Journal of the

OPTICAL SOCIETY of AMERICA

VOLUME 59, NUMBER 4 APRIL 1969

Theoretical Dependence of the Near-Asymptotic Apparent Optical Properties on the Inherent Optical Properties of Sea Water*

GEORGE F. BEARDSLEY, JR., AND J. RONALD V. ZANEVELD
Department of Oceanography, Oregon State University, Corvallis, Oregon 97331
(Received 6 June 1968)

The apparent optical properties of sea water are a set of measurables that describe the geometry of the submarine light field. These properties are related to the inherent optical properties—the volume attenuation coefficient and the volume scattering function—through the process of radiative transfer. A numerical approximation to the equation of radiative transfer was programmed for solution on a high-speed digital computer. The technique is based on resolving the hydrosol into a system of thin slabs, finding approximate transmission and reflection operators for each slab, and then systematically applying these operators to determine the geometry of the light field in the interior of the system of slabs. Thus each set of calculations produces the radiance solid as a function of depth. The apparent optical properties were found by numerical integration of the radiance. A simple three-parameter model was used to simplify study of the dependence of the apparent optical properties on the directional characteristics of the volume scattering function.
INDEX HEADINGS: Oceanography; Scattering; Water; Modulation transfer; Image formation; Spread function; Luminance.

One systematic means of describing the geometry of a monochromatic unpolarized radiance field has been described by Gershun[1]; his results were extended by Preisendorfer[2] for the case of natural light in sea water. Central to Preisendorfer's results are a set of measurables termed the apparent optical properties. Two of these apparent optical properties, the vector and scalar irradiances, are directly related to the total energy carried by the underwater light field, while the remainder are derived properties characterizing the rate of decay of the irradiances, the diffuse reflectivity of the hydrosol, and the angular spread of the sinking and rising radiance fields. It has been shown[3] that these derived apparent optical properties asymptotically approach constant values with increasing depth in homogeneous hydrosols, regardless of the geometry or strength of the radiance field at the surface of the hydrosol. Since the inherent optical properties (IOP) of the hydrosol must be the only factors determining the values of the near-asymptotic derived apparent optical properties (AOP), the possibility exists that measurements of these apparent properties can be used to determine the inherent optical properties of the water column and thus to contribute to the study of water masses, currents, and biological processes. In this paper, we present a set of numerically obtained curves which approximate the theoretical relations between the near-asymptotic values of the derived apparent optical properties and some typical values of the inherent optical properties. Knowledge of these relations should provide information on the utility of measurements of the apparent properties in physical oceanography as well as allow the computation of the near-asymptotic irradiance when the inherent optical properties are known.

THEORY

The theory of radiative transfer is well established; ample discussions are provided in a number of texts.[4,5] The difficulty in the type of problem under consideration is the development of an accurate yet economical method of determining a large number of approximations to the radiance field in the interior of the region

* Work was supported by Office of Naval Research Contract Nonr 1286 (10) Project No. NR 083-102.

[1] A. Gershun, J. Math. Phys. 18, 51 (1939).
[2] R. W. Preisendorfer, *Radiative Transfer on Discrete Spaces* (Pergamon Press (Ltd.), Oxford, 1965).
[3] R. W. Preisendorfer, J. Marine Res. 18, 1 (1959).
[4] S. Chandrasekhar, *Radiative Transfer* (Clarendon Press, Oxford, 1950).
[5] G. M. Wing, *An Introduction to Transport Theory* (John Wiley & Sons, Inc., New York, 1962).

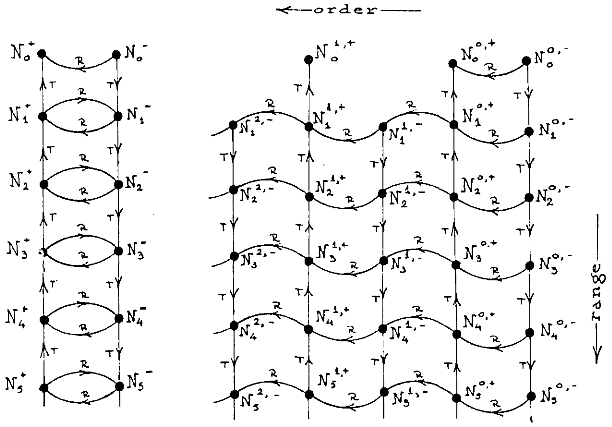

FIG. 1. Flow graphs for the calculation of the radiance field. T and R are the transmission and reflection operators.

being investigated. One technique which appears to meet these specifications is based on invariant bedding and is an extension of a method described earlier by one of us.[6] Since some modifications were made in the method in order to improve the accuracy and decrease the computation time, a short summary of the modified method follows.

The sea water was first considered as a set of thin horizontal plane parallel slabs of equal thickness, as shown on the left side of Fig. 1. This set of slabs is conceptually replaced by a flow graph, as shown on the right side of Fig. 1, with the nodes representing radiance distributions and the paths operators. Inspection of the flow graph yields a pair of equations for the rising (+) and sinking (−) radiances at the ith level,

$$N_i^+ = T[N_{i+1}^+] + R[N_i^-] \quad (1)$$

$$N_i^- = T[N_{i-1}^-] + R[N_i^+]. \quad (2)$$

The incoming surface radiance $N_0(-)$ is considered known, and the operators $T[\cdot]$ and $R[\cdot]$ are fixed by the inherent optical properties and the slab thickness.

Determination of the simultaneous solutions to Eqs. (1) and (2) is a formidable task. Instead, we make use of the rapidly converging solution based on the expanded flow graph shown in Fig. 1(b). Inspection of the flow graph shows that the components of the radiance at any depth and order can be expressed in terms of the radiances of equal or lower order. The total radiance field at any depth i is the sum of all of the orders denoted by the superscript j,

$$N_i^{\pm} = \sum_{j=1}^{\infty} N_i^{j,\pm}. \quad (3)$$

Of particular significance is the fact that an equation for any N_i^j can be written that does not require that particular N_i^j to be known either explicitly or implicitly. Thus it is possible to compute each radiance component by proceeding through the flow chart in a systematic way.

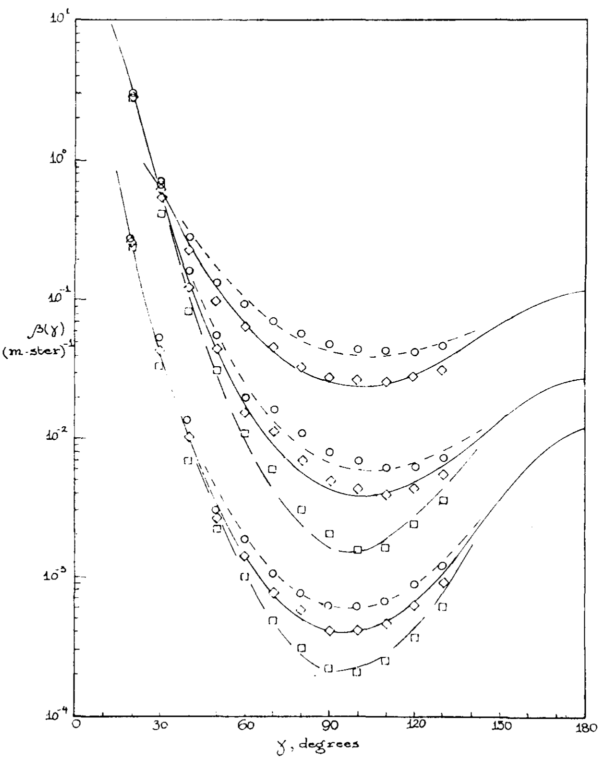

FIG. 2. Some measured scattering functions fitted by the empirical Eq. (4).

Natural hydrosols are characterized by nonzero volume absorption and highly directional scattering. The ever-present absorption requires that the transmission and reflection operators for slabs of finite thickness will always be less than one. Thus the higher-order and deeper radiance components will always be smaller than the low-order components which contributed to them, and all of the partial sums in the series given by Eq. (3) will monotonically approach the limiting value. In practice, we find that a small number of orders is sufficient to obtain a reasonable approximation to the limit.

The transmission and reflection operators for a slab are easily found if the slab is so thin that the likelihood of multiple scattering within the slab is negligible. We took this approach, using the method outlined by Wing.[5] The scattering process was approximated by a scalar model that ignores the partial polarization of submarine

TABLE I. Some observed values of the scattering albedo b/c.

Location	Date	Depth	b/c	Observer
41° N, 70° 55′ W	5/5/66	0 m	0.293	Beardsley
39° 55′ N, 70° 38′ W	4/27/66	0 m	0.258	Beardsley
34° 56.5′ N, 66° 29.5′ W	4/30/66	0 m	0.202	Beardsley
Pend Oreille Lake, Idaho	3/16/57	~30 m	0.697	Tyler, 1960
	4/28/57	~30 m	0.715	Tyler, 1960

[6] G. F. Beardsley, Jr., J. Opt. Soc. Am. 58, 52 (1968).

TABLE II. Derived apparent optical properties as functions of the inherent optical properties at the near-asymptotic depth of 6.5 optical thicknesses.

b/c	0.8	0.8	0.8	0.8	0.2	0.2	0.2	0.2	0.05	0.05	0.05	0.05
ef	0.95	0.95	0.65	0.65	0.95	0.95	0.65	0.65	0.95	0.95	0.65	0.65
eb	0.70	0.30	0.70	0.30	0.70	0.30	0.70	0.30	0.70	0.30	0.70	0.30
F	0.9944	0.99953	0.395	0.881	0.9944	0.99953	0.395	0.881	0.9944	0.99953	0.395	0.881
B	0.0056	0.00047	0.605	0.112	0.0056	0.00047	0.605	0.119	0.0056	0.00047	0.605	0.112
R, %	0.56	0.10	26.6	6.8	0.06	0.005	5.9	1.03	0.013	0.001	1.43	0.223
$D(-)$	1.31	1.31	1.30	1.39	1.16	1.15	1.16	1.20	1.12	1.12	1.12	1.13
$D(+)$	2.52	6.67	1.70	2.61	1.91	4.90	1.58	2.65	1.75	3.83	1.54	2.61
$K(-)/c$	0.316	0.308	0.974	0.671	0.924	0.92	1.13	1.04	1.11	1.11	1.16	1.14
$K(+)/c$	0.372	0.383	0.976	0.678	0.934	1.00	1.13	1.06	1.11	1.17	1.16	1.16
k/c	0.327	0.319	0.980	0.674	0.938	0.935	1.14	1.06	1.12	1.12	1.17	1.15
$\dfrac{a_{calc}}{c}$	0.241	0.228	0.405	0.398	0.797	0.800	0.848	0.880	0.991	0.991	1.00	0.984
$\dfrac{a_{calc}}{a_{true}}$	1.20	1.14	2.02	1.99	0.996	1.00	1.06	1.10	1.04	1.04	1.05	1.03

daylight and introduces an error of a few percent at most.

CALCULATIONS

The general method of the calculations follows that outlined by Beardsley in an earlier paper.[6] The angular resolution of the elevation was increased from 18° to 9°. The angular resolution of the azimuth was 20°.

The general scattering curve found in the oceans is strongly peaked in the forward direction; it has a broad minimum near $\gamma=90°$, and rises again slightly in the backscattering region. A curve of this type can be fitted nicely with the function

$$\beta(\gamma)=\beta_0/[(1-ef\cos\gamma)^4(1+eb\cos\gamma)^4]. \quad (4)$$

The three parameters β_0, ef, and eb have major effects on the magnitude, forward lobe, and backward lobe respectively, but are not completely independent. Figure 2 shows some fits of this empirical function to measured scattering functions.

In our calculations, the beam extinction coefficient c was set equal to 1.0, so that the scattering albedo b/c is numerically equal to b. The slab thickness was taken to be 0.5, in units of $1/c$.

In order to determine the general behavior of the AOP we made calculations by using a wide range of values for the IOP. The calculations were for scattering albedoes equal to 0.05, 0.2, and 0.8. Four shapes of the scattering function were used, corresponding to ef's of 0.95 and 0.65 and eb's of 0.3 and 0.7.

TABLE III. Comparison of computed and measured[2,7] values of apparent optical properties.

	Computed	Measured
$K(+)$, m^{-1}	0.275	0.180
$K(-)$, m^{-1}	0.275	...
k, m^{-1}	0.275	...
$D(+)$	1.26	1.34
$D(-)$	3.17	2.79
R, %	0.80	2.80

Since the values of the near-asymptotic AOP are independent of the incident radiance field,[3] the incident radiance field was taken to be uniform.

The AOP calculated were the $K(+)$, $K(-)$, k, $D(+)$, and $D(-)$ functions and the reflectance. A + sign denotes upwelling light, and a − sign penetrating light. The calculated AOP are defined in terms of the vector irradiances $H(z,\pm)$ and the scalar irradiances $h(z,\pm)$,

$$K(z,\pm)=-\frac{1}{H(z,\pm)}\frac{dH(z,\pm)}{dz} \quad (5)$$

$$k(z)=-\frac{1}{h(z,+)+h(z,-)}\times\frac{d[h(z,+)+h(z,-)]}{dz} \quad (6)$$

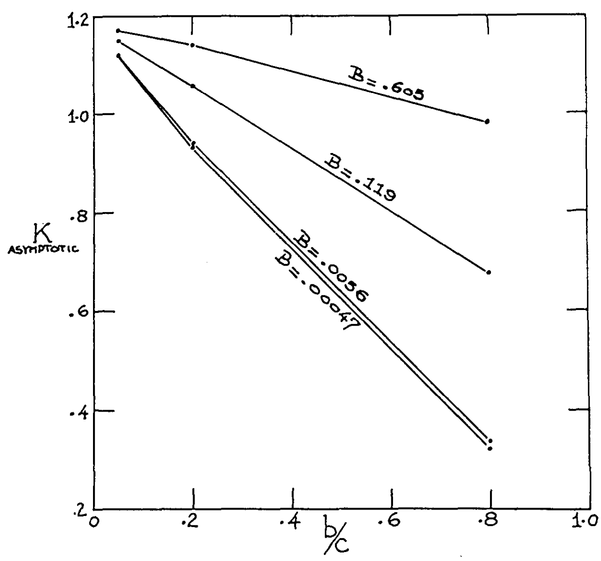

FIG. 3. Asymptotic decay function K as a function of scattering albedo b/c for various values of the relative back-scattering coefficient B.

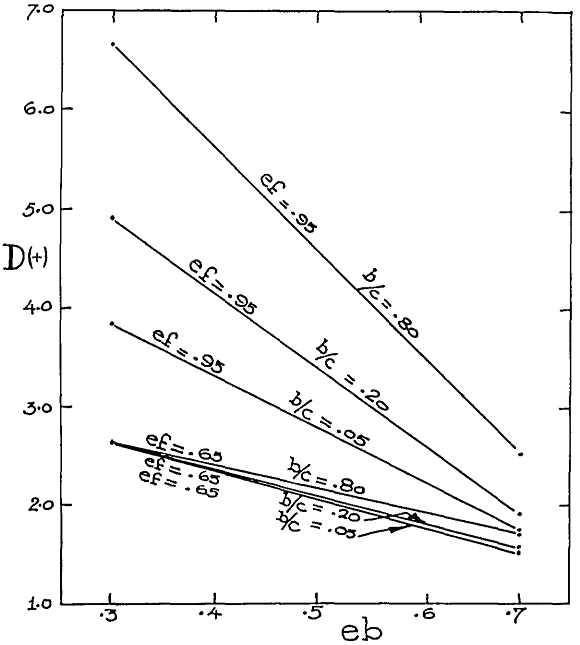

FIG. 4. Asymptotic shape functions $D(-)$ as a function of the scattering albedo b/c for various values of the relative backscattering coefficient B.

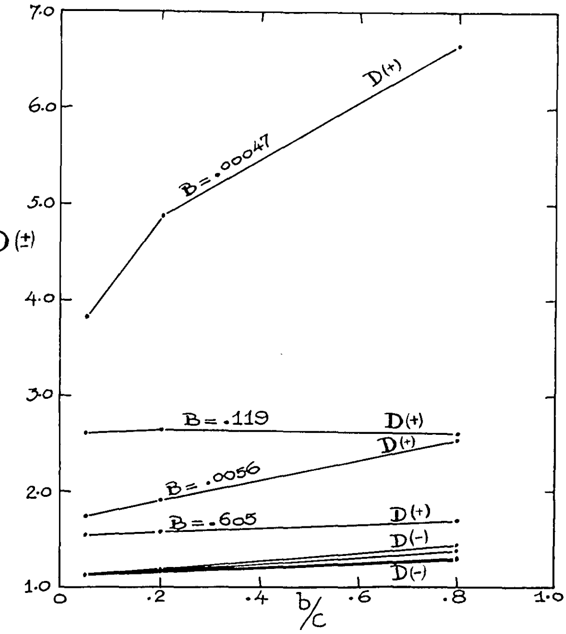

FIG. 5. Details of the dependent of $D(-)$ on the shape of the scattering function.

$$D(z, \pm)=\frac{h(z, \pm)}{H(z, \pm)} \tag{7}$$

$$R(z)=\frac{H(z, +)}{H(z, -)}. \tag{8}$$

We also define

$$F=\frac{1}{b}\int_0^{2\pi}\int_0^{\pi/2}\beta(\gamma)\sin\gamma d\gamma d\phi \tag{9}$$

$$B=\frac{1}{b}\int_0^{2\pi}\int_{\pi/2}^{\pi}\beta(\gamma)\sin\gamma d\gamma d\phi. \tag{10}$$

The calculations were checked for self consistency by using Preisendorfer's[7] equation relating the absorption coefficient and the AOP,

$$a=\frac{K(z, -)-R(z)K(z, +)}{D(z, -)+R(z)D(z, +)}. \tag{11}$$

A test calculation (Table I) was made by using the data taken by Tyler at Lake Pend Oreille, Idaho.[7] The measured scattering curve was used for this hydrosol rather than the empirical fit. The value for b/c was taken from Preisendorfer.[2] The measured radiance field at 6 m was used as the input light field. Although the data are not quite suitable for a test, they are the best known available.

[7] J. Tyler and R. Preisendorfer, in *The Sea*, M. N. Hill, Ed. (Interscience Publ., John Wiley & Sons, Inc., New York, 1962), Vol. 1, p. 397

RESULTS

In general the derived AOP at near-asymptotic depths showed smooth, simple tendencies. Table II is a compilation of the derived AOP for 12 different cases of the IOP. All values were taken at the near-asymptotic depth of 6.5 optical thicknesses which is an actual depth of $z=6.5/c$. Data from this table were used to construct four figures illustrating the functional relations between the inherent and the apparent optical properties. Figures 3 and 4 show the dependence of the $K(+)$ and $D(+)$ functions on the scattering albedo. The apparent complexity of the upwelling $D(+)$ function is resolved by Fig. 5, which indicates that $D(+)$ is primarily dependent upon the eccentricity of the back lobe of the scattering function. Figure 6 shows the dependence of the reflectance on B and the scattering albedo. The calculations were checked for self consistency by computing a from Eq. (12). The ratios of the calculated a's and the input a's were found. The average of these ratios was 1.23, with a standard deviation of 0.36. The relatively small distribution of a(calc)/a(true) is an indication of the reliability of the calculations. The only values of $a_{\text{calc}}/a_{\text{true}}$ which differ significantly from 1.0 are those for $b/c=0.8$. At asymptotic depths $K(+)$, $K(-)$, and k should be equal. The calculations showed this tendency, as the K functions agree to within 25%. The values of the K functions were still converging at the depth at which the numerical calculations were terminated. The K functions differ most for $b/c=0.8$. This might be an indication that the asymptotic level is reached at greater depths for larger values of the scattering albedo, thus explaining the discrepency in the values of $a_{\text{calc}}/a_{\text{true}}$ for $b/c=0.8$.

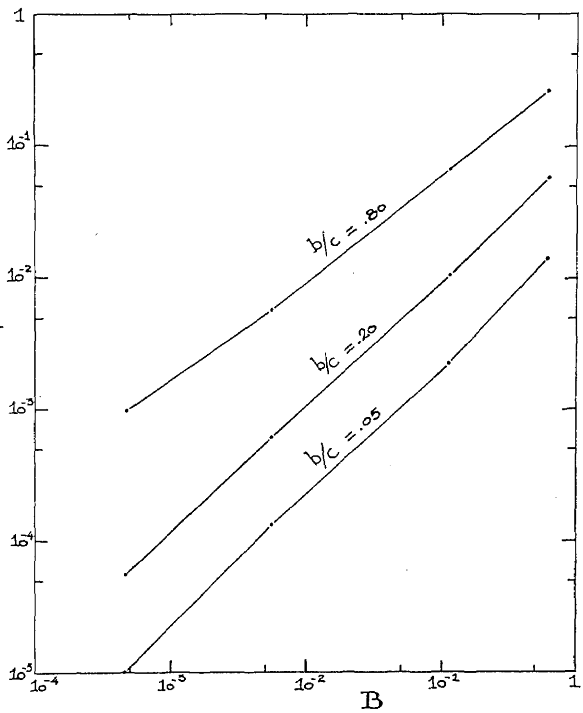

FIG. 6. The reflection coefficient R as a function of the relative back scattering coefficient B for various values of total scattering albedo b/c.

The results of the calculation using Tyler's data are presented in Table III, in which the experimental and calculated values of $K(+)$, $D(+)$, $D(-)$, and R at the near-asymptotic depth of 42.8 m are compared.

Although the $K(+)$ and $D(+)$ functions are in reasonably good agreement, R is not. The error may be due in part to the inconsistency in the data. The value of a was determined by using Preisendorfer's method on values of K, D, and R measured a few weeks before the radiance solid was measured. The scattering function was measured two years after the radiance data were obtained. The scattering function also contained no information or near-forward scattering, which contributes nearly half of b.

Another part of the error may be due to the coarseness of the numerical method. A test with a smaller slab thickness ($\Delta z=0.25$) in the case of $b/c=0.8$, $F=0.88$, and $B=0.22$ showed that although the $K(+)$ and $D(+)$ functions agreed within 5% at near-asymptotic depths, the reflectance was increased nearly 30%. The reflectance seems to be very sensitive to the shape of the lower radiance solid and may be affected by the angular resolution of the numerical integral.

The most interesting feature of the results is an apparently linear relation between a and $K(-)$. Since the reflectance is small, Eq. (11) reduces to

$$a \cong K(-)/D(-). \qquad (12)$$

Since $D(-)$ varies only slightly as the input parameters are varied, Eq. (13) can be further reduced to

$$a \cong 0.8K(-), \qquad (13)$$

with an accuracy of better than 10%. In a study of a restricted region, Eq. (13) could be further improved by selecting a slightly different value for the numerical constant. It must be remembered, however, that the Eq. (13) can be expected to apply experimentally only if very-narrow-band filters are used, so that the grey approximation holds.

The results of the theoretical study may also serve as an indication of the sensitivity of observable properties of the submarine daylight field to variations of biological, chemical, and geological constituents and so aid in planning and interpreting optical measurements.