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HEIGHT–DIAMETER EQUATIONS FOR SIX SPECIES IN THE COASTAL REGIONS OF THE PACIFIC NORTHWEST

by

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Abstract

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Three equations for predicting tree height as a function of diameter (outside bark) at breast height are presented for six species found in coastal regions of the Pacific Northwest. Foresters can use these "height–diameter" equations to avoid the time-consuming task of measuring heights of all individual trees in an inventory, a stand exam, or a timber cruise. Equation coefficients were estimated with weighted nonlinear regression techniques. Because the relationship between a tree's height and diameter depends on the tree's competitive position within the stand, alternative equations, including the average height and average diameter of the 40 largest-diameter trees/ac, are also presented. These equations are used in the Stand Management Cooperative version of ORGANON.

Table of Contents

Introduction

The total height (Ht) of a tree is important for assessing tree volume (Walters et al. 1985; Walters and Hann 1986) and stand productivity through site index (Hann and Scrivani 1987), but accurate measurement of this variable is time-consuming. As a result, foresters often choose to measure only a few trees' heights and estimate the remaining heights with height–diameter equations. Foresters can also use height–diameter equations to indirectly estimate height growth by applying the equations to a sequence of diameters that were either measured directly in a continuous inventory or predicted indirectly by a diameter-growth equation. The diameter-growth prediction approach can be valuable for modeling growth and yield of trees and stands as is done in ORGANON (Hann et al. 1997).

A number of studies of height–diameter relationships in northwestern Oregon, western Washington, and southwestern British Columbia have already been published. Curtis (1967) investigated several equations for Douglas-fir that included tree diameter outside bark at breast height (DBH) as an explanatory variable. Larsen and Hann (1987) and Wang and Hann (1988), using a variant of Curtis's (1967) recommended model, found that an equation which included tree diameter and site index was a better height predictor for 6 of 16 species in the mid-Willamette Valley. Krumland and Wensel (1988) included top height and quadratic mean diameter in their height–diameter equation. Garman et al. (1995) used a Chapman-Richards model to characterize the expected asymptotic behavior of height at large diameters.

Our objective in this study was to develop equations for predicting tree height as a function of DBH by itself or in conjunction with the average diameter and height of the 40 largest-diameter trees/ac (D40 and H40 respectively), for the following six tree species found in the Pacific Northwest's coastal regions:

Douglas-fir	***Pseudotsuga menziesii* (Mirb.) Franco**
Red alder	***Alnus rubra* Bong.**
Sitka spruce	***Picea sitchensis* (Bong.) Carr.**
Western hemlock	***Tsuga heterophylla* (Raf.) Sarg.**
Western redcedar	***Thuja plicata* Donn ex D. Don**
Western white pine	***Pinus monticola* Dougl. ex Laws**

The height–diameter equations developed in this study are being used in the Stand Management Cooperative (SMC) version of the ORGANON stand simulator (Hann et al. 1997). In the SMC model they are used to predict heights of unmeasured trees and to predict height growth in hardwood species.

Data Description

The data for this analysis came from the SMC modeling database, which contains tree measurements from 3387 plots within 378 installations in southwestern British Columbia, western Washington, and northwestern Oregon, west of the Cascades. Our data subset came from 3345 plots within 372 installations. These fixed-area plots ranged in size from 0.1 to 1.2 ac (average size 0.17 ac), and were located between 42.00° N and 50.63° N and between 120.7° W and 127.68° W.

For a given tree to be included in our sample, it had to have a measured DBH, a Ht >4.5 ft, and an undamaged top. By species, Douglas-fir data came from 1309 pure-species plots on 187 installations, and the western hemlock data came from 630 pure-species plots on 83 installations. (Plots with at least 80% of stand basal area in a single species are considered pure-species stands; thus, we also obtained data on other species growing in these plots.) The Douglas-fir site index (SI) (Bruce 1981) averaged 105.7 ft with a range of 56–162 ft; we calculated the individual values by solving Bruce's (1981) dominant height equations for site index and calculating site index from dominant height and age. Douglas-fir breast height age averaged 27.8 yr and ranged from 3 to 108 yr; standing plot basal area averaged 175.4 ft^2/ac and ranged from 3 to 411 ft^2/ac. For western hemlock plots, site index averaged 101.5 ft with a range of 43-131 ft; breast height age averaged 45.7 yr with a range of 8–108 yr; and study plot basal area averaged 225.1 ft^2/ac with a range of 25.7–411.0 ft^2/ac.

In the SMC modeling set, tree diameter at breast height (DBH), measured to the nearest 0.1 in. using a diameter tape, was recorded for all trees. Actual

Table 1. Descriptive statistics for the height–diameter modeling data set.

Tree-level statistics

		DBH (in)			Height (ft)		
Species	Number of trees	Mean	Standard deviation	Range	Mean	Standard deviation	Range
Douglas-fir	71,887	7.54	4.97	0.1–46.0	53.78	32.29	5.0–192.0
Red alder	899	10.0	2.84	2.6–19.4	78.8	10.22	25.6–113.8
Sitka spruce	115	13.9	7.44	1.7–28.9	97.17	41.52	14.4–157.2
Western hemlock	17,984	7.19	4.24	0.1–30.6	58.48	31.28	4.6–159.1
Western redcedar	819	4.6	2.84	0.1–19.8	34.1	18.8	5.0–103.3
Western white pine	191	5.6	2.71	1.0–16.8	38.7	16.77	7.5–107.6

Plot-level statistics

		D40			H40			SI		
Species	Number of plots	Mean	Standard deviation	Range	Mean	Standard deviation	Range	Mean	Standard deviation	Range
Douglas-fir	1,309	11.65	4.49	1.9–31.8	69.8	26.95	13.6–182.1	105.7	4.49	56.1–162
Western hemlock	630	13.6	4.5	3.5–27.0	84.82	29.48	20.3–163.2	101.5	16.12	43–131

tree height measurements were made on a sample of trees from each plot. To calculate H40, unmeasured total tree heights were estimated with plot-level height–diameter fits (Flewelling and de Jong 1994). Table 1 summarizes the resulting height–diameter modeling data set.

Data Analysis

Many of the height–diameter equations presented in the literature use a log-linear model form (Curtis 1967; Wykoff et al. 1982). However, some researchers have found that the residuals of these log-linear equations are not normally distributed (Larsen and Hann 1987), a situation that makes log bias corrections difficult (Flewelling and Pienaar 1981). Therefore, we chose the following nonlinear equation form, recommended by Curtis (1967), to characterize the relationship of *Ht* to *DBH* for this study:

$$\mathrm{Ht} = 4.5 + \exp\left(a_0 + a_1 \mathrm{DBH}^{a_2}\right)$$

where a_0, a_1, and a_2 are parameters to be estimated by nonlinear regression from the data (Table 1).

Krumland and Wensel (1988) proposed that the following model, which includes H40 and D40 as variables, explains more of the residual variation for some species:

$$\mathrm{Ht} = 4.5 + (\mathrm{H40} - 4.5) \times \left[\exp\left(b_0 \times \mathrm{DBH}^{(b_1 + b_2(\mathrm{H40} - 4.5))}\right) / \exp\left(b_0 \times \mathrm{D40}^{(b_1 + b_2(\mathrm{H40} - 4.5))}\right)\right]$$

where b_0, b_1, and b_2 are parameters to be estimated by nonlinear regression from the data (Table 1).

This model constrains the height–diameter curve to equal H40 when DBH equals D40.

Garman et al. (1995) fitted the following Chapman-Richards model to a data set from western Oregon, believing that this model form best accounted for the expected asymptotic behavior in the height–diameter relationship:

$$\mathrm{Ht} = 4.5 + c_1 \times \left(1.0 - \exp\left(c_2 \times \mathrm{DBH}\right)\right)^{c_3}$$

where

c_1 = asymptotic height

c_2 = steepness parameter

c_3 = curvature parameter.

We fitted Eq. [1] to all species, Eq. [2] to Douglas-fir and western hemlock, and Eq. [3] to Douglas-fir—in all cases with weighted nonlinear regression techniques (weight = 1.0/DBH) (Larsen and Hann 1987). However, because users are most interested in predicting unweighted heights, we report the unweighted coefficients of determination here.

Results and Discussion

Tables 2 and 3 contain the regression coefficients, mean square errors (MSEs), and unweighted adjusted coefficients of determination ($\bar{R}^2$) for equations [1] and [2]. Table 4 contains the regression coefficients, MSEs, and 44 $\bar{R}^2$ values for Eq. [3]. The fit for red alder was significantly worse than the fits for any of the conifers. This poor fit is probably due to the greater difficulty in measuring the heights of species with weak apical dominance, and a greater variability in the height–diameter relationship.

Garman et al. (1995) proposed using a Chapman-Richards model form to better extrapolate into larger diameter classes. They reported their results by species, geographic region, and site class. As the SMC modeling data set did not include the latter two categories, we compared the Chapman-Richards

Table 2. Regression coefficients and associated unweighted statistics for all species in the modeling data set, fitted to Eq. [1].

	Coefficients				
Species	a_0 (std. error)	a_1 (std. error)	a_2 (std. error)	MSE	$\bar{R}^2$
Douglas-fir	7.262195456	-5.899759104	-0.287207389	16.453	0.6641
	(0.047807485)	(0.038884463)	(0.00406176)		
Red alder	4.41820972	-12.00274935	-2.13835482	21.7689	0.2975
	(0.01435762)	(2.96861022)	(0.170420047)		
Sitka spruce	5.404491308	-6.570862442	-0.819705048	61.77849	0.6199
	(0.151873318)	(0.872852109)	(0.125177723)		
Western hemlock	6.555344622	-5.137174162	-0.3645508	24.421	0.5348
	(0.07869243)	(0.05409665)	(0.011147923)		
Western redcedar	7.232880669	-5.746899904	-0.271564741	16.54749	0.6792
	(0.529191522)	(0.470945659)	(0.038874446)		
Western white pine	7.946192109	-6.278973035	-0.208892429	21.60565	0.5098
	(2.119025826)	(1.975460938)	(0.099823967)		

Table 3. Regression coefficients and associated unweighted statistics for Douglas-fir and western hemlock, fitted to Eq. [2].

	Coefficients				
Species	b_0 (std. error)	b_1 (std. error)	b_2 (std. error)	MSE	$\bar{R}^2$
Douglas-fir	-2.857232223	-0.393885195	-0.000521583	3.037	0.938
	(0.005046006)	(0.003860332)	(0.000069927)		
Western hemlock	-2.790360488	-0.235470605	-0.002374673	7.2786	0.8613
	(0.009938554)	(0.007385901)	(0.000120054)		

Table 4. Regression coefficients and associated unweighted statistics for Douglas-fir, fitted to Eq. [3].

	Coefficients				
Species	c_1 (std. error)	c_2 (std. error)	c_3 (std. error)	MSE	$\bar{R}^2$
Douglas-fir	167.6806252 (1.4108489)	-0.0700404 (0.0009659)	1.3063557 (0.0057041)	16.438	0.6644

model form to the SMC models by refitting to the SMC data set. The results of the fit are presented in Table 4. For Douglas-fir, the adjusted coefficient of determination for the Chapman-Richards model (0.6644) is greater than that for the Larsen and Hann equation (0.6641) but less than that for the Krumland and Wensel equation (0.938). The coefficient for the asymptotic height from the fit of the SMC modeling data set to the Chapman-Richards model form is 167.68 ft. This value is unrealistic for Douglas-fir (Garman et al. 1995) and probably results from the young age of the trees in the modeling data set. When we fixed the asymptotic height value to the average of the Garman et al. (1995) reported asymptotic height values for Douglas-fir in northern Oregon coastal and Cascade regions, the fit declined ($\bar{R}^2 = 0.6596$). This shows that the model form is sensitive to the limited range of heights in the SMC modeling data set. The Garman et al. (1995) data sets included larger trees, which provided better estimates of the asymptotic height parameter (c_1). The residuals from the Chapman-Richards fit (Eq. [3]) show a trend against H40 (Figure 1). In stands where H40 is large, the Chapman-Richards model under-predicts the height, due to the low estimate for the asymptotic height. The Chapman-Richards model form also exhibits high intrinsic curvature which can produce biased parameter estimates (Ratkowsky 1983).

Including H40 and D40 in the model, as is done in Eq. [2], is another way to account for the tendency of even-aged pure-species stands to have a tighter height–diameter relationship than uneven-aged or mixed-species stands. Scaling the projected height–diameter ratio to the observed H40:D40 point results in a smaller range in predicted heights.

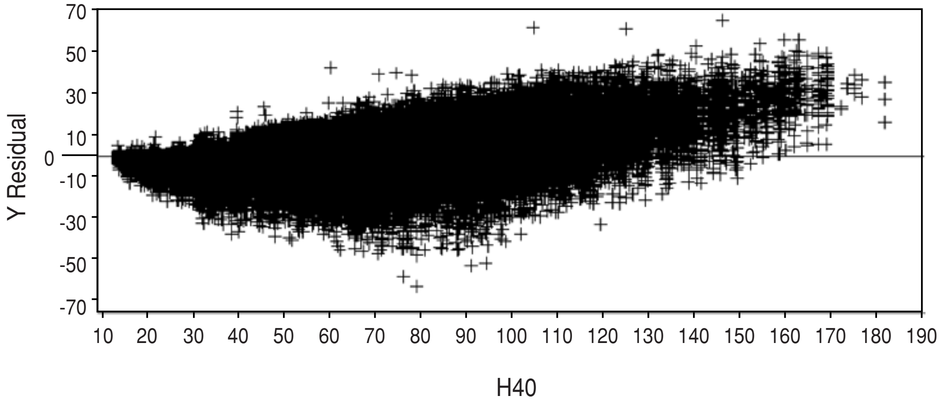

Figure 1. Residuals from the fit of Eq. [3].

Figure 2 shows the height–diameter relationships for Douglas-fir predicted by the three models. Equation [2] is applicable to trees in even-aged, pure-species stands. The DBH measurements in these stands do not normally span the entire range of the modeling data set, but rather fall within a smaller range, as shown by the three curves in Figure 2. For each of the curves, the slope of the predicted Ht values declines as DBH increases; in other words, trees with smaller-than-average DBH values have a larger Ht:DBH ratio. If a stand is not truly even-aged—for instance if it includes a few large residual trees from a previous stand—Eq. [2] may over-predict the heights of the smaller trees. It can also be seen that the predicted heights for trees with small DBH values will be much greater using Eq. [2] than using Eq. [1]. Equations [1] and [3] predict very similar Ht values for DBHs of less than 20 in. For greater DBHs, predicted Ht from Eq. [3] asymptotes to a value of 167 ft, while predicted Ht from Eq. [1] continues to increase, yielding reasonable values of Ht over the full range of DBH in the modeling data set.

In general, we recommend use of Eq. [2] for even-aged stands with at least 80% of the stand basal area in Douglas-fir or western hemlock. However, users should consider the additional expense or effort associated with collecting the additional information this model requires.

In summary, the equations developed in this study show that tree height is strongly correlated to diameter for the species examined. Including D40 and H40 improves the precision of predicting tree height. These three height–diameter equations provide new and useful information about tree species growing in the coastal regions of the Pacific Northwest. They have been incorporated into the SMC version of the ORGANON (Hann et al. 1997) stand simulator.

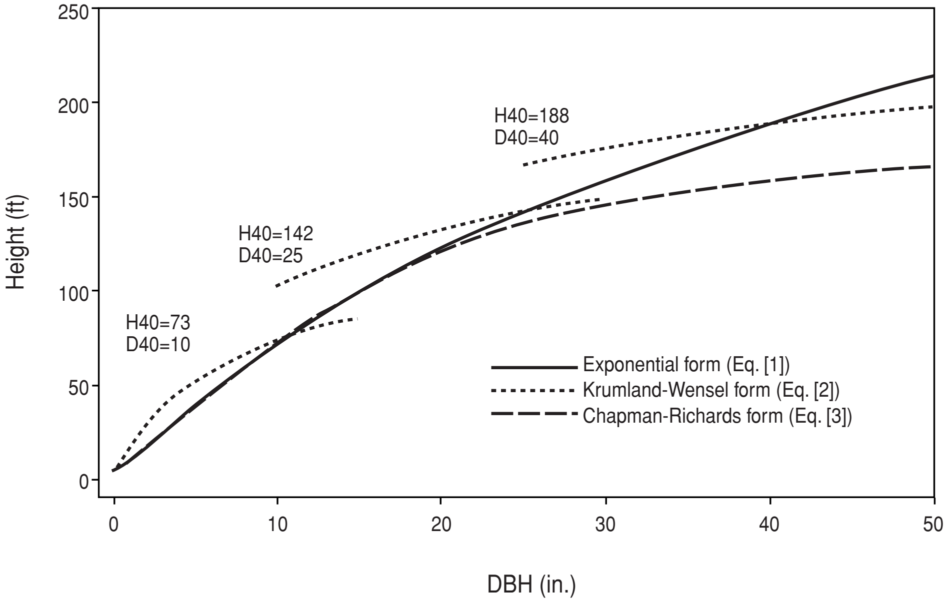

Figure 2. The height–diameter relationships of Eqs. [1], [2], and [3] for Douglas-fir.

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