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BEHROOZ SATVAT for the degree of MASTER OF SCIENCE

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Title: RESIDENCE TIME DISTRIBUTION IN TUBULAR FLOW VESSELS WITH MULTIPLE BAFFLES

Abstract approved: Redacted for Privacy

Thomas J. Fitzgerald

The residence time distribution in a tubular flow vessel with multiple baffles has been investigated. Length of sections between the baffles, orifice diameter to vessel diameter ratio, and the flow rates were varied, with the flow always in the laminar regime.

Conductivity measurements, involving tracer impulse response, were used to evaluate the concentrations in three of the tubular vessels six sections. Laguerre polynomial approximation was used to smooth the time dependent concentration related curves.

A dependency on the flow conditions in the previous sections was found for all twelve cases studied. Accordingly, the individual residence time distributions were dependent upon each other; hence the individual residence time distributions did not convolute to give an overall residence time distribution and large deviation from plug flow within the tubular vessel existed.

RESIDENCE TIME DISTRIBUTION IN TUBULAR FLOW VESSELS WITH MULTIPLE BAFFLES

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INTRODUCTION

Plug flow is the flow for which all fluid elements have the same residence time distribution and there is negligible diffusion relative to the bulk flow. Fluid moves through the reactor with no mixing of earlier and later entering fluid. In other words, there is no backmixing of fluid elements of different ages.

Plug flow is desirable in most cases where chemical or nuclear reactors are used. For most flow reactions where high conversion and good product selectivity are needed, the optimal residence time distribution, (RTD), is that of plug flow, where all molecules spend the same amount of time in the vessel. Plug flow is also required in true countercurrent mass transfer and heat transfer.

In most cases using tubular vessels, circular baffles are used in order to prevent gross backmixing and to approach more closely the condition of plug flow. The effect of these baffles on total residence time distribution is not yet known. The main objective of this project was to study the effectiveness of baffles for improving the plug flow characteristics within tubular vessels.

L. J. Hellinckx et al. (1972) studied this problem for a flow system made up of consecutive compartments. They assumed that

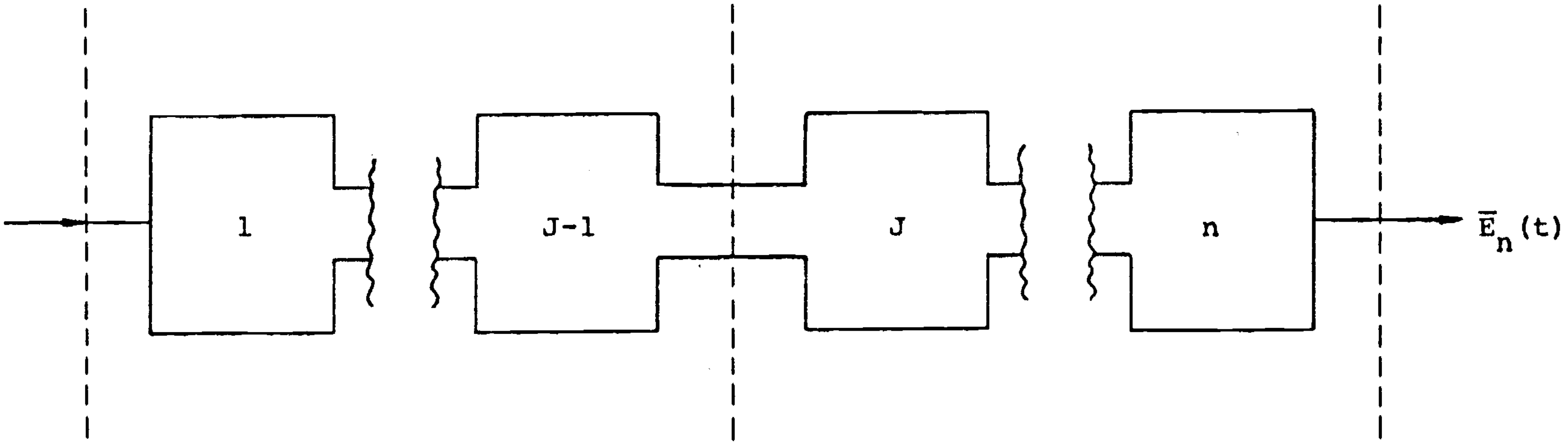

Figure 1. Hellinckx Experiment.

the consecutive flow regions of the system were independent.

They selected a flow system consisting of (n) successive flow regions as shown in Figure 1. For each flow region they defined a function $G_j(t)$ such that the $\overline{E}_j(t)$, the overall age distribution function in the effluent system j, is given as a function of $\overline{E}_{j-1}(t)$, the overall age distribution function in the effluent of system j-1, and of the function $G_j(t)$.

$$\overline{E}_j(t) = \int_o^t G_j(\theta)\overline{E}_{j-1}(t-\theta)d\theta$$

where

$G_j(\theta)$ = the age spectrum generating function.

This integral implies that G_j and $\overline{E}_{j-1}$ are independent; if they are not independent, then the best that can be done is as follows:

Suppose G* is the conditional probability that a molecule remains in the last section an amount of time t-λ, given that the same molecule spent a total time λ in the first (j-1) sections; accordingly,

$$\overline{E}_j(t) = \int_o^t \overline{E}_{j-1}(\lambda)G^*(t-\lambda,\lambda)d\lambda$$

Clearly G* (t-λ, λ) is a 2-place function depending on both (t-λ) and λ and can be integrated over λ but cannot be factored into the product of two independent probability density functions as indicated in Hellinckx article; therefore, Hellinckx approach does not appear to be correct.

S. Veeraraghavan and P. L. Silveston (1971) studied the influence of vessel dimensions and fluid velocity on resident time distribution. They found that the length to diameter ratio, l/D exerts the major influence on the RTD. At $l/D \leq 2.5$, the RTD resembles the response to a pulse signal of a well-stirred vessel with bypassing. Both Reynolds Number, N_{Re}, and nozzle to vessel diameter ratio, d/D, effect the RTD. At $l/D \geq 5.2$, N_{Re} and d/D were found to have negligible affect on the RTD. In this experiment, as will be explained in detail later, the first ratio used was

$$l/D = \frac{30 \text{ inches}}{4 \text{ inches}} = 7.50 \ ;$$

this is greater than 5.2. The second ratio used was

$$l/D = \frac{15 \text{ inches}}{4 \text{ inches}} = 3.75 \ ;$$

this is between the two limiting ratio values of 2.6 and 5.2.

Suppose there are two vessels which are connected to each other; further suppose it takes "t" seconds for a molecule of fluid to go through these two vessels. If each molecule remains τ seconds in the first vessel it will stay in the second vessel $(t-\tau)$ seconds. The only case in which it can be said that the overall residence time distribution is equal to the convolution of the RTD's in each vessel is when the residence time distribution in one vessel is independent of the RTD in the other. In this particular case:

$$E_{overall}(t) = \int_o^t E_1(\tau)E_2(t-\tau)d\tau$$

where:

$E_1(\tau)$ = residence time distribution in the first vessel

$E_2(t-\tau)$ = residence time distribution in the second vessel

$E_{overall}(t)$ = total residence time distribution.

Accordingly a series of n stirred tanks, each with an RTD of $1/T \exp(-t/T)$; has an overall RTD of $(t/T)^{n-1} \exp(-t/T)$, which is the n-fold convolution of $1/T \exp(-t/T)$ with itself. But if the residence time of a molecule in one vessel is dependent on its residence time distribution in a preceding one, then the convolution will not work. For example, in two adjacent sections of pipe through which the fluid flows as laminar streamlines, it can be seen that for a length of ΔL, the RTD will be

$$2\left(\frac{\Delta L}{V_c}\right)^2 \frac{1}{t^3} \cdot U_0\left(t-\frac{\Delta L}{V_c}\right);$$

for the total length of L, the RTD will be

$$2\left(\frac{L}{V_c}\right)^2 \frac{1}{t^3} \; U_0\left(t-\frac{L}{V_c}\right)$$

which is not the multiple convolution of

$$2\left(\frac{\Delta L}{V_c}\right)^2 \frac{1}{t^3} \cdot U_0\left(t-\frac{\Delta L}{V_c}\right) .$$

In this project, as will be discussed in detail later, the residence time distribution in each section will be found. It will then be tested to see if the residence time distributions convolute to each other.

EXPERIMENTAL EQUIPMENT

Water, supplied from a constant head tank located on the roof of the Chemical Engineering Building (about 40 ft above the ground), was fed to the test section in the basement of that building. Since a point probe was to be used in conductivity measurements in the investigation, the water had to be supplied free of dissolved air (which forms bubbles). Accordingly, previously heated water was used to fill the constant head tank. As shown in Figure 2, the hot feet water was cooled to the desired temperature in a countercurrent, tube and shell heat exchanger.

The test section was made of a cylindrical plexiglass tube which consisted of six sections, each separated from one another with baffles as shown in Figure 3. The desired flow rate was measured by a flow meter.

The slide valve which is shown in Figure 4 was used so that the tracer injected would be uniformly distributed into the flowing stream. When the valve was in position (A), sodium chloride tracer was injected into the hole at the top of the slide valve. After the slide valve is suddenly pushed to position (B), the tracer was forced into the tubular vessel by action of the flowing fluids. This action distributed the tracer uniformly into the moving stream.

For measuring the conductivity a platinized platinum (black)

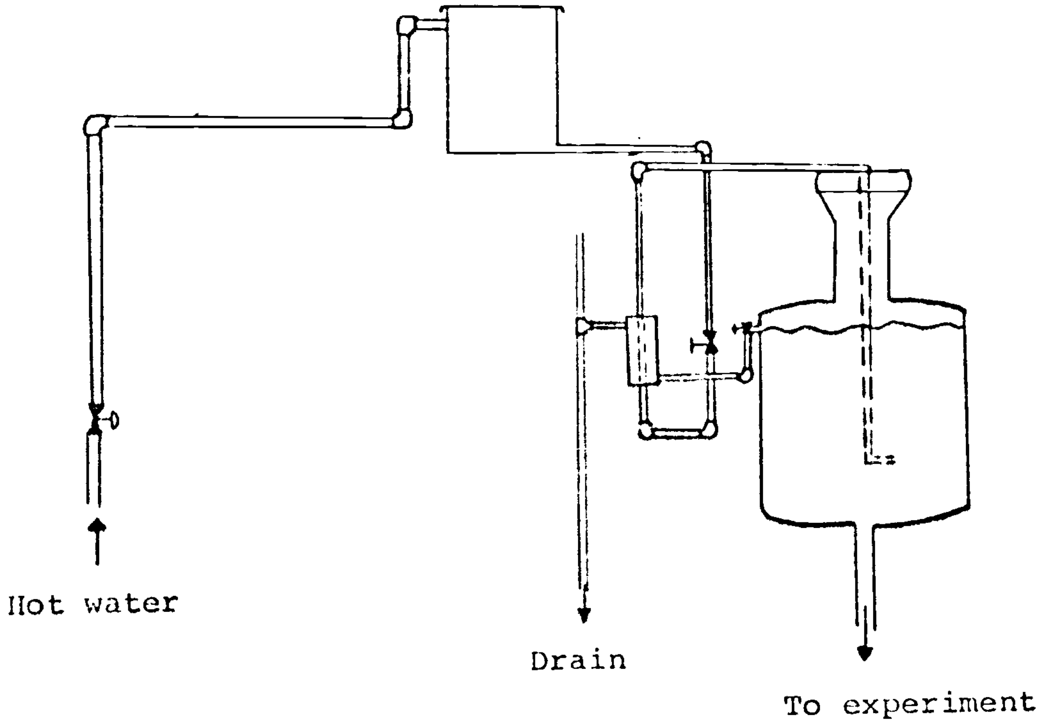

Figure 2. Constant Head Tank.

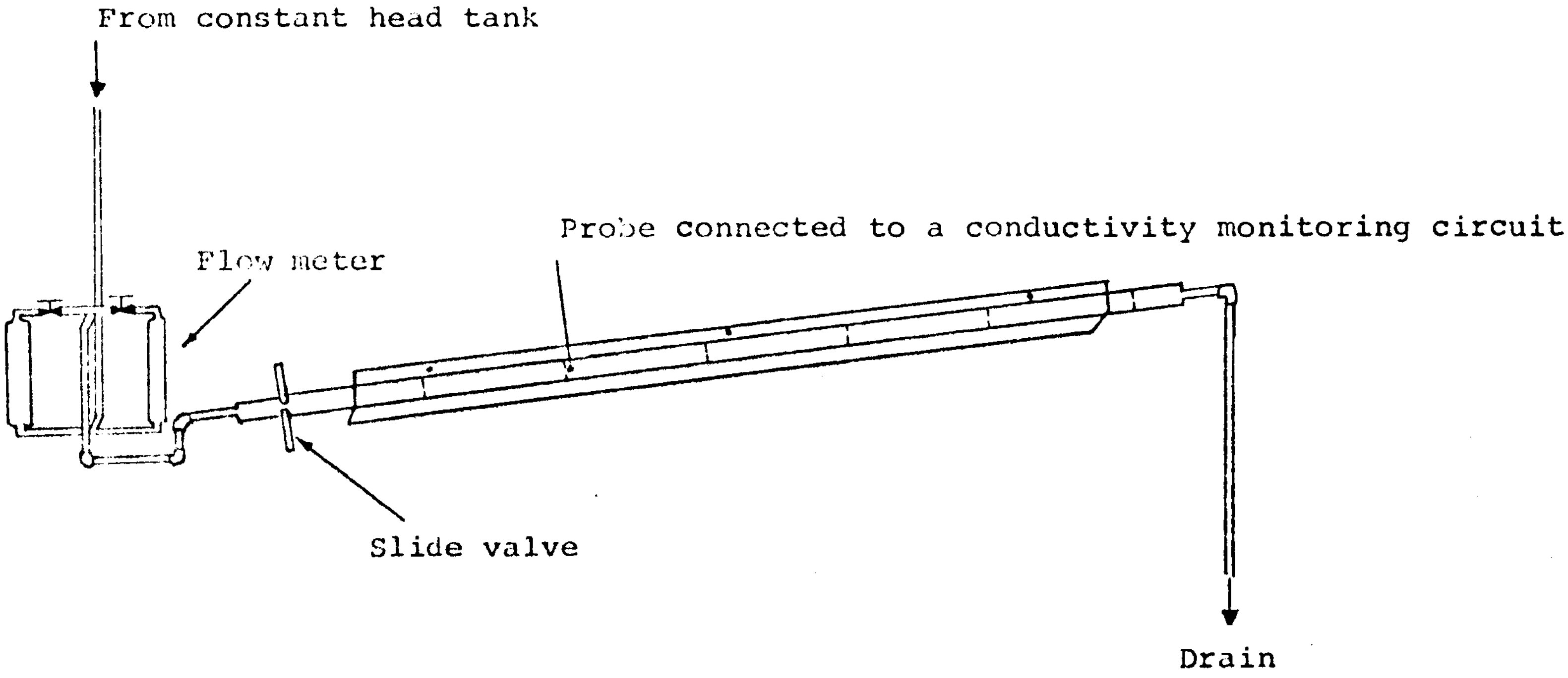

Figure 3. Test Section.

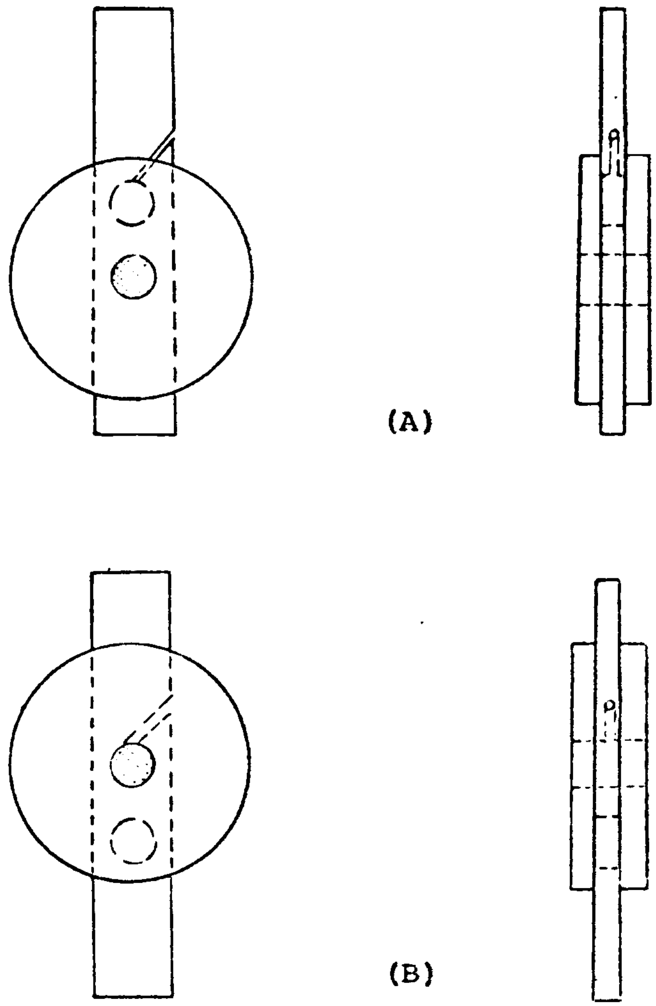

Figure 4. Slide Valve.

electrode was used. This method was introduced by Kohlrausch in the late 1890's for the measurement of the conductivity of electrolytic solutions (Geddes, 1972). Since Kohlrausch's time, the platinum-black electrode has been used with gaseous hydrogen as the standard reference electrode; it is still the most practical and accurate electrode for electrolytic conductivity cells because the electrode-electrolyte impedance is reduced to a very low value by the platinization process. The method advocated by Kohlrausch is still used to produce a platinum black deposit on platinum electrode. The electrode is first cleaned thoroughly and placed in a solution of 0.025N hydrochloric acid containing 3% platinum chloride and 0.025% lead acetate. Failure to include the lead acetate results in a stronger deposit having a velvety black appearance. A direct current is passed through the solution via a large-area platinum anode, and the electrode to be blackened is made the cathode. In these cases both cathode and anode are made of platinum.

A twenty-thousandths inch diameter platinum wire which had been cut into a five-inch length was used to make the conductivity probe. The step-by-step production of the probe is illustrated in Figure 5.

The conductivity probe used in this experiment measured the potential difference when ions were transferred through the water flowing in the tubular vessels. This voltage difference was

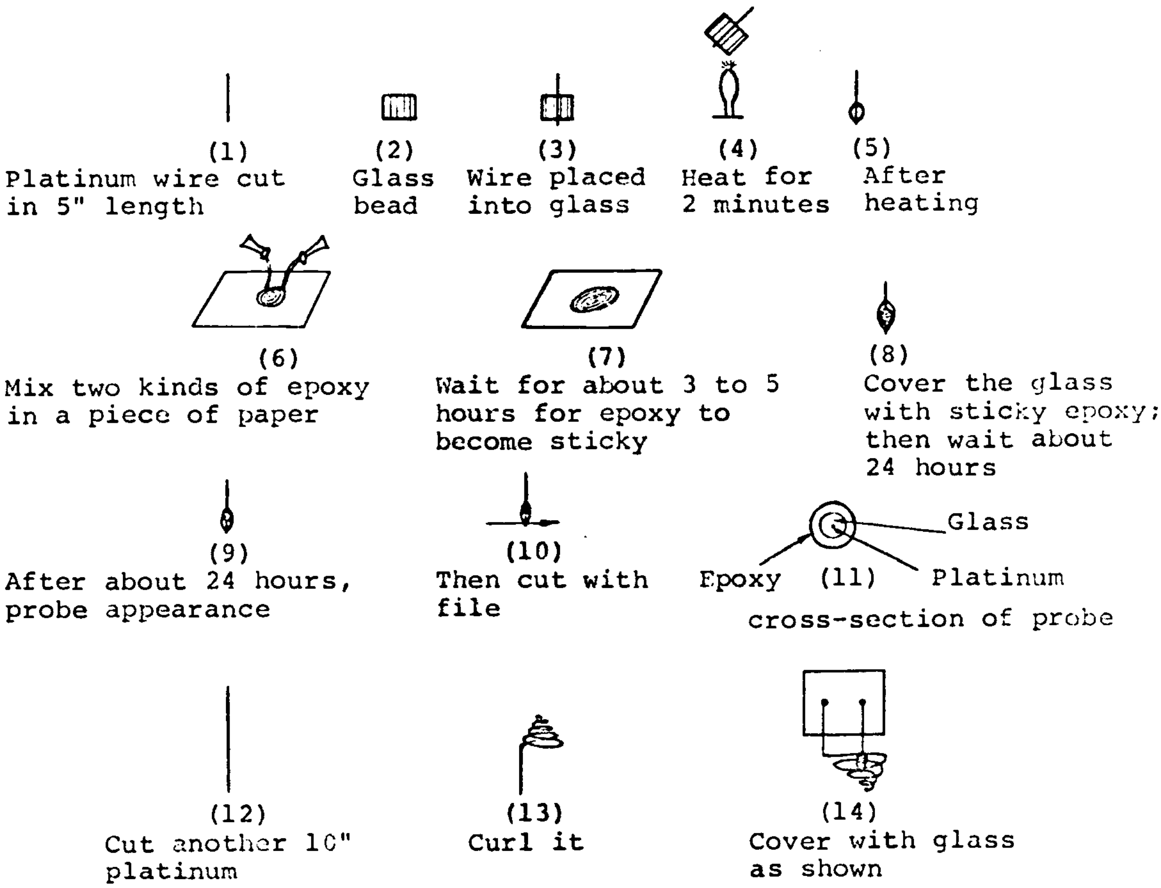

Figure 5. Steps Required to Make Probe.

directly related to the concentration of sodium chloride tracer.

By using a point probe the conductivity can be measured at a point. Since the surface area of a point electrode is much smaller than that of a wire ring electrode, the current flow, and thus the conductance through the probe, will be proportional to the concentration near the point electrode (Khang, 1975). The fluid conductivity near the coiled wire has very little effect on the current flow while the conductance of the fluid close to the point electrode will affect the current flow through the probe.

The putput signal from the amplifier, Number 1 in Figure 6, was amplitude modulated with a carrier frequency given by the sine wave signal generator. The carrier frequency was much higher than the frequencies of the concentration signal. Since the output signal picked up noises whose frequencies were higher and lower than the carrier frequency, a band pass filter was used to block frequencies which were much higher or much lower than the carrier frequency.

Because a single diode produces an offset which is appreciable when input voltage is small, an operational amplifier with two diodes of the same characteristic was used as a precision rectifier.

The output signal still had high frequency components associated with the carrier frequency. By using a low pass filter the high frequency components were removed without adding a

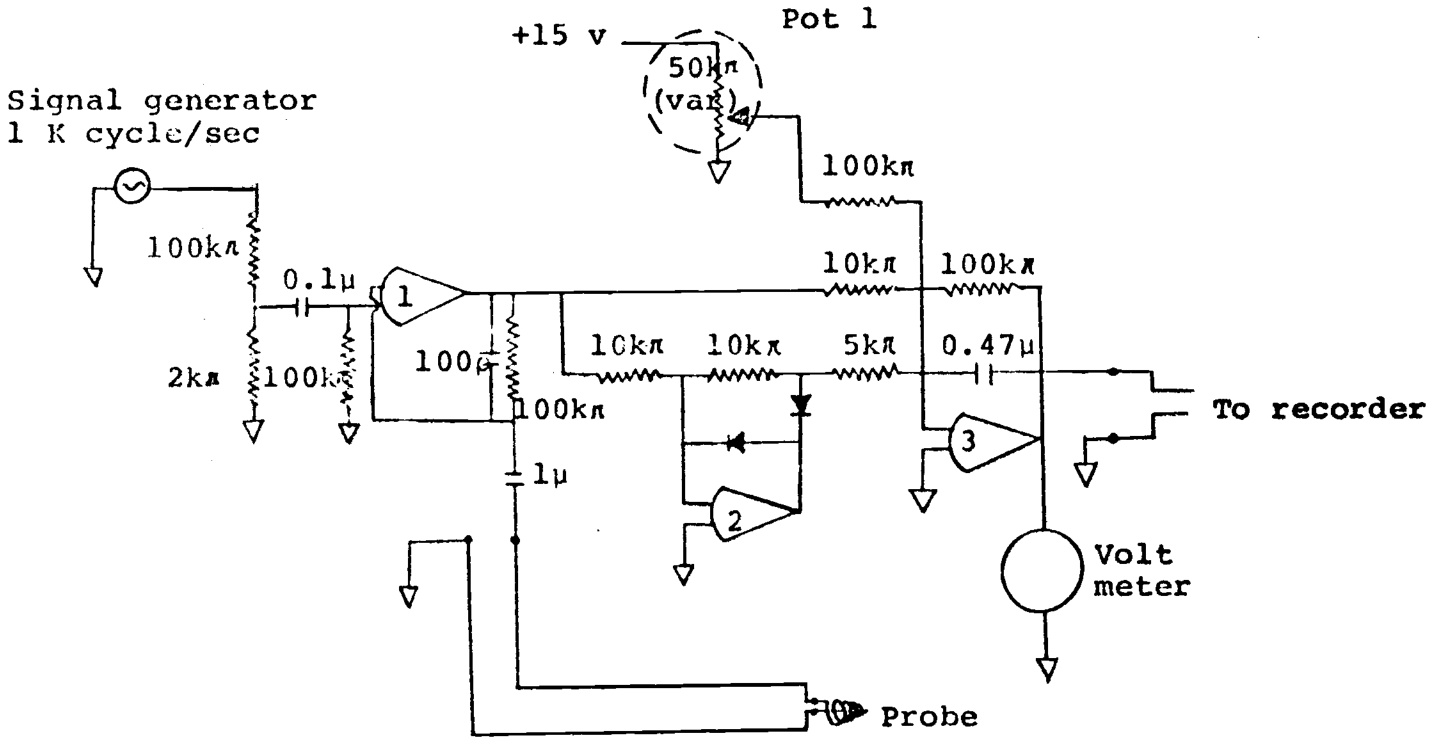

Figure 6. Conductivity Measuring Circuit.

significant time constant to the system. In this experiment the signal generator has an amplitude of 2 volts and a frequency 1 of KHz.

THEORY OF CALCULATIONS

The conductivity was measured at three different points after the first, second, and fifth sections. This permitted the evaluation of the residence time distribution functions after the first, second, and fifth sections.

After finding the tracer output, plots were obtained (for which the equations were unknown to the experimenter). Since there were noises on the curves, it was decided to smooth each of the curves by using Laguerre polynomial.

Approximate each impulse response curve, F(t), by

$$F(t) = A_0L_0 + A_1L_1 + A_2L_2 + A_3L_3 + A_4L_4 + A_5L_5 \cdot \cdot \cdot$$

where L_0, L_1, L_2, L_3, L_4, L_5. . . are orthonormal. For finding A_0 both sides of this equation were multiplied by L_0.

$$L_0F(t) = A_0L_0^2 + A_1L_0L_1 + A_2L_0L_2 + A_3L_0L_3 + \cdot \cdot \cdot$$

By taking an indefinite integral of both sides of the equation, one obtains

$$\int_0^\infty L_0F(t)dt = \int_0^\infty A_0L_0^2dt + \int_0^\infty A_1L_0L_1dt + \int_0^\infty A_2L_0L_2dt + \cdot \cdot \cdot$$

The property of orthonormality reduces this equation to

$$\int_0^\infty L_0 F(t)dt = A_0$$

By the same method, A_1, A_2, . . . A_n can be determined.

$$A_0 = \int_0^\infty L_0(t)F(t)dt$$

$$A_1 = \int_0^\infty L_1(t)F(t)dt$$

$$A_2 = \int_0^\infty L_2(t)F(t)dt$$

$$A_3 = \int_0^\infty L_3(t)F(t)dt$$

$$A_4 = \int_0^\infty L_4(t)F(t)dt$$

$$A_5 = \int_0^\infty L_5(t)F(t)dt$$

According to Solodovnikov (1960), each of the time dependent Laguerre functions can be expressed by:

$$L_v(t) = e^{-t}\left[\frac{2^{v+1/2}t^v}{v!} - \frac{2^{v-1/2}v\,t^{v-1}}{(v-1)!} + \ldots + 2^{v-k+1/2}\right.$$

$$\left.\frac{v!\,(-1)^k}{k![(v-k)!]^2}\, t^{v-k} + \ldots + 2^{1/2}(-1)^v\right]$$

where v equates 0, 1, 2. . . .

This means:

$$L_0(t) = e^{-t} \quad \frac{2^{1/2}}{1} \quad = \sqrt{2}\, e^{-t}$$

$$L_1(t) = e^{-t} \quad \frac{2^{3/2}t}{1} - \frac{2^{1/2}}{1} \quad = (2\sqrt{2}t - \sqrt{2})e^{-t}$$

$$L_2(t) = e^{-t} \quad \frac{2^{5/2}t^2}{2} - \frac{2^{3/2}(2)t}{1} + \frac{2^{1/2}(2)}{2}$$

$$= \quad (4\sqrt{2}\ \frac{t^2}{2} - 4\frac{\sqrt{2}t}{1} + \sqrt{2}\ )\ e^{-t}$$

$$L_3(t) = e^{-t} \quad \frac{2^{7/2}t^3}{6} - \frac{2^{5/2}(3)}{2}\ t^2 + \frac{2^{3/2}(3)}{1}\ t - \frac{2^{1/2}}{1}$$

$$= (\frac{8\sqrt{2}}{6}\ t^3 - \frac{12\sqrt{2}}{2}\ t^2 + 6\sqrt{2}t - \sqrt{2})e^{-t}$$

$$L_4(t) = e^{-t} \quad \frac{2^{9/2}t^4}{24} - \frac{2^{7/2}(4)}{6}\ t^3 + \frac{2^{5/2}(6)}{2}\ t^2 - \frac{2^{3/2}(4)}{1}\ t$$

$$+ \frac{2^{1/2}}{1}$$

$$= \frac{16\sqrt{2}t^4}{24} - \frac{32\sqrt{2}t^3}{6} + \frac{24\sqrt{2}t^2}{2} - \frac{8\sqrt{2}t}{1} + \sqrt{2})e^{-t}$$

$$L_5(t) = e^{-t} \quad \frac{2^{11/2}}{120}\ t^5 - \frac{2^{9/2}(5)}{24}\ t^4 + \frac{2^{7/2}(10)}{6}\ t^3 - \frac{2^{5/2}(10)}{2}\ t^2$$

$$+ \frac{2^{3/2}(5)t}{1} - 2^{1/2}$$

$$= (\frac{32\sqrt{2}t^5}{120} - \frac{80\sqrt{2}t^4}{24} + \frac{80\sqrt{2}t^3}{6} - \frac{40\sqrt{2}t^2}{2} + 10\sqrt{2}t - \sqrt{2})e^{-t}$$

By knowing F(t) data from the experiment and knowing L_1, L_2, L_3, . . . Laguerre polynomial coefficients can be obtained by numerical integration. This provides all of the terms of the Laguerre polynomial:

$$F(t) = A_0 L_0 + A_1 L_1 + A_2 L_2 + \ . \ . \ .$$

Accordingly, the Laguerre polynomial approximation can be obtained for each of the three measured sections of the test tubular vessel.

After finding these equations one can find their Laplace transforms

In general:

$$\int [L_v(t)] = L_v(S) = \sqrt{2} \ \frac{(1-S)^v}{(1+S)^{v+1}}$$

so that:

$$L_0(S) = \frac{\sqrt{2}\,(1-S)^0}{(1+S)^1} = \frac{\sqrt{2}}{(1+S)}$$

$$L_1(S) = \frac{\sqrt{2}\,(1-S)}{(1+S)^2}$$

$$L_2(S) = \frac{\sqrt{s}\,(1-S)^2}{(1+S)^3}$$

$$L_3(S) = \frac{\sqrt{2}\,(1-S)^3}{(1+S)^4}$$

$$L_4(S) = \frac{\sqrt{2}(1-S)^4}{(1+S)^5}$$

$$L_5(S) = \frac{\sqrt{2}\,(1-S)^5}{(1+S)^6}$$

By knowing the measured numerical points, $F(t)$, and the evaluated $L_1(t)$, $L_2(t)$, . . .

$$\int_0^\infty F(t)L_1(t)dt \quad \text{and} \quad \int_0^\infty F(t)L_2(t)dt, \; . \; . \; .$$

were numerically integrated using a newly written computer program. This computer program is given in Appendix A.

ANALYZING THE DATA

Water from the constant-head tank was introduced into the test section at a predetermined flow rate. After the system had reached steady state, a tracer containing 15 grams of NaCl per 200 cubic centimeters of water was injected into the hole provided in the slide valve. This tracer was instantaneously distributed uniformly into the flowing water stream by pushing the slide valve into its second position. By use of the probe which was installed inside the pipe, the conductivity of the water was measured and recorded on a plotter as a plot of the potential difference, v, versus the time, t.

Digitizing produced a set of numbers (almost 50 points per inch). By using the Laguerre function, the curve was smoothed; this resulted in a set of graphs as shown in Figures 10 through 15. To find the convolution integral of two of these graphs the Laplace transform of each graph was taken and then multiplied by the transform of the other graph.

Twelve sets of data were experimentally measured to include two different lengths of individualized sections, two different orifice diameter to tube diameter ratio and three different flow rates.

Table 1 summarizes the experimental parameters for each set of data.

In the digitizing, 50 points per inch were read and the data

Table 1. Summary of Experimental Runs.

Set	Percent of Maximum Flow	Flow Rate (cm^3/sec)	Length of Each sec. (cm)	Diameter of Orifices (cm)	Re Orifice	Re Vessel
A	100%	70	76.2	2.54	4365	1100
C	70%	49	76.2	2.54	3050	765
B	40%	28	76.2	2.54	1745	436
D	100%	70	38.1	2.54	4365	1100
E	70%	49	38.1	2.54	3050	765
F	40%	28	38.1	2.54	1745	436
G	100%	70	76.2	1.27	8727	1100
H	70%	49	76.2	1.27	6100	765
I	40%	28	76.2	1.27	3490	436
K	100%	70	38.1	1.27	8727	1100
L	70%	49	38.1	1.27	6100	765
M	40%	28	38.1	1.27	3490	436

points were stored (for example, set B was stored in *CHEMB). Then by using computer programs "LISTER" and "EDITOR", "GRAPH1B" (data points for Section 1, set B), "GRAPH 2B" (data points for Section 2, Set B) and "GRAPH 5B" (data points for Section 5, Set B) were obtained. Program "LISTER" put a counter for each number. Program "EDITOR" separated the data for each section and put the data for each section in a special file. Both computer programs are given in Appendix A.

To illustrate the calculations, consider Set B:

Set B:	pipe diameter	10.16 cm = 4 inches
Section 1	baffle orifice diameter	2.54 cm = 1 inch
	length of each section	76.2 cm = 30 inches
	40% flow rate	

v = 40% flow rate = 28 cubic centimeters/sec

V = volume of each section = $(\pi D^2/4)L$

$$V = \frac{3.141593}{4}(4^2)\,30 = 3.769911 \times 10^2 \text{ inch}^3$$

or $$V = (3.769911 \times 10^2)(2.54)^3 = 6178 \text{ cm}^3$$

$$\frac{V}{v} = \frac{6178}{28} = 220.64 \text{ sec}$$

N for $\frac{V}{v}$ equal to 220.64 = 551, where N is the equivalent digitized number

$$\theta = \frac{N - N_h}{N_{\frac{V}{v}} - N_h}$$

$$M = N - N_h$$

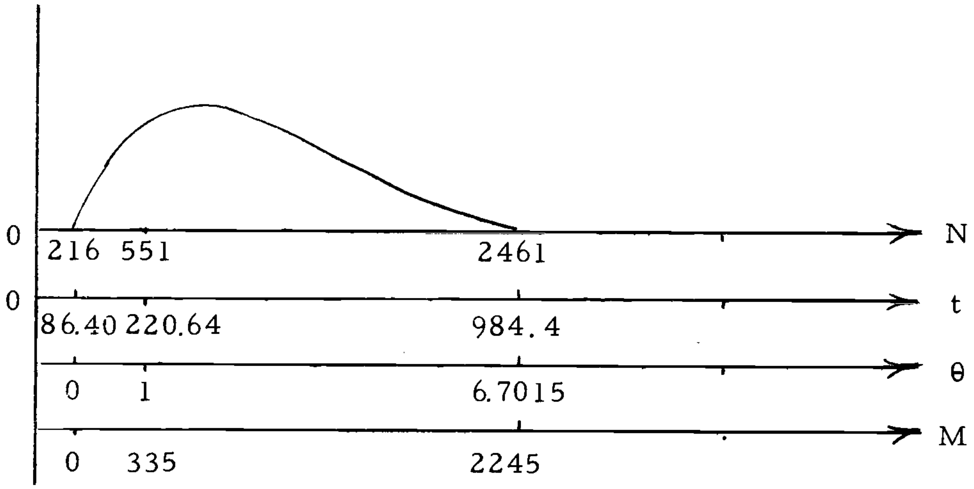

Figure 7. Graphical Presentation of Data Set B, Section 1.

If h_1 is used to designate the dead time, or time after the tracer is first introduced before the first voltage response, the plot indicates:

h_1 = 86.40 sec

N_{h_1} = 216 digitized points

$\bar{t}$ = time of backmixing = $\frac{V}{v} - h_1$

$\bar{t}$ = 220.64-86.40 = 134.24 sec

N for $\bar{t}$ = 335 digitized points

With this information a computer program "SATVAT", was used to obtain A_0, A_1, ... , A_5 for the Laguerre function. Program "SATVAT" involved a trapezoidal rule which calculated

$$A_0 = \int_0^\infty F(t)L_0(t)dt,\ A_1 = \int_0^\infty F(t)L_1(t)dt, \ldots\ldots,$$

$$A_5 = \int_0^\infty F(t)L_5(t)dt \ .$$

This computer program is located in Appendix A with the other programs used in this study.

Set B:

Section 2

$\frac{V}{v}$ = 220.64 Sec

$\frac{2V}{v}$ = 441.28 Sec

N for $\frac{2V}{v}$ = 1103 digitized points from the plot obtained by experiment.

h_2 = 199.20 Sec

N_{h_2} = 498 digitized points

$\bar{t}$ = 441.28-199.20 = 242.08 Sec

N for $\bar{t}$ = 605 digitized points

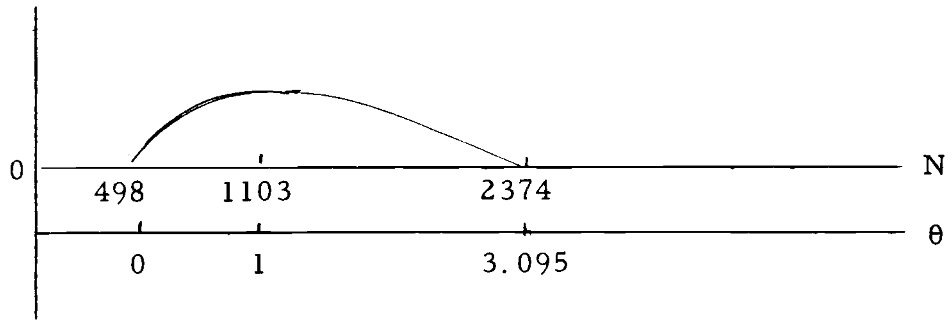

Figure 8. Graphical Presentation of Data Set B, Section 2

Set B:

Section 5

$\frac{V}{v}$ = 220.64 Sec

$\frac{5V}{v}$ = 1103.20 Sec

N for $\frac{5V}{v}$ = 2758 digitized points

h_5 = 603.20 Sec

N_{h_5} = 1508 digitized points

$\bar{t}$ = 1103.20-603.20 = 500 Sec

N for $\bar{t}$ = 1250 digitized points

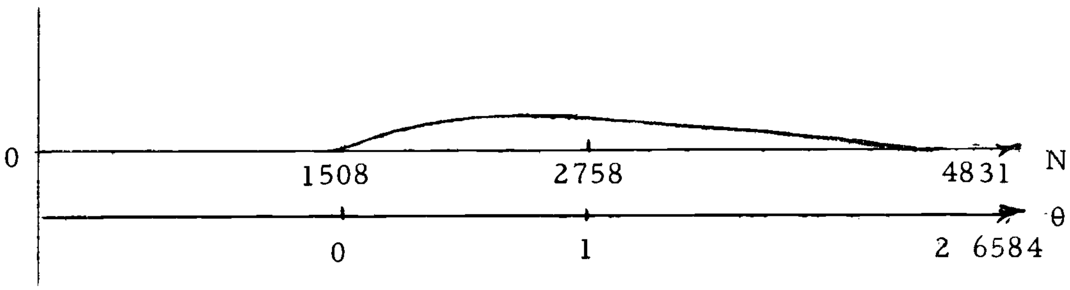

Figure 9. Graphical Presentation of Data Set B, Section 5.

Now for Set B

Section 1 $A_0 = 2.242995053$

$A_1 = 1.761655216$

$A_2 = -0.1237207189$

$A_3 = 0.2948833154$

$A_4 = -0.1475849893$

$A_5 = 0.05262843507$

Section 2 $A_0 = 1.755621812$

$A_1 = 1.025783606$

$A_2 = -0.7096943738$

$A_3 = 0.1847033730$

$A_4 = 0.04202397428$

$A_5 = -0.06748964938$

Section 5 $A_0 = 0.6727787309$

$A_1 = 0.6274949721$

$A_2 = -0.3273751842$

$A_3 = -0.03471244912$

$A_4 = 0.1605360442$

$A_5 = -0.1238418534$

After finding these values for three different curves in Set B, a computer program called "DATAGEN" was used to generate data for these three curves. Then by using two other computer programs called "PLOT" and "PLOTTER", *GRAPH 1B and LAGU 1B (which is its equivalent Laguerre function) were plotted. The same process was repeated for *GRAPH 2B, LAGU 2B and *GRAPH 5B, LAGU 5B. The computer programs are in Appendix A.

Before convoluting the curves, each curve must be normalized. This required that

$$\int_0^{\infty} F(t)dt$$

be equal to one.

For graph 1B each of the coefficients were divided by 5.771202340 which was the value of

$$\int_0^{\infty} F(t)dt$$

when using the old coefficients. This normalized the curve and produced a new set of coefficients:

$$A_0 = 2.242995053/5.771202340 = 0.3886529913$$

$$A_1 = 1.761655216/5.771202340 = 0.3052492552$$

$$A_2 = -0.1237207189/5.771202340 = -0.02143759855$$

$$A_3 = 0.2948833154/5.771202340 = 0.05109564663$$

$$A_4 = -0.1475849893/5.771202340 = -0.02557265897$$

$$A_5 = 0.5262843507/5.771202340 = 0.00911914570$$

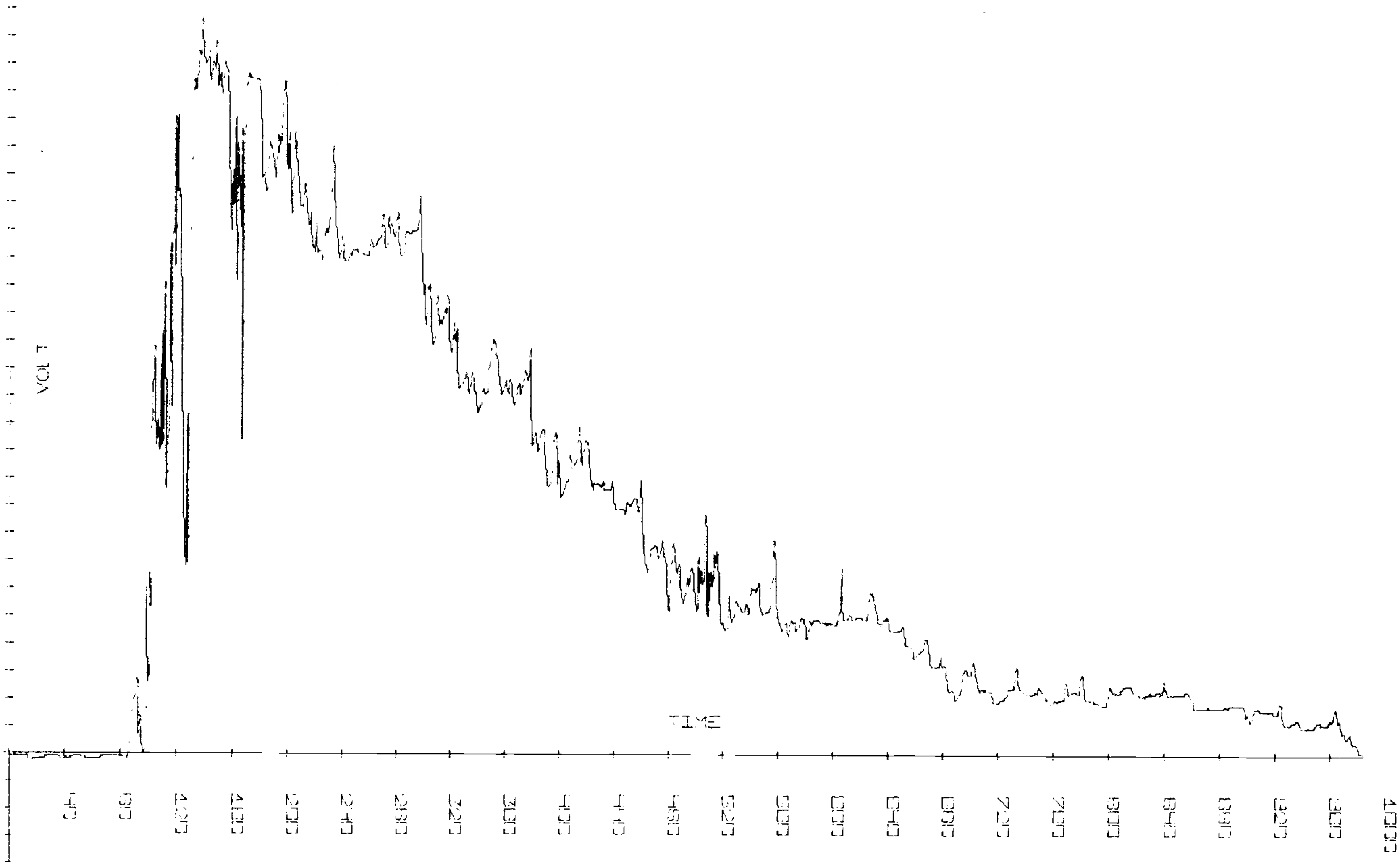

Figure 10. Voltage Versus Time for Section 1, Set B.

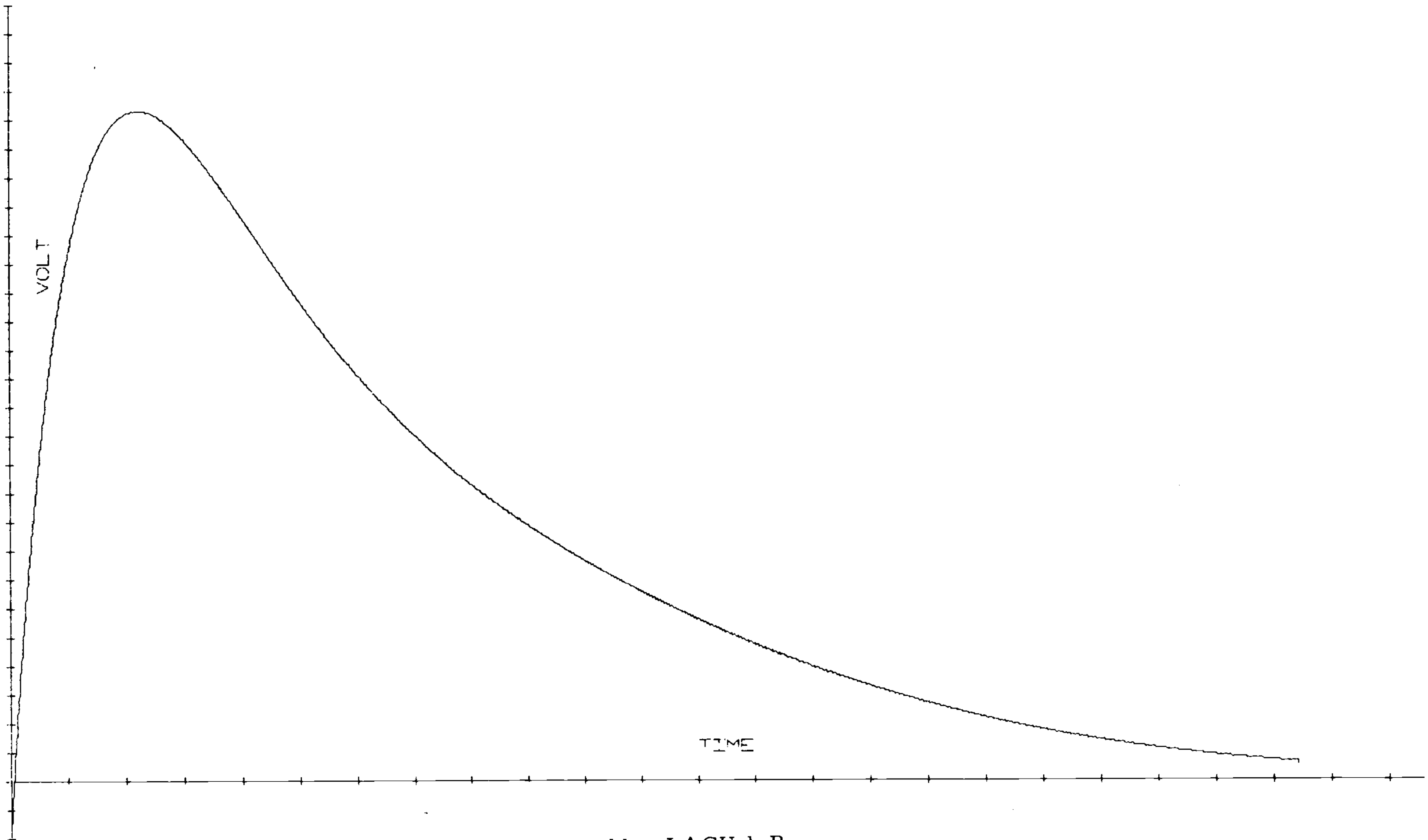

Figure 11. LAGU 1 B

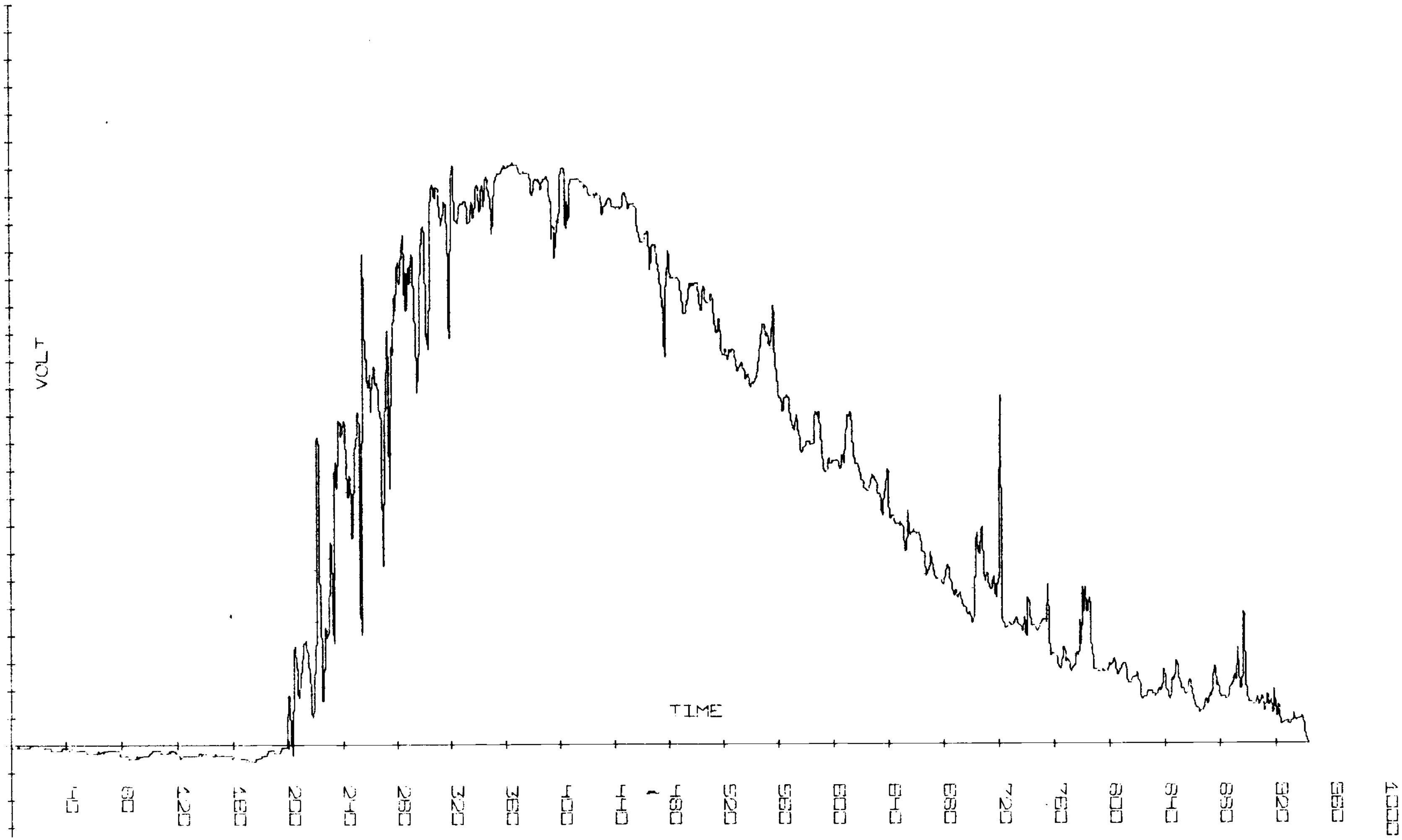

Figure 12. Voltage Versus Time for Section 2, Set B.

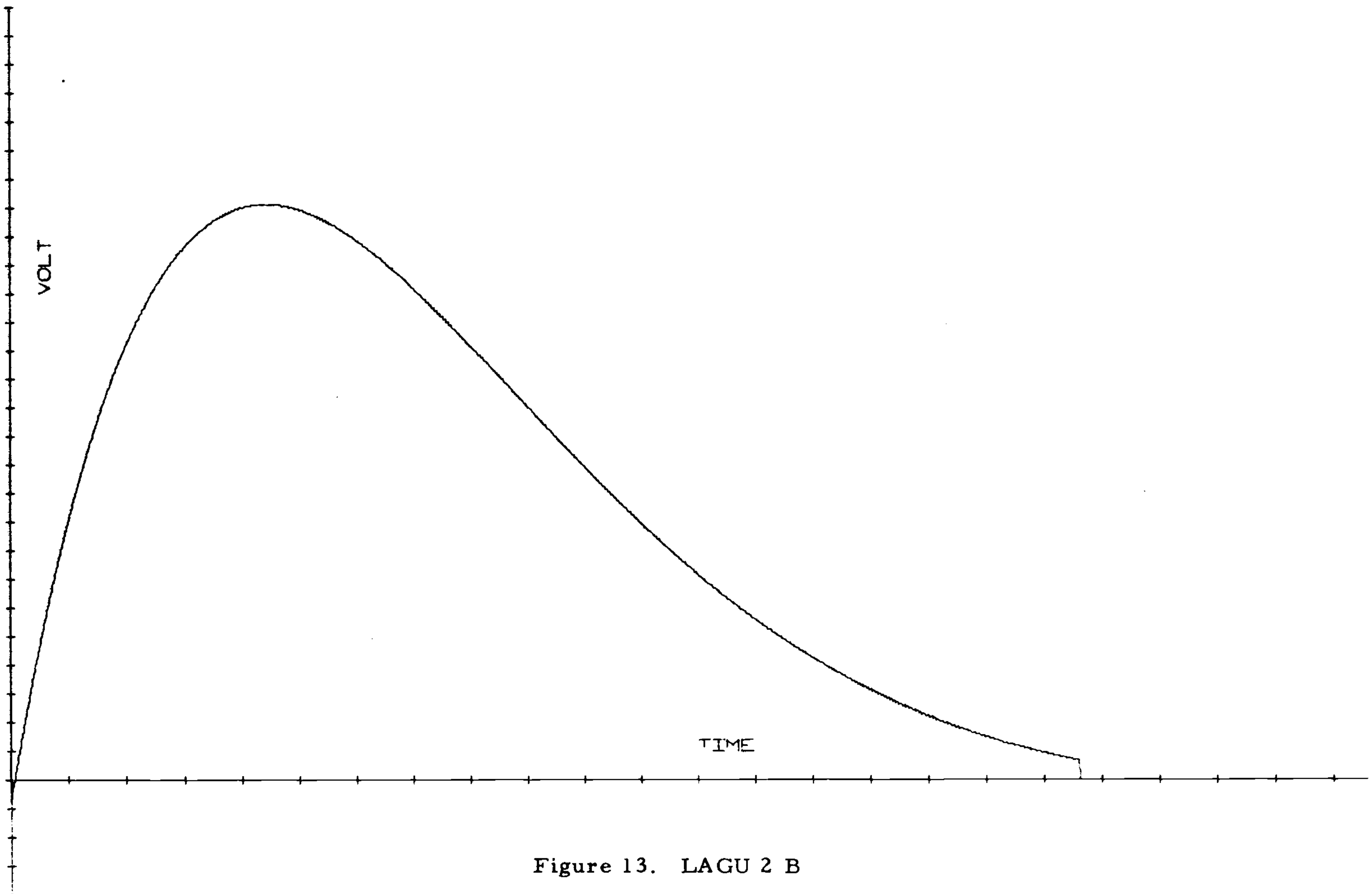

Figure 13. LAGU 2 B

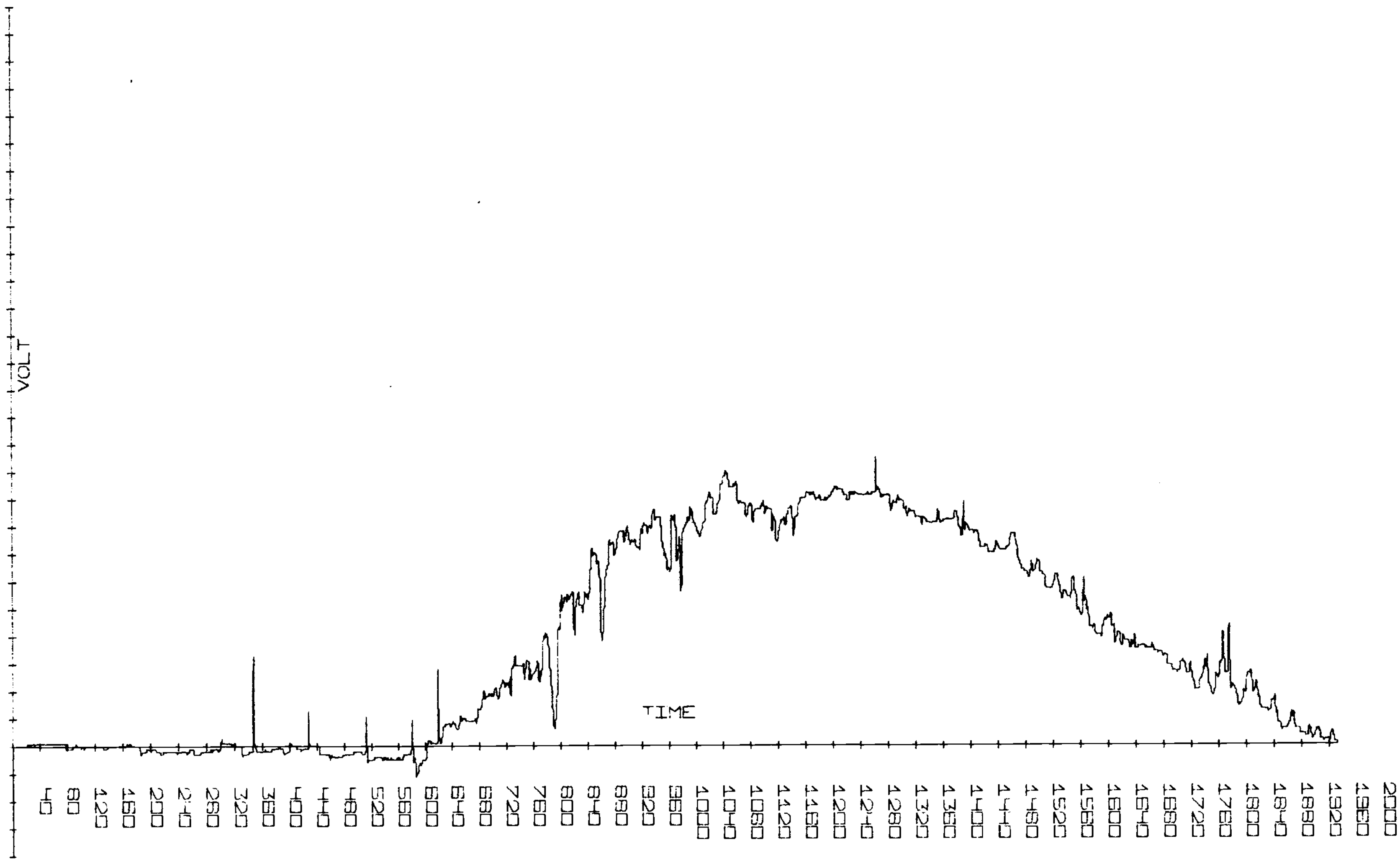

Figure 14. Voltage Versus Time for Section 5, Set B.

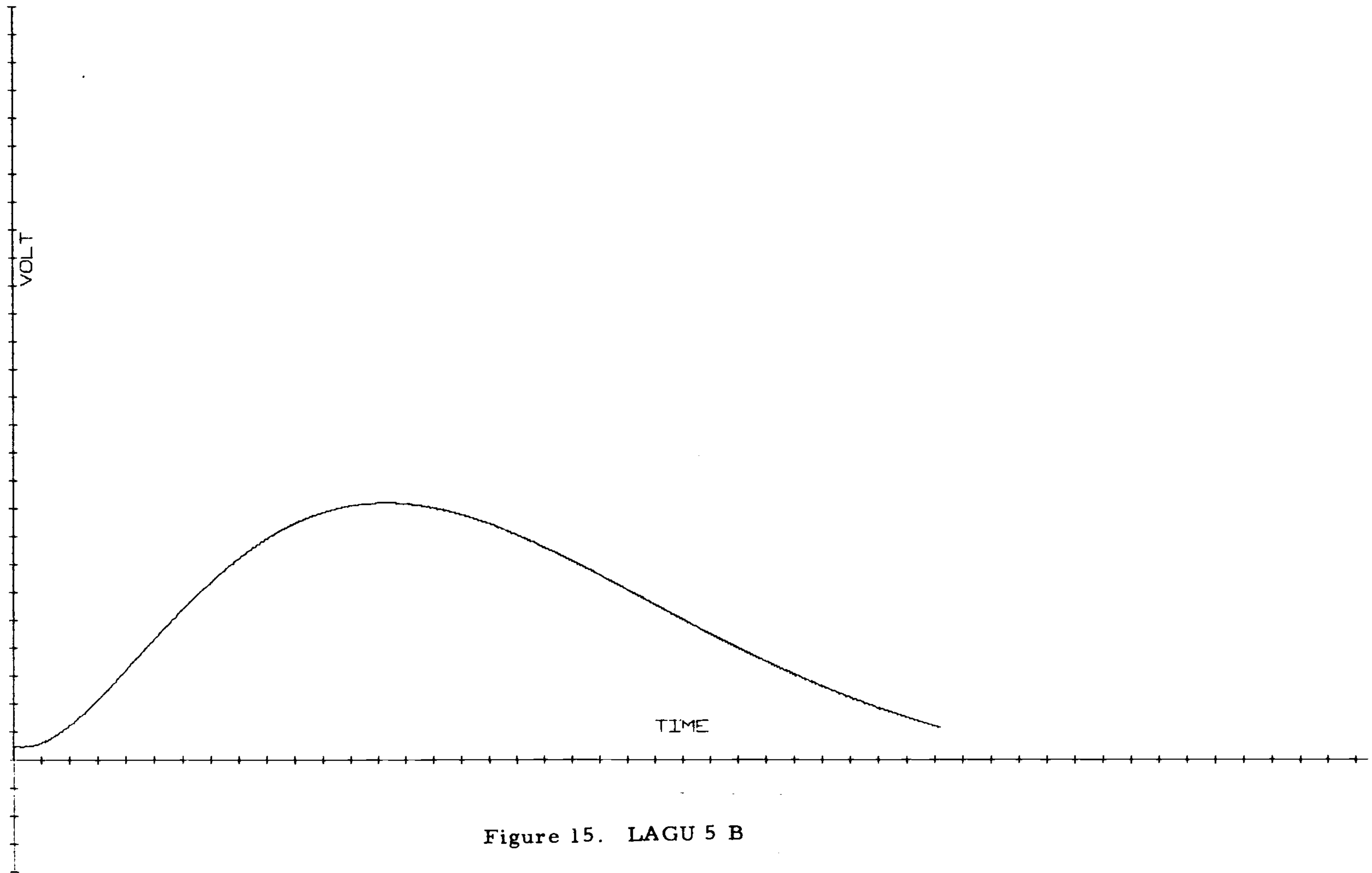

Figure 15. LAGU 5 B

The Laguerre function with these coefficients can then be normalized.

Normalizing Graph 2B; the coefficients became

$A_0 = 1.755621812/3.155037967 = 0.5564502964$

$A_1 = 1.025783606/3.155037967 = 0.3251255981$

$A_2 = -0.7096943738/3.155037967 = -0.2249400423$

$A_3 = 0.1847033730/3.155037967 = 0.05854236143$

$A_4 = 0.04202397428/3.155037967 = 0.01331964136$

$A_5 = -0.06748964938/3.155037967 = -0.02139107358$

Normalizing Graph 5B; the coefficients became

$A_0 = 0.6727787309/1.378688886 = 0.4879844450$

$A_1 = 0.6274949721/1.378688886 = 0.4551389211$

$A_2 = -0.3273751842/1.378688886 = -0.2374539953$

$A_3 = -0.03471244912/1.37868886 = -0.02517786968$

$A_4 = 0.1605360442/1.378688886 = 0.1164410955$

$A_5 = -0.1238418534/1.378688886 = -0.08982581542$

As was mentioned earlier, the Laguerre function is

$$F(t) = A_0L_0=A_1L_1+A_2L_2+A_3L_3+A_4L_4+A_5L_5$$

The Laguerre function for Graph 1B can now be expressed with the evaluated terms as

$$f_1(t) = 0.3886529913\,[\sqrt{2}e^{-t}] + 0.305249552\,[\sqrt{2}(2t-1)e^{-t}]$$

$$-0.02143759855\,[\sqrt{2}(2t^2-4t+1)e^{-t}]$$

$$+0.05109564663\,[\sqrt{2}\,(\tfrac{4}{3}t^3-6t^2+6t-1)e^{-t}]$$

$$-0.02557265897\,[\sqrt{2}\,(\tfrac{2}{3}t^4-\tfrac{16}{3}t^3+12t^2-8t+1)e^{-t}]$$

$$+0.0911914570\,[\sqrt{2}\,(\tfrac{4}{15}t^5-\tfrac{10}{3}t^4+\tfrac{40}{3}t^3-20t^2+10t-1)e^{-t}]$$

The Laplace transform of this function yields

$$\{f_1(t)\} = 0.3886529913\,[\sqrt{2}\frac{1}{(1+S)}] + 0.3052492552\,[\sqrt{2}\frac{(1-S}{(1+S)^2}]$$

$$-0.02143759855\,[\sqrt{2}\frac{(1-S)^2}{(1+S)^3}] + 0.05109564663$$

$$[\sqrt{2}\frac{(1-S)^3}{(1+S)^4}]$$

$$-0.02557265897\,[\sqrt{2}\frac{(1-S)^4}{(1+S)^5}] + 0.0911914570$$

$$[\sqrt{2}\frac{(1-S)^5}{(1+S)^6}]$$

Similarly for Graph 2B, one obtains

$$f_2(t) = 0.5564502964\,[\sqrt{2}e^{-t}] + 0.3251255981\,[\sqrt{2}(2t-1)e^{-t}]$$

$$-0.2249400423\,[\sqrt{2}(2t^2-4t+1)e^{-t}]$$

$$+0.05854236143\,[\sqrt{2}(\tfrac{4}{3}t^3-6t^2+6t-1)e^{-t}]$$

$$+0.01131964136\ [\sqrt{2}(\tfrac{2}{3}t^4 - \tfrac{16}{3}t^3 + 12t^2 - 8t + 1)e^{-t}]$$

$$-0.02139107358\ [\sqrt{2}(\tfrac{4}{15}t^5 - \tfrac{10}{3}t^4 + \tfrac{40}{3}t^3 - 20t^2 + 10t - 1)e^{-t}]$$

and

$$\{f_2(t)\} = 0.5564502964\ [\sqrt{2}(\frac{1}{(1+S)})] + 0.3251255981$$

$$[\sqrt{2}\,\frac{(1-S)}{(1+S)^2}]$$

$$-0.2449400423\ [\sqrt{2}\,\frac{(1-S)^2}{(1+S)^3}] + 0.05854236143$$

$$[\sqrt{2}\,\frac{(1-S)^3}{(1+S)^4}]$$

$$+0.01331964136\ [\sqrt{2}\,\frac{(1-S)^4}{(1+S)^5}] - 0.02139107358$$

$$[\sqrt{2}\,\frac{(1-S)^5}{(1+S)^6}]$$

and for Graph 5B, one obtains

$$f_5(t) = 0.487874450\ [\sqrt{2}e^{-t}] + 0.4551389211\ [\sqrt{2}(2t-1)e^{-t}]$$

$$-0.2374539953\ [\sqrt{2}(2t^2 - 4t + 1)e^{-t}]$$

$$-0.02517786968\ [\sqrt{2}(\tfrac{4}{3}t^3 - 6t^2 + 6t - 1)e^{-t}]$$

$$+0.1164410955\ [\sqrt{2}(\tfrac{2}{3}t^4 - \tfrac{16}{3}t^3 + 12t^2 - 8t + 1)e^{-t}]$$

$$-0.08982581542\ [\sqrt{2}(\frac{4}{15}t^5 - \frac{10}{3}t^4 + \frac{40}{3}t^3 - 20t^2 + 10t - 1)e^{-t}]$$

and

$$\{f_5(t)\} = 0.4879844450\ [\sqrt{2}\frac{1}{(1+S)}] + 0.4551389211\ [\sqrt{2}\frac{(1-S)}{(1+S)^2}]$$

$$-0.2374539953\ [\sqrt{2}\frac{(1-S)^2}{(1+S)^3}] - 0.02517786968$$

$$[\sqrt{2}\frac{(1-S)^3}{(1+S)^4}]$$

$$+0.1164410955\ [\sqrt{2}\frac{(1-S)^4}{(1+S)^5}] - 0.08982581542$$

$$[\sqrt{2}\frac{(1-S)^5}{(1+S)^6}]$$

If $\{f_1\} = A$, and if convolution holds, then

$$\{f_2\} = A.\ B$$

and $\{f_5\} = A.\ B.\ B.\ B.\ B$ (if the other four sections are considered to be identical). So

$$\frac{\{\mathcal{L}[f_2]\}^4}{\{\mathcal{L}[f_1]\}^3} = \mathcal{L}[f_5]$$ in order for the individual RTD's to be independent.

$$f(t) = A_0 L_0 + A_1 L_1 + \ldots\ldots + A_5 L_5$$

$$\mathcal{L}\,[f(t)] \;=\; A_0[\sqrt{2}\,\frac{(1)}{(1+S)}] + A_1[\sqrt{2}\,\frac{(1-S)}{(1+S)^2}] + A_2[\sqrt{2}\,\frac{(1-S)^2}{(1+S)^3}]$$

$$+A_3[\sqrt{2}\,\frac{(1-S)^3}{(1+S)^4} + A_4[\sqrt{2}\,\frac{(1-S)^4}{(1+S)^5}] + A_5[\sqrt{2}\,\frac{(1-S)^5}{(1+S)^6}]$$

or:

$$\mathcal{L}\,[f(t)] \;=\; \frac{\sqrt{6}}{(1+S)^6}\,[\,A_0(1+5S+10S^2+10S^3+5S^4+S^5)$$

$$+ A_1(1+3S-2S^2-2S^3-3S^4-S^5)$$

$$+ A_2(1+S-2S^2-2S^3+S^4+S^5)$$

$$+ A_3(1-S-2S^2+2S^3+S^4-S^5)$$

$$+ A_4(1-3S+2S^2+2S^3-3S^4+S^5)$$

$$+ A_5(1-5S+10S^2-10S^3+5S^4-S^5)\,]$$

or:

$$\{f(t)\} \;=\; \frac{\sqrt{2}}{(1+S)^6}\,[\,(A_0 + A_1 + A_2 + A_3 + A_4 + A_5)$$

$$+ (5A_0+3A_1+A_2-A_3-3A_4-5A_5)S$$

$$+ (10A_0+2A_1-2A_2-2A_3+2A_4+10A_5)S^2$$

$$+ (10A_0-2A_1-2A_2+2A_3+2A_4-10A_5)S^3$$

$$+ (5A_0-3A_1+A_2+A_3-3A_4+5A_5)S^4$$

$$+ (A_0-A_1+A_2-A_3+A_4-A_5)S^5\,]$$

suppose:

$$B_0 = (A_0 + A_1 + A_2 + A_3 + A_4 + A_5)$$

$$B_1 = (5A_0 + 3A_1 + A_2 - A_3 - 3A_4 - 5A_5)$$

$$B_2 = (10A_0 + 2A_1 - 2A_2 - 2A_3 + 2A_4 + 10A_5)$$

$$B_3 = (10A_0 - 2A_1 - 2A_2 + 2A_3 + 2A_4 - 10A_5)$$

$$B_4 = (5A_0 - 3A_1 + A_2 + A_3 - 3A_4 + 5A_5)$$

$$B_5 = (A_0 - A_1 + A_2 - A_3 + A_4 - A_5)$$

so:

$$\mathcal{L}\,[f(t)] = \frac{\sqrt{2}}{(1+S)^6}[B_0 + B_1 S + B_2 S^2 + B_3 S^3 + B_4 S^4 + B_5 S^5]$$

Here again for simplicity a computer program was used to get the Laplace transform for each equation. The computer program used was "CONVO 1" which calculated B_0, B_1, . . . B_5 by knowing A_0, A_1, . . . A_5. The computer program is in Appendix A.

So:

$$\mathcal{L}\,[f_1(t)] = \frac{1}{(1+S)^6}\{1.0000000 + 3.9846905S + 6.3325068S^2 + 4.6368689S^3 + 1.6680492S^4 - 0.033688425S^5\}$$

$$\mathcal{L}\,[f_2(t)] = \frac{1}{(1+S)^6}\{1.0000000 + 5.0079315S + 8.9947915S^2 + 8.0917999S^3 + 2.1122166S^4 - 0.024673770S^5\}$$

$$[f_5(t)] = \frac{1}{(1+S)^6}\{1.0000000 + 5.2225050S + 7.9903209S^2 + 7.8138965S^3 + 0.018980490S^4 + 0.037952272S^5\}$$

Now what will be $\{\mathcal{L}\,[f(t)]\}^2$?

$$\{\mathcal{L}\,[f(t)]\}^2 = \frac{(\sqrt{2})^2}{(1+S)^{12}}[B_0 + B_1S + B_2S^2 + B_3S^3 + B_4S^4 + B_5S^5]^2$$

$$(\frac{(\sqrt{2})}{(1+S)^6})^2[B_0^2 + 2B_0B_1S + (2B_0B_2 + B_1^2)S^2 + (2B_0B_3 + 2B_1B_2)S^3 + (2B_0B_4 + 2B_1B_3 + B_2^2)S^4 + (2B_0B_5 + 2B_1B_4 + 2B_2B_3)S^5 + (2B_1B_5 + 2B_2B_4 + B_3^2)S^6 + (2B_2B_5 + 2B_3B_4)S^7 + (2B_3B_5 + B_4^2)S^8 + 2B_4B_5S^9 + B_5^2S^{10}]$$

Suppose:

$$\alpha_0 = B_0^2$$

$$\alpha_1 = 2B_0B_1$$

$$\alpha_2 = 2B_0B_2 + B_1^2$$

$$\alpha_3 = 2B_0B_3 + 2B_1B_2$$

$$\alpha_4 = 2B_0B_4 + 2B_1B_3 + B_2^2$$

$$\alpha_5 = 2B_0B_5 + 2B_1B_4 + 2B_2B_3$$

$$\alpha_6 = 2B_1B_5 + 2B_2B_4 + B_3^2$$

$$\alpha_7 = 2B_2B_5 + 2B_3B_4$$

$$\alpha_8 = 2B_3B_5 + B_4^2$$

$$\alpha_9 = 2B_4B_5$$

$$\alpha_{10} = B_5^2$$

Now look for $\{\mathcal{L}[f(t)]\}^3$

$$\{\mathcal{L}[f(t)]\}^3 = \left(\frac{\sqrt{2}}{(1+S)^6}\right)^3 [B_0 + B_1S + B_2S^2 + B_3S^3 + B_4S^4 + B_5S^5]^3$$

$$= \left[\frac{\sqrt{2}}{(1+S)^6}\right]^3 \{B_0^3 + (3B_0^2B_1)S + (3B_0^2B_2 + 3B_0B_1^2)S^2$$

$$+ (3B_0^2B_3 + 6B_0B_1B_2 + B_1^3)S^3 + (3B_0^2B_4 + 3B_0B_2^2$$

$$+ 3B_1^2B_2 + 6B_0B_1B_3)S^4$$

$$+ (3B_0^2B_5 + 3B_1^2B_3 + 3B_1B_2^2 + 6B_0B_2B_3 + 6B_0B_1B_4)S^5$$

$$+ (3B_0B_3^2 + 3B_1^2B_4 + 6B_0B_1B_5 + 6B_1B_2B_3 + 6B_0B_2B_4$$

$$+ B_2^3)S^6$$

$$+ (3B_1^2B_5 + 3B_1B_3^2 + 3B_2^2B_3 + 6B_1B_2B_4 + 6B_0B_2B_5$$

$$+ 6B_0B_3B_4)S^7$$

$$+ (3B_0B_4^2 + 3B_2^2B_4 + 3B_2B_3^2 + 6B_0B_3B_5 + 6B_1B_2B_5$$

$$+ 6B_1B_3B_4)S^8$$

$$+ (3B_1B_4^2 + 3B_2^2B_5 + 6B_0B_4B_5 + 6B_1B_3B_5$$

$$+ 6B_2B_3B_4 + B_3^3)S^9$$

$$+ (3B_0B_5^2 + 3B_2B_4^2 + 3B_3^2B_4 + 6B_1B_4B_5 + 6B_2B_3B_5)S^{10}$$

$$+ (3B_1B_5^2 + 3B_3^2B_5 + 3B_3B_4^2 + 6B_2B_4B_5)S^{11}$$

$$+ (3B_2B_5^2 + 6B_3B_4B_5 + B_4^3)S^{12}$$

$$+ (3B_3B_5^2 + 3B_4^2B_5)S^{13} + (3B_4B_5^2)S^{14} + (B_5^3)S^{15}\}$$

suppose:

$$\{\mathcal{L}[f(t)]\}^3 = [\frac{\sqrt{2}}{(1+S)^6}]^3 \{C_0 + C_1 S + C_2 S^2 + \ldots\ldots + \ldots\ldots + C_{14} S^{14} + C_{15} S^{15}\}$$

where

$$C_0 = B_0^3$$

$$C_1 = 3B_0^2 B_1$$

$$C_2 = 3B_0^2 B_2 + 3B_0 B_1^2$$

$$C_3 = 3B_0^2 B_3 + 6B_0 B_1 B_2 + B_1^3$$

$$C_4 = 3B_0^2 B_4 + 3B_0 B_2^2 + 3B_1^2 B_2 + 6B_0 B_1 B_3$$

$$C_5 = 3B_0^2 B_5 + 3B_1^2 B_3 + 3B_1 B_2^2 + 6B_0 B_2 B_3 + 6B_0 B_1 B_4$$

$$C_6 = 3B_0 B_3^2 + 3B_1^2 B_4 + 6B_0 B_1 B_5 + 6B_1 B_2 B_3 + 6B_0 B_2 B_4 + B_2^3$$

$$C_7 = 3B_1^2 B_5 + 3B_1 B_3^2 + 3B_2^2 B_3 + 6B_1 B_2 B_4 + 6B_0 B_2 B_5 + 6B_0 B_3 B_4$$

$$C_8 = 3B_0 B_4^2 + 3B_2^2 B_4 + 3B_2 B_3^2 + 6B_0 B_3 B_5 + 6B_1 B_2 B_5 + 6B_1 B_3 B_4$$

$$C_9 = 3B_1B_4^2 + 3B_2^2B_5 + 6B_0B_4B_5 + 6B_1B_3B_5 + 6B_2B_3B_4 + B_3^3$$

$$C_{10} = 3B_0B_5^2 + 3B_2B_4^2 + 3B_3^2B_4 + 6B_1B_4B_5 + 6B_2B_3B_5$$

$$C_{11} = 3B_1B_5^2 + 3B_3^2B_5 + 3B_3B_4^2 + 6B_2B_4B_5$$

$$C_{12} = 3B_2B_5^2 + 6B_3B_4B_5 + B_4^3$$

$$C_{13} = 3B_3B_5^2 + 3B_4^2B_5$$

$$C_{14} = 3B_4B_5^2$$

$$C_{15} = B_5^3$$

Now look for $\{\mathcal{L}[f(t)]\}^4$

$$\{\mathcal{L}[f(t)]\}^4 = [\frac{\sqrt{2}}{(1+S)^6}]^4\{B_0 + B_1S + B_2S^2 + B_3S^3 + B_4S^4 + B_5S^5\}^4$$

$$= B_0^4 + (4B_0^3B_1)S + (4B_0^3B_2 + 6B_0^2B_1^2)S^2$$

$$+ (4B_0^3B_3 + 12B_0^2B_1B_2 + 4B_0B_1^3)S^3$$

$$+ (4B_0^3B_4 + 12B_0^2B_1B_3 + 12B_oB_1^2B_2 + 6B_0^2B_2^2 + B_1^4)S^4$$

$$+ (4B_0^3B_5 + 12B_0B_1^2B_3 + 12B_0^2B_1B_4 + 12B_0^2B_2B_3 + 12B_0B_1B_2^2 + 4B_1^3B_2)S^5$$

$$+ (6B_0^2B_3^2 + 6B_1^2B_2^2 + 12B_0^2B_1B_5 + 12B_0^2B_2B_4$$

$$+ 12B_0B_1^2B_4 + 24B_0B_1B_2B_3 + 4B_1^3B_3$$

$$+ 4B_0B_2^3)S^6$$

$$+ (12B_0^2B_2B_5 + 12B_0^2B_3B_4 + 12B_0B_1^2B_5 + 12B_0B_1B_3^2$$

$$+ 12B_0B_2^2B_3 + 12B_1^2B_2B_3 + 24B_0B_1B_2B_4$$

$$+ 4B_1^3B_4 + 4B_1B_2^3)S^7$$

$$+ (12B_0^2B_3B_5 + 12B_1^2B_2B_4 + 12B_0B_2^2B_4 + 12B_0B_2B_3^2$$

$$+ 12B_1B_2^2B_3 + 6B_1^2B_3^2 + 6B_0^2B_4^2 + 24B_0B_1B_2B_5$$

$$+ 24B_0B_1B_3B_4 + 4B_1^3B_5 + B_2^4)S^8$$

$$+ (12B_0^2B_4B_5 + 12B_1^2B_2B_5 + 12B_1^2B_3B_4 + 12B_0B_1B_4^2$$

$$+ 12B_1B_2^2B_4 + 12B_0B_2^2B_5 + 12B_1B_2B_3^2$$

$$+ 24B_0B_1B_3B_5 + 24B_0B_2B_3B_4 + 4B_0B_3^3$$

$$+ 4B_2^3B_3)S^9$$

$$+ (6B_0^2B_5^2 + 24B_0B_1B_4B_5 + 24B_0B_2B_3B_5$$

$$+ 24B_1B_2B_3B_4$$

$$+ 12B_1^2B_3B_5 + 12B_0B_2B_4^2 + 12B_1B_2^2B_5 + 12B_0B_3^2B_4$$

$$+ 6B_1^2B_4^2$$

$$+ 6B_2^2B_3^2 + 4B_1B_3^3 + 4B_2^3B_4)S^{10}$$

$$+ (12B_0B_1B_5^2 + 24B_0B_2B_4B_5 + 12B_1^2B_4B_5 +$$

$$+ 12B_0B_3^2B_5 + 12B_1B_3^2B_4$$

$$+ 12B_2^2B_3B_4$$

$$+ 24B_1B_2B_3B_5 + 12B_0B_3B_4^2 + 12B_1B_2B_4^2 + 4B_2^3B_5$$

$$+ 4B_2B_3^3)S^{11}$$

$$+ (24B_0B_3B_4B_5 + 24B_1B_2B_4B_5 + 12B_0B_2B_5^2$$

$$+ 12B_1B_3^2B_5$$

$$+ 12B_1B_3B_4^2 + 12B_2^2B_3B_5 + 12B_2B_3^2B_4 + 6B_1^2B_5^2$$

$$+ 6B_2^2B_4^2 + 4B_0B_4^3 + B_3^4)S^{12}$$

$$+ (12B_0B_3B_5^2 + 12B_1B_2B_5^2 + 12B_0B_4^2B_5 + 12B_2^2B_4B_5$$

$$+ 12B_2B_3B_4^2$$

$$+ 12B_2B_3^2B_5 + 24B_1B_3B_4B_5 + 4B_1B_4^3 + 4B_3^3B_4)S^{13}$$

$$+ (12B_0B_4B_5^2 + 12B_1B_3B_5^2 + 12B_1B_4^2B_5 + 24B_2B_3B_4B_5$$

$$+ 6B_3^2B_4^2 + 6B_2^2B_5^2 + 4B_2B_4^3 + 4B_3^3B_5)S^{14}$$

$$+ (4B_0B_5^3 + 12B_1B_4B_5^2 + 12B_2B_3B_5^2 + 12B_2B_4^2B_5$$

$$+ 12B_3^2B_4B_5$$

$$+ 4B_3B_4^3)S^{15}$$

$$+ (4B_1B_5^3 + 12B_2B_4B_5^2 + 12B_3B_4^2B_5 + 6B_3^2B_5^2 + B_4^4)S^{16}$$

$$+ (4B_2B_5^3 + 12B_3B_4B_5^2 + 4B_4^3B_5)S^{17}$$

$$+ (4B_3B_5^3 + 6B_4^2B_5^2)S^{18} + (4B_4B_5^3)S^{19} + (B_5^4)S^{20}$$

it means:

$$\{ \mathcal{L}\, [f(t)] \}^4 = \left[\frac{\sqrt{2}}{(1+S)^6} \right] \{D_0 + D_1S + D_2S^2 + D_3S^3 + \cdots \cdots + D_{19}S^{19} + D_{20}S^{20}\}$$

where

$$D_0 = B_0^4$$

$$D_1 = 4B_0^3B_1$$

$$D_2 = 4B_0^3B_2 + 6B_0^2B_1^2$$

$$D_3 = 4B_0^3B_3 + 12B_0^2B_1B_2 + 4B_0B_1^3$$

$$D_4 = 4B_0^3B_4 + 12B_0^2B_1B_3 + 6B_0^2B_2^2 + 12B_0B_1^2B_2 + B_1^4$$

$$\begin{aligned} D_5 = {} & 4B_0^3B_5 + 12B_0^2B_1B_4 + 12B_0^2B_2B_3 + 12B_0B_1^2B_3 \\ & + 12B_0B_1B_2^2 + 4B_1^3B_2 \end{aligned}$$

$$\begin{aligned} D_6 = {} & 12B_0^2B_1B_5 + 12B_0^2B_2B_4 + 12B_0B_1^2B_4 + 6B_0^2B_3^2 \\ & 6B_1^2B_2^2 + 24B_0B_1B_2B_3 + 4B_1^3B_3 + 4B_0B_2^3 \end{aligned}$$

$$\begin{aligned} D_7 = {} & 12B_0^2B_2B_5 + 12B_0^2B_3B_4 + 12B_0B_1^2B_5 + 12B_0B_1B_3^2 \\ & + 12B_0B_2^2B_3 + 12B_1^2B_2B_3 + 24B_0B_1B_2B_4 \\ & + 4B_1^3B_4 + 4B_1B_2^3 \end{aligned}$$

$$\begin{aligned} D_8 = {} & 12B_0^2B_3B_5 + 12B_1^2B_2B_4 + 12B_0B_2^2B_4 + 12B_0B_2B_3^2 \\ & + 12B_1B_2^2B_3 + 6B_1^2B_3^2 + 6B_0^2B_4^2 + 24B_0B_1B_2B_5 \\ & + 24B_0B_1B_3B_4 + 4B_1^3B_5 + B_2^4 \end{aligned}$$

$$\begin{aligned} D_9 = {} & 12B_0^2B_4B_5 + 12B_1^2B_2B_5 + 12B_1^2B_3B_4 + 12B_0B_1B_4^2 \\ & + 12B_1B_2^2B_4 + 12B_0B_2^2B_5 + 12B_1B_2B_3^2 \\ & + 24B_0B_1B_3B_5 + 24B_0B_2B_3B_4 + 4B_0B_3^3 \\ & + 4B_2^3B_3 \end{aligned}$$

$$D_{10} = 6B_0^2B_5^2 + 24B_0B_1B_4B_5 + 24B_0B_2B_3B_5 + 24B_1B_2B_3B_4$$
$$+ 12B_1^2B_3B_5 + 12B_0B_2B_4^2 + 12B_1B_2^2B_5 + 12B_0B_3^2B_4$$
$$+ 6B_1^2B_4^2 + 6B_2^2B_3^2 + 4B_1B_3^3 + 4B_2^3B_4$$

$$D_{11} = 12B_0B_1B_5^2 + 24B_0B_2B_4B_5 + 12B_1^2B_4B_5 + 12B_0B_3^2B_5$$
$$+ 12B_1B_3^2B_4 + 24B_1B_2B_3B_5 + 12B_0B_3B_4^2$$
$$+ 12B_1B_2B_4^2 + 4B_2^3B_5 + 4B_2B_3^3 + 12B_2^2B_3B_4$$

$$D_{12} = 24B_0B_3B_4B_5 + 24B_1B_2B_4B_5 + 12B_0B_2B_5^2 + 12B_1B_3^2B_5$$
$$+ 12B_1B_3B_4^2 + 12B_2^2B_3B_5 + 12B_2B_3^2B_4 + 6B_1^2B_5^2$$
$$+ 6B_2^2B_4^2 + 4B_0B_4^3 + B_3^4$$

$$D_{13} = 12B_0B_3B_5^2 + 12B_1B_2B_5^2 + 12B_0B_4^2B_5 + 12B_2^2B_4B_5$$
$$+ 12B_2B_3B_4^2 + 12B_2B_3^2B_5 + 24B_1B_3B_4B_5 + 4B_1B_4^3$$
$$+ 4B_3^3B_4$$

$$D_{14} = 12B_0B_4B_5^2 + 12B_1B_3B_5^2 + 12B_1B_4^2B_5 + 24B_2B_3B_4B_5$$
$$+ 6B_3^2B_4^2 + 6B_2^2B_5^2 + 4B_2B_4^3 + 4B_3^3B_5$$

$$D_{15} = 4B_0B_5^3 + 12B_1B_4B_5^2 + 12B_2B_3B_5^2 + 12B_2B_4^2B_5$$

$$+ 12B_3^2B_4B_5 + 4B_3B_4^3$$

$$D_{16} = 4B_1B_5^3 + 12B_2B_4B_5^2 + 12B_3B_4^2B_5 + 6B_3^2B_5^2 + B_4^4$$

$$D_{17} = 4B_2B_5^3 + 12B_3B_4B_5^2 + 4B_4^3B_5$$

$$D_{18} = 4B_3B_5^3 + 6B_4^2\, B_5^2$$

$$D_{19} = 4B_4B_5^3$$

$$D_{20} = B_5^4$$

Again by use of two computer programs called "CONVO 3" and "CONVO 4" the values were obtained for $[\ f_1(t)]^3$, $[\ f_2(t)]^4$ by knowing $A_0, A_1, \ldots, A_5$ for each section. The computer programs are in Appendix A.

So for Set B:

$$\{\mathcal{L}[f_1(t)]\}^3 = \{\frac{\sqrt{2}}{(1+S)^6}\}^3 \ \{0.35355339 + (4.2264026)S$$

$$+ (23.557544)S^2 + (80.814188)S^3$$

$$+ (190.14209)S^4 + (323.92282)S^5$$

$$+ (410.99906)S^6 + (392.76455)S^7$$

$$+ (281.55377)S^{8} + (148.03460)S^{9}$$

$$+ (54.155596)S^{10} + (12.1655851)S^{11}$$

$$+ (1.0957813)S^{12} - (0.093838570)S^{13}$$

$$+ (0.0020079207)S^{14}$$

$$- (0.000013517523)S^{15}\}$$

$$\{\mathcal{L}[f_2(t)]\}^4 = \{\frac{\sqrt{2}}{(1+S)^6}\}^4 \quad \{0.25 + (5.0079315)S + (46.613858)S^{2}$$

$$+ (268.82350)S^{3} + (1079.0363)S^{4}$$

$$+ (3204.1002)S^{5} + (7288.3963)S^{6}$$

$$+ (12953.243)S^{7} + (18154.257)S^{8}$$

$$+ (200800.821)S^{9} + (17401.922)S^{10}$$

$$+ (11619.7)S^{11} + (5807.7446)S^{12}$$

$$+ (2071.3252)S^{13} + (483.6146)S^{14}$$

$$+ (63.198096)S^{15} + (2.3983112)S^{16}$$

$$- (0.20143417)S^{17} + (0.0039526217)S^{18}$$

$$- (0.0000317 2817)S^{19}$$

$$+ (0.00000009265782 4)S^{20}\}$$

Now calculate $\frac{[\mathcal{L} f_2(t)]^4}{[\mathcal{L} f_1(t)]^3}$

$$\frac{\{\mathcal{L}[f_2(t)]\}^4}{\{\mathcal{L}[f_1(t)]\}^3} = \frac{(\frac{\sqrt{2}}{(1+S)^6})^4}{(\frac{\sqrt{2}}{(1+S)^6})^3}$$

$$[\frac{0.25 + (5.0079315)S + (46.613858)S^2 + \ldots}{0.35355339 + (4.2264026)S + (23.557544)S^2 \ldots}]$$

After dividing and throwing away all the terms after S^5,

$$\frac{\{\mathcal{L}[f_2(t)]\}^4}{\{\mathcal{L}[f_1(t)]\}^3} = \frac{\sqrt{2}}{(1+S)^6} [0.70710678 + (5.7117641)S$$

$$+ (16.449975S^2 + (21.495731)S^3$$

$$+ (13.076777)S^4 - (5.7708116)S^5]$$

or:

$$= \frac{1}{(1+S)^6} [1.0000000 + (8.0776543)S$$

$$+ (23.263778S^2 + (30.399554)S^3$$

$$+ (18.493355)S^4 - (8.16116)S^5]$$

but it was earlier shown that

$$[f_5(t)] = \frac{1}{(1+S)^6}[1.0000000 + (5.2225050)S + (7.9903209)S^2 + (7.8138965)S^3 + (0.018980490)S^4 + (0.037952272)S^5]$$

Now invert

$$\frac{[\mathcal{L} f_2(t)]^4}{[\mathcal{L} f_1(t)]^3}$$

from previous calculations, it was shown that

$$B_0 = A_0 + A_1 = A_2 + A_3 + A_4 + A_5$$

$$B_1 = 5A_0 + 3A_1 + A_2 - A_3 - 3A_4 - 5A_5$$

$$B_2 = 10A_0 + 2A_1 - 2A_2 - 2A_3 + 2A_4 + 10A_5$$

$$B_3 = 10A_0 - 2A_1 - 2A_2 + 2A_3 + 2A_4 - 10A_5$$

$$B_4 = 5A_0 - 3A_1 + A_2 + A_3 - 3A_4 + 5A_5$$

$$B_5 = A_0 - A_1 + A_2 - A_3 + A_4 - A_5$$

so by some basic algebra one can get:

$$A_0 = \frac{1}{32}[B_0 + B_1 + B_2 + B_3 + B_4 + B_5]$$

$$A_1 = \frac{1}{32}[5B_0 + 3B_1 + B_2 - B_3 - 3B_4 - 5B_5]$$

$$A_2 = \frac{1}{32}[10B_0 + 2B_1 - 2B_2 - 2B_3 + 2B_4 + 10B_5]$$

$$A_3 = \frac{1}{32}[10B_0 - 2B_1 - 2B_2 + 2B_3 + 2B_4 - 10B_5]$$

$$A_4 = \frac{1}{32}[5B_0 - 3B_1 + B_2 + B_3 - 3B_4 + 5B_5]$$

$$A_5 = \frac{1}{32}[B_0 - B_1 + B_2 - B_3 + B_4 - B_5]$$

For simplicity, program "INVERSE" was used which gave values of A_0, A_1, . . . , A_5 by knowing B_0, B_1, . . . , B_5. Then again by using program "DATAGEN 5" and "PLOTTER 5" the normal curve for section 5 and also the curve for inverting

$$\frac{[\mathcal{L} f_2(t)]^4}{[\mathcal{L} f_1(t)]^3}$$

were plotted in Figures 16 and 17.

As it is seen for set B, the two curves are completely different so it was concluded that the residence time distribution in one section is not independent of the RTD in the second one.

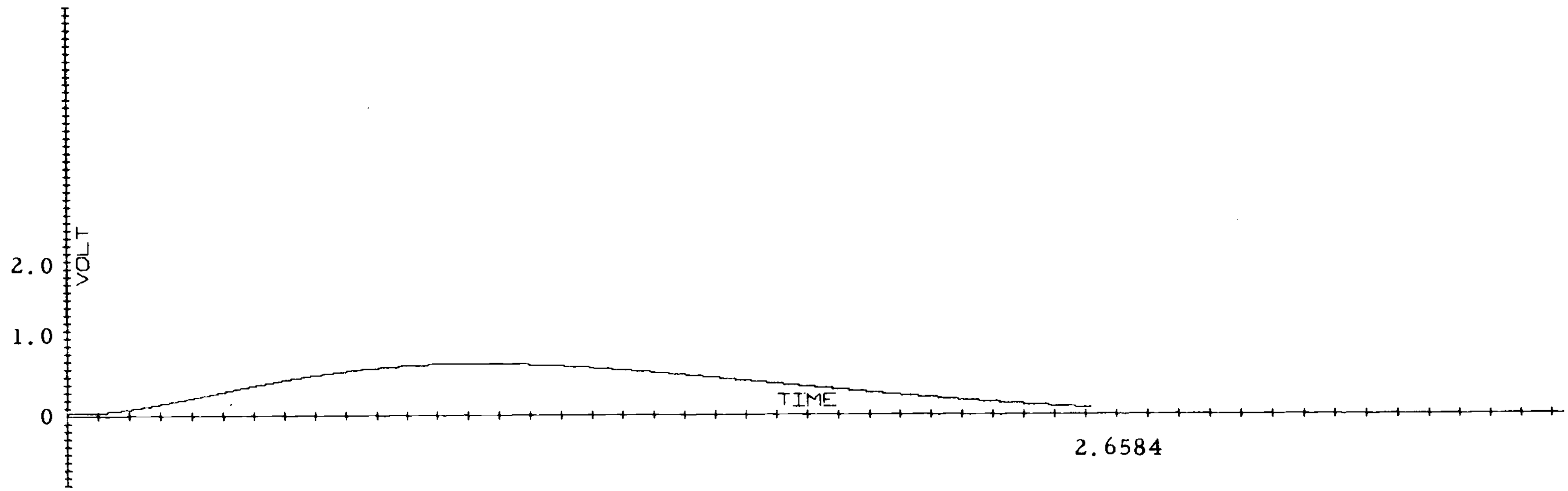

Figure 16. LAGU B NORMAL

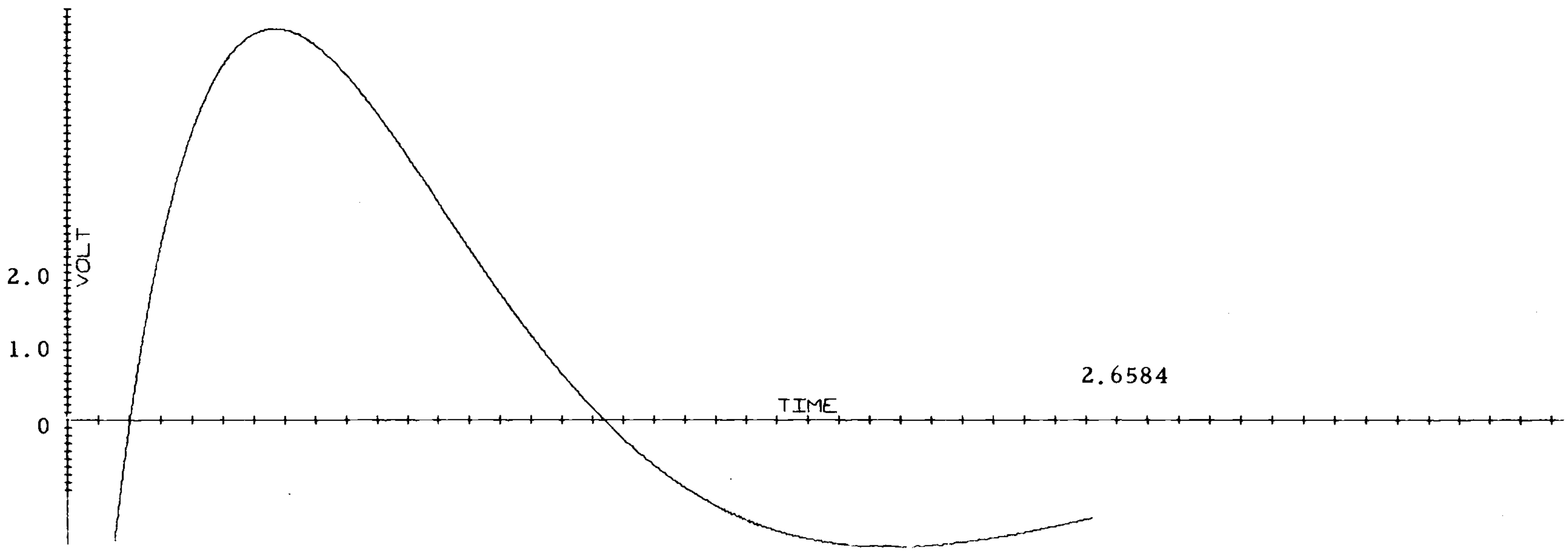

Figure 17. LAGU B CALCULATED

RESULTS

The same data analysis procedure was followed for the other sets and the following results were determined:

Set A	do not convolute
Set B	do not convolute
Set C	do not convolute
Set D	do not convolute
Set E	do not convolute
Set F	do not convolute
Set G	do not convolute
Set H	do not convolute
Set I	do not convolute
Set K	do not convolute
Set L	do not convolute
Set M	do not convolute

the final curves for each set of data are presented in Appendix B.

CONCLUSION

In all of the cases investigated the residence time distribution of each of the individual sections were found to be dependent upon the preceding sections; i.e., not independent of each other. Even though the graphs of set L and M initially indicate independence between the sections, evaluation of the dead time for each graph of these two sets proved that there is dependency between the sections. In the case of independence, (as indicated by the existence of convolution), the following relationship of dead time must be satisfied.

4 dead time for graph obtained after second section

-3 dead time for graph obtained after first section

= dead time for graph obtained after fifth section .

For set L, the dead time relationship is 4(9.6)-3(5.2)=.8 sec, while the actual dead time for 5th section, set L is 27.6 sec, and for set M, 4(26.4)-(13.6) = 64.8 sec, while the actual dead time for 5th section, set M is 166 sec.

Since no independence existed for all of the cases investigated, the residence time distribution of each of the individualized sections were also dependent upon each other. This interdependency indicates that deviation from convolution always existed. Because of this dependency there is no way to evaluate the residence time distribution in a single section. A direct flow visualization will be necessary to find the type of coupling which exists between the flow patterns in adjacent sections.

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APPENDICES

NOMENCLATURE

A	Laguerre coefficient
d	baffle orifice diameter
D	pipe diameter
$\bar{E}$	age distribution function
F,f	Laguerre polynomial
G	age spectrum generating function
G^*	conditional probability
h	dead time
l	length of each section
L	Laguerre function
M	digitized points number starting after dead time
N	digitized points number starting after injecting tracer
t, ,	time
$\bar{t}$	time of backmixing
v	volumetric flow rate
V	volume of each section
θ	shifting time

APPENDIX A

"Computer programs and results for set B"

4 inches diameter of the pipe

30 inches length of each section

1 inch diameter of the orifices

28 cc/sec flow rate

```
      PROGRAM LISTER
C
C ********** BEHROOZ SATVAT / OSU  CHEMICAL ENGINEERING
C ********** JUNE 20, 1974
C
      INTEGER COUNTER, BUFF, OUTUNIT
      DIMENSION BUFF(22)
      DATA (INUNIT=1),
     * (OUTUNIT=2),
     * (BUFF=1,
     *       2,
     *       3,
     *       4,
     *       5,
     *       6,
     *       7,
     *       8,
     *       9,
     *      10,
     *      11,
     *      12,
     *      13,
     *      14,
     *      15,
     *      16,
     *      17,
     *      18,
     *      19,
     *      20,
     *      21,
     *      22)
      WRITE (OUTUNIT,200)BUFF
      COUTNER=0
10    DO 20 I=1,22
      BUFF(I)=FIN(INUNIT)
      IF (BUFF(I).EQ.-99999) GO TO 30
20    CONTINUE
      WRITE (OUTUNIT,100)COUNTER,BUFF
      COUNTER=COUNTER+22
      GO TO 10
30    J=I-1
      WRITE (OUTUNIT,100)COUNTER,(BUFF(I),I=1,J)
      STOP
100   FORMAT (≠ ≠I5≠/≠22I5)
200   FORMAT (≠1≠6X,110(≠-≠)/≠ ≠6X,22I5/≠ ≠6X,110(≠-≠)/)
      END
```

```
      PROGRAM EDITOR
C
C ********** BEHROOZ SATVAT / OSU CHEMICAL ENGINEERING
C ********** JUNE 20, 1974
C
      INTEGER BEGINDX,ENDINDX,POINTER,Y,BUFF,OUTUNIT
      DIMENSION BUFF(22)
      DATA (POINTER=1),(LENGTH=1000)
      WRITE (61,9000)
      NOFG=FFIN(60)
      DO 70 I1=1,NOFG
      OUTUNIT=I1+1
      CALL EQUIP (OUTUNIT,8HFILE     )
      WRITE (61,9001) I1
      NOFS=FFIN(60)
      DO 60 I2=1,NOFS
      WRITE (61,9002) I2
      BEGINDX=FFIN(60)
10    IF (POINTER-BEGINDX) 30,40,20
20    STOP 00001
30    POINTER=POINTER+1
      Y=FIN(1)
      GO TO 10
40    IF (I2.LT.NOFS) GO TO 45
      WRITE (61,9004) I2
      ENDINDX=FFIN(60)
      LENGTH=ENDINDX-BEGINDX+1
45    MAX=POINTER+LENGTH
      FLAG=0
47    DO 50 I4=1,22
      IF (POINTER.LT.MAX) GO TO 49
      FLAG=1
      GO TO 55
49    BUFF(I4)=FIN(1)
50    POINTER=POINTER+1
55    J=I4-1
      WRITE (OUTUNIT,9003)(BUFF(I),I=1,J)
      IF (.NOT.FLAG) GO TO 47
60    CONTINUE
      ENDFILE OUTUNIT
      WRITE (61,9005)I1,OUTUNIT
70    LENGTH=1000
      STOP
9000  FORMAT (≠ NUMBER OF GRAPHS ^≠)
9001  FORMAT (≠ NUMBER OF SETS IN GRAPH≠I2≠ ^≠)
9002  FORMAT (≠ INDEX OF THE 1ST ELEMENT OF SET≠I2≠ ^≠)
9003  FORMAT (22I5)
9004  FORMAT (≠ INDEX OF THE LAST ELEMENT OF SET≠I2
     *   ≠ (LAST SET) ^≠)
9005  FORMAT (≠-EDITTED FILE OF GRAPH≠I2≠ IS WRITTEN≠
     *        ≠ ONTO LUN≠I2)
      END
```

```
      PROGRAM SATVAT1B
      DIMENSION P(2500)
      READ(2,10) (P(N),N=1,2500)
   10 FORMAT(22F5.0)
      WRITE(3,100) (P(N),N=1001,1006)
  100 FORMAT(4X,5E10.3)
      AREA0=0.
      AREA1=0.
      AREA2=0.
      AREA3=0.
      AREA4=0.
      AREA5=0.
      L1=216.
      DO 50 N=L1,2500
      M=N-L1
      XM=M
      THETA=XM/335.
      W=EXP(-THETA)
      Y=SQRT(2.0)
      FL0=(W)*(Y)
      FL1=(W)*(THETA+THETA-1.)*(Y)
      FL2=(W)*(THETA*(THETA+THETA-4.)+1.)*(Y)
      FL3=(W)*(THETA*(THETA*((4./3.)*THETA-6.)+6.)-1.)*(Y)
      FL4=(W)*(THETA*(THETA*(THETA*((2./3.)*THETA-(16./3.))
     $+12.)-8.)+1.)*(Y)
      FL5=(W)*(THETA*(THETA*(THETA*(THETA*((4./15.)*THETA
     $-(10./3.))+(40./3.))-20.)+10.)-1.)*(Y)
      PROD0=(P(N)/100.)*(FL0/335.)
      PROD1=(P(N)/100.)*(FL1/335.)
      PROD2=(P(N)/100.)*(FL2/335.)
      PROD3=(P(N)/100.)*(FL3/335.)
      PROD4=(P(N)/100.)*(FL4/335.)
      PROD5=(P(N)/100.)*(FL5/335.)
      AREA0=AREA0+PROD0
      AREA1=AREA1+PROD1
      AREA2=AREA2+PROD2
      AREA3=AREA3+PROD3
      AREA4=AREA4+PROD4
   50 AREA5=AREA5+PROD5
      WRITE(3,20) AREA0,AREA1,AREA2,AREA3,AREA4,AREA5
   20 FORMAT(3X,≠AREA0=≠,E20.9///3X,≠AREA1=≠,E20.9////3X,
     $≠AREA2=≠,E20.9///3X,≠AREA3=≠,E20.9///3X.
     $≠AREA4=≠,E20.9///3X,≠AREA5=≠,E20.9)
      END
```

```
      PROGRAM SATVAT2B
      DIMENSION P(2500)
      READ(2,10) (P(N),N=1,2500)
   10 FORMAT(22F5.0)
      WRITE(3,100) (P(N),N=1,20)
  100 FORMAT(4X,5E10.3)
      AREA0=0.
      AREA1=0.
      AREA2=0.
      AREA3=0.
      AREA4=0.
      AREA5=0.
      L2=498.
      DO 50 N=L2.2500
      M=N-L2
      XM=M
      THETA=XM/605.
      W=EXP(-THETA)
      Y=SQRT(2.0)
      FL0=(W)*(Y)
      FL1=(W)*(THETA+THETA-1.)*(Y)
      FL2=(W)*(THETA*(THETA+THETA-4.)+1.)*(Y)
      FL3=(W)*(THETA*(THETA*((4./3.)*THETA-6.)+6.)-1.)*(Y)
      FL4=(W)*(THETA*(THETA*(THETA*((2./3.)*THETA-(16./3.))
     $+12.)-8.)+1.)*(Y)
      FL5=(W)*(THETA*(THETA*(THETA*(THETA*((4./15.)*THETA
     $-(10./3.))+(40./3.))-20.)+10.)-1.)*(Y)
      PROD0=(P(N)/100.)*(FL0/605.)
      PROD1=(P(N)/100.)*(FL1/605.)
      PROD2=(P(N)/100.)*(FL2/605.)
      PROD3=(P(N)/100.)*(FL3/605.)
      PROD4=(P(N)/100.)*(FL4/605.)
      PROD5=(P(N)/100.)*(FL5/605.)
      AREA0=AREA0+PROD0
      AREA1=AREA1+PROD1
      AREA2=AREA2+PROD2
      AREA3=AREA3+PROD3
      AREA4=AREA4+PROD4
   50 AREA5=AREA5+PROD5
      WRITE(3,20) AREA0,AREA1,AREA2,AREA3,AREA4,AREA5
   20 FORMAT(3X,≠AREA0=≠,E20.9//3X,≠AREA1=≠,E20.9///3X,
     $≠AREA2=≠,E20.9///3X,≠AREA3=≠,E20.9///3X,
     $≠AREA4=≠,E20.9///3X,≠AREA5=≠,E20.9)
      END
```

```
      PROGRAM SATVAT5B
      DIMENSION P(4890)
      READ(2,10) (P(N),N=1,4890)
   10 FORMAT(22F5.0)
      WRITE(3,100) (P(N),N=2150,2160)
  100 FORMAT(4X.5E10.3)
      AREA0=0.
      AREA1=0.
      AREA2=0.
      AREA3=0.
      AREA4=0.
      AREA5=0.
      L5=1520.
      DO 50 N=L5.4890
      M=N-L5
      XM=M
      THETA=XM/1250.
      W=EXP(-THETA)
      Y=SQRT(2.0)
      FL0=(W)*(Y)
      FL1=(W)*(THETA+THETA-1.)*(Y)
      FL2=(W)*(THETA*(THETA+THETA-4.)+1.)*(Y)
      FL3=(W)*(THETA*(THETA*((4./3.)*THETA-6.)+6.)-1.)*(Y)
      FL4=(W)*(THETA*(THETA*(THETA*((2./3.)*THETA-(16./3.))
     $+12.)-8.)+1.)*(Y)
      FL5=(W)*(THETA*(THETA*(THETA*(THETA*((4./15.)*THETA
     $-(10./3.))+(40./3.))-20.)+10.)-1.)*(Y)
      PROD0=(P(N)/100.)*(FL0/1250.)
      PROD1=(P(N)/100.)*(FL1/1250.)
      PROD2=(P(N)/100.)*(FL2/1250.)
      PROD3=(P(N)/100.)*(FL3/1250.)
      PROD4=(P(N)/100.)*(FL4/1250.)
      PROD5=(P(N)/100.)*(FL5/1250.)
      AREA0=AREA0+PROD0
      AREA1=AREA1+PROD1
      AREA2=AREA2+PROD2
      AREA3=AREA3+PROD3
      AREA4=AREA4+PROD4
   50 AREA5=AREA5+PROD5
      WRITE(3,20) AREA0,AREA1,AREA2,AREA3,AREA4,AREA5
   20 FORMAT(3X,≠AREA0=≠,E20.9///3X,≠AREA1=≠,E20.9///3X,
     $≠AREA2=≠,E20.9///3X,≠AREA3=≠,E20.9///3X,
     $≠AREA4=≠,E20.9///3X,≠AREA5=≠,E20.9)
      END
```

```
      PROGRAM DATAGEN1B
      DIMENSION BUFFER (10)
      FD3=4./3.
      TD3=2./3.
      SIXTD3=16./3.
      FD15=4./15.
      TEND3=10./3.
      FORTYD3=40./3.
      A0=FFIN(48)
      A1=FFIN(48)
      A2=FFIN(48)
      A3=FFIN(48)
      A4=FFIN(48)
      A5=FFIN(48)
      SQRT2=SQRT(2.0)
      T=0.0
10    DO 20 I=1,10
      T2=T*T
      T3=T2*T
      T4=T3*T
      T5=T4*T
      Y=SQRT2*EXP(-T)*
     *      (A0+A1*(T+T-1.0)+A2*(T2+T2-4.0*T+1.0)
     *      +A3*(FD3*T3-6.0*T2+6.0*T-1.0)
     *      +A4*(TD3*T4-SIXTD3*T3+12.0*T2-8.0*T
     *      +1.0)
     *      +A5*(FD15*T5-TEND3*T4+FORTYD3*T3
     *      -20.0*T*T+10.0*T-1.0))
      BUFFER(I)=Y*100.0
      T=T+0.002985
      IF(T.GE.6.701325) GO TO 30
20    CONTINUE
      WRITE (2,100) BUFFER
      GO TO 10
30    J=I-1
      WRITE (2,100)(BUFFER(I),I=1,J)
      STOP
100   FORMAT (5E14.6)
      END
```

```
      PROGRAM DATAGEN2B
      DIMENSION BUFFER (10)
      FD3=4./3.
      TD3=2./3.
      SIXTD3=16./3.
      FD15=4./15.
      TEND3=10./3.
      FORTYD3=40./3.
      A0=FFIN(48)
      A1=FFIN(48)
      A2=FFIN(48)
      A3=FFIN(48)
      A4=FFIN(48)
      A5=FFIN(48)
      SQRT2=SQRT(2.0)
      T=0.0
10    DO 20 I=1,10
      T2=T*T
      T3=T2*T
      T4=T3*T
      T5=T4*T
      Y=SQRT2*EXP(-T)*
     *      (A0+A1*(T+T-1.0)+A2*(T2+T2-4.0*T+1.0)
     *      +A3*(FD3*T3-6.0*T2+6.0*T-1.0)
     *      +A4*(TD3*T4-SIXTD3*T3+12.0*T2-8.0*T
     *      +1.0)
     *      +A5*(FD15*T5-TEND3*T4+FORTYD3*T3
     *      -20.0*T*T+10.0*T-1.0))
      BUFFER(I)=Y*100.0
      T=T+0.0016529
      IF(T.GE.3.07935) GO TO 30
20    CONTINUE
      WRITE (2,100) BUFFER
      GO TO 10
30    J=I-1
      WRITE (2,100)(BUFFER(I),I=1,J)
      STOP
100   FORMAT (5E14.6)
      END
```

```
      PROGRAM DATAGEN5B
      DIMENSION BUFFER (10)
      FD3=4./3.
      TD3=2./3.
      SIXTD3=16./3.
      FD15=4./15.
      TEND3=10./3.
      FORTYD3=40./3.
      A0=FFIN(48)
      A1=FFIN(48)
      A2=FFIN(48)
      A3=FFIN(48)
      A4=FFIN(48)
      A5=FFIN(48)
      SQRT2=SQRT(2.0)
      T=0.0
10    DO 20 I=1,10
      T2=T*T
      T3=T2*T
      T4=T3*T
      T5=T4*T
      Y=SQRT2*EXP(-T)*
     *      (A0+A1*(T+T-1.0)+A2*(T2+T2-4.0*T+1.0)
     *      +A3*(FD3*T3-6.0*T2+6.0*T-1.0)
     *      +A4*(TD3*T4-SIXTD3*T3+12.0*T2-8.0*T
     *      +1.0)
     *      +A5*(FD15*T5-TEND3*T4+FORTYD3*T3
     *      -20.0*T*T+10.0*T-1.0))
      BUFFER(I)=Y*100.0
      T=T+0.0008
      IF(T.GE.2.6584) GO TO 30
20    CONTINUE
      WRITE (2,100) BUFFER
      GO TO 10
30    J=I-1
      WRITE (2,100)(BUFFER(I),I=1,J)
      STOP
100   FORMAT (5E14.6)
      END
```

```
      PROGRAM PLOT1B
      INTEGER TEMP1,YESNO
      WRITE (61,100)
10    READ (60,200)YESNO
      IF (YESNO.EQ.≠NO  ≠) STOP 1
      IF (YESNO.EQ.≠YES ≠) GO TO 15
      GO TO 10
15    CALL PLOTLUN (3)
      CALL ERASE
      CALL FSCALE (.4,2.,5.,80.)
      CALL AXIS (-10.,2471.,-40.,310.,100.,10.)
      REWIND 1
      X=0.0
20    Y=FIN(1)
      CALL PLOT (X,Y,1,0)
      X=X+1.0
      IF (X.LE.2460.) GO TO 20
      CALL PLOT (1200.,10.,0,0)
      CALL WRITEY (1.,0.,8HTIME    )
      X=100.
30    CALL PLOT (X,-10.0,0,0)
      TEMP1=X*0.4
      CALL XLATE (TEMP1,TEMP)
      CALL WRITEY (1.,-90.,TEMP)
      X=X+100.0
      IF (X.LE.2500.0) GO TO 30
      CALL PLOT (70.,130.,0,0)
      CALL WRITEY (1.,90.,8HVOLT    )
      CALL PAGE
      CALL BYENOW
100   FORMAT (≠ WARNING ∨≠/
     *        ≠   BEFORE THE RUN ALL OF THE FOLLOWING FILES≠/
     *        ≠   MUST BE DEFINED AS SHOWN:≠/
     *        ≠     ≥EQUIP,1=<FIRST PLOT DATA>≠/
     *        ≠     ≥EQUIP,3=PLOT≠/
     *        ≠     ≥LABEL,3/<YOUR NAME>≠//
     *        ≠ R E A D Y ∧≠)
200   FORMAT (A4)
      END
```

```
      PROGRAM PLOT2B
      INTEGER TEMP1,YESNO
      WRITE (61,100)
10    READ (60,200)YESNO
      IF (YESNO.EQ.≠NO  ≠) STOP 1
      IF (YESNO.EQ.≠YES ≠) GO TO 15
      GO TO 10
15    CALL PLOTLUN (3)
      CALL ERASE
      CALL FSCALE (.4,2.,5.,80.)
      CALL AXIS (-10.,2371.,-40.,310.,100.,10.)
      REWIND 1
      X=0.0
20    Y=FIN(1)
      CALL PLOT (X,Y,1,0)
      X=X+1.0
      IF (X.LE.2360.) GO TO 20
      CALL PLOT (1200.,10.,0,0)
      CALL WRITEY (1.,0.,8HTIME    )
      X=100.
30    CALL PLOT (X,-10.0,0,0)
      TEMP1=X*0.4
      CALL XLATE (TEMP1,TEMP)
      CALL WRITEY (1.,-90.,TEMP)
      X=X+100.0
      IF (X.LE.2500.0) GO TO 30
      CALL PLOT (70.,130.,0,0)
      CALL WRITEY (1.,90.,8HVOLT    )
      CALL PAGE
      CALL BYENOW
100   FORMAT (≠ WARNING ∨≠/
     *        ≠   BEFORE THE RUN ALL OF THE FOLLOWING FILES≠/
     *        ≠   MUST BE DEFINED AS SHOWN:≠/
     *        ≠     ≥EQUIP,1=<FIRST PLOT DATA>≠/
     *        ≠     ≥EQUIP,3=PLOT≠/
     *        ≠     ≥LABEL,3/<YOUR NAME>≠//
     *        ≠ R E A D Y ∧≠)
200   FORMAT (A4)
      END
```

```
      PROGRAM PLOT5B
      INTEGER TEMP1,YESNO
      WRITE (61,100)
10    READ (60,200)YESNO
      IF (YESNO.EQ.≠NO  ≠) STOP 1
      IF (YESNO.EQ.≠YES ≠) GO TO 15
      GO TO 10
15    CALL PLOTLUN (3)
      CALL ERASE
      CALL FSCALE (.2,2.,5.,80.)
      CALL AXIS (-10.,4841.,-40.,310.,100.,10.)
      REWIND 1
      X=0.0
20    Y=FIN(1)
      CALL PLOT (X,Y,1,0)
      X=X+1.0
      IF (X.LE.4830.) GO TO 20
      CALL PLOT (2300.,10.,0,0)
      CALL WRITEY (1.,0.,8HTIME    )
      X=100.
30    CALL PLOT (X,-10.0,0,0)
      TEMP1=X*0.4
      CALL XLATE (TEMP1,TEMP)
      CALL WRITEY (1.,-90.,TEMP)
      X=X+100.0
      IF (X.LE.5000.0) GO TO 30
      CALL PLOT (70.,130.,0,0)
      CALL WRITEY (1.,90.,8HVOLT    )
      CALL PAGE
      CALL BYENOW
100   FORMAT (≠ WARNING ∨≠/
     *        ≠   BEFORE THE RUN ALL OF THE FOLLOWING FILES≠/
     *        ≠   MUST BE DEFINED AS SHOWN:≠/
     *        ≠     ≥EQUIP,1=<FIRST PLOT DATA>≠/
     *        ≠     ≥EQUIP,3=PLOT≠/
     *        ≠     ≥LABEL,3/<YOUR NAME>≠//
     *        ≠ R E A D Y ∧≠)
200   FORMAT (A4)
      END
```

```
      PROGRAM PLOTTER1B
      INTEGER YESNO
      WRITE (61,100)
10    READ (60,200)YESNO
      IF (YESNO.EQ.≠NO  ≠) STOP 1
      IF (YESNO.EQ.≠YES ≠) GO TO 15
      GO TO 10
15    CALL PLOTLUN (3)
      CALL ERASE
      CALL FSCALE (.4,2.,5.,80.)
      CALL AXIS (-10.,2471.,-40.,310.,100.,10.)
      REWIND 1
      X=0.0
20    Y=FIN(1)
      CALL PLOT (X,Y,1,0)
      X=X+1.0
      IF (X.LE.2244.) GO TO 20
      CALL PLOT (1200.,10.,0,0)
      CALL WRITEY (1.,0.,8HTIME     )
      X=100.
      CALL PLOT (X,-10.0,0,0)
      CALL PLOT (70.,170.,0,0)
      CALL WRITEY (1.,90.,8HVOLT     )
      CALL PAGE
      CALL BYENOW
100   FORMAT (≠ WARNING ∨≠/
     *        ≠   BEFORE THE RUN ALL OF THE FOLLCWING FILES≠/
     *        ≠   MUST BE DEFINEC AS SHCWN!≠/
     *        ≠      ≥EQUIP,1=<PLOT DATA>≠/
     *        ≠      ≥EQUIP,3=PLOT≠/
     *        ≠      ≥LABEL,3/<YOUR NAME>≠//
     *        ≠ R E A D Y ∧≠)
200   FORMAT (A4)
      END
```

```
      PROGRAM PLOTTER2B
      INTEGER YESNO
      WRITE (61,100)
10    READ (60,200)YESNO
      IF (YESNO.EQ.≠NO  ≠) STOP 1
      IF (YESNO.EQ.≠YES ≠) GO TO 15
      GO TO 10
15    CALL PLOTLUN (3)
      CALL ERASE
      CALL FSCALE (.4,2.,5.,80.)
      CALL AXIS (-10.,2371.,-40.,310.,100.,10.)
      REWIND 1
      X=0.0
20    Y=FIN(1)
      CALL PLOT (X,Y,1,0)
      X=X+1.0
      IF (X.LE.1862.) GO TO 20
      CALL PLOT (1200.,10.,0,0)
      CALL WRITEY (1.,0.,8HTIME     )
      X=100.
      CALL PLOT (X,-10.0,0,0)
      CALL PLOT (70.,170.,0,0)
      CALL WRITEY (1.,90.,8HVOLT     )
      CALL PAGE
      CALL BYENOW
100   FORMAT (≠ WARNING ∨≠/
     *        ≠   BEFORE THE RUN ALL OF THE FOLLOWING FILES≠/
     *        ≠   MUST BE DEFINEC AS SHOWN!≠/
     *        ≠      ≥EQUIP,1=<PLOT DATA>≠/
     *        ≠      ≥EQUIP,3=PLOT≠/
     *        ≠      ≥LABEL,3/<YOUR NAME>≠//
     *        ≠ R E A D Y ∧≠)
200   FORMAT (A4)
      END
```

```
      PROGRAM PLOTTER5B
      INTEGER YESNO
      WRITE (61,100)
10    READ (60,200)YESNO
      IF (YESNO.EQ.≠NO  ≠) STOP 1
      IF (YESNO.EQ.≠YES ≠) GO TO 15
      GO TO 10
15    CALL PLOTLUN (3)
      CALL ERASE
      CALL FSCALE (.2,2.,5.,80.)
      CALL AXIS (-10.,4850.,-40.,310.,100.,10.)
      REWIND 1
      X=0.0
20    Y=FIN(1)
      CALL PLOT (X,Y,1,0)
      X=X+1.0
      IF (X.LE.3322.) GO TO 20
      CALL PLOT (2300.,10.,0,0)
      CALL WRITEY (1.,0.,8HTIME     )
      X=100.
      CALL PLOT (X,-10.0,0,0)
      CALL PLOT (70.,170.,0,0)
      CALL WRITEY (1.,90.,8HVOLT     )
      CALL PAGE
      CALL BYENOW
100   FORMAT (≠ WARNING ∨≠/
     *        ≠   BEFORE THE RUN ALL OF THE FOLLCWING FILES≠/
     *        ≠   MUST BE DEFINED AS SHOWN!≠/
     *        ≠      ≥EQUIP,1=<FIRST PLOT DATA>≠/
     *        ≠      ≥EQUIP,3=PLOT≠/
     *        ≠      ≥LABEL,3/<YOUR NAME>≠//
     *        ≠ R E A D Y ∧≠)
200   FORMAT (A4)
      END
```

```
      PROGRAM CONVO1
      A0=TTYIN(15HA0=             )
      A1=TTYIN(15HA1=             )
      A2=TTYIN(15HA2=             )
      A3=TTYIN(15HA3=             )
      A4=TTYIN(15HA4=             )
      A5=TTYIN(15HA5=             )
      B0=A0+A1+A2+A3+A4+A5
      B1=(5.)*(A0)+(3.)*(A1)+A2-A3-(3.)*(A4)-(5.)*(A5)
      B2=(10.)*(A0)+(2.)*(A1)-(2.)*(A2)-(2.)*(A3)+(2.)*(A4)
     ≤+(10.)*(A5)
      B3=(10.)*(A0)-(2.)*(A1)-(2.)*(A2)+(2.)*(A3)+(2.)*(A4)
     ≤-(10.)*(A5)
      B4=(5.)*(A0)-(3.)*(A1)+A2+A3-(3.)*(A4)+(5.)*(A5)
      B5=A0-A1+A2-A3+A4-A5
      WRITE(61,50) B0,B1,B2,B3,B4,B5
   50 FORMAT(//1X,≠B0=≠,E14.7//1X,≠B1=≠,E14.7//1X,≠B2=≠,
     ≤E14.7//1X,≠B3=≠,E14.7//1X,≠B4=≠,E14.7//1X,≠B5=≠,
     ≤E14.7//)
      END
```

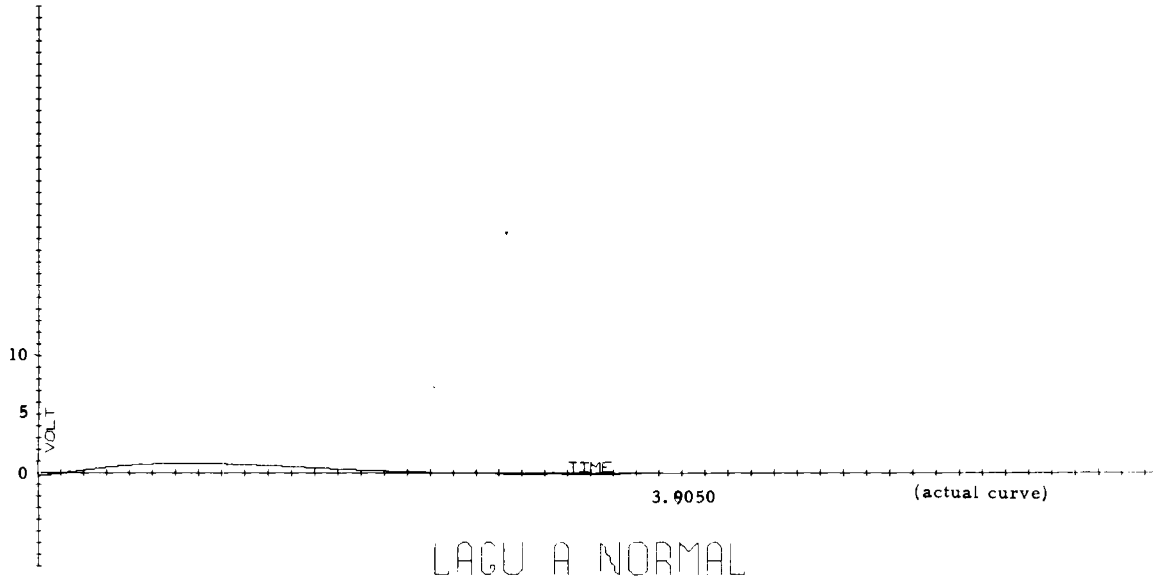
```
      PROGRAM CONVO3
      A0=TTYIN(15HA0=             )
      A1=TTYIN(15HA1=             )
      A2=TTYIN(15HA2=             )
      A3=TTYIN(15HA3=             )
      A4=TTYIN(15HA4=             )
      A5=TTYIN(15HA5=             )
      B0=A0+A1+A2+A3+A4+A5
      B1=(5.)*(A0)+(3.)*(A1)+A2-A3-(3.)*(A4)-(5.)*(A5)
      B2=(10.)*(A0)+(2.)*(A1)-(2.)*(A2)-(2.)*(A3)+(2.)*(A4)
     ≤+(10.)*(A5)
      B3=(10.)*(A0)-(2.)*(A1)-(2.)*(A2)+(2.)*(A3)+(2.)*(A4)
     ≤-(10.)*(A5)
      B4=(5.)*(A0)-(3.)*(A1)+A2+A3-(3.)*(A4)+(5.)*(A5)
      B5=A0-A1+A2-A3+A4-A5
      WRITE(61,50) B0,B1,B2,B3,B4,B5
   50 FORMAT(//1X,≠B0=≠,E14.7//1X,≠B1=≠,E14.7//1X,≠B2=≠,
     ≤E14.7//1X,≠B3=≠,E14.7//1X,≠B4=≠,E14.7//1X,≠B5=≠,
     ≤E14.7///)
      C0=(B0)*(B0)*(B0)
      C1=(3.)*(B0)*(B0)*(B1)
      C2=(3.)*(B0)*(B0)*(B2)+(3.)*(B0)*(B1)*(B1)
      C3=(3.)*(B0)*(B0)*(B3)+(6.)*(B0)*(B1)*(B2)
     ≤+(B1)*(B1)*(B1)
      C4=(6.)*(B0)*(B1)*(B3)+(3.)*(B1)*(B1)*(B2)
     ≤+(3.)*(B0)*(B2)*(B2)+(3.)*(B0)*(B0)*(B4)
      C5=(3.)*(B0)*(B0)*(B5)+(6.)*(B0)*(B1)*(B4)
     ≤+(6.)*(B0)*(B2)*(B3)+(3.)*(B1)*(B1)*(B3)
     ≤+(3.)*(B1)*(B2)*(B2)
      C6=(6.)*(B0)*(B1)*(B5)+(6.)*(B1)*(B2)*(B3)
     ≤+(6.)*(B0)*(B2)*(B4)+(3.)*(B0)*(B3)*(B3)
     ≤+(3.)*(B1)*(B1)*(B4)+(B2)*(B2)*(B2)
      C7=(6.)*(B0)*(B2)*(B5)+(6.)*(B0)*(B3)*(B4)
     ≤+(6.)*(B1)*(B2)*(B4)+(3.)*(B1)*(B1)*(B5)
     ≤+(3.)*(B1)*(B3)*(B3)+(3.)*(B2)*(B2)*(B3)
      C8=(6.)*(B0)*(B3)*(B5)+(6.)*(B1)*(B2)*(B5)
     ≤+(6.)*(B1)*(B3)*(B4)+(3.)*(B0)*(B4)*(B4)
     ≤+(3.)*(B2)*(B2)*(B4)+(3.)*(B2)*(B3)*(B3)
      C9=(6.)*(B0)*(B4)*(B5)+(6.)*(B1)*(B3)*(B5)
     ≤+(6.)*(B2)*(B3)*(B4)+(3.)*(B1)*(B4)*(B4)
     ≤+(3.)*(B2)*(B2)*(B5)+(B3)*(B3)*(B3)
      C10=(3.)*(B0)*(B5)*(B5)+(6.)*(B1)*(B4)*(B5)
     ≤+(6.)*(B2)*(B3)*(B5)+(3.)*(B2)*(B4)*(B4)
     ≤+(3.)*(B3)*(B3)*(B4)
      C11=(3.)*(B1)*(B5)*(B5)+(6.)*(B2)*(B4)*(B5)
     ≤+(3.)*(B3)*(B3)*(B5)+(3.)*(B3)*(B4)*(B4)
      C12=(3.)*(B2)*(B5)*(B5)+(6.)*(B3)*(B4)*(B5)
     ≤+(B4)*(B4)*(B4)
      C13=(3.)*(B3)*(B5)*(B5)+(3.)*(B4)*(B4)*(B5)
      C14=(3.)*(B4)*(B5)*(B5)
      C15=(B5)*(B5)*(B5)
      WRITE(61,200) C0,C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,
     ≤C11,C12,C13,C14,C15
  200 FORMAT(1X,≠C0 =≠,E14.7//1X,≠C1 =≠,E14.7//1X,≠C2 =≠,
     ≤E14.7//1X,≠C3 =≠,E14.7//1X,≠C4 =≠,E14.7//1X,≠C5 =≠,
     ≤E14.7//1X,≠C6 =≠,E14.7//1X,≠C7 =≠,E14.7//1X,≠C8 =≠,
     ≤E14.7//1X,≠C9 =≠,E14.7//1X,≠C10=≠,E14.7//1X,≠C11=≠,
     ≤E14.7//1X,≠C12=≠,E14.7//1X,≠C13=≠,E14.7//1X,≠C14=≠,
     ≤E14.7//1X,≠C15=≠,E14.7)
      END
```

```
      PROGRAM CONVC4
      A0=TTYIN(15HA0=              )
      A1=TTYIN(15HA1=              )
      A2=TTYIN(15HA2=              )
      A3=TTYIN(15HA3=              )
      A4=TTYIN(15HA4=              )
      A5=TTYIN(15HA5=              )
      B0=A0+A1+A2+A3+A4+A5
      B1=(5.)*(A0)+(3.)*(A1)+A2-A3-(3.)*(A4)-(5.)*(A5)
      B2=(10.)*(A0)+(2.)*(A1)-(2.)*(A2)-(2.)*(A3)+(2.)*(A4)
     ≤+(10.)*(A5)
      B3=(10.)*(A0)-(2.)*(A1)-(2.)*(A2)+(2.)*(A3)+(2.)*(A4)
     ≤-(10.)*(A5)
      B4=(5.)*(A0)-(3.)*(A1)+A2+A3-(3.)*(A4)+(5.)*(A5)
      B5=A0-A1+A2-A3+A4-A5
      WRITE(61,50) B0,B1,B2,B3,B4,B5
   50 FORMAT(//1X,≠B0=≠,E14.7//1X,≠B1=≠,E14.7//1X,≠B2=≠,
     ≤E14.7//1X,≠B3=≠,E14.7//1X,≠B4=≠,E14.7//1X,≠B5=≠,
     ≤E14.7///)
      D0=(B0)*(B0)*(B0)*(B0)
      D1=(4.)*(B0)*(B0)*(B0)*(B1)
      D2=(4.)*(B0)*(B0)*(B0)*(B2)+(6.)*(B0)*(B0)*(B1)*(B1)
      D3=(4.)*(B0)*(B0)*(B0)*(B3)+(12.)*(B0)*(B0)*(B1)*(B2)
     ≤+(4.)*(B0)*(B1)*(B1)*(B1)
      D4=(4.)*(B0)*(B0)*(B0)*(B4)+(12.)*(B0)*(B0)*(B1)*(B3)
     ≤+(6.)*(B0)*(B0)*(B2)*(B2)+(12.)*(B0)*(B1)*(B1)*(B2)
     ≤+(B1)*(B1)*(B1)*(B1)
      D5=(4.)*(B0)*(B0)*(B0)*(B5)+(12.)*(B0)*(B0)*(B1)*(B4)
     ≤+(12.)*(B0)*(B0)*(B2)*(B3)+(12.)*(B0)*(B1)*(B1)*(B3)
     ≤+(12.)*(B0)*(B1)*(B2)*(B2)+(4.)*(B1)*(B1)*(B1)*(B2)
      D6=(12.)*(B0)*(B0)*(B1)*(B5)+(12.)*(B0)*(B0)*(B2)*(B4)
     ≤+(12.)*(B0)*(B1)*(B1)*(B4)+(6.)*(B0)*(B0)*(B3)*(B3)
     ≤+(6.)*(B1)*(B1)*(B2)*(B2)+(24.)*(B0)*(B1)*(B2)*(B3)
     ≤+(4.)*(B1)*(B1)*(B1)*(B3)+(4.)*(B0)*(B2)*(B2)*(B2)
      D7=(12.)*(B0)*(B0)*(B2)*(B5)+(12.)*(B0)*(B0)*(B3)*(B4)
     ≤+(12.)*(B0)*(B1)*(B1)*(B5)+(12.)*(B0)*(B1)*(B3)*(B3)
     ≤+(12.)*(B0)*(B2)*(B2)*(B3)+(12.)*(B1)*(B1)*(B2)*(B3)
     ≤+(24.)*(B0)*(B1)*(B2)*(B4)+(4.)*(B1)*(B1)*(B1)*(B4)
     ≤+(4.)*(B1)*(B2)*(B2)*(B2)
      D8=(12.)*(B0)*(B0)*(B3)*(B5)+(12.)*(B1)*(B1)*(B2)*(B4)
     ≤+(12.)*(B0)*(B2)*(B2)*(B4)+(12.)*(B0)*(B2)*(B3)*(B3)
     ≤+(12.)*(B1)*(B2)*(B2)*(B3)+(6.)*(B1)*(B1)*(B3)*(B3)
     ≤+(6.)*(B0)*(B0)*(B4)*(B4)+(24.)*(B0)*(B1)*(B2)*(B5)
     ≤+(24.)*(B0)*(B1)*(B3)*(B4)+(4.)*(B1)*(B1)*(B1)*(B5)
     ≤+(B2)*(B2)*(B2)*(B2)
      D9=(12.)*(B0)*(B0)*(B4)*(B5)+(12.)*(B1)*(B1)*(B2)*(B5)
     ≤+(12.)*(B1)*(B1)*(B3)*(B4)+(12.)*(B0)*(B1)*(B4)*(B4)
     ≤+(12.)*(B1)*(B2)*(B2)*(B4)+(12.)*(B0)*(B2)*(B2)*(B5)
     ≤+(12.)*(B1)*(B2)*(B3)*(B3)+(24.)*(B0)*(B1)*(B3)*(B5)
     ≤+(24.)*(B0)*(B2)*(B3)*(B4)+(4.)*(B0)*(B3)*(B3)*(B3)
     ≤+(4.)*(B2)*(B2)*(B2)*(B3)
      D10=(6.)*(B0)*(B0)*(B5)*(B5)+(24.)*(B0)*(B1)*(B4)*(B5)
     ≤+(24.)*(B0)*(B2)*(B3)*(B5)+(24.)*(B1)*(B2)*(B3)*(B4)
     ≤+(12.)*(B1)*(B1)*(B3)*(B5)+(12.)*(B0)*(B2)*(B4)*(B4)
     ≤+(12.)*(B1)*(B2)*(B2)*(B5)+(12.)*(B0)*(B3)*(B3)*(B4)
     ≤+(6.)*(B1)*(B1)*(B4)*(B4)+(6.)*(B2)*(B2)*(B3)*(B3)
     ≤+(4.)*(B1)*(B3)*(B3)*(B3)+(4.)*(B2)*(B2)*(B2)*(B4)
      D11=(12.)*(B0)*(B1)*(B5)*(B5)+(24.)*(B0)*(B2)*(B4)*(B5)
     ≤+(12.)*(B1)*(B1)*(B4)*(B5)+(12.)*(B0)*(B3)*(B3)*(B5)
     ≤+(12.)*(B1)*(B3)*(B3)*(B4)+(12.)*(B2)*(B2)*(B3)*(B4)
     ≤+(24.)*(B1)*(B2)*(B3)*(B5)+(12.)*(B0)*(B3)*(B4)*(B4)
     ≤+(12.)*(B1)*(B2)*(B4)*(B4)+(4.)*(B2)*(B2)*(B2)*(B5)
     ≤+(4.)*(B2)*(B3)*(B3)*(B3)
      D12=(24.)*(B0)*(B3)*(B4)*(B5)+(24.)*(B1)*(B2)*(B4)*(B5)
     ≤+(12.)*(B0)*(B2)*(B5)*(B5)+(12.)*(B1)*(B3)*(B3)*(B5)
     ≤+(12.)*(B1)*(B3)*(B4)*(B4)+(12.)*(B2)*(B2)*(B3)*(B5)
     ≤+(12.)*(B2)*(B3)*(B3)*(B4)+(6.)*(B1)*(B1)*(B5)*(B5)
     ≤+(6.)*(B2)*(B2)*(B4)*(B4)+(4.)*(B0)*(B4)*(B4)*(B4)
     ≤+(B3)*(B3)*(B3)*(B3)
      D13=(12.)*(B0)*(B3)*(B5)*(B5)+(12.)*(B1)*(B2)*(B5)*(B5)
     ≤+(12.)*(B0)*(B4)*(B4)*(B5)+(12.)*(B2)*(B2)*(B4)*(B5)
     ≤+(12.)*(B2)*(B3)*(B4)*(B4)+(12.)*(B2)*(B3)*(B3)*(B5)
     ≤+(24.)*(B1)*(B3)*(B4)*(B5)+(4.)*(B1)*(B4)*(B4)*(B4)
     ≤+(4.)*(B3)*(B3)*(B3)*(B4)
      D14=(12.)*(B0)*(B4)*(B5)*(B5)+(12.)*(B1)*(B3)*(B5)*(B5)
     ≤+(12.)*(B1)*(B4)*(B4)*(B5)+(24.)*(B2)*(B3)*(B4)*(B5)
     ≤+(6.)*(B3)*(B3)*(B4)*(B4)+(6.)*(B2)*(B2)*(B5)*(B5)
     ≤+(4.)*(B2)*(B4)*(B4)*(B4)+(4.)*(B3)*(B3)*(B3)*(B5)
      D15=(4.)*(B0)*(B5)*(B5)*(B5)+(12.)*(B1)*(B4)*(B5)*(B5)
     ≤+(12.)*(B2)*(B3)*(B5)*(B5)+(12.)*(B2)*(B4)*(B4)*(B5)
     ≤+(12.)*(B3)*(B3)*(B4)*(B5)+(4.)*(B3)*(B4)*(B4)*(B4)
      D16=(4.)*(B1)*(B5)*(B5)*(B5)+(12.)*(B2)*(B4)*(B5)*(B5)
     ≤+(12.)*(B3)*(B4)*(B4)*(B5)+(6.)*(B3)*(B3)*(B5)*(B5)
     ≤+(B4)*(B4)*(B4)*(B4)
      D17=(4.)*(B2)*(B5)*(B5)*(B5)+(12.)*(B3)*(B4)*(B5)*(B5)
     ≤+(4.)*(B4)*(B4)*(B4)*(B5)
      D18=(4.)*(B3)*(B5)*(B5)*(B5)+(6.)*(B4)*(B4)*(B5)*(B5)
      D19=(4.)*(B4)*(B5)*(B5)*(B5)
      D20=(B5)*(B5)*(B5)*(B5)
      WRITE(61,100) D0,D1,D2,D3,D4,D5,D6,D7,D8,D9,D10,D11,D12,
     ≤D13,D14,D15,D16,D17,D18,D19,D20
  100 FORMAT(1X,≠D0 =≠,E14.7//1X,≠D1 =≠,E14.7//1X,≠D2 =≠,
     ≤E14.7//1X,≠D3 =≠,E14.7//1X,≠D4 =≠,E14.7//1X,≠D5 =≠,
     ≤E14.7//1X,≠D6 =≠,E14.7//1X,≠D7 =≠,E14.7//1X,≠D8 =≠,
     ≤E14.7//1X,≠D9 =≠,E14.7//1X,≠D10=≠,E14.7//1X,≠D11=≠,
     ≤E14.7//1X,≠D12=≠,E14.7//1X,≠D13=≠,E14.7//1X,≠D14=≠,
     ≤E14.7//1X,≠D15=≠,E14.7//1X,≠D16=≠,E14.7//1X,≠D17=≠,
     ≤E14.7//1X,≠D18=≠,E14.7//1X,≠D19=≠,E14.7//1X,≠D20=≠,
     ≤E14.7)
      END
```

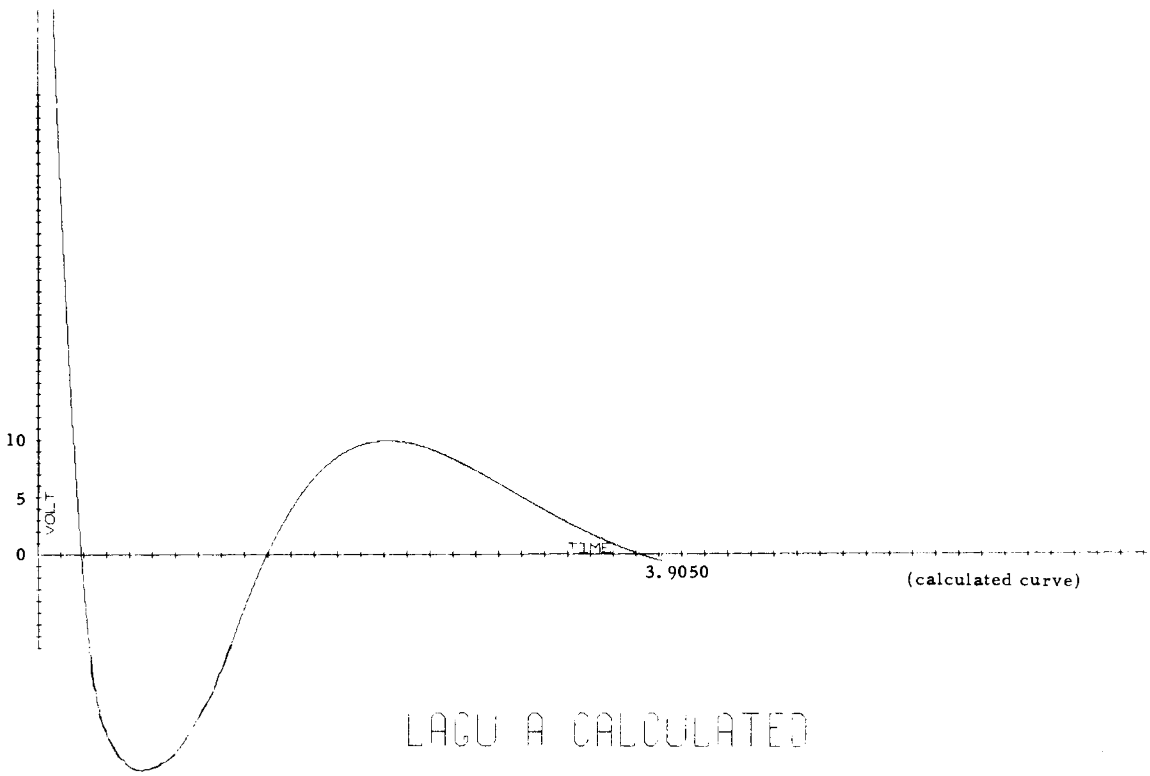
```
      PROGRAM INVERSE
      B0=TTYIN(15HB0=              )
      B1=TTYIN(15HB1=              )
      B2=TTYIN(15HB2=              )
      B3=TTYIN(15HB3=              )
      B4=TTYIN(15HB4=              )
      B5=TTYIN(15HB5=              )
      A0=(1./32.)*(B0+B1+B2+B3+B4+B5)
      A1=(1./32.)*((5)*B0+(3)*B1+B2-B3-(3)*B4-(5)*B5)
      A2=(1./32.)*((10)*B0+(2)*B1-(2)*B2-(2)*B3+(2)*B4+(10)*B5)
      A3=(1./32.)*((10)*B0-(2)*B1-(2)*B2+(2)*B3+(2)*B4-(10)*B5)
      A4=(1./32.)*((5)*B0-(3)*B1+B2+B3-(3)*B4+(5)*B5)
      A5=(1./32.)*(B0-B1+B2-B3+B4-B5)
      WRITE(61,50) A0,A1,A2,A3,A4,A5
   50 FORMAT(//1X,≠A0=≠,E14.7//1X,≠A1=≠,E14.7//1X,≠A2=≠,
     ≤E14.7//1X,≠A3=≠,E14.7//1X,≠A4=≠,E14.7//1X,≠A5=≠,
     ≤E14.7)
      END
```

APPENDIX B

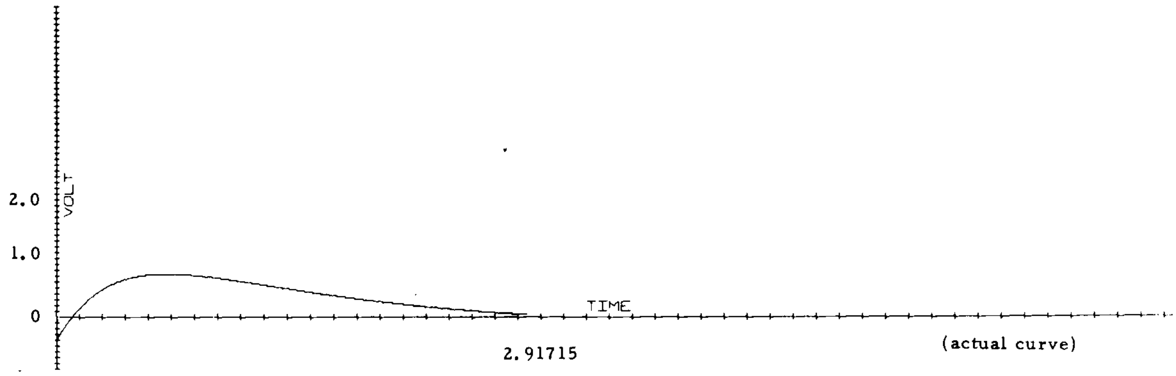
Final Results For All Sets



LAGU A NORMAL



LAGU A CALCULATED



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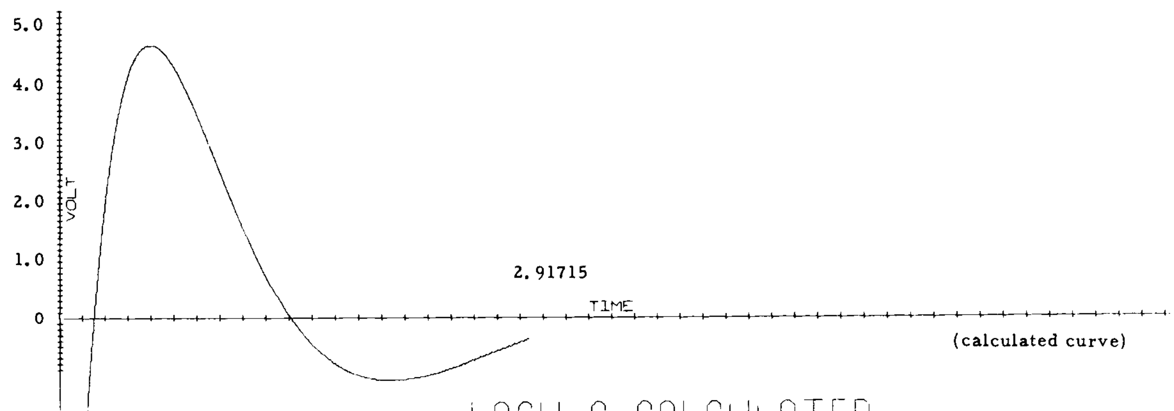
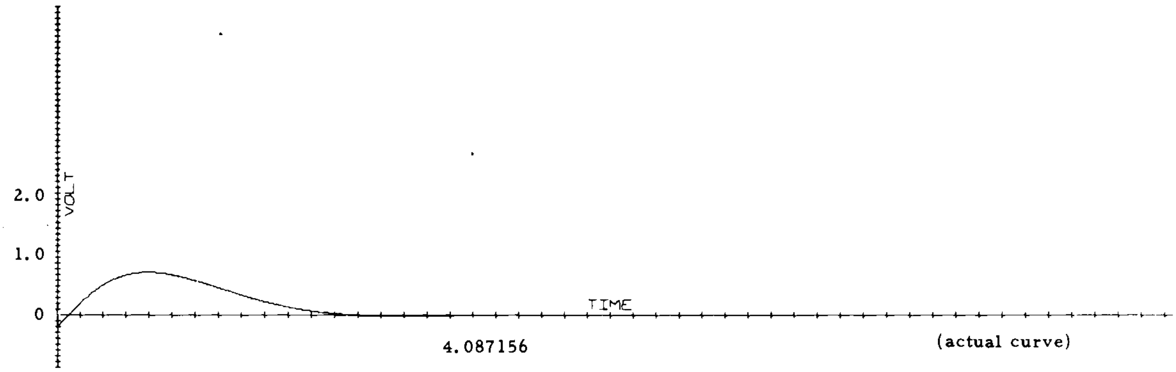
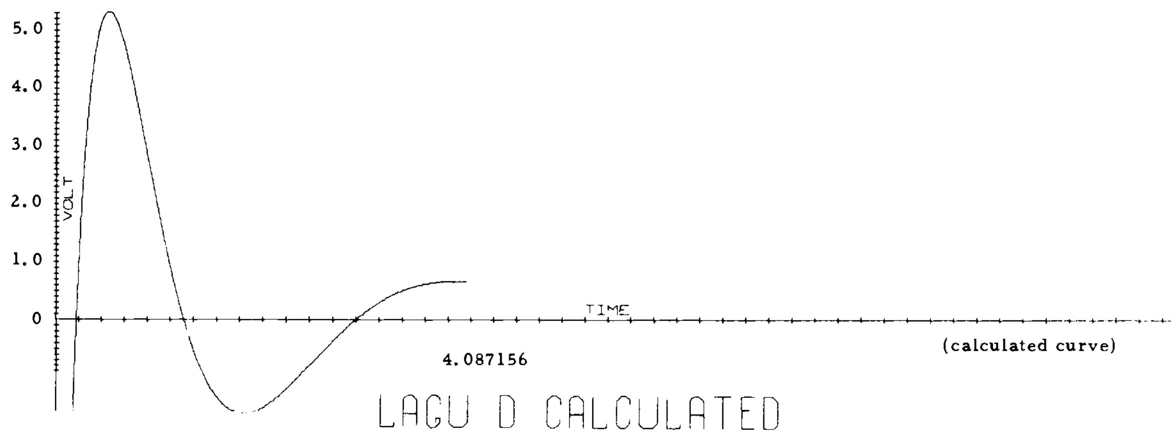


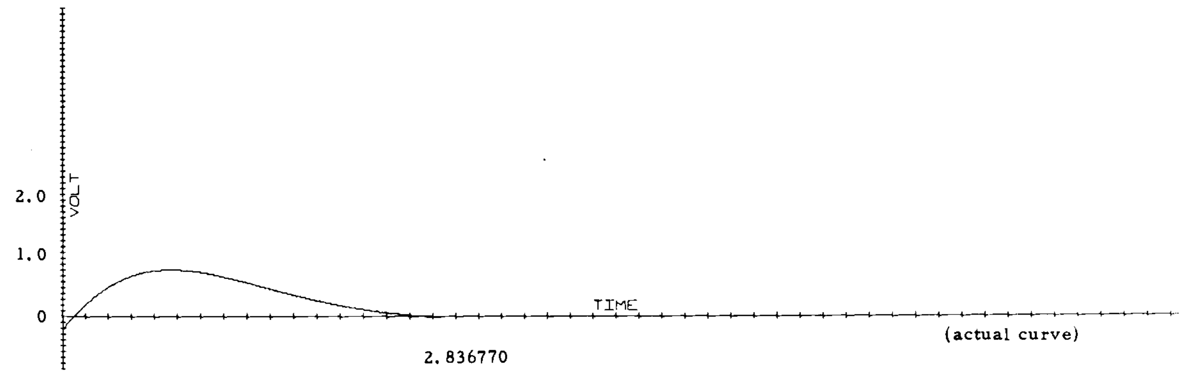
LAGU C CALCULATED



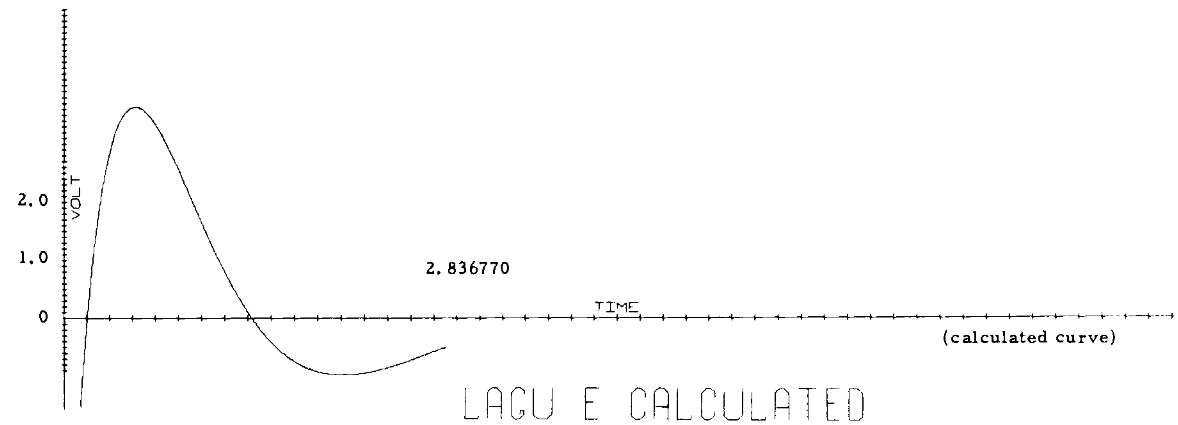
LAGU D NORMAL



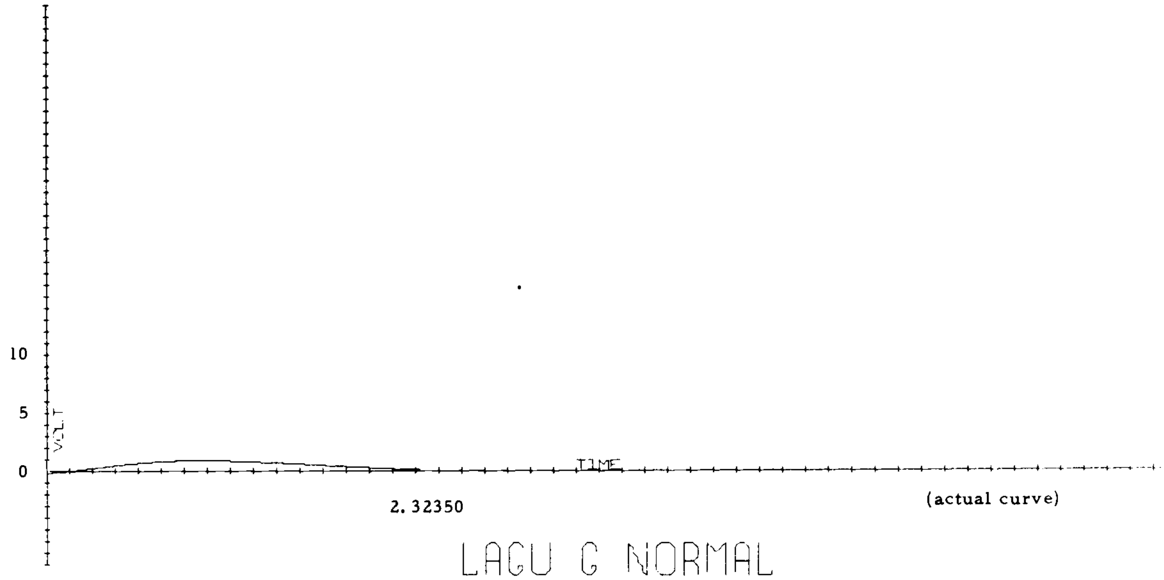
LAGU D CALCULATED

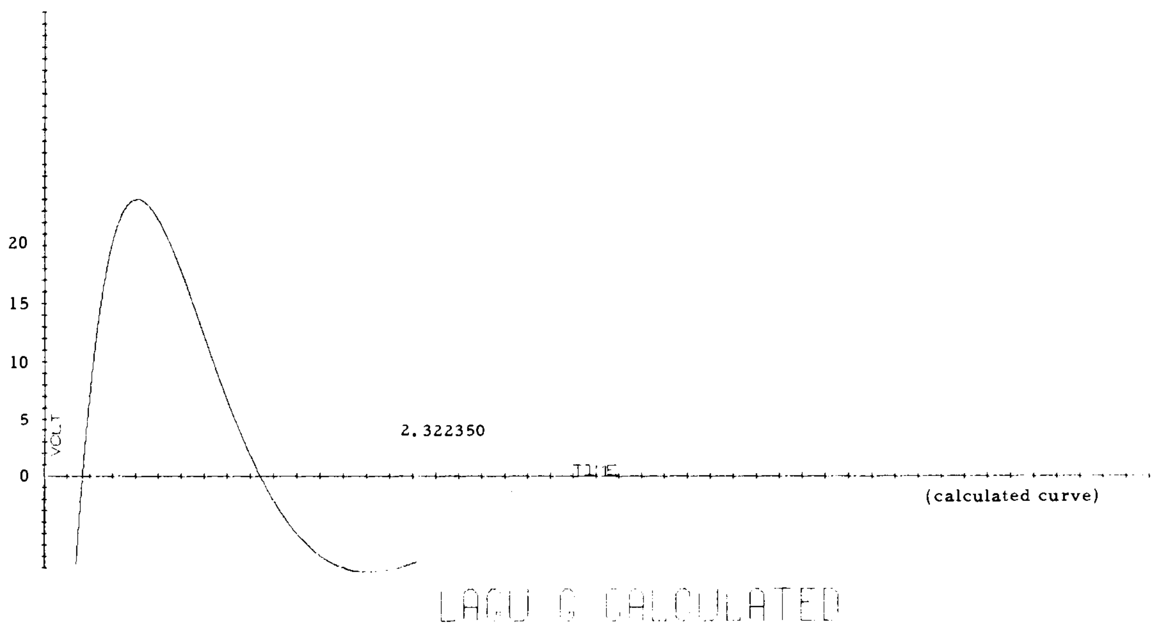


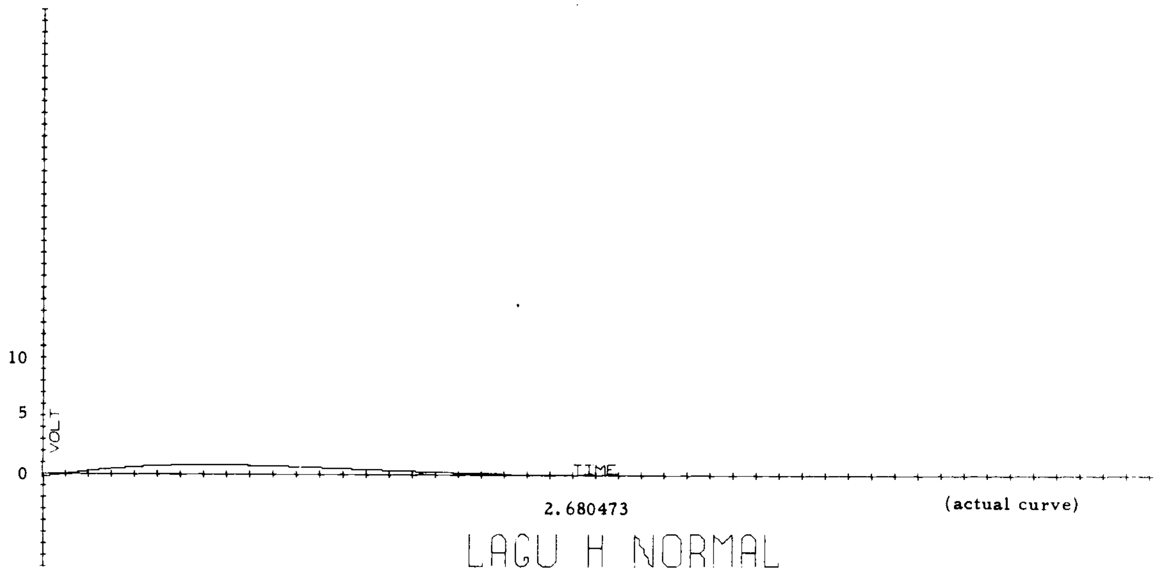
LAGU E NORMAL

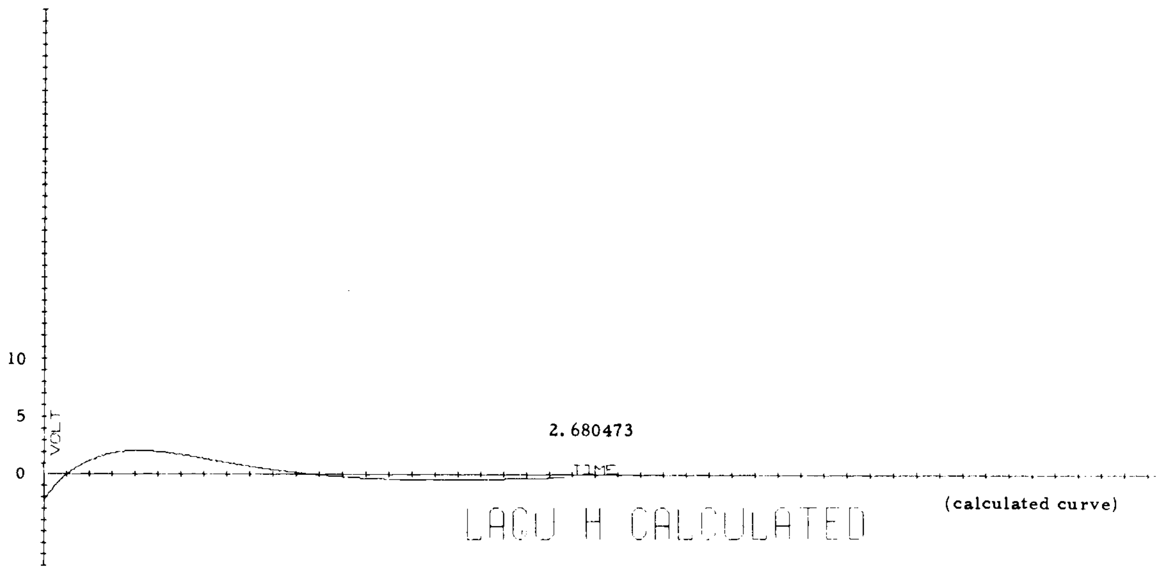


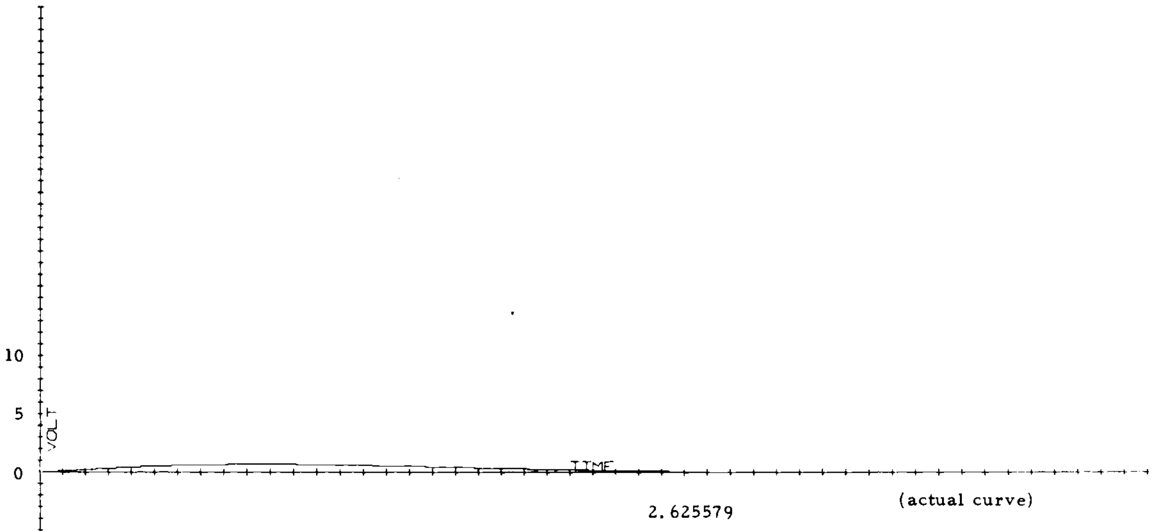
LAGU E CALCULATED


10
5
0
VOLT
TIME
2.32350
(actual curve)
LAGU G NORMAL

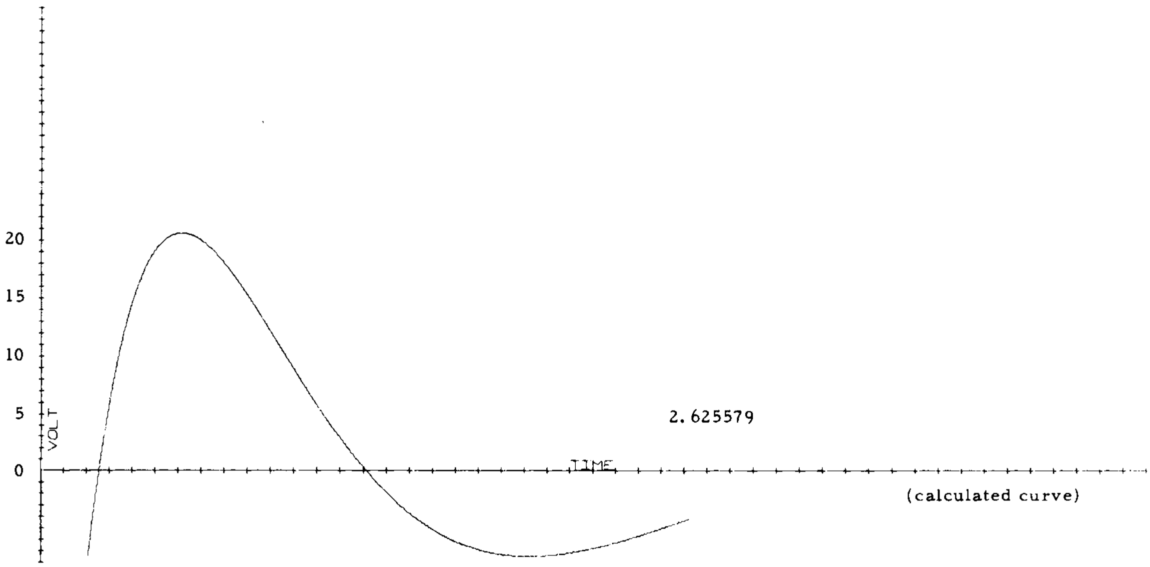

20
15
10
5
0
VOLT
2.322350
TIME
(calculated curve)
LAGU G CALCULATED


10
5
0
VOLT
TIME
2.680473
(actual curve)
LAGU H NORMAL

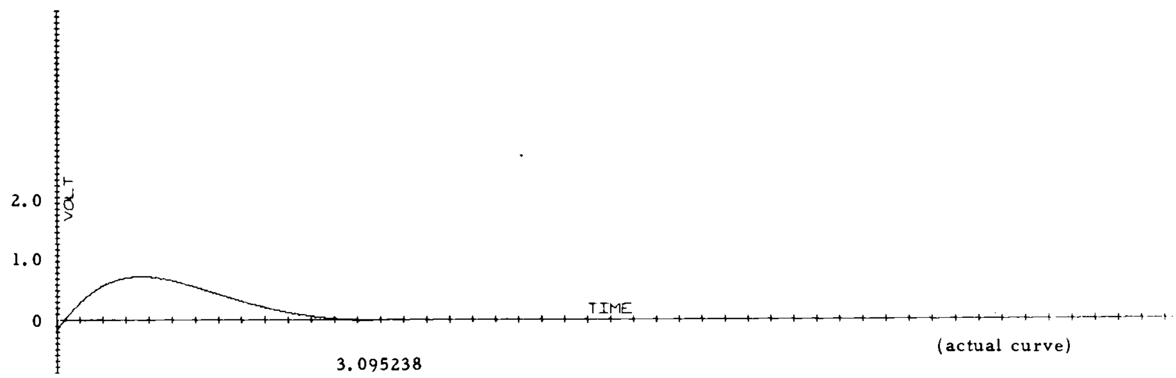

10
5
0
VOLT
2.680473
TIME
(calculated curve)
LAGU H CALCULATED



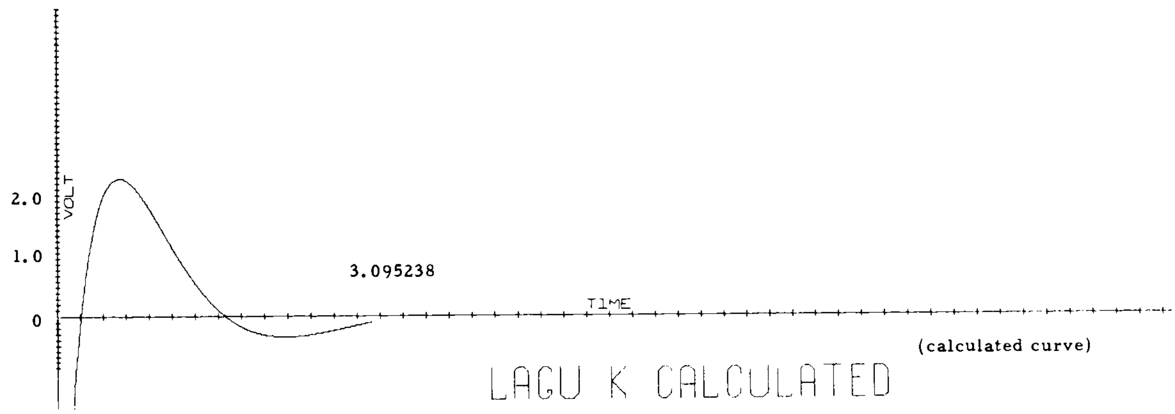
LAGU I NORMAL



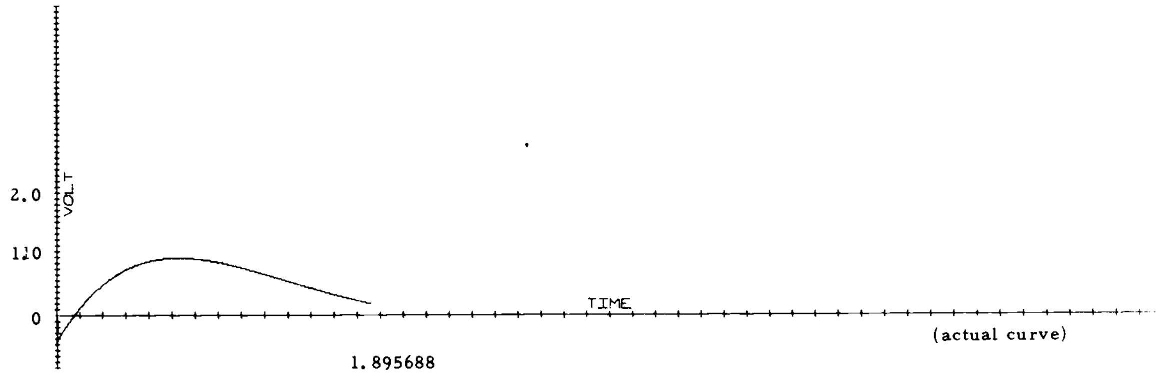
LAGU I CALCULATED



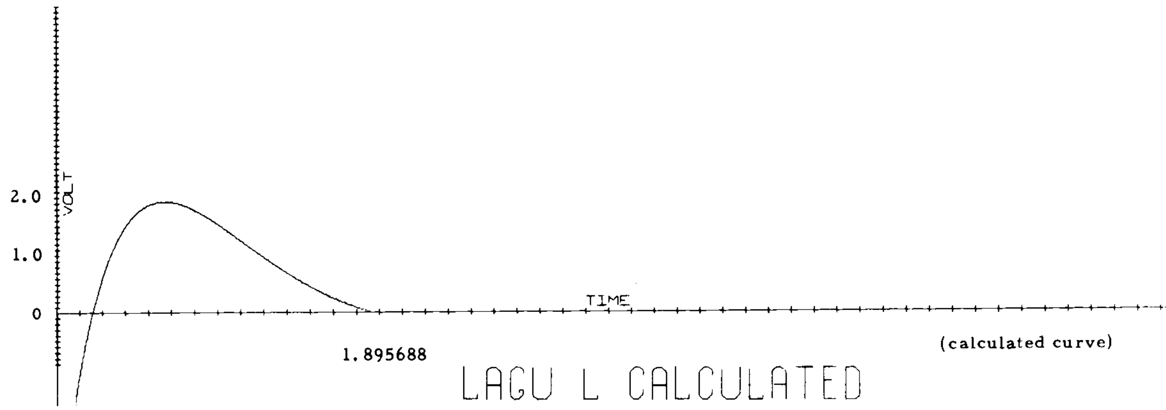
LAGU K NORMAL



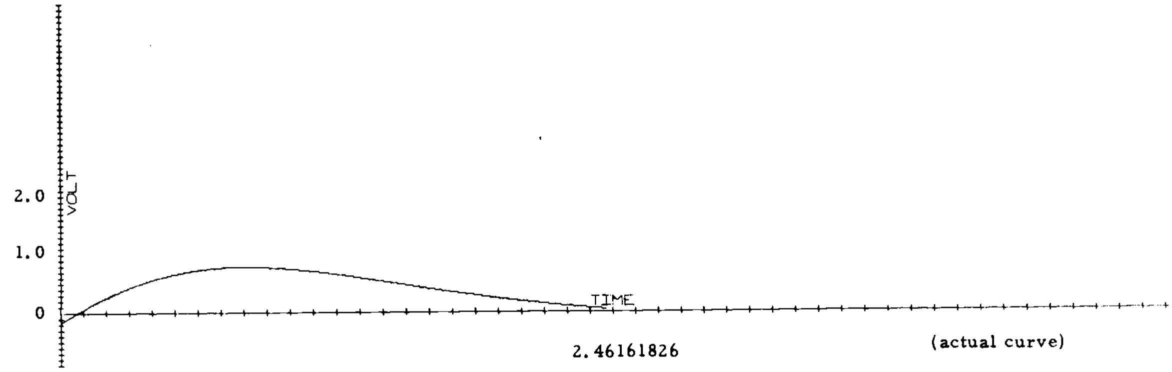
LAGU K CALCULATED



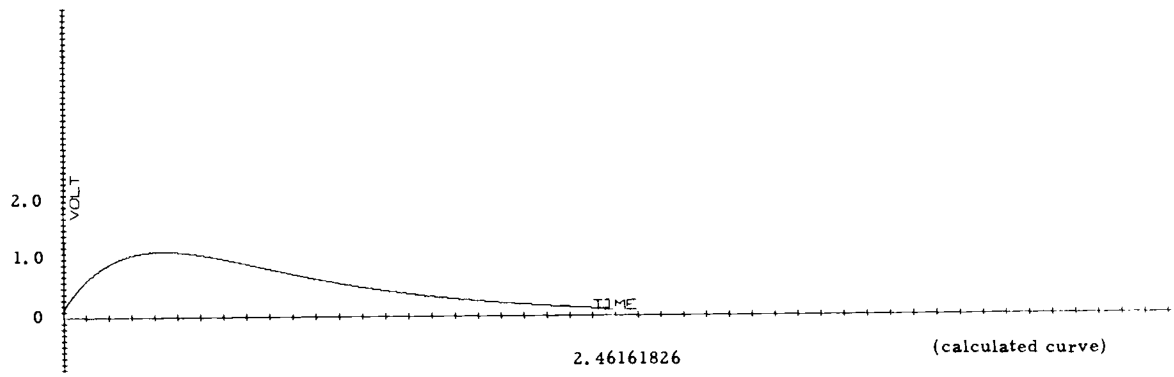
LAGU L NORMAL



LAGU L CALCULATED



LAGU M NORMAL



LAGU M CALCULATED