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ABSTRACT

G. Spencer Brown's book Laws of Form has been enjoying a vogue among social and biological scientists. Proponents claim that the book introduces a new logic ideally suited to their fields of study, and that the new logic solves the problems of self-reference. These claims are false. We show that Brown's system is Boolean algebra in an obscure notation, and that his "solutions" to the problems of self-reference are based on a misunderstanding of Russell's paradox.

INTRODUCTION

In this paper we investigate the logic described by G. Spencer Brown in his book Laws of Form (Brown, 1969). All references to Brown are to this book. Brown claims to have invented a new logic and a superficial perusal of his book with its unusual notation certainly suggests that something different from our usual logic is being described. It is our contention that Brown has merely reinvented Boolean algebra but in an obscure notation. In the first section of this paper we discuss Brown's notation and show how it allows him to obtain implicit axioms "for free." We also show that using Brown's explicit and implicit arithmetic axioms we obtain exactly Boolean arithmetic. In the second section we consider Brown's algebraic axioms and show that they are synonymous with the axioms for Boolean algebra.

A fascinating aspect of Brown's work is his treatment of inconsistent equations. Brown's proponents (e.g., Howe and Von Foerster, 1975) claim that this treatment solves Russell's paradox and successfully handles problems of self-reference. We investigate these claims in the third section of this paper. We argue that what is correct about Brown's treatment is a reinvention of part of the theory of sequential machines. Further we show that Brown cannot solve Russell's paradox since his logic is too weak to even state the paradox. We conclude with some general discussion of the meaning of paradox.

1. NOTATION AND THE ARITHMETIC AXIOMS.

An arithmetic is an equational system (of rules, axioms plus theorems, etc.) containing no variables.

In this section we will show that Brown's arithmetic looks very different from Boolean arithmetic but, in the end, comes to the same thing.

1.1 The Form of Brown'a Original Axioms

Brown admits to only two axioms:

(F 1) The Law of Calling

$$\urcorner\;\urcorner = \urcorner$$

(The value of a call made again is the value of the call.)

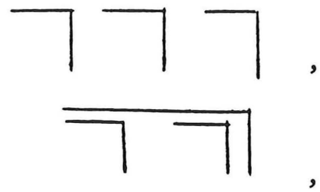
(F 2) The Law of Crossing

$$\overline{\overline{\quad}|\,}| = \quad$$

(The value of a crossing made again is not the value of the crossing.)

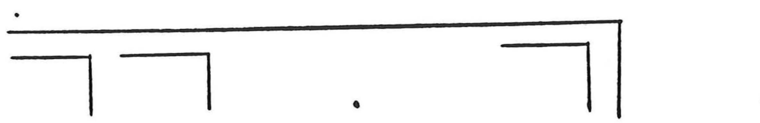
The equals signs in Brown's system are intended as signs of ordinary mathematical identity. He appears to employ only one other symbol, the right corner, "$\urcorner$", but in fact employs at least one other, a blank. Moreover, as we shall see, both the corner and the blank must be available in an indefinitely large number of sizes.

There are two operations in his system, one represented by concatenation, or the writing of terms alongside each other, and the other represented by superimposition or the inscribing of a term above another term. Thus, Brown has terms like



and

The oddest thing about these axioms is the absence of any apparent symbol on the right hand side of (F 2). Here, one might presume, a blank character is intended. In fact, this does not seem to be <u>quite</u> what Brown intends, for he never <u>leaves</u> any blanks in equations. That is, there are no terms like



What is going on here? First of all, the laws of his arithmetic make all terms with a blank equal to similar terms without a blank, so Brown can omit blanks without disturbing the truth-values of his equations. But beyond this, Brown <u>thinks</u>, that he is using only one symbol; he thinks that his language has only one character, the corner, or that if there is a blank in his system, there is only one token of it, simultaneously occupying all the uninscribed paper on which equations are written. While blank characters can be used in a legitimate way, this is not it. In thinking of his language as containing only one contentful character, Brown violates the most basic truth of information theory: at least two symbols are required to convey any information.

We are forced, then, to an interpretation of (F 2) as containing a blank character on the right. Under this interpretation, what looks like a peculiar incomplete equation is merely inperspicuous, since non-digital readers, like people, find it difficult to recognize blanks. Of course a device that reads in discrete pieces, like a digital computer, functions well using a blank character as an unmarked state, even though humans do not. For instance, humans may ask how many blanks they are to notice when they read (F 2). One? One hundred? We must ask how big the blanks are. Noting that the corners vary in size, we must presume that the blanks can too. For a digital reader, answers to all those questions would be presupposed.

The problems caused by using a blank character are compounded by Brown's use of positions instead of symbols to indicate operations. Thus any space of paper left unfilled might indicate any manner of operations applied to sequences of whatever we take the blank character to represent. In fact, Brown treats blank spaces as infinitely ambiguous in just this way, and perhaps brilliantly but certainly illegitimately - thus obtains additional axioms "for free".

Before considering his axioms, we will exchange Brown's notation for a more conventional one. Now proponents of Laws of Form sometimes appear to think that a deep and satisfying truth is revealed through the shapes of Brown's symbols, as adherents of religions often do about the symbols they use, but the metaphysical significance its advocates seem to derive form Brown'a notation should not be confused with its mathematical substance. From a mathematical point of view, notation is insubstantial: notation can be clear or confusing, redundant or concise, convenient or awkward, but it cannot change the subject the language is used to inform us about. Brown merely replaces the ordinary ideographic notation of mathematics with a positional or analytic notation. (See Lyons, 1968.) Again, the shape, nature, etc., of the signs one chooses to convey information with are irrelevant to the mathematical content of what is conveyed. What *is* relevant is the meaning of the symbols and the way they are combined. Brown's system is the same as any other system of axioms employing logical symbols with the same meaning (here, only identity) and the same configuration of axioms, albeit expressed with different characters.

Note that we are not concerned here about the *interpretations* of the axioms, the things for which the non-logical constants are intended to stand, but only about the abstract axiom system itself. For example, Boolean algebra can be interpreted as an algebra of switching circuits, sets, or truth functions. But the language, axioms, and theorems used to describe these interpretations all remain the same. Thus, even if Brown *has* discovered a new interpretation for Boolean algebra, he has not discovered a new mathematical system. Now, since we will show that Brown's systems *are* Boolean, he has at best discovered a new interpretation for such systems. But Brown's informal remarks, such as those associated with the axioms above, cannot be regarded as providing *any* interpretation for his system; they are simply too incredibly vague and abstract. For example, about the symbol "¬", he says

> KNOWLEDGE
>
> Let a state distinguished by the distinction
> be marked with a mark "¬" of distinction.
>
> (page 4)

1.2 The Axioms in Conventional Notation

To return to the main thread of our arguement, we have argued that we may obtain a translation of Brown's axioms into a more conventional language. By allowing " 1" to replace " ⏋" , " 0 " to replace the blank, " $\vee$ " for concatenation and " $\oplus$ " for superimposition, the axioms appear as

(F 1*) $1 \vee 1 = 1$

(F 2*) $1 \oplus 1 = 0$

and when " $\vee$ " and " $\oplus$ " are taken as inclusive and exclusive Boolean addition, we have two familiar laws of Boolean arithmetic.

The truly creative, but, we maintain, suspicious aspect of Brown's axioms is in his use of the identities

(F 3) ⏋ = ⏋

and

(F 4) =

As we have pointed out, blanks and operations performed on blanks are undetectable, so 3 and 4 represent, among a multitude of equations, the fundamental ones

(F 3*) $1 \vee 0 = 0 \vee 1 = 1 \oplus 0 = 0 \oplus 1 = 1$

and

(F 4*) $0 \vee 0 = 0 \oplus 0 = 0$

The conceptual or methodological confusion involved in deriving (F 3*) and (F 4*) from (F 3) and (F 4) is that (F 3) and (F 4) are tautological identities (assuming we can make sense out of (F 4) at all).

That is, (F 3) and (F 4) must be true, simply as a matter of logic, while (F 3*) and (F 4*) are no such thing, but instead give us information about the working of the functions " $\vee$ " and " $\oplus$ ". Of course it's impossible to derive non-tautological information from a tautology, so the fact that Brown seems to have done so is highly curious.

In any case, equations (F 1*) through (F 4*) are translations of equations true in Brown's system, and as these equations serve to uniquely determine the operation of " $\vee$ " and " $\oplus$ " on " 1 " and " 0 " , they complete the Boolean arithmetic for inclusive and exclusive disjunctions. Now it is well known (for example, Klir, 1969) that these two functions, together with the constants, allow us to define all other Boolean functions. Since Brown's laws themselves are familiar laws of Boolean arithmetic, it is clear that the two systems, except for notation, are the same.

Finally, let us note that another translation, perhaps truer to what we suspect Brown's intentions to be, is possible. This translation involves treating the blank as the only constant, concatenation as the only binary function indicator, and treating the corner as a unary function symbol instead of as a constant, writing it over the constants it is being applied to. Thus any occurrence of the corner by itself in an equation would entail the presence of a blank within it for the corner to operate on.

Assigning these grammatical categories to his symbols, " ⅂ " can now be translated as the Boolean complement operation. But because complement is definable in terms of exclusive or and one as

$$-a = a \oplus 1 \quad ,$$

both translations are themselves equivalent.

This second translation may be closer to what Brown has in mind for "⌐", because he sometimes seems to want to use this symbol to stand for an instruction - - which is to say, a function - - rather then an object. Contrast, for instance: "Let any token be intended as an instruction to cross the boundary of the first distinction" on page 5, with the name-like suggestions about the meaning of "⌐" on page 4, quoted above.

In this guise, Brown's axioms appear as follows:

$$-0 \vee -0 = -0$$

$$- -0 = 0$$

$$-0 \vee 0 = 0 \vee -0 = -0$$

$$0 \vee 0 = 0 \quad ,$$

and when the definition

$$1 = -0$$

is employed, they can be re-written as

(F 1*) $1 \vee 1 = 1$

(F 2*) $-1 = 0$

(F 3*) $1 \vee 0 = 0 \vee 1 = 1$

(F 4*) $0 \vee 0 = 0$

These last forms are the ones which will be used in the next section.

2. THE ALGEBRA

According to Lipschutz, 1976, a Boolean algebra is a set containing at least two distinct elements, 0 and 1, two binary operations, $\vee$ and *, and a unary operation, -, satisfying the following axioms:

A1 Commutative Laws

$a \vee b = b \vee a \qquad a * b = b * a$

A2 Distributive Laws

$a \vee (b*c) = (a \vee b) * (a \vee c)$

$a * (b \vee c) = (a * b) \vee (a * c)$

A3 Identity Laws

$a \vee 0 = a \qquad a * 1 = a$

A4 Complement Laws

$a \vee -a = 1; \qquad a*-a = 0$

Brown's arithmetic satisfies these laws and is in fact the two-element Boolean algebra. That there is only one two-element Boolean algebra is well known (Liu, 1977).

2.1 Brown's Algebraic Axioms

Brown offers two algebraic axioms, in addition to his arithmetic principles.

(F 5) $\overline{\overline{p}| \;\; p}| \; =$

and

(F 6) $\overline{\overline{pr}| \;\; \overline{qr}|}| = \overline{\overline{p}| \;\; \overline{q}|}| \; r$.

We will re-write these axioms in standard Boolean notation using the second system of translation suggested in part one. (Alternatively, one could use the first system and the definition of negation.) Thus, they become

$$(F\ 5^*)\quad -(-a \vee a) = 0$$

and

$$(F\ 6^{**})\quad -\left[-(a \vee c) \vee -(b \vee c)\right] = -(-a \vee -b) \vee c\ .$$

Using the definition of conjunction from negation and disjunction,

$$(D1)\qquad a * b = -(-a \vee -b)$$

(F 6 **) becomes

$$(F\ 6^*)\quad (a \vee c) * (b \vee c) = (a * b) \vee c$$

(F 5*) and (F 6*) are of course basic Boolean identities; (F 5*) is a form of excluded middle, and (F 6*) is a form of the distributive laws. Thus Brown's algebraic axioms are <u>implied</u> by the axioms of Boolean algegra.

In fact, in themselves, Brown's axioms are weaker than the Boolean axioms. But as was the case with his arithmetic, Brown uses more principles than he states. First of all, we still have with us the plethora of blanks joinable to either side of any term in any equation. More surprisingly, we find that Brown simply <u>presupposes</u> the commutativity and associativity of his concatenation operation. That is, he commutes terms and omits parentheses in his equations without ever stating as an axiom that such commutations and omissions are permissible.

Brown does not make these presuppositions unknowingly. In his discussion of Sheffer's axioms for Boolesn algebra, p. 109, he says:

> "Sheffer explicitly assumes the restriction of his operator to binary scope ... also, implicitly, assumes the relevance

> of the order in which variables under operation must appear . . . Sheffer was therefore forced to design his initial equations so ingeniously as to contradict them both."

By this Brown seems to mean that Sheffer considered the possibility that his operation was not associative and not commutative, and then showed that his operation was both associative and commutative. In fact, although the Sheffer stroke is commutative, it is not associative. Thus Sheffer had good reason to consider whether these properties held for his operation.

What Brown seems to be saying is that either to assert or to deny these properties is to give them more consideration than they are due. He believes that the fundamental operations of mathematics (whatever they may be) are so necessarily commutative and associative that to assert that they are is to give the very possibility that they might not be more credence than such a possibility deserves. Thus he tells us that the properties are simply not "relevant". This is like objecting to another person's saying "I don't care what color a person is", because such a claim assumes the relevance of color, even as it denies it. The grain of truth in such a view is this: if no one ever had cared what color a person was, it would be odd to deny that you did, and would suggest that you had thought about the possibility of caring more than people who never mention it. But since many people have had a fierce interest in color, just as many mathematical systems do involve non-commutative and non-associative operations, it does not seem untoward to deny either the interest for oneself or the property to the operations.

To state commutativity and associativity is not, as Brown has it, to implicity admit the relevance of order and binary scope; it is to explicitly point out that order and binary scope are irrelevant in the system under consideration, while allowing that they may be relevant in other systems. To treat these two properties as unmentionables is passing strange.

Some defenses for Brown's procedure do exist: one might say that Brown is proposing a logic in which the commutativity and associativity of all operations are rules of inference, so that they need not be stated as axioms of any particular system employing that logic. But even if we were to take this tack, to compare Brown's system with an ordinary mathematical system it would be necessary to add the following unstated axioms:

Associativity of Disjunction

(F 7*) $(a \vee b) \vee c = a \vee (b \vee c)$

Commutativity of Disjunction

(F 8*) $a \vee b = b \vee a$

Because we are treating the corner as complementation, we introduce the maximal element of the algebra through a definition

(D2) $1 = -0$.

But it is also necessary to note that Brown generalizes the power of his blanks to create identities beyond the type (F 3*).

First of all, form algebraic identities of the type

$a = a$

Brown's notation allows him to obtain

(F 9*) $a \vee 0 = a$

Again, in Brown's notation, these two equations cannot be distinguished.

Second, under the interpretation of the corner as a constant, and superimposition as an operation, the same trick ought to be possible for the superimposition operation. That is, from

$$\text{(F 2)} \qquad \overline{\overline{}|\,}| \;=$$

and

$$a = a \quad ,$$

we might obtain

$$\overline{\overline{a}|\,}| = a \quad .$$

Brown does not choose, however, to allow this kind of substitution, and instead proves the above formula as a theorem. This theorem, in our notation

$$- - a = a \quad ,$$

involves a complicated sequence of algebraic manipulations. Brown's proof (pp. 28-31) can easily be reproduced from (F 7*) to (F 8*) using ordinary mathematical modes of inference.

The fact that Brown does not use the superimposition tack to establish double negation and the fact that he never superimposes variables in his algebra, both provide substantial additional evidence that Brown sees the corner as an operation, and not as a constant.

2.2 The Equivalence of the Two Systems

We are now ready to show that all the axioms of Boolean algebra, (A1) through (A4), follow from (F 1*) through (F 9*). Of course, not all of (F 1*) through (F 9*) will be needed, but because all are Boolean identities, the ones not needed are merely redundant.

For reference, Brown's axioms are listed here:

(D1) $a * b = -(-a \vee -b)$

(D2) $1 = -0$

(F 1*) $1 \vee 1 = 1$

(F 2*) $-1 = 0$

(F 3*) $1 \vee 0 = 0 \vee 1 = 1$

(F 4*) $0 \vee 0 = 0$

(F 5*) $-(-a \vee a) = 0$

(F 6*) $(a \vee c) * (b \vee c) = (a * b) \vee c$

(F 7*). $(a \vee b) \vee c = a \vee (b \vee c)$

(F 8*) $a \vee b = b \vee a$

(F 9*) $a \vee 0 = a$

Commutativity

The commutativity of disjunction is (F 8*).

Because " * " is defined as $a * b = -(-a \vee -b)$, the commutativity of " $\vee$ " guarantees that of " * " .

Identity

The identity law for " $\vee$ " is (F 9*).

For conjunction, we again use the definition, obtaining $a * 1 = -(-a \vee -1)$. From (D2), $1 = -0$, we have $a * 1 = -(-a \vee --0)$, and using (F 2*), we now have $a * 1 = -(-a \vee 0)$, but by the identity law for " $\vee$ ", this means $a * 1 = --a$. Double negation now gives us $a * 1 = a$.

Complement

Brown's axiom (F 5*), $-(-a \vee a) = 0$, gives us $--(-a \vee a) = -0$, so by double negation and the definition of " 1 ", we have $-a \vee a = 1$. Because of commutativity, this is the complement law for " $\vee$ ".

From the definition of * , $a * -a = -(-a \vee --a)$, which equals

- (-a V a) by double negation, and by (F 5*), a * -a thus equals 0.

Distribution

By commutativity, a V (b * c) = (b * c) V a. By (F 6*), then, a V (b * c) = (b V a) * (c V a). This last term is (a V b) * (a V c) by another application of commutativity.

The second distributive law requires replacing a, b, and c in (F 6*) with -a, -b, and -c, obtaining (-a V -c) * (-b V -c) = (-a * -b) V -c. Eliminating " * " by (D2), we have

$$- \left[-(-a \vee -c) \vee -(-b \vee -c) \right] = --(--a \vee --b) \vee -c \quad .$$

Negating both sides of this equation and applying double negation leads to

$$-(-a \vee -c) \vee -(-b \vee -c) = -\left[-(a \vee b) \vee -c \right]$$ which,

reapplying the definition of " * " gives us

$$(a * c) \vee (b * c) = (a \vee b) * c \ ,$$

which is the second distributive law when rearranged by commutativity.

We have now shown that (A1) through (A4) are derivable from Brown's axioms plus two definitions. Brown's axioms, moreover, are all well known Boolean identities. The two algebras, then, are synonymous (De Bouvier, 1965). They determine exactly the same class of structures. Brown and his supporters are therefore wrong when they claim he has discovered a new logic. He has reinvented traditional Boolean logic in a notation that would be a printer's nightmare, but with enough obscure and therefore profound-sounding remarks in his commentary to impress non-mathematicians. At best, Brown has produced a new axiomatization for Boolean algebra. Many such equivalent axiomatizations are known (see, for example, Mendelson, 1970) but it is very likely that a minimal subset of (F 1*) through (F 9*) would constitute a new one.

Despite the synonymy of the two axiom sets, one minor difference between Brown's system and Boolean algebra is that the definition of "Boolean algebra" stipulates that all Boolean algebras, in addition to satisfying the listed axioms, must have at least two elements. The true mathematical significance of this difference, we shall see, is minimal, but it looms large in saving Brown from inconsistency.

3. BROWN'S PARADOXICAL EQUATION

The most astounding part of Brown's work is the manner in which he tries to give meaning to relatively inconsistent equations.

3.1 Substitutions and Equations

The valid equations of an algebra are the equations that follow from the axioms defining the algebra - - they are the equations that are true of all structures satisfying those axioms. In a valid equation, any constant can be substituted uniformly for any variable in the equation, and the result must still be a valid equation. Invalid equations, on the other hand, may be true for some substitutions of constants for variables or true in some of the structures satisfying the axioms, but not others. Relatively inconsistent equations -- relative to an axiom system - - are invalid equations that are not true in the axiom system for any assignment of constants to the variables of the equation.

For example,

$$a \vee 1 = a$$

is an invalid equation which is true when "1" is substituted for " a ", while

$$a \vee {-a} = 1$$

is a valid equation, and

$$a \vee {-a} = 0$$

is inconsistent with the definition of Boolean algebra, since from that equation and the valid one immediately before it, we have

$$1 = 0 \quad .$$

As we have noted, this result is not strictly inconsistent with Brown's axioms, but he seems to want to avoid it. He claims, on page 19, for example, that the corner and the blank have distinct values "by definition". But Brown offers no definitions of these symbols, as they are primitives of his algebra, and offers no definition of structure satisfying his system which would require this. We suspect that, in the general spirit of his work, he is confusing use and mention here (see Mates, 1965).

In chapter 11, Brown suggests a way of interpreting inconsistent equations: he allows one to substitute *different* constants for different occurrencts of the *same* variable. For example, in a V-a = 0, we could substitute "0" for the first occurence of "a", and "1" for the second, obtaining 0 V-1 = 0, a valid equation. But changing the rules of logic in this drastic way necessistates not only that 0 = 1, but that there is only one object in the structure to which the logic is being applied. Otherwise, it would not be logically sound to use different constants interchangeably. In any system satisfying the axioms of Boolean algebra, as Brown's does, if 0 = 1, there can be only one object in the system. Such a system is not mathematically exciting because *all* equations are true of it, not only those of Boolean algebra, but those of any algebra.

(Historical note: This system is not original with us or with Brown. It is the Aftermath system of Howland Owl propounded in Walt Kelly's "Pogo" comics, circa 1964.)

Brown treats an equation requiring the technique we have just discussed for its interpretation in great detail, and he feels this equation is the key to new fields of study and to the solution of old paradoxes in mathematics. This equation is

(P1) $f = \overline{f}\rceil$.

First, Brown claims that (P1) can be interpreted as

(P2) $f_t = \overline{f_{t+1}}\rceil$.

To see again that this is nonsense, we could similarly "solve" the arithmetic inconsistency

$x = x + 1$

by claiming it "really" means

$(y + 1) = y + 1$,

by substituting "y + 1" for the first occurrence of "x", and "y" for the second.

3.2 Applications in Automata and Switching Theory

Even though (P1) is inconsistent, (P2) makes perfect sense. It could describe, for example, the behavior of a blinking light. Supposing the two states of the light occur for equal lengths of time. Allowing that unit of time to be assigned to "1", the equation can be read as the true claim that the state of the light at any time is the opposite of what it is after one unit of time has passed. This work of Brown's is reasonable, but it is not, as he seems to think it is, original with him (see Brown's preface).

Such systems are well known. For example, Klir, 1969, describes them as discrete deterministic sequential systems, in a treatment far more detailed and sophisticated than Brown's. Of course, the idea is much older than even Klir's work. Such systems go back at least to McCulloch and Pitts, 1943. Shortly after the publication of their paper von Neumann (see Randell, 1973) recognized that their technique could be used to describe the logical design of a digital computer. McCulloch and Pitts had pointed out that memory could be modeled by sequences of zeros and ones circulating around a sequence of formal neurons arranged in a circle. This idea was quickly implemented. Some of the earliest digital computers had circulating memories which consisted of acoustic signals in delay circuits. These memories were later abandoned on favor of core memories that were faster and did not have to be constantly refreshed, but a faster circulation memory, the so called "bubble" memory, is currently in the developmental stage (Matick, 1975). Thus when Brown claims his equations have application to computer circuitry (page 99) he is certainly correct.

The devices Brown discusses as an extension of his time shift notation also have application in switching theory, but they are a special case of devices that have been studied and used for many years. They are usually called feedback shift registers, and were probably first described by Huffman, 1954. These devices and their generalizations are so important that their study forms a standard part of the computer science and engineering curriculum. They are discussed in detail and with copious references, in Harrison, 1965, and Kohavi, 1970, for example.

3.3 Applications in Set Theory

Brown suggests that his treatment of (P1) provides a solution to Russell's paradox, and an alternative to the theory of types (page 97).

This cry is taken up forcefully in Howe and von Foerster, 1975. This view is entirely mistaken.

First of all, the equation

(P1) $f = \overline{f\rvert}$

can at best be interpreted set-theoretically as

$B = \overline{B}$

which is the claim that the set B is equal to its own complement, or

(S1) $B = \{x \mid x \notin B\}$.

This is to say that B is the set of all objects that are not members of B.

But (S1) has nothing in particular to do with Russell's paradox. Russell's paradox concerns the purported set R which contains all the sets that are not members of *themselves*, i.e.,

(S2) $R = \{x \mid x \notin x\}$.

The Russell set, defined by (S2), and Brown's set, described in (S1), are not described in comparable equations. (S2) is paradoxical precisely because it *looks* like a good definition. (S1) is blatantly inconsistent, as the term to be defined, B, occurs on both sides of the equation.

Russell's paradox is the proof that if R existed, it would both contain and not contain itself, and hence would force us to accept an inconsistency. Nowhere in that proof does one find a claim that corresponds to (S1).

Indeed, in one of the common solutions to Russell's paradox for set theory, namely the von Neumann, 1925, distinction between sets and classes, the Russell set _does_ exist as the universal class, but in no possible set theory could the "Brown set" exist. Investigating (S1), we find it immediately implies that x is in B just in case x is not in B, from which it follows that everything would both be _and_ not be in B. Thus, B would be _both_ the universal class and the empty set. It appears we have only returned to a one object system, but by allowing the relation of membership into the language, we are worse off than when dealing with Brown's purely equational language. B itself can still not exist, for _it_ would both be and not be a member of itself. Hence, (S1) can be true only in "set theories" in which there are _no_ objects.

An important point is being presumed here -- Brown's algebra is a system of _equations_, and as such is in an extremely _weak_ language, in which only small portions of mathematics could _possibly_ be expressed. As a system of equations, Brown's algebra lacks quantifiers, external or genuine negation, and even relational symbols. Almost all statements in even so elementary a branch of mathematics as set theory require these devices, and in particular, every formula involved in Russell's proof employs such symbolism and could not, therefore, even be _expressed_ in the language of _Laws of Form_.

It is the poverty of equational languages which saves Brown from absolute inconsistency. To see this, first note that in saying Brown's language lacks a genuine denial operator, we are pointing out that his corner operates on variables and constants _in_ his language, while genuine denial operates on the complete statements _of_ a language itself. For example, in Boolean algebra, the internal complement operator "-", represents a function on objects of the algebra, as in

$$-0 = 1 \quad ,$$

but a genuine negation operation added to such a language would allow us to say things like

$$" \ 0 \neq 1 \ " \text{ (or "not } [0 = 1] \text{").}$$

This distinction may be clearer in numerical algebra, for here the first equation is nonsense, and the second formula true. Note that absolute inconsistency involves asserting and denying simutaneously a single proposition, or claiming that one statement, given a precise single interpretation, is both true and false. Now, without external negation in a language, there is no way to assert that any of its statements are false, and hence no way to arrive at an absolute inconsistency.

3.4 Paradox and Rationality

There are three kinds of "paradoxes": mathematical-scientific paradoxes, social paradoxes, and inconsistencies. Mathematical-scientific paradoxes are puzzling "paradoxical" truths that are paradoxical in that they are surprising, and that they surprise because there are other truths that appear to be inconsistent with them; or because there are other false beliefs we wish to maintain that are inconsistent with the paradoxical truth. Russell's paradox, that there exists no Russell set, is such a paradox in that it is or appears to be inconsistent with the abstraction principle that every concept determines a set. It requires a solution in that we must now either explain why the abstraction principle, which is quite attractive, is not true, or why the abstraction principle, when stated correctly, is not *really* inconsistent with Russell's result. Such paradoxes are a major source of new directions in the sciences.

We have argued, however, that Brown does not offer such a solution and new direction, since he does not even confront that paradox. He and his supporters, on the other hand (see Brown's preface, and Howe and von Foerster, 1975), suggest that Brown offers the first, one, and only such solution, thus opening up a new field of research and a new way of understanding mathematics and systems in general. We wish to point out that if people *are* troubled by Russell's paradox, the development of alternative solutions to it and the corresponding new logics and languages required in those solutions have by now a long and rich history. (One may consult, for instance, Ramsey, 1925, Schilpp, 1944, or Martin, 1970, to get a sampling of such discussions and more extensive bibliography.).

Similar to this first kind of paradox in mathematics and science are social paradoxes of the kind Marxists and Hegelians are often especially fond -- for example, the disturbing and unacceptable juxtaposition of opulence and squalor. Such paradoxes provide a source of new visions, energy, and creativity, just as the mathematical-scientific paradoxes do. But both of these kinds of paradoxes are often confused with the third kind,of paradox: antinomy or absolute inconsistency. Absolute inconsistency is intolerable in *any rational*, or simply any *language-using* system, for it involves the absolute breakdown of even the possibility of communication. If a statement that things are a certain way does not rule out the possibility that they are <u>not</u> that very way, the statement can communicate no information. If you tell me you are bald, and I accept inconsistencies, even if I believe you I will not yet have any definite beliefs for I may still believe that at the very same time in no sense at all are you bald. Now, it is precisely because this third kind of paradox is intolerable that the other kinds are the source of so much energy. The energy is expended to avoid absolute inconsistency.

Howe and von Foerster, along with many others who see consistency as a bugaboo, confuse these kinds of paradox. In fact, they talk about paradox, inconsistency, self-reference, and the relationship of observer to observed as if all these things were indistinguishable. There is, in fact, no reason whatsoever to see the theory of types as standing in <u>any</u> relation, either supportive or contrary, to the existence of constructive "paradoxical" relationships in psychiatry. (For a discussion of what the theory of types <u>does</u> entail, see Chihara, 1972).

General Systems Theory, if it is to be general, must admittedly involve the discovery of similarities between strikingly different kinds of systems. If it is to be a science, however, and not a branch of magic, the features different systems share in common must be exactly specified; that is, the scientific way to generality is through equivalence in which distinct entities are considered to be the same for given purposes through their participating in a well-defined equivalence relation. In this regard Brown's <u>Laws of Form</u>, and its supporters like Howe and von Foerster, do a disservice to systems theory as a science.

CONCLUSION

We have shown that Brown's axiom system specifies exactly the Boolean algebras. Thus Brown's system has applications to the extent that Boolean algebra has applications. In particular, Boolean algebra has long been used in the design and analysis of computer and communication systems so that Brown is suggesting nothing new when he claims that his system has applications in these fields.

Boolean algebra is only a small fragment of logic. As such it does not contain either quantifiers or membership, and thus it is impossible to even state Russell's paradox within Boolean algebra. Since Brown's system is synonymous with Boolean algebra, it suffers from the same deficiencies. What is claimed to be a solution of Russell's paradox is based on misunderstanding and confusion of terms.

While we recognize the power of paradox to stimulate thinking, we feel that the solution of paradoxes and the clearing up of confusion is progress is science. If General Systems Theory is to become a science, we must avoid confusions without destroying useful analogies.

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