TECHNICAL BULLETIN 67 APRIL 1963

A Multi-stage Stochastic Replacement Decision Model

Application to Replacement of Dairy Cows

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(Application to Replacement of Dairy Cows)

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The Replacement Decision Making Problem

The basic problem of replacement is concerned with what type of remedial action should be taken and when this remedial action should be put into operation in an enterprise with respect to productive units for which diminishing productivity occurs over time. Replacement theory is designed to determine the remedial action and the point in time at which the productive unit should be restored to its original or a more productive position. The decision at any point in time as to whether or not remedial action will be taken and the type of remedial action to be taken is based upon a criterion of optimality. The criterion of optimality merely specifies what is to be maximized or minimized over the life span of the enterprise. An enterprise is made up of more than one productive unit and/or is in operation for more than one time period.

When the decision to replace a productive unit is made at time t_0, a decision is also made regarding the time at which the unit used for replacement will be replaced. This sequence or set of decisions is called a policy. A policy specifies the age of the unit to be replaced and the age of the replacement for the life span of the enterprise. Of the set of all policies, the policy which determines the actions that attain the criterion of optimality is called the optimal policy. The major objective of this bulletin is the presentation of the detailed development of a model for the multistage stochastic replacement decision problem just described. An illustrative example of how the model can be used is given by considering the problem of the replacement of dairy cows.

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* This work was carried out under an Oregon Agricultural Experiment Station project entitled "Economic Replacement Policies in Continually Operating Dairy Enterprises" and appeared in unpublished form as Keith Jenkins' Master's thesis (11).

Replacement decision making problems of continuously operating enterprises are contained in the set of multi-stage decision making problems (3, 4, 5). Thus, replacement problems can be solved as a multi-stage decision making problem, i.e., by considering all replacements for the life span of the enterprise simultaneously.

Examples of Multi-stage Decision Making

Cleland White (16), in a study at the University of Kentucky in 1959, demonstrated the use of multi-stage decision making in determining an optimal policy for the replacement of caged laying hens. Multi-stage decision making was used by White because the past decision determines the age of the hen on hand during the present enterprise period, thus influencing the net returns during that period. White also observed that subsequent decisions affect the net return that can be obtained from replacing the present animal. This is true because of variation in production of hens of different ages, and also because prices and costs will vary through time. Each of these factors will have an effect upon the present decision. Thus, White used multi-stage decision making in determining an optimal policy for replacement of caged laying hens in a continuously operating enterprise.

Single-stage and Multi-stage Decision Making

One of two methods of decision making can be used in making a replacement decision. These two methods are single-stage and multi-stage decision making. In single-stage decision making, the criterion of optimality can be attained by considering a decision at time t_0 independent of the decisions for the preceding and subsequent enterprise periods. In multi-stage decision making, the criterion of optimality can be attained only by considering all decision points simultaneously. A comparison of multi-stage and single-stage decision making for an abstract replacement problem is shown in the following example.

Let an enterprise exist which operates for three periods with three decision points, and two possible actions at each decision point. Let one of these actions represent the keeping of the present productive unit and the other action represent the replacement of the present productive unit by a new productive unit. Also, let the enterprise be terminated at the end of the third period. Suppose the net return from a new production unit is 15 in the first and second periods; due to an increase in the cost of the unit at the third decision point, the net return is 14 in the third period. Further suppose that net returns for units of age one, two, three, and four are 16, 10, 8, and 6, respectively. If the age of the present unit is two then the problem is: what is the optimal replacement policy to follow for the life span of the enterprise?

The restrictions of the problem are presented in Figure 1 where t is number of the enterprise period, a_t is decision point, R_i is replacement with a new unit in the i^{th} enterprise period, K_i^j is the keeping of a unit of age j in the i^{th} enterprise period, $[R_i]$ is net return from the replacement in the i^{th} enterprise period, and $[K_i^j]$ is net return from the present unit of age j in the i^{th} enterprise period.

A single-stage decision making replacement policy for the example would be obtained by looking at each decision point independent

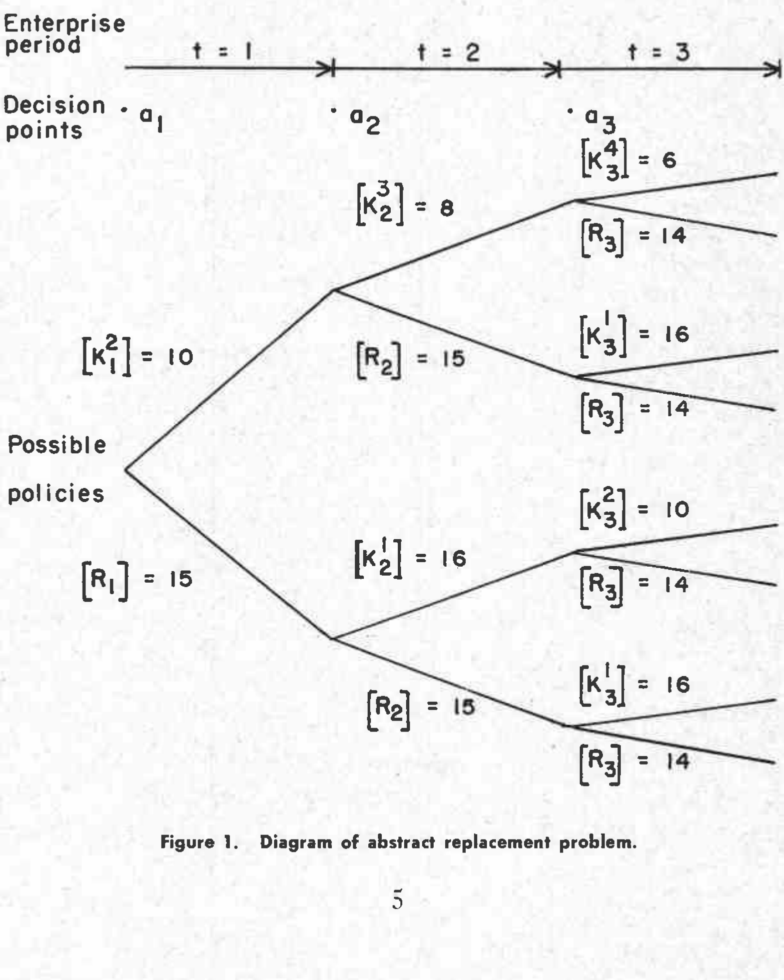

Figure 1. Diagram of abstract replacement problem.

of the other decision points. At decision point a_1, the unit of age two can be kept, $K_1{}^2$, with a net return of 10 or the present unit can be replaced, R_1, with a net return of 15. The most profitable single stage decision is to replace the present unit and obtain the return of 15. At decision point a_2, the present unit is the unit used as a replacement at a_1. This unit can be kept, $K_2{}^1$, with a return of 16 or replaced, R_2, with a return of 15. At a_2 the most profitable single stage decision is $K_2{}^1$ with a return of 16. At decision point a_3, the present unit can be kept, $K_3{}^2$, with a net return of 10 or replaced, R_3, with a return of 14. The most profitable action is to replace the present unit and obtain a net return of 14. Combining the decisions made at a_1, a_2, and a_3, single-stage decision making would generate the replacement policy: replace the unit at a_1, keep the unit at a_2, and replace the unit at a_3. The net return from this policy is 45.

In a small example such as this, a multi-stage replacement policy can be obtained by enumerating all possible replacement policies. The policy which yields the maximum net return will be the optimal sequence of decisions. The eight possible replacement policies and the associated net return of each are $[K_1{}^2, K_2{}^3, K_3{}^4] = 24$, $[K_1{}^2, K_2{}^3, R_3] = 32$, $[K_1{}^2, R_2, K_3{}^1] = 41$, $[K_1{}^2, R_2, R_3] = 39$, $[R_1, K_2{}^1, K_3{}^2] = 41$, $[R_1, K_2{}^1, R_3] = 45$, $[R_1, R_2, K_3{}^1] = 46$, and $[R_1, R_2, R_3] = 44$. The policy $[R_1, R_2, K_3{}^1]$ yields the maximum net returns. Therefore, the optimal sequence of decisions is to replace at a_1, replace at a_2, and keep the present unit at a_3. The optimal replacement policy arrived at by use of multi-stage decision making is a policy which yields higher net returns to the enterprise than the policy recommended by the use of single-stage decision making. Hence, the pitfalls of single-stage decision making and the necessity for the use of multi-stage decision making in obtaining an optimal replacement policy for a continuously operating enterprise are apparent.

The Dairy Cow Replacement Problem

The dairy cow, as other biological productive units, loses efficiency over time. As the present animal loses productive efficiency, the enterprise can be restored by replacing her with an animal with more productive potential. The decision concerning whether the animal is kept or replaced at any decision point in the life span of the enterprise is dependent upon the criterion of optimality, net return from the present animal, and net returns from possible replacement animals.

The dairy cow replacement problem is a multi-stage decision making problem which consists of T decision points, with the decision at any point in the enterprise dependent upon subsequent and preceding decisions. Because the productivity of a present animal of a given lactation may either increase or decrease during the next lactation, returns for the subsequent periods may be affected.

An enterprise period in the dairy cow replacement problem is defined as the time from the beginning of a lactation to the beginning of the next. Hence, for dairy cows it corresponds to essentially a year. In addition, the price of the product or prices of inputs will not remain constant throughout the life span of the enterprise. Therefore, to solve for an optimal replacement policy for a continuously operating dairy enterprise it is necessary to use multi-stage decision making. The criterion of optimality which is assumed for determining the replacement policy for dairy cows is the maximization of expected net return over the life span of the enterprise.[1]

Economic factors which are considered in the decision relative to the replacement of the present animal are: (1) the market value of the present animal, (2) the market value of the possible replacement animal, (3) the nuisance cost associated with replacing the present animal, hereafter called transaction costs, (4) the net market value of production from the present animal, (5) the net market value of production from possible replacements, (6) the maximum net return that can be obtained in subsequent enterprise periods if the present animal is retained, and (7) the maximum net return that can be obtained in subsequent enterprise periods from the replacement of the present animal.

The market value of the present animal and possible replacements is a function of the current production level, the number of lactations remaining for the animal, production in subsequent lactations, the price of beef, and the supply and demand for dairy cows. Transaction costs entail commission charges, transportation costs, and the value of the time and effort involved in making the replacement. The net market value of production for present and replacement animals is determined by considering the market value of production minus the associated costs of production. The maximum net return in subsequent enterprise periods is made up of market value of the animals, transaction costs, and net market value of production.

As with many other productive units, the dairy cow can fail at any point during a given lactation. Failure of a dairy cow is defined as the removal of the animal from the enterprise for sickness, physical injury, or death, i.e., the probability of a failure recovering for dairy purposes is considered to be near zero.

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[1] The problem could be solved using other criteria, e.g., maximum production, maximum return on the dairy cow investment, and others.

In any enterprise period the present animal may be afflicted with mastitis, brucellosis, ketosis, milk fever, or other diseases which will cause production of the animal to diminish to a nonprofitable level. This animal is essentially a failure. Other events which can cause an animal to become a failure are sterility and accidents resulting in physical injury. In fact the animal may just die. If the animal is a failure, and she did not die, then she can be sold for beef. Of course, all of these events can happen to the replacement as well. The possibility of these events occurring gives to an individual animal a stochastic property of failure. The stochastic property of an animal succeeding may be considered as the probability that failure does not occur.

In any enterprise period it will be taken as a certainty that any kind and quality of replacement is available. However, when one considers the replacement of an entire herd or that portion which is of the same age, it is less likely that the same kind and quality of animals for replacement will be available. For example, (1) the owner of a herd of 25 animals will not have 25 raised replacements of the same age on hand during any enterprise period and (2) the owner of a herd of 25 animals will have a difficult and expensive task if he tries to find 25 cows of a given lactation and quality. However, this does not imply that it is an impossibility; it merely implies that there is some probability associated with the finding of a replacement. This is another stochastic factor which influences the dairy cow replacement decision.[2] The probabilities of finding cows in various lactations are shown in Table 1 on page 9.

Thus, the stochastic factors which influence the dairy cow replacement decision are (1) the likelihood of the animal failing, (2) the likelihood of the animal succeeding, and (3) the likelihood of finding (having) a cow in a given lactation.

Development of the Deterministic Replacement Model

The decision concerning whether or not the dairy cow is to be replaced at decision point a_t is a function of net returns during the present enterprise period and the maximum net returns that can be obtained in subsequent time periods. Let NR_j equal the net returns of

[2] Another way of accounting for the supply conditions of replacement animals would be to consider a search cost which would vary for the different lactations. Since this kind of information was not available, the above approach was used.

Table 1. THE NUMBER OF COWS AVAILABLE AND THE PROBABILITY, P_j, OF A COW IN A GIVEN LACTATION

x_j Lactation*	Number of cows available†	Pr[x_j]	P_j
1	31,447	31,447/129,320	.2432
2	22,735	22,735/129,320	.1758
3	18,258	18,258/129,320	.1412
4	14,840	14,840/129,320	.1148
5	12,535	12,535/129,320	.0969
6	9,471	9,471/129,320	.0732
7	7,361	7,361/129,320	.0569
8	4,776	4,776/129,320	.0369
9	3,338	3,338/129,320	.0258
10	1,829	1,829/129,320	.0142
11	1,362	1,362/129,320	.0105
12 and over	1,368	1,368/129,320	.0106
Total	129,320	1.0000	1.0000

* A cow of lactation "1" is considered to be an animal of at least two years of age and not over three years of age. Thus a "1" is an animal ready to begin or in her first lactation; a "2" is an animal ready to begin or in her second lactation, etc. Henceforth, reference to the number of the lactation only will be made.

† Calculated from C. Y. Cannon, and E. N. Hansen. Expectation of life in dairy cows. *Journal of Dairy Science, 22*:1026. 1939.

possible replacement animals in the present enterprise period and $\overline{NR}_j$ equal the maximum net returns that can be obtained in subsequent enterprise periods if an animal of lactation j is used as the replacement.[3] Let $\pi_{t,i}$ equal the maximum net return to the enterprise for the t^{th} and subsequent enterprise periods from a policy which has a present animal of lactation i, then it follows that

$$\pi_{t,i} = \underset{j=1}{\overset{J}{Max}} \; [NR_j + \overline{NR}_j]$$

where j = the lactation of the possible replacements. Now, NR_j equals the market value of the present animal minus the market value of the replacement minus the cost of making the transaction plus the net

[3] If the present animal is considered as a replacement for itself, then j represents the lactation of all possible replacements. Hence, the net return comparison between the present animal and the possible replacements is achieved.

market value of the production of the replacement during the present enterprise period (again i can equal j). This can be demonstrated by considering a present animal of lactation five in the first enterprise period. If animals of lactation $j = 1, 2, \ldots, 12$ are considered as possible replacements, then $\pi_{1,5}$ can be determined.

Let $pv_{1,5}$ = market value of the present animal of lactation five in enterprise period one,

$rc_{1,j}$ = market value of the replacement animal of lactation j in enterprise period one,

$\triangle_j$ = transaction cost,

$r_{1,j}$ = net market value of the production from the replacement animal of lactation j in the first enterprise period, and let

$\overline{NR}_j$ be as previously defined.

It now follows that the maximum net return to the enterprise for the present and subsequent enterprise periods is

$$\pi_{1,5} = Max \begin{bmatrix} pv_{1,5} - rc_{1,1} - \triangle_1 + r_{1,1} + \overline{NR}_1 \\ pv_{1,5} - rc_{1,2} - \triangle_2 + r_{1,2} + \overline{NR}_2 \\ \cdot \\ \cdot \\ \cdot \\ pv_{1,5} - rc_{1,5} - \triangle_5 + r_{1,5} + \overline{NR}_5 \\ \cdot \\ \cdot \\ \cdot \\ pv_{1,5} - rc_{1,12} - \triangle_{12} + r_{1,12} + \overline{NR}_{12} \end{bmatrix}$$

When $j = 5$ then $\triangle_5 = 0$ since the present animal would be replaced by itself and hence no transaction cost would be incurred.

The above set of equations can be rewritten as

$$\pi_{1,5} = pv_{1,5} + \underset{j=1}{\overset{12}{Max}} [- rc_{1,j} - \triangle_j + r_{1,j} + \overline{NR}_j] \quad \text{and}$$

can be generalized to

$$\pi_{t,i} = pv_{t,i} + \underset{j=1}{\overset{J}{Max}} [- rc_{t,j} - \triangle_j + r_{t,j} + \overline{NR}_j] \quad \text{where}$$

t = enterprise period ($t = 1,2, \ldots, T$),
i = lactation of present animal ($i = 1,2, \ldots, J$),
j = lactation of replacement animal ($j = 1,2, \ldots, J$),
and $\triangle_j = 0$ when $i = j$.

Now consider $\overline{NR}_j$, the maximum net return that can be obtained in subsequent enterprise periods. If an animal of lactation j is used as a replacement in enterprise period t, it will be in lactation $j + 1$ in the $t + 1$st enterprise period. In the $t + 1$st enterprise period, the animal will be eligible for replacement. Thus, $\overline{NR}_j$ is the net returns that can be obtained in the $t + 1$st enterprise period by replacing with an animal of lactation j' ($j' = 1,2, \ldots, J$) plus the maximum net return that can be obtained in enterprise periods subsequent to $t + 1$, i.e., stated mathematically.

$$\overline{NR}_j = \underset{j'=1}{\overset{J}{Max}} \, [NR_{j'} + \overline{NR}_{j'}].$$

But $\pi_{t+1,\ j+1} = \underset{j'=1}{\overset{J}{Max}} \, [NR_{j'} + \overline{NR}_{j'}]$ and hence $\overline{NR}_j = \pi_{t+1,\ j+1}$.

The deterministic replacement equation now can be written as

$$\pi_{t,i} = pv_{t,i} + \underset{j=1}{\overset{J}{Max}} \, [- rc_{t,j} + r_{t,j} - \triangle_j + \pi_{t+1,\ j+1}].$$

Development of the Stochastic Replacement Model

The preceding equation presents the replacement model under nonstochastic conditions. To modify the above equation for stochastic conditions one needs to incorporate the probability of failure, success, and acquisition. These stochastic factors are entered into the model by finding the expected value of the net return from the replacement, $r_{t,\ j}$. The expected value of $r_{t,\ j}$ depends upon the probability of finding an animal of lactation j, the probability of success of an animal of lactation j, and the probability of failure of an animal of lactation j. If we let

P_j = probability of finding an animal of lactation j,

q_j = the probability of success of an animal of lactation j, and

p_j = probability of failure of an animal of lactation j,

then the expected value of $r_{t,j}$ is

$E(r_{t,j}) = P_j\,[q_j$ (value of success $_{t,j}$) $+ p_j$ (value of failure $_{t,j}$)].

If the lactation of the present animal is the same as the replacement being considered then the probability of finding the present animal is one ($P_j = 1$). The replacement model now becomes

$$\pi_{t,i} = pv_{t,i} + \underset{j=1}{\overset{J}{Max}} [- rc_{t,j} + P_j\,(q_j VS_{t,j} + p_j VF_{t,j}) - \triangle_j + \pi_{t+1,j+1}]$$

where

$$O \leq P_j < 1, \text{ except when } i = j \text{ then } P_j = 1,$$
$$O \leq q_j \leq 1,$$
$$O \leq p_j \leq 1,$$
$$q_j + p_j = 1,$$

$VS_{t,j}$ = value of success of an animal of lactation j in enterprise period t,

$VF_{t,j}$ = value of failure of an animal of lactation j in enterprise period t,

and the other symbols as previously defined.

Solution of the Replacement Equation

A method of solution which will yield the same results as finding returns for all possible combinations is based upon the mathematical concept of recursion relations. A recursion relation is such that any term of a sequence after a specified term can be obtained as a function of the preceding terms. Thus, an equation which expresses a recursion relation can be solved for the sequence of terms by specifying some initial term.

This method can be demonstrated by considering the abstract example presented on page 4. To solve the example in this manner, the problem is divided into three one-dimensional problems rather than eight individual problems and the solution is initialized at the third enterprise period.

The three problems are as follows:

Problem 1.

If K_2^3 was the unit used in the second enterprise period, then follow the policy which attains

$$Max \begin{bmatrix} [K_3^4] = 6 \\ [R_3] = 14 \end{bmatrix} = [R_3] = 14.$$

If R_2 was the unit used in the second enterprise period, then follow the policy which attains

$$Max \begin{bmatrix} [K_3^1] = 16 \\ [R_3] = 14 \end{bmatrix} = [K_3^1] = 16.$$

If K_2^1 was the unit used in the second enterprise period, then follow the policy which attains

$$Max \begin{bmatrix} [K_3^2] = 10 \\ [R_3] = 14 \end{bmatrix} = [R_3] = 14.$$

Problem 2.

If K_1^2 was the unit used in the first enterprise period, then follow the policy which attains

$$Max \begin{bmatrix} [K_2^3, R_3] = 22 \\ [R_2, K_3^1] = 31 \end{bmatrix} = [R_2, K_3^1] = 31.$$

If R_1 was the unit used in the first enterprise period, then follow the policy which attains

$$Max \begin{bmatrix} [K_2^1, R_3] = 30 \\ [R_2, K_3^1] = 31 \end{bmatrix} = [R_2, K_3^1] = 31.$$

Policies R_3 and K_3^1 were found to return the maximum in the first problem. Now the second solution has added R_2.

Problem 3.

For the present unit at a_1 follow the policy which attains

$$Max \begin{bmatrix} [K_1^2, R_2, K_3^1] = 41 \\ [R_1, R_2, K_3^1] = 46 \end{bmatrix} = [R_1, R_2, K_3^1] = 46.$$

Policy R_2, K_3^1 was found to return the maximum in the second problem. The third solution has added R_1.

The optimal policy R_1, R_2, K_3^1 is the same sequence of decisions that was determined by enumerating all possible policies on page 6. This sequence of decisions, which is the optimal policy, was obtained by organizing the problem in such a manner that a recursive approach could be used. This method of solution is called multi-stage programming.

The replacement equation is a recursive equation, and its solution can be found if an initial position is specified and the enterprise periods are relabeled. The enterprise periods are relabeled from the specified initial position, i.e., instead of indexing the enterprise periods at $t = 1,2,3, \ldots, T$ they are now indexed as $t = T, T\text{-}1, T\text{-}2, \ldots, 3,2,1.$[4] The stochastic replacement equation can now be written as

$$\pi_{t,i} = pv_{t,i} + \underset{j=1}{\overset{J}{Max}} \left[P_j(q_j VS_{t,j} + p_j VF_{t,j}) - rc_{t,j} - \Delta_j + \pi_{t-1,j+1} \right].$$

The solution can be initialized by specifying, $\pi_{t-1,j+1}$, at $\pi_{o,j+1}$ where $t = 1$, the end of the enterprise. In the dairy cow replacement

[4] T should be taken of sufficient length to stabilize the policy. Thus the life span of the enterprise has no effect upon the present replacement policy.

problem, $\pi_{o,j+1}$ is the market value of an animal of lactation $j+1$, since the most profitable alternative at the end of the enterprise is to sell the animal.[5]

A Numerical Example of the Dairy Cow Replacement Equation

The remainder of this bulletin is concerned with a numerical example of obtaining dairy cow replacement policies. It is assumed that over the long-run the probabilities of finding an animal in a given lactation is relevant to the replacement decision. In any specific application of the method, various assumptions and data applicable to the situation would be used.[6]

Probability of finding a dairy cow of a specified age, P_j

Finding of a cow of a given lactation to be used as a replacement can be considered as a random event. Thus a random variable can be defined as the possible lactation of the replacement, i.e., $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, . . . , $x_{12} = 12$. The probability that can be associated with the value of the random variable x_j can be obtained by dividing the number of cows of lactation j that are available by the total number of cows. Data of Cannon and Hansen (7) were used to obtain the probability of finding an animal of a specified lactation. These are shown in Table 1; these data may be out-of-date for conditions today.

Probability of failure and success, P_j and q_j, of dairy cows

The probability of failure of a given animal is conditional upon the butterfat level and the lactation of the cow. These conditional probabilities are shown in Table 2.[7] They were obtained through

[5] The equation was programmed for an I.B.M. 1620 (20K) computer. Program statements are given in the Appendix.

[6] For example, one could assume that all replacements are raised and hence $P_1 = 1$ in Table 1. Another situation would be to determine the price which the herd owner could afford to pay for a replacement provided it is available. The P_j's in Table 1 would not be used at all in this case.

[7] Also see Appendix Table 6 for components of conditional probabilities.

Table 2. PROBABILITY OF FAILURE OF DAIRY COWS GIVEN THE BUTTERFAT LEVEL AND LACTATION*

Lactation	Butterfat level 1 — *Less than 350 pounds* — Probability of failure†	Butterfat level 2 — *350 to 450 pounds* — Probability of failure	Butterfat level 3 — *Above 450 pounds* — Probability of failure
1	.0438	.0543	.0674
2	.0662	.0755	.0835
3	.0825	.0937	.1222
4	.0927	.1196	.1438
5	.1027	.1350	.1678
6	.1393	.1557	.1751
7	.1322	.1576	.1813
8	.1821	.1662	.1947
9	.1614	.1358	.1336
10	.1189	.1578	.1527
11	.1243	.1245	.0927
12 and above	.0452	.0847	.0509

* Calculated from data in Appendix Table 6.
† Probability of success = 1 — Probability of failure.

the Pennsylvania DHIA program which keeps adequate records on IBM cards to allow sorting of actual failures from removals for dairy purposes and low production.

Through the cooperation of Dexter N. Putnam, Dairy Specialist, Pennsylvania State University, over 10,000 cards containing information concerning removal during 1960 were obtained. Cards containing average herd size and herd butterfat production also were obtained. Failures were classified according to the lactation of failure, herd size from which the animal failed, and butterfat level of the herd from which the animal failed. In comparing the proportion of failures in these data with year-end summaries, it was found that cows which failed before 90 days had not been included. It was then learned from Putnam that records of animals failing before they had completed the first 90 days of their lactation were not retained on IBM cards.

To determine a correction factor for this situation, the September 1960 to August 1961 Monthly Reports for 87 herds in Centre County, Pennsylvania, were obtained from Putnam. Failures in these herds were then analyzed using the same classification as was used on other removals. This sample was then used to adjust the original set of data so as to include those animals which had failed before 90 days.

To determine whether or not failure by lactation, herd size, and butterfat levels were independent, several Chi-square contingency tests were run. It was concluded that failure by lactation was independent of herd size, since the calculated value of Chi-square was 34.67 compared to 47.37 at the five percentage point of the Chi-square distribution with 33 degrees of freedom. This result led to the conclusion that the probability of failure of an animal in a given lactation is not conditional upon size of herd. Because this was a cross-sectional analysis, managerial ability was already reflected in the size of the herd. Thus, no dependence between the proportion of failures by lactation and herd size could be expected. However, this does not imply that if the same level of managerial ability was used on different herd sizes the same results would be observed.

Failure of the animal by lactation and butterfat level of the herd were concluded to be dependent, since the calculated value of Chi-square was 119.02, compared to 73.29 at the five percentage point of the Chi-square distribution with 55 degrees of freedom. This result led to the conclusion that the failure of an animal in a given lactation was conditional upon the butterfat level of the herd.

Economic components of the example

Economic components of the dairy-cow replacement model which must be specified numerically are the market value of the present and possible replacement animals, the expected net return of each of the possible replacement animals, transaction costs, and the initial position. For this example the market value of both present and replacement animals with the same lactation number, butterfat level, and condition of health is considered to be the same. The expected net return of the replacement animal is

$$P_j \ (q_j VS_{t,j} + p_j VF_{t,j}).$$

Transaction cost, $\triangle_j$, is equal to zero when the animal being considered for replacement is of the same lactation as the present animal.[8] The initial position, $\pi_{o,j+1}$, is specified as the market value of the animal at the termination of the enterprise.

Market value of the animal, $pv_{t,i}$ *and* $rc_{t,j}$. Market value of the present animal and possible replacement animals in a given enterprise period represent a value for beef plus a value associated with ex-

[8] See footnote 3 on page 9.

pected returns from dairy production. Market value of an animal for this study was estimated using data obtained from a questionnaire mailed to a sample of DHIA herd owners in Oregon.[9] Data were collected on 326 transactions of cows bought or sold for dairy purposes by Oregon DHIA herd owners during 1961. Data collected on each transaction were: (1) the number of lactations the animal had completed when purchased or sold; (2) the price paid or received for the animal; (3) the sale charges incurred in the transaction; (4) the number of miles the animal was transported; and (5) the previous production of the animal or the expected production of the animal if no previous records were available. From these data an equation for estimating dairy cow market value was derived. Estimated market values of animals by lactation and butterfat level as derived by the equation are presented in Appendix Table 1.

Expected net return, $P_j\,(q_j VS_{t,j} + p_j VF_{t,j})$. The expected value of an outcome of an experiment or game is simply the sum of the returns from the various outcomes times the probabilities associated with the outcomes (8). Thus the expected net return of a replacement animal of lactation j can be expressed as

$$\text{E (net returns)} = P_j\,(q_j VS_{t,j} + p_j VF_{t,j}).$$

It is very likely that actual net returns for any given animal and enterprise period will not equal expected net returns; however, average net return for a continuously operating enterprise is equal to expected net return.

Net return from success, $VS_{t,j}$

The estimated net return for an animal of lactation j if she succeeds can be obtained by subtracting certain costs of production from the market value of the production. Costs and returns which are constant to both the present animal and possible replacements need not be considered in the net return, since their inclusion would not change the replacement policy. Costs and returns considered to be constant to animals of all lactations and butterfat levels are labor, barn and facilities charges, value of waste products, value of the calf, and breeding fees.

The value of the production of an animal of a given lactation can be obtained by multiplying the pounds of butterfat times the price of the product. Pounds of butterfat vary by lactation. The relationship between the number of the cow's lactation and production was obtained from data presented by Brody (6). A quadratic equation was

[9] Of the 313 questionnaires mailed, 188 were returned for a response of 60%.

fitted to Brody's data to obtain an index of how production varies by lactation. This equation is plotted in Figure 2. Suppose the index is to be obtained for a cow in the third lactation, then a "3" is substituted into the equation and solved. In the third lactation the index of production is 1.23, i.e., an animal in the third lactation will produce 123% of its first lactation. Thus, the animal which produced 300 pounds of butterfat in its first lactation will produce 369 pounds in its third lactation.

If the price of butterfat is $1.35 per pound, the base level of production is 300 pounds, and the animal is in the third lactation; then the value of production is $481.15.

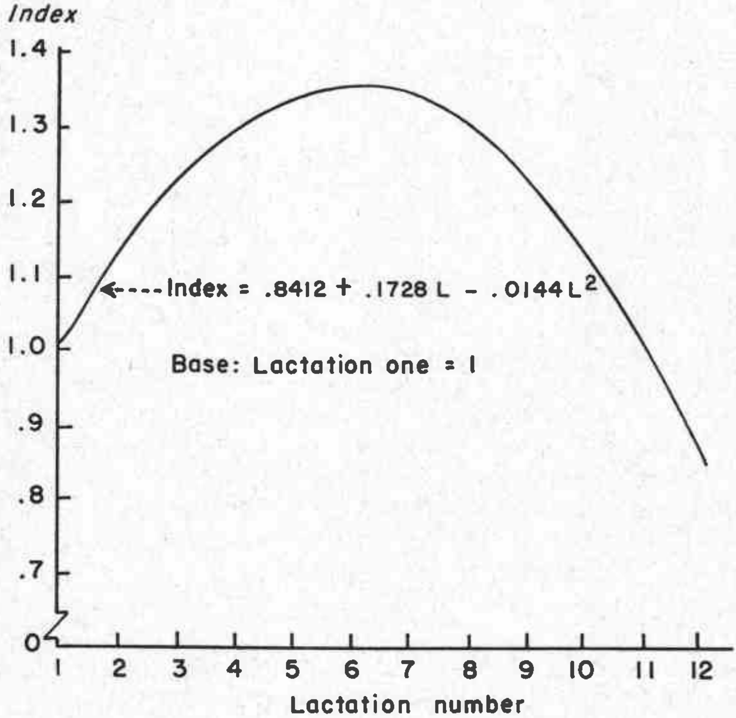

Figure 2. Index of butterfat production by lactation.

Forage costs were estimated for this example to be $11 per month[10] times a weight index to take into consideration that an animal of lactation one in general has not reached her full growth. The weight index used was calculated from a sample of 100 records from the Pennsylvania State DHIA Centre County Monthly Reports and is presented in Appendix Table 5.

For example, an animal in its sixth lactation has a weight index of 1.312; therefore, forage costs would be estimated at ($11) (12) (1.312) = $173.18 for an enterprise period of 12 months.

Cost of concentrates was estimated by fitting a linear equation with cost of concentrates dependent upon the amount of butterfat produced. Data for estimation of the equation were obtained from the Pennsylvania State DHIA Monthly Reports.[11]

Return from failure, $VF_{t,j}$

If an animal fails she can either be sold for beef or removed from the herd because of death. Of animals failing in Pennsylvania in 1960, 10% were deaths and the remaining 90% were sold for beef. Thus, the probability that an animal will be sold for beef given failure is .90. The positive return from a failure is

$$Pr\,[\text{beef}]\ (\text{value of beef}) + Pr\,[\text{death}]\ (\text{value of death})$$
$$= .90\ (\text{value of beef}) + .10\ (\text{value of death}).$$

The value of death is assumed to be near zero and the value of beef is estimated by multiplying the weight of the animal in its first lactation times the previously indicated weight index times the price per pound. Thus, if the animal's base weight in its first lactation is 1,200 pounds, the price is $14 per hundred, and the index is 1.312, then the value of beef is $220.42 and the expected return from the failure is (.90) ($220.42) + (.10) (0) = $198.38. The expected net return of the failure is obtained by subtracting the market value of the animal, $rc_{t,j}$ from the failure's positive return. If the price paid for the animal in the example was $238.24, then

$$VF_{t,j} = \$198.38 - \$238.24 = -\$39.86,$$

i.e., a loss would be incurred if the animal failed.

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[10] The figure of $11 per month is used to estimate cost of forage for the Monthly Report to the herd owners of Pennsylvania DHIA. It is used in this study since it was readily available.

[11] The above relationships are included in the computer program as shown in the Appendix; therefore no specific calculation of a net return is shown here.

Transactions costs, $\triangle_j$. The cost of making the transaction from the present animal to its replacement includes commission charges, transportation, etc. For this example, these costs were obtained from mail-questionnaire data. Average mileage per transaction was 32 miles and the average commission charge was $2.26. If 12 cents per mile were charged, then transportation costs would be $3.84 and the average transportation plus commission cost would be $6.10. The amount of labor used for the transaction would vary and would be of different value to different individuals; hence, for purposes of analysis, transaction cost was set at $10 allowing $3.90 for the value of labor involved.[12]

Initial position, $\pi_{o,j+1}$. As shown previously, some initial position must be specified in order to solve the stochastic multi-stage replacement equation. The initial position is assumed to be the termination of the enterprise. Thus, $\pi_{o,j+1}$ is the sale price of the animal of lactation $j + 1$, i.e., the market value. For purposes of computation $\pi_{o,13}$ is assumed to be equal to $\pi_{o,12}$.

Some optimal dairy cow replacement policies

Replacement policies were determined for animals of three butterfat levels and under various price conditions; however, only the 350- to 450-pound range is presented for illustrative purposes.[13] Prices for 1961 were used as base prices and the effects of variations in feed prices, dairy cow prices, canner and cutter prices, and milk prices were observed. Also, actual prices for the 1950-1961 period were used to determine what the optimal policy for animals in each butterfat level would have been during that particular sequence of enterprise periods.

It is assumed that the probability of finding a cow in a given lactation, the probability of failure, and the probability of success are constant between cows throughout the length of the enterprise. Also, the production relationship between pounds of butterfat and lactation is assumed constant between cows and over time.

Effects of price variations upon optimal policies

Optimal replacement policies were determined under different price conditions. Prices used were 1961 averages and various combi-

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[12] Data were not available to allow a rough approximation to be obtained of costs incurred in searching for an animal of a given lactation. The relative difficulty of finding cows in specified lactations is reflected by P_j.

[13] The other two levels were less than 350 pounds and greater than 450 pounds.

nations of increases and decreases. The optimal policies for the second butterfat level, production between 350 and 450 pounds, are presented in Table 3.

Table 3. OPTIMAL REPLACEMENT POLICIES FOR DAIRY COWS IN BUTTERFAT LEVEL 2 (350-450 POUNDS) UNDER VARIOUS PRICE CONDITIONS

Cow prices	Feed prices	Canner and cutter prices	Milk prices	Replacement policy
1950–1961	1950–1961	1950–1961	1950–1961	2, 3, 4, 5, 1, 2, 3, 4, 5, 6, 7, 8
1961	1961	1961	1961	1, 2, 3, 4, 5, 6
120% 1961	120% 1961	120% 1961	120% 1961	1, 2, 3, 4, 5, 6
80% 1961	80% 1961	80% 1961	80% 1961	1, 2, 3, 4, 5, 6
1961	1961	1961	120% 1961	1, 2, 3, 4, 5, 6, 7
1961	1961	1961	80% 1961	1, 2, 3, 4, 5, 1 or 6
1961	120% 1961	1961	1961	1, 2, 3, 4, 5, 1 or 6
1961	80% 1961	1961	1961	1, 2, 3, 4, 5, 6, 1 or 7
120% 1961	1961	120% 1961	1961	1, 2, 3, 4, 5, 6
80% 1961	1961	80% 1961	1961	1, 2, 3, 4, 5, 6

Effects of price variations are as follows:

1. With dairy cow prices, feed prices, canner and cutter prices, and milk prices constant at the 1961 level, a policy of 1, 2, 3, 4, 5, 6 is obtained for each butterfat level.[14]

2. With dairy cow prices, feed prices, canner and cutter prices, and milk prices each at 120% of the 1961 level, no change in the optimal policy is observed.

3. With dairy cow prices, feed prices, canner and cutter prices, and milk prices each at 80% of the 1961 level, no change in the optimal policy occurred.

4. With dairy cow prices, feed prices, and canner and cutter prices at 1961 levels and milk prices at 120% of the 1961 level, the replacement cycle lengthens.

[14] The policy is read: obtain an animal of lactation one, i.e., ready to begin her first lactation; keep her until she has completed her sixth lactation and then replace her with an animal of lactation one.

5. With dairy cow prices, feed prices, and canner and cutter prices at 1961 levels and milk prices at 80% of the 1961 level, the replacement cycle shortens.
6. With dairy cow prices, milk prices, and canner and cutter prices at 1961 levels and feed prices at 120% of 1961 levels, the replacement cycle shortens.
7. With dairy cow prices, milk prices, and canner and cutter prices at 1961 levels and feed prices at 80% of 1961 levels, the replacement cycle lengthens.
8. With feed prices and milk prices at 1961 levels and an increase or decrease of 20% in 1961 prices of dairy cows and canners and cutters, no change in the optimal policy occurred.

It should be noted that an increase in milk prices tended to lengthen the replacement cycle, while a decrease tended to shorten the cycle. An increase in feed prices resulted in a decrease in the length of the replacement cycle, and a decrease resulted in a longer cycle. Since the prices of dairy cows and canners and cutters generally move together (see Appendix Table 2), it should be noted that when these prices increase or decrease no change is observed in the length of the replacement cycle.

Optimal replacement policies for 1950-1961

It is quite evident that when only one variable changes the replacement policy stabilizes and repeats. However, under actual conditions, milk prices may change in one direction while feed prices and canner and cutter prices may change in an opposite direction. Likewise, milk prices and dairy cow prices need not move in the same direction. To demonstrate that replacement cycles may not be of the same length under actual conditions, the replacement equation was solved using prices from the 12-year period 1950 through 1961 inclusive. Price indexes of dairy cows, feed, canners and cutters, and milk for 1950-1961 are shown in Appendix Table 2. Indexes were determined using 1961 prices as a base.

Optimal replacement policies for the period 1950-1961 were determined for each of the three butterfat levels. For illustrative purposes, only the middle butterfat range is presented. The policy for the 350- to 450-pound butterfat level was (2, 3, 4, 5, 1, 2, 3, 4, 5, 6, 7, 8). This policy is read as follows: Begin with a cow of lactation two in 1951; keep her from 1951 through 1954; then in 1955 replace her with a cow in its first lactation and keep her through 1961 when more data would be necessary to determine the next decision. The replacement animal is assumed to be in the same butterfat level as the present animal when they are in the same lactation.

Conclusions

Although data were obtained from different states, it is of interest and may be significant that in all cases of hypothetical price variations and actual prices for the 12-year period of 1950-1961 the replacement animal was always an animal of lactation one, i.e., an animal ready to begin her first lactation. Results from the mail questionnaire indicated that 80.6% of the Oregon DHIA herd owners sampled did not buy any replacements in 1961; i.e., they raised replacements used in 1961 and this implies that these herd owners used replacements of lactation one.[15]

Actual culling rates observed for Oregon DHIA herds for 1954 to 1961 are shown in Table 4. The culling rate is the percentage of cows removed each year not only for failures as previously defined but also for low production.

The average culling rate for these eight years was 23%. The optimal policy for the eight years as determined by the replacement model was (1, 2, 3, 4, 5, 6, 7, 8) implying a culling rate of 12%.

Hence, herd owners were replacing animals twice as fast as price relationships over the eight-year period would indicate. This is

Table 4. CULLING RATE BY YEARS FOR OREGON DHIA HERDS*

Year	Culling rates
	Percentage
1954	20
1955	19
1956	22
1957	26
1958	24
1959	25
1960	27
1961	26

* Calculated from H. P. Ewalt, and D. E. Anderson. Oregon's dairy herd improvement progress—Annual summary 1954-1961. Corvallis, Oregon State University, Cooperative Extension Service, 1955-62.

[15] This does not imply that P_1 is near one. The probability of any herd owner regardless of the size of his herd having first lactation heifers available for replacement of his entire herd is near zero. Consider that half of the calves born will be heifers, and it would only take a 50% selection to reduce the probability to $\frac{1}{4}$ of having an animal of lactation one available.

not saying that herd owners are irrational, nor that the model is illogical. It is likely that if herd owners in 1954 had had price data for enterprise periods following 1954, they would have lengthened the replacement cycle of their herds, thereby decreasing the number of removals. That is, hindsight is better than foresight. However, this discrepancy could also mean that herd owners are production maximizers and that an animal would be removed as a low producer before she became less profitable than her replacement. Production maximizers are those who may overemphasize the improvement of herd averages at the cost of profit.

Assuming that 1961 price levels and relationships had been anticipated correctly by Oregon DHIA herd owners and that they expected these to prevail in the future, the optimal replacement policy would have been (1, 2, 3, 4, 5, 6). This implies an approximate culling rate of 17%. However, a culling rate of 26% was shown in Table 5 for 1961. While the discrepancy here is not as great as when the 1954 to 1961 period is considered, this much discrepancy implies that it may be profitable to put this model to practical use. To do this, the data-gathering procedure now being used by dairy management advisors would need to be modified only slightly to obtain data that would be of value in providing information to be used in order to make decision making more profitable.

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Appendix

Multi-stage Replacement Program for IBM 1620 (20K)

The following Fortran notation is used:

PI(J+I) $= \pi_{t-1, j+1}$

P(J) $= p_j$

I-P(J) $= q_j$

APV(J) $= \max_{j=1}^{12} \left[\qquad \right]$

SPI(I) $= \pi_{t,i}$

AL(J) = Index of number of cows available by age

AB(K) = Index of number of cows available by butterfat level

TB(J) = Value of butterfat production of animal in lactation j

TC(J) = Cost of production

SP(J) $= P_j$

TEMP(J) $= rc_{t,j} = pv_{t,j}$

FC(J) = Weight index

E = Transaction cost

N = length of enterprise

BF = Butterfat production of animal in first lactation

IB = Butterfat level of the herd

VF $= VF_{t,j}$

R $= VS_{t,j}$

ZA = Price level of dairy cows

ZB = Price level of feed

ZC = Price level of canners and cutters

ZD = Price level of milk

```
DIMENSION PI(13), P(12), APV(12), SPI(12), AL(12), AB(3)
DIMENSION TB(12), TC(12), SP(12), TEMP(13), FC(12)
 50 DO 1 I = 1, 12
  1 READ 201, P(I)
    DO 3 I = 1, 12
  3 READ 201, SP(I)
    DO 6 I = 1, 12
```

```
 6 READ 201, FC(I)
   READ 202, E
   READ 203, N
   READ 202, BF
   READ 203, IB
   BFC = IB
   DO 2 I = 1, 12
 2 READ 202, AL(I)
   DO 4 I = 1, 3
 4 READ 202, AB(I)
30 DO 32 I = 1, 12
   X = I
32 TEMP(I) = 249.14 + .92*X + .5*(X**2) + 17.5864*BFC — 4.75*AL(I)
              — 60.42*AB(IB)
   TEMP(13) = TEMP(12)
36 DO 5 I = 1, 13
 5 PI(I) = TEMP(I)
   DO 40 I = 1, 12
   X = I
   TA = 84.17 + 17.28*X — 1.44*(X**2)
   TB(I) = 1.35*TA*BF*.01
40 TC(I) = 164.256 + .3125*BF*TA*.01
   DO 28 LT = 1, N
   IF (SENSE SWITCH 2) 71, 72
72 PRINT 100, LT
71 PUNCH 203, LT
   READ 204, ZA, ZB, ZC, ZD
   DO 26 J = 1, 12
   DO 22 I = 1, 12
   VF = 156.*ZC*FC(I) — TEMP (I)*ZA
   VBF = ZD*TB(I)
   R = VBF — TC(I)*ZB*FC(I)
   RC = ZA*TEMP(I)
   IF(I — J) 21, 20, 21
20 APV(I) = (1. — P(I) ) * R + P(I)*VF — RC + PI(I + 1)
   GO TO 22
21 APV(I) = SP(I)*((1. — P(I))*R + P(I)*VF) — RC — E + PI(I +1)
22 CONTINUE
   OPT = — 9999.
   DO 25 I = 1, 12
   IF (APV(I) — OPT)25, 25, 23
23 OPT = APV(I)
25 CONTINUE
   PV = TEMP(J)*ZA
   SPI(J) = PV + OPT
   DO 26 I = 1, 12
   IF (APV(I) — OPT)26, 12, 26
12 IF (SENSE SWITCH 2)74, 75
75 PRINT 205, J, SPI(J), I
74 PUNCH 205, J, SPI(J), I
```

```
 26 CONTINUE
    DO 27 J = 1, 12
 27 PI(J) = SPI(J)
 28 CONTINUE
    GO TO 50
201 FORMAT (F6.4)
202 FORMAT (F5.2)
203 FORMAT (I4)
204 FORMAT (F4.3, F4.3, F4.3, F4.3)
205 FORMAT (I4, F18.2, I10)
100 FORMAT (4H LT, I4)
    END
```

Appendix Table 1. MARKET VALUE OF ANIMALS BY LACTATION AND BUTTERFAT LEVEL USED IN THIS STUDY

Lactation	Estimated market value†		
	Butterfat level 1	Butterfat level 2	Butterfat level 3
	Dollars	*Dollars*	*Dollars*
1	227.09	220.56	248.16
2	227.70	221.17	248.76
3	229.51	222.98	250.58
4	232.06	225.54	253.13
5	235.65	229.13	256.72
6	238.24	231.71	259.30
7	241.16	234.64	262.23
8	238.62	232.09	259.69
9	234.62	228.09	255.68
10*	207.97	201.44	229.03
11	191.37	184.84	212.44
12 and above	208.83	202.30	229.89

* Data from the questionnaire did not include any transactions on animals above lactation nine. Market value for animals of lactations 10, 11, and 12 is an extrapolation of the data.

† The regression equation was:

$$re_{t,j} = pv_{t,j} = 249.14 + .92(j) + .50(j^2) + 17.59\ (k) - 4.76(abl_j) - 60.4\ (abbf_k)$$ where

j = Lactation of the animal (1, 2, . . . , 12)

k = Butterfat level (1, 2, 3)

abl_j = Index of number of cows available by lactation based upon Cannon and Hansen data. (Appendix Table 3.)

$abbf_j$ = Index of number of cows available by butterfat level based on Oregon DHIA dairy cows. (Appendix Table 4.)

Appendix Table 2. PRICE INDEXES OF DAIRY COWS, FEED, CANNERS AND CUTTERS, AND MILK, 1950-1961

Year	Price of dairy cow index*	Feed cost index†	Canner and cutter index*	Milk price index‡
1961	1.000	1.000	1.000	1.000
1960	.995	1.000	.983	.976
1959	1.040	.996	1.131	.950
1958	.940	1.003	1.150	.932
1957	.740	1.042	.839	.968
1956	.680	1.042	.695	.976
1955	.650	1.069	.695	.941
1954	.660	1.142	.668	.929
1953	.790	1.183	.742	1.021
1952	1.080	1.301	1.170	1.180
1951	1.103	1.214	1.455	1.171
1950	.884	1.062	1.146	.988

* Calculated from U. S. Department of Agriculture. Economic Research Service. *The Dairy Situation,* February 1962, p. 14 (DS 288).

† Calculated from U. S. Department of Agriculture. *The Dairy Situation,* April 1962, p. 14 (DS 289).

‡ *Ibid.,* p. 7.

Appendix Table 3. INDEX OF THE NUMBER OF COWS AVAILABLE BY LACTATION USED IN REGRESSION EQUATION FOR DAIRY COW PRICES

Lactation	Number of cows*	Index†
1	31,447	1.000
2	22,735	1.383
3	18,258	1.723
4	14,840	2.119
5	12,535	2,509
6	9,471	3.321
7	7,361	4.272
8	4,776	6.588
9	3,338	9.422
10	1,829	17.241
11	1,362	23.152
12 and over	1,368	22.100

* Cannon, C. Y., and E. N. Hansen. Expectation of life in dairy cows. *Journal of Dairy Science, 22*:1026. 1939.

† Index is determined by dividing the number of cows in each lactation into the number of cows available in the first lactation.

Appendix Table 4. INDEX OF THE NUMBER OF COWS AVAILABLE BY BUTTERFAT LEVEL AS OBTAINED FROM ANALYSIS OF 9,576 OREGON COWS COMPLETING A 305-DAY LACTATION IN 1961 USED IN REGRESSION EQUATION FOR DAIRY COW PRICES

Butterfat level	Number of cows	Index*
Less than 350 pounds	2,363	0.6010
350 to 450 pounds	3,932	1.0000
Over 450 pounds	3,281	0.8344
TOTAL	9,576	

* Index is determined using the number of cows in the second butterfat level as a base.

Appendix Table 5. INDEX OF THE WEIGHT OF DAIRY COWS BY LACTATION USED IN THE EXAMPLE

Lactation	Number of cows in sample*	Average weight	Index†
		Pounds	
1	25	1,120	1.000
2	18	1,210	1.080
3	14	1,370	1.223
4	13	1,400	1.250
5	11	1,450	1.286
6 and above	19	1,470	1.312

* Cows used in the sample were selected randomly from the Centre County, Pennsylvania, DHIA Monthly Reports.

† Index was calculated by dividing each weight by the average weight of animals in the first lactation.

Appendix Table 6. NUMBER OF COWS FAILING AND TOTAL NUMBER OF COWS BY LACTATION AND BUTTERFAT LEVEL AS OBTAINED FROM ANALYSIS OF PENNSYLVANIA DHIA DATA

	Number of cows failing by butterfat level			Number of cows by butterfat level*		
Lactation	Less than 350 lbs. butterfat	350-450 lbs. of butterfat	Over 450 lbs. of butterfat	Less than 350 lbs. butterfat	350-450 lbs. of butterfat	Over 450 lbs. of butterfat
1	171	949	556	39,058	17,485	8,251
2	187	955	498	2,826	12,645	5,967
3	187	951	585	2,268	10,149	4,789
4	171	987	560	1,844	8,250	3,893
5	160	941	552	1,558	6,970	3,289
6	164	820	435	1,177	5,265	2,484
7	121	645	350	915	4,092	1,931
8	108	441	244	593	2,654	1,253
9	67	252	117	415	1,856	876
10	27	160	73	227	1,014	478
11	21	94	33	169	755	356
12 and over	8	67	19	177	791	373

* Number of cows by butterfat level was obtained for the Pennsylvania data by sorting herd cards by butterfat levels; summing the number of cows in each level; and applying the Iowa age distribution.