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In communications it is often desirable to recover a signal hidden in noise. The techniques of correlation are often employed for this purpose. Initially, the basic formula and properties of correlation including an approximating formula are presented. Then the description, design, and construction of a DC to 500 KHz electronic correlator that implements the approximating formula is discussed. Experimental results of the correlation of various waveforms including random noise are presented. Finally, the experimental results of the autocorrelation, crosscorrelation, and sample averaging of signals in noise are given for signal to noise ratios of up to -18 dB, -36 dB, and -42 dB respectively.

The Design and Performance of a Wideband Electronic Correlator

by

Gary Edward Spencer

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Professor of Electrical and Electronics Engineering

in charge of major

Redacted for Privacy

Head of Department of Electrical and Electronics Engineering

Redacted for Privacy

Dean of Graduate School

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NOMENCLATURE

ms	Milliseconds
us	Microseconds
ns	Nanoseconds
SNR	Signal to noise ratio
F/H	Follow and hold
DAC	Digital to analog converter
TTL	Transistor-transistor logic
IC	Integrated circuit
mV	Millivolt
FET	Field effect transistor
uF	Microfarads
pF	Picofarads
PRF	Pulse Repetition frequency
sw	Switch
BW	Bandwidth
PRT	Pulse repetition time

THE DESIGN AND PERFORMANCE OF A WIDEBAND ELECTRONIC CORRELATOR

INTRODUCTION

One of the problems in communications is the extracting of a periodic signal buried in noise. Such an occasion may arise, for example, in trying to observe a returning echo originated by a radar. The detection of the presence of a signal transmitted from a low power space probe is another application of correlation techniques.

The correlation function of two time dependent functions is the average value of the product of the two functions, one of which is delayed or shifted in time with respect to the other. However, for the complete correlation function, an infinite number of different delays is required. There are many ways to implement correlation, at least in an approximating form. For example, the use of a tape recorder with movable heads can be used for the variable delay where the resultant signals from the two playback heads are electronically multiplied and integrated. This method has drawbacks because of its low frequency response, variations in playback speed, and expense. Another method is the use of electrical delay lines for delaying one of the signals. This, however, introduces degradation of the signal due to losses in the delay line, in addition to non constant attenuation over a wide frequency range. A hybrid or digital computer may be used, but their size and expense limit their versatility. There are excellent correlators on the market, but their price range is in the $10, 000+ bracket.

This thesis is concerned with the design and performance testing of a wideband, DC to 500 KHz, correlator. The component cost for the system is in the range of one to two hundred dollars. The disadvantage incurred is the time that it takes to generate a correlation function, usually several minutes. Also, the function generated is not a continuous one, but is made up of a maximum of 100 points. The total delay time is from 10 us to 100 ms in steps of 100 ns to 1 ms respectively. The design philosophy and circuitry are considered in moderate detail and then performance and versatility are covered. In autocorrelation, a signal is recovered from noise where the signal to noise ratio (SNR) is -18 dB. With crosscorrelation, a signal is extracted from -36 dB SNR. The added gain of crosscorrelation over autocorrelation is investigated and experimental results are compared with the theory. Sample averaging is implemented by a simple modification to show the capability of this technique to extract a signal from a SNR of -42 dB. Finally, the impulse response of a simple network is obtained by the crosscorrelation of the input with the output when the network is subjected to white noise.

THE MATHEMATICS OF CORRELATION

For stationary random processes the autocorrelation function is defined as

$$R_{11}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f_1(t) f_1(t+\tau)dt$$

where $f_1(t)$ may be some function of a stationary random process, a random function with a hidden periodic component, or possibly just a periodic function. Since the expression requires an averaging process with time as the independent variable, it is called the time average expression for $R_{11}(\tau)$. The crosscorrelation function is defined by the expression

$$R_{12}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f_1(t) f_2(t+\tau)dt.$$

A method for the computation of $R_{11}(\tau)$, the autocorrelation, would consist of the following steps:

1) displace $f_1(t)$ by a small interval τ_1, resulting in $f_1(t+\tau_1)$,

2) multiply the two functions, $f_1(t)$ and $f_1(t+\tau_1)$, continuously,

3) integrate the product over a long duration, and

4) take the average value of the integral over the duration to obtain the point $R_{11}(\tau_1)$ on the autocorrelation curve.

Repetition of this procedure for different values of τ determines as many points on the curve as are necessary. To electronically perform this procedure would require methods for delaying and storing the function of time $f_1(t)$ and $f_1(t+\tau)$. These requirements are not easy to meet.

Instead of working with time averages of a function, the process as a whole can be considered. The process can be considered to consist of an infinite ensemble of functions generated by an infinite number of similar sources. Let y_1 be the value of the random variable representing the values of the functions of the ensemble at some time t, and let y_2 be those observed τ seconds later. Then $P(y_1, y_2; \tau)$ is the joint probability density function of the variables y_1 and y_2 separated by τ seconds. The average or expected value of the product of y_1 and y_2 is then

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y_1 y_2 P(y_1, y_2; \tau) dy_1 dy_2 .$$

This is called the ensemble average.

An important hypothesis in the theory of stationary random processes, known as the ergodic hypothesis (3, 5), states that the time average is equivalent to the ensemble average. Therefore,

$$R_{11}(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y_1 y_2 P(y_1, y_2; \tau) dy_1 dy_2$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} f_1(t) f_1(t+\tau) dt$$

The ensemble average interpretation of the autocorrelation function may be easily implemented. Consider Figure 1 (6) as representing some arbitrary function of time.

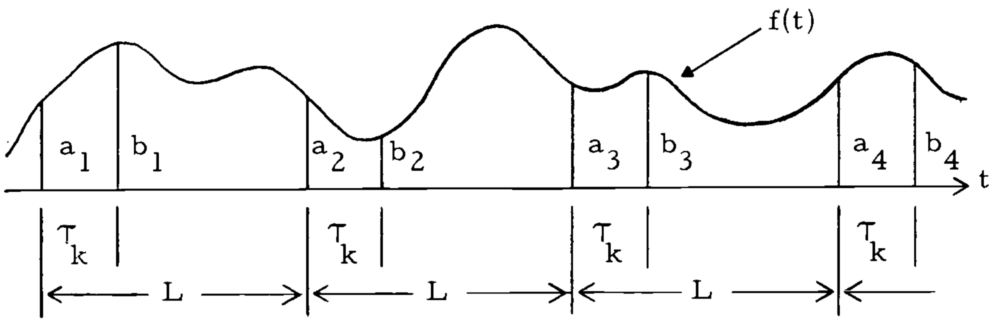


Figure 1. A graphical representation of ensemble averaging.

Let the function be divided into sections of duration such that the values $a_1, a_2, a_3, \ldots$ are independent of one another. The sections L may be considered to be members of an ensemble of functions. The values of $b_1, b_2, b_3, \ldots$ are separated by time τ_k from $a_1, a_2, a_3, \ldots$. Then the mean of the products $a_1 b_1, a_2 b_2, \ldots$ is

$$\frac{1}{n}\sum_{i=1}^{n} a_i b_i$$

If the number of sections is increased to infinity, this average tends to the value of the autocorrelation function at $\tau = \tau_k$. That is;

$$R_{11}(\tau_k) = \lim_{n \to \infty} \frac{1}{n}\sum_{i=1}^{n} a_i b_i .$$

If n is made large, but finite, then the autocorrelation function at $\tau = \tau_k$ will have the approximate value

$$R_{11}(\tau_k) \simeq k\sum_{i=1}^{n} a_i b_i .$$

It is this formula that can be easily implemented electronically.

Properties of Autocorrelation

Some of the properties of autocorrelation are as follows.

1) $R_{11}(\tau) = R_{11}(-\tau)$. The autocorrelation function is symmetrical about $\tau = 0$.

2) $R_{11}(0)$ = mean square of $f(t)$.

3) If a random process has a periodic component of period T, then the autocorrelation function will have a periodic

component of period T.

4) $R_{11}(\infty)$ = square of the mean value of $f(t)$.

5) The maximum value of the autocorrelation function occurs at $\tau = 0$.

THE CORRELATOR

The mathematical formula for the approximation of the correlation function is;

$$R_{11}(\tau) \simeq \frac{1}{n} \sum_{i=1}^{n} a_i b_i$$

where b_i is delayed τ seconds with respect to a_i. A true average is not taken, therefore the correlator output will only be proportional to the sum of the products. That is,

$$R_{11}(\tau) \simeq k \sum_{i=1}^{n} a_i b_i$$

where k is a function of sample size n and integrator time constant.

The necessary steps to electronically implement this formula are:

1) Sample $f(t)$ to get a value a_i,
2) τ seconds later, again sample $f(t)$ to get a value b_i,
3) generate a voltage which is proportional to the product $a_i b_i$,
4) sum the n resultant products.

After processing the n^{th} product, the correlator output voltage will represent an approximation to the magnitude of the correlation function at τ seconds from the origin.

5) Reset the summing circuit to zero and increment the delay τ by $\Delta\tau$ and repeat the previous steps.

The basic block diagram is shown in Figure 2. The pulse generator generates pulses at a fixed rate. When a pulse occurs, the first follow and hold circuit (F/H-1) holds or stores the value of $f(t)$ at that instant. That same pulse simultaneously enters the delay circuit and τ seconds later, a pulse emerges from the delay to trigger F/H-2 which stores the value of $f(t)$ at $t + \tau$ seconds. The two values of $f(t)$, that is $f(t)$ and $f(t+\tau)$, are multiplied and the product is summed in the integrator. The original pulse from the pulse generator is tallied in the sample counter. When 56, 000 samples have been tallied, a single count enters the increment counter. At this point, one point on the correlation curve has been computed and is available at the output of the integrator. The integrator will then be reset and ready for the next value. The increment counter, having been advanced one count, causes the digital to analog converter to increase its output one step. This output voltage determines the amount of delay τ. For the next 56, 000 pulses from the pulse generator, another point on the correlation curve will be computed for the new value of delay τ. After the 56, 000th pulse, the increment counter will again be advanced increasing the DAC output which in turn increases the total delay τ. This cycle is repeated 100 times

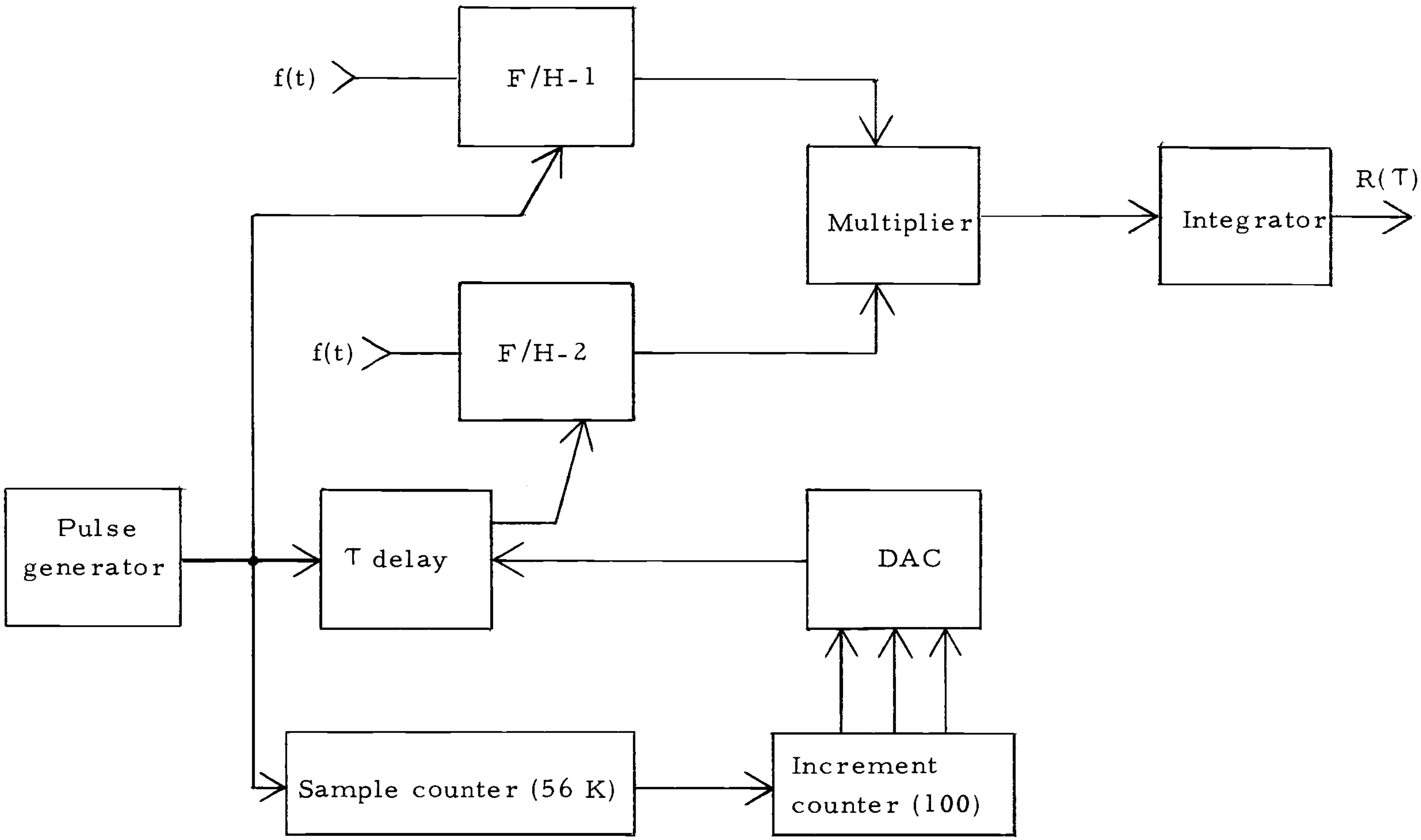


Figure 2. The correlator basic block diagram.

for 100 points on the correlation curve.

System Parameters

One of the system parameters, the number of increments of delay τ used, has been subjectively determined to be 100. If a periodic component appears in a correlation function, three or four cycles of the periodic component should be presented. It is felt that 25 points or so is more than sufficient to accurately determine the shape of one cycle of the correlation function. If more points per cycle are desired, $\Delta \tau$ may be decreased and say one or two periods of the correlation function may be plotted by the 100 points.

The other system parameter, sample size, was set at 56, 000 maximum with two other choices of 24, 000 and 8, 000 selectable by a switch. The larger the sample size, the better a signal can be extracted from noise. However, for autocorrelation, the sample size required to pull a signal out of the noise increases approximately as the fourth power of the noise to signal ratio (5, p. 296). Also the time required to compute only one point on the correlation curve may take minutes for large sample sizes and large values of $\Delta \tau$.

Follow and Hold Circuit

The follow and hold circuit consists of two amplifiers, a switch, and a capacitor as in Figure 3. The first amplifier is a unity gain inverting amplifier with low output impedance capable of driving a large capacitive load with voltage swings of ±2 volts to 500 KHz. The second amplifier is a high input impedance unity gain amplifier using FETs in the input stage. The switch is a high speed FET with 25 ohms "on" resistance and nanosecond switching speeds. When the switch is on, the output of the second amplifier follows the input to the first amplifier. When the "sample" command is given, the switch goes to its off or high resistance state and the output of the second amplifier is whatever voltage was present on the storage capacitor at the instant of "sample." The charge on the capacitor cannot leak off rapidly because the capacitor has very high leakage resistance, the FET switch has very low leakage in the off state (in the nanoamp region), and the second amplifier has an ultra high input impedance achieved by the use of FETs in the first stage. The schematic for the first and second amplifiers are given in Figures 4 and 5 respectively. The switch and drive circuit which is compatible with TTL circuitry is included in Figure 4. In F/H-2, it is required that the output voltage of the second amplifier be raised about 2.5 volts above ground for zero volts in. This is a requirement of the multiplier circuit. The 2.5 volt offset is achieved by passing a constant current through a resistor in the feedback loop. See Figure 6.

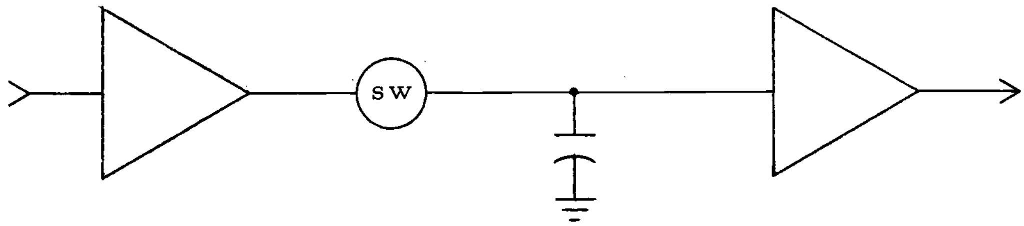


Figure 3. Follow and hold block diagram.

The Multiplier

The multiplication of two functions is carried out in the following manner. Let the voltage sampled by F/H-1 be V_1 and that of F/H-2 be V_2. The product of V_1 and V_2 is achieved by generating a pulse whose amplitude is V_1 and whose duration is proportional to V_2. The area under the pulse, voltage x time, will be proportional to the product $V_1 V_2$. V_1 and V_2 can range ±2 volts about zero volts, therefore it is necessary to raise V_2 2.5 volts above ground so that V_2 will always be positive and thus the pulse duration will always be a positive quantity. If a pulse duration of ten us/volt is chosen the pulse width will be:

$$PW = 10 \text{ us/volt} \times V_2 \text{ volts} = 10V_2 \text{ us}.$$

The area of the pulse will be:

$$\text{Area} = V_1 \times PW = 10V_1V_2 \text{ volt-us}.$$

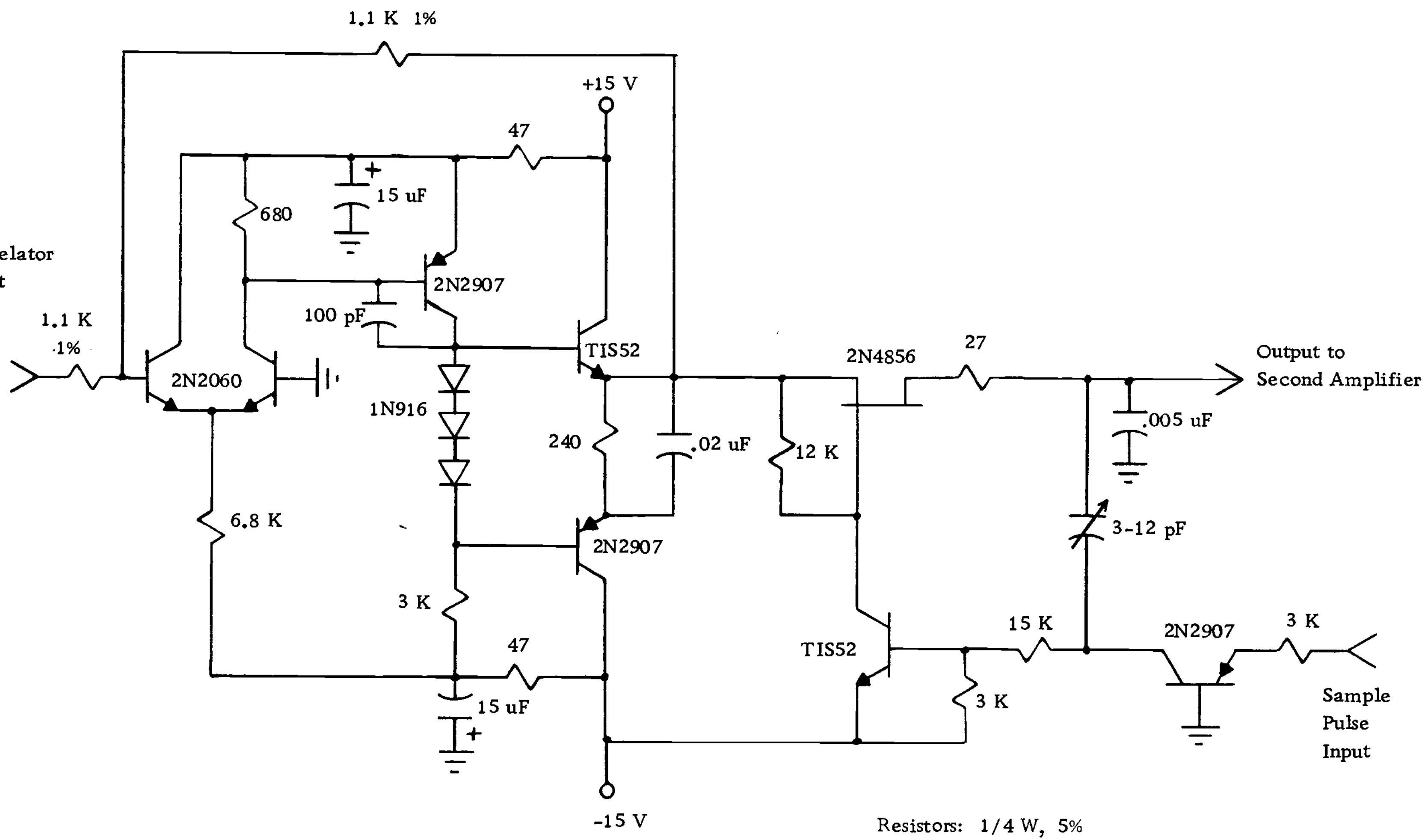


Figure 4. The first amplifier in the follow and hold circuit.

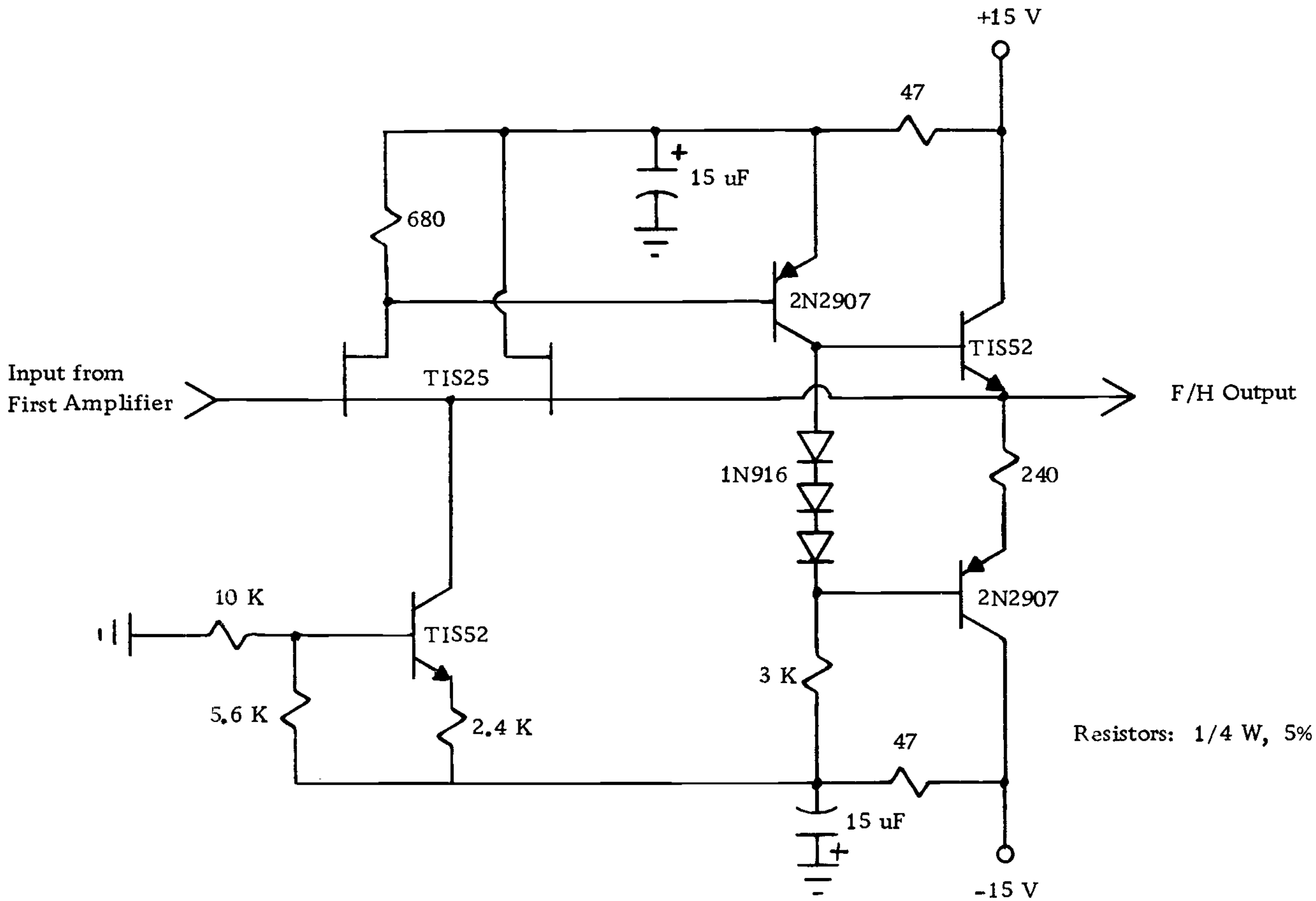


Figure 5. The second amplifier in the follow and hold circuit.

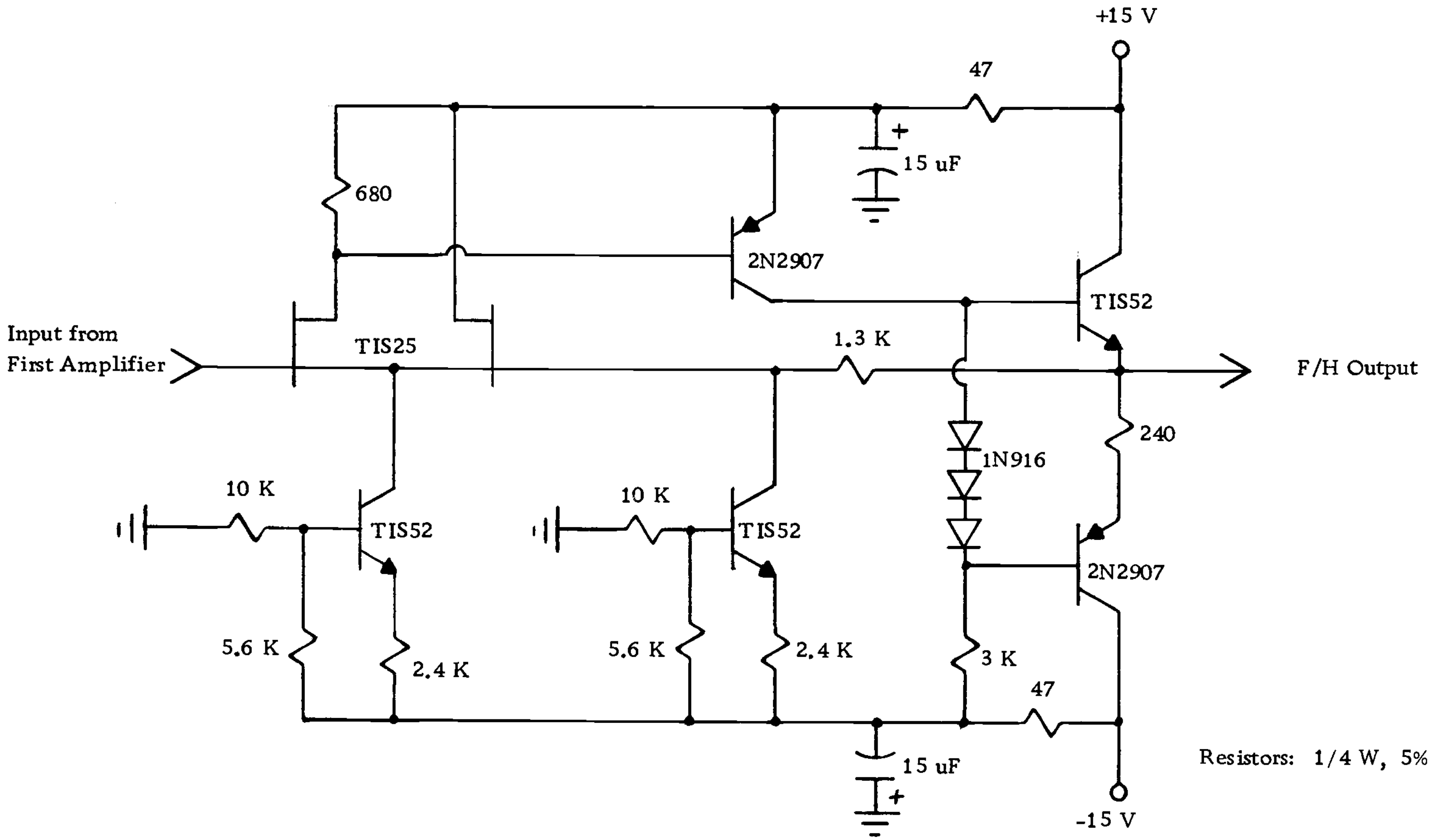


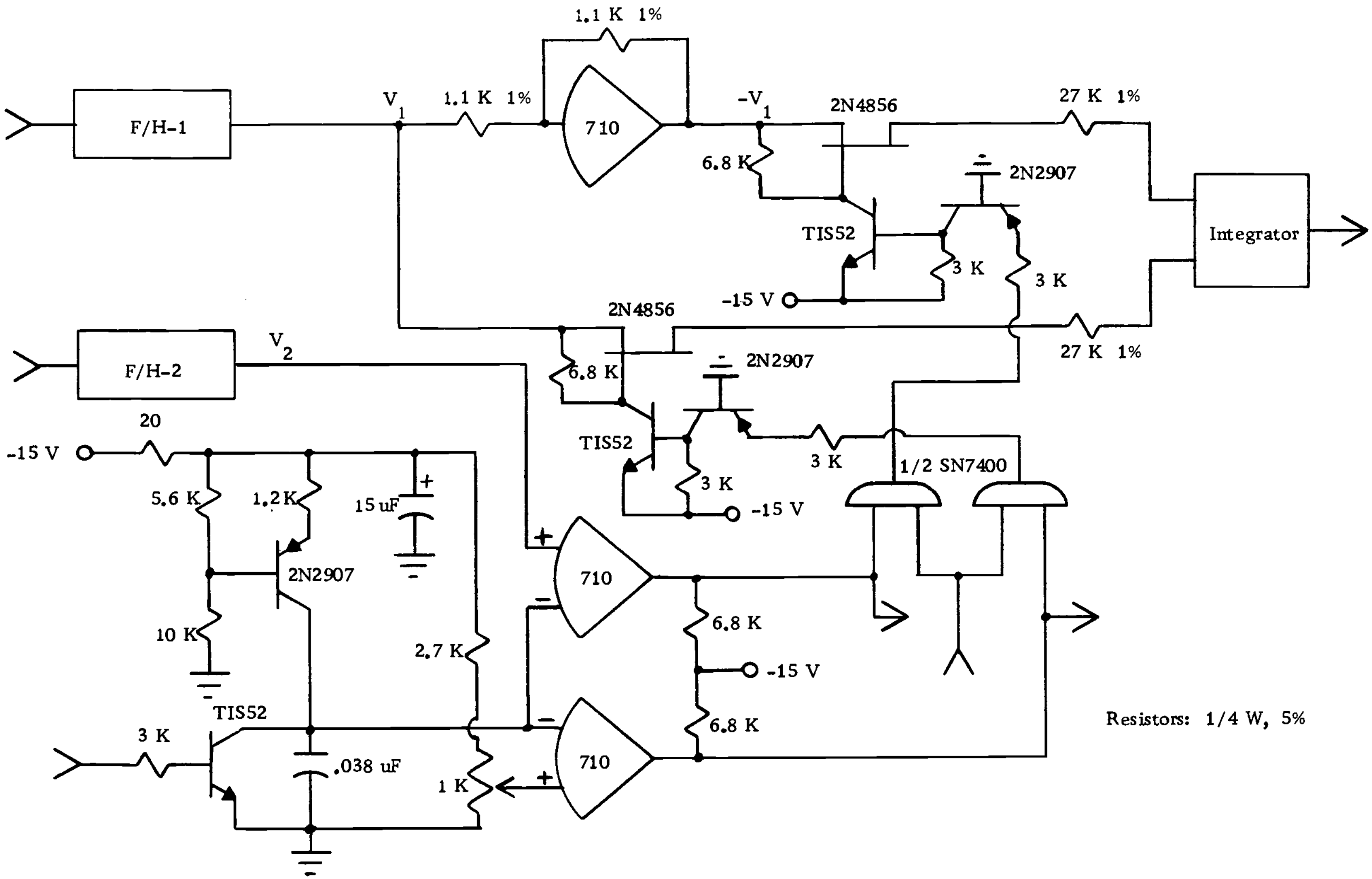
Figure 6. Output amplifier with 2.5 volt offset for F/H-2.

It is desirable that the area under the pulse be zero when either of the inputs to the correlator is zero. When the input to F/H-2 is zero volts, V_2 is 2.5 volts, and the area under the pulse will be 25 V_1 volt-us. It is necessary then to subtract 25 V_1 volt-us from the area. That is:

$$\text{Area} = 10V_1V_2 - 25\ V_1 \text{ volt-us}.$$

The area is summed by the integrator by applying to its input a pulse whose amplitude is V_1 and whose duration is 10 V_2 us. Also, simultaneously, a pulse whose amplitude is $-V_1$ and duration 25 us is summed. The net area summed by the integrator is directly proportional, sign included, to the product of the two sampled voltages of the follow and hold circuits.

Figure 7 is the schematic for the multiplier. The inverting amplifier following F/H-1 is identical to that shown in Figure 4. A constant current source charging a capacitor produces a ramp used in generating a pulse width proportional to V_2, the output voltage of F/H-2. The 710 comparator senses when the ramp voltage exceeds V_2 and turns off the FET switch in series with the integrator input. This circuitry provides the pulse whose area is 10 V_1V_2 volt-us. The second, lower, 710 comparator of Figure 7 switches after 25 us to control the FET switch in series with the other integrator input to provide an opposite polarity pulse whose area is



Resistors: 1/4 W, 5%

Figure 7. Four-quadrant multiplier.

$-25\ V_1$ volt-us.

All of the multiplier's logic controls are compatible with TTL.

The Integrator

The requirements of the integrator are that it have very high input impedance and an ultra low leakage feedback capacitor. This is to insure that the output voltage is not degraded during the several minutes it may take to compute just one point on the correlation curve. A schematic of the integrator and the integrator reset circuit is shown in Figure 8. A 709 linear IC is used as a high gain amplifier and matched FETs provide the high input impedance for the integrator. The reset circuit discharges the feedback capacitor at the end of the computation of a point on the correlation curve. The reset drive is compatible with TTL.

The Sample and Increment Counters

The sample counter counts the number of samples taken per point on the correlation curve. This may range from approximately 8, 000 to 56, 000 selectable by a panel switch. A count of 56, 000 requires 16 flip-flops.

The increment counter records the number of incremental delays or points computed on the correlation curve. Its count is to 100; seven flip-flops are needed. The need for 23 flip-flops resulted in

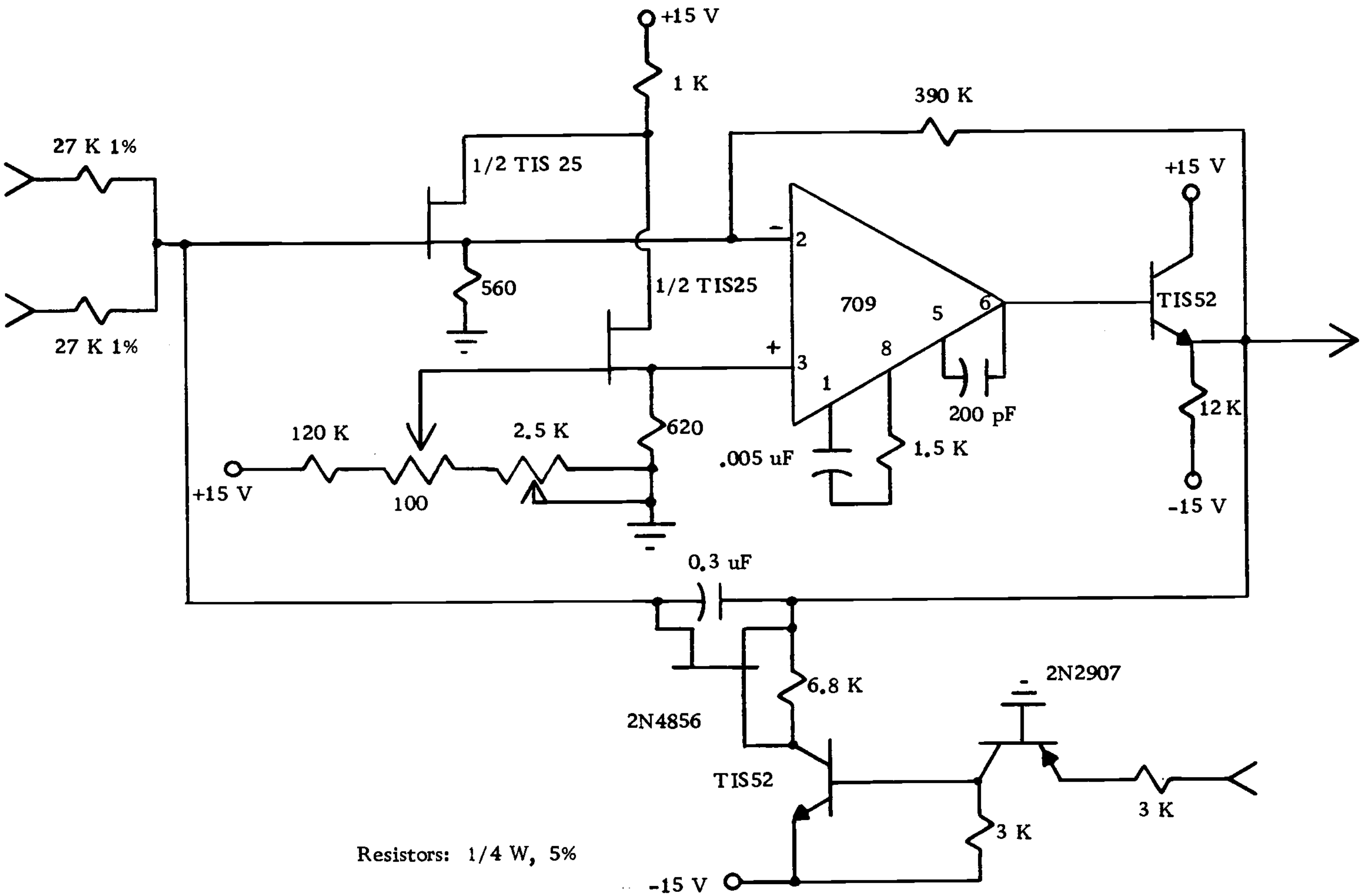


Figure 8. Integrator and reset circuit.

the choosing of the SN7474 dual D-type edge-triggered flip-flop IC. Two flip-flops per package kept the unit count down to 12. The IC's are wired as simple binary counters and they have the ability to be individually cleared or preset. A wiring diagram for the IC and counters is shown in Figure 9.

The Digital to Analog Converter

The DAC converts the binary count in the increment counter into an analog voltage ranging from zero to five volts in 50 mV steps. This voltage is used to control the τ delay. Figure 10, the schematic, illustrates the use of NPN transistors operated in the inverted mode for use as switches. The V_{ec} saturation or offset voltage is less than 5 mV. The switches are driven directly by the $\overline{Q}$ terminal of the increment counter flip-flops. The 709 IC amplifier sums the currents from the various resistor networks when the corresponding transistor switch is off. The amplifier output is used not only to control the τ delay but also to drive the horizontal axis of the XY recorder during a correlation computation.

The Delay Circuit

The τ delay circuit's function is to produce a logic pulse exactly τ seconds after receiving an input logic signal. Referring to Figure 11, the DAC supplies the 710 comparator with a voltage

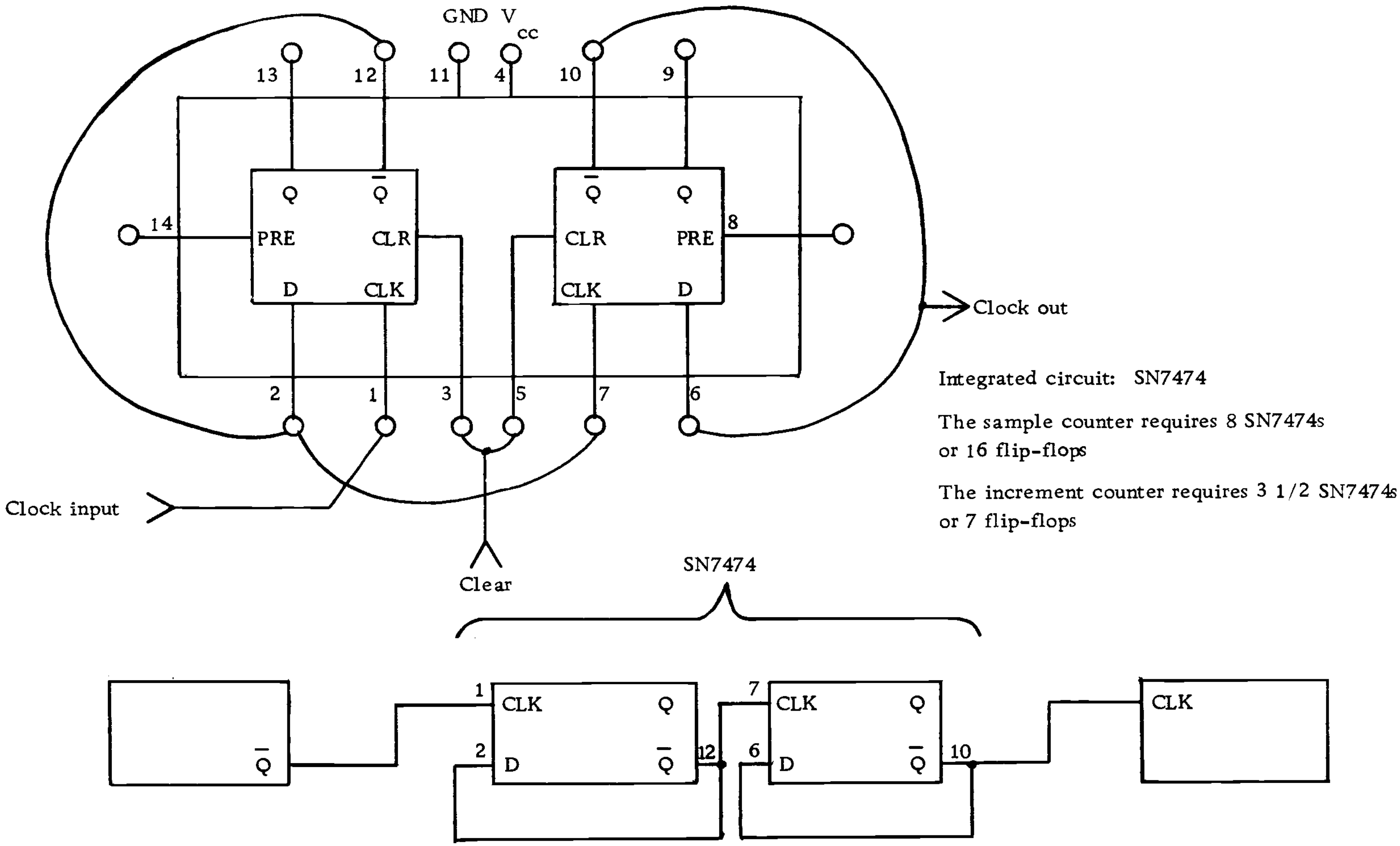


Figure 9. Wiring diagram for the sample and increment counters.

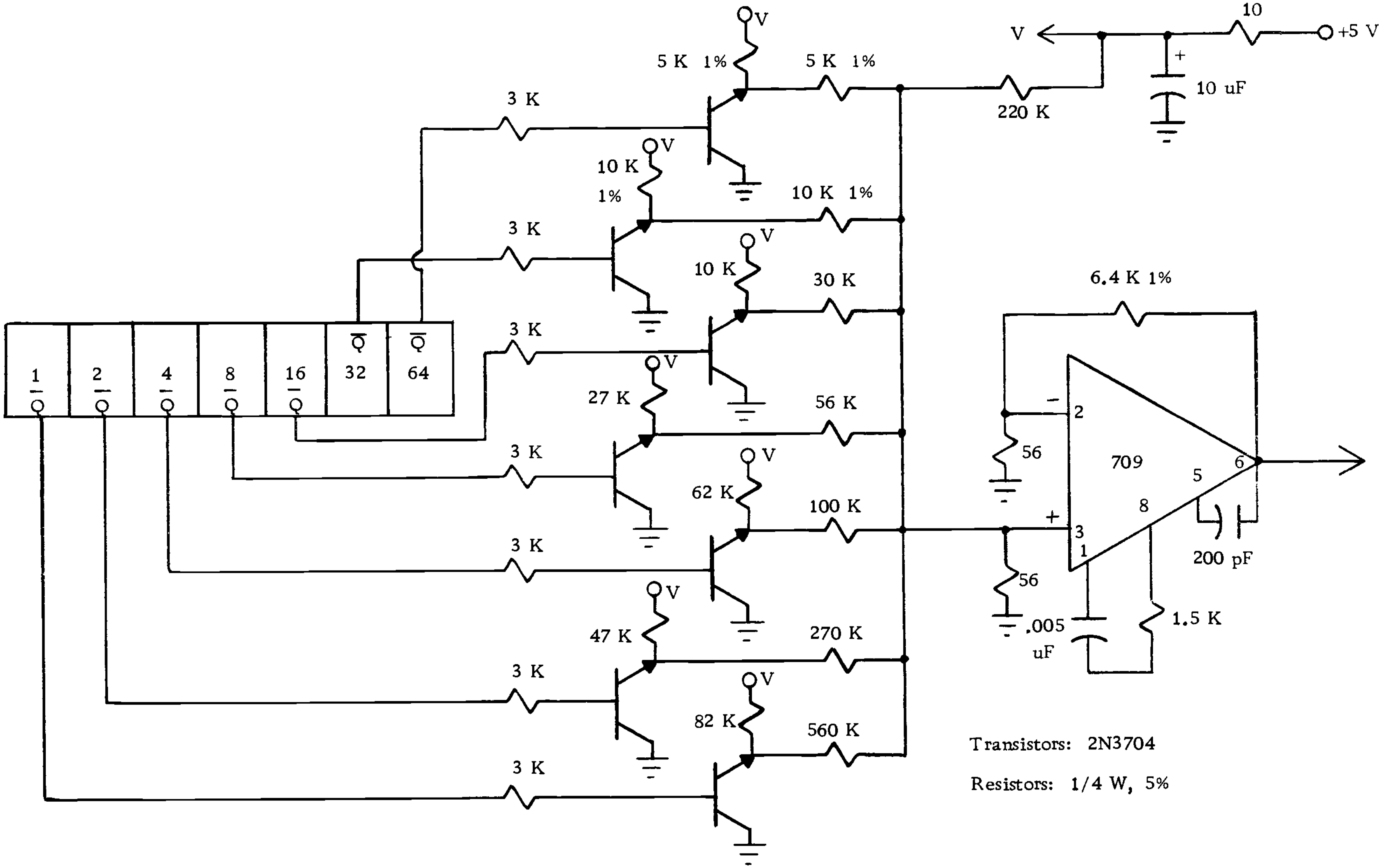


Figure 10. Digital to analog converter schematic.

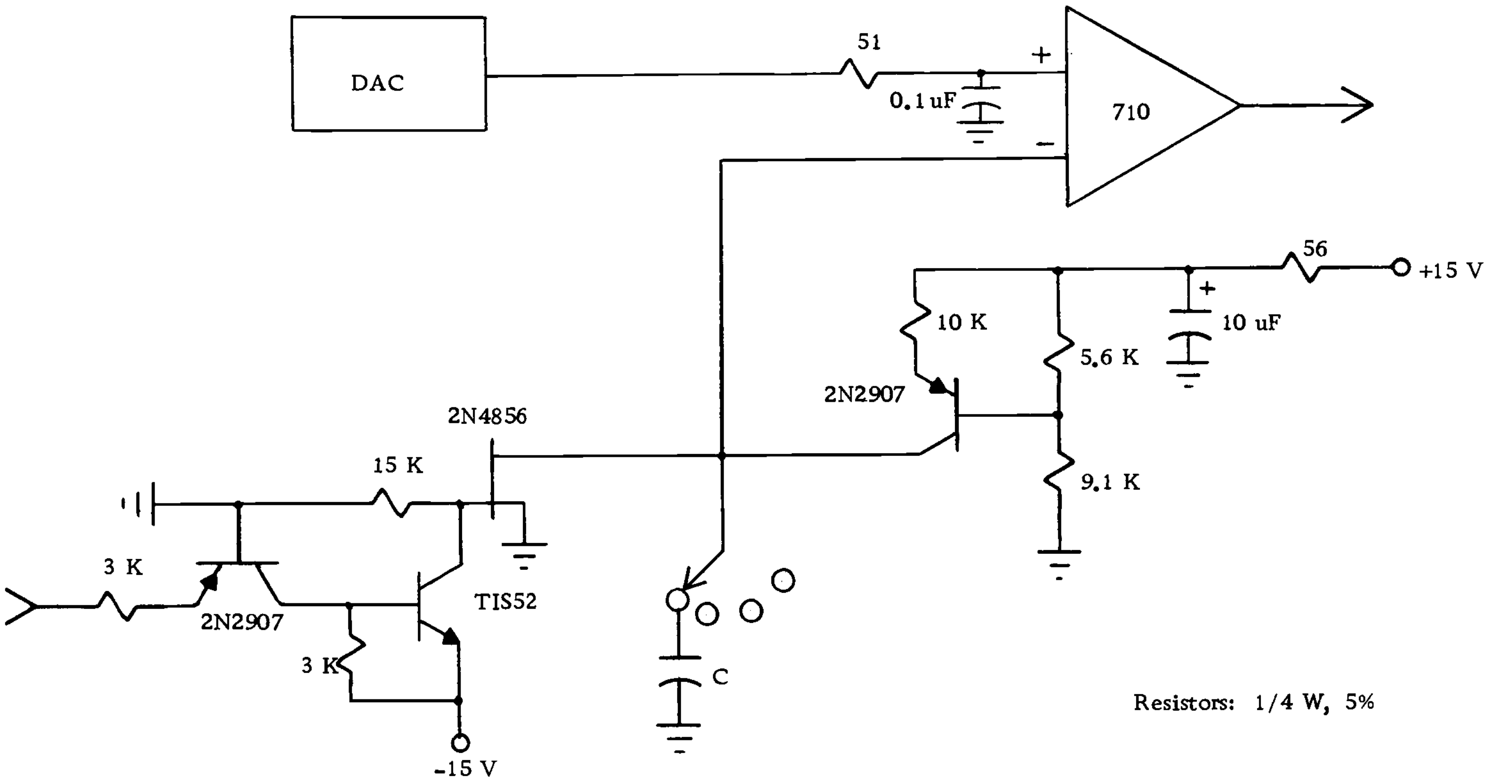
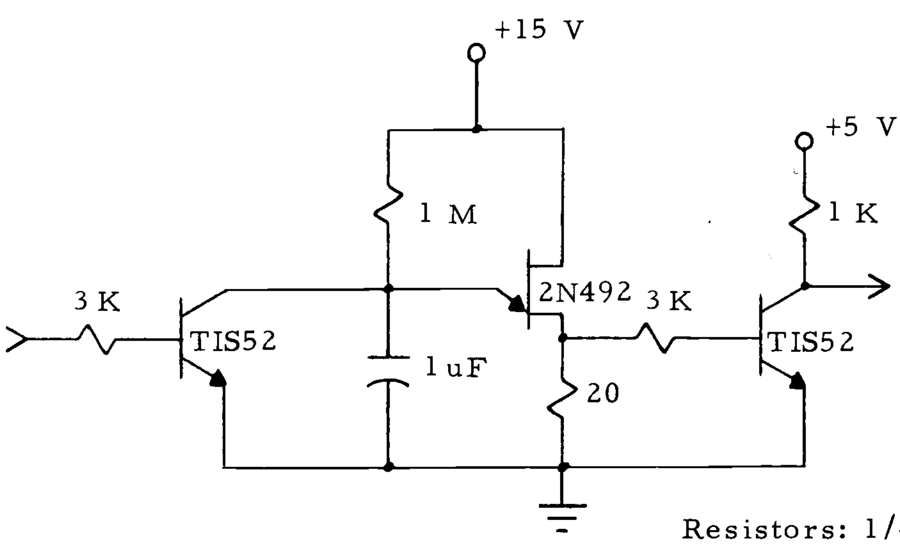


Figure 11. The τ delay circuit.

ranging from approximately zero volts to five volts in 50 mV steps. Each step represents a different value of τ and thus another point on the correlation curve. When a logic signal turns the FET off, a constant current source charges capacitor C until the voltage on C exceeds the DAC voltage. At that point, the comparator signals F/H-2 to hold the present value of the input to the F/H. The value of C for various $\Delta\tau$ increments of delay is listed in Table 1 in the Appendix.

One Second Delay

A one second delay is needed between the computation of points on the correlation curve in order to completely discharge the feedback capacitor in the integrator circuit. Figure 12 is a schematic for such a circuit.



Resistors: 1/4 W, 5%

Figure 12. One second delay circuit.

The Pulse Generator

An external pulse generator is required for the operation of the correlator. Its PRF will determine at what rate the samples will be taken. The pulse width should be from 50 to 80 ns with rise and fall times less than 20 ns and its amplitude from 2.5 to 4.0 volts positive. The PRF can vary greatly for different values of $\Delta\tau$ and two or three frequencies can be used during a correlation run at only one delay setting. If a $\Delta\tau$ is selected and one PRF is to be chosen for the entire run, the highest PRF than can be used must have a period greater than $100 \times \Delta\tau$. If $\Delta\tau$ is one ms, the highest PRF can be 10 Hz because at the 100^{th} increment of $\Delta\tau$, the total delay will be 100 ms and the next sample cannot be taken until the second sample of the previous sample set has been processed. It may be desirable to change the sampling PRF several times during a correlation run. For example, with a $\Delta\tau$ of one ms, a sampling PRF of 100 Hz is satisfactory at least for the first ten points on the correlation curve.

The highest sampling frequency that can be used is 15 KHz. It takes about 50 us to process each sample set not including the τ delay.

The System Diagram

The system diagram shows the interconnection of all the previous circuits discussed. The flip-flops used in the operation control are RS-type flip-flops designed from two conventional gates of a four-gate IC package. Figure 13 illustrates the wiring. Figure 14 is the system diagram.

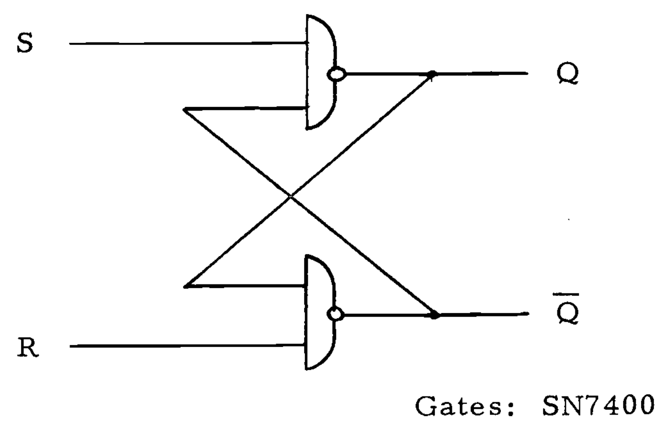


Figure 13. Flip-flop used in system operations control.

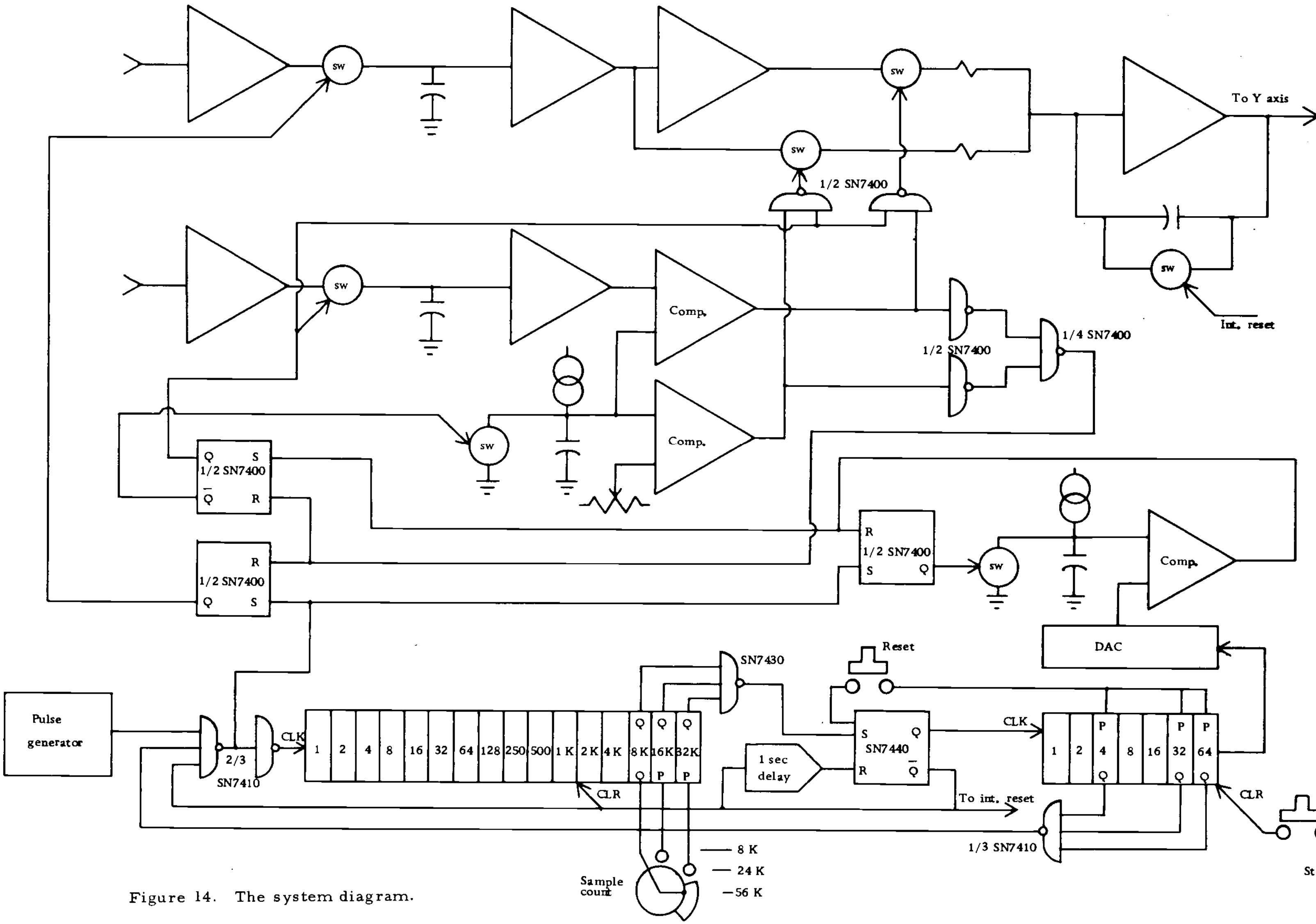


Figure 14. The system diagram.

CORRELATOR PERFORMANCE

System Performance

The frequency response of the correlator has been tested from DC to 500 KHz. A 400 mV rms sine wave was correlated in each instance. A sample size of 8, 000 was taken for each point. For DC, 400 mV was correlated. A plot of the frequency response is found in Figure 15 from the results of Figures 16 and 17.

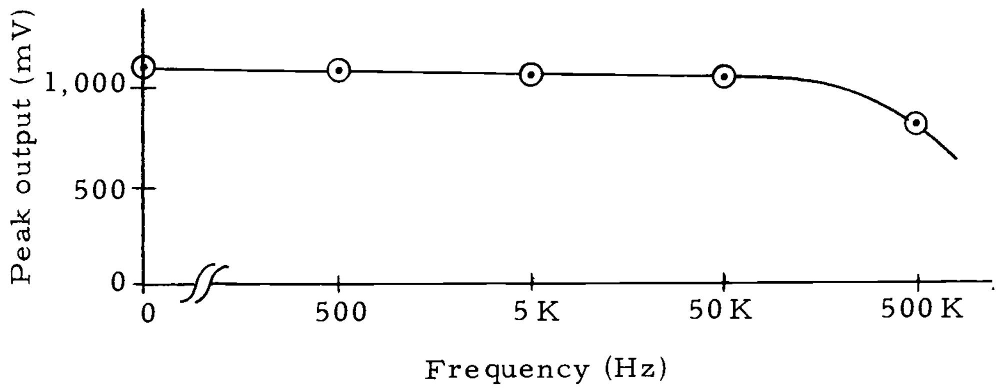


Figure 15. Correlator frequency response.

The gain linearity in autocorrelation is plotted in Figure 18 from the results of Figure 19. The rms value of the input is plotted against the peak value of the correlator output. The peak value is proportional to the mean square of the input, therefore the input output relation plotted on log-log scales should result in a straight line.

The correlator output is directly proportional to the sample

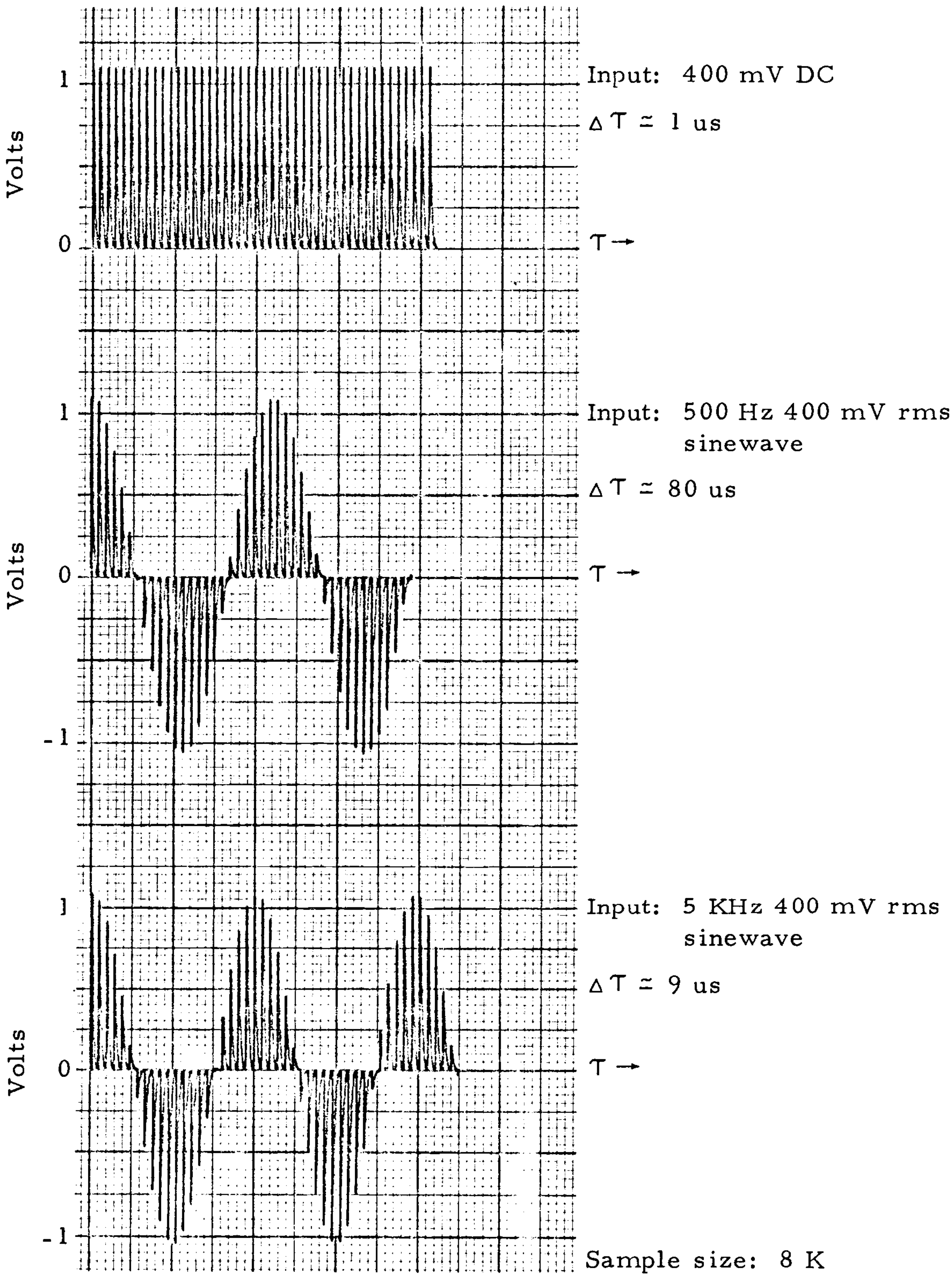
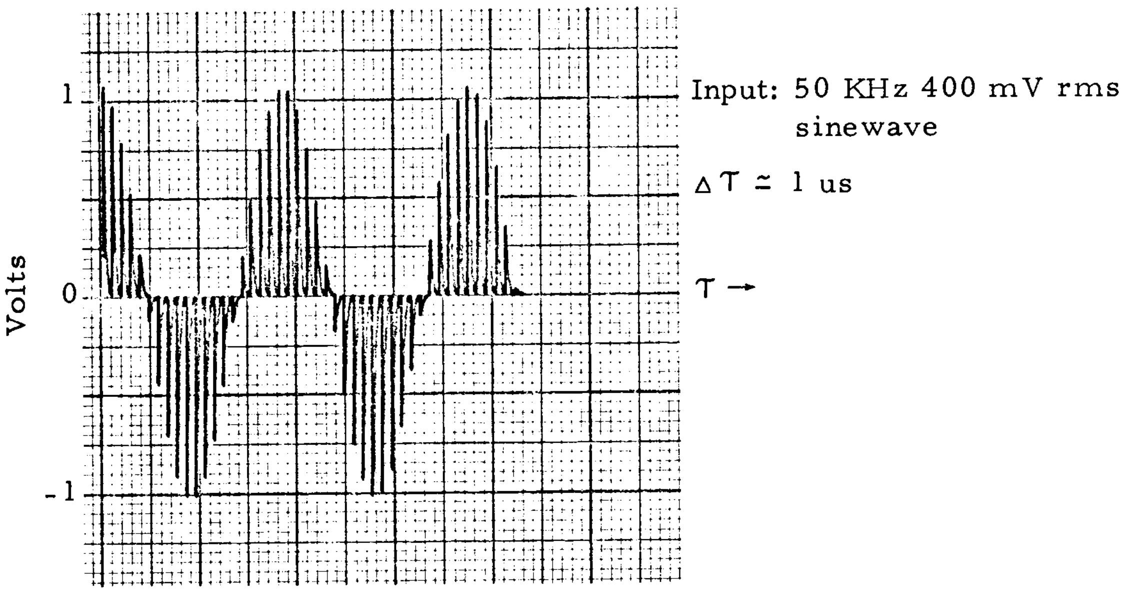


Figure 16. Autocorrelation frequency response.



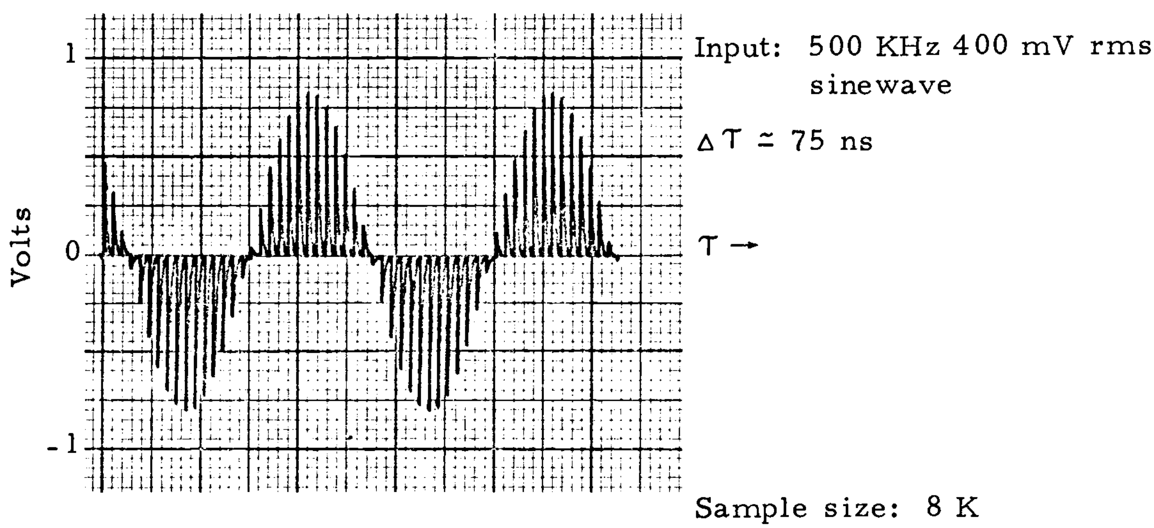


Figure 17. Autocorrelation frequency response.

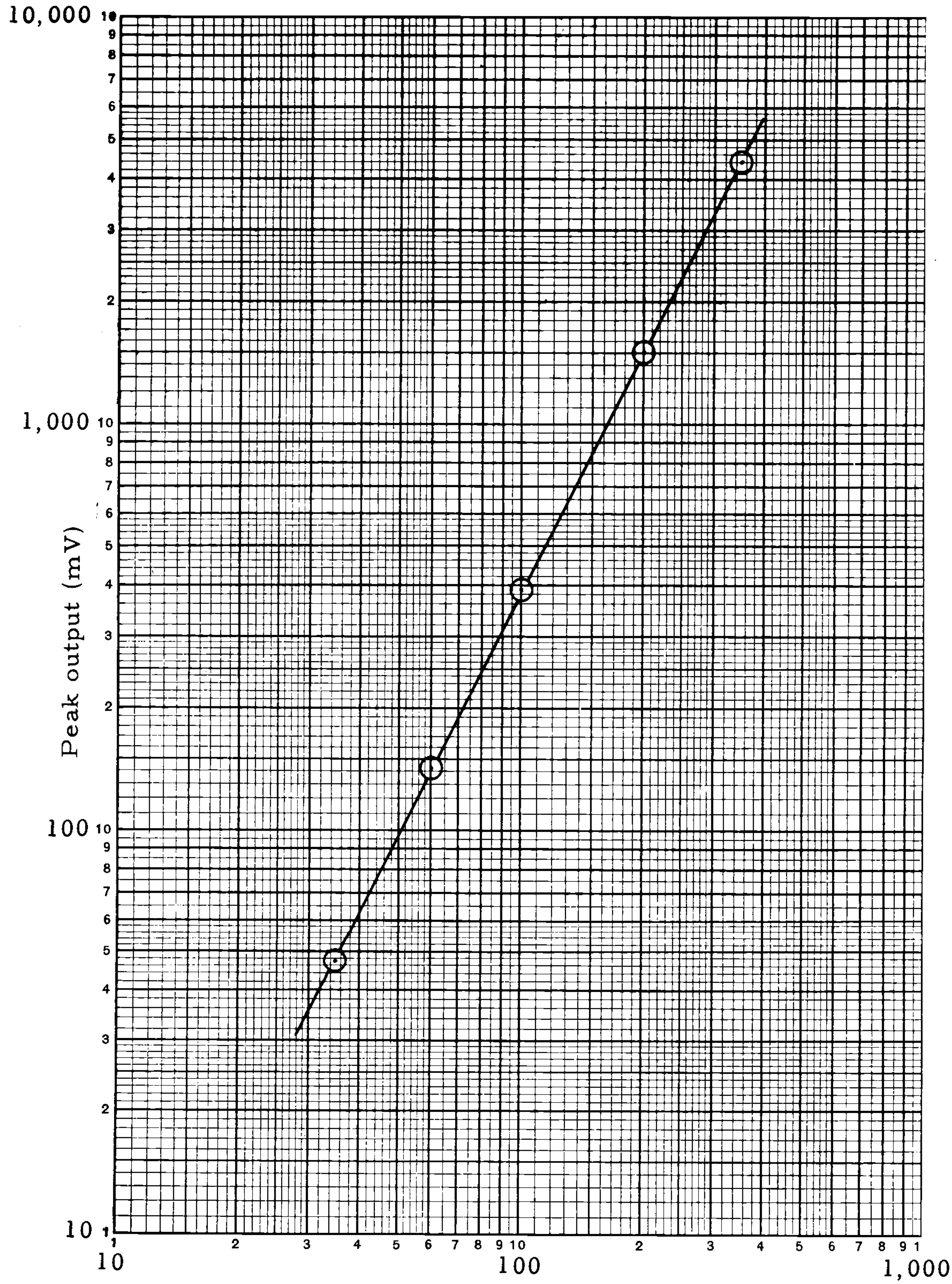


Figure 18. Correlator gain linearity.

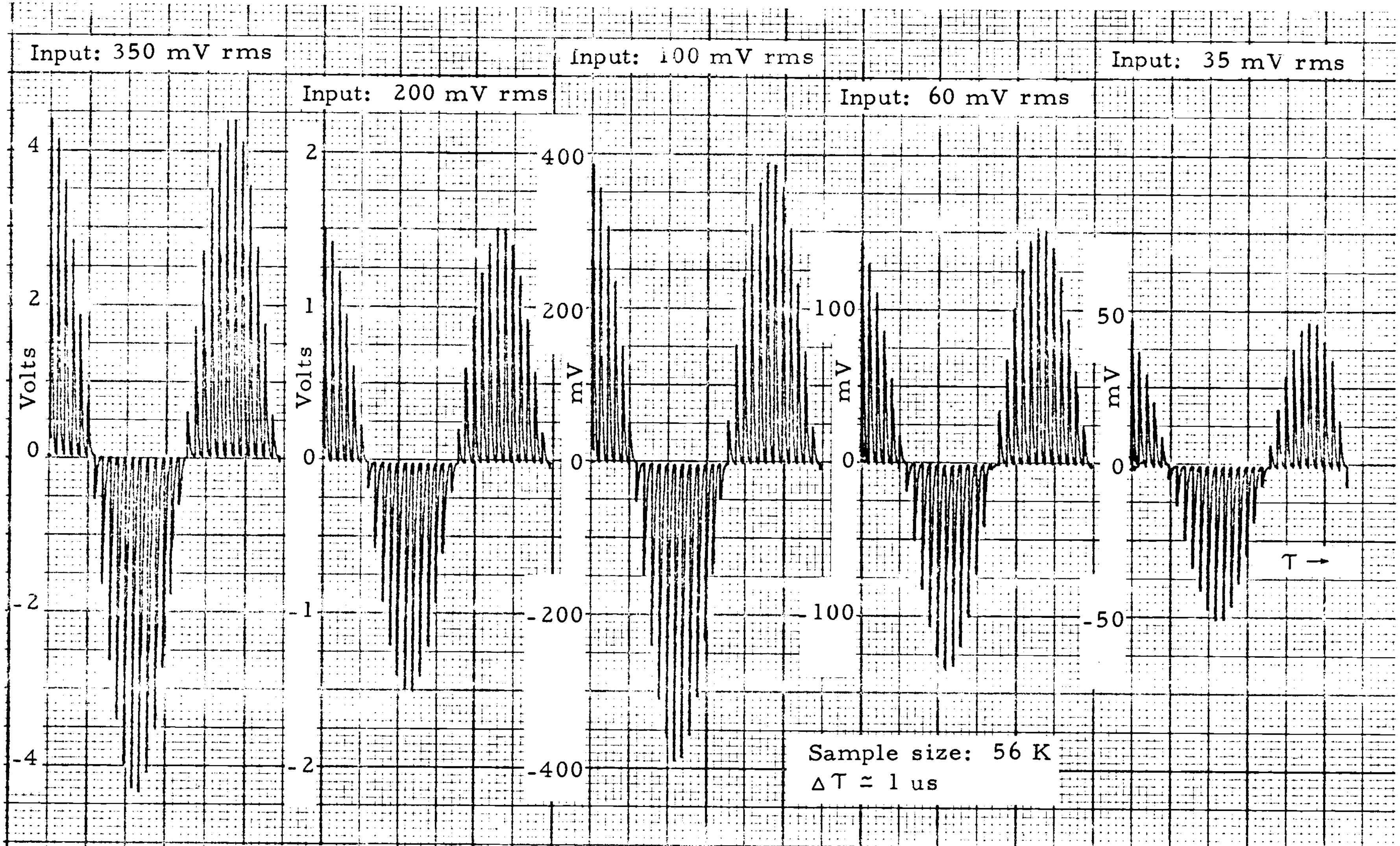


Figure 19. Autocorrelation of 50 KHz sinewave of various amplitudes.

size since a true average is not taken in computing the correlation function. The correlator output is shown in Figure 20 for three sample sizes. Figure 21 illustrates the linear relation between sample size and peak output.

The Autocorrelation of Various Waveforms

A graphical solution of the autocorrelation function of a sawtooth is illustrated in Figure 22. The autocorrelation function of the sawtooth, a triangular wave, and a square wave, as computed by the correlator is shown in Figure 23. A comparison of the graphical and machine computed correlation function of the sawtooth waveform shows the same general trend. An exact comparison cannot be made because of the finite non-negligible fall time and nonsymmetry of the ramp generator output.

Noise in Correlation

The type of noise most often found in communications is gaussian or normally distributed noise. Throughout the remaining experimental results, band-limited gaussian white noise, generated by the General Radio model 1390-B random noise generator, will be used. Its power density spectrum is shown in Figure 24 for a 20 KHz and a 500 KHz bandwidth (1, p. 4).

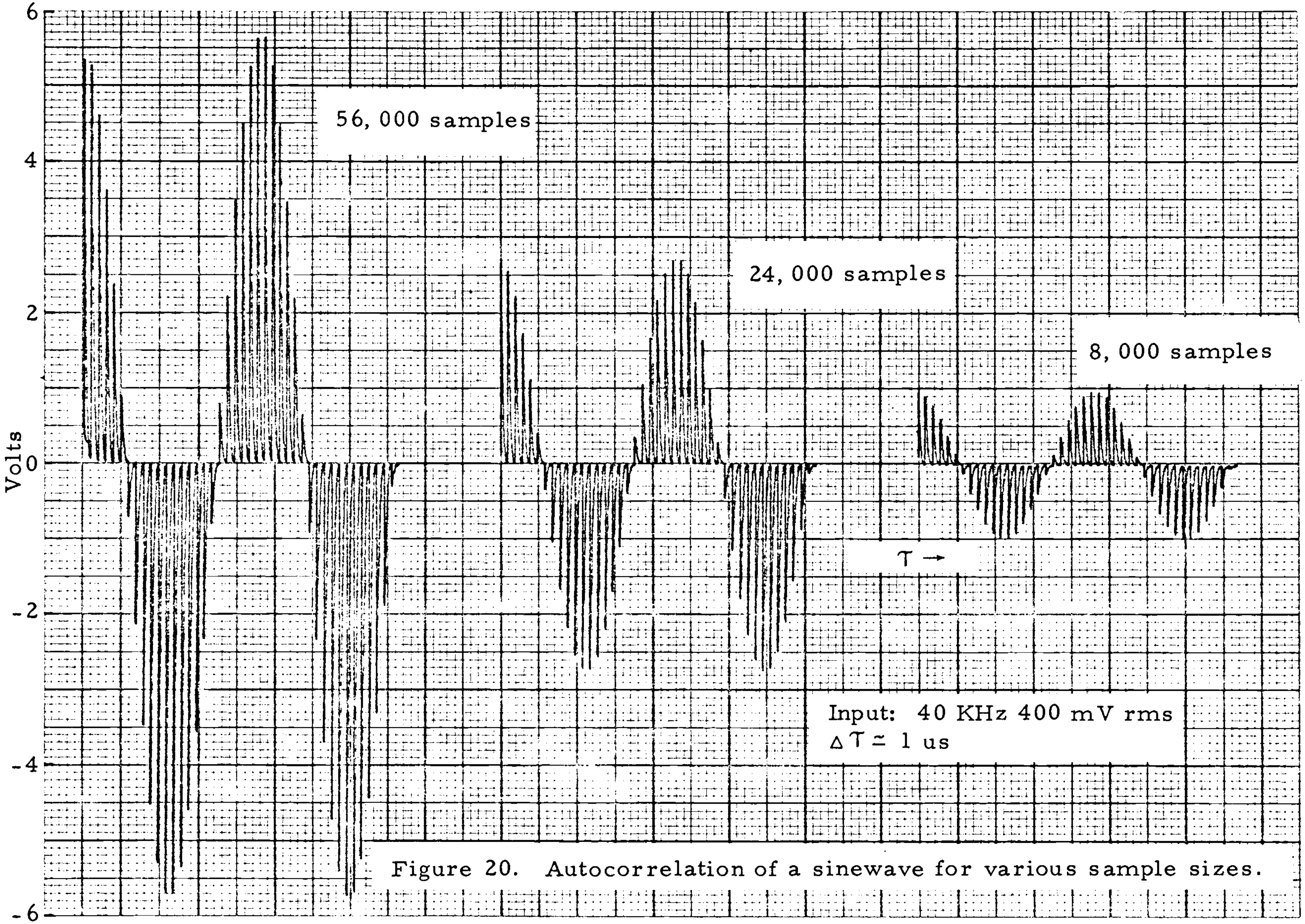


Figure 20. Autocorrelation of a sinewave for various sample sizes.

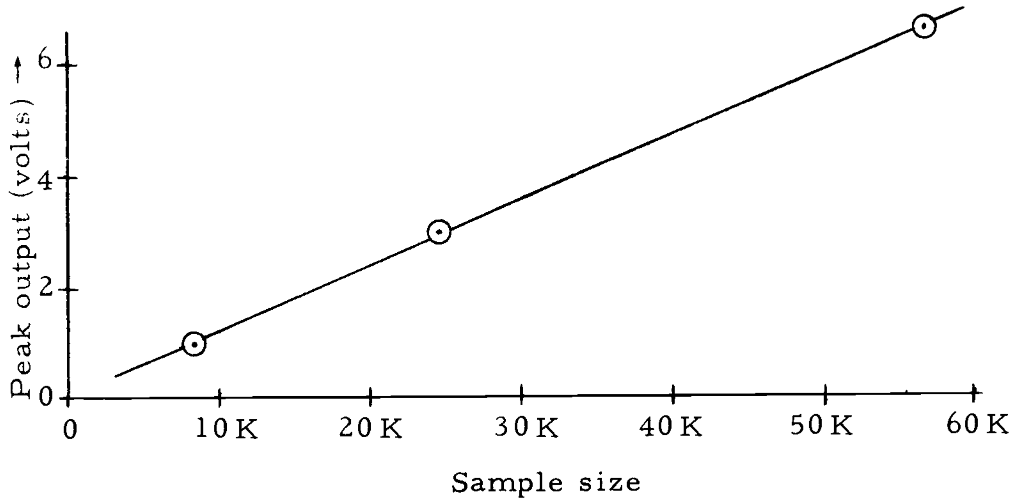


Figure 21. Correlator output voltage as a function of sample size.

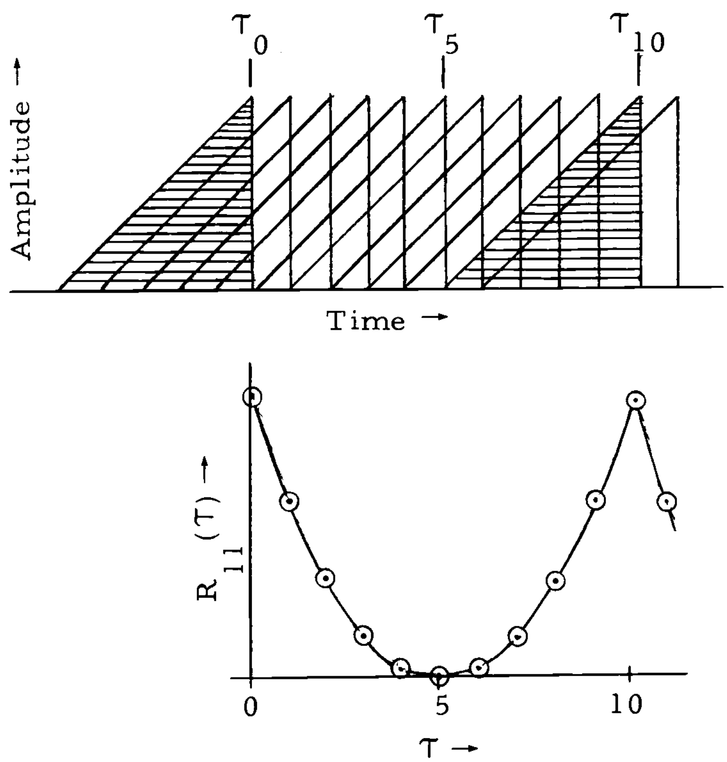
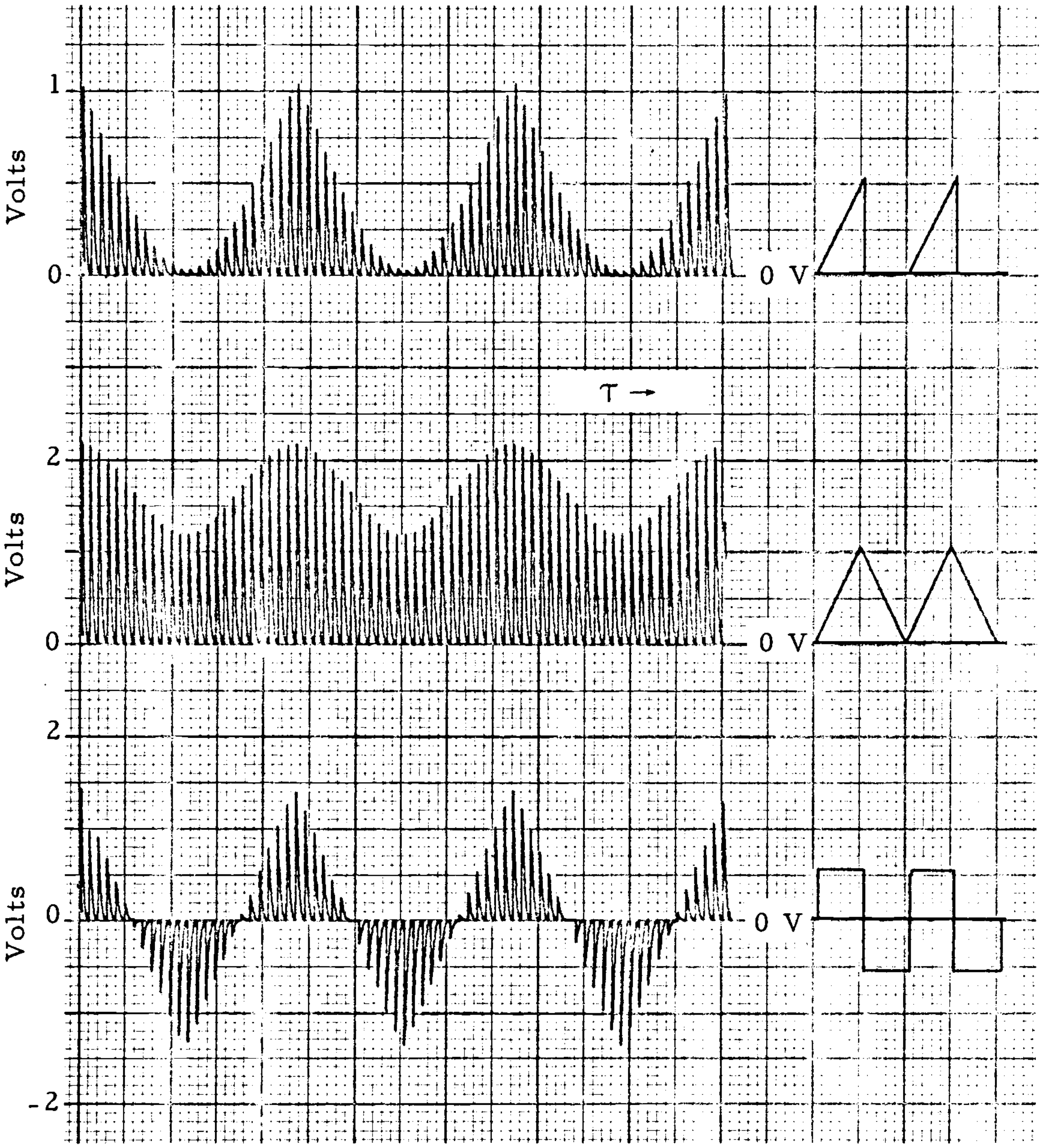


Figure 22. Graphical solution of an autocorrelation function for a sawtooth waveform.



Input: 40 KHz 1 v p-p as shown
$\Delta\tau \simeq 1$ us
Sample size: 8 K

Figure 23. Autocorrelation function for various waveforms.

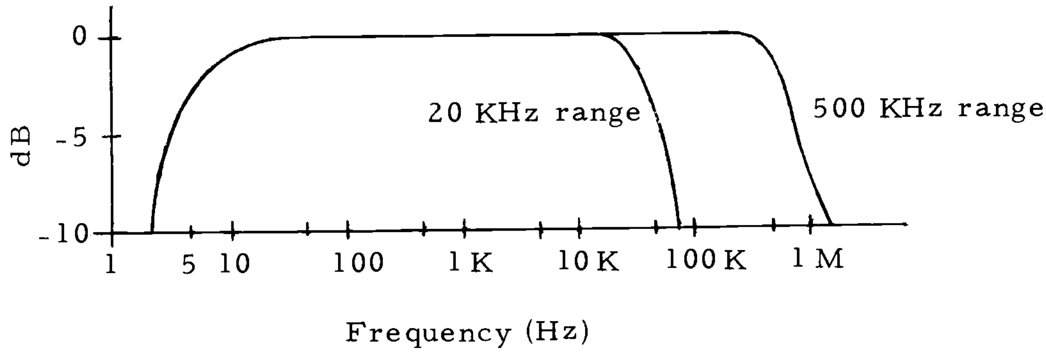
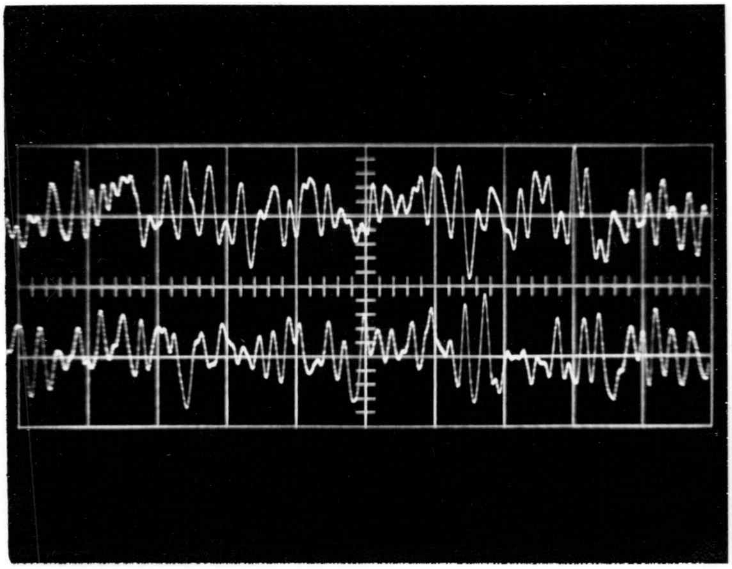


Figure 24. Spectrum level characteristics for the random noise generator.

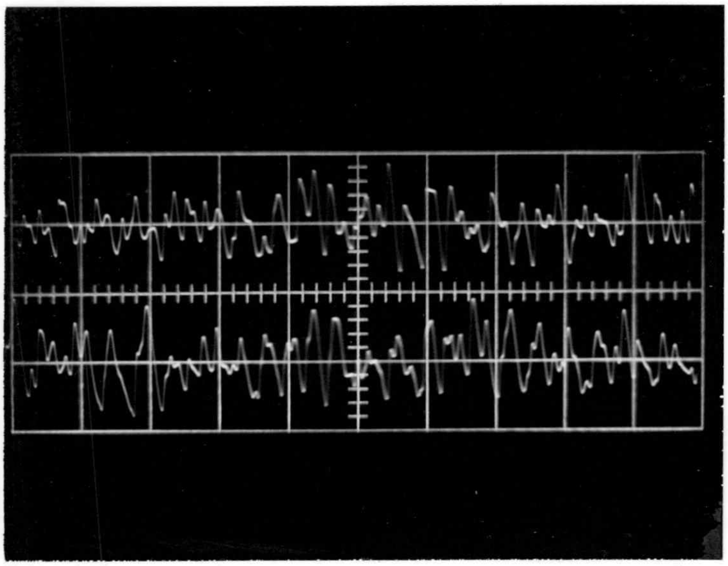
A photograph of a noise waveform appears in Figure 25 for both the 20 KHz and 500 KHz noise BW.

Autocorrelation of Noise

The autocorrelation of band-limited white noise with a zero mean value is graphically presented in Figure 26 for two values of delay. For a small delay, the product of the two noise waveforms over a period of time produces only a small negative quantity toward a final value for the autocorrelation function. The hashed area indicates a negative contribution to the value of the correlation function. Figure 26b indicates that for some larger value of delay, the negative and positive contributions resulting from the product of the two waveforms will cancel and the correlation function for that value of delay will be zero. A further increase in delay will produce a negative value for the correlation function.

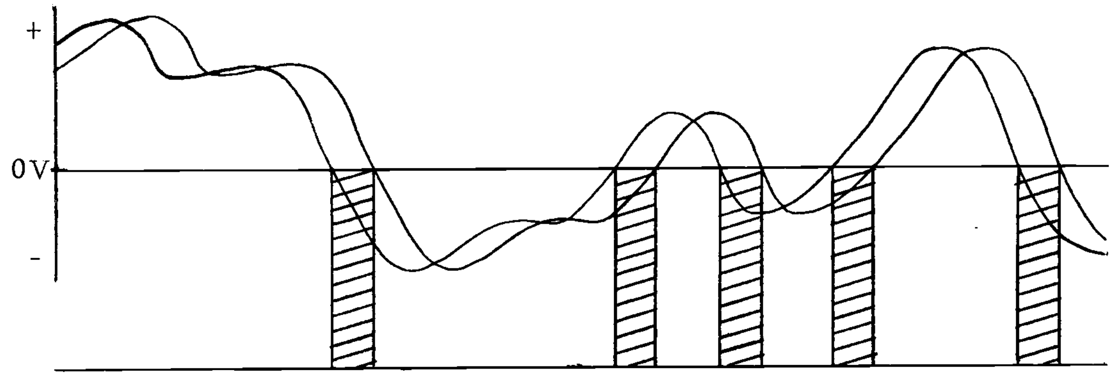


a) White noise band limited to 20 KHz.
Vert: 1 V/cm Horiz: 100 us/cm

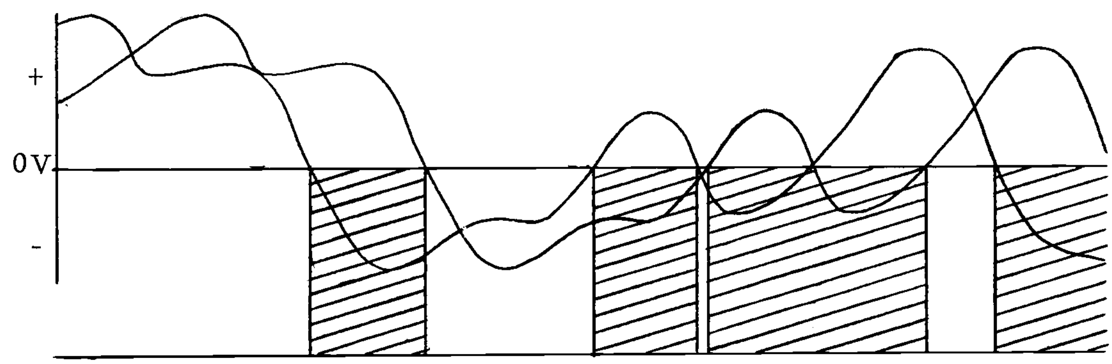


b) White noise band limited to 500 KHz.
Vert: 1/cm Horiz: 5 us/cm

Figure 25. Noise waveforms.



a) small value of delay



b) large value of delay

Figure 26. Graphical representation of autocorrelation for two values of delay.

For the case of an ideally band-limited white noise, the zero crossings of the correlation function are at $\frac{1}{2\,BW}$, and any integral multiple of $\frac{1}{2\,BW}$ (3, p. 211). It can be expected then that the autocorrelation function of band-limited noise as generated by the GR noise generator will have a zero crossing at $\frac{1}{2\,BW}$. By examining the power density spectrum of the generator in Figure 24, the "end" of the band for the 20 KHz BW noise is somewhere between 50 and 100 KHz. The first zero crossing should occur at a delay of from five to ten us. Examining Figure 27a, the autocorrelation of 20 KHz BW noise exhibits a zero crossing at about ten us of delay.

As the delay approaches infinity, $R_{11}(\tau)$ approaches the square of the mean of the input signal. The mean of the gaussian noise is zero, therefore the value of $R_{11}(\tau)$ for large delays approaches zero. Figure 27b and 27c show that the correlation function for noise dampens out to zero with increasing delay.

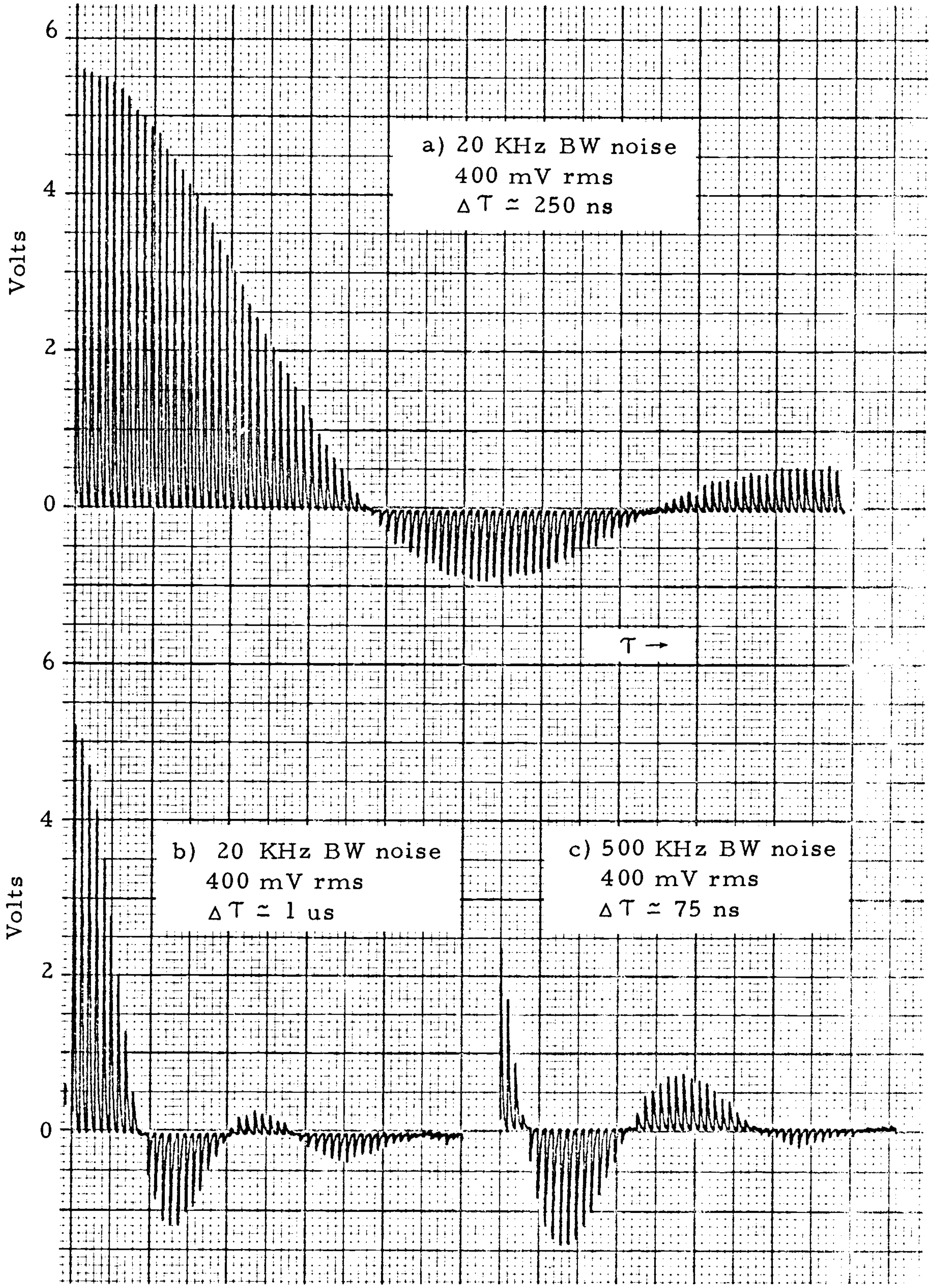
The value of the correlation function for $\tau = 0$ in Figure 27b and 27c should be equal, but the first point in the curve for 50 KHz BW noise is not for $\tau = 0$. The delay circuitry has a 50 to 75 ns residual delay.

The autocorrelation function of 20 KHz BW noise should average about zero for large delay. Figure 28 exhibits the tendency for the correlation function to average about zero.

Autocorrelation of Signals in Noise

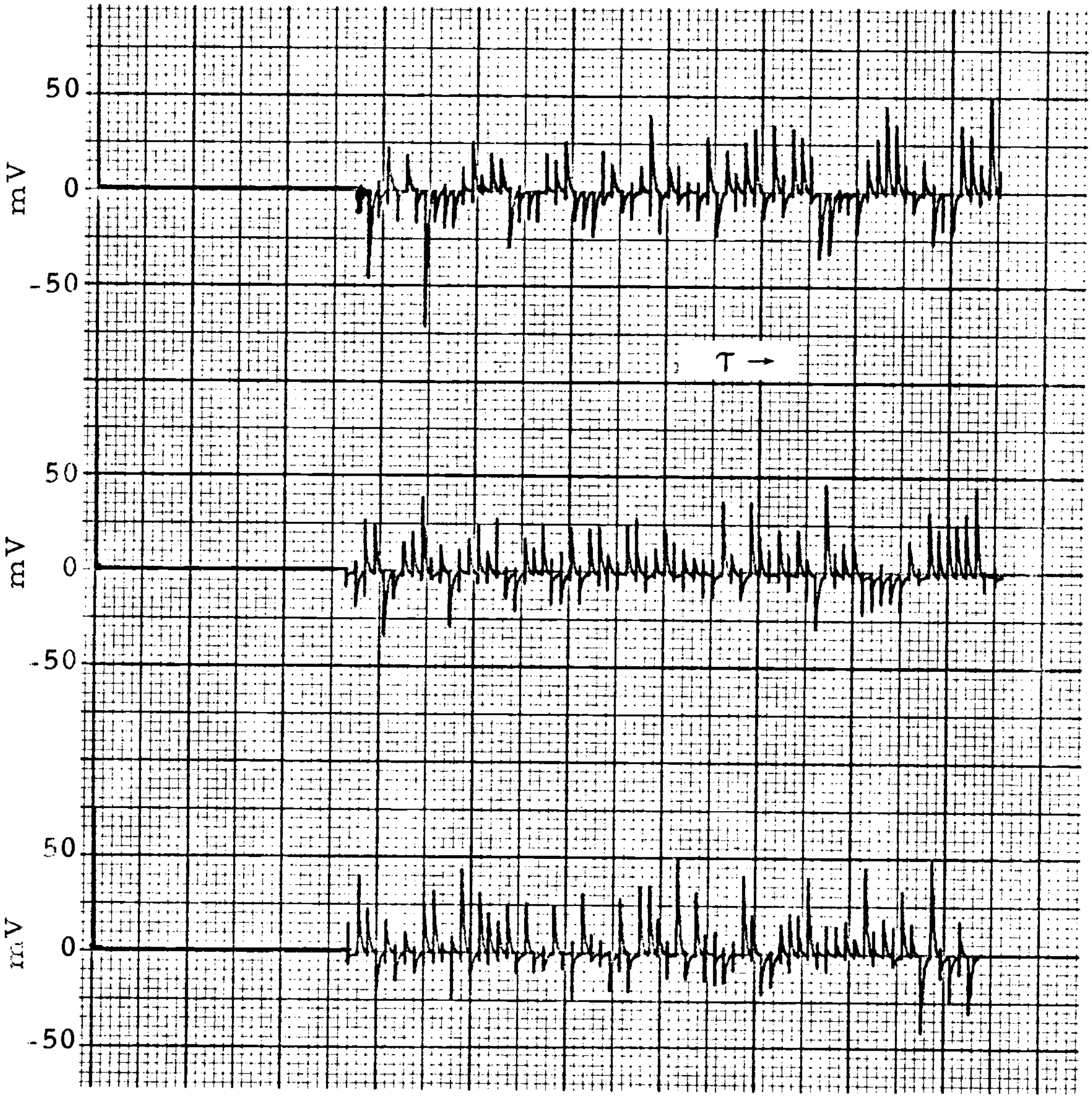
In communication channels, noise is added to the signal being transmitted resulting in the corruption of the original signal. In Figure 29, oscilloscope presentations of the signal, noise, and signal plus noise, are presented for SNR s of 0 and -10 dB. For a SNR of -10 dB, the signal is barely discernible in the noise.

Figure 30 illustrates the autocorrelation of a signal plus noise, the signal, and then noise. A summation of the correlation function



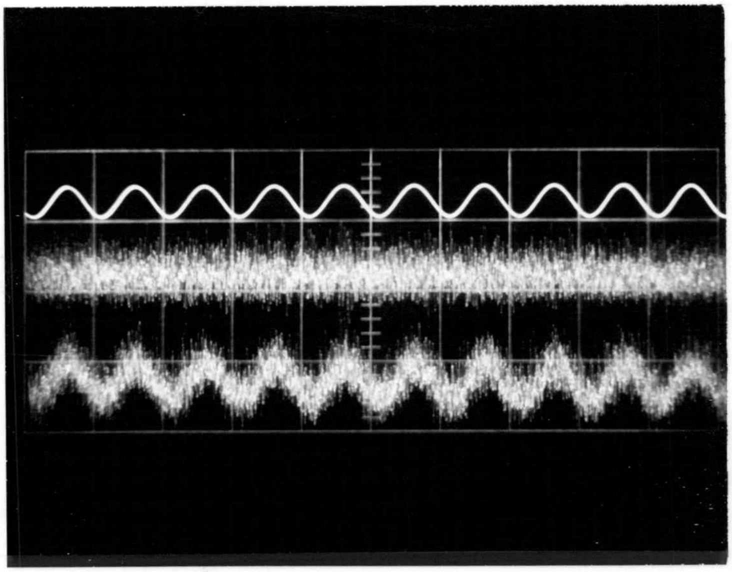
Sample size: 56 K

Figure 27. Autocorrelation of band-limited noise.

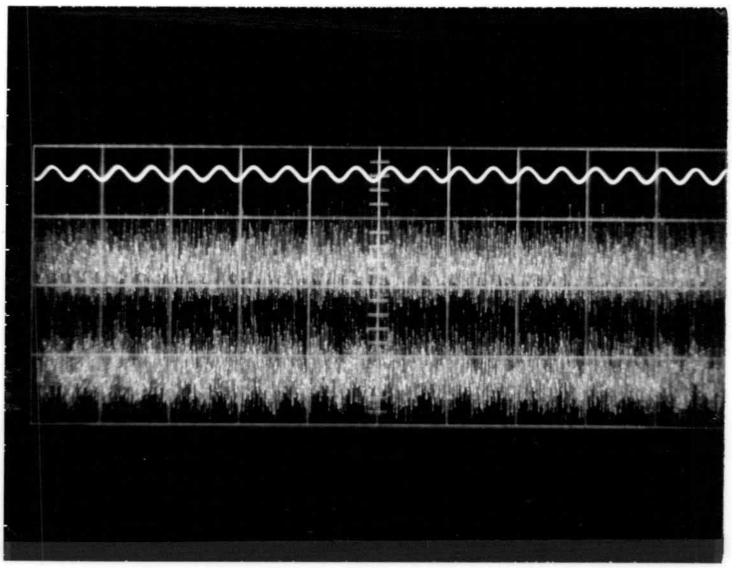


Sample size: 8 K $\Delta\tau \simeq 2.6$ us

Figure 28. The autocorrelation of 20 KHz BW noise for large delay.

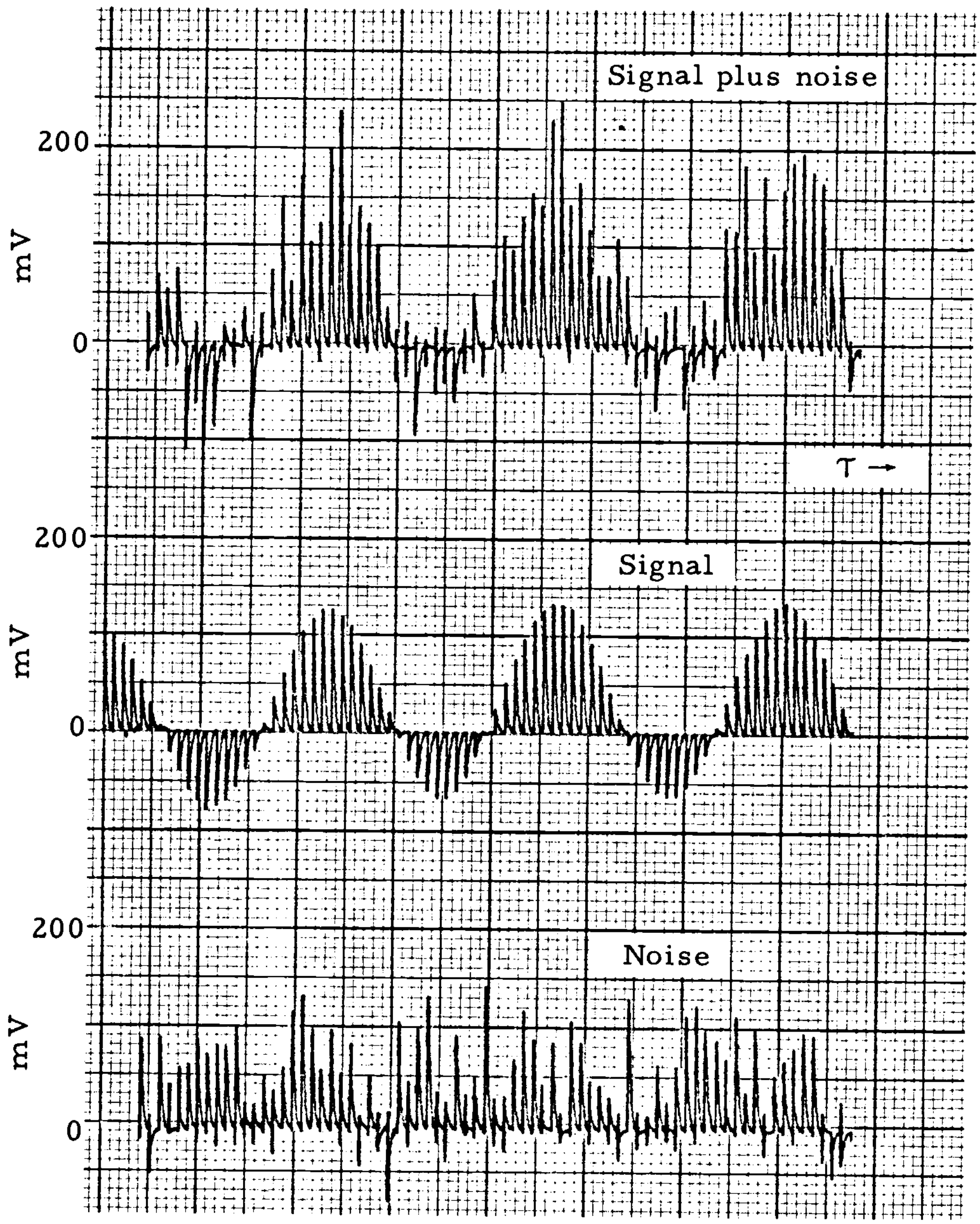


a) Signal, noise, signal plus noise 0 dB SNR.
Vert: 2 V/cm Horiz: 500 us/cm



b) Signal, noise, signal plus noise -10 dB SNR.
Vert: 1 V/cm Horiz: 1 ms/cm

Figure 29. Signal in noise.



Signal: 40 KHz sinewave 55 mV rms
Noise: 500 KHz BW 430 mV rms
SNR: -18 dB

ΔT ≃ 1 us
Sample size: 56 K

Figure 30. Autocorrelation of signal plus noise, signal, noise.

of the signal and the noise will result in the correlation function of the signal plus noise.

Figures 31 and 32 are autocorrelation functions for signals plus noise where the SNR ranges from zero to -18 dB. The offset of the correlator output is due to error in the four-quadrant multiplier when processing low level signals.

Crosscorrelation

In crosscorrelation, a reference signal is used as one of the correlator inputs in attempting to extract a signal from noise. A three volt peak to peak sine wave of the same period as the signal in the noise was used as the reference. Figures 33 and 34 are crosscorrelation results for various SNRs from -12 to -36 dB.

The sample size determines the degree of SNR improvement obtainable through correlation. Figure 35 illustrates the decrease in "noisiness" of the correlation function for increasing sample size.

Crosscorrelation vs. Autocorrelation

Crosscorrelation is a more powerful tool for extracting signals from noise than is autocorrelation. The improvement in output SNR as a function of input SNR for both autocorrelation and crosscorrelation is shown in Figure 36, (6).

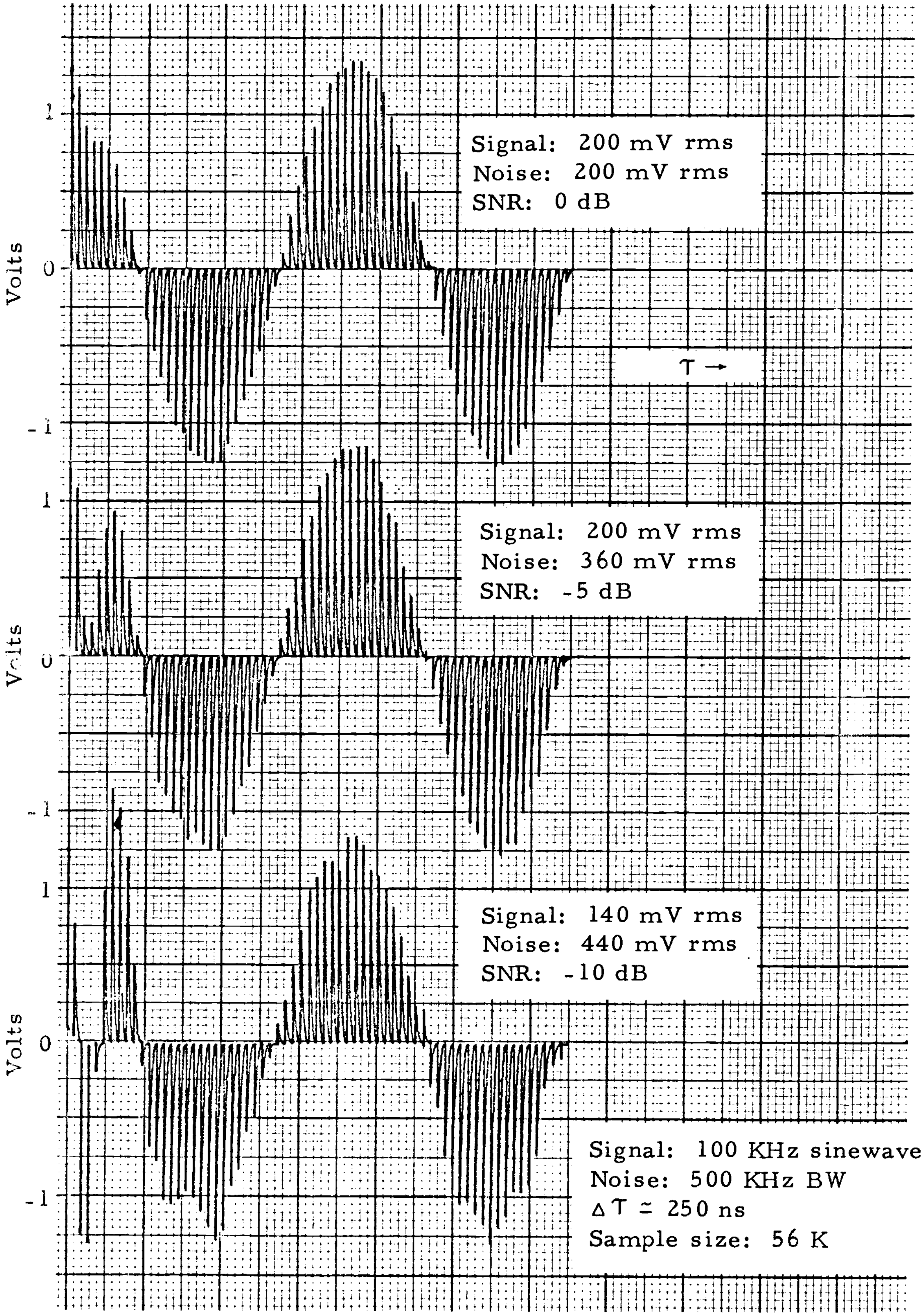


Figure 31. Autocorrelation of signal plus noise.

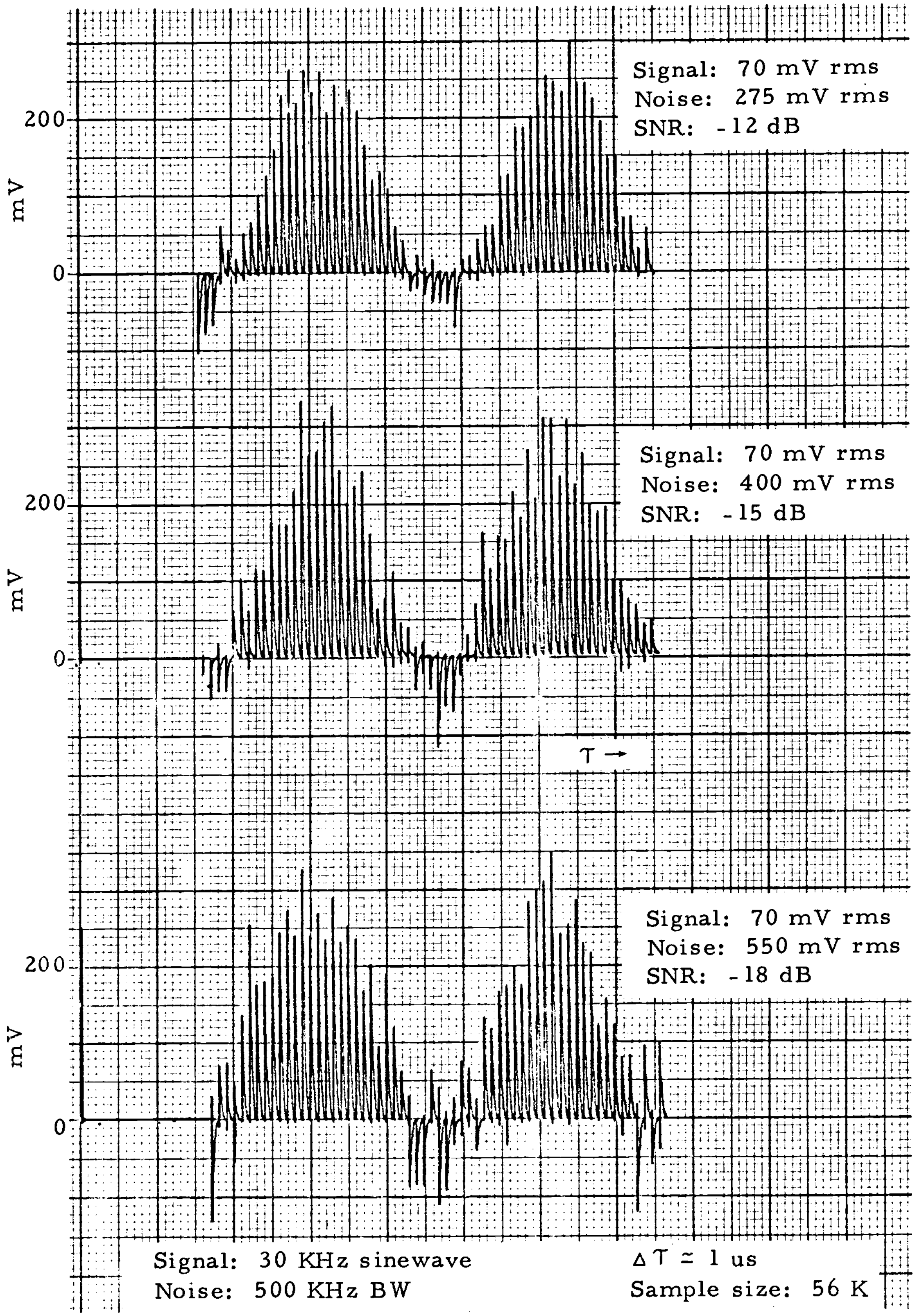


Figure 32. Autocorrelation of signal plus noise.

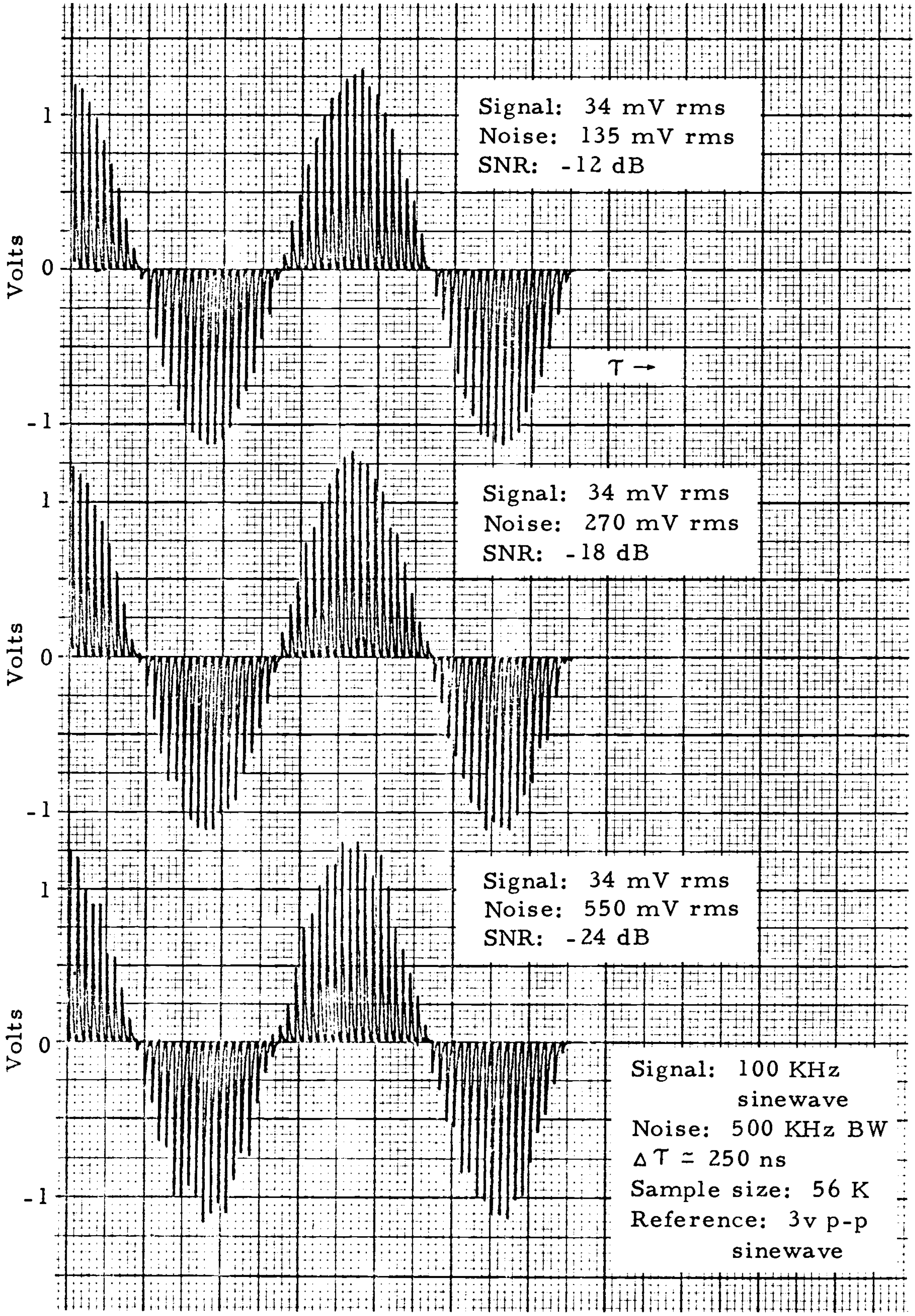


Figure 33. Crosscorrelation of signal plus noise.

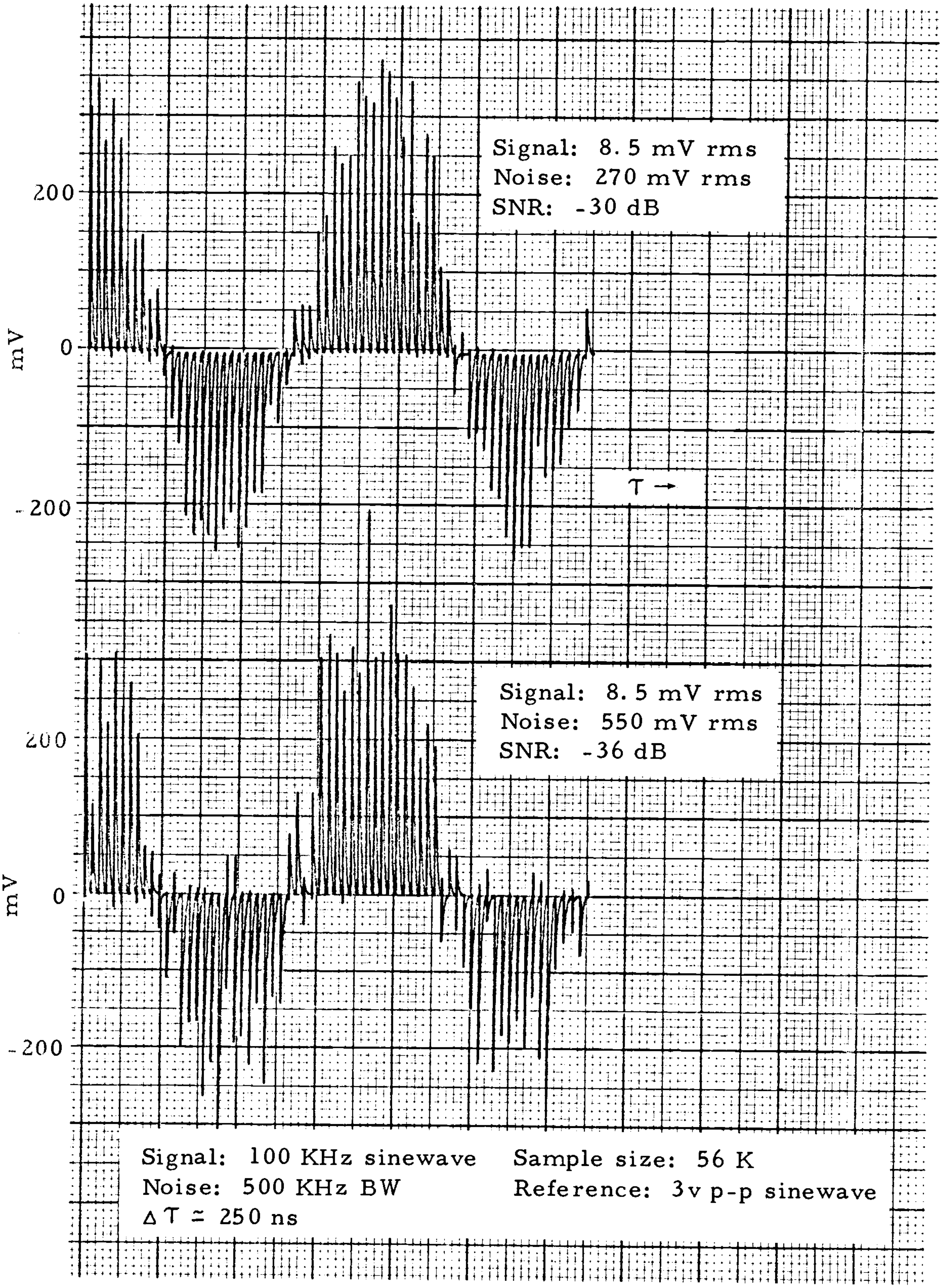


Figure 34. Crosscorrelation of signal plus noise.

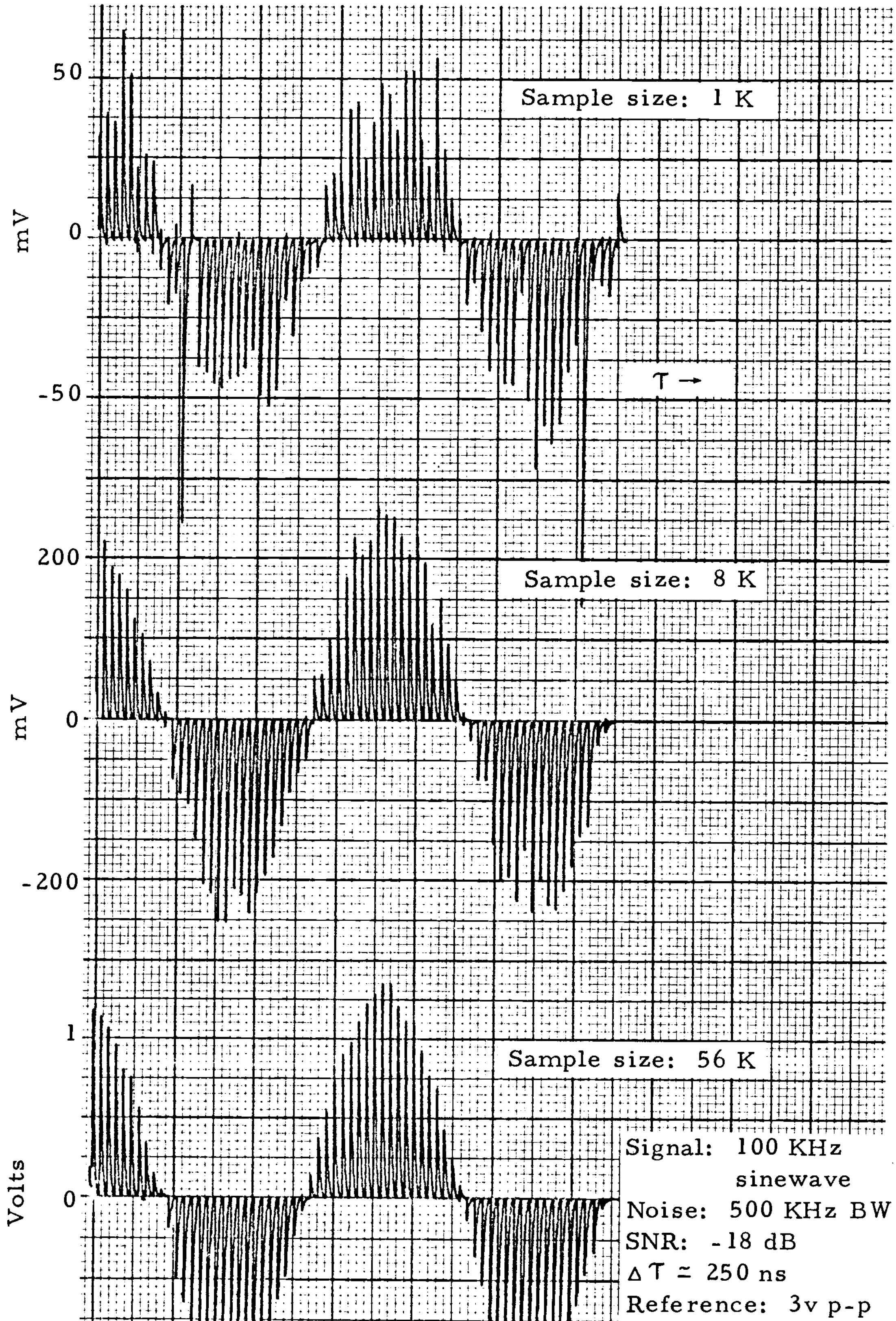


Figure 35. Correlator output as a function of sample size.

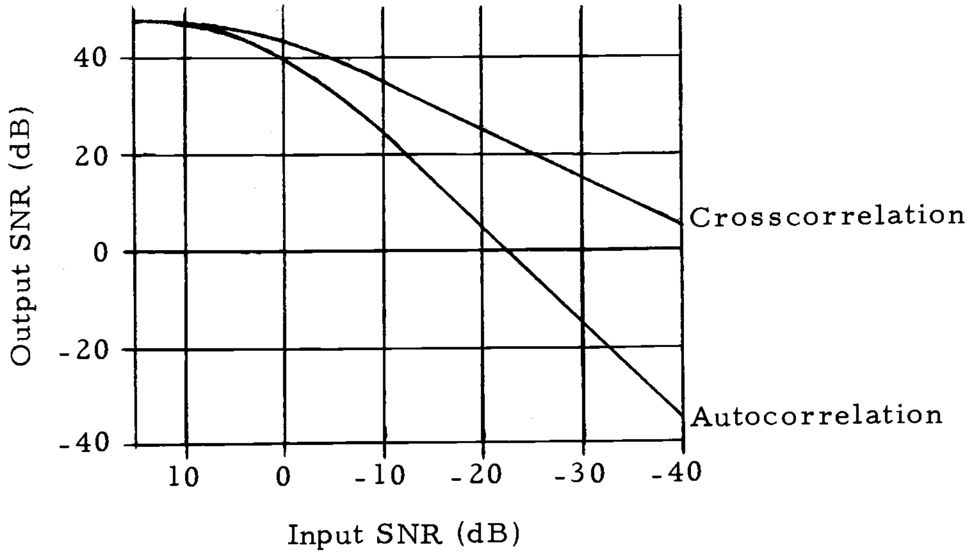
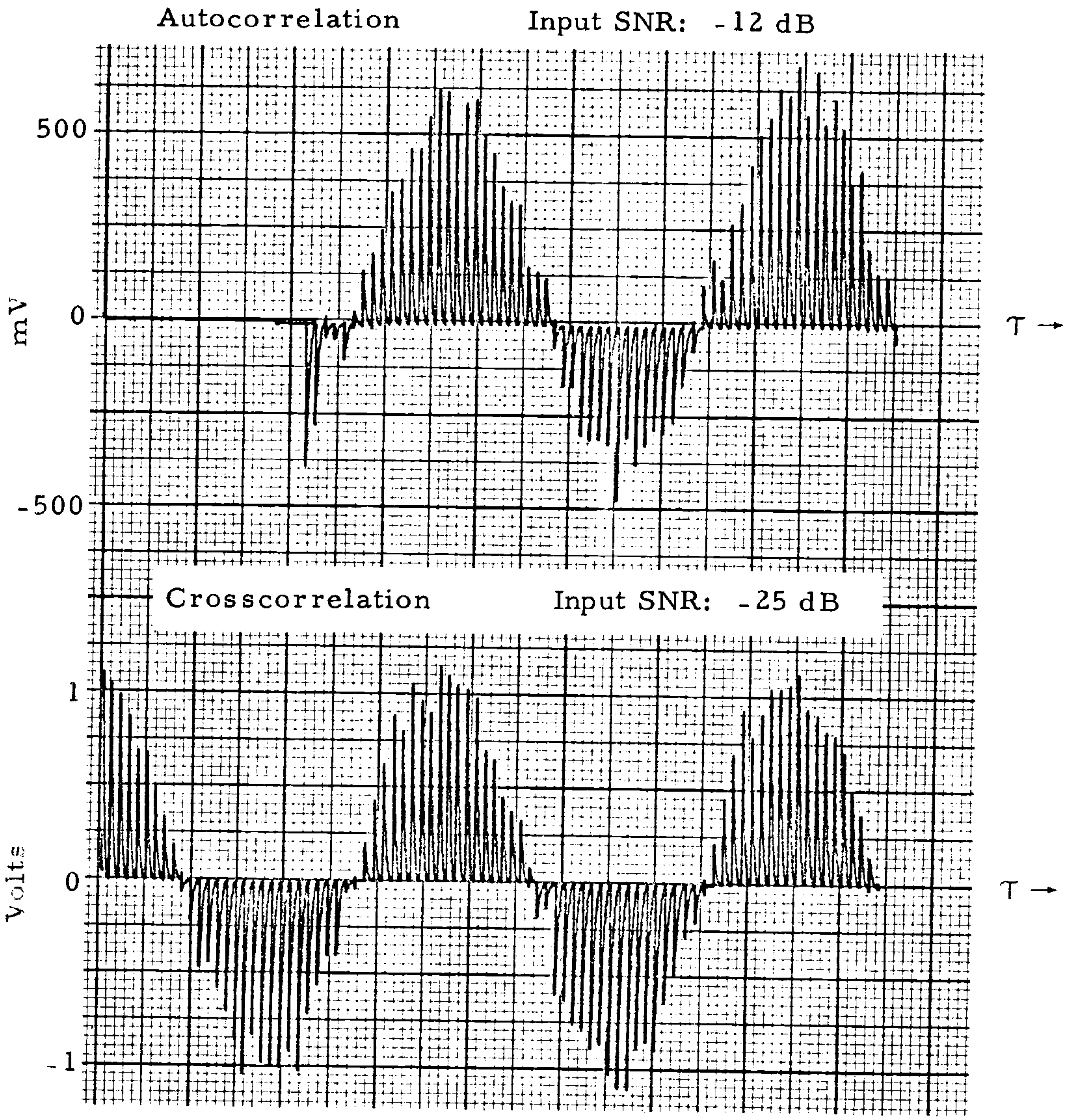


Figure 36. Improvement in SNR through autocorrelation and crosscorrelation.

From Figure 36, it may be found that a -25 dB SNR when crosscorrelated gives the same output SNR as does a -12 dB SNR when autocorrelated. For the sample size of 56, 000, crosscorrelation has a 13 dB gain over autocorrelation for the same SNR output. In Figure 37, the autocorrelation of a SNR of -12 dB and the crosscorrelation of a SNR of -25 dB result in correlator outputs with approximately the same degree of "noisiness."

Sample Averaging

The crosscorrelation of signal plus noise with unit impulses of the same period as the signal results in an output whose shape is that of the signal. If, however, the impulses have the same phase



Signal: 100 KHz sinewave
Noise: 500 KHz BW
$\Delta T \simeq 250$ ns
Sample size: 56 K

Figure 37. Illustrating the gain improvement in SNR of crosscorrelation over autocorrelation.

relationship with the signal throughout the correlation, only one point will be computed on the correlation curve because samples were taken at only one point in the signal's cycle. The correlator can easily be modified to remedy this situation. A DC voltage is applied to one of the inputs and a sample pulse of the same period as the signal is applied to the τ delay circuit. The τ delay output is made to drive both follow and holds such that the DC on the input appears to be a unit impulse of the same period as the signal. The remainder of the circuitry performs as usual taking 56, 000 samples at one point in the signal's cycle and then incrementing the delay $\Delta\tau$. Again samples are taken but now at a different point in the signal's cycle. Repeated operation results in an output as recorded in Figures 38 and 39 where a signal is extracted from noise when the SNR is as low as -42 dB. This method of signal processing is also known as sample averaging.

Impulse Response

The crosscorrelation of white noise and the output response of a network to white noise results in the impulse response of the network (3, p. 240). Shown in Figure 40 is the impulse response obtained by crosscorrelation of the input and output of the RC network whose input is driven by the random noise generator. The time constant of the RC network is 650 ns, and the impulse response appears

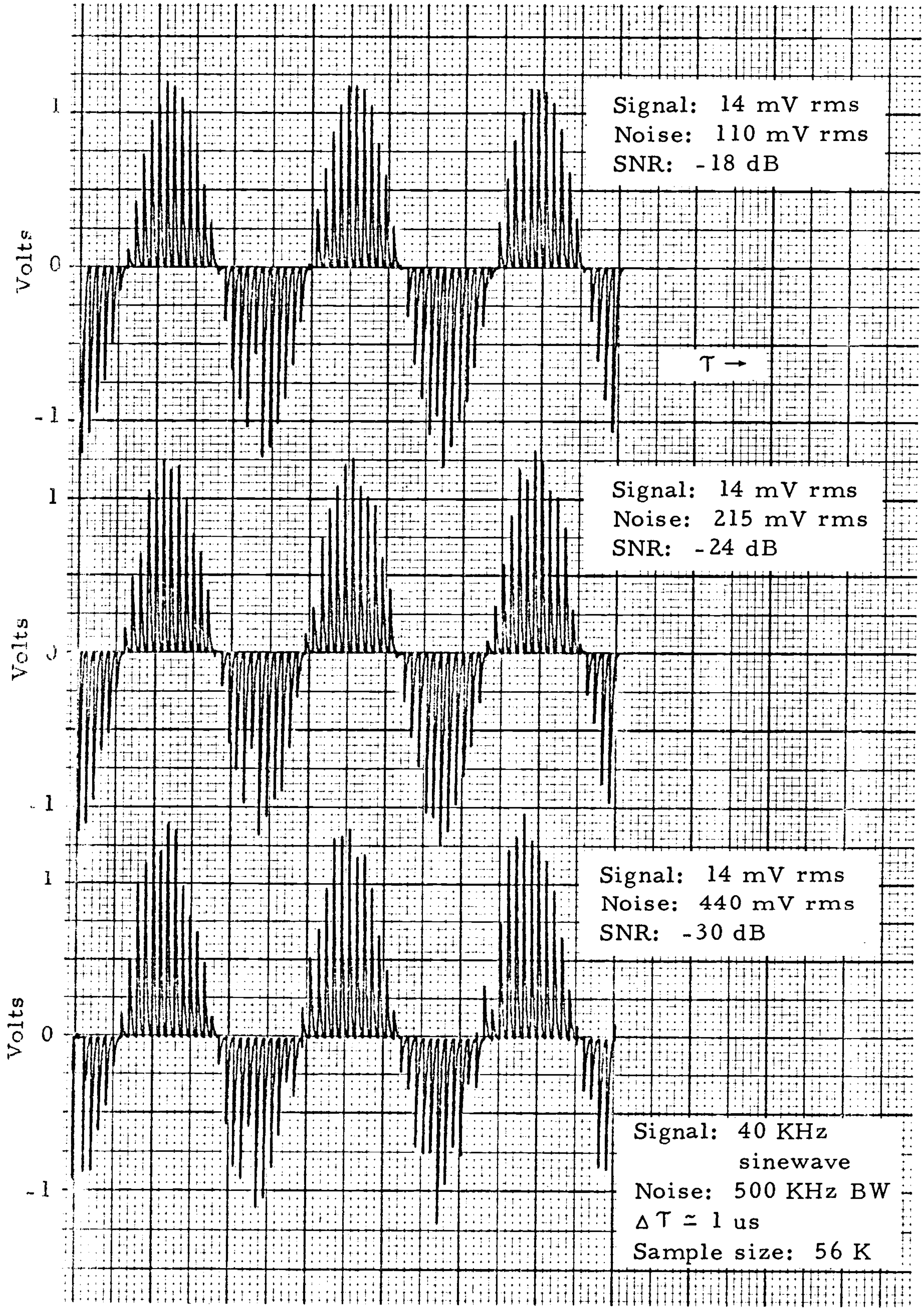


Figure 38. Sample averaging.

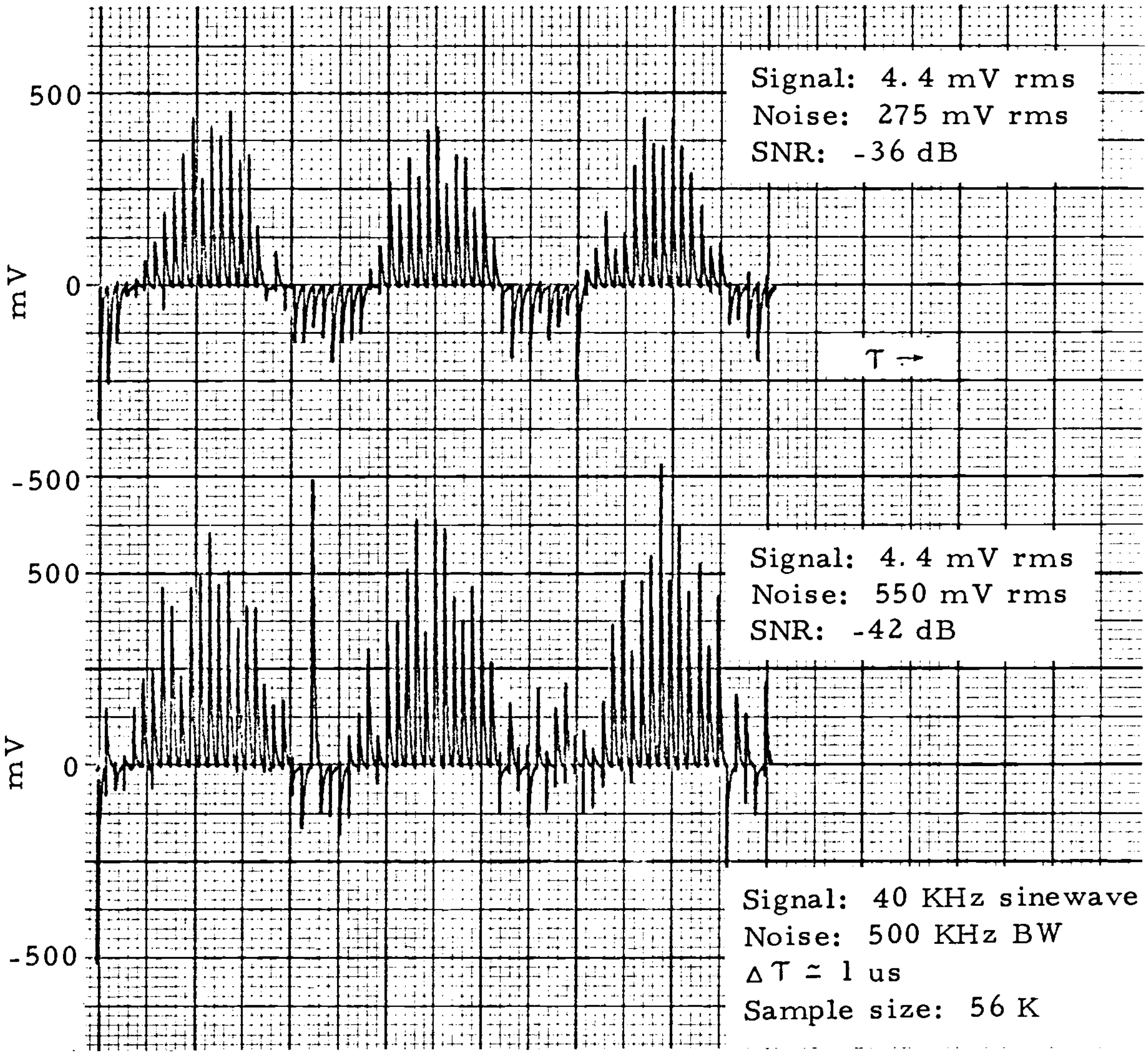
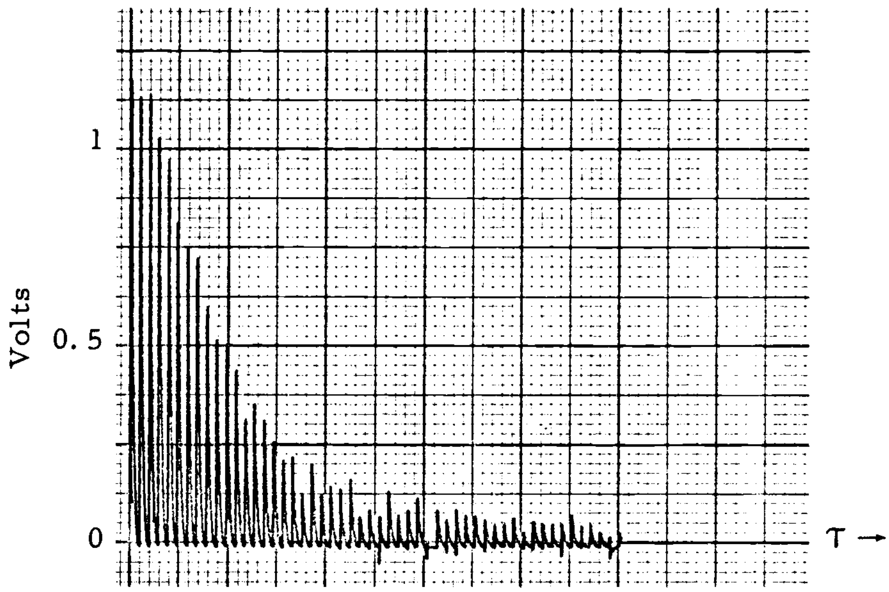


Figure 39. Sample averaging



Noise: 400 mV rms 500 KHz BW
ΔT ≃ 75 ns
Sample size: 56 K

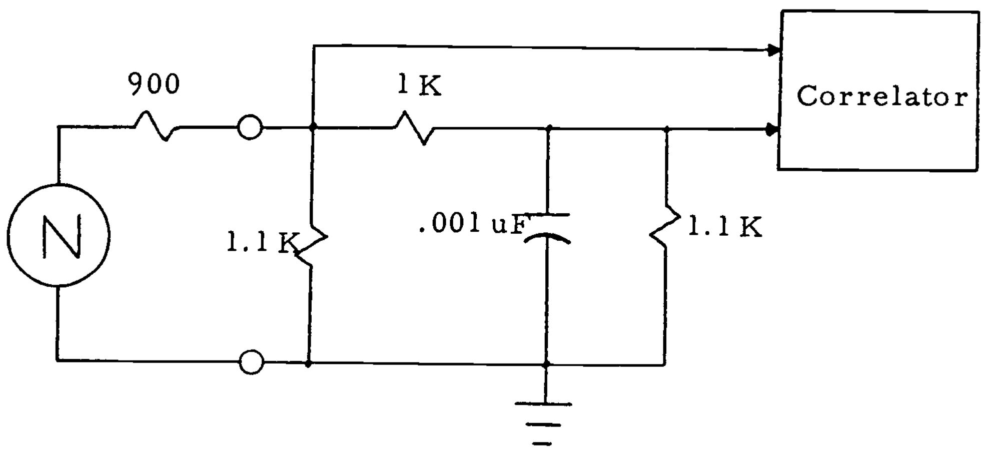


Figure 40. Impulse response from input-output crosscorrelation.

to have approximately this time constant. The 1. 1 K resistors in the network are the input impedances of the correlator.

Operation Problems

A warm-up of about 15 minutes is recommended before using the correlator. Also, before each correlation run, output drift should be nulled. The problem is thermal drift at the input to the integrator. If the integrator has a DC gain of 1, 000, a 10 uV differential drift at the input appears as a 10 mV drift at the output. Since semiconductor devices are extremely temperature sensitive, matched devices on the same chip are needed at the inputs to the unity gain amplifiers and the integrator. At present, the integrator input consists of two FETs matched on the curve tracer. This is the main source of drift. A high gain IC amplifier with a FET input stage would reduce drift considerably. The correlator requires insulation against air currents to reduce drift, and this can be accomplished by enclosing the electronics in a case.

CONCLUSION

A wideband electronic correlator and its versatility at autocorrelation, crosscorrelation, and sample averaging has been successfully demonstrated. The comparison of experimental results with the theory has been made for the increase in gain from crosscorrelation over autocorrelation. If it were possible to measure the SNR of the correlator output, a comparison of gain in SNR from input to output could be compared with the theoretical gains (5).

The power of correlation techniques in processing signals in noise as found in communication systems has been adequately demonstrated.

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APPENDICES

APPENDIX I

Operating Instructions

The following equipment is required to operate the correlator:

1) Three power supplies

   a) +15 V DC ± 0.1 V 100 mA

   b) -15 V DC ± 0.1 V 100 mA

   d) +5 V DC ± 0.1 V 250 mA

2) Pulse generator

3) Output recorder such as:

   a) XY recorder, or

   b) Storage oscilloscope.

After applying power to the correlator, allow 15 minutes for stabilization, then:

1) Adjust pulse generator output for a 2.5 to four volt amplitude and 50 to 75 ns pulse width with rise and fall times of less than 20 ns.

2) With no input signal and sample size set to 56,000, depress "START" and adjust the null potentiometer on the integrator amplifier for less than 10 mV drift per point at the correlator output. (The pulse generator can be set to a PRF of 10 KHz and $\Delta\tau$ to one us.)

3) The X axis output ranges from zero to five volts in 50mV

steps, therefore the horizontal gain should be adjusted for a proper display.

4) Select an appropriate ΔT (e.g., $\Delta T \approx T/20$ where T is the period of the input signal).

5) Set the pulse generator PRT to $100 \times \Delta T$ or 15 KHz, whichever is less.

6) Apply the signal to be correlated to the inputs and depress "RESET" and then "START".

Note: Maximum allowable input signal level is ±2 V about ground. For gaussian white noise, 500 mV rms is the maximum allowable input level.

The time required to arrive at the value of one point on the correlation curve will be

$$\text{time} = \text{sample size} \div \text{pulse generator PRF}$$

Adjustments

In the four-quadrant multiplier of Figure 7, the 1 K potentiometer is adjusted such that the periods of the switch drives from the two IC gates are identical, approximately 25 us.

APPENDIX II

System Specifications

Operations performed: computation of autocorrelation and cross-correlation functions

Number of delay points: 100

Total delay range: 7.5 us to 78 ms

Delay increment range: 75 ns to 780 us

Frequency range: DC to 500 KHz

Input impedance: 1.1 K

Maximum input signal level: 4 V p-p (for gaussian white noise, 500 mV rms)

Output: ±8 volts about ground

X axis output: zero to five volts in 50 mV steps

Power requirements:	15 volts ± 0.1 V	100 mA
	-15 volts ± 0.1 V	100 mA
	5 volts ± 0.1 V	250 mA

APPENDIX III

Delay Circuit Capacitor Values

Table 1 lists the value of capacitance required for the range of incremental delays ΔT and the measured values of ΔT.

Table 1. Delay circuit capacitor values and measured values of incremental delay.

ΔT	C (uF)	Measured ΔT
100 ns	.001	75 ns
300 ns	.003	250 ns
1 us	.01	1 us
3 us	.03	2.6 us
10 us	.1	9 us
30 us	.3	38 us
100 us	1	80 us
300 us	3	270 us
1 ms	10	780 us