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Title: AN ECONOMIC ANALYSIS OF THE SHORT-RUN DEMAND FOR TIMELINESS WITH SPECIAL REFERENCE TO FARM MACHINERY PARTS

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This thesis is an attempt to develop a theoretical microeconomic model which can be used to examine the short-run demand for the timeliness of farm machinery repairs. This analysis focuses on the timing of the repair after a breakdown has occurred.

The nonstochastic model developed allows the incorporation of the timing of the repair as a variable input into a production function. A yield function (a function which gives the instantaneous rate of output in bushels per acre as a function of the date of harvest) is used in deriving this production function. From the production function a demand curve for the timeliness of repairs can be derived.

A constrained input demand curve (CIDC) is used to examine the demand for timely repairs. A specific functional form of the yield function is used in order to allow an easier examination of how various parameters affect the CIDC.

Several testable hypotheses which result from the model are presented. An attempted test of one of the hypotheses is discussed.

An Economic Analysis of the Short-run Demand for Timeliness with Special Reference to Farm Machinery Parts

by

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I. INTRODUCTION

The breakdown of farm machinery is not a new problem. Nor is it a problem that is likely to go away. Machinery breakdowns have been with the farmer since the very beginning of mechanization.

An economic analysis of the repair and maintenance of farm machinery can take on many forms depending upon the issues addressed.[1/] This study uses a microeconomic approach in constructing a model to derive and examine a hypothetical farmer's demand for timeliness of machinery repairs.[2/] Though the problem developed and analyzed in this paper involves only the timing of the application

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[1/] Two issues of the repair and maintenance situation which are not examined in this paper are:

1. Determination of a maintenance policy
2. The choice between repairing or replacing a broken machine

Maintenance policies have been discussed in the literature for some time. Jorgenson, McCall and Radner (1967) provide a good summary of the theory in this area.

The resolution of the choice between repair or replacement of a broken machine is a capital budgeting problem, since most repairs would be expected to last longer than one production period.

The problem of this thesis is a short-run problem, whereas the two issues mentioned above are longer-run in nature.

[2/] The timeliness referred to is the timeliness of the repair after a breakdown has occurred.

of the repair input to the production process, it is believed that the same general approach can be used to examine the short-run timing of the application of other inputs.

When a farmer has a breakdown during a production period, there is an interruption in the production process. The interruption lasts until the equipment is repaired or replaced.[3/] The interruption itself is a short-run phenomenon since it affects only the inputs and output of the current production period. The analysis of the interruption would likewise be short-run.

Since only the interruption is examined, there is no need to be concerned about the particular part which has failed. The effect of the interruption is the same no matter what particular part failed, and it will continue until that part is repaired.

This thesis is an attempt to develop a theoretical model which can be used to examine the short-run demand for the timeliness of farm machinery repairs. The following is an outline of the remaining chapters:

[3/] Replacement may include the rental of equipment services. For the remainder of this thesis it will be assumed that the farmer chooses to repair the equipment. This assumption is made in order to focus the analysis on the short-run interruption and not on the long-run choice of whether to repair or replace.

II Description of the model

This chapter is devoted to the assumptions and the logical development necessary to derive a demand curve for the timeliness of machinery repairs.

III Analytical Examination of the Model

The conditions necessary for a negatively sloped demand curve are examined as its parameters are varied.

IV Empirical Analyses

Attempts to empiricize the model are discussed.

V Summary and Suggested Model Extensions

II. DEVELOPMENT OF THE MODEL

Derivation of an Input Demand Curve

An input demand curve is frequently referred to as a derived demand curve. This is because the input demand is derived from (a) the demand for output faced by the firm and (b) the firm's production function on the assumption of profit maximization.

For the case of a perfectly competitive firm producing one output with one variable input, the demand curve for the input can be derived in the following manner:

q is the quantity of output
x is the quantity of the variable input
p is the price of the output
r is the price of the input
$q = f(x)$ is the production function

The profit function for this firm is:

$$\pi = p f(x) - rx \tag{2.1}$$

The profit maximizing quantity of x may be calculated by setting $d\pi/dx = 0$ and solving for x (this assumes that the second-order condition for maximization is met).

$$d\pi/dx = pf'(x) - r$$

$$d\pi/dx = 0 \quad , \text{ for maximization}$$

$$\therefore \quad r = pf'(x) \qquad (2.2)$$

$f'(x)$ is the marginal physical product of x

Equation (2.2) describes the firm's demand relationship for the input, x.[4/]

Problems with Using Traditional Production Theory to Examine the Timing of Repairs

Carlson's "Mono-periodic production" model provides a useful way of summarizing traditional production theory. In his model

> The production activity starts at a given date and ends at another given date when the output of the production is sold on the market; the time interval between these two dates represents the period under consideration (Carlson, 1969, p. 4).

The mono-periodic production function is a relationship between various quantities of resources used and the output those resources will generate. The production function does not give all the levels of output possible from a given set of resources. It gives only one level of output for each set of resources, and the level of output given is

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[4/] Solving (2.2) for x as a function of r will give the firm's derived demand function for the input x.

"...the maximum product obtainable from the combination (of resources) at the existing state of technical knowledge" (Carlson, 1969, p. 14-15).

The production function contains the implicit assumption that the resources are applied to the production process at the precise moment when they will generate maximum output. Since the timing of resource application is not an explicit variable in the traditional production function, the traditional production function cannot be directly used to examine the timing of resource application.

Development of the Model

Since traditional production theory is inadequate to examine the timing of machinery repairs, a model will be presented which will allow the issues of timely machinery repairs to be examined. It should be noted that the traditional production theory is not being discarded, but only modified so that the timeliness of repair services will appear as an explicit input in a production function. The model is constructed from the situation faced by a crop farmer who experiences an unexpected breakdown during harvest. The model is essentially "short-run" in nature since it assumes that the decision involved does not (a) allow variation in variables determined prior to the harvest season, or (b) have any effect on decisions in subsequent production periods. The model will allow a theoretical examination of the conceptualized situation.

Assumptions

The following assumptions are made to delimit the model and describe more specifically the situation under analysis.

1. All inputs of the production process under the control of the farmer have been applied to the crop prior to the decision period considered in this model except those associated with harvesting.
2. The farmer produces only one crop and he sells his output at the end of the production period.
3. The cost of labor for the production period has been fixed.
4. The farmer does not anticipate any breakdowns, and when he has completed the repair he does not expect any more breakdowns.
5. All model variables are nonstochastic.

The Yield Function

The yield function relates the yield of the crop to two variables. The first variable is a time variable indicating dates during the harvest season when harvesting could occur. The second variable is a summary variable for all of the factors of production used prior to the harvest season and embodied in the standing crop.

YA = y(t, Y) is the yield function

YA is the yield of the crop in bushels per acre. (2.4)

t is the continuous measure of calendar time during the harvest season. The units are hours. t = 0 occurs before harvesting begins and t increases continuously until the harvest season ends.

Y is the variable summarizing all of the factors of production used prior to the harvest season. These factors are embodied in the standing crop.

As stated in the first assumption all of the inputs (machinery, fertilizer, water, labor, weather during the growing season, etc.) used in growing the crop are committed and cannot be added to or subtracted from the standing crop at the beginning of the harvest season. Therefore, for any given harvest season, Y is fixed.

For a particular harvest season (this implies a fixed Y), what does the yield function give? It says that if the total crop could be instantaneously harvested at t_i the yield of the crop in bushels/acre would be $YA_i[YA_i = y(t_i|Y)]$. However, the total crop cannot be harvested instantaneously. Therefore, the factors affecting the harvesting rate will need to be examined. This will be done when the harvesting function is examined. The following notes should be kept in mind about the yield function:

1. The yield function is constructed on the assumption of normal weather[5/] during the harvest season.

[5/] Normal weather is the average weather experience of several years.

2. For the purposes of this model Y acts only as a shift variable, shifting the relationship between YA and t.

The Harvesting Function

The harvesting function measures the capacity of the harvesting equipment in terms of the number of acres harvested per hour.

$$C = \frac{SWE}{825} \text{ is the harvesting function (A. S. A. E., 1963, p. 277)} \tag{2.5}$$

C is the instantaneous rate of harvest in acres per hour.

S is the instantaneous rate of speed of the harvesting machine.

E is the field efficiency[6/] of the harvesting equipment in percent.

W is the width of the harvesting machine in feet.

The capacity of the harvesting equipment increases as any of the three variables--S, W, or E--increase.

Once the harvest season starts it is assumed that the farmer

[6/] "Field efficiency includes the effects of overlap (failure to utilize full rated width of machine) and of time lost in the field as a result of:

Turning and idle travel at ends
Clogging of equipment
Adding seed or fertilizer
Unloading harvested products
Machine adjustments and minor repairs
Lubrication required in addition to daily servicing
Other minor interruptions

It does not include time losses due to daily servicing, traveling to or from the field, or major breakdowns" (A. S. A. E., 1963, p. 227).

cannot alter his equipment. This means that for a given harvest season S, W, and E are fixed.

It is possible to use the harvesting function to calculate how many hours it will take to harvest a particular farmer's crop. If the substitutions H = hours and A = acres are made, the harvesting function can be written as:

$$\frac{A}{H} = \frac{SWE}{825} \tag{2.6}$$

$$\therefore \quad H = \frac{A825}{SWE} \tag{2.7}$$

This equation will give the number of hours required to harvest a particular farmer's crop once his acreage and equipment capacity in terms of S, W, and E are substituted into the function. It should be noted that H is independent of any interruptions which might occur and of when the harvest starts; i.e., H is independent of t.

The Output Function

So far this chapter has developed the yield function which, for a given harvest season, tells the yield in bushels per acre for all acres harvested in a particular instant of time and the harvesting function which tells how many acres the farmer's equipment will harvest in a particular instant of time. By combining these two relationships a third relationship, the output function, can be generated.

$$B = \underbrace{y(t, Y)}_{\text{Yield function}} \underbrace{\frac{SWE}{825}}_{\text{Harvesting function}} \qquad \text{Output function} \qquad (2.8)$$

B is the instantaneous rate of output in bushels per hour

$$B = \frac{\text{Bushels}}{\text{Acre}} \cdot \frac{\text{Acres}}{\text{Hour}} = \frac{\text{Bushels}}{\text{Hour}}$$

The output function is the product of the harvesting function and the yield function. For a particular harvest season the output function is a linear transformation of the yield function. Therefore, in the short-run the shape of the output function is determined by the yield function.

Relationship between the Timing of Repairs and Output

To this point in the development of the model no mention has been made of when the harvesting occurs. Equation (2.7) gives the number of hours of machine operation required to harvest the crop, but it does not specify when during the harvest season these H hours occur. If there are no breakdowns, harvesting would continue uninterrupted from the starting data until all H hours of machine operation are completed and, thus, the starting data would be sufficient to describe when the harvesting takes place. However, if a breakdown occurs additional variables are needed to describe when the harvesting takes place.[7/] These additional variables describe when the breakdown occurs, and when the equipment is repaired. (Since this model is focusing on the timing of repairs, an index of timeliness will be used to examine variations in when the repair is made.) The following variables are used in the analysis of the harvesting operating. .

T_S is the specific value of t, a date, when the harvest started

T_B is the specific value of t, a date, when the breakdown occurred

T_F is the value of t, a date, when the equipment could be repaired at the minimum dollar outlay for the repair.

T_{FA} is the variable representing dates of "speedier" repair service

τ is the index of timeliness; it measures down time avoided by repairing at some T_{FA} instead of T_F.

The relationships between the time variables are given by the following equality and inequalities:

$$0 < T_S \tag{2.9}$$

$$T_S < T_B < T_S + H \tag{2.10}$$

$$T_B < T_F \tag{2.11}$$

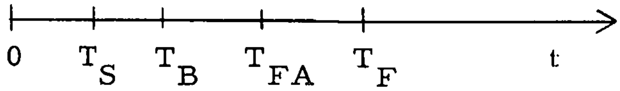
$$T_B < T_{FA} < T_F \tag{2.12}$$

$$\tau = T_F - F_{FA} \tag{2.13}$$

$$0 < \tau < T_F - T_B \tag{2.14}$$

Several of these relationships can be summarized in the following diagram:

7/ As indicated on p. 10, the value of H is unaffected by the occurrence or duration of any breakdown.



Verbally summarizing these relations:

(2.10) States that the farmer's equipment will not break down until after he starts to use it, and that the breakdown occurs before all of the crop is harvested.

(2.11) States that the equipment will not be repaired until after it breaks down.

(2.12) States that the date of "speedier" service must precede the date at which the equipment could have been repaired at minimum dollar outlay, but it must occur after the equipment has broken down.

(2.13) Defines an index of timeliness as the difference between the date the repair could be made at minimum outlay and the date of "speedier" repairs. This index is the number of hours of down time avoided.

(2.14) States that the number of hours of down time avoided must be between zero and the number of hours of down time which could be experienced.

The only argument of the output function which varies during the harvest season is t. Since the output function gives an instantaneous rate of output as a function of time, the integral of the output function over an interval of time will give the bushels of crop harvested during that interval.

The following integral gives the bushels of crop harvested prior to the breakdown.

$$\text{Bushels of output harvested before breakdown} = \int_{T_S}^{T_B} y(t \mid Y) \frac{SWE}{825}\, dt \qquad (2.15)$$

Since the timing of the repair has no effect on the quantity of output harvested prior to the breakdown, this portion of the harvest season need not be considered when the timing of the repair is examined.

The integral (2.16) gives the bushels of crop harvested if the farmer waits until T_F to have the repair made. $[\frac{A825}{SWE} - (T_B - T_S)]$ represents the number of hours required to complete the harvest.

$$\text{Bushels of crop harvested after repair is made at } T_F = \int_{T_F}^{T_F + \frac{A825}{SWE} - (T_B - T_S)} y(t|Y)\frac{SWE}{825}\,dt \qquad (2.16)$$

The integral (2.17) gives the bushels of crop harvested if the farmer has the equipment repaired at the "speedier" date T_{FA}.

$$\text{Bushels of crop harvested after repair is made at } T_{FA} = \int_{T_{FA}}^{T_{FA} + \frac{A825}{SWE} - (T_B - T_S)} y(t|Y)\frac{SWE}{825}\,dt \qquad (2.17)$$

The difference between these two integrals gives the increase in the total product attributable to speedier repairs.

$$Z = \int_{T_{FA}}^{T_{FA} + \frac{A825}{SWE} - (T_B - T_S)} y(t|Y)\frac{SWE}{825}\,dt - \int_{T_F}^{T_F + \frac{A825}{SWE} - (T_B - T_S)} y(t|Y)\frac{SWE}{825}\,dt \qquad (2.18)$$

Z is bushels of output

By substituting $T_{FA} = T_F - \tau$ the above equation can be rewritten as:

$$Z = \int_{T_F-\tau}^{T_F-\tau+\frac{A825}{SWE}-(T_B-T_S)} y(t|Y)\frac{SWE}{825}\,dt - \int_{T_F}^{T_F+\frac{A825}{SWE}-(T_B-T_S)} y(t|Y)\frac{SWE}{825}\,dt \qquad (2.19)$$

Z then can be written in general functional notation as:

$$Z = z(\tau, S, W, E, Y, A, T_S, T_B, T_F) \qquad (2.20)$$

This function describes the output associated with various levels of timely repairs, τ. In the short-run situation it has been argued that all of the variables in the function except τ are fixed. Therefore, it is argued that the above function is a short-run production function with the timing of repairs as an input, and the increase in bushels of crop harvested as the output.

Derivation of a Demand Curve for Timely Repairs

Earlier in this chapter the derivation of an input demand relationship was presented. That derivation depended upon the price of the input, the price of the output and the marginal physical product of the input. Since equation (2.20) is a production function with only one variable input, τ, the same approach can be used to derive the demand relationship for τ. The marginal physical product of τ is

$\partial Z/\partial \mathrm{T}$, and using the same definitions of r and p the profit maximizing condition becomes

$$r = p\,\partial Z/\partial \mathrm{T} \qquad (2.21)$$

Thus equation (2.21) describes the firm's short-run demand relationship for the input: timely repairs.

III. ANALYTICAL EXAMINATION OF THE MODEL

This chapter deals with two main topics. First, the nature of the yield function is examined and second, the demand curve for timely repairs is examined as parameters of the demand curve are changed.

Nature of the Yield Function

If the price of the input, τ, is nonnegative, the economically interesting situation occurs when the use of the input, τ, produces a positive quantity of output, Z.[8/] Positive Z values will be given by the production function (2.20) if $\partial Z/\partial \tau$ is greater than zero when

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[8/] The possible values of Z which can be given by (2.20) may be separated into three groups. The first two groups are trivial from an economic point of view, if τ has a nonnegative price. The third group is extensively examined in the body of the thesis. The groups are:

1. Values of Z less than zero
2. Values of Z equal to zero
3. Values of Z greater than zero

The production function (2.20) gives the quantity of output, Z, generated by various quantities of the input, τ. There is no economic reason to use a quantity of τ which yields no output, or actually decreases output. A profit-maximizing farmer would only consider using quantities of τ which yielded Z values in group #3.

τ is positive.[9]

$$\frac{\partial Z}{\partial \tau} = y([T_F - \tau] | Y) \frac{SWE}{825} - y([T_F - \tau + H - (T_B - T_S)] | Y) \frac{SWE}{825} \qquad (3.1)$$

For $\partial Z/\partial \tau$ to be greater than zero the following inequality must hold.

$$y([T_F - \tau] | Y) > y([T_F - \tau + H - (T_B - T_S)] | Y] \qquad (3.2)$$

Since Y is constant in this analysis[10] the following substitution will be made

$$y_1(t) = y(t | Y) \qquad (3.3)$$

where $y_1(t)$ is the general form of a yield function for a specific value of Y. Writing (3.2) in this new form and substituting T_{FA} for $[T_F - \tau]$ (from (2.13)) yields

$$y_1(T_{FA}) > y_1(T_{FA} + H - (T_B - T_S)) \qquad (3.4)$$

as the condition to be satisfied if $\partial Z/\partial \tau$ is to be greater than zero. This must be true for all possible values of T_{FA} and $[T_{FA} + H - (T_B - T_S)]$ given by (2.9 - 2.14).

A condition such as (3.4) leads to an examination of the possible forms the yield function might have. To examine this question all

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[9] From (2.19) it can be seen that when τ equals zero, Z will also be zero. This means that the production function (2.20) comes out of the (τ, Z) origin. Thus if $\partial Z/\partial \tau > 0$ the values of Z will always be greater than zero for positive τ.

[10] See Chapter II for determinates of Y.

continuous, differentiable functions with

$$y_1(t) > 0 \qquad \text{for} \quad t > 0 \tag{3.5}$$

will be separated into six mutually exclusive groups. Each group will be examined to see if it satisfies condition (3.4). The groups are:

1. Functions which are monotonically increasing
2. Functions which are monotonically decreasing
3. Functions which are constants
4. Functions with one local maximum and no local minima
5. Functions with one local minimum and no local maxima
6. Functions with one or more local maxima and one or more local minima

<u>Group 1</u>

Monotonically increasing functions do not satisfy condition (3.4). For any function, $g(x)$, to be monotonically increasing, it is necessary that

$$g(x) < g(x+k) \qquad \text{where} \quad k > 0 \tag{3.6}$$

Since $[H - (T_B - T_S)]$ is greater than zero (from (2.10)) any monotonically increasing yield function would violate (3.4).

<u>Group 2</u>

Monotonically decreasing yield functions satisfy (3.4). For

Figures depicting examples of each of the six groups of yield functions

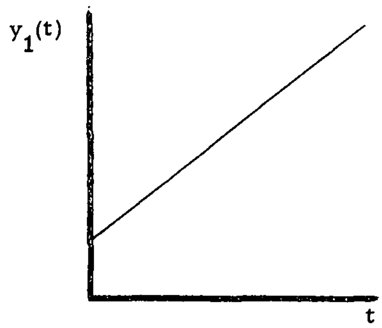

Figure 3. 1. Monotonically increasing

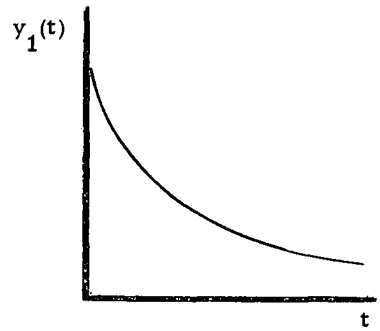

Figure 3. 2. Monotonically decreasing

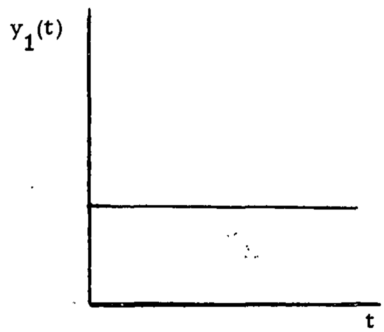

Figure 3. 3. Constant

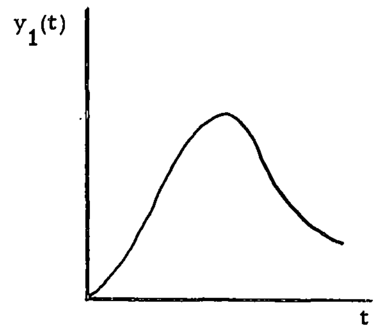

Figure 3. 4. One local maximum, no local minima

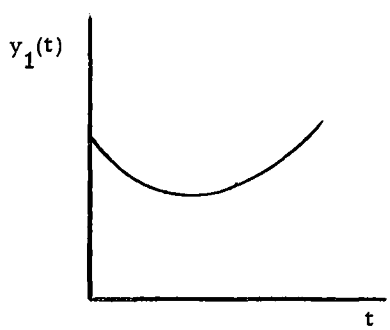

Figure 3. 5. One local minimum, no local maxima

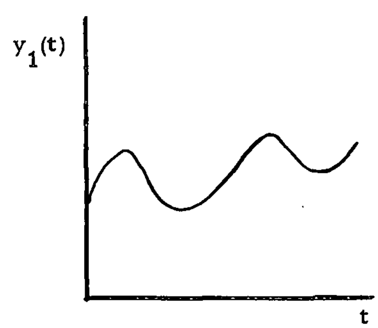

Figure 3. 6. One or more local minima and one or more local maxima

any function, $g(x)$, to be monotonically decreasing, it is necessary that

$$g(x) > g(x+k) \qquad \text{where} \quad k > 0 \tag{3.7}$$

By inspection it can be concluded that any function satisfying (3.7) will also satisfy (3.4).

Group 3

Functions which are constants do not satisfy (3.4). For any function, $g(x)$, to be equal to a constant, it is necessary that

$$g(x) = g(x+k) \tag{3.8}$$

By inspection it can be concluded that any function satisfying (3.8) will not satisfy (3.4).

Group 4

Functions with one local maximum and no local minima satisfy conditions (3.4) provided the following constraint is imposed:

$$T_S \geq T_S^* \text{ 11/} \tag{3.10}$$

11/ T_S is the variable constrained since it is the only short-run variable in (3.4) which can be directly controlled by the farmer. The other variable under the farmer's direct control is H, but its value is the result of long-run harvesting equipment decisions. Since this analysis is focusing on the short-run, H is assumed constant for any particular farmer.

The following argument is presented to demonstrate the necessity and logic of the constraint. Functions of Group 4 have both a rising and a falling portion. This leads to three possible relationships between T_{FA} and $[T_{FA} + H - (T_B-T_S)]$ which will be examined.

Case I

T_{FA} and $[T_{FA} + H - (T_B-T_S)]$ both occur on the rising portion of the function. This case implies the following inequality:

$$y_1(T_{FA}) < y_1(T_{FA} + H - (T_B-T_S)) \tag{3.11}$$

The inequality in (3.11) is in the opposite direction of the inequality in (3.4). Therefore, when T_{FA} and $[T_{FA} + H - (T_B-T_S)]$ both occur on the rising portion of the function, condition (3.4) is violated.

Case II

T_{FA} and $[T_{FA}+ H - (T_B-T_S)]$ both occur on the falling portion of the function. This case imples the following inequality:

$$y_1(T_{FA}) > y_1(T_{FA} + H - (T_B-T_S)) \tag{3.12}$$

i.e., condition (3.4) is satisfied. In fact, the same arguments and conclusions which apply to monotonically decreasing functions apply to this case.

Case III

T_{FA} is on the rising portion of the function and $[T_{FA} + H - (T_B - T_S)]$ is on the falling portion.

Even though T_{FA} and $[T_{FA} + H - (T_B - T_S)]$ are on opposite sides of the local maximum, it is possible for either (3.11) or (3.12) to hold; i.e., (3.4) may or may not hold. Which one is true depends upon the value of the arguments. The critical values for determining whether (3.4) always holds are the minimum values of T_{FA} and $[T_{FA} + H - (T_B - T_S)]$. If (3.4) holds at some minimum values of the arguments, (3.4) will always hold for all values in excess of those minimums. From (2.10) and (2.12) the following lower limits can be obtained for the arguments

$$T_{FA} > T_S \tag{3.13}$$

$$[T_{FA} + H - (T_B - T_S)] > T_S + H \tag{3.14}$$

Combining the information in (3.13) and (3.14) with (3.4), equation (3.15) is deduced.

$$y_1(T_S) = y_1(T_S + H) \tag{3.15}$$

Let T_S^* be the value of T_S which satisfies (3.15). Equation (3.15) establishes a lower limit on T_S [12] which if satisfied will always

[12] Ibid.

make (3.4) hold. If T_S is greater than or equal to T_S^* (3.10), condition (3.4) will always hold for this case.

From the three cases examined under Group 4, it can be concluded that if (3.10) holds, condition (3.4) will always be satisfied. The constraint (3.10) rules out the occurrence of Case I and includes all of Case II and the portion of Case III which always satisfies (3.4).

Group 5

Functions with one local minimum and one local maximum do not always satisfy (3.4). This can be demonstrated by the following example. If T_{FA} occurs at the local minimum then $[T_{FA} + H - (T_B - T_S)]$ must occur on the rising portion of the function. This violates (3.4) since

$$y_1(T_{FA}) < y_1(T_{FA} + H - (T_B - T_S))$$

Group 6

Functions with one or more local maxima and one or more local minima do not always satisfy (3.4). Since conditions (2.9 - 2.14) provide no upper bound on $[T_{FA} + H - (T_B - T_S)]$, there is no reason why the following could not occur.

$$y_1(T_{FA}) < y_1(T_{FA} + H - (T_B - T_S))$$

Thus, this group does not satisfy (3.4).

From the examination of the six groups, two groups emerge as always satisfying (3.4). They are:

> Functions which are monotonically decreasing (Group 2)
>
> Constrained functions with one local maximum and no local minima. (Group 4)

For analytical purposes, constrained functions with one local maximum and no local minima can be viewed as summarizing the other group, since Case II of Group 4 is similar to Group 2. Therefore, throughout the remainder of this chapter the constrained functions with one local maximum and no local minima will be used.

A Profit-Maximizing Farmer's Choice of T_S When Facing a Yield Function with One Local Maximum and No Local Minima

A profit-maximizing farmer selling output and buying inputs in competitive markets, who anticipates no breakdowns, will start his harvest on the date which will give him maximum output. The date the farmer starts harvesting is T_S, and harvesting will end, if there are no breakdowns, on $[T_S + H]$. The integral of the output function between T_S and $[T_S + H]$ gives the bushels of output harvested.

$$OP = \int_{T_S}^{T_S + H} y(t|Y) \frac{SWE}{825} \, dt \qquad (3.16)$$

OP is the total bushels of output harvested between T_S and $[T_S+H]$

The optimal starting date for the farmer must satisfy both the first and second order conditions for output maximization. These conditions are, respectively:

$$\partial OP/\partial T_S = 0 \tag{3.17}$$

$$\partial^2 OP/\partial T_S^2 < 0 \tag{3.18}$$

It should be noted that, as the starting date of harvest, T_S, varies, so does the output, but the date which maximizes the output is the starting date the profit-maximizing farmer chooses.

$$\frac{\partial OP}{\partial T_S} = y(T_S + H|Y)\frac{SWE}{825} - y(T_S|Y)\frac{SWE}{825} \tag{3.19}$$

Setting this derivative equal to zero yields

$$y(T_S + H|Y)\frac{SWE}{825} = y(T_S|Y)\frac{SWE}{825}$$

$$y(T_S + H|Y) = y(T_S|Y) \tag{3.20}$$

And the second order condition requires that

$$\frac{\partial^2 OP_2}{\partial T_S} = y'(T_S + H|Y)\frac{SWE}{825} - y'(T_S|Y)\frac{SWE}{825} < 0 \tag{3.21}$$

i.e., that

$$\frac{SWE}{825}[y'(T_S+H|Y) - y'(T_S|Y)] < 0 \tag{3.22}$$

The first order condition (3.20) states that the optimal T_S is chosen such that the ordinate values at T_S and $[T_S + H]$ are equal. In order to satisfy the second order condition the slope of the yield function must be greater at the critical T_S than at the critical $[T_S + H]$. For the yield functions under consideration, those with one local maximum and no local minima, the second order condition is always satisfied, since, when the first order condition is satisfied, the slope at the critical T_S is always positive and the slope at the critical $[T_S + H]$ is always negative.

It should be noted that the profit-maximizing value of T_S calculated in (3.20) is the same as the level of T_S^* calculated in (3.15). Thus, a profit-maximizing farmer who faces a yield function with one local maximum and no local minima will always have a positively-sloped production function for Z with respect to the input, timely repairs; i.e., $\partial Z/\partial T > 0$, and the constraint (3.10) can be viewed as coming from the profit maximizing assumption. This result means the constraint (3.10) can be derived from the assumptions given in Chapter II. Thus, (3.10) is not just a side condition imposed so that the model " would work" .

The Demand Curve for a Limited-Quantity Input

A firm's input demand curve relates all possible prices of an input to the quantity of that input demanded by the firm at those particular prices. For a perfectly competitive, profit-maximizing firm using only one variable input, the (short-run) demand curve for that input can be represented by the curve AB in Figure 3.7 for prices between O and C. For prices above C the demand curve is identical to the vertical axis. The quantity of input used is dependent upon the price of the input, the production function, and the product price.

However, what does the demand curve look like when there is some maximum quantity of the input available to the firm? The Constrained Input Demand Curve (hereafter CIDC) may differ from the traditional input demand curve described above. Three cases will be used to examine the effect of an input constraint on the demand curve. The cases are defined by the constraint's location in the three stages of production (Ferguson, 1966, p. 122-123).

Case I

The input constraint occurs in stage I (for stage I the domain of the variable input lies between zero and the point at which VAP[13/] curve is a maximum; i.e., O to J in Figure 3.7).

[13/] VAP stands for value of the average product; i.e., product price times average physical product.

Using a specific value for the constraint, G, (this implies that the firm may use any quantity of the input between O and G, but the firm cannot use more than G) the CIDC becomes:

a) for prices between O and I, the quantity of input used by the firm is described by the curve GH,

b) for prices above I the demand curve is identical to the vertical axis.

Case II

The input constraint occurs in stage II (for stage II the domain of the variable input lies between the point at which the VAP curve is a maximum and the point where the VMP[14/] curve is zero; i.e., J to B in Figure 3.7). Using a specific value for the constraint, E, (this implies the firm may use any quantity of the input between O and E, but the firm cannot use more than E) the CIDC becomes:

a) for prices between O and F, the quantity used by the firm is described by the curve ED,

b) for prices between F and C, the quantity used by the firm is described by the curve DA,

c) for prices above C the demand curve is identical to the vertical axis.

[14/] VMP stands for value of the marginal product; i.e., product price times marginal physical product.

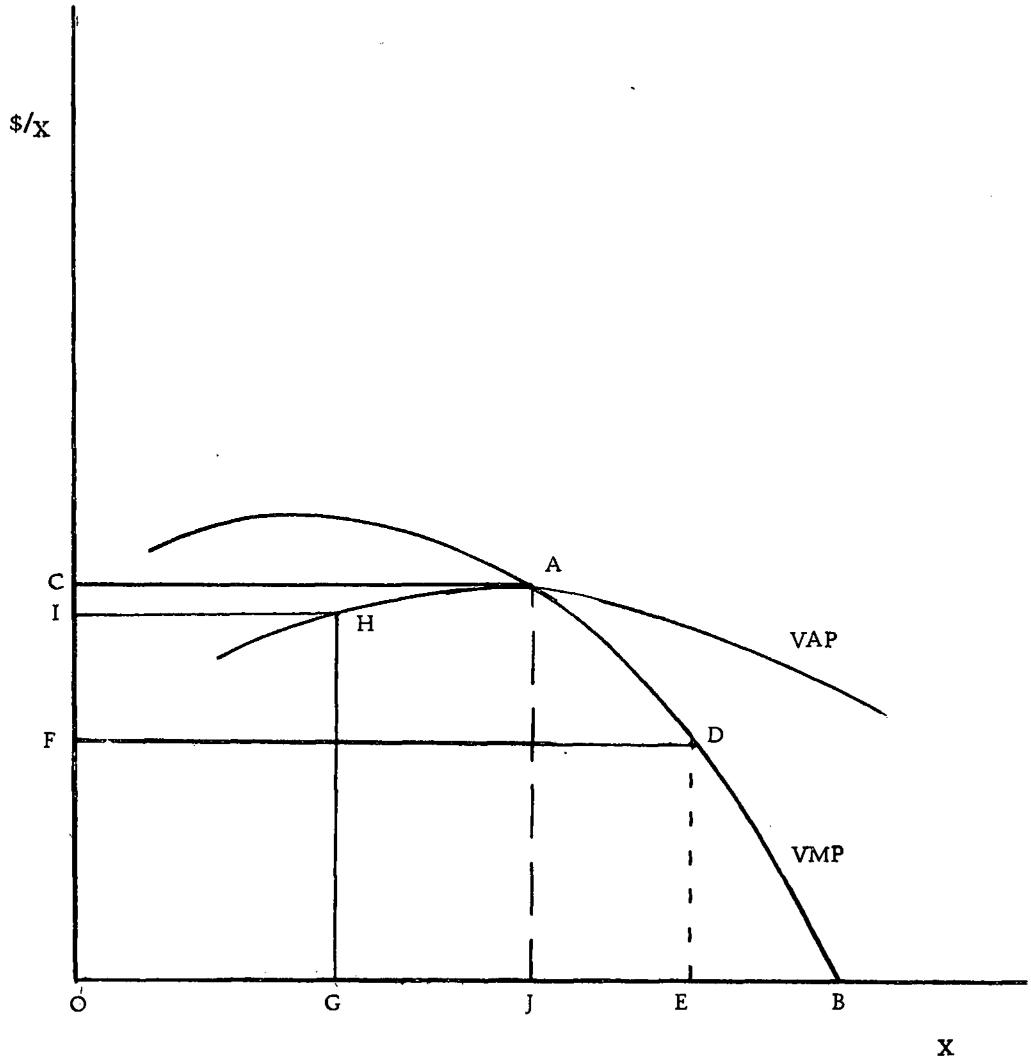

Figure 3.7. VAP and VMP curves

Case III

The input constraint occurs in stage III (for stage III the domain of the variable input lies to the right of the point at which the VMP curve is zero; i.e., to the right of B in Figure 3.7). In this case the constraint does not affect the traditional demand curve since the constraint is greater than the maximum quantity of input the firm would use at nonnegative prices.

It can be seen from the examination of the CIDC that the nature of the curve depends upon the level of the input constraint. When the quantity of input is unconstrained, or the constraint occurs in stage III, the CIDC becomes the traditional input demand curve.

Microeconomic theory texts usually argue that a profit maximizing producer will never operate in stage I of the production function (for an example, see Ferguson, 1966, p. 122). However, the input τ provides a contradiction to this argument. τ is different than most inputs used by the firm since there is an absolute maximum quantity of τ available to the firm. This maximum quantity of τ is given by (2.14). There is no particular reason why the maximum quantity of τ given by (2.14) should occur in any particular one of the three stages of production. Thus, it is possible that the maximum quantity of τ will occur in stage I, and, depending upon the price of τ, the profit maximizing firm will choose to use either none of the input or the maximum quantity of it. By using the CIDC, the demand for the input τ can be examined.

There are four characteristics of the CIDC which will be helpful in determining how the CIDC shifts. They are:

1. Position of the constraint
2. Position of the maximum of the VAP curve
3. Height of the VAP curve
4. Height of the VMP curve

These are the same characteristics used to define and describe the CIDC in the three preceding cases.

Shifts in the CIDC for τ

This section is devoted to an analytical examination of various determinants of the CIDC (demand) for τ. The ultimate purpose of this examination is the generation of testable hypotheses.

The determinants of the CIDC for τ are the variables of the VAP and VMP curves. To this point these variables have been held constant in deriving the CIDC for τ. Now, some of the variables and relationships between variables will be allowed to change so that their effect on the CIDC for τ can be examined. To derive and describe what happens to the CIDC for τ the four characteristics of the CIDC outlined above will be used.

In order to simplify the derivations and exposition in this section, a specific functional form of the yield function will be used. The yield function chosen is

$$y_i(t) = t\,e^{t/-a} \qquad a \geq 1 \tag{3.23}$$

a is the parameter for this family of functions

This particular function was chosen because it satisfies the requirements of having one local maximum and no local minima and it is relatively easy to integrate. Figure 3.8 is a graph of (3.23). This function begins at the origin, reaches a maximum at a, has a point of inflection at $2a$, and is asymptotic to the t axis.

Though the results of this section are deduced from a particular functional form, it is believed that they can be generalized to other yield functions with one local maximum and no local minima. On substituting the yield function (3.23) into (2.19) and on performing the integration operation, the production function for Z becomes

$$Z = \frac{SWE}{825}\Big[-a\,e^{\frac{T_F-\tau+H-(T_B-T_S)}{-a}}\left(a+T_F-\tau+H-(T_B-T_S)\right)+a\,e^{\frac{T_F-\tau}{-a}}\left(a+T_F-\tau\right)$$

$$+a\,e^{\frac{T_F+H-(T_B-T_S)}{-a}}\left(a+T_F+H-(T_B-T_S)\right)-a\,e^{\frac{T_F}{-a}}\left(a+T_F\right)\Big] \tag{3.24}$$

From (3.24) the value of the marginal product and value of the average product curves for τ can be calculated. Denoting the price of the

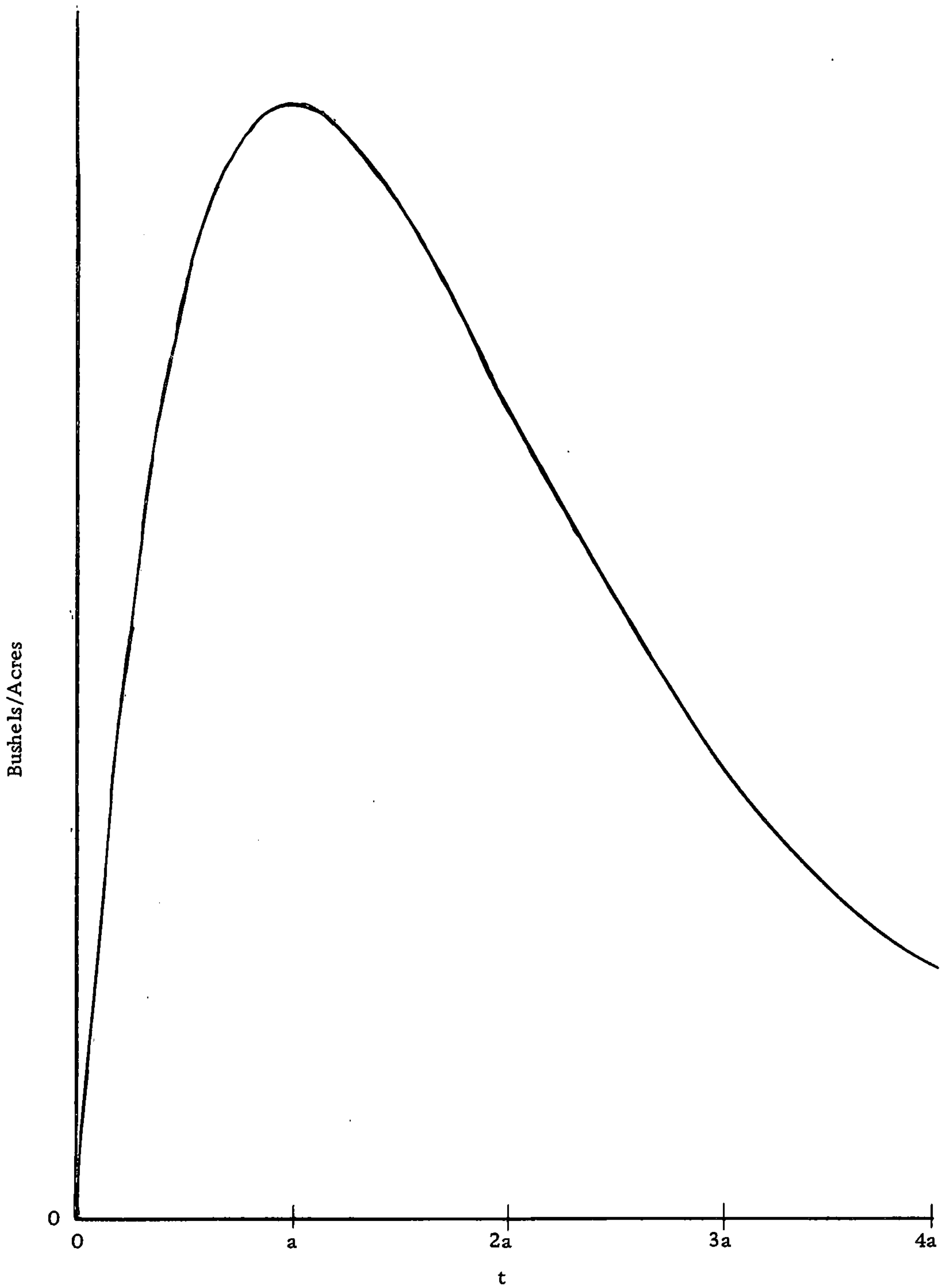

Figure 3.8. Yield function used in analysis

output by P, the value of the marginal product for τ is

$$VMP = \frac{PSWE}{825}\left[-(T_F-\tau+H-(T_B-T_S))e^{\frac{T_F-\tau+H-(T_B-T_S)}{-a}} + (T_F-\tau)e^{\frac{T_F-\tau}{-a}}\right] \tag{3.25}$$

The value of the average product of τ is

$$VAP = \frac{P}{\tau}\,\frac{SWE}{825}\Big[-ae^{\frac{T_F-\tau+H-(T_B-T_S)}{-a}}(a+T_F-\tau+H-(T_B-T_S)) + ae^{\frac{T_F-\tau}{-a}}(a+T_F-\tau) + ae^{\frac{T_F+H-(T_B-T_S)}{-a}}(a+T_F+H-(T_B-T_S)) - ae^{\frac{T_F}{-a}}(a+T_F)\Big] \tag{3.26}$$

Equations (3.25) and (3.26) are basic to the analysis of this section but, because of their complexity, only the results of the analysis will be included in the text.[15/]

Figure 3.9 presents " standard" VAP and VMP curves.[16/] Figures 3.10 to 3.13 use comparisons with the standard as a way of graphically presenting many of the results of this section. It should be noted that the graphs are merely a helpful tool in understanding the economic relationships.

15/ The more difficult derivations are outlined in the Appendix.

16/ Specific parameter values of the output function were used to generate the " standard" curves.

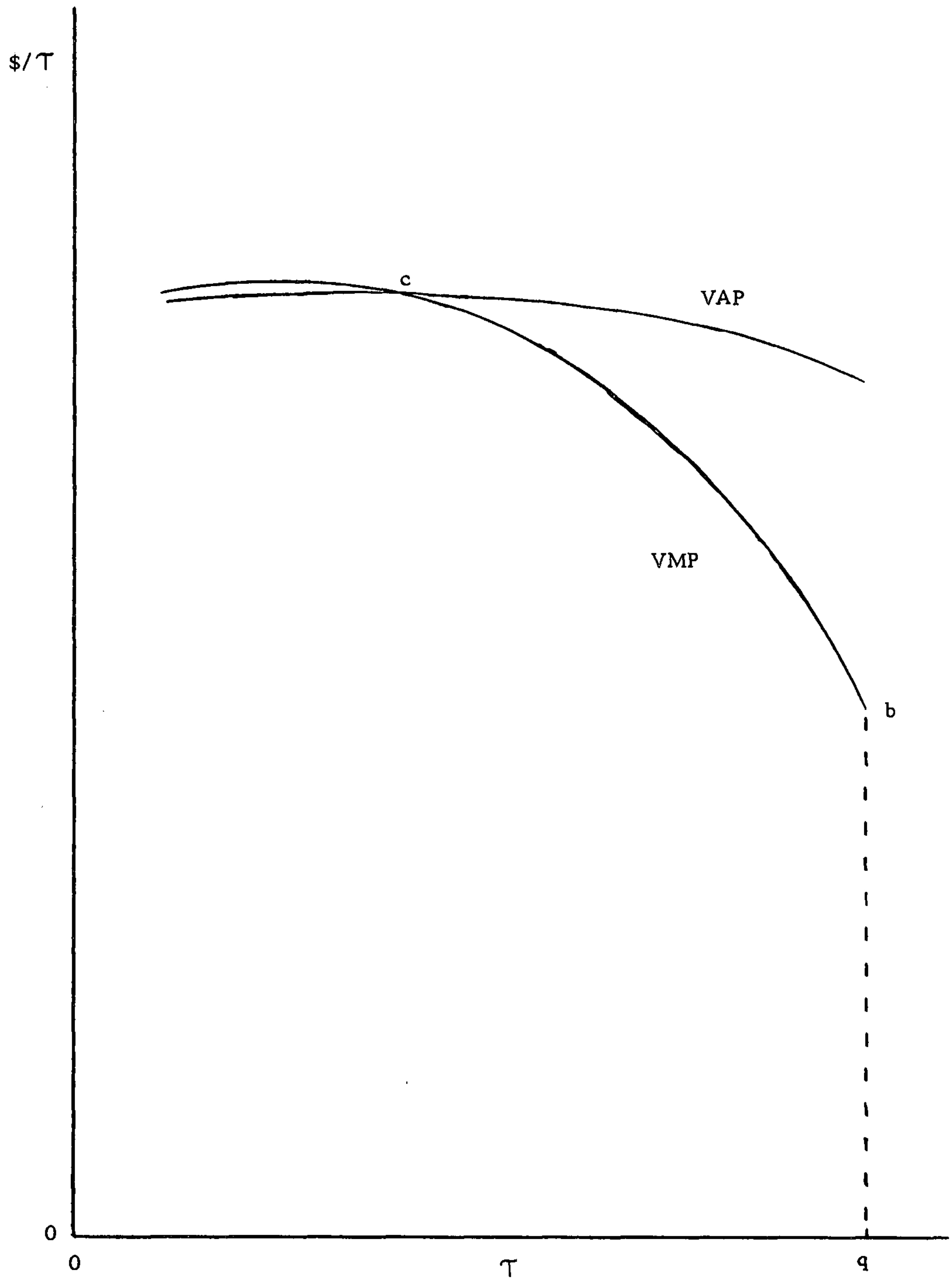

Figure 3.9. Standard VAP and VMP curves

What Happens to the CIDC for T as the Price of the Output Increases?

An increase in the price of the output has no effect on either the position of the constraint or the position where the VAP curve reaches a maximum (i.e., the value of T). But an increase in the price of the output causes the height of both the VAP and VMP curves to increase.

These results are presented in Figure 3.10. The CIDC before the increase in price is given by cbq (for prices above c none of the input T will be used). After the increase in price the CIDC is feq (for prices above f none of the input T will be used). Since it is possible for a farmer to be constrained to operate in stage I, the curve nmh shows the effects of a price increase on a farmer in stage I. Before the price increase the farmer would purchase the quantity oh at any price between h and m. For prices higher than m he would purchase no T. With the increase in price the farmer will now buy the quantity oh at any price between h and n, and he will purchase no T at prices greater than n.

Except for those situations where he uses the same quantity (i.e., either zero or that dictated by the constraint) the farmer (other things equal) will purchase a greater quantity of the input T at every price of T when the price of the output increases.

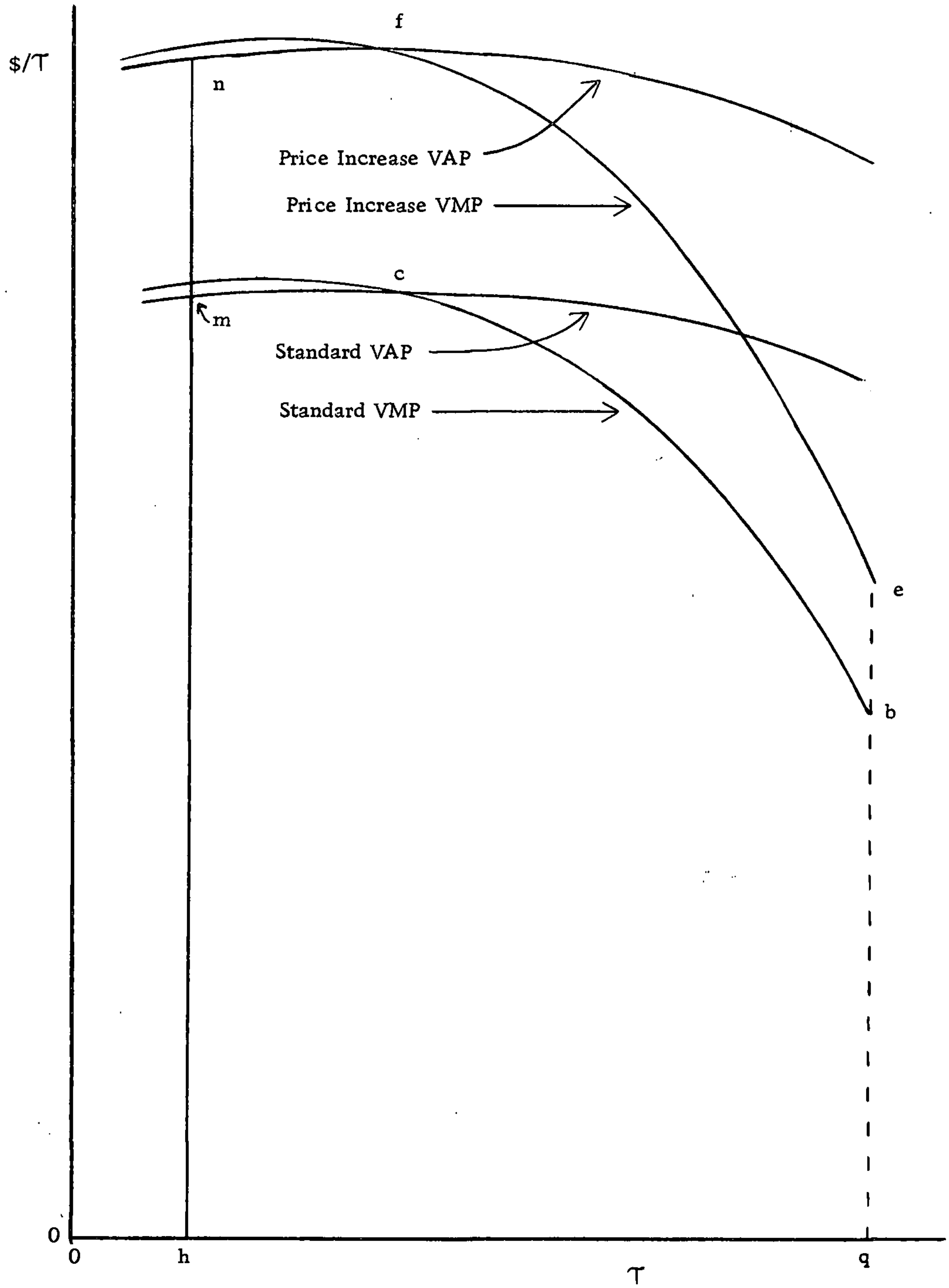

Figure 3. 10. Standard VAP and VMP curves; price increase VMP and VAP curves.

What Happens to the CIDC for τ as the Acreage of the Crop Increases?

It is known from (3.20) and (2.7) that the profit-maximizing farmer will change his starting date as his acreage changes. In order to examine how an increase in acreage affects a profit-maximizing farmer's CIDC for τ, it will be necessary first to examine the effects of a change in acreage on the optimal starting date, T_S^*. Equation (2.7) reveals that as A increases, H increases. As H increases the T_S^* calculated from (3.20) will decrease. Thus, as acreage increases the optimal starting date, T_S^*, becomes smaller (the farmer starts harvest earlier in the season). Since acreage does affect T_S^*, this analysis will require that T_S (the starting date of harvest) always equals T_S^* (the optimal starting date).

The effect of A (acreage) on the four characteristics of the CIDC are examined below:[17/]

1. Position of the constraint

   The position of the constraint is determined by the maximum possible τ. The maximum τ is given by $(T_F - T_B)$. Since the change in acreage does not affect T_F or T_B, the constraint remains the same.

[17/] The more difficult derivations for the section are outlined in the Appendix, equations (A.18 to A.27).

2. Position of the maximum VAP

The position of the maximum VAP, given that $T_S = T_S^*$, always shifts to the right as acreage increases.

3. Height of the VAP curve

The change in the height of the VAP curve at any particular τ will be given by $\left.\frac{\partial VAP}{\partial A}\right|_{T_S = T_S^*}$. This derivative is always positive for positive values of τ. Thus, as acreage increases, given $T_S = T_S^*$, the VAP curve shifts up at every positive value of τ.

4. Height of the VMP curve

The change in the height of the VMP curve at any particular τ will be given by $\left.\frac{\partial VMP}{\partial A}\right|_{T_S = T_S^*}$. This derivative is always positive (except at $H = o$ when the derivative will be zero). Thus, as acreage increases, given that $T_S = T_S^*$, the VMP curve shifts up at every value of τ.

These results are presented graphically in Figure 3.11. The CIDC before the increase in acreage is given by cbq (for prices above c none of the input, τ, will be used). After the increase in acreage the CIDC is feq (for prices above f none of the input, τ, will be used. Since it is possible for a farmer to be constrained to operate in stage I, the curve nmh shows the effect of a price increase on a farmer in stage I. Before the price increase the farmer would purchase the quantity oh at any price between h and m. For

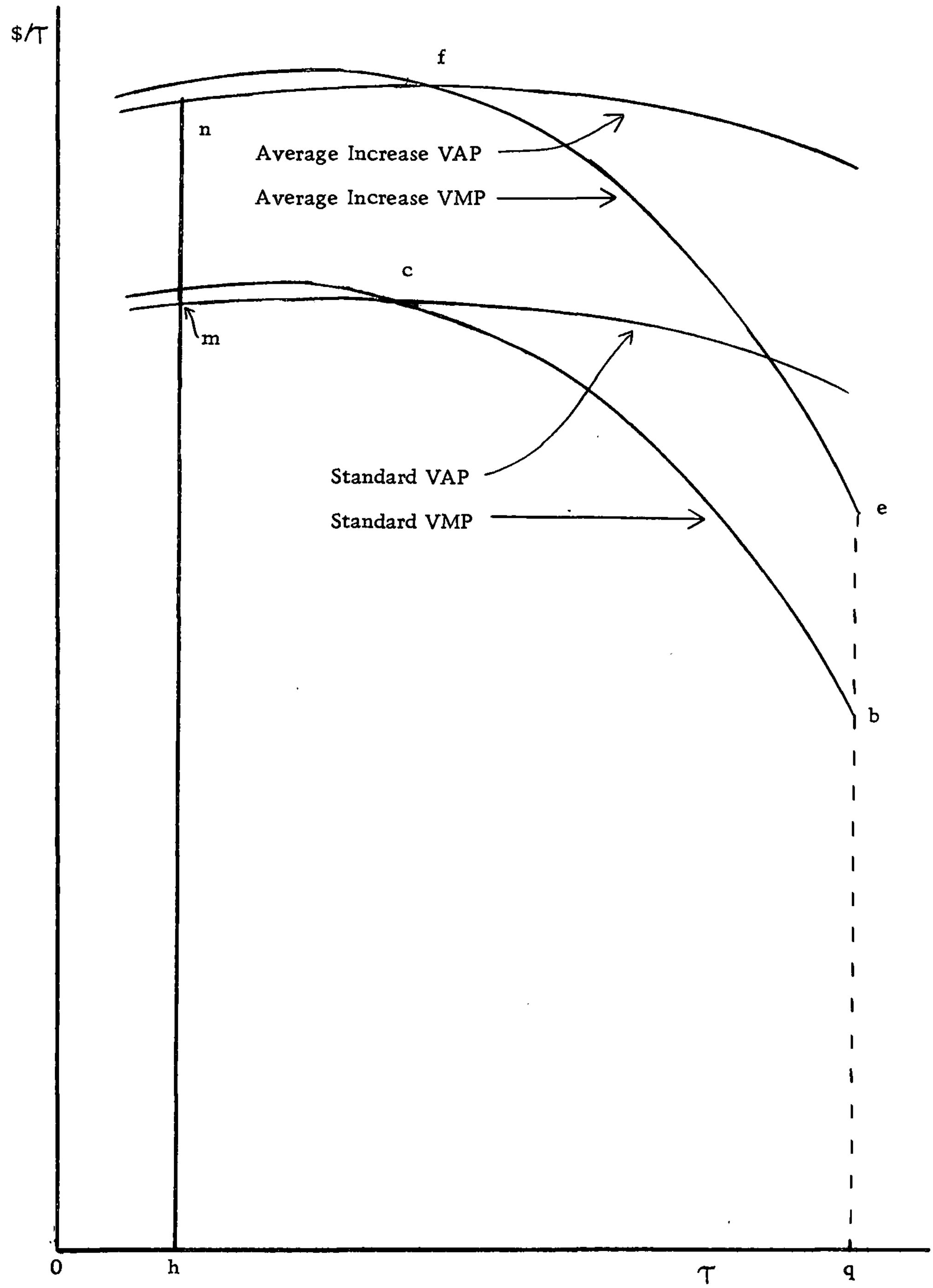

Figure 3.11. Standard VMP and VAP curves; acreage increase VAP and VMP curves.

prices higher than m he would purchase no τ. With an increase in acreage the farmer will now buy the quantity oh at any price between h and n, and he will purchase no τ at prices greater than n.

For two farmers who are in identical circumstances except for their acreage, the farmer with the larger acreage will purchase a greater quantity of the input τ at every price of τ than the farmer with the smaller acreage. The only exception to this occurs when both farmers use the same quantity of τ(i. e., either zero of that dictated by the constraint).

What Happens to the CIDC for τ as Equipment Capacity Increases?

The model variables which measure equipment capacity are S, W, and E (speed, width, and field efficiency). All three are positive, and an examination of equation (2.5) indicates that equipment capacity increases as any one of the three increase. Since it does not matter which one of the three model variables is used to examine the effects of an increase in equipment capacity, S has been arbitrarily chosen. Equations (3.20) and (2.7) reveal that the profit-maximizing farmer will change his starting date as his equipment capacity changes. In order to examine how an increase in equipment capacity affects a profit maximizing farmer's CIDC for τ, it will be necessary for the farmer's T_S to change. The farmer's T_S will always be equal to the T_S^* appropriate to the various equipment capacities. It should be noted that the effect of a change in equipment capacity is the most complex relationship which will be examined. None of the other variables which have been or will be examined affects the output function (2.8). However, S does affect the output function (as do W and E), and this complexity makes the analysis more difficult.

The effects of S on the four characteristics of the CIDC are examined below:[18]

1. Position of the constraint

The position of the constraint is determined by the maximum possible τ. The maximum τ is given by $(T_F - T_B)$. Since it is assumed that a change in S (equipment capacity) does not affect T_F or T_B the constraint remains the same.

2. Position of maximum VAP

The position of the maximum VAP, given $T_S = T_S^*$, always shifts to the left as S (equipment capacity) increases.

3. Height of the VAP curve

The change in the height of the VAP curve at any particular τ will be given by $\left.\frac{\partial VAP}{\partial S}\right|_{T_S = T_S^*}$. A positive value of

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[18] The more difficult derivations of this section are outlined in equations (A.28 - A.32) of the Appendix.

$\frac{\partial VAP}{\partial S}\Big|_{T_S = T_S^*}$, when evaluated at a particular value of τ, means that an increase in S (equipment capacity) will shift the VAP curve up at that point.

Likewise, if $\frac{\partial VAP}{\partial S}\Big|_{T_S = T_S^*}$ is negative, it means that an increase in S (equipment capacity) will shift the VAP curve down at that point. Since a change in S (equipment capacity) affects the harvesting function the sign of $\frac{\partial VAP}{\partial S}\Big|_{T_S = T_S^*}$ may be either positive or negative.

4. Height of the VMP curve

The change in the height of the VMP curve at any particular τ will be given by $\frac{\partial VMP}{\partial S}\Big|_{T_S = T_S^*}$. The derivative, $\frac{\partial VMP}{\partial S}\Big|_{T_S = T_S^*}$, has the same ambiguities with respect to sign as did the VAP curve.

From the above direct analysis only two things are clear about how S (equipment capacity) affects the CIDC. The constraint remains unchanged and the position of the maximum VAP shifts to the left. Thus the analysis does not present an unambiguous conclusion about the CIDC as S (equipment capacity) increases.

In order to obtain a better understanding of these complex relationships, it was decided to evaluate the expressions, $\frac{\partial VAP}{\partial S}\Big|_{T_S = T_S^*}$ and $\frac{\partial VMP}{\partial S}\Big|_{T_S = T_S^*}$, at selected points.[19/]

[19/] The yield function, acreage, and the price of the output were held constant. For each set of H, T_B and K values (since acreage

Figure 3.12 depicts an example of an increase in equipment capacity. The CIDC before the increase in equipment capacity is given by cbq (for prices above c none of the input τ will be used). After an increase in equipment capacity the CIDC is feq (for prices above f none of the input τ will be used). Since it is possible for a farmer to be constrained in stage I, the curve mnh shows the effects of an increase in capacity on a farmer in stage I. Before the increase in equipment capacity the farmer would purchase the quantity oh at any price between h and m. For prices higher than m he would purchase no τ. With the increase in equipment capacity the farmer now purchases the quantity oh at any price between h and n, and he will purchase no τ at prices greater than n.

Figure 3.12 represents only one of several possible shifts in the VAP and VMP curves. The analytical results indicate that the "old" VMP curve and the "new" VMP curves may intersect. An

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is constant H and S vary inversely; $T_F = T_B + K$) the expressions were evaluated at 20 different values of τ from zero to the maximum τ (i.e, K). The yield function parameter used was, a = 10. The value of H, T_B, and K used were:

$$H = 5, 10, 20; \qquad T_B = 7.8, 10, 12; \qquad K = 5, 10, 25$$

This gave 27 sets of H, T_B, and K values. The values of H were chosen to depict a small, medium, and large harvesting capacity relative to the yield function. The values of T_B were chosen to cover as wide a range as possible and still be within the interval T_S^* to $(T_S^* + H)$ for all the sets. The values of K were chosen to depict short, medium and long intervals between T_B and T_F.

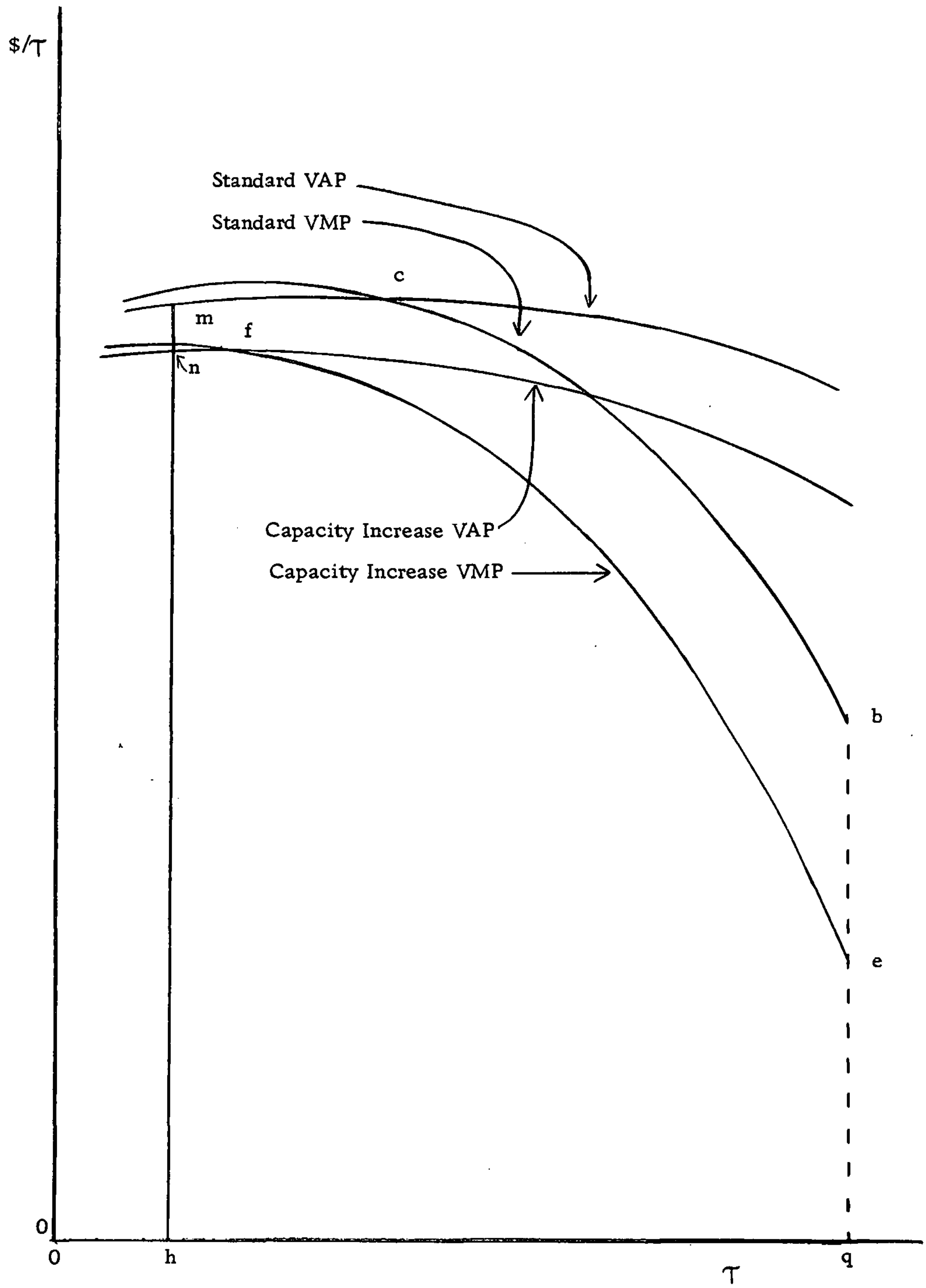

Figure 3.12. Standard VAP and VMP; equipment capacity increase VAP and VMP

intersection will occur if point f is above point c. This leads to ambiguities as to what happens to the CIDC.

However, certain tentative conclusions about the effect of an increase in equipment capacity of the CIDC for τ can be drawn from the combined analytic and numeric results.[20] In general, for two farmers who are in identical circumstances except for equipment capacity, the farmer with the smaller equipment capacity will purchase a greater quantity of the input τ at every price of τ than the farmer with the larger equipment. The exceptions to this general conclusion appear to occur over a narrow range of τ when T_B is less than a, and T_F is greater than a. Over this range of τ the increase in the height of the output function caused by the increase in equipment capacity causes the area between the limits of integration to increase, even though the interval between the limits has decreased.[21]

What Happens to the CIDC for τ as the Date of Breakdown Varies?

From (2.14) it can be seen that the upper limit of τ depends upon both T_F and T_B. The maximum amount of τ is given by

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[20] These conclusions do not have the same rigor as those derived without the aid of numeric examination.

[21] The limits of integration referred to are those used in equation (2.19).

$(T_F - T_B)$. Since both the magnitude of T_B and the difference between T_B and T_F are involved in the CIDC for τ, the following substitution is made:

$$T_F = T_B + K \qquad (3.27)$$

By utilizing this substitution it will be possible to examine the effects of varying T_B while the interval between T_B and T_F remains constant, K.

The effects of T_B, given that $(T_F - T_B)$ is constant, on the four characteristics of the CIDC are examined below:[22/]

1. Position of the constraint

   The position of the constraint is determined by the maximum possible τ. Since the interval $(T_F - T_B)$ is set equal to K and held constant in this analysis, the position of the constraint is independent of the value of T_B.

2. Position of the maximum VAP

   The position of the maximum VAP, given that $[T_F - T_B = K]$, always shifts to the right as T_B increases.

3. Height of the VAP curve

   The change in the height of the VAP curve at any particular τ till be given by $\left.\frac{\partial VAP}{\partial T_B}\right|_{T_F = T_B + K}$.

22/ The derivations of this section are outlined in equation (A. 33) to (A. 43) in the Appendix.

If $T_F \leq a$(i.e., T_B+K) then

$$\left.\frac{\partial VAP}{\partial T_B}\right|_{T_F = T_B + K} \geq 0 \text{ for all } \tau . \tag{3.28}$$

$T_B \geq a$ then

$$\left.\frac{\partial VAP}{\partial T_B}\right|_{T_F = T_B + K} \leq 0 \text{ for all } \tau . \tag{3.29}$$

For those cases which satisfy neither of these conditions the sign of $\left.\frac{\partial VAP}{\partial T_B}\right|_{T_F = T_B + K}$ depends upon τ.

4. Height of the VMP curve

The change in the height of the VMP curve at any particular τ will be given by $\left.\frac{\partial VMP}{\partial T_B}\right|_{T_F = T_B + K}$.

If $T_F \leq a$ (i.e , T_B+K) then

$$\left.\frac{\partial VMP}{\partial T_B}\right|_{T_F = T_B + K} \geq 0 \text{ for all } \tau \tag{3.30}$$

$T_B \geq a$ then

$$\left.\frac{\partial VMP}{\partial T_B}\right|_{T_F = T_B + K} \leq 0 \text{ for all } \tau \tag{3.31}$$

For those cases which satisfy neither of these conditions the sign of $\left.\frac{\partial VMP}{\partial T_B}\right|_{T_F = T_{B+K}}$ depends upon τ.

Figure 3.13 shows an increase in T_B when T_B is greater than or equal to a. The CIDC before the increase in T_B is given by cbq (for prices above c none of the input τ will be used). After

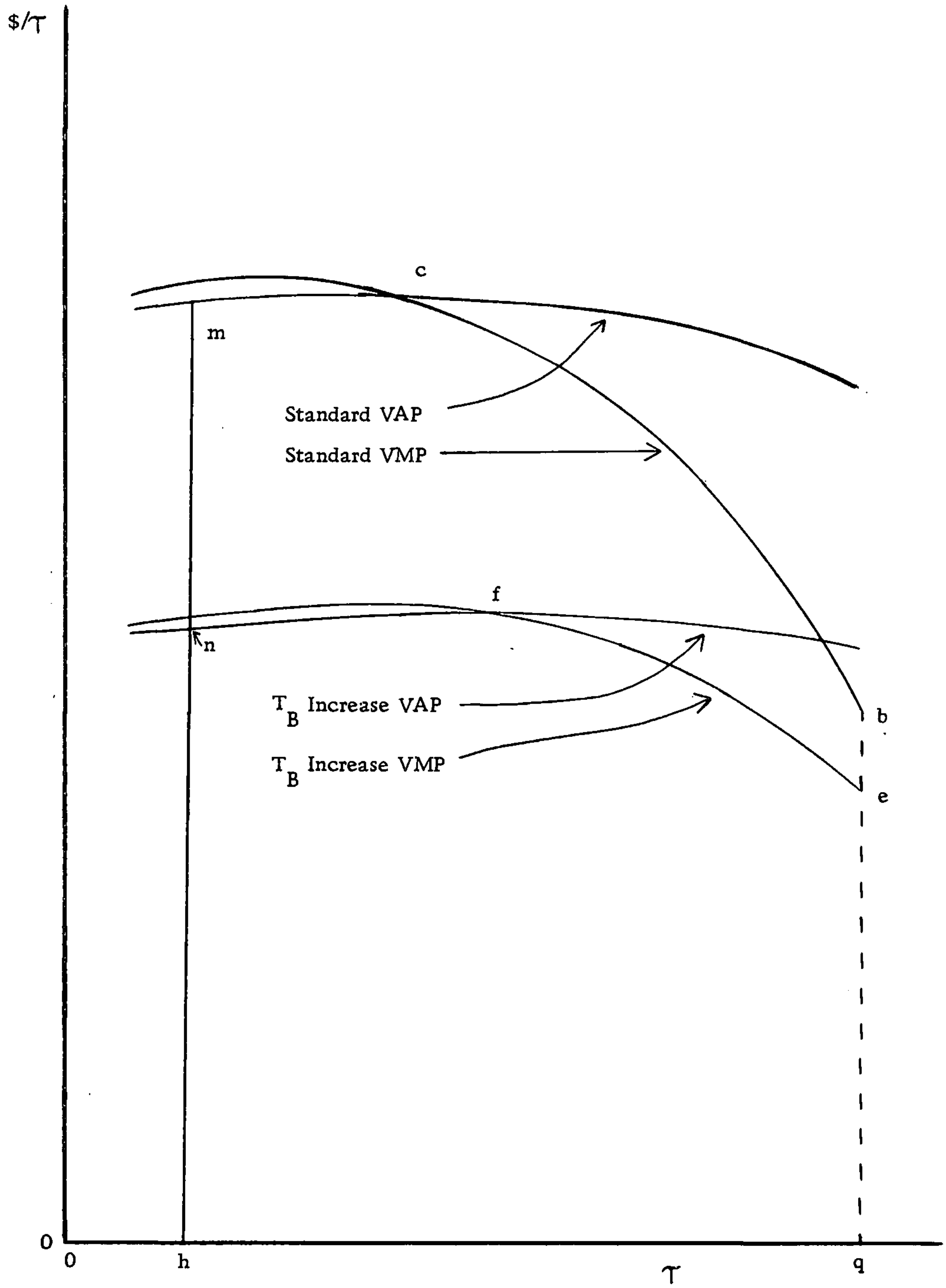

Figure 3.13. Standard VAP and VMP; T_B increase VAP and VMP

the increase in T_B, the CIDC is feq (for prices above f none of the input τ will be used). Since it is possible for a farmer to be constrained to operated in stage I, the curve mnh shows the effects of a price increase on a farmer in stage I. Before the increase in T_B the farmer would purchase the quantity oh at any price between h and m. For prices higher than m he would purchase no τ. With an increase in T_B the farmer will now buy the quantity oh at any price between h and n, and he will purchase no τ at prices greater than n. A similar unambiguous relationship could be depicted when T_F is less than a but the curves for an increase in T_B would lie above rather than below the standard.

The point a, the position of the relative maximum of that yield function, plays an important role in the conclusion about the effects of increasing T_B. For two farmers who are in identical situations (including the same interval between T_B and T_F) except for the time of breakdown, the farmer whose breakdown occurs earlier--provided both of the breakdowns occur after a--will purchase a greater quantity of the input τ at every price of τ than the farmer who breaks down later. The only exception to this occurs when both farmers use the same quantity of τ (i.e., either zero or that dictated by a constraint).

For two farmers who are in identical situations (including the same interval between T_B and T_F) except for the time of breakdown, the farmer whose breakdown occurs later--provided both T_F's occur before a--will purchase a greater quantity of the input τ at every price of τ than the farmer with the earlier breakdown. The only exception to this occurs when both farmers use the same quantity of τ (i.e., either zero or that dictated by the constraint).

What Happens to the CIDC for τ as T_F Varies?

This analysis is the companion of the preceding one. Here, the effects of changing the $(T_F - T_B)$ interval will be examined, since T_B will be held constant and T_F will be allowed to vary.

The effects of T_F on the four characteristics of the CIDC curve are examined below:[23/]

1. Position of the constraint

The position of the constraint is determed by the maximum possible τ. The maximum τ is given by $(T_F - T_B)$. As T_F increase the maximum value of τ increases. Thus, the position of the constraint shifts to the right.

2. Position of the maximum VAP

The position of the maximum VAP always shifts to the right as T_F increases.

[23/] The derivations for this section are outlined in equations (A.44) to (A.53).

3. Height of VAP curve

The change in the height of the VAP curve at any particular τ will be given by $\frac{\partial VAP}{\partial T_F}$.

If $T_F < 2a + T_B - H - T_S$ then

$$\frac{\partial VAP}{\partial T_F} \geq 0 \text{ for all } \tau \tag{3.32}$$

For those cases which do not satisfy this condition the sign of $\frac{\partial VAP}{\partial T_F}$ depends upon τ.

4. Height of the VMP curve

The change in the height of the VMP curve at any particular τ will be given by $\frac{\partial VMP}{\partial T_F}$.

If $T_F < 2a + T_B - H - T_S$ then

$$\frac{\partial VMP}{\partial T_F} \geq 0 \text{ for all } \tau \tag{3.33}$$

For those cases which do not satisfy this condition the sign of $\frac{\partial VMP}{\partial T_F}$ depends upon τ.

Figure 3.14 depicts an increase in T_F. The T_F's of both the standard and the increase are less than $(2a + T_B - H - T_S)$. The standard used for this graph differs from the standard used in figures 3.9 to 3.13. It should be noted that neighter the standard or the increase have a stage I (when T_F is less than $[2a + T_B - H - T_S]$ there will be no stage I). The CIDC before the increase in T_F is

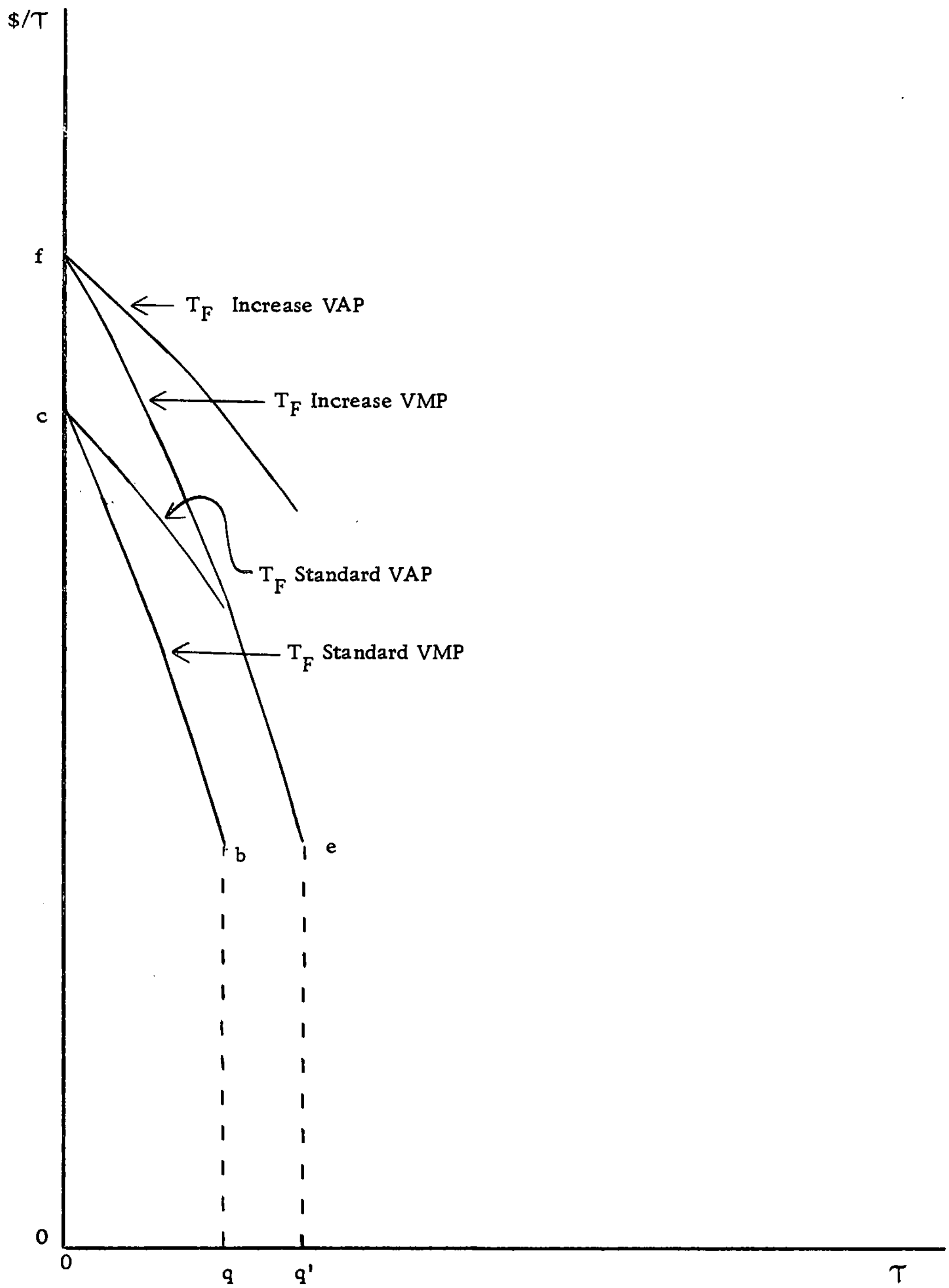

Figure 3.14. T_F standard VAP and VMP; T_F increase VAP and VMP

given by cbq (for prices above c none of the input τ will be used). After the increase in T_F the CIDC is feq' (for prices above f none of the input τ will be used).

For two farmers who are in identical situations except for the date of T_F, the farmer with the later T_F -- provided bothe T_F's are less than $(2a + T_B - H - T_S)$ --will purchase a greater quantity of the input τ at every price below f than the farmer with the earlier T_F. At prices above f, no τ will be used by either farmer.

A Comment on the Conditions Used

The conditions imposed to remove ambiguities in the CIDC were in terms of a, the local maximum of the yield function, and 2a, the point of inflection of the yield function. As the variables being examined are allowed to change, the limits of the integrals in equation (2.19) (the VAP and VMP curves come from (2.19)) change. The conditions imposed are actually conditions on the limits of integration. The position of the limits of integration relative to the position of the local maximum and point of inflection may cause ambiguities if the critical points (a and/or 2a) are sometimes included within the limits of integration and other times are excluded (the inclusion or exclusion depends upon how the variable being examined affects the limits of integration).

Summary of Selected Results

The demand for the timeliness of machinery repairs was examined by using the four characteristics of the CIDC. The results for selected variables are given below:

For other things equal

1. As the price of the output increases the demand curve shifts to the right.
2. As acreage increases the demand curve shifts to the right.
3. As equipment capacity increases the demand curve will generally shift to the left.

IV. EMPIRICAL ANALYSES

Chapters II and III were devoted to the theoretical development of an economic model and an examination of some of its implications. This chapter discusses empirical aspects of the model. The areas covered are:

1. Data on one of the key model components, the yield function
2. An attempted test of the model
3. Other possible tests of the model

Yield Function Data

The number of sources for secondary data on the yield function was quite limited.[24/] The only available study examined the yield functions for several grass seeds in the Willamette Valley of Oregon (Klein, 1967). In this study the time-of-harvest was varied and the resulting effect on yield per acre was measured. Figure 4.1 is a reproduction of the yield function for orchard grass. The yield function for the other grass seeds had this same general form: One local maximum and no local minima.

The yield functions presented by Klein (1967) provide supportive evidence for the theoretically derived yield functions of Chapter III.

[24/] Further work in the estimation of yield functions would provide data helpful in testing the model.

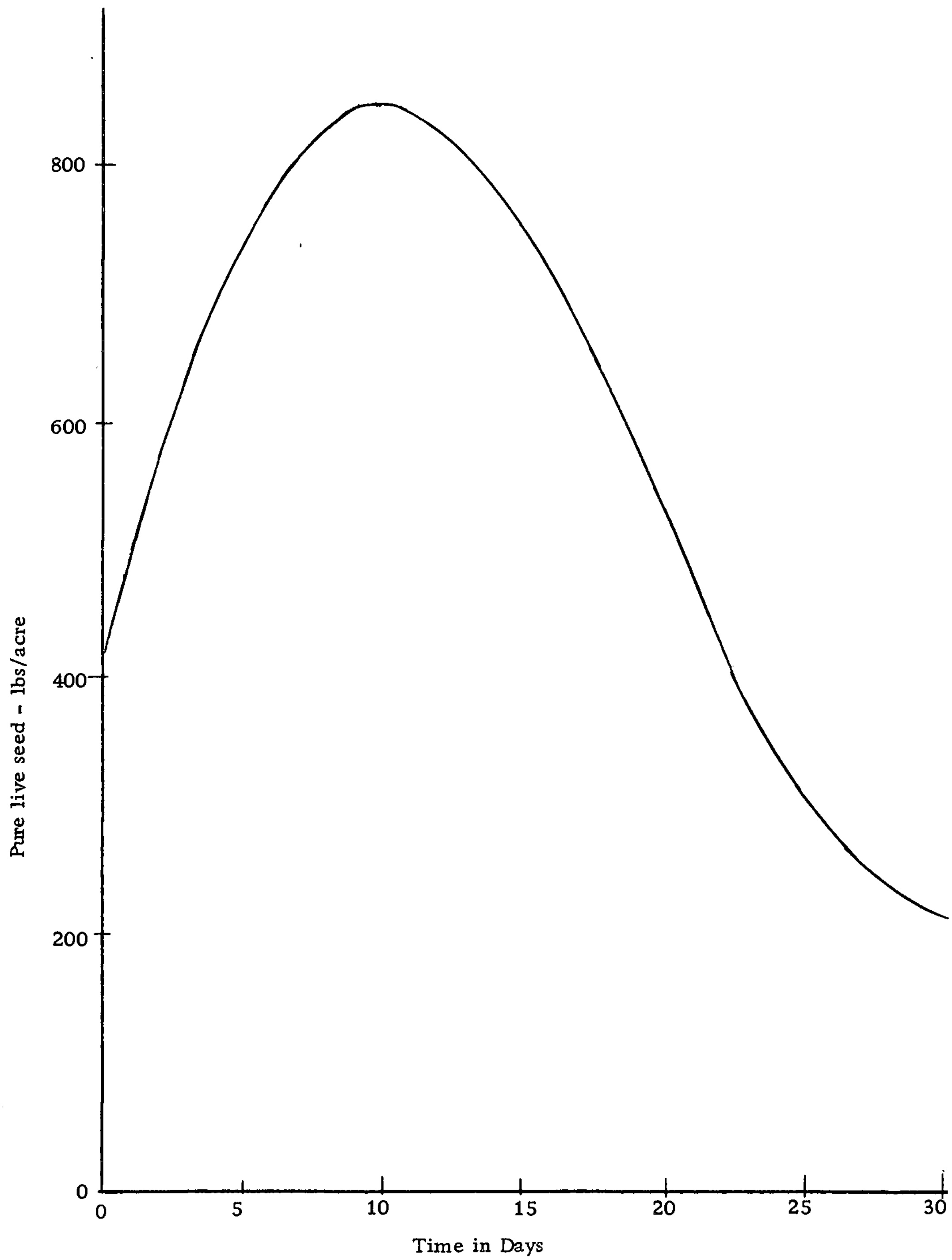

Figure 4.1. Yield function for orchard grass (Klein, 1967, p. 17)

Attempted Test of the Model[25]

The results deduced from the model in the latter part of Chapter III are really predictions generated by the model. These predictions form hypotheses which can be used to test the model.

One of the clearest hypotheses coming from the model is the one concerning acreage. The hypothesis is: other things equal, a farmer with a large acreage would be willing to pay a higher maximum price for any given decrease in the length of interruption caused by a breakdown than would a farmer with a smaller acreage.[26]

There are several alternative ways to test this hypothesis. An attempt was made to use one of these alternatives. The approach used will be discussed in this section. Other possible procedures will be suggested in the following section.

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[25] Though no actual test of the hypothesis resulted from this attempt, discussing it may provide a better understanding of how the model might be tested.

[26] This hypothesis reverses the traditional direction of dependence between price and the quantity. The reversal was made in order to simplify the testing procedure. From Figure 3.11 it can be seen that an increase in acreage shifts both the VAP and VMP curves up at every (nonzero) value of τ. This consistent upward shift allows an arbitrary selection of the intervals used in the questionnaire.

The approach chosen was a mail survey[27/] of rye grass growers in the Willamette Valley of Oregon. This population was chosen because:

1. The number of rye grass growers was large.
2. All of the growers face approximately the same yield function.
3. There was variation in the number of acres per farm.
4. Many of the growers used the same make and model of harvesting equipment.

The purpose of the questionnaire[28/] was the collection of data which could be used to test the hypothesis about the effect of acreage on the willingness to pay for timeliness (i.e., reduction in the interval between breakdown and repair). The hypothesis specified "all other things equal." In the questionnaire this was accomplished by

1. Selecting a population with approximately the same yield function.
2. Specifying in the hypothetical situation all of the model variables except acreage and equipment capacity.
3. Stratifying the returned questionnaires on the basis of harvesting equipment capacity.

[27/] By using a mail survey it was felt that the identification problem which can occur when estimating demand relationships from market data could be avoided.

[28/] A copy of the questionnaire is in the Appendix.

The hypothetical situation to which the farmers were asked to respond was constructed to be as similar as possible to actual situations the farmers might have faced.[29/] The hypothetical situation was individualized to each farmer since it required him to use his own acreage and equipment. The farmer recorded his response to the situation by assigning dollar values to three different alternative reductions in the interval between breakdown and repair.

It was planned that the questionnaire data would be stratified on the basis of equipment capacity and used to fit simple linear regression equations with acreage as the independent variable and the dollars extra (willingness to pay indicated by the farmer) as the dependent variable. A separate regression equation would have been needed for each of the three intervals between breakdown and parts delivery. Examining the slopes of these equations would have provided a test of the hypothesis. A positive slope would have supported the hypothesis.

A pretest of the questionnaire was conducted with a small group of rye grass growers. The pretest indicated a general willingness on the part of these farmers to pay something extra in order to reduce the interval between breakdown and the delivery of parts. However, it also revealed some weakness in the questionnaire. Frequently the

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[29/] By casting the questions in a familiar setting it was hoped that the farmers' responses would indicate their actual behavior if faced with such a situation.

farmers' responses were not in terms of the number of dollars they would be willing to pay but in terms such as "whatever it costs" or "I would trade machines immediately." It was also apparent that several of the farmers who were responding in dollars were giving their estimate of how much they would be charged by the dealer for the faster service rather than how much they would be willing to pay. Since there appeared to be no way of avoiding these two difficulties, this approach was abandoned.[30/]

Other Possible Tests

If a high degree of dealer and/or manufacturer cooperation could be secured, the collection of primary data would be possible. The cooperation which would be needed from the dealers is the offering of various delivery dates and prices for parts which the dealer must order for the farmer. The differences in the delivery dates would measure the reduction in the interval between breakdown and repair, and the price difference would measure the premium for the reduction.[31/] The data collected would include not only the choice

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[30/] In general the difficulty was similar to that encountered whenever a researcher asks a respondent to indicate how he would behave under certain conditions. There is no guarantee that the respondents' belief of how he would behave and his actual behavior will be the same.

[31/] A careful design of the alternative premiums and reductions would be needed in order to generate the data necessary for a test. (For example if all farmers chose the same alternative and are charged the same premium it would be impoosible to use these data for tests.)

the farmer actually makes, but a description of his situation and the delivery alternatives available to him. Though many of the difficulties encountered with the questionnaire could be avoided, other problems present themselves. Examples of these problems are:

1. It may not be easy to locate a cooperative dealer or manufacturer.
2. The number of observations collected from any particular dealer during a season may be small.
3. There is no way of isolating the alternatives available to the farmer (i.e., he may have nondealer sources for reducing the interval such as custom-made parts, etc.).

Another hypothesis suggested by the model involves the level of parts inventory maintained by machinery dealers.[32/] The suggested hypothesis is: as the farmers' ratio of acreage to equipment capacity increases in a region, the level of parts inventory held by dealers in that region would be expected to increase.

Since currently there is little data on yield functions available the identification of comparable regions for a cross-sectional study would be difficult.

[32/] The interval between breakdown and repair can be reduced if the needed parts are in the dealer's inventory. As the level of inventory is increased the cost of providing parts from the inventory would be expected to increase, but the probability of the needed part being in inventory would also be increased. Thus, the dealers would be supplying a greater number of interval reductions with the larger inventory (i.e., providing a greater quantity of timeliness).

V. SUMMARY AND SUGGESTED MODEL EXTENSIONS

Summary

The purpose of this thesis, as set out in Chapter I, was the development of a theoretical economic model which could be used to examine a farmer's short-run demand for timeliness of farm machinery repairs. Chapter II presented such a model. In Chapter III the model was extensively explored. Particular attention was focused upon the effects of several variables on the demand curve for timeliness of farm machinery repairs. Some of the testable hypotheses generated by the model were outlined in Chapter IV.

Generalizing the Model to Other Inputs

The thesis examined the timing of the application of the repair input. This was done by using a production function which was derived in such a way that the timing of the input (repairs) application became an explicit variable of the production function. The same method of analysis seems appropriate for any input which can be applied at various times during the production period. Examples of inputs whose timing (various dates) of application could be examined are irrigation water, cultivation of a crop, and planting of a crop.

Possible Model Extensions Suggested for Further Research

One of the model assumptions is that the price of the output is constant throughout the harvesting season. However, for many crops there is a variation in price during the harvest season (i.e., usually the early harvest brings a higher price). This price variation can be introduced into the model by multiplying the output function by a time-dependent price function. The resulting revenue function (it would give the instantaneous rate of total revenue in dollars per hour) could be used in place of the output function.[33]

It is also possible to modify the model to allow labor costs to vary during the harvest season. This is done through the introduction of a cost function which describes the instantaneous rate of labor costs in dollars per hour as a function of t. By subtracting the cost function from the revenue function, a net revenue function (it would give the instantaneous rate of [total revenue - labor costs] in dollars per hour) is obtained. This net revenue function could then be used in place of the output function.[34]

Though the model of this thesis is nonstochastic, it does provide

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33/ The model of this thesis used a constant price and it was introduced by multiplying the average and marginal productivities by the price of the output. With the above modification the price of the output is entered at an earlier step in the derivation.

34/ Ibid.

the basic structure from which a stochastic simulation model could be built. With a simulation model it would be possible to examine the value of timely repairs as it is affected by the degree of weather uncertainty and the probability of breakdown. Also, uncertainty could be introduced with respect to the repair dates.

The simulation approach would facilitate the extension of the model into a longer-run analysis. Through the simulation approach the effect of equipment age on the probability of breakdown and the resulting demand for timely repairs could be examined. The purchase of new equipment may be one way of avoiding the need for timely machinery repairs.

Implications for Research on the Supply Side

Work on the supply side introduces difficulties not found on the demand side. In this study it was possible to ignore the particular part which had broken. The model was only concerned with the interruption itself and not with its cause. However, to supply a reduction in the length of the interval, the particular part needed must be provided in a speedier way. Since the supply side is part-specific (whereas the demand side is not) the analysis must include the identification of the needed part. One possible method of analysis which could allow the building of a part-specific model is simulation.

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APPENDIX

APPENDIX

QUESTIONNAIRE

1. How many years have you, yourself, operated a farm? ____________ Years

2. How many years have you been growing grass seed? ____________ Years

3. How many acres of the following crops did you (or your partnership) farm last year, and what were their yields?

CROP	ACRES	YIELD
Ryegrass (all varieties)	______	____________ lbs/acre
Orchardgrass	______	____________ lbs/acre
Bentgrass	______	____________ lbs/acre
Bluegrass	______	____________ lbs/acre
Fine Fescue	______	____________ lbs/acre
Tall Fescue	______	____________ lbs/acre
Other grass seeds (specify):		
____________________	______	____________ lbs/acre
____________________	______	____________ lbs/acre
Other crops (specify):		
____________________	______	____________
____________________	______	____________
____________________	______	____________

4. During last year's harvest season, how many hours per day did you generally spend combining?

   ________ Hours/day

5. How many acres, if any, did you custom combine last year? ________ Acres

6. How many acres, of the total acres listed in Question 3, did you have custom-combined by someone else last year?

   ________ Acres

7. Some farmers whose combines have broken down during harvest have had to wait for parts. Have you ever had to wait for parts during harvest?

   _____ Yes _____ No

8. If you ever had to wait, in what year did your most recent wait occur?

   ________ Year

9. If you ever had to wait, how long was your most recent wait? ________ Days

(PLEASE TURN PAGE)

10. Listed below are some self-propelled combines, by make and model. Please indicate the machine or machines you (or your partnership) own, by writing the model year of your combine(s) on the appropriate line(s). If your make or model is not listed, please specify it under "OTHER".

YEAR	MODEL	MAKE	YEAR	MODEL	MAKE
____	G	A-C Gleaner	____	503	I-H
____	F	A-C Gleaner	____	403	I-H
____	E	A-C Gleaner	____	181	I-H
____	C	A-C Gleaner	____	151	I-H
____	105	John Deere	____	510	Massey-Ferguson
____	95	John Deere	____	410	Massey-Ferguson
____	55	John Deere	____	Super 92	Massey-Ferguson
____	1660	Case	____	545	Oliver
____	1060	Case	____	542	Oliver
____	1010	Case	____	431	Oliver
____	1000	Case	____	40	Oliver

OTHERS (Please specify):

____	____	________	____	____	________
____	____	________	____	____	________
____	____	________	____	____	________

(PLEASE TURN PAGE)

IF YOU GROW RYEGRASS, PLEASE ANSWER QUESTIONS 11 THROUGH 14. IF YOU DO NOT GROW RYEGRASS, SKIP QUESTIONS 11 THROUGH 14.

11. Please put YOURSELF in the following situation, and respond as you believe you would if this actually occurred during harvest on YOUR FARM:

You start combining your ryegrass on July 1. The windrowed crop at the beginning of harvest has a potential yield of 1400 lbs/acre, and the price is expected to be 6.5¢/lb. Everything proceeds well, until your biggest capacity combine unexpectedly breaks down on July 5. The combine cannot be operated without the replacement of the broken part. Your local dealer does not have the necessary part in stock, and will not have it until July 12. None of the surrounding dealers have the part either. It is not raining now, but you have no way of being certain what the weather will be.

If it were possible for you to speed up the delivery of the part by paying something extra, how many dollars extra would you be willing to pay?

(a) To get the parts by July 5, I would be willing to pay $____________ extra.

(b) To get the parts by July 7, I would be willing to pay $____________ extra.

(c) To get the parts by July 10, I would be willing to pay $____________ extra.

12. Please comment on the reasoning behind your answer to Question 11:

13. Would your response to Question 11 be the same, or different (please check one) if the question had specified the particular part that had broken?

________ Same ________ Different

14. Please comment on the reasoning behind your answer to Question 13:

Equations (A. 1) to (A. 20) are intermediate results used in the later derivations.

Calculation of T_S^* for the Yield Function (3. 23)

From (3. 15) $$y_1(T_S) = y_1(T_S + H)$$

$$T_S e^{\frac{T_S}{-a}} = (T_S + H)\, e^{\frac{T_S+H}{-a}}$$

$$T_S = \frac{He^{\frac{H}{-a}}}{\left(1-e^{\frac{H}{-a}}\right)} = T_S^* \qquad \text{(A. 1)}$$

Determination of the Position of the Maximum of the VAP Curve

The maximum of the VAP curve occurs where VAP is equal to VMP (Ferguson, 1966, p. 114).

Setting (3. 25) equal to (3.26) and solving for $e^{T/a}$

$$e^{\frac{\tau}{a}} = \left[-ae^{\frac{H-(T_B-T_S)}{-a}} (a+T_F+H-(T_B-T_S))+a(a+T_F) \right] \Bigg/$$

$$\left[-ae^{\frac{H-(T_B-T_S)}{-a}} (a+T_F+H-(T_B-T_S)) + a(a+T_F) + \right.$$

$$\tau \left(-a - T_F + (a+T_F+H - (T_B-T_S)) e^{\frac{H-(T_B-T_S)}{-a}} \right) +$$

$$\left. \tau^2 \left(1 - e^{\frac{H-(T_B-T_S)}{-a}} \right) \right] \qquad \text{(A. 2)}$$

A direct analytic solution of this equation for τ is impossible. However, an indirect approach can be used to determine how the position (i.e., value of τ) at which the VAP curve is a maximum changes.

Equation (A. 2) can be rewritten in the following form.

$$e^{\frac{\tau}{a}} = \frac{-ab}{-ab + b\tau + d\tau^2} \qquad \text{(A. 3)}$$

where

$$b = e^{\frac{H-(T_B-T_S)}{-a}} (a+T_F+H-(T_B-T_S))-(a+T_F) \qquad \text{(A. 4)}$$

$$d = 1 - e^{\frac{H-(T_B-T_S)}{-a}} \qquad \text{(A.5)}$$

To examine what happens at the intersection of

$$e^{\frac{\tau}{a}} \tag{A.6}$$

and

$$\frac{-ab}{-ab + b\tau + d\tau^2} \tag{A.7}$$

(A. 7) will be examined first. Let

$$q = H - (T_B - T_S) \tag{A.8}$$

$$q^* = H - (T_B - T_S^*) \tag{A.9}$$

(A. 7) will have a local maximum at

$$\tau^* = \frac{-b}{2d} = \frac{a}{2} + \frac{T_F}{2} - \frac{q e^{\frac{q}{-a}}}{2(1 - e^{\frac{q}{-a}})} \tag{A.10}$$

τ^* is the value of τ where (A. 7) has a local maximum

if

$$T_F < 3a + \frac{q e^{\frac{q}{-a}}}{(1 - e^{\frac{q}{-a}})} \tag{A.11}$$

It will be assumed that (A. 11) always holds.

Substituting $-b = 2d\tau^*$ (from (A.10)) into (A. 7) yields

$$L = \frac{1}{1 - \frac{\tau}{a} + \frac{\tau^2}{2a\tau^*}} \tag{A.12}$$

$$\frac{\partial L}{\partial \tau^*} = \frac{\frac{\tau^2}{2a\tau^*}}{\left(1 - \tau + \frac{\tau^2}{2a\tau^*}\right)^2} \tag{A.13}$$

Equation (A. 12) will always be nonnegative since τ^* is positive and it will be positive for $\tau > 0$.

Since $e^{\tau/a}$ is an increasing function of τ

$$\frac{\partial e^{\frac{\tau}{a}}}{\partial \tau} = \frac{1}{a} e^{\frac{\tau}{a}} \tag{A.14}$$

and L increases as τ^* increases, the point of intersection between $e^{\tau/a}$ and L shifts to the right as τ^* increases. Thus, τ^* can be used to determine the direction of any shifts in the position of the maximum VAP.

<u>How Does q Affect τ^*</u>

$$\frac{\partial \tau^*}{\partial q} = \frac{2e^{\frac{q}{-a}}\left[-1 + \frac{q}{a} + e^{\frac{q}{-a}}\right]}{4\left(1 - e^{\frac{q}{-a}}\right)^2} \tag{A.15}$$

The sign of $\partial T^*/\partial q$ depends upon the sign of the numerator. Since the sign of $2e^{q/-a}$ is always positive the sign of $\partial T^*/\partial q$ depends upon the term in brackets. The term in brackets is zero when q is equal to zero. Differentiating the term in brackets with respect to q yields

$$\frac{\partial[-1 + \frac{q}{a} + e^{\frac{q}{-a}}]}{\partial q} = \frac{1}{a}[1 - e^{\frac{q}{-a}}] \qquad (A.16)$$

This is always positive, thus

$$\frac{\partial T^*}{\partial q} \geq 0 \qquad (A.17)$$

From (A.1) and (A.8)

$$q = H - \left[T_B - \left[\frac{He^{\frac{H}{-a}}}{1-e^{\frac{H}{-a}}}\right]\right] \qquad (A.18)$$

$$\left.\frac{\partial q}{\partial H}\right|_{T_S=T_S^*} = \frac{1 - e^{\frac{H}{-a}}(1 + \frac{H}{a})}{\left(1-e^{\frac{H}{-a}}\right)^2} \qquad (A.19)$$

Using an argument similar to that used in (A.15-A.17) it can be concluded that

$$\left.\frac{\partial q}{\partial H}\right|_{T_S=T_S^*} \geq 0 \qquad (A.20)$$

Effects of A Given $T_S = T_S^*$

$$\frac{\partial \tau^*}{\partial A}\Big|_{T_S=T_S^*} = \left(\frac{\partial \tau^*}{\partial q}\right)\left(\frac{\partial q}{\partial H}\Big|_{T_S=T_S^*}\right)\left(\frac{\partial H}{\partial A}\right) \tag{A.21}$$

$$\frac{\partial H}{\partial A} = \frac{825}{SWE} \tag{A.22}$$

Since all terms of (A. 20) are positive then

$$\frac{\partial \tau^*}{\partial A}\Big|_{T_S=T_S^*} \geq 0 \tag{A.23}$$

This means as acreage increases the position of the maximum VMP shifts to the right.

Differentiating (3. 26) with respect to A given $T_S=T_S^*$ yields

$$\frac{\partial VAP}{\partial A}\Big|_{T_S=T_S^*} = \frac{SWE}{825}\,\frac{P}{\tau}\left(\frac{\partial q}{\partial A}\Big|_{T_S=T_S^*}\right) e^{\frac{T_F+q^*}{-a}}\left[(T_F+q^*)(e^{\frac{\tau}{a}}-1)-\tau e^{\frac{\tau}{a}}\right] \tag{A.24}$$

Using an argument similar to that used in (A. 15-A. 17) is can be concluded that

$$\frac{\partial VAP}{\partial A}\Big|_{T_S=T_S^*} \geq 0 \tag{A.25}$$

Differentiating (3. 25) with respect to A given $T_S=T_S^*$ yields

$$\frac{\partial VMP}{\partial A}\bigg|_{T_S=T_S^*} = \frac{PSWE}{825}\left(\frac{\partial q}{\partial A}\bigg|_{T_S=T_S^*}\right) e^{\frac{T_F-\tau+q^*}{-a}} \left[-1+\frac{1}{a}(T_F-\tau+q^*)\right] \tag{A.26}$$

Since (T_S^*+H) is greater than or equal to a and $(T_F-T_B-\tau)$ is always nonnegative

$$\frac{\partial VMP}{\partial A}\bigg|_{T_S=T_S^*} \geq 0 \tag{A.27}$$

<u>Effects of S Given $T_S=T_S^*$</u>

From (A.17) and (A.19)

$$\frac{\partial \tau^*}{\partial S}\bigg|_{T_S=T_S^*} = \left(\frac{\partial \tau^*}{\partial q}\right)\left(\frac{\partial q}{\partial H}\bigg|_{T_S=T_S^*}\right)\left(\frac{\partial H}{\partial S}\right) \tag{A.28}$$

$$\frac{\partial H}{\partial S} = -\frac{A825}{S^2 WE} < 0 \tag{A.29}$$

Since one of the terms of (A.28) is negative and the other two terms are nonnegative the product is negative.

$$\frac{\partial \tau^*}{\partial S}\bigg|_{T_S=T_S^*} \leq 0 \tag{A.30}$$

Differentiating (3.26) with respect to S given $T_S=T_S^*$ yields

$$\frac{\partial VAP}{\partial S}\bigg|_{T_S=T_S^*} = \frac{P}{\tau}\frac{WE}{825}\Bigg[-ae^{\frac{T_F-\tau+q^*}{-a}}(a+T_F-\tau+q^*)$$

$$+ae^{\frac{T_F-\tau}{-a}}(a+T_F-\tau)+ae^{\frac{T_F+q^*}{-a}}(a+T_F+q^*)$$

$$-ae^{\frac{T_F}{-a}}(a+T_F)\Bigg]+\frac{P}{\tau}\frac{SWE}{825}\left(\frac{\partial q}{\partial H}\bigg|_{T_S=T_S^*}\right)\left(\frac{\partial H}{\partial S}\right)$$

$$\left[e^{\frac{T_F-\tau+q^*}{-a}}(T_F-\tau+q^*)-e^{\frac{T_F+q^*}{-a}}(T_F+q^*)\right] \quad (A.31)$$

The sign of the first term is always positive and the sign of the second term is always negative. The sign of the derivative may be either positive or negative.

Differentiating (3. 25) with repsect to S given $T_S=T_S^*$ yields

$$\frac{\partial VMP}{\partial S}\bigg|_{T_S=T_S^*} = \frac{PWE}{825}\left[-(T_F-\tau+q^*)e^{\frac{T_F-\tau+q^*}{-a}}+(T_F-\tau)e^{\frac{T_F-\tau}{-a}}\right]+$$

$$\frac{PSWE}{825}\left(\frac{\partial q}{\partial H}\bigg|_{T_S=T_S^*}\right)\left(\frac{\partial H}{\partial S}\right)\Bigg[-e^{\frac{T_F-\tau+q^*}{-a}}$$

$$+\frac{1}{a}(T_F-\tau+q^*)e^{\frac{T_F-\tau+q^*}{-a}}\Bigg] \quad (A.\ 32)$$

The sign of the first term is always positive and the sign of the second term is always negative. The sign of the derivative may be either positive or negative.

Effects of T_B Given $T_F = T_B + K$

From (A.10) substituting $T_F = T_B + K$

$$\left.\frac{\partial T^*}{\partial T_B}\right|_{T_F=T_B+K} = \frac{1}{2} + \frac{\left(e^{\frac{q}{a}} - 1\right) - \frac{q}{a} e^{\frac{q}{a}}}{2\left(e^{\frac{q}{a}} - 1\right)^2} \tag{A. 33}$$

Using an argument similar to that used in (A. 15-A. 16) it can be concluded that

$$\left.\frac{\partial T^*}{\partial T_B}\right|_{T_F = T_B + K} \geq 0 \tag{A. 34}$$

Differentiating (3. 26) with respect to T_B given that $T_F=T_B+K$

$$\left.\frac{\partial VAP}{\partial T_B}\right|_{T_F=T_{B+K}} = \frac{SWE}{825}\frac{P}{\tau} e^{\frac{T_B+K}{-a}} \left[e^{\frac{\tau}{a}}(-T_B-K+\tau)+T_B+K \right] \tag{A. 35}$$

The sign of (A. 35) depends upon the sign of the term in brackets since all the other terms are positive. At $\tau = 0$ the term in brackets is equal to zero. Differentiating it with respect to τ

$$\frac{\partial\left[e^{\frac{\tau}{a}}(-T_B-K+\tau)+T_B+K\right]}{\partial\tau}=\frac{1}{a}e^{\frac{\tau}{a}}\left[-T_B-K+\tau+a\right] \tag{A. 36}$$

If

$$T_B \geq a \tag{A. 37}$$

then (A. 36) will always be negative. If (A. 36) is always negative then (A. 35) will always be negative.

$$\left.\frac{\partial VAP}{\partial T_B}\right|_{T_F=T_B+K} \leq 0 \tag{A. 38}$$

$$\text{if } T_B \geq 0$$

If

$$T_B + K \leq a \tag{A. 39}$$

then (A. 36) will always be positive. If (A. 36) is always positive then (A. 35) will always be positive.

$$\left.\frac{\partial VAP}{\partial T_B}\right|_{T_F=T_B+K} \geq 0 \tag{A. 40}$$

$$\text{if } T_B + K \leq a$$

For values of T_B and K which do not satisfy either (A. 37) or (A. 39) the sign of (A. 35) depends upon the value of τ.

It should be remembered that a is the point where the yield function reaches its local maximum.

Differentiating (3.25) with respect to T_B given that $T_F = T_B + K$

$$\left.\frac{\partial VMP}{\partial T_B}\right|_{T_F = T_B + K} = \frac{PSWE}{825}\left(\frac{1}{a}\right) e^{\frac{T_B + K - \tau}{-a}} \left[a - (T_B + K - \tau)\right] \quad (A.41)$$

The sign of (A.41) depends upon the sign of the term in brackets since all of the other terms are positive.

$$\left.\frac{\partial VMP}{\partial T_B}\right|_{T_F = T_B + K} \leq 0 \quad (A.42)$$

if $T_B \geq a$

$$\left.\frac{\partial VMP}{\partial T_B}\right|_{T_F = T_B + K} \geq 0 \quad (A.43)$$

if $T_B + K \leq a$

For values of T_B and K between these intervals the sign of (A.41) depends upon the value of τ.

<u>Effects of T_F</u>

Differentiating (A.10) with respect to T_F

$$\frac{\partial \tau^*}{\partial T_F} = \frac{1}{2} \quad (A.44)$$

Differentiating (3.26) with respect to T_F

$$\frac{\partial VAP}{\partial T_F} = \frac{P}{\tau}\,\frac{SWE}{825}\, e^{\frac{T_F}{-a}} \left[e^{\frac{-\tau + q}{-a}} (T_F - \tau + q) - e^{\frac{\tau}{a}} (T_F - \tau) - e^{\frac{q}{-a}} (T_F + q) + T_F \right] \quad (A.45)$$

At $\tau = 0$ the term in brackets is zero differentiating with respect to τ yields

$$e^{\frac{T_F-\tau}{-a}} [(1-e^{\frac{q}{-a}}) - \frac{1}{a}(T_F-\tau)(1-e^{\frac{q}{-a}}) + \frac{q}{a} e^{\frac{q}{-a}}] \qquad (A.46)$$

At $q = 0$ the term in brackets is equal to zero. Differentiating with respect to q and substituting for q yields

$$\frac{1}{a^2} e^{\frac{q}{-a}} [2a - T_F + \tau - H - T_S + T_B] \qquad (A.47)$$

This will be positive for all values of τ if

$$T_F < 2a + T_B - H - T_S \qquad (A.48)$$

If T_F satisfies (A.48) then

$$\frac{\partial VAP}{\partial T_F} \geq 0 \qquad (A.49)$$

Differentiating (3.25) with respect to T_F

$$\frac{\partial VMP}{\partial T_B} = \frac{PSWE}{825} e^{\frac{T_F-\tau}{-a}} \left[(1-e^{\frac{q}{-a}}) - (\frac{1}{a})(T_F-\tau)(1-e^{\frac{q}{-a}}) + qe^{\frac{q}{-a}}\right] \qquad (A.50)$$

At $q = 0$ the term in brackets is equal to zero. Differentiating with respect to q and substituting for q yields

$$\frac{1}{a^2} e^{\frac{q}{-a}} [2a - T_F + \tau - H - T_S + T_B] \qquad (A.51)$$

This will be positive for all values of τ if

$$T_F < 2a + T_B - H - T_S \tag{A.52}$$

If T_F satisfies (A.52) then

$$\frac{\partial VMP}{\partial T_F} \geq 0 \tag{A.53}$$