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How Do the Location, Size and Budget of Open Space Conservation Affect Land Values?

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Abstract In this article we present a model to examine the optimal location, size, and budget of open space conservation and the resulting impact on land values and local fiscal conditions in an urban area. Results indicate that open space conservation can transform the defining features of an urban landscape. A well-designed open space conservation program can improve municipal services, increase total property values, and attract households to the city without substantially increasing tax burdens, while an improperly designed open space program can have the opposite effects. Results also reveal the key parameters that determine the optimal location and size of open space conservation and their fiscal and land value effects.

Keywords Open space conservation · Land values · Environmental amenities · Community characteristics

JEL Classification H4 · R3 · Q2

Introduction

Open space is vital to human health and ecosystems. However, an estimated 4 acres of open space are converted to development every minute in the United States (U.S. Forest Service 2014). In response, many communities are developing programs to preserve open space. From 1988 to 2013, 2413 conservation initiatives were placed in

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local and state referenda in the United States, and 75.5 % of those initiatives were approved, providing about $59 billion for land conservation (The Trust for Public Land 2014). How much land should be preserved for open space? Where should open space be preserved? How do the location and size of open space affect urban development and community characteristics? These are critical questions facing conservation managers.

This paper presents a spatially explicit model to examine the optimal location and size of open space and their effects on municipal characteristics (land values, property taxes, etc.). We first use the model to examine how the location and size of open space affect municipal characteristics by taking the conservation budget as given, and then focus on the optimal budget, location, and size of open space. The main novelty of our approach is the explict consideration of local public finance (property tax rates and municipal services) in the classic urban economic model. Because the model determines municipal characteristics endogenously, it provides a useful tool for studying the effects of open space conservation on urban landscapes.

Many studies use spatial city models to explain observed characteristics of urban landscapes. Lee and Fujita (1997) analyze the efficient placement of a ring of undeveloped land, known as a greenbelt, in an urban area and show that land development outside the greenbelt can be optimal. Wu and Plantinga (2003) examine the effects of open space policy on urban development patterns and find that open space designation can lead to more development as well as leapfrog development (i.e., development that skips over vacant land to build in a remote location). Walsh (2007) develops an equilibrium framework to analyze the impact of open space protection and urban growth control policies on the entire metropolitan landscape. He finds that different strategies for open space conservation can have markedly different landscape and welfare implications. Wu (2014) analyzes the fiscal and land value impacts of public open-space conservation in a budget-constrained city and finds that open space conservation will likely increase total land values and municipal services in metropolitan areas that have stringent land use regulations, high development densities, and relatively little open space. An important contribution of our study is that we examine the optimal location, size, and budget of open space conservation - a topic that has received little attention. Furthermore, we examine the interaction between fiscal policies and conservation policies. As local jurisdictions spend more on land conservation, the fiscal impact of land conservation has become an important topic for policy debates (Carruthers and Úlfarsson 2008).

Studies also abound in examining the effect of amenities or disamenties on property values (e.g., Polinsky and Shavell 1976; Cheshire and Sheppard 1995). Many of these studies focus on open space amenities (e.g., Irwin and Bockstael 2001; Irwin 2002; Geoghegan 2002; Geoghegan et al. 2003; Anderson and West 2006; Acharya and Bennett 2001). These studies find that the value of proximity to open space is affected by many factors, including the nature and type of open space (e.g., public vs. private open space) and neighborhood characteristics (e.g., income, crime, and density), and that the type of open space is a critical determinant of its welfare impact (Klaiber and Phaneuf 2010). This paper complements the previous studies by exploring channels through which open space conservation affects property values.

The Model

Consider a local jurisdiction that provides municipal services and open space to local residents and finances these services through property taxes. The land area within the city is denoted by set $D \subset \mathbb{R}^2(u, v)$, with the city center located at (0, 0). The area preserved for public open space is denoted by set $S \subset \mathbb{R}^2(u, v)$. The level and spatial distribution of open-space amenities depend on the nature and spatial configuration of land preserved and are represented by a distribution function $a(u, v; S)$, $(u, v) \in D$. $a(u, v; S)$ captures amenities provided by both public open space and private undeveloped land. There are n income groups living in the city, indexed by $i=1, 2, \ldots, n$.

Open-space amenities and municipal services are capitalized into property values (see, e.g., Yinger 1982; Irwin 2002). We use the Poterba framework (Poterba 1984, 1991) to model the capitalization, but expand it to include municipal services and open-space amenities and also make it spatially explicit. The Poterba framework stipulates that equilibrium requires owners of real properties, such as land, to earn the same return as on other assets. Formally, equilibrium requires $P=R/(c+\tau)$, where P is the value of land for residential development; R is the rental value of housing services obtained from the land; and c is the non-tax cost of home ownership, such as mortgage interest rates; and τ is the local property tax rate.

We expand the Poterba framework to make it spatially explicit. Location determines both commuting costs and amenities, which affect the value of housing services. For example, if commuting is the only difference between locations, the value of housing services would vary by commuting costs tx, where x is the commuting distance from (u, v) to the city center, and t is the commuting cost per unit of distance. Municipal services, such as city water and sewer, and open-space amenities enhance the rental value of housing services. Wu (2010) derives the rental value of housing services that captures all of these effects. Specifically, by assuming households choose residential location and consumption bundle to maximize utility, Wu (2010) derives households' willingness to pay for housing services as $R_i(u, v) = A_i(y_i - t_i x)^{\beta_i} a(u, v; S)^{\gamma_i} g^{\mu_i}$, where $i=1, 2, \ldots, n$, y_i is household income, A_i, β_i, y_i, and μ_i are positive parameters, and $\mu_i<1$.[1] Substituting the rental value of housing services into Poterba's equilibrium condition, we obtain the bid price function for land:

$$p_i(u, v) = \frac{A_i(y_i - t_i x)^{\beta_i} a(u, v; S)^{\gamma_i} g^{\mu_i}}{c + \tau} \tag{1}$$

Households are sorted across the landscape according to their incomes and preferences, which affect their bid prices or willingness to pay for housing. In

[1] The assumption $\mu<1$ is made to ensure that some public services will be provided when they are non-rival public goods (see proposition 1 below).

equilibrium, housing is allotted to those who bid the highest price, and households in income group i live in $D_i \equiv \{(u,v) | p_i(u,v) \geq Max\{p_1(u,v),\ldots,p_n(u,v)\}$. Thus, the equilibrium housing price in the city equals $p(u,v) = Max\{p_1(u,v),\ldots, p_n(u,v)\}$.

The city's annualized cost of open-space conservation equals

$$TC_o = c\iint_S p_0(u,v)dudv, \tag{2}$$

where $p_0(u,v)$ is the cost of land for open space at location $(u,\ v)$. We consider two scenarios as to how $p_0(u,v)$ is determined. In scenario 1, an environmentally conscious landowner is willing to sell his land to the city for a community park at a negotiated price, or the land owner is willing to sell his land at the current price. In this case, $p_0(u,v)$ is exogenous and is not affected by the amount of land conserved.

In scenario 2, we assume that land owners are more sophisticated. They realize that as more land is conserved for open space, land prices may change because open space conservation will affect amenities, municipal services, land supply, and the property tax rate in the city. This scenario is much more difficult to model than scenario 1. In scenario 1, once the total conservation budget is given, the amount of land that can be conserved at a given location will be determined. However, in scenario 2, the amount of land that can be conserved is simultaneously determined with the property tax rate and the level of municipal services because they affect the opportunity cost of conservation and thus the purchasing prices. We adopt scenario 2 in the following analysis. But all results holds for scenario 1 (the proof is available upon request).

Assume all land within the city except the public open space is private and is assessed for property taxes. The total property tax revenue for the city equals

$$TR = \iint_{D-S} \tau p(u,v)dudv. \tag{3}$$

Following Borcherding and Deacon (1972), the cost of municipal services is assumed to be $TC_s = gN^{\lambda}$, where g is the level of municipal services provided to each household, N is the total number of households served, and $\lambda \in [0,1]$ is a parameter indicating the economy of scale in the provision of municipal services. $\lambda = 1$ indicates no economy of scale, and $\lambda = 0$ indicates the largest economy of scale, with all municipal services being pure non-rival public goods.

Open space conservation affects the number of households living in the city by reducing the amount of developable land and by changing amenities and development density. Assuming developers choose development density to maximize profit, Wu (2006) derives household density as a function of rental value of housing services: $d(u,v) = R(u,v)^{\delta}$, where $R(u,v) = Max\{R_1(u,v),\ldots,R_n(u,v)\}$, and $0 < \delta < 1$ is the elasticity of development density with respect to the rental value of housing services. The assumption of $\delta < 1$ is based on previous studies, which consistently find the demand for housing is inelastic. The total number of households living in the city thus equals

$$N = \iint_{D-S} R(u,v)^{\delta} dudv. \tag{4}$$

The city government faces a budget constraint; that is, the total cost of municipal services and open space conservation cannot exceed the total tax revenue:

$$TC_o + TC_s \leq TR. \tag{5}$$

There are two approaches to characterize the public-sector decisions about open space conservation and municipal services provision. One is to assume that the decisions are made by the elected city government, who maximizes the property values of local residents. Alternatively, one can assume that the decisions are made by a majority voting rule. Under the majority voting rule, if each voter chooses (S,τ,g) to maximize his own property value, and relies on the local government to inform them which (S,τ,g)'s are feasible, these two approaches will lead to the same results. Below, we adopt the first approach.

Impacts of Open Space Conservation

In this section, we take the location and size of open space as given and analyze their impact through comparative static analysis. As indicated by land price in Eq. (1), open space conservation affects land values in the city both directly and indirectly. It affects land values directly by changing the level and spatial distribution of amenities in the city. It also affects land values indirectly by influencing the level of municipal services and the property tax rate in the city. Thus, to determine the overall effect of open space conservation, we must determine its effect on the level of municipal services and the property tax rate.

The government chooses the property tax rate and the level of municipal services to maximize property values of local residents. Given the location and size of open space, from Eq. (1), this is equivalent to solving the following maximization problem:

$$\underset{(\tau,g)}{Max} \iint_{D-S} p(u,v)dudv \quad \text{s. t.} \quad TC_s + TC_o \leq TR. \tag{6}$$

Below we first solve this maximization problem for the case of homogenous preferences and then turn to the case of heterogeneous preferences.

Homogeneous Preferences about Municipal Services

When the income groups have homogeneous preference about municipal services (i.e., $\mu_i=\mu$ for $i=1, \ldots, n$), we can solve the maximization problem (6). The results are summarized in the following proposition.

Proposition 1. *Given the location and size of open space, a unique equilibrium exists in the city if and only if*

$$\overline{G} < \frac{r[\alpha-(\lambda-1)\mu]}{\alpha+\lambda\mu}\left[\frac{\mu\widetilde{R}}{(\alpha+\lambda\mu)\widetilde{N}^{\lambda}}\right]^{\frac{\mu}{\alpha-(1-\lambda)\mu}} \tag{7}$$

The equilibrium property tax rate and the level of municipal services that maximize property values of local residents equal

$$\tau^* = \frac{c\mu}{\rho(1+\lambda\delta\mu)-\mu}, \tag{8}$$

$$g^* = \left[\frac{\mu\widetilde{R}}{(1+\lambda\mu\delta)\widetilde{N}^{\lambda}}\right]^{\frac{1}{1+\lambda\delta\mu-\mu}}, \tag{9}$$

where ρ is the share of the total property tax revenue spent on municipal services (as opposed to open space), and

$$\widetilde{R} = \iint_{D-S} A[y(u,v)-tx]^{\beta_i} a(u,v;S)^{\gamma_i\gamma} dudv, \tag{10}$$

$$\widetilde{N} = \iint_{D-S} A[y(u,v)-tx]^{\beta_i\delta} a(u,v;S)^{\gamma_i\delta} dudv, \tag{11}$$

$y(u,v)$ is the level of household income at location (u, v) in equilibrium.

Proof: The proof of proposition 1 and all the subsequent propositions and corollaries are given in the Appendix.

Equations (8) and (9) reveal the key parameters that determine the fiscal impact of open space conservation. Specifically, the property tax rate τ^* increases as a large share of tax revenue is spent on open space conservation (i.e., ρ is smaller). Given the share of conservation spending, τ^* is higher when: i) there is a larger economy of scale in the provision of municipal services (i.e., λ is smaller), ii) the non-tax cost of home ownership is lower (i.e., c is smaller), iii) housing prices are more responsive to municipal services (i.e., μ is larger), and iv) development density is less responsive to housing rent (i.e., δ is smaller). Because τ^* increases with the non-tax cost of home ownership c, such as mortgage interest rates, easy monetary policies can affect not only housing values but also local property tax rates.

Equation (9) reveals that open space conservation can affect municipal services through two channels: i) by changing the tax base (as reflected by $\widetilde{R}$), and ii) by changing the number of people living in the city and thus the cost of municipal services (as reflected by $\tilde{N}$). A smaller tax base tends to reduce the equilibrium level of municipal services, while a larger number of households living in the city tend to have the opposite effect. Open space conservation, however, can increase or decrease the tax base in the city, depending on the two opposite effects. Open space conservation tends to reduce the tax base by reducing the amount of developable land and increase the tax base by increasing amenities and property values. Likewise, open space conservation can have a positive or negative effect on the number of people living in the city; it reduces the number of people living in the city by reducing developable land and increases it by attracting more people to the city and subsequently raising land prices and population densities. When there is a large economy of scale in the provision of municipal services (i.e., λ is small) and land conservation costs are high, open space conservation tends to reduce the level of municipal services because it reduces the tax base more than it reduces the cost of municipal services. In contrast, when there is a little economy of scale in providing municipal services (i.e., λ is large) and the land conserved would have been developed for high-density housing, open space conservation tends to increase the level of municipal services. The final impact depends on its impact on $\left(\widetilde{R}/\widetilde{N}^{\lambda}\right)$.

Differentiating $\left(\widetilde{R}/\widetilde{N}^{\lambda}\right)$ and $\left(\widetilde{G}/\widetilde{R}\right)$ with respect to S (i.e., expand the boundary S parallelly to all directions by an infinitesimal amount) gives the following result:

Corollary 1. *Suppose $a(u,v;S)=a(S)f(d(u,v))$, where $d(u,v)$ is the distance from (u, v) to the open space and $f'(d)<0$.*

$$a)\ \frac{\partial \tau^{*}}{\partial S} \geq 0, \tag{12}$$

$$b)\ \frac{\partial g^{*}}{\partial S} \geq 0\ \ iff\ \ \left(\lambda \varepsilon_S^N - \varepsilon_S^R\right) + \gamma \varepsilon_S^a (1 - \lambda\delta) \geq 0, \tag{13}$$

where $\varepsilon_S^R = -\frac{1}{\widetilde{R}} \frac{\partial \widetilde{R}}{\partial S}\Big|_a$, $\varepsilon_S^N = -\frac{1}{\widetilde{N}} \frac{\partial \widetilde{N}}{\partial S}\Big|_a$, and $\varepsilon_S^a \equiv \frac{1}{a} \frac{\partial a}{\partial S}$.

Corollary 1 provides several insights about the fiscal impacts of open-space conservation. First, fiscal effect of open-space conservation depends on its location. Open space conservation increases the level of municipal services if the preserved land is undevelopable, but provides amenities. In this case, the condition for $\frac{\partial g^{\rightharpoonup}}{\partial S} > 0$ holds because $\varepsilon_S^N = 0$, $\varepsilon_S^R = 0$, and $\varepsilon_S^a > 0$. Intuitively, preserving such land will not change the marginal cost of municipal services, but will increase their marginal benefits because of the increased amenities. Thus, the municipality will increase the level of municipal services with land conservation. This suggests that conserving undevelopable land (such as wetlands or brownfields) that yields little in property tax revenues or preserving land outside the city will increase the level of municipal services. On the other hand, if conservation on undevelopable land offers little amenities, it will have no effect on the optimal level of public services because it does not change either the marginal benefit or the marginal cost of municipal services. Consequently it will have no effect on land prices and the tax base of the city even if it is costless.

Second, open space conservation can increase the level of municipal services even if it does not generate any amenities. This occurs when $\lambda \varepsilon_S^N > \varepsilon_S^R$. Intuitively, open space conservation reduces the marginal cost of public services (due to fewer households receiving the services) more than it reduces the marginal benefit of public services when $\lambda \varepsilon_S^N > \varepsilon_S^R$, causing the optimal level of public services to increase.

Finally, the effect of open-space conservation on municipal services depends on the characteristics of the city. In small cities surrounded by rural land, open space conservation may not generate additional amenities because there are close substitutes for new open space. In those cities, open-space conservation will reduce the level of municipal services because condition (13) for $\frac{\partial g^{\rightarrow}}{\partial S} > 0$ cannot hold when ε_S^N=0 and ε_S^a=0. In contrast, in large cities with high development densities and relatively little open space, large parcels of new open space tend to provide a high level of amenities and reduce costs of municipal services, particularly when the land would be developed for high-density housing. In those cities, open-space conservation is more likely to increase the level of municipal services.

To examine the effect of open space conservation on total land value (*TLV*) within the city, we substitute (8) and (9) into the land value equation to obtain:

$$TLV = \frac{(1 + \lambda\delta\mu - \mu)}{c}\left[\frac{\mu}{\alpha + \lambda\delta\mu}\right]^{\frac{1+\lambda\delta\mu}{(1+\lambda\delta\mu-\mu)}}\left[\frac{\widetilde{R}^{1+\lambda\delta\mu}}{\widetilde{N}^{\lambda\mu}}\right]^{\frac{1}{(1+\lambda\delta\mu-\mu)}}. \tag{14}$$

This suggests that the effect of open space conservation on the total value of land depends on its effect on $\left[\widetilde{R}^{1+\lambda\delta\mu}/\widetilde{N}^{\lambda\mu}\right]$. Differentiating this ratio with respect S, we obtain the following results.

Corollary 2. *Suppose* $a(u,v)=a(S)f(d(u,v))$. *Additional open space conservation increases the total land value within the city if and only if*

$$\left[\lambda\mu\varepsilon_S^N - (1 + \lambda\delta\mu)\varepsilon_S^R\right] + \gamma\varepsilon_S^a \geq 0. \tag{15}$$

From condition (15), we can derive the following results. First, open space conservation increases the total land value if it generates enough amenities (i.e., if ε_S^a is large enough). Second, if additional open space conservation does not generate any amenities (i.e., ε_S^a=0), but diverts public funds away from producing public goods (λ=0), it reduces the total land value. In this situation, open space conservation increases the tax rate and reduces the level of municipal services. Third, open space conservation can increase the total land value even if it does not generate any amenities. In this case, open space conservation serves as a tool for reducing municipal services costs. The effectiveness of open space conservation as such a tool depends critically on its location. To see this, note that when ε_S^a=0, condition (15) reduces to

$$\frac{\varepsilon_S^R}{\varepsilon_S^N} \leq \frac{\lambda\mu}{1 + \lambda\delta\mu}. \tag{16}$$

This suggests that additional open space conservation increases the total property value if the land would be developed for households who would spend less than $\lambda\mu/(1+\lambda\delta\mu)$ on land rents relative to an average household in the city. This result reflects that open space conservation is more effective in reducing municipal services costs if it preserves land that would be developed for high-density housing.

Heterogeneous Preferences about Municipal Services

When the income groups have different preferences about municipal services, they will prefer different combinations of property tax rates and municipal services, and the elected government officials may choose the property tax rate and level of public services preferred by the majority group. To illustrate this, suppose there are two income groups living in the city, and the local government chooses the property tax rate and the level of municipal services to maximize the property values of the major group. This essentially assumes that the property tax rate and the level of municipal services are determined by majority voting rule.

Proposition 2. *Suppose the property tax rate and the level of municipal services are determined by majority voting rule. An equilibrium exists in the city if*

$$\overline{G} < G^*, \tag{17}$$

where $G^* > 0$ is defined in the Appendix. The equilibrium property tax rate equals

$$\tau^* = \frac{\mu_i}{\rho + (\rho\lambda\delta - 1)\mu_i + (\rho\lambda r_{-i} - m_{-i})(\mu_{-i} - \mu_i)}, \tag{18}$$

if income group i is the majority in the city, where $r_i$is the share of households belonging to income group i, and $m_i$is the share of total property tax paid by income group i. The equilibrium level of public services equals the maximum g that satisfies the budget constraint for $\tau = \tau^*$.

Proof: See the Appendix.

When income groups have homogeneous preferences about municipal services (i.e., $\mu_l = \mu_h$), (16) reduces to (8). However, when they have different preferences about the muncipal services, they would choose different property tax rates. To understand the fiscal impact of open space conservation in this situation, consider first the case where low-income households are the majority in the city, and their bid prices for housing are less responsive to the level of municipal services (i.e., $\mu_l < \mu_h$). In this case, the third term in the denominator of (18) becomes $(\rho\lambda r_h - m_h)(\mu_h - \mu_l)$. If the conserved open space would be developed for low-income housing, both m_h and r_h increase with open space conservation. However, because m_h increases faster than r_h and λ_ρ is less than one, $(\rho\lambda r_h - m_h)$ decreases with open space conservation. In this case, the equilibrium tax rate increases with open space conservation, and the increase is larger when the two income groups have heterogeneous preferences. This result holds because the high-income households are willing to give up more lot size for better public services and the low-income majority will set a higher tax rate to take advantage of the higher substitution rate.

On the other hand, if the conserved open space would be developed for high-income housing, both the share of high-income households (r_h) and the share of their tax contribution (m_h) will decrease with open space conservation. Because m_h decreases faster than r_h, $(\rho\lambda r_h - m_h)$ can increase or decrease with open space conservation, depending on the magnitude of ρ. If open space conservation accounts for only a small share of the total budget and there is little economy of scale in the provision of public services (i.e., both ρ and λ are close to one), $(\rho\lambda r_h - m_h)$ will increase with open space conservation, which tends to reduce the property tax rate. Intuitively, when there are a larger number of high-income households living in the city, the low-income majority would have a larger incentive to raise the property tax rate so that they can take advantage of the large tax base of high-income households. However, with open space conservation, there will be fewer high-income households living in the city. Thus, the incentive to impose higher property tax rate becomes smaller. The case where high-income households are the majority can be similarly analyzed.

Optimal Location and Size of Open Space

So far we have examined the effects of open space conservation urban landscape by assuming that the location and size of open space are exogenously determined. In this section we analyze the optimal location and size of open space, which are defined as those that maximize the sum of land values for all households living in the city. The analysis is intended to address the following question. If a city has a time window to preserve any land on the landscape for open space, what would be the optimal location and size of open space in the city? The answer to this question is central for urban planning.

This question is difficult to answer, however. When modeling the optimal location and size of open space, we must take into account the fact that the opportunity cost of open space conservation is affected by the location and size of land conservation. As more land is conserved for open space, the city must devote more tax revenue to open space conservation, which will affect the level of public services and the property tax rate in the city. Changes in the level of public services and the property tax rate in turn affect land values and the opportunity cost of land conservation. Thus, the opportunity cost of open space conservation is endogenous and must be simultaneously determined with the location and size of open space and the level of public services and the property tax rate in the city.

Formally, the optimal location and size of open space are defined by:

$$\begin{aligned} &\underset{(\tau,g,S)}{Max} \iint\limits_{D-S} p(u,v)dudv, \\ &\text{s.t.}\, TC_o + TC_s \leq TR. \end{aligned} \tag{19}$$

There are an infinite number of configurations for any given area of open space. This "dimensionality problem" makes maximization problem (19) unsolvable either analytically or numerically. To overcome this problem, we consider two common forms of publicly conserved open space whose locations, size, and shapes can easily be identified: central parks and greenbelts. These two forms of open space can be described by the radius of their inner and outer boundaries to the city center $[s_0,s_1]$, with $s_0=0$ indicating a central park and $s_0>0$ a greenbelt. With this simplification, we can solve the maximization problem numerically by using parameter values that are broadly consistent with empirical evidence found for the United States. We assume there are two income groups living in the city and simulate both the cases where the two income groups have the same or different preferences.

Parameterizing the Model

According to the 2011 Consumer Expenditure Survey conducted by the U.S. Bureau of Labor Statistics (2014), the average household income for urban residents was $64,986 in 2011, with an annual expenditure of $50,348, including $17,226 on housing and

\$8266 on transportation. This implies that housing accounts for about 40.9 % of total expenditures excluding transportation costs. Because land accounts for approximately 25–30 % of the total housing value, this implies that land accounts for about 10–15 % of total household expenditures excluding transportation costs. Thus, we set $y=\$70{,}000$ and $(1/\beta)=12.5\%$ for high-income households as $(1/\beta)$ represents the share of total expenditures (excluding transportation costs) on land. For low-income households, we set $y=\$35{,}000$ and $(1/\beta)=14.3\%$. The average expenditures on transportation for the second and fourth income quartiles are approximately \$6250 and \$12,500, respectively. Based on this information, we set $(t_l, t_h)=(\$1000, \$1500)$ (per round-trip mile annually).

Land is developed when the bid price for land is above a reservation rent. This implies that the urban boundary is defined by $p(u,v)=p_a$, where p_a is the reservation rent for development, which is set at \$1000/acre.[2] Parameter c is set to one, so the simulated property tax rate should be interpreted as the rate relative to other user costs of home ownership.

The distribution of amenities over the landscape is assumed to take the form of $a(x;s_0,s_1)=a_0+a_1(s_1-s_0)e^{-\delta|x-0.5(s_1+s_0)|}$, where a_0 is the natural amenity level, which is set to one; and a_1 and δ are positive parameters determining the spatial distribution of open space amenities. If $\delta=0$, every household in the city enjoys the same level of amenities from open space regardless of their residential location. Positive parameter values for a_1 and δ indicate that the level of amenities decreases with the distance from the open space. In the baseline, we set $(a_0, a_1, \delta)=(1, 0.5, 1)$. We conduct sensitivity analysis with alternative parameter values and functional forms to test the robustness of simulation results. We have little information about parameters A_l and A_h, and set their values to ensure that the simulated metropolitan area is within the range of a medium-sized city in the United States.[3] Under the parameter values presented in Table 1 for A_l and A_h, the simulated diameter of the urban area is about 17 miles in the baseline, with high-income households living in the suburbs (from 11 miles to the boundary).

Simulation Results

We report two sets of simulation results. The first set of results (Table 2) is based on the model presented in Impacts of Open Space Conservation section and shows the impacts of location, budget, and size of open space (exogenously determined) on city characteristics. The second set of results (Tables 3, 4 and 5) is based on the model presented in Optimal Location and Size of Open Space section and shows the optimal location and size of open space. Each set of results is discussed below.

[2] This number is consistent with the USDA statistics about farmland value. Please see "Land Values and Cash Rents 2007 Summary," available at http://economics.ag.utk.edu/extension/forage/AgriLandVa-08-03-2007.pdf.

[3] For example, Metro Boston has a diameter of approximately 50 miles (from Duxbury, MA to Bedford, MA); Metro Portland Oregon has a diameter of approximately 26 miles (from Wilsonville, OR to Vancouver, WA).

Table 1 Parameter values used in the simulations

Community characteristics	Parameters	Value
Income for Low-income Households ($/year)	y_l	35,000
Income for high-income Households ($/year)	y_h	75,000
Commute costs of low-income Households ($/mile/year)	t_i	1000
Commute costs of high-income Household ($/mile/year)	t_h	1500
Elasticity of bid prices w. r. t. expenditure for low-income households	β_l	7.000
Elasticity of bid prices w. r. t. expenditure for high-income households	β_h	8.000
Elasticity of bid prices w. r. t. municipal services for low-income households	μ_l	0.070
Elasticity of bid prices w. r. t. municipal services for high-income households	μ_h	0.120
Elasticity of bid prices w. r. t. open space amenities for low-income households	γ_l	1.144
Elasticity of bid price sw. r. t. open space amenities for high-income households	γ_h	1.000
The economy of scale parameter	λ	0.85
Agricultural land rent ($/Acre)	p_a	1000
City amenity function parameter	a_0	1.00
City amenity function parameter	a_1	0.50
City amenity function parameter	δ	1.00
Scale parameter for bid-price function for low-income households	A_l	3.210E-25
Scale parameter for bid-price function for high-income households	A_h	7.449E-33

The Impacts of Open Space Conservation

Table 2 shows the impacts of open space conservation on city characteristics.[4] Four possible locations of open space (a central park, a greenbelt in the low-income area, a greenbelt in the high-income area, and a greenbelt at the city boundary) and two levels of conservation budget ($50 million and $100 million) are simulated. The first row of Table 2 describes the characteristics of the city in the baseline (no open space), and the rest of the rows show the city characteristics when open space is designated at different locations. The simulations provide several interesting results, which are summarized as follows.

First, as predicted by the theory, open space conservation increases the property tax rate. For example, when a central park is established with a $50 million budget, the equilibrium property tax rate increases from 7.24 % to 8.77 %.[5] With a $100 million budget, the equilibrium property tax rate increases to 10.38 %. Despite the fact that open space conservation competes for funding with other municipal services, the level of municipal services is higher than in the baseline level for every conservation scenario considered in Table 2.

[4] The simulations assume that open space is purchased at the prevailing market prices before the open space conservation.

[5] The property tax rate is calculated as the rate relative to other user costs of home ownership, which is around 10 % depending on the household's income (Poterba 1991). Thus, the un-normalized property tax rate is between 0.72 and 1.04 %.

Table 2 Effects of location, size, and budget of open space conservation

Location/type	Budget (million $)	Size (s_0, s_1)	Property tax rate (%)	Level of municipal services (index)	Cost of municipal services ($/household)	Total land value in the metro area (million $)
Baseline	–	–	7.24	2378	307	3622
Central park in the city	50	(0, 0.73)	8.77	2382	307	3559
	100	(0, 1.05)	10.38	2381	308	3461
Greenbelt in the low-income area	50	(5.00, 5.14)	8.76	2384	307	3563
	100	(5.00, 5.28)	10.35	2386	308	3496
Greenbelt in the High-income area	50	(15.00, 15.54)	8.76	2382	307	3567
	100	(15.00, 16.14)	10.34	2381	307	3493
Greenbelt at the city boundary	50	(15.75, 18.44)	8.71	2396	308	3591
	100	(17.75, 19.10)	10.24	2403	309	3499

Note: Open space is acquired at the prices in the base case

Table 3 Optimal location, size and budget of open space conservation

Open space/city characteristics	No open space	Central park	Greenbelt in the low income area	Greenbelt in the high income area	Greenbelt at the city boundary
Optimal location, size and budget					
Inner boundary s_0 (mile)	–	0.00	0.01	17.11	17.37
Outer boundary s_1 (mile)	–	0.15	0.18	17.27	17.64
Total acquisition cost (Million $)	–	2.60	3.47	10.59	19.09
Percentage of total budget (%)	–	0.82 %	1.10 %	3.27 %	5.76 %
City characteristics					
Property tax rate (%)	7.2 %	8.7 %	8.7 %	8.9 %	9.2 %
Level of government service (index)	2378	2809	2809	2811	2817
Total land value (million $)	3622	3632	2633	3630	3632

Table 4 Optimal location, size and budget of open space conservation: sensitivity analysis with the level and decay rate of open space amenities

Parameter	Optimal location (s_0, s_1)	Optimal size (acres)	Optimal budget (million $)	Percent of total budget (%)	Percent increase in total land value (%)
Decay rate (δ)					
0.15	(15.24, 21.64)	474,570	1054.0	55.7 %	31.2 %
0.25	(10.35, 11.97)	72,701	432.1	40.7 %	7.6 %
0.40	(0.01, 0.60)	724	64.6	10.4 %	2.7 %
0.60	(0.01, 0.37)	275	23.5	4.1 %	0.9 %
1.00	(0.01, 0.17)	59	4.8	0.9 %	0.2 %
Level of open-space amenities (a_1)					
0.25	(0, 0.08)	13	1.0	0.2 %	0.0 %
0.50	(0.01, 0.17)	59	4.8	0.9 %	0.2 %
0.75	(18.04, 18.57)	39,019	39.0	6.6 %	0.7 %
2.00	(19.89, 20.54)	52,842	51.7	8.4 %	2.9 %
4.00	(4.58, 5.03)	8695	492.4	42.7 %	11.9 %

Note: To isolate the effect of parameter value changes, simulations assumes that two income groups have the same preferences, with the parameter values for both groups set to the values for high-income households in Table 1

Second, open space conservation can increase or decrease the total land value. Open space conservation increases the level of amenities, but must be financed through property taxes. When the negative effect of higher property taxes dominates the positive effect of higher amenities, the total land value will decrease. This is consistent with Cheshire and Sheppard (2002) and Walsh (2007), who found that land-use control could lead to substantial social welfare loss in the form of higher housing prices, smaller houses, and inefficient land use patterns.[6] However, a well-designed open space program can increase the total property values, as demonstrated by the optimal conservation scenarios considered in the next section.

Third, open space conservation can have convoluted equity consequences. Because open space conservation can lead to a higher property tax rate and a lower level of municipal services, it is possible that landowners located farther away from open space may see their property values decreasing with open space conservation because they benefit little from open space amenities. For example, when a greenbelt is established in the high-income area, the average land value in the high-income area increases,

[6] Kopits et al. (2007) examine the tradeoff between private lots and public open space in subdivisions at the urban–rural fringe. They find that households do not value public open space nearly as much as a larger lot. Thus, reducing private acreage to provide more public subdivision open space tends to lead to overall reductions in housing values. More recently, Abbott and Klaiber (2010) find that the interactions between subdivision open space and private open space in the form of lot size change from complementarity at small scales to substitutability at large scales. This paper does not directly model the interactions. However, public open space can affect private lot sizes through its effect on land prices in our model. Because open space conservation tends to increase land prices nearby, it tends to reduce lot sizes in those areas. On the other hand, open space conservation may reduce land prices in areas located farther away because households located in those areas may benefit little from open space amenities, but must pay a higher property tax rate. Thus, open space conservation may lead to larger lot sizes in areas located farther away.

Table 5 The effect of optimal open space conservation in the city on the suburb

Variables	Baseline	Percent change from the baseline				
		δ=0.15	δ=0.25	δ=0.40	δ=0.60	δ=1.0
Total land value (million dollars)	206.3	117 %	−52 %	−4 %	0 %	0 %
Suburban outer boundary (miles)	23.7	4 %	−4 %	0 %	0 %	0 %
Average land rent ($/acre)	1302	36 %	−25 %	−2 %	0 %	0 %
Number of households	45,413	120 %	−53 %	−4 %	0 %	0 %
Lot size (acre/ household)	3.488	−27 %	35 %	2 %	0 %	0 %
Average property tax payment ($/household)	555.18	−1 %	1 %	0 %	0 %	0 %
Level of government service (index)	2773	11 %	−10 %	−1 %	0 %	0 %
Property tax rate	12.2 %	0 %	0 %	0 %	0 %	0 %

whereas the average land value in the low-income decreases. Households in the low-income area benefit little from open space amenities, but pay a higher property tax rate.

Fourth, the simulation results reveal incentives for Tiebout sorting. Low-income households tend to pay less property tax than the cost of their municipal services, while high-income households tend to pay more. Thus, without the economy of scale in providing municipal services, high-income households would have little incentive to be annexed into the city, and local jurisdictions tend to be more fragmented. Preserving land for open space in the city could reduce jurisdictional fragmentation, because it increases land values in the city.

Fifth, both the location and size of open space are important in determining the effects of open space conservation. When the conserved open space is located closer to the city center, the unit acquisition cost is higher because land located near the city center is more expensive. The amount of land that can be purchased with a given budget increases as land located near the city boundary is targeted for conservation. Thus, depending on the location of open space, both the overall level and the spatial distribution of amenities can be quite different for a given conservation budget.

Finally, our results demonstrate the importance of considering public finance impacts when designing open space conservation programs. Without considering the effects of open space conservation on the property tax rate and the level of municipal services, the model would predict a larger city with open space conservation because a higher level of amenities will attract more people to the city. However, when open space conservation is financed through local property taxes, it will affect the property tax rate and the level of municipal services. The change in the property tax rate and municipal services will in turn affect land values and the attractiveness of the city as a place to live. Our results demonstrate that when all these effects are considered, an ill-designed conservation program may actually cause a city to shrink, rather than expand. Indeed, our simulations reveal that the city shrinks when a central park or a greenbelt is preserved in the low-income area.

In summary, open space conservation can change the defining features of an urban landscape, including land prices, development densities, property tax rates, and the level of municipal services. A well-designed open space program can increase the level of municipal services and total property values. But an ill-designed open space program can lead to a higher property tax rate, lower level of municipal services, and lower property values. Open space conservation can cause a city to shrink, rather than expand, because of its public finance impacts.

Optimal Location and Size of Open Space

Results on the optimal location and size of open space are presented in Tables 3 and 4. Specifically, Table 3 presents the optimal budget, location, and size for various forms of open space and the resulting urban characteristics, with a baseline for easy comparison. Among the four types of open space, the greenbelt located in the low-income area is globally optimal, because it leads to the largest increase in land value within the city. The optimal greenbelt has an inner radius of 0.1 mile and outer radius of 0.18 miles, with total acreage of 63.6 acres. The optimal budge for open space is 3.47 million, or a moderate $4.01 per household annually. Establishing such a greenbelt will lead to an increase in total land value within the city by $10.45 million, or 0.3 % compared with the baseline. This suggests that if carefully designed, open space conservation can lead to better public services and higher property values for urban residents. Because of higher amenities and better public services, the city becomes a more desirable place to live, and more people will migrate to the city. As a result, the total demand for land will increase and the property values will go up.

The results in Table 3 show that open space has only a small effect on the total land value, and the effect is insensitive to the location of open space as long as the size and the total budget are optimal. This conclusion, however, depends on the assumptions about the level and decay rate of open space amenities, as shown below. In general, a given budget can allow the purchase of more land in the area located farther away from the city center. Thus, city governments face a tradeoff in determining the optimal location of open space. If a relatively wide greenbelt can generate only a small amount of open space amenities (as determined by parameter a_1 in the amenity function) and the level of amenities decreases fast as one moves farther away from open space (as determined by parameter δ in the amenity function), a greenbelt located near the city center is more likely to be optimal. On the other hand, if a narrow greenbelt can generate a relatively large amount of open space amenities and the level of amenities decreases slowly as one moves farther away from open space, a greenbelt located near the urban boundary is likely to be optimal because more people can live near the greenbelt where land is also less expensive there.

To gain additional insights, we have simulated the optimal location and size of open space (globally optimal) for different values of δ and a_1. The results are reported in Table 4. As δ decreases, the optimal location of open space moves toward the urban boundary, and the optimal size of open space increases. When the level of amenities decreases slowly, more people will benefit from open space, and it is optimal to provide more open space.

The effect of a_1 on the optimal location and size of open space is highly nonlinear. With smaller a_1, it is optimal to locate open space closer to the city center. This reflects that although the land near the city center is more expensive, a given amount of open space can generate a higher level of amenities because the greenbelt is wider. As a_1 increases, the optimal location of open space moves toward the city boundary.

However, after a_1 reaches a certain threshold, the optimal location of open space moves back toward the city center because the increase in the level of amenities from a wider greenbelt outweighs the hike in land acquisition cost. a_1 also affects the optimal size of open space. As a_1 increases, the optimal size of open space increases initially, but decreases after a_1 reaches a critical value. With a larger a_1, a given size of open space can provide more amenities.

The sensitivity analysis suggests that changes in the parameter values can have a significant impact on the optimal location and size of open space. The higher the degree of uncertainty about the distribution of amenities from open space conservation, the more the unintended consequences that the local government will face when preserving land for open space. Changes in urban spatial structure resulting from an open space program could be completely different from those anticipated, and local governments must recognize such complexities when designing land conservation programs.

The Effect of Open Space Conservation on Suburbs

In this section, we extend the model to examine how open space conservation in the city affects its suburb, which is located just outside of the city and has its own local jurisdiction. Residents in the suburb go to the city center to work and, therefore, incur higher commuting costs than urban residents. But the suburbanites enjoy a higher level of amenities, which may be provided by a major geographic feature such as a lake, or by an idiosyncratic history of development such as in a small mill town with reasonable amenities (dining, established neighborhoods). Thus, the amenity distribution function in the suburb is assumed to be:

$$a_s^0(x) = \rho + a(x; s_0, s_1), \tag{20}$$

where $a(x;s_0,s_1)$ is the amenity distribution function in the city, and $\rho>1$ is a parameter reflecting higher levels of amenities in the suburb. However, as the suburb expands beyond a certain threshold, the level of amenities in the suburb may decrease due to a crowding effect or the loss of small-town charm. Thus, the amenity function for the suburb is specified as

$$a_s(x) = \begin{cases} a_s^0(x) & \text{if } b_s - \overline{b} \leq \overline{s} \\ a_s^0(x)\left(\dfrac{\overline{s}}{b_s - \overline{b}}\right)^{\varphi} & \text{if } b_s - \overline{b} > \overline{s} \end{cases} \tag{21}$$

where b_s is the radius of the outer boundary of the suburb, and φ is the elasticity of amenities with respect to the size of the suburb. In the simulations we set $\left(\rho, \overline{b}, \varphi\right) = (7.5, 22, 1.15)$.

Table 5 shows the simulated effect of optimal open space conservation in the city on suburban characteristics under alternative assumptions about the decay rate of open space amenities. The results suggest that when the decay rate of open space amenities is low enough, open space conservation in the city can have a significant effect on the suburb. For example, when the decay rate of open space amenities is η=0.15 or lower, the optimal choice for the city is to preserve a greenbelt at the city boundary. Because the conserved open space is located relatively close to the suburb, it provides a large amount of amenities to suburban residents. As a result, it attracts a large number of households into the suburb. The higher demand for land raises land prices and development densities, which will lead to a larger tax base, increased tax revenue, and a higher level of public services in the suburb. However, as the decay rate of open space amenities increases, less land is conserved for open space, and the land conserved is also located farther away from the suburb. The resulting effect on the suburb decreases. When η is 0.4 or larger, the optimal open space conservation in the city has little effect on the suburb.

Conclusions

In this paper we explore the impacts of open space conservation on urban landscapes. We first consider the case where the location and size of open space are exogenously determined, and then focus on the optimal location, size, and budget of open space conservation.

The analysis leads to several interesting findings. First, open space conservation can change the defining features of an urban environment, including land prices, development densities, property tax rates, and the level of municipal services. A well-designed open space program can increase the level of municipal services and total property values within the city. However, because of the public financial impacts, an ill-designed conservation program can lead to higher property tax rates, lower levels of municipal services, and lower property values.

Second, our results reveal that economy of scale in the provision of municipal services and the level and spatial distribution of open space amenities are key parameters determining the fiscal and land value impacts of open space conservation. If open space conservation diverts tax revenue away from producing highly non-rival municipal services, but provides a relatively low level of amenities, it increases the property tax rate, reduces the level of municipal services, and decreases the total property value within the city. These conditions tend to be satisfied in cities with a large amount of open space, but are struggling to provide essential municipal services. In those cities, open space conservation shrinks the tax base and diverts tax dollars away from essential municipal services.

Third, additional open space conservation can increase total property value only if it increases the level of municipal services. Open space conservation is more likely to increase the level of municipal services when i) it provides a larger amount of amenities, ii) it preserves less expensive land, and iii) it removes land developable for high-density, low-income housing. Parcels satisfying all three conditions may include brownfields in cities that have high development densities and relatively little open space. Preserving such brownfields for open space, instead of developing them for low-income, high-density housing, will likely increase the total property value and the level of municipal services in the city. Indeed, the USDA Forest Service has programs designed to support communities that want to convert existing brownfields into natural open space parks and other land conservation projects.

Fourth, our results reveal trade-offs in targeting open space conservation. If a relatively wide greenbelt can generate only a small amount of open space amenities, and open space amenities decay relatively quickly, a greenbelt located within the city is more likely to be optimal. Conversely, if a relatively narrow greenbelt can generate a large amount of open space amenities, and the open space amenities decay relatively slowly with distance, a greenbelt located near the city boundary is likely to be optimal because the land cost is lower there and more people can live near the greenbelt (due to a larger circumference).

Finally, open space conservation can have severe equity consequences. Although households located near open space benefit from open space amenities, households located farther away may see their property values decreasing because they may have to pay more property taxes and receive less municipal services. This may lead to redistribution of households and changes in property values throughout the city. Local jurisdictions and conservation organization must consider equity consequences when designing appropriate mechanisms to finance their conservation efforts.

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Conflict of Interest The authors declare that they have no conflict of interest.

Appendix

Proof of Proposition 1

When households have homogeneous preferences, the maximization problem (6) can be simplified to

$$\underset{(\tau,g)}{Max} \quad \frac{g^{\mu}}{c+\tau} \text{ s. t. } TC_s + TC_o \leq TR.$$

Substituting Eq. (1) into Eqs. (2)–(4), the total tax revenue and the total costs of municipal services and open space conservation can be written as:

$$TR = \frac{\tau g^{\mu}}{(c+\tau)} \iint_{D-S} A[y(u,v)-tx]^{\beta_i} a(u,v;S)^{\gamma_i} dudv \equiv \frac{\tau g^{\mu}}{(c+\tau)} \widetilde{R}, \tag{A1}$$

$$TC_s = gN^{\lambda} = g^{1+\lambda\mu\delta} \left[\iint_{D-S} A^{\delta}[y(u,v)-tx]^{\beta_i\delta} a(u,v;S)^{\gamma_i\delta} dudv \right]^{\lambda} \equiv g^{1+\lambda\mu\delta} \widetilde{N}^{\lambda}, \tag{A2}$$

$$TC_o = \frac{cg^{\mu}}{(c+\tau)}\iint_S A[y(u,v)-tx]^{\beta_i} a(u,v;S)^{\gamma_i} dudv \equiv \frac{cg^{\mu}}{(c+\tau)}\widetilde{G}, \tag{A3}$$

where $y(u, v)$ denotes the income of the household located at (u, v). Using (A1)–(A3), the city's budget constraint can be written as:

$$\frac{cg^{\mu}}{c+\tau} \leq \frac{M(g)}{\widetilde{R}+\widetilde{G}}, \tag{A4}$$

where $M(g) \equiv g^{\mu}\widetilde{R} - g^{1+\lambda\mu\delta}\widetilde{N}^{\lambda}$. At the optimum, the constraint must hold with equality; otherwise, a smaller τexists that satisfies the budget constraint and improves the objective function. Substituting (A4) into the objective function, the maximization problem (5) can be transformed to

$$\underset{(\tau,g)}{Max}\ \frac{M(g)}{\widetilde{R}+\widetilde{G}}, \tag{A5}$$

with τ being determined by the budget constraint that holds with equality. Note that $M(g)$ is a concave function and reaches its maximum at

$$g^* = \left[\frac{\mu\widetilde{R}}{(1+\lambda\mu\delta)\widetilde{N}^{\lambda}}\right]^{\frac{1}{1-(1-\lambda\delta)\mu}}, \tag{A6}$$

which gives Eq. (8).

Thus, if $\overline{G} < M(g^{\rightharpoonup})$, τ^* defined by the budget constraint at g^* is positive, and (τ^*, g^*) is the optimal solution of (A1). Substituting g^* into the budget constraint and solving for τ, we obtain:

$$\tau^* = \frac{c(1+\lambda\mu\delta)}{1-(1-\lambda\delta)\mu}\left(1+\frac{\widetilde{G}}{\widetilde{R}}\right) - c. \tag{A7}$$

By definition,

$$1-\rho = \frac{TC_o}{TR} = \frac{c\widetilde{G}}{\tau\widetilde{R}} \Rightarrow \frac{\widetilde{G}}{\widetilde{R}} = (1-\rho)\frac{\tau}{c} \tag{A8}$$

Substituting (A8) into (A7) and solving for τ, we obtain Eq. (6).

If $\overline{G} < M(g^{\rightharpoonup}), \tau^{\rightharpoonup}$ defined by the budget constraint at g^* is positive, and (τ^*, g^*) is the optimal solution of (A5). If $\overline{G} \geq M(g^{\rightharpoonup})$, no (τ, g) combination would satisfy the budget constraint, and (A5) has no solution.

Proof of Corollary 1

Differentiating $\log\left(\widetilde{G}/\widetilde{R}\right)$ with respect to S (i.e., expanding the boundary S parallally to all directions by an infinitesimal amount) gives:

$$\frac{d}{dS}\log\left(\frac{\widetilde{G}}{\widetilde{R}}\right) = \frac{1}{\widetilde{G}}\left[\frac{\partial \widetilde{G}}{\partial S}\bigg|_a + \iint_S A(y-tx)^{\beta_i}\gamma a^{\gamma_i}\frac{\partial a}{\partial S}dudv\right] - \frac{1}{\widetilde{R}}\left[\frac{\partial \widetilde{R}}{\partial S}\bigg|_a + \iint_{D-S} A(y-tx)^{\beta_i}\gamma a^{\gamma_i}\frac{\partial a}{\partial S}dudv\right] = \left[\varepsilon_S^G + \gamma\varepsilon_S^a\right] - \left[-\varepsilon_S^R + \gamma\varepsilon_S^a\right] = \varepsilon_S^G + \varepsilon_S^R \geq 0. \tag{A9}$$

Likewise, differentiating $\log\left(\widetilde{R}/\widetilde{N}^{\lambda}\right)$ with respect to S gives

$$\frac{d}{dS}\log\left(\frac{\widetilde{R}}{\widetilde{N}^{\lambda}}\right) = \left[-\varepsilon_S^R + \gamma\varepsilon_S^a\right] - \lambda\left[-\varepsilon_S^N + \gamma\delta\varepsilon_S^a\right] = \left(\lambda\varepsilon_S^N - \varepsilon_S^R\right) + \gamma\,\varepsilon_S^a\,(1-\lambda\delta) \tag{A10}$$

which is greater than or equal to zero if and only if (13) holds.

Proof of Corollary 2

Differentiating $\log\left(\widetilde{R}^{1+\lambda\delta\mu}/\widetilde{N}^{\lambda\mu}\right)$ with respect to S, we obtain:

$$\frac{d}{dS}\log\left(\frac{\widetilde{R}^{1+\lambda\delta\mu}}{\widetilde{N}^{\lambda\mu}}\right) = (1+\lambda\delta\mu)\left[-\varepsilon_S^R + \gamma\varepsilon_S^a\right] - \lambda\mu\left[-\varepsilon_S^N + \gamma\delta\varepsilon_S^a\right] = \left[\lambda\mu\varepsilon_S^N - (1+\lambda\delta\mu)\varepsilon_S^R\right] + \gamma\varepsilon_S^a \tag{A11}$$

which is non-negative if and only if (15) holds.

Proof of Proposition 2

Suppose the high-income households are the majority in the city. The maximization problem (6) can be written as

$$\underset{(\tau,g)}{Max}\ \frac{g^{\mu_h}}{(c+\tau)} \quad \text{s.t.} \quad \frac{M(g)-\overline{G}}{\left(\widetilde{R}_l g^{\mu_l} + \widetilde{R}_h g^{\mu_h}\right)} \geq \frac{1}{c+\tau}, \tag{A12}$$

where $M(g) = \left(\widetilde{R}_l g^{\mu_l} + \widetilde{R}_h g^{\mu_h}\right) - g\left(\widetilde{N}_l g^{\mu_l} + \widetilde{N}_h g^{\mu_h}\right)^{\lambda}$. At the optimum, the constraint must hold with equality. Substituting the budget constraint into the objective function, the maximization problem can be transformed to

$$\underset{(\tau,g)}{Max}\ \frac{M(g)-\overline{G}}{\widehat{R}(g)} \quad \text{s.t.} \quad \frac{M(g)-\overline{G}}{g^{\mu_h}\widehat{R}(g)} = \frac{1}{c+\tau}, \tag{A13}$$

where $\widetilde{R}(g) = \left(\widetilde{R}_l g^{\mu_l-\mu_h} + \widetilde{R}_h\right)$. Note that the objective function is continuous and bounded in $[0, \overline{g}]$, where $\overline{g}$ is the largest g defined by $M(\overline{g}) = 0$, beyond which $M(g) <$

0. Thus, the objective function has a globe maximum in $[0, \overline{g}]$. Denote the maximum point by g^* and $G^* \equiv M(g^*)$. If $\overline{G} < M(g^{\rightharpoondown})$, τ^* defined by the budget constraint at gi is positive, and (τ^*, g^*) is the optimal solution of (A13).

Denote the solution of (A13) by (τ_i^*, g_i^*) when income group i is the majority. (τ_h^*, g_h^*) is indeed the majority group if and only if

$$N_h^*(\tau_h^*, g_h^*) \geq N_l^*(\tau_h^*, g_h^*). \tag{A14}$$

Likewise, (τ_l^*, g_l^*), is indeed the majority group if and only if

$$N_l^*(\tau_l^*, g_l^*) \geq N_h^*(\tau_l^*, g_l^*). \tag{A15}$$

We now prove that either (A14) or (A15) must hold. If (A14) does not hold, then $N_h^*(\tau_h^*, g_h^*) < N_l^*(\tau_h^*, g_h^*)$. In this case, (A15) must hold because $N_h^*(\tau_l^*, g_l^*) \leq N_h^*(\tau_h^*, g_h^*) < N_l^*(\tau_h^*, g_h^*) \leq N_l^*(\tau_l^*, g_l^*)$. Similarly, we can prove that if (A15) does not hold, (A14) must hold. This proves that a majority equilibrium must exist in the city if $\overline{G} < M(g^{\rightharpoondown})$.

To derive the equilibrium property tax rate, we use the transformation $g = [u(c+\tau)]^{1/\mu_h}$ and $\tau = \tau$ to transform the maximization problem (A13) into:

$$\max_{(\tau, u)} \quad u \text{ s.t. } TC_s + \overline{G} = TR_h + TR_l. \tag{A16}$$

where

$$TC_s = g(N_l + N_h)^\lambda = (c+\tau)^{\frac{1}{\mu_h}} u^{\frac{1}{\mu_h}} (N_h + N_l)^\lambda, \tag{A17}$$

$$N_h = [(c+\tau)u]^\delta \widetilde{N}_h, \quad N_l = (c+\tau)^{\frac{\delta\mu_l}{\mu_h}} u^{\frac{\delta\mu_l}{\mu_h}} \widetilde{N}_l, \tag{A18}$$

$$TR_h = \tau u \widetilde{R}_h, \quad TR_l = (c+\tau)^{\frac{\mu_l - \mu_h}{\mu_h}} u^{\frac{\mu_l}{\mu_h}} \widetilde{R}_l, \tag{A19}$$

The Lagrangian function for the maximization problem is

$$L(\tau, u) = u + \xi[TR_h + TR_l - TC_s - TC_o]. \tag{A20}$$

Where ξ is the Lagarangian multiplier. Differentiating (A20) with respect to τ and setting it equal zero, we obtain the following first-order condition:

$$\frac{\partial L}{\partial \tau} = 0 \Rightarrow \frac{\partial TC_s}{\partial \tau} = \frac{\partial TR_h}{\partial \tau} + \frac{\partial TR_l}{\partial \tau}. \tag{A21}$$

Assume household Differentiating (A17) with respect to τ and using (A18) gives

$$\begin{aligned} \frac{\partial TC_s}{\partial \tau} &= \frac{1}{\mu_h}(c+\tau)^{\frac{1}{\mu_h}-1} u^{\frac{1}{\mu_h}} (N_h + N_l)^\lambda + (c+\tau)^{\frac{1}{\mu_h}} u^{\frac{1}{\mu_h}} \lambda (N_h + N_l)^{\lambda-1} \left(\frac{\partial N_h}{d\tau} + \frac{\partial N_l}{d\tau} \right) \\ &= \frac{TC_s}{(c+\tau)} \frac{1}{\mu_h} + \frac{\lambda TC_s}{N} \left(\frac{\partial N_h}{\partial \tau} + \frac{\partial N_l}{\partial \tau} \right) \end{aligned} \tag{A22}$$

Differentiating (A10) with respect to τ and substituting the results into (A22), we obtain

$$\frac{\partial TC_s}{\partial \tau} = \frac{TC_s}{(c+\tau)}\left[\frac{1}{\mu_h} + \frac{\lambda\delta}{N}\left(N_h + \frac{\mu_l}{\mu_h}N_l\right)\right]. \tag{A23}$$

Differentiating (A19) with respect to τ gives

$$\frac{\partial TR_h}{\partial \tau} = \frac{TR_h}{\tau} \tag{A24}$$

$$\frac{\partial TR_l}{\partial \tau} = \frac{TR_l}{\tau} + \frac{\mu_l - \mu_h}{\mu_h}\frac{TR_l}{(c+\tau)} \tag{A25}$$

Substituting (A23)–(A25) into (A21) and noting $TR=TR_h+TR_l$ and $TC_s+TC_o=TR$ gives

$$\frac{TC_s}{(c+\tau)}\left[\frac{1}{\mu_h} + \frac{\lambda\delta}{N}\left(N - N_l + \frac{\mu_l}{\mu_h}N_l\right)\right] = \frac{TR}{\tau} + \frac{\mu_l - \mu_h}{\mu_h}\frac{TR_l}{(1+\tau)} \tag{A26}$$

$$\rho\left[\frac{1}{\mu_h} + \lambda\delta\left(1 + \frac{\mu_l - \mu_h}{\mu_h}r_l\right)\right] = \frac{(1+\tau)}{\tau} + \frac{\mu_l - \mu_h}{\mu_h}m_l, \tag{A27}$$

where $\rho=TC_s/TR$, $r_l=N_l/N$, $m_l=TR_s/TR$. Solving (A27) for τ gives (18). The result for the case where low-income households are the majority can be similarly derived.

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