GENERALIZATION OF AGE-STRUCTURED MODELS IN THEORY AND PRACTICE

Stein Ivar Steinshamn, Center for Research in Economics and Business Administration,
stein.steinshamn@snf.no

ABSTRACT

A generalization of the harvesting functions and the stock updating functions in age-structured bioeconomic models is outlined. Using this generalization everything from completely uniformly distributed fish to extreme schooling is taken care of. The classical Beverton-Holt model comes out as a special case of the generalized model. Both the theoretical outline as well as practical numerical examples are provided, and the generalization can be applied both for simulation as well as optimization purposes given appropriate software.

Here we apply advanced non-linear programming to maximize the net present value in a bioeconomic setting where the new updating and harvesting functions are used as constraints. This is possible thanks to new software, such as KNITRO, for solving highly nonlinear problems. Applications of this generalized model produce interesting new results. One such practical result is that pulse fishing seems to become less and less economically profitable as we move from uniformly distributed fish to schooling species. The main reason why pulse fishing cease to be optimal in schooling fisheries, is that the economies of scale present in search fisheries gradually disappear when we move from search fisheries to schooling fisheries. This may have important implications for how fish stocks ought to be managed in the future, especially with respect to total allowable catches based on bioeconomic criteria.

Keywords: Age-structured modeling, pulse-fishing

SNF

Outline

- I'm going to show:
  - How age-structured models can be generalized.
  - What this generalization can be used for.
  - Results beased on the generalized model compared to classical age-structured modelling.

SNF

- Number of fish at time *t*

$$N(t) = \left\{ \left[N_0^{1-\alpha} + \frac{qE}{M} \right] e^{-M(1-\alpha)t} - \frac{qE}{M} \right\}^{\frac{1}{1-\alpha}}$$

SNF

- Number of fish at time t

$$N(t) = \left\{ \left[N_0^{1-\alpha} + \frac{qE}{M} \right] e^{-M(1-\alpha)t} - \frac{qE}{M} \right\}^{\frac{1}{1-\alpha}}$$

- When $\alpha \to 1$ this reduces to

$$N(t) = N_0 \cdot e^{-(M+F)t}$$

where $F = qE$

SNF

- Total catch:

$$C_t = \int_t^{t+1} qE_t \left\{ \left[N_t^{1-\alpha} + \frac{qE_t}{M} \right] \cdot e^{-M(1-\alpha)\tau} - \frac{qE_t}{M} \right\}^{\frac{\alpha}{1-\alpha}} d\tau$$

- Total catch:

$$C_t = \int_t^{t+1} qE_t \left\{ \left[N_t^{1-\alpha} + \frac{qE_t}{M} \right] \cdot e^{-M(1-\alpha)\tau} - \frac{qE_t}{M} \right\}^{\frac{\alpha}{1-\alpha}} d\tau$$

- Solving the integral and taking the limit $\alpha \to 1$ we get

$$C_t = \frac{F_t}{M + F_t} N_t (1 - e^{-(M+F_t)})$$

where $F_t = qE_t$

Basic equations: definitions

- Instantaneous change in number of fish:

$$\frac{dN(t)}{dt} = -(F + M)N(t)$$

- Instaneous harvest

$$\frac{dC}{dt} = F \cdot N(t)$$

Basic equations

- In the Beverton-Holt model we have

$$F = qE$$

q is defined as a constant catchability coefficient. But it is constant only by hypothesis. Another way to write instantaneous catch is

$$\frac{dC}{dt} = k\rho E_t$$

where k is the selectivity and ρ is the density parameter.

The density parameter

- With uniformly distributed fish (as in Beverton-Holt) the density is proportional to number of fish:

$$\rho = \frac{N}{V}$$

- V is the volume of water that is screened. In this case the catchability coefficient is constant.

SNF

The density parameter

- With perfectly schooling species, on the other hand, the density parameter is constant:

$$\rho = h$$

but then the catchability coefficient becomes a function of number of fish.

SNF

General expression for the density

- Assume $\rho = hN^{\alpha}$ where $0 \le \alpha \le 1$.

- Then $q = \frac{k \cdot \rho}{N} = k \cdot h \cdot N^{\alpha - 1}$

and $F = q \cdot E = k \cdot h \cdot N^{\alpha-1} \cdot E$

Deriving the general expressions

- Substituting the general expression for F into the basic equations yields

$$\frac{dN}{dt} = -(F+M)\cdot N = -(k\cdot h\cdot N^{\alpha-1}\cdot E+M)\cdot N$$

Deriving the general expressions

- Substituting the general expression for F into the basic equations yields

$$\frac{dN}{dt} = -(F+M)\cdot N = -(k\cdot h\cdot N^{\alpha-1}\cdot E+M)\cdot N$$

- And solving this differential equation yields

$$N(t) = \left\{\left[N_0^{1-\alpha} + \frac{khE}{M}\right]e^{-M(1-\alpha)t} - \frac{khE}{M}\right\}^{\frac{1}{1-\alpha}}$$

SNF

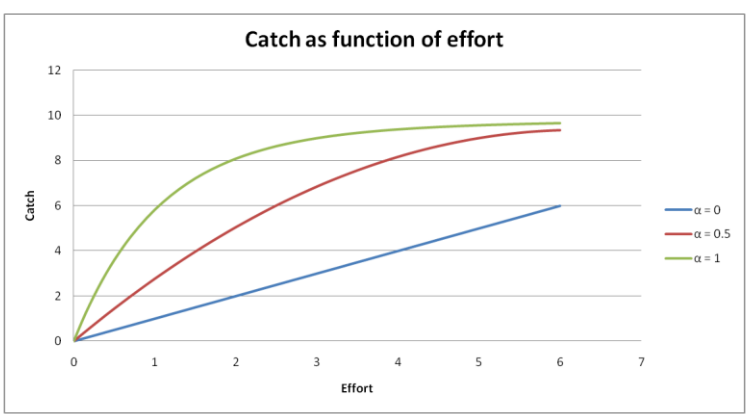
Deriving the general expressions

- Substituting the new expression for $N(t)$ into $\frac{dC}{dt} = F \cdot N$ and taking the integral yields total catch:

$$C_t = \int_t^{t+1} khE \left\{ \left[N_0^{1-\alpha} + \frac{khE}{M} \right] \cdot e^{-M(1-\alpha)\tau} - \frac{khE}{M} \right\}^{\frac{\alpha}{1-\alpha}} d\tau$$

- Unfortunately this integral is not solvable in general for all values of α.

SNF



SNF

Numerical optimization model

- Optimization method: Non-linear programming
- Objective function:
  - Maximize

$$\sum_t \sum_a p_{a,t} C_{a,t} - \sum_t c_t E_t$$

  - subject to

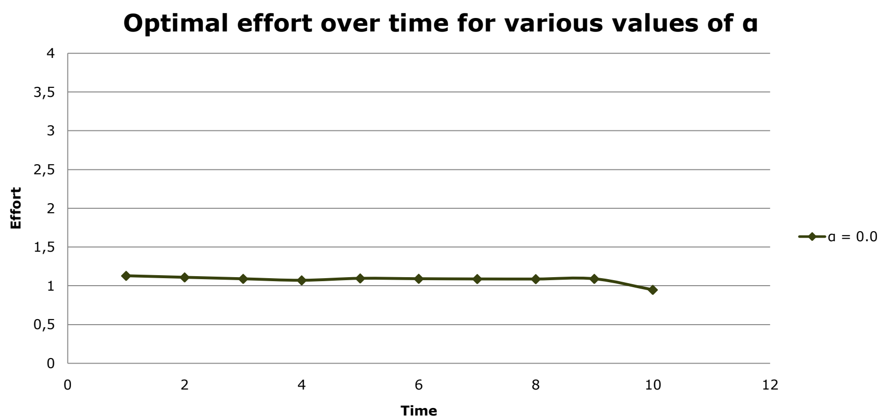
$$C_t = C(N_t, E_t; \alpha)$$
$$N_{t+1} = n(N_t, E_t; \alpha)$$
$$E_t \le E_{\max,t}$$

SNF

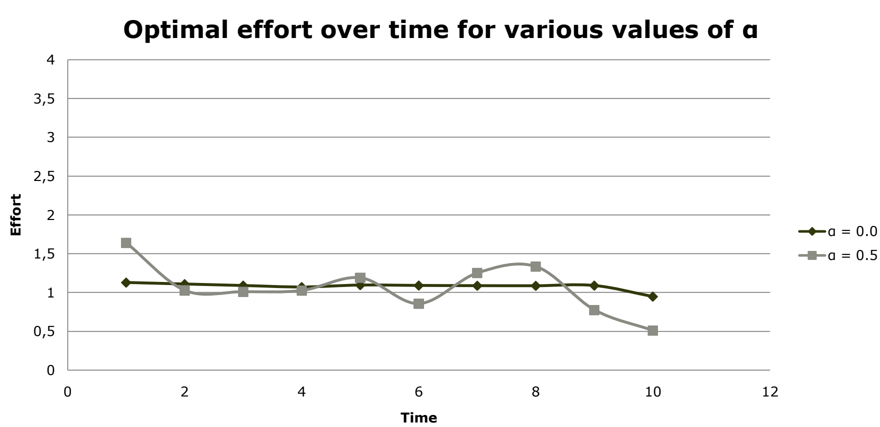
Results from stylized model

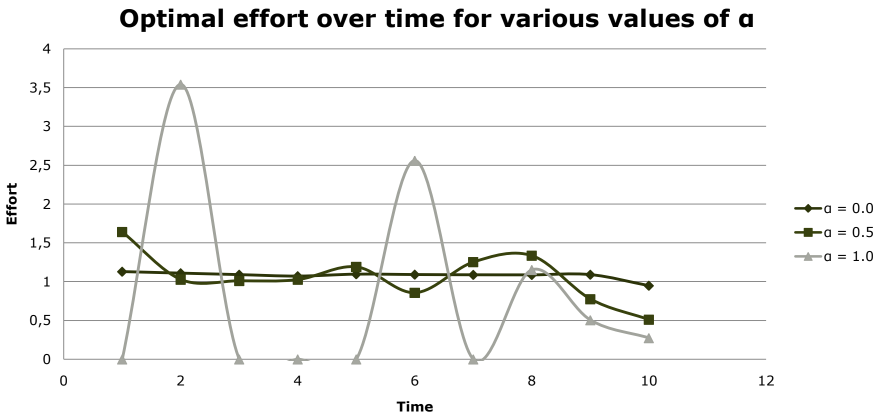
- Hypothetical fish stock
- 4 year-classes
- 10 periods
- No discounting
- Fixed recruitment
- Constant price of catch over time
- Constant cost of effort over time
- Sustainability constraint

SNF


Optimal effort over time for various values of α
4
3,5
3
2,5
2
1,5
1
0,5
0
Effort
0
2
4
6
8
10
12
Time
α = 0.0

SNF


Optimal effort over time for various values of α
4
3,5
3
2,5
2
1,5
1
0,5
0
Effort
0
2
4
6
8
10
12
Time
α = 0.0
α = 0.5



Why does pulse fishing cease to be optimal when we move from search fisheries to schooling fisheries?

- Economies of scale present in search fisheries disappear gradually.
- With uniformly distributed fish the instaneous catch rate is homogenous of degree 2.
- With schooling species the instaneous catch rate is independent of the stock, and hence there cannot be any economies of scale related to builing up the stock.

SNF

Summary

- Have generalized the updating and harvesting functions in age structured models.
- Makes it possible to incorporate everything from completely uniformly distributed fish to extreme schooling along a continuum.
- Pulse fishing typical for uniformly distributed fish cease to be optimal as we move along the continuum towards schooling species.
- This has implications for how optimal TACs ought to be set.