Extending the Mathematics in Qualitative Process Theory

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Abstract

Reasoning about physical systems requires the integration of a range of knowledge and reasoning techniques. P. Hayes has named the enterprise of identifying and formalizing the common-sense knowledge people use for this task "naive physics." Qualitative Process theory by K. Forbus proposes a structure and some of the content of naive theories about dynamics, (i.e., the way things change in a physical situation). Any physical theory, however, rests on an underlying mathematics. QP theory assumes a qualitative mathematics which captures only simple topological relationships between values of continuous parameters. While the results are impressive, this mathematics is unable to support the full range of deduction needed for a complete naive physics reasoner. A more complete naive mathematics must be capable of representing measure information about parameter values as well as shape and strength characterizations of the partial derivatives relating these values. This article proposes a naive mathematics meeting these requirements, and shows that it considerably expands the scope and power of deductions which QP theory can perform.

1 Introduction

Qualitative Process (QP) theory [Forbus, 1984] describes the form and structure of naive theories [Hayes, 1979] about the dynamics of physical systems. A key component of QP theory is the qualitative mathematics used to represent values of continuous parameters and relationships between them. A research strategy for developing this mathematics has been to search for a qualitative mathematics capable of yielding significant results from a minimum of information about the situation being modelled. In the work described here, we ask a slightly different question: what kinds of information can we add to the base theory, and what new questions can we answer with this additional information? We will examine a simple example which reveals two limitations in the current theory. First, the qualitative description of a situation is often ambiguous. Second, QP theory is of limited use in reasoning about the effects of adjustments to continuous control parameters. We will then present an extension to the mathematics used in QP theory which improves its performance in both these areas. We begin with a review of QP theory.

1.1 Mathematics in QP theory

The representation for a continuous parameter in QP theory is a *quantity*. A quantity has four parts:

1. The magnitude of the amount of the quantity.
2. The sign of the amount {-, 0, +}.
3. The magnitude of the derivative.
4. The sign of the derivative.

The use of the sign as a significant qualitative abstraction is adopted from DeKleer [deKleer, 1979] [deKleer and Brown, 1984]. Magnitudes are represented in a *quantity space*. The quantity space for a number consists of all those amounts to which it is potentially related in the situation being modelled. The special value ZERO is always included in every quantity space, and relates the quantity space representation with sign information.

Quantities are related to one another through *Relations*, which can be either ordering relations, functional relations, or influences. Ordering relations include simple statements regarding the relative values of quantities, such as:

$level(p) = level(q)$
$pressure(p)\ Greater\_than\ Zero$

Functional relations are a qualitative analog of continuous monotonic functions whose domain and range are real numbers. The following states that the level of water in a container is qualitatively proportional to the amount in the container:

$level(p)\ Q{+}\ amount\_of(p)$

These are called Qualitative Proportionalities (Qprops). Qprops can be named, permitting the propagation of ordering information through separate instances of the same named relationship. The *Process* is the mechanism of change in QP theory. A process acts to change a situation by *influencing* some parameter(s) of objects in the situation. An *Influence* is similar in information content to a qualitative proportionality, but affects the derivative of the range variable, rather than its amount. For example, the primary effect of a fluid-flow process is on the derivatives of the source and destination fluid quantities.

$amount\_of(destination)\ I{+}\ flow\_rate$
$amount\_of(source)\ I{-}\ flow\_rate$

Qprops are often referred to as *indirect* influences, since they provide pathways through which *direct* influences propagate.

Forbus' implementation of QP theory combines this basic domain information with an initial system description to perform measurement interpretation and envisioning.

The initial description contains only a listing of the basic physical objects in the situation, and need not identify any processes which may be active in that situation. These are automatically determined from descriptions of the conditions under which processes become active. The basic inferences performed are:

1. *Elaboration*, in which all possible process instances which may occur in a situation are added to it.

2. *Process structure determination*, that is, selection from the set of possible process instances, those subsets which are mutually consistent and consistent with the known facts about the situation. Each such subset is a partial description of one or more possible qualitative states of the physical system.

3. *Influence resolution*, in which the set of influences on each situation parameter is closed for each possible process structure, in an attempt to resolve the effect of the influences on the parameter.

4. *Limit analysis*, in which predictions are made regarding possible transitions out of each qualitative state.

For a detailed discussion of these inferences, see [Forbus, 1984]. We will primarily be concerned in this paper with influence resolution and a modification of it we term *Linguistic Perturbation Analysis*. In addition, we will briefly discuss extensions to process structure determination for dealing with uncertain information. For a more detailed discussion of these extensions, see [D'Ambrosio, 1986].

2 Example

We now analyze a hypothetical model of a typical continuous flow industrial process, in order to demonstrate these steps and identify the capabilities and limitations of QP theory. Fig. 1 shows a simplified sketch of the process. Reactants in granular form enter through the port at the top left (a material flow process), and are heated to reaction temperature within the vessel (a heat-flow process). When the reactants reach reaction temperature, they undergo a state change (a reaction), in which they disappear and a fluid product and an off-gas are created. The off-gas exits through the port at the upper right (another material flow process). As the hot off-gas flows out of the reaction vessel, heat is transferred to the cool incoming reactants (counter-current heat flow). We will ignore the processes by which the product is extracted from the vessel and simply allow it to accumulate at the bottom.

Our interest in a system such as this is in reasoning about it for purposes of process control. Many forms of reasoning are needed for process control, from which we

Figure 1: Reaction Vessel

have selected two for initial investigation: measurement interpretation and prediction. Specifically, we would like to determine what might be happening in the system, given some observations of selected measureable parameters (measurement interpretation), and to estimate the effects of possible control actions (prediction). These control actions typically are adjustments to independent continuous parameters.

A well established mathematical theory, control theory, exists for reasoning about systems such as this. Unfortunately, it is not applicable in many situations for any of four reasons:

1. Observational data may be uncertain or incomplete.
2. Precise mathematical models of the underlying physical processes may be unavailable or too complex for efficient reasoning.
3. The results of mathematical modelling must be further interpreted before they can be used by human or automated control systems.
4. Mathematical models carry only part of the modelling burden. Specifically, they cannot conveniently account for the appearrance or disappearance of objects in the situation being modelled, and do not account for the processes by which an appropriate model is formulated.

QP theory, on the other hand, can reason with incomplete data, is computationally tractable, allows for description of system processes in terms familiar to those who

Figure 2: influence graph for furnace active state

actually control such systems today, and can represent and reason about situations in which objects appear and disappear. For these reasons, QP theory offers the promise of significantly extending the scope of automated process control.

The four basic processes crucial to understanding of the system described above, basic heat flow, the reaction, material flow, and counter-current heat flow, are described in detail in [D'Ambrosio, 1986]. Given a suitable initial state description, the first two QP inferences identify three possible states for the situation described, (1) that nothing is happening, (2) that the only thing occurring is that the reaction vessel is being heated, or (3) that all processes are active. The state of interest is the one in which all processes are active. The influence graph in Fig. 2 illustrates a simplified version of the influences and qualitative proportionalities between variables in the state.

Using influence resolution, we can determine various facts about this state, such as (a complete output for the three cases is shown in figure 3):

- If the heat input is increasing, the off-gas generation rate will be increasing also.
- If the incoming reactant temperature is decreasing, the off-gas temp will be decreasing.

However, we cannot determine:

1. Is the product temperature increasing, decreasing, or constant?
2. If the heat input is increasing, is the off-gas exit temperature increasing or decreasing?
3. If we increase the heat input a little, how much will the generation rate increase?
4. If the available observations do not uniquely identify a single state, which of the possible states is more likely?

These limitations are the result of ambiguity in the conclusions derived using QP theory.

3 Ambiguity in QP theory

We identify two types of ambiguity in QP theory, *Internal* and *External* ambiguity. Internal ambiguity occurs when the use of QP theory produces multiple descriptions of a single physical situation. External ambiguity is the dual of this, namely when a single QP theory description corresponds to several possible physical situations which must be distinguished. Internal ambiguity is of two types. First, given a situation description, there may be ambiguity about which of several possible states a system is in (e.g., given a leaky bucket with water pouring in, is the water level rising or falling?). Second, given a specific state, there may be ambiguity about what state will follow it (e.g., given a closed container containing water, and a heat source heating the container, will it explode?).

External ambiguity is the inability to determine, on a scale meaningful to an external observer, the duration of a situation, as well as the magnitude and intra-situation evolution of the parameters of the situation (e.g., how fast is the water rising? How long before the container explodes?)

These ambiguities are the result of four fundamental limitations in QP theory representations and inference mechanisms:

1. Inability to resolve conflicting functional dependencies. That is, if two influences on a parameter are of opposite sign, QP theory has no way to determine the

Parameter	Influence Resolution	QPA(Heater)	QPA(reactants)
P (reactants bin)	-	-	?
A (reactants bin)	-	-	?
T (reactants bin)	0	0	+
P (reactants furnace)	?	-	?
A (reactants furnace)	?	-	?
T (reactants furnace)	?	+	?
P (product furnace)	+	+	?
A (product furnace)	+	+	?
T (product furnace)	?	+	?
P (off-gas furnace)	?	+	?
A (off-gas furnace)	?	+	?
T (off-gas furnace)	?	+	?
P (off-gas offtake)	+	+	?
A (off-gas offtake)	+	+	?
T (off-gas offtake)	?	?	+
Temperature (heater)	0	+	0
Flow-Rate (reactants)	?	+	?
Flow-Rate (off-gas)	?	+	?
Flow-Rate (heat)	?	+	?
G-Rate (reaction)	?	+	?
Temperature-Lost	?	?	-
Temperature-Gained	?	+	-

Note:

1. "?" indicates an ambiguity in the QP analysis.
2. The two independent parameters are in **boldface**.
3. The value shown is the sign of the derivative.

Figure 3: Results of QP analysis of Furnace

resulting composite influence. This is caused by the weak representation for the functional form of dependencies, which captures only the sign but no strength information.

2. Inability to order predicted state changes. This results in the inability to determine which of several possible successor states will be the actual successor of a state. This is caused by lack of ordering information on change rates, as well as lack of quantitative information on the magnitude of change needed for state change.

3. Inability to quantify, even approximately, parameters significant to external observers during times between major state transitions. This is caused by a weak model of *intra*-state situation evolution. Time, quantity values, and functional dependencies are all represented qualitatively in QP theory.

4. Inability to represent non-boolean predicate and state possibilities. This prevents the system from distinguishing between states which are possible, but highly unlikely, and states which are highly likely.

Solving these problems requires extending QP representations to capture more information about the system being modelled. We have studied three classes of extensions: extensions to the quantity representations, the relationship representations, and the certainty representations. Specifically, we have developed an extension to QP theory which utilizes:

- Belief function certainty representations - these will permit capture of partial or uncertain observational data, and estimates of state likelihood.

- Linguistic descriptions of influence sensitivities - to reduce undecidability during influence resolution.

- Linguistic characterizations of parameter values and ordering relationships - to permit capture of partial or uncertain observational data, and enable estimates of the effects of adjustments to continuous control parameters.

It should be noted that these are extensions, not replacement representations. This extension is orthogonal to the quantity-space representation used in QP. The original quantity-space representation is retained, and is assumed in the examples presented in this paper. These extensions reason at the appropriate level of detail for the kinds of control actions typically needed, draw the needed distinctions, are computationally tractable, and can reason with the imprecise or uncertain data typically available. In this paper we concentrate on the second of these extensions, linguistic influence sensitivities, and present a way of annotationing the relationship representation in QP theory to reduce ambiguity. Discussion of the integration of Dempster-Shafer belief functions with QP theory and the underlying ATMS can be found in [D'Ambrosio, 1987b]. A later section of the present paper also discusses our parameter value extensions and shows how they, in combination with the functional extensions, can be used to estimate the effects of potential control actions. Further details can be found in [D'Ambrosio, 1986]. See [Simmons, 1987] for an alternate extended quantity representation.

4 Linguistic Influence Sensitivities

Basic QP theory cannot resolve the conflicting influences on the off-gas temperature parameter in our example. The influence resolution rule used by Forbus states that if opposite influences impinge on a single parameter, then the net influence on the parameter is unknown. In order to reduce the number of situations in which conflicting functional dependencies cannot be resolved, we extend QP theory functional descriptions with a linguistic influence sensitivity. Intuitively, this corresponds to distinguishing between first order, second order, etc., dependencies. With this extension we can now address the second question unanswerable earlier: if we increase the heat input, will the offgas temperature increase or decrease?

Forbus claims that if actual data about relative magnitudes of the influences is available, it can be used to resolve conflicts. We might attempt to achieve this by extending direct and indirect influences with a strength parameter. This is inadequate, however, for two reasons. First, the overriding influence may not be local. Information may have to be propagated through several influences before reaching the parameter at which it is combined. Second, various sources of strength information have varying scopes of validity. In the following sections we first identify two basic influence subgraphs responsible for the ambiguity in our example, and argue that the ambiguity can be eliminated by annotating the subgraphs with influence sensitivity and adding additional situation parameters. We then present extensions to the influence resolution algorithm for utilizing the sensitivity annotations, and finally describe a control structure for managing acquisition and use of annotation information.

4.1 Identifying internal causes of conflict in influence graphs

We have identified two basic patterns of influences which account for the ambiguity previously encountered. These are the conflict triangle (Fig. 4) and the feedback loop (Fig. 5). The reason, for example, that the change in offgas temperature in the offtake cannot be resolved is that there are two conflicting paths through which a single

Figure 4: Conflict Triangle

parameter (offgas temperature in the reaction vessel) affects the target parameter. But the effect on temperature-lost is in this case smaller than the direct effect on the offtake temp, and can be ignored. We can indicate this by adding to the influence arc an annotation indicating temp-lost in counter-current heat flow is relatively insensitive to offgas temperature in the furnace (Fig 4b).

Another ambiguity in the QP theory analysis of the furnace is in the generation rate and associated variables. One of the causes of this ambiguity is the set of influences on product temperature shown in Fig. 3. Since both the generation rate and heat-flow rate are positive, the qualitative derivative of the product temperature is undecidable. This network is similar to one Kuipers [Kuipers, 1986] identifies as introducing a new *landmark value*, not in the original quantity space for the product temperature. This new value represents an equilibrium value towards which the temperature will tend. Recognition of the existence of an equilibrium value permits resolution of the effects of the conflicting influences on product temperature, depending on the assumed ordering between the actual product temperature and the equilibrium value. Kuipers adds the equilibrium value to the set of fixed points in the quantity space for the original variable. We, however, add it as a *new parameter of the model*, subject to influences similar to those of the original quantity. Thus, we can represent and reason about change in both the actual value and the equilibrium value in response to active processes. For example, if the actual temperature is only slightly sensitive to the heat-flow rate, but the equilibrium temperature is very sensitive, then we might conclude that the system will be slow in returning to equilibrium once perturbed. The extended influence diagram for the feedback loop is shown in

Figure 5: Feedback Loops

Fig 5.

4.2 Sensitivity Annotations

An influence is a partial derivative of a controlled variable with respect to a controlling variable. In QP theory, computing a value for a controlled variable takes place in two phases:

1. All of the individual influences on the controlled variable must be identified and the effect of each of these must be computed.

2. The various effects must be combined to determine the composite effect on the controlled variable.

This procedure relies on local propagation to perform influence resolution. If local propagation is to carry the burden of our extended influence resolution, then the propagated value must somehow be extended to represent the sensitivity information. The value being propagated in influence resolution is a quantity, and the representation used is sign abstraction. If we model influences as describing the normalized sensitivity of one variable to changes in another, then we can simply extend the quantity representation for the influence quantity and use a discrete scale of influence magnitudes. We then represent the actual value as a fuzzy set over this value space, to model the imprecision in the available sensitivity information. The following observations lead us to choose a fuzzy set representation for influence sensitivity annotations:

- A discrete representation matches well with the propositional style reasoner underlying our implementation.
- The sensitivity is not always known with precision (recall our comment about lack of precise mathematical models).
- The sensitivity may not be constant over the range of the variables, or may not be independent of the values of other parameters.

An alternate model is described in [Mavrovouniotis, Michael & Stephanopoulos, George; 1987]. A major difference between their work and ours is our assumption that an annotation represents a normalized sensitivity. We show in the next section how this permits us to make semi-quantitative estimates, which we believe their system cannot do. While influence resolution using sensitivity annotations is conceptually simple, two questions arise. First, how can an appropriate discretization for the normalized change value (effect on one variable of changes in another), henceforth referred to as an influence value, can be determined. Second, How are influence values to be propagated through annotated influences.

If we start with an n-level influence value discretization and an m-level sensitivity discretization, then after k influence propagation steps we seemingly might need an $(nm)^k$ influence value discretization to avoid information loss. This worst case complexity can be avoided, however, by the following four observations:

1. We are only interested in the result at a resolution equivalent to the original n-level discretization.
2. Additional detail is only relevant when two annotated influences are being combined, to aid in influence resolution if they conflict.
3. Rather than annotating all influences in a graph, we will only annotate those necessary to disambiguate parameters of interest in a specific query. We can design the propagation algorithm to take advantage of this by treating an unannotated influence as an identity operator for influence values.

4. We use a fuzzy relational algorithm as the basic model for influence propagation. The basic fuzzy relational influence algorithm can be designed so that failure to maintain a fully detailed discretization only increases the ambiguity of the result, rather than produce incorrect results (e.g. if the correct answer is 2.5, and our discretization for influence values contains only the values {1, 2, 3, 4, 5}, we can represent the answer as the set {2,3}).

Given this, we model sensitivity annotations as parameters of a standard *fuzzy relational influence algorithm* [Zadeh, 1973]. We choose a fuzzy representation to allow simple modelling of the imprecision of these annotations[2]. We next detail the algorithms used to compute the consequences of this fuzzy sensitivity.

4.2.1 Computing individual influences

An influence of the form:

(Influenced-variable Q+/- Influencing-variable, Sensitivity)

is taken to specify a fuzzy relation between three amounts: C, the amount of the influencing variable; S, the amount of the influence sensitivity; and Iv, the influence value. The value of Iv can be computed as follows:

$$Iv = \sum_{C,S}(\min(\mu_C, \mu_S, \mu_Q)/Q_{Iv,C,S}(C,S))$$

where $Q_{I,C,S}(C,S)$ is the relation providing a degree of membership for each possible value of Iv for each value of C and S. A short review of fuzzy notation is in order at this point: $\mu_x(y)$ is the degree of membership of element y in the fuzzy set denoted by x, and can take on values in {0,1}. When the argument is omitted, as below, it is assumed that the element is obvious from context. For example, $\sum_C(\mu_C)$ is the sum of the degree of memberships of each element in the fuzzy set C. Also, the notation x/y typically means that y is a member of a set to degree x. The formula above, then, defines the set Iv to consist of all the values of the elements of the relation $Q_{Iv,C,S}$. Each value may appear more than once in Q. The degree to which it is a member of Iv is the maximum of the degrees of membership specified in each appearance. The degree of membership resulting from an appearance of a value in Q is the minimum of the degree of membership of the corresponding value of the influencing variable (C), the degree of membership of the corresponding value of the sensitivity annotation (S), and the degree of membership of the value in $Q_{Iv,C,S}(C,S)$.

[2]The underlying model we assume is of a set of independent, linear influences. Fuzzy set models of sensitivities permit us to allow for the inaccuracies of this model.

This relation ($Q_{Iv,C,S}$) can be customized when specific information is available. As a default, we use the following to generate the table, assuming Influence value and sensitivity annotations are both represented on a { ... -2, -1, 0, 1, 2, ...} scale[3]:

$$Q_{I,C,S}(C_j, S_k) = sign(C_j * S_k) * (abs(C_j * S_k)^{1/2})$$

In cases where the result is not in the original discretization, we use the set representation described earlier. Thus, we get the following default relation table for a five element discretization for sensitivities and influence values:

C/S	-2	-1	0	1	2
-2	2	$\{1,2\}$	0	$\{-2,-1\}$	-2
-1	$\{1,2\}$	1	0	-1	$\{-2,-1\}$
0	0	0	0	0	0
1	$\{-2,-1\}$	-1	0	1	$\{1,2\}$
2	-2	$\{-2,-1\}$	0	$\{1,2\}$	2

4.2.2 Combining influences

Sensitivity annotations provide us with a means of estimating influence magnitudes, which are directly comparable. Below we show an algorithm for computing the combined effect of two influences. A rough translation is that an element is definitely a member of the set of possible values for the combined influence if that element is a member of the value sets for both input values, or if it is a member of the value set for one input, and a weaker element of the same sign is a member of the value set for the other input. Also, an element of the discretization may be an element of the result set under two conditions. First, if it is a member of the value set of one input, and a element of the same magnitude but opposite sign is a member of the value set for the other input. Second, if an element of the same sign but greater magnitude is a member of one value set, and an element of the opposite sign and greater magnitude is a member of the other value set. We formalize this algorithm as follows:

$$\begin{aligned} \mu_{Iv}(i) = \; & (\mu_{Iv1}(i) \wedge \mu_{Iv2}(i)) \\ & \vee(\vee_{j,|j|<|i|}(\mu_{Iv1}(i) \wedge \mu_{Iv2}(j))) \\ & \vee(\mu_{Iv1}(i) \wedge \mu_{Iv2}(-i) \wedge unknown) \\ & \vee(\vee_{j,j>i} \vee_{k,k<-i} (\mu_{Iv1}(j) \wedge \mu_{Iv2}(k) \wedge unknown)) \end{aligned}$$

Subscripts i, j, and k are assumed to be 0 for no influence, increasing positive for positive influence elements, and increasing negative for increasing negative influence

[3]All the algorithms we present are independent of the actual discretization used. We typically use a five or seven element discretization, that is, {-2, -1, 0, 1, 2 } or {-3, -2, -1, 0, 1, 2, 3}.

elements (e.g., -3, -2, -1, 0, 1, 2, 3 for a seven element discrete scale, with -3 the strongest negative influence). The above is only half of the formula actually used. The actual relation is symmetrical in the two infiuences Iv1 and Iv2.

4.3 Annotation Management

In examining the sources of ambiguity in the reaction vessel example, we note that many of the annotations which could resolve the ambiguities are not universally valid. In fact, we identify four levels of validity for an annotation. These validity levels are determined primarily by opportunities in the implementation:

1. An annotation is *universally* valid when it can be incorporated directly into a view or process description, and correctly describes the functioning of a particular influence in all situations in which an instance of the view or process participates. These are rare.

2. An annotation is *scenario* valid when it correctly describes the operation of a particular influence in a particular view or process instance, for all qualitative states in which the instance is active. Product temperature annotations in the example are an instance of this annotation type.

3. An annotation is *state* valid when it correctly describes the operation of a particular influence in a view or process instance, only for a defined subset of the qualitative states of a system.

4. Annotation is *query* valid when it correctly describes the operation of a particular influence in a view or process instance, only for a particular query. The conflict triangle annotation for determining off-gas temperature in the offtake is an example of this type of annotation.

The first type of annotation can simply be part of the basic view or process definition. The other three are added to the QP description of a scenario as needed during problem solving. A four step algorithm extends the basic QP theory influence resolution algorithm:

1. Execute the basic influence resolution.

2. Check results for ambiguities in parameter values of interest. If all interesting parameter values are determined uniquely, then problem solving is complete.

3. Otherwise, search the influence graph for instances of ambiguity causing subgraphs. If one is found, and the parameter for which it might create an ambiguity is ambiguous, then annotate the subgraph with influence sensitivity information if available.

4. Re-execute the basic influence resolution algorithm on the now annotated graph.

This algorithm assumes the extended QP reasoner is embedded in a larger system which has or can obtain the necessary problem specific information to resolve ambiguities. It provides a problem directed way of selecting aspects of the larger system's problem specific knowledge relevant to the query being processed.

5 Linguistic Perturbation Analysis

QP theory cannot directly answer quantitative questions about the effect of changes to independent parameters. Yet, many approaches to process control require a means to estimate the effects of hypothetical actions. The four basic deductions of QP theory do not directly address this problem, even on the qualitative level. However, a relatively simple extension of influence resolution does permit qualitative analysis of the impact of control actions. We use the influence graph for the state of interest and perform a qualitative form of classical small signal or perturbation analysis. This analysis is based on de Kleer's IQ analysis [deKleer and Brown, 1984].

Straightforward application of small signal analysis yields qualitative estimates of the effects of control actions subject to the same limitations as the original QP deductions. Situations can arise in which it is impossible to determine whether a target parameter value will increase or decrease following a control action. Also, many of the control actions which must be reasoned about are adjustments to continuous control parameters. Simple *increase* or *decrease* results are insufficient for reasoning about this kind of control. It is important to be able to estimate *how much* the increase or decrease will be.

The same functional characterizations used to extend influence resolution in the previous section can also remove much of the qualitative ambiguity. Also, these same annotations can be used to obtain linguistic estimates of change magnitudes. This results from our interpretation of function strength annotations as normalized sensitivities. By integrating these annotations with the linguistic quantity space extensions described below we can obtain complete semi-quantitative estimates of the effects of control actions. We call this complete procedure Linguistic Perturbation (LP) Analysis. This section builds up the LP analysis algorithm step by step, starting with the simpler qualitative perturbation analysis.

Parameter	Standard Influence Resolution	Extended Influence Resolution
P (reactants bin)	-	-
A (reactants bin)	-	-
T (reactants bin)	0	0
P (reactants furnace)	?	0
A (reactants furnace)	?	0
T (reactants furnace)	?	+
P (product furnace)	+	+
A (product furnace)	+	+
T (product furnace)	?	0
P (off-gas furnace)	?	0
A (off-gas furnace)	?	0
T (off-gas furnace)	?	0
P (off-gas offtake)	+	+
A (off-gas offtake)	+	+
T (off-gas offtake)	?	-
Temperature (heater)	0	0
Flow-Rate (reactants)	?	-
Flow-Rate (off-gas)	?	-
Flow-rate (heat)	?	0
G-Rate (reaction)	?	+
Temperature-Lost	?	+
Temperature-Gained	?	+

Figure 6: Comparison of Results of Influence Resolution

5.1 Qualitative Perturbation Analysis

Classical small signal analysis determines a delta for a target parameter given a delta for a control parameter. This change is determined by evaluating the partial derivative of the target with respect to the control parameter at the current value of the parameters, and multiplying that value by the control delta. A simple qualitative version of this procedure would be to multiply a qualitative form of this partial derivative by the *sign* of the control delta. This is only valid as long as the change in the control parameter does not result in a change in the view and process structure for the situation. This is the qualitative equivalent of "small signal" analysis. Therefore, a restriction on the application of this technique is that either the current view and process structure must not be dependent on any equality quantity conditions or, if it is, their validity must not be affected by the proposed change.

This procedure is simple to perform, and the partial derivatives are already represented as influences in the influence graph. de Kleer has developed a qualitative procedure for performing this computation which he calls IQ analysis. A problem arises in determining the qualitative change values in de Kleer's confluence formalism, though, because arcs are undirected and he allows cycles in the influence graph. This requires search to find a globally consistent solution, which he performs by introducing assumptions about unknown values, and backtracking as needed.

Forbus eliminates the possibility of cycles in QP theory by fiat, claiming that cycles violate intuitive notions of causality and are unnecessary[Forbus, 1984]. The result of this simplification is that the search intensive IQ algorithm of de Kleer reduces to the simple one-pass influence resolution algorithm of Forbus. An adaptation of this algorithm for qualitative perturbation analysis is shown in Fig. 7. This algorithm uses the derivative of each quantity to store the delta. The basic difference between this and inference resolution is that we initialize certain derivatives to non-zero values.

However, while this algorithm is adequate for Influence Resolution, it is inadequate for Qualitative Perturbation Analysis. The problem is that the sign of the target parameter delta is not always the sign of the first derivative. It is the sign of the *lowest order nonzero derivative*[4]. This fact renders the one-pass influence resolution algorithm only a partial solution to the problem of perturbation analysis. The graph of Fig. 8 shows the problem.

A simple analysis of this diagram according to the above algorithm would lead one to conclude that ((D (Amount product)) = 0). However, the correct answer is ((D (Amount product) > 0). The reason is that, while the first derivative is zero, the second derivative is positive. Therefore, assuming the change persists for a finite period of time, the eventual result will be a positive first derivative, and therefore a positive delta. Increasing the temperature of the reactants will increase

[4]This depends critically on the state persisting for some nonzero time interval.

```
(defun Simple-QPA (delta-list)
  make a list of all quantities (amounts and first derivatives)
    in the situation, ordered by dependencies
    (this is the same as for influence resolution)

  set the value of each quantity in the delta list to the value
    specified, set all others to unknown

  For each quantity:
    If positive direct influences and no negative direct
      influences and no unknown direct influences, set its value
      to positive
    If negative direct influences and no positive direct
      influences and no unknown direct influences, set its value
      to negative.
    If both positive and negative direct influences, or any
      unknown direct influences, set its value to unknown.
    If no direct influences, then:
      If positive indirect influences, and no negative or unknown
        indirect influences, set its value to positive.
      If negative indirect influences, and no positive or unknown
        indirect influences, set its value to negative.
      If both positive and negative indirect influences, set its
        value to unknown.
      Else set its value to zero.
)
```

Figure 7: Simple Qualitative Perturbation Analysis Algorithm

Temp(x) —Q+-> (Rate y)—I+—> Amount(z)

((D (Temp x)) > 0)
((D (Amount z)) ? 0)

Figure 8: QP Sample Influence Graph and Query

the amount of product produced over any non-zero time interval, if all other factors are held constant. Information about second derivatives can be obtained from direct influences (I+/I-) in QP theory.

The inference sanctioned by QP theory, given that the amount of product is directly positively influenced by the generation-rate, is that if the *amount* of the generation-rate is positive, then, in the absence of conflicting influences, the *derivative* of the product amount is also positive. However, we can make the same inference for the next higher order set of values. That is, if the *first derivative* of the generation-rate is positive, than, in the absence of conflicting influences, the *second derivative* of the product amount is also positive. Combining this with the perturbation analysis rule that the result of perturbation analysis is the value of the lowest non-zero derivative, we conclude that the correct result for the example of Fig. 8 is that the amount of product will increase, since the second derivative is the lowest order non-zero derivative. This is equivalent to treating selected direct influences (I+/I-) as indirect influences (Q+/Q-), thereby re-introducing loops into the influence graph and destroying the one-pass nature of Forbus' algorithm. A revised algorithm for QPA is shown in Fig. 9. The key change to the previous algorithm is that, once the previous one-pass algorithm is complete, the new algorithm searches for the earliest (in terms of the quantity dependency partial order) derivative which is zero, and which has a non-zero second derivative. It then assigns that value to the first derivative, thus starting a new round of influence propagation. The procedure repeats until no such zero derivatives can be found. This is the algorithm used to derive the results shown in Fig. 3. Notice that this algorithm uses the same basic procedures for computing individual influences and combining them that are used in influence resolution.

```
defun QPA (delta-list)
 Perform Simple-QPA as described earlier.
 Execute repeatedly until no further changes occur:
   For each quantity in the quantity list:
      If the quantity is a derivative and has a value of Zero
         perform a higher order derivative check and set its
           value according to the results.
         If the value is now not zero, mark a change has occurred.
      Also, if any changes have occurred on this pass through the
        quantity list, then re-execute normal influence check for
        this quantity.


defun high-order-deriv-check (quantity)

 ;;This function is only invoked on derivatives with a current
 ;; value of Zero.

 Set the minus-list to the list of direct negative influences
    (I- with positive derivative of influencing parameter,
     I+ with negative derivative of influencing parameter, or
     I+ or I- with unknown derivative of influencing parameter)
 Set the plus-list to the list of direct positive influences
 If the plus and minus lists are both empty, do nothing.
 If the plus-list is non-empty, and the minus list is empty,
     set the quantity value to plus.
 If the minus-list is non-empty, and the plus-list is empty,
     set the quantity value to minus.
 If neither list is empty, set the quantity value to unknown.
```

Figure 9: Revised Qualitative Perturbation Analysis Algorithm

5.2 Extended Perturbation Analysis

The analysis described in the previous section is capable of deriving many useful results. Our furnace example has only two independent parameters, the temperature of the heat source and the temperature of the incoming reactants. Examination of the results of Qualitative Perturbation analysis shown in Fig. 3 reveals that there are some indeterminacies in the analysis, though. The sources of these ambiguities have already been discussed in the previous section. By replacing the basic influence computation and combination algorithm of QP theory with the extended algorithm discussed in that section, we can eliminate those ambiguities. Figure 5.2 show the results of extended QPA using the annotations described earlier, and compares these results with the results of analysis without annotations. The figures follow the same format as those for influence resolution shown in the preceding section, and the same comments apply, except that the value shown is the computed delta. We compare results both for the increased heat query (labelled "heater") and the increased incoming reactant temperature query(labelled "reactants").

5.3 Final Form: Linguistic Perturbation Analysis

QP theory is limited in the range of "what if" or small signal perturbation analysis questions it can answer by its restricted representations. We have seen that we can reduce the ambiguity in its analyses by adding additional functional characterizations, and providing a semi-quantitative extension representation for influence magnitudes. In traditional small-signal analysis we can obtain an estimate of the final value of a target parameter by adding the computed delta to the initial value for the parameter. QP theory provides no representation for parameter magnitudes to which we can add a computed delta to obtain any meaningful result. The problem is that there is no information within the theory itself which permits us to establish any relevant distinctions beyond those already made in the quantity space, and these distinctions establish an ordinal, not a cardinal, scale. We can do more in reasoning about the consequences of change by providing an extension theory which can represent distinctions relevant to an *external agent.* We use a linguistic variable representation to meet this requirement. For our purposes, we will simply consider a linguistic variable to be a possibility distribution over a discrete set of "interesting" values in some domain. For a more complete discussion, see [Zadeh, 1975].

Using linguistic variables as the needed quantitative representation of parameter magnitudes, we have developed a four step procedure to perform a linguistic version of perturbation analysis. This procedure will need substantial amounts of situation-specific quantitative information, which could be obtained either by default, by observation, or directly from the user. The procedure assumes that we are performing

Parameter	QPA Heater	EQPA Heater	QPA Reactants	EQPA Reactants
P (reactants bin)	-	-	?	-
A (reactants bin)	-	-	?	-
T (reactants bin)	0	0	+	+
P (reactants furnace)	-	-	?	-
A (reactants furnace)	-	-	?	-
T (reactants furnace)	+	+	?	+
P (product furnace)	+	+	?	+
A (product furnace)	+	+	?	+
T (product furnace)	+	+	?	-
P (off_gas furnace)	+	+	?	+
A (off_gas furnace)	+	+	?	+
T (off_gas furnace)	+	+	?	-
P (off_gas offtake)	+	+	?	+
A (off_gas offtake)	+	+	?	+
T (off_gas offtake)	?	+	+	+
Temperature (htr)	+	+	0	0
Flow_Rate (reactants)	+	+	?	+
Flow_Rate (off_gas)	+	+	?	+
Flow_rate (heat)	+	+	+	+
G_Rate (reaction)	+	+	?	+
Temperature_Lost	?	?	-	-
Temperature_Gained	+	+	-	-

Figure 10: Extended QPA Example Summary

the perturbation analysis around some state for which we know the quantitative extension base values for both the source and target parameters. If we have not been able to establish these values either by observation or by influence resolution, then we assume that the extension base values are established by a Correspondence provided to the system. The four steps are listed below and detailed in subsequent paragraphs:

1. Compute input influence.
2. Propagate influence through influence graph.
3. Convert computed influence into new target parameter value.
4. Check reasonableness of result.

For purpose of illustration, we will follow the problem of estimating the effect on reaction rate and off-gas exit temperature if we increase the temperature of the heat source from low to medium. We assume that the extension base values are provided from observation. Computing the input influence is a knowledge-based process. We assume that the system has available a mapping function (parameter-specific) which maps from an old and new input parameter value pair to an influence strength [5]. This mapping function can be expressed as a fuzzy relation between input values and influence magnitudes. Given a matrix IR specifying this mapping function, we can compute the influence equivalent of the delta as:

$$Influence = New\_value \odot Current\_value \odot Influence\_Relation$$

$$\mu_{Iv}(k) = \sum_{i,j} \mu_{New}(i) \wedge \mu_{old}(j) \wedge \mu_{IR}(i,j,k)$$

Propagating this influence through the view and process structure of the state uses the extended perturbation analysis algorithm described earlier. The coarseness of the influence representation needed is in general a function of the coarseness of the discretization of the goal parameter as well as the number of influence annotations made and their coarseness. We provide no procedure for computing this, but assume the user has selected an appropriate discretization. In general this must be maintained as relatively coarse: as it gets finer and finer, it approaches assuming a linear model, which is an invalid assumption in most cases. One problem arises, however,

[5] A Temperature increase from 90 degrees C to 110 degrees C might be a big or little change - there is no information internal to the theory which can be used to determine this. Since all parameters in a quantity space share a single value discretization, number of discrete steps in the change does not directly provide this information.

in using the extended qualitative perturbation analysis algorithm to estimate quantitative changes. Influence relations (I+/I-) are qualitative abstractions of equations with a temporal aspect[deKleer and Bobrow, 1984]. In general, in order to obtain a quantitative estimate of the effect of propagating a delta through a direct influence, we must know *how long* the delta is in effect. We could ignore this problem in the qualitative analysis for three reasons:

1. dt is simply a scaling parameter which affects all direct influences proportionately, and therefore does not affect their relative magnitudes when combining conflicting direct influences.

2. Most directly influenced variables, as we saw in section 4, exist in feedback loops which control their values. When considering only the change in equilibrium due to a control action, time no longer need be considered. The amount of the shift in the equilibrium value is determined solely by the relative strengths of the direct and indirect influences in the feedback loops around the equilibrium variable.[6]

3. When combining direct and indirect influences for a parameter not in equilibrium, we can assume that the strength annotation on a direct influence is chosen to reflect its effect after some *nominal* time period.

The first and second of these three assumptions are still valid for quantitative (Linguistic) analysis, but the third might not be. The extended qualitative perturbation analysis algorithm presented earlier must therefore be extended to scale influences propagated through direct influence arcs (I+/I-) by a user specified time delta. However, in accordance with assumption two above, this must only be done for influences which are not part of feedback loops for variables in equilibrium. The example in fig. 11 does not incorporate this extension, and must be viewed as estimating results for some standard nominal time delay after the change action is taken.

The next step is the inverse of the first step, combining the resulting influence with the initial goal parameter value to obtain a final result. Assuming the influence relation (IR) is represented in matrix form, this inverse relation is straightforward to compute:

$$\mu_{New}(i) = \vee_{j,k}\mu_{Influence}(k) \wedge \mu_{Old}(k) \wedge \mu_{IR}(i,j,k)$$

Fig. 11 shows the result of a linguistic perturbation analysis for a moderate increase in heater temperature. The system is able to make several interesting distinctions, such as the fact that, while the temperature of the product in the furnace

[6]Note that while this argument and the previous one are each independently reasonable, each is based on assumptions which contradict the assumptions of the other!

is unlikely to change much, the off-gas temperature in the offtake may change significantly. Such approximations are subject to substantial inaccuracy, and a person who attempts such rough estimates will try to verify their reasonableness somehow. We can do this by testing whether the results satisfy the quantity restrictions[7]. for the state.

We can perform the reasonableness test in either of two ways. First, we can directly test whether or not the estimated value for the target parameter is within its quantity restriction. The restriction values are automatically derived by our implementation for each possible system state. Alternately, a more extensive test can be performed by estimating the final values of all parameters of the state, and testing whether or not all parameter pairs satisfy all quantity conditions imposed on them by the state. This is a more restrictive test, since testing each parameter in isolation may miss relational constraints of the state.

5.4 Summary

In this section we have developed a technique for using QP theory to reason about the consequences of continuous control actions within a qualitative state. Starting from the classical method of small signal analysis, we developed a qualitative notion of small signal analysis, and extended the basic influence resolution algorithm of Forbus to perform this analysis. We then combined this basic algorithm with the quantitative extensions for parameter values developed earlier, and presented a technique for estimating the effects of continuous control actions within a qualitative state.

6 Evaluation

Each of the above algorithms requires different information from the user and makes certain assumptions about the information provided which might limit the applicability of the algorithm.

Linguistic influence resolution derives its power from two sources, functional strength annotations and an appropriate discretization of the influence propagation parameter. We have already identified four classes of annotation: universal, system specific, state specific, and query specific. Of these, query-specific annotations are potentially the most troubling. While we have specifically excluded from consideration here the source of annotation information, it still remains to demonstrate that it is at least feasible that some external knowledge source could provide the required information. Recognizing that an annotation is appropriate requires that sufficient information

[7]A quantity restriction is the possibility distribution representing the union of all possible values the parameter can have in a particular system state.

Values prior to control action:

Domain for Heater and Product Temperature (in degrees C):
50, 100, 150, 200, 1000, 1500, 2000, 2500, 3000, 3500

Domain for generation rate (in arbitrary mass units per minute):
100, 125, 150, 175, 200

Domain for off-gas Temperature in offtake (in degrees C):
100 200 300 400 500

Observations are expressed as D/S belief distributions over possible values: $[belief, plausibility]/value + [bel, plaus]/val + \ldots$ *for all values with plausibility* > 0:

(Temp heater): $[1,1]/2500$
(Temp product): $[0,0.01]/1000 + [.84,1]/1500 + [0,.01]/2000$
G-Rate: $[.95,.97]/125 + [.03,.05]/150$
(Temp (c-s off-gas offtake)): $[.95,.97]/200 + [.03,.05]/300$

Now do LP analysis - new heater temp is one discretization element higher than before:

(lpa h new_value = $[1,1]/3000$)

Estimates of system parameter values following control action:

(Temp heater): $[1,1]/3000$
(Temp product):
$[0,.01]/1000 + [0,1]/1500 + [0,1]/2000 + [0,.01]/2500$
G-rate: $[0,1]/125 + [0,1]/150 + [0,1]/175$
(Temp (c-s off-gas offtake): $[0,1]/200 + [0,1]/300 + [0,1]/400$

Figure 11: Linguistic Perturbation Analysis Example

be available at the time the annotation is requested. A query specific annotation is requested when preliminary application of the influence resolution algorithm reveals an ambiguity in the result for some interesting parameter. At that time, several facts are available to aid the search for relevant query-specific annotations:

1. The subgraph causing the ambiguity can be recognized using graph matching techniques.

2. The entry point of the change into the graph is readily identifiable by examining the change values of the nodes influencing subgraph nodes.

3. The component(s) which generated the subgraph can be identified as long as this information is recorded when the influence graph is constructed (the current implementation does not do this).

These facts seem to be exactly those which might be expected to trigger recognition of the relevant query-specific patch.

The second problem is the choice of an appropriate discretization for the influence parameter. This has been discussed earlier and shown not to be as critical as it might seem. The worst result of choosing too coarse a discretization is that some ambiguity remains in the final result which might have been eliminable. Also, a discretization finer than that of the control and observable variable values is only necessary when combining annotated influences, not whenever computing individual annotated influences. Thus, the minimum discretization needed to maintain full information grows more slowly than might be expected. We have obtained adequate results on the example used in this paper using an influence discretization with the same coarseness as the discretization for functional strength annotations.

Finally, we note an implicit assumption that annotations are intended to resolve local ambiguities. There may be a danger when an influence computed using a strength annotation propagates outside the locale where it is valid and combines with an influence from another, unrelated annotation. It is partially for this reason that we have adopted the approach of only adding those annotations actually needed to resolve ambiguity, rather than all possible annotations. This procedure has been sufficient for all examples studied so far. Should it turn out inadequate in other applications, another alternative would be to "age" influences, that is, to broaden or fuzzify them at each propagation step. This would serve to nullify the effect of strength annotations outside the immediate environment for which they are intended.

The second and more complex of the procedures we have described in linguistic perturbation analysis. It depends on the ability of the user to establish suitable discretizations for parameters of interest, in addition to all of the information needed for linguistic influence resolution. It also places special demands on the functional annotations. These annotations must now describe quantized partial sensitivities of one parameter with respect to another. While this use of annotations is consistent with the use made by linguistic influence resolution (and in fact solves the problems created by propagation beyond local domain of applicability described above), it makes far more intensive use of the annotation mechanism. This therefore raises again, and even more strongly, the problem of establishing an appropriate discretization for the influence parameter. However, the same comments made earlier apply here - the only danger of an insufficient discretization is an increase in the ambiguity of the result. In fact, establishing a fairly coarse discretization can be a useful way to prevent over reliance on the linearity assumption inherent in LPA. Our development of the LPA algorithm depended on equilibrium assumptions. An extension of this to nonequilibrium situations is necessary. Finally, it is not yet clear that the use of the same annotations is valid when a direct influence is used to estimate a second derivative in the basic QPA algorithm.

7 Summary

7.1 Review

We began this work with an interest in pursuing a symbolic, knowledge-based approach to the control of complex engineered systems. The work presented here has been based on two premises. First, we believe that Qualitative Process (QP) theory offers potential for reasoning about the control of complex engineered physical systems, especially when they are poorly understood or the capability for making observations of the systems is limited. Second, we surmise that problem solving in this domain proceeds by an iterative process of building, applying, and patching models of the system under consideration.

On the basis of these premises we examined QP theory and found it severely limited in its present form. First, it is often unable to determine unambiguously the qualitative value of system parameters. Often, when information is available which could potentially serve to disambiguate results, there is no way to express the information within QP theory. Second, QP theory in its current form provides no facility for performing quantitative reasoning. Many of the tasks involved in control of engineered systems involve adjustments of continuous control parameters, or estimation of effects relative to some scale external to the system under consideration. Both of these reasoning tasks require some form of quantitative capability.

In order to surmount these problems, we have developed a set of extensions to QP theory which reduce internal ambiguity and expand the scope of QP theory, by providing an extension theory which can reason semi-quantitatively about consequences of external control actions. These extensions are based on the use of linguistic variables to represent the uncertain or imprecise system-specific information typically available to supplement models built from a domain theory. The extension set has three basic components:

1. A *Linguistic Quantity Space*, which can represent partial information about quantity conditions and relate quantity orderings with linguistic descriptions of parameter magnitudes. These linguistic parameter values make user relevant distinctions.

2. *Linguistic functional strength annotations* and an extension to the influence resolution algorithm which makes use of these annotations to resolve ambiguity.

3. *Linguistic Perturbation* analysis, which builds on all of the above mechanisms and provides a way to estimate the effects of hypothetical control actions.

We have shown, at least for the example problem, that these extensions can be used to derive answers to several qualitative and quantitative questions which cannot be answered using basic QP theory. Specifically, we have demonstrated:

- The unambiguous determination of qualitative parameter values given linguistic functional strength characterizations (*linguistic influence resolution*).

- The semi-quantitative estimation of the effects of adjustments to continuous control parameters (*linguistic perturbation analysis*).

7.2 Further Research

Much work remains to be done. Most importantly, the work described here must be extended to include limit analysis, the fourth basic deduction in QP theory. With semi-quantitative estimates of both parameter values and change rates, it should be possible to choose between possible future states which basic QP theory cannot disambiguate, as well as estimate state durations.

Also, we have given only the briefest sketch of possible kinds of additional functional description which could be used to reduce the ambiguity of the results of the basic QP theory deductions. Functional relationships often have a temporal character as well as relative strengths. What characteristics of relationships between continuous parameters of a situation do people observe? How are these characteristics remembered, and how are they used in problem solving?

This last question touches on another major research area, the subject of our second basic premise. What is the nature of the overall problem solving architecture? What other kinds of knowledge are available during reasoning about physical systems besides domain theories of the kind representable in QP theory? How are they combined? We have suggested ambiguity-based model patching as one possible mechanism for interaction of different kinds of knowledge: there must be others.

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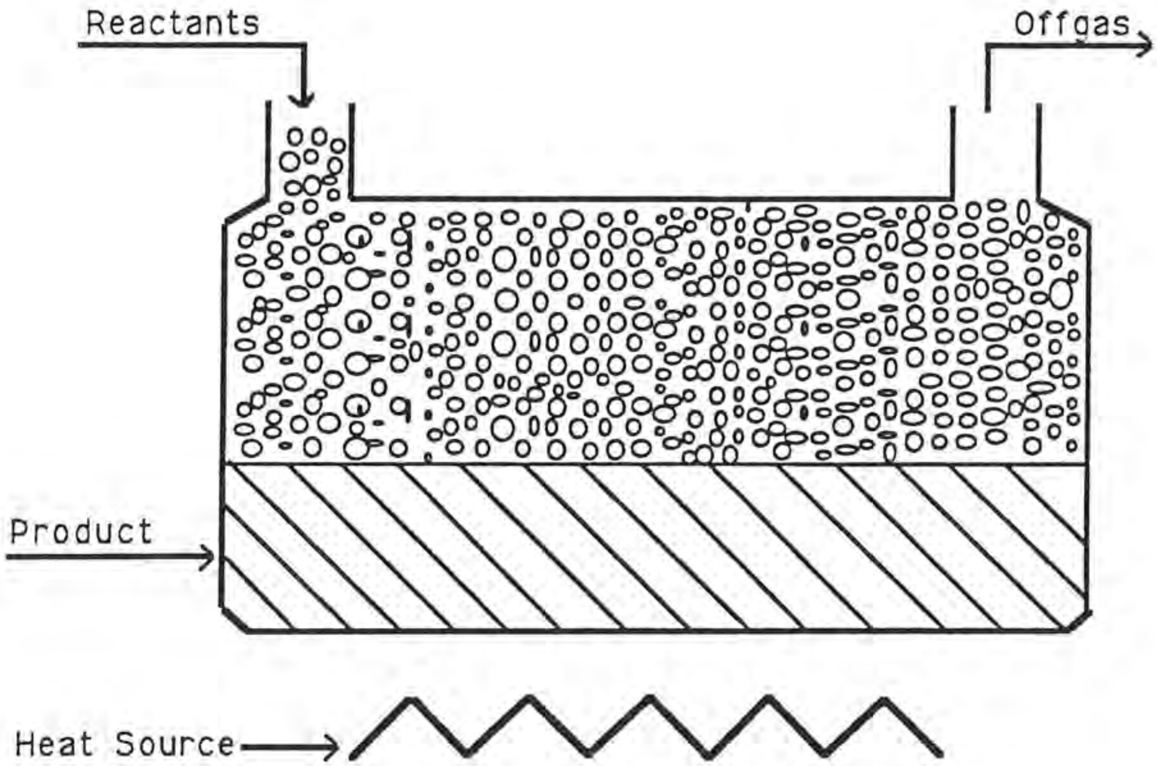

Figure 1: Reaction Vessel

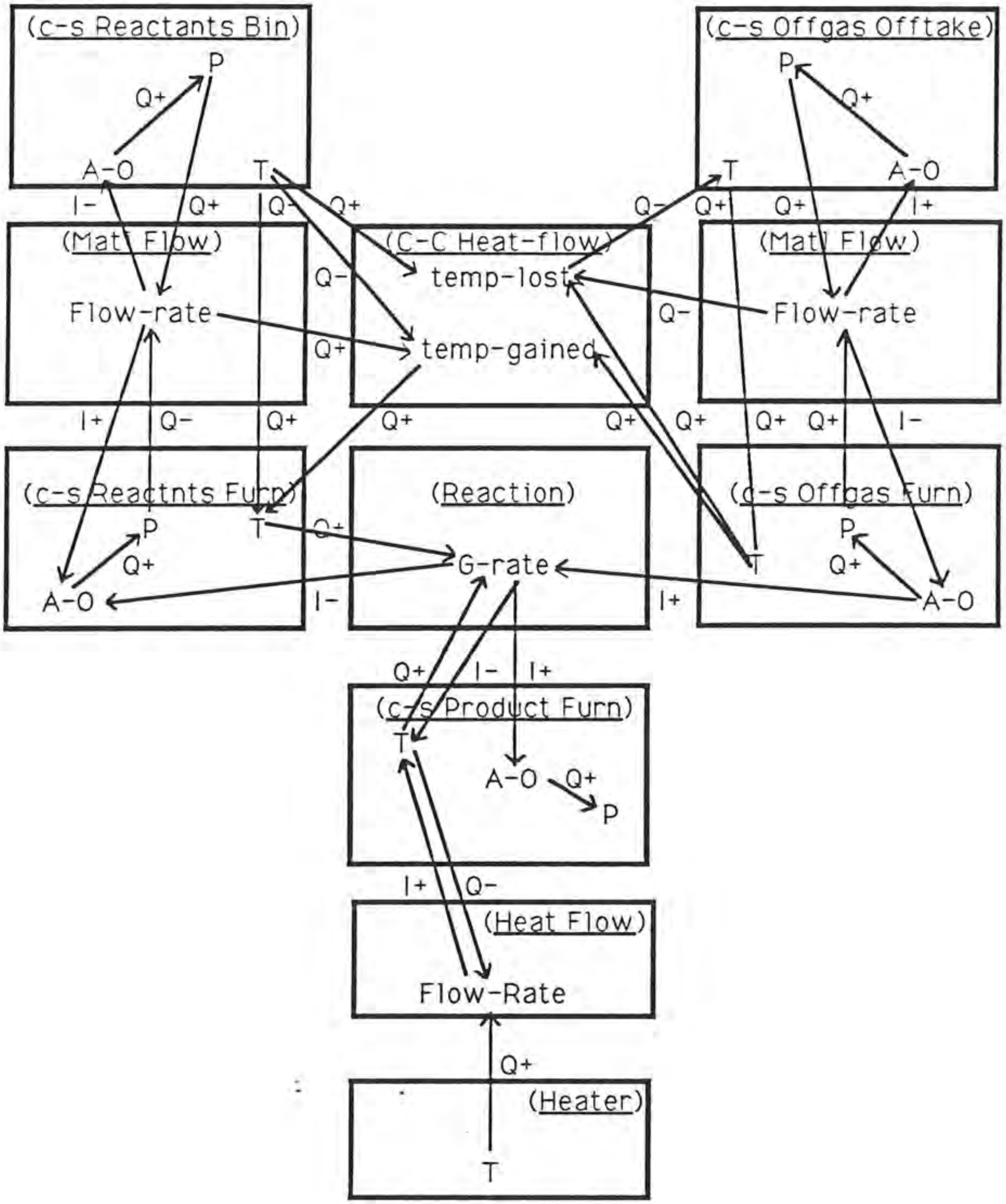

Figure 2: Influence Graph for Furnace

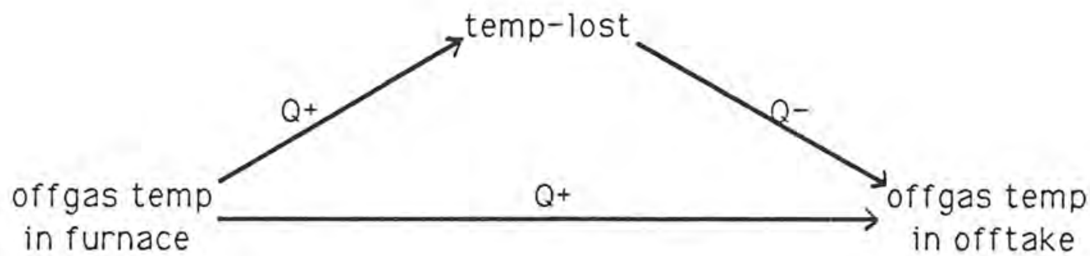

Fig. 4a

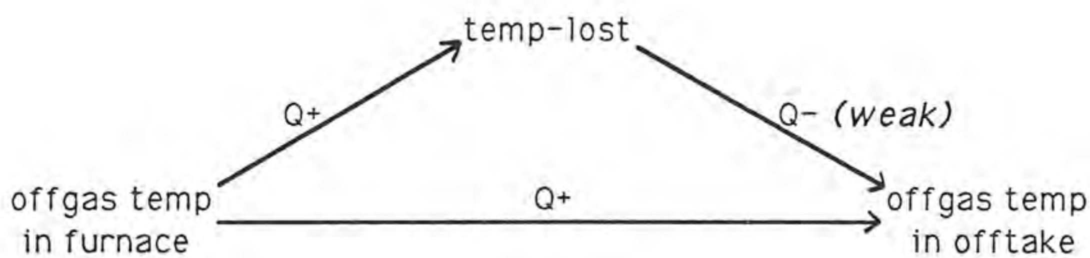

Fig. 4b

Figure 4: Conflict Triangle

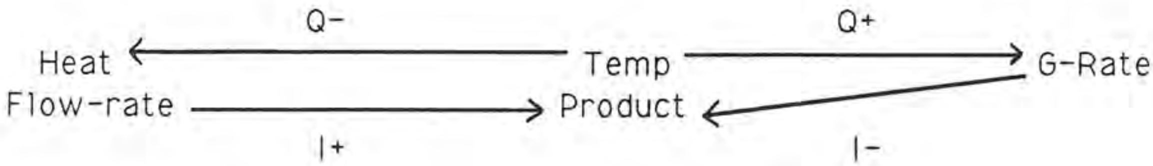

Fig. 5a

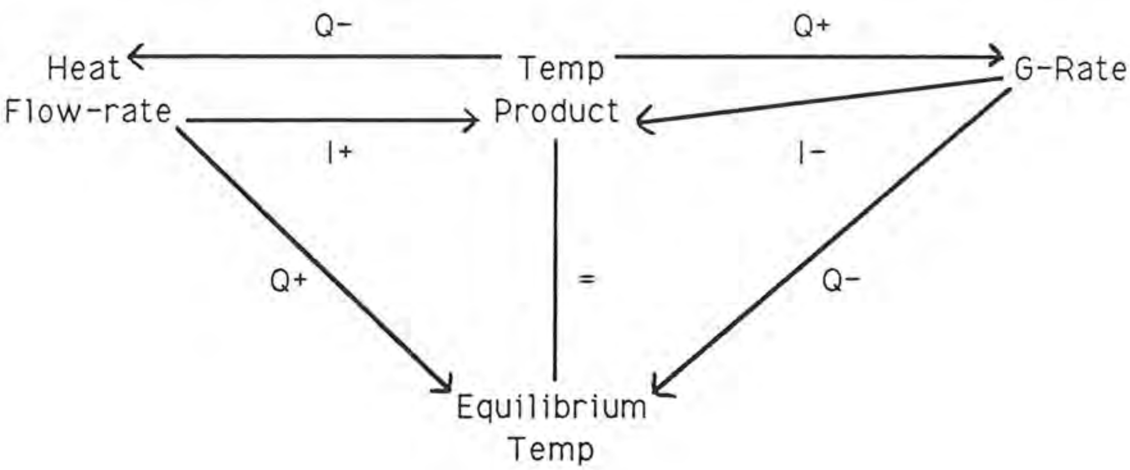

Fig. 5b

Figure 5: Feedback Loops