

AN ABSTRACT OF THE THESIS OF

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Abstract approved:

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Abstract Body

The aim of this Physics Education Research project was to implement a modified version of Calibrated Peer Review and measure the impacts in junior-level students' scientific writing. The project developed and implemented a set of guiding questions, instructor written examples, and a rubric which were implemented in a multi-staged process, relying on student evaluation of the examples and of peer work using the developed rubric. Quantitative data obtained using the rubric itself, as well as free-form student reflection on the process suggests that students' writing improved in three distinct ways: judgment as to the appropriate level of detail to include, explanations and structuring of mathematics, and organization of thought. Suggestions for future classroom use and further studies of the effects of rubrics and peer evaluation on scientific writing are provided.

Key Words: physics, writing, rubrics, peer review, education

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The Impact of Guiding Questions and Rubrics in the  
Scientific Writing of Middle-Division Physics Students

by

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# **The Impact of Guiding Questions and Rubrics in the Scientific Writing of Middle-Division Physics Students**

## **Introduction**

Writing, the task of transferring ideas from the brain down onto paper, is a difficult skill to master for nearly everyone. Coupling the task of writing with the field of physics, the process only becomes even more intimidating. Most students struggle with writing, as it is a skill that comes slowly and only with diligent practice. By the time a physics undergraduate enters upper-divisional classes, they have usually had multiple writing classes focusing on honing their grammar, sentence structure, word choice and spelling skills, all important components of any type of writing. However, very few have had any formal training or practice writing scientific papers. The writing process is important in many ways, not limited solely to communicating ideas and findings from one individual to another. The process of structuring arguments, physical concepts and their problem solving procedures helps students clarify their own understanding of the subject material. Scientific writing is also an important evaluation tool; when students provide full written solutions – as opposed to traditional homework problem solutions that are rarely comprised of a coherent structure – a deeper level of understanding of the subject can be determined by the instructor. However, physics



instructors do not generally have the luxury of being able to spend much (if any) time on facilitating the writing process. This project seeks to develop a writing unit based on peer evaluation using a rubric that can be implemented in the future to facilitate the scientific writing process with minimal load on the instructor.

The project was motivated by the fact that undergraduate physics students entering the paradigms have not had much experience writing scientific papers or had the opportunity to clearly explain a problem solving process pertaining to physics. Throughout the first two paradigm courses, Symmetries (PH 320) and Vector Fields (PH 421), students work in small groups solving physics problems and then discuss their findings with the class. These activities are guided by the instructor, but the students are responsible for carrying out their own work and clearly communicating their ideas and problem solving strategies to one another, as well as considering and expressing the physical meaning of their final solution. This unique in-class activity structure allows students to become more proficient in verbally communicating the problem solving process and analysis of the problem. This project took the in-class activities one step further by having students write formal solutions to the in class activities.

The structure of this project was inspired by the Calibrated Peer Review (CPR) developed by UCLA. The original CPR is fully described in the Background Section (Section 2) of this paper. To facilitate our students' understanding of the requirements of scientific writing, a set of guiding questions was developed and distributed to the students at the beginning of the project. These guiding questions consisted of general prompts with short explanations, aimed at cuing students to what was generally considered important in

scientific writing. A rubric was then developed based on these guiding questions. Each of the rubric criteria were not repeated verbatim from the guiding questions, but were derived from them so that assessing a paper with the rubric would give quantitative data as to how well a paper had addressed each of the guiding questions. Three writing samples based on an in-class activity were also developed, each of varying quality. The guiding questions, rubric, and the writing samples were distributed at varying times throughout the process, to slowly build on the writing process. The unit included five steps, each of which are outlined in the Methodology Section (Section 3) of this paper.

Using the rubric to acquire data from each of the student writing samples, the project was able to determine the change in student writing. Data acquired in each step of the process allowed for determining where and when the change occurred, and provided insight as to which stages of the process best facilitated change. Full data collection procedures are described in the Methodology Section (Section 3). Collected data is presented in the Methodology Section, analyzed and further explored in the Data Analysis Section (Section 4), and summarized and explained in the Results Section (Section 6). A brief discussion of the implications of the data and the importance of the problem are expressed in the Discussion Section (Section 7).

The collaborating project members each brought their own biases to the research. The main author of this paper, an undergraduate senior in physics, scored each of the papers and played a large role in writing the project materials. His bias was towards looking largely at the content and style of the students' writing, and not the veracity of what was actually written. Having only recently gone through the paradigm courses, he did not bring a mastery

of the physics concepts that Corinne Manogue, professor of the course and leader of the research, would have brought to scoring each of the papers. Professor Manogue brought her own bias to the research, which consisted largely of concern for deepening students' understanding of physical concepts through the practice of writing; while the improvement of the writing itself was an important goal, it was not the sole goal. Professor Manogue's experience in writing professional scientific papers also brought professional bias as to what she believed a good scientific paper was, which may or may not have been an appropriately scaled goal for a junior level physics course.

Ultimately, this project aims to set up the framework for future study into the use of rubrics to facilitate physics writing capabilities, and ultimately develop a unit that can be implemented in future classrooms with minimal load on the instructor.

# Background

This section explores Calibrated Peer Review and physics education research pertaining to the use of group activities, student writing, and using a rubric.

## Calibrated Peer Review

Calibrated Peer Review is a writing process allowing for in-depth writing assignments with minimal load on the instructor. To ease the load on instructors, CPR has students peer review one another's work using a calibrated rubric. The steps for setting up a writing activity using calibrated peer review requires three essential components.

First, the instructor creates an instruction sheet for the writing assignment, including a list of source material from which students draw to address the writing prompt as well as a set of guiding questions to direct students when addressing the assignment. The instructor then creates a rubric for students to use in peer evaluations. The rubric typically include yes or no questions, such as, “Are there at least three reasons given to support the use of SI in the scientific community?” Finally, the instructor creates three benchmark samples of varying quality and scores each with the rubric that the students will eventually use. Each of these components are developed online using the website provided by UCLA, which is as of now free.

At this point, students are ready to use the CPR developed writing assignment. Students start by reading the instruction sheet, providing them with guidance on what they need to write and the source material from which to draw their information. Students then begin the text entry stage of CPR where each submit their own essay using the online program.

After submitting their own essay, the students are given access to the rubric and the benchmark samples and asked to score each paper following the rubric criteria. This is known as the calibration and review stage. Students submit their scores and receive instant feedback on how well they were able to score each paper. The students then use the rubric to score other students' work, and then score their own submitted essay. Grades for each paper are assigned based on a weighted average of the peer evaluations. Students who followed the rubric well when scoring the benchmark papers will have a greater weight in the average score, and students who did not score the benchmark papers well will have very little weight in the score of the paper. This aspect of CPR mitigates the common problem of “the blind leading the blind” when using peer evaluation as a form of grading and feedback to each student.

When the assignment has been completed by all students, the results stage of CPR begins. In this stage, the program determines how well the student scored their peers by calculating their deviation from the average. The instructor chooses a maximum allowed deviation; students below this threshold are considered to have mastered the evaluation process. Each student can then read each of their peer reviews on their own paper, and compare it with their self evaluation. An overall score is assigned by the program which

compiles the quality of four criteria: text entry, calibration, reviews and self-assessment. The average weighted text rating is the score for the text entry criterion, and the students' deviations are used for assessing the quality of the other three criterion. Deviations are turned into a score and averaged with the other criteria for an overall paper score. The table below is an example of what the student sees at the end of the review stage.

<b>Scores and Overall Grade</b>		
<b>Stage</b>	<b>Performance</b>	<b>Score</b>
<b>Text Entry</b>	Avg. Weighted Text Rating = <b>7.33</b>	<b>22.00</b> out of 30
<b>Calibrations</b>	Avg. Calibration Deviation = <b>0.00</b>	<b>30.00</b> out of 30
<b>Reviews</b>	Avg. Review Deviation = <b>0.33</b>	<b>30.00</b> out of 30
<b>Self-Assessment</b>	Self-Assessment Deviation = <b>0.33</b>	<b>10.00</b> out of 10
<b>Overall Score</b>		<b>92.00</b> out of 100

It should be noted that in a standard Calibrated Peer Review process, the instructor is never required to read any student papers; all of the grading comes from the students. It is also important to note that a large portion of the grade comes from evaluating other work, not the quality of the paper the student writes. A student could submit a very poor paper, and it would only bring down the average overall score marginally as compared to if the quality of the paper were the only grading criterion.

## **Prior Research**

There has been a great amount of research done on student's ability to write about science, use rubrics to evaluate writing, small group activity work, as well as research determining what affects student ability to understand and communicate an understanding of

science. Research in a school in South Africa found that writing about science improves when nonlinguistic means (such as demonstrations) are used in conveying ideas, and that explicit guidelines are important for students to be able to produce good writing (Kaundra, 1998). This investigation also reports that the quality of scientific writing is proportional to the amount of time available for the completion of the assignment. The authors of this investigation assert that, "...an understanding of the procedural aspects of an experiment is as crucial to the production of a good report as an understanding of concepts is in the production of a good essay." Writing assignments implemented in the paradigms courses did not involve any experimentation, but it is important to note that without a good understanding of the concepts with which each activity involved, a student will never be able to produce very good writing.

Physics education research carried out in an introductory physics course found that "better problem solutions emerged through collaboration than were achieved by individuals working alone." (Heller, 2008) This is important to the paradigms project because students worked in small groups when completing the activities on which they based their writing. Because more complete and expert-like solutions are produced by students when working in groups, it seems to suggest students come away with a better understanding than when working alone, and definitely are able to more clearly express their solution which is integral to the writing process.

One final important piece of research involved implementing traditional CPR in an introductory zoology lecture class. What researchers found no data to support that students' technical-writing skills improved, nor did their abilities to convey scientific understanding

improve. Furthermore, grades received via the CPR process were consistently higher than instructor feedback scores alone. Though implementation of CPR in this research seemed to not aid in goals the paradigms project seeks to achieve, the authors of the paper have noted that, “The best uses of CPR may be for assigning questions requiring more objective answers (especially in introductory classes); in coordination with group work in some hybrid setup; and/or for refining already “good” writing skills in upper-divisional courses (where more subjective scaling could be used).” The authors also stress the importance of writing rubric questions that “are concise and cover all important criteria for the writing assignment” and “to craft writing prompts that support course goals.” These reflective comments are especially important to this research since the paradigms students worked together in groups when completing their activities, are all upper-divisional, and usually have at least one course in honing technical writing skills prior to entering the paradigms classes.



# Methods

This section describes the creation of project materials, adapting traditional CPR, the five stages of the project, and the data collection methods employed.

## Creation of Project Materials

The set of guiding questions (see appendix A) was the first material created for the project. In CPR, the guiding questions are the prompts that an instructor uses to cue the students as to what is important to address in writing a paper. They are traditionally topic specific to a given paper, but the guiding questions created for this project were developed to be useful for all scientific writing. They took the form of nine concise prompts, each with questions for students to consider when addressing each prompt. The purpose of the guiding questions was to help give students a broad understanding of what goes into any good scientific writing, so that they could take them and use them when writing any paper, not simply the papers due for this project.

The next material created for the project was the rubric. This was derived from, but did not directly copy, the guiding questions. Each rubric criteria addressed one of the guiding questions in such a way that it would be quantifiable in one of three categories: Poor, Fair, or Very Good. In general, a poor attempt reflected very little or no effort at all on the part of the author in addressing that particular criterion. A fair attempt usually reflected an attempt that

was flawed in an important way. A very good attempt conveyed a flawless or near flawless implementation of a criterion. The rubric changed slightly throughout the process as further collaboration with physics faculty occurred; when a change was made to the rubric the students were alerted. Most changes were minor, however, the final two criteria (including a diagram and mathematical or logical veracity) were added to the rubric after stage 3 of the process.

Three written examples were produced (Appendix B), each covering a single in-class activity, and each addressed the rubric criteria to varying degrees of quality. Each paper was written so that certain rubric criteria were addressed well, others addressed fairly, and some not addressed at all. There was no single paper which met all criteria well. This was a deliberate choice so that students would not simply find the best paper and realize that it should be scored with all Very Good marks. Akin to CPR, students were given these examples and used the rubric to score each of them. The members of the project also scored each paper using the rubric so that data on how well students were scoring each of the rubric criteria could be obtained.

The only other materials created for the project were the various writing prompts.

## **Adaptation of Traditional CPR**

The process of our project was modified in many important ways from the published CPR procedure. The first important change was the generality of the guiding questions. The

guiding questions of a traditional CPR activity are topic specific, guiding students to address very specific concerns in a given paper. The guiding questions developed for use in this project were created to be general so that they could be useful for students in any scientific writing and not for each specific activity.

Another important change came when structuring the writing process stages. In traditional CPR, each stage of the writing process occurs over the same paper. Students write a paper, read instructor examples of the same topic, peer review other student papers, and end with scoring their own paper. In our project, each of these stages occurred over separate class activity write-ups. This is an important difference because although the in-class activities were related and shared similar geometry, they gradually built in conceptual difficulty.

Each of the three papers were based on small group activities that took place during class. Students worked in groups of three or four students, using small whiteboards to draw diagrams, ideas and equations to solve various problems. Instructor guidance was provided only when needed. At the end of each activity, groups shared their findings with the entire class, and the instructor lead a brief reflection session drawing attention to any important conclusions and aspects. Students thus had opportunities to fully explore the material they were to write about with other individuals in the class as well as with instructor guidance. In traditional CPR assignments, students are given source material to explore outside of class, and the topics may or may not be linked to those covered in lecture.

## Stages of the Project

The project was carried out over five stages as outlined below.

### Stage 1

Students were given the guiding questions (Appendix A) and the following prompt:

Using the handout “Guiding Questions for Science Writing” to suggest topics that you should address, write up your “analysis” of the activity entitled **Electrostatic Potential From Two Charges**. You do not need to do the calculations from every case, but your analysis should include some comparison of different cases, as we discussed in class after the activity. To help us with the grading process, please turn in this writing assignment stapled separately from your other homework.

Each student then wrote and submitted their first paper, without having seen an instructor example or the rubric.

### Stage 2

The project's scientific writing examples (Appendix B) were uploaded to the class

website. Students were given the original version of the rubric (Appendix C) and used the Rubric Evaluation Worksheet (Appendix F) on the class website to submit their scores for each of the three examples. Students followed this prompt on the website:

You will find three files attached below. These are sample write-ups for the ring activity that you have just completed. Your task is to read each of the sample write-ups and evaluate them on a Very Good-Fair-Poor scale, based on the criteria listed in the Rubric. Fill out the Evaluation Submission Page for each of the write-ups (in the end, you will have filled it out three times). Identify the sample you are evaluating indicating the paper code (D1, D2, or D3) at the top of the Evaluation Submission Page. If there is an evaluation criterion you feel is inappropriate to include in the evaluations (i.e. items about handling experimental data), indicate this on the Evaluation Submission Page by selecting the “Not Applicable” radio button.

Data were sent to the course instructor and stored in Excel spreadsheet format. Each of the project collaborators scored the examples as well, allowing for statistical calculations determining the level of agreement students had with professional scoring. Ideally, students would have received feedback informing them of how well they agreed with instructor scoring; due to time constraints this never occurred. Statistical analysis was carried out on the data received in this stage of the process.

### **Stage 3**

Students wrote a second paper based on the following prompt:

Writing Assignment: Write a complete solution to the following problem. Keep the Guiding Questions and the Evaluation Rubric in mind when writing this assignment.

(a) Find an integral expression (that Maple could evaluate) for the electric field everywhere in space due to a ring of charge. Assume the ring has a uniform charge density, a radius  $R$ , and that the total charge on the ring is  $Q$ . (b) Find the electric field and the electric potential due to this ring of charge for all locations on the  $z$ -axis. Comment on your answers. (c) In your discussion section, explore the similarities and differences between your answer for the potential and your answer for the electric field. There are many things one might discuss such as the relationship of the physics to the mathematics and/or various limiting cases. Make your own professional choice about what to discuss. Since you don't have the results of other students, it would not be relevant to comment about them.

Students submitted these papers in hard copy along with a standard homework assignment.

### **Stage 4**

In this stage, students anonymously scored three other student writings and submitted

their scores online. The following prompt was given:

You can find the files containing Writing Assignment 2, written by your fellow students on Blackboard. You are being asked to evaluate three of these files, following the writing rubric we have been using in class. The files are numbered. If your file is numbered N, please review the files numbered N+1, N+2, N+3. If your number is near the end of the list, please cycle back to the beginning in the obvious way. You can find your own file quickly, if you know your letter code, which is written on your final exam from last term. We have revised the Rubric, and the Evaluation page. Please use the updated versions below.

Students used an updated version of the Rubric Evaluation Worksheet from stage 2 of the project, as well as an updated rubric to submit their peer reviews. After everyone had uploaded peer reviews, the data was made available to the students so that everyone in the class could receive feedback in the form of the peer reviews.

A problem occurred during this stage in implementation of the updated Rubric Evaluation Worksheet, and a number of peer evaluation scores were never actually collected. Many students only received peer feedback from one other student, when the intent was to have three peer review scores available to each student. When the glitch in the web page was located and corrected, many students had already submitted the scores and asking them to do so again would have been unfair.

## Stage 5

Students wrote a third paper after having had further experience with the rubric in the peer review stage. They were given the following prompt:

Use the Guiding Questions and Rubric to write up the small group activity where you calculated the magnetic vector potential due to a spinning ring.

Students were also given the following prompt asking them to reflect on the writing process:

Write a page or so, answering the following questions: (a) Evaluate your writing. How did it change between the first writing assignment and the last one? (You might want to refer to some of the rubric evaluation criteria and/or you might want to discuss how your ability to express your ideas has changed.) (b) What events in the past six weeks have had the greatest impact on your writing?

This reflection piece of the project was important because it asked students to analyze their writing and identify the changes that had occurred. Not only was this important for the students' sake of solidifying positive change that may have occurred, but it could also allow for triangulation of the quantitative data on how student writing had changed.



## **Data Collection**

This section contains detailed information of the data collected, as well as an exploration into the validity of the data.

### **Procedure**

The data that were examined came from the following sources: Student Notebooks from 2006, Student writing samples (stages one, three and five), Student evaluations of instructor examples (stage two), Student evaluations of one another's writing (stage four) and student reflections of the process (stage five).

#### **2006 Student Notebooks**

Data were obtained from scoring the previous year's group of physics students scientific notebooks. These notebooks were write-ups of the in class activities, similar in content to the writing students in this research project covered. The notebooks from previous years, however, did not receive very explicit instruction or guidance. Professor Manogue assigned students these notebooks with the general directions to write about the activity, how they solved it, and to identify what had been learned. Ideally, student notebooks would look like well written scientific papers, addressing each of the rubric criteria, even though the

rubric had not yet been developed when the notebooks were assigned. These notebooks were scored to provide context on how the presence of the guiding questions themselves may have had an impact on student writing (assuming students from the previous year had roughly the same writing capabilities as the students involved in this research project).

### **Student Paper Scores**

To collect data from the student writing samples, each paper was read twice and scored using the rubric in each criteria. The use of the rubric allowed for quantitative data showing how each criteria differed across papers for a given student, as well as how overall criteria scores differed for all students across the three papers. Each criteria was given a score during a second reading of a given paper. The first reading of a student's sample was uninterrupted, and provided information on the style, content, and flow of the paper. The second reading was briefly interrupted each time a score for a particular rubric criteria could be assigned.

### **Evaluation of Instructor Samples**

Students used an online submission form when evaluating the instructor examples in stage two of the process. After having been prompted to read through each sample, students

were able to assign a letter grade to each criteria with a short justification of their score. These responses were stored in electronic spreadsheet format. The instructor examples were also scored using the rubric by the professor of the course, an undergraduate senior who had completed the paradigms courses in the previous year, a collaborating graduate student, and a collaborating post-doctoral associate. These instructor evaluations allowed for calibration of the rubric, extrapolating to what degree of accuracy students were able to understand the rubric and apply consistent scores to the writing samples.

### **Evaluation of Other Student's Writing Samples**

In stage four, each student paper was assigned a code and uploaded electronically to a secure database. Students were given the codes of 3 papers to score, and using the web-based scoring method from step two uploaded their score to a spreadsheet for instructor evaluation purposes. The codes were assigned randomly so the grading between students were anonymous.

### **Reflections of the Process**

After having produced their final writing sample, students were asked to compare their final work to their original paper and comment on the process. As part of another

homework assignment for the class, they were prompted to identify how their writing had changed, and to describe the parts of the course or process that had lead to this change. Reflections were to be approximately one page in length. Below is the actual prompt:

## **Validity of Data**

### **Rubric Calibration**

For a rubric to capture quantitative data about the quality of students' writing accurately, a given paper should be assigned the same scores by separate evaluators. However, due to the subjective nature of grading, and the brief period of development time for the rubric itself, it is expected that not all evaluators would assign the same exact scores for a given paper. To measure the deviance of evaluations, the instructor developed samples were scored by four parties: the professor of the course, the undergraduate evaluator, a graduate student, and a Postdoctoral associate. The results of this calibration can be found in appendix E.

The rows in bold show the deviations of the undergraduate evaluator from the average scores. Because only integer values are possibly obtained using the rubric, it may be more valuable to round each average to the nearest integer and obtain deviations from that. Having four evaluators score three separate examples shows that for any given paper or any given criteria, the deviation of the undergraduate evaluator from the average is never more than 0.75 (which only occurred for one criteria in one paper), and on average, is less than 0.25 points different from the average. This calibration shows that data obtained by the undergraduate

evaluator has an average uncertainty of values that are less than 0.5 per criteria. Because the data is discrete in integer values, it is likely that when assessing a given individual student's paper, only one criterion score would differ from a group of evaluator scores. However, this calibration does suggest that average changes over many criteria or many papers of less than 0.25 may be meaningless.

# Data Analysis

This section explores each of the data sets acquired and outlines how the rubric was used on the student data to obtain quantitative information of the quality of the writing.

## Student Paper Scores

Quantitative data obtained using the rubric are available upon request, or by accessing the wiki (prior permission required).

## Criteria Scoring

To demonstrate how the rubric was used in scoring each paper, the following excerpts from student writing exemplify meeting the requirements of a particular rubric criteria to varying degrees of success, with a short justification of how it was scored.

**Criteria 1 - Did the writer have a clear, concise description of the problems being solved?**

Sample Writing:

*The problem presented to our team involved a set up of two charges of equal magnitude  $+Q$ . These charges were arranged a distance of  $D$  in the opposite*

*directions from the origin along the x-axis. The objective was for our team to calculate the electric potential at any point on the x-axis so long that  $|x| \gg D$ . We were then meant to use the approximations to predict the behavior of a test charge placed anywhere along the x-axis.*

Evaluation:

This received a score of 3 (Very Good). The author clearly described the physical set up of the problem (value of charges and positions of the charges), followed by a brief explanation of what he/she would be solving. The author could have defined exactly what “calculating the electric potential” meant (students were specifically asked to approximate  $V(x)$  using a fourth-order series expansion). If there were a higher score available, this would have distinguished between the two. As it stands this is a succinct, descriptive explanation of the problem being solved.

Sample Writing:

*The general problem assigned was to determine, to the fourth order, a series expansion of the equation for the electric potential (in two dimensions, specifically the xy-plane) of essentially a dipole system with varying parameters. The conditions that remained the same from case to case were that the two charges were situated a distance apart of  $2D$ , lying along the x-axis, with the axial point of the dipole lying at  $x=0$ . The conditions that differed from one situation to another were: both charges being either positive, or of equal yet opposite charge value; and that the electric potential was being determined for  $V(x)$  (sic), where  $|x| \ll D$  or  $|x| \gg D$ , or for  $V(x,y)$  (sic),*

*where  $x=0$  and  $|y|\ll D$  or  $|y|\gg D$ . The specific conditions of the situation which was asked to be solved were those where both charges were positive and the electric potential was determined for  $V(x,y)$ , where  $x=0$  and  $|y|\ll D$ .*

Evaluation:

This received a score of 2 (Fair). Though the author has set up the physical system, and defined the questions he/she will be answering in the rest of the paper, there is a large amount of extraneous information and the wording is often unclear. When the author writes, "...of essentially a dipole system with varying parameters," it seems like the system is not actually a dipole (the author probably intended to distinguish this system from a point dipole), and that the system itself has varying parameters (the author meant to describe that there were various different problems being asked of different groups, which he/she goes on to explain). The author went on to identify all of the varying similar physical problems that other groups would solve, only identifying his/her own in the final sentence. This makes identifying the problem being addressed specifically by the author a more difficult task than it should be.

**Criteria 2 - Did the writer use professional judgment on how much detail to provide in writing the solution to the physics problem?**

Sample Writing:

*1) Also important, will be to find a way in which to expand the final equation for potential as a series expansion, or in other words, to recognize a similarity between our equation for potential and a previously known power series expansion formula.*



2) *The importance of the resultant series expansion can be appreciated and better understood when compared to the resultant expansions for each other scenario.*

3) *Symmetry and anti-symmetry, in brief terms, are defined by the relations  $f(-x)=f(x)$ , and  $f(-x)=-f(x)$ , respectively. For a physical example, one can think of a sphere, which is symmetrical about all three axes of Cartesian space; unlike a pyramid, which is symmetrical only about the z-axis of Cartesian space, and anti-symmetrical about the xy-plane.*

Evaluation:

The three excerpts above came from the same paper, receiving a score of 1 (Poor) in this criteria. The paper had many components similar to the excerpts above that demonstrate poor professional judgment to detail. In the first excerpt, the writer should have omitted everything after “as a series expansion,” because the second part of the sentence is redundant, if not somewhat confusing. The second excerpt is an example of a statement which is not necessarily true, as a series expansion could be appreciated and fully understood on its own without comparison to other scenarios. The author probably meant to write that it would be interesting to compare the results to other scenarios. The final excerpt is an example of the author of too much information which is already obvious to the reader. The author could have stopped after defining the mathematical relationships. Furthermore, a pyramid is not symmetric about the z-axis. The author probably had a misconception about what axial-symmetry meant, or perhaps meant to say cone and not pyramid.

**Criteria 3 - Did the writer convey a complete understanding of the relationships and meanings in the symbols of the equations used in solving the problem?**

Sample Writing:

$$V(x) = \{kQ / (x-D)\} + \{kQ / (x+D)\}$$

*Obviously it is necessary to use the equation  $V(x) = kQ / x$  to find the approximation for the potential. In this problem it is necessary to divide the formula into two parts using the super position principle:  $V(x) = kQ/(x-D) + kQ/(x+D)$  ...where  $V(X)$  is the potential with respect to position  $x$ ,  $k$  represents Boltzmann constant,  $Q$  represents charge and  $D$  represents distance from the origin on either side of the  $y$ -axis. The reason it is necessary for a two part equation is so that each charge is represented. The symmetry of the situation allows one formula to represent both sides of the axis because the charges are identical in value and in distance from the origin.*

Evaluation:

This paper earned a score of 1 (Poor) for criteria 3. The author did explain the terms within the equation he/she has referenced in this excerpt, however, the equation used was itself not the original equation for finding electric potential, and  $k$  is not the Boltzmann constant. Furthermore, the distances in the denominators should have absolute values. Finally, the last sentence is erroneous; the superposition principle does not require that the magnitudes of the distances between an axis and a charge be identical. This sentence seems to imply, incorrectly, that given charges need to be of equal magnitude and be the same distance from an axis to allow for one equation to describe electrostatic potential.

Sample Writing:

*One of the physical concepts necessary to solving the problem is that of the superposition of potential, due in part to the fact that electric potential is not a vector quantity and can simply be summed together considering the relation  $V = \sum (Q_i / (4\pi\epsilon_0 |r - r_i|)) \dots$  [on a separate page of calculations the author begins with]  $V = \sum (Q_i / (4\pi\epsilon_0 |r - r_i|))$ ,  $V = Q_1 / (4\pi\epsilon_0 r_1) + Q_2 / (4\pi\epsilon_0 r_2)$  [the author included a diagram explicitly labeling  $r_1$  and  $r_2$ , as well as  $Q_1$  and  $Q_2$ ]*

Evaluation:

This paper received a 1 (Poor) for this criteria. The author correctly identified the equation necessary to solve the problem, but did not describe what physical quantity each variable or constant represented. Furthermore, the magnitudes of  $r_1$  and  $r_2$  were presented in the diagram, but the author did not describe how he or she had explicitly introduced these magnitudes in the equation. Ultimately, the author made no attempt to describe the equation in words.

**Criteria 4 - Did the writer completely explain tricky parts of the calculations, clearly explaining each mathematical manipulation carried out that wasn't algebraically trivial?**

Sample Writing:

*The next step involved using the familiar power series:*

*$(1+z)^p$  for  $|z| < 1$  to find the fourth order approximation for  $V(x)$ . In order to accomplish this, we needed to put the denominators in the form  $(1+z)^p$ :*

[here the author factors out an  $x$  from each term]

*Now considering the  $|D/x| < 1$  because we already know that  $|x| \gg D$  we now have the power series in the correct form. The series' can then be expanded as follows:*

[here the author does the series expansion for  $(1+D/x)^{-1}$  and  $(1-D/x)^{-1}$ ]

*Combining these two fourth order approximations yields:* [here the author sums the two expansions together] *Now by adding in the constants that were set aside we completed the approximation:*

[the author writes the final solution to fourth order accuracy]

Evaluation:

This paper received a 3 (Very Good) for this criteria. Each mathematical step was explained and, when necessary, justified. The important things for students to note in this activity was what he/she needed to do in order to expand the function in a series. The author correctly identified and explained factoring out an  $x$ , since the set-up of the problem had  $|x| \gg D$ .

**Criteria 7 - Did the writer explain what was learned or what insights were gained in solving this problem?**

Writing Sample:

*By observing the approximation it can be concluded that a positive test charge would be accelerated towards +infinity if located at  $+x$  or -infinity if at  $-x$ . Noting that there is an  $x$  in the denominator is [sic] can be concluded that as  $x$  approaches infinity, the*

*electric potential approaches zero, thus the power series converges for  $|x| \gg D$ . This equation will only work for  $|x| > D$  and breaks down otherwise.*

Evaluation:

This excerpt represents the entire conclusion section of a student's paper. Because there isn't any mention of what was learned or which insights were gained in solving the problem, this paper received a score of 1 for this criteria. To receive a score of 3 in this section, the author should have reflected on using a power series to make an approximation, and what he/she needed to do to apply a power series to the equation.

**Criteria 8 - Did the writer convey an understanding of what the final results tell about the physics?**

Writing Sample:

*By observing the approximation it can be concluded that a positive test charge would be accelerated towards  $+\infty$  if located at  $+x$  or  $-\infty$  if at  $-x$ . Noting that there is an  $x$  in the denominator it can be concluded that as  $x$  approaches infinity, the electric potential approaches zero, thus the power series converges for  $|x| \gg D$ . This equation will only work for  $|x| > D$  and breaks down otherwise.*

Evaluation:

This conclusion section did an excellent job at discussing what the final results told about the physics of the problem, receiving a score of 3 in this criteria. The author clearly described what would happen if a test charge were introduced at various locations, taking the limit as  $x$  grew infinitely large, and discussing the situations under which their solution is valid.

## **Student Evaluations of Instructor Created Samples**

In stage 2 of the project, students submitted scores for three scientific writing examples based on an in-class activity they had completed. The project team (including the instructor, doctoral students, and the collaborating undergraduate senior) also scored the examples using the same rubric and evaluation form. This provided some data as to how well students could adhere to the rubric when evaluating scientific writing. Root-mean-square calculations were carried out using the data obtained. Using these calculations, it was found that student evaluations of the project examples were not in very good agreement with project collaborator evaluations. Calculating the root-mean-square of the difference between the average of the students' scores and project collaborator scores, it was found that for each example paper there was an RMS value of 0.72-0.9 (a score of 0 would indicate perfect agreement with collaborator evaluations). Averaging RMS differences between each student's evaluation and the collaborator evaluation scores across all criteria for all papers yielded a median RMS of 0.64 with a standard deviation of 0.23. Essentially, student scores were usually in disagreement with collaborator evaluations, and varied widely in how they disagreed.

## **Student Evaluations of Anonymous Student Papers**

In stage 4 of the process, students uploaded their evaluations of their peers' papers. Ideally, students were to score three other papers so that every student in the class would have three unique peer reviews on which to reflect. There were two problems that occurred during this stage of data collection, however. The first problem was that of student participation. Not all students submitted three peer reviews. Furthermore, there was an error with the online submission form, and an unknown amount of peer reviews were lost before the error was detected and corrected. Most students in the class still received at least one peer review from another student.

## **Student Reflections**

There was a consensus amongst students that content organization, amount of detail to include, and mathematical explanations improved from the first writing sample to the final writing sample. Of the 17 reflections received, 12 students identified improvement in their attention to detail, 6 noted improvement in their papers' organization, and 6 noted that explaining their mathematical steps improved.

There were 3 students who reported improvement in understanding of the concepts, 2 students who believed they improved in their ability to analyze their solution, and one student who reported an improvement in the ability to express their ideas. One student

reported that having to explain things in words had solidified their understanding of the physics of the problems being addressed.

When identifying the factors that had lead to changes in their writing, 7 students reported that peer evaluations had helped improve their writing. The next most reported cause of change (4 students) was the repetition, or practice of writing again and again.

There were two students who reported a better understanding of the physics as being a reason for why their writing had improved. Two students also reported that the in class discussions were important to explaining how their writing had changed. There were also two students who identified the rubric as being an agent of change in the quality of their writing. One student felt that the guiding questions had helped improve their writing, and one other student cited working on the in-class activities had improved their writing.

There were two students who issued criticisms of the process, each identifying a lack of instructor feedback on their papers as a hindrance to improvement in the quality of their writing. This is an important criticism because an important aspect of the writing unit developed in this project is that it should require minimal load on the instructor.

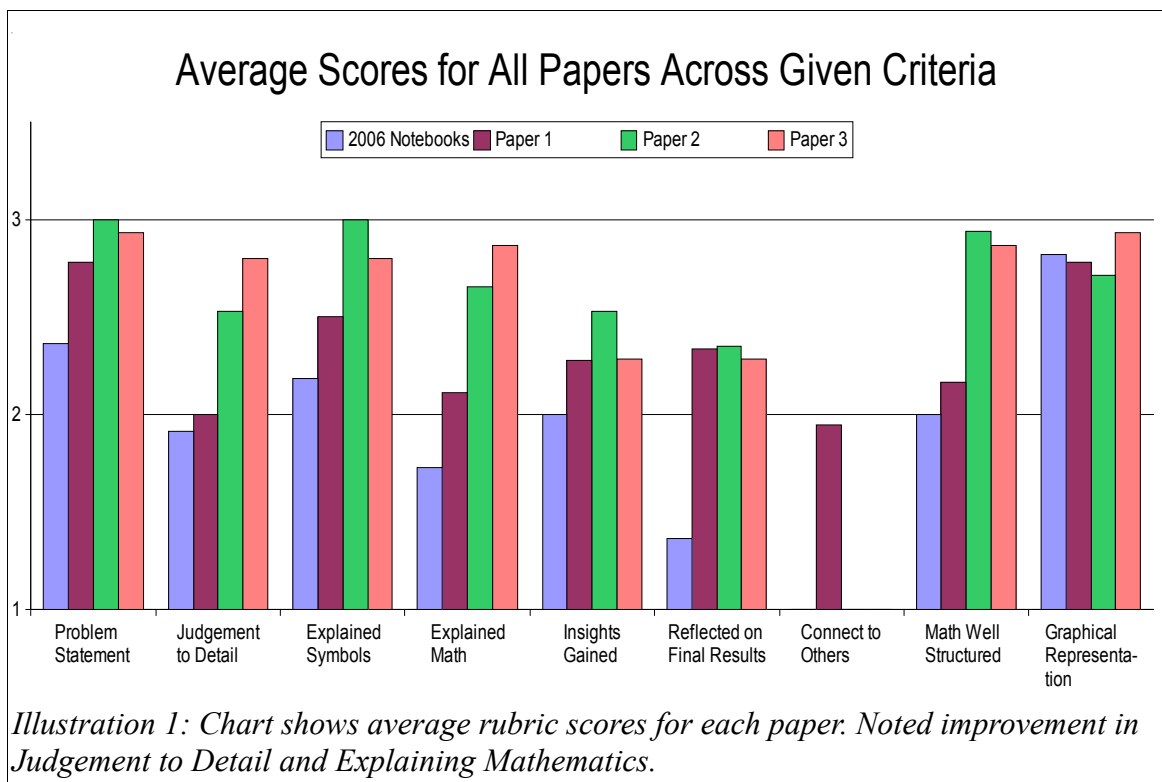


# Results

This section seeks to explore how student writing changed from the beginning of the process to the end of the process.

## Quantitative Data Acquired with Rubric

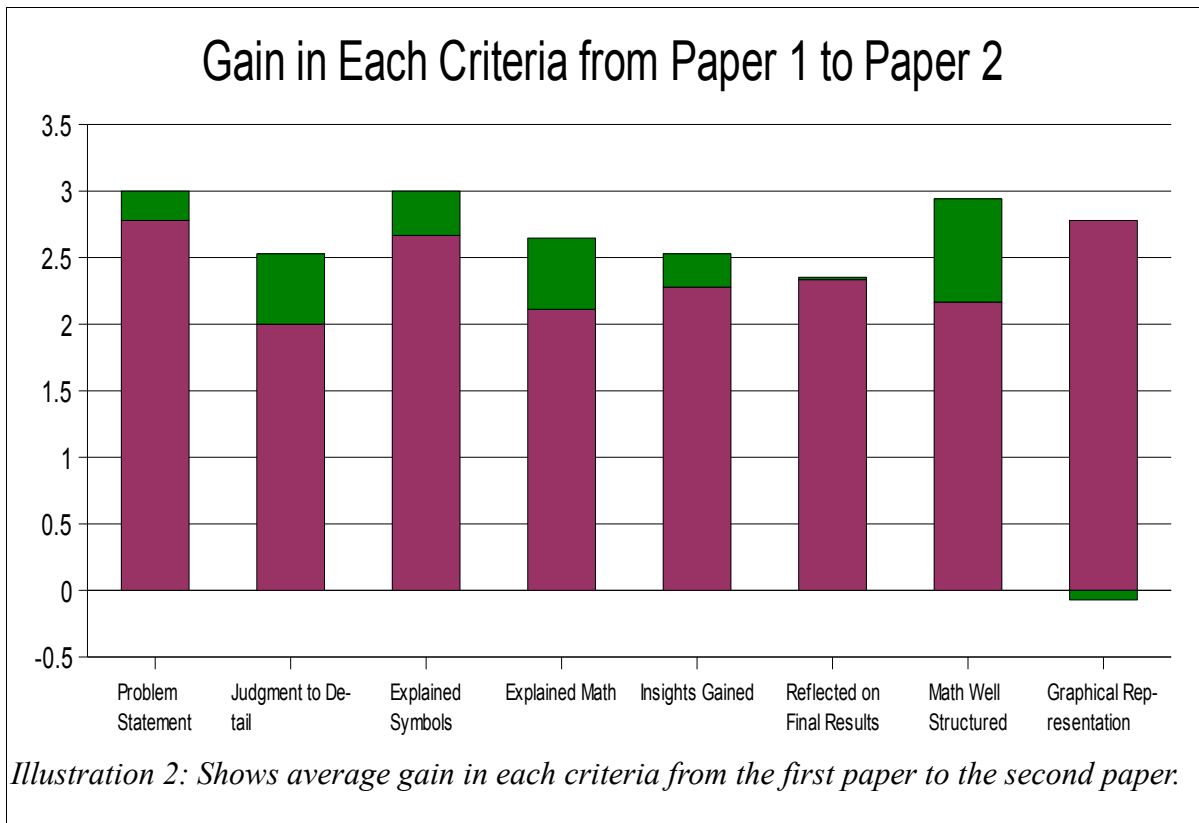
To measure how student writing changed throughout the process, each paper had been scored using the rubric. The graph below shows the average scores for a given rubric criteria across all three papers students produced. It also includes scores from notebook samples taken from the previous year of students. Note that a 3 is the highest possible average score for a given criteria. Note that each of the rubric criteria are available in Appendix C.

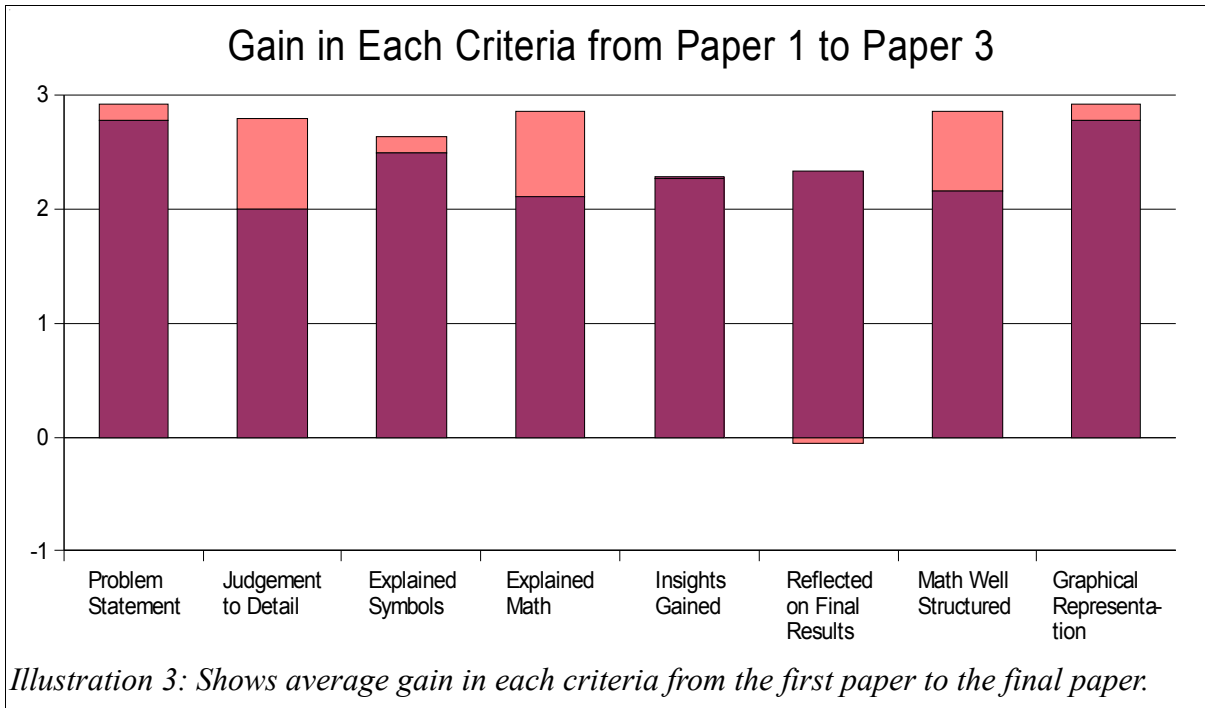


It can be seen that there was substantial growth in most rubric criteria in comparison to the notebook writing samples from the previous year (with the exception of criteria 11). Note that the nature of criteria 9 made it impossible to score for most papers, only paper number 1 had relevant information to score in that criteria, and is thus only represented with one bar. In the cases of criteria 1, 3, and 8, most of the change in comparison to the previous year's samples came from the very first writing sample. This change could best be explained by the presence of the guiding questions, which may have bettered students' understanding of what they were expected to produce. Criteria 2, 4 and 10 showed marked improvement from the first writing sample to the final sample.

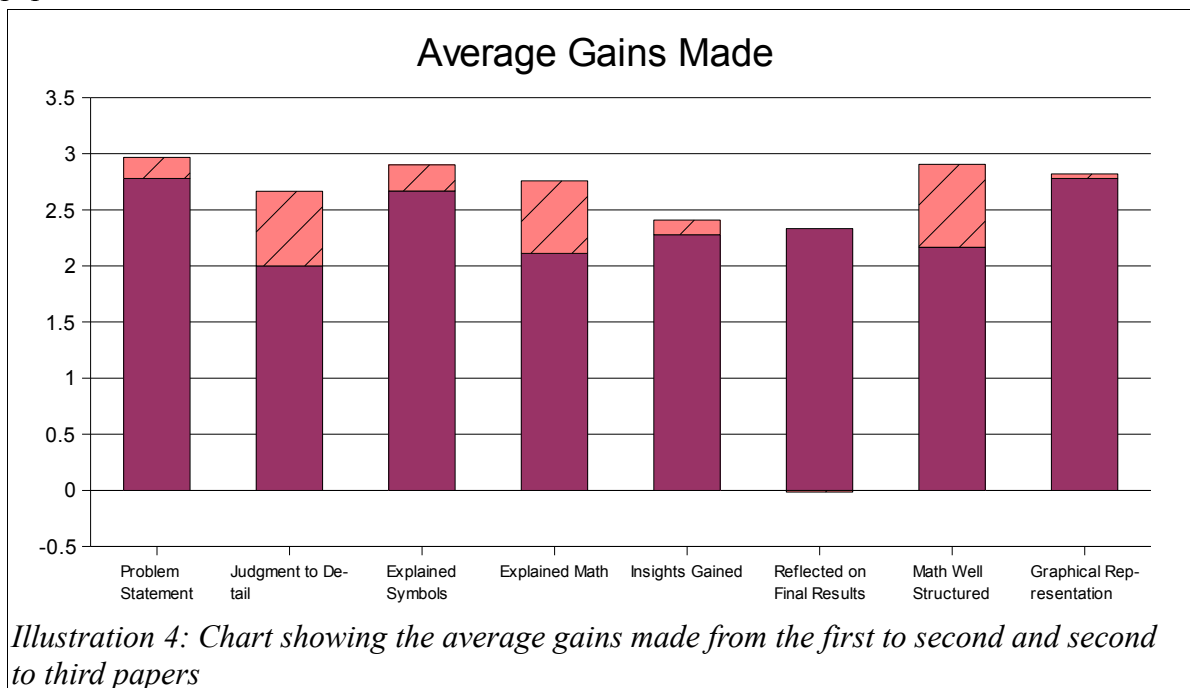
To measure how much improvement was made for each criteria, the change in averages of the scores from the first paper to the other papers were added to the average

criteria scores of the first writing sample. The maroon portion of a bar corresponds to the average class score for a given criteria, whereas the salmon/green portions represents the change. Note that if the two portions add to 3, the class as a whole improved the maximal amount. A portion of salmon/green in the negative direction reflects a net loss in the average score of a criteria.



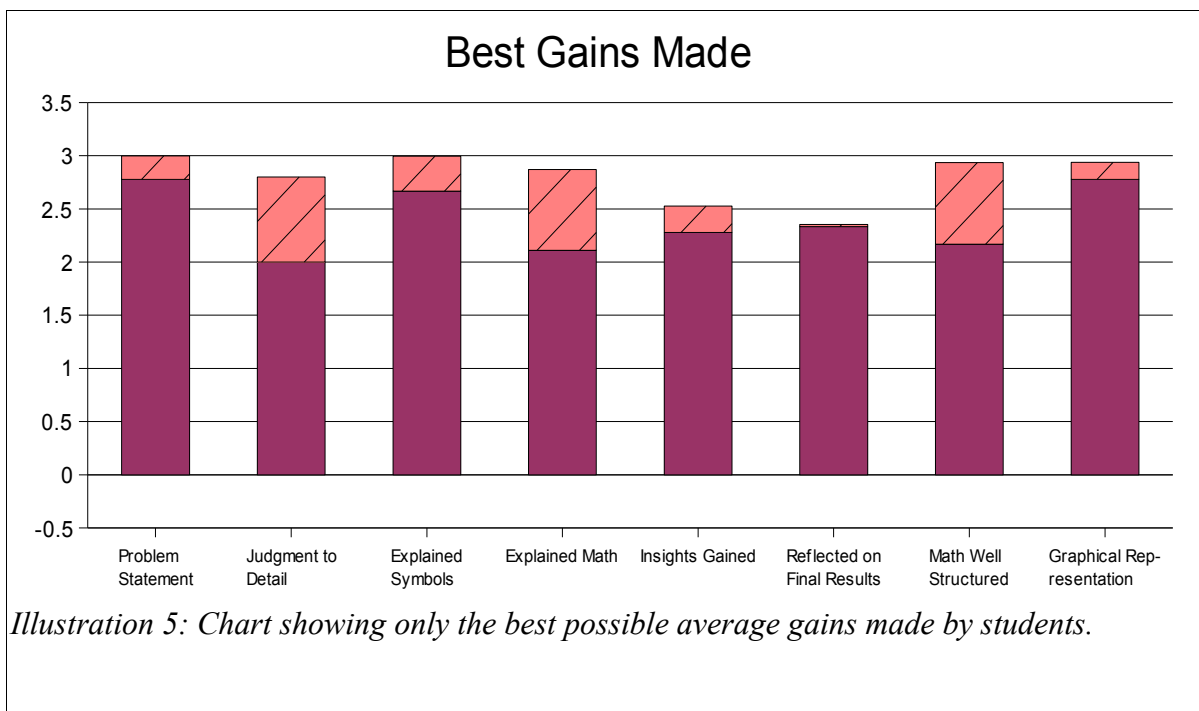


By taking the average of these changes, the following graphs are obtained. They reflect an average in the change from paper 1 to paper 2, and the change from paper 1 to paper 3.



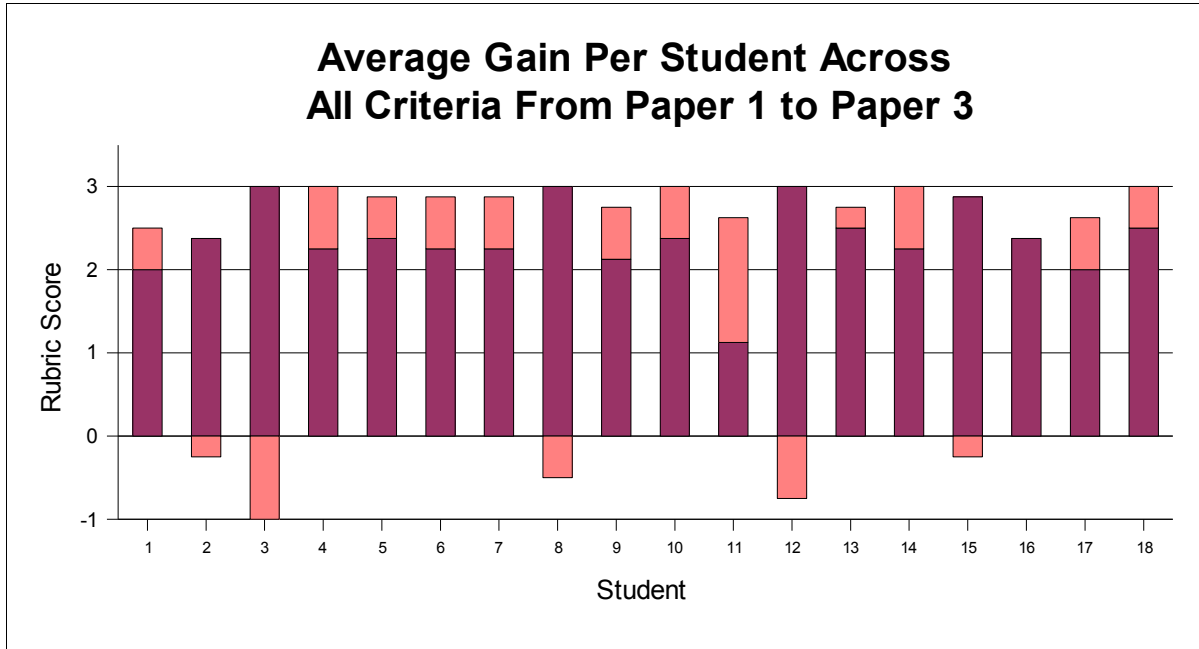
Because the 2nd and 3rd activities may have had conceptually more difficult material about which to write, an average of the two changes smooths out aberrations due to student misconceptions and poor understanding of the activity on which the writing was based. Note however that if there were improvement in a criteria from paper 2 to paper 3, this average will minimize the apparent overall gain in that criteria.

By giving students the benefit of the doubt and taking only the class' best gains, the following graph is obtained. This illustration may be useful if students were able to produce their best writing when they had better understanding of differing portions of the second and third activities.



By taking the average of all criteria scores for paper one per student and adding the average change in all criteria from paper 1 to paper 3 for each student, the following graph is obtained. This gives a good idea of how much each student changed individually; a score of

three on this graph represents perfect scores across all rubric criteria. Once again, a salmon portion in the negative portion of the axis represents a net loss instead of net gain.



## Exemplar Paper

As can be seen in the graphs above, most students made gains in being able to address rubric criteria successfully. Observing the graph of Average Scores for All Papers Across Given Criteria shows that the largest gains were in criteria 2, 4, and 10, each of which directly involve how stylistically well written the paper is. There are many writing samples that show improvement in the overall structure of the paper and writing style. Included in Appendix D are two papers from a student (Electric Dipole and Magnetic Vector Potential), the first and third

writing samples produced.

Note first the vast improvement in layout and formatting. This was a change unique to this student. There were other students who altered slightly the way they laid out sections and formatted their text, but only this student made any vast improvements. However, format changes aside, there were significant gains made in how well the content of the paper was structured.

### **Change in Professional Judgment to Detail**

To observe the change in professional judgment, consider the following excerpts from the first paper:

*In this problem we're dealing with electrostatic potential. It should be brought to attention before going into the problem that electrostatic potential is a scalar quantity and not a vector quantity.*

The author never clearly describes why it is important to consider this detail, and its inclusion does not benefit the reader's understanding of the problem or its intricacies.

*The length  $r$  is found by a simple application of the Pythagorean theorem which states that for any right triangle  $a^2 + b^2 = c^2$  where  $a$  and  $b$  are the opposite and adjacent sides of the triangle respectively and  $c$  is the hypotenuse.*

It would have sufficed to have simply stated that  $r$  was obtained using the

Pythagorean theorem; the application of the theorem is important to the reader, but the theorem itself will already be obvious.

The final writing sample produced by the student shows a marked improvement in professional judgment on how much detail to include in the write-up. Not only did the author exclude extraneous information like in the excerpts above, but the paper was also much more meticulous in describing every motivation of each step in the solution.

### **Change in Explanations of Mathematical Manipulations**

The explanation of mathematical manipulations were much better presented in the third paper. Consider the following excerpts taken directly after a mathematical manipulation in the first paper:

*Simply factor out a  $D^2$  out of the denominator to obtain*

*Next, separate the constants from the variables*

*In addition to separation of constants, take the denominator and change it to the numerator by making the power negative*

Compare them to the following excerpts from the final paper:

*Since we already know how to find the current  $\mathbf{I}$  in terms of  $\lambda$  and  $\mathbf{v}$  we can now simply plug in our values for  $\lambda$  and  $\mathbf{v}$  to derive an equation for the current in the ring.*



*Looking at figure 1 again, we need to find vector equations for the  $\mathbf{r}$  and  $\mathbf{r}_0$  vectors.*

*We know that at all points along the  $\mathbf{r}_0$  vector the  $z_0$  component is equal to zero. Thus the general equations for the two vectors in space are below*

In the mathematical steps from the final writing sample, each mathematical manipulation was motivated and justified. In the first paper the math that had been carried out was simply described in words. In this way, the writer improved significantly at describing the tricky math behind solving the problem.

### **Change in the Structuring of the Mathematics**

For this student, there were mathematical manipulations carried out within the text of the solution. However, there are multiple pages of disassociated mathematics attached at the end of the paper. In the first writing sample produced, students often included mathematical procedures completely separately from the rest of the solution in this fashion. The math of the problem was often sandwiched in between the introduction on the conclusion, or attached to the end with a reference somewhere in the paper reading “see attached calculations.” In the final paper produced by this student, the math seems like a part of the solution to the physics, rather than a disembodied section of its own.

## Other Changes

There is a lack of an “outline” section in the final paper that was present in the first. Outlining problem solving methods is an important part of successfully answering a question in physics, but it does not serve a purpose in the write-up of the solution itself. There were a significant amount of students who included such information which detracted from the purpose of the write-up in their first writing sample, but had removed such sections by their final write-up.

There was nearly a complete lack of analysis on the author's part in the first paper (save a few diagrams with short explanations on the last few pages) on what their answer told about the physics and nothing on what they had gained in solving the problem. In the final paper, however, there were paragraphs dedicated to analysis of the problem. This change was not typical in most students, as most students did not show evidence for change in their ability to analyze what the solution physically meant.

## Student Reflections

There was a general consensus amongst students that improvements in their ability to write occurred primarily in their organization, attention to detail, and their ability to explain

mathematical explanations. These three student-made observations correspond strongly with rubric criteria 2 and 4, which showed the most change using the quantitative data derived from the rubric. An interesting point to note is that even with the generality of the prompt (given in the Methodology Section) there was still fairly widespread consensus amongst the students as to how writing had improved. The fact that the prompt did not ask for them to identify specific changes through use of a survey of questions addressing each of the rubric criteria allowed students to reflect on any part of the process and how, in general, their writing had improved. Many of the students relied on the rubric criteria to identify change in their writing, but others did not.

# Discussion

This project was an ambitious undertaking, consisting of both curriculum development and education research components. From the start of the project, there was one overarching goal: create a writing unit that could be implemented at minimal instructor load that would utilize a peer review process to improve scientific writing and understanding of concepts learned in class. The tools and stages used in this research were modeled around the materials and processes of Calibrated Peer Review, specifically refined for use in a junior-level physics course.

It was found that the guiding questions themselves positively impacted the writing of many students in many of the rubric criteria. Simply outlining exactly what is important to include in a scientific paper resulted in decent attempts by the students to produce a well written paper. Over the course of the research, as students familiarized themselves with the rubric through review of both instructor created examples and their peers' work, student writing continued to improve. While this seems to attest to the fact that the rubric and peer evaluation themselves positively impacted the students' writing, other factors may have been at play. The simple task of having students write iteratively may have been a cause for improvement, as addressed by some of the students in their reflections. Also, students may have improved by virtue of the fact that they were engaging in a process that was new to them, not that there was anything inherently beneficial in the specific new process.

One goal of both our project and traditional Calibrated Peer Review is to be able to

assign writing tasks without having direct instructor feedback. In this project, students received no feedback whatsoever from either the instructor or project collaborators. This was an issue of concern for at least two of the students, and poses an interesting dilemma for future implementation and research of a similar unit. It would probably be most appropriate to combine the process of guiding questions, rubrics and peer evaluation with some level of direct instructor feedback. One suggestion is to have the students engage in the peer review process and have the instructor provide detailed feedback on the student writing in the final stage. This means that students have had a chance to grow in their ability to communicate, producing a more refined paper, before the instructor reads their work. It is important to note, however, that even without instructor feedback, there was improvement in the students' writing capabilities.

One interesting question to address in future research would be to examine separate groups, one where students use a rubric, guiding questions and peer review, one where students simply write iteratively with no instructor feedback, one where students write iteratively with instructor feedback, and one where students use the rubric, guiding questions, and peer review with instructor feedback during some point in the process. Such a research project could more accurately ascertain the impact of guiding questions, rubrics, and peer review.

# Conclusions

This project sought to improve not only students' ability to communicate science effectively, but to help further deepen understanding of physical concepts with minimal instructor load. By adapting Calibrated Peer Review and implementing it so that students wrote papers based on group activities completed during class, the project hoped to facilitate these goals. What was found was that not only did student writing noticeably improve in their attention to detail and their ability to explain the mathematics involved in solving the physics problem, but also that students in general were able to identify this change themselves. It was also discovered that explicit instructions outlining what should be included when writing about science, provided in the form of the guiding questions, were largely responsible for improvement over the previous year's students who had received very little instruction on what to write.

There is no overwhelming data to support that deeper understanding of the physics involved in writing the papers definitely occurred. However, while data showed no improvement in students' ability to convey an understanding of the physics of the problem, or to explain what was learned and which insights were gained, the lack of improvement does not necessarily mean that deeper understanding of the physics did not occur. Because each activity was conceptually more difficult than the previous, and the mathematics involved more complex, the constancy of the quantitative data acquired may attest to the fact that the project actually did facilitate a better understanding of the physics involved in each assignment. Furthermore, three of the seventeen students openly identified that a deeper

understanding of physical concepts had occurred over the course of the project.

Implementation of this project required intense effort on the part of project collaborators. Development of the guiding questions, rubric, writing prompts, data collection and analysis, and the structuring of the stages required a dedicated amount of time on the part of the course instructor. To achieve the goal of minimal instructor load for potential adopters of this writing unit, several considerations and revisions should be made. Data show that the guiding questions themselves had a positive impact on student writing, and the rubric and peer evaluation process were effective in facilitating certain change in student writing, attesting to the fact that they are both useful in the state that they are in. Question 8 of the guiding questions asks students to explain their mathematical manipulations. While the other criteria which showed improvement were more vaguely derived from the rubric, this question corresponds directly with a rubric criteria. Students were not addressing this question well in the beginning of the process, but by the final paper, this guiding question was addressed in a much more complete way, suggesting that the rubric and the peer review process played an important role in bettering students' ability to write.

Time could be taken to adapt these tools for specific course goals, but an interested instructor would not have to devote considerable time to such changes. Further work is needed to improve the online submission tools used in stages 2 and 4 to automate the feedback process. Ideally, students should be able to receive instant feedback on how well they scored instructor examples as well as instant feedback from their peers. The original CPR handled this extremely effectively; adopting the official CPR on line structure may be the answer to addressing these concerns. Criticisms from two students identified a lack of

instructor feedback as being a hindrance to their learning, so it may be impossible to maximize student learning while completely removing instructor feedback of papers.



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# Appendices

## Appendix A

# Guiding Questions for Physics Writing

1. State the problem.

*What is the problem that you are trying to solve, and what – if any – assumptions or idealizations are being made about the physical situation.*

2. Outline the general strategy.

*What physics concepts are relevant? Which general physical equations will be useful in solving this problem? Explain how the physical quantities are related to one another?*

*Connect the dots between any quantities in any ways that you can.*

3. Explain your terminology.

*What is the role of each of the symbols in these equations? For constants, just list their names and values if used in numerical calculations. For variables, briefly describe what they represent.*

4. Set-up your equations.

*How did you apply the information in your problem to the general equations? How did your example fit into and change the general equation. Think about how you went about putting in the information from the example you cared about, and any raw data taken, into the general equations.*

5. Explain any data taking procedures used in collecting information needed to solve the physical problem.

*Remember to include all pertinent information, including how to setup any apparatus used and detailed instructions on how data was acquired.*

6. Organize your data.

*List any raw data taken. Use graphs and charts to show concisely the relevant quantities in relationship to one another.*

7. Analyze your data.

*Explain how the data fits into the theory governing the problem you are solving. Comment on any unusual or anomalous data, providing an explanation of how it may have come about being recorded.*

8. What were the mathematical manipulations used in the process of solving the problem?

*Show the steps of algebra used to solve any tricky parts of the problem, write a short sentence for each explaining why they are true, and include any areas of difficulty that may have lead to dead ends.*

9. Reflect on your final answer.

*What is it that this answer tells you about the physical quantities involved, and how they are related to each other? Is this a limiting case, or are there limiting cases to this answer for which it is valid? Were there any better ways to solve the problem that you could consider? How did your solution compare and tie into work that others have done in this field of work? What was the most important, significant finding made in solving the problem?*

## **Appendix B**

# **Instructor Examples of Scientific Writing**

## Electrostatic Potential Due to a Ring of Charge (Code:1D)

The problem I was asked to solve was to find the electrostatic potential due to a ring of charge. I was told that the ring had a radius  $R$  and a total charge  $Q$ . In order to solve this problem I started with the general equation:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{q_i}{|\vec{r} - \vec{r}_i|} \quad (1)$$

Where  $q_i$  is the individual charge,  $|\vec{r} - \vec{r}_i|$  is the distance between the point we are measuring the potential at ( $\vec{r}$ ) and the charge ( $\vec{r}_i$ );  $\epsilon_0$  is the permittivity of free space.

Dr. Manogue gave us the next equation which was  $V$  for a linear charge distribution

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(r')|dr'|}{|\vec{r} - \vec{r}'|} \quad (2)$$

Where  $\vec{r}'$  is the position of the piece of charge, and  $|dr'|$  is the little distance used to integrate around the ring. I also knew the charge distribution was constant, so I had:

$$\lambda = \frac{Q}{2\pi R} \quad (3)$$

After plugging this into Eqn (2) I had:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{2\pi R} \int \frac{|dr'|}{|\vec{r} - \vec{r}'|} \quad (4)$$

I used cylindrical coordinates because of the geometry of the ring. In this system  $|dr'|$  becomes  $Rd\phi'$  and the limits of integration then become  $[0, 2\pi]$ . Applying this to Eqn (4) yeilds:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{2\pi} \int_0^{2\pi} \frac{d\phi'}{|\vec{r} - \vec{r}'|} \quad (5)$$

Because  $\vec{r}$  and  $\vec{r}'$  won't always point in the same direction, I needed to write them out explicitly. Using the solution from our homework assignment to write out  $|\vec{r} - \vec{r}'|$  in cartesian coordinates converted to polar components I had:

$$V(r, \phi, z) = \frac{1}{4\pi\epsilon_0} \frac{Q}{2\pi} \int_0^{2\pi} \frac{d\phi'}{\sqrt{(r^2 + R^2 + z^2 - 2rR \cos(\phi - \phi'))}} \quad (6)$$

This is an elliptic integral that can be evaluated numerically with computer software. I was then asked to find an expression for  $V$  along the  $z$ -axis. This makes  $r$  equal to 0 and Eqn (7) becomes:

$$V(r=0, \phi, z) = \frac{1}{4\pi\epsilon_0} \frac{Q}{2\pi} \int_0^{2\pi} \frac{d\phi'}{\sqrt{R^2 + z^2}} \quad (7)$$

This is easily integrable to give:

$$V(r=0, \phi, z) = \frac{Q}{4\pi\epsilon_0} \frac{1}{\sqrt{R^2 + z^2}} \quad (8)$$

I was then prompted to expand this in a power series to approximate  $V$  at points very close to zero. After recognizing that I needed to use the power series

$$(1 + c)^p = 1 + pc + \frac{p(p-1)}{2!}c^2 + \dots \quad (9)$$

I factored out an  $R$  from the denominator so that  $c \ll 1$ . I then had:

$$V = \frac{Q}{4\pi\epsilon_0 R} \left(1 + \frac{z^2}{R^2}\right)^{-\frac{1}{2}} \quad (10)$$

Using Eqn (10) and recognizing that  $p = -\frac{1}{2}$  and  $c = \frac{z^2}{R^2}$ , I obtained the following:

$$V(z) = \frac{Q}{4\pi\epsilon_0 R} \left(1 - \frac{z^2}{2R^2} + \frac{3z^4}{8R^4} + \dots\right) \quad (11)$$

I learned that applying information to get the equation you want is really hard, and that you have to know a lot of tricks or else you will get stuck along the way. I discovered that working in a group can also make things a lot easier, because up until this assignment I didn't have much difficulty with our group activities.

## Electrostatic Potential Due to a Ring of Charge (Code: 2D)

To solve this problem, I started with:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{q_i}{|\vec{r} - \vec{r}_i|} \quad (1)$$

This equation gives the electrostatic potential due to  $N$  point charges. In this equation,  $q_i$  represents the individual charges,  $|\vec{r} - \vec{r}_i|$  is the distance between the point we are measuring the potential at ( $\vec{r}$ ) and location of the charge ( $\vec{r}_i$ ) and  $\epsilon_0$  is the permittivity of free space. From this equation we can see that  $V$  is directly proportional to the amount of charge, and inversely proportional to the distance between  $\vec{r}$  and  $\vec{r}_i$ .

This equation took on a similar form for a linear charge distribution:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\vec{r}')|d\vec{r}'|}{|\vec{r} - \vec{r}'|} \quad (2)$$

The prime notation used here is a convenient way to denote variables that are related to the position of the charge. Thus,  $\vec{r}'$  is the position of the piece of charge, and  $|d\vec{r}'|$  is the little distance used to integrate around the ring. Since I was given the total charge and radius of the ring and told that it was a *constant* charge density, I had the following expression:

$$\lambda = \frac{Q}{2\pi R} \quad (3)$$

After plugging this into Eqn (2) I had:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{2\pi R} \int \frac{|d\vec{r}'|}{|\vec{r} - \vec{r}'|} \quad (4)$$

I used cylindrical coordinates because of the geometry of the ring. In this system  $|d\vec{r}'|$  becomes  $Rd\phi'$  and the limits of integration then become  $[0, 2\pi]$  to sum over the entire ring. Applying this to Eqn (4) yields:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{2\pi} \int_0^{2\pi} \frac{d\phi'}{|\vec{r} - \vec{r}'|} \quad (5)$$

It seemed to me that I was ready to integrate now, but because  $\vec{r}$  and  $\vec{r}'$  won't always point in the same direction, I needed to write them out explicitly. Using the solution from our homework assignment to write out  $|\vec{r} - \vec{r}'|$ 's components in cartesian form and converting them to in polar form, I had

$$V(r, \phi, z) = \frac{1}{4\pi\epsilon_0} \frac{Q}{2\pi} \int_0^{2\pi} \frac{d\phi'}{\sqrt{(r^2 + R^2 + z^2 - 2rR \cos(\phi - \phi'))}} \quad (6)$$

This is an integral that can't be solved by hand.

The next step was to set  $r = 0$  and Eqn (7) became:

$$V(r=0, \phi, z) = \frac{1}{4\pi\epsilon_0} \frac{Q}{2\pi} \int_0^{2\pi} \frac{d\phi'}{\sqrt{R^2 + z^2}} \quad (7)$$

I integrated to get:

$$V(r=0, \phi, z) = \frac{Q}{4\pi\epsilon_0} \frac{1}{\sqrt{R^2 + z^2}} \quad (8)$$

After recognizing that I needed to use the power series

$$(1 + c)^p = 1 + pc + \frac{p(p-1)}{2!}c^2 + \dots \quad (9)$$

I factored out an R from the denominator so that  $c \ll 1$ . I then had:

$$V = \frac{Q}{4\pi\epsilon_0 R} \left(1 + \frac{z^2}{R^2}\right)^{-\frac{1}{2}} \quad (10)$$

Using Eqn (10) and recognizing that  $p = -\frac{1}{2}$  and  $c = \frac{z^2}{R^2}$ , I obtained the following:

$$V(z) = \frac{Q}{4\pi\epsilon_0 R} \left(1 - \frac{z^2}{2R^2} + \frac{3z^4}{8R^4} + \dots\right) \quad (11)$$

I discovered that unless I focused on a specific axis, the simplest form of an expression can come as an unsolvable integral. I probably would not have recognized this at first. I also discovered that changing the position vectors into rectangular coordinates and then describing each of their rectangular components in polar form can allow for easier manipulation. After focusing



on the  $z$ -axis I saw that an otherwise difficult integral to calculate can become manageable. After expanding my solution in a power series that was familiar to me, I also saw that the electrostatic potential contained only even powers of  $z$ . The group that evaluated points far from 0 along the  $z$ -axis had an answer that was similar to mine, but with the  $z$  terms in the denominator and the  $R$  terms in the numerator.

## Electrostatic Potential Due to a Ring of Charge (Code:3D)

The problem I was asked to solve was to find the electrostatic potential due to a ring of charge. The ring had a radius  $R$  and a total charge  $Q$ .

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{q_i}{|\vec{r} - \vec{r}_i|} \quad (1)$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(r') |dr'|}{|\vec{r} - \vec{r}'|} \quad (2)$$

$$\lambda = \frac{Q}{2\pi R} \quad (3)$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{2\pi R} \int \frac{|dr'|}{|\vec{r} - \vec{r}'|} \quad (4)$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{2\pi} \int_0^{2\pi} \frac{d\phi'}{|\vec{r} - \vec{r}'|} \quad (5)$$

$$V(r, \phi, z) = \frac{1}{4\pi\epsilon_0} \frac{Q}{2\pi} \int_0^{2\pi} \frac{d\phi'}{\sqrt{(r^2 + R^2 + z^2 - 2rR \cos(\phi - \phi'))}} \quad (6)$$

$V$  along the  $z$ -axis:

$$V(r=0, \phi, z) = \frac{1}{4\pi\epsilon_0} \frac{Q}{2\pi} \int_0^{2\pi} \frac{d\phi'}{\sqrt{R^2 + z^2}} \quad (7)$$

$$V(r=0, \phi, z) = \frac{Q}{4\pi\epsilon_0} \frac{1}{\sqrt{R^2 + z^2}} \quad (8)$$

I was then prompted to expand this in a power series to approximate  $V$  at points very close to zero. After recognizing that I needed to use the power series

$$(1 + c)^p = 1 + pc + \frac{p(p-1)}{2!} c^2 + \dots \quad (9)$$

$$V = \frac{Q}{4\pi\epsilon_0 R} \left(1 + \frac{z^2}{R^2}\right)^{-\frac{1}{2}} \quad (10)$$

$$V(z) = \frac{Q}{4\pi\epsilon_0 R} \left( 1 - \frac{z^2}{2R^2} + \frac{3z^4}{8R^4} + \dots \right) \quad (11)$$

I discovered that unless I focused on a specific axis, the simplest form of an expression can come as an unsolvable integral. I probably would not have recognized this at first. I also discovered that changing the position vectors into rectangular coordinates and then describing each of their rectangular components in polar form can allow for easier manipulation. After focusing on the  $z$ -axis I saw that an otherwise difficult integral to calculate can become manageable. After expanding my solution in a power series that was familiar to me, I also saw that the electrostatic potential contained only even powers of  $z$ . After letting  $z$  approach infinity, the expression for potential became  $V(z) = \frac{Q}{4\pi\epsilon_0 R}$  which is the potential due to a point charge. This makes sense because as you get further and further away, the ring appears to vanish to a single point.

The group that evaluated points far from 0 along the  $z$ -axis had an answer that was similar to mine, but with the  $z$  terms in the denominator and the  $R$  terms in the numerator. The group that had evaluated the expression on the  $x, y$ -plane at points close to 0 had the expression  $V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{2\pi R} \left[ 2\pi + \frac{z^2}{2R^2} \right]$ . The group that evaluated it on this plane for points far outside the ring had a similar expression with the  $R$ 's and  $r$ 's swapped. If you look at all of these results collectively you will see that at points very far from the ring you approach the expression for electrostatic potential due to a point charge, and for the point at the center of the ring you get the electrostatic potential due to a point charge a distance  $R$  away with charge  $Q$ .

## Appendix C

# Rubrics

<b>Content Criterion</b>	<b>Very Good</b>	<b>Fair</b>	<b>Poor</b>
Did the writer have a clear, concise description of the problems being solved?	The problems being solved were included seamlessly within the write-up of the solution. Writer included a diagram when appropriate.	There was some mention of the problems being solved within the write-up, but they were organized poorly or were worded in a way that made them difficult to understand for the reader.	There was no attempt made at describing the problems being solved within the paper.
Did the writer use professional judgment on how much detail to provide in writing the solution to the physics problem?	Detail in the problem solving process was ample and not overly wordy.	There was a minor lack of detail in portions of the write up to the solution, or there were minor places where the problem solving process was too wordy or contained unnecessary information.	There was a major lack of detail in the explanation of the write up, or the problem solving process contained far too much information or was overly wordy.
Did the writer convey a complete understanding of the relationships and meanings in the symbols of the equations used in solving the problem?	Writer clearly presented the meaning of the symbols in each equation, including important relationships between them.	Some symbols were not clearly explained, and/or some relationships between them were omitted. Statements may not have been worded clearly due to writing style or poor sentence structuring.	The reader could not understand what many of the symbols represented in the equations, nor could an understanding of how they were related be reached. The writing style may have been very difficult to follow.
Did the writer completely explain tricky parts of the calculations, clearly explaining each mathematical manipulation carried out that wasn't algebraically trivial?	Calculations and mathematical manipulations were explained thoroughly so that the reader could follow each progression in the solution.	Tricky mathematical manipulations were not explained clearly so that the reader's understanding of the mathematical process was somewhat lacking. Most of the manipulations were explained thoroughly, but a few were either omitted or unclear.	Mathematical steps taken in reaching the solution were omitted entirely, or so incomplete that the reader could largely not follow the progressions made in reaching a solution.
Did the writer present data in a clear, efficient manner, explaining the relevance of the data to the problem solving process?	Data was clearly presented in a meaningful way that showed relevance of the physical quantities to one another. Any tables or graphs had clear labels, giving the reader a complete understanding of what quantities were involved and how they were related.	Data was wholly included, but arranged in such a way that it was not completely clear to the reader what quantities were involved, or how they were related to one another. For the most part it is obvious to the reader which quantities are being discussed, but properly labeled units or axes may have been omitted.	Data was arranged in such a way that the reader could not understand what quantities were being displayed, how they were related to one another, and in what units of measurement they were made.
Did the writer analyze their data, explaining how it fit in to the theory (or did not fit), and also give a reason for any anomalous data that had occurred?	Data was analyzed to show how it fit in with the theory or predicted model and is easily understandable to the reader. Plausible reasons were given for any anomalous data that had occurred.	The data analysis generally described how the particular findings fit into the predicted model or theory, but were lacking in an explanation of anomalous data or did not completely explain how the collected data differed from the expected model.	There was either no detailed analysis of the data presented, or the analysis was so lacking that it did not present any relevance to the theory behind the experiment or how it fit into a predicted model.
Did the writer explain what was learned or what insights were gained in solving this problem?	There was a complete statement of what was learned in answering the posed question, and why it was educational or important.	Writer mentioned a physics or mathematical concept learned, but did not clearly describe it.	The writer does not describe what was learned, or describes overly general things, such as, "Learned to work in a group."
Did the writer convey an understanding of what the final results tell about the physics?	Writer clearly explained what the final results tell about the physics of the problem and described what is physically interesting or unique about the solution to the problem.	An attempt is made to relate the mathematical manipulations to the physical concepts, but the physical situation is weakly related to these results.	The writer made no attempt at describing how their final solution related to the physical concepts.
Was the writer able to connect the solution to similar work done by others, tying together how the writer's efforts support and make contributions to the field?	The writer explained how their work was connected to other endeavors in the field, and how it contributed to the total scientific process. There was a good comparison and contrast between their own work and the work of other, similar physical problems	There was an attempt made at comparing and contrasting the work done by the writer to others, but it was either apparently lacking or not clearly worded such that the reader had difficulty in understanding how this particular endeavor fit in with others' work.	There was no attempt made at connecting the writer's work to others; there was no comparison made to the work of others.

Criteria Number	Content Criterion	Very Good	Fair	Poor
1	Did the writer have a clear, concise description of the problems being solved?	The problems being solved were included seamlessly within the write-up of the solution.	There was some mention of the problems being solved within the write-up, but they were organized poorly or were worded in a way that made them difficult to understand for the reader.	There was no attempt made at describing the problems being solved within the paper.
2	Did the writer use professional judgment on how much detail to provide in writing the solution to the physics problem?	Detail in the problem solving process was ample and not overly wordy.	There was a minor lack of detail in portions of the write up to the solution, or there were minor places where the problem solving process was too wordy or contained unnecessary information.	There was a major lack of detail in the explanation of the write up, or the problem solving process contained far too much information or was overly wordy.
3	Did the writer convey a complete understanding of the relationships and meanings in the symbols of the equations used in solving the problem?	Writer clearly presented the meaning of the symbols in each equation, including important relationships between them.	Some symbols were not clearly explained, and/or some relationships between them were omitted. Statements may not have been worded clearly due to somewhat poor writing style.	The reader could not understand what many of the symbols represented in the equations, nor could an understanding of how they were related be reached. The writing style may have been very difficult to follow.
4	Did the writer completely explain tricky parts of the calculations, clearly explaining each mathematical manipulation carried out that wasn't algebraically trivial?	Calculations and mathematical manipulations were explained thoroughly so that the reader could follow each progression in the solution.	Tricky mathematical manipulations were not explained clearly so that the reader's understanding of the mathematical process was somewhat lacking. Most of the manipulations were explained thoroughly, but a few were either omitted or unclear.	Mathematical steps taken in reaching the solution were omitted entirely, or so incomplete that the reader could largely not follow the progressions made in reaching the solution.
5	Did the writer present data in a clear, efficient manner, explaining the relevance of the data to the problem solving process?	Data were clearly presented in a meaningful way that showed relevance of the physical quantities to one another. Any tables or graphs had clear labels, giving the reader a complete understanding of what quantities were involved and how they were related.	Data was wholly included, but arranged in such a way that it was not completely clear to the reader what quantities were involved, or how they were related to one another. For the most part it is obvious to the reader which quantities are being discussed, but properly labeled units or axes may have been omitted.	Data was arranged in such a way that the reader could not understand what quantities were being displayed, how they were related to one another, and in what units of measurement they were made.
6	Did the writer analyze the data, explaining how it fit in to the theory (or did not fit), and also give a reason for any anomalous data that had occurred?	Data was analyzed to show how it fit in with the theory or predicted model and is easily understandable to the reader. Plausible reasons were given for any anomalous data that had occurred.	The data analysis generally described how the particular findings fit into the predicted model or theory, but were lacking in an explanation of anomalous data or did not completely explain how the collected data differed from the expected model.	There was either no detailed analysis of the data presented, or the analysis was so lacking that it did not present any relevance to the theory behind the experiment or how it fit into a predicted model.
7	Did the writer explain what was learned or what insights were gained in solving this problem?	There was a complete statement of what was learned in answering the posed question, and why it was educational or important.	Writer mentioned a physics or mathematical concept learned, but did not clearly describe it.	The writer does not describe what was learned, or describes overly general things, such as, "Learned to work in a group."
8	Did the writer convey an understanding of what the final results tell about the physics?	Writer clearly explained what the final results tell about the physics of the problem and described what is physically interesting or unique about the solution to the problem.	An attempt is made to relate the mathematical manipulations to the physical concepts, but the physical situation is weakly related to these results.	The writer made no attempt at describing how the final solution related to the physical concepts.
9	Was the writer able to connect the solution to similar work done by others, tying together how the writer's efforts support and make contributions to the field?	The writer explained how their work was connected to other endeavors in the field, and how it contributed to the total scientific process. There was a good comparison and contrast between their own work and the work of other, similar physical problems	There was an attempt made at comparing and contrasting the work done by the writer to others, but it was either lacking or not clearly expressed such that the reader had difficulty in understanding how this particular endeavor fit in with others' work.	There was no attempt made at connecting the writer's work to others; there was no comparison made to the work of others.
10	Were the mathematical steps included within the structuring of the paper, and did the writer technically proficient spelling and grammar?	Equations were part of the grammatical structure of the sentence. Words were spelled correctly and punctuation was used properly.	Paper contains grammar and spelling errors, but maintains the status of equations being a part of the sentence structuring.	Language used was very unclear and sentence structure was not in any way included along with mathematical procedures.
11	Was there a graphical representation of the problem?	The author included a diagram or drawing of the involved physical objects that included clearly labeled any involved variables and constants.	A graphical representation of the problem was included in the paper, but any involved quantities were not clearly labeled.	Author did not include a diagram at all, or any included diagram was disorganized and untidy to the point that it was neither understandable nor helpful.
12	Were the mathematical manipulations correct and physical reasoning valid?	There were no mathematical mistakes in the paper and all physical reasoning was logical and valid.	There were minor errors in the physical arguments or the mathematical processes, but the reader could understand the mistakes and fill in the gaps themselves.	The paper contained many incorrect mathematical manipulations and invalid physical arguments. The reader could not understand the writer's arguments because they were so incorrect.

## Appendix D

# Exemplar Papers

## Electric Potentials for Two Charges

Two charges  $+Q$  and  $+Q$  are placed on a line at  $x = -D$  and  $x = +D$  respectively. What is a fourth order approximation to the electric potential  $V(x,y)$  for  $x = 0$  and the magnitude of  $y$  much less than the distance  $D$ ? For what values of  $y$  does your series converge? For what values of  $y$  is your approximation a good one? Which direction would a test charge move under the influence of this electric potential?

The general steps in solving this problem are as follows

1. Draw a diagram containing the information given in the question
2. Recognize the problem type and choose equations accordingly
3. Set up equations for the problem including the superposition principle to reduce from two equations into a single equation
4. Convert the equations from the problem into equations that mirror common power series equations
5. Convert the equation for electrostatic potential from the problem into a power series expansion
6. Analyze the power series for ranges of convergence and divergence

In this problem we're dealing with electrostatic potential. It should be brought to attention before going into the problem that electrostatic potential is a scalar quantity and not a vector quantity. We will be using the formula for electrostatic potential ( $V$ ) and the superposition principle to solve the first few steps of this problem.

Super position principle as it applies to electrostatic potential states that the total potential ( $V_T$ ) is equal to the sum of the potentials involved ( $V_1 + V_2 + \dots$ ).

The formula for electrostatic potential is simply  $V=kQ/r$  where  $k$  is a constant with a value of  $(1/4\pi\epsilon_0)$ ,  $Q$  is in this case a point charge, and  $r$  is the distance between a test charge and the point charge  $Q$ . The total distance in this problem  $r = (D^2+y^2)^{1/2}$  where  $D$  is a fixed value and is the length along the  $x$  axis the charge is from the origin. (See Figure 1 for a schematic view of the problem). The length  $r$  is found by a simple application of the Pythagorean theorem which states that for any right triangle  $a^2 + b^2 = c^2$  where  $a$  and  $b$  are the opposite and adjacent sides of the triangle respectively and  $c$  is the hypotenuse.

A point charge located along the  $y$  axis will have two separate potentials acting upon it, however, due to the super position principle we can sum these potentials into a single equation for potentials in the problem.

$$V(Q_1) = kQ/((-D)^2 + y^2)^{0.5} \quad \text{and} \quad V(Q_2) = kQ/(D^2 + y^2)^{0.5}$$



Notice that the two scalar quantities  $V(Q_1)$  and  $V(Q_2)$  are equal since  $(-D)^2 = D^2$  so using the superposition principle we can now obtain an equation for  $V_T$

$$V_T = 2kQ/((D^2 + y^2)^{0.5})$$

Now that we have an equation for our electric potential in the problem we need to find a way to relate it to a power series. One way would be to simply create a power series for the form that  $V_T$  is in above. However, in this case it is easier to just manipulate the equation for  $V_T$  into the form of a familiar power series. In this case it is the power series for anything of the form  $(1 + Z)^p$  which can be found on the Common Power Series handout.

This power series converges for all  $|Z| < 1$  which comes in to play when we decide which variable to factor out of the denominator.

$V_T = 2kQ/((D^2 + y^2)^{0.5})$  Simply factor a  $D^2$  out of the denominator to obtain

$V_T = 2kQ/((D(1+(y^2/D^2)))^{0.5})$  Next, separate the constants from the variables

In addition to separation of constants, take the denominator and change it to the numerator by making the power negative

$$V_T = (2kQ/D)((1+(y^2/D^2))^{-0.5})$$

So now we clearly have a problem of the form  $C(1 + Z)^p$ , where  $C = 2kQ/D$   
By observation our  $p = -0.5$  and  $Z = (y^2/D^2)$

Now plug these values into the power series for  $(1 + Z)^p$

$$(1 + Z)^p =$$

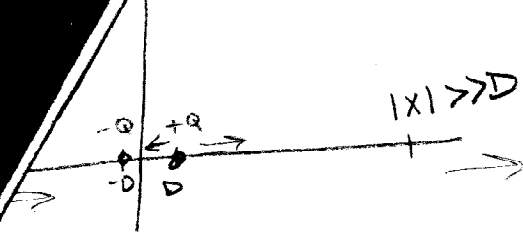
Now plugging in our values we obtain the following solution

Notice that our series converges for all  $|y^2/D^2| < 1$  and in our problem  $y \ll D$  therefore our series is completely convergent over the domain of our problem.  
Additional Problems in Electric Potentials for Two Charges (From Electric Potentials from two charges worksheet)

I.) **Problem one.**

Follow the same steps as in the first example. In this problem however, notice that the charges are only located along the  $x$ -axis and the potentials  $V(Q_1)$  and  $VQ_2$  vary with respect to  $x$  only. When you're setting up your equations you get expressions for distance along the  $x$ -axis in terms of  $x$ , and  $D$ . Simply apply superposition just as in the other problems and then convert to the same power series as before, ie  $(1 + Z)^p$ . You end up with the first equation on the PH320: Day 5 Handout.

Note: Effects of charges on the power series expansion.

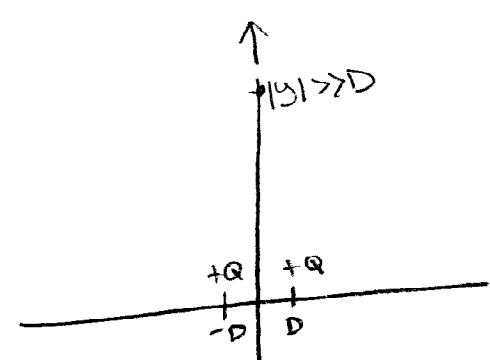


$$\frac{Q}{2\pi\epsilon_0 |x|} \left( 1 + \frac{D^2}{x^2} + \frac{D^4}{x^4} + \dots \right) \text{ for } \begin{matrix} |x| \gg D \\ y=0 \end{matrix}$$

Converges for all  $|x| \gg D$  since  $x$  is in the numerator

5. (MAIN PROBLEM - SEE TYPED)

7.



$$\frac{Q}{2\pi\epsilon_0 |y|} \left( 1 - \frac{1}{2} \frac{D^2}{y^2} + \frac{3}{8} \frac{D^4}{y^4} + \dots \right)$$

Converges for all  $|y| \gg D$  since  $y$  is in the denominator.

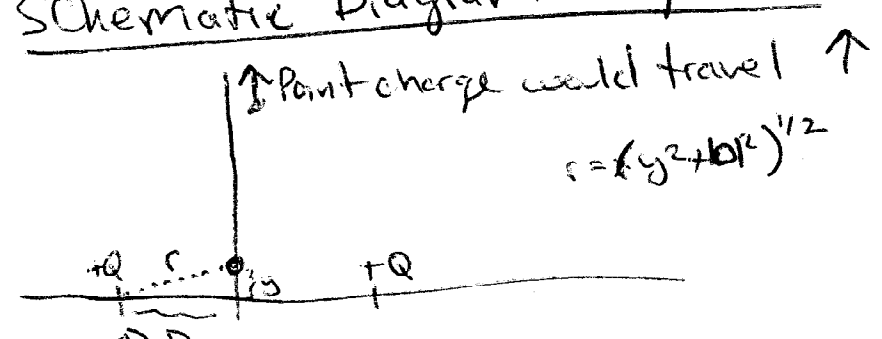
Note: Mirrors Problem 5

8. NOT SURE IF #8 was included

PROBLEM 5 Solution (See Printout for Details)

$$V(x,y,z) = \frac{Q}{2\pi\epsilon_0 D} \left( 1 - \frac{1}{2} \frac{y^2}{D^2} + \frac{3}{8} \frac{y^4}{D^4} + \dots \right) \text{ for } \begin{matrix} |y| \ll D \\ \text{and } x=0 \end{matrix}$$

Schematic Diagram Figure 1.



$$r = (y^2 + D^2)^{1/2}$$

Put Here

Find  $V_T$  4<sup>th</sup> order approximation for  $|x| \ll D$

$$V_T = V_1 + V_2 = \frac{kQ}{(D-x)} + \frac{kQ}{(D+x)}$$

⇒ Summation Similar To  $(1+z)^0$

After some algebraic manipulation

$$V_{(x,y,z)} = \frac{Q}{2\pi\epsilon_0 D} \left( 1 + \frac{x^2}{D^2} + \frac{x^4}{D^4} + \dots \right) \text{ for } |x| \ll D \text{ and } y=0$$

Series converges for all  $x$  since  $|x| \ll D$

for  $|x| \ll D$

Similar To Previous Example so I'm going to skip setting up  $V_T$  function. Note, the difference in charge (+/-)

and observe the differences between solution 1 and solution 2.

$$V_{(x,y,z)} = \frac{Q}{2\pi\epsilon_0 D} \left( \frac{x}{D} + \frac{x^3}{D^3} + \frac{x^5}{D^5} + \dots \right) \text{ for } |x| \ll D \text{ and } y=0$$

Series converges for all  $x$  since  $|x| \ll D$  is numerator

$$V_T = \frac{kQ}{(x-D)} + \frac{kQ}{(-x-D)}$$

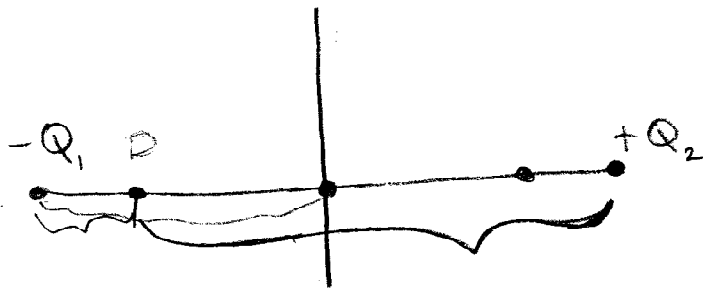
⇒ Series expansion

$$V_{(x,y,z)} = \frac{Q}{2\pi\epsilon_0 |x|} \left( \frac{D}{x} + \frac{D^3}{x^3} + \frac{D^5}{x^5} + \dots \right) \text{ for } x \gg D, y=0$$

Series converges for all  $x \gg D$  since  $x$  is in the denominator

\* Arrows Denote Direction at a given location A  $\oplus$  Test Charge Would Travel \*

the Power Series Expansion as above



Come Up with Equations for both

$$V(Q_1) = \frac{-Q \cdot K}{|(-x-D)|}$$

$$V(Q_2) = \frac{QK}{|(x-D)|}$$

Finding Equations for Potentials is the most difficult part.

∴ Sum The Potentials  $V(Q_1)$  and  $V(Q_2)$

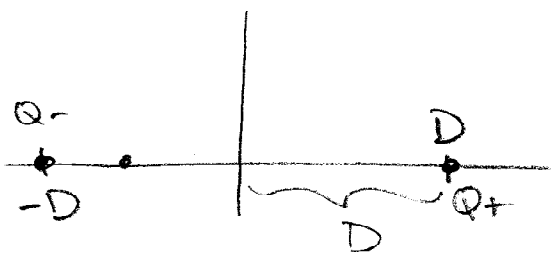
$$V_T = QK \left( \frac{-1}{|(-x-D)|} + \frac{1}{|(x-D)|} \right)$$

Now sum  $V(Q_1)$  and  $V(Q_2)$  expansions separately

To obtain  $\frac{Q}{4\pi\epsilon_0} \cdot \frac{2}{|x|} \left( 1 + \frac{D^2}{x^2} + \frac{D^4}{x^4} + \dots \right)$  for  $|x| > D$  and  $y=0$

3.) Exactly The Same as Problem Two with a different charge ( $Q+$ , and  $Q+$ ) which changes the series as follows (Work it out on your own using the previous guidelines. for  $|x| \gg D$  which lies to the left and right of your points respectively.

$$V(x,y,z) = \frac{Q}{4\pi\epsilon_0} \frac{2}{|x|} \left( \frac{D}{x} + \frac{D^3}{x^3} + \frac{D^5}{x^5} + \dots \right)$$
 for  $|x| > D$  and  $y=0$

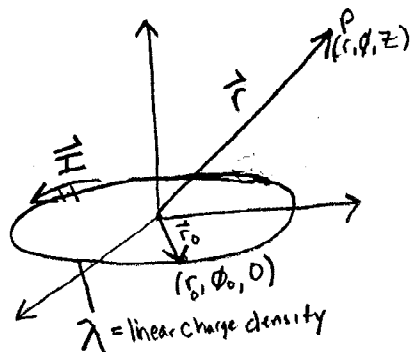


## Magnetic Field Due to a Ring of Charge

### Question

Calculate the magnetic vector potential  $\vec{A}$  due to a spinning ring of charge at some point  $P$  in space. Given that the ring has a constant linear charge density  $\lambda$ , a constant angular velocity  $\omega$ , and a constant tangential velocity  $\vec{v}$  for any point along the ring. The ring has a total charge  $Q$ . See Figure I for a schematic diagram of the problem. Simplify the solution into an equation that a mathematical program like Maple can compute.

Figure I: Ring of Charge in Space



### Solution

There is a right hand rule that states if you point your thumb in the direction of current around a ring and curl your fingers around the ring, your fingers will point in the direction of the magnetic field  $B$  induced by the current flowing through the wire. In this case we're given no direction of current, so I'm going to say that it is flowing in the counterclockwise (CCW) direction. Thus, the right hand rule for the ring of charge in our given coordinate system can be seen in figure II.

Figure II: Magnetic Field Lines due to Ring of Charge in Space



In general the equation for the magnetic vector potential  $\vec{A}$  is as follows,

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{I}(\vec{r}_0) |d\vec{r}_0|}{|\vec{r} - \vec{r}_0|}$$

The Current  $I$  for the ring is equal to the tangential velocity  $v$  multiplied by the charge per unit length of the ring  $\lambda$ .

$$\vec{I} = \lambda \vec{v} \quad (2)$$

The linear charge density of the ring  $\lambda$  can be found very easily in knowing

$$\lambda = \frac{Q}{2\pi r_0} \quad (3)$$

In other words, if it is known for some line of charge that the linear charge density is constant throughout then the linear charge density is simply the total charge  $Q$  on the ring divided over the length of the ring.

The velocity can be found with respect to some period  $T$  at constant angular momentum. Since velocity is simply the distance travelled per unit time and velocity is in the  $\hat{\phi}_0$  direction. Thus tangential velocity is simply the circumference of the ring divided by the period. Where the period  $T$  is the time it takes any point on the ring to complete a single revolution.

$$\vec{v} = \frac{2\pi r_0}{T} \hat{\phi}_0 \quad (4)$$

Since we already know how to find the current  $I$  in terms of  $\lambda$  and  $v$  we can now simply plug in our values for  $\lambda$  and  $v$  to derive an equation for the current in the ring.

$$\vec{I} = \lambda \vec{v} = \left(\frac{Q}{2\pi r_0}\right) \left(\frac{2\pi r_0}{T}\right) \hat{\phi}_0 = \frac{Q}{T} \hat{\phi}_0$$

The  $|d\vec{r}_0|$  component of our integrand can easily be obtained by symmetry arguments. In

cylindrical coordinates,

$$d\vec{r} = dr\hat{r} + r d\phi\hat{\phi} + dz\hat{z}$$

In the case of our current problem, as seen in figure I. The  $dr_0$  is zero because there is no change in  $r_0$  as the vector traces out every point on the ring. In addition the way we have our problem set up, the ring is strictly in the  $xy$ -plane which means  $dz_0$  is zero as well. Thus

$$d\vec{r}_0 = dr_0\hat{r} + r_0 d\phi_0\hat{\phi} + dz_0\hat{z} = 0 + r_0 d\phi_0\hat{\phi} + 0$$

And so our  $|d\vec{r}_0|$  component of the integrand is simply  $r_0 d\phi_0$ .

Looking at figure 1 again, we need to find vector equations for the  $\vec{r}$  and  $\vec{r}_0$  vectors. We know that at all points along the  $\vec{r}_0$  vector the  $z_0$  component is equal to zero. Thus the general equations for the two vectors in space are below

$$\vec{r} = r\hat{r} + r\phi\hat{\phi} + z\hat{z}$$

$$\vec{r}_0 = r_0\hat{r} + r_0\phi_0\hat{\phi} + 0\hat{z}$$

Now we can simply plug all of the parts we calculated into equation 1 as follows

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\left(\frac{Q}{T}\right)\hat{\phi}_0 r_0 d\phi_0}{(r^2 + r_0^2 - 2rr_0 \cos(\phi - \phi_0) + (z - z_0)^2)^{3/2}}$$

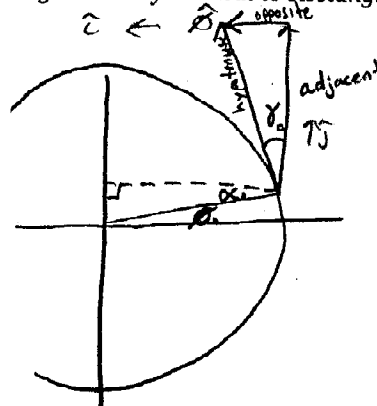
Note: The denominator is simply the solution of  $|\vec{r} - \vec{r}_0|$  in cylindrical coordinates which is as follows

$$|\vec{r} - \vec{r}_0| = \sqrt{r^2 + r_0^2 - 2rr_0 \cos(\phi - \phi_0) + (z - z_0)^2}$$

We can now simplify our problem by inserting things that we know about the problem. We know that  $r_0, Q$ , and  $T$  are all constants and can be treated like such throughout the problem. We also know that  $z_0$  is equal to zero. We incur a problem though. Even a mathematically calculating program like maple cannot compute this integral in the form we currently have it. A simple fix is to change our coordinate systems. We will convert from the cylindrical coordinate system to the rectangular coordinate system. The only thing we need to convert is the  $\hat{\phi}_0$  into rectangular coordinates. First we will draw a picture of what the  $\hat{\phi}_0$  direction looks like and

convert it to rectangular coordinates. In general a table that gives you this conversion can be found. We will rely on symmetry arguments however. If we draw some tangent line along the unit circle, arbitrarily choosing a small angle for  $\phi$  we can draw a line parallel to the  $x$  axis and perpendicular to the  $y$  axis that intersects our circle at the point  $M$ . We know the angle  $\alpha$  by the rule of opposite interior angles is equal to  $\phi$  (thus  $\phi = \alpha$ ). We then use geometry arguments to determine that the angle  $\gamma$  is also equal to  $\phi$  as well. So we have an end result of  $\phi = \alpha = \gamma$ . See Figure III below.

Figure III: Cylindrical to Rectangular



Solve for  $x$  and  $y$  components using the SOH CAH TOA rules

$$\cos(\phi) = \frac{a}{h} \quad \sin(\phi) = \frac{o}{h}$$

$$\tan(\phi) = \frac{o}{a}$$

$$\therefore -\sin(\phi)\hat{i} + \cos(\phi)\hat{j}$$

is the direction of  $\hat{\phi}$

We now take the  $x$  and  $y$  components of our  $\hat{\phi}_0$  direction vector. These point in the  $\hat{i}$  and  $\hat{j}$  rectangular vector directions respectively. Using simple trigonometry in the form of SOH CAH TOA we find these components to be as follows.

$$\hat{\phi}_0 = -\sin(\phi_0)\hat{i} + \cos(\phi_0)\hat{j}$$

Thus we obtain two much simpler integrals to compute. The final integral components plugging in all of the components we've solved for. The end result after plugging in our unknowns is as follows

$$\bar{A} = \frac{\mu_0}{4\pi} \int_0^{2\pi} \frac{\left(\frac{Q}{T}\right)((-\sin(\phi_0)\hat{i} + \cos(\phi_0)\hat{j})r_0 d\phi_0}{(r^2 + r_0^2 - 2rr_0 \cos(\phi - \phi_0) + (z)^2)^{1/2}}$$

This is an equation that a program designed to compute mathematical equations can obtain a solution to.

The magnetic vector potential decreases in size as the radius away from the ring in any given direction is increased. This makes physical sense because it is an analog of the magnetic field which decreases as the distance away from the ring of constant current is increased. If we analyze the integrand and say that it is for a point along the positive x axis a radius  $r$  away from the ring our expression simplifies down since our  $r$  components of our vectors are constants, our  $z$  is equal to zero, and we are left with two simple integrals, one in the  $x$  direction and one in the  $y$  direction as follows

$$\bar{A} = \frac{\mu_0}{4\pi} \int_0^{2\pi} \frac{\left(\frac{Q}{T}\right)(-\sin(\phi_0)r_0 d\phi_0}{(r^2 + r_0^2 + 2rr_0 \cos(\phi_0))^{1/2}} \hat{i}$$

And the other vector component of the magnetic potential along the  $x$  axis would be

$$\bar{A} = \frac{\mu_0}{4\pi} \int_0^{2\pi} \frac{\left(\frac{Q}{T}\right)\cos(\phi_0)r_0 d\phi_0}{(r^2 + r_0^2 + 2rr_0 \cos(\phi_0))^{1/2}} \hat{j}$$

Let Maple compute these integrals and it is simple to analyze the behavior of the magnetic potential due to a ring of current in the  $xy$ -plane along the  $x$  axis. If you perform similar tricks for a few different points in space you can see that the general behavior of the magnetic potential simply points in the  $\hat{\phi}_0$  direction at all points and is therefore tangent to the ring at all given points in space. To be clearer, the magnetic potential is tangent to the current flowing through the ring in a given direction. This makes physical sense because the magnetic field is perpendicular to the ring and the magnetic potential compared to the magnetic field is an analog of the electric potential compared to the electric field which is perpendicular.



## Appendix E

# Rubric Calibration Data

		Criteria 1	Criteria 2	Criteria 3	Criteria 4	Criteria 7	Criteria 8	Criteria 9	Average Deviation
Course Professor	1D	3	3	2	2	1	1	1	
Thesis Author	1D	3	3	3	3	1	1	1	
Postdoc Assoc.	1D	3	3	2	3	1	1	1	
Graduate Student	1D	2	3	3	3	1	1	1	
Average	1D	2.75	3	2.5	2.75	1	1	1	
Average (rounded to Integers)	1D	3	3	3	3	1	1	1	
<b>My Deviation from The Average</b>	<b>1D</b>	<b>0.25</b>	<b>0</b>	<b>0.5</b>	<b>0.25</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0.14</b>
<b>My Deviation from Integer Average</b>		<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>

Course Professor	2D	1	3	3	2	2	1	1	
Thesis Author	2D	1	3	3	3	3	2	2	
Postdoc Assoc.	2D	1	2	3	3	3	1	2	
Graduate Student	2D	1	3	2	3	3	1	2	
Average	2D	1	2.75	2.75	2.75	2.75	1.25	1.75	
Average (rounded to Integers)	2D	1	3	3	3	3	1	2	
<b>My Deviation from Average</b>	<b>2D</b>	<b>0</b>	<b>0.25</b>	<b>0.25</b>	<b>0.25</b>	<b>0.25</b>	<b>0.75</b>	<b>0.25</b>	<b>0.29</b>
<b>My Deviation from Integer Average</b>	<b>2D</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>0.14</b>

Course Professor	3D	3	2	1	1	3	3	3	
Thesis Author	3D	3	1	1	1	3	3	3	
Postdoc Assoc.	3D	2	1	1		3	3	3	
Graduate Student	3D	2	1	1	1	3	3	3	
Average	3D	2.5	1.25	1	1	3	3	3	
Average (rounded to Integers)	3D	3	1	1	1	3	3	3	
<b>My Deviation from Average</b>	<b>3D</b>	<b>0.5</b>	<b>-0.25</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0.04</b>
<b>My Deviation from Integer Average</b>	<b>3D</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>

<b>Average Deviation of Each Criteria</b>	<b>All Papers</b>	<b>0.25</b>	<b>0</b>	<b>0.25</b>	<b>0.17</b>	<b>0.08</b>	<b>0.25</b>	<b>0.08</b>	<b>0.15</b>
<b>Average Integer Deviation of Each Criteria</b>	<b>All Papers</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0.33</b>	<b>0</b>	<b>0.05</b>

**Rubric Evaluation Worksheet**

## Rubric Evaluation Worksheet

Your Name	<input type="text"/>
Code for Paper Reviewed	<input type="text"/>
1. Did the author have a clear description of the questions being asked?	
<input type="radio"/> Very Good <input type="radio"/> Fair <input type="radio"/> Poor <input type="radio"/> Not Applicable	
<input type="text"/>	
2. Did the author use professional judgement on how much detail to provide in writing the solution to the physics problem?	
<input type="radio"/> Very Good <input type="radio"/> Fair <input type="radio"/> Poor <input type="radio"/> Not Applicable	
<input type="text"/>	
3. Did the author convey a complete understanding of the relationships and meanings of the symbols in the equations used to solve the problem?	
<input type="radio"/> Very Good <input type="radio"/> Fair <input type="radio"/> Poor <input type="radio"/> Not Applicable	

--

4. Did the author completely explain any tricky parts of the calculations, clearly explaining each mathematical manipulation that was not algebraically trivial?

- Very Good       Fair       Poor       Not Applicable

--

5. Did the author present data in a clear and consistent manner, explaining the relevance of the data to the problem solving process?

- Very Good       Fair       Poor       Not Applicable

--

6. Did the author analyze their data, explaining how it fit (or did not fit) into the theory, and also give a reason for any anomalous data that occurred?

- Very Good       Fair       Poor       Not Applicable

--

7. Did the author explain what was learned or what insights were gained by solving this problem?

- Very Good       Fair       Poor       Not Applicable

--

8. Did the author convey an understanding of the physical implications (implications about physics) of the final results?

- Very Good       Fair       Poor       Not Applicable

--

9. Was the author able to connect the solution to similar work done by others, tying together how the author's efforts support and make contributions to the field?

- Very Good       Fair       Poor       Not Applicable

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