is a limitation of RO analysis in that it cannot value any uncertain variables. Therefore, we choose allocated flexibility as a decision variable which is deterministic to be valued successfully. However, the decision makers also need to know the optimal solution in terms of outflows to achieve the desired energy; however, putting outflows as the decision variable will cause allocated flexibility to be an uncertain variable. As noted, it is not desirable to use an uncertain decision variable. Also, we cannot use both outflows and allocated flexibility as the decision variable in the same stage, as the later will be treated as a dependent variable of outflows with uncertainties a result of inflow uncertainty. Additionally, this will highlight a limitation of the RO analysis of allocated flexibility in that it can be done in terms of energy but not in terms of outflows. Therefore, we need to formulate a solution which will fulfill all the requirements of the decision makers while making sure the integrated model is conceptually correct.

This problem of allocation (upper level) versus strategy or design (lower level) is present is many design problems. In such scenarios, we need to focus on the design using both an economic standpoint that considers cost of manufacturing and labor etc. while also finding a safe and feasible design. To model a design problem of different criteria, one approach is to optimize each criteria of design individually. In our problem of scheduling of hydropower, we adopt the same idea of focusing on maximizing economics in one level and finding a feasible strategy, or design, in the other level to ensure safety and technical feasibility. Therefore, a bi-level optimization method is used in our model to apply this concept as a remedy to overcome all the limitations in the previous simplified version of the model. The upper level will strictly focus on maximizing revenue and the lower level will ensure a safe feasible design. Also, the two levels in Bi-level programming can have different decision variables as shown in the general formulation. This addresses the limitations in RO valuation. Thus, we provide a complete **Two-Stage Bi-Level Flexible-Robust Optimization** model as an approach to a general Hydropower Scheduling problem of large scale systems. In this paper, we have limited our case study to single reservoir of Columbia River system: Grand Coulee which is managed by Bonneville Power Administration (BPA). The descriptions of Grand Coulee are provided by BPA.

Lower Columbia River: Grand Coulee Reservoir

An optimization model is proposed which considers the reservoirs of the Columbia River system per the problem description below. We will have a similar approach as our previous paper where we started with a two-stage model. The goal of the first stage of optimization is to provide an optimal robust solution of maximum energy generation capacity for each time period for each reservoir. Previously this kind of multi-stage model has been presented [35] in optimization of Real-time Hydrothermal system operations where 3 sub-models (hourly, daily and monthly) have been coupled with different time-steps (hourly, daily and monthly) and optimization period (daily, monthly and yearly) with an objective to maximize revenue. The optimal result of the monthly model acts as a boundary constraint to the daily model and the optimal result of the daily model is applied as a boundary in the hourly model. In our model, Stage 1 optimization helps BPA to know the maximum total flexibility that can be generated during the optimization period. This total flexibility acts as a boundary constraint in Stage 2 optimization and is allocated optimally for better revenue and to address any negative shocks. Bi-level programming is introduced in the Stage 2 model.

Stage 1

In the first stage of optimization the goal is to maximize electricity generation capacity keeping the physical and operational constraints of Water Balance Constraints, Reservoir WSE Constraints, Turbine Flow Constraints, output constraints and reservoir WSE Constraints on the end of the period. The decision variable for Stage 1 model is total *outflow* as each time-step. We have considered daily time-steps and an optimization period of 14 days in both stages of the proposed model. The Stage 1 model will provide maximum energy generation capacity at each time period as the output. Detailed information about the formulation of the objective function, constraints for Stage 1 is already presented in Chapter 2 and is not repeated here.

We will now provide a detailed mathematical formulation of the Stage 2 Flexible-Robust Optimization with Bi-Level Programming.

Stage2 (Bi- level Optimization):

In this section, we will discuss the goals of the two levels of Stage 2 model and how they are integrated by presenting the mathematical formulation. The Stage 1 optimization provides the maximum power generation capacity, $\tilde{E}^{max,1}$, and thereby the maximum total flexibility and also the *maximum* flexibility can be achieved each day; the Stage 2 optimization is designed to provide a solution of optimally *allocating* the flexibility for better revenue. The Stage 2 optimization is divided into two levels: The *Upper Level* provides decisions of allocating flexibility which are set as the *target* allocation. To guarantee the target meets the physical and operational constraints of the reservoir, the *Lower Level* optimization is introduced which is treated as a constraint in the Upper level. The objective function is to minimize the deviation of allocation of flexibility (results from lower level) from the target allocated flexibility set in the upper level. The lower level optimization will be solved first and will provide us an achievable solution of allocated flexibility closest to the target set in the upper level satisfying all the physical and operational constraints. The objective function of the upper level is to find the optimal target by maximizing the net revenue through allocation of flexibility, and to minimize the value of holding the flexibility for future use. The valuation is done by the Real Option model as described in Chapter 2. A few probabilistic constraints (described in mathematical formulation below) are included in the upper level to ensure the targets are not unrealistic. Also, the same probabilistic constraints which were used in the upper level are included in the lower level to ensure the outflow solution does not violate those constraints. This will guarantee that lower level results are also optimal to the upper level. Thus, the bi-level optimization is formulated to handle two different purposes of the problem. The upper level strictly focuses on the economic part, whereas the lower level aims at the feasible designs and operations of the problem, i.e. the set of actual reservoir outflows to achieve the desired energy target.

Note:

In the below mathematical formulation,

- Variables having the overscript ~ are uncertain variables.
- Variables without the overscript ~ are deterministic variables.
- Variables having superscript * are optimal solutions.

Upper Level Optimization Problem:

Model Input:

- Demand, D = $[d_1, d_2, \dots \dots \dots \dots, d_{14}]$
- Flexibility each day $\tilde{F}$= $\tilde{E}^{max,1}$ - D = $[\,\tilde{e}_1^{max,1} - d_1, \tilde{e}_2^{max,1} - d_2, \dots \dots ..]$ = $[\tilde{f}_1, \tilde{f}_2 \dots \dots \dots \tilde{f}_{14}]$
- Total Flexibility in 14 day period, $\tilde{F}_{total} = \sum_{t=1}^{14} \tilde{f}_t$
- Price, P

Model Decision Variables:

We propose the decision variable to be the allocated flexibility *H*. Since it is daily-based model, the number of the decision variables is 14 for a single reservoir, given as an array of decision variables, *H*:

$$[h_1, h_2, \dots \dots \dots, h_{14}]$$

Model Objective:

The Flexible-Robust Objective:

Maximize Expected Increase in Net Revenue through allocation:

$$\max_{h_t} \sum_{t=1}^{14}(((\mathrm{p}_t * (\mathrm{h}_t - \tilde{f}_t(\tilde{e}_t^{max,1}, d_t))) * 24) - \widetilde{\mathrm{V}}(\mathrm{h}_t * 24)) \qquad (1)$$

where $\tilde{f}_t$ is the flexibility obtained in period *t* in MWh; p_t is the price/MWh to sell electricity in each day over 14 days period; and h_t is the allocated flexibility in period *t* in MWh. $\widetilde{\mathrm{V}}(\mathrm{h}_t * 24)$ is the value of the allocated flexibility $(\mathrm{h}_t * 24)$ in MW-day. Uncertainty of the value of allocated flexibility is incorporated from actual availability of flexibility in future periods i.e. $\tilde{A}_{t+1}, \tilde{A}_{t+2}$, ... which is a result of inflow uncertainty. Thus during optimization, there will be a comparison between the revenue generated by the allocated flexibility, and the expected loss of dollar value due to the allocated flexibility, i.e., the value of holding or the future value of allocated flexibility.

Subject to:

Model Constraints:

1. *Maximum Allocation of Flexibility (does not require lower level optimization):*

This constraint is applied to validate that allocated flexibility is less than or equal to the actual availability of flexibility each day. It is unrealistic to allocate more flexibility than actually will be available on a given day:

$$\mathrm{h}_t - \tilde{A}_t(\tilde{A}_{t-1}, \mathrm{h}_{t-1}, \tilde{f}_t) \leq \delta \qquad (2, 3)$$

where,

$$\tilde{A}_t = \tilde{A}_{t-1} - \mathrm{h}_{t-1} + \tilde{f}_t,$$

$\tilde{A}_t$ is the actual availability of flexibility after allocating flexibility for t-1 days as per decision made in MWh, and h_{t-1} is the allocated flexibility on Day (t -1) in MWh, $\tilde{f}_t$ is the flexibility obtained in period *t* in MWh, δ is the maximum tolerance of the violation and is set as 1 MWh. We treat equation 2 as a *Reliability Constraint*. Therefore, equation 2 becomes:

$$Pr\{ \mathrm{h}_t - \tilde{A}_t(\tilde{A}_{t-1}, \mathrm{h}_{t-1}, \tilde{f}_t) \ \leq \ \delta \} \geq \ R, \ 0 \leq R \leq 1$$

where *R* is the reliability factor (i.e., the probability of meeting the constraint).

2. Total Flexibility (does not require lower level optimization):

This constraint is applied to validate that the total allocation of flexibility during the optimization period is equal to the maximum total flexibility we get from Stage 1 optimization results. It is unrealistic to allocate more flexibility than is actually available during the optimization period. Also, it is not beneficial economically to BPA if we keep some flexibility without allocating at the end of optimization period. Storing extra water will increase water level in storage and forebay elevation which needs to be at a fixed range at the end of optimization period, as given by:

$$0 \ \leq \ \sum_{t=1}^{14}(\tilde{f}_t(\tilde{e}_t^{max,1}, d_t) \ - \ \sum_{t=1}^{14} \mathrm{h}_t \ \leq \ tol \qquad (4)$$

where tol is maximum allowable deviation and is set as 5% of total flexibility $\sum_{t=1}^{14}\tilde{f}_t$. In future we can further tighten the allowance. We consider equation 4 as a *Reliability Constraint*. Therefore, equation 4 becomes:

$$Pr\{ \ 0 \ \leq \ \sum_{t=1}^{14}(\tilde{f}_t(\tilde{e}_t^{max,1}, d_t) \ - \ \sum_{t=1}^{14} \mathrm{h}_t \ \leq \ tol \ \} \geq \ R, \ 0 \leq R \leq 1$$

where *R* is the reliability factor.

3. Minimum Allocation of Flexibility (does not require lower level optimization):

We assume that there will be sufficient energy generation each period to meet the demands at a minimum: the energy sectors must meet contracted demand. They cannot hold the water for future use without meeting current demand. Therefore, there will not be instances of negative flexibility, as quantified by:

$$\mathrm{h}_t \leq d_t + \delta \qquad (5)$$

where h_t is the allocated flexibility in MWh; d_t is the demand on Day t in MWh; and δ is a very small value set as 1 MWh. We decided to constrain to have a minimum allocated flexibility $\mathrm{h}_t \geq \delta$ (instead of $\mathrm{h}_t \geq 0$) so that in the lower level optimization, we do not encounter any negative achievable allocated flexibility ($\tilde{\grave{\mathrm{h}}}_t < 0$).

4. *Deviation from target allocation (**requires result of lower-level optimization**):*

This constraint is given by:

$$|\,(\mathrm{h}_t - \tilde{\grave{\mathrm{h}}}_t(\tilde{\mathrm{e}}_\mathrm{t}, d_t))\,| \leq \delta \qquad (6), (7)$$

where,

$$\tilde{\grave{\mathrm{h}}}_t = \tilde{\mathrm{e}}_\mathrm{t}(Q_{out,t}, \widetilde{hd}_t(\widetilde{FB}_t, \mathrm{TW_t}), \xi_t) - d_t$$

$\tilde{\grave{\mathrm{h}}}_t$ is the achievable allocated flexibility in MWh closest to the target h_t satisfying all the constraints (Physical, Operational and other Probabilistic constraints) in the Nested Optimization with reliability factor R. δ is the maximum deviation from the target and is set as 1 MWh. We consider equation 6 as a *Reliability Constraint*. Therefore, equation 6 becomes:

$$Pr\{\,|(\mathrm{h}_t - \tilde{\grave{\mathrm{h}}}_t(\tilde{\mathrm{e}}_\mathrm{t}, d_t))\,| \leq \delta\,\} \geq R, \;\; 0 \leq R \leq 1$$

where *R* is the reliability factor.

Below we describe the **Lower Level Optimization** mathematical formulation to achieve $\tilde{\grave{\mathrm{h}}}_t(\tilde{\mathrm{e}}_\mathrm{t}, d_t)$

Lower Level Optimization

Model Inputs:

- Demand, D = $[d_1, d_2, \dots \dots \dots \dots, d_{14}]$
- Maximum energy generation capacity $\tilde{E}^{max,1}$ = $[\,\tilde{e}_1^{max,1}, \tilde{e}_2^{max,1}, \dots\dots, \tilde{e}_{14}^{max,1}]$
- Allocated Flexibility (target), H = $[h_1, h_2, \dots\dots\dots, h_{14}]$

- Price
- $\tilde{Q}_{in}$

Model Decision Variables:

The total outflows of GCL reservoir at each time step are defined as decision variables in the optimization. We have considered an optimization period of 14 days with daily a time-step; therefore, the number of decision variables is 14 for the single reservoir and is given as an array of flow rate, $Q`_{out,Stage2}$, decision variables as follows:

$$[Q_{out,1}\ Q_{out,2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ Q_{out,14}]$$

Model Objective:

Minimize the total deviation from target:

$$\min_{Q_{out,t}} \sum_{t=1}^{14} (\mathrm{e}_{\mathrm{d,t}}(\mathrm{h_t}, \mathrm{d_t}) - \tilde{\mathrm{e}}_{\mathrm{t}}(\mathrm{Q}_{\mathrm{out,t}}, \widetilde{\mathrm{hd}}_{\mathrm{t}}(\widetilde{\mathrm{FB}}_{\mathrm{t}}, \mathrm{TW_t}), \xi_t))^2 \qquad (8, 9, 10)$$

Where,

$$\mathrm{e}_{\mathrm{d,t}} = \mathrm{h_t} + \mathrm{d_t} \ ;$$

$$\tilde{\mathrm{e}}_{\mathrm{t}} = \eta * 9.81 * \widetilde{hd}_t(\widetilde{FB}_t, \mathrm{TW_t}) * Q_{out,t} * 8.6310 * 10^{-3} * \xi_t$$

In this equation, $\mathrm{e}_{\mathrm{d,t}}$ is the target energy in MWh should be generated to meet the target allocation of flexibility on day t, $\tilde{\mathrm{e}}_{\mathrm{t}}$ is the actual energy generated in MWh on day t, $\mathrm{h_t}$ is the target allocation of flexibility in MWh, $\mathrm{d_t}$ is the demand on day t in MWh, η is the efficiency of the reservoir, taken as 0.75, $\widetilde{hd}_t$ is the head in ft, and ξ_t is taken as 1 hour.

Subject to:

Model Constraints:

a. Water Balance Constraints

$$0 \le \tilde{S}_t\ (\tilde{Q}_{in,t}, \tilde{Q}_{in,t-1}, Q_{out,t}, Q_{out,t-1}, \mathrm{delt_t}) \le S_{max} \qquad (11, 12)$$

Where,

$$\tilde{S}_{t+1} = ((\tilde{Q}_{in,t} + \tilde{Q}_{in,t+1})/2 - (Q_{out,t} + Q_{out,t+1})/2).\,\mathrm{delt_t} + \tilde{S}_t$$

In these equations, $\tilde{S}_t$ is reservoir storage in kcfs-day. S_{max} is the maximum storage capacity, $\tilde{Q}_{in}$ and Q_{out} are inflow and outflow to reservoir in kcfs, respectively, and $\mathrm{delt_t}$ is time (day) between each time step. At this stage, the water leakage and natural water loss is not considered. We consider equation 11 as a *Reliability Constraint*. Therefore, equation 11 becomes:

$$Pr\{0 \leq \tilde{S}_t\,(\tilde{Q}_{in,t}, \tilde{Q}_{in,t-1}, Q_{out,t}, Q_{out,t-1}, \mathrm{delt_t}) \leq S_{max}\} \geq R, \quad 0 \leq R \leq 1$$

where *R* is the reliability factor.

b. Reservoir Water Surface Elevation (WSE) Constraints

$$FB_{min} \leq \widetilde{FB}_t(\tilde{S}_t) \leq FB_{max} \qquad (13, 14)$$

Where,

$$\widetilde{FB}_t = c_1 * (\tilde{S}_t)^2 + c_2 * (\tilde{S}_t) + c_3$$

where $\widetilde{FB}_t$ is the reservoir water level in ft at time *t*; FB_{min} and FB_{max} are the allowable minimum and maximum reservoir water elevation respectively; c_1 = $-3.63*10^{-6}$, c_2 = 0.0406 and c_3 =1208. The constants are determined by fitting actual forebay elevation observations with a polynomial regression model.

c. *Turbine Flow Constraints*

$$Q_{tb-min} \leq Q_{tb,t} \leq Q_{tb-max} \quad (15)$$

In this constraint, $Q_{tb,t}$ is turbine flow for power generation in kcfs at each time step and Q_{tb-min} and Q_{tb-max} are allowed minimum and maximum discharge respectively.

Since we are not ignoring spill flow, Turbine flow and Outflow will be same. Therefore, we can re-write equation 15 as below:

$$Q_{tb-min} \le Q_{out,t} \le Q_{tb-max} \quad (16)$$

d. *Output Constraints*

$$N_{d-min} \le \widetilde{N}_{d,t}(Q_{out,t}, \widetilde{hd}_t) \le N_{d-max} \qquad (17, 18)$$

Where,

$$\widetilde{N}_{d,t} = \eta * 9.81 * \widetilde{hd}_t(\widetilde{FB}_t, \mathrm{TW_t}) * Q_{out,t} * 8.6310 * 10^{-3}$$

In the output constraint, $\widetilde{N}_{d,t}$ is power output in MW at time *t*. N_{d-min} and N_{d-max} are the minimum and maximum output capacity respectively.

e. *Reservoir Water Surface Elevation (WSE) Constraints on the end-of-period*

The optimization is conducted for 14 days, which is a relatively short term for the reservoir operations. To be consistent with middle-term or long-term operation, the water surface elevation (WSE) in the reservoir at the end of optimization period is expected to stay within a target WSE to fulfill future requirements. In the example problem we have formulated, the historical data from the actual operation scheme is used as the target WSE for the optimization model. To avoid equality constraints, a small range on the target WSE is used to restrain the WSE on the end-of-period to be close to the target WSE:

$$FB_{tar,end} - \Delta \le \widetilde{FB}_t(\tilde{V}_t) \le FB_{tar,end} + \Delta \quad (19)$$

where $FB_{tar,end}$ is the target WSE on the end-of-period and Δ is the deviation from the target WSE. The Δ is set as 1% in the model and $FB_{tar,end}$ is taken as 1280 ft.

f. Maximum Allocation of Energy

Though a similar type of constraint (2) has been applied in the main problem, we still need to place this constraint on the lower level to ensure that the deviated solution from the target still satisfies the constraint:

$$\tilde{\mathrm{e}}_{\mathrm{t}}(\mathrm{Q}_{\mathrm{out,t}}, \widetilde{\mathrm{hd}}_{\mathrm{t}}, \xi_t) - \tilde{E}_{A,t}(\tilde{E}_{A,t-1}, \tilde{\mathrm{e}}_{\mathrm{t-1}}, \tilde{e}_t^{max,1}) \le \delta \qquad (20, 21)$$

Where,

$$\tilde{E}_{A,t} = \tilde{E}_{A,t-1} - \tilde{\mathrm{e}}_{\mathrm{t-1}} + \tilde{e}_t^{max,1}$$

$\tilde{E}_{A,t}$ is the actual availability of energy in MWh at day t after allocating achievable flexibility for t-1 days obtained from lower level optimization, $\tilde{\mathrm{e}}_{\mathrm{t-1}}$ is the energy generated in MWh on Day (t-1), $\tilde{e}_t^{max,1}$ is the maximum energy that can be generated in MWh on day t, δ is the maximum tolerance for violation and is set as 1 MWh. We consider equation 20 as a *Reliability Constraint*. Therefore, equation 20 becomes:

$$Pr\{ \tilde{\mathrm{e}}_{\mathrm{t}}(\mathrm{Q}_{\mathrm{out,t}}, \widetilde{\mathrm{hd}}_{\mathrm{t}}, \xi_t) - \tilde{E}_{A,t}(\tilde{E}_{A,t-1}, \tilde{\mathrm{e}}_{\mathrm{t-1}}, \tilde{e}_t^{max,1}) \le \delta\} \ge R, \ 0 \le R \le 1$$

where *R* is the reliability factor.

g. Total Energy during the Optimization period

$$0 \le \sum_{t=1}^{14} \mathrm{e}_{\mathrm{d,t}}(\mathrm{h}_{\mathrm{t}}, \mathrm{d}_{\mathrm{t}}) - \sum_{t=1}^{14} \tilde{\mathrm{e}}_{\mathrm{t}}(\mathrm{Q}_{\mathrm{out,t}}, \widetilde{\mathrm{hd}}_{\mathrm{t}}, \xi_t) \le tol \qquad (22)$$

tol is maximum allowable deviation and is set as 0.2% of total energy $\sum_{t=1}^{14} \mathrm{e}_{\mathrm{d,t}}$. We consider equation 22 as a *Reliability Constraint*. Therefore, equation 22 becomes:

$$Pr\{ 0 \le \sum_{t=1}^{14} \mathrm{e}_{\mathrm{d,t}}(\mathrm{h}_{\mathrm{t}}, \mathrm{d}_{\mathrm{t}}) - \sum_{t=1}^{14} \tilde{\mathrm{e}}_{\mathrm{t}}(\mathrm{Q}_{\mathrm{out,t}}, \widetilde{\mathrm{hd}}_{\mathrm{t}}, \xi_t) \le tol \} \ge R, \ 0 \le R \le 1$$

where *R* is the reliability factor.

h. Deviation from target:

$$|\ e_{d,t}(h_t, d_t) - \tilde{e}_t(Q_{out,t}, \widetilde{hd}_t, \xi_t)|\ \leq\ \delta \qquad (23)$$

where $e_{d,t}$ is the target energy in MWh should be generated to meet the target allocation of flexibility on day t, $\tilde{e}_t$ is the actual energy generated in MWh on day t, δ is the maximum deviation from the target and is set as 1 MWh. We consider equation 23 as a *Reliability Constraint*. Therefore, equation 23 becomes:

$$Pr\{\ |\ e_{d,t}(h_t, d_t) - \tilde{e}_t(Q_{out,t}, \widetilde{hd}_t, \xi_t)|\ \leq\ \delta\ \} \geq\ R,\ \ 0 \leq R \leq 1$$

where *R* is the reliability factor.

***Lower Level Optimization*: Model Output**:

- Optimal Outflow corresponding to achievable allocation of flexibility, $Q^{`*}_{out,Stage2}$
- Deviation from target allocated flexibility, ε_h = $[(\ e_{d,1} - \tilde{e}_1)\ , (\ e_{d,2} - \tilde{e}_2)\ , \dots \dots ., (\ e_{d,14} - \tilde{e}_{14})]$ = $[(h_1 - \widetilde{h`}_1), (h_2 - \widetilde{h`}_2), \dots \dots \dots, (h_{14} - \widetilde{h`}_{14})\ ]$; (from equation 15)
- Achievable allocated flexibility $\widetilde{H`}$ closest to the target allocation H, $\widetilde{H`}$ = $[\widetilde{h`}_1, \widetilde{h`}_2, \dots\dots \widetilde{h`}_{14}]$

***Upper Level Optimization*: Model Output**:

- Optimal Outflow corresponding to achievable allocation of flexibility, $Q^{`*}_{out,Stage2}$
- Achievable allocated flexibility $\widetilde{H`}$ closest to the optimal target allocation H*; $\widetilde{H`}$ = $[\widetilde{h`}_1, \widetilde{h`}_2, \dots\dots \widetilde{h`}_{14}]$
- Maximum Net Revenue in selling the Flexibility, $\tilde{R}^{max}$

4. SOLUTION METHODOLOGY

Figure 1 shows the complete structure of our proposed model. In this section, we will discuss in detail the methodology used and the convergence criteria of the model. Also, we will talk about some compatibility issues raised during the formulation of the model and how they have been solved. The methodology and the convergence criteria for the Stage 1 model has already been discussed in Chapter 2. In this paper, we will focus on the methodology of Stage 2 for the model which uses bi-level programming.

Stage2 (Upper Level)

We have applied Robust Optimization for the Stage 2 upper level optimization, converting the objective function (Eq.1) into robust objective and handled all the Constraints (Eq -2, 4,6) as Probabilistic Constraints. For the risk aversion coefficient, we used a value of 10e5 (this parameter can be set by the user). Another limitation of the RO analysis is the assumption of no uncertainty in the current state. Therefore, in the objective function (Eq. 1), to evaluate the value of allocated flexibility in the current period, the only uncertainty considered is for the available flexibility in the future periods i.e. $\tilde{A}_{t+1}, \tilde{A}_{t+2}, \ldots.$ No uncertainty has been considered for the available flexibility in the current period i.e $\tilde{A}_t$. However, there is uncertainty in the current period and, therefore, we handle the uncertainty of the current period outside the RO model by converting Eq. 1 into a Robust Objective function. While the RO model evaluates the future risks based on the uncertainty of the future available flexibility, uncertainty on current available flexibility is quantified using a Full Tensor Numerical Integration (FTNI) method. The Probabilistic constraints are handled by Inverse Reliability Method and the Reliability factor can be set by the user, such as 50% or 99 %. We applied Sequential Quadratic Programming (SQP) to solve the optimization problem. The convergence criteria are step size tolerance, constraint violation tolerance, function tolerance, which are all set as $\xi = 10^{-5}$, and maximum function evaluations set to 40000. Though Bi-level programming has been criticized for being computationally expensive and hard to solve, particularly using classical methods due to difficulties such as non-linearity, discreteness, non-differentiability, non-convexity etc. [36]. However, SQP is probably one of the best tools for solving non-linear programming model using classical methods. It has been a hot topic over the years for researchers to overcome this drawback, and an evolutionary algorithm has been attempted as a solution [37]. Therefore, we will solve our problem using a Genetic Algorithm as well and compare the results in terms of computational cost and accuracy of optimal solutions.

Stage2 (Lower Level)

We have handled the uncertainty of the objective function by quantifying it using a Gaussian Quadrature method and converting into a robust objective and we have handled the Constraints (Eq -11, 20, 22, 23) as Probabilistic constraints. The algorithm used for Stage 2 lower level optimization is Sequential Quadratic Programing (SQP). The convergence criteria are step size tolerance, constraint violation tolerance, function tolerance; all set as $\xi = 10^{-5}$ and maximum function evaluations set as 40000. We have also set $\xi = 10^{-8}$ to see if we get better results; that is, less deviation of achievable allocated flexibility from the specified target allocated flexibility. The objective function at the optimal point observed in both the cases is similar; however, when $\xi = 10^{-8}$, the number of iterations and function evaluations are 46 and 841, respectively, whereas for $\xi = 10^{-5}$, they are 37 and 701, respectively. Thus, by tightening the convergence criteria we get the same result with increased computational cost which is not desirable. Therefore, we have used $\xi = 10^{-5}$.

Techniques used for reducing the computational cost

As bi-level programming is generally computationally expensive, we have used some simplifications in the mathematical formulation to provide a faster solution. Since we set a target in the upper level, we want to know in the lower level how close we can get to the set target without violating any physical and operational constraints. Therefore, we can reduce our design space close to the set target, with some allowance. Thus, we use constraint Eq. 23. This constraint will reduce the design space as it bounds the feasible region close to the target, thereby helping to find the optimal feasible solution quickly (if there is one) by exploiting the advantage of classical methods like SQP, reducing the computational cost. Also, this constraint avoids the chances of local convergence far from the target. As we are using SQP, which is fast but does not guarantee global convergence (only guaranteed to find a local optima), this constraint proved significantly useful to provide a fast and accurate solution. However, we are doing further research to make the model more computationally efficient, and this is one of our future works for the project.

Challenges in the integration of two levels

In the Stage 2 upper level optimization, the allocated flexibility $\mathrm{h_t}$ is a decision variable and is a deterministic value. This allocated flexibility is set as the *target* when we pass this value as input in the lower level optimization. In the lower level optimization, the *achievable* allocated flexibility closest to the target value $\mathrm{h_t}$ is an uncertain variable, $\widetilde{\mathrm{h}`}_t$. The uncertainty in $\widetilde{\mathrm{h}`}_t$actually comes from the uncertainty in inflows $\tilde{Q}_{in}$. Inflows create uncertainty in Storage $\tilde{S}_t$, Forebay $\widetilde{FB}_t$, Head $\widetilde{\mathrm{hd}}_\mathrm{t}$, energy $\tilde{\mathrm{e}}_\mathrm{t}$ and finally in $\widetilde{\mathrm{h}`}_t$ which is the function of $\tilde{\mathrm{e}}_\mathrm{t}$. Due to the uncertainty, we cannot treat $\widetilde{\mathrm{h}`}_t$ as a decision variable and use this achievable flexibility to calculate the objective function in the upper level. Also, this will again highlight the limitations RO analysis which has been discussed previously in the problem description. Therefore, we use the target value $\mathrm{h_t}$ as the decision variable in the upper level, and $\widetilde{\mathrm{h}`}_t$ is the achievable flexibility in the lower level. This raises another issue in that $\widetilde{\mathrm{h}`}_t$ ensures a feasible solution in terms of operations of reservoir, not $\mathrm{h_t}$. So how can we solve this compatibility issue in the integration of two levels?

In answer to the above issues, we have provided a solution to ensure the mathematical model is conceptually correct. We have applied a probabilistic constraint (Eq. 6) to bind the maximum allowable deviation of the feasible solution from the target solution by a small value δ, so that the deviated solution will have negligible effect on the net revenue. Anything beyond that small deviation with less than the desired Reliability factor will be an infeasible solution. Thus, we can still use the target value as the decision variable in the upper level optimization to calculate the objective function, as the deviated value will meet the target at a desired reliability level (e.g. 95%, 99%). We will show in our result that the achievable allocated flexibility, $\widetilde{\mathrm{h}`}_t$ has negligible effect in the Real Option valuation and also on Net Revenue when compared with its respective target allocated flexibility, $\mathrm{h_t}$. We need to validate this new probabilistic constraint for every iteration in the upper level instead of only for the optimal target allocated flexibility. This is because the achievable feasible solution closest to the optimal target may be beyond the maximum allowed deviation δ. Thus the deviated solution will affect the net revenue significantly as this is too far away from the target and no longer guarantee san optimal solution in terms of revenue. Therefore, we will lose the actual optimal solution which maximizes the revenue ensuring feasibility. Therefore, in this approach, an optimal target of allocated flexibility from upper level optimization will help us to provide an optimal feasible allocated flexibility very close to target (≤ δ) and its respective optimal outflow in the lower level optimization.

5. RESULTS FOR THE SINGLE RESERVOIR MODEL

In this section, we will show the results of the **Two-Stage Bi-Level Flexible-Robust Optimization** model having daily time steps for the 14 day optimization period. In this paper, we will show the results for a Single Reservoir, Grand Coulee. We will compare the results of our Model using non-linear constrained gradient algorithm, SQP, and using an evolutionary algorithm, a GA.

Figure 2 shows the optimal decisions for achievable allocation of flexibility using SQP, as denoted by the green line. The blue lines in the figure are the flexibility after Stage 1 and the dashed red line is the maximum availability flexibility (cumulative) at each day, which can be *allocated* or *held* for future use. In our example problem, we have considered the highest price to be on Day 8. We assume the uncertainty increases each day; therefore, the highest uncertainty is on Day 14. Based on these assumptions, it is reasonable to have most of the allocation on Day 8 as this is the highest selling price, and on Day 14 as this is the highest uncertainty and is most highly valued by the Real Option Model. Figure 3 shows the deviation of the achievable allocated flexibility obtained in the lower level from the optimal target allocated flexibility in the upper level. We can see the maximum deviation is 0.65 MWh from the target, which is very close. The changes in the Real Option value and the Net Revenue due to the difference between the optimal target allocated flexibility and the respective achievable allocated flexibility is shown in Table 1. We can see there is a slight decrease in Net Revenue ($1577) and RO value ($573), respectively, in the achievable solution. This is because the achievable solution provided a robust solution, thereby, not allocating the entire total flexibility at the end of the optimization period. On the other hand, the target solution is deterministic and uses all the flexibility as the end of optimization period; however, we can see the difference ($1577) is very negligible to BPA or any large energy sectors as they deal in millions of dollars. Therefore, it holds to our assumption that the slight deviation from target allocated flexibility does not affect the optimal solution. Table 2 shows the comparison of the optimization result using SQP and GA. We can clearly see the number of function evaluation for SQP is much lower than that for GA. Even with so many function evaluations, the GA provides much a worse result than the SQP result. Therefore, we confirm that SQP is the best choice in this problem, even considering the bi-level programming. Figure 4 shows the optimal outflows that will provide the optimal allocation of flexibility as shown in Figure 2. Figure 2 provides an in-depth understanding of the model and the significance of Flexible Robust Objective by integrating Real Option model; however, Figure 4 will be useful for the operators as they require the actual water release information for each reservoir to obtain the Maximum Net Revenue provided by the bi-level programming. Also, unlike in the simplified model in the previous chapter, the bi-level programing in Stage 2 of the proposed model of this paper took care of the operational constraints of the complex river system, and provided an optimal robust solution. Also, this model is not restricted to only discrete decision variables in Stage 2 and considers continuous decision variables in both the levels of Stage 2. Table 3 shows the significant improvement (8 %) in the solutions when compared the results with our previous chapter. This improvement is likely due to gaining more design points due to continuous decision variables. This shows the current model is not only able to handle the constraints better using bi-level programming, but is also highly efficient in providing better solutions.

Note: In the Tables, the ***Net Revenue*** is the Net revenue achieved from selling the flexibility only. The ***Total Net Revenue*** is the Net revenue achieved from selling the flexibility and the demand.

6. CONCLUSION

An approach to solve the challenges in the integration of Robust optimization with the Real Option Model and to introduce more practical constraints of reservoirs (operational and physical constraints) has been presented; we have adopted a bi-level programming technique and proposed a **Two-Stage Bi-Level Flexible-Robust Optimization** model. The proposed model proved more efficient than the simplified version of Chapter 2. Although Bi-level optimization is computationally expensive in general, we have made this model efficient using good mathematical formulation and inheriting the computational efficiency of one of the best tools for classical methods, SQP. However, the problem solved is a simplified solution and if will be considered as a future scope to increase the computational efficiency of the model. Though evolutionary algorithms are encouraged over SQP for Bi-level optimization in the literature, we have shown that SQP is more efficient in this problem, in terms of providing faster and better solutions. The GA fails is inferior to SQP in terms of speed and solution quality. As future scope, we will consider more accurate distribution of uncertainties and incorporate price and demand uncertainties. Also, we will consider in future to run the model for the entire Lower Columbia River system (multi-reservoir system) instead of the single reservoir, Grand Coulee.

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TABLES OF CHAPTER 3

Optimal Decision			
Optimal Target (Upper Level D.V)		***Achievable (Lower Level Solution)***	
RO Value $	***Net Revenue $ (A)***	***RO Value $***	***Net Revenue $ (B)***
$ 10591	$ 673325	$ 10018	$ 671748
Difference in Net Revenue $ (A – B)		***Percentage % ((A – B) * 100) / A***	
$ 1577		0.2 %	

Table 1. Comparison of Real Option Value and Net Revenue between target (Upper Level) and Achievable (Lower Level) optimal allocation of Flexibility

Algorithm	***Fun Eval***	***Net Revenue $***	***Total Net Revenue $***
SQP	405	$ 671748	$ 13103244
GA	36296	$ 308084	$ 12739580

Table 2. Comparison of Optimization Results using SQP and GA algorithm

	Net Revenue $	***Increase in Net Revenue $***	***Percentage of Increase***
Proposed Model (Bi-Level Programming)	$ 671748	$ 48790	8 %
Simplified Model (Chapter 2)	$ 622958		

Table 3. Comparison of Optimization Results between Previous Model (Chapter 2) and Proposed Model (Chapter 3)

FIGURES OF CHAPTER 3

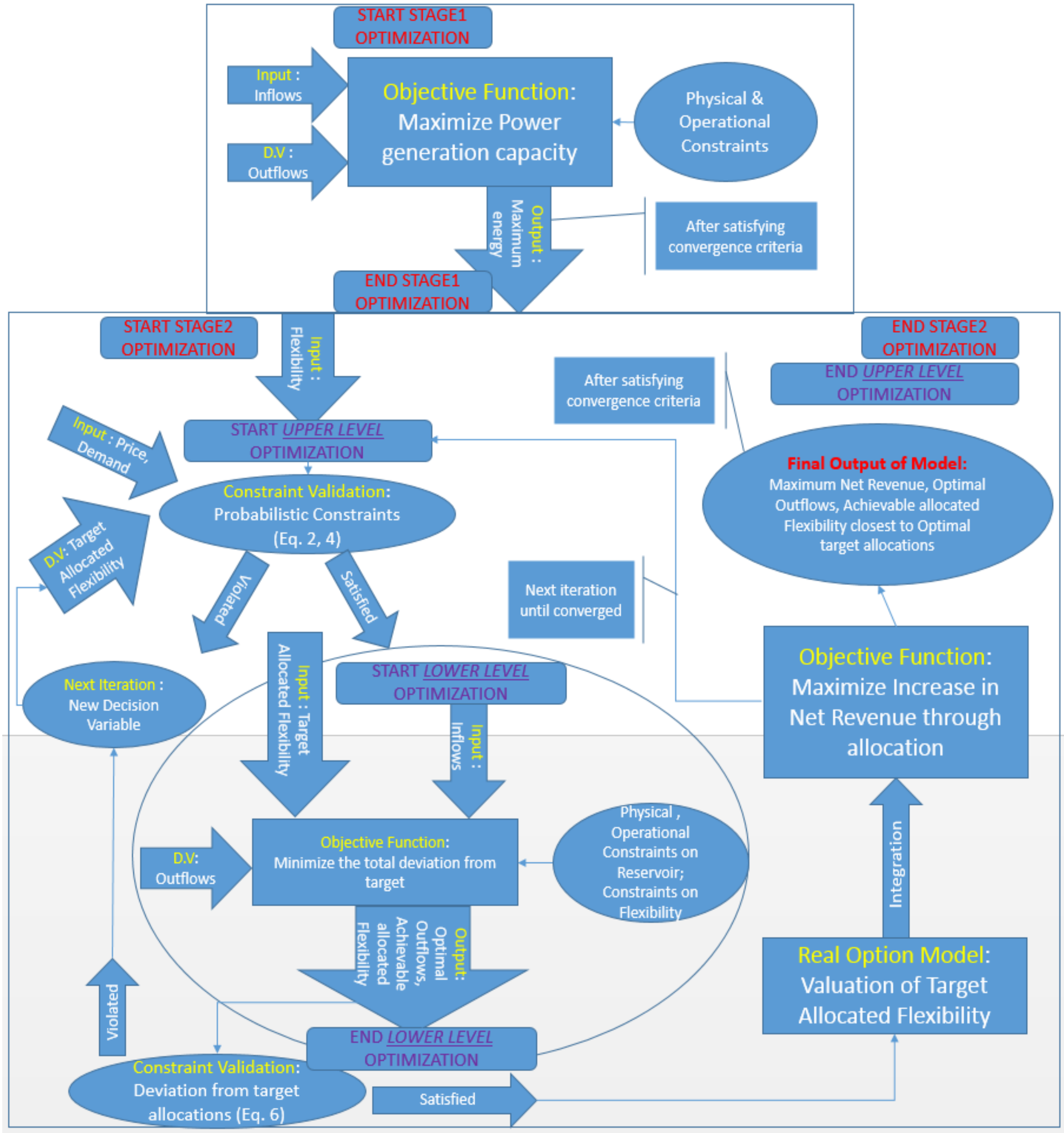

Figure 1. Bi-Level Flexible Robust Optimization Approach

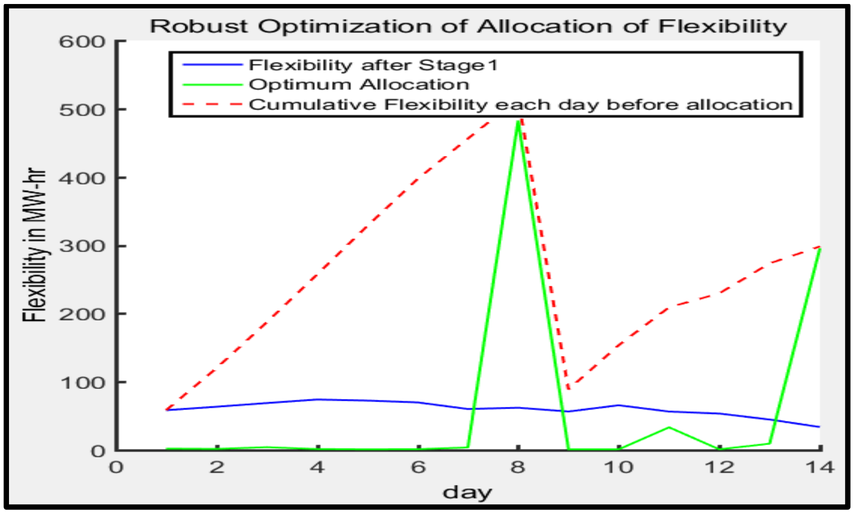

Figure 2. Optimal Decision of Achievable Allocated Flexibility

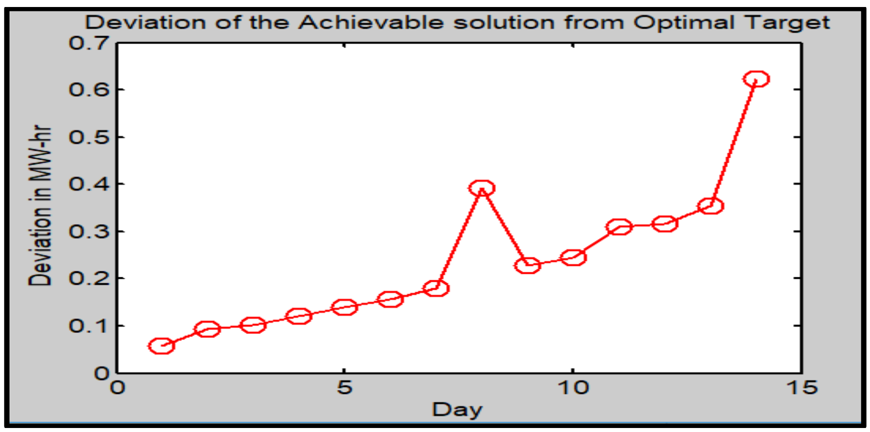

Figure 3. Deviation of Optimal Achievable Allocated Flexibility (Lower Level) from the Optimal Target (Upper Level)

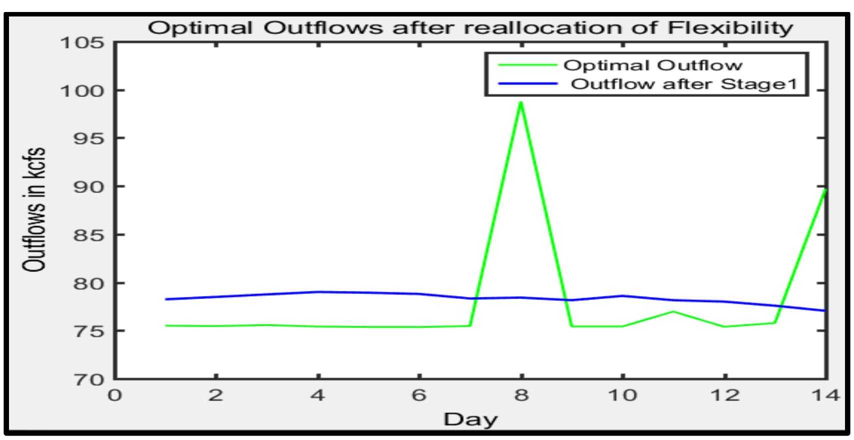

Figure 4. Optimal Outflows to provide optimal allocation of flexibility

CHAPTER 4
GENERAL CONCLUSION

A Multilevel Flexible Robust Optimization method has been presented to optimize the allocation of flexibility by maximizing revenue and minimizing the value of holding the flexibility, ensuring the solution also minimizes physical and operational violation of the reservoir. Chapter 2 shows a better optimal solution by integration of Real Option analysis with the Robust Optimization framework. Chapter 3 successfully eliminated the limitations of the simplified model of Chapter 2 by introducing Bi-level Programing, where the Upper Level focuses on maximizing net revenue and the lower level ensures the feasibility of the target solution in each iteration. There were some compatibility issues and limitations identified with the introduction of Bi-Level programming in handling the uncertain parameters. However, the challenges have been overcome successfully and the evidence has been provided for the justification of any assumptions made. Though the approach has been inplemented on single reservoir, the mathamatical formulation is not restricted to a single reservoir and can be applicable to the entire river system. This is a future scope of the research: to test the model on multi-reservoir systems. Due to the large complexity of the system, the computational cost of the model will eventually increase to increase the accuracy of result. In energy allocation problems, as the operators are requied to run the model in a daily or hourly basis due to the changes in the input data over time, they need a model result in limited time period. Therefore it is likely for them to place more importance on the computational time rather than accurac, as a few hundred or thousand dollars of loss will not impact much in their million dollar net revenue. Therefore, future research will be focussed on reducing the computational cost of the model while enuring the loss in accuracy does not provide a significant negative impact on the revenue obtained for the energy sector.

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