AN ABSTRACT OF THE THESIS OF

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The acquisition and elimination of products and the resources needed to create them constitutes an important part of the business decision-making process. *Activity-based costing* (ABC) supports this process by providing a tool for evaluating the relative profitability of various products. It accomplishes this by allocating costs to products based on the activities, and in turn the resources demanded by those activities, required to produce them. In allocating indirect costs traditionally considered "fixed," such as equipment, administrative overhead, and support staff salaries, ABC treats all costs as variable in the long-run.

However, many costs can only vary in discrete steps. For example, one usually cannot purchase a fractional piece of equipment; one chooses either to buy it or not to buy it. Also, in adding support staff, one will typically find that people demand full-time positions, so increments will come in discrete amounts. This *stairstep semivariable* nature

of many costs runs counter to ABC's treatment of all costs as variable. In addition, different products often draw upon the same resources. This creates complex interactions, making it difficult to predict the ultimate consequences of adding or eliminating a particular product.

Mixed integer programming (MIP) provides another tool for making these *product/resource mix* decisions. Unlike ABC, however, it can handle variables that take on integer values, and hence deal appropriately with stairstep semivariable costs. It also ensures that the decision recommended by the model will optimize profitability, given that a solution exists and the underlying assumptions hold true. In doing this, MIP automatically adjusts for all of the complex interactions that exist among the various products and resources.

Using a simplified two product/two resource model, one can detail the mathematics behind ABC and MIP, and then link the two approaches through a common variable. This allows one to establish the conditions under which ABC and MIP will yield the same results, and those under which they will differ. Since MIP produces an optimal solution, the fact that ABC yields a different result under specific circumstances underscores the danger of relying solely on the product margins generated by an ABC model.

A Mathematical Analysis and Critique of Activity-Based Costing using Mixed Integer Programming

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I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

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Kevin Hamler-Dupras, Author

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For Sara and her dreams.

A MATHEMATICAL ANALYSIS AND CRITIQUE OF ACTIVITY-BASED COSTING USING MIXED INTEGER PROGRAMMING

STATEMENT OF THE PROBLEM

Introduction

The successful operation of a business enterprise involves a variety of decisions made under conditions of limited information and limited resources. In microeconomics, the theory of the firm defines the goal of this decision-making as the "maximization of the value of the firm," with *value* defined as "the present value of the firm's expected future cash flows."[1] To this end, the firm's management "combines various inputs to produce a stipulated output in an economically efficient manner."[2]

This output can take the form of either goods or services, but for simplicity, we will refer to this output as the firm's *product mix*. Although this terminology generally implies the production of goods, the product mix could just as easily include services. The inputs, or *resources*, include the labor, materials, equipment, and other expenses or assets, both tangible and intangible, used to generate the firm's product mix. Taken together, the inputs and outputs chosen form a *product/resource mix*.[3]

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[1]James L. Pappas and Eugene F. Brigham, *Managerial Economics*. (Hinsdale, Illinois: The Dryden Press, 1979), p. 11.

[2]J.P. Gould and C.E. Ferguson, *Microeconomic Theory*. (Homewood, Illinois: Richard D. Irwin, Inc., 1980), p. 122.

[3]This terminology closely parallels that used by Wernerfelt in his discussion of resource-product matrices (Birger Wernerfelt "A Resource-based View of the Firm." *Strategic Management Journal* 5 (April-June 1984): 171-80.)

This leads to the issue of how, given a sequence of time periods under consideration, a firm can determine the product/resource mix to deploy in each time period that will maximize the present value of the resulting cash flows. We will refer to this as the *product/resource mix problem*.

Activity-based costing (ABC) provides one approach to the problem of selecting what products to sell in order to maximize profitability.[4] Since the analysis of how different products utilize resources plays a central role in this method, it serves as a tool for choosing the appropriate product/resource mix, and hence provides a method for addressing the product/resource mix problem. Given its recent popularity as a decision tool, ABC's effectiveness in addressing this problem presents an important topic for investigation.

Activity-Based Costing

In ABC, one examines the activities involved in supporting each product, along with the costs generated by those activities. By allocating these costs to *cost pools* associated with the activities, and then further allocating them to the products generating those activities, one arrives at a total cost for each product.[5] These costs include not only the direct costs involved in production, but also the so-called "fixed" costs created by a product's existence. Cooper and Kaplan enumerate them as follows:

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[4]Although the term "profitability" often appears in the literature without any clarifying definition, in the absence of such clarification one can assume it to take on the meaning generally ascribed to it in microeconomic theory, i.e., "the present value of the firm's expected future cash flows."

[5]Raef A. Lawson, "Beyond ABC: Process-Based Costing" *Cost Management*, Fall 1994, p. 3.

Logistics
Production
Marketing and Sales
Distribution
Service
Technology
Financial Administration
Information Resources
General Administration.[6]

For example, one might designate the acquisition of raw materials as an activity, setting up an associated cost pool called "Raw Materials Acquisition." Since buying the materials and bringing them into the plant requires the efforts of both purchasing and receiving, the *first-stage allocation* would involve taking a portion of the costs from each of these departments and putting it in the cost pool. The *second-stage allocation* would then distribute these costs to the various products based on raw material usage. Figure 1 depicts this for three products: A, B, and C.

The basis for these allocations comes from an analysis which identifies the relevant *cost drivers.*[7] Continuing with the preceding example, one could use the number of purchase orders as the cost driver for allocating purchasing department overhead; the number of truck loads received could serve as the cost driver for receiving department costs. Thus, if the purchasing department can handle 10,000 purchase orders, and 8,000 of them relate to raw materials purchases, then eighty percent of the purchasing department's cost would go to the Raw Materials Acquisition cost pool.

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[6]Robin Cooper and Robert Kaplan, "Measure Costs Right: Make the Right Decisions," *Harvard Business Review*, September-October 1988, p. 97.
[7]Lawson, p. 33.

Likewise, if the receiving department can handle 1,000 truck loads, and 700 of them involve raw materials purchases, then one would allocate seventy percent of the receiving department's cost to the raw materials acquisition pool. Then, for the second stage of the allocation process, it might make sense to let the quantity of raw material used act as the cost driver. So, if Product A consumes one-half of the raw material, then it would receive one-half of the costs in the raw materials acquisition pool.

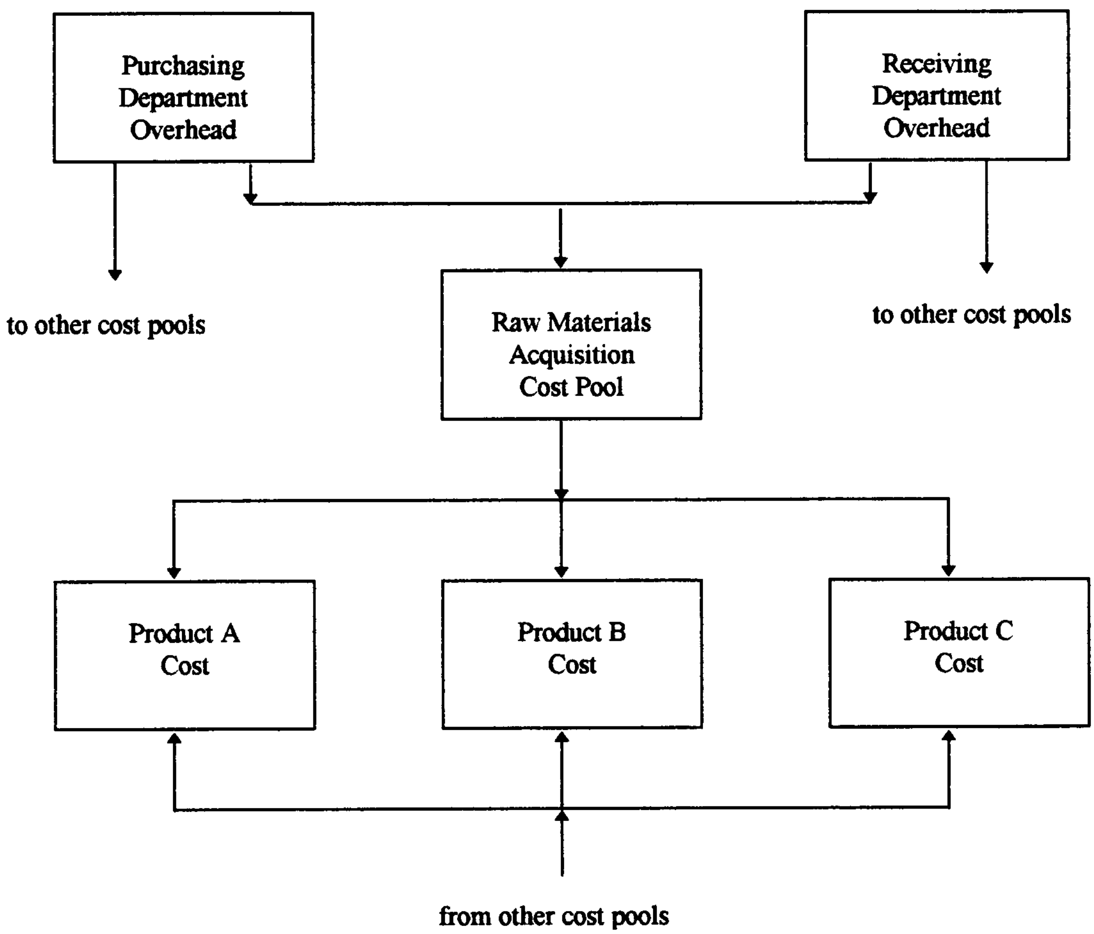

Figure 1. Cost flow using activity-based cost allocation

This simplified example belies the fact that the web of allocations created in an ABC system can take on a high level of complexity in the attempt to match reality as closely as possible. In the above example, one could substantially increase the usefulness of the model by setting up a separate acquisition cost pool for each raw material. That way, products that use raw materials requiring a high level of purchasing and receiving activity per unit would reflect that in their cost.

Having traced as many costs as possible to each product, one arrives at product costs per unit, which, when compared to product prices, provide a basis for judging product profitability. Using this information, a firm's management can make decisions regarding which products to keep and which to eliminate. These decisions may in turn result in a shifting or elimination of resources. Also, by including new products and their projected costs in the analysis, management could also consider the impact of adding products or resources.

Cooper and Kaplan developed ABC in response to what they saw as a tendency for existing cost accounting systems to provide incorrect information, leading to poor decisions regarding product mix and resource deployment. As they have observed:

> Managers in companies selling multiple products are making important decisions about pricing, product mix, and process technology based on distorted cost information. What's worse, alternative information rarely exists to alert these managers that product costs are badly flawed. Most companies detect that problem only after their competitiveness and profitability have deteriorated.[8]

They have offered the following explanation for this situation:

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[8]Cooper and Kaplan, p. 96.

> Conventional economics and management accounting treat costs as variable only if they change with short-term fluctuations in output. We (and others) have found that many important cost categories vary not with short-term changes in output but with changes over a period of years in the design, mix, and range of a company's products and customers. An effective system to measure product costs must identify and assign to products these costs of complexity.[9]

In criticizing traditional costing methods, Cooper and Kaplan have assumed that product costs can provide valid information for product/resource mix decisions in the first place. As we will see, even though ABC may provide an answer to the question of how to develop more useful product costs, one can justifiably argue that the proponents of this approach have actually asked the wrong question.

Product Costing as a Decision Tool

Beneath the criticism of traditional cost systems and the search for better product cost information lies a fundamental objective: to maximize profitability. As stated earlier, this involves maximizing the present value of the firm's expected future cash flows. Therefore, in assessing the usefulness of a decision tool, one must consider how well the information provided by the tool assists management in achieving this objective.

Although ABC differs from other costing methods in terms of the costs allocated and the basis for those allocations, all approaches to product costing adhere to the same underlying mathematics. Hence, we will focus on examining product costing in general as a decision tool, making special note of how activity-based ideas relate to this examination.

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[9]Ibid., p. 97

Traditional cost accounting assumes that certain costs vary with output, while others remain unchanged over a given range of output. The literature refers to these costs as *variable* and *fixed*, respectively. Assuming that prices remain constant over the given output range, one can calculate the *contribution margin*[10] for product i using the following equation:

$$C_i = Q_i(p_i - v_i) \tag{1}$$

where C_i represents the contribution margin, Q_i the quantity, p_i the price, and v_i the variable cost per unit for product i. Summing these elements over all products, $i = 1, 2, \ldots, n$ yields the total contribution margin, C, so:

$$C = \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \left[Q_i(P_i - V_i)\right] \tag{2}$$

Subtracting the fixed cost, F, yields net income, expressed in the following equation:

$$I = C - F \tag{3}$$

Economic theory states that given a set of fixed resources, management should "produce and sell so as to maximize the total contribution margin of the firm."[11] In this situation, we treat F in equation 3 as a constant, so increasing income, I, depends solely on increasing C. Since the total contribution margin simply equals the sum of the individual product contribution margins (expressed in equation 2), one should make only products with positive contribution margins, eliminating all those with contribution margins less than or equal to zero.

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[10]Don. T. DeCoster and Elson L. Schafer, *Management Accounting: A Decision Emphasis.* (New York: John Wiley & Sons, Inc., 1979), pp. 10-11.

[11]DeCoster and Schafer, p. 11.

However, emphasizing products with the highest contribution margins per unit will not necessarily yield the greatest total contribution. For example, suppose Product A has twice the contribution per unit of Product B, but that we can produce three times as much Product B given the same set of resources. The total contribution generated by producing A will equal two thirds that obtained from making B.

Since ABC takes into account how products draw upon resources through activities, it offers a possible tool for addressing the difficulty just described. In addition, the use of contribution margin per unit assumes a limited *relevant range* of activity.[12] Advocates of ABC would argue that many of the important decisions entertained by management require an expansion of the relevant range to the point where so-called "fixed" costs become variable, and hence relevant to the decision-making process.

Moving away from the traditional terminology which distinguishes between "fixed" and "variable" costs, activity-based methods split costs into three new categories:

> 1. Volume-based costs: dependent on volume-based activities such as production, direct labor hours or machine hours. Examples would be direct materials (production) and machining costs (machine hours per unit). These can be classified as unit level activities, i.e., they are performed each time a unit is produced. Each product may have volume (variable) costs that are dependent on different cost drivers.
>
> 2. Nonvolume-based traceable costs: incurred each time a batch of goods is produced. They are generally fixed with respect to individual units of product but are traceable to product lines. Examples of this type of cost are setup costs, which are dependent upon number of setups; and shipping costs, dependent upon the number of orders received.
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> 3. Nonvolume-based, non-traceable costs: facility level activities that sustain a facility's operations; also referred to as common costs. They

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[12]DeCoster and Schafer, p. 12.

are also fixed and cannot be traced to individual products and are therefore arbitrarily allocated for product costing purposes.[13]

The definition given for the second category leaves out an important point. Nonvolume-based traceable costs occur not only with each production run, but also with the decision to add or eliminate "fixed" resources solely associated with the product line in question. In a general sense, this category relates to production levels, and not just to batches. For example, 2,000 units might represent one production level, 1,000 another, and zero production another.

Let v_i represent the *volume-based cost* associated with product i and F the total *nonvolume-based cost*. Taking an activity-based approach, we allocate F to the individual products, yielding F_i for each product. Then, replacing the contribution margin C_i with a *product margin*,[14] M_i, we have equation 1 restated as follows:

$$M_i = C_i - F_i = Q_i(p_i - v_i) - F_i \qquad (4)$$

Likewise, equation 2 becomes:

$$M = \sum_{i=1}^{n} M_i = \sum_{i=1}^{n} \left[Q_i(p_i - v_i) - F_i\right] \qquad (5)$$

Finally, we reformulate equation 3 to the following:

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[13]Lawrence M. Metzger "The Power to Compete: The New Math of Precision Managemen," *The National Public Accountant*, May 1993, p. 15.

[14]The literature does not appear to use the term "product margin" in a formally-defined sense, although it appears frequently in professional discourse. Depending on the user's intent and perspective, it could refer to a product's contribution margin, gross margin or net margin, based on any number of cost allocation schemes. We have formally defined it here through equation 4 in order to distinguish the margin for a product as derived by activity-based cost allocation from that given by the traditionally-defined contribution margin.

$$I = C - F = \sum_{i=1}^{n} \left[Q_i(p_i - v_i)\right] - \sum_{i=1}^{n} F_i = \sum_{i=1}^{n} \left[Q_i(p_i - v_i) - F_i\right] = M \qquad (6)$$

The fixed cost portion, F, has disappeared from the income equation, having melded into the total product margin, M.

In practice, the sum of the F_i's will not necessarily equal F. This stems from a modification suggested by Cooper and Kaplan to compensate for underutilized capacity. Rather than burdening products with the total cost of available capacity, this modification only allocates the cost of utilized capacity, allocating the cost of excess capacity to a separate "dummy" product.[15]

Putting equation 6 on a per unit basis, we divide by Q_i to yield the per unit product margin, m_i:

$$m_i = p_i - v_i - (F_i / Q_i) \qquad (7)$$

The ABC decision rule says to eliminate product i if $m_i \leq 0$. Since $Q_i \geq 0$ and M_i = $(Q_i)(m_i)$ for all i, this infers that we will eliminate product i if $M_i \leq 0$. Given this, equation 4 demands that $C_i \leq F_i$.

In order for this decision rule to work, the elimination of product i must truly result in a savings at least equal to F_i to compensate for the lost contribution, C_i. This means that F_i must vary with changes in Q_i. However, this variability will take the form

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[15]Cooper and Kaplan, p. 101.

of a stairstep function, since nonvolume-based costs do not, by definition, vary directly with output. In other words, F_i consists of *stairstep semivariable costs.*[16]

Table 1 details an example where this stairstep feature creates problems for the ABC decision rule. It involves two products, Product A and Product B, each of which draws upon a Common Resource. In real life, this Common Resource might encompass a number of stairstep semivariable costs, related to things such as support staff, facilities, and so on. This example consolidates them into a single resource for the sake of simplicity and clarity. In addition to the Common Resource, Product B also requires a Special Resource, such as a specialized piece of equipment.

Using ABC to allocate a portion of the Common Resource cost to Product B results in a zero margin for that product. Hence, we discontinue its production, which allows us to sell off the Special Resource plus 14 units of the Common Resource. However, the resulting $29,000 in savings falls short of the $30,000 contribution loss, diminishing net profit by $1,000.

In eliminating Product B, we have increased the Common Resource's excess capacity to 104 from a figure of only two. Disposing of an additional unit of Common Resource would recover $2,000, but the corresponding loss of contribution would amount to ($2.00)(207 - 104)/0.10 = $2,060.

The difficulties just discussed result from the interactions among the various products and resources. When management uses product costs to make decisions, those decisions will quite likely change the product costs, and those new costs could easily lead

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[16]DeCoster and Schafer, p. 35.

Table 1. Product Elimination Example

Product Information	**Product A**	**Product B**
Contribution per Product Unit	$2.00	$3.00
Common Resource Activity Units per Product Unit	0.10	0.30
Special Resource Activity Units per Product Unit	0.00	1.00
Total Contribution	$22,758	$30,000
ABC Common Resource Allocation	(11,000)	(29,000)
ABC Special Resource Allocation	0	(1,000)
ABC Margin	**$11,758**	**$ 0**
ABC Margin per Unit	**$1.0333**	**$0.0000**
Resource Information	**Common Resource**	**Special Resource**
Beginning Resource Units	20	1
Cost per Resource Unit (Beginning)	$2,000	$1,000
Value per Resource Unit (Sold)	$2,000	$1,000
Activity Units per Resource Unit	207	10,000
Decision Results	**Keep Product B**	**Eliminate Product B**
Product A Units Produced	11,379	11,379
Product B Units Produced	10,000	0
Common Resource		
Resource Units Eliminated	0	14
Total Capacity in Activity Units	4,140	1,242
Activity Units used by Product A	1,138	1,138
Activity Units used by Product B	3,000	0
Excess Capacity in Activity Units	2	104
Special Resource		
Resource Units Eliminated	0	1
Total Capacity in Activity Units	10,000	0
Activity Units used by Product B	10,000	0
Excess Capacity in Activity Units	0	0
Product A Contribution	$22,758	$22,758
Product B Contribution	30,000	0
Common Resource Cost	(40,000)	(12,000)
Special Resource Cost	(1,000)	0
Net Profit	**$11,758**	**$10,758**

to decisions that contradict the original decision, as we have seen in the preceding example. This leads to a trial-and-error approach, which, although somewhat systematic, does not engender a high degree of confidence in the final conclusion. If one could truly depend on the allocated cost, F_i, to vary in a single, known fashion for each product, then ABC would work. However, the likelihood of this occurring with any regularity seems remote.

This example raises serious questions as to the usability and dependability of the product margin information derived from ABC. Given the extensive analysis required in even these simplified scenarios, one can imagine the formidable task of sorting out the complexities of a real-world application. ABC takes a system of interactive components and attempts to isolate the behavior of certain components, i.e., products. However, as Lawson has pointed out:

> Processes in a business form a system of interdependent components. The goal of management should be to optimize the system, not to maximize the returns to individual components of the system.[17]

Although Lawson made this statement with respect to ABC's inadequacy as a tool for process improvement, our analysis has shown that it also applies to ABC as a decision tool in general.

Issues in Dynamic Modeling

Besides its failure to adequately address the interdependence of components within a system, ABC suffers from an additional flaw. One will recall that in order for us to

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[17]Lawson, p. 34.

assume variability in nonvolume-based costs, we had to expand the relevant range of activity from a short-term to a long-term focus. Long-range decision-making differs substantially from its short-range counterpart, as DeCoster and Schafer explain:

> Long-range decisions have two unique characteristics. First, they involve changes in the productive or service potential of the firm. Second, and equally important, they cover a relatively long time span, so their effect on the firm is best measured in terms of cash flow, adjusted for the time value of money.[18]

So, even though ABC justifies treating nonvolume-based costs as variable by taking a long-range approach, it does not take into account an essential ingredient of such an approach: the time value of money.

ABC has an additional limitation in terms of dynamic modeling. Although it allows one to generate alternative product costs based on different scenarios, each scenario represents a slice of time taken to represent all time periods. This static viewpoint severely limits one's ability to realistically model the dynamics involved in actual business environments. In particular, this approach neglects the important concept of *product life cycle*.[19]

Products typically go through four stages from the time they first appear to their waning years. In the *startup* stage, features which differentiate the product take precedence. The sales price tends to run high, in keeping with the high costs of producing and promoting a new product, which understandably results in low volume. Limited

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[18]DeCoster and Schafer, p. 8.

[19]The concept of product life cycle figures prominently in marketing theory, and its meaning here matches that found in the literature. The particular terminology and ideas used here come from Sammy G. Shina, *Concurrent Engineering and Design for Manufacture of Electronics Products* (New York: Van Nostrand Reinhold, 1991), pp. 24-27.

information about the product and risk aversion on the part of potential customers also contribute to keeping volume low at this stage.

In the second, or *growth* stage, the product experiences a rapid increase in volume as it gains market acceptance. The costs of producing and promoting it drop, with an accompanying decrease in price, which further increases volume and market acceptance. In this stage, the costs fall more rapidly than the price, so profitability tends to run high.

This rapid growth eventually slows as the market becomes saturated and interest wanes due to the entrance of new products. In this *maturity* stage, the product reaches a plateau, with the price typically falling in an attempt to maintain volume. Cost saving improvements generally cannot keep up with the fall in price, leading to an erosion of the profit margin.

Finally, the product enters either a *decline* or a *commodity* stage. In the case of decline, profitability and volume fall to the point where the product eventually leaves the market. Alternatively, enough profit margin and demand may remain to justify keeping the product indefinitely, in which case the product becomes a commodity. As a commodity, the product maintains a large, stable volume, but suffers from low profitability and a lack of product differentiation.

In going through these four stages, a product can experience a variety of changes in terms of cost, pricing, and resource requirements. Hence, the assessment of a product's profitability and the burden it places on an organization's resources depends, to a large degree, on the dynamics of its product life cycle. The static approach of ABC fails to address this issue of a product's characteristics changing with time.

Extensions of Activity-Based Costing

As with any decision modeling technique, ABC has undergone an evolution and synthesis with other methods since its inception. Accordingly, we must assess the impact of these developments on ABC's suitability as a tool for making product/resource mix decisions. Mecimore and Bell have developed a useful framework for analyzing this evolution, in which they identify four distinct generations of ABC.[20]

First-generation methods of ABC focus on activities and whether they add value or not. In second-generation ABC, the focus shifts to processes, with activities combining to form a process. Third-generation approaches take a broader view by linking processes to business units. Fourth-generation methods go one step further by combining business units to form a representation of the entire organization. Mecimore and Bell offer the following explanation for this evolution:

> Most of the attention [of first-generation ABC] was directed toward best use of resources, not processes. While this focus led to better product costing, it did little to help implement JIT, continuous improvement systems, zero defect philosophies, and other current management concepts.[21]

Although these "current management concepts" certainly provide useful tools for improving profitability, the relevance of ABC with respect to them seems rather tenuous. Ironically, the proponents of total quality management (TQM) and just-in-time (JIT) have generally placed very little emphasis on costing methods, reasoning that if one focuses on improving processes, doing things right the first time, keeping inventories low, and

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[20]Charles D. Mecimore and Alice T. Bell, "Are We Ready for Fourth-Generation ABC?" *Management Accounting*, January 1995, pp. 22-30.

[21]Ibid., p. 24.

fulfilling customer requirements, then low costs and high profitability will follow.[22] In fact, detailed cost measurement creates additional transactions which, under the JIT philosophy, one should generally avoid as much as possible.[23]

Advancing ABC through generations which simply combine activities into progressively higher levels (first process, then business unit, then organization) overlooks the fact that process improvement and product/resource mix decisions serve fundamentally different purposes. One can make an existing product more profitable by improving the processes associated with it, but perhaps replacing it with another product would increase profits even more. This does not mean that management should abandon continuous process improvement. It simply means that taking a process-oriented approach does not necessarily provide all the answers for running a successful business.

Firms meet customer needs through the creation of products, so even a company with a strong process orientation must ultimately address the issue of product mix. Products require activities, so even though one might combine activities into processes, processes into business units, and business units into an organization, the critical analysis occurs at the activity level. Resources come into play because activities require them. Thus, all generations of ABC rely on an analysis of how activities draw upon resources in the creation of products, and hence generate product costs in much the same way.

This becomes quite clear when one examines the "process-based costing" system presented by Lawson.[24] In a previous reference, we noted Lawson's argument that

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[22]Arthur R. Tenner and Irving J. DeToro, *Total Quality Management: Three Steps to Continuous Improvement* (New York: Addison-Wesley Publishing Company, Inc., 1992), p. 127.

[23]Thomas E. Vollmann, William L. Berry, and D. Clay Whycark, *Manufacuring Planning and Control Systems*, 3d. ed. (Burr Ridge, IL: Richard D. Irwin, Inc., 1992), p.72.

optimization efforts should focus on the system as a whole and not just its separate components. For Lawson, this entails analyzing the interactions between activities, with costs flowing from resources to "micro activities," and then from these micro activities to various "macro activities." As he puts it:

> In general, if a product requires the consumption of a given activity, the product is consuming not only that activity but also all activities supporting it and preceding it in the process. Thus, in a multistage process, the cost of a unit of output from an activity includes not only the cost of resources consumed in that activity but also the cost of resources consumed in all prior stages of the business process. A superior cost management system must include recognition of this fact.[25]

Although Lawson attempts to distance process-based costing from ABC, it clearly matches what Mecimore and Bell would describe as "second-generation ABC." Despite the added complexity of having activities interact with one another to form processes, this approach still relies on the allocation of costs to activities, and then from activities to products. Also, even though it accounts for interactions among activities, it does not compensate for interactions among products and resources. As we have seen, the stairstep semivariable nature of many resource costs creates difficulties in trying to isolate the impact of altering the product mix. One could feasibly discontinue a product that has a low margin based on ABC, only to find that the margins of the remaining products have gone down due to absorbing the cost of resources which did not change in the face of the product elimination.

Thus, in spite of the evolution of ABC in terms of merging activity analysis with process improvement, its use in product/resource mix decision modeling still depends on

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[24]Lawson, pp. 33-43.
[25]Ibid., p. 36.

the allocation of costs to products based on how those products draw upon resources through the activities they require. This basic feature defines ABC and distinguishes it from all other management tools.

Problem Definition

The analysis of how products draw upon resources through the activities they require can provide useful information for evaluating the desirability of various combinations of products and resources. By incorporating it within the context of schemes such as TQM or reengineering, management can achieve the benefits of process improvement while simultaneously acquiring a firm foundation on which to base product/resource mix decisions. However, ABC does not actually add anything to the latter that did not already exist, and its one distinguishing feature, activity-based cost allocation, does not adequately address the needs of the latter.

We have already seen how product costing can provide valuable information for short-range decisions, where nonvolume-based costs do not vary over the relevant range of activity. Under such circumstances, volume-based costs provide a basis for determining the contribution margin of each product, which establishes product profitability. ABC attempts to improve on this by taking into account the manner in which various products utilize resources which have traditionally represented "fixed" costs. It does this by extending the relevant range to the point where nonvolume-based costs become variable (more specifically, stairstep semivariable), and hence relevant to the assessment of product profitability. However, careful analysis of this approach has revealed that in doing this, ABC creates a situation where confounding interactions can pose difficulties in isolating an individual product's impact on profitability. It also makes the time-value of money a relevant issue, yet offers no provision for addressing this. In addition, it fails to incorporate the dynamics of product life cycles.

Although ABC has serious limitations as a decision tool, the resource utilization analysis underlying this technique can provide important information for evaluating a firm's product/resource mix. Thus, if one could devise a way to incorporate process and activity-based thinking into an approach that addresses the shortcomings of ABC, this would result in a desirable tool for making product/resource mix decisions. In light of the conclusions developed in the preceding examination, this decision tool should:

1) utilize information regarding product contribution margins;
2) incorporate an analysis of how products draw upon a firm's resources through an activity-based approach;
3) address the stairstep variable nature of costs related to these resources;
4) generate results essentially free of confounding interactions among product/resource components;
5) provide the ability to examine a long-range time horizon by accounting for the time-value of money; and
6) further take advantage of a long-term perspective by incorporating the dynamics of product life cycles.

Through the investigation which follows, we will attempt to identify currently available techniques which address these issues and, if necessary, develop new techniques, with the purpose of arriving at an appropriate tool for making product/resource mix decisions. Then, having arrived at such a model, we will use it to analyze and evaluate the effectiveness of ABC as a tool for making product/resource mix decisions.

REVIEW OF THE LITERATURE

Traditional Financial Analysis

The preceding chapter touched upon traditional cost accounting methods and how they relate to or differ from activity-based methods. This discussion involved the assignment of costs to individual products in order to determine product profitability. As we saw, one can determine a product's contribution margin by taking the difference between its price and the variable costs associated with its production. We also found that this contribution margin provides useful information for making short-run decisions.

The product/resource mix problem, however, requires a long-run perspective. As previously noted, this requires using the time value of money. *Traditional financial analysis*, which draws heavily upon the theory of the firm, provides a systematic method for examining the impact on profitability of both short-range and long-range product/resource mix decisions. Although the presentations of this method found in the literature do not make direct use of ABC concepts and terminology, they implicitly depend on the idea that products utilize resources to different extents. In order to understand the financial impact of acquiring or disposing of a particular resource, one must identify the extent to which various products draw upon it.

We have already seen the application of traditional financial analysis in the use of contribution margins for short-term product mix decisions. In order to incorporate the acquisition or disposition of fixed resources in the analysis, one must examine the change in product mix that would result from a specific change in resources. The acquisition of resources would generally lead to increased production, implying an increase in total contribution per time period, while the disposition of resources would result in the opposite. Discounting this impact for each future time period by an appropriate interest rate would yield the net present value of the product/resource mix change. Similarly, the money expended or recovered in acquiring or disposing of fixed resources would generate an associated stream of positive or negative cash flows, which, when discounted, would provide a net present value for the change in fixed resources. The sum of the two net present values would yield the decision's projected impact on profitability.

For example, suppose management can add a product offering a contribution of $45,000 per year by purchasing a machine for $240,000 that will last for ten years and have no salvage value. At an interest rate of ten percent, $45,000 per year for ten years would yield a present value of $276,506. Subtracting the purchase price of the machine produces a net present value of $36,506.

This approach fits situations where a single action (or set of actions) results in a single, identifiable result. In the example just described, acquiring the machine (the action) leads to an increase in contribution (the result). Even when faced with more complex circumstances, one can often divide the problem into separate scenarios, each of which possesses this single action, single result characteristic. Each scenario represents a particular mix of resources and products, each with a corresponding net present value. In order to maximize profitability, one would choose the scenario with the greatest net present value.

In pursuing this sort of analysis, one can either specify the product mix and then use that to determine the appropriate resource additions or subtractions, or specify a change in resources and then identify the product mix or mixes possible within the new resource constraint. The impact of resource changes on cash flow depends greatly on the *permanency* of the resources involved. We have already discussed the pitfalls of labeling expenses related to equipment and the like as "fixed" while treating support staff and similar expenses as "variable." However, these two types of resources do differ in a very significant way. At the heart of this difference lies the issue of permanency.

To illustrate this, suppose the purchase of a certain piece of equipment would lead to the sale of an existing unit and an increase in support staff. In the context of traditional financial analysis, the purchase and the sale would each appear only once in the stream of cash flows.[26] The cost of additional support staff, however, would appear in each time period for as long as the new resource remains in use. In other words, resources such as equipment and other "fixed assets" have a higher level of permanency than "indirect operating costs" such as support staff.

One can find this distinction and the associated notion of permanency deeply rooted in traditional accounting theory. As Mosich and Larsen explain:

> Initial expenditures that are included in the cost of assets are called *capital expenditures,* and such expenditures are commonly said to be *capitalized*; expenditures treated as expenses of the current accounting period are called *revenue expenditures*. This terminology, while not ideal, is satisfactory and is widely used.[27]

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[26]Although the purchase would also typically imply purchases in the future for replacment units, one could incorporate these costs into the intitial purchase price by adding the present value of a perpetuity representing those future purchases.

[27]A. N. Mosich and E. John Larsen, *Intermediate Accounting*, 5th ed. (New York: McGraw-Hill Publishing Co., 1982), p. 442.

They elaborate on this as follows:

> The theoretical test to distinguish between a capital expenditure from a revenue expenditure is simple: Have the services acquired been consumed entirely within the current accounting period, or will there be a carryover of services to future periods?[28]

Having established the inappropriateness of the terms "fixed" and "variable" for distinguishing between the two categories of resources, we might choose to borrow from the terminology used by Mosich and Larsen. In doing this, we would refer to a piece of equipment as a "capital resource" and support staff as a "revenue resource." Although this would provide consistency with standard accounting terminology, the term "revenue resource" does not adequately convey the meaning intended. Even the term "capital resource" could potentially create confusion, since those who deal with financial matters often think of funds available from creditors and investors as capital resources.

Given the lack of suitable existing nomenclature, we will use the terms *permanent* and *temporary* to distinguish between the two types of resources. Admittedly, even such "permanent" resources as equipment eventually require replacement as firms consume their useful life. However, the terminology proposed here has the useful traits of simplicity, clarity, and familiarity, which easily overcome the minor issues just mentioned.

These ideas will come to greater light in the context of a mathematical framework. Accordingly, let u_{itz} represent the units of resource i in time period t under scenario z, where $i = 1, 2, \ldots, r$ for permanent resources and $i = r+1, r+2, \ldots, m$ for temporary

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[28]Ibid.

resources, with $t = 1, 2, \ldots, p$ and $z = 1, 2, \ldots, s$. The variable v_{itz} will represent the units of permanent resource i disposed of in time period t under scenario z, where $i = 1, 2, \ldots, r$. Also, let x_{jtz} represent the change in the quantity of product j produced in time period t under scenario z, where $j = 1, 2, \ldots, n$. For each time period, t, each permanent resource has a discounted purchase cost per unit, d_{it}, and discounted salvage value per unit, e_{it}, where $i = 1, 2, \ldots r$. Likewise, each temporary resource has a discounted cost per unit, d_{it}, for each time period, where $i = r+1, r+2, \ldots m$. However, since temporary resources provide service for only one period, they have no salvage value. Finally, each product has a discounted contribution margin per unit associated with each time period, denoted by c_{jt}. Letting NPV_z represent the overall net present value for scenario z, we have:

$$NPV_z = \sum_{t=1}^{p} \sum_{j=1}^{n} c_{jt} x_{jt} - \sum_{t=1}^{p} \sum_{i=1}^{m} d_{it} u_{it} + \sum_{t=1}^{p} \sum_{i=1}^{r} e_{it} v_{it}$$

$$\text{for } z = 1, 2, \ldots, s \qquad (8)$$

In developing each scenario, one specifies the changes in resources by setting the values for the u_{it}'s, and v_{it}'s. Changing the resources constitutes an action which alters the activity units available to support various products. This action results in a set of possible changes in product mix. Each action/result combination defines a scenario, with the x_{jt}'s coming from the u_{it}'s, and v_{it}'s through the link formed between products and resources by activities.

Product contribution margins play a key role in this sort of analysis. They provide essential information for determining the cash flows associated with various product mixes. Also, in order to ascertain the impact of purchasing or eliminating resources on product mix, one must understand how products draw upon those resources. The stairstep variable nature of resource costs also comes into play. Having different scenarios allows resource costs to vary, but only in a stepwise fashion. In addition, traditional financial analysis examines a sequence of time periods over some relevant range. Hence, it can take into account the dynamics of product life cycles, as well as incorporate the time value of money.

The preceding comments address five of the six criteria we established for evaluating product/resource mix decision tools. The fourth criterion, however, presents a problem. In order to avoid difficulties with confounding interactions, one would typically have to enumerate many combinations of actions and results. Although spreadsheet software can make such analysis relatively palatable in many cases, problems often reach a level of complexity that makes this approach impractical.

This calls for a technique that can evaluate a large number of possible scenarios and identify the most profitable one. *Linear programming* provides a reasonably straightforward means of accomplishing this.

Linear Programming

Linear programming has long served as an effective tool for finding the optimal allocation of resources among competing activities.[29] In particular, it has proved its usefulness with respect to identifying the most profitable product mix given a set of resource constraints.[30] Hence, one can readily see its applicability to the product/resource mix problem under consideration here.

The mathematical formulation of a linear programming problem has the following *standard form*:[31]

Maximize $$Z = \sum_{j=1}^{n} c_j x_j \qquad \text{(9a)}$$

subject to $$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad \text{for } I = 1, 2, \ldots, m \qquad \text{(9b)}$$

$$x_j \geq 0 \qquad \text{for } j = 1, 2, \ldots, n \qquad \text{(9c)}$$

In the above set of expressions, which we will refer to collectively as formulation 9, Z represents the value of a linear function, called the *objective function*, that one wishes to optimize. This objective function, specified by equation 9a, consists of *decision variables*, represented by the x_j's, and *parameters* (input constants), denoted by the c_j's. The a_{ij}'s and b_i's appearing in inequality set 9b represent additional parameters.

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[29]Frederick S. Hillier and Gerald J. Lieberman, *Introduction to Operations Research*, 5th ed. (New York: McGraw-Hill Publishing Co., 1990), p. 29.

[30]Ibid., p. 31.

[31]Ibid., p. 35.

Inequality sets 9b and 9c specify restrictions called the *functional constraints* and *nonnegativity constraints*, respectively.[32]

Equation 9a bears a striking resemblance to equation set 8. The c_{jt}, d_{it} and e_{it} terms in equation set 8 correspond to the c_j's in equation 9a, while the x_{jt}, u_{it}, and v_{it} terms parallel the x_j's. In essence, rather than having a fixed number of equations with set values for the decision variables, each corresponding to a different scenario, equation 9a incorporates an infinite number of scenarios by allowing the the x_j's to take on any value. Rather than a finite number of NPV_z's, one has an infinite number of Z values from which to choose.

As originally defined, the x_{jt}'s represent changes in product quantities. However, since linear programming can evaluate every possible product mix, it makes sense to redefine them as total product quantity. This makes the analysis more straightforward by eliminating the need to work from some initial product mix. It also makes x_{jt} consistent with the nonnegativity constraints. The nonnegativity constraints for u_{it}, and v_{it} already followed naturally from the reality that physical quantities cannot take on negative values.

In the financial analysis approach associated with equation set 8, the ways in which different products utilize resources create limitations that define the various scenarios. Linear programming can use the same information, but rather than incorporating the information on resource utilization directly into the objective function, it imposes these

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[32]Ibid., pp. 35-36.

restrictions through the functional constraints. Taking an activity-based point of view, we need to define resource utilization in terms of activities. This means finding the relationship between *product* units and *activity units,* then between *activity units* and *resource capacity units*, and finally between *resource capacity units* and *resource units.*

As an example, suppose a given resource can support activities A, B, and C, with each unit of activity A requiring one resource capacity unit, each unit of B taking up two resource capacity units, and each unit of C using three units of resource capacity. Relating this to a particular product, assume each unit of product requires 20 units of activity A, 15 units of activity B, and 10 units of activity C. Accordingly, each product unit will require (1)(20) + (2)(15) + (3)(10) = 80 resource capacity units. If the resource capacity units available each period equals 16,000, then one could produce up to 16,000/80 = 200 units of this product each period.

From this sort of analysis, one can establish a parameter, a_{ijt}, for each of resource i, product j, and period t, that specifies the units of resource capacity required for each unit of product. One can also establish a variable, b_{it}, representing the capacity of resource i in period t. Combining these with the notation used in equation set 8, we can restate formulation 9 as follows:

$$\text{Maximize} \quad Z = \sum_{t=1}^{p} \sum_{j=1}^{n} c_{jt} x_{jt} - \sum_{t=1}^{p} \sum_{i=1}^{m} d_{it} u_{it} + \sum_{t=1}^{p} \sum_{i=1}^{m} e_{it} v_{it} \quad (10a)$$

$$\text{subject to} \quad \sum_{j=1}^{n} a_{ijt} x_{jt} \le b_{it} \quad \text{for } i = 1, 2, \ldots, m;\ t = 1, 2, \ldots, p \quad (10b)$$

$$x_{jt} \ge 0 \quad \text{for } j = 1, 2, \ldots, n;\ t = 1, 2, \ldots, p \quad (10c)$$

$$u_{it} \geq 0, \; v_{it} \geq 0 \qquad \text{for } i = 1, 2, \ldots, m; t = 1, 2, \ldots, p \qquad \text{(10d)}$$

Unfortunately, formulation 10 does not express the problem in standard form, since the b_{it}'s do not represent parameters with constant values. Instead, each b_{it} depends on the total resource i capacity at time t, which equals the initial number of resource units, plus the units purchased up to that time, less the units sold up to that time, all times the capacity per resource unit. This will require additional parameters.

We can derive the needed restatement of inequality set 10b by setting up a constraint for each resource i, where $I = 1, 2, \ldots, m$, and time period t, where $t = 1, 2, \ldots, p$. We will let b_{it} denote the resource capacity units and q_i: the resource activity units for each unit of resource i. Since permanent resources provide capacity beyond the period of acquisition and temporary resources do not, the form of their associated constraints will differ. Thus, each permanent resource i will also have a u_{i0} representing the initial units of that resource, where $i = 1, 2, \ldots, r$. This gives us the following expression for permanent resources:

$$\sum_{j=1}^{n} a_{ijt} x_{jt} \leq b_{it} \rightarrow \sum_{j=1}^{n} a_{ijt} x_{jt} \leq q_i u_{i0} + \sum_{T \leq t} q_i u_{iT} - \sum_{T \leq t} q_i v_{iT}$$

$$\rightarrow \sum_{j=1}^{n} a_{ijt} x_{jt} - \sum_{T \leq t} q_i u_{iT} + \sum_{T \leq t} q_i v_{iT} \leq q_i u_{i0}$$

$$\text{for } i = 1, 2, \ldots, r; t = 1, 2, \ldots, p \qquad \text{(11a)}$$

Modifying this for temporary resources yields:

$$\sum_{j=1}^{n} a_{ijt} x_{jt} - q_i u_{it} \leq 0 \qquad \text{for } i = r+1, \; r+2, \ldots, m; t = 1, 2, \ldots, p \qquad \text{(11b)}$$

Replacing inequality set 10b with inequalities 11a and 11b allows us to restate formulation 10 in standard form:

Maximize $$Z = \sum_{t=1}^{p} \sum_{j=1}^{n} c_{jt} x_{jt} - \sum_{t=1}^{p} \sum_{i=1}^{m} d_{it} u_{it} + \sum_{t=1}^{p} \sum_{i=1}^{r} e_{it} v_{it} \quad (12a)$$

subject to $$\sum_{j=1}^{n} a_{ijt} x_{jt} - \sum_{T \leq t} q_i u_{iT} + \sum_{T \leq t} q_i v_{iT} \leq q_i u_{i0}$$

for $i = 1, 2, \ldots, r$; $t = 1, 2, \ldots, p$ (12b)

$$\sum_{j=1}^{n} a_{ijt} x_{jt} - q_i u_{it} \leq 0$$

for $i = r+1, r+2, \ldots, m$; $t = 1, 2, \ldots, p$ (12c)

$x_{jt} \geq 0$ for $j = 1, 2, \ldots, n$; $t = 1, 2, \ldots, p$ (12d)

$u_{it} \geq 0$, $v_{it} \geq 0$ for $I = 1, 2, \ldots, m$; $t = 1, 2, \ldots, p$ (12e)

Linear programming, then, can augment traditional financial analysis in a way that avoids confounding interactions among product/resource components. However, in taking such an approach, one loses the ability to treat resource costs as stairstep semivariable, since the u_{it} and v_{it} decision variables can take on noninteger values. Overcoming this difficulty requires the addition of constraints which restrict decision variables to integer values. Doing this transforms a linear programming (LP) model into an *integer programming* (IP) model.[33]

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[33]Ibid., p. 457.

Integer Programming

Moving to an IP formulation of the problem satisfies all six of our criteria for decision tool selection. However, this also introduces computational hurdles, due to the ironic reality that examining a large but finite number of solutions usually presents a more complex problem than confronting an infinite number of solutions.[34] Accordingly, one should keep the number of integer decision variables to a minimum, and also limit the range of values that each of those variables can take.

Although firms usually sell products in discrete units, the large numbers typically involved make the use of integer values superfluous. For example, one might question the practicality of distinguishing between 9,999 units and 9,998 units, or even between 99 units and 98 units. Thus, the integer restrictions applied to formulation 12 would normally only relate to the the u_{it} and v_{it} decision variables, and not to the the x_j's. Restricting only some of the variables to integer values creates a *mixed integer programming* (MIP) model.[35]

The relevant IP models found in the literature generally center around *capital budgeting* issues. Capital budgeting relates to the current discussion in that it addresses the question of allocating limited resources to competing projects. By treating products as "projects," one can see a close parallel between the product/resource mix problem and capital budgeting.

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[34]Ibid., p. 466.
[35]Ibid., pp. 457-458.

In one presentation, Karabakal, Lohmann, and Bean use a zero-one integer program to solve what they refer to as "the replacement problem."[36] Just as we have emphasized the need to isolate product/resource interactions, they underscore the importance of addressing "economic interdependencies":

> In serial replacement, it is common to assume that the firm has sufficient capital so that, for all individual assets, indicated capital replacement expenditures can be financed in any time period over the planning horizon. In practice, however, firms frequently use *budgets* to control their expenditures. In this case, it is necessary to consider all replacement decisions in each time period together since competition for the limited funds creates interdependent problems.[37]

As one might expect, Karabakal et. al. have incorporated net present values in their objective function. Although their research objective focused on the development and demonstration of a particular algorithm, it bears relevance to the issues addressed here in that it:

1) confirms the need to account for interdependencies;
2) gives an example of using net present values in an IP objective function; and
3) demonstrates a practical application of IP to a large scale capital budgeting problem.

One finds the theme of interdependencies echoed in a study by Kumar and Lu.[38] It also provides an additional example of using net present values in a IP objective function, and provides another demonstration of IP applied to a large scale capital

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[36]Nejat Karabakal, Jack R. Lohmann, and James C. Bean "Parallel Replacement under Capital Rationing Constraints," *Management Science* 40 (March 1994): 305.

[37]Ibid., p. 306.

[38]P.C. Kumar and Trami Lu "Capital Budgeting Decisions in Large Scale Integrated Projects: Case Study of a Mathematical Programming Application," 36 *The Engineering Economist* (Winter 1991): 127-50.

budgeting problem. It also provides an example of a MIP model, with product output treated as continuous and resources treated as integer.

The IP models related to capital budgeting that appear in the literature, including the two just mentioned, involve integer variables that can only take on the values zero or one. Linear programming models that restrict the decision variables in this way fall into a category called binary integer programmiing (BIP).[39] The capital budgeting problem, then, has focused on decisions where management needs to decide whether to acquire a particular resource or not. The "yes/no" nature of this decision process makes BIP a suitable approach.

Our concern, however, involves developing a model that can entertain the possibility of acquiring or disposing of *multiple* units of a resource. This raises the question of whether the articles cited give an adequate indication of the practicality of a MIP model based on formulation 12. The answer lies in turning to a technique that translates general integer variables into binary representations.

To illustrate this, consider the variable u_{it}, which represents the number of units of resource i purchased at time t. For the time being, we will leave v_{it} (resource units sold) out of the model. Now, suppose that u_{it} can take on any integer value from zero to seven, and we define three binary variables, u_{it1}, u_{it2}, and u_{it3}, creating the following relationship:

$$u_{it} = u_{it1} + 2u_{it2} + 4u_{it3} \tag{13}$$

[39]Hillier and Lieberman, p. 458.

Since the various combinations of u_{it1}, u_{it2}, and u_{it3} can produce any value from zero to seven, substituting the right side of equation 13 wherever u_{it} appears will create a binary integer formulation of the problem. We can use the same technique to form a binary representation for v_{it}.

In order to approximate the CPU time that a real-world problem would require (based on formulation 12 with integer constraints on u_{it} and v_{it}), we can translate the integer variables into a binary representation, determine the number of variables and constraints that one might expect, and then note the results obtained by Karabakal et. al. for a similar sized problem. For the purposes of making such an approximation, suppose that we have five time periods and ten resources, and that we can purchase or sell as many as seven units of each resource. This yields (5)(10)(3)(2) = 300 binary integer decision variables. Since the complexity comes from the integer variables, the continuous variables representing product quantities should not have that great an impact on CPU time, so for the purposes of this approximation, we will only count the integer variables. The number of functional constraints equals (5)(10) = 50. Thus we have 50 constraints and 300 binary variables, for a problem of size 50 X 300.

One of the problem configurations presented by Karabakal et. al. yielded a median CPU time of 15.50 seconds and a maximum CPU time of 17.79 seconds for a problem with 54 constraints and 362 binary variables (54 X 362). Another configuration produced median and maximumu times of 55.56 and 114.84 seconds, respectively, for a 62 X 309 problem. A third configuration yielded median and maximum times of 89.73 and 141.66 seconds, respectively, for a 62 X 303 problem. Karabakal et. al. used 33.33 minutes as the maximum CPU time limit, and only hit that limit for the median time for one of the five configurations when it reached a size of 98 X 565.[40] They obtained these results using an algorithm coded in Pascal running on an IBM 3090-600E under the MTS operating system.[41]

In practice, one would probably formulate the problem using general integer rather than binary integer variables, since this would provide a more straightforward interpretation of the solution. The technique of binary representation used here has served the sole purpose of estimating the amount of CPU time that one might expect to encounter. Although highly speculative, this approximation at least offers some evidence that the type of model contemplated here has the potential to serve as a practical decision tool for solving product/resource mix problems. As we have seen, this approach satisfies all six of the criteria set forth in the first chapter. Also, as we have just demonstrated, it has the potential to solve realistically sized problems within a satisfactory span of computer processing time.

Additional Issues

So far, we have assumed *perfect competition*, in which the firm accepts a price dictated by the market no matter what quantity it produces.[42] Although we wish to limit our scope to businesses operating in competitive markets, this does not necessarily imply

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[40]Karabakal et. al., p. 317.

[41]Ibid., p. 316.

[42]Walter Nicholson, *Microeconomic Theory: Basic Principles and Extensions*, 6th ed. (Fort Worth, TX: The Dryden Press, 1982), p. 447.

perfect competition. Even in the face of competition, firms often have the ability to alter their market share through pricing strategy. Under such conditions, price enters the profit equation as a variable rather than a constant, leading to a nonlinear model.[43]

In practical application, a model does not necessarily have to represent price as a continuous variable in order to account for pricing strategy. For example, in applying *game theory* to pricing decisions, a firm works from a finite set of strategies.[44] One could feasibly limit this set to just three strategies: "charge higher than the competition," "charge lower than the competition," or "meet the competition." Using this approach in the context of a MIP model, one could simply run the model three times: once for each strategy. The overall optimal solution would equal the best of the optimal solutions generated for each of the three strategies.

Another issue that complicates the use of a MIP model relates to the possibility of multiple optimal solutions. In identifying an optimal solution, a MIP model can only guarantee that no better solution exists. Given that business decisions often involve subjective considerations that do not find their way into the mathematical model, one could benefit from the identification of possible alternative solutions. Unfortunately, the intensive computation typically involved in running a MIP model makes such identification impractical in most cases. Hence, the model should address as many relevant issues as possible so the one optimal solution obtained will suffice.

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[43]Ibid., p. 640.
[44]Ibid., p. 672.

MIP Model for Evaluating ABC

The activity-based analysis encompassed by ABC methods provides a useful means for identifying and specifying relationships between products and resources. Although these methods have found wide applicability in the areas of product costing and process improvement, they have not adequately addressed the issue of optimizing profitability through product/resource mix decisions. The idea that one can optimize net present value (and hence profitability) through a MIP model that treats product quantities as continuous variables and resources as integer variables has already appeared in the literature. However, the relevant research has dealt primarily with capital budgeting issues, which characterize resource decisions in a "yes/no" vein. This does not quite match the needs of the product/resource mix problem under consideration here, which allows for the possibility of adding or deleting multiple units of any given resource. These capital budgeting models also do not explicitly incorporate ABC tools or terminology.

One can address the limitations of the methods previously described by using the following MIP model as a tool for supporting real-world product/resource mix decisions:

$$\text{Maximize} \quad Z = \sum_{t=1}^{p} \sum_{j=1}^{n} c_{jt} x_{jt} - \sum_{t=1}^{p} \sum_{i=1}^{m} d_{it} u_{it} + \sum_{t=1}^{p} \sum_{i=1}^{r} e_{it} v_{it} \qquad (14a)$$

$$\text{subject to} \quad \sum_{j=1}^{n} a_{ijt} x_{jt} - \sum_{T \le t} q_i u_{iT} + \sum_{T \le t} q_i v_{iT} \le q_i u_{i0}$$

for $i = 1, 2, \ldots, r$; $t = 1, 2, \ldots, p$ (14b)

$$\sum_{j=1}^{n} a_{ijt} x_{jt} - q_i u_{it} \le 0$$

for $i = r+1,\ r+2, \ldots, m;\ t = 1, 2, \ldots, p$ (14c)

$x_{jt} \le D_{jt}$ for $j = 1, 2, \ldots, n;\ t = 1, 2, \ldots, p$ (14d)

$x_{jt} \ge M_{jt}$ for $j = 1, 2, \ldots, n;\ t = 1, 2, \ldots, p$ (14e)

$x_{jt} \ge 0$ for $j = 1, 2, \ldots, n;\ t = 1, 2, \ldots, p$ (14f)

$u_{it} \ge 0$ and general integer, $v_{it} \ge 0$ and general integer

for $I = 1, 2, \ldots, m;\ t = 1, 2, \ldots, p$ (14g)

In this restatement of formulation 12 with integer constraints on u_{itz} and v_{itz}, the variables represent the following:

x_{jt} quantity of product j produced in time period t

u_{it} units of resource i acquired in time period t

v_{it} units of resource i disposed of in time period t

c_{jt} discounted contribution margin per unit of product j at time t

d_{it} discounted acquisition cost per unit of resource i at time t

e_{it} discounted salvage value per unit of resource i at time t

a_{ijt} capacity units of resource i required per unit of product j in time t

q_i resource activity units per unit of resource i

u_{i0} initial units of permanent resource I

D_{jt} maximum demand for product j in time period t

M_{jt} minimum demand for product j in time period t.

As discussed in the section on pricing issues, one can incorporate pricing strategies into the model by running a different version for each strategy and then selecting the most profitable version. The above model includes constraints for market demand, which would typically vary depending on the pricing strategy. The model could also incorporate additional constraints to account for other relevant issues that may arise.

Our examination of the literature, then, has led to a mathematical model, expressed in formulation 14, that can serve as a sound basis for product/resource mix decisions. This makes it an appropriate point of reference for judging the effectiveness of ABC as a decision tool. However, one cannot realistically expect to conduct this evaluation with the general form of the model, given the extreme complexity of the relationships involved. Therefore, we must find an appropriate set of simplifying assumptions that will produce a practical model for evaluating ABC.

METHODOLOGY

Simplifying Assumptions

As noted at the end of the preceding chapter, we need to simplify the MIP model given by formulation 14 in a manner that will create a practical means for evaluating ABC as a decision tool. We will approach this simplification from a variety of tacks.

In attempting to describe complex processes, economic models often apply the concept of *ceteris paribus*.[45] This generally involves isolating the behavior of one part of the system and treating everything else as an aggregate. The duality that this creates makes for a much clearer analysis, and also facilitates graphic representations by reducing the problem to two dimensions. In the present context, we can apply this principle by:

1) dividing the product mix into two products: one representing a product under consideration for addition or elimination, and another representing all other products;
2) dividing the resource mix into two resources: one associated strictly with the product of interest, and the other corresponding to a common resource pool.

Another area for simplification lies in the categorization of costs. As previously discussed, all costs vary in the long run. However, costs differ in how they vary. Some increase proportionally with units produced. Others vary in a stairstep fashion. Those in the latter group do not all vary in the same way, though. Some relate to the cost of setting up a batch, run or lot, bearing a close relationship to short run production volume, but not varying continuously. Others have more of a distinct disconnect from short run

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[45]Ibid., p. 7.

production volume, such as full-time support staff or capital assets. Within this last grouping, one can distinguish between "temporary" expenditures that one can discontinue on short notice, such as staff salaries, and "permanent" expenditures associated with assets that one must sell or scrap. This presents us with four cost categories, each with potentially different behaviors.

We can make the analysis more manageable by reducing the cost categories to two: *continuous* and *discrete*. We accomplish this by treating both volume-based and batch-related costs as continuous, and all other costs as discrete.

The first consolidation of cost categories requires that we make an assumption about the size and frequency of batches. For time frames associated with short run decisions, one often finds that the number of units in a batch accounts for a small portion of the total volume produced. In such cases, muliplying cost per unit times the number of units will yield a total cost not significantly different from that obtained by multiplying cost per batch times the number of batches. Thus, for most practical purposes, we can greatly simplify the analysis without harming its relevance by treating batch-related costs the same as volume-based costs. In a similar fashion, one can argue that even though products often come in discrete units, treating them as continuous in a mathematical sense does not harm the usefulness of the analysis. Using the assumptions just discussed, we will model both volume-based and batch-related costs using continuous variables.

The other consolidation requires an assumption regarding the cost of capital assets. In the case of new assets, one can usually arrange some form of lease in lieu of an outright purchase, either through the vendor or a third party financing company. For existing assets, one could feasibly sell them to a company that would lease them back to the firm on a monthly payment schedule. By assuming that one can translate any set of asset-related costs into an equilavent set of lease payments, the difference between temporary and permanent costs disappears.

In order to facilitate our examination, then, we will work from a two-product, two-resource model, treating costs as either continuous or discrete. In assembling this simplified model, we must take care that we adequately address the key issues examined earlier.

Simplified Model

In our previous analysis, we identified four main issues:

1) the tendency for ABC to distort profitability in situations of excess capacity;
2) the limitations of ABC in dealing with confounding interactions among products and resources;
3) the problems inherent in treating stepwise variable costs as continuously variable; and
4) the failure of traditional ABC to incorporate product life cycles and the time value of money.

Our present challenge lies in reconciling the two *ceteris paribus* assumptions (two products and two resources) and the assumptions regarding cost categorization with these four issues.

With respect to capacity utilization, the examples given earlier related only to the elimination of a product in the face of underutilized capacity. In order to acquire a more general understanding of the problem, our examination will focus on two key dimensions:

1) whether the decision involves the elimination or addition of a product or product line; and

2) whether or not the firm can adapt capacity utilization in the face of product additions or deletions.

Although we could examine the general case where we have two products for which we want to determine the most profitable product mix, we can address the relevant issues in a simpler yet satisfactory manner by considering two scenarios:

1) we have two existing products, A and B, and wish to determine whether to keep or eliminate product B; or

2) we have an existing product, A, and wish to determine whether or not to add product B.

We can further simplify the model by assuming that Product A and Product B require a set of common resources, consolidated into what we will call Resource 1. Product A will require no other resources, while Product B will require additional resources, consolidated into Resource 2. Since both products will draw upon a common resource, the model will enable us to investigate both the issue of capacity utilization and the issue of confounding interactions. These correspond to the first two issues listed above.

By treating costs as either continuous or discrete, we obtain a simplified model that still provides a means for studying the impact of stepwise semivariable costs. Thus, our simplified model will also enable us to address the third issue. This leaves one issue remaining, which involves product life cycles and the time value of money.

Although ABC provides a static framework for modeling product profitability, this framework does not preclude the use of a dynamic approach in setting up the model. Just as we translated a capital expenditure into a periodic expense by equating it to a series of lease payments of an equal net present value, one can treat each of the static parameters of an ABC model as the net present value of that parameter's values through time. For example, rather than using depreciation expense to measure the cost of a piece of equipment, one could use the net present value of retaining and replacing that piece of equipment. One could take a similar approach to reflect changes occurring through a product's life cycle.

Given that one could feasibly implement ABC using a dynamic approach, the fourth issue does not possess the same level of significance as the other three. Thus, our investigation will not suffer unduly from its omission. Restating formulation 14 to match the simplified model just discussed yields:

Maximize $Z = c_A x_A + c_B x_B - d_1 u_1 - d_2 u_2 + e_1 v_1 + e_2 v_2$ (15a)

subject to $a_{1A} x_A + a_{1B} x_B - q_1 u_1 + q_1 v_1 \leq q_1 b_1$ (15b)

$a_{2B} x_B - q_2 u_2 + q_2 v_2 \leq q_2 b_2$ (15c)

$x_A \leq D_A$ (15d)

$x_B \leq D_B$ (15e)

all variables nonnegative (15f)

b_1, b_2, u_1, u_2, v_1 and v_2 general integer (15g)

These variables represent the following:

x_A quantity of Product A produced

x_B quantity of Product B produced

u_1 units of Resource 1 acquired

u_2 units of Resource 2 acquired

v_1 units of Resource 1 eliminated

v_2 units of Resource 2 eliminated

c_A contribution margin per unit of Product A

c_B contribution margin per unit of Product B

d_1 acquisition cost per unit of Resource 1

d_2 acquisition cost per unit of Resource 2

e_1 salvage value per unit of Resource 1

e_2 salvage value per unit of Resource 2

a_{1A} capacity units of Resource 1 required per unit of Product A

a_{1B} capacity units of Resource 1 required per unit of Product B

a_{2B} capacity units of resource 2 required per unit of Product B

q_1 resource activity units per unit of Resource 1

q_2 resource activity units per unit of Resource 2

b_1 beginning units of Resource 1

b_2 beginning units of Resource 2

D_A demand for Product A

D_B demand for Product B.

We can further simplify the problem by dividing it into two scenarios. In one, which we will refer to as the *elimination scenario*, the hypothetical operation has both Resource 1 and Resource 2 available and produces both Product A and Product B. Within this context, management must decide whether to keep or eliminate Product B. In the second, which we will call the *acquisition scenario*, the decision involves whether or not to acquire Resource 2 in order to produce Product B.

Within each scenario, management has two alternatives: make Product B or not make Product B. Suppose management chooses between these two alternatives using ABC. Doing this effectively imposes an additional constraint on formulation 15. Deciding to produce Product B implies $x_B > 0$, while deciding against Product B means $x_B = 0$. For each of the two alternatives, we can solve the corresponding MIP model, then compare the resulting optimal *Z* values.

When a constraint becomes active, it limits the profit that one would have obtained otherwise. One could liken it to a broad jumper wearing weights; he or she will never achieve the same distance as without the weights. Thus, imposing an active constraint necessarily reduces the optimal value of *Z*, so one should view ABC in a positive light only as long as the added constraint remains inactive. As we will see, this concept provides a basis for evaluating the effectiveness of ABC as a tool for making product/resource mix decisions.

Evaluation Methodology

The MIP model provides an optimal solution given the assumptions specified by management. Given realistic assumptions, a solution should exist. Also, even if multiple solutions exist, management at least knows that it cannot do better under the assumptions given. Implementing such a model, however, could prove daunting, and possibly not cost-effective. It does, nonetheless, provide a viable point of reference for evaluating the effectiveness of other approaches.

The ABC approach may provide a more accessible and possibly cost-effective solution. As we have seen, though, it has a potential for misleading results which can lead to inappropriate decisions. Before applying it, then, one should have a clear understanding of its shortcomings. This involves identifying the conditions under which it will or will not function in a useful way.

As mentioned previously, one can use ABC to choose between $x_B > 0$ and $x_B = 0$. Doing this implies the imposition of a constraint on formulation 15. If this constraint has no impact on the optimal MIP solution, then we can judge the ABC-based decision as proper and useful. However, if it becomes active, it can only reduce the optimal solution. Thus, if requiring $x_B > 0$ diminishes profitability, then we can say that $x_B = 0$ would have yielded a preferable solution. Likewise, if ABC specifies that $x_B = 0$ and this constraint becomes active, then it follows that $x_B > 0$ would have generated a solution with better profitability.

For each scenario, we can solve formulation 15 under two different assumptions, corresponding to the choices $x_B > 0$ and $x_B = 0$. We will refer to the optimal solutions as Z and Z',[46] respectively, and their difference as:

$$P = Z - Z' \tag{16}$$

Letting m_B represent the ABC product margin per unit of Product B, ABC will function effectively when the following conditions hold:

1) $m_B = 0 \rightarrow P = 0$;

2) $m_B > 0 \rightarrow P > 0$; and

3) $m_B < 0 \rightarrow P < 0$.

In the first instance, ABC dictates indifference between keeping Product B and eliminating it, or between adding Product B and not adding it. Hence, Z and Z' should come out the same, which means:

$$Z = Z' \rightarrow Z - Z' = 0 \rightarrow P = 0 \tag{17}$$

In the second instance, a positive ABC margin implies that setting $x_B = 0$ would constrain profitability. This means that :

$$Z > Z' \rightarrow Z - Z' > 0 \rightarrow P > 0 \tag{18}$$

Finally, a negative ABC margin indicates that adding the constraint $x_B > 0$ should reduce profitabilty, implying that:

$$Z < Z' \rightarrow Z - Z' < 0 \rightarrow P < 0 \tag{19}$$

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[46]By convention, one would typically use Z^* to represent a specific optimal solution. However, since we want to compare two different optimal solutions, Z and Z', we have left off the asterisk in order to keep the notation clean. Given this, one needs to take special care to keep in mind that Z and Z' represent particular optimal solutions under two different conditions.

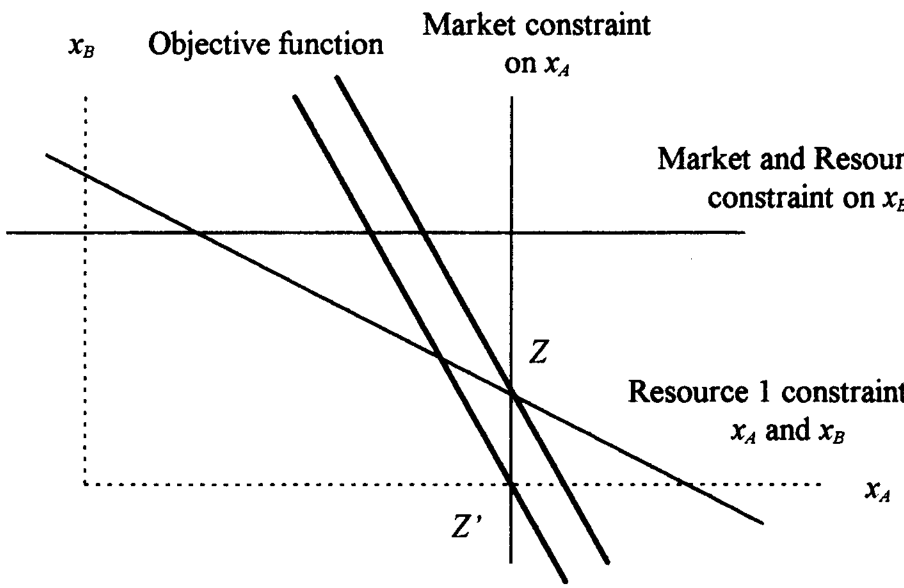

Figure 2. Graph comparing optimal solutions: $P > 0$

Figures 2, 3 and 4 provide graphic representations of these conditions. Focusing on just the decision variables correponding to the quantities produced of Products A and B, they depict three different arrangements of the constraints and objective functions. In each figure, one can see the position of the objective function line for each optimal solution, *Z* and *Z'*.

In Figure 2, we start with an optimal solution, *Z*, at a corner point satisfying $x_B > 0$. Then, if we impose the constraint $x_B = 0$ on the system, we see the optimal solution drop to a corner-point on the x_A axis. This new optimal solution, *Z'*, results in lower net profit, so $Z > Z' \rightarrow Z - Z' > 0 \rightarrow P > 0$. Thus, if ABC yields a positive Product B margin, then one will correctly choose $x_B > 0$, and hence obtain the preferable solution, *Z*, over the less profitable solution, *Z'*.

Figure 3 shows the opposite situation. Here, we begin with an optimal solution, Z', where $x_B = 0$. If, however, we impose the constraint $x_B > 0$, then the objective

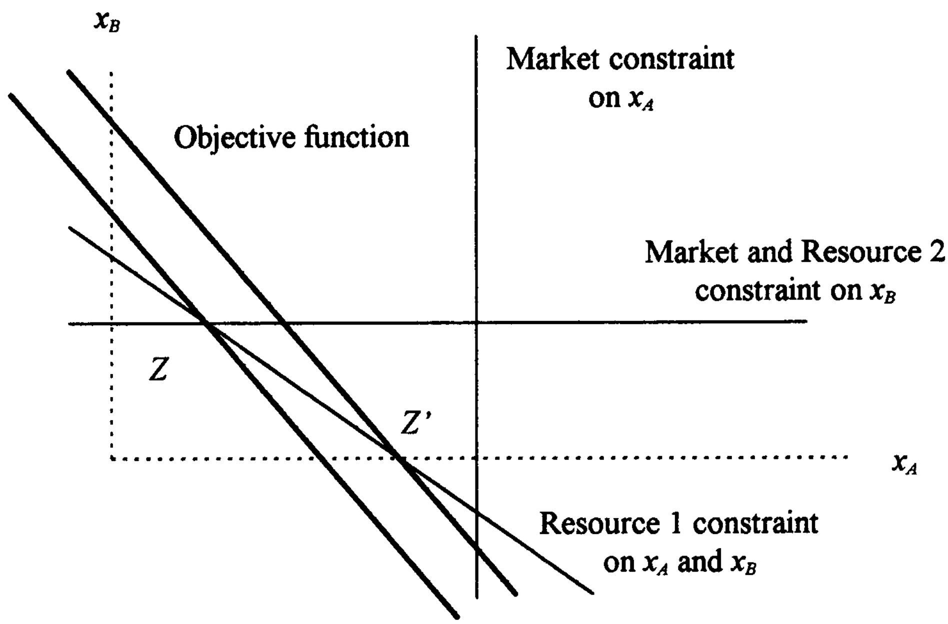

Figure 3. Graph comparing optimal solutions: $P < 0$

function line will shift to the left, reducing net profit. The graph exaggerates this by placing Z at the next corner-point solution, which assumes that management will produce enough Product B to meet market demand. Even without this exaggeration, we still have $Z < Z' \rightarrow Z - Z' < 0 \rightarrow P < 0$. In order for ABC to produce the desired result, then, this situation must correspond to the case where Product B has a negative margin.

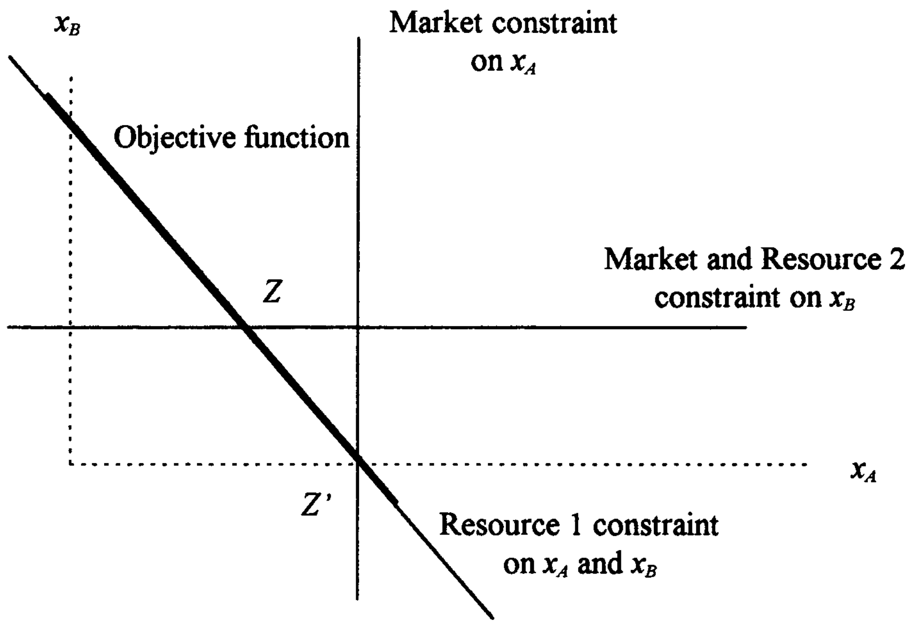

Figure 4. Graph comparing optimal solutions: $P = 0$

Finally, Figure 4 illustrates an example where $Z = Z' \rightarrow Z - Z' = 0 \rightarrow P = 0$. Again, we have placed Z where the production of Product B meets market demand, even though it may lie anywhere along the line segment connecting Z and Z'. Here, ABC must produce a Product B margin of zero in order to yield a proper decision.

Identifying these conditions will require an expression that relates m_B to P. This will entail merging the MIP equations for the two alternatives with the equation for Product B's ABC margin. We have as our goal, then, the development of equations for both the elimination and acquisition scenarios that will relate the ABC margin per unit for Product B to the difference in profit resulting from making Product B. Besides relating m_B to P, these equations should also reveal how common resource capacity utilization impacts this relationship.

MATHEMATICAL DEVELOPMENT AND ANALYSIS

Evaluation Model for the Elimination Scenario

Analyzing the impact of ABC on the original MIP model a requires a slight modification of the objective function. The way we have set up the model, the number of units of a resource consists of the beginning units plus or minus the units acquired or sold. Thus, in calculating the net profit, the cost associated with the beginning units will not change as we add or eliminate resource units. Accordingly, we have left these fixed costs out of the objective function in the MIP model. However, since ABC uses these intial costs in determining the per unit product margins, m_A and m_B, we need to restate the objective function to include them. Letting f_1 and f_2 represent the cost per beginning unit of Resource 1 and Resource 2, respectively, we have:

$$Z = c_A x_A + c_B x_B - d_1 u_1 - d_2 u_2 + e_1 v_1 + e_2 v_2 - f_1 b_1 - f_2 b_2 \tag{20}$$

In the elimination scenario, we will limit changes in resource units to reductions, since the decision of whether or not to eliminate Product B centers around the savings obtained through reduced resource costs. Also, for the case where management keeps Product B, we will assume no changes in resource units. We make these assumptions based on the notion that management has already optimized its resources for the existing operation, which includes making Product B. Replacing equation 15a with equation 20 and discarding resource unit changes, we can solve formulation 15 for the elimination scenario with $x_B > 0$ and obtain the following system of equations:

$$Z = c_A x_A + c_B x_B - f_1 b_1 - f_2 b_2 \tag{21a}$$

$$a_{1A} x_A + a_{1B} x_B + s_1 = q_1 b_1 \tag{21b}$$

$$a_{2B} x_B + s_2 = q_2 b_2 \tag{21c}$$

Although the variables appear in general form, they actually correspond to values for a particular optimal solution. The slack variables, s_1 and s_2, represent the excess capacity in activity units for Resource 1 and Resource 2, respectively. For Resource 1, this excess capacity stems from the market demand constraints imposed by inequalities 15d and 15e. With respect to Resource 2, it could also result from the MIP model selecting greater Product A production at the expense of Product B due to their relative contributions. In either case, the slack variables embody the effects of the market demand constraints, which explains why we do not have corresponding equations for 15d and 15e.

Eliminating Product B makes x_B equal to zero, v_2 equal to b_2, and the Resource 2 constraint superfluous. Also, distinguishing its formulation from that of the other choice requires new variables for Z, x_A, s_1 and s_2. In keeping with the notation used earlier, we will denote them by Z', x'_A and s'_1. This yields:

$$Z' = c_A x'_A + e_1 v_1 + e_2 v_2 - f_1 b_1 - f_2 b_2 \tag{22a}$$

$$a_{1A} x'_A + q_1 v_1 + s'_1 = q_1 b_1 \tag{22b}$$

We set up the equation for the change in profit by merging 21a and 22a as follows:

$$P = Z - Z'$$

$$\rightarrow P = \left(c_A x_A + c_B x_B - f_1 b_1 - f_2 b_2\right) - \left(c_A x'_A + e_1 v_1 + e_2 v_2 - f_1 b_1 - f_2 b_2\right)$$

$$\rightarrow P = c_A x_A - c_A x'_A + c_B x_B - e_1 v_1 - e_2 v_2 \tag{23}$$

The ABC margin for Product B consists of its contribution, less its allocated portion of the Resource 1 cost, less the Resource 2 cost, all divided by the units of Product B. In terms of the variables defined in the MIP model, this translates to the following:

$$m_B = \frac{c_B x_B - \dfrac{a_{1B} x_B f_1 b_1}{q_1 b_1 - s_1} - f_2 b_2}{x_B} = c_B - \frac{a_{1B} f_1 b_1}{q_1 b_1 - s_1} - \frac{f_2 b_2}{x_B} \tag{24}$$

The variables x_A and x'_A provide the means for linking equation 23 with equations 21b and 22b, respectively. Rearranging the terms in equations 21b and 22b, and then substituting for x_A and x'_A in equation 23, we have:

$$a_{1A} x_A + a_{1B} x_B + s_1 = q_1 b_1 \rightarrow x_A = \frac{q_1 b_1 - a_{1B} x_B - s_1}{a_{1A}} \tag{25a}$$

$$a_{1A} x'_A + q_1 v_1 + s'_1 = q_1 b_1 \rightarrow x'_A = \frac{q_1 b_1 - q_1 v_1 - s'_1}{a_{1A}} \tag{25b}$$

$$P = c_A \frac{q_1 b_1 - a_{1B} x_B - s_1}{a_{1A}} - c_A \frac{q_1 b_1 - q_1 v_1 - s'_1}{a_{1A}} + c_B x_B - e_1 v_1 - e_2 v_2$$

$$= \frac{c_A}{a_{1A}} \left(q_1 b_1 - a_{1B} x_B - s_1 - q_1 b_1 + q_1 v_1 + s'_1 \right) + c_B x_B - e_1 v_1 - e_2 v_2$$

$$= \frac{c_A}{a_{1A}} \left(s'_1 - s_1 + q_1 v_1 - a_{1B} x_B \right) + c_B x_B - e_1 v_1 - e_2 v_2 \tag{25c}$$

A comparison of equations 24 and 25c reveals that x_B provides the means for linking the two. Thus, we need to rearrange the terms in these equations to obtain two expressions for x_B, then set the two equal to one another. Doing this yields the following:

$$P = \frac{c_A}{a_{1A}}\left(s'_1 - s_1 + q_1 v_1 - a_{1B} x_B\right) + c_B x_B - e_1 v_1 - e_2 v_2$$

$$\rightarrow P = +\frac{c_A}{a_{1A}}\left(s'_1 - s_1 + q_1 v_1\right) - \frac{c_A a_{1B} x_B}{a_{1A}} + c_B x_B - e_1 v_1 - e_2 v_2$$

$$\rightarrow P - \frac{c_A}{a_{1A}}\left(s'_1 - s_1 + q_1 v_1\right) + e_1 v_1 + e_2 v_2 = c_B x_B - \frac{c_A a_{1B} x_B}{a_{1A}}$$

$$\rightarrow \left(c_B - \frac{c_A a_{1B}}{a_{1A}}\right) x_B = P + \frac{c_A}{a_{1A}}\left(s_1 - s'_1 - q_1 v_1\right) + e_1 v_1 + e_2 v_2$$

$$\rightarrow x_B = \frac{P + \frac{c_A}{a_{1A}}\left(s_1 - s'_1 - q_1 v_1\right) + e_1 v_1 + e_2 v_2}{\left(c_B - \frac{c_A a_{1B}}{a_{1A}}\right)} \tag{26a}$$

$$m_B = c_B - \frac{a_{1B} f_1 b_1}{q_1 b_1 - s_1} - \frac{f_2 b_2}{x_B} \rightarrow \frac{f_2 b_2}{x_B} = c_B - \frac{a_{1B} f_1 b_1}{q_1 b_1 - s_1} - m_B$$

$$\rightarrow x_B = \frac{f_2 b_2}{c_B - \frac{a_{1B} f_1 b_1}{q_1 b_1 - s_1} - m_B} \tag{26b}$$

$$\frac{P + \frac{c_A}{a_{1A}}\left(s_1 - s'_1 - q_1 v_1\right) + e_1 v_1 + e_2 v_2}{\left(c_B - \frac{c_A a_{1B}}{a_{1A}}\right)} = \frac{f_2 b_2}{c_B - \frac{a_{1B} f_1 b_1}{q_1 b_1 - s_1} - m_B} \tag{26c}$$

Rearranging the terms in equation 26c to isolate the change in net profit gives us the following:

$$P = \frac{f_2 b_2 \left(c_B - \frac{c_A a_{1B}}{a_{1A}}\right)}{c_B - \frac{a_{1B} f_1 b_1}{q_1 b_1 - s_1} - m_B} - \frac{c_A}{a_{1A}}\left(s_1 - s'_1\right) + \frac{c_A q_1 v_1}{a_{1A}} - e_1 v_1 - e_2 v_2 \tag{27}$$

This provides a basis for evaluating the effectiveness of ABC in handling the elimination scenario. However, before pursuing an analysis of the relationships expressed in equation 27, we will develop the corresponding expression for the acquisition scenario. That way, the analysis can highlight any similarities or differences between the two scenarios.

Evaluation Model for the Acquisition Scenario

In the elimination scenario, we considered the decision of whether to keep Product B or eliminate it. In the acquisition scenario, the decision involves whether or not to add Product B. In developing the evaluation model for this scenario, we start with a reformulation of equations 21a through 21c. In this situation, we have no beginning units of Resource 2, and rather than the cost savings from eliminating Resource 2 and units of Resource 1, we have the added costs of acquiring units of these resources. This gives us:

$$Z = c_A x_A + c_B x_B - d_1 u_1 - d_2 u_2 - f_1 b_1 \qquad (28a)$$

$$a_{1A} x_A + a_{1B} x_B - q_1 u_1 + s_1 = q_1 b_1 \qquad (28b)$$

$$a_{2B} x_B - q_2 u_2 + s_2 = 0 \qquad (28c)$$

Deciding against Product B yields equations similar to 22a and 22b, but with some slight modifications. Since it involves maintaining the status quo, we can discard the variables related to resource elimination. Thus, we have:

$$Z' = c_A x'_A - f_1 b_1 \qquad (29a)$$

$$a_{1A} x'_A + s'_1 = q_1 b_1 \qquad (29b)$$

As in the elimination scenario, we determine the change in net profit by subtracting Z' from Z:

$$P = Z - Z'$$

$$\rightarrow P = \left(c_A x_A + c_B x_B - d_1 u_1 - d_2 u_2 - f_1 b_1\right) - \left(c_A x'_A - f_1 b_1\right)$$

$$\rightarrow P = c_A x_A - c_A x'_A + c_B x_B - d_1 u_1 - d_2 u_2 \tag{30}$$

Since the Resource 2 cost involves units acquired rather than beginning units, the expression for the ABC margin will also differ slightly from that of the elimination scenario:

$$m_B = \frac{c_B x_B - \dfrac{a_{1B} x_B \left(f_1 b_1 + d_1 u_1\right)}{q_1 b_1 + q_1 u_1 - s_1} - d_2 u_2}{x_B} = c_B - \frac{a_{1B}\left(f_1 b_1 + d_1 u_1\right)}{q_1 b_1 + q_1 u_1 - s_1} - \frac{d_2 u_2}{x_B} \tag{31}$$

Again paralleling the elimination scenario, the variables x_A and x'_A provide the link between net profit and the Resource 1 constraints. Rearranging the terms in equations 28b and 29b, and then substituting for x_A and x'_A in equation 30, we have:

$$a_{1A} x_A + a_{1B} x_B - q_1 u_1 + s_1 = q_1 b_1 \rightarrow x_A = \frac{q_1 b_1 + q_1 u_1 - a_{1B} x_B - s_1}{a_{1A}} \tag{32a}$$

$$a_{1A} x'_A + s'_1 = q_1 b_1 \rightarrow x'_A = \frac{q_1 b_1 - s'_1}{a_{1A}} \tag{32b}$$

$$P = c_A \frac{q_1 b_1 + q_1 u_1 - a_{1B} x_B - s_1}{a_{1A}} - c_A \frac{q_1 b_1 - s'_1}{a_{1A}} + c_B x_B - d_1 u_1 - d_2 u_2$$

$$= \frac{c_A}{a_{1A}}\left(q_1 b_1 + q_1 u_1 - a_{1B} x_B - s_1 - q_1 b_1 + s'_1\right) + c_B x_B - d_1 u_1 - d_2 u_2$$

$$= \frac{c_A}{a_{1A}}\left(s'_1 - s_1 + q_1 u_1 - a_{1B} x_B\right) + c_B x_B - d_1 u_1 - d_2 u_2 \tag{32c}$$

An examination of equations 31 and 32c reveals that they closely parallel equations 24 and 25c. Equation 24 only requires adding $d_1 u_1$ to $f_1 b_1$ and $q_1 u_1$ to $q_1 b_1$, plus changing f_2 to d_2 and b_2 and u_2, to transform it into equation 31. Similarly, to change equation 25c into 32c, one need only replace e_1, e_2, v_1 and v_2 with d_1, d_2, u_1 and u_2, respectively. Thus, we can change the final evaluation equation for the elimination scenario (equation 27) into its acquisition scenario counterpart as follows:

$$P = \frac{d_2 u_2\left(c_B - \frac{c_A a_{1B}}{a_{1A}}\right)}{c_B - \frac{a_{1B}\left(f_1 b_1 + d_1 u_1\right)}{q_1 b_1 + q_1 u_1 - s_1} - m_B} - \frac{c_A}{a_{1A}}\left(s_1 - s'_1\right) + \frac{c_A q_1 u_1}{a_{1A}} - d_1 u_1 - d_2 u_2 \tag{33}$$

Generalized Evaluation Model

Equations 27 and 33 provide the means for analyzing the key relationships for the elimination and acquisition scenarios, respectively. In order to facilitate this analysis, we define the following set of new variables for the elimination scenario:

$$\Delta s_1 = s_1 - s'_1 \tag{34a}$$

$$k_1 = f_2 b_2\left(c_B - \frac{c_A a_{1B}}{a_{1A}}\right) \tag{34b}$$

$$k_2 = c_B - \frac{a_{1B} f_1 b_1}{q_1 b_1 - s_1} \tag{34c}$$

$$k_3 = \frac{c_A}{a_{1A}} \tag{34d}$$

$$k_4 = \frac{c_A q_1 v_1}{a_{1A}} - e_1 v_1 - e_2 v_2 \tag{34e}$$

For the acquistion scenario, we have the following:

$$\Delta s_1 = s_1 - s'_1 \tag{35a}$$

$$k_1 = d_2 u_2 \left(c_B - \frac{c_A a_{1B}}{a_{1A}} \right) \tag{35b}$$

$$k_2 = c_B - \frac{a_{1B}(f_1 b_1 + d_1 u_1)}{q_1 b_1 + q_1 u_1 - s_1} \tag{35c}$$

$$k_3 = \frac{c_A}{a_{1A}} \tag{35d}$$

$$k_4 = \frac{c_A q_1 u_1}{a_{1A}} - d_1 u_1 - d_2 u_2 \tag{35e}$$

We can gleen additional meaning from k_2 by restating equation 34c in terms of equation 24 and equation 35c in terms of equation 31. For the elimination scenario, we have:

$$k_2 - m_B = c_B - \frac{a_{1B} f_1 b_1}{q_1 b_1 - s_1} - m_B \frac{f_2 b_2}{x_B}$$

$$= c_B - \frac{a_{1B} f_1 b_1}{q_1 b_1 - s_1} - \left(c_B - \frac{a_{1B} f_1 b_1}{q_1 b_1 - s_1} - \frac{f_2 b_2}{x_B} \right) = \frac{f_2 b_2}{x_B} \rightarrow k_2 = \frac{f_2 b_2}{x_B} + m_B \tag{36}$$

The acquisition scenario yields a similar result:

$$k_2 - m_B = c_B - \frac{a_{1B}(f_1 b_1 + d_1 u_1)}{q_1 b_1 + q_1 u_1 - s_1} - m_B$$

$$= c_B - \frac{a_{1B}(f_1 b_1 + d_1 u_1)}{q_1 b_1 + q_1 u_1 - s_1} - \left(c_B - \frac{a_{1B}(f_1 b_1 + d_1 u_1)}{q_1 b_1 + q_1 u_1 - s_1} - \frac{d_2 u_2}{x_B} \right)$$

$$= \frac{d_2 u_2}{x_B} \rightarrow k_2 = \frac{d_2 u_2}{x_B} + m_B \tag{37}$$

Using these new variables, we can rewrite equations 27 and 33, which represent the final evaluation equations for the two scenarios, into a single simplified formula:

$$P = \frac{k_1}{k_2 - m_B} - k_3 \Delta s_1 + k_4 \tag{38}$$

Thus, we have established a generalized evaluation model, given by equation 38, which will enable us to analyze how P, m_B, and Δs_1 relate to one another in both the elimination and acquisition scenarios. We will do this for each criterion by assuming values of P and m_B that meet the criterion, then determining algebraically the relationships among the other variables that must follow.

Analysis of the Generalized Evaluation Model

We previously specified the following criteria for evaluating the effectiveness of the ABC decision model:

1) $m_B = 0 \rightarrow P = 0$;

2) $m_B > 0 \rightarrow P > 0$; and

3) $m_B < 0 \rightarrow P < 0$.

Beginning with the first criterion, we set $m_B = 0$ and $P = 0$ in equation 38 as follows:

$$0 = \frac{k_1}{k_2 - 0} - k_3 \Delta s_1 + k_4 \rightarrow k_3 \Delta s_1 - k_4 = \frac{k_1}{k_2} \tag{39}$$

Moving on to the second criterion, where $m_B > 0$ and $P > 0$, we have:

$$P = \frac{k_1}{k_2 - m_B} - k_3 \Delta s_1 + k_4 > 0 \tag{40}$$

In order to move terms between the right and left sides of the inequality, we need to distinguish between positive and negative variables. We already know from equations 36 and 37 that k_2 equals the Resource 2 cost per unit of Product B plus the Product B ABC margin per unit. We will assume that Resource 2 costs something, however minimal, so this ensures that:

$$k_2 - m_B > 0 \rightarrow k_2 > m_B \tag{41}$$

for all possible values of m_B. This result will facilitate the analysis considerably, for not only does it restrict the range of possibilities for equations 37 and 38, it also prevents division by zero.

Even with this information, though, we still must assume something about the direction of $k_3 \Delta s_1 - k_4$. We begin by assuming that $k_3 \Delta s_1 - k_4 = 0$. Imposing this assumption on equation 40 produces:

$$P = \frac{k_1}{k_2 - m_B} - k_3 \Delta s_1 + k_4 > 0 \rightarrow \frac{k_1}{k_2 - m_B} > 0 \rightarrow k_1 > 0 \tag{42}$$

For $k_3 \Delta s_1 - k_4 > 0$, we have:

$$P = \frac{k_1}{k_2 - m_B} - k_3\Delta s_1 + k_4 > 0 \rightarrow \frac{k_1}{k_2 - m_B} > k_3\Delta s_1 - k_4$$

$$\rightarrow \left(\frac{k_2 - m_B}{k_3\Delta s_1 - k_4}\right)\left(\frac{k_1}{k_2 - m_B}\right) > \left(\frac{k_2 - m_B}{k_3\Delta s_1 - k_4}\right)(k_3\Delta s_1 - k_4) \rightarrow \frac{k_1}{k_3\Delta s_1 - k_4} > k_2 - m_B$$

$$\rightarrow m_B > k_2 - \frac{k_1}{k_3\Delta s_1 - k_4} \qquad (43)$$

In contrast, $k_3\Delta s_1 - k_4 < 0$ yields:

$$P = \frac{k_1}{k_2 - m_B} - k_3\Delta s_1 + k_4 > 0 \rightarrow \frac{k_1}{k_2 - m_B} > k_3\Delta s_1 - k_4$$

$$\rightarrow \left(\frac{k_2 - m_B}{k_3\Delta s_1 - k_4}\right)\left(\frac{k_1}{k_2 - m_B}\right) < \left(\frac{k_2 - m_B}{k_3\Delta s_1 - k_4}\right)(k_3\Delta s_1 - k_4) \rightarrow \frac{k_1}{k_3\Delta s_1 - k_4} < k_2 - m_B$$

$$\rightarrow m_B < k_2 - \frac{k_1}{k_3\Delta s_1 - k_4} \qquad (44)$$

For the third criterion, we have $m_B < 0$ and $P < 0$. As before, we start by assuming that $k_3\Delta s_1 - k_4 = 0$, yielding:

$$P = \frac{k_1}{k_2 - m_B} - k_3\Delta s_1 + k_4 < 0 \rightarrow \frac{k_1}{k_2 - m_B} < 0 \rightarrow k_1 < 0 \qquad (45)$$

Moving on to $k_3\Delta s_1 - k_4 > 0$, we have:

$$P = \frac{k_1}{k_2 - m_B} - k_3\Delta s_1 + k_4 < 0 \rightarrow \frac{k_1}{k_2 - m_B} < k_3\Delta s_1 - k_4$$

$$\rightarrow \left(\frac{k_2 - m_B}{k_3\Delta s_1 - k_4}\right)\left(\frac{k_1}{k_2 - m_B}\right) < \left(\frac{k_2 - m_B}{k_3\Delta s_1 - k_4}\right)(k_3\Delta s_1 - k_4) \rightarrow \frac{k_1}{k_3\Delta s_1 - k_4} < k_2 - m_B$$

$$\rightarrow m_B < k_2 - \frac{k_1}{k_3 \Delta s_1 - k_4} \tag{46}$$

Finally, assuming $k_3 \Delta s_1 - k_4 < 0$ produces:

$$P = \frac{k_1}{k_2 - m_B} - k_3 \Delta s_1 + k_4 < 0 \rightarrow \frac{k_1}{k_2 - m_B} < k_3 \Delta s_1 - k_4$$

$$\rightarrow \left(\frac{k_2 - m_B}{k_3 \Delta s_1 - k_4}\right)\left(\frac{k_1}{k_2 - m_B}\right) > \left(\frac{k_2 - m_B}{k_3 \Delta s_1 - k_4}\right)(k_3 \Delta s_1 - k_4) \rightarrow \frac{k_1}{k_3 \Delta s_1 - k_4} > k_2 - m_B$$

$$\rightarrow m_B > k_2 - \frac{k_1}{k_3 \Delta s_1 - k_4} \tag{47}$$

Inequalities 42 through 47 reveal the basic relationships between the Product B ABC margin per unit, m_B,and the change in Resource 1 excess capacity, Δs_1, under the conditions where ABC meets the criteria for effective decision modeling. In each case, $k_2 - k_1/(k_3 \Delta s_1 - k_4)$ plays a pivotal role, except when $k_3 \Delta s_1 - k_4 = 0$. This also holds true for equation 39, which relates to the first criterion, where $m_B = 0$ and $P = 0$. Rearranging terms produces the following:

$$k_3 \Delta s_1 - k_4 = \frac{k_1}{k_2} \rightarrow \left(\frac{k_2}{k_3 \Delta s_1 - k_4}\right)(k_3 \Delta s_1 - k_4) = \left(\frac{k_2}{k_3 \Delta s_1 - k_4}\right)\left(\frac{k_1}{k_2}\right)$$

$$\rightarrow k_2 = \frac{k_1}{k_3 \Delta s_1 - k_4} \rightarrow k_2 - \frac{k_1}{k_3 \Delta s_1 - k_4} = 0 = m_B \tag{48}$$

Having established these relationships, the task remains to examine their practical implications. This will include analyses in both general terms and with respect to specific examples.

RESULTS

Summary of Mathematical Results

The preceding analysis of the generalized model has provided us with seven mathematical conditions under which ABC will function as an effective decision-making tool. ABC has only one condition under which it functions properly given a zero ABC margin for Product B. For a positive ABC margin, any of three different conditions will produce desirable results. A negative ABC margin also has three alternative conditions. We summarize them as follows:

1. $m_B = 0$

   $$k_2 - \frac{k_1}{k_3 \Delta s_1 - k_4} = 0$$

2. $m_B > 0$

   a) $k_3 \Delta s_1 - k_4 = 0$ and $k_1 > 0$

   b) $k_3 \Delta s_1 - k_4 > 0$ and $m_B > k_2 - \dfrac{k_1}{k_3 \Delta s_1 - k_4}$

   c) $k_3 \Delta s_1 - k_4 < 0$ and $m_B < k_2 - \dfrac{k_1}{k_3 \Delta s_1 - k_4}$

3. $m_B < 0$

   a) $k_3 \Delta s_1 - k_4 = 0$ and $k_1 < 0$

   b) $k_3 \Delta s_1 - k_4 > 0$ and $m_B < k_2 - \dfrac{k_1}{k_3 \Delta s_1 - k_4}$

c) $k_3 \Delta s_1 - k_4 < 0$ and $m_B > k_2 - \dfrac{k_1}{k_3 \Delta s_1 - k_4}$

In order to interpret the practical implications of these conditions, we need to translate them into the original decision variables. For the elimination scenario, we have:

$$k_1 = f_2 b_2 \left(c_B - \frac{c_A a_{1B}}{a_{1A}} \right) \tag{49}$$

$$k_3 \Delta s_1 - k_4 = \frac{c_A}{a_{1A}}(s_1 - s'_1) - \frac{c_A q_1 v_1}{a_{1A}} + e_1 v_1 + e_2 v_2 \tag{50}$$

$$k_2 - \frac{k_1}{k_3 \Delta s_1 - k_4} = c_B - \frac{a_{1B} f_1 b_1}{q_1 b_1 - s_1} - \frac{f_2 b_2 \left(c_B - \dfrac{c_A a_{1B}}{a_{1A}} \right)}{\dfrac{c_A}{a_{1A}}(s_1 - s'_1) - \dfrac{c_A q_1 v_1}{a_{1A}} + e_1 v_1 + e_2 v_2} \tag{51}$$

Doing the same for the acquisition scenario yields the following:

$$k_1 = d_2 u_2 \left(c_B - \frac{c_A a_{1B}}{a_{1A}} \right) \tag{52}$$

$$k_3 \Delta s_1 - k_4 = \frac{c_A}{a_{1A}}(s_1 - s'_1) - \frac{c_A q_1 u_1}{a_{1A}} + d_1 u_1 + d_2 u_2 \tag{53}$$

$$k_2 - \frac{k_1}{k_3 \Delta s_1 - k_4} = c_B - \frac{a_{1B}(f_1 b_1 + d_1 u_1)}{q_1 b_1 + q_1 u_1 - s_1} - \frac{d_2 u_2 \left(c_B - \dfrac{c_A a_{1B}}{a_{1A}} \right)}{\dfrac{c_A}{a_{1A}}(s_1 - s'_1) - \dfrac{c_A q_1 u_1}{a_{1A}} + d_1 u_1 + d_2 u_2} \tag{54}$$

In both equation 51 and equation 54, which relate to the elimination and acquisition scenarios, respectively, the change in excess Resource 1 capacity appears in the denominator of the term that one one subtracts from the other. Since s_1 represents excess capacity with Product B and s'_1 without, the denominator will become less positive or more negative as Product reduces excess capacity.

In both scenarios, the term $k_3\Delta s_1$ multiplies the change in excess capacity in Resource 1 activity units by Product A contribution dollars per Resource 1 activity unit, translating the change in excess capacity into dollars. The term corresponding to k_4 consists of three components. The first multiplies the activity units gained or lost from acquiring or eliminating Resource 1 units by the Product A contribution dollars per Resource 1 activity unit. The second represents the cost of acquired Resource 1 units or the savings from disposing of Resource 1 units. The third corresponds to the cost or savings from a change in Resource 2 units. The combined term $k_3\Delta s_1$ - k_4 values the usable activity units gained or lost due to changes in resource units or capacity utilization, and nets this against the cost of acquired resources or savings from eliminating resources.

The term k_2 takes the contribution margin per unit for Product B less the ABC allocation of Resource 1 cost per unit of Product B. The k_1 term, though similar, differs slightly. In this case, we take the Product B contribution margin per unit, then subtract a term that adjusts the per unit contribution margin for Product A by the ratio of the Product B and Product A Resource 1 utilization rates; we then multiply this result by the cost associated with Resource 2. Adjusting the Product A contribution for utilization rates puts it on a comparable basis with the Product B contribution. If Product B requires more of Resource 1 per unit, then we adjust Product A's margin upward. Conversely, if Product B requires less of Resource 1, then we adjust Product A's margin downward.

The term c_A / a_{1A} deserves special notice. It takes the contribution per unit of Product A, divided by the Resource 1 activity units required per unit of Product A. In other words, it provides the value of Resource 1 activity units measured in terms of gained or lost contribution from Product A. In the context of a linear programming model, one would call this the *shadow price*[47] of Resource 1 activity units in terms of Product A: it gives the increase in profit that would result from having another Resource 1 activity unit available and usable.

The preceding analysis reveals three key components that influence whether or not ABC will yield appropriate decisions. One corresponds to k_2, another to $k_3 \Delta s_1 - k_4$, and a third to the $c_B - c_A\, a_{1B} / a_{1A}$ portion of k_1. The first compares Product B's contribution margin to its Resource 1 ABC allocation. The second compares the value of usable activity units gained or lost due to changes in resource units or capacity utilization against the cost of acquired resources or savings from eliminating resources. The third compares Product B's contribution margin per unit with that of Product A, adjusted for the relative rates at which they draw upon Resource 1.

In the first chapter, we noted how the complexity of product/resource interactions can make the practical application of ABC problematic. The myriad of relationships expressed in the seven cases we have identified supports this assertion. We also proposed

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[47]Hillier and Lieberman, p. 95.

that changes in excess capacity play an important role in these interactions. As one can see from the seven sets of conditions for the effective functioning of ABC, the variables related to excess capacity did not cancel out in the process of algebraic manipulation. Hence, we have a mathematical confirmation of our second assertion.

Model Validation

Through algebraic manipulation, we have arrived at seven sets of mathematical conditions under which ABC will theoretically lead to appropriate decisions. In order to attain a greater comfort level with their validity, it will help to examine some specific examples.

We begin by noting the strong parallel that exists between the elimination and acquisition scenarios. In fact, with only a few minor modifications, we can translate the Chapter One example from an elimination to an acquisition scenario. Table 2 shows the results of performing this transformation on the data contained in Table 1. One can readily see that although they assume opposite perspectives, mathematically they yield essentially the same results.

Since the two scenarios yield the same results from the standpoint of the equation 40 generalized model, we will focus our attention on just the elimination scenario. Table 3 presents the results for a number of variations on the original Table 1 example.[48] The first column gives the results for the original example. Although Table 1 indicates that $m_B = 0$,

[48]Tables 4 through 8 in the Appendix offer the details to these variations in the same format as Table 2.

Table 2. Product Addition Example

Product Information	**Product A**	**Product B**
Contribution per Product Unit	$2.00	$3.00
Common Resource Activity Units per Product Unit	0.10	0.30
Special Resource Activity Units per Product Unit	0.00	1.00
Total Contribution	$22,758	$30,000
ABC Common Resource Allocation	(11,000)	(29,000)
ABC Special Resource Allocation	0	(1,000)
ABC Margin	**$11,758**	**$ 0**
ABC Margin per Unit	**$1.0333**	**$0.0000**
Resource Information	**Common Resource**	**Special Resource**
Beginning Resource Units	6	0
Cost per Resource Unit (Beginning)	$2,000	$1,000
Cost per Resource Unit (Acquired)	$2,000	$1,000
Activity Units per Resource Unit	207	10,000
Decision Results	**Add Product B**	**Do Not Add Product B**
Product A Units Produced	11,379	11,379
Product B Units Produced	10,000	0
Common Resource		
Resource Units Acquired	14	0
Total Capacity in Activity Units	4,140	1,242
Activity Units used by Product A	1,138	1,138
Activity Units used by Product B	3,000	0
Excess Capacity in Activity Units	2	104
Special Resource		
Resource Units Acquired	1	0
Total Capacity in Activity Units	10,000	0
Activity Units used by Product B	10,000	0
Excess Capacity in Activity Units	0	0
Product A Contribution	$22,758	$22,758
Product B Contribution	30,000	0
Common Resource Cost	(40,000)	(12,000)
Special Resource Cost	(1,000)	0
Net Profit	**$11,758**	**$10,758**

Table 3 shows that this only holds true to four decimal places. As in any practical application, one must determine what level of precision makes sense. For the purposes of this analysis, we will treat any number equal to zero within four decimal places as equal to zero.

Table 3. Comparison of Examples using the Generalized Evaluation Model

	Case A ABC Fails	**Case B ABC Fails**	**Case C ABC Works**	**Case D ABC Works**	**Case E ABC Works**
Product A Contribution/Unit	2.000000	2.000000	3.029592	3.029592	3.029592
Resource 1 Activity Units per Product A Unit	0.1000000	0.050000	0.173034	0.173034	0.173034
Product B Contribution/Unit	3.000000	3.530000	2.100000	2.110000	2.090000
Resource 1 Activity Units per Product B Unit	0.300000	0.350000	0.196900	0.196900	0.196900
Resource 1 Units Eliminated	14	17	10	10	10
m_A	1.033326	1.508473	1.271989	1.271989	1.271989
m_B	0.000022	(0.010691)	(0.000023)	0.009977	(0.010023)
Z	11,758.00	17,058.00	14,473.73	14,573.73	14,373.73
Z'	10,758.00	16,758.00	14,473.73	14,473.73	14,473.73
P	1,000.00	300.00	0.00	100.00	(100.00)
$k_3\Delta s_1 - k_4$	(31,000.00)	(105,000.00)	(13,474.54)	(13,474.54)	(13,474.54)
$k_2 - \dfrac{k_1}{k_3\Delta s_1 - k_4}$	0.003204	(0.010405)	(0.000023)	0.010719	(0.010766)
k_1	(3,000.00)	(10,470.00)	(1,347.45)	(1,337.45)	(1,357.45)
k_2	0.100000	0.089309	0.099977	0.109977	0.089977
k_3	20.00000	40.000000	17.508651	17.508651	17.508651
k_4	28,960.00	105,760.00	15,242.91	15,242.91	15,242.91
Δs_1	(102.00)	19.00	101.00	101.00	101.00

In case A, $m_B = 0$, so condition set 1 must hold in order for ABC to function effectively. Checking against the actual numbers, we have:

$$k_2 - \frac{k_1}{k_3 \Delta s_1 - k_4} = 0.1000 - \frac{-3{,}000}{(20)(-102) - 28{,}960} = 0.0032 \qquad (55)$$

In order for condition set 1 to have held, the result in equation 55 should have equaled zero. As we have already seen, ABC fails in this case because it recommends indifference to Product B, even though the elimination of Product B reduces net profit by $1,000.

Case B gives another example where ABC fails to work correctly. Here, we have:

$$m_B = -0.0107 < 0 \qquad (56)$$

We also have:

$$k_3 \Delta s_1 - k_4 = (40)(19) - 105{,}760 = -105{,}000 < 0 \qquad (57)$$

This means that condition set 3c provides the basis for judging ABC's effectiveness. Performing the appropriate calculations as in equation 55 yields:

$$k_2 - \frac{k_1}{k_3 \Delta s_1 - k_4} = 0.0893 - \frac{-10{,}470}{(40)(19) - 105{,}760} = -0.0104 \qquad (58)$$

Condition set 3c requires a positive result for equation 58, so ABC does not function properly in this case. We verify this by noting that even though Product B has a negative ABC margin, eliminating it leads to a $300 reduction in profit.

The two preceding examples both put ABC in an unfavorable light. However, one can just as easily produce scenarios where ABC works fine. Cases C, D and E in Table 3 all provide examples of ABC functioning effectively. One can readily see that they meet condition sets 1, 2c and 3c, respectively.

Although these sample calculations offer some verification that the seven sets of mathematical conditions developed here do indeed work, the various equations and inequalities appearing in them would probably not prove useful in the context of an actual decision modeling application. In practice, one would validate the results of ABC by recomputing the product margins based on the decisions derived from the initial run. However, they do point out the critical need for validation of any results derived from ABC analysis.

Conclusions

We have drawn upon concepts from operations research and economics to evaluate the effectiveness of ABC as a tool for modeling product/resource mix decisions. Using a simplified MIP model as the standard, we have derived seven sets of conditions under which ABC will lead to appropriate decisions. The mathematical relationships expressed in these seven sets of conditions provide a means for generating, at will, examples in which ABC works and ones in which it fails.

Although one can readily produce these examples, in practice the complexity of the mathematical relationships will usually prevent one determining whether ABC will function correctly or not. This points to the crucial need for validating the decision recommended by ABC by running the model again assuming the implementation of that decision.. It also reveals a serious flaw in ABC from a conceptual standpoint.

In developing the MIP model, we made a variety of assumptions. As long as the assumptions mesh reasonably well with reality, the model will produce useful results. In addition, management has an opportunity to judge the reasonableness of those assumptions and act accordingly. For ABC to function properly, though, certain complex conditions must hold. These conditions amount to assumptions, yet the complexity of the relationships will generally deprive management of the ability to judge whether they might reasonably apply.

Before using a model, one should have the ability to evaluate the reasonableness of its assumptions. As we have seen, though, ABC depends on assumptions that one cannot readily evaluate. This constitutes a severe shortcoming.

As improvements in technology lead to more powerful computer hardware and more user-friendly software, advanced techniques such as MIP should become more accessible. In fact, one can easily develop complex models that include integer constraints using existing spreadsheet applications: the *Solver* feature of Microsoft® Excel provided the means for generating the examples presented here. As a result, one will eventually have little excuse for accepting ABC's shortcomings.

We conclude, then, that the use of activity-based concepts within a MIP model provides a preferable approach over the use of product costing using ABC. If one does opt to use product costs developed through ABC methods, one should *always* validate the result by explicitly determining the impact of the decision on net profit.

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APPENDIX

Table 4. Case A Detail

Product Information	**Product A**	**Product B**
Contribution per Product Unit	$2.00	$3.00
Common Resource Activity Units per Product Unit	0.10	0.30
Special Resource Activity Units per Product Unit	0.00	1.00
Total Contribution	$22,758	$30,000
ABC Common Resource Allocation	(11,000)	(29,000)
ABC Special Resource Allocation	0	(1,000)
ABC Margin	**$11,758**	**$ 0**
ABC Margin per Unit	**$1.0333**	**$0.0000**
Resource Information	**Common Resource**	**Special Resource**
Beginning Resource Units	20	1
Cost per Resource Unit (Beginning)	$2,000	$1,000
Savings per Resource Unit (Eliminated)	$2,000	$1,000
Activity Units per Resource Unit	207	10,000
Decision Results	**Keep Product B**	**Eliminate Product B**
Product A Units Produced	11,379	11,379
Product B Units Produced	10,000	0
Common Resource		
Resource Units Eliminated	0	14
Total Capacity in Activity Units	4,140	1,242
Activity Units used by Product A	1,138	1,138
Activity Units used by Product B	3,000	0
Excess Capacity in Activity Units	2	104
Special Resource		
Resource Units Eliminated	0	1
Total Capacity in Activity Units	10,000	0
Activity Units used by Product B	10,000	0
Excess Capacity in Activity Units	0	0
Product A Contribution	$22,758	$22,758
Product B Contribution	30,000	0
Common Resource Cost	(40,000)	(12,000)
Special Resource Cost	(1,000)	0
Net Profit	**$11,758**	**$10,758**

Table 5. Case B Detail

Product Information	**Product A**	**Product B**
Contribution per Product Unit	$2.00	$3.53
Common Resource Activity Units per Product Unit	0.05	0.35
Special Resource Activity Units per Product Unit	0.00	1.00
Total Contribution	$22,758	$35,300
ABC Common Resource Allocation	(5,593)	(34,407)
ABC Special Resource Allocation	0	(1,000)
ABC Margin	**$11,758**	**($ 107)**
ABC Margin per Unit	**$1.05085**	**($0.0107)**
Resource Information	**Common Resource**	**Special Resource**
Beginning Resource Units	20	1
Cost per Resource Unit (Beginning)	$2,000	$1,000
Savings per Resource Unit (Eliminated)	$2,000	$1,000
Activity Units per Resource Unit	207	10,000
Decision Results	**Keep Product B**	**Eliminate Product B**
Product A Units Produced	11,379	11,379
Product B Units Produced	10,000	0
Common Resource		
Resource Units Eliminated	0	14
Total Capacity in Activity Units	4,140	1,242
Activity Units used by Product A	1,138	1,138
Activity Units used by Product B	3,000	0
Excess Capacity in Activity Units	2	104
Special Resource		
Resource Units Eliminated	0	1
Total Capacity in Activity Units	10,000	0
Activity Units used by Product B	10,000	0
Excess Capacity in Activity Units	0	0
Product A Contribution	$22,758	$22,758
Product B Contribution	35,300	0
Common Resource Cost	(40,000)	(6,000)
Special Resource Cost	(1,000)	0
Net Profit	**$17,058**	**$16,758**

Table 6. Case C Detail

Product Information	**Product A**	**Product B**
Contribution per Product Unit	$3.03	$2.10
Common Resource Activity Units per Product Unit	0.1730	0.1969
Special Resource Activity Units per Product Unit	0.00	1.00
Total Contribution	$34,474	$21,000
ABC Common Resource Allocation	(20,000)	(20,000)
ABC Special Resource Allocation	0	(1,000)
ABC Margin	**$14,474**	**$ 0**
ABC Margin per Unit	**$1.2720**	**$0.0000**
Resource Information	**Common Resource**	**Special Resource**
Beginning Resource Units	20	1
Cost per Resource Unit (Beginning)	$2,000	$1,000
Savings per Resource Unit (Eliminated)	$2,000	$1,000
Activity Units per Resource Unit	207	10,000
Decision Results	**Keep Product B**	**Eliminate Product B**
Product A Units Produced	11,379	11,379
Product B Units Produced	10,000	0
Common Resource		
Resource Units Eliminated	0	10
Total Capacity in Activity Units	4,140	2,070
Activity Units used by Product A	1,969	1,969
Activity Units used by Product B	1,969	0
Excess Capacity in Activity Units	202	101
Special Resource		
Resource Units Eliminated	0	1
Total Capacity in Activity Units	10,000	0
Activity Units used by Product B	10,000	0
Excess Capacity in Activity Units	0	0
Product A Contribution	$34,474	$34,474
Product B Contribution	21,000	0
Common Resource Cost	(40,000)	(20,000)
Special Resource Cost	(1,000)	0
Net Profit	**$14,474**	**$14,474**

Table 7. Case D Detail

Product Information	**Product A**	**Product B**
Contribution per Product Unit	$3.03	$2.11
Common Resource Activity Units per Product Unit	0.1730	0.1969
Special Resource Activity Units per Product Unit	0.00	1.00
Total Contribution	$34,474	$21,100
ABC Common Resource Allocation	(20,000)	(20,000)
ABC Special Resource Allocation	0	(1,000)
ABC Margin	**$14,474**	**$ 100**
ABC Margin per Unit	**$1.2720**	**$0.0100**
Resource Information	**Common Resource**	**Special Resource**
Beginning Resource Units	20	1
Cost per Resource Unit (Beginning)	$2,000	$1,000
Savings per Resource Unit (Eliminated)	$2,000	$1,000
Activity Units per Resource Unit	207	10,000
Decision Results	**Keep Product B**	**Eliminate Product B**
Product A Units Produced	11,379	11,379
Product B Units Produced	10,000	0
Common Resource		
Resource Units Eliminated	0	10
Total Capacity in Activity Units	4,140	2,070
Activity Units used by Product A	1,969	1,969
Activity Units used by Product B	1,969	0
Excess Capacity in Activity Units	202	101
Special Resource		
Resource Units Eliminated	0	1
Total Capacity in Activity Units	10,000	0
Activity Units used by Product B	10,000	0
Excess Capacity in Activity Units	0	0
Product A Contribution	$34,474	$34,474
Product B Contribution	21,100	0
Common Resource Cost	(40,000)	(20,000)
Special Resource Cost	(1,000)	0
Net Profit	**$14,574**	**$14,474**

Table 8. Case E Detail

Product Information	**Product A**	**Product B**
Contribution per Product Unit	$3.03	$2.09
Common Resource Activity Units per Product Unit	0.1730	0.1969
Special Resource Activity Units per Product Unit	0.00	1.00
Total Contribution	$34,474	$20,900
ABC Common Resource Allocation	(20,000)	(20,000)
ABC Special Resource Allocation	0	(1,000)
ABC Margin	**$14,474**	**($ 100)**
ABC Margin per Unit	**$1.2720**	**($0.0100)**
Resource Information	**Common Resource**	**Special Resource**
Beginning Resource Units	20	1
Cost per Resource Unit (Beginning)	$2,000	$1,000
Savings per Resource Unit (Eliminated)	$2,000	$1,000
Activity Units per Resource Unit	207	10,000
Decision Results	**Keep Product B**	**Eliminate Product B**
Product A Units Produced	11,379	11,379
Product B Units Produced	10,000	0
Common Resource		
Resource Units Eliminated	0	10
Total Capacity in Activity Units	4,140	2,070
Activity Units used by Product A	1,969	1,969
Activity Units used by Product B	1,969	0
Excess Capacity in Activity Units	202	101
Special Resource		
Resource Units Eliminated	0	1
Total Capacity in Activity Units	10,000	0
Activity Units used by Product B	10,000	0
Excess Capacity in Activity Units	0	0
Product A Contribution	$34,474	$34,474
Product B Contribution	20,900	0
Common Resource Cost	(40,000)	(20,000)
Special Resource Cost	(1,000)	0
Net Profit	**$14,374**	**$14,474**