How Far Are Current Advisory Speeds from being Optimal? An Analysis Based on Safety Performance

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ABSTRACT

Posting advisory speed signs at sharp horizontal curves to provide the driving public with a safe speed is a practice well established in the United States. The operational effectiveness of these signs has long been questioned in the current literature. The authors of this paper recently developed a function to model the expected safety effect of these signs. The function stems from a statistical analysis on crash data from 2-lane rural highways in the state of Oregon.

In general, that research effort found that advisory speed signs tend to enhance safety. However, the authors also determined that advisory speed signs may not be displaying the value with the greatest potential safety benefit. Since the derived function proved meaningful from the engineering and human factors perspectives, these authors then extend the use of this function to compute and recommend the theoretically "optimal" advisory speed. A new posting procedure resulted from this effort. The authors compared the expected performance of advisory speeds from the proposed procedure to the speeds derived from current posting guidelines. A comparable performance suggests that current guidelines are close to the hypothetically "optimal" advisory speed. In general, both the current and new computational methods performed better than speeds determined by the ball bank indicator method.

This paper also presents a field validation analysis of the engine function of the new posting method. The results confirmed the meaningfulness of the function, and therefore, of the potential benefit for determining safety-based advisory speeds with the method proposed in this paper.

INTRODUCTION

The authors of this paper have recently proposed a new crash modification function (CMF) to account for advisory speeds in their recently completed effort for the Oregon Department of Transportation (ODOT) *(1)*. The authors coined this CMF the "Advisory Speed Crash Factor" or ASCF. The ASCF models how the displayed advisory speed relative to the speed limit and the associated side friction demand jointly associate with the likelihood of crashes. A recent paper by these authors *(2)* discusses a plausible human factors interpretation of the ASCF and establishes some of its basic mathematical properties. Because the main objective of the current paper is to use the proposed ASCF concept to develop a new posting procedure, it is necessary to provide a brief overview of that paper as presented in the section following the literature review.

This paper also further defines the mathematical properties of the ASCF. The authors only highlighted key issues as they pertain to the derivation of a basic equation for an "optimal" advisory speed.

The main section of this paper focuses on the proposed procedure (named the OSU method) and compares the resulting advisory speeds to advisory speeds from currently established procedures.

This paper includes a section on the robustness and field validation of the ASCF function. The material in this section is presented as a review of the relationship between the ASCF function and its meaningfulness for the proposed engineering application.

The authors performed all statistical procedures summarized in this paper using an open source statistical package *(3)*, *(4),* and *(5)* but similar analyses can be performed with comparable software.

BACKGROUND

The practice of posting advisory speed signs is well established in the United States. The procedures to determine advisory speeds have been evolving since the 1930s, and the practice has been standardized since 1948. The *Manual on Uniform Traffic Control Devices* (MUTCD) states that advisory speeds shall be determined by an "engineering study that follows established engineering practices" (*4* Section 2C.08). The document mentions three commonly accepted such practices: the use of a ball bank indicator (the most widely implemented), geometric design equation, and the use of an accelerometer. The thresholds for the ball bank method have been continually updated through subsequent editions of the MUTCD *(6).*

There is a wide variety of advisory speed-posting thresholds currently in use in the United States. ODOT has recently adopted the thresholds suggested by the latest edition of the MUTCD. Previously, Oregon used more conservative thresholds *(1)*.

LITERATURE REVIEW

A recent research effort *(7)* performed at the Texas Transportation Institute (TTI) for the Texas Department of Transportation observed that there were considerable inconsistencies for advisory speed posting procedures. This shortcoming appeared linked to the ball-bank and accelerometer approaches. Ultimately the TTI team recommended the use of the design speed equation approach, which yields more consistent values. The TTI team modified this approach to also incorporate a speed variable.

A recent study by Dixon and Rohani *(8)* suggests other sources of variation, since they found that a large proportion of curve sites in the state of Oregon do not comply with the state policy. Various authors argue that such lack of consistency results in poor adherence to advisory speeds *(7), (9), (10)* and *(11)*.

In an operational evaluation, Chowdhury et al. *(10)* argued that posting criteria are not adequate, since modern vehicles can generate a side friction ranging up to 0.90 before skidding out. Such vehicle performance amply exceeds the side friction demands associated with the ball bank indicator thresholds. Along those lines, Lyles and Taylor in their report "Communicating Changes in Horizontal Alignment" *(9)* argue that advisory speed signs are largely ineffective if the goal of the signs is that drivers adhere to the posted speed. They report that practitioners and the driving population perceive advisory speeds to be too low. This premise was also suggested on an operational assessment by Avelar *(11)*.

In the area of currently accepted safety modelling *(12)*, Elvik and Vaa *(13)* suggest a flat CMF for advisory speeds ranging from 0.71 to 0.87, depending on crash severity.

This literature review found only one paper by Ritchie *(14)* suggesting that advisory speeds may, contrary to expectation, incite drivers to higher speeds. The authors speculates that overconfidence may result from availability of information about the "sharpness" of curves immediately downstream, as the plaques convey.

THE SAFETY EFFECT OF ADVISORY SPEED SIGNS

Recent work by Oregon State University (OSU) researchers found a link between advisory speed signs and their hypothetical long term safety benefit *(2)*. The findings of the Oregon research effort do not necessarily contest current literature, which has repeatedly documented poor adherence. On the contrary, the authors of this paper deem a safety improvement possible, despite poor operational compliance, if these signs are successful in conveying meaningful information about the severity of downstream horizontal alignments. Drivers may then adjust their driving thus reducing their chances to be in a crash.

The authors proposed the ASCF function to model the safety impact of advisory speed signs. The function is directly derived from a statistical analysis performed on a probability sample of 210 directional horizontal curve sites representative of rural two-lane two-way state highways in Oregon. The data included geometric and signage features collected from field visits for a previous work by Dixon and Rohani *(8)*, as well as the crash record for the years 2000 through 2004 at the study sites. The authors proposed a full statistical model for non-intersection crashes, that is, excluding turn, rear and angle crash types. The data set included curves with radii ranging from 100 to 2150 ft, and deflection angles ranging from 1.5° to 200°. The next sub-sections sumarize the highlights of that work and further advances regarding the ASCF function.

Model Selection

Since crash data is random by nature, modelling techniques must be appropriately based on their stochastic variability. The authors applied a step-wise selection procedure based on the Akaike Information Criterion (AIC) to select a statistical model considering both the Poisson and Negative Binomial (NB) specifications for fitting a Generalized Linear Model (GLM). If the variance to mean ratio in the model is close to one, then two model specifications are adequate: the classical Negative Binomial (NB2) and the simpler Poisson. A ratio larger than one indicates

Poisson-overdispersion and in that case only the NB2 specification is appropriate. Since a goodness-of-fit evaluation from the regression output indicated no evidence of Poisson-overdispersion, both candidate specifications are equally adequate to describe the data. After some consideration, the authors endorsed the Poisson specification for its simplicity and because it allowed an alternative goodness-of-fit assessment. Equation 1 shows the resulting full model for the mean.

Equation 1 Full model.
$$E(\#Crashes) = 0.1554 \times AADT^{0.931} \times CurveLength^{-0.956} \times \exp[-0.282(LaneWidth) + 0.892(Angle) + 0.001(Radius) + 0.002(Angle \times Radius) - 0.004(AdvSpdPresent \times Radius) - 1.211(AdvSpdPresent \times Angle) + 4.026(AdvSpdPresent) + \{5.799(SFD) + 0.024(ASD) - 0.553(ASD \times SFD)\}]$$

Where:

#Crashes	=	Total non-intersection crash frequency (no units);
AADT	=	Annual Average Daily Traffic (vpd);
CurveLength	=	Length of the Curve (ft);
LaneWidth	=	Width of travel lane (ft)
Radius	=	Horizontal Radius (ft);
Angle	=	Horizontal Curve Central Angle (Radians)
SFD	=	Side Friction Demand at Advisory Speed (no units);
ASD	=	Advisory Speed Differential, defined as speed limit minus posted advisory speed (mph); and
AdvSpdPresent	=	Indicator variable equals to one when advisory speed signs are present, otherwise the value is zero.

All variable coefficients in Equation 1 satisfied at least a 0.95 level of confidence, except Radius, Angle and ASD. The authors retained these coefficients because the model includes statistically significant interactions for their variables (confidence levels of 0.995 or better). Based on the magnitudes of Variance Inflation Factors (VIFs), the authors determined that the standard errors in the final model were stable. The result is said to balance a minimum level of multicollinearity with the meaningfulness of the predictors from the engineering standpoint. Further details on the scope of application and statistical modeling can be found in references *(1)*, *(2),* and *(15).*

Interpretation of the Full Model

Although the proposed model specification is relatively simple, there is some complexity in the model interpretation emerging from the use of interaction terms among the covariates. However, the inclusion of interaction terms was crucial to reducing the model entropy (per the AIC statistic), increasing the goodness of fit, and ultimately, to developing the ASCF function.

Some predictors are inevitably interrelated in this case, even without modelling interactions. For instance, the horizontal radius and the deflection angle determine the curve length, and thus these three variables are correlated. It is no simple task to isolate the effect for any of these variables from the full model because it includes them simultaneously. However, characterizing such behaviour is not the focus of this research. The authors interpret the inherent

complexity in the model as a necessary mathematical way around the very probable case of non-linear underlying structures. Linear models are useful and powerful tools as far as they reasonably fit real world data. The actual relationship between crash occurrence and the relevant covariates, however, is likely not the convenient linear combination of relatively independent terms. Covariates that are expected to have more direct effects on crash occurrence shall be accounted for, but their simple interpretation becomes more challenging, as noted above. Further details on marginal effects for a model with interactions may be found in a previous work by these authors *(2)* and in Brambor et al. *(16)*.

The Advisory Speed Crash Factor

Equation 2 represents the functional form of the ASCF. Basically, the ASCF is a multiplicative factor applied to the "baseline" number of crashes, which is determined by the rest of variables in the statistical model. This is the reason why the ASCF is referred to as a sub-model throughout this paper.

Equation 2 Functional form of the ASCF.
$$ASCF = exp[5.799(SFD) - 0.553(ASD \times SFD) + 0.024(ASD)]$$

The concept of the ASCF is analogous to what existing literature refers to as a crash modification function (CMF). Two variables are involved in the ASCF functional form: the Advisory Speed Differential, or ASD (defined as the speed limit minus the advisory speed) and the Side Friction Demand associated with the advisory speed, or SFD *(17)*. In the case of sites not displaying advisory speeds, both the ASD and the SFD were computed using an advisory speed of 5 mph below the speed limit.

The ASCF proved a useful tool to estimate the safety benefit of the advisory speed signs in Oregon. These signs may be responsible for an average of 27% crash reduction at curve sites *(2)*. The Oregon study indicates that advisory speeds are, in general, safety enhancing elements at horizontal curve sites. Such results confirm the safety benefit associated with the signs, as long time assumed by the transportation community. To a certain extent, the results also abide current posting practices, despite of well documented consistency issues in the case of the ball bank indicator *(7), (8)*.

A closer examination of the mathematical properties of the ASCF function suggests an opportunity to develop a new computational posting procedure based on safety performance. The next section briefly explores such properties and their potential use for a posting procedure.

MATHEMATICAL PROPERTIES OF THE ASCF

The two variables that constitute the ASCF function are not mathematically independent. Both variables include the advisory speed in their formulation, though the ASD also includes the speed limit, while the SFD incorporates the radius and superelevation. For posting purposes, the authors considered the speed limit, radius, and superelevation as fixed parameters.

After applying a natural logarithm transformation, the ASCF can be expressed as a third degree polynomial representation of the advisory speed, as shown in Equation 3. This equation results from re-arranging Equation 2 as a polynomial of Adv.Speed when expressing ASD and SFD in terms of speed limit, advisory speeds, radius and superelevation.

Equation 3 Advisory speed univariate parameterization of ln(ASCF).

$$ln(ASCF)_{(Adv.Speed)} = (\beta_1 \times SpLim - \beta_2 \times SE - \beta_3 \times SE \times SpLim)$$

$$+(-\beta_1 + SE \times \beta_3) \times Adv.Speed$$

$$+\left(\frac{\beta_2 + SpLim \times \beta_3}{15 \times R}\right) \times (Adv.Speed)^2 - \left(\frac{\beta_3}{15 \times R}\right) \times (Adv.Speed)^3$$

Where:

β_1 = ASD coefficient from the ASCF function ($\frac{1}{mph}$);

β_2 = SFD coefficient from the ASCF function (no units);

β_3 = ASD x SFD coefficient from the ASCF function ($\frac{1}{mph}$);

SpLim = Speed Limit (mph);

Adv.Speed = Advisory Speed (mph);

R = Radius (ft); and

SE = Superelevation (no units);

Equation 3 directly links the advisory speed to a factor associated on the expected number of crashes. Most important is the known mathematical relationships of polynomials of the second or higher order to their local maximum and minimum values. Such local extremes are referred to as "optimal" values in the operations research literature.

The Theoretically "Optimal" Advisory Speed

If two different potential advisory speeds are compared using Equation 3, the "safer" advisory speed would be the one associated with the smaller ASCF. This observation leads to the following question: Is there an advisory speed such that the ASCF has a practical minimum value? From this point on, this particular advisory speed is referred to as the optimal advisory speed.

A relatively simple application of univariate calculus imposes the convexity and extreme point conditions on Equation 3 if an optimal advisory speed actually exists. These conditions can be expressed as:

$$\frac{d^2}{d(AdvSpeed)^2} Ln(ASCF)_{(AdvSpeed)} > 0; \text{ and}$$

$$\frac{d}{d(AdvSpeed)} Ln(ASCF)_{(AdvSpeed)} = 0.$$

The convexity is independent of the radius and the curve superelevation. Mathematically, it only requires the advisory speed be lower than the speed limit and that both variables have positive values. This condition holds for all advisory speed and speed limit candidate values. Therefore, the optimal advisory speed exists for virtually every 2 lane, 2 way rural highway situation.

There are two points satisfying the extreme point condition, but only the result shown in Equation 4 also achieves the convexity condition as discussed above. Equation 4, therefore, is the closed functional form of the theoretically optimal advisory speed.

Equation 4 Theoretically optimal advisory speed.

$$AdvSpeed_{Optimal} = \frac{-2\left(\frac{\beta_2 + SpLim \times \beta_3}{15R}\right) + \sqrt{\frac{4(\beta_2 + SpLim \times \beta_3)^2}{225R^2} + \frac{4\beta_3 \times (SE \times \beta_3 - \beta_1)}{5R}}}{-\frac{2\beta_3}{5R}}$$

It is important to note that the solution for Equation 4 depends on the coefficient estimates empirically determined. The Oregon State University posting method, presented in the next section, results in values directly applicable to Oregon rural highways. The authors later demonstrate that this equation tends to agree more with the national guidelines for posting signs than with the historically conservative Oregon policy values.

THE OREGON STATE UNIVERSITY POSTING METHOD

In order to propose a posting procedure based on Equation 4, the authors addressed relevant shortcomings inherent to the process of translating a purely theoretical result into a specific engineering application. In this section, the shortcomings are discussed and the solutions outlined. The emerging procedure is coined "The OSU method", named after the Oregon State University, the institution of affiliation for the authors.

The first shortcoming lies in the functional form of Equation 4. There is a mathematical singularity when the radius of the curve approaches zero. This mathematical caveat is verified when testing the equation at small radii curves. Large SFDs can be expected for sharp curve (smaller radii) locations. Because of modern vehicle performance, SFDs of 0.5 or more are not unfeasible for many passenger cars, but these larger SFDs would introduce a safety concern of other vehicle types such as trucks and trailers.

The authors then implemented a practical solution to this issue: If the side friction demand resulting from Equation 4 exceeds an acceptable threshold (e.g. 0.23, 0.25 or 0.3), then the preferred advisory speed shall be the largest speed that does not exceed that threshold.

The second issue of concern is the role of the regression coefficient estimates in Equation 4. The posting application of the ASCF coefficients is limited by the fact that Equation 1does not only account for these three ASCF terms, but also includes an indicator variable for the presence of advisory speeds. This means that Equation 1 assigns a different baseline of crashes to curves without advisory speeds than it does to posted curves. This makes sense in safety evaluation, where the ASCF is an effect of the advisory speed on expected crashes when comparing similar curves.

For the ASCF to accommodate the case of no-advisory speed needed (which occurs if the recommended advisory speed is within 5 mph of the posted speed limit), the authors repeated the statistical estimation of the function coefficients after removing the indicator variable for advisory speed from Equation 1. Doing this forces the only advisory speed coefficients remaining in the equation (i.e. the three ASCF coefficients) to account for as much variation associated with advisory speeds as structurally possible. The cost of this procedure, naturally, is a reduced goodness of fit. However, the authors advocate for the modified model because the meaning of the coefficients is more appropriate for a posting procedure; in that case two decisions are being made explicitly: the appropriate advisory speed, and if such advisory speed should be posted. Conversely, the coefficients from Equation 1 are estimated discounting that the

effect of the later decision is accounted for somewhere else in the model. The coefficients resulting from the reduced model are shown in Equation 5.

Equation 5 ASCF from reduced model.
$$ASCF = exp[3.98(SFD) - 0.399(ASD \times SFD) + 0.065(ASD)]$$

The authors are aware that these coefficients differ from those shown in Equation 2, as well as do the advisory speeds resulting from the two sets of coefficients. Even so, the authors deem each set useful for differentiated applications: Equation 2 for safety performance evaluation and Equation 5 for the proposed posting procedure.

The third and final issue with Equation 4 is the simplest to solve. Since posted advisory speeds are multiples of 5 mph, the new procedure shall recommend the advisory speed as such multiple of 5 value with the smallest ASCF possible, which occurs in the vicinity of the optimal advisory speed.

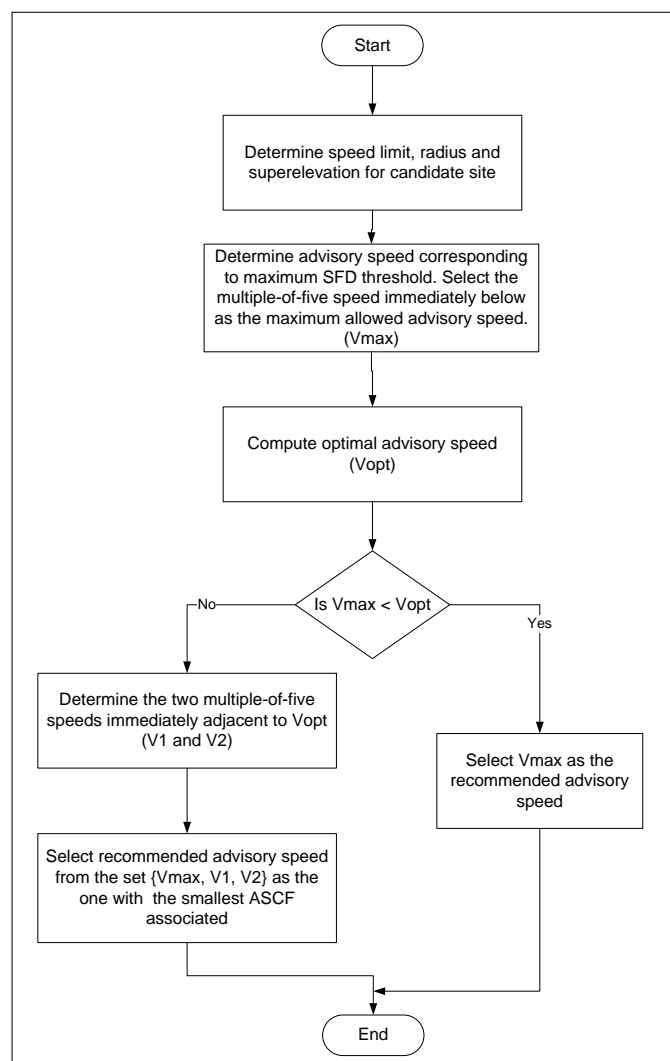The logical steps to implement the proposed posting procedure are represented in Figure 1.

FIGURE 1 Logical steps to the OSU posting method.

Relationship with Current Posting Criteria

The authors computed the OSU method advisory speed for the entire available Oregon state-wide sample using a maximum SFD of 0.23. Similarly, the authors also computed the theoretical MUTCD 2009 values for the sample of sites and compared the performance of these values to the current advisory speeds. This section reviews these comparisons.

   Both computational methods yielded larger values than currently posted advisory speeds in Oregon. The OSU method recommended, on average, higher advisory speeds. This observation can be summarized by comparing the raw averages: 42.69 mph for the current plaques, 44.95 mph for MUTCD and 45.16 mph for OSU. This result is not surprising, since a previous study identified the historic Oregon advisory speed policy as among the most conservative across the United States *(8)*. It is interesting to note, however, that the OSU and the MUTCD trends are more similar to each other than they are to the historic Oregon policy.

   When the authors contrasted the advisory speeds from the three methods to their associated SFDs, they observed that the current Oregon values were smaller than those obtained using the two computational methods (0.101 for the current plaques, 0.121 for MUTCD, and 0.124 for OSU).

   When comparing how the associated SFD varied by curve radius, the authors observed that the three sets of speeds tended to exhibit larger SFDs at smaller radii. On average, the MUTCD and OSU speeds result in SFDs 0.03 above the current values all across the radii range, as also suggested by the raw averages.

   Figure 2 shows a comparison of posting methods using the contour map of the ASCF function (a higher number of crashes correspond to the higher points in this surface).
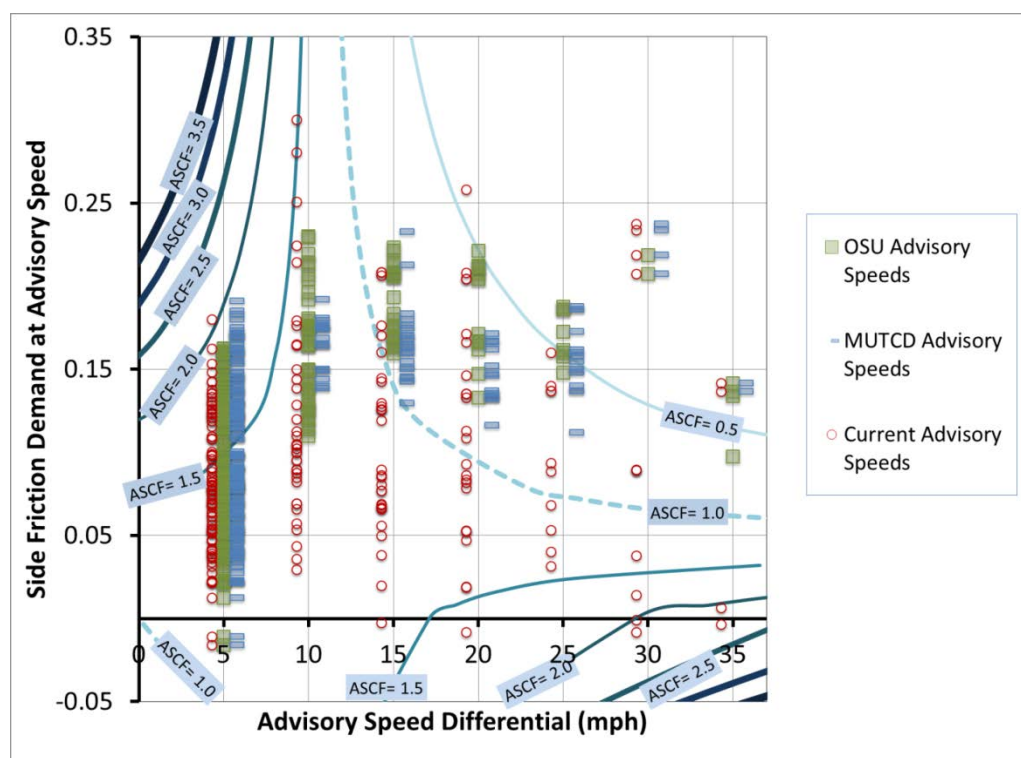

FIGURE 2 Comparison of posting methods over the ASCF contour map.

It is important to notice that OSU advisory speeds do not land along the diagonal of symmetry for the surface (as they would be expected) precisely because these speeds were calculated using Equation 5 but the contour correspond to the Equation 2 coefficients for the reasons exposed when deriving the OSU method. Current advisory speeds are notably more dispersed than any of the two computational methods. It is also obvious that current advisory speed tend to favour low advisory speeds that are coupled with lower SFDs, and as a result, they fall closer to the "horizontal ASCF hill" that is located along the ASD axis. Advisory speeds from the MUTCD method tend to fall in a roughly horizontal line when they are explicitly posted (i.e. ASD>5mph). This trend is probably reflecting that this method mostly ponders SFD as a criterion disjoint from the corresponding ASD to certain extent. Finally, though OSU speeds tend to favour larger SFDs but this trend also draws very close to the MUTCD set. Though this comparison is somehow informative, the authors consider that the posting methods should better be contrasted to the theoretical scenario when advisory speeds are not present.

Figure 3 displays the theoretical safety performance as it relates to the Advisory Speed Differential (Speed Limit minus Advisory Speed). The Absolute ASCF or AASCF is the ratio of the ASCF at the advisory speed to the ASCF resulting if the advisory speed was set just below the speed limit. This reference ASCF is particularly meaningful for advisory speeds close to the speed limit. Figure 3 demonstrates that regardless of the posting criterion, lower advisory speeds tend to be more beneficial.
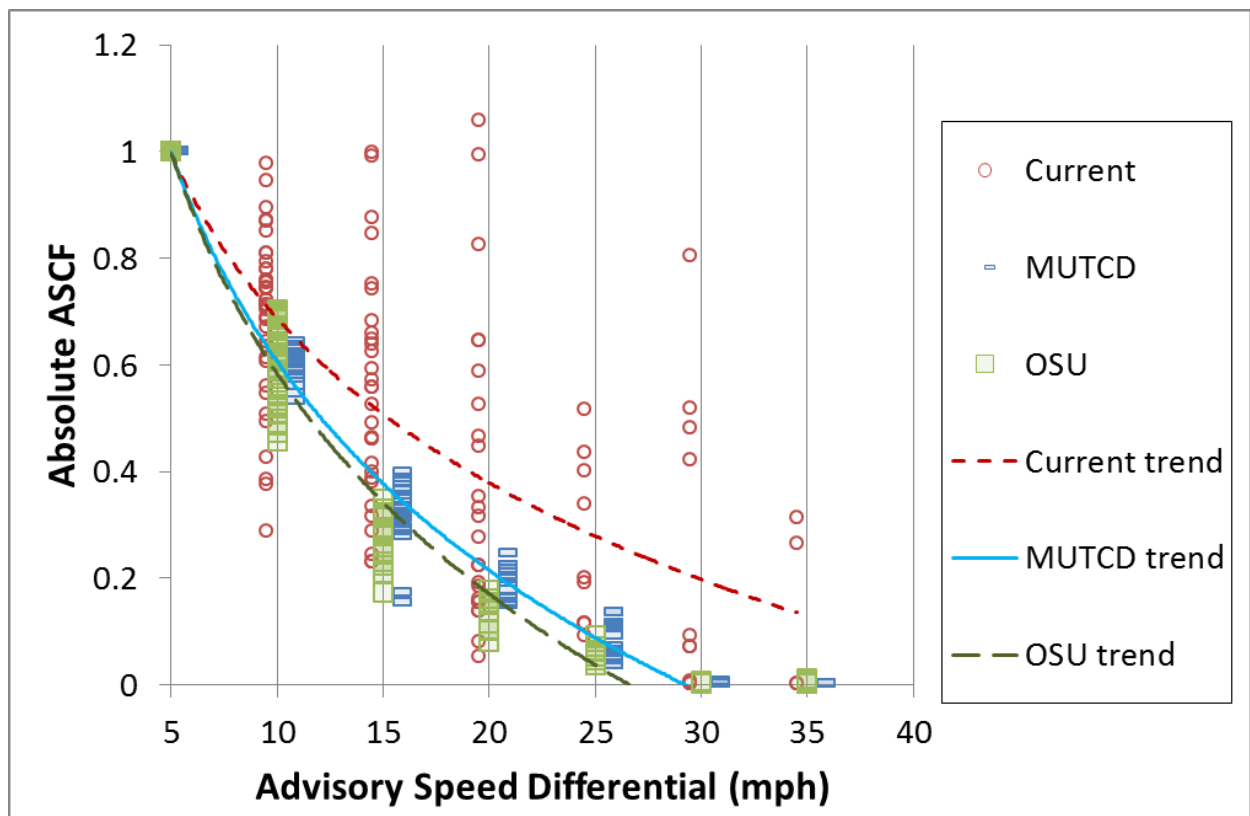

FIGURE 3 Absolute ASCF by posting method vs. curve radius.

The trend is less distinct for the case of currently posted speeds, as they relate to more disperse AASCF values as the ASD increases. Interestingly, as in Figure 2, both the OSU and MUTCD criteria do not exhibit excessive variation. This observation resonates with previous work that suggested consistency issues may be associated with the use of the Ball Bank indicator method *(5, 8)*.

Finally, the trends in Figure 3 suggest that both the MUTCD and the OSU criteria are expected to have a safety performance better than the currently posted speeds. Although the trend lines are very comparable, the OSU criterion appears to improve its safety performance at a slightly faster rate as the advisory speed differential increases.

Given these comparisons, it is not surprising how closely the OSU and the MUTCD methods performed. It is possible to map both constituent elements of the ASCF directly to the current posting guidelines. Table 2C-5 of the MUTCD 2009 encourages the inclusion of advisory speeds and other signage, such as chevrons, based on the difference between the advisory speed and the speed limit (the ASD in this analysis). At certain thresholds, their postage becomes mandatory. It is also possible to theoretically establish a cause-effect relationship between the Ball-Bank indicator angle and the SFD through the articulation of vehicle dynamics and road geometry (the original basis for the ball-bank application).

Discussion of Results and their Scope

The authors recognize that, similar to determining speed limits, posting criteria for advisory speeds are affected by technical and social trends. The authors expect that Equation 4 incorporates such elements implicitly through the use of ASCF empirically determined coefficients.

The authors rely on the fact that the statistical analysis was performed based on a probability sample and believe that the coefficients in this paper are not biased towards particular site characteristics, and that they represent a balanced average of factors such as the various levels of laxity in posting advisory speeds, severity of law enforcement at different jurisdictions, current vehicle fleet, curve sharpness, proportion of crashes by type, and severity of weather conditions among others. The authors recognize that operating speeds upstream of horizontal curves are influential on traffic operations at the curves. The authors expect, however, that operating speeds are roughly accounted for in the ASD by the speed limit, as operating speed would rise or fall to a significant extent as a response to higher or lower speed limits. In this regard the authors consider that the fact that advisory speeds obtained from the OSU method positively correlates with speed limit, as verified in a sensitivity analysis, is an indication that the OSU method is sensitive to traffic operations prevailing upstream the curve, as has been suggested by other researchers *(7)* and *(18)*.

ROBUSTNESS OF THE ASCF FUNCTION: FIELD VALIDATION ANALYSIS

This section presents the field validation analysis of the full-model and the ASCF sub-model, and is provided as supplemental evidence of the substance behind the ASCF model.

Field Validation Based on a New Sample of Sites

During July of 2011, the authors collected another independent sample consisting of 44 new curve sites so as to field validate both the full model and the ASCF sub-model. These sites were selected randomly from the state-maintained rural highways in Oregon including a regional subset distributed across four counties. This new sample comprised a wide variety of geometric and operational characteristics: radii ranging from 110 through 1800 ft, superelevations between 1% and 15%, and AADTs between 474 and 6160 pcph. The data set also included six sites without speed plaques and three sites that were located at 45 mph speed zones.

The researchers obtained crash records for the period 2003 to 2007 and identified a total of 29 crashes at the validation sample sites. The largest number of crashes at a particular location was five. The authors could not locate any recorded crashes at 27 of the sites.

Overall Goodness-of-fit

Some literature cautions that traditional goodness of fit criteria may be misleading for count models where the mean is predicted as a small value in combination with a small sample size *(19)*. Due to this concern, the authors developed an alternative goodness-of-fit test metric so as to relax the assumption of large sample sizes that the Maximum Likelihood Estimation methods rely upon.

The regression model is the parameterization of the expectation of a random "response" variable conditioned to the values of a vector of covariates. Statistical theory *(20)* relates the conditional, joint, and marginal expectations of random variables as shown in Equation 6.

Equation 6 Conditional, marginal and joint probabilities relationship for random variables.

$$P(Y = y, \vec{X} = \vec{x}) = p(y, \vec{x}) = p_{\vec{x}}(\vec{x}) \times p(y|\vec{x})$$

Where:

y	=	predicted variable;
$\vec{x}$	=	vector of predictors;
$p(Y = y, X = \vec{x})$	=	joint probability of y and $\vec{x}$;
$p_{\vec{x}}(\vec{x})$	=	marginal probability of $\vec{x}$; and
$p(y|\vec{x})$	=	conditional probability of y given $\vec{x}$.

In the frame of this proposed test, every data point is equally weighed. It is simple then to obtain the joint probability of both the response variable and the vector of predictors. The marginal probability of *y*, the response variable, can be obtained in turn by integrating the joint probability over all the available realizations of $\vec{x}$, the vector of predictors.
This logic is valid without any assumptions regarding the relationships between the variables, and it may be applied to any given conditional probability distribution, such as Equation 1.

Finally, it is possible to predict the expected frequencies for values of *Y* by substituting the Poisson probability function in Equation 6 and solving as described, by integration, for the marginal distribution of '*y*'. This marginal distribution is then used to predict the frequency of sites with '*y*' crashes, for a sample of size of *n*. Equation 7 shows this result.

Equation 7 Expected frequency of Sites with '*y*' crashes in the validation sample.

$$E.F._{\cdot(y)} = \sum_{i=1}^{n} \frac{e^{\left[y\vec{\beta}.\vec{x}_i - \left(\exp(\vec{\beta}.\vec{x}_i)\right)\right]}}{y!}$$

Where:

$E.F.(y)$ =   Expected Frequency of sites with 'y' crashes; and

n   =   Number of sites in the validation sample.

TABLE 1 Goodness-of-fit Test over the Observed Distribution of Crashes

Observed Number of Crashes	Actual Frequency of Sites	Expected Frequency of Sites
0	27	30.3398
1	9	9.9078
2	6	2.5815
3 and more	2	1.1709
Total	44	44
	Chi-Squared Statistic	5.5649
	p-value	0.1348

This equation may be used to assess the overall goodness of fit of the model without the need to mandate any assumptions about the sample size or the size of the predicted mean. In fact, the concern about a low count in the response variable is now removed, because the new count variable is in this context the number of sites with a particular number of crashes, as opposed to evaluating Equation 1, where the corresponding count is the number of crashes. Table 1 shows the results of the goodness of fit test just outlined. The resulting p-value supports the hypothesis of the model adequately fitting the validation data set.

Validation of the ASCF

The authors developed and performed a GLM estimation for a partition of the vector of covariates in order to find the statistical significance of the predicted values for the ASCF and the associated crash baselines.

The analysis revealed that both partition coefficient estimates were statistically different from zero (p-values of 0.012 and $<2\times10^{-16}$ respectively). The estimation found no significant evidence of Poisson-overdispersion (p-value of 0.09 for a 54.812 residual deviance on 42 degrees of freedom, for a dispersion parameter estimate of 1.3), which is also evident at an aggregate level from Table 1.

The authors computed a global estimate of the probability of a type I error. A very small p-value of 2.03×10^{-11} from a Hotelling's T^2 test (which considers simultaneously both the partition coefficients) increases the confidence on the validity of the full model. This p-value represents the probability of both the baseline and ASCF terms being as significant under the assumption that they were significant in the original sample only by chance (this is the default assumption, the null hypothesis).

Additionally, it was possible to estimate the statistical power of this analysis, since it is testing specific coefficient expectations, which implies a single point alternative hypothesis. The probability that the analysis would result in a type-II error was found as a p-value of 0.084. This value was computed from a Hotelling's T^2 test, which considers both the partition coefficients simultaneously. The corresponding statistical power of the validation is 91.6%. The statistical power is the probability that this analysis rejects the null hypothesis when both the baseline and ASCF terms are in fact as found in the original analysis (this is the alternative hypothesis).

Similarly, both type-I and type-II errors can be obtained for the ASCF partition alone. In this case, the probability of a type-I error was 0.0057, which indicates that it is unlikely that the ASCF sub-model effect may be attributed to chance only. However, the statistical power in this case is moderate, 71.7%, which indicates the need of a larger sample to increase the confidence on the actual ASCF coefficients.

Final Remarks on the Validation Analysis

Based on the field validation analysis, the authors are confident about the relevance and validity of the model as a safety performance function. Adequate goodness of fit on a second independent sample of curve sites indicates a good predictive power.

This confidence also extends to partitioning the model in baseline crashes versus the ASCF sub-model. Although this further analysis deems the ASCF contribution to the full model statistically significant (i.e. its coefficients are statistically different from zero), a mild statistical power for the given sub-model indicates that such a result may not be almost certainly expected as are the overall fit and predictive power of the full model. However, the authors embrace the postulate of an actual ASCF effect, considering the favourable evidence in the modelling and validation samples (both rejecting the hypothesis of a null ASCF), as well as the plausible human factors articulation of such an effect, as described in a previous work by these authors *(2)*.

Because of space limitations, this section did not include the mathematical and statistical procedures in their full extension. Such procedures will soon be published as part of the doctoral dissertation of one of the authors of this paper, including: a formal demonstration of Equation 7 with its statistical properties, the GLM analysis on the partition of the vector of predictors, and specifications of the null and alternative hypothesis for such analysis.

CONCLUSIONS AND RECOMMENDATIONS

The main objective of this paper is to develop a procedure to post advisory speed plaques directly based on their expected safety performance. Such a procedure is based on the main criterion of the Advisory Speed Crash Factor. The ASCF describes, to the authors' satisfaction, how safety performance is statistically related to the two covariate functions associated with the advisory speed: the Advisory Speed Differential and the Side Friction Demand.

The authors derived a closed-form equation to determine the theoretically optimal advisory speed. Such a theoretical optimal speed is thought to balance the human factors effects that the authors induce underlie the ASCF: Drivers adjust their behaviour considering and balancing the information the ASD and SFD variables carry jointly. The ASD is thought to indicate how much slower drivers should be navigating the curve while the SFD is thought to indicate the level of discomfort the driver will experience for a given advisory speed.

The authors identified and addressed the issues naturally expected from deriving an engineering application from a theoretical concept. The resulting procedure is the OSU method. They then contrasted this newly developed method with both the MUTCD recommended values as well as the currently displayed advisory speeds in Oregon. Both the OSU method and the MUTCD produced advisory speed values that are believed to perform better than currently posted speeds. In that comparative analysis, it became apparent that advisory speed values based on a computational method (either the OSU or MUTCD) offer, in general, more consistent values than actual advisory speeds that most likely have been determined by the ball bank indicator method. As a result, the authors share the opinion of researchers who encourage the use of computational alternatives *(7)*. The authors deem the safety-performance-based OSU formulation a viable alternative among other computational methods already available in the literature.

The close performance of the OSU method and the MUTCD criterion is not surprising. It is possible to link the ASCF components to the MUTCD posting guidelines in a meaningful way. This finding suggests that MUTCD procedures yield values that are almost optimal, if indeed there is an "optimal" advisory speed under the current conditions of generalized driver understanding of the signs.

The authors performed a field validation analysis in order to test the robustness of the ASCF function. The analysis verified the predictive power of the function over the number of crashes of an independent sample of curve sites. As a consequence, the authors recommend two direct engineering applications stemming from the ASCF function: for safety assessment, as previously demonstrated in the case of Oregon rural highways, and the determination of advisory speed values for new sites, by using the OSU method, as outlined in this paper.

Finally, the authors recommend future work to explore the link of the ASCF to field operational data. Specifically, future research should explore how the operating speed relates to the components of the ASCF bivariate function. The authors also recommend future research exploring alternative analysis tools to verify these results, as well as calibrating the OSU method using data from other states because general driver awareness and understanding of the signs probably varies significantly across jurisdictions.

ACKNOWLEDGEMENTS

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