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Title: A STUDY OF THE EFFECT OF THE ROTOR COIL COUPLINGS OF A SYNCHRONOUS MACHINE ON ITS PREDICTED TRANSIENT RESPONSE

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The effect of the mutual inductances between the rotor coils of a synchronous machine on its defined reactances and predicted transients are investigated. Two linearized models of the machine are considered. One uses the Laplace-Transform approach, with minimum number of assumptions on the inter-coil couplings, in developing a model for the machine with short-circuited stator terminals. The other model is obtained by modifying the commonly used model and equivalent circuits given by Adkins (4).

In the first model, the root-locus technique of classical control theory is used as a tool to investigate the effect of mutual inductances on the short-circuit eigenvalues of the general Park model. Transfer functions for the short-circuit stator and field

current responses are derived in terms of the mutual inductances. Numerical examples are presented for two (solid rotor) turbogenerators and one (salient pole) hydrogenerator unit with dampers.

In the second model, the effect of the rotor-coil mutual inductances on the dynamic behavior of the synchronous machine are investigated for the machine connected to an infinite-bus via a transmission line. Small displacements around a fixed operating point are assumed in order to linearize the nonlinear model; and the state space approach is used for eigenvalue analysis and simulation of the model on a digital computer. A numerical example is given for a hydrogenerator unit.

When the models studied were required to yield a fixed set of time constants and short-circuit reactances, it was noticed that the mutual couplings between the rotor coils did not affect the model eigenvalues. The rotor coil mutual couplings also hold no effect on the contribution of the eigenvalues to the short-circuit stator current components (i_d, i_q) when the field excitation voltage was fixed. On the other hand, the rotor currents were significantly affected by the mutual couplings between the rotor coils, and their effect should be included in any detailed investigation of synchronous machine.

A Study of the Effect of the Rotor Coil Couplings of a Synchronous Machine on its Predicted Transient Response

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Mohammad Hashem Nehrir

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Associate Professor of Electrical and Computer Engineering
in charge of major

Redacted for privacy

Head of Department of Electrical and Computer Engineering

Redacted for privacy

Dean of Graduate School

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D = damping coefficient

H = inertia constant (sec.)

I = current in the Laplace-Transform Domain

i = current in the time domain

L = inductance

L_1 = leakage inductance

L_d, L_d', L_d'' (X_d, X'_d, X''_d) = direct-axis synchronous, transient and subtransient inductances (reactances)

L_q, L'_q, L''_q(X_q, X'_q, X''_q) = quadrature-axis synchronous, transient and subtransient inductances (reactances)

L'_x, L''_x(X'_x, X''_x) = newly defined direct-damper transient and sub-transient inductances (reactances)

L'_y, L''_y(X'_y, X''_y) = quadrature-damper transient and subtransient inductances (reactances)

L_{ab}(X_{ab}) = mutual inductance (reactance) between any two coils "a" and "b".

M = inertia coefficient ($sec.^2$)

p = time domain operator, d/dt

r = resistance

s = independent variable in the Laplace-Transform Domain

T = torque (p.u.)

t = independent variable in the time domain

T_d(T_q) = direct- (quadrature-) axis time constant (sec.)

LIST OF SYMBOLS (Continued)

$T'_d, T''_d (T'_q, T''_q)$ = direct- (quadrature-) axis short-circuit transient and subtransient time constants (sec.)

$T'_{d0}, T''_{d0} (T'_{q0}, T''_{q0})$ = direct- (quadrature-) axis open-circuit transient and subtransient time constants (sec.)

V = voltage in the Laplace Transform Domain

v = voltage in the time domain

V_B = magnitude of voltage at the infinite-bus (p. u.)

X = reactance

Greek Letters

θ = instantaneous rotor angle (rad.)

δ = rotor torque-angle

ω = rotor speed (rad./sec.)

ω_b = rotor synchronous speed

λ = flux linkage

σ_d (σ_q) = direct- (quadrature-) axis leakage factor

Prefix Δ indicates small displacement

Subscripts

a = armature (phase a)

d = direct-axis

e = external (refers to transmission line)

f = field

x = direct-axis amortisseur winding

LIST OF SYMBOLS (Continued)

g, y = quadrature-axis amortisseur windings

md (mq) = direct- (quadrature-) axis rotor-stator mutual

q = quadrature-axis

$$\Delta_d = x''_d \sigma_d T_f T_x$$

$$\Delta_q = x''_q \sigma_q T_g T_y$$

A STUDY OF THE EFFECT OF THE ROTOR COIL COUPLINGS OF A SYNCHRONOUS MACHINE ON ITS PREDICTED TRANSIENT RESPONSE

I. INTRODUCTION

With the demand for electric power ever increasing, stability studies of large-scale power systems have become of more concern, making it necessary to represent the model power system components more accurately.

The theory of the synchronous machine, the most important power system component, is based on the two-reaction theory developed by Park (1). Several authors have discussed the two-reaction theory in great detail (2-5). The developed theory has been widely used to study the stability of power systems in recent years (6-10). However, the effect of such factors as iron loss and the mutual couplings between the coils of the synchronous machine were not investigated in the literature cited. The effect of iron losses on the behavior of a synchronous machine has been investigated by other authors (11-14), and the conventional equivalent circuits have been modified to take the iron losses into account. However, such models are too complex to be used for large-scale power system studies and will not be considered in this thesis.

The mutual couplings between the rotor and the stator coils of the synchronous machine were first considered in 1966 by Canay (15). In his work, he developed a transfer function block diagram for the synchronous machine in which the mutual coupling between all the possible combinations of the rotor and stator coils were explicitly considered. His work was later used to study the torque-angle loop analysis of the synchronous machine (16), and also became a cause for modifying the conventional equivalent circuits of the synchronous machine by different methods (13, 14, 17-24). However, because of the complexity involved in representing all the mutual inductances explicitly, and also, because the mutual inductances cannot be measured, conveniently, the models that include the mutuals are usually not used in power system studies.

In this work, first the Laplace-Transform technique is used to develop a Park Domain model for the synchronous machine with short circuited stator terminals. The model is then used to study the effect of the rotor coil mutual couplings on the internally defined reactances and time constants of the model.[1] The rotor-stator mutual inductances are taken to be equal on each axis ($L_{af} = L_{ax}$ and $L_{ay} = L_{ag}$) by rescaling the equivalent damper winding currents. These mutual inductances are, however, not used explicitly in the

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[1] The internally defined reactances and time constants of the model will here on be called the defined model reactances and time constants (or defined model parameters).

model equations, but they are used in defining the model reactances, as previously discussed in the literature. Hence, only two mutual inductances remain whose effects should be investigated. They are the mutual inductance between the field and the direct-axis amortisseur windings (L_{fx}) and that between the two equivalent quadrature-axis rotor coils (L_{gy}). In the cases that only one equivalent rotor coil is assumed on the quadrature-axis, only one mutual inductance (L_{fx}) remains, whose effect is to be investigated. These mutual inductances are given in terms of the direct-and quadrature-axis leakage factors σ_d and σ_q as defined in Appendix II.

The root-locus technique of classical control theory is used as a tool to study the characteristic equation of the general linearized short-circuit model of the synchronous machine. The data for the open-circuit time constants and the short-circuit reactances, made available by the manufacturer or owners of the machines, are taken as the basis for obtaining the mutual effect of the rotor coils on the defined model reactances and time constants, which are in general unknown. It is assumed that the open-circuit time constants and short-circuit reactances, provided by the manufacturer or owners of the machines, are accurately measured according to IEEE Standards (25) or else found by sophisticated analytical techniques, such as those given in the literature (14, 17, 19, 24).

Transfer functions for the field current and the stator current components of the short-circuited machine model are developed in terms of the defined leakage factors, with the field voltage as input for the transfer functions. The effects of the leakage factors on these current responses are also investigated.

Secondly, the widely used synchronous machine model presented in the literature (2-4), in which it is assumed that the mutual coupling between the rotor coils on each axis is equal to the rotor-stator mutual coupling, is modified to make the rotor coil couplings different from the rotor-stator couplings. A state space approach is used to study the effect of variation of the rotor coil coupling of a synchronous machine model on its dynamic behavior when connected to an infinite-bus via a transmission line. The nonlinear model is linearized by assuming that small displacements occur around a fixed operating point. A digital computer was used for simulation of the linearized model. The effect of the rotor coil mutual coupling was obtained on the field and stator currents as well as on the rotor angle and frequency when the line reactance was suddenly increased.

Although both, the transform domain and time domain models obtained are based on the original Park's equations for the synchronous machine, they are different in the way their model reactances and time constants are defined. In the transform domain approach, the Park Domain open-circuit time constants and short-circuit reactances are assumed to be known and the unknown model parameters (reactances and time constants) are calculated in terms of the given open-circuit time constants and short-circuit reactances. But, in the model for which time domain analysis is performed, the Park Domain short-circuit reactances are defined in terms of the open- and short-circuit time constants, which are assumed to be known.

From the studies of the transform domain and time domain models, the following important observations are made:

> The rotor coil mutual couplings do not have significant effects on the stator-current components, rotor angle and frequency. The mutual couplings, on the other hand, have significant effect on the rotor coil currents.

Hence, it can be concluded that the conventional models are sufficient for studies in which the stator currents and mechanical oscillations in the rotor are of interest; but the mutual effects between the rotor coils should be taken into account when the rotor electrical quantities are of concern. The conventional machine equivalent circuits should also be modified to make the rotor coil couplings different from the couplings between the rotor and stator coils, when they are to be used for the study of rotor electrical quantities.

II. EFFECT OF THE ROTOR COIL MUTUAL COUPLINGS ON THE MODEL PARAMETERS AND RESPONSES - A TRANSFORM DOMAIN APPROACH

2.1 Introduction

In this chapter, the synchronous machine equations in the direct- and quadrature-axis reference frame, as derived by Park (1), are used to develop a LaPlace-Transform Domain model for the synchronous machine with short-circuited stator windings. The short-circuit model is then used for investigation of the effect of the mutual couplings between the equivalent rotor coils of the machine on its defined parameters and current responses.

The root-locus technique of classical control theory is used in the eigenvalue analysis of the short-circuit model. The results of this analysis are used to derive an algorithm for determining the effect of the variation of the mutual couplings (expressed as leakage factors, σ_d and σ_q)[2] on the defined model parameters. The conventional (Park Domain) short-circuit reactances and open-circuit time constants, made available by the manufacturers or owners of the machines, are used in the derivation of the defined parameters.

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[2] σ_d and σ_q are defined in terms of the self inductances of the rotor coils and the mutual inductance between them; i.e.,

$$\sigma_d = 1 - \frac{L^2_{fx}}{L_f L_x}, \quad \sigma_q = 1 - \frac{L^2_{gy}}{L_g L_y}$$

Numerical examples for two turbogenerators and one hydrogenerator units are presented to show the effect of the mutual couplings on the model parameters and responses.

2.2 Generalized Machine Equations

The equations for a synchronous machine can be derived straight-forwardly from a linear two-pole model. The stator is considered to be three identical, symmetrically placed, lumped windings called "a, b, c". The rotor windings are four unequal lumped windings. Two of the rotor windings (f, x) are considered to be on one axis (direct) and the other two (g, y) are placed on another axis (quadrature) 90 electrical degrees from the direct-axis. Winding "f" represents the field winding while windings "x", "g", "y" are fictitious windings which account for the damper bars and current paths through the iron parts of the rotor. Salient pole machines with damper windings are normally modeled with only one winding on the rotor quadrature-axis accounting for the dampers. However, two quadrature-axis rotor windings are usually used for solid rotor machines, accounting for the effect of iron paths, and they can with some justification be replaced with one winding. The assumption of two rotor windings on the quadrature-axis yields a six-winding model; and when only one quadrature-axis rotor winding is assumed, a five-winding model is obtained.

The development will start with the symmetrical (six-winding) model, and will later be simplified to consider a five-winding model.

The assumptions made in the development of the model are as follows:

1) all inductances are independent of current (saturation is neglected);

2) all distributed windings may be adequately represented as lumped windings;

3) the effects of currents flowing in the iron parts of the rotor may be represented by three (or two) lumped rotor coils as previously described.

In order to eliminate the position dependent parameters in the equations describing the synchronous machine, the stator quantities can be transformed from the "a, b, c" representation to a "d, q, o" representation. In the work to follow, zero sequence components will be dropped from the transformation; because only balanced disturbances will be considered. The rotor quantities are invariant in the transformation, and the three-phase stationary stator windings are transformed into two windings (d, q) in quadrature which rotate at rotor speed and in the direction of rotation of the rotor.

Figure 2.1 shows a symmetrical (six-winding) model circuit for the synchronous machine after transformation to the d-q representation. The polarities shown in Figure 2.1 are used in Appendix I for derivation of the Park Domain machine equations.

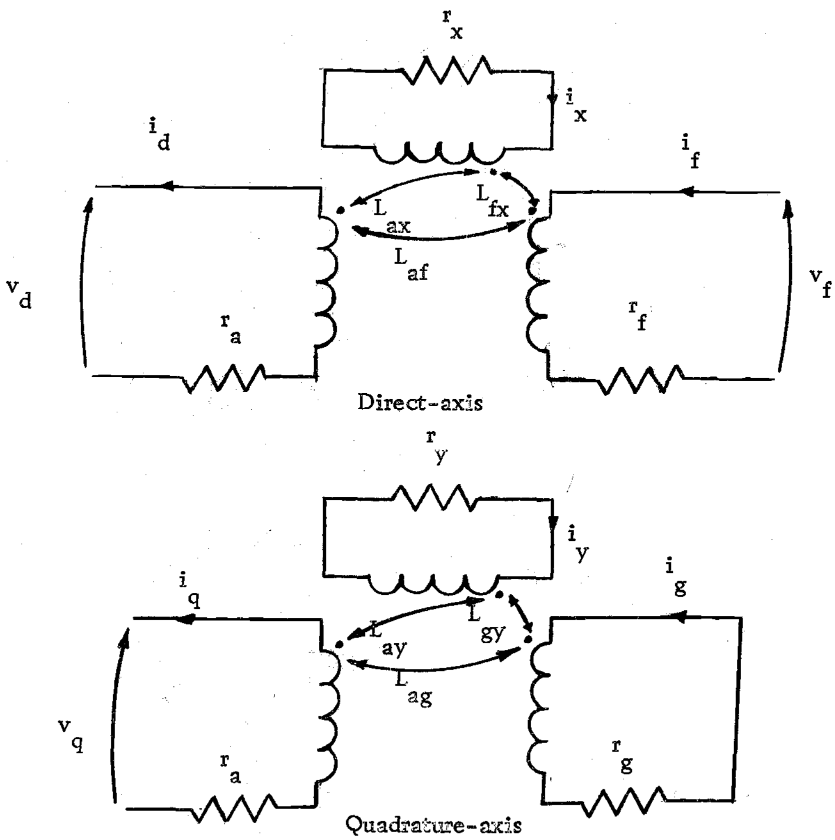

Figure 2.1 Synchronous machine model circuit.

Let the machine be operating at no-load with a fixed field voltage V_f, and a sudden three-phase short-circuit be applied on the stator at time $t = 0$. Because of the time continuity of flux linkages and the presence of the leakage inductances, the currents cannot change instantaneously. Hence, immediately after the short-circuit, the value of all currents but i_f remain at zero, while the initial value of the field current is obtained from, V_f/r_f. From equation (A1-5), the stator terminal voltages before the short-curcuit are:

$$v_d(0^-) = -\omega\lambda_q(0) = 0 \tag{2-1}$$

$$v_q(0^-) = \omega\lambda_d(0) = \omega L_{af}\, i_f(0) \triangleq E_0 \tag{2-2}$$

Equations for the winding currents of Figure 2.1 can be written in the Laplace-Transform Domain, with the initial conditions taken into account, as given by equation (2-3).

$$\begin{bmatrix} -(r_a+sL_d) & \omega L_q & sL_{af} & sL_{ax} & -\omega L_{ay} & -\omega L_{ag} \\ -\omega L_d & -(r_a+sL_q) & \omega L_{af} & \omega L_{ax} & sL_{ay} & sL_{ag} \\ -3/2sL_{af} & 0 & r_f+sL_f & sL_{fx} & 0 & 0 \\ -3/2sL_{ax} & 0 & sL_{fx} & r_x+sL_x & 0 & 0 \\ 0 & -3/2sL_{ay} & 0 & 0 & r_y+sL_y & sL_{yg} \\ 0 & -3/2sL_{ag} & 0 & 0 & sL_{yg} & r_g+sL_g \end{bmatrix} \begin{bmatrix} I_d \\ I_q \\ I_f \\ I_x \\ I_y \\ I_g \end{bmatrix} = \begin{bmatrix} L_{af}i_f(0) \\ 0 \\ V_f+L_fi_f(0) \\ L_{fx}i_f(0) \\ 0 \\ 0 \end{bmatrix} \tag{2-3}$$

In equations (2-3) s is the operator in the Laplace-Transform Domain, and the capital letters I and V indicate the transformed electrical quantities (current, voltage).

The current variations following the short-circuit can be determined from equations (2-3), provided the inductances in the above equations are known. The inductances are combined to define the model reactances[3] (X'_d, X''_d, X'_x, X''_x, X'_q, X''_q, X'_y, X''_y). The expression for these defined reactances are given in Appendix II.

In order to reduce the algebra involved in the work that follows, equations (2-3) are put in the following form:

$$\begin{bmatrix} a_{11} & \cdots & a_{16} \\ \vdots & & \vdots \\ a_{61} & \cdots & a_{66} \end{bmatrix} \begin{bmatrix} I_d \\ I_q \\ I_f \\ I_x \\ I_y \\ I_g \end{bmatrix} = \begin{bmatrix} U_1 \\ \vdots \\ U_6 \end{bmatrix} \qquad (2\text{-}4)$$

Equation (2-4) is used in Appendix II for simplification of the characteristic equation.

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3 Defined model reactances and time constants are: X_d, X'_d, X''_d, X'_x, X''_x, X_q, X'_q, X''_q, X'_y, X''_y, T_f, T_x, T_y, T_g, T'_d, T''_d, T'_q, T''_q. The measured reactances and time constants are X_{dm}, X'_{dm}, X''_{dm}, X_{qm}, X'_{qm}, X''_{qm}, T'_{d0}, T''_{d0}, T'_{q0}, T''_{q0}, where the subscript "m" denotes "measured" value.

2.3 Root-Locus Study of the Characteristic Equation

At this point, the characteristic equations of six-winding and five-winding machine models will be considered.

a. Six-winding machine model

The poles of the short-circuit current transfer functions are the roots of the characteristic equation of equation (2-4). The characteristic equation can be factored as[4]

$$\prod_{j=1}^{6} (s+p_j) = \omega^2 \prod_{m=1}^{4} (s+z_m) + \prod_{n=1}^{6} (s+p_n) \qquad (2\text{-}5)$$

where ω is the electrical frequency in rad./sec. By inspection of the work in Appendix II, it can be seen that the polynomials on the right-hand side of equation (2-5) can be factored as

$$\prod_{m=1}^{4} (s+z_m) = (s^2 + A_d s + B_d)(s^2 + A_q s + B_q) \qquad (2\text{-}6)$$

and

$$\prod_{n=1}^{6} (s+p_n) = (s^3 + C_d s^2 + D_d s + E_d)(s^3 + C_q s^2 + D_q s + E_q) \qquad (2\text{-}7)$$

---

[4] The characteristic equation is derived in Appendix II.

where, expressions for the parameters A through E are given in Appendix II.

The eigenvalues of equation (2-4) can be obtained when equation (2-5) is set equal to zero The resultant equation can be written in the following form:

$$1 + \frac{\omega^2 \prod_{m=1}^{4} (s + z_m)}{\prod_{n=1}^{6} (s + p_n)} = 0 \qquad (2\text{-}8)$$

Equation (2-8) represents a polynomial ratio suitable for application of the root-locus technique of classical control theory. This technique will be used to investigate the general behavior of the eigenvalues.

The zeros of the polynomial ratio of equation (2-8), which are the roots of equation (2-6) can be written as follows:

$$s^2 + A_d\, s + B_d = (s + z_1)(s + z_2) \qquad (2\text{-}9)$$

$$s^2 + A_q\, s + B_q = (s + z_3)(s + z_4) \qquad (2\text{-}10)$$

By expanding the formula for the roots of a quadratic by using the binomial theorem, the roots of equations (2-9, 10) can be written as follows:

$$z_1 \cong \frac{-X_d}{\alpha_d}\left(1 + \frac{X_d \Delta_d}{\alpha_d^2} + 2 \cdot \frac{X_d^2 \Delta_d^2}{\alpha_d^4} + \ldots\right) \quad (2\text{-}11)$$

$$z_2 = -\frac{\alpha_d}{\Delta_d} - z_1 \quad (2\text{-}12)$$

$$z_3 = -\frac{X_q}{\alpha_q}\left(1 + \frac{X_q \Delta_q}{\alpha_q^2} + 2 \cdot \frac{X_q^2 \Delta_q^2}{\alpha_q^4} + \ldots\right) \quad (2\text{-}13)$$

$$z_4 = -\frac{\alpha_q}{\Delta_q} - z_3 \quad (2\text{-}14)$$

where

$$\alpha_d = X'_x T_x + X'_d T_f \quad (2\text{-}15)$$

$$\Delta_d = X''_d \sigma_d T_f T_x \quad (2\text{-}16)$$

$$\alpha_q = X'_y T_y + X'_q T_g \quad (2\text{-}17)$$

$$\Delta_q = X''_q \sigma_q T_y T_g \quad (2\text{-}18)$$

The expressions for the model parameters appearing in equations (2-15) through (2-18) are given in Appendix II.

It has been found that for most machine parameter values, each of the expressions inside the parentheses in equations (2-11) and (2-13) are much less than unity. Hence, z_1 and z_3 can be approximated as $(-X_d/\alpha_d)$ and $(-X_q/\alpha_q)$, respectively. Also,

Δ_d and Δ_q are normally much smaller than α_d and α_q, respectively. It can therefore be concluded that, for most machine parameter values, $|z_1| << |z_2|$ and $|z_3| << |z_4|$.

The poles of the polynomial ratio are the roots of equation (2-7), which can be written as

$$s^3 + C_d s^2 + D_d s + E_d = (s + p_1)(s + p_2)(s + p_3) \qquad (2\text{-}19)$$

$$s^3 + C_q s^2 + D_q s + E_q = (s + p_4)(s + p_5)(s + p_6) \qquad (2\text{-}20)$$

The roots of the above polynomials are approximated in Appendix III using the root-locus technique.

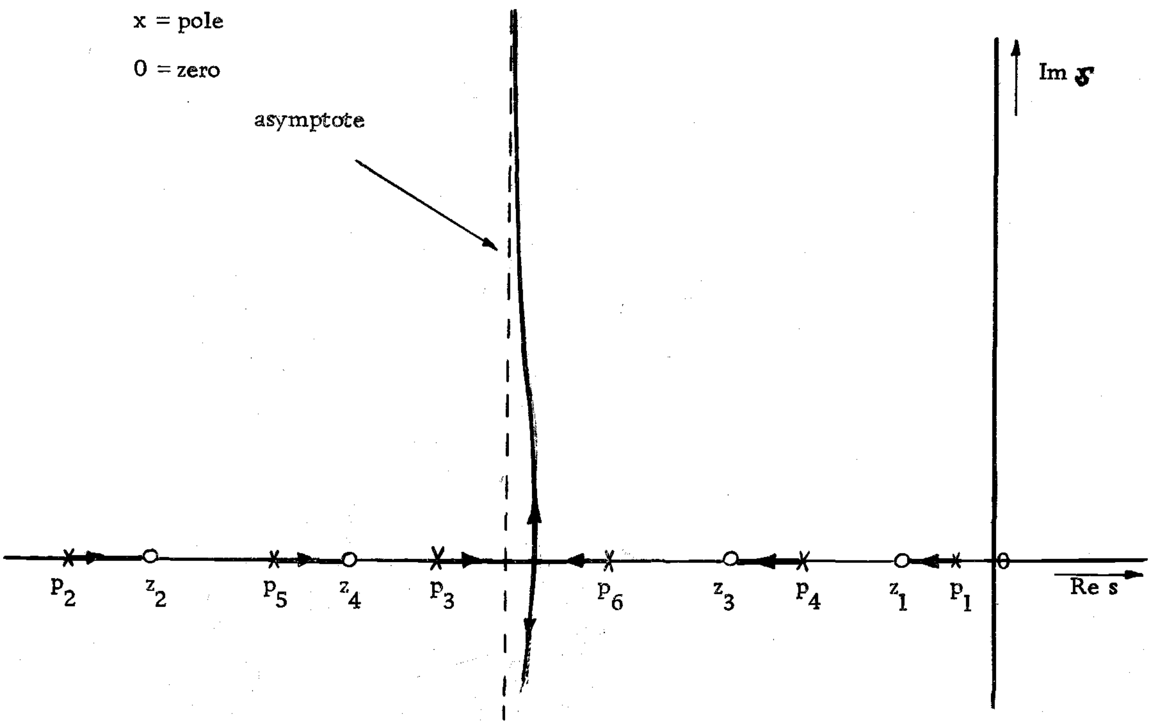

Figure 2.2 Root-locus plot for the characteristic equation of a six-winding machine model.

The estimated locations of the zeros and poles of the polynomial ratio in equation (2-8) are shown plotted on the s-plane in Figure 2.2. In this figure, the heavy lines indicate the loci of the values of s which satisfy equation (2-8) as ω is varied. As ω is increased from zero, the solutions move from the poles to the zeros, as indicated in Figure 2.2. Since ω^2, the gain-factor of the root-locus polynomial ratio, is very large compared to unity for the synchronous machine (when running at normal operating conditions), four eigenvalues are expected to lie very near the zeros (z_1 to z_4). The other two eigenvalues will form a complex pair and approach the asymptote shown in Figure 2.2. Physically, the effect of the complex pair of eigenvalues is observed in the field current wave form obtained during a short-circuit test. The real part of the complex eigenvalues indicates the rate of decay of the direct-current offset component in the stator current components, and the imaginary part is proportional to the field current frequency resulting from an actual short-circuit test.

The study of the characteristic equation of the five-winding machine model will now follow.

b. Five- winding machine model

When only one equivalent damper coil is assumed on the quadrature-axis, as for a salient pole (or hydro) generator, the order of synchronous machine model becomes five. The characteristic equation and the current transfer function can then be obtained by deleting one of the two equivalent quadrature-axis damper circuits in Figure 2.1, say the circuit with coil "g". The characteristic equation will then be obtained by simplifying and factoring equations (2-6) and (2-7). The result in simplified form is as follows:

$$\prod_{j=1}^{5} (s + p_j) = \omega^2 \prod_{m=1}^{3} (s + z_m) + \prod_{n=1}^{5} (s + p_n) \tag{2-21}$$

where z_m and p_n are roots of the following polynomials:

$$\prod_{m=1}^{3} (s + z_m) = (s + \frac{X_q}{X''_q T_y}) (s^2 + A_d s + B_d) \tag{2-22}$$

$$\prod_{n=1}^{5} (s + p_n) = (s^2 + \frac{X_q (T_q + T_y)}{X''_q T_q T_y} s + \frac{X_q}{X''_q T_q T_y}) \times$$

$$(s^3 + C_d s^2 + D_d s + E_d) \tag{2-23}$$

The solution of equation (2-2) occurs when,

$$1 + \frac{\omega^2 \prod_{m=1}^{3} (s + z_m)}{\prod_{n=1}^{5} (s + p_n)} = 0 \tag{2-24}$$

The polynomial ratio in equation (2-24) has three zeros. Two zeros are the same as those for the polynomial ratio of the six-winding machine model (z_1, z_2), and one zero is at $z_3 = - X_q/(X''_q T_y)$. Three poles of the above polynomial ratio are also the same as those for the six-winding machine model (p_1, p_2, p_3), and the other two, which are functions of the quadrature-axis parameters, can be approximated, as done previously, using the binomial expansion. The expression for p_4 and p_5 then become,

$$p_4 \cong \frac{-1}{T_y + T_q} \tag{2-25}$$

$$p_5 \cong - \frac{T_y + T_q}{T_y \; T_q} \cdot \frac{X_q}{X''_q} - p_4 \tag{2-26}$$

It is noticed that for most model paramater values, each of the three zeros are near a pole as follows:

z_1 near p_1 and $|z_1| > |p_1|$

z_2 near p_2 and $|z_2| < |p_2|$

z_3 near p_5 and $|z_3| < |p_5|$

The estimated locations of the poles and zeros on the s-plane are shown plotted in Figure 2.3. Thus, when ω is increased from zero, the solutions move, on the root-locus plot, from the poles to the zeros. At normal operating conditions since the angular speed of the rotor is again large compared to unity, three eigenvalues lie very near the zeros and the other two form a complex conjugate pair, as in the case of the six-winding model.

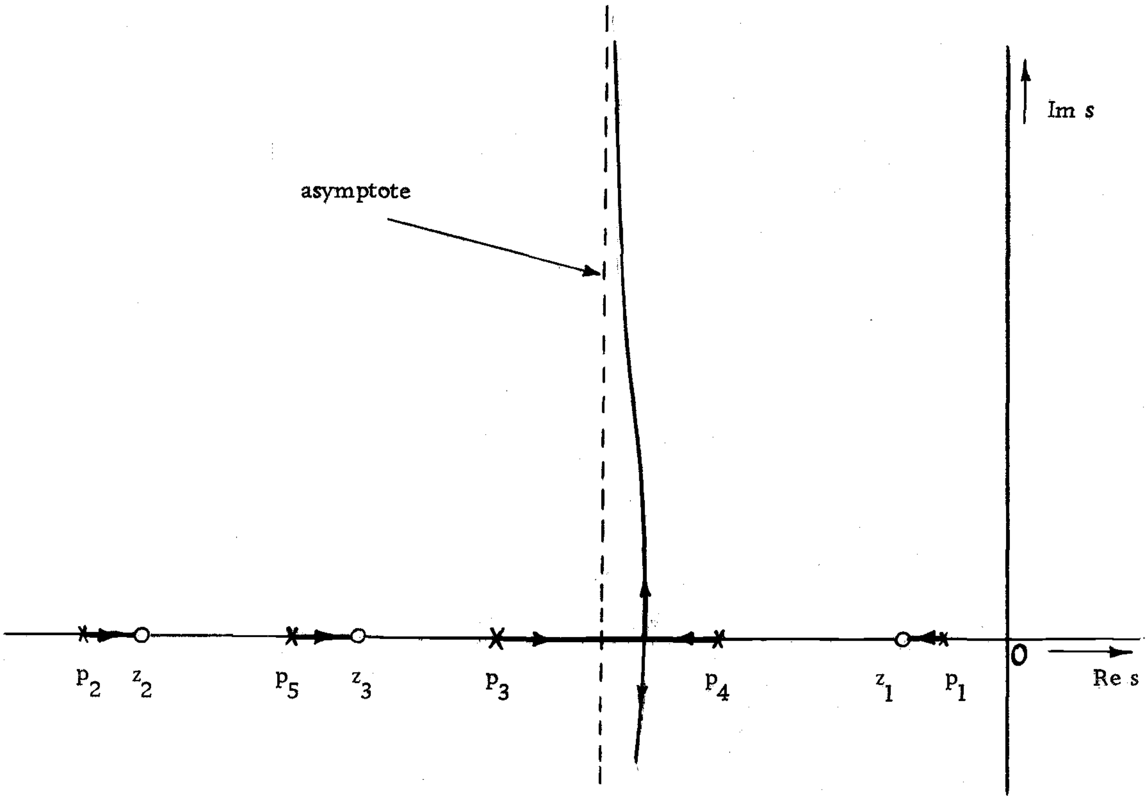

Figure 2.3 Root-locus plot for the characteristic equation of a five-winding machine model.

The assumption that the real roots of the characteristic equation are very near the zeros of the root-locus polynomial ratio at normal operating conditions of the machine will be used in the next section to develop an algorithm for derivation of the unknown model parameters as a function of the leakage factors.

The five-winding, short-circuited stator- and field-current transfer functions can be obtained from equation (2-4) by deleting the equations for the quadrature-axis rotor coil "g". Assuming that the short-circuit occurs at time t = 0 and the field voltage remains fixed during the short-circuit, the current expressions can be written as follows:

$$I_d(s) = \frac{\omega^2 E_0\left(s+\frac{X_q}{X''_q T_y}\right)\left[s^2+\frac{T_f+T_x}{\sigma_d T_f T_x}\, s+\frac{1}{\sigma_d T_f T_x}\right]}{sX''_d \prod_{j=1}^{5} (s+p_j)} \tag{2-27}$$

$$I_q(s) = \frac{E_0\omega\left(s+\frac{1}{T_y}\right)\left[s^3+\left(\frac{X_d}{X''_d T_d}+\frac{X'_x T_x+X'_d T_f}{\sigma_d X''_d T_f T_x}\right)s^2+\frac{X_d(T_f+T_x+T_d)}{\sigma_d X''_d T_f T_x T_d}\, s+\frac{X_d}{\sigma_d X''_d T_f T_x T_d}\right]}{s\, X''_q \prod_{j=1}^{5} (s+p_j)} \tag{2-28}$$

where, $E_0 = \omega L_{af}\, i_f(0)$ is the stator open-circuit voltage. And the expression for the field current becomes

$$I_f(s) = \frac{\prod_{K=1}^{5} (s+z_k)}{s \prod_{j=1}^{5} (s+p_j)} \cdot i_f(0) \tag{2-29}$$

where

$$\prod_{K=1}^{5}(s+z_k) = s^5 + \left[\frac{X'_x T_x + X'_d T_f}{\sigma_d X''_d T_f T_x} + \frac{X_d}{X''_d T_d} + \frac{X_q(T_q+T_y)}{X''_d T_q T_y}\right] s^4 +$$

$$\left[\frac{X_q(T_q+T_y)}{X''_q T_q T_y} \cdot \frac{X_d}{X''_d T_d} + \frac{(X'_x T_x + X'_d T_f) + X_d(T_f+T_x+T_d)}{\sigma_d X''_d T_f T_x T_d} + \frac{X_q}{X''_q T_q T_y} + \frac{\omega^2 X''_x}{X''_d}\right] s^3$$

$$+ \left[\frac{X_q(T_q+T_y)}{X''_q T_y T_q} \cdot \frac{X_d(T_f+T_x+T_d)}{\sigma_d X''_d T_f T_x T_d} + \frac{X_q}{X''_q T_q T_y}\left(\frac{X_d}{X''_d T_d} + \frac{X'_d T_f + X'_x T_x}{\sigma_d X''_d T_f T_x}\right)\right.$$

$$\left. + \frac{X_d}{\sigma_d X''_d T_f T_x T_d} + \omega^2 \left(\frac{X'_x T_x + X'_d T_f}{\sigma_d X''_d T_f T_x} + \frac{X''_x X_q}{X''_d X''_q T_y}\right)\right] s^2 +$$

$$\left[\frac{X_q(T_q+T_y)}{X''_q T_q T_y} \cdot \frac{X_d}{\sigma_d X''_d T_f T_x T_d} + \frac{X_q}{X''_q T_y T_q} \cdot \frac{X_d(T_f+T_x+T_d)}{\sigma_d X''_d T_f T_x T_d} + \right.$$

$$\left. \omega^2 \left(\frac{X_q}{X''_q T_y} \cdot \frac{X'_x}{\sigma_d X''_d T_f} + \frac{X_d}{\sigma X''_d T_f T_x}\right) + \frac{X_d X_q (1+\omega^2 T_x T_q)}{\sigma_d X''_d X''_q T_x T_y T_q T_d}\right] s +$$

$$\frac{X_d X_q (1+\omega^2 T_d T_q)}{\sigma_d X''_d X''_q T_f T_x T_y T_d T_q} \tag{2-30}$$

The newly defined reactances X'_x and X''_x are given as follows:

$$X'_x = \omega \left(L_d - \frac{3}{2} \cdot \frac{L_{ax}^2}{L_x}\right) \tag{2-31}$$

$$X''_x = \omega\,[L_d - \frac{3}{2\sigma_d}\,(\frac{L^2_{ax}}{L_x} - (1 - \sigma_d)\,\frac{L_{af}\,L_{ax}}{L_{fx}}\,] \qquad (2\text{-}32)$$

Note that by definition, the quadratic equation in the numerator of the transfer function for $I_d(s)$, equation (2-27), is the same as the open-circuit characteristic equation; i.e.,

$$s^2 + \frac{T_f + T_x}{\sigma_d T_f T_x}\,s + \frac{1}{\sigma_d T_f T_x} \triangleq (s + \frac{1}{T'_{d0}})\,(s + \frac{1}{T''_{d0}})$$

where, T'_{d0} and T''_{d0} are the open-circuit transient and subtransient time constants, respectively.

When the field excitation voltage is not fixed, the assumption $v_f = R_f\,i_f\,(0)$, will no longer hold, after the short circuit is applied and the Laplace-Transform of $v_f(t)$ will not be V_f/s. For this condition, the expressions for the short-circuit stator current components of a five-winding machine model become

$$I_d(s) = \frac{\omega\,L_{af} V_f(s)\,[\,s(1 - \frac{L_{fx} L_{ax}}{L_x L_{af}}) + \frac{1}{T_x}\,]\,[\omega^2 + s^2 + \frac{s\,X_q}{X''_q T_q}\,(s + \frac{1}{T_y}) / (s + X_q / X''_q T_y)]}{X''_d L_f \sigma_d \prod_{i=2}^{5} (s + p_i)} \qquad (2\text{-}33)$$

$$I_q(s) = \frac{\omega^2 L_{af} X_d\, V_f(s)(s + \frac{1}{T_y})\left[s(1 - \frac{L_{fx} L_{ax}}{L_x L_{af}}) + \frac{1}{T_x}\right]}{X''_d X''_q T_d L_f \sigma_d (s + X_q / X''_q T_y) + \prod_{i=2}^{5} (s + p_i)} \qquad (2\text{-}34)$$

Note that the transfer functions given by equations (2-33, 34) will be non-minimum phase; i.e., zero being in the right half s-plane, if the quantity $L_{fx} L_{ax}/L_x L_{af}$ is greater than unity. Further investigation is therefore in order, to determine whether a non-minimum phase condition is physically possible for the problem at hand; and therefore put restriction on the numerical value of the quantity $L_{fx} L_{ax}/L_x L_{af}$. This investigation is out of the scope of the present work.

The model parameters appearing in equations (2-27) through (2-34) are considered to be unknown. An algorithm is presented in the next section to express the unknown parameters in terms of the open-circuit time constants and the short-circuit reactances, which are given for a machine and assumed to be measured or otherwise well-defined. Numerical examples will then follow to show how the defined parameters match their corresponding measured values, and how they are affected by the variation of the leakage factors.

2.4 Derivation of Model Parameters From Measured Data

In this section, an algorithm will be presented for determination of the unknown model parameters from the measured open-circuit time constants and short-circuit reactances. The above time constants and reactances are used in contrast to the short-circuit time constants; because, for most machines, the short-circuit time constants given by manufacturers are not obtained from test. The given open-circuit time constants and short-circuit reactances are considered to be more reliable to use, as the former can be obtained from a load rejection test, and the latter can be reasonably accurately computed from the machine geometry. It is assumed that the open-circuit time constants are either accurately determined by the manufacturer or measured according to IEEE standards (25).

Figure 2.4 shows a flow-chart form summary of the calculation sequence of the direct-axis model parameters in terms of the assumed known time constants and reactances. The calculation sequence for the quadrature-axis model parameters is obtained by replacing all the known direct-axis time constants and reactances in Figure 2.4 with the corresponding quadrature-axis values. The details of the calculation sequence are discussed in Appendix IV. The assumption that the eigenvalues of the short-circuit model are very near the zeros of the root-locus polynomial ratio (discussed in section 2.3) is used in the derivation of the model parameters. It is also assumed that the mutual couplings between the rotor-stator coils are equal; i.e., $L_{af} = L_{ax}$ and $L_{ag} = L_{ay}$. This assumption can be implemented by scaling the damper-winding currents such that the flux linkages and the winding time constants

Start

$$X_d = X_{dm}^{*}$$

$$X''_d = X''_{dm}$$

find T'_d from

$$T'^{\,2}_d - \frac{X'_{dm}}{X_{dm}}(T'_{d0}+T''_{d0})T'_d + T'_{d0}T''_{d0}(X'_{dm}-X''_{dm}+X'_{dm}X''_{dm}/X_{dm})/X_{dm} = 0$$

find T_x from

$$T_x^2 - (T'_{d0}+T''_{d0})T_x + T'_{d0}T''_{d0}/\sigma_d = 0$$

$$T_f = T'_{d0} + T''_{d0} - T_x$$

$$Q = (X_{dm}T'_d + T'_{d0}T''_{d0}X''_d/T'_d - X'_d T_f)/T_x$$

Use an iterative method to find X'_d from

$$\sigma_d^{1/2}(X'_d - X''_d)^{1/2} + \sigma_d^{1/2}(X_d - X'_d)^{1/2} - (X_d - Q + X'_d T_f/T_x)^{1/2} = 0$$

$$X'_x = Q - X'_d T_f/T_x$$

$$X''_x = \left[X'_x - (1-\sigma_d)X_d + ((1-\sigma_d)(X_d - X'_d)(X_d - X'_x))^{1/2}\right]/\sigma_d$$

Figure 2.4 Summary of calculation sequence of direct-axis parameters.

*Subscript "m" indicates the measured or given values.

remain unchanged. The above scaling requires changes in the inductance and resistance values of the effective damper coils, to leave the individual coil time constants unchanged.

Using the calculated value of the model parameters obtained from the calculation sequence in Figure 2.4, the effect of the leakage factors, σ_d and σ_q, on the model eigenvalues can be obtained by solving the characteristic equations (2-5) and (2-21) for the six- and five-winding machine models, respectively. The effect of the leakage factors on the observable currents (i_d, i_q and i_f) can also be obtained by finding the zeros of the current transfer functions from equations (2-27) through (2-29).

The eigenvalues nearly have the same value as the zeros of the root-locus polynomial ratio, given by equations (2-11) through (2-14). The zeros are expressed in terms of α_d, $\triangle_d$, α_q, and $\triangle_q$, given by equations (2-15) through (2-18). However, these terms are expressed in terms of the known open-circuit time constants[5] as follows:

$$\alpha_d = X'_x T_x + X'_d T_f = T'_{d0} + T''_{d0} = \text{constant} \qquad (2\text{-}35)$$

$$\alpha_q = X'_y T_y + X'_q T_g = T'_{q0} + T''_{q0} = \text{constant} \qquad (2\text{-}36)$$

---

[5] See Appendix IV.

$$\Delta_d = \sigma_d T_f T_x = T'_{d0} T''_{d0} = \text{constant} \qquad (2\text{-}37)$$

$$\Delta_q = \sigma_q T_g T_y = T'_{q0} T''_{q0} = \text{constant} \qquad (2\text{-}38)$$

Hence, the short-circuit model eigenvalues will remain unaffected by the variation of the leakage factors as long as a given set of measured values are to be yielded by the model. The contribution of each eigenvalue to the two stator current components (i_d, i_q) will also remain unaffected with the variation of leakage factor(s), when the field excitation voltage is fixed, as in the standard short-circuit test. This is because the expression for the currents i_d and i_q, given by equations (2-27, 28), are also given in terms of the constants defined in equations (2-35) through (2-38). However, the stator current components are functions of the rotor coil mutual couplings when the field excitation voltage is not constant. This is because their transfer function expressions, equations (2-33, 34), are given in terms of the quantity $L_{ax} L_{fx}/L_{af} L_x$. However, when it is assumed that the damper winding currents initially completely compensate for sudden changes in the armature current, Olive (8) has shown that the ratio $L_{ax} L_{fx}/L_{af} L_x$ is unity. In this case, the effect of the rotor coil mutual coupling on the stator current components is zero.

In Chapter III, a time domain simulation of the synchronous machine connected to an infinite bus is performed, in which it is shown that the effect of the rotor coil mutual coupling on the stator current components is quite negligible.

The contribution of the eigenvalues to the field current also vary with the leakage factor(s); because, as shown by expression (2-30), the zeros of the field current transfer function are functions of the defined reactance X''_x, and this reactance, as given by equation (2-32), is a function of the direct-axis leakage factor, σ_d. Hence, the field current expression is affected by the leakage factor.

Numerical examples presented in the next section will support the theory developed so far.

2.5 Numerical Examples

The calculation sequence presented in section 2.4 is used here to compute the model parameters of two turbogenerators and one hydrogenerator unit. The turbogenerators are represented by the six-winding model, and the five-winding model is used to represent the hydro unit. The data for the above units are given in Table 2.1. They are standard case data given to the Western System Coordinating Council (WSCC) by the machine owners. For all three units, the data are given in per-unit on the machine MVA base. Note that T'_{q0} is equal to zero for the hydrogenerator unit.

a. The model parameters

Tables 2.2 through 2.4 show the defined model parameter values as a function of the leakage factors, σ_d and σ_q, for the three units considered. Note that σ_d and σ_q affect only the direct- and quadrature-axis parameters, respectively. Also, note that the numerical value of X''_d does not appear in the tables, but as shown in the calculation sequence of Figure 2.4 and discussed in Appendix IV, the value of defined X''_d equals to its measured value.

The numerical values of the model eigenvalues and the predicted short-circuit time constants, which are defined to be the inverse of the real part of the eigenvalues, are given in Table 2.5 for the three units considered. The short-circuit time constants provided by WSCC are also shown for comparison with the calculated time constants. The fact that the imaginary part of the pair of complex eigenvalues is not exactly equal to the rotor synchronous speed (120π rad./sec.) indicates that the net magnetic field is moving with respect to the rotor during transient.

As predicted earlier in this chapter, the variation of the leakage factors did not affect the eigenvalues of the short-circuit model with fixed excitation voltage. Hence, the eigenvalues and the calculated short-circuit time constants remained unchanged as the leakage factors were varied, and they are given independent of

Table 2.1. W.S.C.C. data for three generators used in the numerical examples.*

Parameter**	Machine 1	Machine 2	Machine 3
rated MVA	834.60000	1300.0000	142.1000
rated KV	20.0000	25.0000	13.8000
X_d	2.1830	2.1290	0.9299
X'_d	0.4130	0.4670	0.3570
X''_d	0.3390	0.3150	0.2480
X_q	2.1570	2.0740	0.6510
X'_q	1.2850	1.2700	0.6510
X''_q	0.3320	0.3080	0.2970
X_0	0.1740	0.1838	0.1480
r_a	0.0017	0.0020	0.00360
T'_d (sec.)	0.9500	1.3400	3.0600
T''_d (sec.)	0.0350	0.0350	0.0200
T'_{d0} (sec.)	5.6900	6.1200	8.0000
T''_{d0} (sec.)	0.0410	0.0520	0.0300
T'_q (sec.)	0.1540	0.1850	0.0000
T''_q (sec.)	0.0350	0.0350	0.0200
T'_{q0} (sec.)	1.5000	1.5000	0.0000
T''_{q0} (sec.)	0.1440	0.1440	0.0600
T_A (sec.)	0.4400	0.4000	0.2000

*Machines 1, 2 are turbogenerators (Pittsburgh 7 and Diablo Canyon 1) owned and operated by the Pacific Gas and Electric Company, and machine 3 is a hydrogenerator unit installed at the John Day Project on the Columbia River.

**Resistance and reactance values are given in per unit and the time constants are in seconds.

Table 2.2 Predicted parameters for Pittsburg 7 turbogenerator (Machine 1 of Table 2.1).

σ_d	T_f(sec)	T_x(sec)	X'_d	X'_x	σ_q	T_g(sec)	T_y(sec)	X'_q	X'_y
0.1	5.2900	0.4410	0.4258	0.3585	0.4	1.1904	0.4536	1.4418	0.4110
0.2	5.5197	0.2113	0.4193	0.4549	0.5	1.3156	0.3284	1.3614	0.3395
0.3	5.5919	0.1391	0.4166	0.5840	0.6	1.3839	0.2601	1.3111	0.3395
0.4	5.6274	0.1036	0.4149	0.7329	0.7	1.4279	0.2161	1.2723	0.3974
0.5	5.6484	0.0826	0.4137	0.8980	0.8	1.4589	0.1851	1.2384	0.5182
0.6	5.6623	0.0687	0.4127	1.077	0.9	1.4821	0.1619	1.2040	0.7299

Table 2.3. Predicted parameters for Diablo Canyon 1 turbogenerator (Machine 2 of Table 2.1).

σ_d	T_f(sec)	T_x(sec)	X'_d	X'_x	σ_q	T_g(sec)	T_y(sec)	X'_q	X'_y
0.1	5.6041	0.5679	0.4864	0.3152	0.4	1.1904	0.4536	1.4243	0.4033
0.2	5.9024	0.2696	0.4756	0.3619	0.5	1.3156	0.3284	1.3474	0.3220
0.3	5.9951	0.1769	0.4711	0.4554	0.6	1.3839	0.2609	1.2989	0.3107
0.4	6.0403	0.1317	0.4683	0.5769	0.7	1.4279	0.2161	1.2616	0.3561
0.5	6.0671	0.1049	0.4663	0.7207	0.8	1.4589	0.1851	1.2288	0.4626
0.6	6.0848	0.0872	0.4647	0.8844	0.9	1.4821	0.1619	1.1956	0.6576

Table 2.4. Predicted parameters for John Day hydrogenerator unit (Machine 3 of Table 2.1).

σ_d	T_f(sec)	T_x(sec)	X'_d	X'_x	X''_x
0.1	7.7191	0.3109	0.3616	0.2546	0.0542
0.2	7.8777	0.1523	0.3595	0.2495	0.3140
0.3	7.9291	0.1009	0.3586	0.2665	0.4354
0.4	7.9546	0.0754	0.3580	0.3018	0.5204
0.5	7.9698	0.0602	0.3576	0.3411	0.5733
0.6	7.9799	0.0501	0.3572	0.3908	0.6171
0.7	7.9871	0.0429	0.3569	0.4529	0.6576
0.8	7.9925	0.0375	0.3566	0.5324	0.6999

Table 2.5. Predicted eigenvalues and the defined and given short-circuit time constants for machines 1, 2 and 3.

	Model eigenvalues	Calculated s.c. time constants (sec.)	Given s.c. time constants (sec.)
Machine 1	$-1.9065 \pm j376.9281$	$T_A = 0.5245$	$T_A = 0.440$
	-29.5492	$T''_d = 0.0338$	$T''_d = 0.035$
	-25.3516	$T''_q = 0.0394$	$T''_q = 0.035$
	-1.1868	$T'_q = 0.8426$	$T'_q = 0.154$
	-0.9342	$T'_d = 1.0704$	$T'_d = 0.950$
Machine 2	$-2.4145 \pm j376.8941$	$T_A = 0.4142$	$T_A = 0.400$
	-28.2216	$T''_d = 0.0354$	$T''_d = 0.035$
	-27.0989	$T''_q = 0.0369$	$T''_q = 0.035$
	-1.1511	$T'_q = 0.8687$	$T'_q = 0.185$
	-0.7525	$T'_q = 1.3289$	$T'_d = 1.340$
Machine 3	$-5.0001 \pm j376.7652$	$T_A = 0.1999$	$T_A = 0.200$
	-47.9149	$T''_d = 0.0208$	$T''_d = 0.020$
	-36.5786	$T''_q = 0.0273$	$T''_q = 0.020$
	-0.3262	$T'_d = 3.0656$	$T'_d = 3.060$

leakage factors in Tables 2.5.

Comparison of the calculated and given short-circuit time constants in Table 2.5, shows an inconsistency between the calculated and given values. The inconsistency is more apparent in the quadrature-axis time constants, T'_q and T''_q. This is because the measurement or computational methods used in obtaining the quadrature-axis time constants are less reliable, especially because of the iron paths in the rotor body.

In the preliminary work leading into this thesis, the values of the short-circuit time constants, given in Table 2.1, were used to determine the defined model parameters. However, the resultant short-circuit model had two complex pairs of eigenvalues, which contradicts all the available measured short-circuit data for synchronous machines. On the other hand, when the short-circuit time constants were calculated from the algorithm of Figure 2.4, and used to compute the eigenvalues in the manner described, only a single pair of complex eigenvalues was obtained. As a result, it is concluded that the machine short-circuit time constants, given in Table 2.1, are in error, or at least inconsistent with all the other parameters in that table.

Using the predicted value of the short-circuit reactances and time constants, given in Tables 2.2 through 2.5, it is noticed that the conventional expressions relating the reactances and time

Table 2.6. Reactance values as a function of the leakage factors for machines 1, 2 and 3.

	Machine 1				Machine 2				Machine 3			
	X'_d		X''_d		X'_d		X''_d		X'_d		X''_d	
σ_d	New	Conv.	New	Conv.	New	Conv.	New	Conv.	New	Conv.	New	Conv.
0.1	0.426	0.410	0.339	0.351	0.486	0.462	0.315	0.331	0.362	0.356	0.248	0.251
0.2	0.419	0.410	0.339	0.346	0.476	0.462	0.315	0.324	0.360	0.356	0.248	0.249
0.3	0.417	0.410	0.339	0.343	0.471	0.462	0.315	0.321	0.359	0.356	0.248	0.249
0.4	0.415	0.410	0.339	0.342	0.468	0.462	0.315	0.319	0.358	0.356	0.248	0.248
0.5	0.414	0.410	0.339	0.341	0.466	0.462	0.315	0.317	0.357	0.356	0.248	0.248
	X'_q		X''_q		X'_q		X''_q		X'_q		X''_q	
σ_q	New	Conv.	New	Conv.	New	Conv.	New	Conv.	New	Conv.	New	Conv.
0.4	1.442	1.212	0.332	0.395	1.424	1.201	0.308	0.365	Not Applicable		0.297	Not Applicable
0.5	1.361	1.212	0.332	0.373	1.347	1.201	0.308	0.345			0.297	
0.6	1.311	1.212	0.332	0.359	1.299	1.201	0.308	0.333			0.297	
0.7	1.272	1.212	0.332	0.339	1.262	1.201	0.308	0.323			0.297	
0.8	1.238	1.212	0.332	0.329	1.196	1.201	0.308	0.315			0.297	

constants, as used in the literature (2-5) and given by equations (2-39) through (2-42), are only approximate and become more accurate as the leakage factors approach unity.

$$X'_d \cong X_d \, T'_d / T'_{d0} \tag{2-39}$$

$$X''_d \cong X'_d \, T''_d / T''_{d0} \tag{2-40}$$

$$X'_q \cong X_q \, T'_q / T'_{q0} \tag{2-41}$$

$$X''_q \cong X'_q \, T''_q / T''_{q0} \tag{2-42}$$

The calculated reactances obtained from the derived algorithm and those obtained from equations (2-39) through (2-42) are given in Table 2.6 for comparison. Since the difference between the field and the direct-axis time constants (T_f, T_x) is quite large as compared to the difference between the two equivalent quadrature-axis time constants (T_g, T_y),[6] the expressions for the direct-axis reactances given by equations (2-39) and (2-40) hold more accurately than those for the quadrature-axis reactances, when the leakage factors are varied.

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[6] See the numerical value of the time constants in Tables 2.2 and 2.3.

b. Poles and zeros of the root-locus polynomial ratio

The numerical values of the poles and zeros of the polynomial ratio, equation (2-8), used in the eigenvalue analysis of the model are given in Table 2.7 for the three machines considered. From these values it is noticed that the poles of the root-locus polynomial ratio are all real, as predicted in Appendix III. Also, in calculating the defined model reactances (X'_d, X''_d, etc.), it was assumed that the eigenvalues were close to the zeros of the root-locus polynomial ratio in equation (2-8) or (2-24). The fact that the calculated eigenvalues are quite close to, but not exactly at the zeros (see Table 2.7), would indicate that the calculation method, with its inherent approximations gives consistent results; i.e., the eigenvalues are practically on the top of the zeros.

c. Short-circuit current responses

As shown in section 2.4, the zeros of the transfer functions for the short-circuited stator current components are not functions of the leakag factors when the field voltage is fixed. However, it was shown for a five-winding machine model, that the zeros of the field current transfer function are functions of the direct-axis leakage factor. Table 2.8 gives the values for the zeros of the

Table 2.7. Short-circuit model eigenvalues and the zeros and poles of the root-locus polynomial ratio.*

Machine 1			Machine 2			Machine 3		
eigenvalues	zeros (z_1 to z_4)	poles (p_1 to p_6)	eigenvalues	zeros (z_1 to z_4)	poles (p_1 to p_6)	eigenvalues	zeros (z_1 to z_3)	poles (p_1 to p_5)
- 0.9342	- 0.9342	- 0.1152	- 0.7525	- 0.7525	- 0.1175	- 0.3262	- 0.3262	- 0.1185
-29.5492	-29.5475	-29.9056	-28.2216	-28.2245	-29.0504	-47.9149	-47.9025	-49.7127
- 1.1868	- 1.1868	- 2.3517	- 1.1511	- 1.1511	- 2.2026	-36.5786	-36.5544	- 3.8714
-25.3516	-25.3451	- 0.9986	-27.0989	-27.0834	- 0.8235	- 5.0001 ± j2376.7652		- 1.9441
- 1.9065 ± j376.9281		-26.8044	- 2.4145 ± j376.8941		-28.9771			-39.1806
		- 1.4237			- 1.4323			

*The subscripted numbers on p's and z's correspond to those on Figures 2.2 and 2.3.

model field current transfer function for the John Day hydro unit (machine 3). The values in Table 2.8 are obtained when the calculated model parameters are used in equation (2-30), and indicate that the contribution of each eigenvalue to the field current varies with the value of the leakage factor.

Table 2.8. Zeros of the field current fransfer function as a function of σ_d for the John Day hydrogenerator unit.

σ_d	Zeros of the field current transfer-function			
0.1	83.3996±j257.3399,	-245.2739,	-16.0636,	-0.2815
0.2	17.5685±j431.1761,	-123.5641,	- 5.7829,	-0.6100
0.3	5.6119±j502.7568,	-102.2290,	- 2.6126,	-1.2022
0.4	0.2289±j548.1696,	- 92.6026,	- 1.3376 ± j1.0619	
0.5	-2.3662±j574.8103,	- 86.2818,	- 1.0312 ± j1.3143	
0.6	-4.1987±j596.0419,	- 84.7493,	- 0.8367 ± j1.4127	
0.7	-5.6830±j614.9998,	- 82.0501,	- 0.7019 ± j1.4569	
0.8	-7.0621±j634.2583,	- 79.4919,	- 0.6020 ± j1.4750	

The zeros of the field current transfer function can be obtained from Table 2.8 by predicting an approximate value for the leakage factor σ_d. As discussed in section 2.4, Olive (8) has shown that, when it is assumed that the damper winding currents initially completely compensate for sudden changes in the armature current, the following relations hold:

$$\frac{L_{ax} L_{fx}}{L_{af} L_x} = 1 \tag{2-43}$$

$$\frac{L_{ay} L_{gy}}{L_{ag} L_y} = 1 \tag{2-44}$$

When the above assumptions are used, the defined expression for X''_d becomes the same as X'_x and that for X''_q becomes equal to X'_y. Hence, the approximate values of σ_d and σ_q, for a given machine, are those for which the values of X''_d and X''_q are respectively the same as X'_x and X'_y. It should however be noted that because of the assumption made in this thesis, that $L_{af} = L_{ax}$ and $L_{ay} = L_{ag}$, equations (2-43) and (2-44) yield $L_{fx} = L_x$ and $L_{gy} = L_y$, which will put restriction on the values of the mutual inductances L_{fx} and L_{gy}. Hence, the predicted value of σ_d and σ_q obtained from equations (2-43) and (2-44) are only approximate values.

If the assumption to make $X'_x = X''_d$ and $X'_y = X''_q$ is imposed, it is noticed from the values of X'_x and X'_y in Tables 2.2 through 2.4 that the mutual coupling between the rotor coils are quite strong (weak leakage factor) for the machines considered and the coupling is stronger on the direct-axis.

2.6 Conclusions

In this chapter, a linear model is developed for the synchronous machine with short-circuited stator terminals which takes the effect of the rotor coil mutual couplings into account and makes a minimum of assumptions on the inter-coil couplings. An algorithm is developed to evaluate the defined model parameters in terms of the defined leakage factors and the machine short-circuit reactances and open-circuit time constants given by the manufacturers or owners of the machines.

From the comparison of the calculated short-circuit reactances and time constants, given in Tables 2.2 through 2.5, with their corresponding given values in Table 2.1, inconsistency in the available machine data, particularly, in the quadrature-axis reactances and time constants is noticeable. As explained in section 2.5, the inconsistency is due to the error in the given shor-circuit time constant values appearing in Table 2.1.

It is also noticed that when the field voltage is kept constant, the leakage factors (σ_d and σ_q) have no effect on the model short-circuit time constants and the short-circuit stator current components. It is also established that the field and damper bar current responses are affected by the leakage factors. In order to obtain the proper leakage factor(s) for a generator, it is necessary to observe the response of the generator field current to a disturbance and compare it with the field current response of the model, when a similar disturbance is applied to the model.

If the assumption proposed by Olive (8) is imposed, from which X'_x becomes equal to X''_d and X'_y equal to X''_q, it is noticed from the values of X'_x and X'_y in Tables 2.2 through 2.4, that the mutual couplings between the rotor coils are quite strong and the coupling is stronger on the direct-axis.

It is also noticed that the open-circuit field time constant, T_f, is approximately equal to the machine open-circuit time constant, T'_{d0}. If this assumption is made, the product of σ_d and T_x will be equal to T''_{d0}. It can therefore be concluded that as σ_d approaches unity, T_x approaches the value of T''_{d0}. However, a similar conclusion can not be drawn for the quadrature-axis, except when σ_q approaches unity. This is because T_y and T_g are in the same order of magnitude, where as T_f is much larger than T_x. T_g is, in general, not close to T'_{q0} except for values of σ_q near unity.

In summary, it can be concluded that:

1. the leakage factor(s) have no effect on the short-circuited stator current components when the field voltage is fixed and the model is to yield a fixed response.
2. The leakage factor(s) affect the stator current responses when the field excitation voltage is not fixed, whether the

stator terminals are shorted or not. However, as will be shown in Chapter III for a one machine infinite-bus system, the effect of the leakage factor on the stator current components is very small.

3. The leakage factor(s) do affect the field and equivalent damper currents. Hence, in order to observe the proper field current response from the model, the leakage factor(s) must be known.

III. EFFECT OF THE ROTOR COIL MUTUAL COUPLING ON THE MACHINE DYNAMIC BEHAVIOR— A TIME DOMAIN APPROACH

3.1 Introduction

In the conventional linear models and equivalent circuits of the synchronous machine presented in the literature (2-4) and widely used in the investigation of machine transients, it is assumed that the mutual coupling between the rotor coils on each axis is the same as the rotor-stator mutual couplings on the corresponding axis. However, as shown in Chapter II, variation of the mutual couplings between the rotor coils have significant effect on the rotor current responses. It therefore becomes necessary to make the rotor coil mutual couplings different than those between rotor and stator.

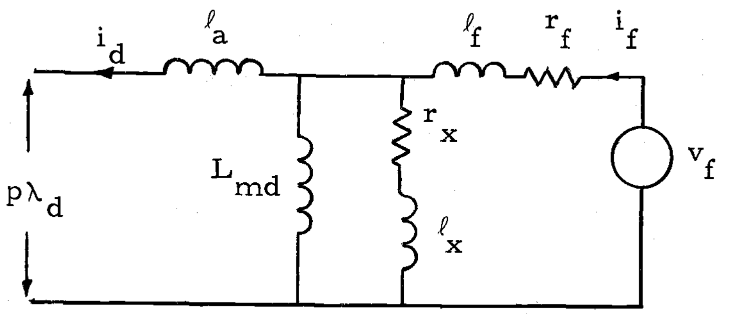
Although the original Park's Equations are also used in the derivation of the model considered in this chapter, the present model differs from that discussed in Chapter II in that the model discussed herein considers the short-circuit reactances to be defined in terms of the conventional open- and short-circuit time constants, which are assumed to be known; whereas, in the model discussed in Chapter II the conventional open-circuit time constants and short-circuit reactances were assumed to be known and the defined model reactances and time constants were calculated in terms of the known reactances and time constants.

In this chapter, the conventional model given in the literature (2-4) is modified to make the rotor coil mutual inductance (L_{fx}, L_{gy}) the sum of the rotor-stator mutual inductances (L_{md}, L_{mq}) and the unknown inductances (ℓ_{fx}, ℓ_{gy}). The modified model is then used to investigate the effect of variation of the rotor coil mutual coupling on the predicted transients of a synchronous machine with short-circuited stator terminals, and a synchronous machine infinite-bus system. A five-winding machine model is used in this investigation. The state space approach is used in the development and digital simulation study of the model, and the given data for the John Day hydrogenerator units (Table 2.1) are used in presenting a numerical example.

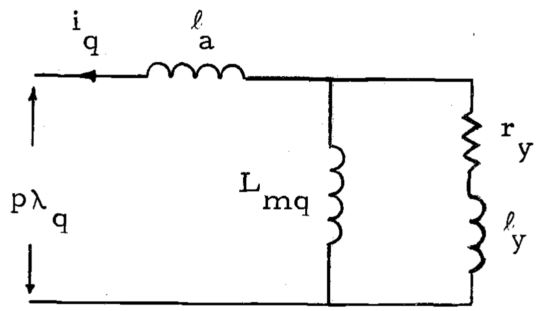
The eigenvalues of the machine model with short-circuited stator terminals are compared with those obtained for the model derived in Chapter II. Rotor angle, frequency and current responses of the one machine infinite-bus system to a sudden change in the transmission line reactance are also presented as a function of the rotor coil mutual coupling.

3.2 Equivalent Circuits

The direct- and quadrature-axis equivalent circuits, which represent the classical machine model developed in the literature (2-4) are shown in Figure 3.1. The basic equations from which the equivalent circuits are derived, are given in Appendix V. The equivalent circuit elements represent the armature resistance and leakage inductance, r_a and ℓ_a; rotor-stator mutual inductances, L_{md} and L_{mq}; field resistance and leakage inductance, r_f and ℓ_f; and the amortisseur winding resistances and leakage inductances r_x, r_y, ℓ_x, and ℓ_y. Note that in the derivation of the equivalent circuits of Figure 3.1 it is assumed that the rotor-stator and the rotor coil mutual inductance are equal; i.e., $L_{af} = L_{ax} = L_{fx} = L_{md}$ and the armature resistance r_a is external to the networks.



Direct-axis



Quadrature-axis

Figure 3.1 Equivalent circuits of classical machine model.

However, the conventional formulation used in deriving the above equivalent circuits represent only the stator circuit reasonably accurately (15). Hence, when the rotor coil current responses of the machine are to be studied, the mutual coupling inductance between the field and the direct-axis damper winding (L_{fx}) should be different from the rotor-stator mutual inductance. In what follows, it will be assumed that the direct-axis rotor coil mutual inductance (L_{fx}) is the sum of the rotor-stator mutual inductance (L_{md}) and an extra mutual inductance (ℓ_{fx}), which is normally unknown. The reactances corresponding to ℓ_{fx}, L_{md} and L_{fx} are respectively x_{fx}, X_{md} and X_{md}, such that $X_{fx} = X_{md} + x_{fx}$.

The basic equations, which represent a classical machine model are modified in Appendix V to include the effect of the extra mutual coupling inductance ℓ_{fx}. Figure 3.2 shows the modified equivalent circuits for a five-winding machine model, which are derived on the basis of the modified machine equations given in Appendix V.

It should be noticed that if a six-winding machine model, with two lumped-equivalent amortisseur windings on the quadrature-axis is desired, the extra mutual coupling inductance between these two windings (ℓ_{gy}) should also be taken into account. Addition of the dotted parts to Figure 3.2 makes that figure the modified equivalent circuit for a six-winding machine model.

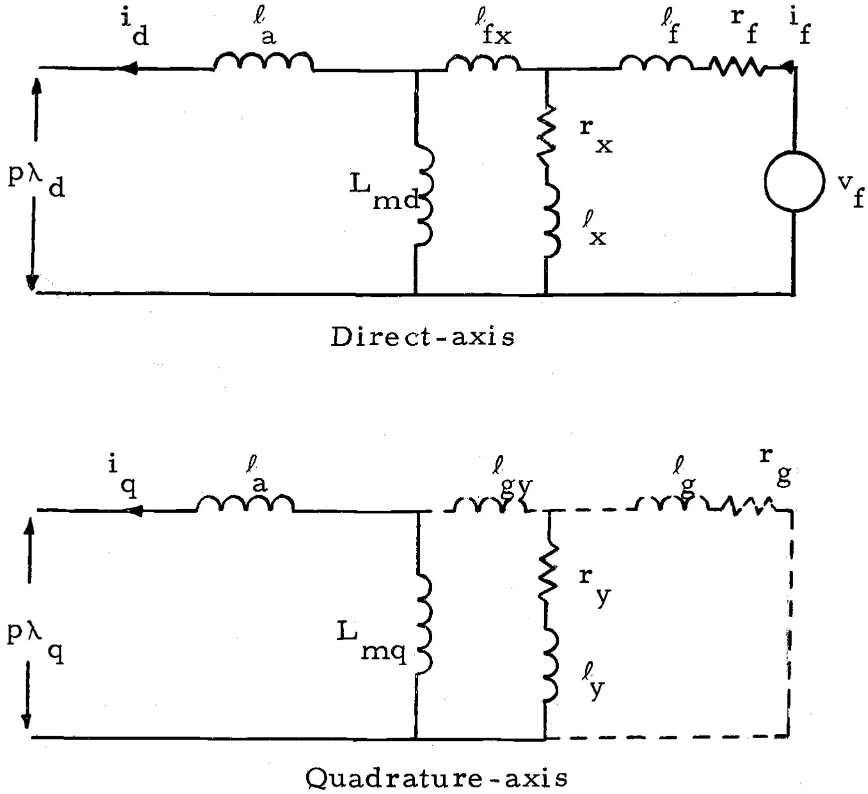


Figure 3.2 Modified equivalent circuits.

The expressions for the circuit parameters of Figure 3.2 are derived in Appendix V for the five-winding machine model. As shown in that appendix, the circuit parameters for the modified equivalent circuit of Figure 3.2 are obtained in terms of the given or measured open- and short-circuit time constants and the direct-axis leakage factors σ_d.[7] The leakage factor, which is a measure

---

[7] $\sigma_d \triangleq 1 - \frac{L_{fx}^2}{L_f L_x}$, where $L_f = \ell_f + \ell_{fx} + L_{md}$, $L_x = \ell_x + \ell_{fx} + L_{md}$, $L_{fx} = L_{md} + \ell_{fx}$

of the mutual coupling between the rotor coils, is introduced to modify the conventional equivalent circuit of Figure 3.1, by assuming the mutual inductance between the rotor coils (f and x) to be different from the rotor-stator mutual inductance L_{md}. This assumption is true, because the mutual coupling between the rotor coils (f and x) is stronger than the rotor-stator mutual couplings. Hence, the modified Park's Equations and the equivalent circuits are developed by using the expression $(L_{md} + \ell_{fx})$ as the mutual inductance between the coils "f" and "x" on the rotor, instead of using just the term "L_{md}", which is used in the conventional machine models given in the literature (2-4).

Once the mutual inductance ℓ_{fx}, or its corresponding leakage factor σ_d, is determined for a given machine, all the equivalent circuit parameters become known.

Formulations similar to those in Appendix V will also yield the equivalent quadrature-axis parameters as a function of σ_q when a six-winding machine model is to be used.

In the derivation of the above equivalent circuit parameters, it is assumed that the armature leakage reactance, $X_{\ell a}$, is equal to the zero sequence reactance of the machine. It is further assumed that, as mentioned in Chapter II, the following equalities hold true between the direct-axis reactances and time constants:

$$X'_d \cong X_d \frac{T'_d}{T'_{d0}} \tag{3-1}$$

$$X''_d \cong X'_d \frac{T''_d}{T''_{d0}} \tag{3-2}$$

3.3 System Studied

The one-machine infinite-bus system considered consists of a synchronous generator connected to an infinite-bus through a transmission line, as shown in the one line diagram of Figure 3.3. It is assumed that voltage and frequency of the infinite-bus are not altered by real and reactive power flow. Excitation control is not incorporated in this study.

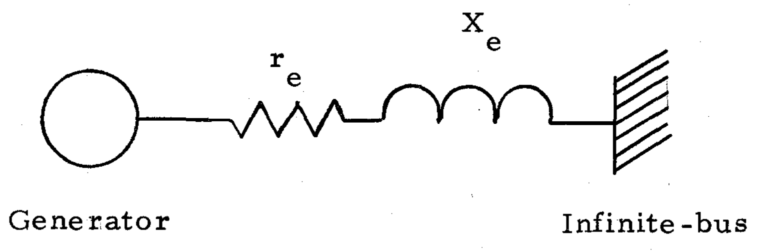


Figure 3.3 One line diagram of the synchronous machine infinite-bus system.

Park's equations for the five-winding synchronous machine model connected to an infinite-bus, and including the effect of the leakage factors are given in Appendix VI. These equations are non-linear and the small displacement technique is employed to linearize them around an operating point by letting all variables change from their steady state value by a small amount Δ; e. g., $i_d = I_d + \Delta i_d$, where the capital I denotes the steady state value. Since Δ is assumed to be small, terms including Δ^2 and higher order may be neglected without losing accuracy. Since the machine equations are expressed in a reference frame fixed in the rotor, it is convenient to transform the transmission line equations into the same rotor reference frame by using Park's transformation. The resulting linearized small displacement equations of the one-machine infinite-bus system are given by equations (3-3). A step voltage for Δe_f is used. Constant prime mover torque is considered, making $\Delta T_m = 0$.

$$
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \Delta e_f \\ \Delta T_m \\ 0 \end{bmatrix}
=
\begin{bmatrix}
-(r_a+r_e) & (X_q+X_e) & 0 & -X_{mq} & 0 & (X_q+X_e)I_q & -V_B \cos\delta_0 \\
-(X_d+X_e) & -(r_a+r_e) & X_{md} & 0 & X_{md} & X_{md}I_f - (X_d+X_e)I_d & V_B \sin\delta_0 \\
0 & 0 & r_x & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & r_y & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & X_{md} & 0 & 0 \\
(X_q-X_d)I_q & (X_q-X_d)I_d + X_{md}I_f & X_{md}I_q & -X_{mq}I_d & -X_{md}I_q & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0
\end{bmatrix}
\begin{bmatrix} \Delta i_d \\ \Delta i_q \\ \Delta i_x \\ \Delta i_y \\ \Delta i_f \\ \frac{\Delta\omega}{\omega_b} \\ \Delta\delta \end{bmatrix}
$$

$$
+ \begin{bmatrix}
-(X_d+X_e) & 0 & X_{md} & 0 & X_{md} & 0 & 0 \\
0 & -(X_q+X_e) & 0 & X_{mq} & 0 & 0 & 0 \\
-X_{md} & 0 & X_x & 0 & X_{mfx} & 0 & 0 \\
0 & -X_{mq} & 0 & X_y & 0 & 0 & 0 \\
-\frac{X_{md}^2}{r_f} & 0 & \frac{X_{md}X_{mfx}}{r_f} & 0 & \frac{X_{md}X_f}{r_f} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & M\omega_b & D_I \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\frac{P}{\omega_b}
\begin{bmatrix} \Delta i_d \\ \Delta i_q \\ \Delta i_x \\ \Delta i_y \\ \Delta i_f \\ \frac{\Delta\omega}{\omega_b} \\ \Delta\delta \end{bmatrix}
$$

(3-3)

where,

$X_{mfx} = X_{md} + x_{fx}$

$X_x = X_{\ell x} + X_{mfx}$

$X_f = X_{\ell f} + X_{mfx}$

$D_I = D + \frac{X_q - X_d}{\omega_b} I_d I_q + \frac{X_{md}}{\omega_b} I_f I_q$

Equation (3-3) is not in standard state variable form (px=Ax+Bu); and in order to arrange it in this form, it is convenient to define the following terms:

$$\Delta V = [\,0 \quad 0 \quad 0 \quad 0 \quad \Delta e_f]^T \tag{3-4}$$

$$\Delta T = [\Delta T_m \quad 0]^T \tag{3-5}$$

$$V_1 = \begin{bmatrix} (X_q+X_e)\, I_q & X_{md} I_f - (X_d+X_e) I_d & 0 & 0 & 0 \\ -V_B \operatorname{Cos} \delta_0 & V_B \operatorname{Sin} \delta_0 & 0 & 0 & 0 \end{bmatrix}^T \tag{3-6}$$

$$V_2 = \begin{bmatrix} (X_q - X_d)I_q & (X_q - X_d)I_d + X_{md}I_f & X_{md}I_q & -X_{mq}I_d & X_{md}I_q \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3-7}$$

$$V_3 = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \tag{3-8}$$

$$Z = \begin{bmatrix} -(r_a + r_e) & (X_q + X_e) & 0 & -X_{mq} & 0 \\ -(X_d + X_e) & -(r_a + r_e) & X_{md} & 0 & X_{md} \\ 0 & 0 & r_x & 0 & 0 \\ 0 & 0 & 0 & r_y & 0 \\ 0 & 0 & 0 & 0 & X_{md} \end{bmatrix} \tag{3-9}$$

$$\Delta I = \begin{bmatrix} \Delta i_d & \Delta i_q & \Delta i_x & \Delta i_y & \Delta i_f \end{bmatrix}^T \tag{3-10}$$

$$\Delta A = \begin{bmatrix} \frac{\Delta\omega}{\omega_b} & \Delta\delta \end{bmatrix}^T \tag{3-11}$$

$$G = \begin{bmatrix} M\omega_b & D_I \\ 0 & 1 \end{bmatrix} \tag{3-12}$$

$$X = \begin{bmatrix} -(X_d+X_e) & 0 & X_{md} & 0 & X_{md} \\ 0 & -(X_q+X_e) & 0 & X_{mq} & 0 \\ -X_{md} & 0 & X_x & 0 & X_{mfx} \\ 0 & -X_{mq} & 0 & X_y & 0 \\ -\frac{X_{md}^2}{r_f} & 0 & \frac{X_{md}X_{mfx}}{r_f} & 0 & \frac{X_{md}X_f}{r_f} \end{bmatrix} \quad (3\text{-}13)$$

$$y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad (3\text{-}14)$$

Using the above definitions and rearranging, equations (3-3) can be written in the form, $px = Ax + Bu$, as follows:

$$p\begin{bmatrix} \Delta I \\ \Delta A \end{bmatrix} = \begin{bmatrix} -\omega_b X^{-1}Z & -\omega_b X^{-1}V_1 \\ -\omega_b G^{-1}V_2 & -\omega_b G^{-1}V_3 \end{bmatrix} \begin{bmatrix} \Delta I \\ \Delta A \end{bmatrix} + \begin{bmatrix} \omega_b X^{-1} & y \\ y^T & \omega_b G^{-1} \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta T \end{bmatrix} \quad (3\text{-}15)$$

where, the system matrix A is

$$A = \begin{bmatrix} -\omega_b X^{-1}Z & -\omega_b X^{-1}V_1 \\ -\omega_b G^{-1}V_2 & -\omega_b G^{-1}V_3 \end{bmatrix} \quad (3\text{-}16)$$

and the control matrix B is

$$B = \begin{bmatrix} \omega_b X^{-1} & y \\ y^T & \omega_b G^{-1} \end{bmatrix} \tag{3-17}$$

Equations (3-15) give a seventh order system in the small displacement form; and they are arranged in the desired state variable form, where the system eigenvalues are calculated from $\det[\lambda I - A] = 0$, I being the identity matrix. The short-circuited synchronous machine eigenvalues can also be calculated by neglecting the state variables in equation (3-11), and the control variables in equation (3-5). Equations (3-15) will then become

$$p[\Delta I] = [-\omega_b X^{-1} Z]\,[\Delta I] + [\omega_b X^{-1}]\;\Delta V \tag{3-18}$$

where, r_e and X_e are set equal to zero in the matrices X and Z, and the eigenvalues are obtained from $\det[\lambda I + \omega_b X^{-1} Z] = 0$. From equations (3-18), it is clear that the order of the machine model is five, and the eigenvalues of the above equations correspond to the conventional machine short-circuit time constants.

3.4 System Data

John Day per-unit machine parameters, given in Table 2.1, are used to study the effect of the leakage factor, σ_d, on the eigenvalues of the new short-circuit model and the response of a synchronous machine infinite-bus system, with a sudden change in the transmission line reactance. The per-unit inertia constant and damping coefficient for the John Day unit, and the transmission line parameters used in the above studies are as follows:

$$M = 0.01833$$

$$D = 2.0$$

$$X_e = 0.3$$

$$r_e = 0.0$$

3.5 Analysis of Short-Circuit Model Eigenvalues

Equations (3-18) represent the linearized model of the short-circuited synchronous machine in state variable form. The eigenvalues of the above system of equations are obtained as a function of the leakage factor for the John Day generators, using a digital computer. The eigenvalues are given in Table 3.1 for three values of the leakage factor.

As noticed from Table 3.1, the model eigenvalues, which represent the machine transients, are not significantly affected by the leakage factor. These eigenvalues are very close to those obtained for the short-circuit model derived in Chapter II, although a different approach has been used in developing the present model.

Table 3.1 Short-circuit model eigenvalues for the John Day hydro-generator unit.

σ_d	Eigenvalues			
0.1	-4.998±j376.766,	-0.324,	-36.541,	-48.259
0.5	-4.998±j376.766,	-0.324,	-36.541,	-48.217
1.0	-4.998j376.766,	-0.325,	-36.541,	-48.058

3.6 Eigenvalue and Time Response Analysis of the One Machine Infinite-Bus System

When a synchronous machine infinite-bus system is represented in full detail with only one damper coil on each axis, a seventh order system results. Equations (3-15) represent the linearized, small displacement differential equations given in state variable form. The A-matrix given by equation (3-16) is solved, using a digital computer, to give the seven eigenvalues at the chosen operating point. Table 3.2 shows the eigenvalues of the John Day machine connected to an infinite-bus, via an external reactance (with the machine operating at full capacity of 142.1 MVA and with 0.95 lagging power factor) as a function of the leakage factor σ_d.

Table 3.2 Eigenvalues of one machine infinite bus system as a function of σ_d.

σ_d					
0.05	-6.07±j358.46	-54.97±j141.12	-0.1375	-23.98	-34.81
0.5	-6.07±j358.46	-54.97±j141.12	-0.1380	-23.97	-34.78
1.0	-6.07±j358.46	-54.97±j141.12	-0.1386	-23.96	-34.59

From Table 3.2 it is noted that variation in the leakage factor does not affect the system eigenvalues significantly.

A digital computer simulation of the system is also performed according to equations (3-15). In the simulation study, the effect of variation of the leakage factor on the stator and rotor current responses are observed when the transmission line reactance is suddenly increased from 0.3 to 2.0 (p.u.) Although this sudden change in the transmission line reactance is not small, the changes in the responses shown in Figures 3.4 and 3.5 are all within one percent from the operating point, which is small for all practical purposes. Hence, the assumption of small displacement around the operating point, which was used in linearizing the nonlinear one-machine infinite-bus system, holds.

Figure 3.4 shows the change in the peak stator current responses, Δi_d and Δi_q, the normalized rotor frequency, $\Delta\omega/\omega_b$, and rotor angle, $\Delta\delta$, as a function of time. It is noticed that variation of leakage factor has negligible effect on the above quantities. This effect that only showed in the first swing, was very small for different values of σ_d and could not be detected on the continuous responses. This is to be expected, as it was shown in Chapter II, that the stator current responses are not significantly affected by the leakage factor σ_d. The rotor angle and frequency are also expected to be unaffected by the leakage factor, as they represent the energy balance in the system. Since the input mechanical energy is only a function of the prime mover, the final rotor frequency and angle should not be affected by the leakage factor.

Figure 3.5 shows the peak values of the direct-axis rotor currents, Δi_f and Δi_x, for three values of σ_d. From this figure it is noticed that the leakage factor has significant effect on the rotor currents, i_f and i_x. Similar observation was made from the short-circuit model derived in Chapter II. It is therefore essential to obtain the proper value of the leakage factor for studies in which the rotor coil currents have important parts.

3.7 Conclusions

In this chapter, it was again established that the mutual coupling between the direct-axis rotor coils of a five-winding synchronous machine model largely affects the rotor current responses during transients. However, the stator current components and the rotor angle and frequency remain essentially unaffected by the variation of the above mutual coupling when the model was to yield a fixed set of time constants. These observations were made by modifying the widely used model of the synchronous machine to have a rotor coil mutual inductance different from that between the rotor and the stator. Modified equivalent circuits for the direct- and quadrature-axis equations were also derived. These equivalent

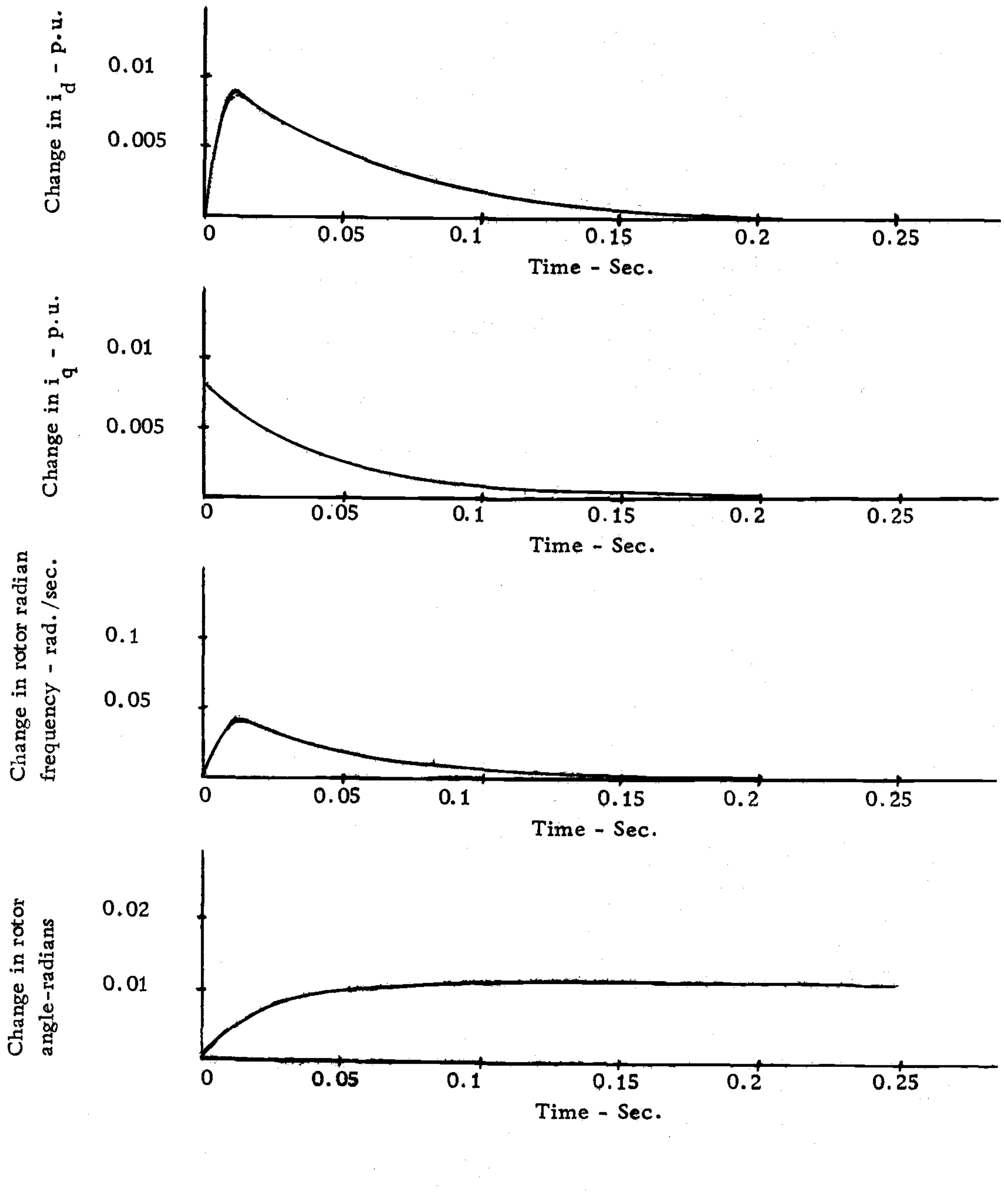


Figure 3.4. Time response of stator current components, rotor frequency and angle to a sudden increase in the transmission line reactance reactance as a function of the leakage factor. (Change in the responses is not noticeable as leakage factor varies.)

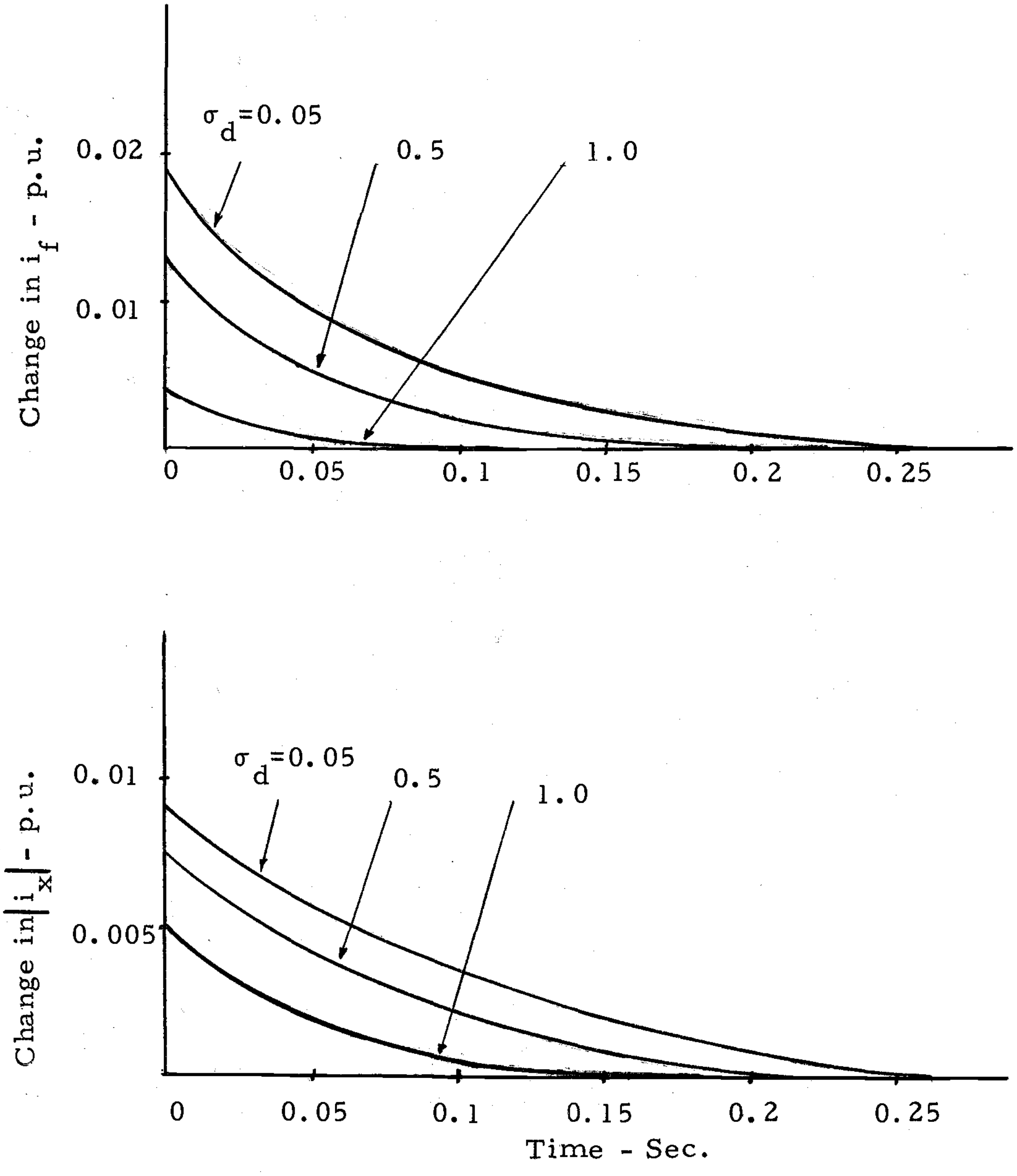


Figure 3.5. Time response of direct-axis rotor currents to a sudden increase in the transmission line reactance for different values of the leakage factor.

circuits must be used when a detailed study of the rotor electrical quantities are desired during transients. The conventional equivalent circuits are, however, sufficient for the study of the stator quantities and rotor oscillations, as it was observed that these quantities are not significantly affected by the rotor coil mutual coupling. However, the mutual inductance between the rotor coils is not normally known. It is therefore essential that the value of the above mutual inductance (or its corresponding leakage factor) be determined by comparing the field current response of the actual machine and that of the model, with similar disturbances applied to the machine and the model.

IV. CLOSURE

4.1 Conclusions

In this research it has been shown that in order to obtain a detailed model of a synchronous machine, the mutual inductances between the concentrated coils of the model should be accounted for. The rotor-stator mutual inductances on each axis were made equal ($L_{af} = L_{ax}$ and $L_{ay} = L_{ag}$) by rescaling the damper winding currents, and only the effect of the rotor coil mutual inductances (L_{fx} and L_{gy}) on the behavior of the synchronous machine were investigated.

Park's Equations and a Laplace-Transform Domain approach were first used to develop a model for the synchronous machine with short-circuited stator terminals. In this model, the numerical value of the rotor-stator mutual inductances, or their corresponding reactances, were not necessary, as they were only used in defining expressions for the transient and subtransient reactances. The defined model reactances and time constants were then obtained in terms of the rotor coil mutual inductances. The given or measured short-circuit reactances and open-circuit time constants were assumed to be known and were used in investigating the effect of the rotor coil mutual couplings on the defined model reactances and time constants as well as the observable current responses (i_d, i_q, i_f).

Park's Equations were also used to modify the widely used synchronous machine model presented in the literature (2-4). The modification was made to make the rotor-coil mutual inductances different from that between the rotor and stator coils. The rotor-stator mutual reactances of each axis were approximated in terms of the corresponding axis synchronous reactance and the armature leakage reactance. The direct-axis rotor coil mutual reactance was assumed to be different from the rotor-stator mutual reactance, and its effect on the machine behavior was investigated. The effect of the rotor coil mutual coupling was also investigated on the current responses of the machine, when connected to an infinite-bus. In this model, the short-circuit reactances were defined in terms of the conventional open- and short-circuit time constants, which were assumed to be known. The state space approach, in the time domain, was used in this investigation.

Although the two models considered are based on Park's equations, they are different in the way their inductances are defined. However, the results obtained from the study of both models were quite in agreement and can be stated as follows:

The rotor coil mutual couplings largely affect the rotor coil currents during transients. On the other hand, the stator current components, the rotor angle and rotor frequency remain essentially unaffected by the rotor coil mutual couplings when the models were to yield a fixed set of parameters. Hence, in the studies that only the stator quantities, the rotor torque-angle, or the rotor frequency are of concern, the conventional models provide sufficient information; but, when the rotor electrical quantities are of concern, proper value of the leakage factor(s) should be used in the models considered.

Precise knowledge of the field current transients is important in the stability investigations of the synchronous machine with regard to the action of the regulator if the output characteristics of the exciter are taken into account. The above knowledge is also very important in the selection of the proper discharge resistor for the main circuit breaker in the field circuit. Hence, when such information is necessary, it is important to have the rotor coil mutual coupling effect included in the model.

Modified equivalent circuits for the synchronous machine were also obtained on the basis of the modification made to the conventional equations derived in the literature (2-4). These equivalent circuits can be used in calculation of the effective damper currents.

Through a digital simulation of the machine model and the eigenvalue studies, it can be concluded that, in order to obtain the proper mutual couplings for the rotor coils of a synchronous machine, the field current response of the machine, under a given disturbance, should be compared with the model field current response when the model is also disturbed with a similar model disturbance. The leakage factor values that make the model field current response the same as that of the actual machine represent the mutual couplings between the rotor coils.

4.2 Further Research

Further research in the following areas will enhance the present study:

1) In the transfer functions obtained for the stator current components of the synchronous machine, it was noticed that the transfer functions have one zero that was a function of the quantity $L_{ax} L_{fx}/L_{af} L_x$. It was further noticed that these zeros could be positive or negative depending on the value of the quantity $L_{ax} L_{fx}/L_{af} L_x$. Further studies could be conducted to determine if positive zeros are physically realistic.
2) Consideration of the induced voltages in the field winding of the synchronous machine, due to the mutual couplings between the rotor and stator coils, when a disturbance is applied to the machine. This study will be helpful in making the present excitation system models more accurate.
3) Parameter identification of the synchronous machine, with the effect of the mutual couplings taken into account.

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APPENDICES

APPENDIX I

Park's Equations for a Generalized Six-Winding Synchronous Machine Model

The quantities such as flux-linkage, voltage, or current are transformed to the corresponding Park Domain quantities by the following transformation:

$$[U_{Park}] = [T]^{RT}\,[U_{real}] \tag{A1-1}$$

where U represents flux-linkage (λ), voltage (v), or current (i). The variable vectors for a six-winding machine model are defined as follows:

$$[U_{Park}] = \begin{bmatrix} u_d \\ u_q \\ u_f \\ u_x \\ u_y \\ u_g \end{bmatrix} \qquad [U_{real}] = \begin{bmatrix} u_a \\ u_b \\ u_c \\ u_f \\ u_x \\ u_y \\ u_g \end{bmatrix}$$

and the transformation matrix in (A1-1) is defined to be

$$[T]^{RT} = 2/3 \begin{bmatrix} \cos\theta & \cos(\theta-120^{o}) & \cos(\theta+120^{o}) & 0 & 0 & 0 \\ -\sin\theta & -\sin(\theta-120^{o}) & -\sin(\theta+120) & 0 & 0 & 0 \\ 0 & 0 & 0 & 3/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3/2 \end{bmatrix} \qquad \text{(A1-2)}$$

The transformation of the Park variables to the "real" variables is performed by

$$[U_{real}] = [T]\,[U_{Park}] \qquad \text{(A1-3)}$$

where

$$[T] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 & 0 \\ \cos(\theta-120^{o}) & -\sin(\theta-120^{o}) & 0 & 0 & 0 \\ \cos(\theta+120^{o}) & -\sin(\theta+120^{o}) & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{(A1-4)}$$

Applying the above transformation and assuming that the amature resistance in the three phases are equal, say r_a, and with reference quantities as chosen in Chapter II, it follows that

$$\begin{bmatrix} v_d(t) \\ v_q(t) \\ v_f(t) \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} p & -\omega & 0 & 0 & 0 & 0 \\ \omega & p & 0 & 0 & 0 & 0 \\ 0 & 0 & p & 0 & 0 & 0 \\ 0 & 0 & 0 & p & 0 & 0 \\ 0 & 0 & 0 & 0 & p & 0 \\ 0 & 0 & 0 & 0 & 0 & p \end{bmatrix} \begin{bmatrix} \lambda_d(t) \\ \lambda_q(t) \\ \lambda_f(t) \\ \lambda_x(t) \\ \lambda_y(t) \\ \lambda_g(t) \end{bmatrix} +$$

$$\begin{bmatrix} -r_a & & & & & \\ & -r_a & & & 0 & \\ & & r_f & & & \\ & & & r_x & & \\ & & & & r_y & \\ & 0 & & & & r_g \end{bmatrix} \begin{bmatrix} i_d(t) \\ i_q(t) \\ i_f(t) \\ i_x(t) \\ i_y(t) \\ i_g(t) \end{bmatrix} \qquad \text{(A1-5)}$$

where p is the operator $\frac{d}{dt}$ and $\omega = p\theta$. The direct and quadrature damper winding equivalents are taken to be short circuited; hence, their terminal voltages have been set to zero.

With the reference direction adopted above, the flux-linkages can be written in terms of the terminal currents, in the Park Domain, as follows:

$$[\lambda_{Park}] = [L]\,[I_{Park}] \qquad \text{(A1-6)}$$

where

$$[L] = \begin{bmatrix} -L_d & 0 & L_{af} & L_{ax} & 0 & 0 \\ 0 & -L_q & 0 & 0 & L_{ay} & L_{ag} \\ -\frac{3}{2} L_{af} & 0 & L_f & L_{fx} & 0 & 0 \\ -\frac{3}{2} L_{ax} & 0 & L_{fx} & L_x & 0 & 0 \\ 0 & -\frac{3}{2} L_{ay} & 0 & 0 & L_y & L_{yg} \\ 0 & -\frac{3}{2} L_{ag} & 0 & 0 & L_{yg} & L_g \end{bmatrix} \qquad \text{(A1-7)}$$

Substituting (A1-7) in (A1-5) and taking Laplace Transform yields

$$\begin{bmatrix} -(r_a+sL_d) & \omega L_q & sL_{af} & sL_{ax} & -\omega L_{ay} & -\omega L_{ag} \\ -\omega L_d & -(r_a+sL_q) & \omega L_{af} & \omega L_{ax} & sL_{ay} & sL_{ag} \\ -\frac{3}{2} sL_{af} & 0 & r_f+sL_f & sL_{fx} & 0 & 0 \\ -\frac{3}{2} sL_{ax} & 0 & sL_{fx} & r_x+sL_x & 0 & 0 \\ 0 & -\frac{3}{2} sL_{ay} & 0 & 0 & r_y+sL_y & sL_{yg} \\ 0 & -\frac{3}{2} sL_{ag} & 0 & 0 & sL_{yg} & r_g+sL_g \end{bmatrix} \begin{bmatrix} I_d \\ I_q \\ I_f \\ I_x \\ I_y \\ I_g \end{bmatrix} =$$

$$\begin{bmatrix} V_d \\ V_q \\ V_f \\ 0 \\ 0 \\ 0 \end{bmatrix} + [L]\,[I_{Park}(0)] \qquad \text{(A1-8)}$$

where the capital letters for voltage and current indicate the quantities in the Laplace Transform Domain. The vector $[I_{Park}(0)]$ contains the initial values of the untransformed, Park Domain currents.

APPENDIX II

Derivation of the Characteristic Equation for a Six-Winding Machine Model with Short-Circuited Stator

From Equation (2-4), the unobservable rotor coil currents (I_x, I_y, I_g) can be obtained in terms of I_d, I_q, I_f as follows:

$$I_x = (u_4 - a_{41} I_d - a_{43} I_f) / a_{44} \tag{A2-1}$$

$$I_y = -a_y I_q \tag{A2-2}$$

$$I_g = -a_g I_q \tag{A2-3}$$

where

$$a_y = (a_{52} a_{66} - a_{62} a_{56})/(a_{55} a_{66} - a_{56} a_{65}) \tag{A2-4}$$

$$a_g = (a_{62} a_{55} - a_{52} a_{65})/(a_{55} a_{66} - a_{56} a_{65}) \tag{A2-5}$$

Substituting expressions (A2-1) through (A2-3) in (2-4), yields

$$\begin{bmatrix} a_{11} a_{44} - a_{41} a_{14} & a_{44} [a_{12} - a_{15} a_y - a_{16} a_g] & a_{13} a_{44} - a_{43} a_{14} \\ a_{21} a_{44} - a_{41} a_{24} & a_{44} [a_{22} - a_{25} a_y - a_{26} a_g] & a_{23} a_{44} - a_{43} a_{24} \\ a_{31} a_{44} - a_{41} a_{34} & 0 & a_{33} a_{44} - a_{43} a_{34} \end{bmatrix} \begin{bmatrix} I_d \\ I_q \\ I_f \end{bmatrix} = \begin{bmatrix} a_{44} u_1 - a_{14} u_4 \\ a_{44} u_2 - a_{24} u_4 \\ a_{44} u_3 - a_{34} u_4 \end{bmatrix} \tag{A2-6}$$

The characteristic equation, which is the determinant of the matrix on the left hand side of (A2-6), can be forced into the following form:

$$\omega^2(s^2+A_d s+B_d)(s^2+A_q s+B_q) + (s^3+C_d s^2+D_d s+E_d) \times (s^3+C_q s^2+D_q s+E_q) \qquad \text{(A2-7)}$$

where

$A_d = (X'_x T_x + X'_d T_f)/\Delta_d$ $\qquad$ $A_q = (X'_y T_y + X'_q T_g)/\Delta_q$

$B_d = X_d/\Delta_d$ $\qquad$ $B_q = X_q/\Delta_q$

$C_d = \dfrac{X_d}{X''_d T_d} + A_d$ $\qquad$ $C_q = \dfrac{X_q}{X''_q T_q} + A_q$

$D_d = \dfrac{X_d (T_f + T_x + T_d)}{\Delta_d T_d}$ $\qquad$ $D_q = \dfrac{X_q (T_g + T_y + T_q)}{\Delta_q T_q}$

$E_d = \dfrac{X_d}{\Delta_d T_d}$ $\qquad$ $E_q = \dfrac{X_q}{\Delta_q T_q}$

and

$\Delta_d = \sigma_d x''_d T_f T_x$ $\qquad$ $\Delta_q = \sigma_q X''_q T_y T_g$

$T_d = \dfrac{X_d}{\omega r_a}$ $\qquad$ $T_q = \dfrac{X_q}{\omega r_a}$

and

σ_d = direct-axis leakage factor $\triangleq 1 - \frac{L_{fx}^2}{L_f L_x}$

σ_q = quadrature-axis leakage factor $\triangleq 1 - \frac{L_{gy}^2}{L_g L_y}$

$$X_d \triangleq \omega L_d$$

$$X_q \triangleq \omega L_q$$

$$X'_d \triangleq \omega (L_d - \frac{3}{2} \frac{L_{af}^2}{L_f})$$

$$X'_q \triangleq \omega (L_q - \frac{3}{2} \frac{L_{ag}^2}{L_g})$$

$$X''_d \triangleq \omega [L_d - \frac{3}{2\sigma_d} (\frac{L_{af}^2}{L_f} + \frac{L_{ax}^2}{L_x} - \frac{2L_{af}L_{ax}L_{fx}}{L_f L_x})]$$

$$X''_q \triangleq \omega [L_q - \frac{3}{2\sigma_q} (\frac{L_{ag}^2}{L_g} + \frac{L_{ay}^2}{L_y} - \frac{2L_{ag}L_{ay}L_{yg}}{L_g L_y})]$$

$$X'_x \triangleq \omega (L_d - \frac{3}{2} \frac{L_{ax}^2}{L_x})$$

$$X'_g \triangleq \omega (L_q - \frac{3}{2} \frac{L_{ax}^2}{L_y})$$

and

$$T_f = \frac{L_f}{r_f}$$

$$T_x = \frac{L_x}{r_x}$$

$$T_y = \frac{L_y}{r_y}$$

$$T_g = \frac{L_g}{r_g}$$

APPENDIX III

Derivation of Poles of Root-Locus Polynomial Ratio

Root-locus technique can also be used to approximate the poles of the root-locus polynomial ratio in equation (2-8). The poles of the above polynomial ratio are the roots of equations (2-19) and (2-20), rewritten below.

$$s^3 + C_d s^2 + D_d s + E_d = (s + p_1)(s + p_2)(s + p_3) \qquad \text{(A3-1)}$$

$$s^3 + C_q s^2 + D_q s + E_q = (s + p_4)(s + p_5)(s + p_6) \qquad \text{(A3-2)}$$

Equation (A3-1) can be rewritten as,

$$1 + \frac{D_d (s + E_d/D_d)}{s^2 (s + C_d)} = 0 \qquad \text{(A3-3)}$$

Equation (A3-3) is suitable for application of the root-locus technique. The estimated locations of the zeros and poles of the polynomial ratio in (A3-3) are shown plotted on the s-plane in Figure A3.1.

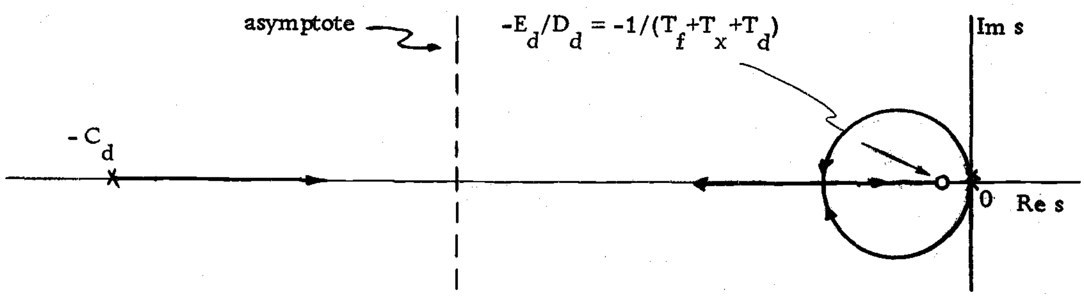

Figure A3.1. Root-locus plot for approximating the roots of a third order polynomial.

Since for most machine-parameter values, the gain factor of the root-locus polynomial ratio ($D_d = X_d/\Delta_d T_d$) is much larger than unity, and the zero lies very near the origin, one root of equation (A3.1) is expected to lie very near the zero. Once one root (p_1) is determined, the remaining two roots can be obtained by dividing ($s + p_1$) into the polynomial in equation (A3-1), from which a second order polynomial is obtained. Solution of the second order polynomial can then be obtained using the binomial expansion of the quadratic formula. Since for most machine parameters the zero at $-E_d/D_1$ and the pole at $-C_d$ are very far apart, the roots (p_2 and p_3) of equation (A3-1) are real.

Based on the above discussion, the roots of equation (A3-1) are approximated as follows:

$$p_1 \cong Z_d \left(1 + \frac{2Z_d P_d}{K_d}\right) \tag{A3-4}$$

$$p_2 \cong P_d - \frac{K_d}{P_d+Z_d} - \frac{K_d^2}{(P_d + Z_d)^3} - \frac{Z_d K_d^3}{(P_d + Z_d)^5} \tag{A3-5}$$

$$p_3 \cong -p_1 + \frac{K_d}{P_d + Z_d} + \frac{K_d^{\,2}}{(P_d + Z_d)^3} + \frac{Z_d K_d^{\,3}}{(P_d + Z_d)^5} \qquad \text{(A3-6)}$$

and

$$Z_d = \frac{-1}{T_f + T_x + T_d} \qquad \text{(A3-7)}$$

$$P_d = \frac{X_d}{X''_d \, T_d} + \frac{\alpha_d}{\Delta_d} \qquad \text{(A3-8)}$$

$$K_d = \frac{-X_d}{Z_d \, T_d \, \Delta_d} \qquad \text{(A3-9)}$$

Expressions similar to those for p_1 through p_3 can be written for p_4 to p_6, if D_q, Z_q, P_q, K_q are respectively substituted for D_d, Z_d, P_d, K_d in equations (A3-7) through (A3-9), where

$$D_q = \frac{X_q}{\Delta_q \, T_q} \qquad \text{(A3-10)}$$

$$Z_q = \frac{-1}{T_g + T_y + T_q} \qquad \text{(A3-11)}$$

$$P_q = - \frac{X_q}{X''_q \, T_q} + \frac{\alpha_q}{\Delta_q} \qquad \text{(A3-12)}$$

$$K_q = \frac{-X_q}{Z_q \, T_q \, \Delta_q} \qquad \text{(A3-13)}$$

It should be noted that, the assumption that one pole of the root-locus polynomial ratio in equation (A3-3) lies very near the zero, holds better for the direct-axis than for the quadrature-axis, as the gain factor D_d is larger than D_q for most machine parameter values.

APPENDIX IV

Derivation of Defined Short-Circuit Model Parameters from the Measured Data

From Figure 2.1, the rotor direct-axis circuit equations can be written, in the Laplace-Transform domain, as

$$\begin{bmatrix} V_f \\ 0 \end{bmatrix} = \begin{bmatrix} r_f + sL_f & sL_{fx} \\ sL_{fx} & r_x + sL_x \end{bmatrix} \begin{bmatrix} I_f \\ I_x \end{bmatrix} \qquad \text{(A4-1)}$$

The characteristic equation of (A4-1) can be simplified and written in terms of the Park Domain open-circuit time-constants as,

$$s^2 + \frac{1}{\sigma_d}\left(\frac{1}{T_f} + \frac{1}{T_x}\right) s + \frac{1}{\sigma_d T_f T_x} = \left(s + \frac{1}{T'_{d0}}\right)\left(s + \frac{1}{T''_{d0}}\right) = 0 \qquad \text{(A4-2)}$$

From equation (A4-2)

$$\sigma_d T_f T_x = T'_{d0} T''_{d0} \qquad \text{(A4-3)}$$

$$T_f + T_x = T'_{d0} + T''_{d0} \qquad \text{(A4-4)}$$

Solving (A4-4) for T_f and substituting in (A4-3), it yields,

$$T_x^2 - (T'_{d0} + T''_{d0})\, T_x + \frac{T'_{d0} T''_{d0}}{\sigma_d} = 0 \qquad \text{(A4-5)}$$

T_x can be obtained from (A4-5), and it is real if,

$$\sigma_d \geq \frac{4T'_{d0}\ T''_{d0}}{(T'_{d0} + T''_{d0})^2} \qquad \text{(A4-6)}$$

Equation (A4-6) gives the lower limit for σ_d and the upper limit is $(1 - \sigma_{d_{min}})$, where $\sigma_{d_{min}}$ is the lower limit of σ_d.

Similarly, T_y can be obtained from the following quadratic equation:

$$T_y^2 - (T'_{q0} + T''_{q0})\, T_y + \frac{T'_{q0}\ T''_{q0}}{\sigma_q} = 0 \qquad \text{(A4-7)}$$

Solution of (A4-7) is also real if

$$\sigma_q \geq \frac{4T'_{q0}\ T''_{q0}}{(T'_{q0} + T''_{q0})^2} \qquad \text{(A4-8)}$$

Assuming that the real roots of the characteristic equation for the short-circuit model are very near the zeros of the root-locus polynomial ratio, as discussed in section 2.3, equation (2-6), whose roots are the zeros of the root-locus polynomial ratio, can be written as,

(A4-9)

$$(s^2 + A_d s + B_d)(s^2 + A_q s + B_q) = (s + \frac{1}{T'_d})(s + \frac{1}{T''_d})(s + \frac{1}{T'_q})(s + \frac{1}{T''_q})$$

Since the direct- and quadrature-axis parameters affect only their respective time-constants, the direct-axis short-circuit time constants can be written as follows:

$$(s+\frac{1}{T'_d})(s+\frac{1}{T''_d}) = s^2+A_d s+B_d = s^2+\frac{X'_x T_x+X'_d T_f}{\sigma_d X''_d T_f T_x} s + \frac{X_d}{\sigma_d X''_d T_f T_x} \quad \text{(A4-10)}$$

Assuming that the quadrature-axis pole(s) make negligible contribution to the direct-axis current component, from equation (2-27), expression for the above current (in the Laplace-Transform domain) can be approximated as,

$$I_d(s) \cong \frac{E_0 \omega^2}{sX''_d} \cdot \frac{(s+\frac{1}{T'_{d0}})\ (s+\frac{1}{T''_{d0}})}{(s^2+Ks+\omega^2)\ (s+\frac{1}{T'_d})\ (s+\frac{1}{T''_d})} \quad \text{(A4-11)}$$

where K is a constant.

From the definition of transient reactance it follows that,

$$\frac{E_0}{X'_{dm}} \triangleq I_{dss} + i_{dt}(0) = \frac{E_0}{X_{dm}} + i_{dt}(0) \quad \text{(A4-12)}$$

or

$$W = \frac{1}{X'_{dm}} - \frac{1}{X_{dm}} \cong \frac{1}{X''_d} \cdot \frac{(1 - T_d'/T_{d0}')(1 - T_d'/T_{d0}'')}{(1 - T_d'/T_d'')} \quad \text{(A4-13)}$$

where, $i_{dt}(t)$ and I_{dss} indicate the transient and steady-state components of the direct-axis current, respectively and subscript m along with the parameters indicates their measured or given values.

From (A4-12) and application of (A4-13), it follows that

$$\frac{1}{T_d''} = \frac{X_d}{X_d''} \cdot \frac{T_d'}{T_{d0}' \; T_{d0}''} \qquad \text{(A4-14)}$$

Substituting (A4-14) in (A4-13), it yields

$$T_d'^2 - \frac{X'_{dm}}{X_{dm}} (T_{d0}' + T_{d0}'')T'_d + T'_{d0} \; T''_{d0}(X'_{dm} - X''_d + X'_{dm} X''_d / X_{dm})/X_{dm}$$

$$= 0 \qquad \text{(A4-15)}$$

Also, from the definition of subtransient reactance, it follows that

$$\frac{E_0}{X''_{dm}} = I_{dss} + i_{dt}(0) + i_{dst}(0) = \frac{E_0}{X'_{dm}} + i_{dst}(0) \qquad \text{(A4-16)}$$

where $i_{dst}(t)$ is the subtransient component of the direct-axis current. From (A4-16), it follows that

$$W'' = \frac{1}{X''_{dm}} - \frac{1}{X'_{dm}} \cong \frac{1}{X''_d} \cdot \frac{(1 - T_d''/T_{d0}')\,(1 - T_d''/T_{d0}'')}{(1 - T_d''/T_d')} \qquad \text{(A4-17)}$$

Adding (A4-13) and (A4-17) and using (A4-14) it yields

$$(W' + W'')\, X''_d = 1 - \frac{X''_d}{X_{dm}} \qquad \text{(A4-18)}$$

From (A4-18), it follows that

$$X''_d = X''_{dm} \tag{A4-19}$$

Equation (A4-19) indicates that the defined value of the subtransient reactance is the same as its measured value. T'_d and T''_d can also be obtained from (A4-15) and (A4-14), respectively.

When the damper winding currents are scaled such that $L_{af} = L_{ax}$, from the expressions for X'_d and X'_x it follows that,

$$\frac{L_f}{L_x} = \frac{X_d - X'_x}{X_d - X'_d} \tag{A4-20}$$

Using the definition of σ_d, equation (A4-20) becomes

$$L_{fx}/L_x = \left[\frac{(1-\sigma_d)\,(X_d - X'_x)}{X_d - X'_d} \right]^{1/2} \tag{A4-21}$$

Using (A4-15) in the expression for X''_d, it follows that

$$[\sigma_d(X'_d - X''_d)]^{1/2} + [(1-\sigma_d)(X_d - X'_d)]^{1/2} = (X_d - X'_x)^{1/2} \tag{A4-22}$$

Since the defined value of X_d is also the same as its measured or given value, X'_d can be obtained from (A4-22), once X'_x is known in terms of X'_d.

From equation (A4-10), it is noticed that

$$\frac{1}{T'_d}+\frac{1}{T''_d} = \frac{X'_x T_x + X'_d T_f}{\sigma_d T_f T_x X''_d} = \frac{X'_x T_x + X'_d T_f}{X''_{dm} T'_{d0} T''_{d0}} \qquad \text{(A4-23)}$$

Substituting expression (A4-14) in (A4-23) and solving for X'_x, it yields;

$$X'_x = Q - X'_d\, T_f/T_x \qquad \text{(A4-24)}$$

where

$$Q = \frac{X_{dm} T'_d + X''_{dm} T'_{d0} T''_{d0}/T'_d}{T_x} \qquad \text{(A4-25)}$$

X'_d can then be obtained by substituting (A4-24) in (A4-22) and solving by an iterative method. X'_x can also be determined by inserting the value of X'_d in (A4-24). From equation (2-32), expression for X''_x can also be written in terms of the previously calculated parameters as

$$X''_x = \frac{1}{\sigma_d}\left[X'_x - (1-\sigma_d)X_d + \sqrt{(1-\sigma_d)(X_d - X'_d)(X_d - X'_x)}\right] \qquad \text{(A4-26)}$$

A summary of calculation sequence of the above parameters is presented in section 2.4.

Similar expressions to above can also be obtained for the quadrature-axis parameters by changing all the direct-axis model parameters in (A4-1) to (A4-26) by their corresponding quadrature-axis parameters; because no assumptions involving the magnitude of the ratio of the open-circuit time constants is involved.

APPENDIX V

Derivation of Synchronous Machine Parameters

The per-unit equations which describe the behavior of a conventional synchronous machine (Park's Equations) may be expressed for generator action, for a five-winding model, by

$$\begin{bmatrix} \lambda_d \\ v_f \\ v_x \end{bmatrix} = \begin{bmatrix} -(L_{md}+\ell_a) & L_{md} & L_{md} \\ -L_{md}p & r_f(\ell_f+L_{md})p & L_{md}p \\ -L_{md}p & L_{md}p & r_x+(\ell_x+L_{md})p \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_x \end{bmatrix} \qquad \text{(A5-1)}$$

$$\begin{bmatrix} \lambda_q \\ v_y \end{bmatrix} = \begin{bmatrix} -(\ell_a+L_{mq}) & L_{mq} & i_q \\ -L_{mq}p & r_y+(\ell_y+L_{mq})p & i_y \end{bmatrix} \qquad \text{(A5-2)}$$

where the damper coils ("x," "y") are assumed to be short-circuited, making $v_x = v_y = 0$.

When the mutual inductance between the direct-axis rotor coils ("f" and "x") is assumed to be $L_{md} + \ell_{fx}$, equations (A5-1) become

$$\begin{bmatrix} \lambda_d \\ v_f \\ v_x \end{bmatrix} = \begin{bmatrix} -(L_{md}+\ell_a) & L_{md} & L_{md} \\ -L_{md}p & r_f(\ell_f+L_{md}+\ell_{fx})p & (L_{md}+\ell_{fx})p \\ -L_{md}p & (L_{md}+\ell_{fx})p & r_x+(\ell_x+L_{md}+\ell_{fx})p \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_x \end{bmatrix} \qquad \text{(A5-3)}$$

and equations (A5-2) remain unchanged.

From equations (A5-3), the direct-axis current, i_d, can be written as

$$i_d = \frac{D_1}{D_2}$$

where

$$D_1 = \begin{bmatrix} \lambda_d & L_{md} & L_{md} \\ v_f & r_f+(\ell_f+L_{md}+\ell_{fx})p & (L_{md}+\ell_{fx})p \\ v_x & (L_{md}+\ell_{fx})p & r_x(\ell_x+L_{md}+\ell_{fx})p \end{bmatrix}$$

$$= r_f r_x [1+(T_1+T_2)p+T_1T_3p^2]\,\lambda_d - r_x L_{md}(1+T_{\ell x}p)v_f \qquad \text{(A5-4)}$$

and

$$D_2 = \begin{bmatrix} -(\ell_a+L_{md}) & L_{md} & L_{md} \\ -L_{md}p & r_f+(\ell_f+L_{md}+\ell_{fx})p & (L_{md}+\ell_{fx})p \\ -L_{md}p & (L_{md}+\ell_{fx})p & r_x+(\ell_x+L_{md}+\ell_{fx})p \end{bmatrix}$$

$$= -r_f r_x L_d [1+(T_4 T_5)p+T_4 T_6 p^2] \qquad \text{(A5-5)}$$

The time constants used in equations (A5-4, 5) are defined in terms of the leakage and mutual reactances as follows:

$$T_1 = \frac{1}{\omega r_f} (X_{\ell f}+X_{md}+x_{fx}) \qquad \text{(A5-6)}$$

$$T_2 = \frac{1}{\omega r_x} (X_{\ell x}+X_{md}+x_{fx}) \qquad \text{(A5-7)}$$

$$T_3 = \frac{1}{\omega r_x} [X_{\ell x}+\frac{X_{\ell f}(X_{md}+x_{fx})}{X_{\ell f}+X_{md}+x_{fx}}] \qquad \text{(A5-8)}$$

$$T_4 = \frac{1}{\omega r_f} [X_{\ell f}+x_{fx}+\frac{X_{\ell a} X_{md}}{X_{\ell a}+X_{md}}] \qquad \text{(A5-9)}$$

$$T_5 = \frac{1}{\omega r_x} (X_{\ell x}+x_{fx}+\frac{X_{\ell a} X_{md}}{X_{\ell a}+X_{md}}) \qquad \text{(A5-10)}$$

$$T_6 = \frac{1}{\omega r_x} [\frac{(X_{\ell a}+X_{md})[X_{\ell f} X_{\ell x}+(X_{md}+x_{fx})(X_{\ell f}+X_{\ell x})] - X^2_{md}(X_{\ell f}+X_{\ell x})}{(X_{\ell a}+X_{md})(X_{\ell f}+x_{fx})+X_{\ell a} X_{md}}] \qquad \text{(A5-11)}$$

$$T_{\ell x} = \frac{X_{\ell x}}{\omega r_x} \qquad \text{(A5-12)}$$

where, $X = \omega L$ with proper subscript, and $x_{fx} = \omega\ell_{fx}$.

The direct-axis principle time constants are determined by the following identities (see reference 4):

$$(1+T'_{d0}p)\,(1+T''_{d0}p) \equiv 1+(T_1+T_2)p+T_1T_3p^2 \qquad \text{(A5-13)}$$

$$(1+T'_{d}p)\,(1+T''_{d}p) \equiv 1+(T_4+T_5)p+T_4T_6p^2 \qquad \text{(A5-14)}$$

Since the per-unit resistance of the damper winding is much larger than the field winding (4), T_2 and T_3 are then much less than T_1 and the right-hand side of (A5-13) differs very little from $(1+T_1p)(1+T_3p)$. Hence

$$T'_{d0} \cong T_1 \;,\quad T''_{d0} \cong T_3$$

Similarly, from equation (A5-14),

$$T'_{d} \cong T_4, \qquad T''_{d} \cong T_3$$

The quadrature-axis time constants can also be derived similarly. They are

$$T''_{q0} = \frac{1}{\omega r_y}\,(X_{\ell y} + X_{mq}) \qquad \text{(A5-15)}$$

$$T''_q = \frac{1}{\omega r_y} \left(X_{\ell y} + \frac{X_{\ell a} X_{mq}}{X_{\ell a} + X_{mq}} \right) \tag{A5-16}$$

The known reactances are defined as follows:

$$X_d = X_{\ell a} + X_{md} \tag{A5-17}$$

$$X'_d = \frac{T'_d}{T'_{d0}} \cdot X_d = X_{\ell a} + \frac{X_{md}(X_{\ell f} + x_{fx})}{X_{\ell f} + X_{md} + x_{fx}} \tag{A5-18}$$

$$X''_d = \frac{T'_d T''_d}{T'_{d0} T''_{d0}} \cdot X_d = X_{\ell a} + \frac{X_{md}[X_{\ell f} X_{\ell x} + x_{fx}(X_{\ell f} + X_{\ell x})]}{X_{\ell f} X_{\ell x} + (X_{md} + x_{fx})(X_{\ell f} + X_{\ell x})} \tag{A5-19}$$

$$X_q = X_{\ell a} + X_{mq} \tag{A5-20}$$

$$X''_q = \frac{T''_q}{T''_{q0}} \cdot X_q = X_{\ell a} + \frac{X_{mq} X_{\ell y}}{X_{mq} + X_{\ell y}} \tag{A5-21}$$

Where, $X_{\ell a}$, $X_{\ell f}$, $X_{\ell x}$, $X_{\ell y}$ are the leakage reactance of the armature, field, direct- and quadrature-axis damper windings, respectively.

With the assumption that the armature leakage reactance, $X_{\ell a}$, is equal to the zero sequence reactance, X_0, the unknown resistances and reactances in equations (A5-15) through (A5-21) can be obtained in terms of the additional direct-axis rotor-coil mutual reactance, x_{fx}, and other known reactances as follows:

$$X_{\ell a} = X_0 \tag{A5-22}$$

$$X_{md} = X_d - X_0 \tag{A5-23}$$

$$X_{\ell f} = \frac{X_{md}(X'_d - X_0)}{X_d - X'_d} - x_{fx} = X'_f - x_{fx} \tag{A5-24}$$

$$X_{\ell x} = \frac{(X_d - X''_d)x_{fx}^2 + [(X''_d - X_0)(X'_f - X_{md}) - X'_f X_{md}]\, x_{fx} + X'_f X_{md}(X''_d - X_0)}{X_{md}X'_f - (X''_d - X_0)(X_{md} + X'_f)} \tag{A5-25}$$

$$r_f = \frac{1}{\omega T'_{d0}} \quad (X_{\ell f} + X_{md} + x_{fx}) \tag{A5-26}$$

$$r_x = \frac{1}{\omega T''_{d0}} \quad [X_{\ell x} + \frac{X_{\ell f}(X_{md} + x_{fx})}{X_{\ell f} + X_{md} + x_{fx}}\,] \tag{A5-27}$$

and

$$X_{mq} = X_q - X_0 \tag{A5-28}$$

$$X_{\ell y} = \frac{X_{mq}(X''_q - X_0)}{X_q - X''_q} \tag{A5-29}$$

$$r_y = \frac{1}{\omega T''_{q0}} \quad (X_{\ell y} + X_{mq}) \tag{A5-30}$$

The mutual reactance, x_{fx}, can be written in terms of the leakage factor, σ_d, and the direct-axis rotor-coil reactances as

$$X_{fx} = \sqrt{(1 - \sigma_d)\, X_f\, X_x} \tag{A5-31}$$

where

$$X_f = X_{\ell f} + X_{md} + x_{fx} \tag{A5-32}$$

$$X_x = X_{\ell x} + X_{md} + x_{fx} \tag{A5-33}$$

Equation (A5-31) can be used to solve for x_{fx} in terms of σ_d, once the expressions for $X_{\ell f}$ and $X_{\ell x}$, equations (A5-23, 24), are substituted in equation (A5-31). Hence, for a given value of σ_d, x_{fx} is obtained from equation (A5-31) and the other unknown reactances and resistances are obtained from equations (A5-24) through (A5-30)

APPENDIX VI

Park's Equations for a Synchronous Machine Infinite-Bus System

Using the generator conventions for the stator currents, Park's Equations in matrix form can be written as

$$
\begin{bmatrix} v_d \\ v_q \\ 0 \\ 0 \\ e_f \end{bmatrix}
=
\begin{bmatrix}
-r_a - p\frac{X_d}{\omega_b} & \frac{\omega}{\omega_b} X_q & \frac{X_{md}}{\omega_b} p & -\frac{\omega}{\omega_b} X_{mq} & \frac{X_{md}}{\omega_b} \\
-\frac{\omega}{\omega_b} X_d & -r_a - p\frac{X_q}{\omega_b} & \frac{\omega}{\omega_b} X_{md} & \frac{X_{mq}}{\omega_b} p & \frac{\omega}{\omega_b} X_{md} \\
-X_{md}\frac{p}{\omega_b} & 0 & r_x + \frac{X_{\ell x} + X_{md} + x_{fx}}{\omega_b} p & 0 & \frac{X_{md} + x_{fx}}{\omega_b} p \\
0 & -\frac{X_{mq}}{\omega_b} p & 0 & r_y + \frac{X_{\ell y} + X_{mq}}{\omega_b} p & 0 \\
-\frac{X_{md}^2}{\omega_b r_f} p & 0 & \frac{X_{md}^2}{\omega_b r_f} p & 0 & X_{md}\left(1 + \frac{X_{\ell f} + X_{md} + x_{fx}}{\omega_b r_f} p\right)
\end{bmatrix}
$$

$$
\times \begin{bmatrix} i_d \\ i_q \\ i_x \\ i_y \\ i_f \end{bmatrix} \qquad \text{(A6-1)}
$$

in a reference frame fixed in the synchronous machine rotor. The transmission line equations can also be written, in the Park Domain, as

$$v_d = (r_e + \frac{X_e}{\omega_b} p)\, i_d - \frac{\omega}{\omega_b} X_e\, i_q + v_{dB} \qquad \text{(A6-2)}$$

$$v_q = \frac{\omega}{\omega_b} X_e\, i_d + (r_e + \frac{X_e}{\omega_b} p)\, i_q + v_{qB} \qquad \text{(A6-3)}$$

Where v_{dB} and v_{qB} are the infinite-bus voltages in the direct- and quadrature-axis, respectively. Also

$$v_{dB} = v_B \text{ Sin } \delta \qquad \text{(A6-4)}$$

$$v_{qB} = v_B \text{ Cos } \delta \qquad \text{(A6-5)}$$

Electromagnetic torque equation is expressed in per-unit as

$$T_e = \psi_d\, i_q - \psi_q\, i_d \qquad \text{(A6-6)}$$

where $\psi = \omega_b \lambda$, and λ is the flux-linkage in per-unit. The mechanical and electrical torque, and rotor speed and angle are related by

$$T_m - T_e = Mp \frac{\omega}{\omega_b} + Dp \frac{\delta}{\omega_b} \qquad \text{(A6-7)}$$

and

$$p\,\delta = \omega - \omega_b \qquad \text{(A6-8)}$$

Equations (A6-1) through (A6-8) give the synchronous machine and transmission line equations in a reference frame fixed in the rotor.