AN ABSTRACT OF THE DISSERTATION OF

Parnian Hosseini for the degree of Doctor of Philosophy in Civil Engineering presented on November 11, 2016.

Title: Multi-objective Optimization of Reservoir Operation Under Uncertainty with Robust and Flexible Decision Variables

Abstract approved:____

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Optimization of reservoir operation is involves various competing objectives for a scarce resource (water). To find the optimal operation of reservoirs, it is essential to consider multiple objectives simultaneously. There are various sources of uncertainty associated with the reservoir operation problem that should be considered as well.

The overarching goal of this research is to develop a framework for finding flexible and reliable solutions to the reservoir operation problem with competing objectives. Because some sources of uncertainty are not well quantified, providing flexible decision variables lets the decision maker choose accordingly from a range of options knowing that all the flexible decision variables are feasible with a specified probability of failure and that are relatively optimal. To accomplish this goal, each flexible decision variable is represented by a random variable within a specific range instead of a single deterministic decision variable. An additional objective is added to the optimization problem, in order to maximize the flexibility of decision variables. The proposed methodology is tested for two mathematical test problems and the operation of the Grand Coulee reservoir, which is located on the Columbia River in the Northwestern United States. The Stochastic Collocation (SC) method is used to sample the random variables and approximate the expected values of the objectives.

For the Grand Coulee reservoir, the decision variables are the daily turbine outflows. The first objective of the optimization is to minimize the forebay elevation deviation at the end of the optimization period. The second objective is to maximize the revenue from the hydropower production. The results show that the proposed methodology could find some flexible decision variables with 45% coefficient of variation. The corresponding expected objectives have less than 20% deterioration from the deterministic Pareto solutions. However, the number of function evaluations increases exponentially with the number of decision variables. Therefore, this methodology is suggested for problems with a few decision variables.

For finding flexible decision variables in problems with many decision variables, a dimension reduction method called Karhunen Loeve (KL) expansion is implemented in the optimization problem. By extracting useful information from the decision variables, the decision space can be represented with merely a few random variables using a set of deterministic decision variables. The results show that three random variables are sufficient to generate decision variable realizations which have mean and variance less than 1% and 5% different from the original decision variable realizations, respectively. The proposed methodology is capable of efficiently finding flexible decision variables that lead to expected objective values close to the Pareto deterministic solutions. To force the generated decision variable realizations to stay within the feasible bounds and therefore reduce the number of constraints that need to be checked, the data is transformed to be within bounds first, and then the KL-expansion is performed. Using the transformed data decreases the computational time but the decrease in computational time is not significant.

The inflow uncertainty is also considered as the only source of input uncertainty. Forecast inflow ensembles can be used as the source of inflow uncertainty. However in this study due to lack of information, historical inflows are used instead. The inflow uncertainties are represented using the KL-expansion. Robust optimization is performed by optimizing the weighted sum of the expectation and standard deviation of the objective due to uncertain inflows. The weights in the robust objective formulation can be changed based on the decision maker's preference of robustness versus performance.

Finally, the combined framework to find robust and flexible decision variables is tested on a reservoir operation problem and the results were compared to the deterministic case.

Multi-objective Optimization of Reservoir Operation Under Uncertainty with Robust and Flexible Decision Variables

by
Parnian Hosseini

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APPROVED:

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I understand that my dissertation will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my dissertation to any reader upon request.

Parnian Hosseini, Author

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CONTRIBUTION OF AUTHORS

Dr. Arturo Leon (Associate Professor at University of Houston), Dr. Nathan Gibson (Associate Professor at Oregon State University, Department of Mathematics), and Dr. Chris Hoyle (Associate Professor at Oregon State University, School of MIME) are the lead PIs of the project funded by Bonneville Power Administration (BPA) and their collaboration is reflected through the development of this work.

Dr. Nathan Gibson contributed with the overall design and writing of technical mathematical concepts and results and overall structure of this research.

Dr. Duan Chen (Post-doc at University of Houston) contributed in the framework of constrained multi-objective optimization problem.

Dr. Hoyle and Dr. Gibson contributed on the concept of robust optimization. Dr. Veronika Vasylkivska (Post-doc at Oregon State University, Department of Mathematics) contributed the codes for the Stochastic Collocation and Karhunen Loeve Expansion method.

Matthew McIntire (Ph.D. Student at Oregon State University, Department of Mechanical Engineering) contributed on the code for creating polynomial surrogate.

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1. Introduction

1.1. Literature Review

Due to the rapid increase in global population (United-Nations, 2011) the importance of water resources management is crucial. Optimization is a tool to find the best decisions to a complex problem, and to get the optimal objectives. Multiple review studies (e.g., (Singh, 2012)) exist that provide an extensive review of applications of optimization techniques in water resources management. Reservoirs have been built all around the world for different purposes and it is important to find reservoirs' operational strategies that balance and meet all water demands (Ahmad et al., 2014). Managing the operation of a reservoir system can be very complex as it is dependent on many variables, such as, inflows, storage, and water and hydroelectric power demands (Rani and Moreira, 2010). The works of (Labadie, 2004) and (Rani and Moreira, 2010) reviewed various optimization and simulation models for optimizing reservoir systems including both classical methods and evolutionary algorithms. The popularity of evolutionary algorithms in optimization problems has rapidly increased over the last few years partly due to their potential to solve problems that are difficult to solve by some of the classical search techniques (Nicklow et al., 2009).

In reservoir problems, conflicts may happen in cases when improving one objective is only possible by degrading other objectives. Evolutionary algorithms can be the favorable tool for these types of problems as they can find trade-offs of the optimal solutions for the competing objectives in a single optimization run (Golberg, 1989). A comprehensive review of the use of evolutionary algorithms in water related problems is presented in (Reed et al., 2013). Genetic Algorithm (GA) based methods are the most popular evolutionary algorithms in water resources management applications and have been reviewed in (Nicklow et al., 2009). Among these methods, the Non-dominated Sorting Genetic Algorithm-II (NSGA-II) (Deb et al., 2000) can be considered as the primary technological breakthrough for the generation of advanced multi-objective evolutionary algorithms that incorporate elitism and efficient non-domination sorting when finding the Pareto solutions (Reed et al., 2013). Some of the applications of NSGA-II in multi-objective reservoir operation problems and a proposed method to rank the solutions are discussed in (Malekmohammadi et al., 2011).

To have reliability and robustness, it is essential to consider uncertainties in any optimization problem and this is specifically important in water resources management problems (Nicklow et al., 2009). Despite the recurring use of flexibility in recent water resources management literature, the precise meaning of the term "flexibility" is elusive (DiFrancesco and Tullos, 2014). The importance of decision makers' preferences in the optimization process for water management problems has been addressed in numerous studies. In some cases, the decision makers tend to disregard the global optimal solutions due to local and short-term benefits (e.g., (Babbar-Sebens et al., 2013; Babbar-Sebens, 2017; Kaini et al., 2012)). In this study, flexibility refers to alternative options in decision space rather than a specific decision variable that corresponds to an optimal objective. Flexible decision variables in water distribution systems dealing with future demand uncertainty has been considered using decision tree analysis in (Basupi and Kapelan, 2013). Although, flexibility can be provided to some extent by considering future scenarios, it is desirable to find more flexible approaches (Maier et al., 2014).

The popularity of the term "robust optimization" for different applications makes the meaning of the term unclear and the results are difficult to interpret (Ray et al., 2010). Minimizing the effects of variation in objectives due to uncertain behavior of some parameters is known as the principle of robust optimization and can be categorized into two types (Chen et al., 1996):

- Type I: objective variation due to unavoidable parameter uncertainties, such as input parameters.
- Type II: objective variation due to variation in decision variables.

Various approaches have been used in the literature including: optimization of objective expectation due to parameter variation (Sahinidis, 2004) and minimizing the deviation of objectives due to parameter variations (Ray et al., 2010). To guarantee robust solutions found by optimization, solutions must have feasibility robustness and sensitivity robustness. Feasibility robustness refers to making sure the problem parameters remain in the feasible region despite the occurrence of the variations. Sensitivity robustness implies the insensitiveness of the objective function to the parameter variation (Jung and Lee, 2002).

1.2. Research Questions

In this study, three research questions are investigated:

1. How can flexibility in decision variables be incorporated in reservoir optimization problems and how does flexibility affect the Pareto solutions? (Type II robust optimization considers the feasibility robustness)
2. How is computational efficiency increased with reduction of the dimension of the search space in reservoir operation?
3. How can we incorporate flexibility in decision variables in reservoir operation under uncertainty and how does flexibility affect the Pareto solutions? (Type I and II robust optimization consider feasibility and sensitivity robustness simultaneously)

1.3. Outline

The main goal of this study is to develop a framework for finding flexible decision variables. Flexibility in this study refers to having more than one option (either a range of options or multiple feasible scenarios) that are ensured to fulfill feasibility robustness for a given risk threshold. The probability of failure of the problem constraints due to the flexible decision variables is guaranteed to be less than a given threshold. This threshold is selected based on the risk attitude of the decision maker (Gibson et al., 2014).

Chapter 2, introduces the flexibility concept by defining each decision variable as a random variable and clarifying the proposed concept through simple test problems and a reservoir operation case study. In Chapter 3, the concept of flexible decision variables is discussed specifically for reservoir operation and a new approach is introduced for improving the computational efficiency of the problem. Namely, a dimension reduction method is used to decrease the number of random variables that are necessary to approximately regenerate the decision space.

In Chapter 4, the concept of robust optimization under input uncertainty (type I) is investigated and both feasibility robustness and sensitivity robustness are considered in the process of optimization. To ensure the sensitivity robustness, a linear combination of objective expectation and objective variation is considered as the objective function of the robust optimization (Arora, 2004; McIntire et al., 2014). Then the two concepts of robust optimization under uncertainty and flexible decision variables are combined in the proposed framework. Although the expected value of the optimal objective may be sacrificed to some extent due to flexibility and robustness, the feasibility and robustness are guaranteed for a given risk threshold. This framework can be extended to other decision support tools and also other operating problems.

1.4. Case Study

The proposed framework in this study is tested on a simplified version of the Grand Coulee reservoir problem. The Grand Coulee reservoir is one of 10 large reservoirs on the Columbia River in Northwestern United States that are operated by Bonneville Power Administration (BPA) (Figure 1.1). Hydropower production is one the main objectives of reservoir operation for the BPA.

Model characteristics are adapted from a more comprehensive model studied by (Chen et al., 2014). The reservoir problem studied in this research is described in a simple Sketch (Figure 1.2). Turbine outflows in daily time-steps are the decision variables. A period of two weeks (August 25th-September 8th) is selected for this problem.

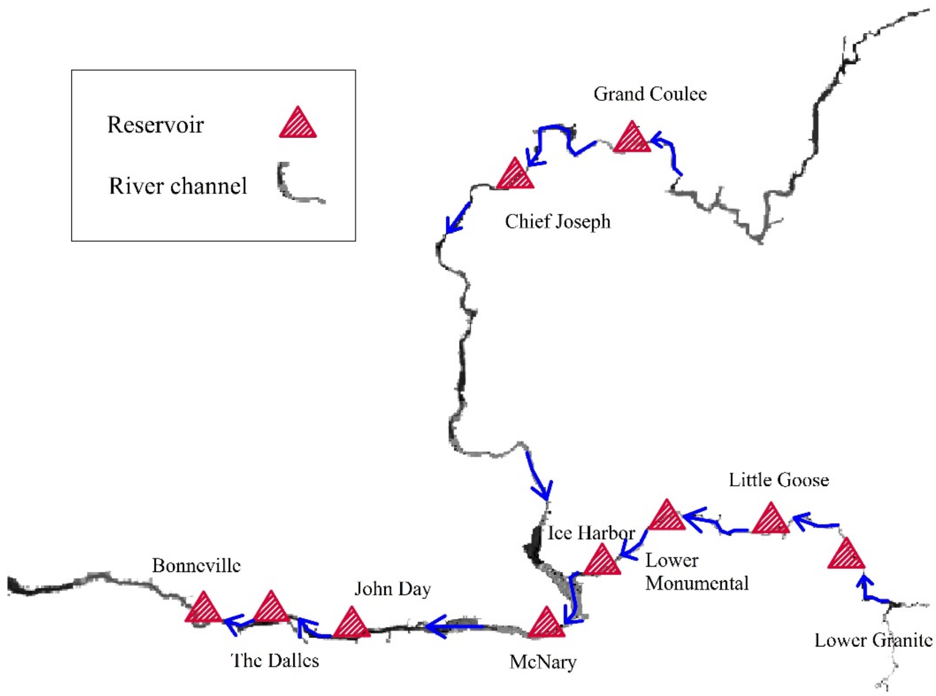

Figure 1.1. The Sketch map of the reservoirs on the Columbia River

The main reason for choosing this period for the optimization problem is because some of the 10 reservoirs on the Columbia river shift their objectives from maximizing hydropower generation and minimizing fish flow violation to only maximizing power production (however, the Grand Coulee does not have a fish flow violation objective). Data availability is the other reason for choosing these 14 days. However, the methods described here are general and can be applied to any other time frame. It is assumed that turbine flow is the only outflow from the reservoir; spill flow is considered zero. The inflow hydrograph is the input of the optimization problem and inflow uncertainty is considered using historical inflow hydrographs (Chen et al., 2016).

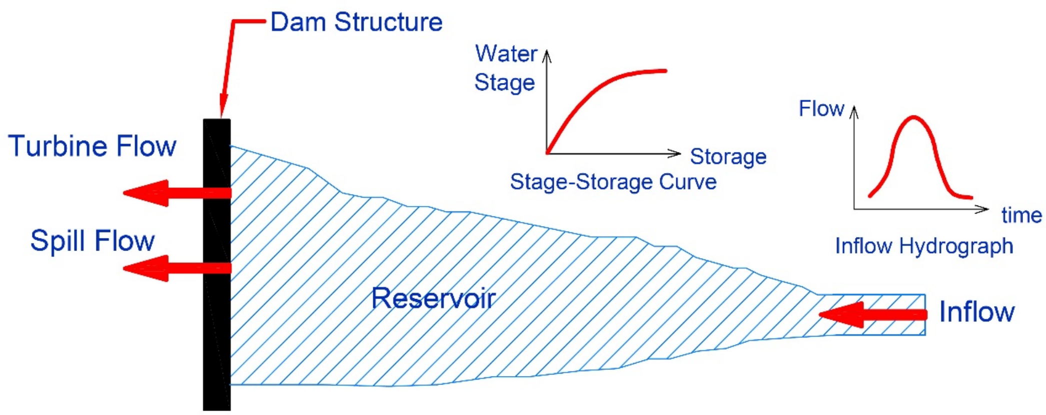

Figure 1.2. Simple sketch of reservoir problem and its parameters

One of the objectives of the problem is maximizing the revenue from hydropower production. This objective is designed to make sure the system meets the power demand while maximizing the revenue from power generation. The difference of generated hydropower and demand, net electricity, is multiplied by the price (from the power market) to calculate revenue. Predicted hydropower prices can be another source of input uncertainty. However for simplicity, price is assumed to be deterministic and is pre-determined by an economic model (Chen et al., 2014).

Another objective of this reservoir operation problem is minimizing the forebay elevation (reservoir water surface elevation) deviation at the end of the optimization period. To satisfy future planning requirements, forebay elevation is expected to remain within a certain elevation range by the end of optimization.

The constraints of this reservoir operation problem include:

1.the maximum and minimum allowable value of turbine flows

2.the maximum allowable difference between any two consecutive daily out-flows (ramp constraint)

3.the maximum allowable forebay elevation deviation

2. Flexible Decision Variables in Multi-Objective Optimization of Reservoir Operation

2.1. Abstract

This study explores optimization in the case when certain input uncertainties cannot be well quantified. In these scenarios the operator (decision maker) prefers to have the most flexibility, which is defined to be the range of options for decision variables, and still achieve the objectives of the reservoir operation. The proposed framework determines upper and lower bounds on each decision variable value by treating the amount of flexibility as an additional objective. In finding these bounds, there is a trade-off between the amount of flexibility that can be allowed and the values of the expected objectives. The Multi-objective Genetic Algorithm NSGA-II is used for the optimization and each decision variable is represented by a random variable. To compute the expectations of objectives, the Stochastic Collocation (SC) method is used, which deterministically samples the random variables at strategically chosen points and uses the corresponding weights to find the expected value of the objectives. The effectiveness of the proposed approach to find optimal and flexible decision variables is shown in three problems including two mathematical tests and a simplified single reservoir operation. Comparison of deterministic solutions to those with flexible decision variables shows that the expectations of objectives are close to the deterministic solutions especially for the reservoir operation problem.

2.2. Introduction

It is very important to study how to operate and release water from a reservoir system over time to maximize the goals and benefits, such as hydropower production, while satisfying water demands (e.g., irrigation, domestic water use and environmental requirements). Most of these objectives conflict with each other (Labadie, 2004) thus, an optimal trade-off of solutions must be found. A comprehensive review of the use of evolutionary algorithms for several water resources management optimization problems can be found in (Nicklow et al., 2009; Reed et al., 2013).

Different sources of uncertainty affect the reservoir operation problems. Sources of uncertainty are classified as: knowledge deficiency and natural variability (Tung and Yen, 2005). Uncertainties caused by knowledge deficiency can be decreased by gathering more data. To consider the effects of uncertainties due to natural variability, robust optimization has been implemented to find solutions that are less sensitive to small deviations in variables (Deb and Gupta, 2006). The work of (Kerachian and Karamouz, 2006) and (Ganji et al., 2007) implemented a stochastic evolutionary algorithm to account for natural variability in reservoir inflow. The coupling of a dimension reduction method within a multi-objective evolutionary algorithm was investigated by (Chen et al., 2016). Robust optimal solutions were found considering decision variables' variability as the source of uncertainty (Bernardo and Saraiva, 1998). However, the variability in decision space was decreased to ensure robustness which is different from the approach proposed in this study.

Flexibility in decision making is a fairly recent approach in water resources management problems. A methodology for a flexible design in water distribution systems was formulated by (Basupi and Kapelan, 2013). They combined a Multi-Objective Genetic Algorithm, Monte Carlo sampling, and decision tree analysis to find a flexible design considering uncertain future demand. They represented possible future demand scenarios by random variables. Decision tree analysis was used to portray the possible scenarios and to study the consequences for each option. Flexible designs using the decision tree analysis were also studied by (Marques et al., 2015). They used a real option methodology to find flexible design variables in a water distribution network. A decision tree was presented to predict different scenarios that may arise. In both of these studies, certain scenarios for realizations that may happen were considered, and the optimal design for each scenario was found. Only few particular scenarios were examined in the decision tree approach. Moreover, they evaluate constraints discretely. On the other hand, the proposed method in this paper attempts to find a range of options for each decision variable and the constraints are calculated in a continuous probability space.

Some sources of uncertainty are not well predicted (e.g. price variations). Therefore, the operator may need to modify some of the assumed optimal decision variables in response to the actual varying circumstances. These modifications may result in violations of some constraints or deterioration of some objectives from their predicted optimal values.

The main contribution of this paper is the development of a method for incorporating flexibility in decision variables within optimization problems. In other words, the proposed method finds a range of options for each decision variable instead of a single deterministic value. Each decision is represented by a uniform random variable and the decision maker can choose any value of the decision variables within the optimal range provided by this optimization framework, and be confident that the probability of constraint violations will be within pre-established bounds. The current study seeks to find the largest possible range of options for each decision variable. Moreover, the preference of the decision maker for a different distribution of random variables representing each decision variable can be implemented. For example, if the turbine outflow from a reservoir can be selected from a range of options and the decision maker has the tendency to choose from the center of the interval rather than at the boundaries, a Beta distribution can be used instead of Uniform distribution.

This paper further aims to assess the influence of flexible decision variables on the resulting Pareto solutions. A Pareto curve is a set of non-dominated solutions of a multi-objective optimization problem (Van Veldhuizen and Lamont, 1998). Note that the flexibility concept proposed in this paper is not the same as finding multiple optimal solutions for a multi-objective optimization problem (called non-dominated, non-inferior or Pareto solutions). In fact flexible decision variables can be found for each of the Pareto solutions in a multi-objective optimization problem. It is also different from sensitivity analysis in which the variation of an objective (output) due to uncertainty of decision variables is studied. In this paper, variability of the decision variables (inputs) is not assumed (like the sensitivity analysis case), therefore the proposed method allows variability in decision variables without assuming any pre-specified ranges. Interval arithmetic (Hanss, 2005) is another method to consider the effect of a prescribed interval for each input and study the resulting outputs due to the intervals. Unlike the sensitivity analysis and the interval arithmetic method, in the proposed approach the goal is to find feasible ranges for decision variables by optimization.

This paper is organized as follows. Section 2.3, explains the proposed methodology. Section 2.4, shows the efficacy of the proposed method on three test problems: a single-objective mathematical test problem (section 2.4.1), a multi-objective mathematical test problem (section 2.4.2) and then a reservoir operation problem (section 2.4.3). The optimal flexible decision variables are found and compared with the deterministic optimal decisions. Section 2.5, presents the discussion and conclusion of the proposed methodology.

2.3. Methodology

For the multi-objective optimization, the non-dominated sorting multi-objective optimization method (NSGA-II) (Deb et al., 2002) is used. Each decision variable is replaced by lower and upper bounds, which doubles the number of decisions in the optimization. An additional objective is also added to maximize the flexibility of decisions, which is thus represented by the summation of the standard deviations for all m decision variables ($\{\sigma_i\}_{i=1}^{m}$).

For each variable, the decision maker can choose any value within the optimal range. A uniform random variable represents the potential solutions within the range of the lower and upper bounds of each decision variable. Each decision variable is represented by a random variable:

$$x(t_i) = \mu_i + \sigma_i \xi_i \tag{2.1}$$

where $x(t_i)$ represents the i^{th} decision variable. Uniform random variables are assumed in this study (ξ_i). The upper and lower bounds of each decision variable are used to calculate the mean (μ_i) and standard deviation (σ_i) of the decision variable with equation (2.2) and equation (2.3). Each random variable is shifted according to the prescribed upper and lower bounds:

$$\mu_i = \frac{u_i + l_i}{2}, \tag{2.2}$$

$$\mu_i = \frac{u_i - l_i}{\sqrt{12}}, \tag{2.3}$$

where (l_i) and (u_i) are the lower and upper bounds of the i^{th} uniform random variable.

Each random variable (ξ_i) can be sampled multiple times (Ξ_{ik}) and the value of the objective function$f(\{x_i(\Xi_{ik})\}_{i=1}^{m})$, can be calculated for multiple samples $\overrightarrow{\xi_k} = \{\Xi_{ik}\}_{i=1}^{m}$of the random variable leading to a sample mean. Alternatively, to decrease the number of function evaluations without sampling the random variable many times, the Stochastic Collocation (SC) Method is used to find the expected value of the objective function (Gibson et al., 2014; Leon et al., 2012; Xiu and Hesthaven, 2005).

The Stochastic Collocation method is used instead of Monte Carlo (MC) method for decreasing the number of function evaluations and therefore increasing the convergence rate. The set of collocation points $\vec{Z} = [z_1, z_1, \dots, z_{NC}]$ is chosen as the roots of appropriate orthogonal polynomials (one-dimensional) (Gibson et al., 2014; Xiu, 2010). The expectation of the objective function is calculated as

$$E[f(\{\xi_i\}_{i=1}^{m})] = \sum_{k=1}^{NC} f(z_k)\omega_k, \tag{2.4}$$

where z_k, represents the collocation points, ω_k represents the corresponding weights of the collocation points and NC is the total number of collocation points, which is problem dependent (Sankaran et al., 2010) (Figure 2.1).

In constrained optimization problems, to make sure that the results are feasible, the probabilistic constraints concept is implemented. To calculate the probability of failure (PF) of a constraint, it is assumed that the constraint can be represented by a polynomial of random variables referred to as polynomial surrogate (McIntire et al., 2014). The polynomial surrogate of the constraint is created using the constraint values calculated at the aforementioned collocation points. These polynomial surrogates can be sampled efficiently (without sampling the whole system many times) to calculate the probability of failure (PF). The probability of failure of a constraint's surrogate is used as the representation of the constraint's violation (McIntire et al., 2014).

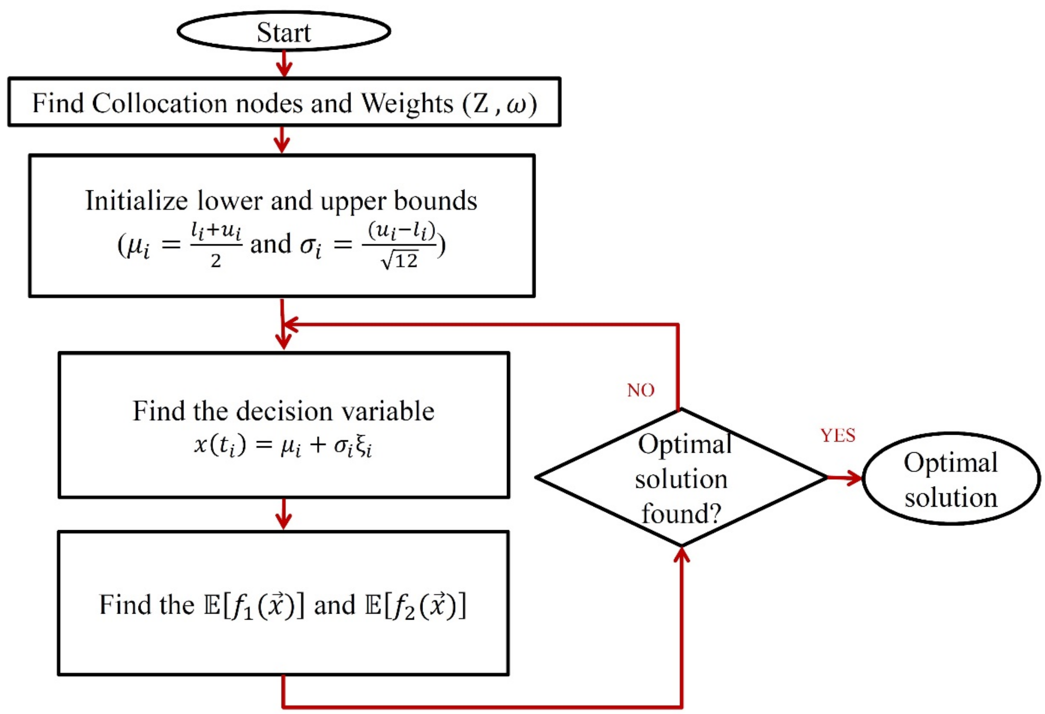

Figure 2.1. Flowchart for the proposed method

2.4. Application

A simple mathematical function, with known optimal solutions, is used to demonstrate the proposed method.

2.4.1. Test 1: Single-Objective Mathematical Test Problem

The goal is to find the optimal values with flexible decision variables for a quadratic function using the proposed methodology. We start with the following deterministic problem.

Problem 1A: Find x_i, in order to

Minimize $f(x) = \sum_{i=1}^{m}(x_i^2)$, (2.5)

subject to $0 \leq x_i \leq 1$,

where $x_i$is the i^{th}decision variable and m is the number of decision variables of the quadratic function. The known deterministic optimal decision variables and the resulting optimal objective are $x_i = 0$ and $f(x_i) = 0$, respectively.

To find the flexible decision variables, the bounds of the uniform random variables representing each decision variable are found by optimization (i.e. $\xi_i \in [l_i, u_i]$). We state this as the following problem.

Problem 1B: Find $\vec{l} = [l_i]_{i=1}^{m}$ and $\vec{u} = [u_i]_{i=1}^{m}$, in order to

Minimize $E\left[f_1(\vec{\xi})\right] = E[\sum_{i=1}^{m}(\xi_i)^2]$, and (2.6)

Maximize $f_2\left(\vec{\xi}\right) = \|\vec{\sigma}\|$, (2.7)

Subject to $0 \leq l_i \leq u_i \leq 1$ for all i, (2.8)

where $\vec{\xi} = [\xi_i]_{i=1}^{m}$, $\vec{\sigma} = [\sigma_i]_{i=1}^{m}$ as defined in equations (2.1), (2.2) and (2.3). The additional objective equation (2.7) is for maximizing the decision variables' flexibility by maximizing the standard deviation of all the random decision variables. For exact solution to problem 1B go to Appendix.

Results

We demonstrated the approach on the test problem for $m = 2$. The optimal results for two conflicting objectives can be demonstrated as the trade-off of the objectives (Figure 2.2). The first objective is the expected value of the quadratic function of the random variables (2.6) and the second objective is the norm of the standard deviations of the random variables, which represents the flexibility of each Pareto solution calculated by (2.7). The comparison of the deterministic solution of the quadratic function (Problem 1A) and the Pareto solutions with flexible decision variables (Problem 1B) shows that the higher is the flexibility of one scenario, the farther it will deviate from the deterministic optimal solution (Figure 2.2 A).

The solutions with warmer colors, have more flexibility in decision variables while the expected objective values are higher than the deterministic minimum solution. The flexible decision variables (the mean and the range of options) corresponding to each of the Pareto solutions, testify that the ranges of options are more for the solutions with warmer colors (Figure 2.2 C, D). Due to randomness of the flexible decision variables, the value of the first objective may vary from its mean value, as is shown by horizontal lines in Figure 2.2 B.

2.4.2. Test 2: Multi-Objective Mathematical Test Problem

As most water resources problems are multi-objective, a multi-objective problem is tested in this section.

Using test problems is an effective way to validate a methodology for the following reasons: The exact optimal deterministic solutions to the test problem are known and the objectives are simple mathematical functions, which makes them more straightforward to understand. (Deb, 1998) proposed using test problems to measure some characteristics of multi-objective algorithms. In this study, test problem ZDT1 suggested by (Zitzler et al., 2000) is considered. The reason for choosing this specific test problem is the similarities of its objective functions with the reservoir problem case study (Section 2.4.3).

The number of decision variables in ZDT1 can vary from 2 to 30 (this range is according to (Deb et al., 2002), however it can be any other number as well). ZDT1 has two objectives that should be minimized and the Pareto is convex. The characteristics of ZDT1 are shown in Table 2.1 (Deb et al., 2002).

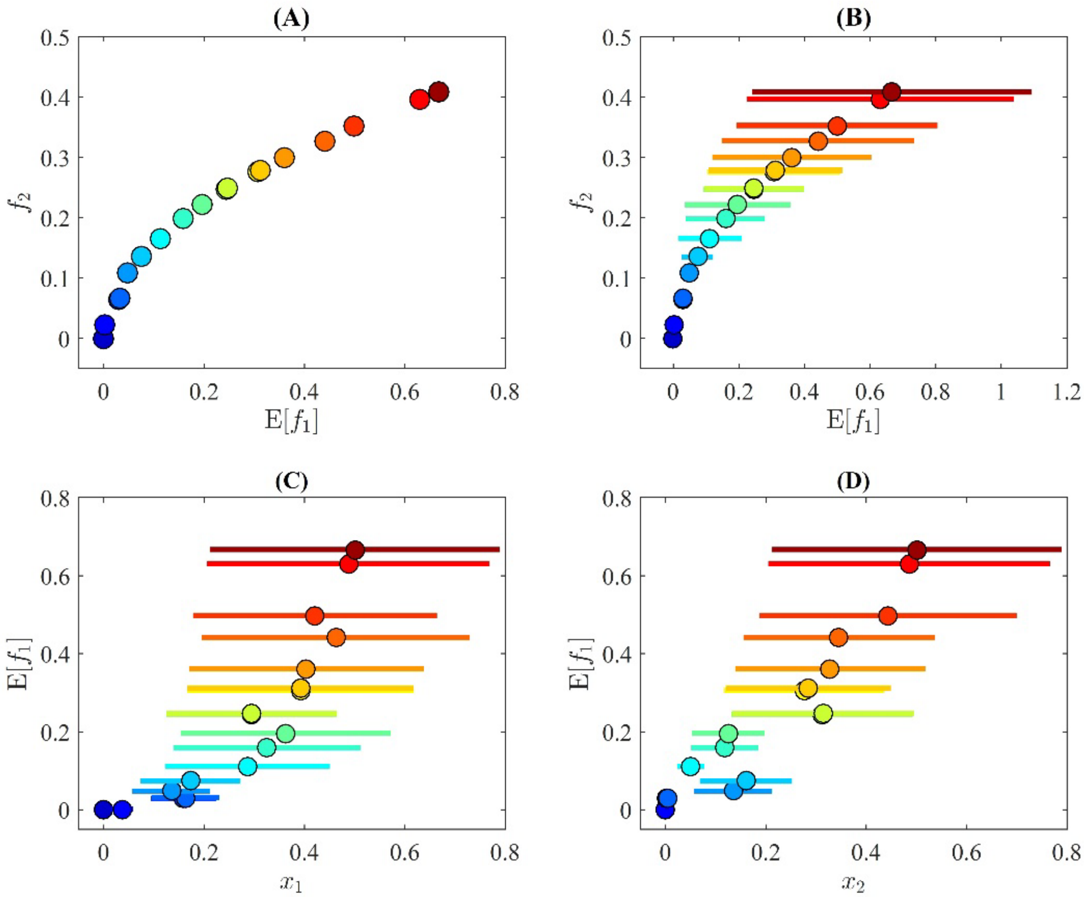

Figure 2.2. Optimal solutions for single-objective test problem with two decision variables A) Pareto solutions; B) The range of changes of the objective due to randomness of decision variables; C) Flexible bounds for first decision variable for all the Pareto solutions; and D) Flexible bounds for second decision variable for all the Pareto solutions (Note that these are the standard deviations, $\mu_i \pm \sigma_i$, not the entirety of the range $[l_i, u_i]$)

We state the deterministic (non-flexible) test problem as follows,

Problem 2A: Find $\vec{x} = [x_i]_{i=1}^{m}$ in order to

Minimize $f_1(x) = x_1$, and (2.9)

Minimize $f_2(x) = g(x)\left[1 - \sqrt{x_1/g(x)}\right]$, (2.10)

where $g(x) = 1 + 9\left(\sum_{i=2}^{m} x_i\right)/(m-1)$,

Subject to $0 \leq x_i \leq 1$ for all i. (2.11)

To find flexible decision variables for ZDT1 problem, with an additional objective for flexibility, there are three objectives.

Problem 2B: Find $\vec{l} = [l_i]_{i=1}^{m}$ and $\vec{u} = [u_i]_{i=1}^{m}$ in order to

Minimize $E[f_1(\vec{\xi})] = E[\xi_1]$, and (2.12)

$$\text{Minimize } E[f_2(\vec{\xi})] = E\left[g(\vec{\xi})\left(1 - \sqrt{\xi_1/g(\vec{\xi})}\right)\right], \tag{2.13}$$

Subject to $0 \leq l_i \leq u_i \leq 1$ for all i, (2.14)

Minimize $f_2(x) = g(x)[1 - \sqrt{x_1/g(x)}]$, (2.15)

where $\xi_i \in [l_i, u_i]$.

Table 2.1. ZDT1 (All objective functions are to be minimized)

Problem	m	Variable bounds	Objective functions	Optimal solutions
ZDT1	2	[0,1]	$f_1(x) = x_1$ $f_2(x) = g(x)\left[1 - \sqrt{x_1/g(x)}\right]$ $g(x) = 1 + 9\left(\sum_{i=2}^{m} x_i\right)\Big/(m-1)$	$x_1 \in [0,1]$ $x_i = 0$ $i = 2, \dots, m$

Results

The first two objectives are the minimization of the expected values of f_1 and f_2, respectively (Figure 2.3 A). The third objective is the maximization of the flexibility of the decision variables. The 3D surface shown in Figure 2.3 A is created with the Pareto solutions for visualization purpose although these Pareto solutions not represent the whole surface. The closer the flexible solutions are to the deterministic Pareto solutions (Figure 2.3 B), the smaller are the ranges of flexible decision variables. For example, solutions shown with warmer colors are closer to the deterministic Pareto solutions and the decision variables have less flexibility (Figure 2.3 C, D); to have more flexibility the solutions shown with green color, have wider range of options in their both decision variables, however the corresponding Pareto solutions are dominated by the deterministic Pareto solutions. Optimizing three objectives in problem 2B is the reason for the difference in the shape of Pareto solutions in problem 2A and 2B (Figure 2.3 B). The decision makers can make their decisions based on the flexibility for each decision variable (Figure 2.3 C, D) and the corresponding Pareto solution and its variation (Figure 2.3 B). In this case, each Pareto solution corresponds to the expected value of the objectives due to randomness of the chosen flexible decision variables. The objective values can vary for both objectives. The standard deviation of each objective is represented by the ellipse's radius in each dimension and is increased by larger flexible ranges in decision variable.

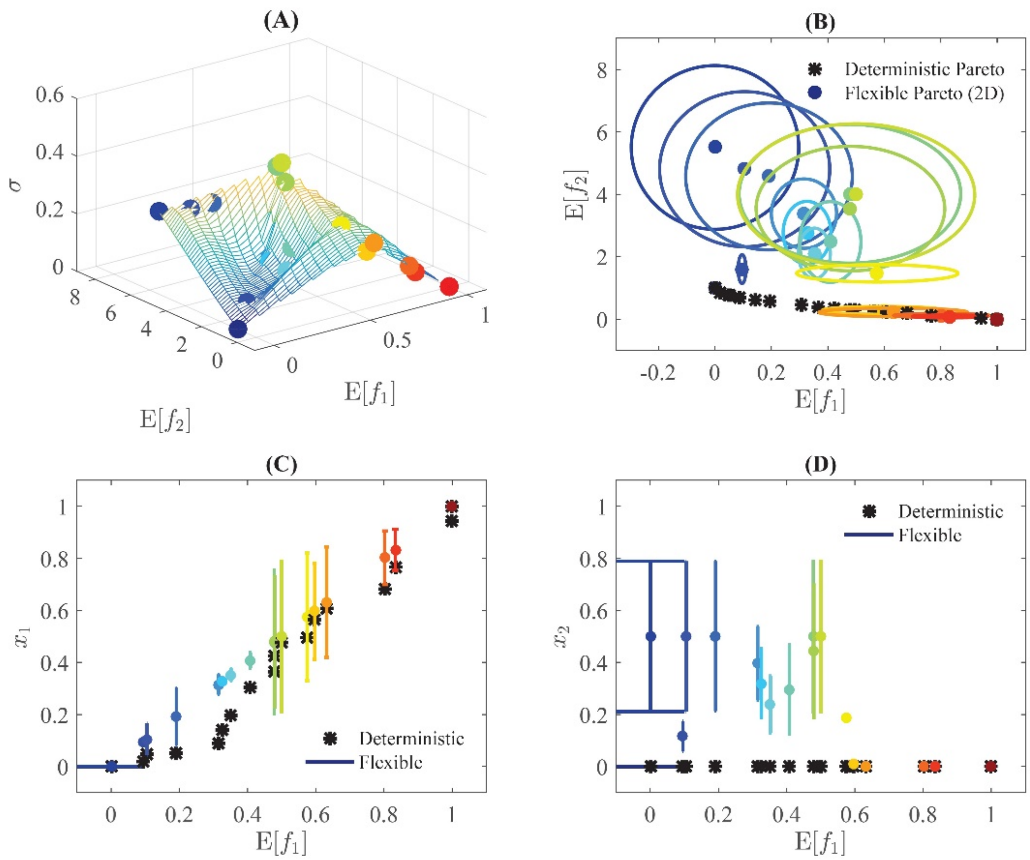

Figure 2.3. Comparing scenarios for multi-objective test problem with two decision variables for deterministic and optimal solutions with flexible decision variables A)3D Pareto solutions B) Comparison of deterministic scenarios (in black) and the flexible solutions (in color) in 2D; The range of changes of the objective due to randomness of decision variables are shown with ellipses; C) Flexible bounds for first decision variable for all the Pareto solutions and the deterministic first decision variables (in black); and D) Flexible bounds for second decision variable for all the Pareto solutions and the deterministic second decision variables (in black)

2.4.3.Test 3: Reservoir Operation Problem

A simplified model of the Grand Coulee reservoir on the Columbia River is used as the test case (Table 2.2), which is based on the problem studied by (Chen et al., 2014, 2016). The desirable decision variables are the daily turbine outflows. To simplify the reservoir test problem, a period of two days was considered[1]. Minimizing the deviation of forebay elevation[2] at the end of the optimization period is the first

[1] It was also tested for problems with 4 and 6 decision variables (time-steps)
[2] Reservoir's water surface elevation

objective. This objective is required to maintain the forebay elevation at a desired elevation for consistency with long-term planning. To further restrict the deviation of the forebay elevation, the forebay elevation deviation at the end of the optimization period constrained as well. The second objective is to maximize the revenue due to hydropower generation. The difference of generated hydropower and demand is called net electricity. The net electricity multiplied by the hydropower price determines the revenue at each time-step and the summation of the produced revenue at all time-steps is considered as the second objective value. The daily hydropower price for this period is considered deterministic and is pre-determined by an economic model (Chen et al., 2016).

The flexible and non- flexible problems are stated as follows:

Problem 3A: Find $\vec{Q} = [Q_n]_{n=1}^{N_t}$ in order to

Minimize $f_1(\vec{Q}) = \frac{(|FB_{end}(\vec{Q}) - FB_{target}|)}{U_r - L_r}$, and (2.16)

Minimize $f_2(\vec{Q}) = -\left(\frac{\sum_{n=1}^{N_t}(PG_n(\vec{Q}) - PL_n) * Pr_n}{\sum_{n=1}^{N_t}(Pr_n * PL_n)}\right)$, (2.17)

Subject to $Q^{min} \leq Q_n \leq Q^{max}$ for all n, (2.18)

Subject to $f_2(\vec{Q}) \leq \delta FB$. (2.19)

Problem 3B: Find $[l_n]_{n=1}^{N_t}$and $[u_n]_{n=1}^{N_t}$in order to

Minimize $E[f_1(\vec{\xi})] = \frac{\left(\left|FB_{end}(\vec{\xi}) - FB_{target}\right|\right)}{U_r - L_r}$, and (2.20)

Minimize $E[f_2(\vec{\xi})] = -\left(\frac{\sum_{n=1}^{N_t}\left(PG_n(\vec{\xi}) - PL_n\right) * Pr_n}{\sum_{n=1}^{N_t}(Pr_n * PL_n)}\right)$, (2.21)

Maximize $f_3(\vec{\xi}) = \|\vec{\sigma}\|$, (2.22)

Subject to $Q^{min} \leq Q_n \leq Q^{max}$ for all n, (2.23)

Subject to $PF(C) \leq \alpha$, (2.24)

$C: f_2(\vec{\xi}) \leq \delta FB$. (2.25) where $\vec{\xi} \in [l_n, u_n]$ and is the set of random variables representing the decision variables, $FB_{end}$is the forebay elevation at the end of the optimization period, which depends on the turbine outflows (Q), $FB_{target}$is the desired forebay elevation at the end of optimization period, which is pre-determined for each reservoir (δFB), U_r and $L_r$are the maximum and minimum allowable reservoir's forebay elevation.

Table 2.2. Reservoir operation problem (All objective functions are to be minimized)

Problem	n	Variable bounds	Objective functions
Reservoir operation problem	2	[30,290]	$f_1(\vec{Q}) = \frac{(\lvert FB_{end}(\vec{Q}) - FB_{target} \rvert)}{U_r - L_r}$ $f_2(\vec{Q}) = -\left(\frac{\sum_{n=1}^{N_t}(PG_n(\vec{Q}) - PL_n) * Pr_n}{\sum_{n=1}^{N_t}(Pr_n * PL_n)}\right)$

where N_t is the number of time-steps in the optimization. In the proposed methodology the objectives of the optimization are formulated as a minimization. Therefore, the second objective is formulated as a minimization of revenue loss. PG_n is the hydropower produced in n^{th} time-step, PL_n is the hydropower demand (load) and Pr_n is the price of hydropower. The constraints of this problem are designed to maintain the outflows within the allowable boundaries ($[Q^{min}, Q^{max}] = [30,290]$ kcfs for this problem). For problem 3B, $\vec{\sigma}$ is the set of standard deviation of the decision variables, the probability of failure (PF) of the constraint is calculated and the solutions with PF less than an allowable failure threshold (α) are considered as feasible solutions. The value of α is chosen based on the risk attitude of the decision maker.

Results

A surface is created using the Pareto solutions in 3D (Figure 2.4 A) which demonstrates the spread of Pareto solutions, however this surface is not the exact representation of the Pareto surface. In this figure, x-axis is the expectation of the first objective (minimizing the deviation of forebay elevation at the end of the optimization period), y-axis is the expectation of the second objective (maximizing the revenue from hydropower production) and z-axis is the flexibility ($\|\vec{\sigma}\|$). Each decision variable's standard deviation is bounded by the maximum and minimum constraints. Therefore, in this problem, the maximum standard deviation can be almost 75 kcfs (in Uniform distribution, $\sigma = \frac{u-l}{\sqrt{12}}$). Therefore, in Figure 2.4 A the maximum flexibility happens when both decision variables have the maximum allowable standard deviation. The analogy of the deterministic Pareto solutions of problem 3A and the projected Pareto solutions problem 3B in 2D, shows that the Pareto solutions corresponding to flexible decision variables are dominated by the deterministic Pareto solutions (Figure 2.4 B). In other words, each solution on the deterministic Pareto is better than the flexible solutions at least for one of the objectives. As it was expected, the objective values are sacrificed (in comparison to deterministic Pareto solutions) to some extent to allow flexibility in the decision variables. As an example, the solution showed with yellow color ($E[f_1] \approx 0.06$ and $E[f_2] \approx -2.2$) is dominated by the deterministic solution with $E[f_1] \approx 0.05$ and $E[f_2] \approx -2.4$. But this solution offers flexibility for both decision variables ($\sigma_1 = 50$ and $\sigma_2 = 10$). Moreover, the variations of the objectives due to random behavior of the flexible decisions are more restricted in comparison to the objective variations in test 1 (sections 2.4.1) and test 2 (section 2.4.2). The complex structure of the reservoir operation problem and the impact of the higher number of non-linear constraints may be the reason that the Pareto solutions are less different from the deterministic Pareto solutions.

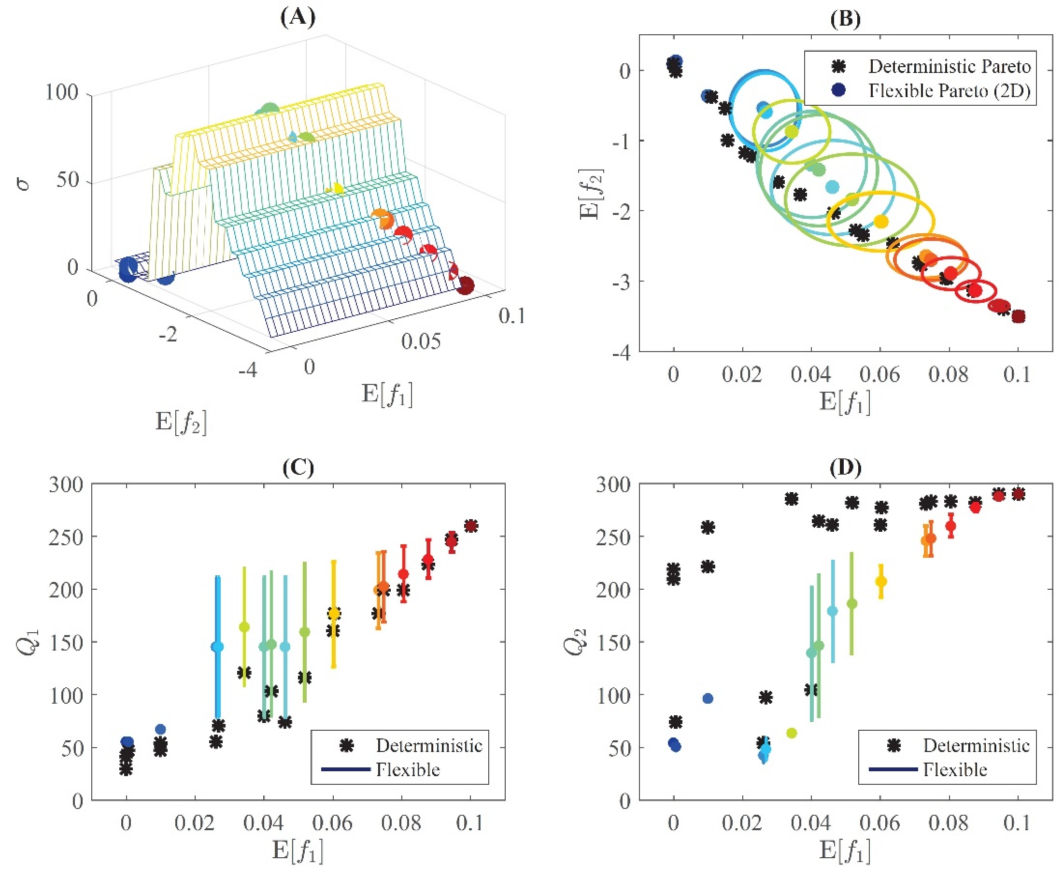

Figure 2.4. Optimal solutions for reservoir operation problem with two decision variables A) Pareto solutions; B)The range of changes of the objective due to randomness of decision variables; C) Flexible bounds for first decision variable for all the Pareto solutions; and D) Flexible bounds for second decision variable for all the Pareto solutions ($PF < 1\%$)

Each of the flexible Pareto optimal solutions can be desirable based on the decision maker's preference. The solutions with higher flexibility (e.g., solution with green color in Figure 2.4) may be preferred by a certain decision maker over the solutions with lower forebay elevation deviation or even lower revenue losses. All of the solutions are assured to have less than 1% probability of failure of their constraints. The results of the optimization with α equal to 5, 10 and 20% are not very different from those shown here with $\alpha = 1\%$ (Figure 2.5).

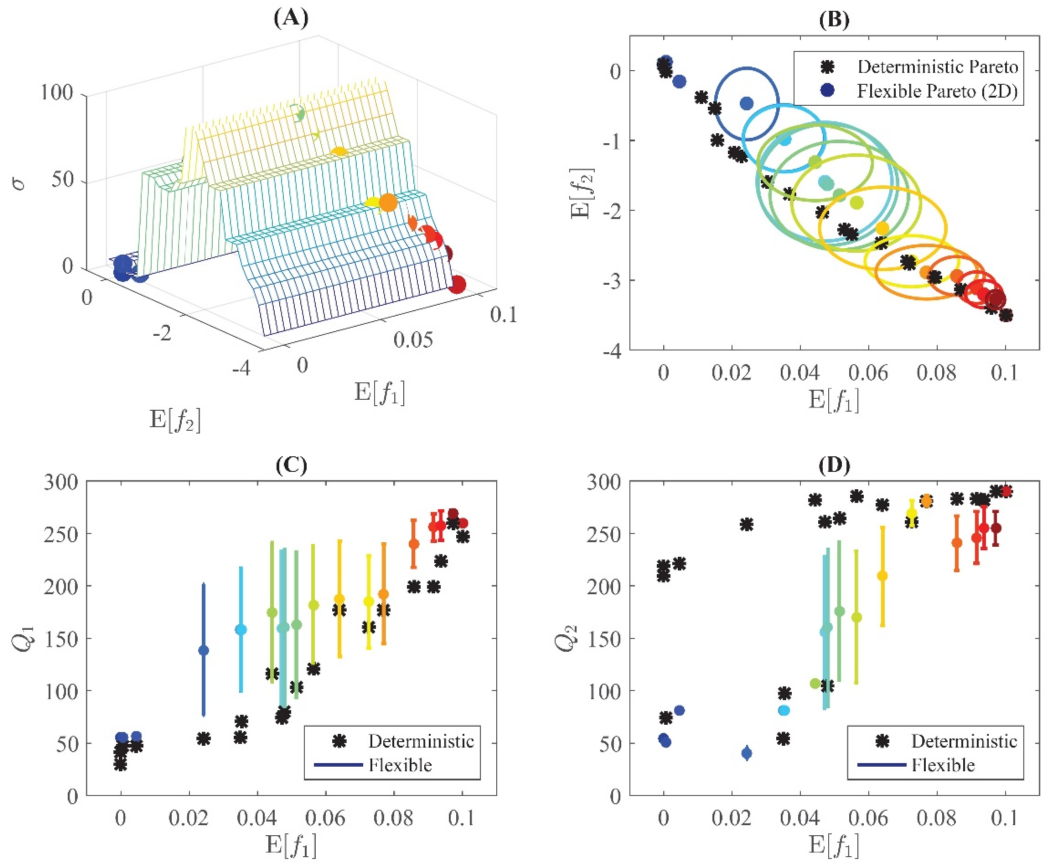

Figure 2.5. Optimal solutions for reservoir operation problem with two decision variables A) Pareto solutions; B)The range of changes of the objective due to randomness of decision variables; C) Flexible bounds for first decision variable for all the Pareto solutions; and D) Flexible bounds for second decision variable for all the Pareto solutions ($PF = 20\%$)

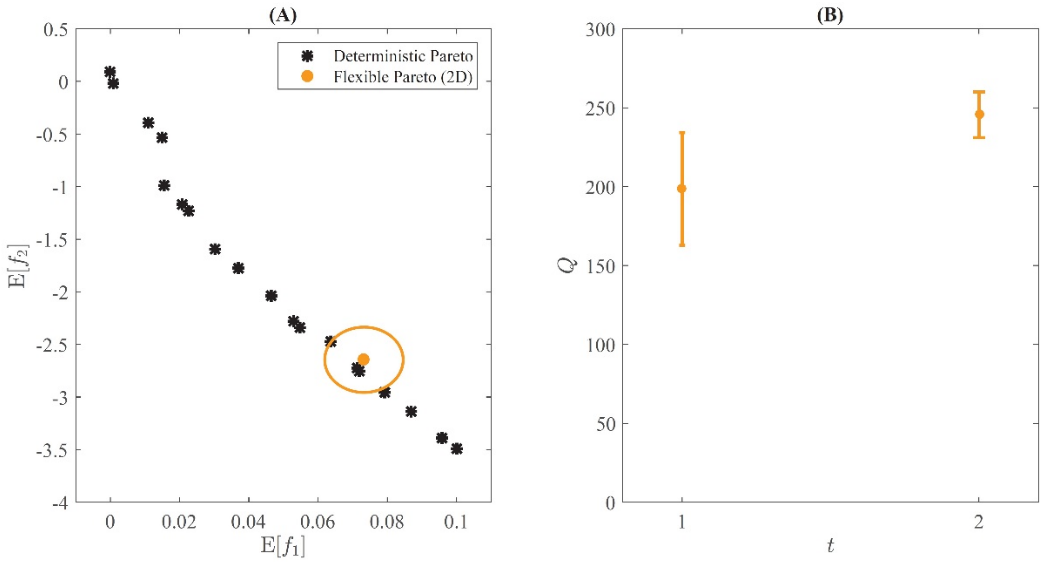

Figure 2.6. Example 1 of the decision that the decision maker may choose in decision and objective space

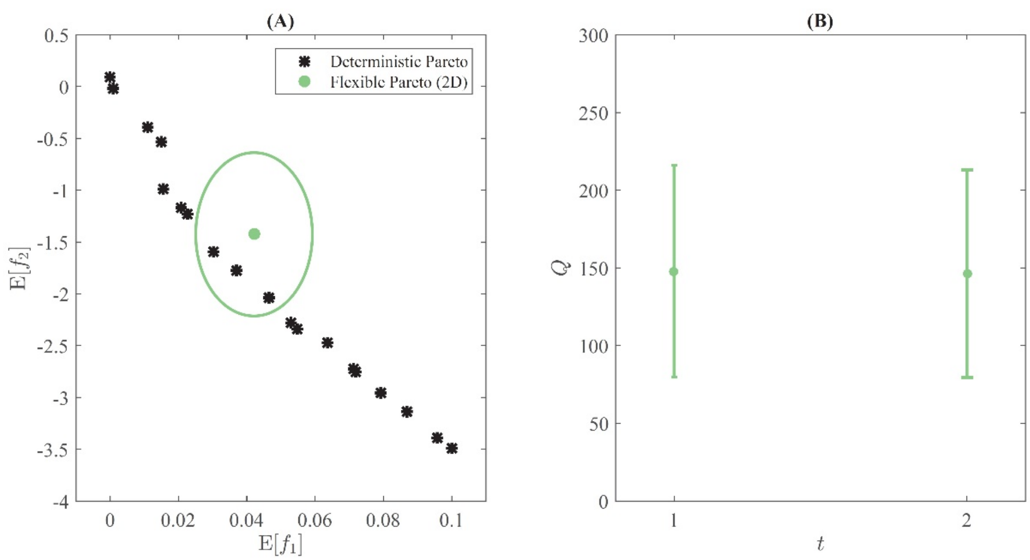

Figure 2.7. Example 2 of the decision that the decision maker may choose in decision and objective space

The decision makers can choose different options from the Pareto solutions based on their preferences. If flexibility in the first decision variable is more desirable for the decision maker, the solution shown in Figure 2.6 can be selected. The coefficient of variation (CV) is used to show the relative flexibility of each decision variable (σ_Q/μ_Q), which is 18% CV for the outflow in the first time-step and 5.9% CV for the outflow in the second time-step. The flexibility in decision space leads to average 1.9% difference for the first objective and 4.1% decrease in revenue. However a decision maker who preferred high flexibility in both decision variables may choose the solution shown in Figure 2.7. The flexibility of the outflow in both time-steps are the same ($CV = 46\%$) and corresponds to an average 11.3% difference for the first objective and 17.2% less generated hydropower from what the deterministic optimization would suggest to provide.

2.5. Conclusions And Future Directions

The proposed methodology finds the optimal flexible decision variables, with a pre-specified probability of failure. Each decision variable is represented by a range and the operator can choose any value in that range.

Due to high computational cost, this methodology can be suitable for short-term operation of reservoirs with a small number of decision variables (e.g., 6) as the function evaluations will increase exponentially by increasing the number of decision variables (NC^m where NC is the number of collocation points in each dimension and m is the number of decision variables). Although the proposed framework is not efficient for reservoir operation problems with many decision variables (e.g., larger than 6), it can be useful when the number of decision variables is relatively small (e.g., 6) and the operator needs flexibility in decision making. To improve resolution, one needs to decrease the time-steps which increases the dimension. Using a basis function representation would allow arbitrarily fine resolution, which is independent from the decision space dimension. Therefore to make this methodology applicable to problems with multiple decision variables, future work is required to implement a dimension reduction method to increase the computational efficiency, which is the topic of ongoing research.

Moreover, to find solutions with limited variation in the objective space, the concept of robust objective can be implemented in addition to flexibility but is beyond the scope of this paper. Robustness refers to minimizing the objectives variation due to variable uncertainties while optimizing the expected of the objective. This can be done by robust objective concept which is a weighted sum of objective expectation and its standard deviation. The weights are chosen based on the risk attitude of the decision maker (Arora, 2004; McIntire et al., 2014).

Acknowledgments

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3. Flexible Decision Variables in Reservoir Operation Using Dimension Reduction Approach

3.1. Abstract

This paper presents a framework for using dimension reduction approach to find flexible decision variables in a multi-objective optimization reservoir operation problem. The use of flexible decision variables provides the operator with options if unforeseen variations occur (e.g., uncertainty sources that are not included in the process of optimization). If each decision variable is represented by a random variable, then problems with many decision variables can get computationally expensive. Therefore, to decrease the number of random variables and consequently reduce the computational time, a dimension reduction technique (Karhunen-Loeve expansion) is implemented within the process of optimization. A multi-objective evolutionary algorithm (NSGA-II) is used to optimize the expected value of the objectives and simultaneously maximize the flexibility of decision variables.

The Grand Coulee reservoir is used as the case study. The decision variables of this simplified reservoir problem are the daily turbine outflows. The objectives are chosen to be maximizing revenue from hydropower production and minimizing the forebay elevation deviation by the end of optimization period (Chen et al., 2014).

The proposed framework is capable of decreasing the number of random variables and then finding flexible decision variables while having a pre-specified probability of failure of constraints.

3.2. Introduction

Operation of a reservoir system is complex due to many competing objectives, requirements and constraints. Optimization methods are used to effectively maximize beneficial aspects of a reservoir system (such as hydroelectric power production) and manage water utilization. Different optimization techniques have been applied to problems such as groundwater, irrigation and reservoir operation management to name a few (Singh, 2012). The use of optimization techniques for reservoir operation systems has been reviewed comprehensively (Ahmad et al., 2014; Labadie, 2004). The application of evolutionary algorithms in various water resources optimization problems was reviewed in (Nicklow et al., 2009; Reed et al., 2013). Different sources of uncertainty can affect the solutions in an optimization problem. Optimization of reservoir operation under uncertainty was investigated in (Sahinidis, 2004). Consideration of input uncertainty was addressed by several studies (e.g., Escudero, 2000; Faber and Stedinger, 2001; Gibson et al., 2014; Karamouz et al., 2009; Leon et al., 2012). Numerous studies have demonstrated the importance of decision makers' preferences in the process of optimization for water management problems and that not all decision makers are able to identify their preferences accurately (e.g., Babbar-Sebens et al., 2013; Babbar-Sebens, 2017; Kaini et al., 2012). Although, optimization is a strong tool to find optimal solutions, decision makers may alter the optimal decisions or choose the solutions with better local-benefits rather than solutions identified by optimization for a large and complex decision space (Piemonti et al., 2013).

The main focus of this paper is to find flexible decision variables. The importance of flexibility in decision making and the consideration of decision maker preference in the trade-off between performance and robustness, has been discussed extensively (Bernardo and Saraiva, 1998; DiFrancesco and Tullos, 2014; Herman et al., 2015; Marques et al., 2015). A methodology for finding optimal flexible decision variables was proposed by (Hosseini et al., 2017b), representing each decision variable by a random variable. This methodology can be computationally expensive for problems with many decision variables and can suffer from the "Curse of dimensionality" (Bellman, 2015) as it would lead to a multi-dimensional decision space.

The main contribution of this paper is to find flexible decision variables for a reservoir operation problem using a dimension reduction method. To have a representation of the decision space a set of optimal decision variables found by deterministic optimization is used. The Karhunen-Loeve (KL) expansion method is then used to reduce the dimension of the decision space (Chen et al., 2016; Gibson et al., 2014; Xiu, 2010) so that the decision space can be approximated by a few random variables.

3.3. Methodology

To find decision variables in a reservoir operation problem, the primary decision space can be approximated by either,

1. a set of deterministic optimal decision variables (decisions leading to deterministic Pareto solutions in the objective space) or,
2. a set of historical decision variables or,
3. a combination of both deterministic and historical optimal decision variables.

Option 1 is investigated in this study and is briefly explained in section 3.3.1. The dimension reduction method (KL-expansion) is implemented to reduce the dimension of the decision space to a manageable number of random variables, and is discussed in section 3.3.2 (Gibson et al., 2014; McIntire et al., 2014).

3.3.1.Multi-Objective Optimization (NSGA-II)

The non-dominated sorting multi-objective optimization method (NSGA-II) (Deb et al., 2002) is used. NSGA-II is one of the most popular evolutionary algorithms (Reed et al., 2013) for finding a set of non-dominated optimal solutions to a multi-objective optimization problem. NSGA-II is an improved version of the NSGA method proposed by (Srinivas and Deb, 1994) which uses non-dominated ranking to classify the population in the process of selection. An elitist mechanism and a crowded comparison operator are used in NSGA-II to find more diverse Pareto solutions with higher computational efficiency. The elitism in this method refers to the coupling of the best parents and the best offspring obtained (Malekmohammadi et al., 2011).

3.3.2.Karhunen-Loeve Expansion

The Karhunen-Loeve (KL) expansion method is a dimension reduction method (Hernández-Andrés et al., 1998; Xiu, 2010), which is conceptually similar to Principle Component Analysis (PCA). This method uses to obtain the covariance structure of a given set of realizations and extracts the orthogonal eigen-pairs (eigenvalues and eigen-functions). The random process can be represented by a series expansion (the KL-expansion) of the eigen-pairs and their corresponding random coefficients (Chen et al., 2016). It is assumed that there are M decision variable realizations corresponding to the deterministic Pareto solutions and each realization has n time-steps.

The decision space is assumed to be a Gaussian process. The deterministic realizations $[Q_i(t)]_{i=1}^{M}$ are used to determine the sample mean (equation (3.1)) and covariance structure (equation (3.2)) of the decision space (Gibson et al., 2014; Xiu, 2010)

$$\bar{Q}(t) = \frac{\sum_{i=1}^{M} Q_i(t)}{M}, \tag{3.1}$$

$$C(t_j, t_k) = \frac{\sum_{i=1}^{M} \left(Q_i(t_j) - \bar{Q}_j\right)\left(Q_i(t_k) - \bar{Q}_k\right)}{M-1}, \tag{3.2}$$

We let $[\lambda_k, \psi_k]_{k=1}^{\infty}$ be the eigenvalues and eigen-functions of the integral equation ($\lambda\psi(t) = \int_{t_0}^{n} C(x,t)\psi(x)dx$), and let $\vec{\xi} = [\xi]_{k=1}^{\infty}$ be uncorrelated Gaussian random variables with mean 0 and standard deviation 1. Then the KL-expansion is given by:

$$Q^g\left(t, \vec{\xi}\right) = \bar{Q}(t) + \sum_{k=1}^{\infty} \sqrt{\lambda_k}\psi_k(t)\xi_k, \tag{3.3}$$

The KL-expansion can be truncated and approximated by a few random variables as follows:

$$Q^g\left(t, \vec{\xi}\right) = \bar{Q}(t) + \sum_{k=1}^{N_{rv}} \sqrt{\lambda_k}\psi_k(t)\xi_k, \tag{3.4}$$

where N_{rv} is the number of terms in the truncated KL-expansion and $Q^g(t)$ is the random vector of decision variables.

3.3.3. Stochastic Collocation (SC) Method

The Stochastic Collocation (SC) is a method used to approximately calculate the expectation of a function depending on a multi-dimensional random space. The SC method exploits properties of Lagrange interpolation polynomials and involves simple deterministic sampling at discrete points (Xiu and Hesthaven, 2005).

The Monte Carlo (MC) method is one of the traditional approaches to generate realizations and estimate the expected system output. However, the MC method is known to have a slow convergence rate (e.g., Babuska et al., 2004; Babuška et al., 2010; Xiu and Hesthaven, 2005) requiring a relatively high number of function evaluations to estimate the expected value of the output with high precision. While the MC method involves a sample mean, the SC method uses a weighted sum of an objective function solved deterministically (non-intrusive method) at the chosen collocation points,

$$E\left[f(t,\vec{Y})\right] = \sum_{nc=1}^{N_c} f(t, Y_{nc})\omega_{nc}, \tag{3.5}$$

where $\vec{Y} = [Y_{nc}]_{nc=1}^{N_c}$ is the set of collocation nodes and $\vec{\omega} = [\omega_{nc}]_{nc=1}^{N_c}$ is the set of corresponding weights for the collocation nodes. The collocation nodes are the tensor product of the roots of suitable one-dimensional orthogonal polynomials (Gibson et al., 2014; Xiu, 2007). This method turns out to be less computationally expensive in comparison to the MC method and it only needs the function evaluations at a set of pre-determined points.

3.3.4. Probabilistic Constraints

Due to the stochastic nature of the problem, the constraints cannot be evaluated deterministically. Sampling the problem can be very computationally expensive (McIntire et al., 2014). Since sampling a polynomial surrogate is computationally cheaper than solving the whole system at each sample, a polynomial surrogate of each constraint is constructed using the constraint values calculated at the collocation nodes (Gibson et al., 2014). The constructed polynomial surrogate can be sampled to approximate the probability of failure (PF) of that constraint. The PF is used as the representation of constraint violation in the process of optimization.

3.3.5.Flexible Decision Variables

Using the concept of dimension reduction, the decision space can be approximated by a few random variables given the mean and covariance structure of realizations from the decision space. As discussed in section 3.3.1, the realizations are taken to be deterministic optimal solutions. In the proposed methodology the flexible decisions are defined by the choice of coefficients in the KL-expansion. Thus, the optimization method finds the mean and standard deviation of each KL-expansion random variable ($[\xi_k]_{k=1}^{N_{rv}}$). These parameters are referred to as Control Variables to avoid confusion with the deterministic decision variables (which are the turbine outflows in daily time-steps in this study). The objectives of this optimization problem are the expected value of the problem objectives with an additional objective to maximize flexibility. Flexibility of decision variables is represented by the sum of squares of the standard deviations of the random variables in the KL-expansion multiplied by the corresponding eigenvalues. By including the eigenvalues in this calculation, the actual influence of each random variable's standard deviation is taken into account. The approach is summarized in Figure 3.1.

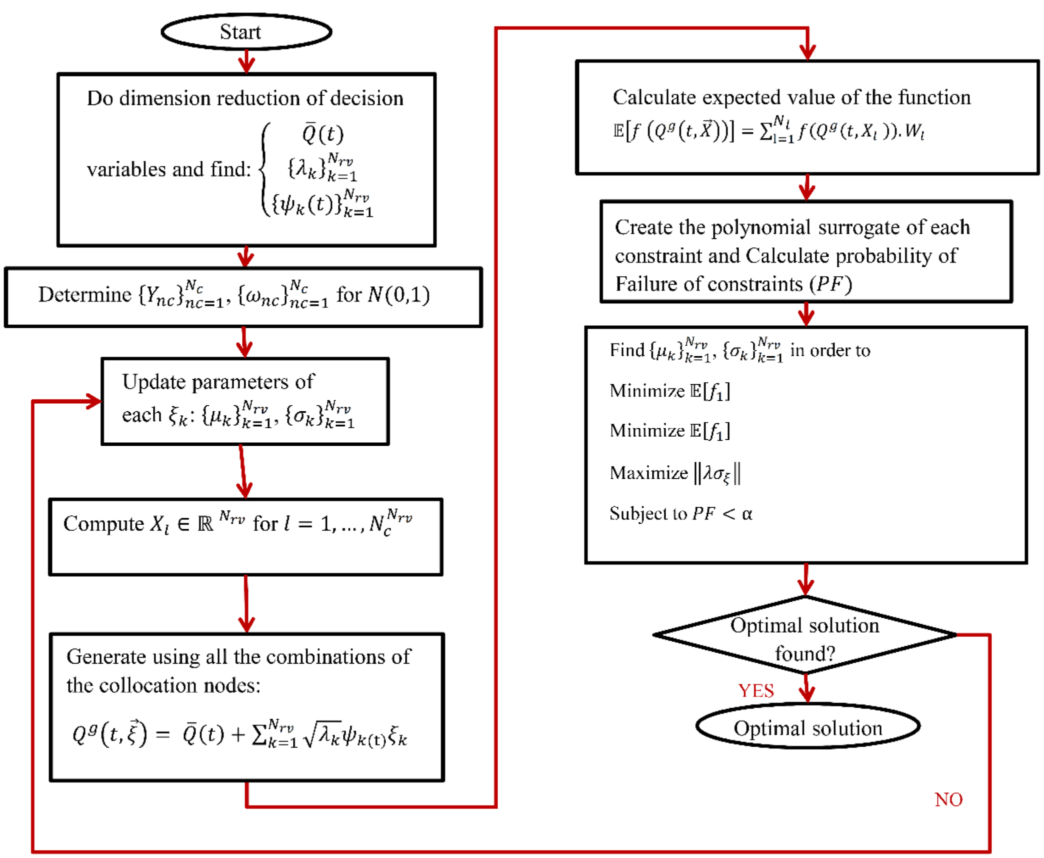

Figure 3.1. Flowchart for the proposed method

3.4. Case Study

The efficacy of the proposed methodology is tested on a reservoir operation problem described by (Hosseini et al., 2017b) and is a simplified version of the Grand Coulee reservoir problem studied by (Chen et al., 2016). The Grand Coulee reservoir is one of the largest reservoirs on the Columbia River in the Northwestern United States and hydropower production is one of its main purposes. The goal is finding the optimal turbine outflows in daily time-steps while minimizing the revenue loss[1] and minimizing the forebay elevation[2] and a given target by the end of optimization period. Forebay elevation is expected to be within a target elevation range by the end of the optimization period for future planning requirements (Chen et al., 2016). The deterministic optimal decision variables are assumed to be representative of the decision space. A period of 14 days starting August 25th is selected for this optimization problem. Data availability and the importance of optimizing the reservoirs on Columbia River in these two particular weeks (Chen et al., 2014) are the main reasons to choose this period[3].

The following scenarios are studied in this paper:

1. Deterministic optimization
2. Flexible decision variables using dimension reduction of the data
3. Flexible decision variables using dimension reduction of transformed data

3.4.1. Deterministic Optimization

This first scenario is the deterministic optimization of reservoir operation to find $\vec{Q} = [Q_n]_{n=1}^{N_t}$, in order to

$$\text{Minimize } f_1(\vec{Q}) = \frac{(|FB_{end}(\vec{Q}) - FB_{target}|)}{U_r - L_r}, \text{ and} \tag{3.6}$$

$$\text{Minimize } f_2(\vec{Q}) = -\left(\frac{\sum_{n=1}^{N_t}(PG_n(\vec{Q}) - PL_n) * Pr_n}{\sum_{n=1}^{N_t}(Pr_n * PL_n)}\right), \tag{3.7}$$

---

[1] maximizing revenue due to hydropower generation
[2] reservoir's water surface elevation
[3] the proposed framework can be extended to any other period and any other reservoir network

Subject to $Q^{min} \leq Q_n \leq Q^{max}$ for all n, (3.8)

Subject to $f_1(\vec{Q}) \leq \delta FB$, (3.9)

Subject to $|Q_n - Q_{n+1}| \leq Q^{ramp}$ for all n. (3.10)

where $\vec{Q}$ is the set of turbine outflows from the reservoir for N_t time-steps, f_1 represents the forebay elevation variation, FB_{end} is the forebay elevation by the end of the optimization period, FB_{target} is the desired forebay elevation at that time (which is a known value in the reservoir operation problem), U_r and L_r are the allowed maximum and minimum forebay elevations, PG_n is the hydropower produced for time-step n, PL_n is the load and Pr_n is the price of power. This problem has constraints for maximum and minimum allowable turbine outflows ($[Q^{min}, Q^{max}] = [30{,}290]$ kcfs for this problem (3.4.1)). Ramping constraints are also considered (equation (3.10)). The purpose of a ramping constraint is to restrict the outflow variation in two consecutive time-steps ($Q^{ramp} = 70$ kcfs for this problem). Also, a constraint is included to control the maximum allowable forebay elevation deviation (δFB equation (3.9)). Forebay elevation deviation, in addition to being one of the objectives, is also a constraint to prevent the forebay elevation to change drastically from its target.

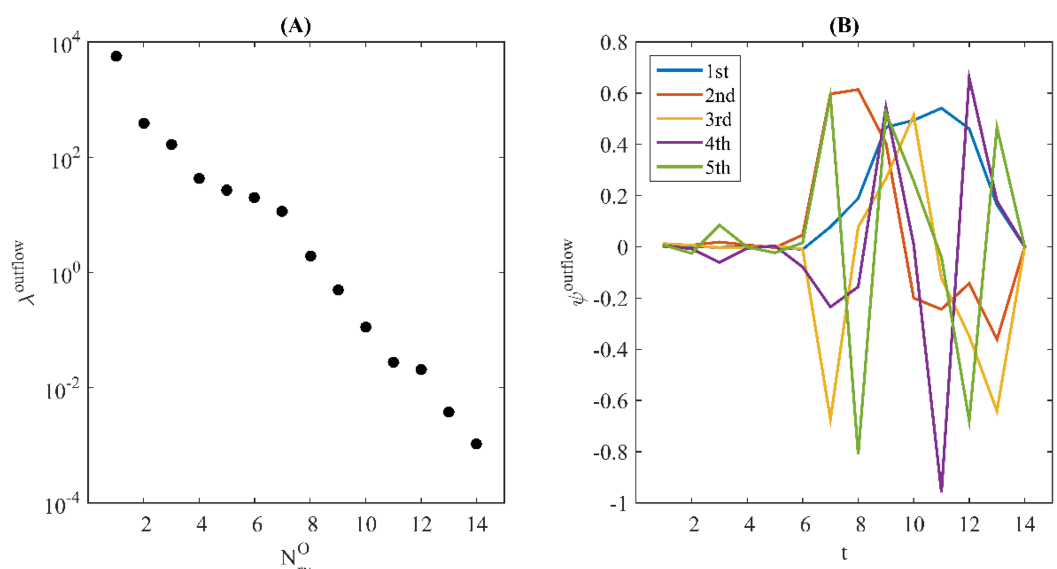

Figure 3.2. (A) Eigenvalues and (B) eigen-functions of the original (deterministic) decision variables (Semi-logarithmic scale)

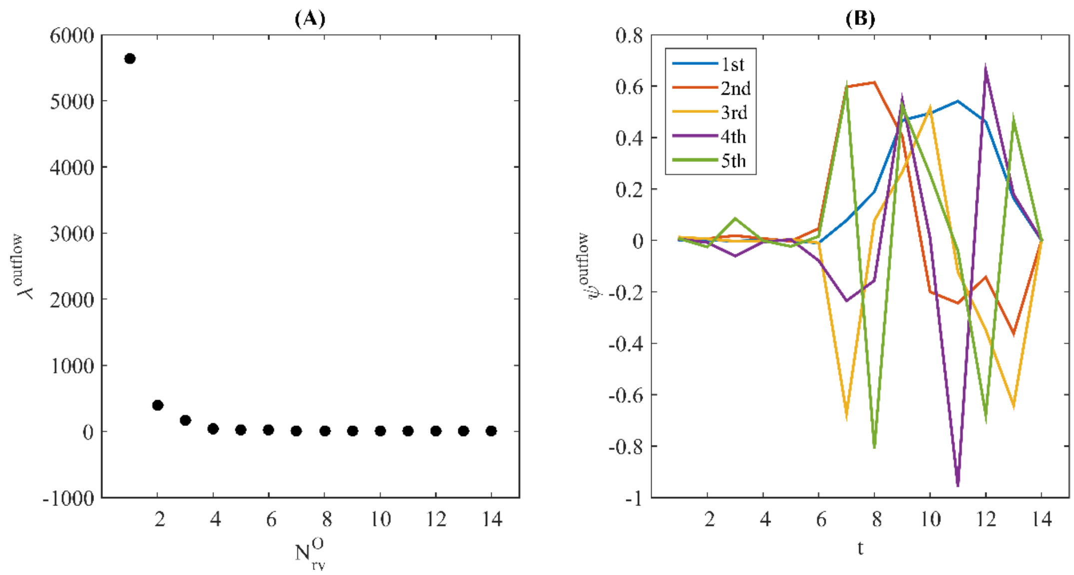

Figure 3.3. (A) Eigenvalues and (B) eigen-functions of the original (deterministic) decision variables

The dimension reduction method is applied to the optimal deterministic decision variables. The eigenvalues decrease exponentially when they are sorted in descending order (Figure 3.2). Therefore, the effect of the first few eigen-pairs are more important than the rest of them (Figure 3.3). The comparison of the randomly generated realizations (Q^g) using only a few versus all the eigen-pairs (equation (3.4)), demonstrates that even 3 random coefficients can be sufficient for the purpose of representing the decision space and generating flexible decision variables (Figure 3.4 C, D).

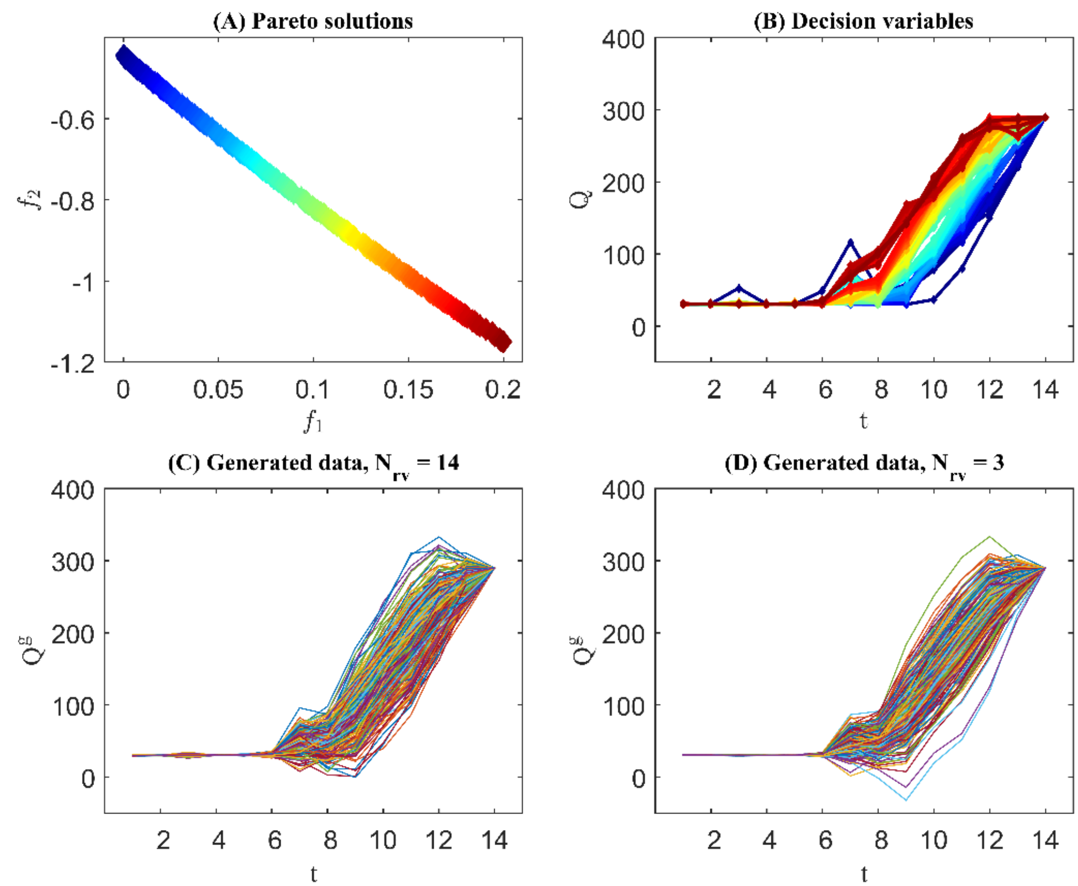

Figure 3.4. (A) The original (deterministic) optimal Pareto solutions, (B) the corresponding decision variables, (C) the generated decision variable realizations using 14, (D) the generated decision variable realizations using 3 random variables in the KL-expansion (equation (3.4))

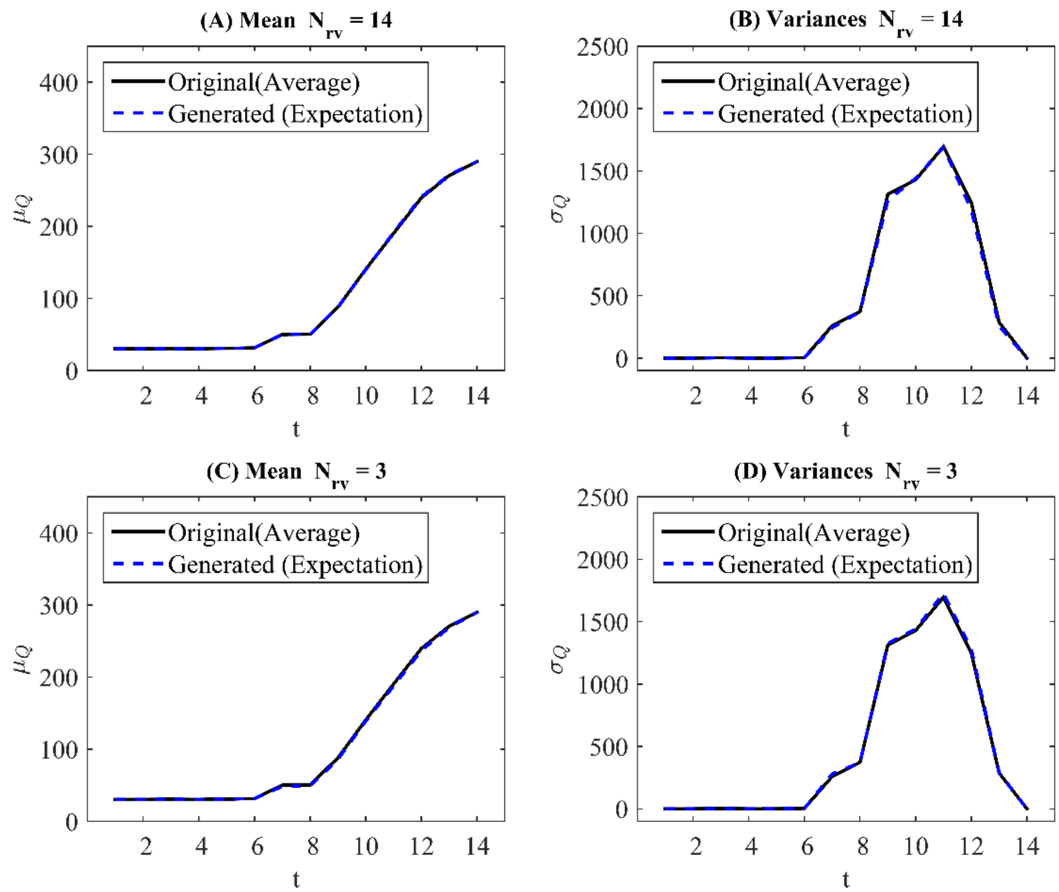

Figure 3.5. Comparison of (A) the expected of the original and generated decision variables using 14, (B) the variance of the original and generated decision variables using 14, (C) the expected of the original and generated decision variables using 3, (D) the variance of the original and generated decision variables using 3 random variables in the KL-expansion

A comparison of the original data and the generated realizations' means, indicates that the average of the original data and the expected value of the generated realizations are the same using 14 and 3 random variables (Figure 3.5 A and C, respectively). The same comparison is made to investigate the extent of deviation of the original and generated realizations. The standard deviation of the generated realizations using the truncated KL-expansion is as good as standard deviation of the ones generated using complete covariance structure information (Figure 3.5 B and D, respectively). Although the agreement of the standard deviations is not as precise as the means, it still has less than 10 % error (maximum relative error in the 11th time-step). To investigate the effect of the generated realizations on the objective space, the original

Pareto solutions are compared to a randomly generated solution set using complete and truncated KL-expansions (Figure 3.6). Because of the random nature of the KL-expansion the generated decision variables may fall outside the boundary of the original data. Therefore, there are scenarios in the objective space in addition to the original deterministic Pareto solutions. However, they may be infeasible and are disqualified and eliminated in the process of optimization. Note that the solution sets shown in Figure 3.6 are not Pareto optimal solutions. Each scenario on these sets are the result of a randomly generated realization of the decision variables in the objective space. All the randomly generated outflows are calculated using random variables with normal distribution.

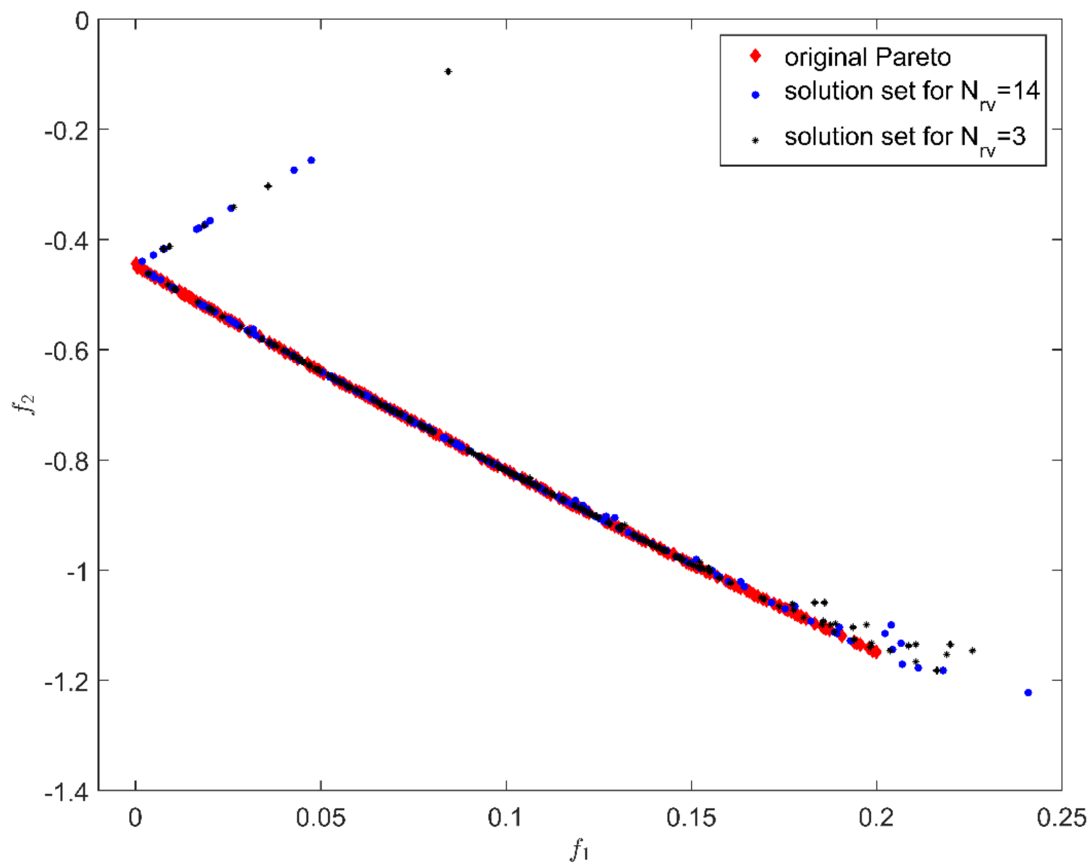

Figure 3.6. Comparison of the original deterministic Pareto solutions and the objectives corresponding to a solution set of randomly generated decision variables by KL expansion using 14 and 3 random variables

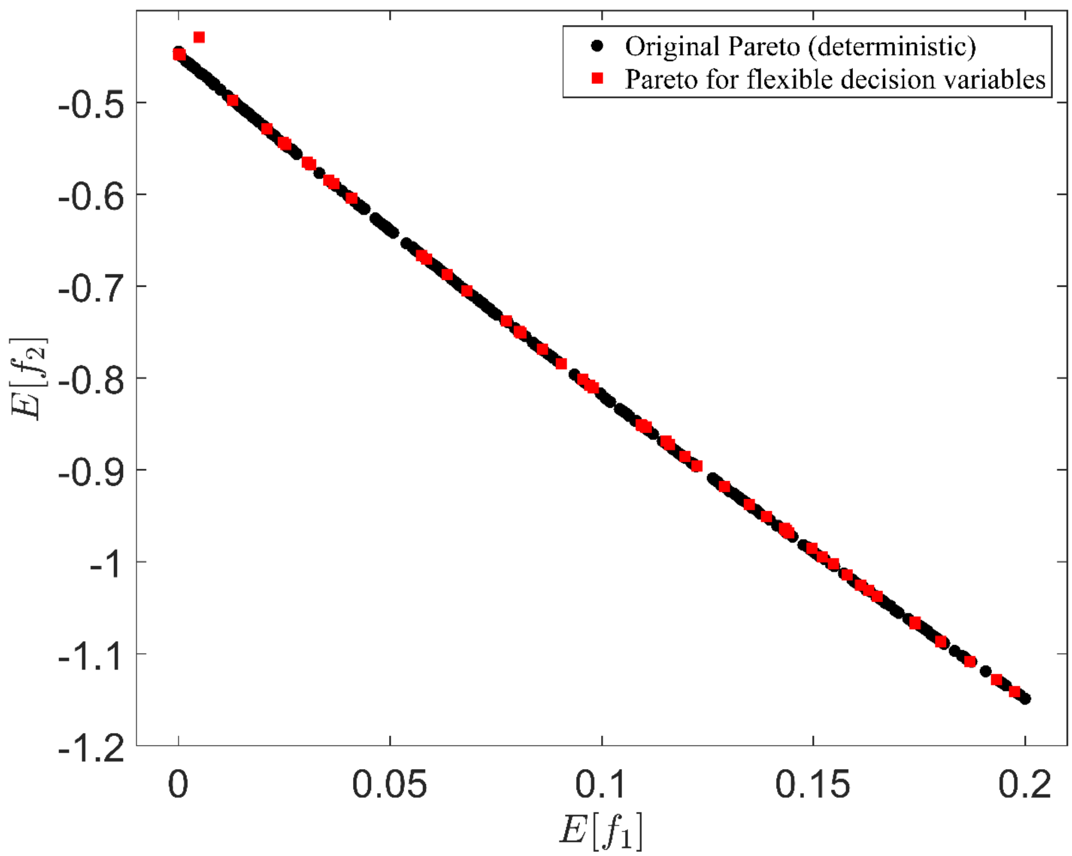

Figure 3.7. Comparison of the original deterministic Pareto solutions and the Pareto corresponding to the flexible decision variables (in 2D)

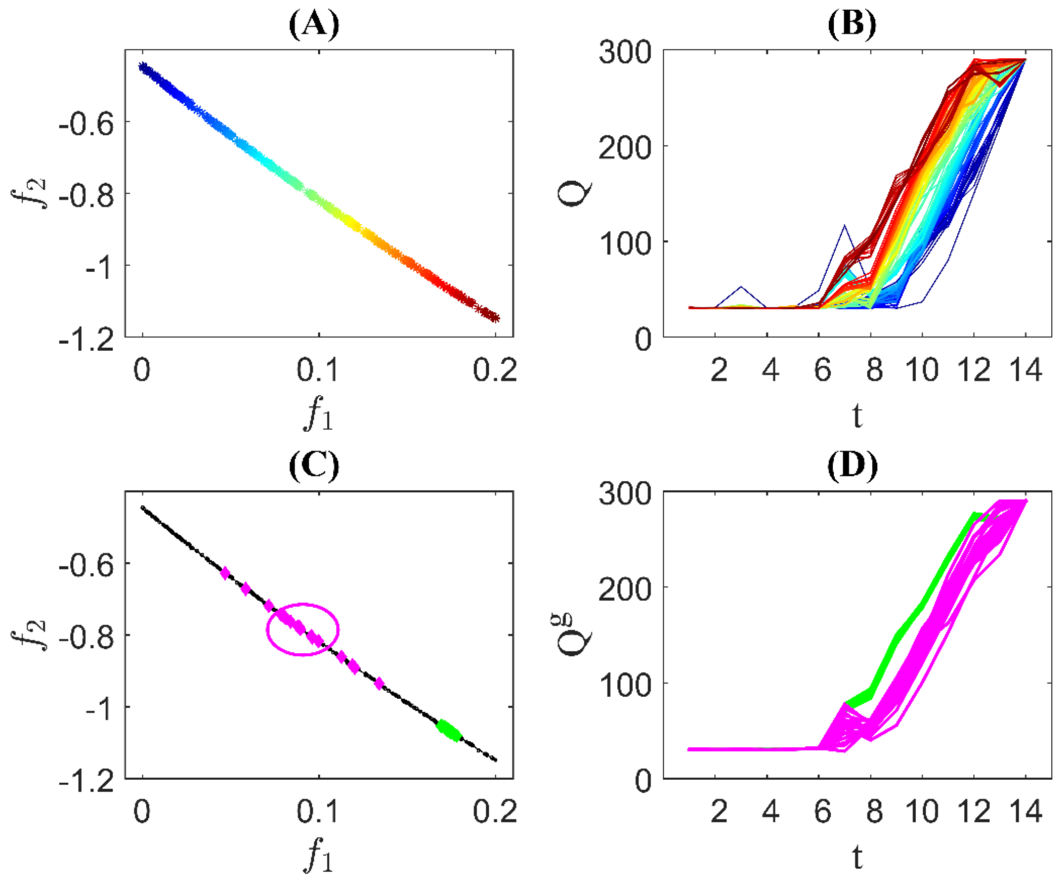

Figure 3.8. (A) The original deterministic optimal Pareto solutions, (B) the corresponding decision variables (with the same color as A), (C) comparison of the original deterministic Pareto solutions and the samples corresponding to 2 of the flexible Pareto solutions; each ellipse represents the standard deviation of f_1 and f_2 by its radii in horizontal and vertical dimension, respectively, (D) samples of the decision variable (outflow) realizations for 2 samples showed in C

3.4.2. Flexible Decision Variables Using Dimension Reduction Of The Data

The second scenario explored involves optimizing expected values of operation objectives while also maximizing flexibility in decisions. The control variables are the statistical parameters of the random variables in the KL-expansion representation of the decision variable.

The problem is stated as follows:

Find $[\mu_{\xi k}]_{k=1}^{N_{rv}}$ and $[\sigma_{\xi k}]_{k=1}^{N_{rv}}$ and compute $\overrightarrow{X} = [X_l]_{l=1}^{N_l}$, in order to

$$\text{Minimize } E\left[f_1(Q^g(t,\vec{X}))\right] = \frac{\left(\left|FB_{end}\left(Q^g(t,\vec{X})\right)-FB_{target}\right|\right)}{U_r-L_r}, \text{ and} \tag{3.11}$$

$$\text{Minimize } f_2\left(Q^g\left(t,\vec{X}\right)\right) = -\left(\frac{\sum_{n=1}^{N_t}\left(PG_n\left(Q^g(t,\vec{X})\right)-PL_n\right)*Pr_n}{\sum_{n=1}^{N_t}(Pr_n*PL_n)}\right), \tag{3.12}$$

$$\text{Maximize } f_3(\vec{X}) = \left\|\lambda\sigma_\xi\right\|, \tag{3.13}$$

$$\text{Subject to } PF(\text{constraint 1, constraint 2, constraint 3}) \leq \alpha, \tag{3.14}$$

$$\text{Constraint 1 : } Q^{min} \leq Q_n^g \leq Q^{max} \text{ for all } n, \tag{3.15}$$

$$\text{Constraint 2 : } C_2\text{: } f_1(Q^g) \leq \delta FB, \tag{3.16}$$

$$\text{Constraint 3 : } \left|Q_n^g - Q_{n+1}^g\right| \leq Q^{ramp} \text{ for all } n, \tag{3.17}$$

where $N_l = {N_c}^{N_{rv}}$ and X_l are all the adjusted stochastic collocation nodes corresponding to the parameters of the random variables ($\mu_{\xi k}$ and $\sigma_{\xi k}$).

To ensure feasibility, the joint probability of failure (PF) of the constraints should be less than the allowable probability of failure (α is chosen based on the decision maker's risk attitude). For each constraint, a polynomial surrogate of the constraint values is approximated. The PF of each constraint is calculated by sampling each polynomial surrogate and a single violation of one constraint is considered a failure for that sample. The solutions which violate ($PF < \alpha$) are considered as infeasible solutions.

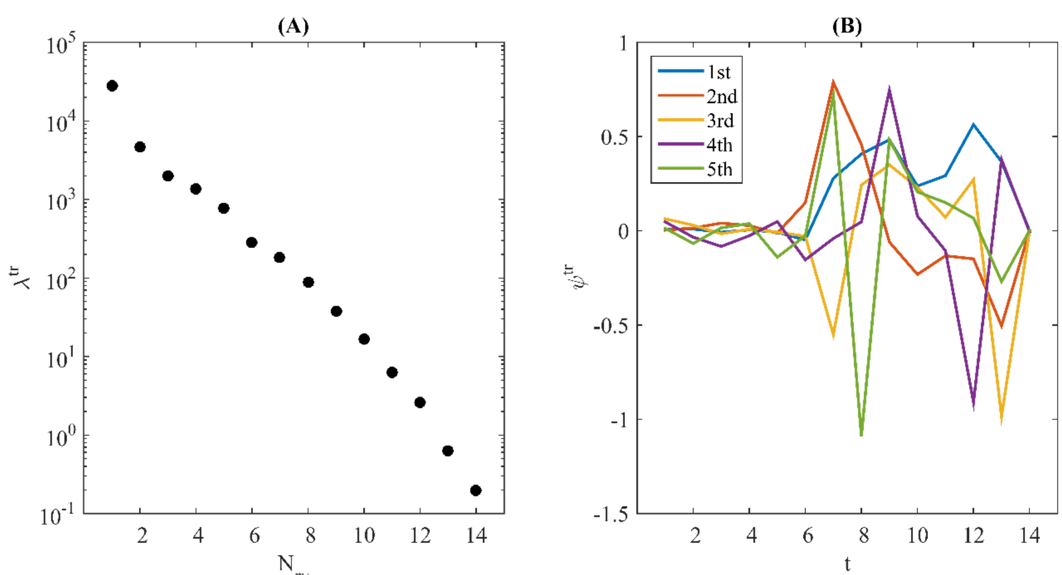

Figure 3.9. (A) Eigenvalues and (B) eigen-functions of the transformed deterministic decision variables (Semi-logarithmic scale)

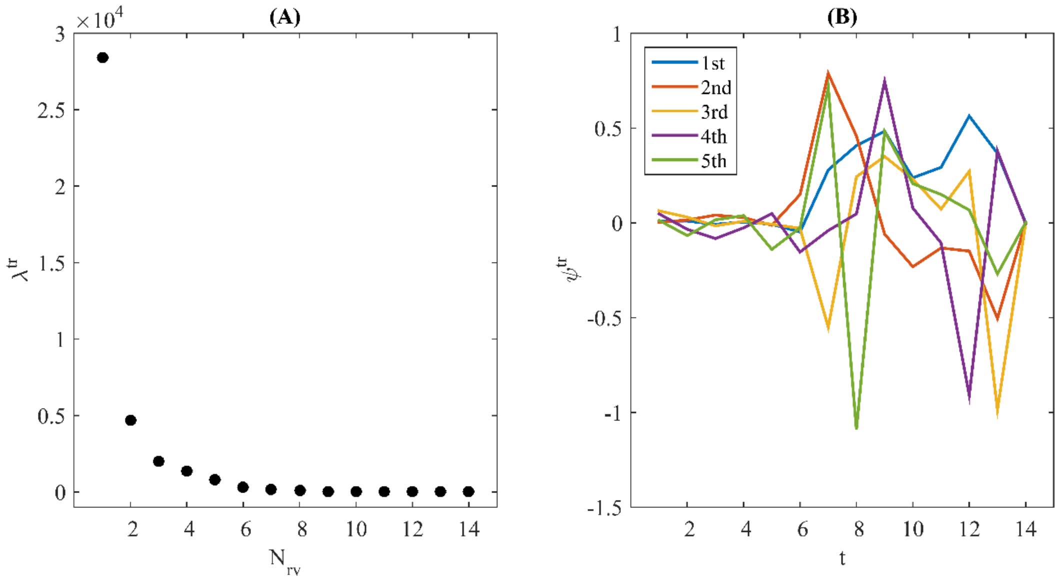

Figure 3.10. (A) Eigenvalues and (B) eigen-functions of the transformed deterministic decision variables

Discussion

Each Pareto solution for the flexible decision variables is the expectation of all the flexible scenarios (Figure 3.7). Therefore, each Pareto solution is a representation of what the expected value would be; the variation of the objective value in the objective space is not shown in this figure. To show the variability of the objective values due to random samples of KL-expansion, a few examples of the flexible decision variables (20 random samples) and the corresponding objectives and their standard deviations are shown (Figure 3.8 D and C). Only two of the Pareto solutions are selected to demonstrate the flexibility in decision space and the resulting variation in objective space more clearly. Each optimal control variable found by the proposed methodology, lead to a cluster of possible decision variables that give the decision maker flexibility in decision space.

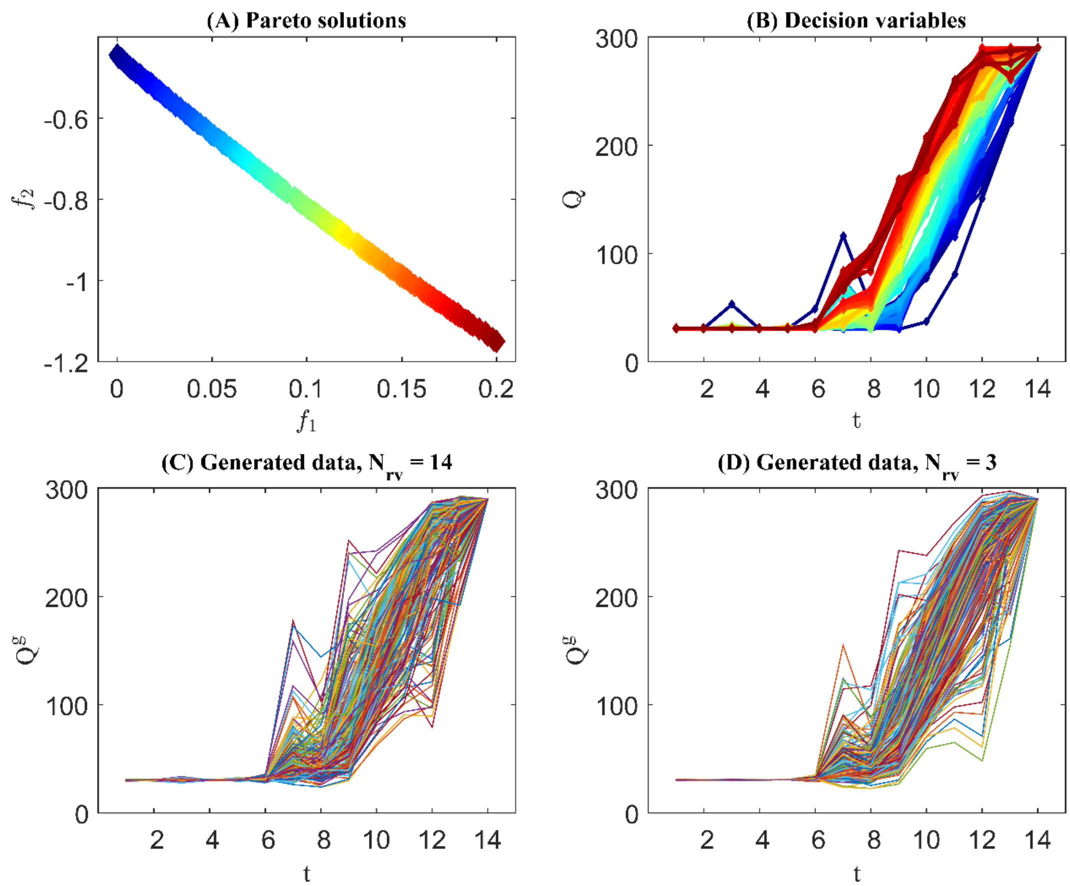

Figure 3.11. (A) The original deterministic optimal Pareto solutions, (B) the corresponding decision variables, (C) the generated decision variable realizations using 14, (D) the generated decision variable realizations using 3 random variables in the KL-expansion with transformed data (equation (3.4))

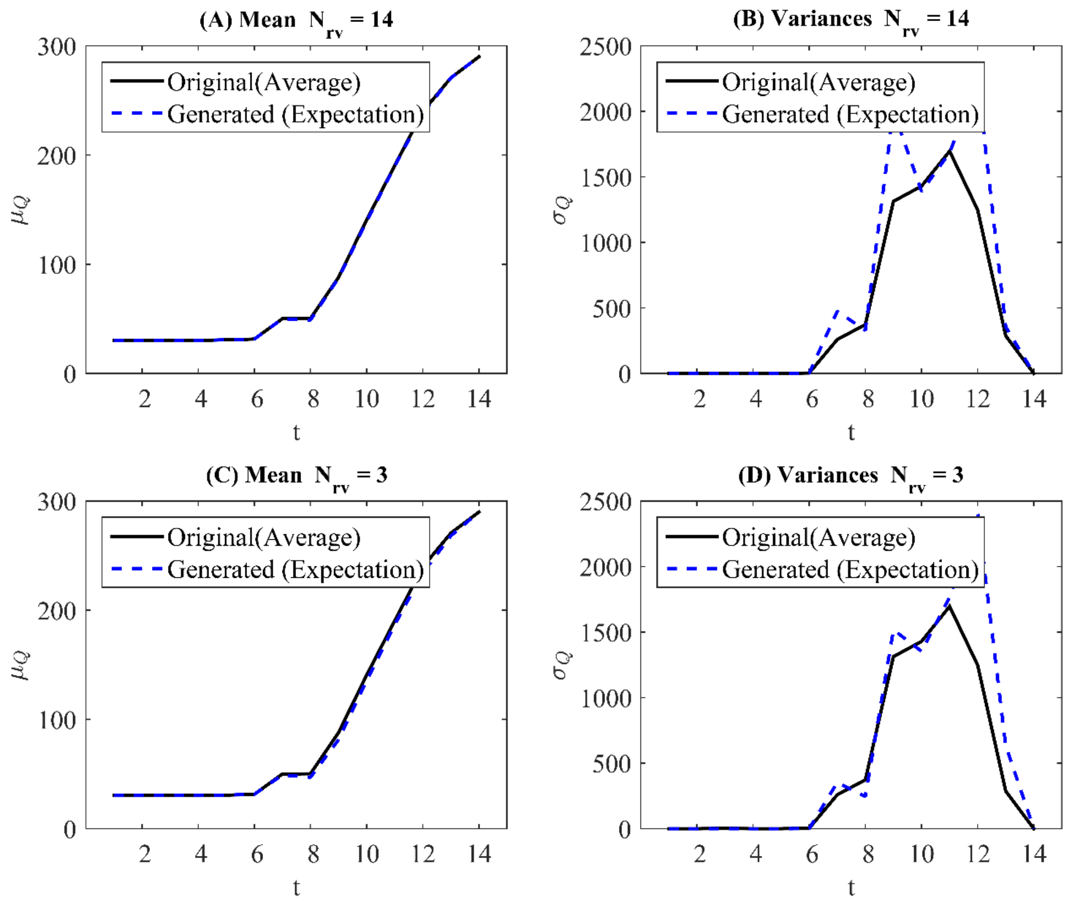

Figure 3.12. Comparison of (A) the expected of the original and generated decision variables using 14, (B) the variance of the original and generated decision variables using 14, (C) the expected of the original and generated decision variables using 3, (D) the variance of the original and generated decision variables using 3 random variables in the KL-expansion with transformed data

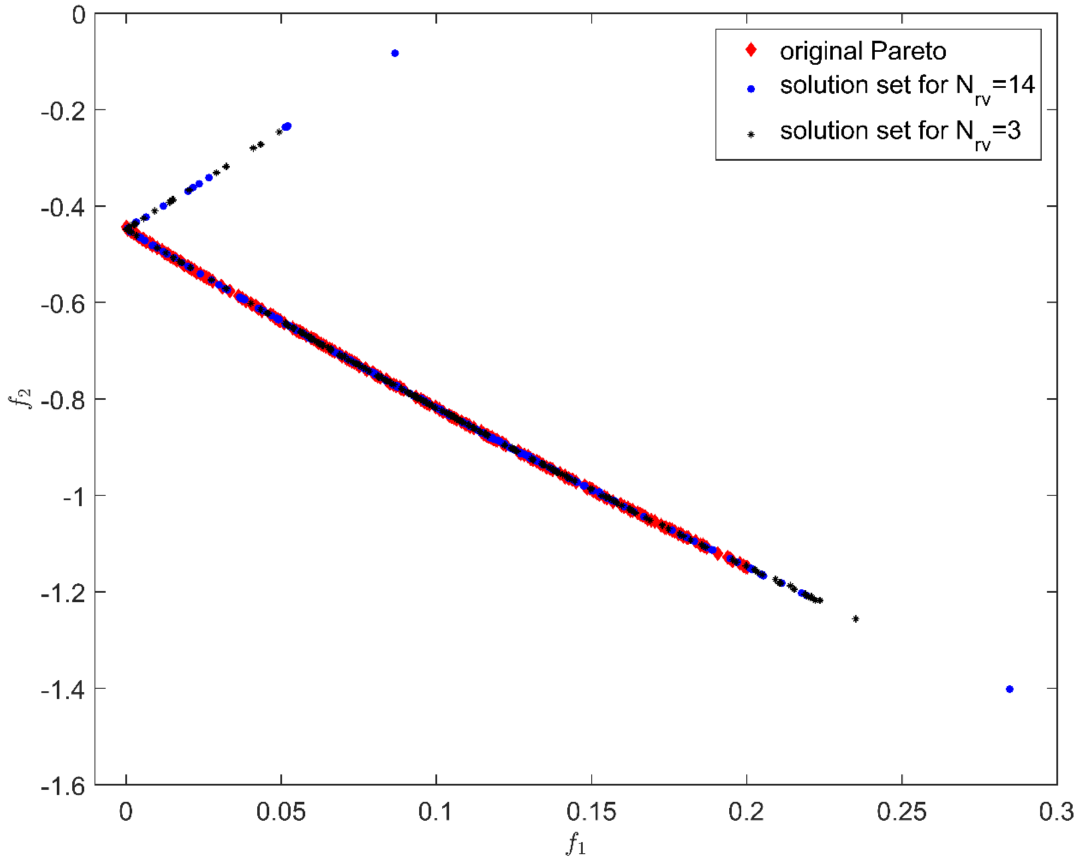

Figure 3.13. Comparison of the original deterministic Pareto solutions and the objectives corresponding to a solution set of randomly generated decision variables by KL-expansion with transformed data using 14 and 3 random variables

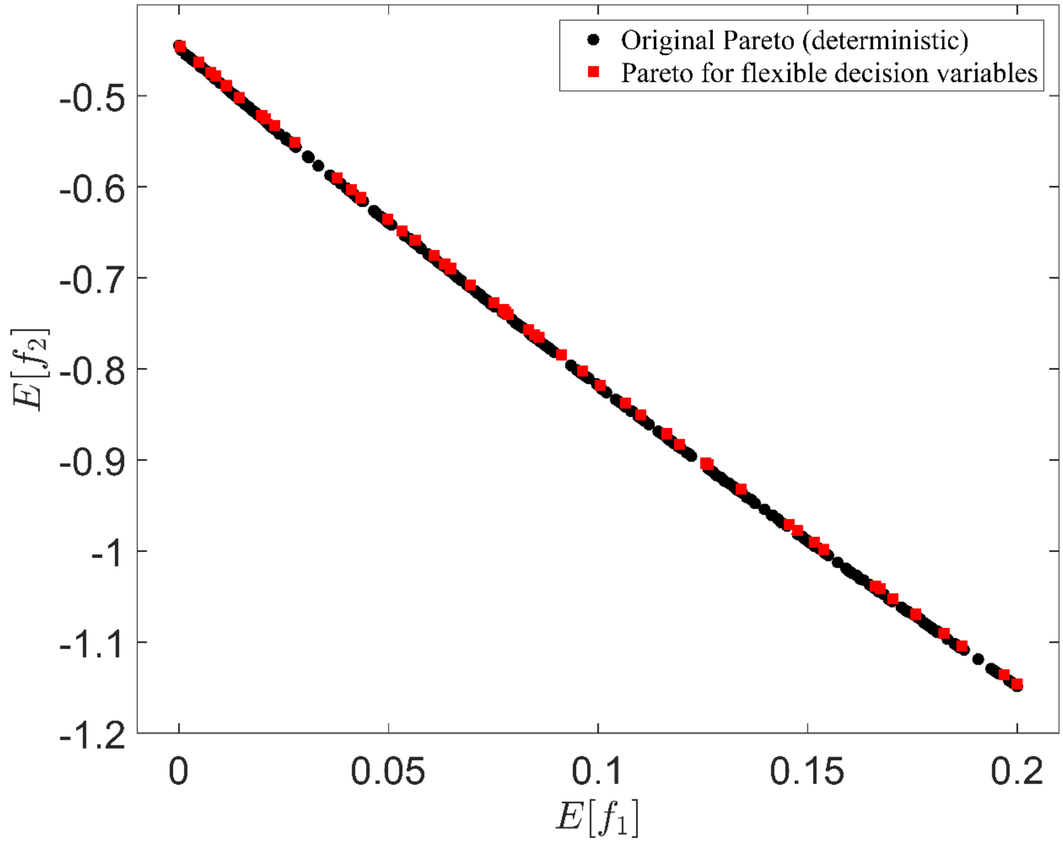

Figure 3.14. Comparison of the original deterministic Pareto solutions and the Pareto corresponding to the flexible decision variables (in 2D) using transformed data

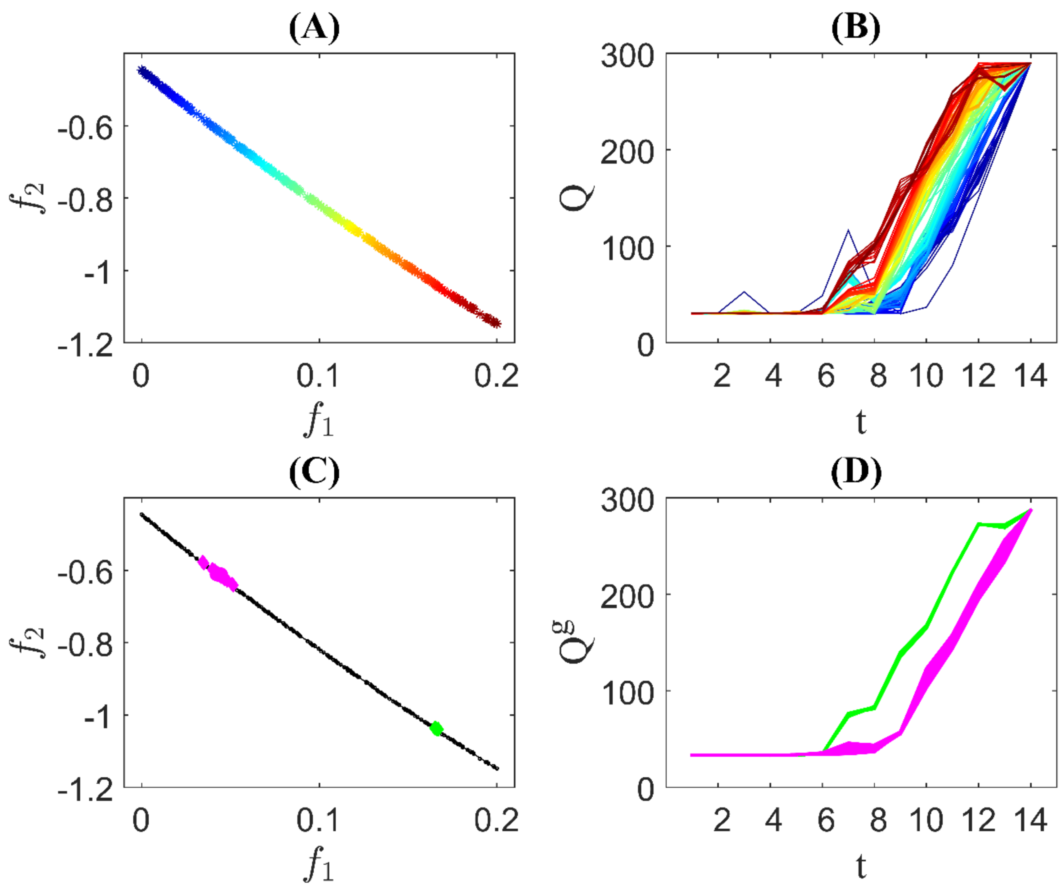

Figure 3.15. (A) The original deterministic optimal Pareto solutions, (B) the corresponding decision variables (with the same color as A), (C) comparison of the original deterministic Pareto solutions and the samples corresponding to 2 of the flexible Pareto solutions; each ellipse represents the standard deviation of f_1 and f_2 by its radii in horizontal and vertical dimension, respectively, (D) samples of the decision variable (outflow) realizations for 2 samples showed in C using transformed data

3.4.3. Flexible Decision Variables Using Dimension Reduction Of Transformed Data

Due to the use of random variables in the representation, the generated realizations using KL-expansion may not always be within the feasible boundaries ($[Q^{min}, Q^{max}]$, Figure 3.5Figure 3.4). Therefore, constraints are imposed to make sure each generated turbine flow is in the feasible region (equation (3.15)). Constructing the polynomial surrogates of these constraints can be computationally expensive. An alternative is to transform the data with a function designed to restrict the generated solutions to stay in the feasible region. The original data is transformed with $Q^{tr}(t) = \left(Q^{avg}_{conservative}\right) + Q^{bound}_{conservative}\tan\left(\frac{Q(t)-Q^{avg}_{conservative}}{Q^{bound}_{conservative}}\right)$ and the covariance structure of the transformed data is used to generate the KL-expansion formulation, instead. Where $Q^{avg}_{conservative} = \frac{Q^{min}_{conservative}+Q^{max}_{conservative}}{2}$ and $Q^{bound}_{conservative} = \frac{Q^{min}_{conservative}-Q^{max}_{conservative}}{2}$. To keep the solutions in the feasible region, a conservative boundary is considered ($Q^{max}_{conservative} = Q^{max} - Q^{min}$). The transformed data is used in the process of dimension reduction thus implicitly enforcing bounds, so the boundary constraints for the decision variables (equation (3.15)) can be disregarded.

Discussion

The data Transformation approach reduces the computational time since the generated solutions are automatically in the feasible region and there is therefore no need to calculate PF of the boundary constraints. By using the transformed data rather than the original data, some constraint calculations can be avoided and this leads to a 7% reduction in CPU time. The reduction in computational time can be larger (83% less CPU time) for the problems with no ramping constraints as there is no need for constructing the polynomial surrogates of the decision variables and calculating the probability of failure of the constraints.

The convergence rate of the eigenvalues of the transformed data to zero is slower than the convergence rate of eigenvalues of the original data (Figure 3.10). However, the generated decision variables using the transformed data are bounded in a desirable range (Figure 3.11). Further, the generated realizations are more diverse (Figure 3.11 D) compared to the generated realizations in Figure 3.4. The objective values corresponding to generated decision variables are not always similar to the original Pareto solutions (Figure 3.13). Although the variance of the generated data using the transformed data does not match the original variance (Figure 3.12), the generated data can be useful for the purpose of finding flexible decision variables because the mean of the original and generated data are very close. Also the comparison of the mean and variance using truncated KL-expansion and the KL-expansion using 14 random variables, shows that the use of only a few random variables to generate realizations is sufficient as the variance and especially the mean value are not very different (Figure 3.12).

As the Pareto solutions of section (3.4.3) are in 3-dimensional objective space, the 2D projection of Pareto (Figure 3.14) is compared to the original deterministic Pareto solutions (3.4.1). The comparison indicates that the expectation of the objectives corresponding to the flexible decision variables are very similar to the original Pareto solutions (Figure 3.14). The samples of the flexible decision variables shows that for each Pareto solution there is a cluster of possible decision variables; the corresponding objective values shows larger variations can happen for more flexible decision variables (Figure 3.15 D). The variability in objective space is mostly along the deterministic Pareto solutions.

3.5. Conclusion

The goal of this study is to find flexible decision variables for a reservoir operation problem that give decision makers more options in the process of optimization. Additionally, the variation of objective space could be minimized by the implementation of robust objectives, but this is not within the scope of this paper. The proposed methodology finds flexible decision variables using dimension reduction. This method is more computationally efficient than other methods that represent each decision variable with a random variable and is more practical for problems with multiple decision variables (time-steps). While, the results of this study are illustrated for probability of failure less than 10%, the problem has also been solved for 20% and no probability of failure ($\alpha < 1\%$). Decreasing the allowable probability of failure results in less flexible options.

The expected values of the objective functions are approximated by the SC method using only a few collocation points in each random dimension. The error in the approximation by the SC method is less than 1% in comparison to the calculated expectation by the MC method.

The optimal decision variables from the original data (section 3.4.2) are more flexible than the decisions found using the transformed data (section 3.4.3) (Figure 3.8 and (3.14)), however the use of transformation ensures that the constraints are satisfied.

While a Gaussian distribution is assumed for the distribution of random coefficients in KL-expansion, this choice can be the decision maker's preference and in this study does not have a significant effect on the results.

Acknowledgments

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4. Robust multi-objective optimization of reservoir operation under uncertain inflows with flexible decision variables

4.1. Abstract

This study presents a robust and flexible optimization framework for the operation of a single reservoir. Robust optimization considers the effects of uncertainty in the process of finding optimal solutions. The main source of input uncertainty considered in this study is due to uncertain inflows. Uncertainties of outflows can be correlated to the uncertainties of inflows via the dynamics of the reservoir system. Because other sources of uncertainty can also influence the reservoir operation, instead of determined decision variables, the reservoir operator may prefer flexible decision variables, which ensures that no constraints are violated as long as the reservoir operation is within the range of these decision variables. A multi-objective evolutionary algorithm is used for maximizing flexibility while also maximizing hydropower generation and minimizing variation in forebay elevation of the Grand Coulee reservoir, which is located in the Columbia River (U.S. Pacific Northwest). The results of this study are compared against a purely robust optimization framework and the advantages of this robust and flexible optimization approach are discussed.

4.2. Introduction

Optimization under uncertainty has been studied in various applications. To have reliability and robustness, it is essential to consider uncertainties in water resources planning and management problems (Nicklow et al., 2009). Some of the parameters of an optimization problem may not be well predicted, however the statistics may be known. Therefore the expectation of the problem objective may be calculated with respect to the random parameter, and can be used to replace the original objective of the problem (Houda, 2006). However, there are scenarios for which considering only the expectation of an objective is not an adequate representation of the possible realizations of the objective, as large and unacceptable negative and positive deviations can cancel out and inaccurately lead to an expected value that seems acceptable (Jin and Sendhoff, 2003). Different approaches have been studied in literature for addressing robustness in the process of optimization, including but not limited to the minimization of the expectation of the objective simultaneously with the minimization of objective variation due to uncertain parameters (Sahinidis, 2004).

To find robust and reliable optimal decisions, the variation in the objective space due to input and parameter uncertainties should either be confined or at least remain within the feasible region (Jung and Lee, 2002). A concise review of utilization of robust optimization in water resources management problems has been provided and the different definitions of robustness in each problem were described in (Ray et al., 2010).

Reservoir operation is one of the important areas in water resources management in which the operator needs to determine how to release water from the reservoir in desired time-steps (e.g., hourly, daily) to optimize the objectives that the reservoir is meant to fulfill (Ahmadi et al., 2014). According to multiple studies, the main source of uncertainty associated to a reservoir system is the uncertain inflows (e.g., Escudero, 2000; Faber and Stedinger, 2001; Gibson et al., 2014; Karamouz et al., 2009; Kerachian and Karamouz, 2006; Leon et al., 2012; Rani and Moreira, 2010). Considering uncertainties is essential in optimizing the operation of a reservoir system, as these variations may lead to inadequate objectives and violation of constraints. Optimal scenarios may become less desirable due to uncertainties and even become infeasible. Uncertain inflows can be represented by a linear stochastic perturbation of the expectation of inflow realizations rather than assuming a stochastic behavior, as that may lead to large impractical uncertainties (Gibson et al., 2014; Leon et al., 2012).

On the other hand, some situations may occur, for instance, due to an unforeseen source of uncertainty, in which the decision maker prefers to alter the expected optimal decision variables (e.g., Babbar-Sebens, 2017; Babbar-Sebens et al., 2013; Kaini et al., 2012) calculated either deterministically or through a robust optimization framework considering input uncertainty. In this case, flexible decision variables that will allow deviation from its expected value and ensure feasibility and relative optimality, are desirable (Hosseini et al., 2017b). The concept of finding flexible decision variables using a dimension reduction method to find optimal solutions with flexible decision variables was proposed in (Hosseini et al., 2017a). A set of feasible decision variables was utilized to represent the decision space and a KL expansion was used to approximate the decision space with a few number of control variables. Then in the optimization process, the parameters of the control variables were found in order to optimize the expected value of the objectives and also maximize the flexibility in decision space.

The main contribution of this paper is to develop a framework for finding flexible and robust decision variables in a multi-objective reservoir operation problem under uncertain inflows. The proposed methodology is able to utilize the historical and/or deterministic optimal decision variables to represent and regenerate the decision space with a manageable number of control variables (Hosseini et al., 2017a). Moreover, forecast inflow hydrographs are considered as the source of input uncertainty. Input uncertainty can be represented by a continuous random framework and subsequently the dimension can be reduced to a small number of random variables (Gibson et al., 2014).

The expected value of each objective can be approximated with respect to these two different sources of uncertainty (flexible decision variables and random inflows). To ensure robustness the weighted sum of the expected objective and its variance is considered as the robust objective of the optimization problem (Arora, 2004; McIntire et al., 2014). The optimal solutions are chosen to be robust with respect to uncertain inflows (type I robust design) and also flexible within the decision space (type II robust design) (Chen et al., 1996).

4.3. Methodology And Case Study

The concept of robust objectives is implemented to minimize the variation in objective space from uncertain inputs and flexible outputs. The stochastic behavior of the problem is due to the uncertain inflows coming into a reservoir and the randomness of the flexible decision variables. Therefore, the proposed methodology is a combination of type I and II robust design problems (Chen et al., 1996). To simplify the explanation of the model, the methodology is explained in 3 sections. First, the robust optimization under uncertain inputs is explained (section 4.3.1). Then the implementation of a dimension reduction method with optimization for finding flexible decision variables is described (section 4.3.2). At last, the structure of the overall proposed methodology for finding robust and flexible decision variables is explained (section 4.3.3).

Operation of Grand Coulee reservoir in Columbia River (located in Northwestern United States) is chosen as the case study to test the efficacy of the proposed method. This reservoir problem is a simplified version of the problem studied by (Chen et al., 2016; Hosseini et al., 2017b).

4.3.1. Robust Optimization Under Uncertain Inputs

The uncertain input in this problem can described by an ensemble of forecasted inflows to the reservoir. However, in the absence of this data, historical inflow hydrographs can be used instead. Historical inflows for Grand Coulee are used in this study. It is assumed that the inflows are represented by a Gaussian process[1]. A Karhunen-Loeve (KL) expansion is used to reduce the dimension of random inflows and represent inflow uncertainty with a few random variables. The average and the covariance structure of the historical inflows are calculated using

$$\bar{Q}^{inflow}(t) = \frac{\sum_{i=1}^{M_l} Q_i^{inflow}(t)}{M_l}, \tag{4.1}$$

$$C^{inflow}(t_j, t_k) = \frac{\sum_{i=1}^{M_l}\left(Q_i^{inflow}(t_j) - \bar{Q}_j^{inflow}\right)\left(Q_i^{inflow}(t_k) - \bar{Q}_k^{inflow}\right)}{M_l - 1}, \tag{4.2}$$

where $\bar{Q}^{inflow}$inflow and $C^{inflow}$are the average and covariance of the inflow realizations, respectively and M_l is the number of the historical inflow realizations. Then the KL expansion is as follows

---

[1] A distributional sensitivity test has shown that there is not significant sensitivity to different types of distributions of inflow (Gibson et al., 2014).

$$I^g(t,\vec{\xi}) = \bar{Q}^{inflow}(t) + \sum_{k=1}^{\infty}\sqrt{\lambda_k^{inflow}}\psi_k^{inflow}(t)\xi_k^I, \quad (4.3)$$

$$\lambda^{inflow}\psi^{inflow}(t) = \int_{t_0}^{n} C^{inflow}(x,t)\psi^{inflow}(x)dx, \quad (4.4)$$

where $[\lambda_k^{inflow}]_{k=1}^{\infty}$ are the eigenvalues and $[\psi_k^{inflow}]_{k=1}^{\infty}$ are the eigen-functions of the integral equation (4.4), and $\vec{\xi}^I = [\xi_k^I]_{k=1}^{\infty}$ are uncorrelated random variables with mean 0 and standard deviation 1. The eigenvalues converge rapidly to zero when sorted in decreasing order (Figure 4.1, Figure 4.2 A). Therefore only a few eigenvalues and eigen-functions are sufficient to regenerate inflow realizations using the KL expansion (equation (4.5)). The eigenvalues are negligible after the fourth eigenvalue (Figure 4.1), however, even 2 random variables can suffice (Figure 4.2). The reduced order model is given by the truncated KL expansion

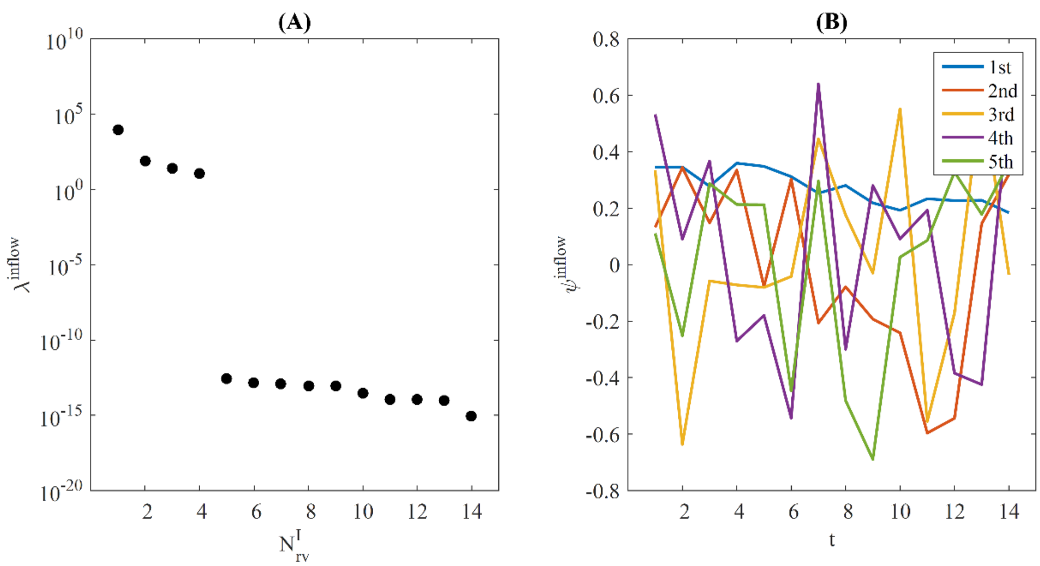

Figure 4.1. Eigenvalues and eigen-functions of the historical inflow ensembles in semi-logarithmic scale

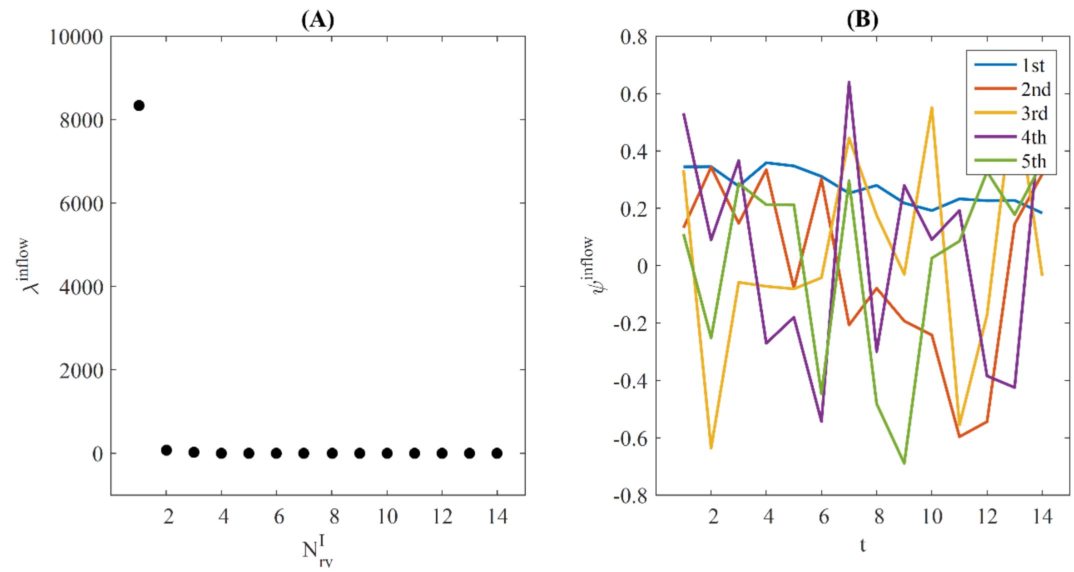

Figure 4.2. Eigenvalues and eigen-functions of the historical in flow ensembles

$$I^g(t,\vec{\xi}) \cong \bar{Q}^{inflow}(t) + \sum_{k=1}^{N_{rv}^I} \sqrt{\lambda_k^{inflow}} \psi_k^{inflow}(t)\xi_k^I, \tag{4.5}$$

where N_{rv}^I is the number of random variables needed to approximate the uncertain inflows. For a realization of the random vector $\vec{\xi}^I$, the quantity I^g is the generated inflow realization. Using the generated inflow realizations, the objective functions can be evaluated non-intrusively for each sample and the expectation and standard deviation of the objective function (with respect to random inflows) can be used to formulate the robust objective of the optimization problem.

To decrease the number of function evaluations while calculating the expected value of the objectives, the Stochastic Collocation (SC) method is used. In SC method, the system is calculated at strategically chosen samples of the random variables called collocation points. These collocation points and their corresponding weights are used to approximate the expected value of a function

$$E[f(t,\xi)] \cong \sum_{nc=1}^{N_c} f(t,\xi_{c_{nc}})\omega_{nc}, \tag{4.6}$$

The number of function evaluations required for a given error tolerance is significantly less compared to other sampling techniques such as Monte Carlo Method.

Robust optimal solutions to the reservoir operation problem are found using the robust objectives concept (Arora, 2004). A robust objective is implemented to simultaneously optimize the expected value of the objective (weighted summation) and minimize the variation (standard deviation) of the objective due to input uncertainty. A robust objective ensures that the robust optimal solutions are relatively less sensitive to the input uncertainty and are more reliable in their constraints considering a pre-specified probability of failure (McIntire et al., 2014). A robust objective for a minimization problem can be defined by

$$\text{Minimize } F = \left(1 - \omega_f\right)\frac{E[f]}{\mu_{f_{max}}} + \omega_f \frac{\sigma_f}{\sigma_{f_{max}}}, \quad (4.7)$$

where $E[f]$ and σ_f are the expected value and standard deviation of function f due to uncertain input uncertainties, respectively. The expectation and the standard deviation are normalized by their maximum values ($\mu_{f_{max}}$ and $\sigma_{f_{max}}$), ω_f is the weight of robust objective and indicates the decision maker's preference of robustness versus performance. This weight helps in choosing solutions in the trade-off between robustness and optimal objective values.

Problem Description

The decision variables of the optimization problem are the daily turbine outflows that are released from the reservoir. To ensure the achievement of the reservoir's future requirements, the water surface elevation of the reservoir is expected to remain within an elevation range by the end of optimization period. Therefore, the first objective of the optimization problem is the minimization of forebay elevation (reservoir's water surface elevation) deviation.

The second objective is the maximization of revenue due to hydropower production[1]. The power demand (load) is extracted from the generated hydropower and the difference is called the net electricity. The revenue is calculated from multiplication of price by the net electricity. The price of hydropower is assumed to be known and pre-determined by an economic model (Chen et al., 2016). Grand Coulee reservoir does not have the fish flow requirements, and other objectives are neglected in this study.

The decision variables are the daily turbine outflows for a period of 14 days starting from August 25th. This period is chosen due the importance of these14 days for BPA. For some of the operated reservoirs by BPA, the objectives of reservoir operation shifts in this period.

The statement of the robust optimization framework is as follows:

Find $\vec{Q} = [Q_n]_{n=1}^{N_t}$, in order to

$$\text{Minimize } F_1 = \left(1-\omega_{f_1}\right)\frac{E\left[f_1\left(\vec{Q},\vec{I}^g(\vec{\xi}^I)\right)\right]}{\mu_{f_1max}} + \omega_{f_1}\frac{\sigma_{f_1}}{\sigma_{f_1max}}, \tag{4.8}$$

$$\text{where } E[f_1(\vec{Q},\vec{I}^g(\vec{\xi}^I))] = \frac{\left(\left|FB_{end}\left(\vec{Q},\vec{I}^g(\vec{\xi}^I)\right)-FB_{target}\right|\right)}{U_r-L_r}, \tag{4.9}$$

$$\text{Minimize } F_2 = \left(1-\omega_{f_2}\right)\frac{E\left[f_2\left(\vec{Q},\vec{I}^g(\vec{\xi}^I)\right)\right]}{\mu_{f_2max}} + \omega_{f_2}\frac{\sigma_{f_2}}{\sigma_{f_2max}}, \tag{4.10}$$

$$\text{where } E[f_2(\vec{Q},\vec{I}^g(\vec{\xi}^I))] = -\left(\frac{\sum_{n=1}^{N_t}\left(PG_n\left(\vec{Q},\vec{I}^g\left(\vec{\xi}^I\right)\right)-PL_n\right)\times Pr_n}{\sum_{n=1}^{N_t}(Pr_n\times PL_n)}\right), \tag{4.11}$$

$$\text{Subject to } C_1, C_2, PF(C_3) \leq \alpha\ , \tag{4.12}$$

$$C_1\text{: } Q^{min} \leq Q_n \leq Q^{max} \text{ for all } n, \tag{4.13}$$

$$C_2\text{: } |Q_n - Q_{n+1}| \leq Q^{ramp} \text{ for all } n, \tag{4.14}$$

$$C_3\text{: } f_2\left(\vec{Q},\vec{I}^g\left(\vec{\xi}^I\right)\right) \leq \delta FB. \tag{4.15}$$

where $\vec{Q}$ is the set of turbine outflows from the reservoir, $\vec{I}^g$ is the generated inflow hydrograph with N_t time-steps using a set of random variables ($\vec{\xi}^I = [\xi_k^I]_{k=1}^{N_{rv}^I}$), $E[f_1]$

[1] the framework is designed for minimization problems, therefore this objective is converted to minimization of revenue loss

represents the expectation of forebay elevation variation, $FB_{end}$is the forebay elevation at the last time-step, FB_{target} is the desired forebay elevation at the last time-step (which is a known value in the reservoir operation problem). U_r is the maximum water level of the reservoir and L_r is the minimum water level and are used to normalize the forebay elevation values. PG_n is the hydropower produced at time-step n, PL_n is the load and Pr_n is the price for the hydropower. Constraints of this problem consists of box constraints for the allowable turbine flows in each time-step (equation (4.13)), the ramp constraint for the outflow in two consecutive time-steps (equation (4.14)) and the maximum forebay elevation (equation (4.15)). The forebay elevation constraint should be satisfied for the most likely uncertain inflows. To enforce this, a polynomial surrogate of the constraint is approximated and the solutions with Probability of Failure (PF) less than a pre-specified threshold (δFB), are considered to be feasible.

Discussion

The inflow realizations can be generated using 2 random coefficients in KL expansion and in comparison to the original historical inflows (Figure 4.3), the mean and the variance are almost the same. The difference of original inflow realizations' average and the expectation of generated samples is less than 2%. The difference of the original and generated inflows' variances is more than the difference of the mean values (approximately 10%), because the KL expansion generally predicts the mean value more precisely than the other modes.

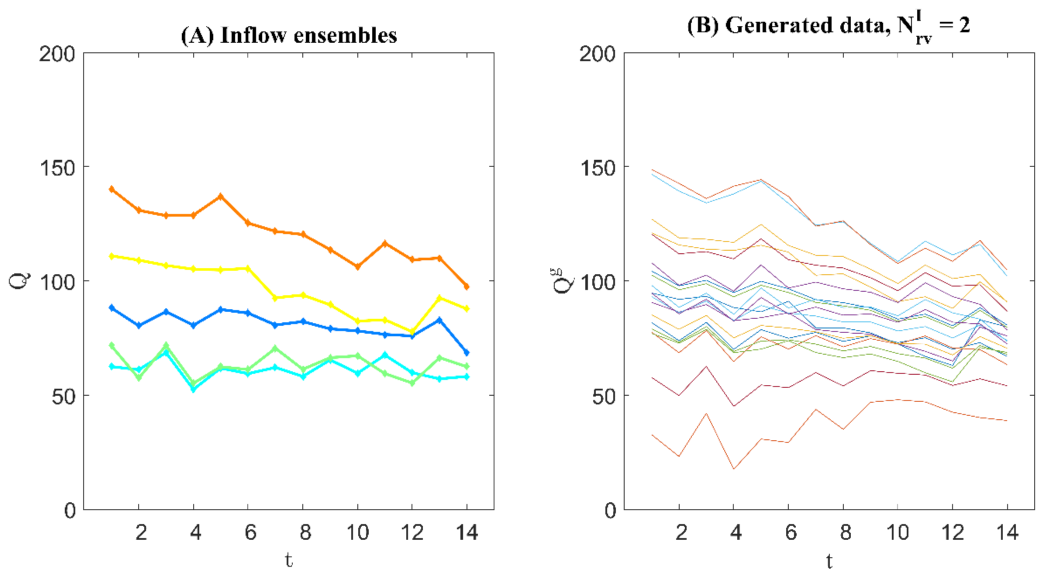

Figure 4.3. Comparison of the historical inflow ensembles (A) and the generated inflows using 2 random coefficients of KL-expansion (B)

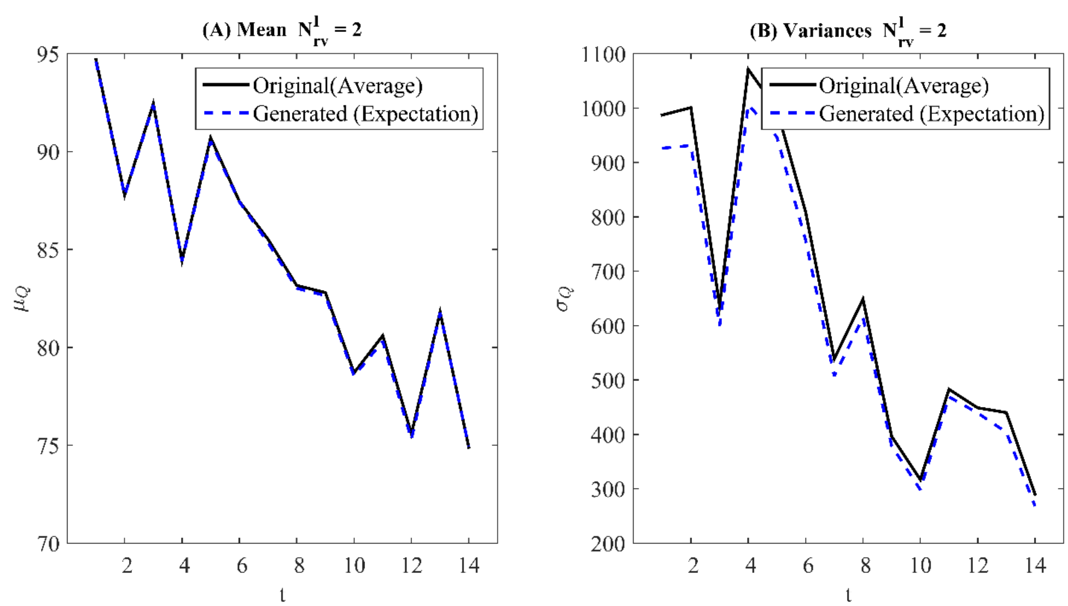

Figure 4.4. Comparison of the mean (A) and variance (B) of the historical inflow ensembles and the generated inflows using 2 random coefficients of KL-expansion

The performance of a multi-objective problem can be evaluated by Hypervolume Index. This index measures the volume of objective space dominated by the Pareto solutions. Since the hypervolume index combines the convergence and diversity metrics in one index, is found to be a good index for comparison purpose (Chen et al., 2016; Zitzler et al., 2003). The comparison of the Pareto solutions in different generations shows that the optimization is converged after 45000 generations as the variation in the hypervolume index is less than 15% (Figure 4.5).

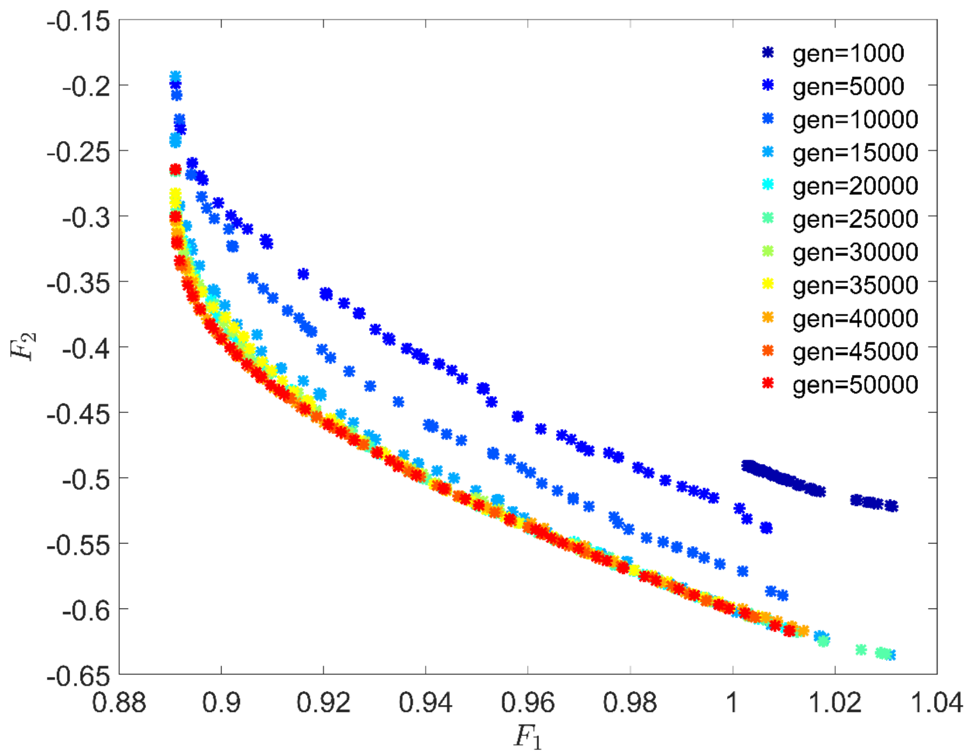

Figure 4.5. The comparison of robust Pareto solutions in different generations

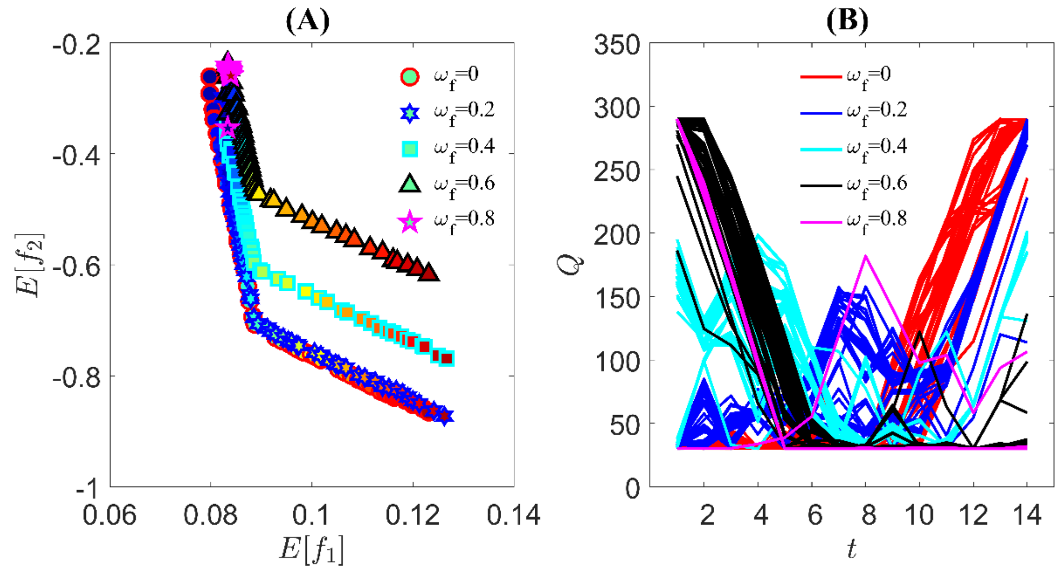

Figure 4.6. (A) Comparison of the objective expectations and (B) comparison of decision variables for different weights $\omega_f = [0,0.2,0.4,0.6,0.8]$ (ω_f was assumed equal for both objectives, α=20%, $N_{rv}^I = 2$)

The comparison of the results using different weights in the robust objective (equation (4.7)) shows that by increasing the weight, the objective values deteriorates (Figure 4.6 A) while the variation of the objectives improves (Figure 4.7). The influence of objective value is less than the standard deviation only for very high weight values (i.e. ω=0.8). Therefore, all the Pareto solutions are relatively close in the objective and decision space (Figure 4.6 A and B).

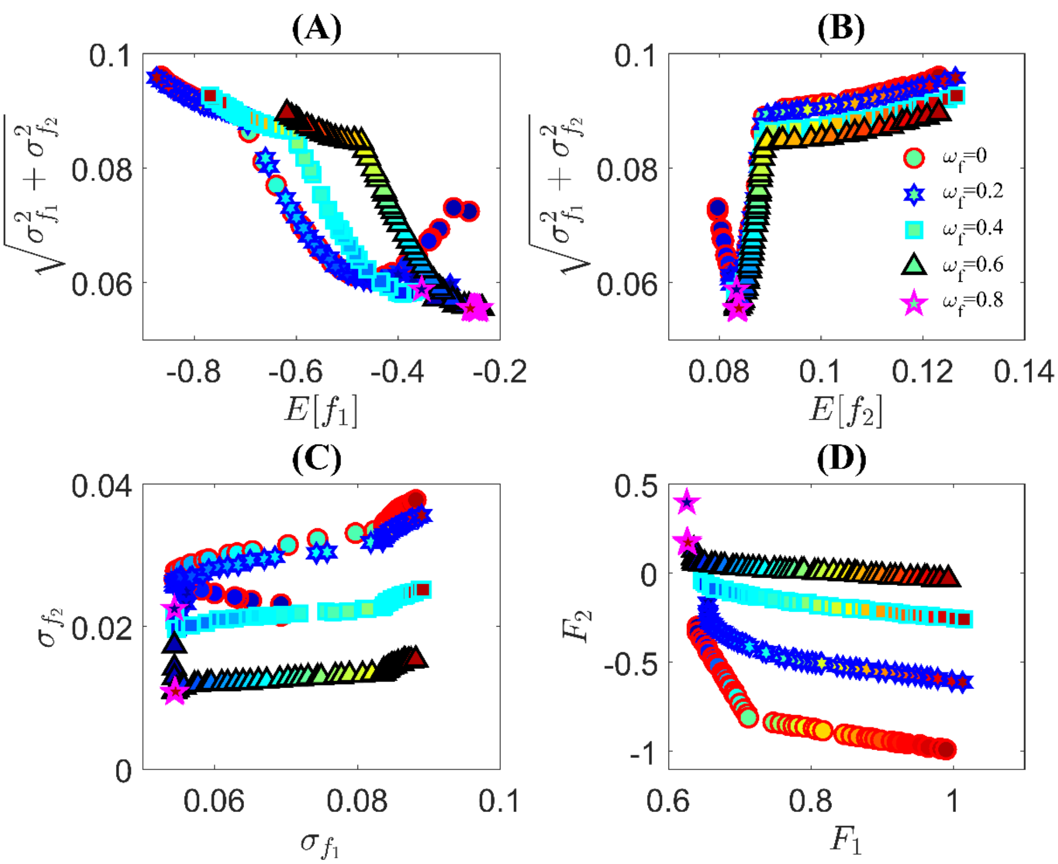

Figure 4.7. Comparison of the results for different weights $\omega_f = [0,0.2,0.4,0.6,0.8]$ (A) total variability of objectives versus expectation of first objective, (B) total variability of objectives versus expectation of second objective, (C) standard deviation of first objective versus expectation of first objective, (D) standard deviation of second objective versus expectation of second objective due to inflow uncertainty (ω_f was assumed equal for both objectives, α=20%, $N_{rv}^I = 2$)

4.3.2. Flexible Decision Variables Using Dimension Reduction Method

In this section the goal is to describe a method to find flexible decision variables as proposed in (Hosseini et al., 2017a). Representing each decision variable by a random variable leads to a multi-dimensional random space. Sampling the random space can become impractical when the number of decision variables increases. Therefore, a dimension reduction method is applied to a set of deterministic decision variables to represent the decision space by a few random variables. Then the optimization problem is designed to find the control variables which are the means and the standard deviations of the aforementioned random variables. The desired optimization solutions should be feasible and have optimal expected objectives and maximum flexibility in decision variables. A brief explanation follows.

A set of decision variables corresponding to deterministic Pareto solutions are used as the original representation of decision space. KL expansion is the method used to reduce the dimension of decision space. The mean (equation (4.16)) and the covariance structure (equation (4.17)) of the deterministic decision variables is extracted and used for generating decision variable realizations (equation (4.18))

$$\bar{Q}^{outflow}(t) = \frac{\sum_{i=1}^{M} Q_i^{outflow}(t)}{M}, \tag{4.16}$$

$$C^{outflow}(t_j, t_k) = \frac{\sum_{i=1}^{M_l}\left(Q_i^{outflow}(t_j) - \bar{Q}_j^{outflow}\right)\left(Q_i^{outflow}(t_k) - \bar{Q}_k^{outflow}\right)}{M-1}, \tag{4.17}$$

$$Q^g\left(t, \vec{\xi}^O\right) \cong \bar{Q}^{outflow}(t) + \sum_{k=1}^{\infty} \sqrt{\lambda_k^{outflow}}\, \psi_k^{outflow}(t) \xi_k^O, \tag{4.18}$$

where $\left[\lambda_k^{outfow}, \psi_k^{outfow}\right]_{k=1}^{\infty}$ are the eigenvalues and eigen-functions (eigen-pairs) of the integral equation (4.19), respectively. $\vec{\xi}^O = [\xi_k^O]_{k=1}^{\infty}$ are the uncorrelated Gaussian random variables.

$$\lambda^{outflow} \psi^{outflow}(t) = \int_{t_0}^{n} C^{outflow}(x, t) \psi^{outflow}(x) dx, \tag{4.19}$$

By using finite number of random coefficients (N_{rv}^O), samples of decision variables realizations could be approximated (equation (4.20)).

$$Q^g\left(t, \vec{\xi}^O\right) \cong \bar{Q}(t) + \sum_{k=1}^{N_{rv}^O} \sqrt{\lambda_k^{outflow}} \psi_k^{outflow}(t) \xi_k^O, \tag{4.20}$$

The SC method is used to calculate the expected value of the objective functions.

Problem Description

The control variables[1] of this optimization problem are the mean and standard deviation of each KL-expansion's random coefficient. The number of daily-time steps are 14 days and the number of optimization control variables are assumed (Hosseini et al., 2017b). However, a third objective is considered to maximize the flexibility of the decision variables. Due to randomness of the turbine outflows, polynomial surrogate of each constraint is created. The feasible solutions have less than probability of failure. α reflects the risk attitude of the decision maker.

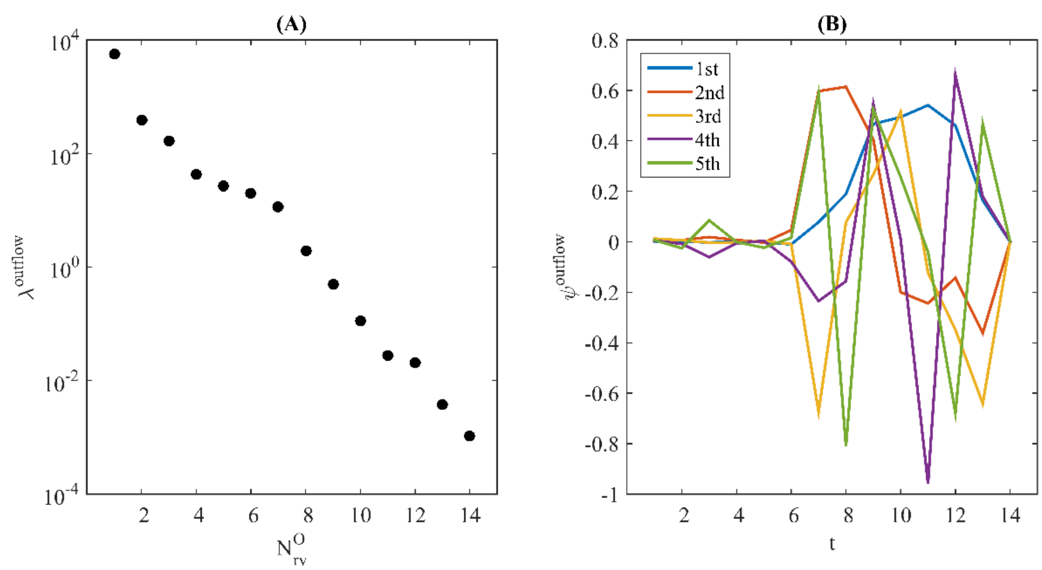

Figure 4.8. Eigenvalues and eigen-functions of the optimal deterministic decision variables in semi-logarithmic scale

[1]Control variables are the same as the decision variables that the optimization is supposed to find in order to optimize the problem's objectives. However, in this section these variables are referred to control variables not to be mistaken as deterministic decision variables (turbine outflows)

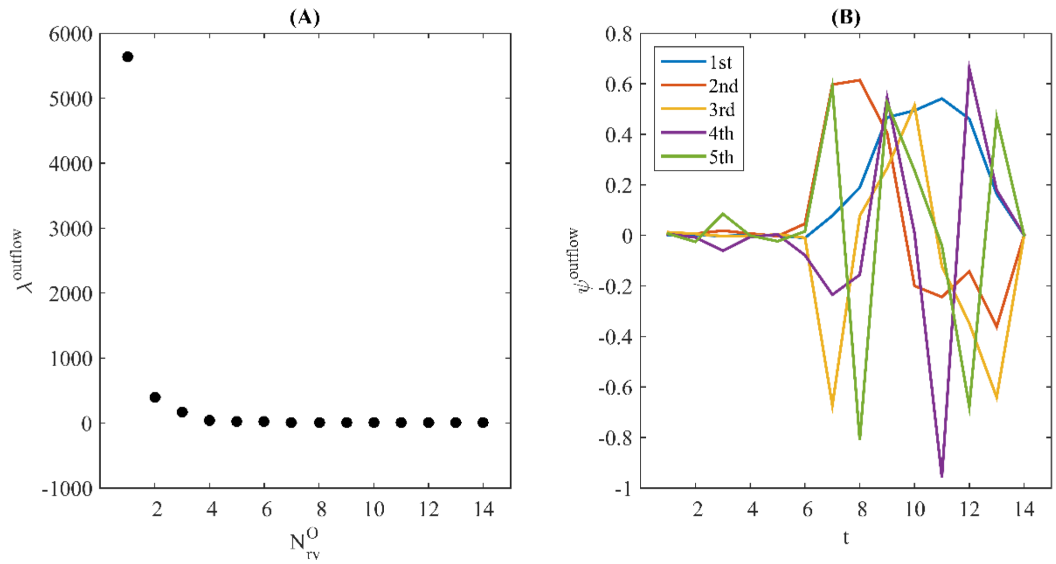

Figure 4.9. Eigenvalues and eigen-functions of the optimal deterministic decision variables

Find $[\mu_{\xi k}]_{k=1}^{N_{rv}^O}$ and $[\sigma_{\xi k}]_{k=1}^{N_{rv}^O}$, in order to

$$\text{Minimize } E[f_1(Q^g(t,\vec{\xi}^O))] = \frac{\left(\left|FB_{end}\left(Q^g(t,\vec{\xi}^O)\right) - FB_{target}\right|\right)}{U_r - L_r} \tag{4.21}$$

$$\text{Minimize } E[f_2(Q^g(t,\vec{\xi}^O))] = -\left(\frac{\sum_{n=1}^{N_t}\left(PG_n\left(Q^g(t,\vec{\xi}^O)\right) - PL_n\right)\times Pr_n}{\sum_{n=1}^{N_t}(Pr_n \times PL_n)}\right) \tag{4.22}$$

$$\text{Maximize } f_3(\vec{\xi}) = \|\lambda\sigma_{\xi}\| \tag{4.23}$$

$$\text{Subject to } PF(C_1, C_2, C_3) \leq \alpha \tag{4.24}$$

$$C_1: Q^{min} \leq Q_n^g \leq Q^{max} \text{ for all } n \tag{4.25}$$

$$C_2: f_1(Q^g) \leq \delta FB \tag{4.26}$$

$$C_3: \left|Q_n^g - Q_{n+1}^g\right| \leq Q^{ramp} \text{ for all } n \tag{4.27}$$

Discussion

The decrease of eigenvalues while sorted in a descending order, shows that a finite number of random coefficients can be enough for representing the decision space (Figure 4.8). The number of random coefficients (N_{rv}^O) in the KL-expansion is assumed 3 as the eigenvalues converged to zero after the third eigenvalue (Figure 4.9). The randomness of the KL-expansion may cause the generated decision variables to be outside of the feasible region. However, by using only 3 random coefficients infinite samples of the decision space can be generated (Figure 4.10). The mean and the variance of the generated decision variables are close to the mean and the variance of the original data (mean less than 1 and variance less than 5 percent difference (Figure 4.11)).

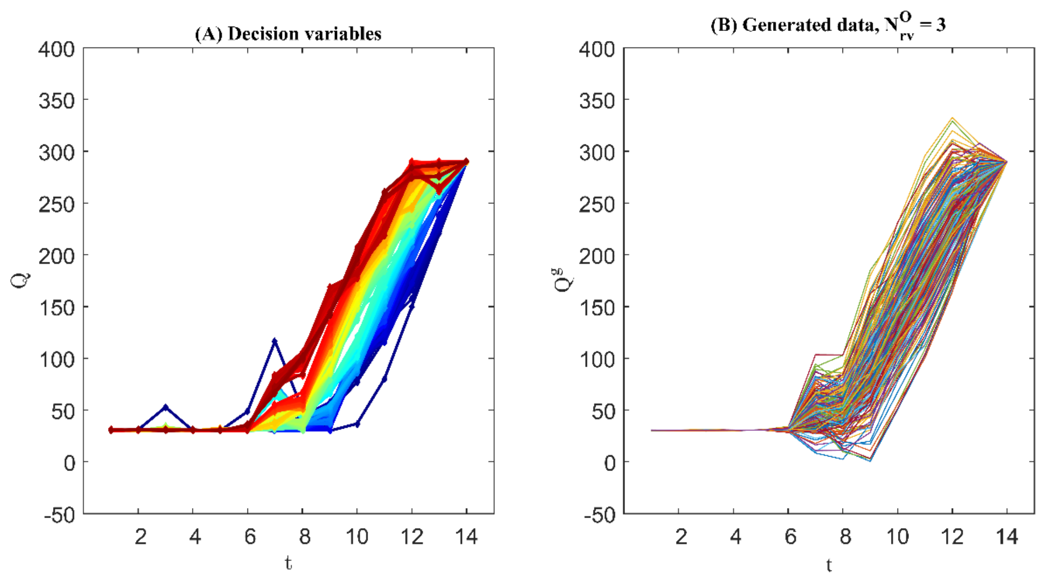

Figure 4.10. (A) Comparison of the original deterministic decision variables and (B) generated outflow realizations using 3 random coefficients of the KL-expansion

The flexible decision variables are found by the proposed methodology have the same expected Pareto solutions. However, the randomness of the flexible decision variables leads to variation in objective space (ellipses in Figure 4.12). To simultaneously observe the variability in the objective space (Figure 4.13 A) and the decision space (Figure 4.13 B), two solutions from the flexible Pareto are selected. There are two clusters of decision variable samples for each of the solutions and the variability of corresponding objectives are mostly along the deterministic Pareto. The decision maker has multiple decision variable realization options, for two given expected objective values.

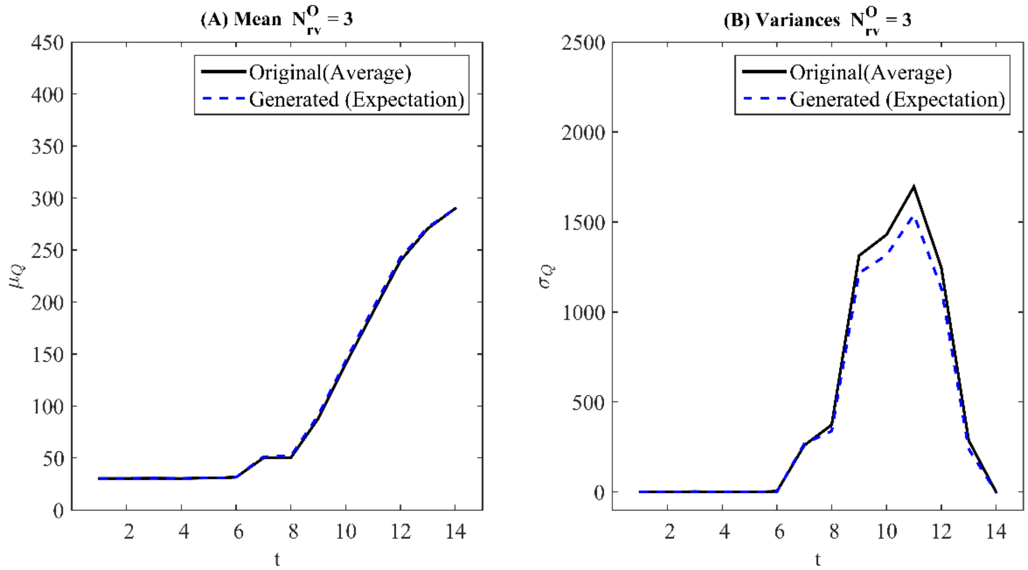

Figure 4.11. (A) Comparison of the mean and (B) variance of the original deterministic decision variables and the generated outflow realizations using 3 random coefficients of the KL-expansion

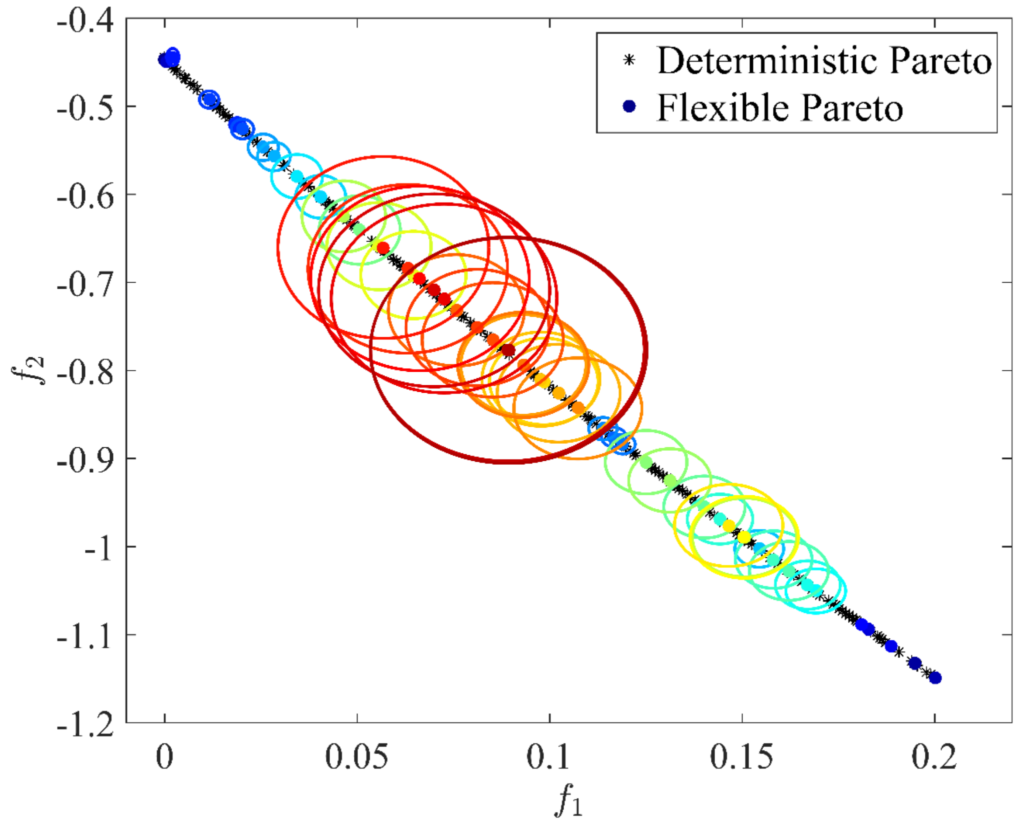

Figure 4.12. Comparison of the original Pareto solutions (deterministic) and the Pareto for flexible decision variables with 20% probability of failure, each ellipse represents the standard deviation of f_1 and f_2 by its radii in horizontal and vertical dimension, respectively

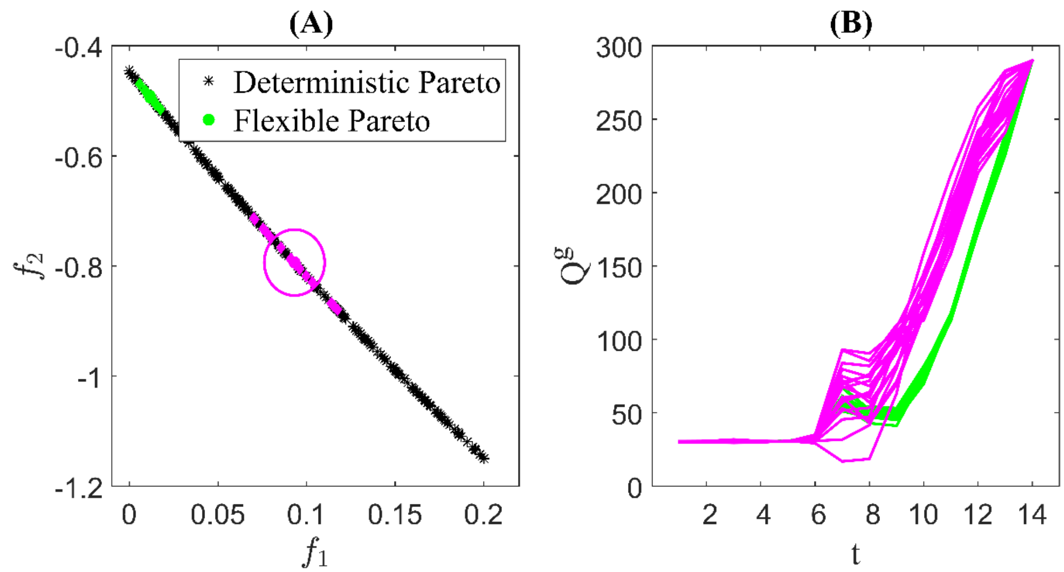

Figure 4.13. (A) Comparison of the original Pareto solutions (deterministic) and the samples corresponding to 2 of the flexible Pareto solutions; each ellipse represents the standard deviation of f_1 and f_2 by its radii in horizontal and vertical dimension, respectively, (B) samples of the decision variable (outflow) realizations for 2 samples ($\alpha = 20\%$)

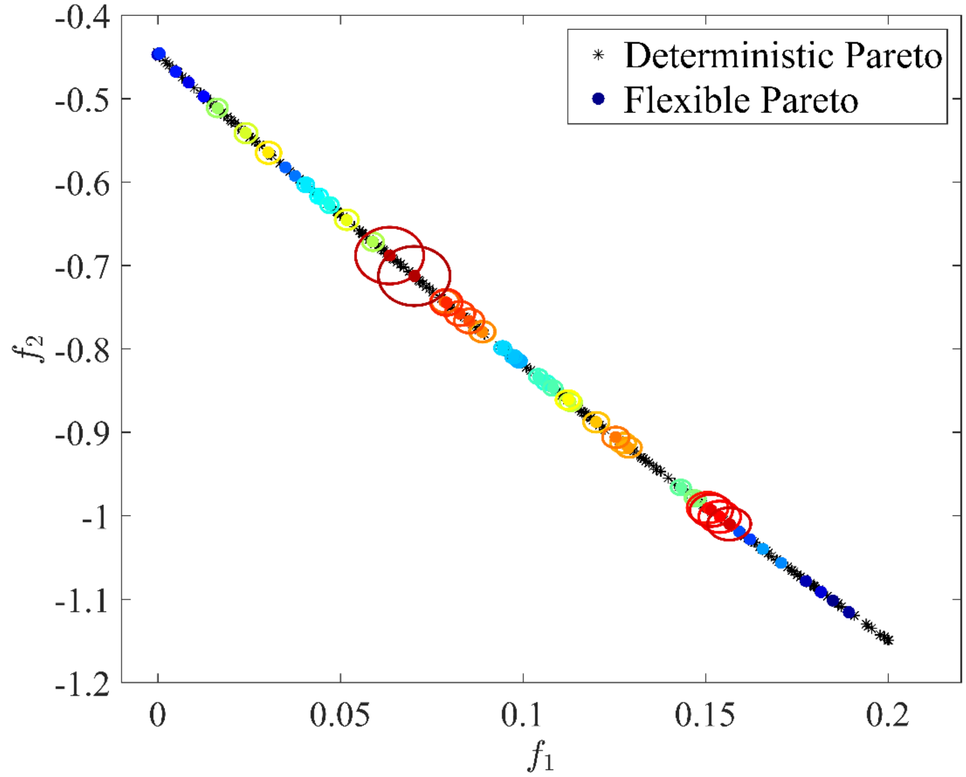

Figure 4.14. Comparison of the original Pareto solutions (deterministic) and the Pareto for flexible decision variables with less than 1% probability of failure, each ellipse represents the standard deviation of f_1 and f_2 by its radii in horizontal and vertical dimension, respectively

For the decision maker with risk averse attitude, the probability of failure can be decreased. The results of probability of failure less than 1% ($\alpha < 1$) shows less variability in objective space as it is expected that no random samples would violate any constraints (Figure 4.14). Therefore, consequently the flexibility in decision space will be also less.

4.3.3.Robust And Flexible Decision Variables

To find robust and optimal solutions with flexible decision variables, the two concepts of robust optimization (section 4.3.1) and flexible decision variables (section 4.3.2) are implemented in the proposed framework. The steps of the proposed method are as follows:

1. Perform the dimension reduction of inflow ensembles to determine the number of necessary KL expansion coefficients (N_{rv}^{I}) to represent the input uncertainty.
2. Perform the dimension reduction of the deterministic decision variables to determine the number of necessary KL expansion coefficients (N_{rv}^{O}) to represent the decision space (outflow realizations) and determine the flexible decision variables.
3. Perform the multi-objective optimization method to find control variables in order to optimize the robust objectives and maximize flexibility.

Problem Description

The control variables of this optimization problem are the parameters of KL-expansion's random coefficients ($[\mu_{\xi k}]_{k=1}^{N_{rv}^{O}}, [\sigma_{\xi k}]_{k=1}^{N_{rv}^{O}}$). These control variables are used to generate samples of turbine outflow realizations in daily time-steps (two weeks). The decision variables corresponding to the deterministic Pareto are used to construct the KL-expansion. To generate outflow realizations using the truncated KL-expansion (equation (4.20)), 3 random coefficients ($N_{rv}^{O} = 3$) are used (section 4.3.2).

The inflow uncertainty is introduced to the optimization problem using a finite series representation of historical inflows (equation (4.5)). 2 random coefficients ($N_{rv}^{O} = 2$)

are used in the truncated KL expansion (section 4.3.1). Therefore the problem objectives are dependent on two different sets of random variables. The SC method is used to strategically sample the random variables in few points. Using the evaluated function values for the collocation nodes and their corresponding weights, the expected values of the objective functions are approximated.

The randomness of the problem variables leads to variability of objective values. To ensure the reliability of the optimal solutions, robust objectives are calculated. The variability of each objective due to input uncertainty (type I robust design) and control variable uncertainty (type II robust design) is implemented in the objective calculation. The weighted sum in robust objective equation (4.7) lets the optimization minimize both the objective and its variability due to dependent random variables, simultaneously.

$$\text{Minimize } F_1 = \left(1-\omega_{f_1}\right)\frac{E\left[f_1\left(\vec{Q}^g(\vec{\xi}^O),\vec{I}^g(\vec{\xi}^I)\right)\right]}{\mu_{f_1max}} + \omega_{f_1}\frac{\sigma_{f_1}}{\sigma_{f_1max}}, \tag{4.28}$$

$$\text{where } E[f_1(\vec{Q}^g(\vec{\xi}^O),\vec{I}^g(\vec{\xi}^I))] = \frac{\left(\left|FB_{end}\left(\vec{Q}^g(\vec{\xi}^O),\vec{I}^g(\vec{\xi}^I)\right)-FB_{target}\right|\right)}{U_r-L_r}, \tag{4.29}$$

$$\text{Minimize } F_2 = \left(1-\omega_{f_2}\right)\frac{E\left[f_2\left(\vec{Q}^g(\vec{\xi}^O),\vec{I}^g(\vec{\xi}^I)\right)\right]}{\mu_{f_2max}} + \omega_{f_2}\frac{\sigma_{f_2}}{\sigma_{f_2max}}, \tag{4.30}$$

$$\text{where } E[f_2(\vec{Q}^g(\vec{\xi}^O),\vec{I}^g(\vec{\xi}^I))] = -\left(\frac{\sum_{n=1}^{N_t}\left(PG_n\left(\vec{Q}^g(\vec{\xi}^O),\vec{I}^g(\vec{\xi}^I)\right)-PL_n\right)\times Pr_n}{\sum_{n=1}^{N_t}(Pr_n\times PL_n)}\right), \tag{4.31}$$

$$\text{Maximize } f_3(\vec{\xi}) = \left\|\lambda^{outflow}\sigma_{\xi}\right\|, \tag{4.32}$$

$$\text{Subject to } PF(C_1, C_2, C_3) \le \alpha, \tag{4.33}$$

$$C_1: Q^{min} \le Q_n^g \le Q^{max} \text{ for all } n, \tag{4.34}$$

$$C_2: f_1(Q^g) \le \delta FB, \tag{4.35}$$

$$C_3: \left|Q_n^g - Q_{n+1}^g\right| \le Q^{ramp} \text{ for all } n, \tag{4.36}$$

To ensure the feasibility of solutions, the probability of failure of constraints is calculated by creating and sampling a polynomial surrogate of each constraint. Some of the constraints only depend on the variability of control variables (such as C_1 and C_2) while the others depend also on the input uncertainty (C_3).

Discussion

The results show that the framework can find some optimal and feasible solutions. However the difference of Pareto solutions are not as notable as the robust case with no flexible decision variables. The diversity of the solutions in both objective space and decision space is less than the case with no flexibility (Figure 4.15). By changing the weight in the robust optimization formulation (equations (4.28)) and (4.30)) it is expected to have more robust solutions. The comparison of the Pareto solutions in 3D (Figure 4.16) shows that the Pareto surface when $\omega_f = 0.4$ is below the Pareto surface when $\omega_f = 0$. However this is not true for all the Pareto solutions as this problem is actually a 5-dimensional optimization problem.

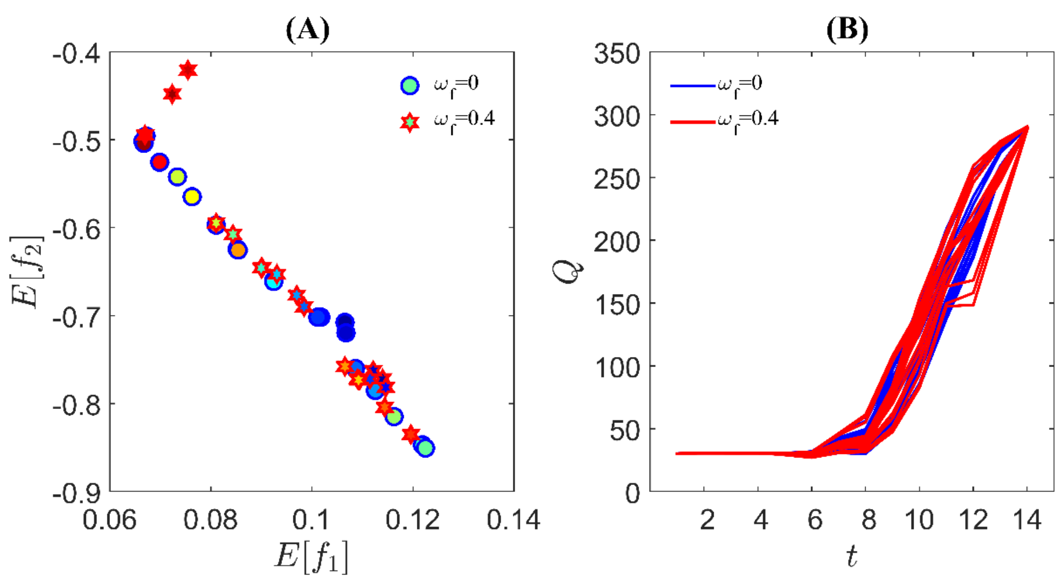

Figure 4.15. Results with flexible and robust decision variables: A) Pareto solutions projected in 2D for different weights; B) the expected decision variables corresponding to each Pareto solution

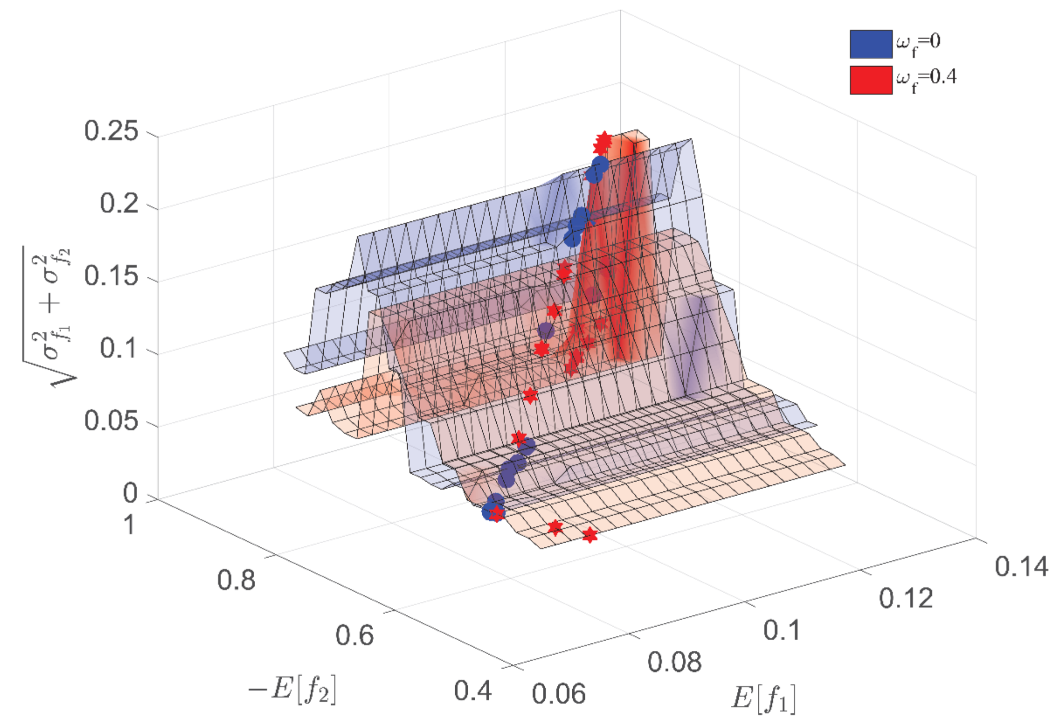

Figure 4.16. The Pareto solution in 3D for different weights indicating the robustness in objective space

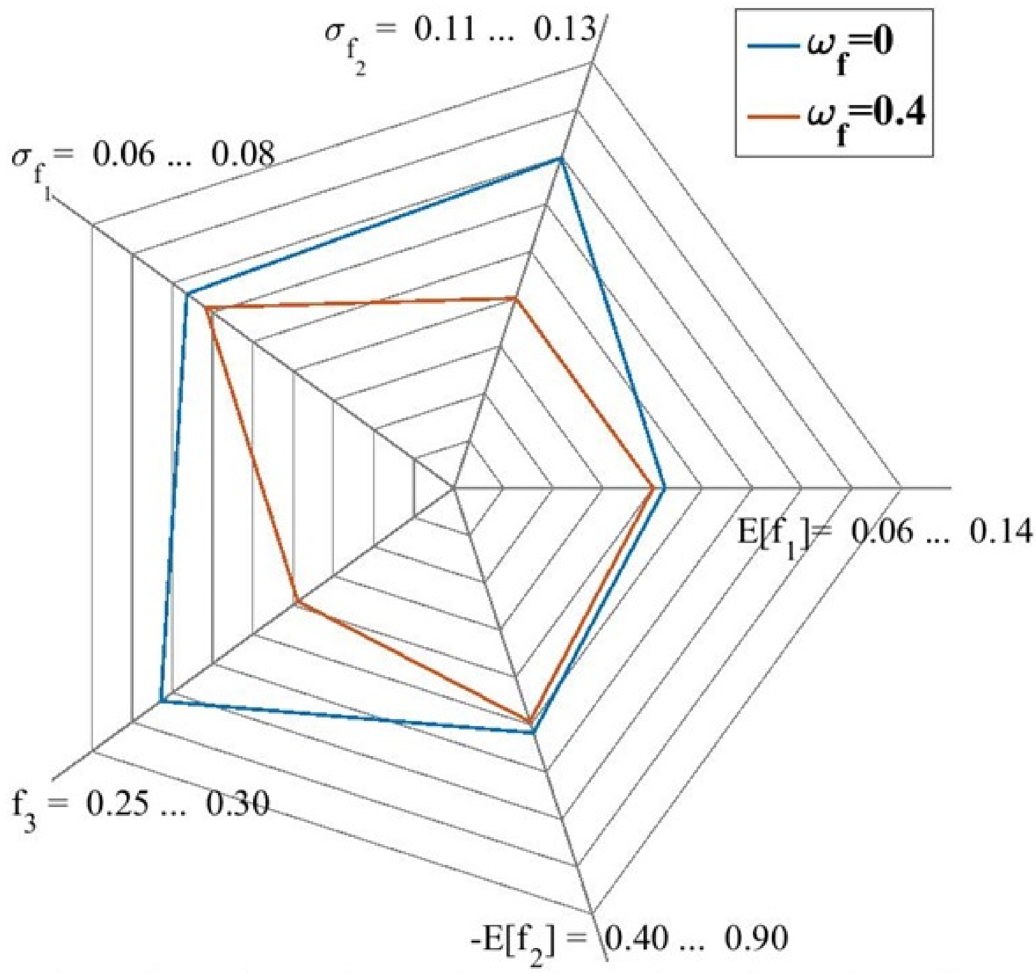

Figure 4.17. The spider plot showing two samples from the two Pareto solutions with $\omega_f = 0$ and $\omega_f = 0.4$ in 5D

To better compare the results, an example of both Pareto solutions (which have the closest value of expected f_1 and f_2) are demonstrated in a 5-D plot (Figure 4.17). The results shows that by increasing the weight more robust solutions can be achieved as the standard deviation of the objectives are decreased. However the flexibility is also decreased for this solution. The comparison of the flexibility and robustness of all the Pareto solutions for these two scenarios shows the trade-off between these two objectives which are not competing objectives (Figure 4.18).

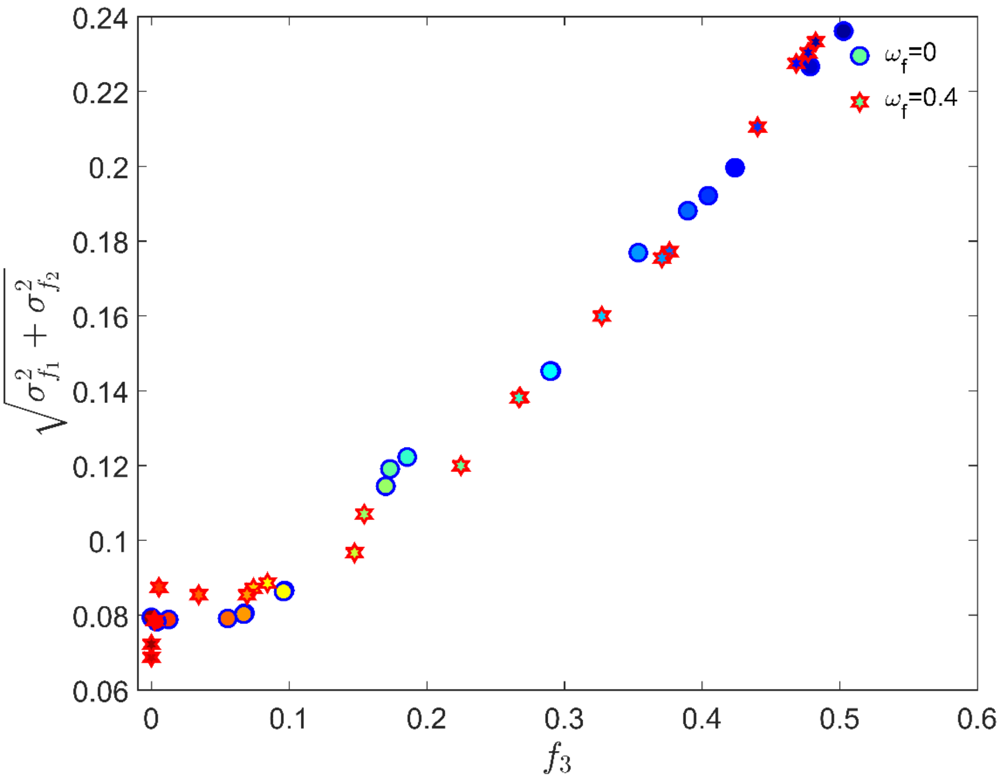

Figure 4.18. The flexibility of decision variables versus robustness for two different weights in robust optimization

4.4. Conclusion

The robust objective concept is used to ensure that the optimal decision variables are not sensitive to the uncertain inflows. Using a KL-expansion, some realizations of inflow hydrographs are generated. The results shows that by using 2 random variables, the inflow realizations have the same mean (less than 2% relative error) and variance (less than 10%) as the original data.

The robust objective concept is used to simultaneously minimize the expectation as well as the variation of the objectives due to random inflows by using a weighted average. Comparison of the results for various weights shows that the performance of the objective functions is sacrificed by decreasing the standard deviation of the objectives. The weights are the decision maker's preference of robustness versus performance.

Next the optimal robust and flexible decision variables of the reservoir problems are found. The variation of the objective functions stem from both uncertain inflows and the randomness of the flexible decision variables. The optimization problem here also has an additional objective to maximize the flexibility. The solutions show that robustness and flexibility are not exclusively competing in this example which can be the reason for the less diverse Pareto solutions in comparison to the case with only robust solutions.

Acknowledgments

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5. Final Discussion

5.1. Summary

Study of uncertainties in water resources management problems and considering the effects of these variabilities on the system has never been more critical. Optimization of reservoir operation under uncertainty for competing objectives was the main focus of this study. Two types of uncertainties were addressed in this research: Flexibility in decision variables and Input uncertainties (i.e. inflow).

The concept of flexibility in decision making is a fairly recent approach especially in optimization of reservoir operation. In this research, flexibility refers to a range of options that could be chosen by the decision maker and the feasibility of choosing any of those options randomly, in that it has been determined that feasible optimal solutions satisfy a certain probability of failure tolerance. Therefore, the decision maker could have more than one optimal solution and could act according to real-time circumstances.

The first approach for finding flexible decision variables was to represent each decision variable (time step) by a random variable. The methodology was tested in two mathematical test problems and then was extended to a simple multi-objective reservoir problem. The results indicated the capability of the proposed methodology for finding Pareto optimal solutions while maximizing the flexibility in decision space.

The results of the reservoir problem showed that by flexibility in decision space, the objectives' expectations were close to the deterministic Pareto solutions. Therefore, for a certain probability of failure, the decision maker could be confident that the any choice of the flexible decision variables were feasible and the expectation of these options would not be very different from the deterministic Pareto solutions. In other words, although the objectives might be sacrificed from their deterministic values, the decision maker would have a range of feasible options. For example in a specific case, the method would allow almost 45% flexibility (Coefficient of Variation) in the reservoir turbine flow in both time-steps and had less than 20% deterioration of the objectives.

The Stochastic Collocation (SC) method was used for sampling the random variables. This method was much faster than Monte Carlo method as the system was only evaluated on a few strategically chosen points called collocation points. To ensure the feasibility of each flexible decision variable, the evaluated constraint values for each collocation point were used for constructing a Polynomial Surrogate representing that constraint. This method was computationally efficient since rather than evaluating the whole system for many samples of the random variables, only the Polynomial Surrogate would be sampled to approximate the Probability of Failure. Although using the SC method made this approach much faster, the function evaluations would increase exponentially by adding to the number of decision variables ($NC^{N_{rv}}$). This approach would only work well for problems with few decision variables. For example, for short-time optimization reservoir operation this methodology could be useful.

To develop a framework that could be extended to problems with many decision variables, a dimension reduction method was used to represent the decision space by a manageable number of random variables. The decision variables corresponding to the deterministic Pareto solutions were used as the representation of the decision space. The Karhunen-Loeve (KL) expansion was used to extract the mean and covariance structure of the deterministic decision variables and regenerate decision variable realizations using a few random coefficients. For example, in the specific problem discussed in section 3.4.2, by using only three random coefficients, the mean and variance of the deterministic and generated realization were almost the same (less than 10% relative error). The results showed the capability of the proposed approach for finding flexible decision variable solutions for systems with many decision variables. Some of the flexible solutions might lead to a wide range of objective values. Although the expected objective values were in the same range of the deterministic Pareto solutions, the variability in objective space might not be desirable.

The Robust optimization concept was used to confine the variation of objectives due to randomness of dependent variables by implementing the robust objectives approach. The weighted sum of objective expectation and standard deviation of objective due to random variables were used as the robust objectives. The comparison of the robust optimization results using different weight showed that the standard deviation of the objectives were decreased by increasing the weight in the robust objective. The Pareto solution consisted of the objective expectations. The decision maker could specify the weight based on their preference on robustness and objective performance.

Finally a framework was developed to find robust solutions with flexible decision variables. The variation of the objectives due to random inflows and also randomness of flexible decision variables were minimized in the robust objectives. The results showed the efficacy of the framework to find ranges of flexible decision variables with optimal expected objectives.

5.2. Future Research

Implementing the SC method for approximating the expected value of the objectives decreased the computational time. For multi-dimensional random space, the full-tensor grid was used in this research and it is recommended to use the sparse-grid and investigate if for a given accuracy the computational time can be decreased.

The deterministic decision variables were used as the representation of the decision space and it is suggested to use a combination of deterministic and the historical decision variables to have a more diverse decision space to study and compare the results to the results of the current research.

Inflow uncertainty was considered as the sole source of input uncertainty and other sources such as price uncertainty can be considered in the proposed framework.

Implementation of the KL-expansion method to decrease the decision space helped to decrease the computational efforts of the optimization problem with a single reservoir and can be used for a multi-reservoir system.

Although the Pareto optimal concept was considered in the multi-objective optimization problem, to decrease the number of objectives, the weighted sum concept was used to define robust objectives. In the robust objective calculation, the weights of all objectives were assumed equal. However it is recommended to find the solutions for various weights of interest or Pareto optimal solutions without weights.

5.3. Conclusions

The proposed framework is capable of finding flexible decision variables considering an allowable probability of failure of constraints. Two types of robust optimization (uncertain inflow and flexible outflow) are considered. These solutions may help the decision maker have more information and options in their decision making process. This framework can be useful in other decision support tools.

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Appendices

Exact Solution to Test Problem (1B)

Note: $E[\sum_{i=1}^{m} \xi_i^2] = \sum_{i=1}^{m} E[\xi_i^2]$ by independence and $E[\xi_i^2] = \mu_i^2 + \sigma_i^2$, thus min $E[f_1]$ =min $\sum_{i=1}^{m} \mu_i^2 + \sigma_i^2$.

Also, max f_2 is equivalent to max $\|\vec{\sigma}\|^2 = \sum_{i=1}^{m} \sigma_i^2$, which are clearly contradictory objectives.

Consider $m = 2$, to find the Pareto curve, assume $[\sigma_i]_{i=1}^{2}$is known; Thus f_2 is determined.

For a given optimal solution with a known $\sigma_1 + \sigma_2$, the corresponding optimal $\mu_1 + \mu_2$ must merely be min $\mu_1^2 + \mu_2^2$ while satisfying constraints:

$\mu_i = \frac{u_i + l_i}{2}$ and $\sigma_i = \frac{u_i - l_i}{\sqrt{12}}$ becomes:

I: $2\mu_i = u_i + l_i$

II: $\sqrt{12}\sigma_i = u_i - l_i$

By adding and subtracting (I) and (II) and considering constraint:

$$\begin{cases} 2\mu_i + \sqrt{12}\sigma_i = 2u_i \leq 2, \text{ then } \mu_i \leq 1 - \frac{\sqrt{12}}{2}\sigma_i \\ 2\mu_i - \sqrt{12}\sigma_i = 2l_i \geq 0, \text{ then } \frac{\sqrt{12}}{2}\sigma_i \leq \mu_i \end{cases}$$

i.e. min μ_i^2 subject to $\frac{\sqrt{12}}{2}\sigma_i \leq \mu_i \leq 1 - \frac{\sqrt{12}}{2}\sigma_i$, then $\mu_i^* = \frac{\sqrt{12}}{2}\sigma_i$

Thus, in general $f_1^* = \sum_{i=1}^{m} 3\sigma_i^2 + \sigma_i^2 = \sum_{i=1}^{m} 4\sigma_i^2 = 4(f_2^*)^2$ as seen in Figure 2.2 A. We note that every feasible f_2 value is attainable by the Pareto curve via the appropriate choice of f_1 namely $f_1^* = 4(f_2^*)^2$; given an (f_1, f_2) pair, there is no unique $(\mu_1, \mu_2, \sigma_1, \sigma_2)$ which attains it, as demonstrated by the apparent randomness in Figure 2.2 C and D, except in cases of $f_2 = 0$ and $f_2 = 1/\sqrt{12}$ (left and right-most points in the Pareto, i.e. when $(l_i, u_i) = (0,0)$ and (0,1), respectively. In the latter case, $\mu_i = \frac{1}{2}$ and $\sigma_i = \frac{1}{\sqrt{12}}$ so that $f_2^2 = \sum_{i=1}^{2} \sigma_i^2 = \frac{1}{6}$ and $f_1 = 4\left(\frac{1}{6}\right) = \frac{2}{3}$).