Combining Opinions from Several Experts

Xiaoning Ling
W. G. Rudd
Department of Computer Science
Oregon State University
Corvallis, Oregon 97331-3902

89-30-4

Combining Opinions from Several Experts

Xiaoning Ling and W. G. Rudd
Department of Computer Science
Oregon State University
Corvallis, Oregon 97331-3902

Abstract This paper addresses the problem of aggregating a number of expert opinions which have been expressed in some numerical form. An important feature of sets of expert opinions is the possibility of stochastic dependence between members of the sets. We develop an approach for combining expert opinions which formally allows for such dependence. This approach is based on an extension of the Dempster-Shafer theory, a well-known calculus for reasoning with uncertainty in artificial intelligence.

1. Introduction

Consider a project manager who must solve a problem for a project, and suppose that the manager consults several experts who offer him their opinions. What solution to the problem should the manager derive from these expert opinions? This is the problem of solving problems using opinions from more than one expert. This important and basic issue in decision-making processes appears in many application domains of artificial intelligence (AI).

We do not restrict ourselves to combining expert opinions from human experts; we can readily envision the need to combine expert advice from several distributed computer-based expert systems, or from mixtures of human and non-human opinion creators. In the following, we shall not distinguish between human and non-human "experts."

How should the manager arrive at a solution? First, because opinions provided by experts generally involve some degree of uncertainty, he or she should be able to "reason with uncertainty." Experts often use phrases like "probably true", "might be", "likely", and "most likely" to qualify their opinions. Most expert systems that include reasoning with uncertainty assign numerical values to measures of uncertainty or to strengths of beliefs.

In the rare situation in which each expert expresses precisely the same opinion as the others, then the manager might reasonably accept this opinion as a very plausible solution, particularly if the experts differ in their backgrounds, methodology, and in the knowledge they use in reaching their decisions. But in most cases experts disagree because they possess different sets of information and expertise. Therefore, the manager must reconcile differences between expert opinions and reach a composite solution that takes all different opinions into account.

Experts differ in levels of expertise. Some experts are more expert than the others because of better training, more experience, and greater intelligence. Therefore, our manager, when she or he combines expert opinions, should treat opinions of different experts differently. We should pay the most attention, or weight most heavily, the opinions of the best experts.

While experts do not share precisely the same information and expertise, they often share some of these. For example, they may access certain data from the same information sources and they might base their opinions at least partially on this body of data. In cases like this, the opinions of experts are statistically dependent upon each other. Morris observed that the degrees of dependence between experts significantly affect combined results [Morris 1986]. He claims that "The issue of nonindependence among experts is critically important .... It is the single most important issue in practical applications. Yet, it is often ignored in many expert combination formulas, probably because it is extremely difficult to think about, much less quantify"

This paper addresses these problems for an automatic manager of systems of multiple experts. The key method employed here is the Dempster-Shafer theory (D-S theory) [Shafer 1976, Dempster 1967], which is a well-known calculus for reasoning with uncertainty in AI. There are two reasons why we chose this method: (1) the D-S theory can be used to handle epistemic information -- information that is not necessarily constructed from precise mathematical information, which, for example, might be constructed from sometimes vague and sometimes confused perception [Shafer 1976]; (2) the theory naturally and easily handles ignorance or lack of knowledge [Bhatnagar and Kanal 1986]. Clearly, these properties are desired for our purpose of combining expert opinions since experts do offer their opinions with certain degrees of ignorance and, in most situations, at least some of the information they use is epistemic in nature.

Unfortunately, the combination rule of the D-S theory is mathematically justified only if the information sources to combine are statistically independent [Dempster 1967]. Since dependencies between experts generally exist, we must extend the combination rule must to deal with dependent evidence. We have developed a formal approach for combining dependent opinions of multiple experts.

In the next section, we review some basic concepts of the D-S theory, illustrate how the theory can be used to combine results from independent experts, and suggest a method for weighting expertise. We then propose an approach for combining dependent experts for the situations where expert opinions can be represented as simple support functions. This method is an extension of the combination rule of the D-S theory. Finally, we discuss an approach for combining more complex opinions of experts and the problem of higher-order dependence.

2. Combining Opinions from Independent Experts

In this section, we briefly review some basic concepts of the D-S theory and illustrate these concepts by simple examples of medical reasoning. In these examples, we consider situations in which several physicians make diagnoses on a patient. Our objective is to combine opinions from experts who offer differing diagnoses to produce a composite diagnosis. For now we assume that these experts base their opinions on distinct observations (independence assumption). The examples we describe are extensions of those in Gordon and Shortliffe [1985]. For details of D-S theory, see that article, [Shafer 1976], and [Dempster 1967].

Suppose that propositions h_1, h_2, ..., h_n are all possible actual answers to the question Q that manager M asks experts to answer. The set $H = \{h_1, h_2, ..., h_n\}$, is called the **frame of discernment**; each h_i is a **singleton hypothesis**. The elements of H are to be mutually exclusive and exhaustive. The power set of the frame of discernment H, denoted by 2^H, is a **hypothesis space** with respect to H, which is a set of all possible answers to the question Q. A non-singleton hypothesis, a subset of H with more than one member, should be interpreted as the hypothesis that one of the singletons in it is the correct answer to the question Q.

Example 2.1. Suppose a physician is considering a case of cholestatic jaundice. This problem is caused by an inability of the liver to excrete bile normally. In a typical case of this type, the diagnostic hypothesis set might include hepatitis (Hep), cirrhosis (Cirr), Gallstones (Gall), and Pancreatic cancer (Pan). In terms of D-S theory, the set {Hep, Cirr, Gall, Pan} is the frame of discernment H. The hypothesis space is the power set of the frame of discernment: { ø, {Hep}, {Cirr}, {Gall}, {Pan}, {Hep Cirr}, {Hep, Gall}, {Hep, Pan}, {Cirr, Gall}, {Cirr Pan}, {Gall, Pan}, {Hep, Cirr, Gall}, {Hep, Cirr, Pan}, {Hep, Gall, Pan}, {Cirr, Gall, Pan}, H}. The hypothesis space is the set of all possible hypotheses.

The D-S theory uses a number m(A) in the range [0,1] inclusive to indicate belief in the hypothesis A. This number is a measure of that portion of the total belief committed exactly to the hypothesis A. Thus the belief concerning the frame of discernment H can be represented as a function m: 2^H-->[0,1]. This function m is called a **basic probability assignment** (bpa) that must satisfy $m(\emptyset) = 0$ and $\sum m(A) = 1$. We use this function to represent opinions of experts. A **focal element** F of a bpa m is the hypothesis F that satisfies $m(F) > 0$. Note that m(H) is a measure of the extent to which we can make no decision at all regarding any of the hypotheses; m(H) is a measure of our ignorance about the problem.

Example 2.2. Let H be {Hep, Cirr, Gall, Pan}. Suppose a physician P_1 observes some symptoms from a patient and he decides that these symptoms support the diagnosis of {Hep, Cirr} to the degree 0.6, but do not support a choice between cirrhosis and hepatitis. The remaining belief, 1 - 0.6 = 0.4, denotes the degree of belief at which P_1 has not gathered any evidence that could be used to assign to any hypothesis in 2^H. The unknown degree 0.4 is assigned to the frame H. Thus $m_1(\{Hep, Cirr\}) = 0.6$, $m_1(H) = 0.4$, and the value of m_1 for every other hypothesis in 2^H is 0.

Given several bpas over the same frame of discernment but based on distinct independent pieces of evidence, Dempster's rule of combination [Dempster 1967] enables us to compute a new bpa which is a composite effect of the original bpas. Suppose m_1, with focal elements $A_1, \dots, A_k$, and m_2, with focal elements $B_1, \dots, B_m$, are bpas over the same frame H. The new bpa is defined by $m(\emptyset) = 0$, and

$$m(C) = \frac{\Sigma_{(A_i \cap B_j)=C}\ m_1(A_i) * m_2(B_j)}{1 - \Sigma_{(A_i \cap B_j)=\emptyset}\ m_1(A_i) * m_2(B_j)} \qquad (2.1)$$

for all non-empty C in 2^H. This combination rule can be repeatedly applied to any number of bpas and the final combined result does not depend on the order in which the combination is done ([Shafer, 1976], p62).

Example 2.3. Suppose a different physician P_2 observes some symptoms from the same patient as that in Example 2.2. These symptoms lead her/him (independently) to a diagnosis of {Cir, Gall, Pan} with degree 0.7. Thus $m_2(\{Cir, Gall, Pan\}) = 0.7$, $m_2(H) = 1\text{-}0.7 = 0.3$, and the other values of m_2 are 0. Combining m_2 with m_1, from Example 2.2, using (2.1), we obtain a new bpa m_3:
$m_3(\{Cirr\}) = 0.42$, $m_3(\{Hep, Cirr\}) = 0.18$,
$m_3(\{Cirr, Gall, Pan\}) = 0.28$, and $m_3(H) = 0.12$.
Notice that the bpa m_3 assigns the highest degree to the hypothesis {Cirr}. This seems reasonable since both diagnoses agree that Cirr might be the correct diagnosis.

If we completely trust P_1 and P_2 and if we value their their opinions equally, we can claim that m_3 is a composite opinion of P_1 and P_2 (We are implicitly assuming that P_1 and P_2 are independent. We discuss issues of dependence below). As we mentioned earlier, however, opinions of experts should be treated differently according to the levels of their expertise. To do this, we assign weights s $(0 \le s \le 1)$ to experts to indicate our level of trust in their opinions. If $s = 1$ is assigned to an expert, that expert is considered to be completely reliable or always correct in his/her opinion. If $s = 0$, his/her opinions are without value. If $(0 < s < 1)$, the opinion of the expert is partially to be trusted. We propose that a bpa that indicates the opinion of such an expert be modified as follows:

$$m'(A_i) = s * m(A_i), \quad \text{for all} A_i \text{ in } 2^H - H, \tag{2.2}$$

$$m'(H) = 1 - \sum m'(A_i). \tag{2.3}$$

The resulting bpa m' is the weighted opinion of the expert. This weighting formula is motivated by the idea that if one only partially trusts in the expert, the discounted portion of the degree of belief in this expert should be considered to be "unknown."

Example 2.4. Let m_1 and m_2 be the opinions of physicians P_1 and P_2 in Example 2.3. Suppose that P_1 is completely trusted because of his good reputation and that P_2 is highly trusted because of his frequent successes and occasional failures. We could assign $s_1 = 1$ to P_1 and $s_2 = 0.7$ to P_2. By (2.2) and (3.3), we obtain the weighted bpas m_1' and m_2':

$m_1'(\{Hep, Cirr\}) = 0.6$, $\quad m_1'(H) = 0.4$,

$m_2'(\{Cirr, Gall, Pan\}) = 0.49$, $\quad m_2'(H) = 0.51$.

Combining m_1' with m_2', we obtain m_3', the composite opinion of P_1 and P_2:

$m_3'(\{Cirr\}) = 0.294$, $\quad m_3'(\{Hep, Cirr\}) = 0.306$,

$m_3'(\{Cirr, Gall, Pan\}) = 0.196$, $\quad m'(H) = 0.204$.

Comparing m_3' with m_3 of Example 2.3, we discover that the beliefs in {Cirr} and {Cirr, Gall, Pan} have been significantly reduced and the belief in {Hep, Cirr} and the unknown belief $m_3'(H)$ have increased. This seems reasonable since Cirr, Gall, or Pan are the solutions that P_2 suggested, who has been discounted, and {Hep, Cirr} is the suggestion of P_1, who received higher weight.

An alternative representation of opinions of experts is the belief function. A belief function represents the total confidence in the truth of a hypothesis and any larger hypotheses that contain the hypothesis. Belief functions are interconvertible with their corresponding basic probability assignments. Let m be a bpa and Bel be its corresponding belief function. Then

$Bel(A) = \sum_{B \text{ in } A} m(B)$, and (2.4)

$m(A) = \Sigma_{B \text{ in } A} (-1)^{|A-B|} Bel(B)$. (2.5)

Example 2.5. By (2.4), the belief function corresponding to m_3' in Example 2.4 is

Bel(H) = 1, Bel({Cirr, Gall, Pan}) = 0.49,

Bel({Hep, Cirr}) = 0.6, Bel({Cirr}) = 0.294.

The fact that the belief in H is 1 simply means that one of the hypotheses in H must be true. The belief in {Cirr, Gall, Pan} is the sum of all m(A)s such that A is a subset of {Cirr, Gall, Pan}.

Some of the other important concepts in the D-S theory, such as commonality numbers, degrees of doubt, upper probabilities, and weight of conflict are to be found in [Shafer 1976] and [Gordon and Shortliffe 1985].

3. Combining Dependent Opinions of Experts

As we mentioned earlier, the combination rule (2.1) of the D-S theory can be used only for combining independent opinions of experts, and we have assumed that the experts were independent in the discussions above. We now propose an extension to the combination rule so that we can handle dependent expert opinions. For simplicity, we assume that opinions of experts to combine can be represented as simple support functions (simple support functions have unique foci). In the next section, we will discuss more general situations.

The need to account properly for dependence between opinions is indicated most simply when we observe that if the D-S combining rule is applied twice to same opinion, the resulting beliefs are increased, even though no new information has been added. In example 3.6 below we explore this situation in more detail. We describe our approach for pair-wise dependence here. We discuss higher order dependences later.

Let E_1 and E_2 be two bodies of evidence collected by P_1 and P_2, respectively. Suppose that P_1 and P_2, using E_1 and E_2, offer bpas m_1 and m_2 as their opinions about the frame of discernment H. If m_1 and m_2 are not independent, we could imagine that their derivations are based on certain common evidential sources, that is, E_1 and E_2 are overlap somehow. Let E be the overlapping evidence. Thus, we could view E_1 as a composition of two parts, the part E that is shared by E_2, and and a part E_1' that is completely disjoint from E_2. Similarly, E_2 could be viewed as a combination of E and E_2' where E_2' is completely disjoint from E_1.

The weight of a piece of evidence is a positive measure of how strongly a piece of evidence supports the hypothesis it points to. Let w_1, w_2, w_1', w_2', and w be the weights for E_1, E_2, E_1', E_2', and E respectively. We make the following definition to connect weights of evidence with dependence:

Definition 3.1. The dependence parameters between two information sources E_1 and E_2, denoted by D_{12}, which means how strongly E_1 depends upon E_2, and D_{21}, which means how strongly E_2 depends upon E_1, are defined as

$$D_{12} = w / w_1 \text{ and} \tag{3.1}$$

$$D_{21} = w / w_2. \tag{3.2}$$

According to the addition law of weights ([Shafer 1976], p77), we have

$$D_{12} = w / (w_1' + w), \text{ and} \tag{3.3}$$

$$D_{21} = w / (w_2' + w). \tag{3.4}$$

From (3.3) and (3.4), we can easily deduce the following properties of dependence parameters:

Theorem 3.1.

(1) $0 \le D_{12}, D_{21} \le 1$,

(2) $D_{12} = 0$ iff $D_{21} = 0$ iff E_1 and E are independent,

(3) $D_{12} = 1$ iff E_1 logically implies E_2 and

$D_{21} = 1$ iff E_2 logically implies E_1, and

(4) $D_{12} = D_{21} = 1$ iff E_1 and E_2 are logically equivalent.

From these properties, we can see the intuitive appeal of our definition of dependence parameters. If two experts use completely distinct bodies of evidence, the weight of shared evidential source w must be 0 (since no evidence is shared at all). Therefore, E_1 and E_2 are independent. It immediately follows that the opinions of P_1 and P_2, m_1 and m_2 must be independent. On the other hand, the more information sources P_1 and P_2 share, the larger w is, and therefore, the larger the dependence parameters are. When $E_1 = E_2$, that is, P_1 and P_2 share the exact same evidential sources for their judgements about H, it must be true that $w = w_1 = w_2$, which yields $D_{12} = D_{21} = 1$.

In the general case, the determination of the dependence parameters D_{ij} is not a trivial matter. Here we take a stochastic approach. We present a more detailed analysis of our approach to this problem in [Ling, Rudd, et al 1989].

Armed with dependence parameters, we can combine m_1 and m_2 by a four-step computation:

(1) decomposing m_1 and m_2 into independent pieces;

(2) reconcile disagreements between P_1 and P_2;

(3) weight P_1 and P_2; and

(4) combine those independent weighted pieces by our combination rule (2.1).

Step One. Decomposing m_1 and m_2.

Recall that m_1 is the opinion of P_1, which is derived based on E_1. E_1 is a composition of E_1' and E. We would like to decompose m_1 into two bpas, m_{11} and m_{12}. This decomposition should satisfy:

(1) combining m_{11} with m_{12} produces m_1;

(2) m_{11} is supported by E_1'; and

(3) m_{12} is supported by E.

We call the decomposition with these properties a **legal decomposition.**

Theorem 3.2. The decomposition defined by

$m_{11} = 1 - (1 - m_1)^{1/(k+1)}$, and

$m_{12} = 1 - (1 - m_1)^{(1 - 1/(k+1))}$

where $K = D_{12} / (1 - D_{12})$, is a legal decomposition.

Proof. We construct the decomposition step by step, which shows that the resulting decomposition is a correct one. Clearly, the foci of m_{11} and m_{12} must be the same as the focus of m_1 because the combination of m_{11} and m_{12} must yield m_1. According to Shafer [Shafer 1976], the relationship between the weight of evidence w and the degree of belief b supported by this evidence should be

$w = - \ln (1-b)$.

Therefore, we have

$$w_1' = - \ln(1- m_{11}) \quad (3.5)$$

$$w = - \ln(1- m_{12}) \quad (3.6)$$

$$w = K * w_1' \quad (3.7)$$

where $K = D_{12} / (D_{12} - 1)$ deduced from Definition 2.1. Thus we have

$(1 - m_{11})^k = (1 - m_{12})$.

Let

$$V_{11} = 1 - m_{11} \quad \text{and} \quad (3.8)$$

$$V_{12} = 1 - m_{12}. \quad (3.9)$$

We obtain $V_{11}^k = V_{12}$. (3.10)

Combining m_{11} and m_{12} by combination rule (2.1), we have

$$m_{11} + m_{12} - m_{11} * m_{12} = m_1. \quad (3.11)$$

Let $V_1 = 1 - m_1$, (3.12)

and notice (3.8) and (3.9). (3.11) becomes

$$V_{11} * V_{12} = V_1. \quad (3.13)$$

Solving (3.10) and (3.13), we obtain the decomposition formula described in the theorem. Notice that the resulting m_{11} and m_{12} satisfy (3.11). Therefore, combining them indeed yields m_1. And also notice that the definition of the dependence parameters D_{ij} guarantees that the decomposition satisfies the other two properties of a legal decomposition. Thus we have proved the theorem.

End of proof

Example 3.1. Again, suppose that physicians P_1 and P_2 are considering a case of cholestatic jaundice with the frame of discernment {Hep, Cirr, Gall, Pan}. Suppose that physician P_1 collects distinct symptoms of the patient, e_1, e_2, e_3, and e_4, and he judges that these symptoms support the diagnosis of {Hep, Cirr} to the degree 0.6 (m_1({Hep, Cirr}) = 0.6). And suppose that P_2 gathers distinct symptoms from the same patient, e_3, e_4, e_5, e_6, and e_7 and he decides that these symptoms support hypothesis {Cirr, Gall, Pan} with degree 0.7 (m_2({Cirr, Gall, Pan}) = 0.7). Thus, $E_1 = \{e_1, e_2, e_3, e_4\}$, $E_2 = \{e_3, e_4, e_5, e_6, e_7\}$, $E_1' = \{e_1, e_2\}$, $E = \{e_3, e_4\}$, and $E_2' = \{e_5, e_6, e_7\}$. If we assume that every symptom equally supports the diagnosis (this assumption could be relaxed if we have the medical knowledge to distinguish between the strengths of these symptoms), we infer that $D_{12} = 1/2$ and $D_{21} = 2/5$. Applying Theorem 3.2 to m_1 and m_2, we obtain the following decomposed pieces of opinions of P_1 and P_2:

m_{11}({Hep, Cirr}) = 0.3675, m_{12}({Hep, Cirr}) = 0.3675,

m_{21}({Cir, Gall, Pan}) = 0.382, m_{22}({Cir, Gall, Pan}) = 0.5144.

To check the decomposition, we combine m_{11} with m_{12}, and m_{21} with m_{22}. Indeed, these combinations reproduce m_1 and m_2. As a check, we use Shafer's formula

$w = -\ln(1 - m)$ to recover the weights w_{11}, w_{12}, w_{21}, w_{22} and compute dependence parameters D_{12} and D_{21} by Definition 2.1, we discover that they indeed equal 1/2 and 2/5, respectively.

Step Two. Reconciling Disagreement Between P_1 and P_2.

Though m_{12} and m_{21} produced in the first step are based on the same evidential source E, they are not equal in general because they are derived by different experts. In this step, we must combine m_{12} and m_{21} into a single level of belief m' to be associated with the single underlying evidential source E. While there is no *a priori* reason for selecting a specific arithmetic rule for computing m', we suggest that a weighted average of m_{12} and m_{21} would fit most applications. Suppose that P_1 is assigned weight s_1 and

P_2 is assigned weight s_2. m' can be computed by

$$m'(A) = (s_1/(s_1 + s_2)) * m_{12}(A) + (s_2/(s_1 + s_2)) * m_{21}(A) \quad (3.13)$$

for each A in 2^H and $A \neq H$, and

$$m'(H) = 1 - \Sigma_{A \neq H}\ m'(A) \quad (3.14)$$

Example 3.2. Applying (3.13) to m_{12} and m_{21} produced in the last step, and again assigning weight $s_1 = 1$ to P_1 and $s_2 = 0.7$ to P_2, we obtain the reconciled bpa m':

m'({Hep Cirr}) = (1/1.7) * 0.3675 + (0.7/1.7) * 0 = 0.2162,

m'({Cirr Gall Pan}) = (1/1.7) * 0 + (0.7/1.7) * 0.3820 = 0.1573,

m'(H) = 1 - 0.2162 - 0.1573 = 0.6265.

Step Three. Weighting Opinions of Experts

Apply the weighting formulae (2.2) and (2.3) to m_{11} and m_{22} produced in Step One. This yields the weighted expert opinions m_{11}' and m_{22}'.

Example 3.3. Let m_{11} and m_{22} be those in Example 3.1. Weighting m_{11} and m_{22} yields

m_{11}'({Hep Cirr}) = 0.3675 * 1 = 0.3675, m_{11}'(H) = 1 - 0.3675 = 0.6325,

m_{22}({Cir Gall Pan}) = 0.5144 * 0.7 = 0.3601, m_{22}(H) = 1 - 0.3601 = 0.6399.

Step Four. Combining Weighted Independent Pieces of Opinions.

Now, we have three bpas m_{11}', m_{22}', and m' which are based on independent evidential sources E_1', E_2', and E. We can use the D-S combination rule (2.1) to combine them. The resulting bpa is a composite effect of dependent opinions of the two experts.

Example 3.4. To continue our example, we combine m_{11}', m_{22}', and m' produced in Example 3.2 and 3.3 . This combination gives the resulting bpa m:

m({Cirr}) = 0.22, m({Hep, Cirr}) = 0.29,

m({Cirr, Gall, Pan}) = 0.244, m(H) = 0.248.

The belief function corresponding to the bpa m is

Bel({Cirr}) = 0.22, Bel({Hep. Cirr}) = 0.51,

Bel({Cirr, Gall, Pan}) = 0.464, Bel(H) = 1.

The following are examples illustrating that if we treat dependent opinions of experts as if they are independent and combine them by the Dempster rule, we overestimate the degrees of beliefs.

Example 3.5. Compare the belief function in Example 3.4 with the belief function in Example 2.5. Both of the two belief functions are produced from the exact same opinions of experts. But the belief function in Example 2.5 represents a higher degree of belief than that of Example 3.4 because it is combined under the independence assumption.

Example 3.6. Suppose a physician P makes a diagnosis of {Cirr, Gall} with belief 0.45 for a patient (m_1({Cirr, Gall}) = 0.45). A few days later, the physician P tells the patient the exact same diagnosis when he meets the patient in a supermarket (m_2({Cirr, Gall}) = 0.45). Clearly, m_1 and m_2 are completely dependent on each other ($D_{12} = D_{21} = 1$). Suppose that we trust the physician completely (weight = 1). Combining the two opinions of P by our approach proposed above yields a bpa that stays as same as the original bpa m_1 and m_2. On the other hand, if we ignore the dependency and combine m_1 and m_2 directly by Dempster rule, we produce a degree of belief 0.7 in {Cirr, Gall}. Notice that the combination greatly increases the degree of belief. This is not reasonable since we should not increase degree of belief only by stating the belief twice. In fact, by repeatedly combining a diagnosis with itself, we can make drive our belief in that diagnosis as close to certainty (m=1) as we wish. While repetition is an important tool we use to convince others of the validity of our views, it should have no impact on the results that are produced by formal or automated reasoning systems.

Our presentation has concentrated on situations in which only two expert opinions are involved. Our approach can be applied pairwise to reconcile opinions from any number of experts, provided that there are no dependencies that cannot be decomposed into pairs of dependent sets.

4. Combining Separable Support Functions

The approach we propose above is limited to the expert opinions that can be represented as bpas with unique foci (simple support functions). In this section, we extend this approach to handle more complex expert opinions that can be represented as separable support functions. A **separable support function** is a simple support function or a support function produced by combining simple support functions.

Let m_1 and m_2 be expert opinions about H that represent two separable support functions. Suppose that they are based on information sources E_1 and E_2. And suppose that the dependence parameters between E_1 and E_2 are estimated as D_{12} and D_{21}. We might want to use our approach for simple support functions to combine m_1 and m_2. But in the case of non-simple separable support functions, the decomposition could be really difficult because such support functions have more than one focus. This might force us to solve a high order equation system. Fortunately, Shafer shows ([Shafer 1976], p90) that separability means that there exists a collection of simple support functions that can be combined to yield the original separable support function. Under certain conditions, this collection of simple support functions, a **canonical decomposition**, is uniquely determined.

In order to combine m_1 and m_2 which correspond two separable support functions, we first use Shafer's canonical decomposition method to decompose m_1 into simple support functions $m_{11}, m_{12}, ..., m_{1k}$, and decompose m_2 into simple support functions $m_{21}, m_{22}, ... , m_{2p}$. Given the dependence parameters D_{12} and D_{21} between E_1 and E_2, the dependence parameters between m_{1i} and m_{2j} could be defined as

$D_{1i,2j} = D_{12}, \quad (1 \le i \le k, \ 1 \le j \le p)$ and

$D_{2j,1i} = D_{21} \quad (1 \le i \le k, \ 1 \le j \le p)$.

These definitions are imply that each smaller part of E_1 depends to the same degree on every part of E_2 and vice versa. This is a natural choice if we have no knowledge about how dependence is distributed internally between E_1 and E_2.

Now the problem of combining two separable support functions is changed to the problem of combining several simple support functions. We can use the approach described above to combine those simple support functions. The combination yields a composite effect of two separable support functions.

5. Higher Order Dependencies

For the sake of simplicity, in the discussion above we defined the dependence parameters in such a way that only two evidential sources are involved in each pairwise dependence. Theoretically, there is no inherent difficulty in extending the definitions to represent simultaneous dependencies among many evidential sources. But practically there would be two difficulties with applying this definition if all possible dependencies must be explicitly represented. First, the number of dependencies is exponential in the number of sources. And it is not clear that humans have the ability to assess and detect high order dependencies among a large number of sources. But these two difficulties are not unique to our approach. For instance, probability theories also have the same problems [Henrion, 1986]. In any event, it is apparently not necessary to consider every possible dependence; fortunately, it is not true that every event directly depends upon all others in the real world [Pearl 1986]. Therefore we could view world we are dealing with as sparsely connected, and extend our definition of dependence parameters without worrying too much about the complexity. For example, we can define the dependence parameter that represents how much E_1 depends on E_2 and E_3 as $D_{1|2,3} = w_{123}/w_1$ where w_{123} is the weight of the evidential source that is shared by E_1, E_2, and E_3; and w_1 is the weight of E_1.

With the extended definition, we can use the approach discussed in the last section to combine multiple expert opinions. The procedure involves more decompositions that decompose original opinions into independent pieces and more combinations that combine independent pieces into a final result.

6. Conclusions

The approach investigated above provides a formal way to combine expert opinions which might be stochastically dependent. The initial evidence indicates that the combined results are quite sensitive to the degree of dependence. The higher the degree of dependence is, the lower the degrees of belief are obtained in comparison with degrees of belief derived under the assumption that the opinions are independent.

The efficacy of our approach for practical problems in AI has yet to be fully established. We are implementing a system that will enable us to apply our results to realistic systems.

One reason for combining expert opinions is to generate belief values in which we can have increased confidence over those produced by single experts. Another reason for combining opinions is that by doing so, we could, in principle, broaden the base of possible solutions to problems by including opinions from experts with differing frames of discernment. Our methodology applies to systems in which all the experts use the same frame of discernment (or frames that are at least compatible, see [Shafer, 1976]). We are working to extend our results to more general situations in which the experts have arbitrary frames of discernment.

In summary, we have developed a framework for combining the opinions of experts. Further work will concentrate on methods for determining dependencies, on extending our results to more general situations, and on demonstrating the utility of our approach.

References

Bhatnagar, R. K. and Kanal, L. N., 1986, Handling Uncertain Information: A Review of Numeric and Non-numeric Methods, *Uncertainty in Artificial Intelligence*, 3-26.

Dempster, A. P., 1967, Upper and Lower Probabilities Induced by a Multivalued Mapping, *Annals of Math. Statistics*, 325-339.

Gordon, J. and Shortliffe, E. H., 1985, A Method for Managing Evidential Reasoning in a Hierarchical Hypothesis Space, *Artificial Intelligence* 26, 323-357.

Henrion, M., 1986, Uncertainty in Artificial Intelligence: Is Probability Epistemologically and Heuristically Adequate ? *Uncertainty in Artificial Intelligence.*

Ling, Xiaoning, W. G. Rudd, Thomas G. Dietterich, Bruce D'Ambrosio, and Cho-Chun Hsu, 1989, Combining Dependent Evidence Using Dempster-Shafer Theory, in preparation.

Morris, P. A., 1986, Comment on Combining Distributions, *Statistical Science* Vol. 1, No. 1, 141-144.

Pearl, J., 1986, Fusion, Propagation, and Structuring in Belief Networks, *Artificial Intelligence* 29, 241-288.

Shafer, G., 1976, *A Mathematical Theory of Evidence*. Princeton, NJ: Princeton University Press.