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R. R. Mohler

In recent years, considerable attention has been focused on the application of orthogonal expansions to system analysis, parameter identification, model reduction and control system design. However, little research has been done in applying their useful properties to Power System analysis and control. This research attempts to make some inroads in applying the so called " orthogonal expansion approach " to analysis and control of Power systems, especially the latter.

A set of orthogonal functions commonly called Walsh functions in system science after it's discoverer J.L. Walsh [1923] have been successfully used for parameter identification in the presence of severe nonlinearities. The classical optimal control problem is applied to a synchronous machine infinite bus system via the orthogonal expansion approach and a convenient method outlined for designing PID controllers which can achieve prespecified closed loop response characteristics.The latter is then applied for designing a dynamic series capacitor controller for a single machine infinite bus system.

Analysis and Control of Power Systems using Orthogonal Expansions

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Stephen Ronald Fernandes

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Redacted for Privacy

Head of Department of Electrical and Computer Engineering

Redacted for Privacy

Dean of Graduate School

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Chapter 1 Introduction

1.1 Applications of Orthogonal Expansions in System Science

In recent years, considerable attention has been focused on the application of orthogonal expansions to system analysis, parameter identification, model reduction and control system design. Two sets of such functions namely the Walsh Function and Block Pulse functions, first received considerable attention in system science (Chen and Hsiao [1975 a]). The key point to the application was the derivation of the operational matrix of integration of the orthogonal functions which was used to convert linear differential equations to a set of algebraic equations which were solved using Kronecker products for unknown parameters. Subsequently , the Walsh Function method was applied to parameter identification of lumped systems (Rao and Sivakumar, [1975]), distributed systems (Paraskevopoulos [1978], Sinha et al [1980]), bilinear systems (Chen and Shih [1978], Karanam et al [1978]) time delay syatems (Rao and Sivakumar [1979]) and multi input-multioutput systems (Paraskevopoulos [1978],Rao and Sivakumar [1981].

Shieh et al [1978] applied block pulse functions to the analysis of time varying and non-linear networks. Ganapathy & Ram Mohan Rao [1978], employed block pulse functions to find piecewise constant gains to linear time invariant systems. Zhu and Lu [1987] presented a method of heirarchial control for large scale time varying systems via block pulse transformation. Rao et al [1980] proposed the single term Walsh series approach. Palanisamy [1981] applied the single term Walsh series approach to the analysis and optimal control of linear systems, Palanisamy & V.R Arunachalam [1985] to the analysis of bilinear systems. Zhu and Lu [1988] proposed a heirarchical recursive algorithm for non-linear optimal control systems using the single term Walsh series approach with the improved new predication method of large scale system theory.Palanisamy & Balachandran [1987] presented amethod for the analysis of linear singular systems with the single term Walsh series approach.

Cameron et al [1980,83], Kouvaritakis and Cameron [1981] used Walsh functions for predicting the existence of limit cycles in nonlinear feedback systems.

The use of orthogonal polynomials to represent a given function was well known in engineering mathematics.Solutions of differential equations by orthogonal polynomial approximation have been extensively studied by Villadsen and Michelson [1978], Finlayson [1972]. Recently, orthogonal polynomials have been successfully applied to a host of problems in analysis, identification & control.

Razzaghi & Arabshahi [1989] proposed a method for finding an approximate solution of linear time-varying systems and bilinear systems via Fourier series based on the utilization of the operational matrix of integration & the product operational matrix.Taylor series (Lee & Tsay [1987]) and Fouriers series (Paraskuopolous et al 1985, Paraskuopolous 1987, Razzaghi & Razzaghi [1988]) had been applied earlier to analysis. Margallo & Bejarno [1989] used the method of harmonic balance with generalized Fourier series and Jacobian elliptic functions to find approximate solutions of non-linear differential equations (to second order). Ardekani & Keyhani [1989] presented a new algorithm for the identification of non-linear lumped time invariant SISO systems based on the differentiation properties of the exponential Fourier series. Hersh, M.A [1988] introduced an approach to the description of the time variation of the parameters of time varying systems based on periodic functions and outlined an adaptive identification and control algorithm valid for a large class of time-varying systems.

Laguerre polynomial approximations (Wang et al [1984] , Hwang and Shih [1982,1983], Ranganathan et al [1984]) & Chebychev polynomial approximations (Liu and Shih [1983], Chou & Horng [1985 b,c]) were successfully applied to system analysis and control system design. Legendre polynomial approximations (Wang & Chang [1982,1983] ,Chou & Horng [1985a], Shih & Kung [1985]) were also successfully applied to solving control system problems such as parameter identification,model reduction and control system design. Hwang & Guo [1984] applied the shifted Legendre Polynomials for transfer function matrix identification in MIMO systems.Horng & Chou [1987] applied the shifted Jacobi series to analysis and identification of nonlinear systems described by a Hammerstein model consisting of a single-valued nonlinearity followed by a linear plant. Chang & Yang [1986] and Chang et al [1986] proposed the so-called generalized orthogonal polynomials

(GOP's) approach. Wang, Chang, Yang [1987] applied the same to the analysis & parameter identification of a bilinear system and introduced a single form of the integration operational matrix of GOP's which represented all kinds of individual orthogonal polynomials and non-orthogonal taylor series, to simplify the computational algorithm.Tsu-Tsian Lee and Yih-Fong Chang [1987] applied the operational matrix of integration, together with the operational matrix of linear transformation of general orthogonal polynomials to analyse time varying delay systems. Hwang, Chen & Shih [1987] applied Hahn polynomials for the analysis and parameter estimation of linear discrete time single input/single output systems described by difference equations. Szczepaniak [1987] applied the orthogonal series technique for the approximate determination of sensitivity functions of linear dynamic systems. Lahouaoula [1987] used double general orthogonal polynomials for identifying the parameters of linear parabolic distributed systems. Ding & Frank [1989, 1991] presented criterias to test observability and controllability in terms of the coefficent matrix of the orthogonal expansions. Recently there has been a growing interest in H^∞ identification of systems [H^∞ (Re s >0) the Hardy space of bounded analytic functions in the right half plane] , one main reason being the fundamental role played by the H^∞ norm in robust controller synthesis and the requirements this places on system identification. Makila [1990] approximated stable systems by Laguerre filters. The classicalLaguerre series approximation of infinite dimensional systems has also been studied for the L_2 norm, Glader et al [1991] for the L_1 , H^∞ and Hankel norms. Related results have been given for H^∞ approximation by Gu et al [1989] usiing Fourier series methods on the unit circle. Makila [1991] studied H^∞ identification of stable continuous-time systems using generalized Laguerre series methods. Zervos & Dumont [1988] modelled a plant by an orthonormal Laguerre network put in state space form, proposed a simple predictive control law and designed an explicit deterministic adaptive controller.

The application of discrete orthogonal functions such as the discrete Fourier transform, discrete Walsh transformation (Kak [1974],Chou and Horng [1986],Horng et al [1987], Lewis & Fountain [1991]),discrete Legendre (Horng and Ho [1986], Hwang & Shyu [1987]) discrete Laguerre (King and Paraskoupulos [1979], Hwang & Shih [1983], Marionne & Turchiano [1985],Horng & Ho [1985a]) and discrete Chebychev(Hwang & Shih [1984], Horng and Ho [1985b]) polynomials in system analysis, signal processing,model reduction, identification and optimal control have been discussed too. Horng & Chou [1988] developed a new and simple method for designing a single loop PID controller using a computer oriented algebraic approach which is based on general discrete orthogonal polynomials eliminating in the process any trial & error procedures in determining the parameters of the PID controller.Hwang & Shyu [1988] applied a finite series [discrete Legendre] expansion method for the analysis and parameter identification of a discrete nonlinear system described by a Hammerstein model.

Another concept was to use piecewise linear polynomial functions which have the advantages of piecewise orthogonal functions and classical orthogonal polynomials.Signficant advantages of using piecewise linear polynomial functions for solving problems of analysis and identification have been demonstrated by Lio & Chou [1987]. Liou & Chou [1987] applied the piecewise linear polynomial function approach to the minimum energy control of linear systems with time delay.

Though Orthogonal functions have been applied extensively in System science little research has been done in applying them to power system analysis & no research in power system control.

Deb & Datta [1987] introduced the concept of the Walsh Operational transfer function (WOTF) which was conceptually a new way of looking at Chen & Hsiao's [1975] operational matrix. The approach was applied to analyse a D.C chopper circuit. Deb & Datta [1992] used the approach to analyze a continuously variable pulse width modulated system. Thyagarajan & Sankarnarayanan [1989] proposed a Walsh function approach for equivalencing the controllers in multi-machine power systems. The method proposed is based on the principle of obtaining the equivalent time constants from the unit step response of the individual blocks for constant time ranges & finally averaging them. Bhattacharya & Basu [1992] used the Walsh Transform for forecasting of monthly energy demand in large scale power systems using autoregressive model fitting.

1.2 Overview

Among these approaches to parameter identification the ones based on the shifted Legendre polynomials (Wang and Chang [1982]) have been shown to be more effective than the others since it merely requires arbitrary but active length of input-output data to identify the system and is immune from zero mean additive noise to some extent. In addition, the shifted Legendre functions could give a faster convergence rate for many general problems. However, the error of shifted Chebychev functions is distributed nearly uniformly in the time interval of interest.This characteristic of shifted Chebychev series is of importance in cases where it is desirable that the error involved in the approximation should not be concentrated only in certain portions of the time interval of interest but should be uniformly distributed over the interval.

The common procedure in analysis, identification, model reduction and control, adopted by these orthogonal function / polynomial expansion methods are as follows:

(a) Convert the differential equation to an integral equation through multiple integration.

(b) Approximate the various variables involved in the integral equation by a truncated orthogonal series.

(c) Replace the integral of the basis vector in the integral equation with the product of an operational matrix of integration and the basis vector.

(d) Equate the coefficents of like basis functions to get a set of algebraic equations.

(e) Solve the resultant algebraic equations to give unknown coefficent vectors of the system variables. The differential equation involved in the problem is reduced to a set of algebraic equations , the problem is thus greatly simplified.

The main disadvantages of the above approaches (orthogonal functions and orthogonal polynomials) are : their applicability restrictions (their solutions are satisfactory for certain systems only) , their analyticity requirements for both the input and the output signals and their relative inflexibility for approximation purposes. The computational work is laborious owing to the algebraic equations with enlarged dimensions. Also the use of piecewise-continuous basis functions (Walsh Functions) leads to piecewise-constant gains.The inversion of large dimension matrices as a result of the Kronecker matrix product is required in some problems and much computer time is consumed by this matrix inversion.

1.3 Outline of the Thesis

Chapter 2 deals with the theory of orthogonality, orthogonalization and orthogonal systems. Common orthogonal expansions ranging from Haar functions to Chebychev polynomials are studied and comparisons made on their ability to represent time-varying functions (assumed measurable in L_2 space). Chapter 3 deals with the delineating properties of a versatile set of orthogonal expansions namely Walsh functions. Their delineating property is extended to nonlinear systems and then used for parameter identification of nonlinear systems.

Chapter 4 deals with analysis of a perturbation model of a single machine infinite bus power system using orthogonal expansions. The classical Linear Quadratic Regulator (LQR) problem is solved through the orthogonal function approach (This deals with transformation into the Orthogonal domain and consequent simplification of the problem using the elegant properties of orthogonal expansions). The application of Walsh functions to the control problem led to piecewise constamnt optimal feedback gains.

Chapter 5 deals with the application of orthogonal functions for the design of compensators / controllers capable of achieving prespecified dynamic responses for the closed loop system. A PID controller is designed for series capacitor control of a SMIB system. The method has not been fully developed. Further research could yield an advanced outlook to this application.

Chapter 2 Orthogonal Expansions

2.1 Orthogonality

The notion of orthogonality may be introduced by means of the Stieltjes - Lebesgue integral. Let $\mu(x)$ be a positive bounded monotone increasing function in the interval of orthogonality [a,b] whose derivative $\mu'(x)$ vanishes at most in a set of measure zero (in the sense of Lebesgue). The (real) function $f(x)$ is called L_μ integrable, if it is μ measurable and morover the condition

$$\int_a^b |f(x)|\, d\mu(x) < \infty \qquad .....(2.1)$$

is fulfilled.

If $\mu(x)$ is absolutely continuous and let $\rho(x) = \mu'(x)$, then for any L_μ integrable function $f(x)$ the relation

$$\int_a^b f(x)\, d\mu(x)\,) = \int_a^b f(x).\, \rho(x)\, dx \qquad .....(2.2)$$

is valid i.e $f(x)$ is an $L_{\rho(x)}$ integrable function and $\rho(x)$ is called a weight function. If in particular $\rho(x) = 1$ then $f(x)$ is said to be L - integrable. A function $f(x)$ is called L^2_μ or $L^2_{\rho(x)}$ integrable, if it is L_μ or $L_{\rho(x)}$ integrable, respectively and if furthermore

$$\int_a^b f^2(x)\, d\mu(x) < \infty \quad \text{or} \quad \int_a^b f^2(x).\, \rho(x)\, dx < \infty \qquad .....(2.3)$$

holds respectively. If $\rho(x) = 1$ the function $f(x)$ is called L^2 integrable.

A finite system $\{ \varphi_n(x) \}$ of L^2_μ integrable functions is said to be orthogonal with respect to the distribution $d\mu(x)$ or simply orthogonal if

$$\int_a^b \varphi_m(x).\varphi_n(x)\, d\mu(x) = 0 \qquad (m \neq n) \qquad .....(2.4)$$

holds and none of the functions $\varphi_n(x)$ vanishes almost everywhere. The system $\{ \varphi_n(x) \}$ is called orthonormal if besides the condition of orthogonality the conditions

$$\int_a^b \varphi^2_n(x)\, d\mu(x) = 1 \qquad (n = 0, 1...) \qquad .....(2.5)$$

are also fulfilled.

A system of functions $\{ f_n(x) \}$ is called linearly independent in [a,b], if the validity of a relation of the form

$$\sum_{k=0}^{n} a_k f_k(x) = 0 \qquad .....(2.6)$$

for μ almost every $x \in [a,b]$ necessarily implies

$$a_o = a_1 = a_2 = ... = a_n$$

Every orthogonal system $\{ \varphi_n(x) \}$ is linearly independent. If we multiply , by $\varphi_k(x)$, both sides of the equation

$$\sum_{k=0}^{n} a_k \varphi_k(x) = 0 \qquad .....(2.7)$$

(valid μ almost everywhere) and integrate the interval of orthogonality, we obtain on account of the orthogonality

$$a_k . \int_a^b \varphi^2_k(x)\, d\mu(x) = 0 \qquad ....(2.8)$$

and consequently $a_k = 0$ for any k.

Any series $\Sigma\, c_n\, \varphi_n(x)$ constructed with the functions of an orthogonal system and an arbitrary set of real numbers c_0, c_1, ... is called an orthogonal series. However, if the coefficents c_n are, according to Fourier's manner, representable in the form

$$c_n = \frac{1}{\int_a^b \varphi^2_n (x)\, d\mu(x)} \cdot \int_a^b f(x)\, \varphi_n (x)\, d\mu(x) \qquad ......(2.9)$$

then $\Sigma\, c_n\, \varphi_n (x)$ is the orthogonal expansion of the function f(x) expressed as

$$f(x) \sim \sum_{k=0}^{\infty} c_n\, \varphi_n (x) \qquad .....(2.10)$$

The difference between orthogonal expansions & orthogonal series is essential. For instance, the partial sums

$$S_n(x) = \sum_{k=0}^{\infty} c_k\, \varphi_k (x) \qquad .....(2.11)$$

of the expansion (2.10) above are distinguished among the partial sums of any arbitrary orthogonal series constructed with the functions of { $\varphi_k (x)$ } by the following minimum property.

Let f(x) denote an L^2_μ integrable function and { $\varphi_n (x)$ } an arbitrary orthonormal system. Among all the expansions of the form

$$S_n(x) = \sum_{k=0}^{n} a_k\, \varphi_k (x) \qquad ......(2.12)$$

the integral

$$I(S_n) = \int_a^b [f(x) - S_n(x)]^2 \, d\mu(x) \qquad ......(2.13)$$

attains for $S_n(x) = s_n(x)$ the least value.
For with regard to the orthonormality we obtain

$$I(S_n) = \int_a^b f^2(x) \, d\mu(x) - 2. \sum_{k=0}^{n} a_k c_k + \sum_{k=0}^{n} a_k^2$$

$$= \int_a^b f^2(x) \, d\mu(x) + \sum_{k=0}^{n} (a_k - c_k)^2 - \sum_{k=0}^{n} c_k^2) \qquad ..(2.14)$$

The expression on the R.H.S is minimal if and only if, its middle term vanishes i.e

$a_k = c_k$, $k = 0, 1$ ..... what is equivalent to

$$S_n(x) = s_n(x) \qquad ......(2.15)$$

Completeness of an orthogonal system :

A system of functions $\{ g(x) \}$ from L^2_μ is called <u>complete</u> (in L^2_μ) if there is only one function $f \in L^2_\mu$ for which the values of the scalar products

$$(f, g) = \int_a^b f(x) \, g(x) \, d\mu(x) \qquad ......(2.16)$$

are given numbers satisfying $\sum_{k=0}^{n} c_n^2 < \infty$

A simple characterization of the necessary and sufficient condition for the completeness of an orthonormal system is that

$$\int_a^b f^2(x)\, d\mu(x) \quad = \quad \sum_{k=0}^{n} c_n^2 \qquad \text{.....(2.17)}$$

where c_0, c_1, ... are the expansion coefficients of the function f(x), should be valid for all $f \in L_\mu^2$

Investigation of the convergence behaviour of orthogonal series by methods belonging to the general theory of series

The concern here is to determine the convergence features of a general orthogonal series

$$\sum_{n=0}^{\infty} c_n . \varphi_n(x) \qquad \text{......(2.18)}$$

from the properties of the coefficents c_0, c_1, ... It is seen however that the values of the functions $\varphi_n(x)$ can be chosen arbitrarily in a set of μ measure zero N without disturbing the orthonormality of the system { $\varphi_n(x)$ } while 2.18 can also be rendered divergent also in N, putting, for instance, $\varphi_n(x) = \infty$ for every $x \in N$ and every $n = 0,1 ...$.

It can be seen at once that the condition $\sum_{n=0}^{\infty} |c_n| < \infty$ implies the absolute convergence of 2.18 almost everywhere since by Schwarz's inequality (see appendix) we have

$$\sum_{n=0}^{\infty} \int_a^b | c_n . \varphi(x) | \, d\mu(x) \leq \sqrt{\mu(b) - \mu(a)} . \sum_{n=0}^{\infty} | c_n | \sqrt{\int_a^b \varphi^2{}_n(x)\, d\mu(x)}$$

$$= \sqrt{\mu(b) - \mu(a)} . \sum_{n=0}^{\infty} | c_n | < \infty$$

.....(2.19)

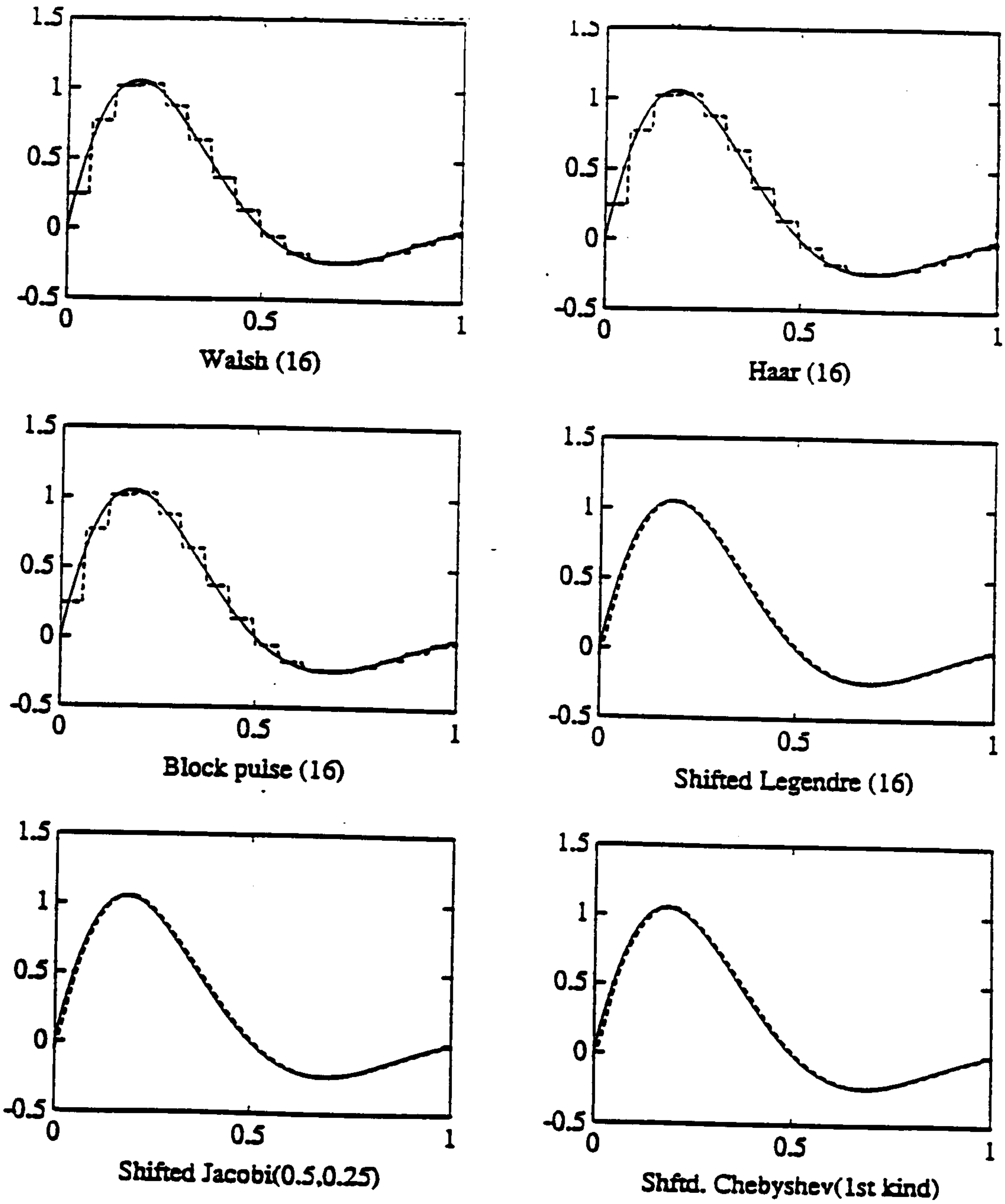


Fig 2.1 Some Orthogonal Approximations of an Arbitrary Signal
$f(t) = 0.5 + 2.\ \sin(2\pi t).\exp(-3t)$

& thus the absolute convergence everywhere of the orthogonal series 2.18 results immmediately from B.Levi'theorem. (See Appendix A).

On the otherhand the requirement

$$\sum_{n=0}^{\infty} c_n^2 \quad < \quad \infty \qquad \text{....(2.20)}$$

is indispensable for the purpose of obtaining convergence tests for general orthogonal series, because if it were not fulfilled the series (2.18) would diverge almost everywhere in the special case $\varphi_n(x) = r_n(x)$ i.e in the case of the Rademacher functions. Thus the useful convergence tests lie somewhere between the conditions $\sum c^2_n < \infty$ and $\sum | c_n | < \infty$. The condition $\sum c^2_n < \infty$ does not at all secure the convergence almost everywhere of every orthogonal series. It is sufficient merely for the convergence of special series like Rademacher & Haar series.

2.2 Orthogonal Systems

2.2.1 Haar's orthogonal system:

This consists of step functions, defined in the interval of orthogonality [0,1] as

$$\chi_0^{(0)}(x) = 1 \qquad \chi_0^{(1)}(x) = \begin{cases} 1 & x \in [0,\frac{1}{2}) \\ 0 & x = \frac{1}{2} \\ -1 & x \in [0,\frac{1}{2}) \end{cases} \qquad \text{....(2.21)}$$

These are the first two Haar functions; the other functions are defined by putting for every natural m (≥ 1) and $1 \leq k \leq 2^m$

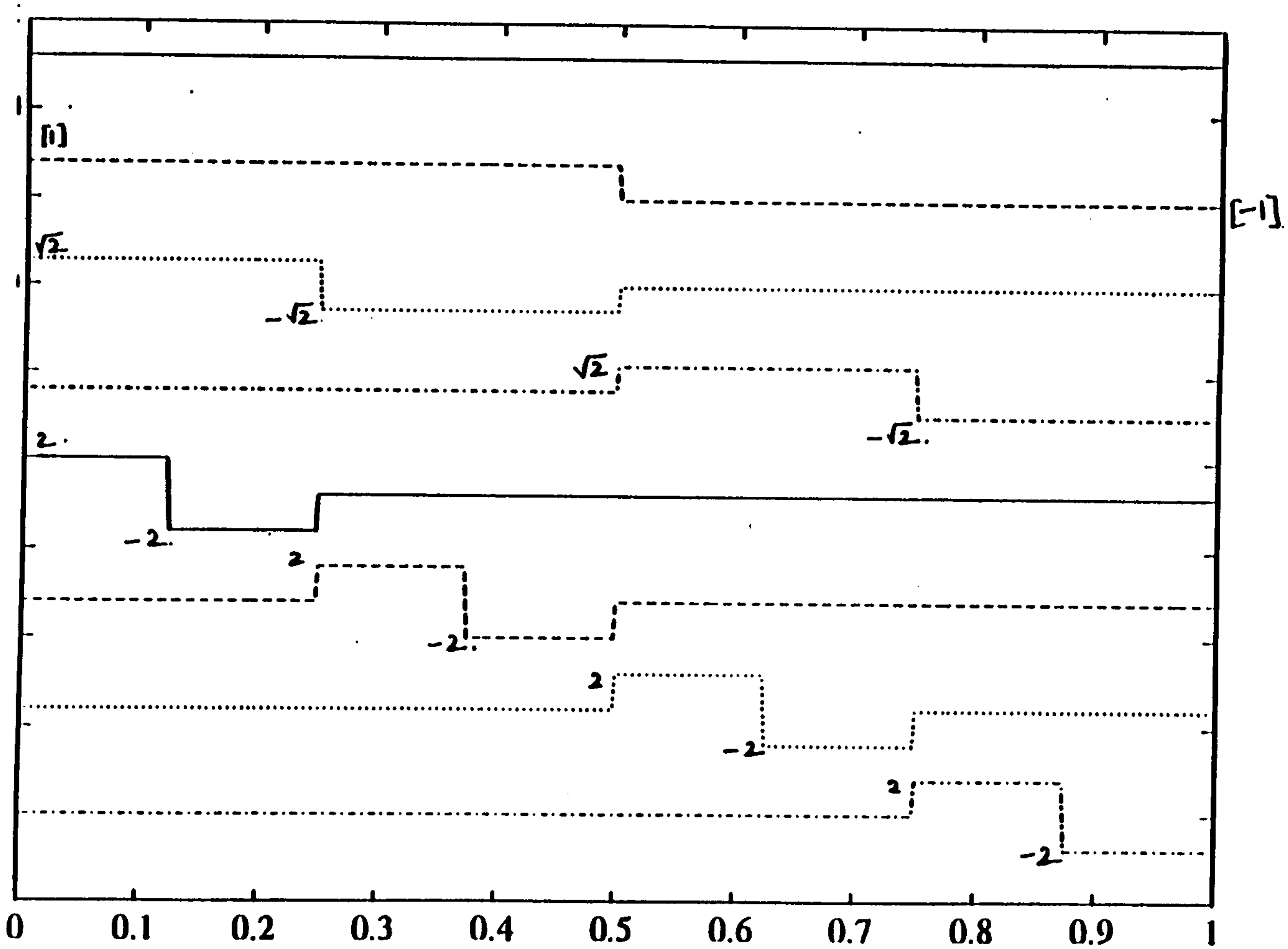


Fig 2.2 First Eight Haar Functions

$$\chi_m{}^{(k)}(x) = \begin{cases} \sqrt{2^m} & x \in \left(\frac{k-1}{2^m}, \frac{k-\frac{1}{2}}{2^m}\right) \\ -\sqrt{2^m} & x \in \left(\frac{k-\frac{1}{2}}{2^m}, \frac{k}{2^m}\right) \\ 0 & x \in \left(\frac{n-1}{2^m}, \frac{n}{2^m}\right) \end{cases} \qquad \text{......(2.22)}$$

with $n \neq k$

$1 \leq n \leq 2^m$

At the points of discontinuity let $\chi_m{}^{(k)}(x)$ be equal to the arithematic mean of the values taken by $\chi_m{}^{(k)}(x)$ in the two adjacent intervals. At the points 0 and 1 let $\chi_n{}^{(k)}(x)$ takes the same value as in the interval $(0, \frac{1}{2^{m+1}})$ and $(1- \frac{1}{2^{m+1}}, 1)$ respectively. As is easily seen the totality of Haar's functions is an orthonormal system. The functions $\chi_n{}^{(k)}(x)$ are normed and orthogonal too. Clearly $\chi_0{}^{(k)}(x)$, (k=0,1,...) is orthogonal to all others. If $m \geq 1$, $i \leq i$, $j \leq 2^m$ for $i \neq j$ already the product $\chi_m{}^{(i)}(x) \cdot \chi_m{}^{(j)}(x)$ itself vanishes, while for $n > m$ the interval, wherein $\chi_n{}^{(i)}(x)$ does not vanish, is contained in an interval of constancy of $\chi_m{}^{(j)}(x)$ and therefore

$$\int_0^1 \chi_n{}^{(i)}(x).\chi_m{}^{(j)}(x)dx = \pm\sqrt{2^m} \int_0^1 \chi_n{}^{(i)}(x)dx = 0 \qquad \text{.....(2.23)}$$

Let f(x) be an L- integrable function in [0,1], it's expansion in the Haar functions

$$f(x) \sim c_0 \chi_0(x) + \sum_{m=0}^{\infty} \sum_{k=1}^{2^m} c_m{}^{(k)} \chi_m{}^{(k)}(x) \qquad \text{....(2.24)}$$

has the remarkable property of representing the function f(x) very well. The following are true.

If f(x) is L-integrable, then the Haar expansion of f(x) converges to f(x) almost everywhere.

The Haar orthogonal system is complete for if every expansion coefficent of the function f(x) vanishes, then we have the partial sums $S_n(x) = 0$ (of f(x) formed by breaking off the Haar expansion of the function f(x) at some term); therefore it follows from (2.18) that f(x) = 0 almost everywhere.This result signifies even still more,since it asserts the completeness not merely in L^2 but for all L integrable functions on the whole.

2.2.2 Rademacher and Walsh Orthogonal Systems

Rademacher's orthonormal system is the result of collecting the Haar functions $\chi_m^{(k)}(x)$ with equal lower indices to a single function. The nth Rademacher function is defined by putting

$$r_0(x) = \chi_0^{(0)}(x)$$

$$r_1(x) = \chi_0^{(1)}(x) \qquad .....(2.25)$$

$$r_{n+1}(x) = \frac{1}{\sqrt{2^n}} \cdot \sum_{k=1}^{2^n} \chi_n^{(k)}(x) \qquad (n = 1, 2, ...; \quad 0 < x < 1)$$

As is easily seen Rademacher's functions alternatively assume the values 1 and -1. They may as well be defined by the relation

$$r_n(x) = \text{sign}\ (\sin(2^n \pi x)) \qquad ....(2.26)$$

where the symbol sign a means, as usual , 1 for $a > 0$, -1 for $a < 0$ and 0 for $a=0$ Rademacher's system is incomplete, since for every n

$$\int_0^1 r_n(x)\, r_1(x)\, r_2(x)\, dx = 0 \qquad ....(2.27)$$

Therefore we cannot expect, even in the case of convergence, that the Rademacher expansion of a function should represent it. Thus the representation capacity of Rademacher's orthonormal system is "bad" but in spite of this the convergence circumstances of Rademacher series are "good" and rather interesting If

$$\sum_{n=0}^{\infty} c_n^2 < \infty$$

the Rademacher series

$$\sum_{n=1}^{\infty} c_n r_n(x)$$

converges almost everywhere. To prove this, we put

$$c^{(k)}{}_n = \frac{c_n}{\sqrt{2^n}} \qquad (k = 1 , ..., 2^n)$$

then we have

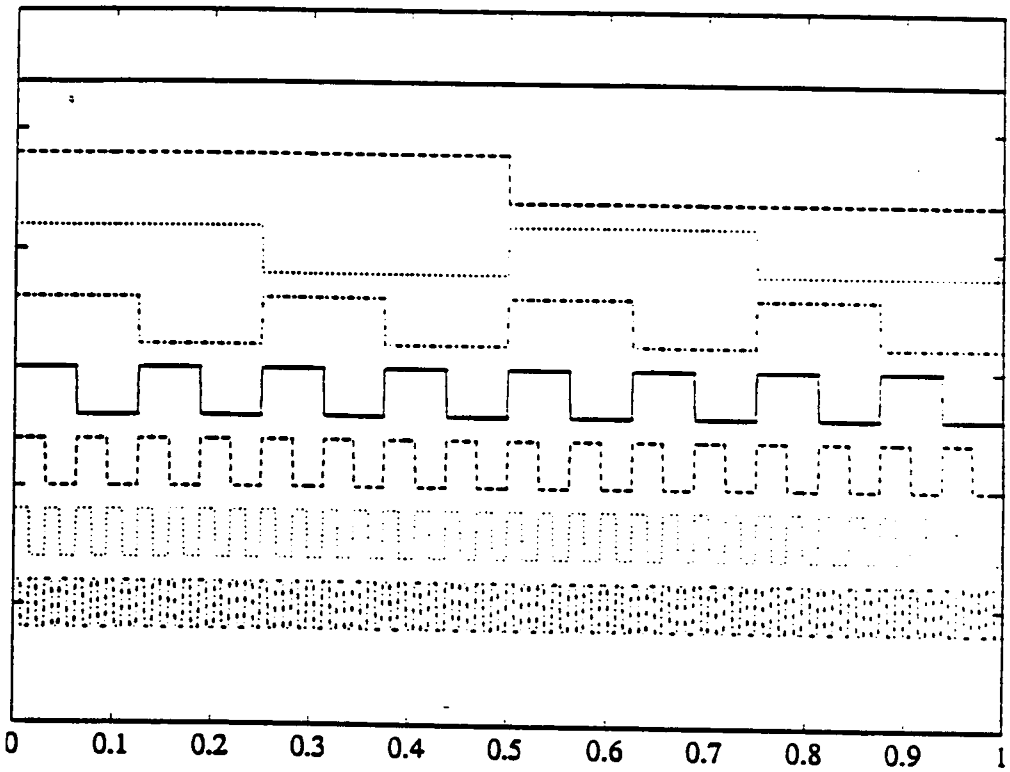
$$\sum_{n=0}^{\infty} \sum_{k=1}^{2^n} (c_n^k)^2 = \sum_{n=1}^{\infty} c_n^2 < \infty \qquad ....(2.28)$$

and consequently it follows from the Riesz-Fisher theorem (see appendix) that the series]

$$\sum_{n=0}^{\infty} \sum_{k=1}^{2^n} c_n^k \chi_n^{(k)}(x) \qquad ...(2.29)$$

is the Haar expansion of a functionf $\in L^2$. This entails the convergence almost everywhere of this Haar expansion and therefore that of the Rademacher series

$$\sum_{n=1}^{\infty} c_n r_n(x) = \sum_{n=0}^{\infty} \sum_{k=1}^{2^n} c_n^k \chi_n^{(k)}(x) \qquad ....(2.30)$$



First eight Rademacher Functions

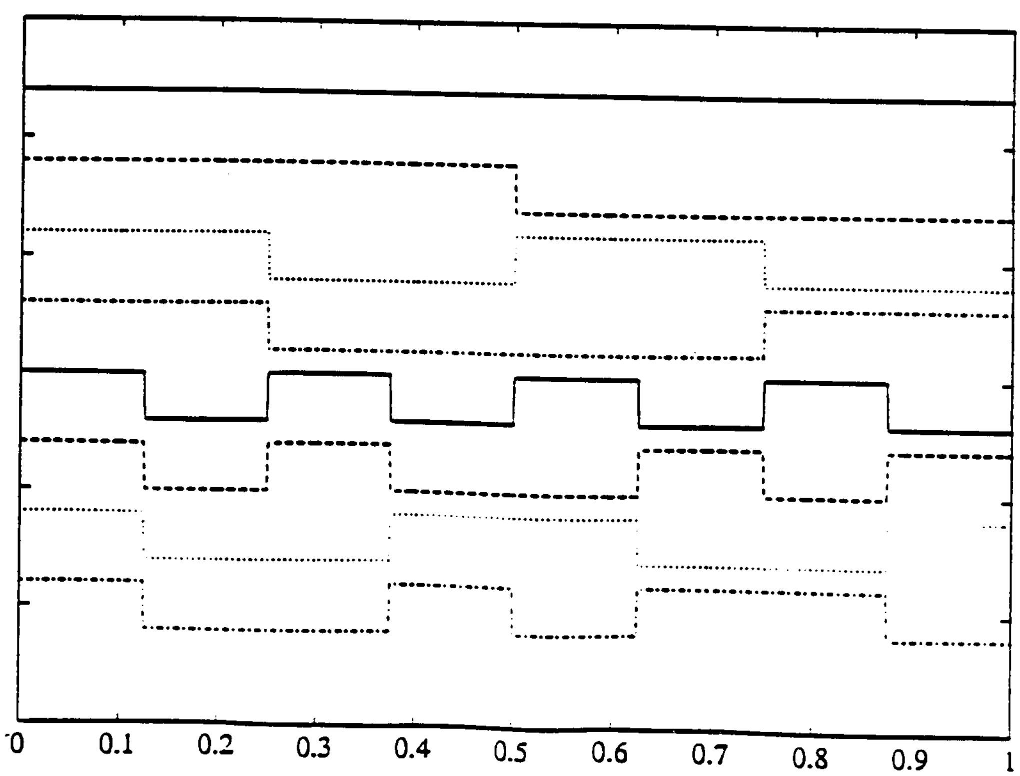


Fig 2.3 First Eight Walsh functions

Walsh orthogonal system:

The property of Rademacher's system $\{ r_n(x) \}$ of any two different finite products of its functions being orthogonal to each other

$$\int_0^1 (r_{k1}(x)\, r_{k2}(x) \;\ldots\; r_{kn}(x))\,(r_{p1}(x)\, r_{p2}(x) \;\ldots\; r_{pn}(x))\, dx \;=\; 0 \qquad \ldots(2.31)$$

for finite different sequences $(k_1,\ldots k_n)$ and $(p_1,\ldots p_m)$ gives rise to the construction of the Walsh orthogonal system considered by Walsh[1923]

$$w_o(x) = 1 \qquad \text{in } [0, 1)$$

if $n \geq 1$ and $2^{v1} + 2^{v2} + 2^{v3} + \;\ldots\; 2^{vp} \quad (v_1 < v_2 < v_3 \ldots < v_p)$

is the dyadic representation of n, then

$$w_n(x) = r_{v1+1}(x)\, r_{v2+1}(x) \;\ldots\; r_{vp+1}(x) \qquad \ldots(2.32)$$

Hence the Rademacher's system constitutes a part of Walsh's system

$$w_{2n}(x) = r_{n+1}(x) \qquad \ldots(2.33)$$

The Walsh orthogonal system is complete in L^2

To prove this, let us suppose that $f \in L^2$ (or let f be at least L integrable) and

$$\int_0^1 f(x)w_n(x)dx = 0 \;(n=0,1, \;\ldots\;) \qquad \ldots(2.34)$$

morover

$$F(x) = \int_0^x f(t)\, dt \qquad (0 \leq x \leq 1) \qquad \ldots(2.35)$$

It then follows that

$$\int_0^1 f(x)w_o(x)dx = \int_0^1 f(x)\,dx = F(1) - F(0) = 0$$

and

$$\int_0^1 f(x)w_1(x)dx = \int_0^1 f(x)\,r_1(x)dx = F(\tfrac{1}{2}) - F(0) - [\,F(1) - F(\tfrac{1}{2})\,] = 0$$

Taking into consideration relations established above we have

$$\int_0^1 f(x)w_2(x)dx = \int_0^1 f(x)\,r_2(x)dx = 2[\,F(\tfrac{1}{4}) + F(\tfrac{3}{4})\,] = 0$$

$$\int_0^1 f(x)w_3(x)dx = \int_0^1 f(x)\,r_1(x)\,r_2(x)dx = 2\,[\,F(\tfrac{1}{4}) - F(\tfrac{3}{4})\,] = 0$$

and consequently $F(\frac{1}{4}) = F(\frac{3}{4}) = 0$

Continuing in this way, it is easily seen that at every dyadically rational point the relation

$$F(\frac{k}{2^n}) = 0 \qquad (\,n = 0,1 \,....\, j,\ k = 0,\,1,\,....\,2^n\,) \qquad ......(2.36)$$

holds. The function $F(x)$ is, however, continuous and therefore $f(x) = F'(x)$, this result implies that $f(x) = 0$ too is true almost everywhere, in accordance with our statement.

There is an interesting connection between the system of Haar and Walsh.

Putting $\chi_0^{(0)}(x) = w_o(x)$ $\qquad$ $\chi_0^{(1)}(x) = w_1(x)$

and then

$$\chi_1^{(1)}(x) = \frac{1}{\sqrt{2}}\,[\,w_2(x) + w_3(x)\,] \qquad ....(2.37)$$

$$\chi_1^{(2)}(x) = \frac{1}{\sqrt{2}}\,[\,w_2(x) - w_3(x)\,]$$

we generally define $\chi_n^{(k)}(x)$ by induction. Let $\chi_{n-1}^{(k)}(x)$ be represented in the form

$$\chi_{n-1}^{(k)}(x) = \frac{1}{\sqrt{2^{n-1}}} \sum_{v=2^{n-1}}^{2^n-1} a^{(n-1)}_{kv} w_v(x) \qquad (k=1,2 \ldots . 2^{n-1}) \qquad \ldots..(2.38)$$

where $a^{(n-1)}_{kv} = \pm 1$.The matrix $\| a^{(n)}_{kv} \|$ is constructed as follows; we write each row of the matrix $\| a^{(n-1)}_{kv} \|$ twice below another (forming thereby the left half of $\| a^{(n)}_{kv} \|$ and then we prolong every row by writing it once more, firstly with the same sign, secondly however with opposite sign (forming thus the right half of $\| a^{(n)}_{kv} \|$ We then define $\chi_n^{(k)}(x)$ as

$$\chi_n^{(k)}(x) = \frac{1}{\sqrt{2^n}} \sum_{v=2^n}^{2^n-1} a^{(n)}_{kv} w_v(x) \qquad \ldots.(2.39)$$

The system $\{ \chi_n^{(k)}(x) \}$ is, apart from the dyadically rational points, identical with Haar's system.

The convergence features of the expansions in Walsh's functions are by far not as favourable as in the case of Haar's functions.There are for instance, continuous functions whose Walsh expansion diverges at a point (as proved by Walsh(1923)) whereas this cannot happen in case of Haar expansions.

2.2.3 Orthogonal Polynomials

An orthogonal system can be obtained by orthogonalization of the linearly independent system $\{ x^n \}$ of the powers of the variable x with integer exponents with respect to a distribution $d\mu(x)$ in an interval [a,b]. We then have for $\varphi_n(x)$ a polynomial $\rho(x)$ of degree exactly equal to n, whose sign can be determined in such a way that we choose the sign of the coefficent of the highest power of x (the leading coefficent) to be positive

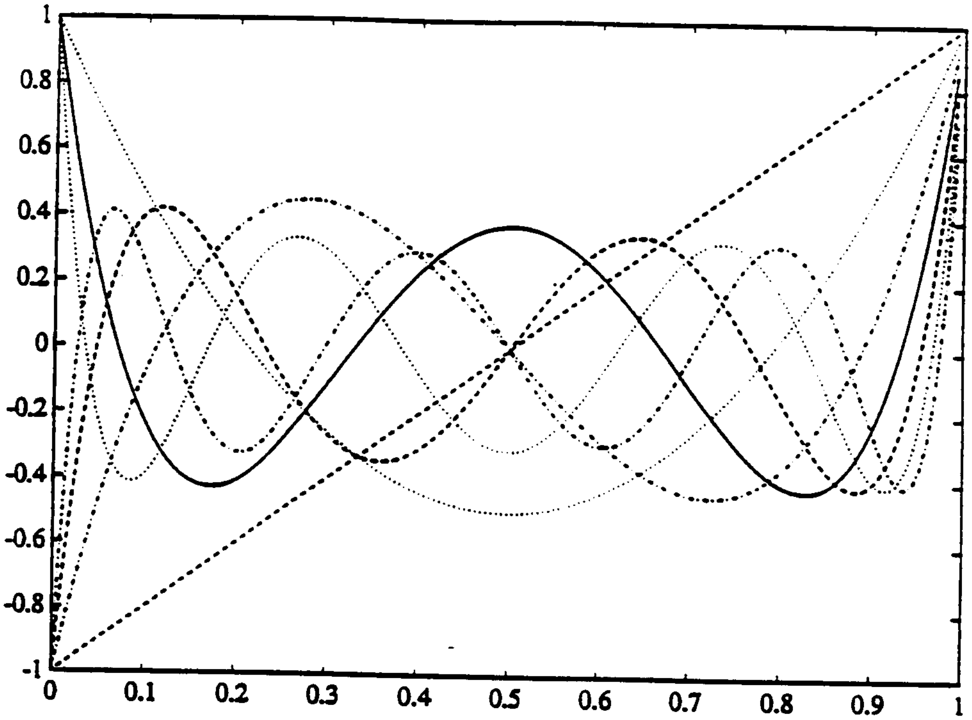


Fig 2.4a First Eight Shifted Legendre Polynomials

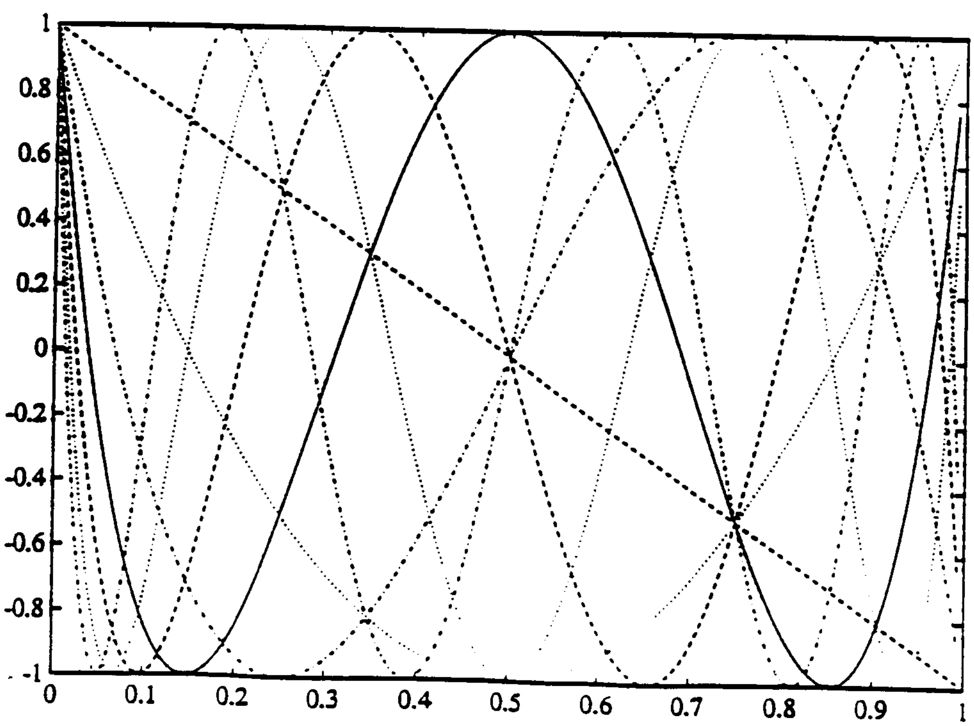


Fig 2.4b First Eight Shifted Chebyshev Polynomials

$$\varphi_n(x) = \sum_{k=0}^{n} r_k \rho_k(x) \qquad .....(2.40)$$

Conversely, the polynomials $\rho_k(x)$ can also be represented as linear combinations of their polynomials .

$$\varphi_0(x), \varphi_1(x), \varphi_2(x)................\varphi_k(x) \qquad ....(2.41)$$

It follows from this that for $k < n$ the $\rho_k(x)$ are orthogonal to the polynomials $\varphi_k(x)$. Thus multiplying both sides of the preceding equation by $\rho_r(x)$ where $r < n$ and then integrating and using the orthogonality of $\rho_r(x)$ and $\varphi_k(x)$
we find that

$$\sum_{k=0}^{n} r_k \cdot \int_a^b \rho_k(x)\rho_r(x)d\mu(x) = r_r = \int_a^b \varphi_n(x)\rho_r(x)d\mu(x) = 0 \qquad ...(2.42)$$

for $v = 0,1, \ldots n-1$. From this we infer that

$$\eta_0 = \eta_1 = ...= \eta_{n-1} = 0 \quad \text{i.e} \quad \varphi_n(x) = r_n . \rho_n(x) \qquad ....(2.43)$$

and consequently

$$1 = \int_a^b \varphi^2{}_n(x)\, d\mu(x) = r^2{}_n \int_a^b \rho^2{}_n(x)\, d\mu(x) = r^2{}_n \qquad ....(2.44)$$

We see that $\gamma_n = \pm 1$ and since we have assumed the leading coefficents of both $\varphi_n(x)$ and $\rho_n(x)$ to be positive, it is necessary to take $\gamma_n = 1$ i.e $\varphi_n(x) = \rho_n(x)$ in accordance with the assertion.

The following is also true and is stated without proof.

1. The orthogonal system of polynomials $\{ \rho_n(x) \}$ belonging to the distribution $d\mu(x)$ is complete in $L_\mu{}^2$

2. Among all polynomials $\pi_n(x)$ of degree not higher than n the integral

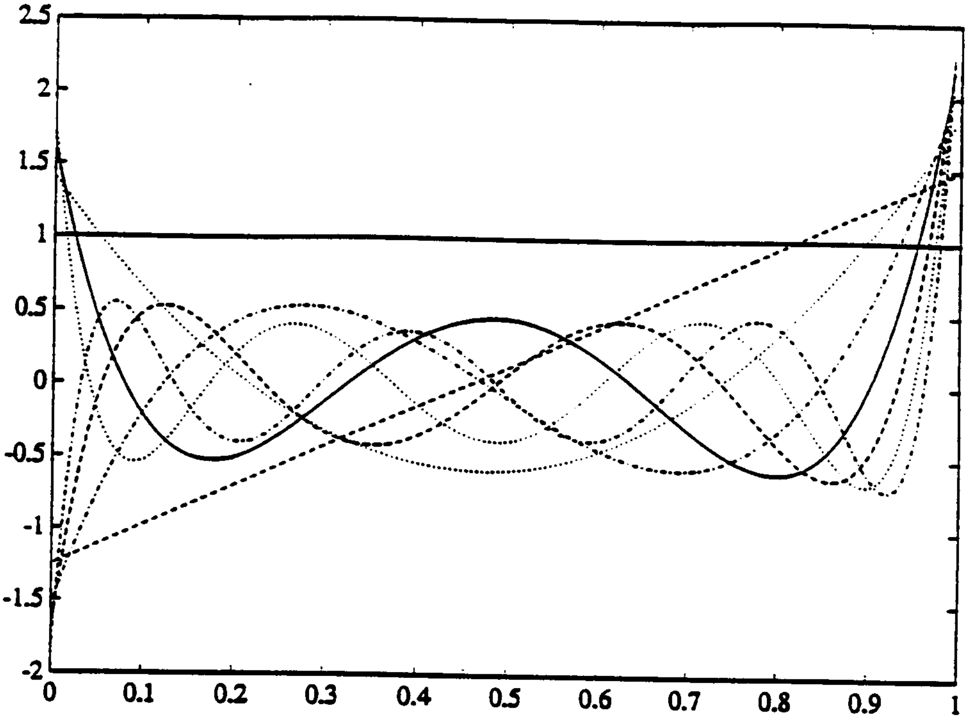


Fig 2.5a First Eight Shifted Jacobi (0.5, 0.25) Polynomials

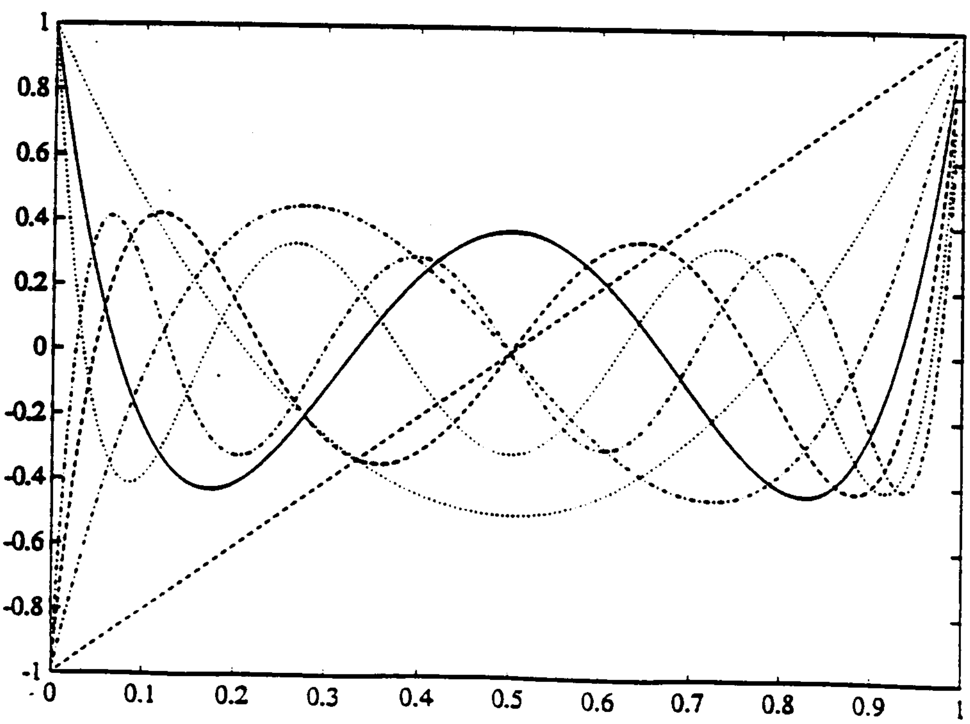


Fig 2.5b First Eight Laguerre Polynomials

$$\int_a^b [\, f(x) - \pi_n(x)]^2 \, d\mu(x) \qquad \text{....(2.45)}$$

attains its least value for $\pi_n(x) = S_n(x)$
where $S_n(x)$ is the nth partial sum of the expansion in the polynomials $\rho_n(x)$ of $f(x)$ where

$$f(x) \sim \sum_{n=0}^{\infty} c_n \, \rho_n(x) \qquad \text{....(2.46)}$$

Orthogonal polynomials have the following basic properties

1. Recurrence relation:

$$\varphi_{i+1}(x) = (a_i x + b_i)\, \varphi_i(x) - c_i\, \varphi_{i-1}(x) \qquad \text{.....(2.47)}$$

with $\varphi_0(x) = 1$, $\varphi_1(x) = a_0 x + b$ where a_i, b_i & c_i are the recurrence coefficents, whose values are specified by the particular classical orthogonal polynomials under consideration

2. Differential recurrence relation

$$\varphi_i(x) = A_i \varphi'_{i+1}(x) + B_i \varphi'_i(x) + C_i \varphi'_{i-1}(x) \qquad \text{...........(2.48)}$$

where A_i, B_i & C_i are the differential recurrence coefficents, whose values again are specified by the particular classical orthogonal polynomials under consideration, table 1.

The orthogonal polynomials $\varphi_i(x)$ with respect to the weight function $w(x)$ over the interval $a \leq x \leq b$ (or $a < z < b$ when a,b are infinite) are defined to be of degree i in x and to satisfy the condition

$$\int_a^b w(x)\, \varphi_i(x)\, \varphi_j(x)\, dx = r_i\, \delta_{ij} \qquad \text{.....(2.49)}$$

and the recurrence relation (2.47).

where δ_{ij} is the Kronecker Delta given as

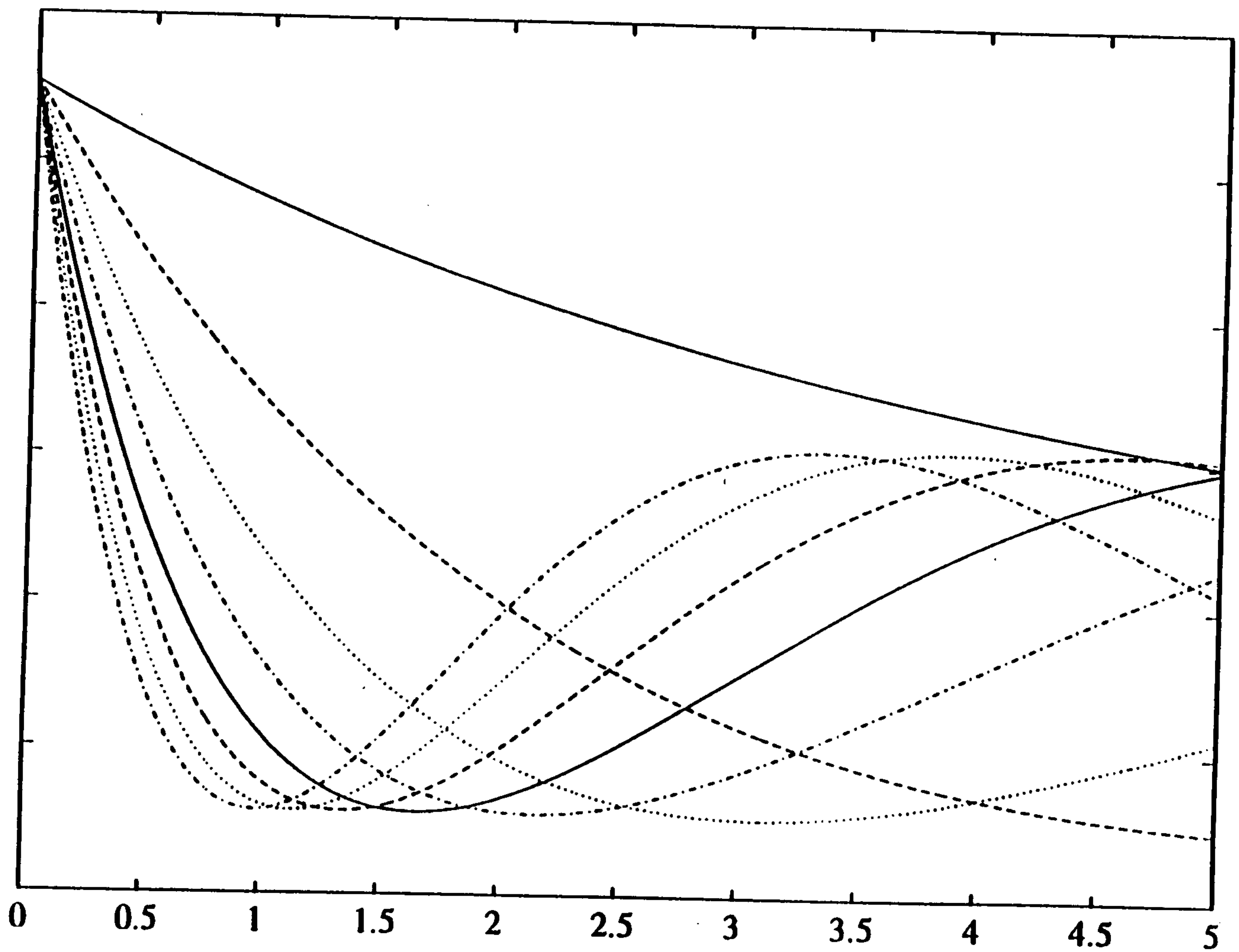


Fig 2.6 First Eight Laguerre Functions (p=0.25)

$$\delta_{ij} = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$$

Some intervals on which orthogonal polynomials are defined are not suitable for solving practical problems so that shift transformations are necessary. The general shifted orthogonal polynomials may be obtained by putting

$$x = pt + q$$

which transforms the domain [a,b] into the domain

[min (a', b') , max(a', b')] where $p = \frac{(a-b)}{(a' - b')}$ and $q = \frac{(a'b - ab')}{(a' - b')}$

Thus the shifted general orthogonal polynomial becomes

$$\varphi^*_{i+1}(t) = (a_i^* t + b_i^*)\, \varphi^*_i(t) - c^*_i\, \varphi^*_{i-1}(t) \qquad \text{......(2.50)}$$

where

$$a_i^* = a_i p \qquad b_i^* = b_i + a_i q$$
$$c_i^* = c_i \qquad \text{for} \quad i = 0, 1 \ldots.$$

$$\varphi^*_0(t) = 1 \qquad \varphi^*_i(t) = a^*_0 t + b^*_0$$

The new polynomials $\varphi^*_i(t)$ with the above recurrence relation are orthogonal with respect to the weight function

$$w^*(t) = w(pt+q)$$

over the interval [min (a', b') max(a', b')] . For example again see figures 2.4 to 2.6.

Operational matrix of integration:

Common Orthogonal expansions have the following useful property

$$\int_0^t \varphi(x)\,dx = P.\varphi(t) \qquad ....(2.51)$$

where P is called the operational matrix of integration. The coefficents of the recurrence formulae of common orthogonal polynomials used in system science along with some of their useful properties and their operational matrices are as shown in appendix B.

2.2.4 Comparative Study

A noisy sample signal namely power variation during a perturbance in a single machine infinite bus system (modelled by a 11 th order differential equation model) with Gaussian noise superimposed is used for comparing the representation properties of some common orthogonal polynomials/ functions.

The square error at each sampled point is plotted. Note the different scales on the Y-axis. This gives an indication of the distribution of error and hence the effectiveness of the orthogonal approximation. Note the pronounced error at the initial and final points for each. The Walsh function represenation yields the least maximum error with the error well distributed over the whole normalised time interval among the orthogonal functions while the shifted Legendre polynomial representation yields the least error for the orthogonal polynomials.

The error becomes less pronounced with increase in the number of functions used except in the case of the 3 polynomials which tend to become unstable in this particular analysis.

The mean square error for different number of orthogonal functions/ polynomials are as shown in the following table

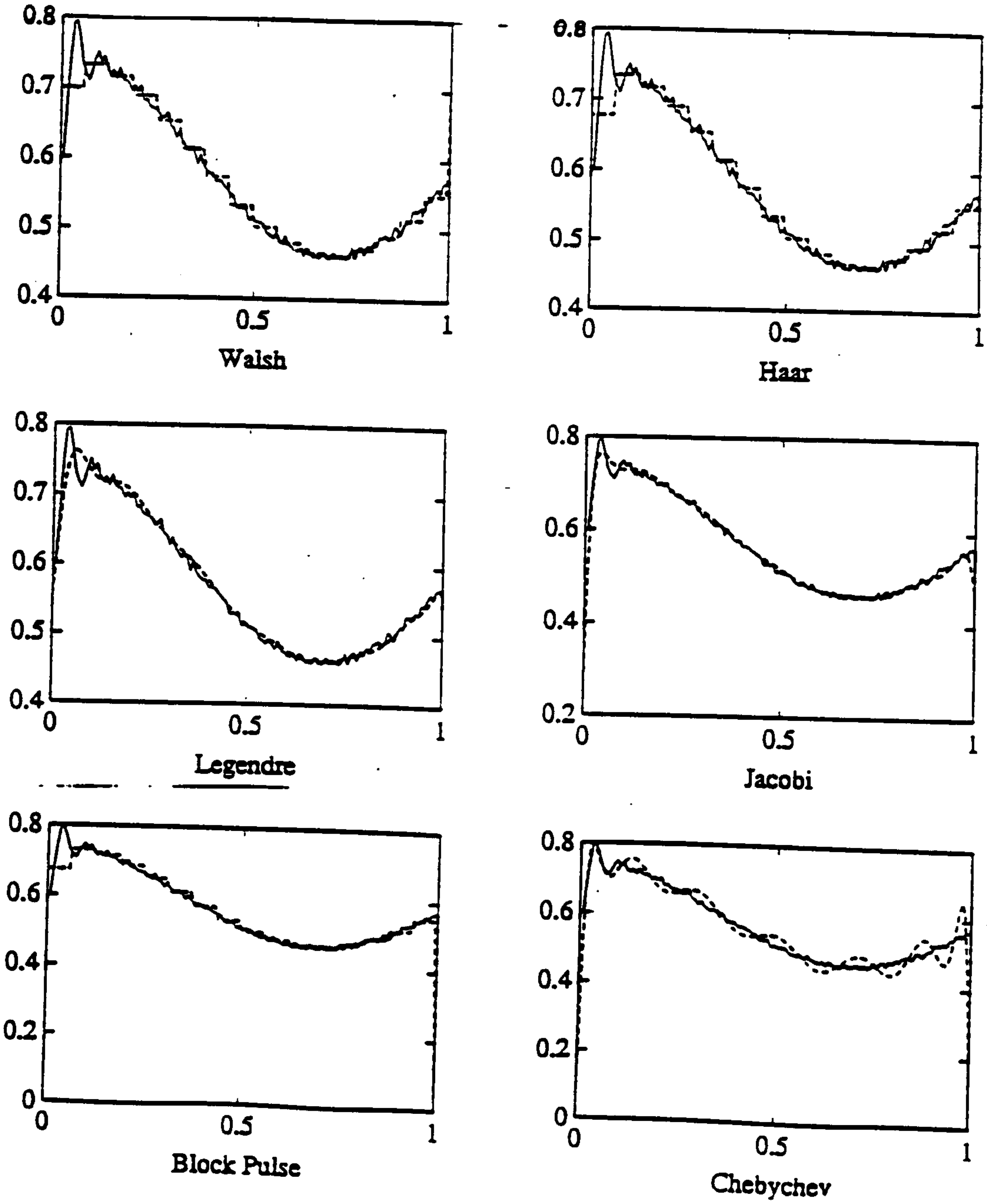


Fig 2.7a 16 Orthogonal Function Approximation of Noisy Power Measurement (Low Frequency Analysis)

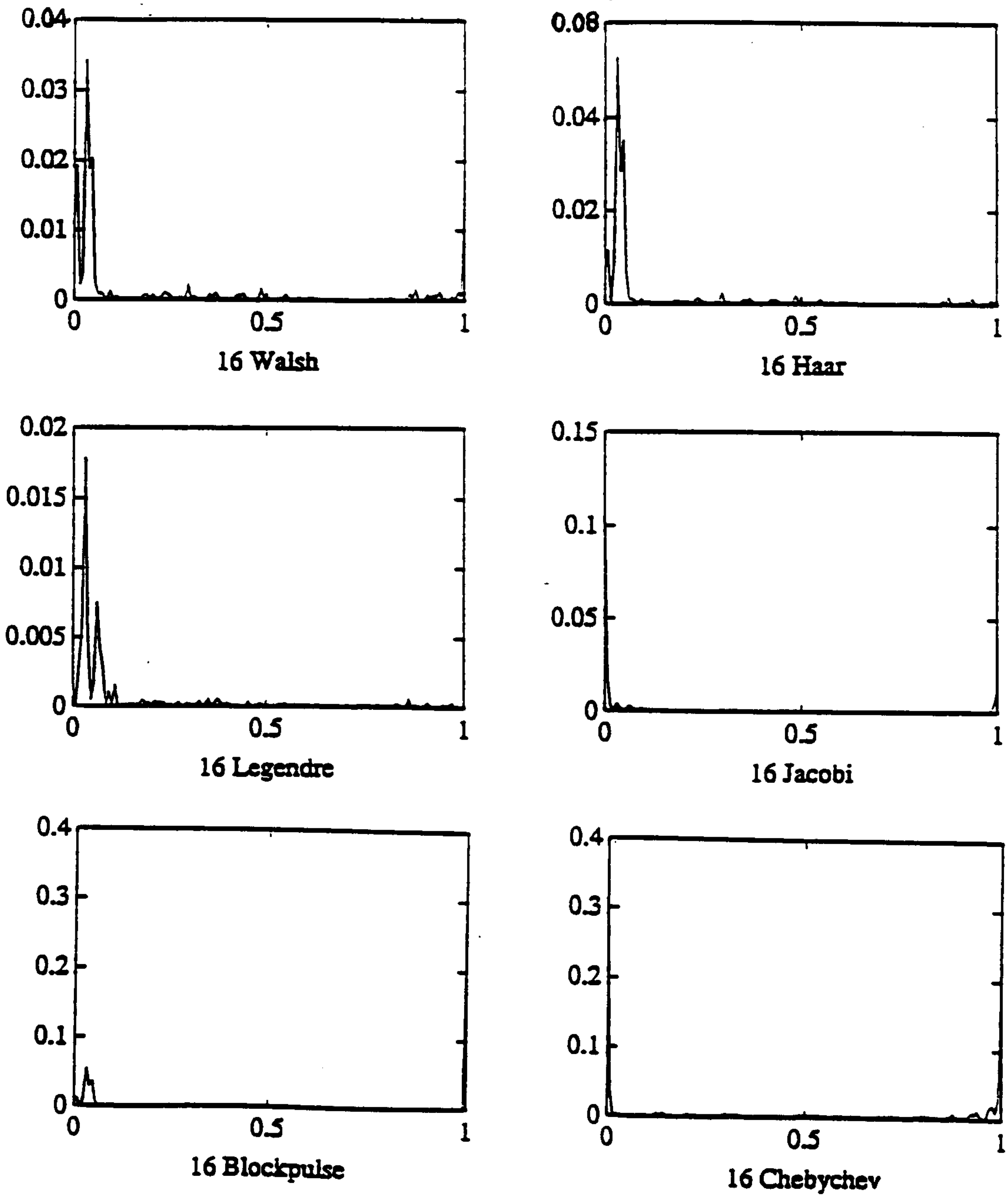


Fig 2.7b Error^2 Distribution for 16 Function Approximation of Power Measurement.

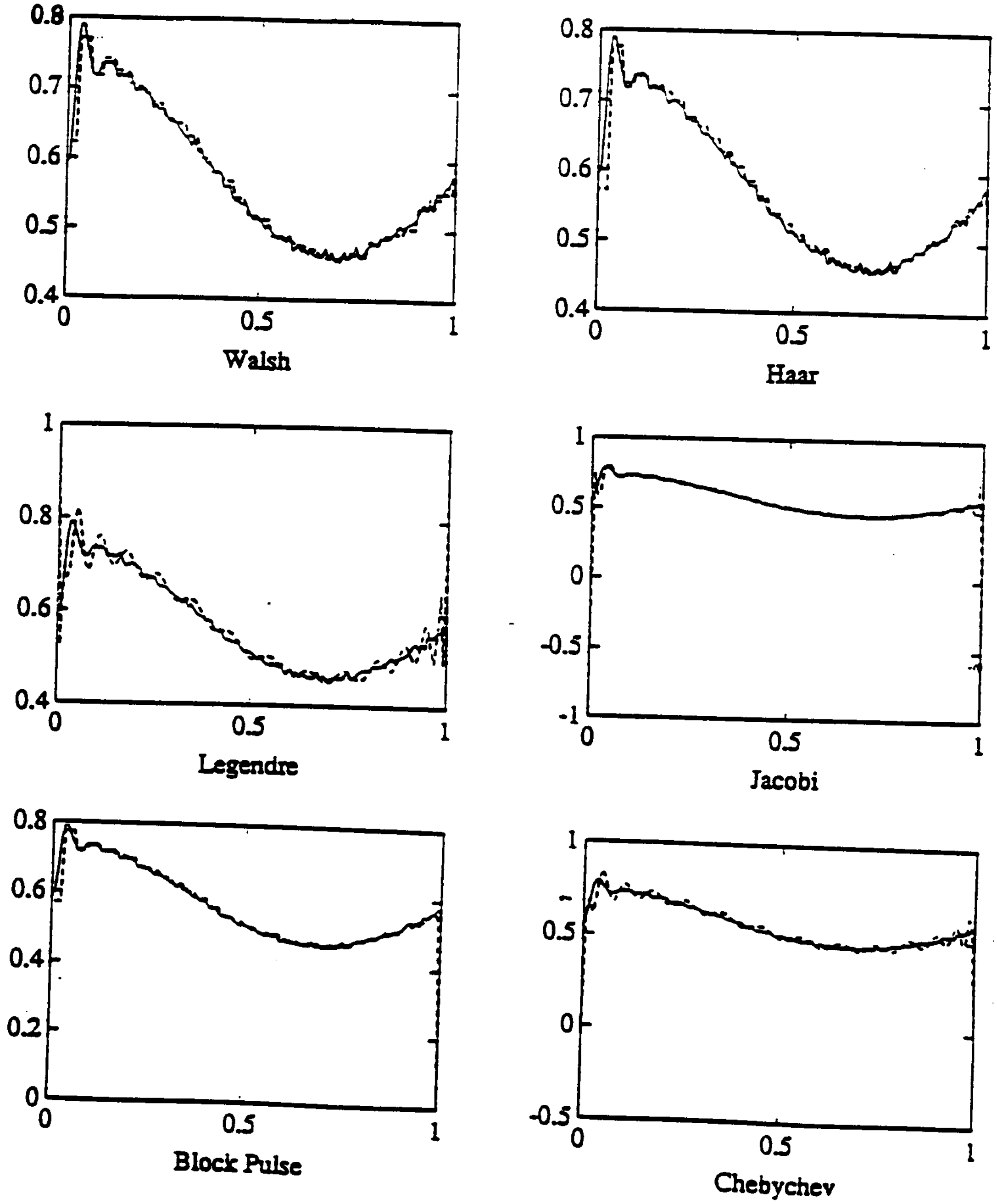


Fig 2.7c 32 Orthogonal Function Approximation of Noisy Power Measurement.

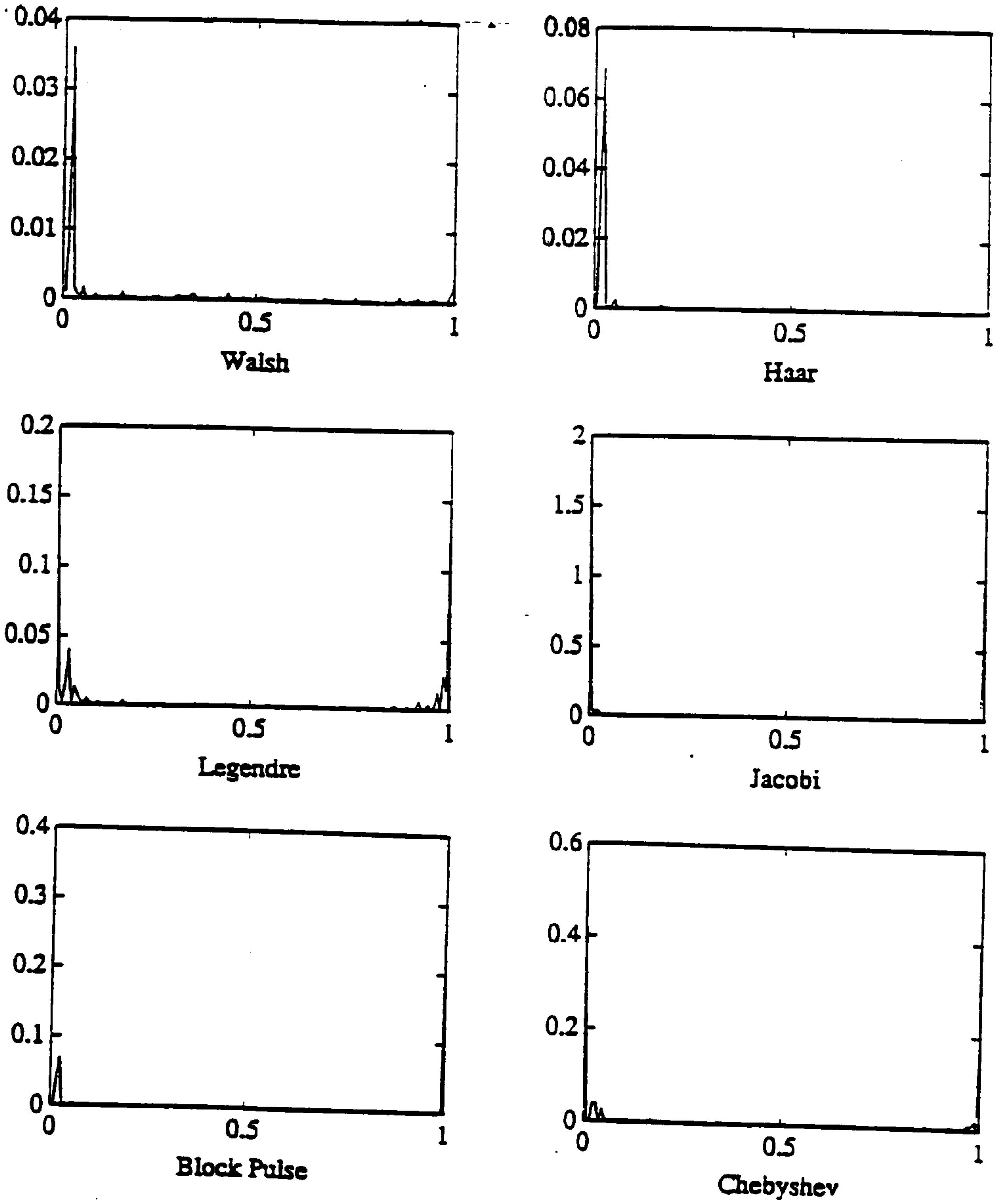


Fig 2.7d Error ^{2} Distribution for 32 Function Approximation of Power Measurement.

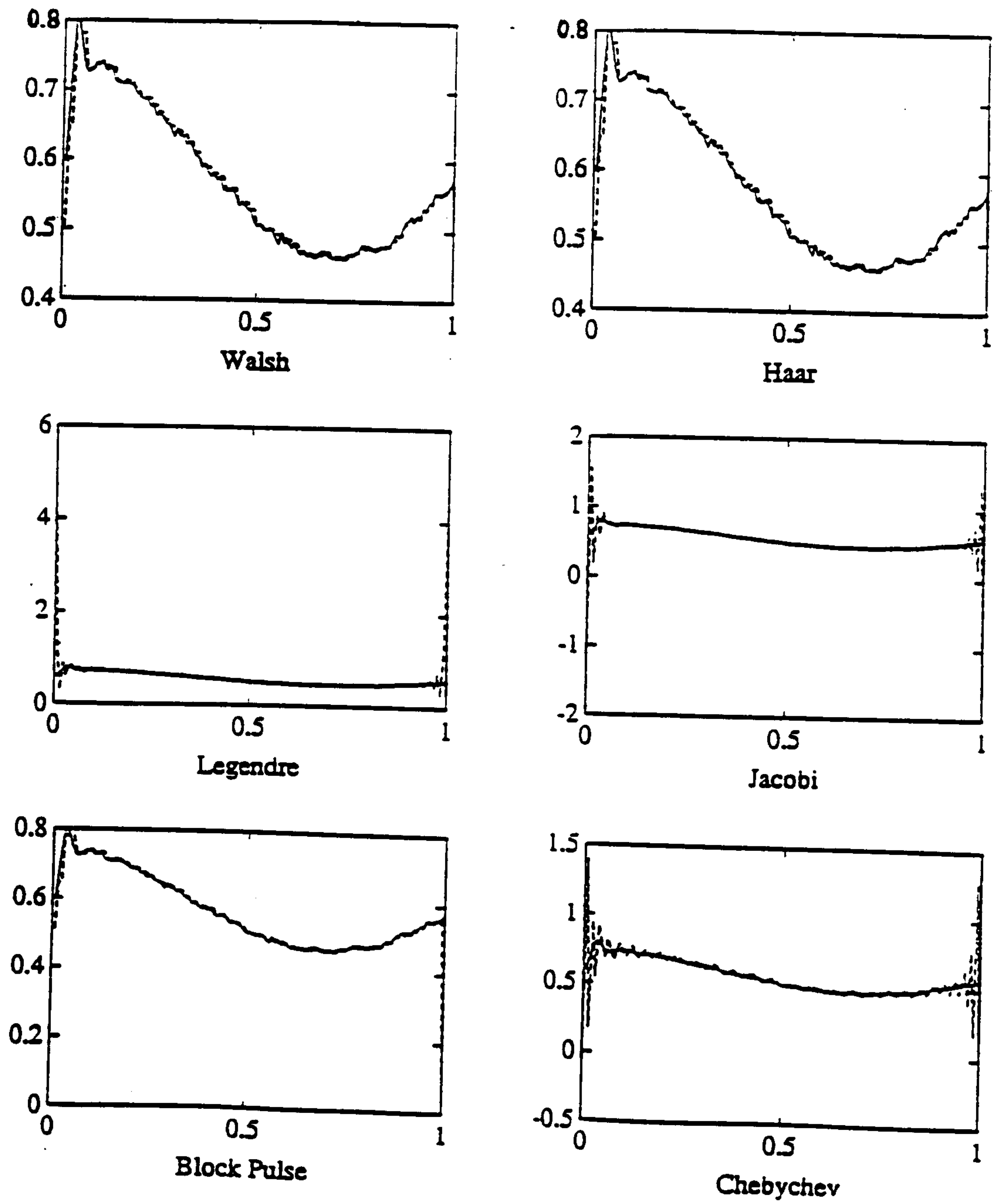


Fig 2.7e 64 Orthogonal Function Approximation for Noisy Power Measurement.

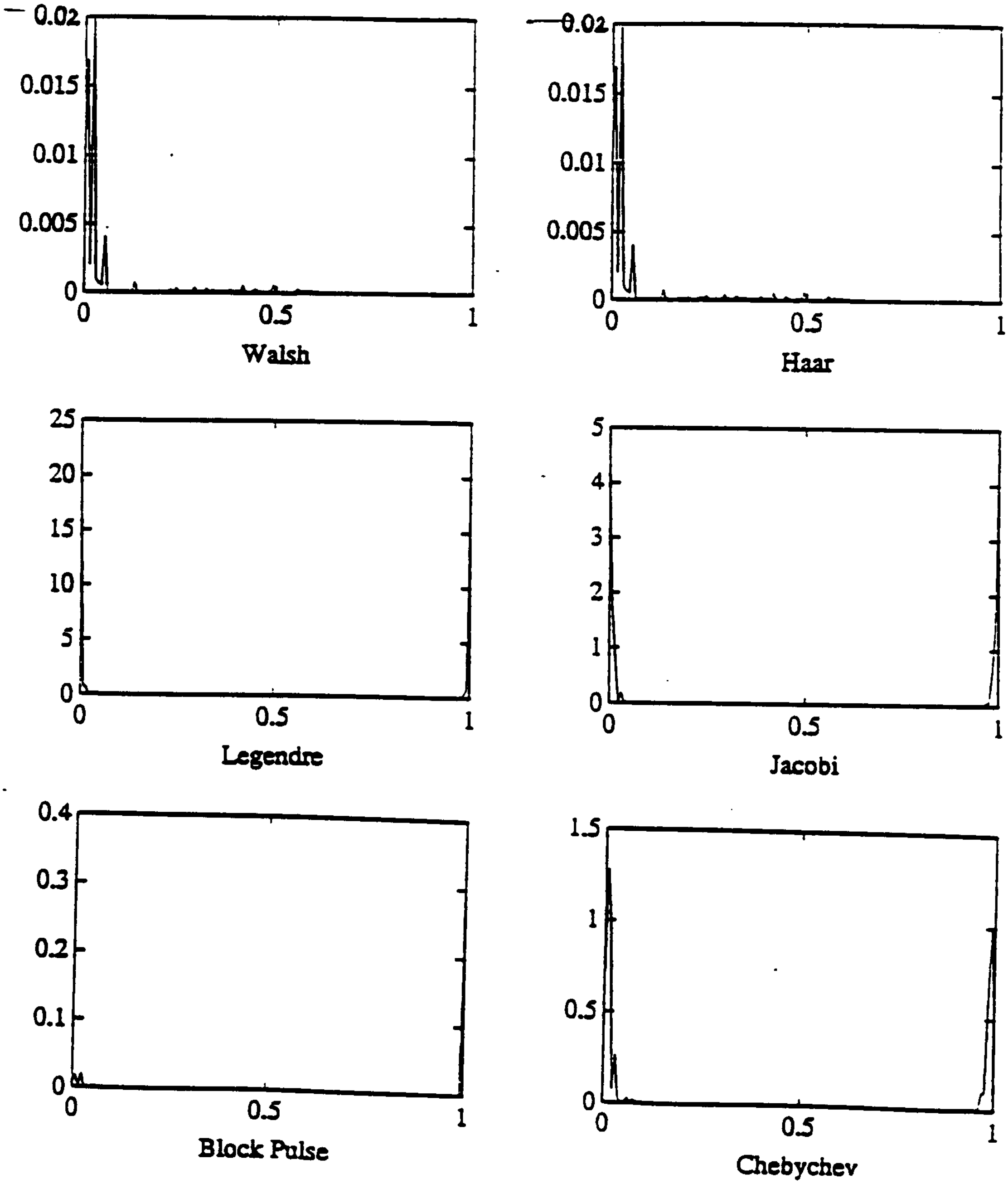


Fig 2.7f Error ^{2} Distribution for 64 Function Approximation of Power Measurement.

Table 2.1 Mean Square Error for Different Number of Orthogonal Functions/Polynomials

NOS	WALSH	HAAR	BLOCKP	LEGENDRE	JACOBI	CHEBYCHEV
For Power measurements with Gaussian noise						
16	0.0004	0.0005	0.0014	0.1891	0.1907	0.1893
32	0.0002	0.0003	0.0012	0.1897	0.1919	0.1996
64	0.0001	0.0002	0.0010	0.3024	0.1950	0.2163
For voltage measurements						
16	0.0000	0.0001	0.0028	0.3324	0.3371	0.3326
32	0.0000	0.0002	0.0029	0.3344	0.3404	0.3528
64	0.0000	0.0003	0.0029	0.6566	0.3455	0.4026
For Rotor Angle measurements						
16	0.0001	0.0001	0.0017	0.2681	0.2713	0.2682
32	0.0000	0.0001	0.0018	0.2693	0.2735	0.2886
64	0.0000	0.0002	0.0019	0.4992	0.2775	0.3172

Chapter 3: Parameter Identification of Severely Nonlinear Systems Using Walsh Functions

The first part shows some of the delineation properties of Walsh functions for some common nonlinearities. Most derivations can easily be derived using the useful properties of the walsh product matrices and are based on Taylor series expansions & the binomial theorem.

The second part uses the above delineated results for parameter identification of severely non-linear systems.

Consider the following nonlinearities & their infinite series/binomial expansions.

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\ldots$$

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\ldots$$

$$\cosh(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} \ldots\ldots$$

$$\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \ldots\ldots$$

$$\ln(x) = (x-1) - \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3!} \ldots\ldots$$

if $|x-1| < 1$

$$a^x = 1 + x\ln(a) + \frac{(x\ln(a))^2}{2!} + \ldots\ldots$$

if $a > 0$

$$x^n = 1 + n(x-1) + n(n-1)\frac{(x-1)^2}{2!} + \dots\dots\dots\dots$$
$$+ n(n-1)\dots(n-r+1)\frac{(x-1)^r}{r!} + \dots\dots\dots\dots$$

1) The above is valid for all x if n is a +ve integer

2) Valid for $|x-1| < 1$ if n is -ve or a fraction.

Let x (t) be a time varying function ($t \in [0, 1)$) and let it's Walsh function approximation be given by

$$x(t) = C^T \Phi(t)$$

where $\Phi(t)$ is the Walsh function column vector containing the first m Walsh functions as it's entries and C^T is the Walsh coefficient vector with the coefficients found in the least square sense, Walsh [1923].

$$\Phi(t)\,\Phi^T(t)\,C = \Lambda_C\,\Phi(t)$$ Karanam, Frick & Mohler [1978]

$$\Lambda_C = [\,C\;\; \Lambda_1^m C\;\; \Lambda_2^m C\;\; \Lambda_3^m C\;\; \dots\; \Lambda_{m-1}^m C]$$

where $\Lambda_i^m = \begin{bmatrix} \Lambda_i^{m/2} & O^{m/2} \\ O^{m/2} & \Lambda_i^{m/2} \end{bmatrix}$ & $\Lambda_{i+m/2}^m = \begin{bmatrix} O^{m/2} & \Lambda_i^{m/2} \\ \Lambda_i^{m/2} & O^{m/2} \end{bmatrix}$

For $m > 1$ and $i = 0, 1, 2 \dots\dots(\frac{m}{2} - 1)$

and $\Lambda_0^{m/2} = I_{(m/2)}$ For additional properties of Λ_i matrices refer to Karanam et al [1978]

Proof:

We have for

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$

or

$$\sin(C^T.\Phi(t)) = [C^T.\Phi(t) - \frac{(C^T.\Phi(t))^3}{3!} + ..].$$

$$= C^T.[I - \frac{\lambda_C^{\,2}}{3!} + \ldots\ldots].\Phi(t)$$

$$= C^T.\lambda_C^{\,-1}.[\lambda_C - \frac{\lambda_C^{\,3}}{3!} \quad \frac{\lambda_C^{\,5}}{5!} \quad \ldots].\Phi(t)$$

$$= k^T.\ \sin(\lambda_C)\,\Phi(t)$$

provided the inverse of λ_C^{-1} exists and is unique. By construction the first row of Λ_C matrix is equal to C^T from which we have

$$k^T = [1\,0\,0\,..\,0] \qquad = C^T.\lambda_C^{\,-1}$$

which is true for any m provided λ_C^{-1} exists. It then follows that the Walsh function approximations for the above nonlinearities can be written in terms of the entries of C^T as

$$\sin(x(t)) = [k^T.\ \sin(\lambda_C)].\ \Phi(t)$$

$$\cos(x(t)) = [k^T.\ \cos(\lambda_C)].\ \Phi(t)$$

Table 3.1 Walsh coefficent vectors found by two methods

Method1: Walsh vector found in the least square sense
Method2: Walsh vector found from the series expansion

Parent Vector	Sin(x(t)) Mthd1	Sin(x(t)) Mthd2	Cos(x(t)) Mthd1	Cos(x(t)) Mthd2	Sinh(x(t)) Mthd1	Sinh(x(t)) Mthd2	Cosh(x(t)) Mthd1	Cosh(x(t)) Mthd2	Exp(x(t)) Mthd1	Exp(x(t)) Mthd2
0.7436	0.6005	0.6027	0.6610	0.6686	0.9158	0.9129	1.4336	1.4342	2.3494	2.3470
0.3847	0.2503	0.2524	-0.2726	-0.2652	0.5481	0.5453	0.3654	0.3662	0.9135	0.9115
0.0476	0.0072	0.0084	-0.0583	-0.0514	0.1011	0.0996	0.0880	0.0905	0.1891	0.1900
0.0762	0.0341	0.0352	-0.0679	-0.0611	0.1314	0.1299	0.0980	0.1006	0.2294	0.2305
-0.0038	0.0072	0.0088	0.0014	0.0085	-0.0212	-0.0234	-0.0279	-0.0263	-0.0492	-0.0497
-0.0048	0.0059	0.0075	0.0008	0.0078	-0.0219	-0.0240	-0.0272	-0.0254	-0.0491	-0.0494
-0.1016	-0.0330	-0.0315	0.0949	0.1023	-0.1898	-0.1916	-0.1686	-0.1666	-0.3584	-0.3582
-0.1588	-0.0863	-0.0848	0.1155	0.1227	-0.2510	-0.2527	-0.1901	-0.1880	-0.4411	-0.4407
-0.0054	-0.0040	-0.0031	-0.0018	0.0051	-0.0067	-0.0077	-0.0091	-0.0060	-0.0158	-0.0137
-0.0073	-0.0057	-0.0048	-0.0010	0.0058	-0.0088	-0.0097	-0.0099	-0.0068	-0.0187	-0.0164
-0.0519	-0.0226	-0.0217	0.0419	0.0490	-0.0884	-0.0893	-0.0771	-0.0740	-0.1655	-0.1633
-0.0805	-0.0491	-0.0482	0.0528	0.0597	-0.1193	-0.1201	-0.0886	-0.0854	-0.2079	-0.2054
-0.0274	-0.0198	-0.0191	0.0143	0.0212	-0.0351	-0.0358	-0.0278	-0.0244	-0.0629	-0.0602
-0.0419	-0.0329	-0.0322	0.0206	0.0274	-0.0511	-0.0518	-0.0345	-0.0310	-0.0856	-0.0828
-0.0275	-0.0072	-0.0062	0.0229	0.0300	-0.0534	-0.0543	-0.0520	-0.0490	-0.1054	-0.1033
-0.0420	-0.0209	-0.0200	0.0278	0.0348	-0.0688	-0.0697	-0.0571	-0.0540	-0.1260	-0.1237

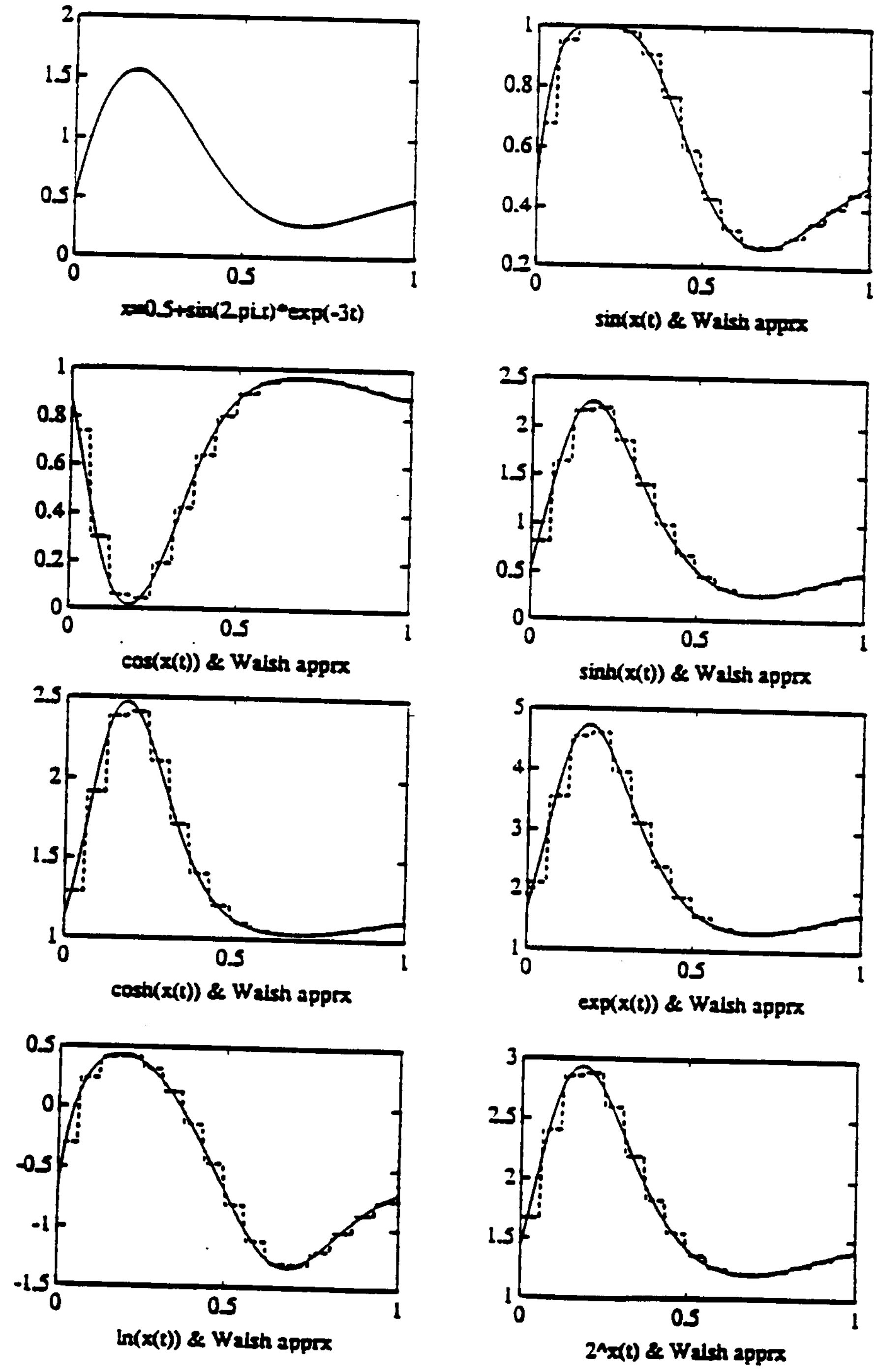


Fig 3.1 Walsh Approximation of Common Nonlinearities by the Method Outlined.

$$\sinh(x(t)) = [k^T . \sinh(\lambda_C)] . \Phi(t)$$

$$\cosh(x(t)) = [k^T . \cosh(\lambda_C)] . \Phi(t)$$

$$\exp(x(t)) = [k^T . \exp(\lambda_C)] . \Phi(t)$$

$$\ln(x(t)) = [k^T . \ln(I + \lambda_C)] . \Phi(t)$$

$$\text{where } D = C^T - [1\ 0 \ldots\ldots 0]$$

$$a^{x(t)} = [k^T . \exp(\lambda_C)] . \Phi(t)$$

$$\text{where } E = \ln(a) . C$$

$$X(t)^n = [k^T . (I + \lambda_C)^n] . \Phi(t)$$

$$\text{where } D = C^T - [1\ 0 \ldots 0]$$

An arbitrary signal $x(t) = 0.5 + 2 . \sin(2 . pi . t) . \exp(-3t)$ was used for verifying the results for these common nonlinearities as shown in fig . The results are convincing. Note : the same method can be extended to other nonlinearities provided their series expansions exist.

Karanam et al [1978] have expressed some doubt about the possibility of expressing fractional nonlinearities in terms of the walsh expansion of the parent function x(t) since their lemmas 3.1 & 3.2 do not hold as correctly pointed out. However the binomial series expansion circumvents the problem though the parent function has to be normalised such that $|x(t)-1| < 1$ for negative and fractional nonlinearities.This can be done by appropriate scaling since the Walsh transform preserves linearity.

Parameter identification of severely nonlinear systems :

Consider the following variable structure system (Karanam et al [1978]) with a sinusoidal modification.

$$y'(t) = a\ y(t) + b\ y^2(t) + c\ y(t)\,u^2(t) + d\ \sin(u(t))$$

with $a = -1,\ b = 0.5,\ c = 0.3,\ d = 1.5,$

Let $u(t) = \exp(-0.5t) = Z^T\ \Phi(t)$

$x(t) = E^T\ \Phi(t)$

$k^T = [\,1\ 0 \ldots 0\,]$

and zero initial conditions. The output y(t) is generated using a 4 th order Runge Kutta method with the integration step h =1/256

We then have the above eqn represented in terms of the Walsh expansions of its various terms as

$$E^T\ \Phi(t) = a\ E^T P\ \Phi(t) + b\ E^T \lambda_E P\ \Phi(t) + c\ E^T\ \lambda_Z{}^2\ P\ \Phi(t) + \\ + d\ [\,k^T.\sin(\ \lambda_Z\ )\,].\ P\ \Phi(t)$$

or

$$E^T\ \Phi(t) = [\,a\ b\ c\ d\,] \begin{matrix} E^T P\ \Phi(t) \\ E^T\ \lambda_E\ P\ \Phi(t) \\ E^T\ \lambda_Z{}^2\ P\ \Phi(t) \\ [\,k^T.\ \sin(\ \lambda_Z\ )\,].\ P\ \Phi(t) \end{matrix}$$

Sampling $\Phi(t)$ at 4 different time instants , the parameters are uniquely determined. The number of Walsh functions used are varied too.

Example 2

Consider the following exponential AR model of Ozaki [1985] modelled without the noise (e(t))

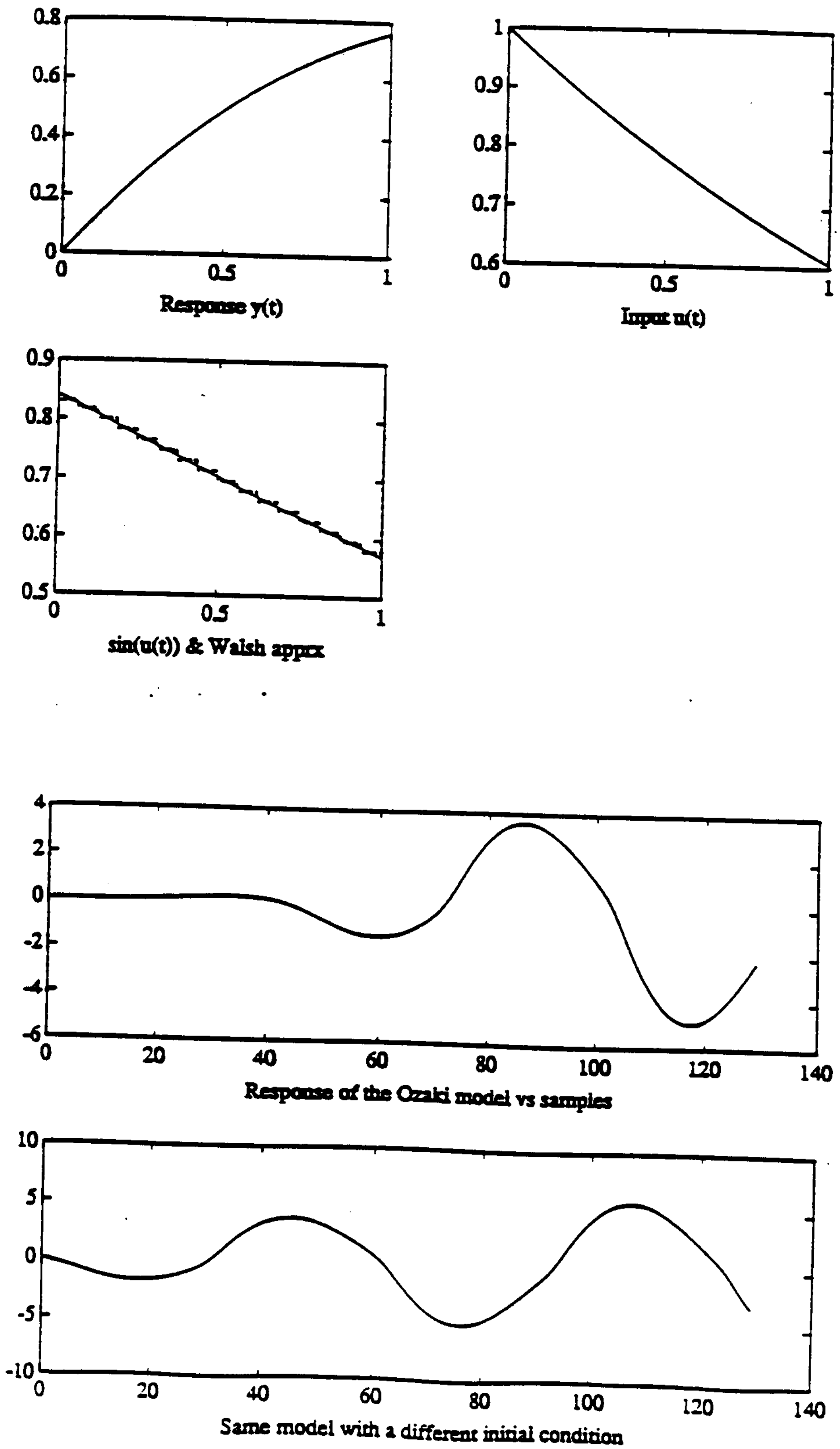


Fig 3.2 Responses for Examples 1 & 2 as Outlined in Chapter 3.

$$x(t) = \sum_{i=1}^{n_y} \{ a_i + b_i . \exp [-x(t-1)^2]\} x(t-i)$$

Let ny =2

and the parameters chosen in accordance with Ozaki's model

$a1 = 1.95 \qquad b1 = -0.96$

$a2 = 0.23 \qquad b2 = -0.24$

$$x(0) = x(-1) = 0$$

$$x(t) = E^T \Phi(t)$$

Using the above outlined delineation method we have

$$E^T \Phi(t) = a1 . E^T \Phi(t-1) + b1 . (k^T .\exp (\lambda_C)) \lambda_E \Phi(t-1) + a2\ E^T \Phi(t-2) +$$

$$+ b2 ([k^T . \exp (\lambda_C)) \lambda_E \Phi(t-2)$$

where $k^T = [1\ 0...0]$, $C^T = - E^T$

or $C^T T^2 \Phi(t) = a1\ E^T T + b1. [k^T . \exp(\lambda_C)] \lambda_E T \Phi(t)$

$$+ a2. E^T \Phi(t) + b2 . [k^T . \exp(\lambda_C)] T \lambda_E \Phi(t)$$

where $\Phi(t+1) = T \Phi(t)$. T is the forward shift transformation matrix.

$$C^T \; T^2 = [\,a1\; b1\; a2\; b2\,] \qquad C^T \; T$$

$$[\,k^T . \exp(\; \lambda_C\;)]\;\; \lambda_E \; T$$

$$E'$$

$$[\,k^T \; . \; \exp(\; \lambda_C\;)\, \lambda_E\,]$$

The Least Square Estimate for the parameter vector is used for identification. For m=32 . The estimated parameters are as shown in the adjoining table..

Example 3

Consider the system below with a fractional nonlinearity

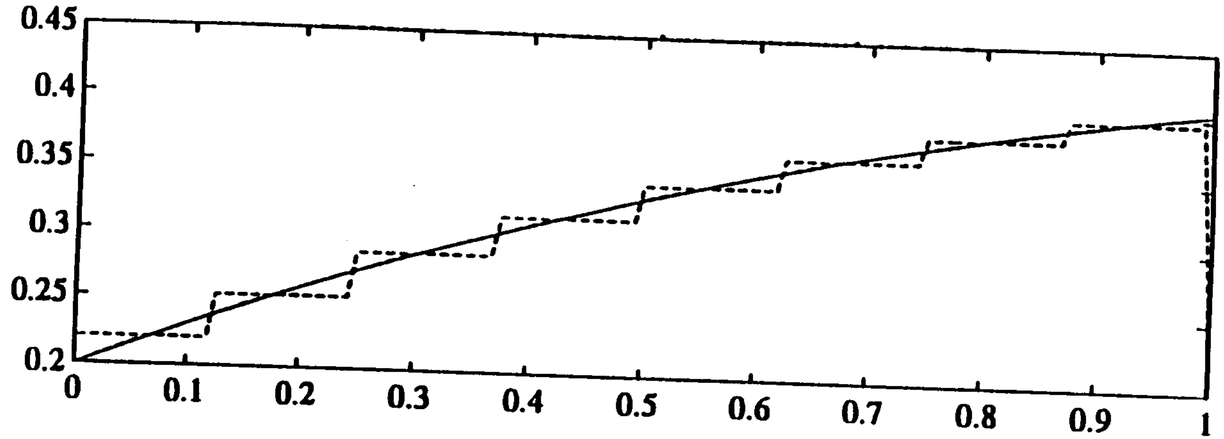
$$x(t) = a\;\; x(t)^{\frac{3}{2}} + n\, x(t)\, u(t) \;\; + b\, u(t)^{\frac{1}{2}}$$

we have for

$a = -1$	$x(0) = 0.2$
$b = 0.3$	$u(0) = 1.0$
$n = 0.5$	$u(t) = \exp(-0.5t)$

here $\quad | x(t) - 1| < 1$ for $\quad t \in [0,1)$

Let $\quad x(t) = E^T \; \Phi(t)$



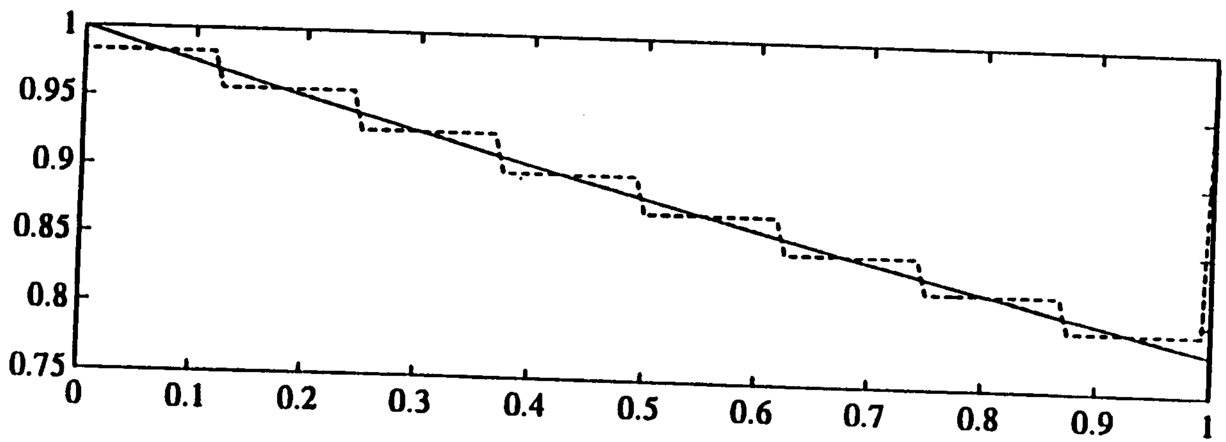


Fig 3.3 Responses for Example 3 as Outlined in Chapter 3

and $[\, k^T . (I + \lambda_E)^{\frac{3}{2}}]^T \; = r$

and $[k^T \,.\, (I+ \lambda_E)^{\frac{1}{2}}]^T \; = s$

where $k^T =[1\ 0...0]$, $C^T = E^T - k^T$

$$u(t) = Z^T \Phi(t)$$

then we have

$$[E^T - x(0)\, k^T \,]\, \Phi(t) = a\ r^T P\, \Phi(t) + n\ E^T \lambda_Z P\, \Phi(t) + b\ s^T P\, \Phi(t)$$

$$E^T\ \Phi(t) = [a \quad n \quad b] \begin{bmatrix} r^T\ P\,\Phi(t) \\ E^T\ \lambda_Z P\, \Phi(t) \\ s^T\ P\, \Phi(t) \end{bmatrix}$$

Sampling at 4 different time instants and using different number of Walsh functions the parameters are uniquely determined and are as shown for different number of functions used.

Conclusion:

The useful properties of the Walsh product matrices were used for finding the Walsh coefficents of nonlinear functions of the parent signal in terms of the Walsh expansion coefficents of the parent signal. Based on the findings parameter identification of systems with severe nonlinearities were performed with just the input and output Walsh function representations being used. The problem of fractional nonlinearities posed in Karanam et al[1978] was circumvented too.

Table 3.2 Parameter Estimates

For example 1.

m	a	b	c	d
4	-0.9543	0.4798	0.2896	1.4923
8	-0.9782	0.4912	0.2945	1.4977
16	-0.9911	0.4909	0.2956	1.4979
true values	-1.0000	0.5000	0.3000	1.5000

For Example 2 $m = 32$

	a1	a2	b1	b2
estimated	1.9483	0.2265	-0.9498	-0.2367
true values	1.9500	0.2300	-0.9600	-0.2400

For example 3:

m	a	n	b
4	-0.9232	0.4777	0.2879
8	-0.9456	0.4856	0.2954
16	-0.9854	0.4967	0.2971
true values	-0.1000	0.5000	0.3000

Chapter 4 Analysis and Optimal Control of a Single Machine Infinite Bus Power System Using Orthogonal Expansions

Linearized models have long been used for analysis and optimal control of power systems. Yu, Vongsurya & Wedman [1970] used a 8th order linearized model of a synchronous machine infinite bus system to derive an optimal control to improve the dynamic response of a power system. Davison E.J & Rau N.S [1971] developed a computational method for determining the optimal constant feedback gains between the manipulated inputs and the measurable outputs and applied it to a synchronous machine connected to an infinite bus. Iyer S.N & Cory B.J [1971] applied a second order optimising algorithm to a simple model of a turbo-alternator to determine the optimum control inputs under transient conditions. Moussa H.A.M & Yu Yao-nan[1972] developed a techinique for determining the weighting matrix Q of an optimal linear regulator in conjunction with the dominant eigenvalue shift of the closed loop system and applied it to a typical power sytem. Yu Yao-Nan & Moussa H.A.M [1972] applied the above design techinique for the optimal stabilization of a multimachine system. Elangovan S & Kuppurajulu A [1972] proposed a suboptimal control policy using simplified models which reduced the number of states to be measured thus simplifying the control structure. Anderson et al [1977] presented a general method for the synthesis of linear, optimal, dynamic control system compensating networks (for excitation system designs) by parameter optimisation.

Analysis using Walsh Functions

A 4th order nonlinear model of a SMIB system with series capacitor compensation (Appendix B) is used to obtain a linearized model.

$$X(t) = A.X(t) + B.U(t) \qquad ....(4.1)$$

$$Y(t) = C.X(t)$$

where $X(t) = [\ \Delta\delta(t)\ \ \Delta w(t)\ \ \Delta Eq(t)\ \ \Delta Ef(t)]^T$ is the state vector
and $U(t) = \Delta Xc(t)$ is the series capacitor input perturbation.

For the initial conditions in Appendix B, A , B and C are given as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -26.9746 & -0.2418 & -33.1372 & 0 \\ -0.0469 & 0 & -0.2531 & 0.2 \\ 0 & 0 & 0 & -4 \end{bmatrix} \quad ......(4.2)$$

$$B = [\,0\ \ -177.249\ \ -0.02671\ \ \ 0\,]^T$$

$$C = [\,0\ \ 1\ \ 0\ \ 0\,]$$

$$Xo = [1.08313\ \ 0.0003\ \ 1.28367\ \ \ 1.5]^T$$

The rotor speed is chosen as the output. The linearized model is then analysed using Walsh functions (recursive analysis also included) and an optimal control generated based on the analysis by Chen & Hsiao [1975] for linear time invariant systems with quadratic performance criteria. The optimal control leads to piecewise constant gains.

Let

$$X(t) = C_1 \,.\, \Phi(t) \quad ....(4.3)$$

$$U(t) = Z.\ \ \Phi(t) \quad ....(4.4)$$

where C1 is a n x m matrix & Z, a 1 x m vector of Walsh coefficents

Since $X(t) = \int_0^t X(t)\,dt + Xo$ where $X(0) = Xo$ we have

$$X(t) = C_1.P.\Phi(t) + [\,Xo\ 0\ 0\ ....\ 0].\Phi(t) \quad ......\ (4.5)$$

where P is the Walsh operational matrix of integration.

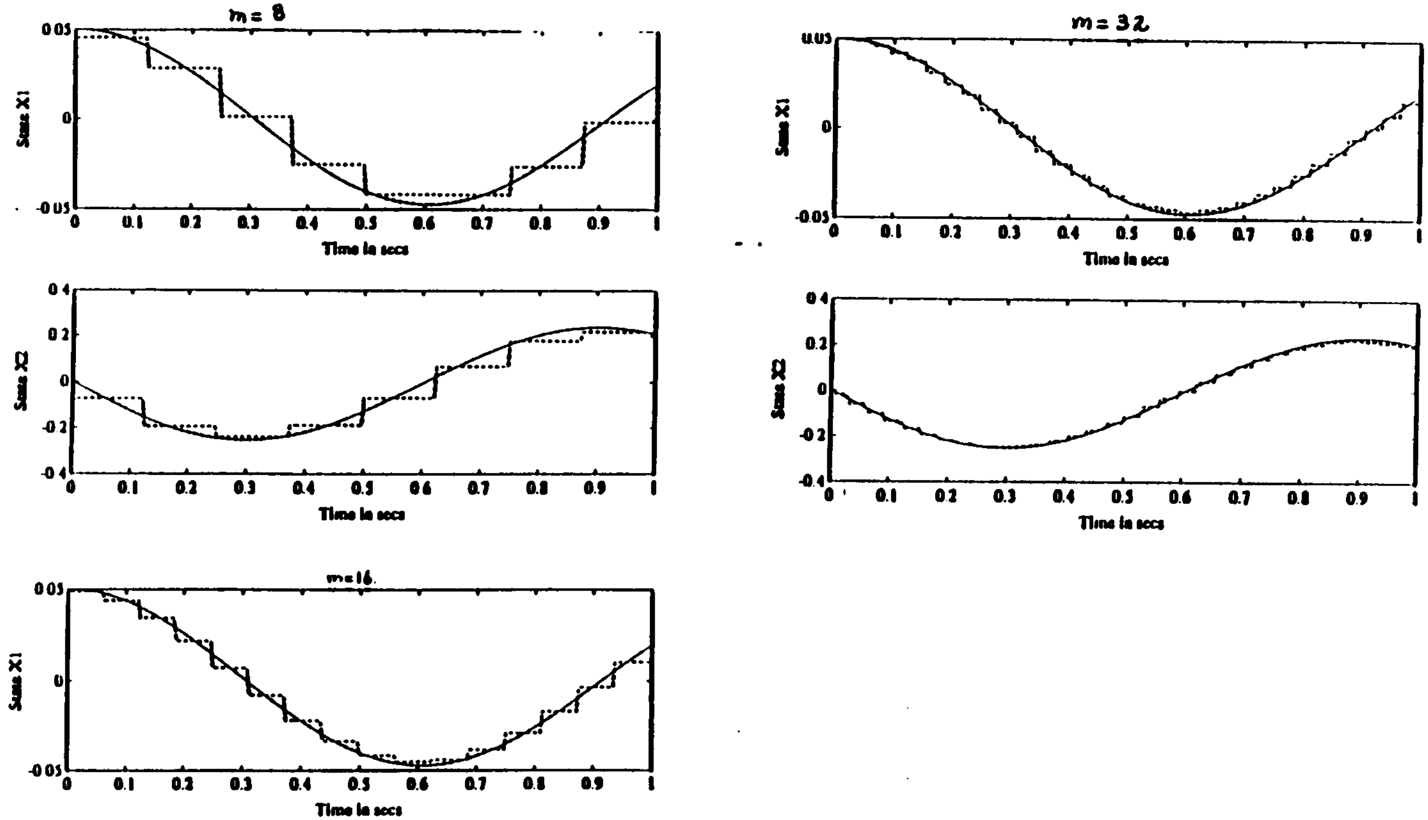


Fig 4.1 Perturbation Analysis Using Walsh Functions (Kronecker Product Solution for Rate Vector).

Substituting (4.3) & (4.2) in (4.1) we have

$$C_1.\,\Phi(t) = A.[\,C_1.P + [Xo\ 0\ 0...0]].\Phi(t) + B.Z.\Phi(t)$$

or $$C_1 = A.C_1.P + [A.Xo\ 0\ 0\ ..\ 0] + BZ \qquad .....\ (4.6)$$

Let $$[\,AXo\ 0\ 0...0] + BZ = M \qquad .....\ (4.7)$$

then $$C_1 = A.C_1.P + M \qquad ......(4.8)$$

Using Kronecker products the solution of the above can be given as

$$\mathbf{c1} = [\,I\ -\ A\,f\,P']^{-1}\,.\,\mathbf{m} \qquad ....\ (4.9)$$

where **c1** is an nm vector obtained by rearranging the columns of the n x m matrix C_1 one next to the other & **m** is found in a similar manner from M. Chen & Hsiao[1975].

The rate variable is thus determined. The state variable vector is then found by substitution.

$$X(t) = D.\,\Phi(t) = [\,C_1.P + [Xo\ 0\ 0..0]].\Phi(t) \qquad .....(4.10)$$

Recursive analysis of the linearized system using Walsh functions

Rao et al [1980] have presented a recursive method for extending computation beyond the limit of the initial normal interval in Walsh series analysis to any limit. The main advantage of this method is the avoidance of using operational matrices of prohibitively large size & the reduction of the computational efforts to a minimum while retaining accuracy.

The method outlined in Chen & Hsiao[1975] , of reducing the computational effort (Recursive procedure (Appendix C)) while constructing the Kronecker product and avoiding direct inversion of (mn x mn) matrices, is quite complex. In a

2^k term approximation the Kronecker product reduction still involves at least k inversions of n x n matrices, 3k multiplications of n x n matrix with an n-vector.

To increase the accuracy of the solution in the initial normal interval [0,1) itself & to compute the solution for large t beyond the normal interval while maintaining the same accuracy would require increasing m, the number of Walsh functions used & corresponding renormalization of the interval of interest. In this case the computational requirement becomes manifold.

The method outlined by Rao et al [1980] leads to a qualitative solution. Let Y represent the n-vector of average values of the rate variable X in the interval ((i-1)/m, i/m), then we have

$$Y = [Y1\ Y2 \ldots\ldots Ym] = W.C_1 \qquad \ldots..(4.11)$$

where W is the m X m Walsh matrix, see phase I report

At the end of the interval [0,1) i.e at t=1 the state is given by

$$X(1) = \frac{1}{m} \sum_{i=1}^{m} Yi + X(0) \qquad \ldots..(4.12)$$

Rao et al[1980]

Let R = BZ &

$$M = [A\,Xo\ 0\ 0 \ldots 0] + R = [\,M_0\ M_1\ M_2\ M_3 \ldots M_{m-1}\,] \qquad \ldots\ldots(4.13)$$

The recursive formula connecting solutions in two contiguous unit intervals [i-1, i) & [i , i+1) yields

$$M^i = [\,A\,X(i-1)\ 0\ 0 \ldots..0] + R^{(i)} \qquad \ldots\ldots(4.14)$$

where $R^{(i)} = B.Z^{(i)}$

Let $$Q = [\,I - P' \otimes A]^{-1} \qquad \ldots\ldots(4.15)$$

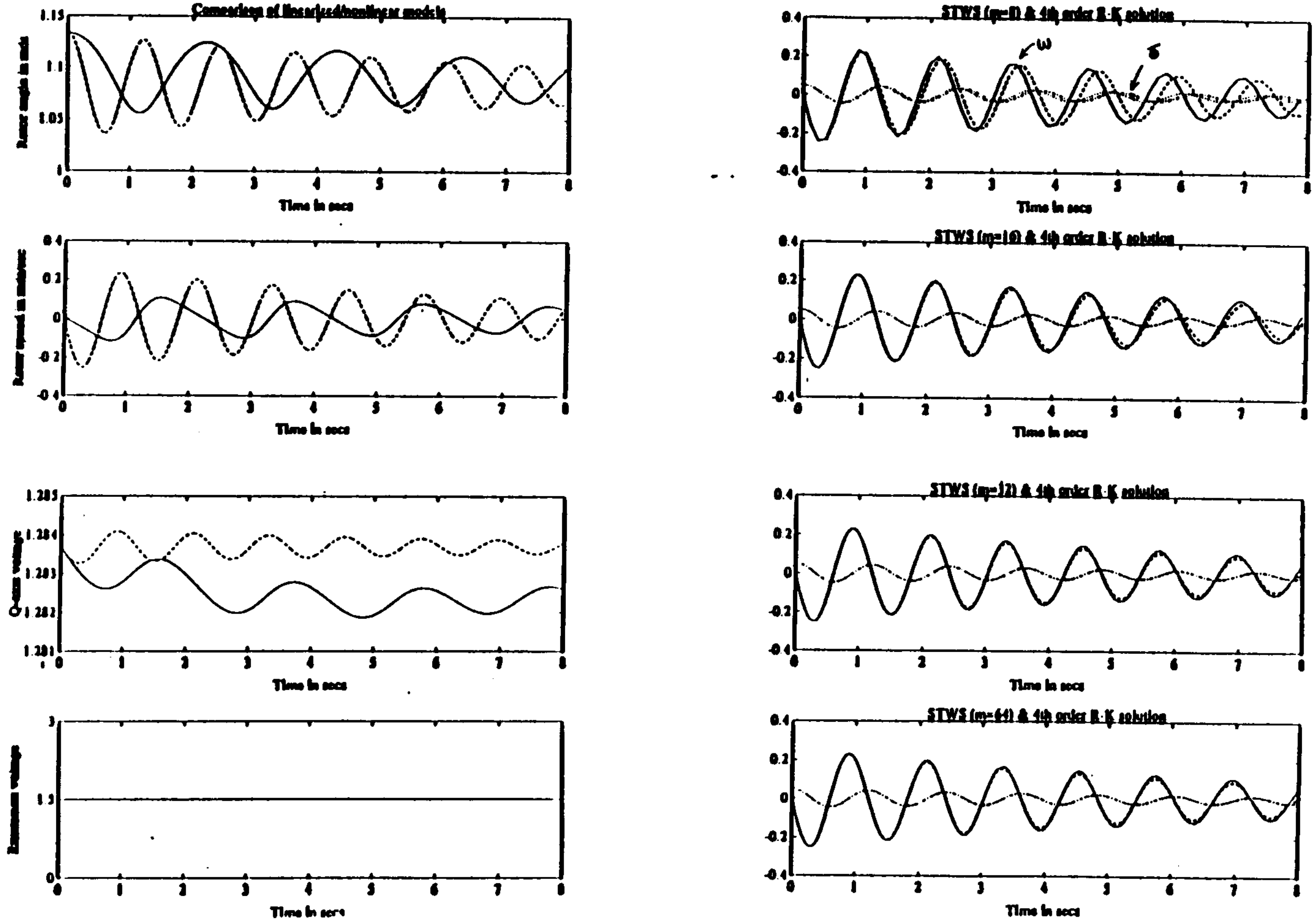


Fig 4.2 Perturbation Analysis Results

then $c^{(i)} = Q\,m^{(i)}$

$$X(i) = \frac{1}{m} \sum_{i=1}^{m} Mj^{(i)} + X(i-1) \qquad i=1, 2...$$

$$X(i) = X(t) \,|_{t=i}$$

The superscript i indicates that the quantities belong to the ith unit interval.In the above method the term containing the Kronecker product Q is computed once only. In the case when the interval $[0\frac{1}{m})$ is stretched to unit length the above reduces to

$$M^{(i)} = (I - A)^{(i)} \; [R^{(i)} + A\,X(j-1)]\,. \qquad .....(4.16)$$

$$D^{(i)} = V^{(i)}\,P + X(j-1)$$

$$X(i) = X(j-1) + M^{(i)} \qquad i=1,2 ....$$

where $R^{(i)} = B\,U(i)$

A single term approximation continued over 2^k suitably normalized contiguous intervals give the same results as an approximation that handles 2^k Walsh components en-bloc over the whole interval at a time.

Optimal control of the linearized model :

For the system described by (3.1) let the quadratic performance index to be minimized be represented as

$$J = \frac{1}{2} \int_{0}^{tf} (X'.Q.X + u'R\,u)\,dt \qquad ......(4.17)$$

The optimal solution for the control is well known as

$$U^* = R^{-1}B^T P(t) \qquad \text{......(4.18)}$$

where P(t) satisfies the following equation

$$\begin{bmatrix} X \\ P \end{bmatrix} = \begin{bmatrix} A & BR^{-1}B^T \\ Q & -A \end{bmatrix} \begin{bmatrix} X \\ P \end{bmatrix} \qquad \text{......(4.19)}$$

& the boundary conditions are specified as

$$X(0) = Xo \qquad \text{......(4.20)}$$
$$P(tf) = 0$$

changing the independent variable by defining

$$\tau = tf - t$$

then (4.19) becomes

$$\begin{bmatrix} X(\tau) \\ P(\tau) \end{bmatrix} = -M \begin{bmatrix} X(\tau) \\ P(\tau) \end{bmatrix} \qquad \text{......(4.21)}$$

where $$M = \begin{bmatrix} A & BR^{-1}B^T \\ Q & -A \end{bmatrix} \qquad \text{......(4.22)}$$

The transition matrix of the above is given as

$$\exp(-st) = \begin{bmatrix} V_{11}(\tau)\, V_{12}(\tau) \\ V_{21}(\tau)\, V_{22}(\tau) \end{bmatrix} \qquad \text{......(4.23)}$$

since $P(\tau = 0) =)$ the solution of (3.19) can be written as

$$X(\tau) = V_{11}(\tau)\, X\,(\tau{=}0) \qquad \text{..... (4.24)}$$
$$P(\tau) = V_{12}(\tau)\,.\, X(\tau{=}0) \qquad \text{......(4.25)}$$

From (3.24) we have

$$X(\tau{=}0) = V_{11}{}^{-1}(\tau).\, X(\tau) \qquad \text{......(4.26)}$$

Substituting (3.26) into (3.25) yields

$$P(\tau) = V21(\tau)\; V11^{-1}(\tau).X(\tau) \qquad \text{......(4.27)}$$

Then the optimal control is reduced to

$$U^{*}(t) = R^{-1}B'\, V_{12}(tf - t)\, V_{12}{}^{-1}(tf - t)\, X(tf - t)$$

$$= -L\,(tf - t)\, X(tf - t) \qquad \text{......(4.28)}$$

where L (tf -t) is the optimal feedback gain matrix.

Walsh series solution :

Normalizing $\tau' = \tau\, /\, tf$ yields from (3.23)

$$\begin{bmatrix} X(\tau') \\ P(\tau') \end{bmatrix} = -\, tf.\, M. \begin{bmatrix} X(\tau') \\ P(\tau') \end{bmatrix} \quad 0 <= \tau' < 1 \qquad \text{....(4.29)}$$

Let

$$\begin{bmatrix} X(\tau') \\ P(\tau') \end{bmatrix} = C\,.\,\Phi(\tau') \qquad \text{....(4.30)}$$

where C is an 2n X m matrix & $\Phi(\tau')$ an m-vector then we have

$$\begin{bmatrix} X(\tau') \\ P(\tau') \end{bmatrix} = C\,.\,P\,.\,\Phi(\tau') + \begin{bmatrix} X(\tau=0) \\ On \end{bmatrix} \qquad ....(4.31)$$

Substituting (3.31) and (3.30) in (3.29) yields

$$C = -tf.\,M.\,\{\,C.P + \begin{bmatrix} X(\tau'=0) & & \\ & O2n \ldots\ldots O2n \\ On & & \end{bmatrix} \qquad ......(4.32)$$

Defining k as

$$k = \begin{bmatrix} -tf\,A\,X(\tau'=0) \\ -tfQX(\tau'=0) \\ O2n \end{bmatrix} \qquad ....(4.33)$$

then (3.33) is simplified into

$$C = [\,I + tf.M \otimes P'\,]^{-1}.k \qquad .....(4.34)$$

Solving for C, the Walsh coefficent for $X(\tau')$ & $P(\tau')$ are determined. Substituting in (3.31) yields the Walsh coefficents of $X(\tau')$ & $P(\tau')$. The optimal control is generated as

$$U^*(\tau') = R^{-1}.B'T\,\Phi(\tau')$$

where $\qquad P(\tau') = T\,\Phi(\tau')$

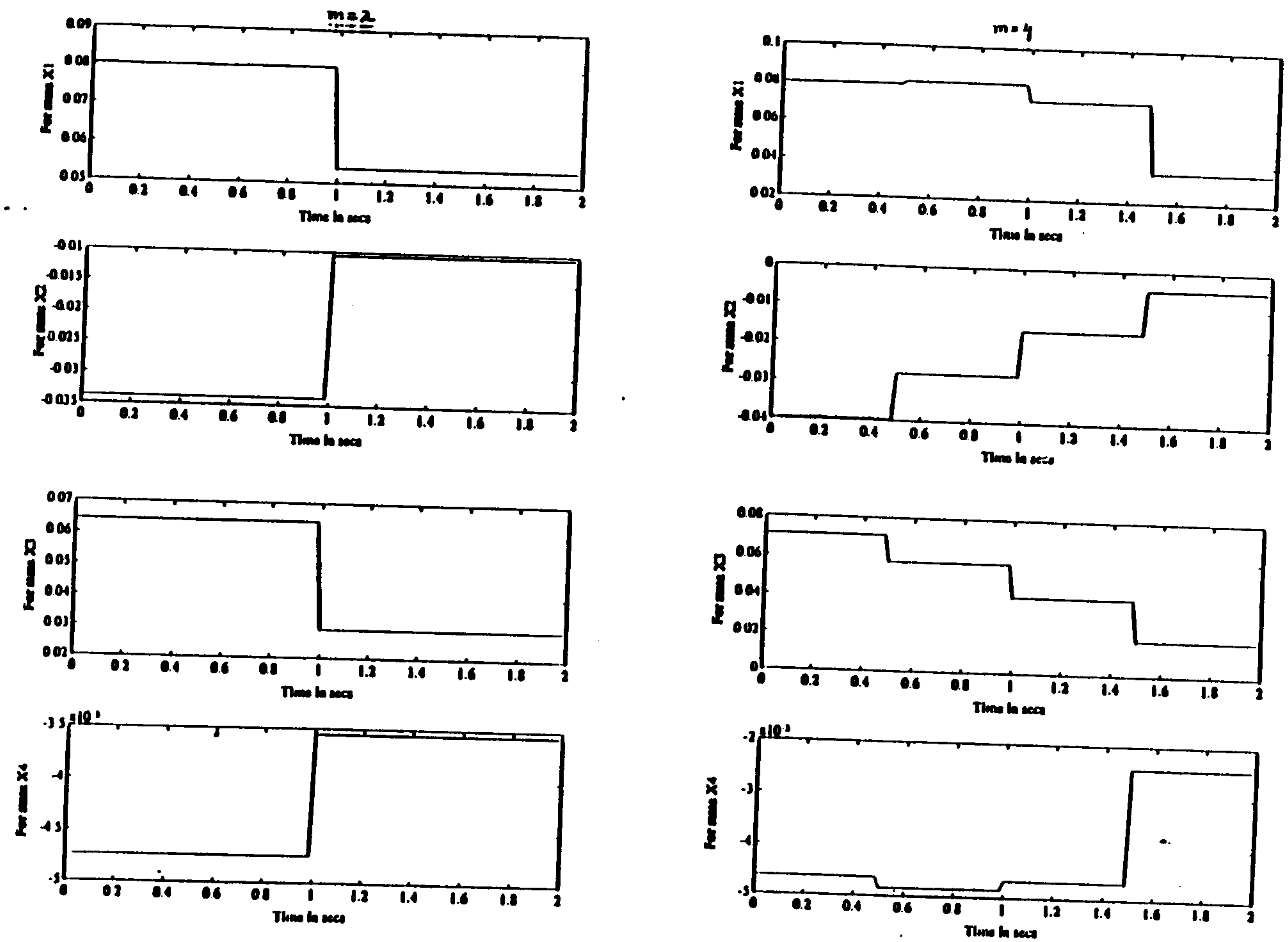


Fig 4.3 a Optimal Feedback Gain Plots for Different Number of Subintervals (m=2 and m=4)

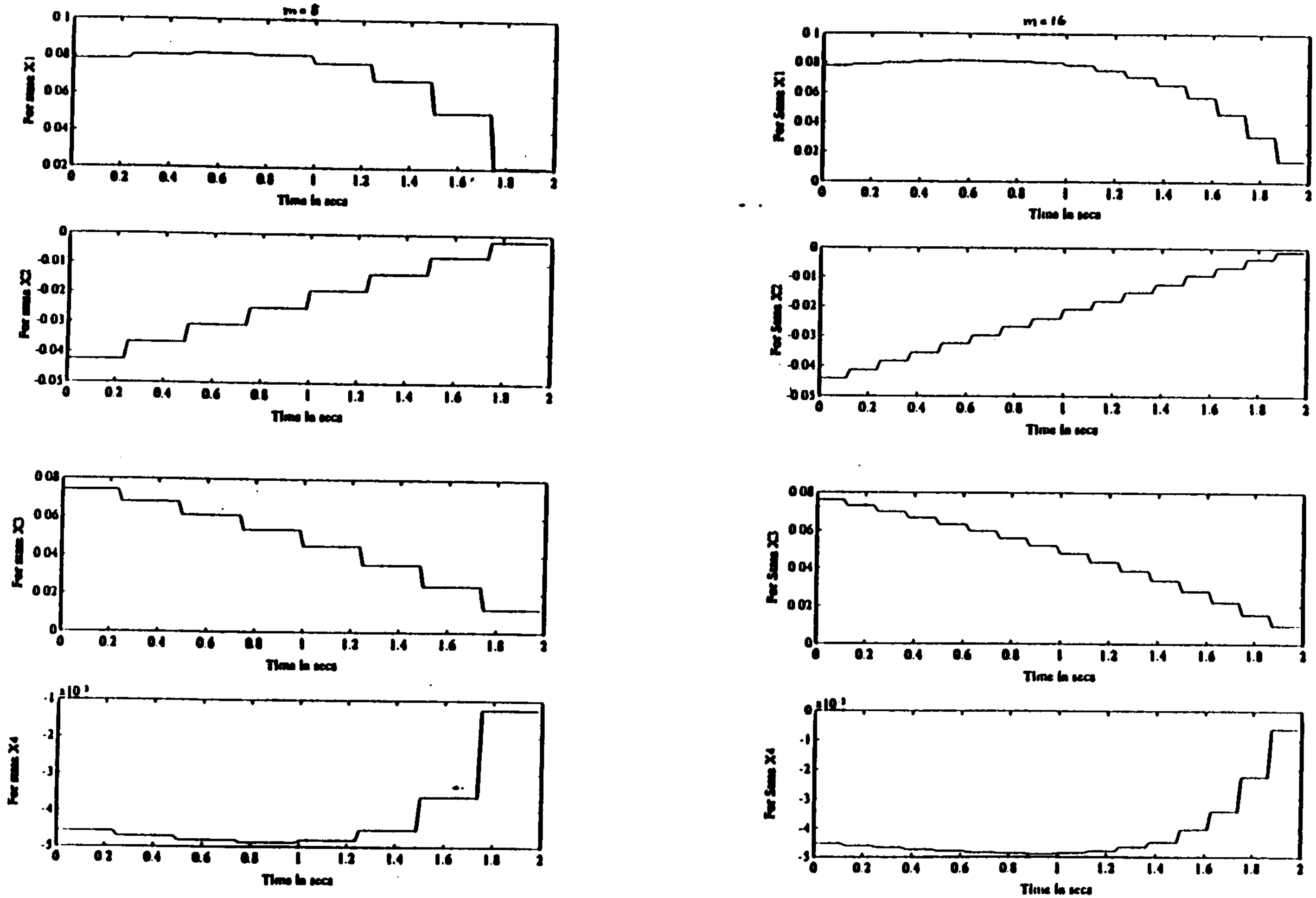


Fig 4.3 b Optimal Feedback Gain Plots for Different Number of Subintervals (m=8 and m=16)

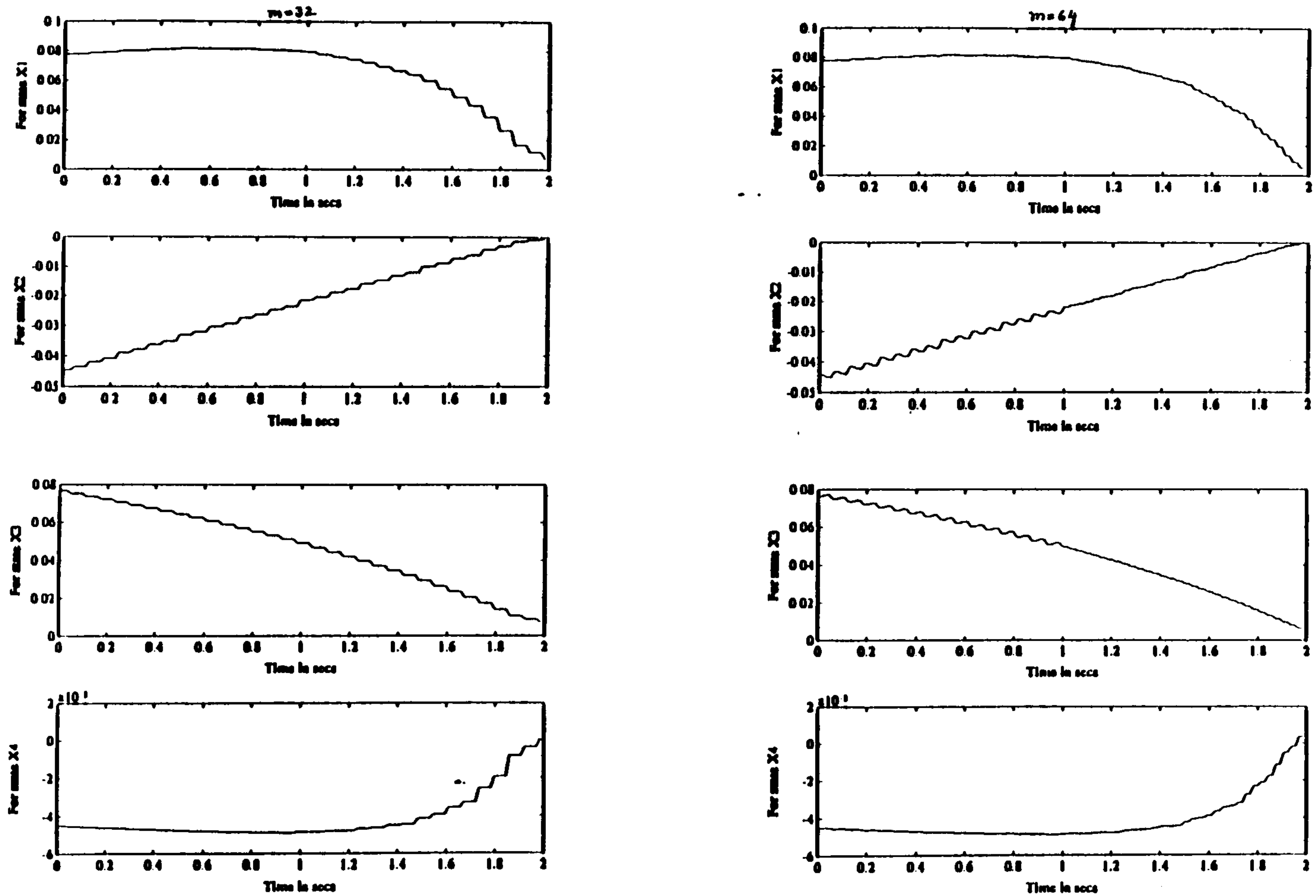


Fig 4.4 c Optimal Feedback Gain Plots for Different Number of Subintervals (m=32 and m=64)

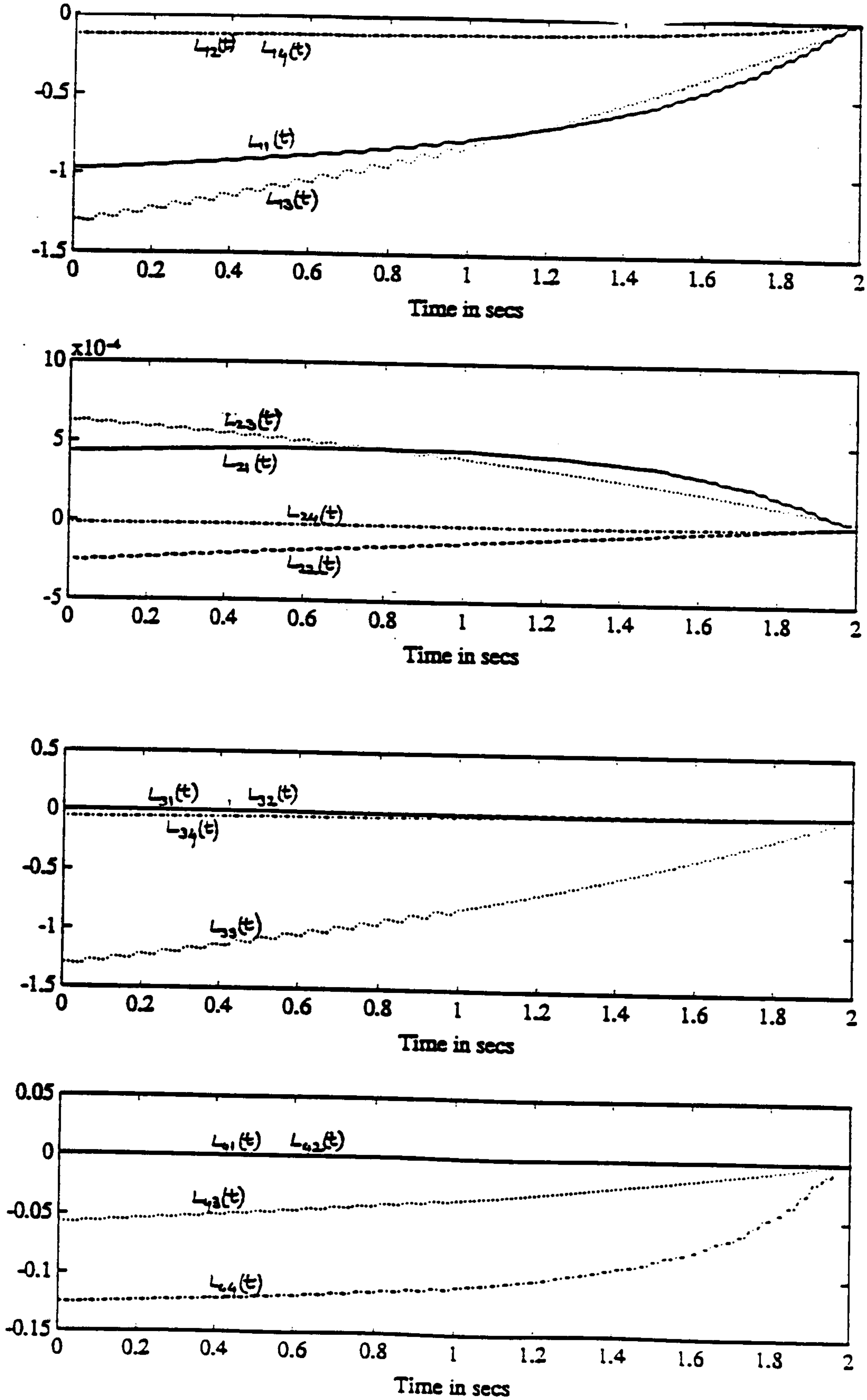


Fig 4.4 Time Varying Elements of L(t)

Simulation Results :

The results are as shown in the subsequent figures. The linearized system model is compared with the actual nonlinear model for small perturbations. The linearized system is then analysed using different number of Walsh functions using the Kronecker product based Walsh analysis outlined in Chen & Hsiao[1975]. The linearized system is analysed using the single term Walsh series approach outlined in Rao et al [1980] and compared with the 4 th order Runge Kutta method. Less number of integration steps are required using the Walsh function approach to yield the same accuracy as the R.K method.

The optimal piece wise feedback gain for each state is then determined using the Walsh series approach outlined in Chen et al [1975]. The feedback gains for different number of subintervals are as shown. The weighting matrices for the cost functional are taken as $Q = I$ & $R=1$.

Chapter 5. PID Controller Design for Series Capacitor Control Using Orthogonal Expansions

PID controllers have been used extensively for power system control. Dhaliwal N.S & Wichert H.E [1978] investigated the effect of derivative gain and other governor parameters on the stability of single & multimachines. Hagihara et al [1979] investigated the effect of derivative gains and other governor parameters on the stability boundaries of a hydraulic turbine generating unit supplying an isolated load and provided some general guidelines for optimum adjustment of derivative gains. Malik O.P et al [1983] studied the influence of digital PID voltage regulators on the transient performance of a synchronous machine. Wang & Lee[1991] used a linearized model for performing an eigenvalue analysis for different loading conditions & sensitivity analysis of controller parameters of a power system containing series-capacitor compensation & proposed a shunt reactor PID control scheme for effectively suppressing unstable torsional modes.

Based on a particular controller / compensator configuration and a set of prespecified required dynamic responses for the controlled system a simple scheme is derived to identify the controller / compensator parameters based on orthogonal expansions of the input and output signals.

For the initial conditions and parameters shown in Appendix A, the A & B matrices are shown again as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -26.9746 & -0.2418 & -33.1372 & 0 \\ -0.0469 & 0 & -0.2531 & 0.2 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

$$B = [\,0 \quad -177.249 \quad -0.02671 \quad 0\,]^T$$

$$Xo = [1.08313 \quad 0.0003 \quad 1.28367 \quad 1.5]^T$$

The eigenvalues of A are given as

-0.1497 + j 5.1918	mode 1	
-0.1497 - j 5.1918	mode 2	.....(5.1)
-0.1955	mode 3	
-4.0000	mode 4	

Modes 1 & 2 are the mechanical modes of the system while modes 3 & 4 are the electrical modes.The damping provided by each mode is indicated by the damping coefficent for each case defined as

$$\xi = \frac{s}{\sqrt{s^2 + w^2}} \qquad \text{for a typical mode} \quad s + jw$$

we then have

$$\xi_{1,2} = -0.029$$

....(5.2)

$$\xi_{3,4} = -1.000$$

Orthogonal\ expansion analysis of the transfer function :

The corresponding transfer function relating the perturbed rotor speed output to the perturbed series capacitor input is given as

$$T(s) = \frac{\Delta w(s)}{\Delta Xc(s)} = \frac{-177.2490\, s.\ (s + 0.2481)}{(s^2 + 0.2994\ s + 26.9772)(s + 0.1955)}$$

............(5.3)

The above can also be represented as

$(s^2 + 0.2499s + 26.9772).(s + 0.1955).\Delta w(s) = -177.2490\, s\,(s + 0.2481).\Delta Xc(s)$

Transforming to the time domain & noting that the initial conditions for the linearized system are zero we have

$$\Delta w'''(t) + 0.4949.\Delta w''(t) + 27.0357.\Delta w'(t) + 5.2740.\Delta w(t) = -177.2490.\Delta Xc''(t) - 5.2740.\Delta Xc'(t) \qquad ......(5.4)$$

Integrating the above equation successively (initial conditions zero) yields

$$\Delta w(t) + 0.4949.\int_0^t \Delta w'(t)\,dt + 27.0357.\int_0^t\int_0^t \Delta w(t)\,dt + 5.2740.\int_0^t\int_0^t\int_0^t \Delta w(t)\,dt = -177.2490\int_0^t \Delta Xc(t)\,dt - 5.2740\int_0^t\int_0^t \Delta Xc(t)\,dt \qquad ......(5.5)$$

Let the orthogonal expansion approximation of $\Delta w(t)$ & $\Delta Xc(t)$ in the initial interval be given as

$$\Delta w(t) = C^T \Phi(t)$$
$$\Delta Xc(t) = R^T \Phi(t) \qquad ......(5.6)$$

where C and R are the orthogonal coefficent vectors whose elements are determined according to Fourier 's manner in the form

$$C_i = \frac{1}{\int_0^t \Phi_i^2(t)\,p(t)\,dt} . \int_0^t p(t).\Delta w(t).\Phi(t).dt$$

$$R_i = \frac{1}{\int_0^t \Phi_i^2(t)\,p(t)\,dt} . \int_0^t p(t).\Delta Xc(t).\Phi(t).dt \qquad .....(5.7)$$

where p(t) is an appropriate weighting function & $\Delta w(t)$ and $\Delta Xc(t)$ are assumed L_2 integrable functions approximated in the time interval $t \in [0, tf)$

Then, using the following operational property of some common orthogonal expansions

$$\underbrace{\int_0^t \int_0^t \int_0^t \cdots \int_0^t}_{n \text{ times}} \Phi(\tau)\, d\tau.d\tau...d\tau \approx P^n . \Phi(t)$$

where P is the corresponding operational matrix of integration. Eqn 5.5 can be expressed as

$$C^T.[I+0.4949.P+27.0347.P^2+5.274.P^3].\Phi(t) = [-177.2490.R^T P-5.274.R^T.P^2]\,\Phi(t)$$

or $C^T . K = M$ so that $C^T = M . K^{-1}$ .....(5.8)

where
$$K = [\, I + 0.4949.\, P + 27.0347.\, P^2 + 5.2740\, P^3 \,]$$
$$M = [\, -177.2490.\, R^T P - 5.2740.\, R^T .\, P^2 \,]$$

Given the input signal $\Delta Xc(t)$ the coefficent vector R is found & substituted in the above eqn to get the output coefficent vector C. The output is then generated as
$C^T \cdot \Phi(t)$ for $t \in [0, tf)$. The step response for the above is then compared with the exact solution as shown in fig 5.1 for different orthogonal expansions. The exact solution for the step input of amplitude A is found by inverse Laplace tranforming $\frac{T(s)}{s}.A$

The exact solution is found to be

$$Y(t) = A\,[\, 0.3458595.\,(\,1- \exp(-0.1955.t\,) - 0.342639622 + 0.342639622\,.\exp(-0.1497.\,t\,).\cos(\,5.1918.t\,) - 33.98218084.\exp(-0.1497.t\,).\sin(\,5.1918.t\,)\,]$$
.......(5.9)

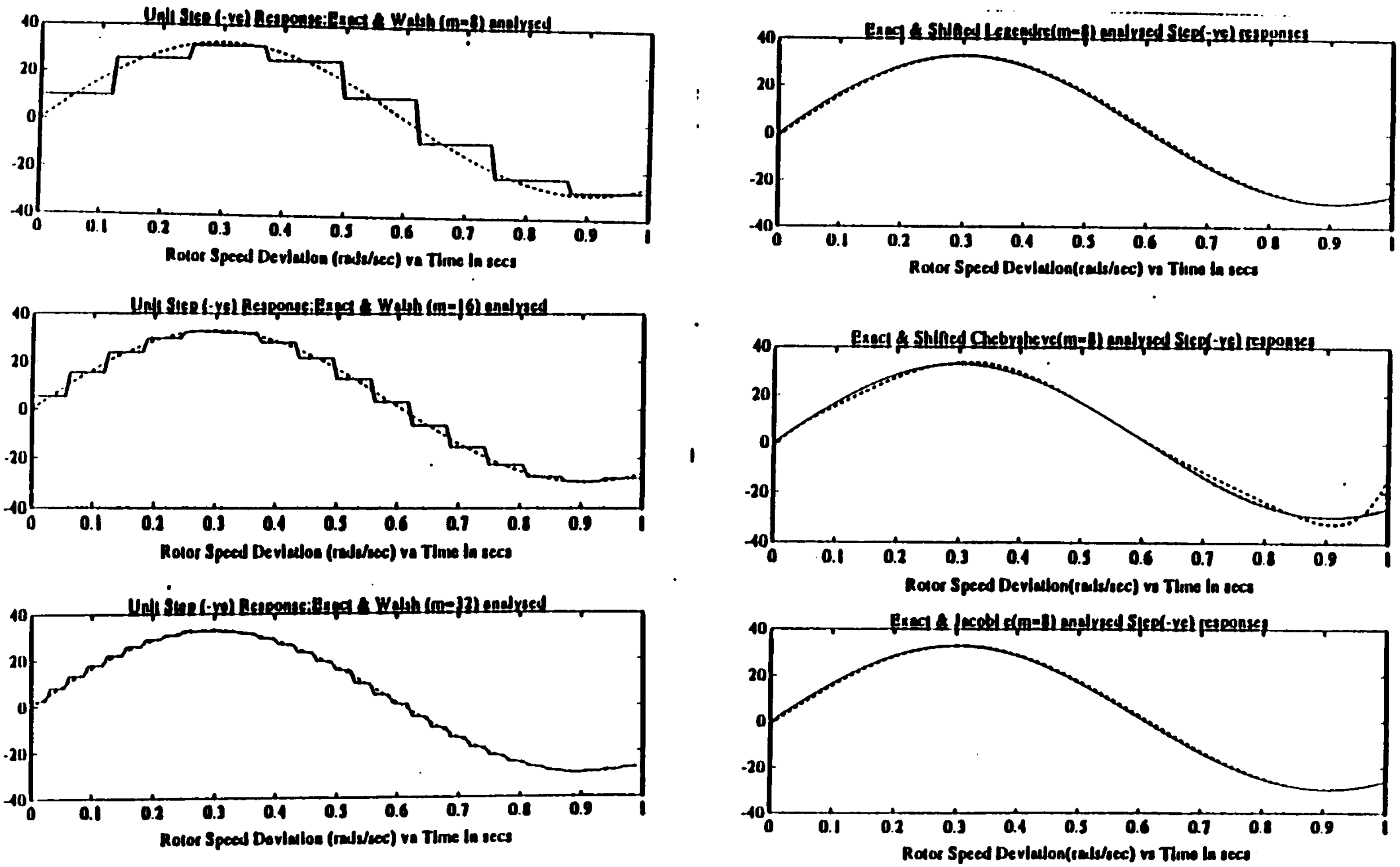


Fig 5.1 Step Responses: Exact & Walsh Analysed

PID Controller Design :

Electromechanical damping can be provided by changing the parameters of the power system like series capacitor control, braking resistor control , shunt reactor control.

A simple method is developed for designing a PID controller for improving the mechanical damping (dynamic compensation) of a SMIB system using orthogonal expansions.

The common controller configuration is as shown in fig 5.2 . $G_R(s)$ in the feedback loop is the reset block (wash out term) which removes the steady state offset error.

The PID controller transfer function is given as

$$Gc(s) = KP + \frac{KI}{s} + s\,KD \qquad \text{......(5.10)}$$

where the parameters KP , KI, KD are to be determined so that the resulting closed loop response of the complete system meets prespecified dynamic responses.

Controller design:

The dynamic characteristic of the above and most other compensators / controllers are fixed invariably on a trial and error basis using conventional control theory. The method outlined below removes this tedious yet important process of design. The compensator / controller parameters are chosen accurately to yield the required response characteristics. Compensator / controller parameters are identified by solving a set of linear equations resulting from the transformation of the transfer function model into the orthogonal domain.

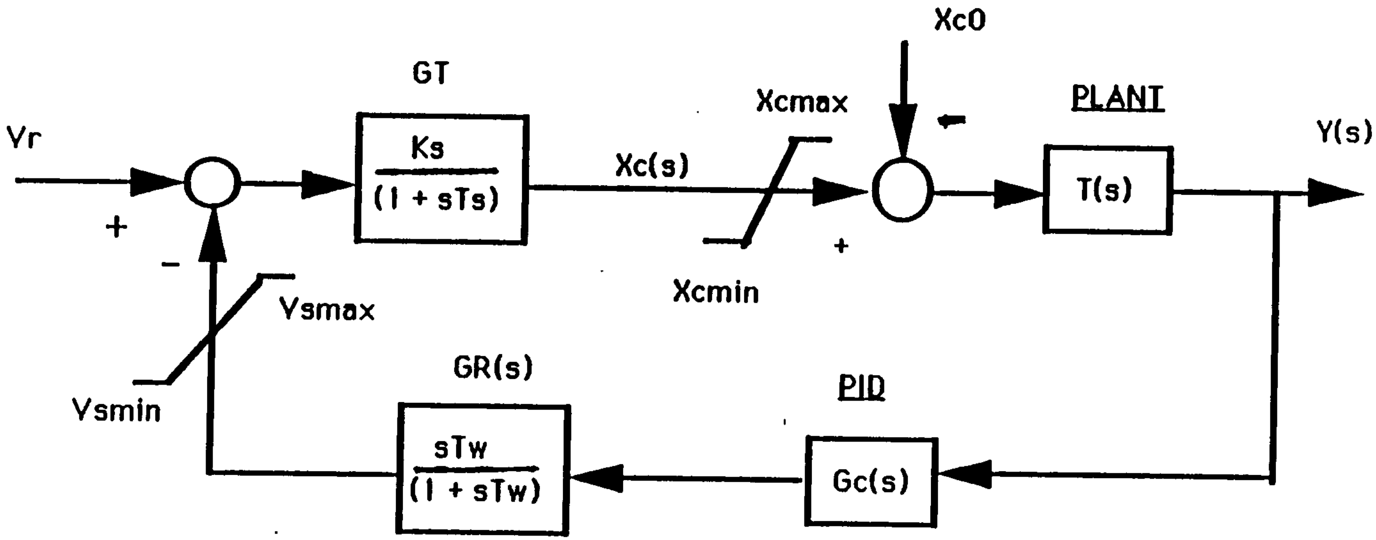


Fig 5.2 Closed Loop System with PID controller

The resulting closed loop system has to satisfy a set of given specifications such as bounds on the step responses, demands of a certain degree of non-interaction between different signals, operating point stability & for a given class of common inputs and external disturbances, asymptotic regulation (in the mean for stochastic external signals)

The process of design involves

a) Creating a known perturbation at a summing point (or a command input perturbation)

b) Defining a required response characteristic (based on the response of the uncompensated system)

c) Using the orthogonal expansion representation of the transfer function to find the unknown controller parameters which would give the desired response characteristic.

d) Repeating the above process for different types of perturbations as desired.

e) Given the possible range of controller parameters the choice of a suitable set depends on the specific task the controller has to achieve.

Let Vr(t) be the reference input voltage which is varied as a function of time assumed so solely for the purpose of designing the dynamic characteristic of the closed loop system. The closed loop transfer function relating the input Vr(t) to the output, perturbed rotor speed, is given by

$$\frac{\Delta w(s)}{Vr(s)} = \frac{GT(s) \, . \, T(s)}{(1 + GT(s). \, GR(s) \, . \, GC(s) \, . \, T(s))} = \frac{NUM(s)}{DEN(s)} \quad .....(5.11)$$

where $NUM(s) = -177.249\, Ks\, (Tw\, s^3 + (1 + 0.2481.Tw)s^2 + 0.2481 s)$

and $DEN(s) = d1 - KD.\, d2 - KP.\, d3 - KI.\, d4$

$$d1=(s^2+0.2994s+26.9772)(s+0.1955)(1+sTs)(1+sTw)$$
$$d2=(177.249\, s^3.(s+0.2481).Tw.Ks)$$
$$d3=(177.249\, s^2.(s+0.2481).Tw.Ks)$$
$$d4=(177.249\, s.(s+0.2481).Tw.Ks)$$

Transforming into the orthogonal domain we have

$$\mathbf{K}.\mathbf{D} = \mathbf{G} \qquad \text{....(5.12)}$$

where the parameter vector

$$\mathbf{K} = [\, KD \quad KP \quad KI \,]$$

and

$$\mathbf{D} = \begin{bmatrix} C^T.[177.249.\mathbf{P}^{-3}(\mathbf{P}^{-1}+0.2481.\mathbf{I}).Tw.Ks] \\ C^T.[177.249\ \mathbf{P}^{-2}(\mathbf{P}^{-1}+0.2481.\mathbf{I}).Tw.Ks] \\ C^T.[177.249\ \mathbf{P}^{-1}(\mathbf{P}^{-1}+0.2481.\mathbf{I}).Tw.Ks] \end{bmatrix}$$

$$\mathbf{G} = (\mathbf{P}^{-2}+0.2994.\mathbf{P}^{-1}+26.9772.\mathbf{I}).(\mathbf{P}^{-1}+0.1955.\mathbf{I}).(\mathbf{I} + \mathbf{P}^{-1}Ts)(\mathbf{I} + \mathbf{P}^{-1}Tw)$$

where **I** represents an identity matrix of dimension (m x m)

The Least Square Estimate of the parameter vector **K** such that the cost function

$$J = [\, \mathbf{KD} - \mathbf{G}\,].[\mathbf{KD} - \mathbf{G}]^T \qquad \text{...(5.13)}$$

is minimized yields

$$\mathbf{K} = \mathbf{G}.\mathbf{D}^T(\mathbf{D}.\mathbf{D}^T)^{-1}$$

$\mathbf{D}^T(\mathbf{D}.\mathbf{D}^T)^{-1}$ is the pseudoinverse of **D** which can be found by adjusting m, the number of Orthogonal functions/polynomials used. If the pseudoinverse cannot be determined then the controller configuration can be changed. The parameters of the PID controller for some desired dynamic performance can thus be found

Simulation Results :

The step response of the system is used for designing the PID controller in the present case. The amplitude of the step input to the uncompensated system was A= -0.1. The exact output was generated using the inverse Laplace transform obtained time domain solution. This output was then used to specify a damped reference trajectory as shown in fig 5.3 . The reference trajectory was transformed into the orthogonal domain and the parameters of the PID controller identified by the scheme outlined above. For m=32 & employing different Orthogonal Expansions the following parameters for the PID controller configuration were obtained.The ones obtained by the shifted Legendre's series expansions are seen to be better than the shifted Chebyshev & Walsh orthogonal approximations. For a set of controller values the complete nonlinear model including the controller (6th order differential equation, see Appendix) is simulated for a single cycle fault. The faulted line was opened after 17msec , the fault cleared and the line reclosed after 250 msec. The uncontrolled and controlled rotor angle and speed responses along with the control input (Xc) are shown.

PID Controller Parameters

m=32	KD	KP	KI
Walsh	-0.0001	-0.0309	-1.5756
Legendre (shifted)	-0.0001	-0.0282	-1.2535
Chebyshev (shifted)	-0.0001	-0.0249	-1.3561

Eigenvalues of the closed loop system for each set of parameter values

Walsh	Legendre (Shifted)	Chebyshev (Shifted)
-73.3618 + j45.6835	-68.0374 + j37.5955	-57.3863 + j54.3185
-73.3618 - j45.6835	-68.0374 - j37.5955	-57.3863 - j54.3185
-0.2482	-0.2482	-0.2482
-0.0008	-0.0010	-0.0009
-3.2643 E06	-3.2115 E06	-3.3619 E06

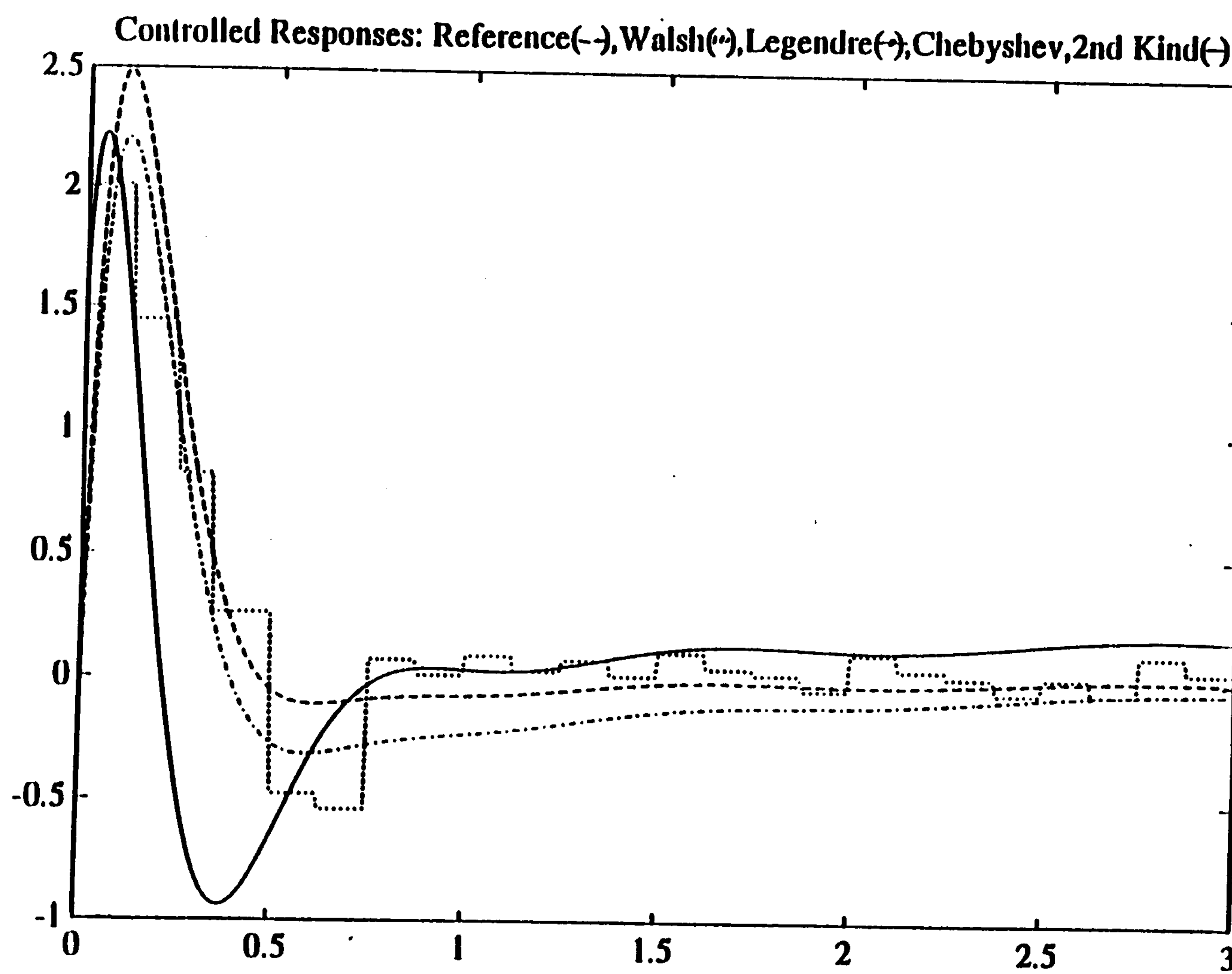


Fig 5.3 Rotor Speed Deviation (Rads/sec) vs Time in Secs

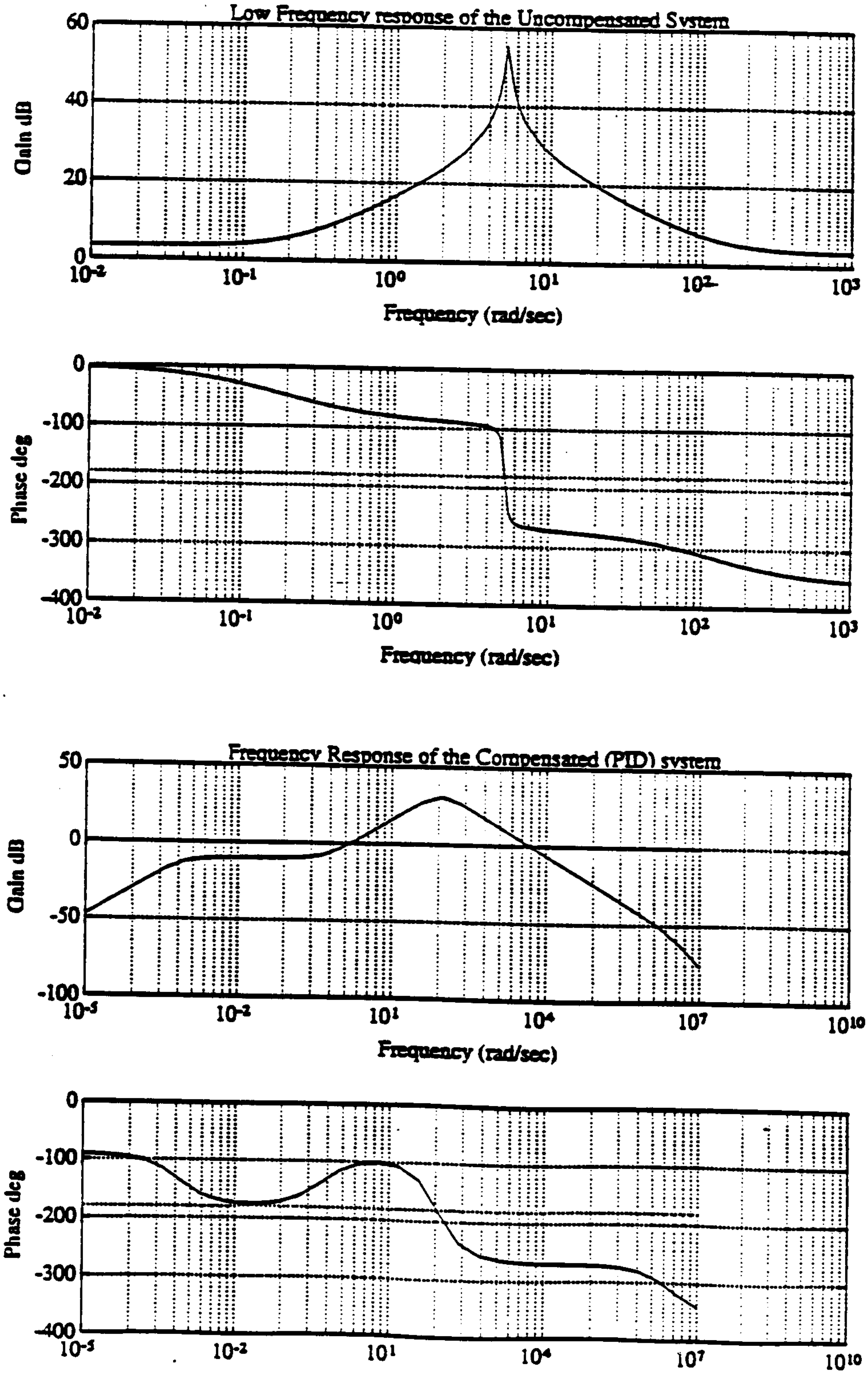


Fig 5.4 Low Frequency Bode Plots for Uncompensated and Compensated Systems

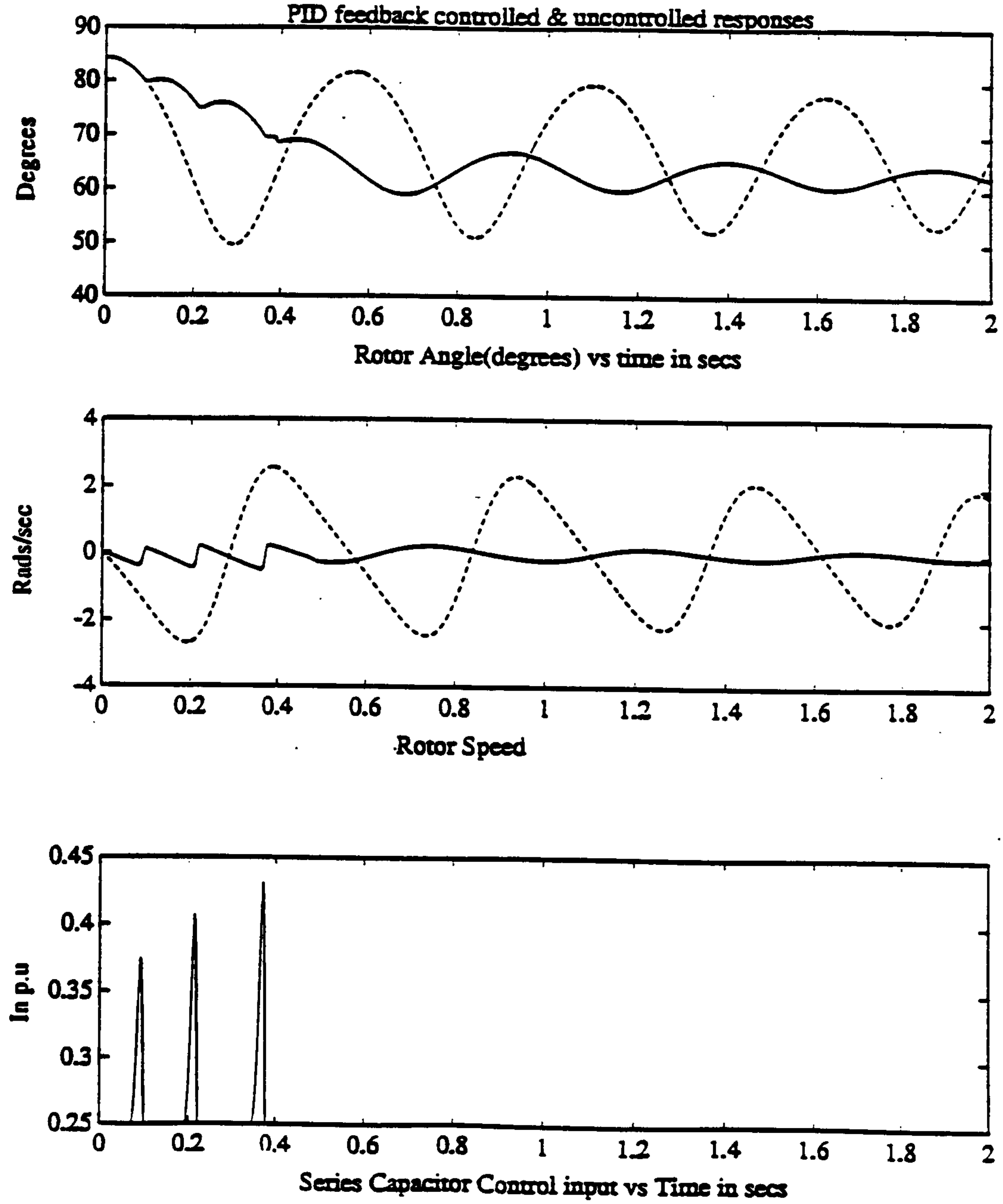


Fig 5.5 PID Feedback Controlled & Uncontrolled Responses for Perturbed Model

The first two modes are the mechanical modes of the controlled system. Compared with the uncontrolled mechanical modes these are pushed well to the left of the eigenvalue plane with a corresponding damping coefficient of ξ = -0.875 for the shifted Legendre series. The third mode is the electrical mode corresponding to the q-axis voltage of the generator and has a slightly increased damping coefficent

ξ = - 0.2482 The electrical mode corresponding to the exciter voltage Ef remains at -4 and is unaffected by the controller. The fourth and fifth modes shown above are the controller modes.

Chapter 6 Conclusion

In this research work, an overview of the so called " orthogonal expansion' approach is provided. Common orthogonal expansion representations were compared and their usefulness made apparent.

The classical optimal control problem was solved through the orthogonal expansion approach and applied to a perturbation model of a synchronous machine infinite bus system. Walsh functions, a convenient orthogonal series representation due to the binary values [1 or -1] each of the functions takes, were used for analysis and optimal control. The optimal control led to piecewise constant feedback gains.

Common nonlinear operations on time varying functions were delineated in the Walsh orthogonal domain and this useful property was used for parameter identification.These delineating properties of Walsh functions could be an area of further research. Useful applications could result from such research for the analysis and control of nonlinear systems.

A convenient method was developed for designing compensators/ controllers which would achieve prespecified closed loop dynamic responses using the orthogonal expansion approach and applied to the design of a PID series capacitor controller for a single machine infinite bus system.

Orthogonal functions like Laguerre functions and Legendre functions have been used for adaptive control, Zervos and Dumont [1988], with preliminary success for deterministic systems. The authors use orthogonal ladder networks for devising deterministic controllers.These controllers, unlike regressive model based controllers do not depend on structured models. Regressive model (ARMA , BARMA or NARMA) based controllers have distinct disadvantages since model uncertainities may lead to potential destabilization. Morover, significant transients are induced when the model order is changed.

Inspite of some disadvantages like their analyticity requirements, matrix manipulation etc, orthogonal expansions continue to be used for a large number of applications. This is seen by the fact that more than 150 publications have been presented since Chen and Hsiao [1975] first used Walsh functions for analysis and control. A variety of orthogonal functions ranging from Fourier series to piecewise linear polynomial functions have been used largely for parameter identification. A few publications have presented convenient methods for control synthesis and design but on the whole this area is still largely unresearched. Power system control as a subset of system science still has potential applications of orthogonal expansions in the area of identification and control but on the whole would depend on the developments which would take place in this area of system science in the future.

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APPENDICES

Appendix A: Miscellaneous Theorems

B. Levi's Theorem: *

If $\{ f_n(x) \}$ is a monotone increasing sequence of L_μ integrable functions and furthermore

$$\left| \int_a^b [f_n(x)] \, d\mu(x) \right| \leq C \quad (n = 0, 1 \ldots)$$

then the limiting function $f(x) = \lim_{n \to \infty} f_n(x)$ is also L_μ integrable and the relation

$$\lim_{n \to \infty} \int_a^b [f_n(x) \, d\mu(x) = \int_a^b [f(x) \, d\mu(x)$$

holds. If in particular $\mu_0(x)$, $\mu_1(x)$ are L_μ integrable functions such that

$$\sum_{n=0}^{\infty} | U_n(x) | \, d\mu(x) < \infty$$

then the series $\sum_{n=0}^{\infty} U_n(x)$ is (absolutely) convergent almost everywhere

Proof: The sequence $\{ f_n(x) - f_0(x) \}$ is monotone increasing, $f(x) - f_0(x)$ is its limiting function and $f_n(x) - f_0(x) \geq 0$.

Furthermore, our assumptions imply that

$$\int_a^b [f_n(x) - f_0(x)] \, d\mu(x) \leq \lim_{n \to \infty} \int_a^b [f_n(x) - f_0(x)] \, d\mu(x)$$

* Reference : Alexits G, International Series of Monographs in Pure & Applied Mathematics ,vol 20,1961

$$\leq C + \int_a^b | f_0(x) | \, d\mu(x)$$

so that we have

$$\int_a^b [f(x) - f_0(x)] \, d\mu(x) \leq \lim_{n \to \infty} \int_a^b [f_n(x) - f_0(x)] \, d\mu(x)$$

where f(x) is the limiting function & L_μ integrable (By Fatou's theorem)

Fatou's Theorem:

Let $\{ f_n(x) \}$ denote a sequence of positive L_μ integrable functions tending to the function $f(x)$, μ almost everywhere. If a constant C exists such that

$$\int_a^b f_n(x) \, d\mu(x) \leq C \qquad (n=1, 2, ...)$$

then the limiting function f(x) is also L_μ integrable and furthermore

$$\int_a^b f_n(x) \, d\mu(x) \leq C$$

Schwarz's inequality

$$\int_a^b | f(x) g(x) | \, d\mu(x) \leq \left\{ \int_a^b f^2(x) \, d\mu(x) \int_a^b g^2(x) \, d\mu(x) \right\}^{\frac{1}{2}}$$

The set of all $L_\mu{}^2$ integrable functions can be converted to a metric space $L_\mu{}^2$ .The distance between two points

$$f \in L_\mu^2 \quad , \quad g \in L_\mu^2$$

is defined by

$$\| f - g \| = \{ \int_a^b [f(x) - g(x)]^2 \, d\mu(x) \}^{\frac{1}{2}}$$

It satisfies the requirements, usually imposed on the idea of a metric:

1^0 $\quad \| f - g \| = \| g - f \| \geq 0$

2^0 $\quad \| f - g \| = 0$

is equivalent to $f(x) = g(x)$ μ almost everywhere; 3^0 if, g, h denote three points of L_μ^2 then

$$\| f - g \| \leq \| f - h \| + \| h - g \| \quad \text{holds}$$

The properties 1^0 and 2^0 result immediately from the definition of distance while 3^0 is a consequence of Schwartz's inequality above.

__Riesz-Fisher theorem (stated without proof)__

Let $\{ \varphi_n(x) \}$ denote an arbitrary orthonormal system and $\{ c_n \}$ a sequence of real numbers. A necessary and sufficient condition that $\{ c_n \}$ be the sequence of the expansion coefficents of an L_μ integrable function $f(x)$ is

$$\sum_{n=0}^{\infty} c_n^2 < \infty$$

The partial sums $S_n(x)$ of the expansion of $f(x)$ then converge to $f(x)$ in the sense of the L_μ^2 metrics

Appendix B: Useful Properties of Some Orthogonal Expansions

Shifted Legendre   Lee L and Kung F.C [1985]

$$L_0(t) = 1$$

$$L_1(t) = \frac{2t}{tf} - 1$$

$$L_{n+1}(t) = \frac{2n+1}{n+1}\left(2\frac{t}{tf} - 1\right).L_n(t) - \frac{n}{n+1} L_{n-1}(t)$$

Orthogonality relation :

$$\int_0^t L_i(t).L_j(t)\,dt = \begin{cases} 0 & i \neq j \\ \dfrac{tf}{2i+1} & i = j \end{cases}$$

Orthogonal coefficent vector elements

$$C_i = \frac{2i+1}{tf}\int_0^{tf} f(t)\,L_i(t)\,dt \qquad i = 1,2\ \ldots\ m-1$$

Where f (t) is the L_2 integrable function which is approximated in the interval [0 , tf)

Operational matrix of integration

$$P = tf.\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & .. & 0 & 0 & 0 \\ \frac{-1}{6} & 0 & \ldots & 0 & 0 & 0 \\ .. & .. & .. & .. & .. & \frac{1}{2m+3} \\ 0 & 0 & 0 & .. & \frac{-1}{2(2m-1)} & 0 \end{bmatrix}$$

Shifted Chebyshev polynomials (2nd kind)   Lee T.T and Tsay S.C [1986]

$$U_0(t) = 1$$

$$U_1(t) = 2 - \frac{4t}{tf}$$

$$U_{i+1}(t) = 2(1 - \frac{2t}{tf})\, U_i(t) - U_{i-1}(t)$$

Weight function : $w(t) = (tf.\, t - t^2)^{\frac{1}{2}}$

Orthogonality relation : $\int_0^{tf} w(t).\, U_i(t).U_j(t)\, dt = \begin{cases} 0 & i \neq j \\ (tf)^2 \cdot \pi & i = j \end{cases}$

Orthogonal coefficent vector elements

$$Ci = \frac{8}{(tf)^2 \cdot \pi} \int_0^{tf} w(t).\, f(t).U_i(t).\, dt \quad i = 0,1\ 2 \ldots m\text{-}1$$

Operational matrix of integration :

$$P = tf. \begin{bmatrix} \frac{1}{2} & \frac{-1}{4} & 0 & .. & 0 & 0 & 0 \\ \frac{3}{8} & 0 & \frac{-1}{8} & .. & 0 & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. \\ \frac{1}{2(m-1)} & 0 & 0 & .. & \frac{1}{4(m-1)} & 0 & \frac{-1}{4(m-1)} \\ \frac{1}{2m} & 0 & 0 & ... & 0 & \frac{1}{4m} & 0 \end{bmatrix}$$

<u>Laguerre Polynomials</u> Jha A.N, Zaman S , Ranganathan V [1986]

$$G_0(t) = 1$$

$$G_1(t) = 1 - t$$

$$G_k(t) = (k!)^2 . \sum_{n=0}^{k} \frac{(-1)^n}{(n!)^2 (k-n)!} t^n$$

Weight function : $w(t) = \exp(-t)$

Orthogonality relation : $\int_0^{tf} w(t).\, G_i(t).G_j(t)\, dt = 0 \qquad i \neq j$

$= 1 \qquad i = j$

Operational matrix of integration:

$$P = \begin{bmatrix} 1 & -1 & 0 & 0 & .. & 0 & 0 \\ 0 & 1 & -1 & 0 & .. & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 0 & .. & 1 & -1 \\ 0 & 0 & 0 & 0 & .. & 0 & 1 \end{bmatrix}$$

<u>Walsh Functions</u> Karanam V.R, Frick P.A, Mohler R.R [1978]

$P(1) = 1 \qquad P_{(m)} = \begin{bmatrix} P_{(\frac{m}{2})} & I_{(\frac{m}{2})} \\ I_{(\frac{m}{2})} & P_{(\frac{m}{2})} \end{bmatrix} \qquad ; \; m = 2^n \qquad n = 1, 2 \; ...$

<u>Blockpulse Functions</u> Jaw Y.G, Kung F.C [1984]

$B_i(t) = 1 \qquad \text{for} \quad \frac{(i-1)}{m}.T \leq t < \frac{i}{m}.T$

$0 \qquad \text{otherwise}$

Orthogonal coefficent vector elements :

$$Ci = \frac{m}{T} \int_0^{tf} f(t)\, B_j(t)\, dt$$

Operational matrix of integration:

$$P = \frac{T}{m} \cdot \begin{bmatrix} \frac{1}{2} & 1 & 1 & .. & 1 \\ 0 & \frac{1}{2} & 1 & .. & 1 \\ 0 & 0 & \frac{1}{2} & .. & 1 \\ 0 & 0 & .. & .. & \frac{1}{2} \end{bmatrix}$$

<u>Hermite Polynomials</u> Tsay Y.F, Lee T.T [1986]

$$H_0(t) = 1$$

$$H_1(t) = 2t$$

$$H_{n+1}(t) = 2t.\ H_n(t) - 2n.\ H_{n-1}(t) \qquad n = 1, 2 ...$$

Orthogonality condition

$$\int_0^{tf} \exp(-t^2)\, H_n(t) H_m(t) dt = \begin{cases} 0 & n \neq m \\ \sqrt{(\pi.2^n.\ n!)} & n = m \end{cases}$$

Orthogonal coefficent vector elements :

$$Cn = \frac{1}{\sqrt{(\pi.2^n.\ n!)}} \int_0^{tf} \exp(-t^2)\, f(t)\, H_n(t) dt$$

$$n = 0,1, ...$$

Operational matrix of integration:

$$P = tf.\begin{bmatrix} 0 & \frac{1}{2} & 0 & 0 & .. & 0 \\ \frac{1}{2} & 0 & \frac{1}{4} & 0 & .. & 0 \\ 0 & 0 & 0 & \frac{1}{6} & .. & 0 \\ .. & .. & .. & .. & .. & \frac{1}{2(m-1)} \\ pm & 0 & 0 & 0 & .. & 0 \end{bmatrix}$$

where $pm = (-1)^{\frac{m}{2}+1} \cdot (m-1).(m-2).(m-3) \ldots \frac{1}{2}(\frac{m}{2}+1)$ if m is even

$pm = 0$ if m is odd

<u>Delayed Unit Step functions</u> Hwang C. [1983]

$$U_i(t) = \begin{cases} 1 & t \geq ih \\ 0 & t < 0 \end{cases}$$

Operational matrix of integration:

$$P = tf.\begin{bmatrix} \frac{h}{2} & h & h & .. & h & h \\ 0 & \frac{h}{2} & h & .. & h & h \\ .. & .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 0 & 0 & \frac{h}{2} \end{bmatrix}$$

<u>Fourier series</u> Paraskevopoulos P.N, Sparis P.D, Mouroutsos S.G [1985]

$F_i(t) = \cos(\frac{2.i.\pi.t}{T})$ $i = 0,1,2 \ldots$

$F_i^*(t) = \sin(\frac{2.i.\pi.t}{T})$ $i = 1,2,,3 \ldots$

Operational matrix of integration:

$$P = \begin{bmatrix} T & 0 & -T.Er' \\ 0 & \mathbf{0}_{(rxr)} & \frac{T}{2\pi}.\mathbf{I}_{(rxr)} \\ \frac{T.Er}{2\pi} & \frac{-T.\mathbf{I}_{(rxr)}}{2\pi} & \mathbf{0}_{(rxr)} \end{bmatrix}$$

where $Er' = [1 \quad 1/2 \quad 1/3 \quad 1/4 \; .. \; 1/r]$

and $\mathbf{I}_{(rxr)} = \begin{bmatrix} 1 & 0 & 0 & .. & 0 \\ 0 & \frac{1}{2} & 0 & .. & 0 \\ 0 & 0 & \frac{1}{3} & .. & 0 \\ .. & .. & .. & .. & .. \\ 0 & 0 & 0 & .. & \frac{1}{r} \end{bmatrix}$ where $m = 2r + 1$

<u>Piecewise Linear polynomials</u> Lee C.T, Chou Y.S [1987]

$$N_0(t) = 1 - \frac{mt}{T} \qquad 0 \le t \le T$$

$$0 \qquad \text{otherwise}$$

$$N_i(t) = (1 - i) + \frac{m.t}{T} \qquad (i-1)T \le t \le iT$$

$$= (1+i) - \frac{mt}{T} \qquad iT \le t \le (i+1)T$$

$$= 0 \qquad \text{otherwise}$$

for i = 1,2 .... n-2 and

$$N_{n-1}(t) = (1-m) + \frac{m.t}{T} \qquad (m-1)T \leq t \leq T$$
$$= 0 \qquad \text{otherwise}$$

where m = n-1

Operational matrix of integration:

$$P = \frac{T}{m} \cdot \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & .. & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & .. & \frac{1}{2} \\ .. & .. & .. & .. & .. \\ 0 & 0 & 0 & .. & \frac{1}{2} \end{bmatrix}$$

<u>Taylor series</u> Sparis P.D, Mouritsos S.G [1988]

$T_0(t) = 1$

$T_1(t) = (t - to)$

$T_2(t) = (t-to)^2$

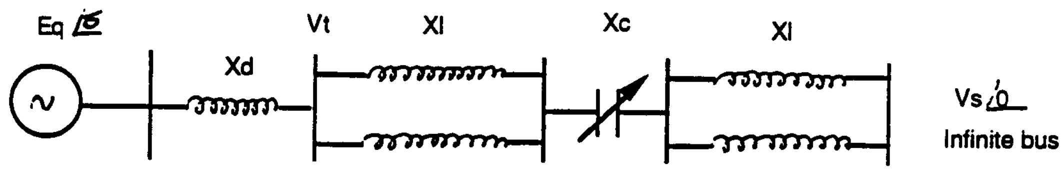
......

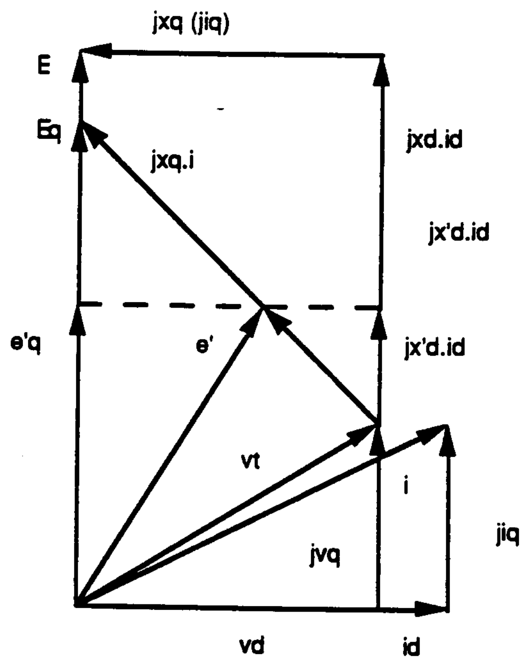
$T_n(t) = (t-to)^n$

Operational matrix of integration:

$$P = \begin{bmatrix} 0 & 1 & .. & 0 & 0 \\ 0 & 0 & .. & 0 & 0 \\ .. & .. & .. & .. & .. \\ 0 & 0 & .. & \frac{1}{(r-2)} & 0 \\ 0 & 0 & .. & 0 & \frac{1}{(r-1)} \\ 0 & 0 & .. & 0 & 0 \end{bmatrix}$$

Appendix C 4th order Model of a Single Machine Infinite Bus System Power System, Machine Parameters and Complete Model of the Power System with the PID Controller





Phasor diagram for 4th order model

Fig C 1. Single Machine Infinite Bus System Model and Phasor diagram for 4th order model.

The configuration above of the power system is modelled by a 4th order differential equation as:

$$\delta = \omega$$

$$\omega = \frac{\omega b}{H}.Pm - \frac{D}{H}\omega - \frac{\omega b}{H}\left[\frac{Vs}{X'_{d\Sigma}} Eq \sin(\delta) + \frac{Vs\,(Xd' - Xd)}{2.\,X'_{d\Sigma}.\,X_{d\Sigma}} \sin(2\delta) \right]$$

$$Eq = \frac{Ef}{Tdo} - \frac{Xd\Sigma.\,Eq}{Tdo.X'_{d\Sigma}} + \frac{(Xd - X'd)}{T'do.\,X'_{d\Sigma}} Vs.\cos(\delta)$$

$$Ef = -\frac{Ef}{Te} + \frac{V}{Te}$$

δ - rotor angle corresponding to the infinite bus system

ω - deviation of rotor speed from synchronous speed ωb

H - Inertia of the generator

D - equivalent damping factor

Vs - Infinite bus voltage

Eq - transient q- axis voltage

Ef - transient excitation voltage

V - excitation control signal

Tdo - equivalent transient rotor time constant

Te - equivalent time constant of the main excitation winding of the generator

$X'_{d\Sigma} = X'd + Xl = X'd1 + Xt + Xl$

X'd - d-axis transient reactance

Xt - transformer reactance

Xl - transmission line reactance

$X_{d\Sigma} = Xd + Xl$

Xd - d-axis reactance

For series capacitor control the modified equations for U = Xc become

$$\delta = \omega$$

$$\omega = \frac{\omega b}{H}.Pm - \frac{D}{H}\,\omega - \frac{\omega b}{H}\left[\frac{Vs\,Eq}{(X'_{d\Sigma} - U)}\sin(\delta) + \frac{Vs\,(Xd' - Xd)}{2.\,(X'_{d\Sigma} - U).\,(X_{d\Sigma} - U)}\sin(2\,\delta)\right]$$

$$Eq = \frac{Ef}{Tdo} - \frac{(X_{d\Sigma} - U)}{Tdo.(X'_{d\Sigma} - U)}.\,Eq + \frac{(Xd - X'd)}{T'do.\,(X'_{d\Sigma} - U)}\;Vs.\cos(\delta)$$

$$Ef = -\frac{Ef}{Te} + \frac{V}{Te}$$

The linearized equations or perturbation equations about the operating point given by

(δo , wo , Eqo , Efo , Uo) are given by

$$X(t) = A.\,X(t) + B\,U(t)$$

$$Y(t) = C\,X(t)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a1 & -\frac{D}{H} & a2 & 0 \\ a3 & 0 & a4 & \frac{1}{Tdo} \\ 0 & 0 & 0 & \frac{-1}{Te} \end{bmatrix}$$

and $B = [\,0 \quad b1 \quad b2 \quad o\,]$ $\qquad C = [\,0 \quad 1 \quad 0 \quad 0\,]$

where

$$a1= -\frac{\omega b}{H}\left[\frac{Vs.Eqo}{(X'_{d\Sigma} - Uo)}\cos(\delta o) + \frac{Vs^2.(X'd-Xd)}{(X'_{d\Sigma}-Uo)\,(X_{d\Sigma}-Uo)} \cos(2.\delta o)\right]$$

$$a2= -\frac{\omega b}{H}\frac{Vs \sin(\delta_o)}{(X'_{d\Sigma} -Uo)}$$

$$a3= -\frac{Vs\,(Xd-X'd) \sin(\delta o)}{Tdo\,(X'_{d\Sigma}-Uo)}$$

$$a4= -\frac{(X_{d\Sigma} - Uo)}{Tdo\,(X'_{d\Sigma} - Uo)}$$

$$b1= -\frac{\omega b}{H}\left[\frac{Vs\,Eqo \sin(\delta o)}{(X'_{d\Sigma}-Uo)^2} + \frac{Vs^2(X'd-Xd)\sin(2\delta o)}{2\,(X'_{d\Sigma}-Uo)^2\,(X_{d\Sigma}-Uo)} + \right.$$

$$\left. + \frac{Vs\,(X'd-Xd) \sin(2\delta o)}{2(X'_{d\Sigma}-Uo)\,(X_{d\Sigma}-Uo)^2}\right]$$

$$b2= \frac{(X_{d\Sigma}-X'_{d\Sigma}).\,Eqo}{Tdo\,(X'_{d\Sigma} - Uo)^2} + \frac{(Xd - X'd)\,Vs \cos(\delta o)}{Tdo\,(X'_{d\Sigma} - Uo)^2}$$

Complete 6 th order nonlinear model of the PID controlled system.

The first four equations are the same as above. The other two equations are given by

$$us = Ks.\frac{Vr}{Ts} - Ks.\frac{p}{Ts} - \frac{us}{Ts}$$

Auxiliary equation : $U= Uo + us$;

$$p = (KP - D.\,KD)\left[\frac{\omega bPm}{H} - \frac{D\,\omega}{H} - \frac{\omega b}{H}\left(\frac{Vs\,Eq \sin(\delta)}{(X'_{d\Sigma} - U)} + \right.\right.$$

$$+ \frac{Vs\,(Xd' - Xd)\sin(2\delta)}{2(X'_{d\Sigma}-U).\,(X_{d\Sigma}-U)}]$$

$$- \omega b.\, KD\, [\frac{Vs.\sin(\delta)}{(X'_{d\Sigma}-U)}\{ Ef - \frac{(X_{d\Sigma}-U).\,Eq}{(X'_{d\Sigma}-U)} + \frac{(Xd - Xd')\,Vs.\cos(\delta)}{(X'_{d\Sigma}-U)}\}$$

$$+ \frac{Vs.Eq.\cos(\delta).\omega}{(X'_{d\Sigma}-U)} + \frac{Vs.Eq.\sin(\delta)}{(X'_{d\Sigma}-U)^2}.\{ Ks.Vr - Ks.\,p - us \}$$

$$+ \frac{Vs^2.(Xd' - Xd)\cos(2\delta).\omega}{(X'_{d\Sigma}-U)(X_{d\Sigma}-U)}$$

$$+ \frac{Vs^2\,(Xd' - Xd)\sin(2\delta)}{(X'_{d\Sigma}-U)^2(X_{d\Sigma}-U)^2}.(X'_{d\Sigma} + X_{d\Sigma} - 2U)\,\{ KsVr - Ks\,p - us \}\,]$$

$$+ KI.\,\omega - \frac{p}{Tw}$$

<u>Machine parameters and initial conditions</u>

wb=120.π , Xl=0.8 , Xd=1.5 , Xt=0 , Xdd=1.07 ,
Xddsum =1.87 , Xdsum=2.3, Vs=1.0 , H=6.204
D=1.5 , Xc(0)=0.25 , V=1.5, Pm=0.7, tdo=5.0, te=0.25, delo=1.0831277 rad,
wo=0.00030996 rads/sec, eqo =1.2837 , efo=1.5

Closed loop parameters

Ts = 5.0E-07 sec, Ks = 50 , Tw = 2 sec.
Xco=0.0;

Appendix D Kronecker Products & Recursive Method of Chen & Hsiao [1975]

Definition: Let A be a m x n matrix & B a k x l matrix , the Kronecker product of A, B is a (mk) x (nl) matrix

$$A \ \Theta \ B = \begin{bmatrix} a_{11}B & a_{12}B & .. & .. & a_{1n}B \\ .. & .. & .. & .. & .. \\ a_{m1}B & a_{m2}B & .. & .. & a_{mn}B \end{bmatrix}$$

Properties: For appropriate dimensional matrices C & D and identity matrix I

(1) $A \ \Theta \ B \neq B \ \Theta \ A$

(2) $A \ \Theta \ 1 \neq 1 \ \Theta \ A$ (1 is a scalar)

(3) $(A+B) \Theta C = A \ \Theta \ C + B \ \Theta \ C$

$A \ \Theta \ (B+C) = A \ \Theta \ B + A \ \Theta \ C$

(4) $(AB) \ \Theta \ (CD) = (A \ \Theta \ C)(B \ \Theta \ D)$

If C is a m x n matrix, then

$$(AB) \ \Theta \ C = (A \ \Theta \ I_m)(B \ \Theta \ C) = (A \ \Theta \ I_m)(B \ \Theta \ C)$$

(5) $(A \Theta B)^{-1} = A^{-1} \Theta B^{-1}$

If there exists i A matrices & j B matrices ; the Kronecker products of any combinations of A's & B's have the same dimension. Let $\sum \{ A^i , B^j \}$ denote the summation of all possible Kronecker product combination of i A's and j B's

For example

$$\Sigma \{ A^2 \} = A \Theta A$$

$$\Sigma \{ AB \} = A \Theta B + B \Theta A$$

$$\Sigma \{ AB^2 \} = A \Theta B \Theta B + B \Theta A \Theta B + B \Theta B \Theta A$$

....

Eigenvalues of Kronecker Product

(1) Suppose { ra_1 ....rb_n } are respective eigenvalues of n x n matrices A and B; then { $ra_1 . rb_j$, i, j = 1, ... , n } are the eigenvalues of $A \Theta B$

In effect there exists proper matrices P and Q such that

$$A = P \aleph (ra_1).P^{-1} \qquad B = Q \aleph (rb_j).Q^{-1}$$

where $\aleph$ denotes a diagonal matrix with diagonal elements composed of the eigenvalues r_1

Now $$A \Theta B = (P \Theta Q)(\aleph (ra_1) \Theta \aleph (rb_j))(P^{-1} \Theta Q^{-1})$$

$$= (P \Theta Q)(\aleph (ra_1 rb_j))(P \Theta Q)^{-1}$$

(2) The eigenvalues of ($A \Theta I_n$) consist of n duplicate ra_1 , i = 1, ... , n

(3) The eigenvalues of $\Sigma \{ A I_n)$ are { $ra_1 + ra_j$, i, j = 1, ... , n }

In fact

$$\Sigma \{ A I_n) = A \Theta I_n + I_n \Theta A$$

$$= P \aleph (ra_1).P^{-1} \Theta I_n + I_n \Theta P \aleph (ra_j).P^{-1}$$

$$= P(\aleph(ra_1, i, j = 1, ..,n)).P^{-1} + P(\aleph(ra_j, i, j = 1, ..,n)).P^{-1}$$

$$= P(\aleph(ra_1 + ra_j, i, j = 1, ..,n)).P^{-1}$$

Recursive Algorithm for solving $c1 = [I - A \Theta P^T]^{-1}.\ m$

For $m = 2^\alpha$, α any positive integer we have

$$G_\alpha = I, \quad R_\alpha = -\frac{A}{2^{\alpha+1}}, \quad F_\alpha = -\frac{A}{2^{\alpha+1}} \cdot R_\alpha$$

$$F_{\alpha+1} = 0, \quad m_{i,\alpha+1} = m_i \qquad i = 0,1,2, ...,2^{\alpha-1} - 1$$

Then R_β & $m_{i,\beta}$ for $\beta = \alpha, \alpha - 1, \ldots, 1$, $i = 0,1, \ldots 2^{\beta-1} - 1$ are calculated by the recursive formulas :

$$G_\alpha = I - F_{\beta+1}$$

$$R_\beta = -2^{-\beta-1} G_\beta^{-1} A$$

$$F_\beta = F_{\beta+1} + 2^{-\beta-1} A. R_\beta$$

$$m_{1, i+2^{\beta+1}} = m_{i,\beta+1} + 2^{-\beta-1} A. G_\beta^{-1} m_{(i+2^{\beta}-1), \beta+1}$$

The elements of c1 are obtained as column vectors as

$$c_{10} = G_0^{-1} m_{0,1} \qquad G_0 = I - \frac{A}{2} - F_1$$

All the other vectors c_j, $j = 0,1, \ldots (m-1)$ are found by substituting them with

$$c_{1, i+2^{\beta+1}} = R_\beta c_{1i} + G_\beta^{-1} m_{(i+2^{\beta}-1), \beta+1}$$

$$\beta = 1, 2, \ldots, \alpha \qquad i = 0,1, \ldots (2^{\beta-1}-1)$$

In the algorithm matrices (G, F, R) are n x n matrices and hence reduction of computing time m_i & c_{1i} are the column vectors of the column vector m & c1 respectively.