

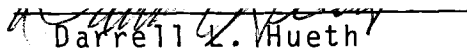
An Abstract of the Thesis of

Mario E. Niklitschek for the degree of Master of Science in Agricultural and Resource Economics presented on August 23, 1985.

Title: Exact Welfare Measurement: Theory and Application
to Recreation Economics

Redacted for privacy

Abstract Approved:


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Although the limitations of consumer surplus have become widely known, there exists a lack of studies which present in a coherent framework recent developments in estimating Hicksian welfare measures. This study attempts to fill partially this gap by analyzing some theoretical and empirical aspects in the estimation of exact welfare measures using market data. Analytical expressions of the "exact" welfare measures, compensating variation (CV) and equivalent variation (EV), for some frequently used single demand equations as well as flexible functional form demand systems are presented.

The different magnitudes of benefit estimates generated for the same valuation problem have been a source of important criticism of market related valuation studies. This study compares different functional forms consistent

with the restrictions required for the aggregation of consumers and commodities.

The results for steelhead rivers in Oregon indicate that the definitional differences in welfare measure (ordinary versus compensated measures) are quite small, which is consistent with the small budget shares that these activities account for a representative household. The results also suggest that the welfare measures may be very sensitive to the formulation of the model. However, the almost ideal demand system (AIDS) and translog indirect utility function demand system (TLOG) produce very close welfare estimates when catch rate, trips and a composite good are considered the commodities yielding utility.

The treatment of endogenous quality variables has been a widely discussed issue in the recreation economics and labor economics literatures. In this study, unlike that of Bockstael and McConnell (1981), the demand system is specified as a function of exogenous parameters rather than endogenous implicit prices, avoiding the simultaneity of the demand system and the identification problem. Our empirical results support the comparative static predictions on the price and income elasticities for catch rate per trip and number of trips.

A limitation of this study is the restrictive form assumed for the production technology. However, following Strong (1984), our model can be extended to take into

account substitution among inputs, non constant returns to scale, and exogenous quality variables.

Future research should be directed to study the properties of "exact" welfare measures associated with different flexible functional forms. For large economic welfare changes, a global approximation to the underlying utility function as provided by Fourier series might be preferable. The comparisons among different model specifications should be done using rigorous statistical methods including the construction of confidence intervals for the welfare estimates.

EXACT WELFARE MEASUREMENT: THEORY AND APPLICATION
TO RECREATION ECONOMICS

by

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EXACT WELFARE MEASUREMENT: THEORY AND APPLICATION TO RECREATION ECONOMICS

CHAPTER I

INTRODUCTION

Welfare economics has a long history of controversy, beginning with the loosely defined concepts of rent and consumer surplus in the last century by Ricardo (1829) and Dupuit (1844) respectively. After the contribution of Marshall (1930) in the beginning of the current century, consumer and producer surplus developed as the basic tools actually used in most empirical economic welfare studies. These concepts have, however, remained controversial from a theoretical perspective.

With the work of Samuelson (1942) part of the controversy was focused on the limitations, on and welfare interpretation, of the geometric area definition of consumer surplus given by Dupuit. Samuelson demonstrated that consumer surplus unlike producer surplus does not provide a unique welfare measure except under extremely restrictive assumptions on consumer preferences. Also, Samuelson established that, the conditions under which consumer surplus is a meaningful money measure of utility, as was proposed by Marshall, are at least as restrictive as the

conditions required to have a unique welfare measure of an economic change.

Fortunately, the alternative measures of consumer welfare proposed by Hicks, compensating variation and equivalent variation, have been accepted widely and form the foundation of recent theoretical developments. Since the Hicksian concepts can be given a "willingness to pay" interpretation rather than the utility concept proposed by Marshall, and are based on compensated demand curves, they are not subject to the Samuelson criticism of consumer surplus.

The controversy still remained, however, turning only to new grounds. The "willingness to pay" measures were considered by most to be unobservable in market data. Applied economists continued pursuing empirical studies based on consumer surplus, which theoreticians continued to attack.

This state of disruption of welfare economics lasted until Willig's work appeared in the mid-1970's. Willig (1976) has claimed that consumer surplus is a "reasonable" approximate of the Hicksian measures in many practical cases. The conditions given by Willig (1979) under which consumer surplus is a good approximation of the Willingness to pay measures can be summarized as: a) the price variation under study must have a relatively small impact on welfare; b) all price variations must be in the same

direction c) the income effects associated with the commodities under study must be small or d) preferences must be homothetic.

In the last few years the discussion has focused on whether these conditions are satisfied in circumstances of empirical relevance. McKenzie (1979) argues that the non-uniqueness problem of consumer surplus remains as a considerable difficulty in many cases. Some investigators have attempted to avoid the problem of non-uniqueness by restricting the matrix of uncompensated price effects rather than the Slutsky matrix, to be symmetric. For example, Cicchetti et al. (1975) imposed symmetry on the Marshallian system to calculate a unique measure of consumer surplus in a demand recreation study, where multiple-sites were considered. This procedure of course cannot be given theoretical justification. Hausman (1981) emphasized that there also is a shortcoming of consumer surplus in measuring the welfare effects of a project or policy which has a substantive impact on consumers, such as taxation of labor income. The most important criticism however to the Willig approximation argument given by Hausman emerges in measuring dead weight losses. In the latter case consumer surplus can be a poor approximation, even if the Willig's conditions hold.

Most recent research has been directed toward the direct estimation of the theoretically accepted welfare

measures. The results of integrability theory, known for a long time by economic theorists, have been used as the foundation to estimate compensating variation and equivalent variation directly from Marshallian demand equations. Hausman (1981) provided analytical solutions for some simple demand equations when a single price changes. Vartia (1983) goes further giving an algorithm for any demand system when multiple price changes are considered. A more standard approach, which has been long used, is the direct calculation of welfare effects based on the specification of an utility function. In this context the use of duality theory has become popular in attaining greater flexibility to the demand system. Most of the effort however has been oriented toward testing the properties derived from consumer theory rather than the evaluation of consumer's welfare changes. One of the main objectives of this study is to present these more recent developments under a coherent framework, emphasizing their potential applications to estimate welfare measures of consumers and in particular consumers of non-market goods.

As one would expect, non-market valuation is a difficult field and is without well established accepted principles. The difficulty of non-existence of direct observations to provide welfare measures has been approached by considering models of observable behavior in markets related to environmental goods. This technique is based on

the argument that if the benefit measures are to be taken seriously, they must be calculated from a theory capable of being tested against observations of consumer's behavior. This market related valuation approach, unlike that of contingent valuation techniques, prefers to estimate benefits from real actions of consumers in markets rather than from hypothetical answers regarding the expected benefits of a project or policy. Since the indirect approach appears to be preferable, whenever possible, it will be adopted in this study.

The most popular of these demand models, applied mainly to recreational valuation, has been the travel cost model. Hotelling (1949) suggested that demand curves could be derived by observing the rate of participation of population groups located in various distance zones from a given recreational site, as a function of the average travel cost required to transport the participants from their zone of residence to the site. Thus, travel costs were proposed as a proxy for market price, with recreation participation at a site falling as distance from the site and travel cost rose. Clawson (1959) extended the travel cost idea into an operational model by estimating demand for a recreational site and measuring the total value or benefits of the site.

In the 1970's more general models of individual behavior, such as the household production function, have provided a more direct linkage of travel models to welfare

economics. One of the most important extensions of the traditional travel cost model has been the incorporation of quality. Perhaps, the first study to consider quality endogenously was Brown, Singh and Castle (1964). This study examined the effect of fishing success on the demand for fishing days in a simultaneous equations framework. They found that fish catch per capita was influenced significantly by the number of trips per capita. McConnell (1981) in the context of household production model shows that the endogeneity of catch rate appears naturally, since anglers can influence their catch rates by employing various combinations of fishing inputs. Also, he pointed out that considering two utility-yielding commodities, catch rate per trip and annual fishing trips, the respective implicit price depended on the quantity of the other commodity. Thus, as was indicated by Brown et al., a simultaneity problem emerges. This is exactly the same conceptual problem discussed in the labor economics literature with respect to quantity and quality of children, which was first analyzed formally by Becker and Lewis (1973). Also, Bockstael and McConnell (1983) show that in this case the Marshallian demand equations expressed as functions of implicit prices are not uniquely determined. This study provides some insights into this issue, by specifying a demand system to explain catch rate per trip and number of trips.

Another difficulty associated with market related valuation is that the same valuation problem can generate radically different benefit estimates. Ziemer et al. (1980) found that consumers surplus is quite sensitive to the choice of functional forms. This study compares benefit estimates obtained from different functional forms which are theoretical consistent with the restrictions required for aggregating consumers over zones.

Objectives of Thesis

The major objectives of this study can be summarized as threefold:

- 1) To present an analytical framework based on recent advances in applied welfare economics and consumer theory, for estimating exact welfare measures.
- 2) To develop a recreational demand model considering catch rate and trips as commodities yielding utility.
- 3) To obtain and compare the values of fishing experiences, using different specifications for the demand system, at 11 steelhead fishing rivers in Oregon.

Organization of Thesis

The first and major objective is pursued in chapters 2 and 3. Chapter 2 discusses the basic principles of welfare economics and consumer theory. Chapter 3 deals with the

estimation of exact welfare measures from market observations. Chapter 4 provides the demand model and its empirical implementation. Chapter 5 presents the estimates of the parameters, along with comparisons of the price and income elasticities for the different functional forms. This chapter also calculates the welfare estimates from participating in fishery activities. Chapter 6 summarizes the main conclusions and provides some implications for future research.

CHAPTER II
WELFARE MEASUREMENT AND DUALITY THEORY FOR
AN INDIVIDUAL HOUSEHOLD

This chapter reviews the basic principles and concepts of welfare economics. The main purpose is the description of appropriate welfare measures of economic change.

The last part of this discussion shows how the Hicksian welfare measures can be defined formally using duality theory when both a change in prices and quantity of a rationed commodity are considered.

Basic Concepts in Welfare Economics

Welfare economics can be differentiated from positive economics in that it is normative, since it is concerned with what "ought" to be rather than what "is". "As with positive economics, the propositions of welfare economics are also logical deductions from a set of definitions and assumptions which, may or may not themselves be realistic. Unlike positive economics, however, a difficulty with welfare economics is that economic 'welfare' is not an observable variable such as the number of machines, market prices, or profits. The economic welfare status of an individual is formally given by his or her utility level

which is unobservable", thus welfare propositions cannot be tested (Just, Hueth and Schmitz, p. 3).

Since one cannot measure the increase in utility obtained from consumption increases, welfare gains and losses are usually expressed in monetary terms. Namely, the amount of money the individual is willing to pay or accept for different states of the world. We can distinguish on the basis of the techniques used to measure the consumer's welfare effects of an economics change between the "old" and the "new" welfare economics.

The old welfare economics holds that the triangle like area called consumer surplus to the left of the ordinary demand curve and above price is a serviceable money measure of utility to consumers. The old-fashioned geometry can thus be used to evaluate economic welfare changes. The price on the demand curve is interpreted as the maximum price the consumer is willing to pay for the last unit consumed. Therefore, in order to evaluate willingness to pay for his current consumption of the commodity at a given price, the area under the demand curve and above the price which the consumer actually pays is determined. The consumer surplus for a good is the difference between the maximum a consumer would be willing to pay for his current consumption of the good and the amount he actually pays.

In spite of the fact that consumer surplus is a widely used tool in applied welfare economics, it is a

controversial concept. Samuelson (1942) concluded that consumer surplus is generally not a unique money measure of utility. It requires constancy of the marginal utility of income, with respect to the prices and/or income that change. The utility function must be homothetic with respect to the goods for which the price changes in order to guarantee path independence. The conditions required to have a meaningful money measure of utility change are still more restrictive.

Yet another problem that arises is with interpersonal comparisons of utility. Assume that it is feasible to attain a money measure of utility change for two persons. If utilities are to be comparable between the two individuals, the money measures of utility is not a common unit of measurement of intensity of preferences that can be applied to both consumers, unless their marginal utilities of income were identical.

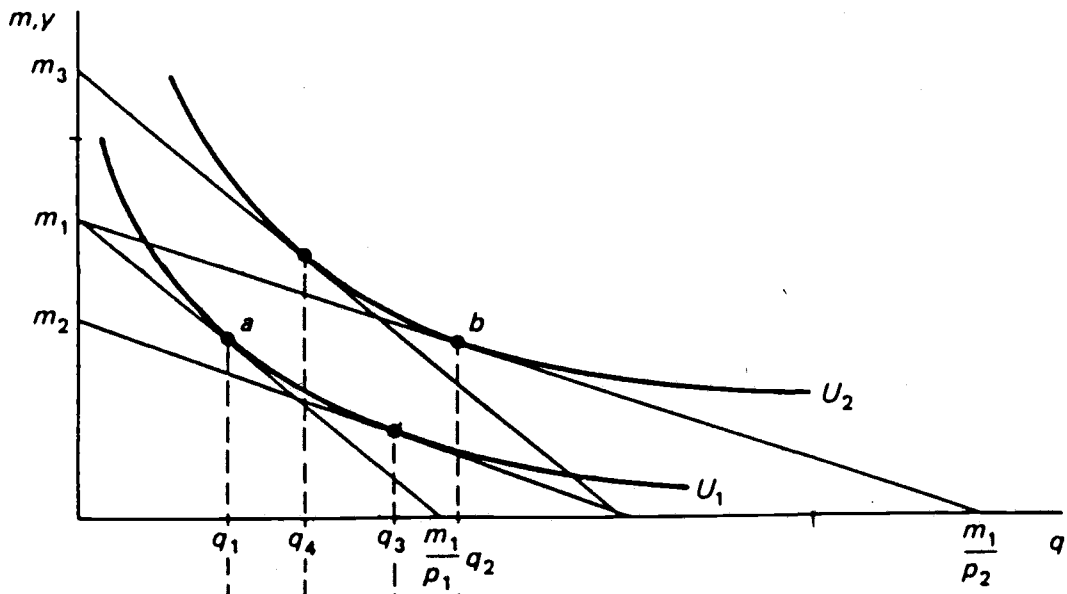
Since it is seemingly impossible to develop a unique and generally applicable money measure of utility change, the new welfare economics focuses on measures of consumer change that have simple but plausible willingness-to-pay interpretation. The most widely accepted welfare measures, compensating variation and equivalent variation, were first proposed by Hicks (1943).¹ "Compensating variation is the amount of income which must be taken away from a consumer (possibly negative) after a price and/or income change to

restore the consumer's original welfare level. Equivalent variation is the amount of income that must be given to a consumer (again possibly negative) in lieu of price and income changes to leave the consumer as well off as with the change" (Just, Hueth and Schmitz, p. 85). Note that unlike the definition given by Willig (1976) compensating variation and equivalent variation as defined here have the same sign on the direction of the change in welfare in the instance of a welfare gain both would be positive.

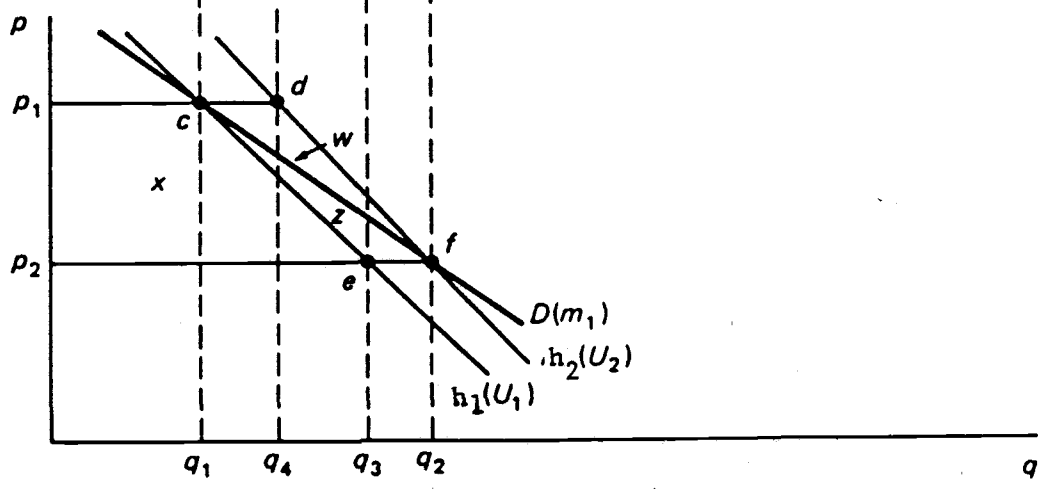
To analyze the relationship of compensating variation, equivalent variation and consumer surplus, consider a consumer with preferences over the good q with price p and the good y (which can be regarded as a Hicksian composite good) with a price of 1.

The consumer is faced with the budget constraint $p \cdot q + y = m$, where m is income and the good y serves as a numeraire. Consider a change in the price from p^0 to p^1 in Figure 1. With a lower price the consumer increases the consumption from q^1 to q^2 and decreases the consumption of y from y^1 to y^2 . To find the compensating variation associated with the price fall we shift the budget line downward until it is just tangent to the original welfare level. Thus, the compensating variation welfare measure is the distance $m_1 - m_2$ in Figure 1(a). It can be shown, considering the price-quantity space in Figure 1(b),

Figure 1



(a)



(b)

that the compensating variation associated with a price fall from p^1 to p^2 is also equal to the change in area under the Hicksian demand curve given the initial level of utility (area X). Similarly, equivalent variation can be found as $m_3 - m_1$ in quantity-quantity space, and the change in area under the Hicksian demand curve given the subsequent level of utility in the quantity-price space (area X + Z + W). The change in consumer surplus by definition, is equal to the change in area under the Marshallian demand curve (X + Z). Thus, for a normal good and a single price reduction, consumer surplus is greater than compensating variation and less than equivalent variation. Also, it can be shown that compensating variation is bounded by the initial level of income m_1 , but that equivalent variation is unbounded. These conditions are reversed for a price increase.

If preferences are such that the income effect is zero for good q (that is, the marginal rate of substitution between q and y is constant on a parallel line to the vertical axis) compensating variation, equivalent variation and consumer surplus are all equal. For multiple price changes, a sufficient condition for an equivalence of compensating and equivalent variation are zero income effects for all of the commodities which experience price changes. Also, it can be shown that the compensating variation of move from state 0 to state 1 is equal to the negative of equivalent variation of moving from 1 to 0. The

move from 0 to 1 can be a reduction or an increase in prices.

It may appear at this point that the willingness to pay measures are almost as limiting as the old approach of consumer surplus. Moreover, the equivalence of EV and CV requires no less restrictive conditions on preferences than the sufficient conditions to obtain a unique money measure of utility change. However in the application of these concepts only one of these measures (EV or CV) is usually appropriate.

Most practical problems involve gainers and losers and thus inter-personal comparisons are necessary. How can we add individual welfare economic changes? Pareto (1896) argued that the only objective basis under which one can say that society is better off is when no one is made worse off. The compensation criteria proposed by Kaldor and Hicks (1939) tries to deal with a wider class of cases. A Kaldor-Hicks improvement exists if a change from state 0 to state 1 would enable the gainers to compensate the losers while continuing to gain themselves. Since the compensation need only be hypothetical, a Kaldor-Hicks improvement is only a potential Pareto improvement.

The willingness-to-pay measures can be used to evaluate a proposed policy action under the Kaldor-Hicks criterion. For example, if the individuals have the explicit right to the initial situation, and if the Kaldor-Hicks criterion

were to be used, this would be equivalent to the criterion that: $\sum CV > 0$. The relevant welfare economic measure of the change is the algebraic sum of compensating variation through the individuals involved in the policy. If individuals have an explicit right to the subsequent situation then $\sum EV > 0$ is the appropriate criterion.

For example, consider a mitigation and enhancement project for salmon and steelhead fish runs.² The Northwest Power Planning Council has established the 1953, pre-McNary dam run level as the reference point for mitigation and enhancement projects which are mandated by the conservation act of 1980. Thus, the beneficiaries (commercial fisherman, recreationists, and American Indians) at present do not have rights to increases in runs beyond the 1953 level.

For enhancement projects which increase fishery runs beyond the pre-McNary level, the relevant question is what they are willing to pay for the increase in fish production, since they do not have rights to increases in runs beyond those levels. The question associated with the economic evaluation of the effects on fish runs of a new hydro-electric project will be, what minimum amount of money are fisherman willing to accept as compensation for reduction in runs. Thus, in the former case the appropriate measure is compensating variation; while in the latter case the appropriate measure would be equivalent variation. Since the current runs are less than pre-McNary run levels the

fishermen are not willing to pay anything for increases in runs up to the levels established by the Northwest Council (these levels have been mandated). Instead, the evaluation of increases in runs up to those levels must be viewed as compensation for past losses, that is, the amount fishermen would have been willing to accept to tolerate such losses. It makes no difference at present whether a project to increase the size of the run is called enhancement or mitigation. The appropriate measure of benefits to fishermen is willingness to accept compensation in lieu of the increase in runs, that is, equivalent variation. Note that this measure is equal to the negative of compensating variation for reduction in runs due to construction of a new hydroelectric plant.

Robert Willig (1976) contends that consumer's surplus is often a good approximation to the true measure, compensating variation or equivalent variation. He derives bounds for the percentage difference between the correct measure of either compensating variation, or equivalent variation and the consumer surplus. His bounds, as was first pointed out by Hotelling (1938) depend on the proportion of the consumer's income spent on the commodity of interest and on the income elasticity. He concludes that in most of the cases the approximation error will be less than the errors involved in estimating the demand curve.

However, Hausman (1981) shows that problems arise in measuring welfare changes associated with taxation of labor income and dead weight losses. He derived exact welfare measures for a single price change, which is also the situation in which consumer's surplus is often used in applied research. Thus, since the exact welfare measures (compensating variation and equivalent variation) can be easily obtained, there exists no reason for use of such an approximated method as consumer surplus.

Exact Welfare Measures for Non-Restricted Commodities

The basic tools used to obtain formal definitions of exact welfare measures emerge from the dual approach to consumer behavior.

Consider a utility maximizing consumer with an ordinal utility function assumed to be strictly increasing quasi-concave and twice differentiable defined over an n goods vector $q = (q_1 \dots q_n)$. The standard utility maximization problem considers the maximization of a well-behaved ordinal utility function subject to a budget constraint:

$$\text{Max}_q u(q) : pq = m, \quad q \geq 0 \quad 2-1$$

where $p = (p_1 \dots p_n)$ and m denote positive exogenous prices and expenditure respectively. Ignoring corner

solutions, the solution to this problem is a system of Marshallian demand functions:

$$q_i = q_i(p, m), \quad i = 1 \dots n \quad 2-2$$

The properties of the Marshallian demand system, given that it arises from utility maximization are:

Homogeneity. The ordinary demand functions q_i are homogeneous of degree zero in prices and income. That is for scalar $\lambda > 0$, $q_i(\lambda p, \lambda m) = q_i(p, m) = q_i(p/m, 1)$.

Adding-up. The total value of demands are equal to the total expenditure.

$$\sum_{i=1}^n p_i q_i(p, m) = m$$

Symmetry. The Slutsky matrix or the matrix of substitution terms is symmetric, that is, for all $i \neq j$.

$$\partial q_i / \partial p_j + q_j (\partial q_i / \partial m) = \partial q_j / \partial p_i + q_i (\partial q_j / \partial m)$$

Negativity. The Slutsky substitution matrix must be negative semidefinite. Which insures that the Hicksian demand curves slope downward, that is

$$\partial q_i / \partial p_i + q_i (\partial q_i / \partial m) \leq 0 \text{ for all } i$$

where $q_i = q_i(p, m)$ are the Marshallian demand functions.

The indirect utility function $v(p,m)$ is defined as the maximum utility achievable at given prices and income. That is:

$$v(p,m) = \max_q [u(q) : pq = m] \quad 2-3$$

The indirect utility function can be shown to be non-increasing in prices and non-decreasing in income, quasi-convex in prices and homogeneous of degree 0 in prices and income.

The dual approach to the utility maximization problem is to consider the associated minimization problem which defines the expenditure (cost) function

$$c(p,u) = \min_q [pq : u(q) = u]$$

The expenditure function is non-decreasing in prices and increasing in utility. It is concave in prices and homogeneous of degree 1 in prices.

The expenditure function and the indirect utility function are intimately related. Since the indirect utility function is monotonically increasing in income while the expenditure function is monotonically increasing in utility, either function can be inverted to derive the other corresponding function.

There exists two extremely useful properties of these functions. First, the partial derivatives of the

expenditure function with respect to prices provide the Hicksian demand function.

$$\partial c(u,p)/\partial p_i = h_i(u,p) \quad 2-5$$

Differentiating the compensated demand curve (Hicksian) h_i with respect to p_j the symmetry and negativity properties can be established directly. The substitution matrix is symmetric by Young's theorem, that is

$$\begin{aligned} \partial h_i(u,p)/\partial p_j &= \partial^2 c(u,p)/\partial p_i \partial p_j \\ &= \partial^2 c(u,p)/\partial p_j \partial p_i = \partial h_j/\partial p_i(u,p) \end{aligned} \quad 2-6$$

Since the cost function is concave in prices it immediately follows that the Hicksian demand curves are downward sloping. That is

$$\partial h_i(u,p)/\partial p_i = \partial^2 c/\partial p_i^2(u,p) \leq 0 \quad 2-7$$

Also, the Marshallian demand equations can be derived from the indirect utility functions as follows:

$$q_i(p,m) = \frac{-\partial v(p,m)}{\partial p_i} / \frac{\partial v(p,m)}{\partial m} \quad 2-8$$

This, known as Roy's identity, allows us to find the ordinary demand system if we know the indirect utility function.

We can now formally define the exact welfare measures (compensating variation and equivalent variation) using the indirect utility function and the expenditure function. Consider a change in prices from p^0 to p^1 and where adjustments in consumption are possible, which leads to a change in utility from u^0 to u^1 . The implicit definition of compensating variation is given by

$$v(p^1, m^0 - CV) = v(p^0, m^0) = u(q^0) \quad 2-9$$

The implicit definition of equivalent variation is:

$$v(p^1, m^0) = v(p^0, m^0 + EV) \quad 2-10$$

Also, compensating variation is directly defined using the expenditure function as:

$$CV(p^0, p^1, m^0) = c(p^0, u^0) - c(p^1, u^0) \quad 2-11$$

Equivalent variation likewise is defined directly as:

$$EV(p^0, p^1, m^0) = c(p^0, u^1) - c(p^1, u^1) \quad 2-12$$

Compensating and equivalent variation are unique welfare measures for any set of price changes.

$$EV(p^0, p^1, m^0) = c(p^0, u^1) - c(p^1, u^1)$$

$$= - \int_L \sum_{i=1}^n [\partial c(p, u^1) / \partial p_i] dp_i \quad 2-13$$

$$= - \int_L \sum_{i=1}^n h_i(p, u^1) dp_i$$

where L denotes any path of integration from p^0 to p^1 . We can choose the path of integration arbitrarily since what we began with was the exact differential dc . In addition, as mentioned above, it can be proven that equivalent variation and compensating variation are equal if the income effects for all commodities for which prices change are zero. (The properties are established in Just, Hueth, and Schmitz, pp. 370-373).

Exact Welfare Measures for Restricted Commodities

The discussion so far treats all purchases as subject to consumer choice. However, many cases exist where the quantities of some goods are exogeneously fixed. This is especially true when we are dealing with public goods, such as environmental quality.

Following Deaton and Muellbauer (1980) we can define the restricted cost function, c^* , as the minimum income to achieve the level of utility u at given prices p when quantity x of a good must be bought. For example, x_1 can be the supply of an environmental service. Denote \bar{p} and \bar{q} the

price and quantity vectors excluding the rationed good (q_1).
Formally:

$$c^*(u, p, x_1) = \min_q [pq : v(q_1, q) = u, q_1 = x_1]$$

$$= p_1 x_1 + \min_q [\overline{pq} : v(q_1, q) = u] \quad 2-14$$

The restricted expenditure function has to be greater or equal to the unrestricted expenditure function, since the additional constraint cannot make the consumer better off and may make him worse off. Maler (1974) showed that the properties of the cost function hold in the case with rationing and that the restricted expenditure function is convex with respect to x_1 . We can derive the restricted Hicksian demand functions by differentiation of the restricted cost function with respect to p_i

$$\partial c^*(u, p, x_1) / \partial p_i = h_i(u, p, x_1) \quad 2-15$$

By considering the relationship between the restricted and unrestricted cost functions, it can be shown that

$$\partial c^*(u, p, x_1) / \partial x_1 = p_1 - p_1^*(u, p, x_1) \quad 2-16$$

Where p_1^* is the shadow price, or the money valuation (reduction in minimum cost) of a unit of x_1 provided at zero price. Maler has also shown that the shadow price of a

restricted good with price equal to zero can be expressed as the price of any private good times the marginal rate of substitution between that good and x_1 , that is

$$\partial c^*(u, p, x_1) / \partial x_1 = -p_i \left(\frac{\partial u}{\partial x_1} / \frac{\partial u}{\partial q_1} \right) \quad 2-17$$

We can now define the compensating variation associated with a change in quantity of the restricted good and the vector of prices from (p^0, x_1^0) to (p^1, x_1^1) as:

$$\begin{aligned} CV(x_1^0, x_1^1, p^0, p^1, m^0) \\ = c^*(u^0, p^0, x_1^0) - c^*(u^0, p^1, x_1^1) \end{aligned} \quad 2-18$$

Similarly, equivalent variation can be defined

$$\begin{aligned} EV(x_1^0, x_1^1, p^0, p^1, m^0) \\ = c^*(u^1, p^0, x_1^0) - c^*(u^1, p^1, x_1^1) \end{aligned} \quad 2-19$$

The geometric interpretation of change in a quantity rationed is the change in area below the Hicksian demand curve above price, and left of quantity. The change in price of a restricted quantity is given simply by $q_1(p^1 - p^0)$, since adjustments are not possible. (See Just, Hueth and Schmitz, pp. 399-401).

CHAPTER III

ESTIMATION OF EXACT WELFARE MEASURES USING MARKET DATA

This chapter describes the problems encountered in obtaining welfare measures from market data. A major purpose is to derive welfare estimates of price changes for different functional forms of demand systems. The discussion begins with a problem originating in empirical estimations that is, under what conditions can estimation of a subset of Marshallian demands be used to estimate welfare measures. Moreover, since market data are involved, the restrictions on the structure of preferences to get aggregate demand curves consistent with the individual utility maximization problem must be considered. A specific kind of preferences which allows aggregation over consumer is presented by a cost function called PIGL (Deaton and Muellbauer, 1980). Different specifications of this cost function are used to derive some well known demand systems such as: the linear expenditure system (LES), the almost ideal demand system (AIDS) and the translog indirect utility function system (TLOG). Also, two methods to incorporate demographic characteristics into a demand are described along with some implications for welfare economics.

At the end of the chapter the integrability problem is considered, that is, the derivation of utility functions from a well specified demand system. In this context the Vartia iterative method which provides an approximation to true welfare measures is described. Also, this method is compared to the result of a single linear demand equation by solving analytically an ordinary differential equation. Finally the estimation of welfare measures from market data when a rationed commodity exists is considered.

Aggregation over Commodities and the Estimation of a Complete Demand System

There are two basic approaches to estimate exact welfare measures. The first involves deriving demand specifications in terms of parameters of a known underlying direct or indirect utility specification. The parameter estimates can then be used to evaluate welfare effects based on the functional form of the utility function. The second approach involves the specification of a demand system based on the theoretical properties and on statistical considerations (good fit of the data). To obtain estimates of the willingness-to-pay measures, the second approach involves the solution of a system of partial differential equations. However, Hausman pointed out (1981) that, for one price

change, an ordinary differential equation can be used to solve for the expenditure function.

Both methods involve the estimation of a complete demand system. Although it seems an unfeasible requirement in many empirical studies, there are some standard approaches to this problem. In a cross-sectional study it would be reasonable to assume fixed prices of a sub-set of commodities which would allow the use of Hicksian aggregation.⁴ Then, the Hicksian composite good can be interpreted as a numeraire which is used to normalize income and commodity prices. From the consumer maximization problem:

$$\text{Max } [u(q); \bar{p}\bar{q}_n + q_{n+1} = m] \quad \bar{q}_n, q_{n+1} > 0 \quad 3-1$$

where \bar{q}_n is a vector of n commodities, \bar{p} is the vector of corresponding prices and $q = (\bar{q}_n, q_{n+1})$ is the vector of $n+1$ commodities. Prices and income have been normalized by the price p_{n+1} . The estimated system is a complete set of demand equations if $\bar{q}_n(\bar{p}, m)$ is observed. Note that the homogeneity condition is automatically satisfied and adding-up requires $\bar{p}\bar{q}_n(p, m) \leq m$ (See Epstein, 1982).

The other assumption almost always made explicitly or implicitly is weak separability. A demand study which considers a one period world, needs to be justified on weakly intertemporal separable preferences. The omission of such restricted goods as environmental quality and goods

provided by the government without use charges (public and nonpublic goods) in a demand system, also has to be rationalized by assuming separability of preferences.

Weakly separable preferences are represented by a direct utility function of the form

$$u = u(u(q_1), u_2(q_2) \dots u_g(q_g) \dots u_n(q_n))$$

3-2

for subvectors of commodities $q_1 \dots q_g \dots q_n$. A group of goods is said to be weakly separable from all other goods, if $\frac{\partial u_i / \partial u_j}{\partial q_k} = 0$ where i and j are commodities of the same group and k refers to commodities of other groups. Weak separability allows the analysis of the consumer maximization problem in two stages.⁵ In the first stage expenditure is allocated to broad groups of goods, while at the second stage, group expenditures are allocated to the individual commodities. The second stage involves maximization of a subutility function $u_g(q_g)$ subject to the total expenditure on group g (i.e. $P_g q_g$). Group indirect utility functions and cost functions can be defined in the usual way and these functions keep the properties described in the latter chapter.

Indirect Utility Function Specifications

The choice of a functional form for the underlying direct or indirect utility function is not an easy matter. It is aggravated because classical statistic procedures cannot be employed to choose a system since the functional forms are generally non-nested, i.e., one is not a special or limiting case of the other.

One further limitation on the class of functional forms is to require that any candidate satisfy the aggregation conditions. Exact aggregation allows generation of market demand functions consistent with individual utility maximizing behavior. One specific set of expenditure functions, which by the theorems of Muellbauer (1975, 1976) permit exact aggregation are known as the PIGL class.⁶ The PIGL individual expenditure function is given by:

$$c^h(u^h, p) = k^h [a(p)^\alpha (1-u^h) + b(p) u^h]^{1/\alpha} \quad 3-3$$

Where $a(p)$ and $b(p)$ are linear homogeneous concave functions, h denotes the household, and α and k are scalars.

When α is equal to one, the cost function takes the linear form:

$$\begin{aligned} c^h(u^h, p) &= a(p) k^h + [b(p) - a(p)] u^h k^h \\ &= a(p) k^h + b^*(p) u^h k^h. \end{aligned} \quad 3-4$$

Equation 3-4, is a special case of the expenditure function for quasi-homothetic preferences known as the Gorman polar form which allows exact linear aggregation. The term $a(p)$ represents the cost of living when u is zero, and may thus be interpreted as a subsistence expenditure.

Inverting 3-4 to obtain the indirect utility function:

$$v^h(p, m) = \frac{m - k^h a(p)}{k^h b^*(p)} \quad 3-5$$

The individual Marshallian demand system can be derived using Shephard lemma on 3-4 and substituting the indirect utility function into the Hicksian systems (equation 3-5) to obtain:

$$q_i^h = a_i(p) k^h + b_i^*(p)/b^*(p) [m^h - a(p) k^h] \quad 3-6$$

where $a_i(p)$ and $b_i^*(p)$ are the i th partial derivatives of $a(p)$ and $b^*(p)$, respectively. We observe from 3-6 that the Engel Curves are straight lines but need not go through the origin. This demand function specification is quite restrictive since although the income elasticities are not equal to 1 (the homothetic case) they approach unity as total income increases, which is unlikely to be true for narrowly defined goods. If it is assumed that all consumers face the same prices, the aggregate demand curves are given by

$$\begin{aligned}
 q_i &= 1/H \sum_h q_i^h \\
 &= a_i^*(p) + b_i^*(p)/b^*(p) [m - a^*(p)]
 \end{aligned}
 \tag{3-7}$$

where H denotes the total number of households, m the average expenditures and $a^*(p)$ is equal to the average of $a(p)$ k^h . Note that 3-7 does not depend on the distribution of expenditures. Thus, the marginal propensity for spending must be identical for all consumers, that is, a reallocation of income from any one to any other individual leaves market demands unchanged. This condition is satisfied since $b_i(p)/b^*(p)$ in 3-6, is only a function of prices. Some variation in preferences is allowed however, since k^h is indexed on h .

If α approaches zero in 3-3 and inserting for simplicity $k^h = 1$ we get the cost function form, known as PIGLOG.

$$\log c(u^h, p) = (1-u^h) \log a(p) + u^h \log b(p)
 \tag{3-8}$$

Using Shephard lemma and multiplying both sides by $p_i/c(u^h, p)$ we find the expression for the budget share of good i . That is:

$$\begin{aligned}
 w_i^h &= \partial \log c(u^h, p) / \partial \log p_i \\
 &= p_i q_i / c(u^h, p)
 \end{aligned}$$

The logarithmic differentiation of 3-8, and the substitution of the indirect utility function gives the budget share as a function of prices and expenditures.

$$w_i^h = a_i(p) + b_i^*(p)[\log m^h - \log a(p)]/\log b^*(p) \quad 3-9$$

Where $b^*(p)$ is equal to $b(p)/a(p)$ and i indicates the partial derivative with respect to prices. For notational simplicity, 3-9 can be written as

$$w_i^h = \alpha_i(p) + \beta_i(p) \log(m^h) \quad 3-10$$

Where $\alpha_i(p)$ and $\beta_i(p)$ are functions only of prices.

The average aggregate budget for the i th good is given by:

$$\begin{aligned} w_i &= p_i \sum_h q_i / \sum_h m^h \\ &= \sum_h (m^h/M) w_i^h \end{aligned} \quad 3-11$$

Where M is the expenditure of all households $\sum_h^m m^h$.

Substituting 3-10 in 3-11, we get the aggregate demand function in share form.

$$w_i = \alpha_i(p) + \beta_i(p) \sum_h (m^h/M) \log m^h \quad 3-12$$

If we define implicitly the aggregate index K by $\log(m/K) = \sum_h (m_h/M) \log m_h$, where m is the average level of total expenditure, 3-12 becomes

$$w_i = \alpha_i(p) + \beta_i(p) \log(m/K) \quad 3-13$$

Where m/K is called representative expenditure by Deaton and Muellbauer (1980). Unlike the linear case, representative expenditures are used rather than average expenditures.

Assuming that each household has the same preferences ($k^h = 1$), K can be interpreted as an index of the distribution of household budgets. It can be shown that $K = Z/H$, where $Z = -\sum_h m_h/M \log(m_h/M)$. Z represents a measure of income equality as given by Theil (1972). Z decreases as inequality increases, thus $K = Z/H$ decreases and the representative budget level increases.⁷

We can now give special functional forms to the PIGLOG expenditure function class. This allows us to estimate exact welfare measures for a representative consumer.

Linear Expenditure System (LES)

The representative expenditures function associated with the individual cost function 3-4, when exact linear aggregation exists is given by:

$$c(u,p) = a^*(p) + b^*(p) u$$

3-14

This expression is the cost function corresponding to the linear expenditure system when

$$a^*(p) = \sum_k p_k \gamma_k \quad \text{and} \quad b^*(p) = \beta_0 \prod_k p_k^{\beta_k}$$

where γ_k and β_k are parameters, $\sum_k \beta_k = 1$, and where

$\beta_0 = \prod_k \beta_k^{\beta_k}$ is a non-estimable cost parameter.

Differentiating partially $a^*(p)$ and $b^*(p)$ with respect to p and substituting in 3-7, we have the linear expenditure system.

$$q_i = \gamma_i + \beta_k / p_i [m - \sum_k p_k \gamma_k] \quad 3-15$$

It can be checked that the restrictions of adding up, homogeneity and symmetry are satisfied. The negativity condition is also satisfied if $0 < \beta_i < 1$ and $q_i > \gamma_i$.

To evaluate compensating variation and equivalent variation we can use the direct definition in 2-10 and 2-11. The compensating variation of a change in the price vector from p^0 to p^1 for the LES is:

$$\begin{aligned}
 CV(p^0, p^1, m^0) \\
 = m^0 - p^1 - \prod_k (p_k^1/p_k^0)^{\beta_k} (m^0 - p^0)
 \end{aligned}
 \tag{3-16}$$

Similarly equivalent variation is given by:

$$\begin{aligned}
 EV(p^0, p^1, m^0) \\
 = p^0 - m^0 + \prod_k (p_k^0/p_k^1)^{\beta_k} (m^0 - p^0)
 \end{aligned}
 \tag{3-17}$$

where $P = p_k \gamma_k$

Almost Ideal Demand System (AIDS)

The representative cost function associated with the individual PIGLOG (Equation 3-8) is given by:

$$\log c(u, p) = (1-u) \log a(p) + u \log b(p)
 \tag{3-18}$$

The AIDS cost function developed by Deaton and Muellbauer (1980) is a special case of 3-18, when:

$$\log a(p) = \alpha_a + \alpha_k \log p_k + 1/2 \sum_k \sum_j \gamma_{kj}^* \log p_k \log p_j$$

and:

$$\begin{aligned}
 \log b(p) = \log a(p) + \beta_0 \prod_k p_k^{\beta_k} \quad \text{for} \\
 j, k = 1, \dots, n.
 \end{aligned}$$

Following a similar procedure to that used to derive 3-13, we can obtain the AIDS aggregate demand system:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \log p_i + \beta_i \log (m/kP) \quad 3-19$$

Where P is a price index defined by:

$$\log p = \alpha_0 + \sum_k \alpha_k \log p_k + 1/2 \sum_k \sum_j \gamma_{kj} \log p_k \log p_j$$

and $\gamma_{ij} = 1/2 (\gamma_{ij}^* + \gamma_{ji}^*)$, m is the average level of total expenditure and K is defined as in 3-13.

We need to impose some restrictions on the parameters to make the AIDS consistent with the theory of demand and hence to get meaningful welfare measures. First, for adding up, the budget share sums to 1 if $\sum_i \alpha_i = 1$, $\sum_j \gamma_{ij} = 0$ and $\sum \beta_i = 0$. Second, the homogeneity condition holds if

$$\sum_j \gamma_{ij} = 0 \text{ and third, the symmetry restriction holds if}$$

$$\gamma_{ij} = \gamma_{ji} \quad 8$$

Although the negativity condition cannot be ensured by any restrictions on the parameters alone, it can be checked for any given estimates by calculating the estimated Slutsky matrix.

Given that the demand system is expressed as a function of γ_{ij} instead of the cost function parameters γ_{ij}^* , we need to make the expenditure function dependent on estimable

parameters (γ_{ij}). Imposing the condition $\gamma_{ij} = 1/2 (\gamma_{ij}^* + \gamma_{ji}^*)$ as well as the symmetry conditions, the cost function becomes:

$$\log c(u, p) = \log P + u \beta_0 \prod_k p_k^{\beta_k} \quad 3-20$$

Where the term $\log P$ is the price index defined in 3-19.9

The welfare measures can be evaluated from 3-20. Thus, compensating variation of a price change from p^0 to p^1 is given by

$$\begin{aligned} CV(p^0, p^1, m^0) & \quad 3-21 \\ & = m^0 - \text{Exp} [\log P^1 + \sum_k (p_k^1/p_k^0)^{\beta_k} \log (m^0/P^0)] \end{aligned}$$

and the AIDS equivalent variation is

$$\begin{aligned} EV(p^0, p^1, m^0) & \quad 3-22 \\ & = \text{Exp} [\log P^0 + \sum_k (p_k/p_k)^{\beta_k} \log (m^0/P^1)] - m^0 \end{aligned}$$

Where $\log P_0$ and $\log P_1$ correspond to the price index evaluated at initial and subsequent prices, respectively.

The Translog Indirect Utility Function System (TLOG)

The aggregate cost function PIGLOG in 3-18 can be written as:

$$\log c(u, p) = \log a(p) + u \log b_*(p) \quad 3-23$$

Where $b_*(p) = b(p)/a(p)$

The expenditure function associated with the translog indirect utility function, developed by Jorgenson and Lau (1970), is also of the PIGLOG class when an additional restriction is imposed. The functional forms in this case for $a(p)$ and $b_*(p)$, are (see Appendix A):

$$a(p) = \frac{-(\alpha_0 + \sum \alpha_i \log p_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \log p_i \log p_j)}{-1 + \sum_i \beta_{Mi} \log p_i}$$

and

$$b_*(p) = 1 / (-1 + \sum_i \beta_{Mi} \log p_i)$$

where $\beta_{Mi} = \sum_k \beta_{ki}$

The TLOG Marshallian Demand System can be obtained from 3-13 or starting from the representative cost function (see Appendix A). The solution is given by:

$$w_i = \frac{\alpha_i + \sum_j \beta_{ij} \log p_j - \beta_{Mi} \log (m/K)}{-1 + \sum_i \beta_{Mi} \log p_i} \quad 3-24$$

K was defined as Z/H where Z is an income distribution indicator and H denotes the number of households. If it is

feasible to assume a composite Hicksian commodity the only additional restrictions required to have a demand system consistent with economic theory would be symmetry and negativity. Symmetry can be imposed through the parameters in the following way:

$$\beta_{ij} = \beta_{ji} \quad \text{for } i \neq j$$

The compensating variation welfare measure corresponding to the TLOG system of a price change from p^0 to p^1 is given by:

$$\begin{aligned} & CV(p^0, p^1, m^0) \qquad \qquad \qquad 3-25 \\ & = m^0 - \text{Exp} \left[\frac{(\log p^1/p^0) + (1 - \sum_i \beta_{Mi} \log p_i^0) \log m^0}{1 - \sum_i \beta_{Mi} \log p_i^1} \right] \end{aligned}$$

and the TLOG equivalent variation by

$$\begin{aligned} & EV(p^0, p^1, m^0) \\ & = \text{Exp} \left[\frac{(\log p^1/p^0) + (1 - \sum_i \beta_{Mi} \log p_i^1) \log m^0}{1 - \sum_i \beta_{Mi} \log p_i^0} \right] - m^0 \end{aligned}$$

where $\log P = \alpha_0 + \sum_i \log p_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \log p_i \log p_j$,

which is equal to the AIDS price index but with the TLOG parameters.

Demographic Characteristic Specifications
for the Demand Systems

Demographic variables such as age composition, educational level and number of children are important determinants of household consumption patterns. In general, we can model differences in behavior by making demand depend not only on prices and total expenditures, but, also on some list of household characteristics. Modeling those effects is not only useful because more precise estimates of the price and budget responses can be obtained but, because it allows welfare comparison between households due to an economic change. We may be interested in knowing the economic welfare effect of a policy on a specific group of households.

We are concerned with incorporating household characteristic variables into a complete demand system. We discuss demographic scaling, a procedure first employed by Barten (1964). The Barten model assumes a direct utility function such as:

$$u = u (q_1/S_1, q_2/S_2 \dots q_n/S_n) \quad 3-27$$

where S_i is a function of demographic variables. If we interpret S_i as reflecting the number of "equivalent adults" in the household then we can interpret the household's preferences as depending, not on the number of gallons of

milk it consumes, but on gallons per equivalent adult.

Making $q_{i*} = q_i/S_i$ the consumer maximization problem can be written in vector form as:

$$\text{Max } [u(q_*) : p_*q_* = m] \quad q_{i*} \geq 0 \quad 3-28$$

where $p_* = pS$ and S is a vector of demographic parameters which depend on demographic variables.

The cost function is thus given by:

$$c(u, p_*) = m \quad 3-29$$

and the indirect utility function by:

$$v(p_*, m) = u \quad 3-30$$

It can be easily verified that the modified Marshallian Demand system has the form:

$$q_i = S_i q_i(p_*, m) \quad 3-31$$

and multiplying both sides of this equation by p_{i*}/m , the demand system in budget share form becomes:

$$w_i = w_i(p_*, m) \quad 3-32$$

(See Appendix B for another specification).

Demographic scaling introduces n parameters ($S_1 \dots S_n$) to the original demand system which is equivalent to $q_j = q_j(p_*, m)$ and postulates that only these parameters depend on the demographic variables. The specification of demographic scaling is completed by postulating a functional form relating the scaling parameters to the demographic variables. Assuming linear demographic scaling the S parameters are expressed by:

$$S_j(z) = 1 + \sum_{r=1}^{\infty} \xi_{jr} z_r \quad 3-33$$

where ξ and z_r denote scalars and demographic variables, respectively.

Thus, we can incorporate some household characteristics in each of the complete demand systems considered (LES, AIDS and TLOG). Since, the cost function is now defined as the minimum cost of attaining u at prices p given characteristics S , we can obtain welfare measures due to a price change (quantity) for a group of households with certain characteristics. Compensating variation takes the form:

$$\begin{aligned} CV(p^0, p^1, S^0, m^0) \\ = c(p^0, S^0, u^0) - c(p^1, S^0, u^0) \end{aligned} \quad 3-34$$

For example, we may be interested in knowing the welfare effects of a price increase on retired people. Then

making $S_1 = 1 + \xi \text{RET}$, where RET is a dummy variable with 1 if the household head is retired, 0 elsewhere. Compensating variation can be evaluated for $S_1^0 = 1$ nonretired, and $S_1^1 = 1 + \xi$ retired.

Integration of the Demand System

In many cases the complete ordinary demand system $q(p,m)$ of the utility maximizing consumer is known, or it can be estimated empirically, although the utility function is not. Integrability theory shows that demand functions provide all the information needed to determine the indifference surfaces and thus the utility function. Chipman and Moore (1976) show that if a continuously differentiable complete demand system $q(p,m)$ satisfies the properties consistent with consumer theory, then a twice differentiable, indirect utility function exists, which represents $q(p,m)$.

Hurwicz and Uzawa (1971) show that if a merely differentiable $q(p,m)$ satisfies along with a Lipschitz condition, then an increasing and strictly quasi-concave direct utility function exists that represents the demand system. The Lipschitz condition, the weak condition that all income derivatives $\partial q_i(p,m) / \partial m$ exist and are uniformly bounded by a constant. This integrability result provides the complete set of restrictions that must be imposed on functional forms for demand functions if it is

desired to impose utility maximization in the estimation. Alternatively, if the integrability conditions are tested empirically and found to be consistent with the data, a utility function exists. In that case the demand functions may be used for welfare analysis.

Although it is often not possible to find an exact or analytic solution for the integration problem, Hausman (1981) shows that exact welfare measures for a single price change can be calculated for several functional forms of demand functions frequently used in empirical studies. Vartia (1983) provides a numerical method of solving differential equations to approximately determine the value of compensating and equivalent variation for a change in the price vector when it is not possible to find an exact analytical solution.

Vartia's procedure begins by introducing an auxiliary variable t such that $0 \leq t \leq 1$, letting $p(t)$ be a differentiable curve in the price space connecting $p_0 = p(0)$ to $p_1 = p(1)$ and $m(t)$ is any expenditure starting from $m_0 = m(0)$. Given the indirect utility function $v(t) = v(p(t), m(t))$ constrained to remain on an indifference curve along a path of price.

$$dv(t)/dt = \sum_{i=1}^n \frac{\partial v(p(t), m(t))}{\partial p_i(t)} \frac{dp_i(t)}{dt} + \frac{\partial v(p(t), m(t))}{\partial m(t)} \frac{dm(t)}{dt} = 0$$

using Roy's theorem we obtain¹⁰

$$dv(t)/dt = \lambda(p(t), m(t)) \quad 3-36$$

$$\left[\frac{dm(t)}{dt} - \sum_i q_i(p(t), m(t)) \left(\frac{dp_i(t)}{dt} \right) \right]$$

where λ is the marginal utility of income which is positive, and $q_i(p(t), m(t))$ is the Marshallian demand equation of good i at t . Since to remain on an indifference curve 3-35 must be satisfied, one obtains the first order differential equation in the expenditure function $c(t)$.

$$dc(t)/dt = \sum_i q_i(p(t), c(t)) \left(\frac{dp_i(t)}{dt} \right) \quad 3-37$$

with the initial value $c(0) = c^0 = p^0 q^0 = p^0 q(p^0, m^0)$.¹¹

Note that 3-35 is directly related to the definition of an expenditure function. Equation 3-35 indicate the least expenditure needed to attain the original utility level when prices have changed. Also, note that $q_i[p(t), c(p(t), u^0)] = h_i(p(t), u^0)$ corresponds to a compensated (Hicksian) demand curve instead of the Marshallian demand curve as 3-36.

Using the uniqueness property of first order differential equations, the expenditure function $c(t) = c(p(t), u^0) = c(p(t), q^0)$ is the only solution having this initial value. Compensating variation for a price change

from p^0 to p^1 is uniquely given by $c(0) - c(p(1), q^0) = c^0 - c(p^1, u^0)$. Integrating 3-37 can be written as a line integral in the price space, this line integral is independent of the path of integration since the symmetry condition is satisfied. The line integral choosing a particular path, can be rewritten as a sum of ordinary Raiman integrals as follows:

$$c^0 - c(t) = -\sum_i \int_0^t q_i(p(t), c(t)) \frac{dp_i(t)}{dt} dt \quad 3-38$$

When 3-38 is evaluated at $t = 1$ ($p(1) = p^1$) it becomes the compensating variation definition given in Chapter II, p. 22.

Since 3-37 is a first order, nonlinear, differential equation, it must usually be solved numerically. Following Vartia, choosing $t_0, t_1 \dots t_k$ so that $0 = t_0 < t_1 < \dots < t_N = 1$ we derive from 3-38 the equation:

$$\begin{aligned} c^0 - c^1 &= \sum_{k=1}^N [c(t_{k-1}) - c(t_k)] \\ &= \sum_{k=1}^N \left[\int_{t_k}^{t_{k-1}} q_i(p(t), c(t)) dp_i(t) \right] \end{aligned} \quad 3-39$$

Although the integration of $q_i(p(t), c(t))$ cannot be performed explicitly the integral can be approximated by different methods. In the simplest polygon method $q_i(p(t),$

$c(t)$) is replaced by its value at the higher integration limit t_{k-1} , i.e., by the constant $q_i(p(t_{k-1}), c(t_{k-1}))$.

A more efficient method is provided by approximating the integrands $q_i(p(t), c(t))$ by the average of their end point values; we get for $k = 1, 2, \dots, n$.

$$c(t_{k-1}) - c(t_k) = \sum_i \frac{1}{2} [q_i(p(t_k), c(t_k)) + q_i(p(t_{k-1}), c(t_{k-1}))] [(p_i(t_{k-1}) - p_i(t_k))] \quad 3-40$$

By making $p(t) = p^0 + t(p^1 - p^0)$, $0 \leq t \leq 1$, the linear price curve connecting p^0 to p^1 for a given number of steps N where $t_k = k/N$, and $p_k = p(t_k)$. The main algorithm given by Vartia (1983) to solve 3-37 is given by:

$$c_{k-1} - c_k = \frac{1}{2} \sum_i (q_k^i + q_{k-1}^i) (p_{k-1}^i - p_k^i) \quad 3-41$$

where $q_i = q_i(p_k, c_k)$, $k = 1, \dots, N$ and the starting values are $(p^0, q^0) = (q(p^0, c^0), c^0)$. The solution C_k of 3-41 is determined by iteration from:

$$c_k^{(m)} = c_{k-1} - \frac{1}{2} \sum_i (q_i^i(p_k, c_k^{(m-1)}) + q_{k-1}^i) \cdot (p_{k-1}^i - p_k^i) \quad 3-42$$

where $c_k^{(0)} = c_{k-1}$, $k \geq 1$. When $|c^{(m)} - c^{(m-1)}|$ is negligible we set $c_k = c_k^{(m)}$ and $q_k = q_k^{(m)}$ and the calculation of the next k is started. Appendix C shows an example for a single price change and a linear demand equation.

Dividing 3-37 by $c(t)$ and multiplying each term in the summation by $p_i(t)/p_i(t)$ we get:

$$d \log c(t) = \sum w_i(p(t), c(t)) \frac{d \log p_i(t)}{dt} \quad 3-43$$

where $w_i(p, c) = p_i q_i(p, c)/c$ is the i th value share.

Proceeding similarly with 3-41 we get:

$$\log(c_{k-1}/c_k) = 1/2 \sum (w_i(p_k, c_k) + w_i(p_{k-1}, c_{k-1})) \cdot \log(p_{k-1}^i/p_k^i) \quad 3-44$$

Then, the same iteration procedure, which can be done with a computer, is used to calculate the numerical approximation of compensating and equivalent variation.

Aside from the rather difficult computational problems encountered with this approach, another problem must be considered. The method is useful if a demand system, which fits the data well and is consistent with consumer theory, can be estimated without the explicit specification of an indirect or direct utility function. An example of a demand system with these properties is the nonlinear system used by

Tyrell and Mount (1982). The budget share has the logistic form:

$$w_i = \text{Exp} [f_i(p, z, m)] / \sum_{j=1}^n \text{Exp} [f_j(p, z, m)]$$

where $f_j ()$ is the general notation for an unspecified function and z is the household characteristic vector.

The Hausman contribution can be considered as the particular case of 3-37 when a single demand equation exists. The complicated first order differential equation becomes:

$$dc(p, u)/dp = q(p, c(p, u)) \quad 3-45$$

With initial value $c(p^0, u^0) = c^0 = m^0$ or equivalently $u^0 = v(p^0, m^0)$. Consider a single demand equation assuming a composite Hicksian commodity with the frequently used functional forms $q = \alpha p + \beta m + \gamma z$ and $q = \gamma z \text{Exp}(\alpha p + \beta m)$ where z is a household characteristic vector. Solving the ordinary differential equations:

$$dc(p, u^0)/dp = \alpha p + \beta c + \gamma z \quad \text{and}$$

$$dc(p, u^0)/dp = \gamma z \text{Exp}(\alpha p + \beta c)$$

With the initial value $c(p^0, u^0) = c^0 = m^0$, we find that the expenditure function for the linear demand function is¹²:

$$c(p, u^0) = u^0 \text{Exp} (\beta p) - 1/\beta (z\gamma + \alpha p + \alpha/\beta) \quad 3-46$$

and for the semi-log demand functions is:

$$c(p, u^0) = -1/\beta \log [-\beta((\gamma z/\alpha) \text{Exp} (\alpha p) + u^0)] \quad 3-47$$

For the demand equations to be legitimate it is necessary that the consumer theory properties are satisfied. The homogeneity condition holds by normalization using the composite commodity price. The negativity condition, or equivalently the Slutsky condition, is satisfied if:

$$\begin{aligned} \partial^2 c(p, u^0) / \partial p^2 &= \partial h(p, u^0) / \partial p \\ &= \alpha + \beta (\alpha p + \beta c + \gamma z) \leq 0 \end{aligned} \quad 3-48$$

and

$$\begin{aligned} \partial^2 c(p, u^0) / \partial p^2 &= \partial h(p, u^0) / \partial p \\ &= \alpha + \beta \gamma z \text{Exp} (\alpha p + \beta m) \leq 0 \end{aligned} \quad 3-49$$

for the linear and semi-log functions, respectively. If the demand functions to be considered are aggregations over consumers, the conditions for exact aggregation must be verified. It is observed that only the linear demand equation allows the calculation of a representative

expenditure function consistent with individual utility maximizing behavior.

Inverting 3-46 for the indirect utility function corresponding to the linear demand equation

$$u = v(p, m) = \text{Exp}(-\beta p) [m + 1/\beta (\alpha p + \alpha/\beta + \gamma z)] \quad 3-50$$

and from the definitions of compensating variation and equivalent variation we get for a price change from p^0 to p^1 .

$$\begin{aligned} CV(p^0, p^1, m^0) &= 1/\beta [q^1(p^1, m^0) + \alpha/\beta] \\ &\quad - 1/\beta \text{Exp}(\beta(p^1 - p^0)) [q^0(p^0, m^0) + \alpha/\beta] \end{aligned} \quad 3-51$$

and

$$\begin{aligned} EV(p^0, p^1, m^0) &= 1/\beta \text{Exp}(\beta(p^0 - p^1)) [q^1(p^1, m^0) + \alpha/\beta] \\ &\quad - 1/\beta [q^0(p^0, m^0) + \alpha/\beta] \end{aligned} \quad 3-52$$

The formulas for the semi-log demand function using the indirect utility function

$$u = v(p, m) = -(\gamma z (\text{Exp}(\alpha p))/\alpha) - (\text{Exp}(-\beta m))/\beta \quad 3-53$$

are given by:

$$CV (p^0, p^1, m^0) = 1/\beta \log [1 + \beta/\alpha (q^0 (p^0, m) - q^1 (p^1, m))] \quad 3-54$$

and

$$EV (p^0, p^1, m^0) = -1/\beta \log [1 + \beta/\alpha (q^1 (p^1, m) - q^0 (p^0, m))] \quad 3-55$$

Estimation of Welfare Measures for Restricted Commodities

This section examines a method for observing preferences for certain classes of restricted goods in market data. The approach takes advantage of the fact that the level of restricted goods enter as arguments of demand functions for nonrestricted goods. By observing the effect on the demand curve for nonrestricted goods we can infer the value consumers place on changes in the levels of restricted goods. One should recall that direct evaluation of quantity change requires knowledge of the Marshallian demand curve for the restricted good in inverse form, i.e., the curve of the marginal value of consuming X_1 .

The indirect method can be used only if the value of the restricted goods is contingent upon the availability of some nonrestricted goods. This assumption, known as weak complementarity, is defined by Maler (1974, pp. 183-189). It requires that when the quantity demanded of a nonrestricted good q is zero, the marginal utility or shadow

price of the restricted good x_1 is zero.¹³ Formally the restricted expenditure function can be determined from equations 2-14 and 2-15 (Chapter II) solving the partial differential equation system:

$$\begin{aligned} \partial c^*(p, x_1, u) / \partial p_i \\ = q_i (\bar{p}, x_1, c(p, x_1, u)) \end{aligned} \quad 3-56$$

and

$$\begin{aligned} \partial c^*(p, x_1, u) / \partial x_1 \\ = \psi(p, x_1, c(p, x_1, u)) \end{aligned} \quad 3-57$$

with the initial value condition

$$c^0 - \bar{p}^0 \bar{q} (p^0, x^0, m^0) + p_1^0 x_1^0$$

and where $\psi(p, x_1, c(p, x_1, u)) = p_1 - p_1^* (p, x_1, c(p, x_1, u))$ is the marginal value of consuming x_1 given that price p_1 , must be paid for it.

Both methods described for welfare evaluation, that is, specification of a utility function and integration of the demand system, can be used if equation 3-57 can be estimated by specifying x_1 in the same way as a commodity price.

However, since equation 3-57 normally cannot be estimated from market data an alternative procedure is required. Assuming weakly complementarity, equation 3-57 becomes

$$\partial c^*(\hat{p}, x_1, u)/\partial x_1 = 0 \quad 3-58$$

where \hat{p} is a price vector that makes the compensated demand for activity 1 equal to zero. This condition is equivalent to $\partial u/\partial x_1 (0, \bar{q}, x_1) = 0$ (the marginal utility of x_1 is zero when the quantity demanded of a nonrestricted good is zero), from equation 2-17 in Chapter II.

If we have a simple equation for 3-56, the system can be solved analytically. For example, for the single linear and semi-log demand equations the method of characteristics could be used.

CHAPTER IV
ESTIMATION OF A DEMAND SYSTEM FOR RECREATIONAL
COMMODITIES USING A ZONAL MODEL

In this chapter, a demand model for sport fishing is developed, which allows the use of the demand systems presented in chapter III to estimate benefits from participating in fishing activities. The special case of the household production theory with a Leontief fixed-proportions technology is applied to 1977 data on steelhead sport fishing at 11 rivers in Oregon. The interdependence between quality and quantity discussed in Bockstael and McConnell (1981) and first pointed out formally by Becker (1973) is considered. The zonal travel cost approach for estimate the market demand curve for fishing trips is analyzed in the context of the aggregation theory discussed in Chapter III.

The Theoretical Model

Consider an individual that derives utility from catch rate (Z_q), trips (Z_n), a bundle of commodities with no significant time cost (Z_1), a bundle of commodities with time but not significant money cost (Z_2), and as in Chapter

III, household characteristics (z). Then the utility function is denoted by:

$$u = u (Z_n, Z_q, Z_1, Z_2, z) \quad 4-1$$

In order to specify the constraints we must be careful in how the endogenous quality variable is defined (catch rate). When quality is measured per season or per year, production technology is joint because the time spent must enter each production function, when quality (Z_q) is measured as a catch rate per unit of trip (Z_n), the production function for quality and quantity are separate but a product term exists in the cost function which creates jointness.

Defining endogenous quality as the catch rate per trip and assuming fixed proportions the technological restrictions can be written as:

$$Z_i = \text{Min} (t_i, X_i) \quad 4-2$$

for $i = n, q, 1, 2$ and where t_i, X_i are time and market good inputs required in the production of commodity i . The technological coefficients are $a_i = t_i/Z_i$ and $b_i = X_i/Z_i$ for time and market goods, respectively. The definition of Z_1 and Z_2 allows us to set their technological coefficients: $a_1 = 0, b_1 = 1, a_2 = 1$ and $b_2 = 0$. Moreover, the consumer

is restricted by the time and non wage income available. The time constraint is given by

$$t_n + t_q Z_n + t_2 = T - h \quad 4-3$$

where h is labor time supplied and T is the total time available. The budget constraint is

$$p_n X_n + p_q X_q Z_n + p_1 X_1 = V + hW \quad 4-4$$

where V is nonlabor income and p_k is the respective market good price.

Both constraints can be written as a function of commodities (Z). That is:

$$a_n Z_n + a_q Z_q Z_n + a_2 Z_2 = T - h \quad \text{and}$$

$$p_n b_n Z_n + p_q b_q Z_q Z_n + p_1 Z_1 = V + hW \quad 4-5$$

The first term in both restrictions indicates the amount of input (time or market goods) required to consume Z_n trips per unit of time independently of the level of quality. The second term indicates the inputs required to consumer $Z_n Z_q$ (total catch per quarter). The Hicksian composite good Z_1 serves as a numeraire such that the money prices of quantity and quality are normalized with respect

to p_1 . Similarly the Hicksian composite good Z_2 can be used as a numeraire to normalize the time prices (time technological coefficients a_i) with respect to a_2 . The definition of Z_2 implies that $a_2 = 1$. Also, set $p_1 = 1$.

To combine both constraints 4-5 into one, rather strong assumptions are required. We need the individual's marginal value of time equal to the wage rate. However, as the labor supply literature demonstrates, institutional constraint can prevent interior solutions in the labor market (where the marginal rate of substitution between leisure and goods equals the wage rate); destroying the presumed relationship between an individual's wage and the value of his time (Bockstael, Strand and Haneman, 1984). Furthermore, endogenous labor supply is not a sufficient condition to have one constraint. It is also necessary for the opportunity cost of time through different commodities to be equal. However, considering that at least some component of work time can be traded for money at the margin; and ignoring the second problem, the time constraint can be substituted into the income constraint yielding:

$$\pi_n Z_n + \pi_q Z_q Z_n + Z_0 = V + TW = m \quad 4-6$$

where $\pi_k = (W a_k + p_k b_k)$ and $Z_0 = Z_1 + Z_2$ W is an aggregate commodity expressed in units of the free time cost commodity

(Z_1). W can be defined more precisely as the wage rate applicable to a discretionary employment, V is the income from non-discretionary sources and m is the full income.

The maximization problem of the consumer can be summarized by the indirect utility function as follows:

$$\begin{aligned} v(\pi(p,d), m) &= \max_{Z_n, Z_q, Z_0} [u(Z_n, Z_q, Z_0) : m \\ &= \pi_n(p,d) Z_n + \pi_q(p,d) Z_n Z_q + Z_0] \end{aligned} \quad 4-7$$

where d denotes the technological coefficients and p input prices. Since the budget constraint is nonlinear, standard duality cannot be applied directly. However, we can redefine a new variable $Z_q^* = Z_n Z_q$ and a new utility function such that:

$$\begin{aligned} u^*(Z_n, Z_q^*, Z_0) &= u(Z_n, Z_q^*/Z_n, Z_0) \\ &= u(Z_n, Z_q, Z_0) \end{aligned} \quad 4-8$$

The equivalent maximization problem which defines the same indirect utility function from a linear budget constraint is given by:¹⁴

$$\begin{aligned} v(\pi(p,d), m) &= \max_{Z_n, Z_q^*, Z_0} [u^*(Z_n, Z_q^*, Z_0) : m \\ &= \pi_n(p,d) Z_n + \pi_q(p,d) Z_q^* + Z_0] \end{aligned} \quad 4-9$$

And since the π 's are a function of exogenous parameters the standard duality theory described in Chapter II applies. Thus, using Le Roy identity the Marshallian demand function is given by:

$$q_i(\pi, m) = \frac{\partial v(\pi, m)}{\partial \pi_i} / \frac{\partial v(\pi, m)}{\partial m} \quad 4-10$$

Note that unlike the "commodity shadow-price approach" (Barnett, 1977), equation 4-10 is defined over exogenous parameters (a function of input prices) rather than shadow prices. (See Appendix D for a more general formulation of the household production model.)

Becker and Lewis (1973), comparing the comparative static response of a nonlinear quantity/quality constraint with those which would have been observed if the constraint were linear, pointed out some interesting properties of income and price elasticities for quality and quantity. Here the shadow price concept becomes useful, since it allows the comparison of the effect of change in parameters (π 's and m) on Z_q and Z_n with the effect which would exist if the shadow price were exogenous. The endogeneity of shadow prices is obvious since:

$$\partial m / \partial Z_n = \pi_n^* = \pi_n + Z_q \pi_q \quad 4-11$$

and

$$\partial m / \partial Z_q = \pi_q^* = \pi_q Z_q$$

Following Becker and Lewis, it seems plausible to assume that the income elasticity with respect to quality is larger than that with respect to quantity, holding the shadow prices constant (π^* 's). Then the observed income elasticity for quantity holding the π 's constant may be negative because of the change in relative shadow prices. We observe in 4-10 that quantity becomes relatively more expensive when the direct effect on quality is greater than on quantity. Now consider the substitution effect of an increase in π_n , for example: a higher on-site entry fee. The relative increase in the shadow price of quantity (π_n^*) decreases the quantity demanded (number of trips). But the fall in Z_n reduces the shadow price of quality (catch per trip) which induces more substitution in favor of quality.

The specific model used in modeling consumer demand for sport-fishing considers fish catch rate as an endogenous quality commodity and thus corresponds to Z_q . But, sport-caught fish depends also on exogenous variables such as fish density and water quality, which can be policy controlled. The appropriate method to measure welfare impact on anglers of water quality improvement and number of fish releases from hatcheries was described by Strong (1984). The method includes the estimation of a production function for Z_q

depending on exogenous technology variables. Then, assuming constant return to scale the marginal cost of fish catch rate per trip $\pi_q(p,d)$ can be used to estimate the economic welfare changes (d denotes exogenous technology variables).¹⁵

There is controversy over the appropriate definition of the opportunity cost of a trip in the travel cost model literature (what are the expenditures to be included as travel costs). Also, the interaction between on-site time and number of trips has been recognized as a problem (see Desvousges, Smith and McGivney, 1983). Mendelson and Brown (1983) argue that the prices of other inputs used to produce a recreation experience such as: time on the site, eating and lodging, or equipment expenses, generally should not be included in the travel cost analysis, unless there is some reason to believe that the marginal utility of eating, lodging and so forth is zero. The avoidable expenditures to make a trip can be rationalized in the model described, as inputs (for example, on-site time) used to produce endogenous quality commodities or services, such as: sumptuous food, comfortable accommodations and in general an enjoyable recreational experience. Since these quality variables are quite difficult to define, a feasible treatment could be the definition of an aggregate quality commodity similar to the procedure widely used in the estimation of demand systems with time series data (for

example, see Berndt, Darrough and Diewert, 1977). However, a difficulty still remains if we define two endogenous quality commodities for sport fishing (catch rate per trip and trip comfort, for example). This is how we should allocate the inputs between both activities. A criterion may be that all the inputs (apart from the travel inputs) which are not statistically significant in the catch rate production function are allocated to the aggregate endogenous quality.

Finally, if the fish catch rate were treated as an exogenous utility variable, and if it were assumed that the consumer does not have discretionality to spend on a trip, then the model would collapse to the travel cost model (where the opportunity cost of a trip includes the total expenditure, such as on-site time).

Aggregation over Consumers and the Zonal Travel Cost Model

In the traditional travel cost model the dependent variable is defined as the number of trips per capita for each zone, and independent variables are expressed in terms of zonal averages. Brown et al. (1983) supports this approach, arguing that the estimation of a travel cost model from individual observations can lead to a downward bias in the travel cost coefficient, unless individual observations on the dependent variables are adjusted to a per capita

basis. Even if the dependent variable is adjusted, a bias in the travel cost coefficient may still result from errors in individual recreationists' estimates of travel costs (Brown et al., 1983). Since we have sample data in an empirical study, the total number of trips for each zone can only be approximately determined. The expected total number of fishing trips for each zone (EFT) is estimated by multiplying the sample number of trips per zone by the sample population (eligible anglers per zone for the data used in this study) and dividing by the number of computed questionnaires for each zone. That is, sample trips per zone times the inverse of the sample-population ratio.

Now, it is straight forward to use the Muellbauer exact aggregation theory described in Chapter III, to obtain the zonal travel cost model. Since EFT is equivalent to $\sum_{h=1}^H q_i^h$ (equation 3-7) then $q_i = \sum_{h=1}^H q_i^h / H$ corresponds to expected per capita number of trips for each zone (q_i becomes Z_i in this chapter). Exact linear aggregation allows the demand equation to be written: $Z_i = Z_i(\pi, M)$, where M is the average expenditure per zone and the vector π represents the common input prices and technology to all consumers of the same zone (p, a_i, b_i are the same).

Instead of working with quantities, we can aggregate over the different expenditure patterns of consumers. The average budget share $w_i = p_i (\sum_{h=1}^H q_i^h / \sum_{h=1}^H m^h)$ (equation 3-11) can be used as an alternative approach to obtain aggregation

per zone. Using this chapter notation, the average budget share per zone becomes $w_j = \pi_j \text{EFT}/(M H)$ and the share form demand equation is given by:

$$w_j = w_j (\pi_n, \pi_q, M^*) \quad 4-12$$

where M^* denotes the representative expenditure, which can be expressed as the average expenditure per zone (M) divided by an income distribution coefficient (K) (defined in Chapter III).

The latter specification of the zonal travel cost model involves in a sense less restrictive requirements for the preference structure, since it does not require linear Engel curves. It allows the estimation of more flexible demand equations and the differences in income distribution through different zones can be included in the model.

Unfortunately, the last method is still quite restrictive and it is not appropriate for commodities with low participation rates. Recreation commodities as with most goods are not bought by all consumers, although the proportion purchasing any one good can be expected to rise as its price falls. Consequently, a decrease in price not only causes individuals who already buy the good to buy more, but also causes new consumers to purchase for the first time. "A correct treatment requires that both these effects be adequately modeled, but it is impossible if

aggregate demand is treated as coming from a representative consumer who buys some of all the goods" (Deaton and Muellbauer, pp. 148-149).

As is shown by Bockstael and McConnell (1984), a more appropriate method includes (when individual data are available) modeling explicitly the discrete and continuous aspects of the consumer maximization problem into two stages. The first stage would be the qualitative choice of whether or not to participate in a recreational activity, and the second stage would be the quantitative decision of how many recreational trips to make, given that the decision to participate has already been made. This method (first considered by Heckmann, 1976) is based on the specification of a random utility function and the discrete choices specified using logit, generalized logit or probit models, depending on the probability distribution used for the random disturbances of the utility function. These procedures have been used by Haneman (1984) and Bockstael, Strand and Haneman (1984). However, there is more work to be done to develop a utility theoretical foundation which would allow the estimation of exact welfare measures.

The Empirical Implementation for Steelhead Fishing

An important objective of this study is to apply well developed demand systems to estimate benefits for recreation commodities. The source of data is a subset of 191

questionnaires of a larger random sample of Oregon angling licenses purchases during 1977. Detailed information about the survey design, along with copies of the questionnaire, has been reported by Sorhus, Brown and Gibbs (1981) and by Strong (1984).

The questionnaire was designed to obtain data on angler expenditures and fishing activities on a quarterly basis. Respondents reported the total number of times they went fishing, primarily for salmon and steelhead during the previous quarter. To avoid potential problems with memory bias, the anglers were only asked to provide information such as: expenditures, fish catch, river names, and on-site time for the last three trips taken during the previous quarter. Therefore, in order to use expenditures and catch rate per trip and per river we must assume that the last trips are representative of all of the trips taken during a quarter. Then, the average of the per trip data for a river are considered representative of all the relevant trips.

Because of the way in which the questionnaire was constructed, it becomes difficult to obtain enough observations to estimate a multiple site model. Instead, following Strong (1984) the data across sites are pooled such that the demand equations for a typical site are estimated.

The questionnaires from steelhead anglers were divided into 49 zones with most of the zones having five respondents who had actually fished during the quarter.

The two demand equations for a representative angler and a representative river, ignoring income distribution differences among zones, are specified as a function of implicit prices as follows (using a notation more adequate for computational work):

$$W_N = W_N (PTM, SE, FINQ)$$

and

$$W_Q = W_Q (PTM, SE, FINQ)$$

4-12

where (the equivalent notation is shown in parenthesis):

$$W_N = (EFT/H) \times (PTM/FINQ) \quad (W_n)$$

EFT = expected fishing trips for each zone and per river (EFT).

H = Zone population (H).

FINQ = $1274 \times WAGE + INCQ$, full income per quarter (m).

INCQ = Reported household income per quarter.

PTM = $WAGE \times TRIT + EXM (\pi_n)$.

WAGE = The questionnaire does not provide information about exogenous income and wage rate. The wage rate was computed following Brown (1984) by dividing the average yearly income by 2,000. It was assumed that the relevant wage (opportunity cost of time) is 50% of that wage calculated before (w).

TRIT = $AMT/50$, time spent traveling, assuming a speed of 50 miles per hour and AMT denotes average round trip miles per steelhead trips (t_n).

EXM = $0.10 \times \text{AMT}$, 10 cents per mile is used as car or camper operation cost ($p_n b_n$).

SE = EXPE/CR, cost per fish (π_q).

EXPE = EXTO + WAGE \times STT ($\pi_q Z_q$).

STT = on-site time (t_q).

EXTO = EXTN + SSEQ expenditures in market foods related to catch ($p_q b_q Z_q$).

EXTN = EXT ($0.75/(1 + \text{NPA}) + 0.25$), household expenditure pre trip such as: gas used in the boat, lodging, food, guide service, bait and lures. The fishing parties expenditures were adjusted adding 25 percent of the group expenses (less the respondent's own expenses). The respondent's expenses are estimated dividing the total group expenses by the number of people in the fishing party ($1 + \text{NPA}$).

SSEQ = $0.03 \text{ SSE}/\text{AT}$, the use cost per trip of fishing equipment used for steelhead angling. It is assumed a depreciation rate of 7% and interest rate of 5% per year. SSE denotes the average replacement value per respondent in zone i of fishing and related equipment used for salmon steelhead angling.

AT = Number of trips per quarter.

CR = Catch rate per trip (Z_q).

WQ = SE \times CR \times TPC/FINQ.

TPC = EFT/H, denotes steelhead fishing trips per capita from distance zone i to the specified river (Z_n).

Note that all the variables defined denote average for each zone. If the only endogenous variable considered is the number of trips 4-12 becomes the traditional travel cost model. The single equation for trips per capita is given by:

$$\text{TPC} = q_n (\text{TPTR}, \text{FINQ})$$

As was described in Chapter III we can also include a household characteristics variable in both models. For example, a dummy variable (MET) for metropolitan effect, zero if the respondents of zone i resided in Multnomah, Washington or Clackamas counties, and 1 otherwise (Z_1).

CHAPTER V
EMPIRICAL RESULTS AND COMPARISONS OF
WELFARE ESTIMATES

The empirical results are presented in two parts. The first section covers estimation of different demand equation specifications. The estimated parameters of alternative functional forms, assuming that the only commodity (apart from the commodity which serves as a numeraire) is the number of trips, are reported. Then the case when the recreationist is viewed as being able to increase the quality of the trip (catch rate) through increased expenditures is presented. In the second section, the welfare benefits from participating in steelhead fishing activities are given for the different demand functional forms.

Single Demand Equations for Fishing Trips

As is usually done in the travel cost literature, we estimate the demand parameters for the linear and semilog functional forms. Specifying the disturbance terms, these demand equations can be written as:

$$TPC = \alpha_L^0 + \alpha_L TPTR + \beta_L FINQ + \gamma_L MET + \epsilon_L^t \quad 5-1$$

and

$$TPC = \alpha_S^0 \text{ Exp } [\alpha_S TPTR + \beta_S FINQ + \gamma_S MET + \epsilon_S^t]$$

where TPTR is total expenditure per trip, FINQ is full income, MET is a dummy variable with zero for metropolitan area, and ϵ_t denotes the error term at observation t . The standard assumption that ϵ_t is normally distributed with zero expected value and a constant variance was made.

The OLS estimates for equation 5-1 are given by Table 5-1.

Table 5-1
Semilog and Linear Equation Estimates

<u>Explanatory Variables</u>	<u>Functional Forms</u>	
	<u>Linear</u>	<u>Semilog</u>
TPTR	-.527134 ^b (2.061)	-7.736690 ^c (3.695)
FINQ	-0.005158 (.976)	-.039464 (.912)
MET	.108744 ^b (2.412)	.986757 ^b (2.673)
Constant	.156545 ^b (2.471)	-2.47572 ^c (4.774)
R ²	.279	.430
F Statistic	5.809 ^c	11.315 ^c

m = 49

- o The coefficients corresponding to TPTR and FINQ are the figures shown by the first 2 rows multiplied by 10^{-3} .
 - o t values are given in parenthesis.
 - o a, b and c denotes significance at the .10, .05 and .001 level, respectively.
-

The results suggest that the semilog functional form fits the data substantially better than the linear form. Thus, the data seems to reject the assumption of a linear Engel curve, as required for exact linear aggregation (see Chapter III).

We also observe in Table 5-1 that the full income coefficient, is not significant at the 10% level for the two equations estimated. The fact that the income effect is not significantly different from zero for trips per capita seems to confirm the hypothesis given by Becker and Lewis (1973) and described in Chapter IV.

To illustrate the relationship between number of fishing trips and expenditure (for example on-site time) per trip, the following equation is estimated:

$$\begin{array}{rcl}
 \text{EXPE} = & -14.432 & -115.657^b \text{ TPC} + 1.271 \text{ AGE} \\
 & (.251) & (2.005) \quad (1.000) \\
 & 0.00564^b \text{ FINQ} & \\
 & (2.627) & \\
 & & R^2 = .276 \\
 & & F = 5.727^c
 \end{array}
 \qquad 5-2$$

The same notation as in Chapter IV is used. EXPE is the average expenditures per trip, TPC is fishing trips per capita, AGE is average age and FINQ, average full income. Equation 5-2 shows a significant income effect at the 5% level on the expenditures per trip. Besides, the negative coefficient of TPC confirms the inverse relationships between quantity and endogenous quality stated in Chapter

IV. It must be noted that 5-2 cannot be strictly interpreted as a demand equation. Instead, we should estimate the complete demand system for quantity and quality, where the symmetry condition is imposed.

We also estimate single equations for trips per capita in share form corresponding to the AIDS and TLOG demand systems. The variable MET (defined in page 73) was specified following the demographic scaling method, which was described in Chapter III. Defining the additive disturbance in the K-th expenditure share equation on observation t as ϵ_k^t , the AIDS and TLOG:

$$\begin{aligned} WN &= \alpha + \gamma \log (TPTR (1 + \xi MET)) + \beta \log (FINQ) \\ &\quad - \alpha \log (TPTR (1 + \xi MET)) \\ &\quad - 0.5 \gamma (\log (TPTR (1 + \xi (MET)))^2 + \epsilon_A^t \end{aligned} \quad 5-3$$

and

$$WN = \frac{\alpha + \beta \log (TPTR (1 + \xi MET)) - \beta \log FINQ}{-1 + \beta \log (TPTR (1 + \xi MET))} + \epsilon_T^t$$

We use the Gauss iteration procedure in TSP techniques to obtain the parameters for these non-linear equations in 5-3. The TLOG model for this specification produced unexpected signs and insignificant results and thus it is not reported. The LES system was not fitted because is too restrictive to be applied in explaining the demand for such a disaggregated commodity as sport fisheries.

The AIDS estimates are given by Table 5-2. The t statistics of γ seems to indicate that the simpler demand system where the γ 's are ignored, as developed by Deaton (1978), is a good approximation for this application. To compare the three estimated demand equations, the uncompensated price and expenditure elasticities are calculated. The expenditure and own price elasticities for a demand equation in shares form can be written as:

$$E_{im} = 1 + (m/w_i) (\partial w_i / \partial m) \quad 5-4$$

and

$$E_{ij} = -1 + (m/q_i) (\partial w_i / \partial p_i)$$

Table 5-2

AIDS Single Equation Estimates

Parameter	Estimate
α	8.846750 (3.070)
γ	0.121126 (.413)
β	-0.984963 (2.490)
ξ	63.113300 (0.094)

Log likelihood function

- ° The first three coefficients correspond to the figures on the table multiplied by 10^{-3}
- ° Asymptotic t statistic are given in parenthesis.

The AIDS own-price and expenditure elasticities are given by:

$$E_{im} = 1 + \beta_j/w_j \quad 5-5$$

and

$$E_{ij} = -1 + 1/w_j (\gamma_{ji} - \beta_j (\alpha_j + \sum_k \gamma_{jk} \log p_k))$$

for $i, k = 1, \dots, n$, respectively. Table 5-3 contains the elasticity values estimated for single demand equations.

Table 5-3

Elasticities for Single Demand Equations

	<u>Own-price</u>	<u>Expenditures</u>
Linear	-0.38	-0.39
Semilog	-0.73	-0.39
AIDS	-0.83	-0.22

° The elasticities are calculated at sample mean values.

Note that the expenditure elasticities for all the different functional forms are negative, although the parameters involved in the calculation for linear and semilog equation are not statistically significant. Moreover, the AIDS equation shows higher price elasticity in

absolute terms and higher expenditure elasticity than both the linear and semilog equations.

Since the negativity condition cannot be imposed directly in a demand system, it must be verified after the estimation. The negative compensated price affect requirement is satisfied for the three demand equations, because both price and expenditure elasticities are negative. More precisely from the Slutsky equation expressed in terms of elasticities $E_{ij} = E_{ij}^u + w_j E_{im}$, where u indicates the compensated effect, the negativity of price and expenditure elasticities guarantees a negative compensated price elasticity.

System of Demand Equations for Trips and Catch Rate per Trip

In this section we report empirical results from estimating consumer demand functions, derived from the reciprocal indirect translog (TLOG) and almost ideal demand system equations (AIDS) in share form, for trips and catch rate. Since it is expected that the error in both equations would be correlated we used a multivariate regression procedure. Parameter estimates, when the symmetry restriction is imposed, are presented in Table 5-4.

The t-statistics and the maximum value of the likelihood function are substantially greater for the AIDS system.

Also the number of iteration to reach convergence was lower for AIDS.

Table 5-4

Parameter Estimates of AIDS and TLOG,
Trips and Catch Rate Demand Equations

AIDS		TLOG	
<u>Parameter</u>	<u>Estimate</u>	<u>Parameter</u>	<u>Estimate</u>
α_1	10.166200 (4.859)	α_1	-.203442 (1.620)
α_2	7.295850 (2.676)	α_2	-.334675 (.519)
γ_{11}	-0.034328 (2.144)	α_{11}	-.036514 (2.432)
γ_{12}	-0.032875 (3.944)	β_{12}	.0383622 (2.076)
γ_{22}	-0.001339 (0.011)	β_{22}	.0230347 (.179)
β_1	-0.167456 (3.944)		
β_2	-.720371 (2.210)		
Log (L)	662.274		653.033

- The coefficients are the figures in the table multiplied by 10^{-3} .

- Asymptotic t statistics are given in parenthesis.

To calculate the elasticities corresponding to the demand system in 5-4, we use the TLOG expressions for expenditure and price elasticities. It can be easily shown that these elasticities are given by:

$$E_{im} = 1 - \beta_{Mi}/Dw_i \quad 5-6$$

and

$$E_{ij} = -1 + \beta_{ij}/Dw_i - \beta_{Mi}/D$$

Where $D = (-1 + \sum \beta_{mi} \log p_i)$

In Table 5-5 we present the estimated elasticities which have been calculated for the AIDS and TLOG demand systems.¹⁶

Table 5-5

Estimated Elasticities for AIDS and TLOG
Demand Systems with Endogenous Z_n and $Z_n Z_q$ Variables

<u>Elasticity</u>	<u>AIDS</u>	<u>TLOG</u>
E ₁₁	-1.29	-0.69
E ₁₂	-0.05	-0.32
E ₂₁	-0.27	-0.06
E ₂₂	-0.99	-1.03
E _{1m}	-0.42	1.02
E _{2m}	-0.05	1.09

The elasticities are calculated at sample mean values.¹⁷

The income elasticity signs for the two demand systems estimated are erratic. The income elasticities are positive for TLOG and negative for AIDS. The cross price elasticities seems to indicate that both goods are complements (we should compare compensated cross elasticities to conclude complementarity in the Hicksian sense). As was indicated before, the elasticities can be used to verify the negativity condition. It is easy to show that the compensated own price elasticities are negative for both demand systems at sample mean values.

In comparative static analysis we may be more interested in knowing the elasticities for catch rate per trip (Z_q), rather than the elasticities for catch per quarter. (See Chapter IV). A property of elasticities is that the elasticity of a product of any two variables is equal to the sum of the two elasticities. Thus the elasticity of catch rate per trip with respect to any variable is given by:

$$E_{z_q} = E_{z_n z_q} - E_{z_n} \quad 5-7$$

Table 5-6 shows the new elasticities calculated using 5-7. (Note that E_{21} in table 5-5 must be read $E_{z_n z_q \pi_n}$).

The AIDS system confirms the two hypothesis described in Chapter III. An increase in π_n has a large positive effect on Z_q , and the income elasticity for quality is substantially greater than the income elasticity for

quantity. The TLOG system only confirm the first hypothesis due to the large positive estimate for the income elasticity of Z_n . It should be kept in mind that the fit of the AIDS is considerable better than TLOG.

Table 5-6

Estimated Elasticities for AIDS and TLOG, Demand Systems with Endogenous Variables Z_n and Z_q

<u>Elasticity</u>	<u>AIDS</u>	<u>TLOG</u>
$Ez_n\pi_n$	-1.29	-0.69
$Ez_n\pi_q$	-0.05	-0.32
$Ez_q\pi_n$	1.02	0.63
$Ez_q\pi_q$	-0.85	-0.71
Ez_nm	-0.42	1.02
Ez_qm	0.37	0.07

Sport Fishing Benefit Estimates

Recreation benefits are defined as the welfare gains enjoyed by recreationists as a result of the consumption of recreational commodities. They could be also defined as the negative of welfare losses if the recreationists can no longer participate. Thus, the value of an existing site, is given by the welfare impacts on recreationist of eliminating the site. This is equivalent to measuring the welfare impacts for an increase in prices of site related commodities such that the demands are reduced to zero.

For the case when only one utility yielding commodity (trip to a fishing site) is considered, we are interested in estimating compensating variation and equivalent variation for an increase in π_n from π_n^0 to the minimum price $\hat{\pi}_n$ at which Z_n is zero.

When two commodities are considered, trips (Z_n) and catch rate (Z_q), we must estimate the Hicksian welfare measures for an increase in shadow prices from (π_n^0, π_q^0) to the minimum prices $(\hat{\pi}_n, \hat{\pi}_q)$ at which both demands Z_n and Z_q are reduced to zero.

There is some ambiguity as to which $(\hat{\pi}_n, \hat{\pi}_q)$ is the appropriate one in calculating the entire benefits enjoyed due to the existence of a set of commodities.

The more sustainable theoretical interpretation is to consider the prices at which the compensated demands become zero assuming such a price exist. However, this approach is difficult to use in applied research, since the vector $\hat{\pi}$ is quite sensitive to the functional form employed. Instead, it seems preferable to compare CV and EV for different functional forms on identical price changes.

One criteria to choose an unique vector $\hat{\pi}$, independent of the functional form, is to take the maximum value of prices observed in the sample. Since the maximum prices of the entire population should be the relevant one, the use of $\hat{\pi}$ (the sample maximum) generates conservative estimates for recreation benefits.

In Table 5-7 the benefit estimates for a representative household, when all the on-site expenditures are considered exogenous, are presented.

Table 5-7

Single Demand Equation Welfare Estimates for a
"Representative" Household During One Quarter
(1977 dollars)

	Linear	AIDS
Compensating Variation	-25.397	-7.018
Ordinary Surplus	-25.421	--
Equivalent Variation	-25.436	-7.023

- The estimates were calculated at sample mean values and sample maximum values.¹⁸

As was expected from the small income effect observed the definitional differences in welfare measures are very small in each case. And the negative income effect (negative income elasticity) is consistent with the equivalent variation estimate being greater in absolute terms.

The extreme high benefit estimation given by the linear equation,¹⁸ (more than three times the AIDS estimates) are similar to the results found by Ziemer et al. (1980). In their study of warm water fishing in Georgia, the consumer

surplus estimated from the linear equation was 3.08 of the semilog equation welfare estimate.

Table 5-8 shows benefit estimates when trips and catch rate are the recreation commodities yielding utility. It is observed, that the AIDS welfare estimates in Table 5-8 are not substantially different from the values estimated from the AIDS single demand equation (Table 5-7). Thus, if we are interested in knowing the total benefits from sport fisheries activities (the values of the recreation experience), the single equation method considering all the expenditures on related recreation activities appears to provide a reasonable approximation, at least in this case.

Table 5-8

Two Demand Equation Welfare Estimates for a
Representative Household During One Quarter Period
(1977 dollars)

	<u>AIDS</u>	<u>TLOG</u>
Compensating Variation	-8.350	-8.112
Equivalent Variation	-8.359	-8.105

- The estimates were calculated at sample mean values and sample maximum values.¹⁸

Like the single demand case the two demand equation system predicts very similar values for CV and EV. Note in table 5-5 that, the smaller equivalent variation in absolute

value for the TLOG demand system in table 5-8 is consistent with the positive TLOG income elasticity shown.

An interesting result is the similarity of the welfare estimates given by the two flexible functional forms, even though the estimated elasticities and the fit of the data are quite different.

CHAPTER VI

SUMMARY AND CONCLUSIONS

The purpose of this thesis was to analyze the estimation of Hicksian welfare measures based on a neo-classical model of consumer behavior. The theoretical framework provided by the literature of duality theory and applied consumer theory was employed to discuss some important issues in non-market valuation.

It was shown that the widely used zonal model in recreational demands is just a particular case of linear aggregation over households as was presented by Gorman (1959). A limitation of the zonal travel cost model for highly disaggregated commodities was emphasized. The low participation rates observed for disaggregated recreation activities are incompatible with the similarity of preferences assumed to exist among households. This assumption is required to obtain welfare estimates for a representative consumer. In this study we aggregate household consumption shares in zones, which suffer from the same kind of problems as linear aggregation. It is hoped, however, that the comparisons of the empirical estimates are not greatly affected by this shortcoming.

It was shown that demographic characteristics can be incorporated in a demand system using a non ad hoc specification. Additionally, a possible application to obtain welfare estimates for an specific group of people, subject to special public concern was illustrated.

The treatment of endogenous quality variables has been a widely discussed issue in the recreation economics and labor economics literature. This study provides a simple method to deal with endogenous quality under the specification of a standard demand system. In this approach, the maximization problem as well as the demand equations are expressed as functions of exogenous parameters rather than endogenous implicit prices. Thus, the demand equations are not simultaneous and the identification problem does not arise. Also, the theoretical properties of the demand system are the same as in the case when the budget constraint is linear. Since a homogeneous cost function is not required, it can be considered as a generalization of Barnett's approach.

The predictions of comparative static analysis regarding price and income elasticities for quantity and quality are tested, using data for 11 steelhead rivers in Oregon. Both income and price elasticities for catch rate per trip are found to be greater than those corresponding to number of trips, supporting the theoretical results.

A major objective of this study was the estimation and comparison of exact welfare measures of sport fishing commodities using theoretically admissible demand systems. Analytical expressions of Hicksian welfare measures were presented for the linear expenditure system (LES), almost ideal demand system (AIDS) and the translog indirect utility function system (TLOG). Also, compensating (CV) and equivalent variation (EV) formulas were derived for the linear and semilog single demand equations.

The estimates produced by the iterative method presented by Vartia (1983) were compared with the "exact" estimates for the linear demand equation. Vartia's method produced quite close estimates for even a small number of steps.

To keep the focus on demand system specifications and welfare analysis, some strong assumptions were made about the production technology. Not only were substitution among inputs and non-constant return to scale ruled out, but other possible exogenous variables such as water quality and hatchery releases were ignored. These limitations may explain the low explanatory power of the cost per fish variable. A more flexible specification following Strong (1984) can be made however, estimating the production functions for trips and catch rate per trip where exogenous quality variables are considered.

Our empirical results suggest that the definitional differences in welfare measures (i.e., the differences between ordinary and compensated measures) are not an important problem. These differences never exceeded 0.2% throughout the various demand systems estimated. This is not a surprising result if we take into account that the highest welfare estimate was less than 0.3% of the full income for a representative household, and that the income elasticities are in absolute terms very small. (The income was above 1 for only the translog indirect utility system.)

However, we should not conclude necessarily from here that sophisticated models based on economic behavior do not pay the extra cost of implementation. Our results indicated that the welfare measures may be very sensitive to the formulation of the model. The linear demand equation, when the on site expenditures are considered exogenously, estimates compensating variation as high as two times the CV of the almost ideal demand system when expenditures are associated to catch rate. A more surprising result was the quite close estimates between the TLOG and AIDS system, even when the elasticities estimated and the fit of the data were very different. A subject for a future study that follows directly from this study is the comparison of welfare measures for other flexible functional forms. An interesting further extension would be the comparison of flexible functional forms based on Taylor series expansion

with those based on Fourier series expansion (see Gallant, 1981). When one is interested in the evaluation of large economic welfare changes, a global approximation to the underlying utility function as provided by Fourier series might be preferable to the local approximation entailed in functional forms such as AIDS and TLOG.

The sensitivity of welfare estimates to model specifications, found in this study and many others, limits to a large extent their usefulness for policy considerations. Future research should be directed to test rigorously for appropriate functional forms and model specifications. Ideally, confidence intervals for the welfare estimates as was proposed by Hausman (1981) should also be calculated.

ENDNOTES

1. An exception is McKenzie (1983). He argues that the "money metric" approach developed by Samuelson is superior to compensating variation. The "money metric" is equivalent variation plus base expenditures.
2. This example is from a paper presented at the National Marine Fisheries Service Workshop by Darrell Hueth and Mario E. Niklitschek.
3. These properties are proved by Just, Hueth and Schmitz, 1982.
4. The composite commodity theorem is owed to Hicks (1936), which asserts that if a group of prices move in parallel, then the corresponding group of commodities can be treated as a single good.
5. Weak separability, is both necessary and sufficient for the second stage of two-stage budgeting (Deaton and Muellbauer, p. 124).
6. PIGL is a special case of generalized linearity (GL) when the representative expenditure level is independent of prices. (See Deaton and Muellbauer, pp. 155-156).
7. From $3-12 \log (m/K) = \sum_h (m^h/M) \log m^h$, we can write the left side terms as $\log (m/K) = \log M - (\log H + \log K)$ and doing $\log Z = \log H + \log K$ we have $\log Z = - \sum_h m^h/M \log (m^h/M)$ and $K = Z/H$.
8. As was indicated before, if we use a Hicksian composite commodity as a numeraire the only conditions required are symmetry, negativity and adding-up.
9. Consider the case of two commodities, the AIDS cost function is given by:

$$\begin{aligned}
 C(u, p) = & \alpha_0 + \alpha_1 \log p + \alpha_2 \log p_2 + \frac{1}{2} [(\gamma_{11} (\log p_1)^2 \\
 & + \gamma_{12}^* \log p_1 \log p_2 + \gamma_{21}^* \log p_1 \log p_2 \\
 & + \gamma_{22}^* \log (p_2)^2)] + \beta_0 p_1^{\beta_1} p_2^{\beta_2} u.
 \end{aligned}$$

if we make

$$\gamma_{ij} = \frac{1}{2} (\gamma_{ij}^* + \gamma_{ji}^*) \quad \text{or}$$

$$\gamma_{12} = \frac{1}{2} (\gamma_{12}^* + \gamma_{21}^*) \text{ and } \gamma_{11}^* = \gamma_{11}, \gamma_{22}^* = \gamma_{22}$$

The cost function becomes

$$\begin{aligned} c(u, p) = & \alpha_0 + \alpha_1 \log p_1 + \alpha_2 \log p_2 \\ & + \frac{1}{2} [\gamma_{11} (\log p_1)^2 + 2\gamma_{12} \log p_1 \log p_2 \\ & + \gamma_{22} (\log p_2)^2] + \beta_0 p_1^{\beta_1} p_2^{\beta_2} u. \end{aligned}$$

and since, if symmetry is imposed in the demand system

$$\log p \text{ is equal to } \alpha_0 + \alpha_1 \log p_1 + \alpha_2 \log p_2$$

$$+ \frac{1}{2} [\gamma_{11} (\log p_1)^2 + 2\gamma_{12} \log p_1 \log p_2$$

$$+ \gamma_{22} (\log p_2)^2] \text{ the cost function becomes:}$$

$$c(u, p) = \log P + \beta_0 p_1^{\beta_1} p_2^{\beta_2} u.$$

10. That is, $\partial u(p(t), c(t)) / \partial p_i = [\partial u(p, m) / \partial m] q_i(p, m)$
 $= -\lambda(p(t), m(t)) q_i(p, m).$

11. It can be shown that the first order differential equation defined in 3-37 is equivalent to the system of partial differential equations given by:

$$(\partial c(p, u^0)) / \partial p_i = q_i(p, c(p, u))$$

and the same initial condition

$$c^0 = p^0 q(p^0, m^0) = p^0 q^0$$

12. Maler (1974) developed an equivalent procedure from the Slutsky equation we can write the second order partial differential equation:

$$(\partial^2 c / \partial p^2) - (\partial q / \partial m) (\partial c / \partial p) - (\partial q / \partial p) = 0$$

The expenditure function can be calculated solving the second order differential equation with the initial conditions:

$$c^0 = m = c(p^0, u^0) \text{ and}$$

$$\partial c(p^0, u^0) / \partial p = q(p^0, m)$$

13. Bradford and Hildebrandt (1977) gave a wide spectrum of possible applications to economic welfare evaluation of public goods as follows:

<u>Public Good</u>	<u>Private Good</u>
Public highways	transportation vehicles
air safety	air travel
public recreational areas	transportation to areas
local public goods	residences
public television	television sets

14. To illustrate the equivalent maximization problem, consider a Cobb-Douglas direct utility function

$$u = q_1^\alpha q_2^\beta$$

The solution to the maximization problem with the constraint $m = \pi_1 q_1 + \pi_2 q_1 q_2$

is given by the indirect utility function:

$$v = \left[\left(\frac{\alpha - \beta}{\alpha} \right) \frac{m}{\pi_1} \right]^\alpha \left[\left(\frac{\beta}{\alpha - \beta} \right) \frac{\pi_1}{\pi_2} \right]$$

Redefining the variable $q_2^* = q_1 q_2$ the

direct utility function becomes $u^* = q_1^{\alpha - \beta} q_2^{*\beta}$

and the budget constraint $m = \pi_1 q_1 + \pi_2 q_2^*$.

and the solution provides the same indirect utility function than before. It is assumed that second order conditions are satisfied in the first case and therefore they are also satisfied in the second case.

15. Note, that in other kinds of recreation activities as: kayaking and rafting, water-quality becomes an argument of the utility function, and the weakly complementary condition described in Chapter III is required to evaluate benefits of improvement in water quality as flow level.
16. The uncompensated cross elasticity for a demand equation in share form is given by:

$$E_{ij} = (1/w_i) (\partial w_i / \partial p_j)$$

and the formulas for the TLOG and AIDS cross price elasticities are:

$$E_{ij} = (\beta_{ij}/Dw_i) - (\beta_{Mj}/D)$$

and

$$E_{ij} = (1/w_i) (\gamma_{ij} - \beta_i (\alpha_i + \sum_k \gamma_{jk} \log p_k))$$

respectively. Where $D = (-1 + \sum_i \beta_{Mi} \log p_i)$

17. The sample mean values used in the elasticity computations are:

$$WN = 0.000118314$$

$$WQ = 0.000689585$$

$$PTM = 18.534$$

$$SE = 272.998$$

18. The sample values used in the welfare estimates are:

	Mean	Maximum
TPTR	94.151	427.264
PTM	18.534	144.564
SE	272.998	2262.950
FINQ	9960.270	--

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APPENDICES

APPENDIX A

THE TRANSLOG COST FUNCTION AND THE DEMAND SYSTEM

a. The Representative Cost Function

The Jorgenson and Lau translog indirect utility function is:

$$u = \alpha_0 + \sum_i \alpha_i \log (p_i/m) \\ + \frac{1}{2} \sum_i \sum_j \beta_{ij} \log (p_i/m) \log (p_j/m)$$

for $i, j = 1, \dots, n$. Which can be written as:

$$u = \alpha_0 - \log m \sum_i \alpha_i + \sum_i \alpha_i \log p_i \\ + \frac{1}{2} \sum_i \sum_j \beta_{ij} \log p_i \log p_j - \frac{1}{2} \log m \sum_i \sum_j \beta_{ij} \log p_j \\ - \frac{1}{2} \log m \sum_i \sum_j \beta_{ij} \log p_i + (\log m)^2 \sum_i \sum_j \beta_{ij}$$

We simplify the notation by writing $\sum_k \beta_{ki} = \beta_{Mi}$ and choosing the normalizations $\sum_i \alpha_i = -1$ and $\sum_i \beta_{Mi} = 0$ we get the indirect utility function with individual utility maximization behavior.

$$u = \alpha_0 + \sum_i \alpha_i \log p_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \log p_i \log p_j \\ + (-1 + \sum_i \beta_{Mi} \log p_i) \log m$$

Thus the cost function becomes:

$$\log c(u, p) =$$

$$\frac{u - \frac{1}{2} \sum_i \sum_j \beta_{ij} \log p_i \log p_j - \sum_i \alpha_i \log p_i - \alpha_0}{-1 + \sum_i \beta_{Mi} \log p_i}$$

b. The TLOG Marshallian Demand System

The logarithmic differentiation of representative cost function in (a) with respect to price.

$$\partial \log c / \partial \log p_i = w_i \\ = [-(\alpha_i + \sum_j \beta_{ij} \log p_j) (-1 + \sum_i \beta_{Mi} \log p_i) \\ + \beta_{Mi} (u - \alpha_0 - \frac{1}{2} \sum \sum - \sum \alpha_i \log p_i)] / (-1 + \sum_i \beta_{Mi} \log p_i)^2$$

Substituting the indirect utility function we get

$$w_i = [\alpha_i + \sum_j \beta_{ij} \log p_j - \beta_{Mi} \log m^-] / (-1 + \sum_i \beta_{Mi} \log p_i)$$

Where m^- is the representative expenditure which is equal to the average total expenditures over the income distribution factor K (which was defined in 3-13).

APPENDIX B

DEMOGRAPHIC TRANSLATING

An alternative specification is demographic translating which was employed by Pollak and Wales (1978). It introduces a direct utility function $u = u(q-d) = u(q^*)$ where d denotes the vector of translating parameters. The consumer problem becomes

$$\text{Max}_{q^*} [u(q^*): pq^* = m - pd = m^*] \quad q^* \geq 0$$

The cost function is given by $c(u,p) - pd = m^*$. Thus the indirect utility function is:

$$v(p, m - pd) = u$$

Finally the demand system for demographic translating is:

$$q_i = q_i(p, m^*)$$

or

$$q_i = d_i + q_i(p, m - pd)$$

And multiplying by p/m both sides and by m^*/m^* the second term on the right side, the demand system in budget share form is equal to

$$w_i = d_i p_i / m + (1 - p_i d_i / m) w_i^* (p, m - p d)$$

A linear function form for d can be postulated as:

$$d_i(n) = d_i^* + \sum_{r=1}^{\xi} \xi_{ir} z_r$$

where z_r denotes a household characteristic.

APPENDIX C

ILLUSTRATIVE CALCULATIONS OF THE NUMERICAL PROCEDURE
TO ESTIMATE CV AND EV

We illustrate the algorithm given by Vartia choosing a simple example. We take a demand system with one linear demand equation $q = a + bp + cm$ with parameters $a = 2$, $b = -1$ and $c = .01$. The following initial and final values are considered:

Table C-1

Variable	P	q	m
(0) Initial Values	5	67	7000
(1) Final Values	72	0	7000

Using 10 steps, i.e., $N = 10$, the price path $p(t) = p^0 + t(p^1 - p^0)$ is given by $p_k = 5 - 6.7k$. Since CV of move from 0 to 1 equals minus EV of move from 1 to 0, we can estimate EV in the same way but from p^1 to p^0 . Table C-2 shows the calculations starting from the final situation ($p^1 = 5$, $q^1 = 67$, $m^1 = 7000$). Column p_k indicates the price path starting from $p^0 = 5$. Using the demand equations $q(1) = q(p^1, c(0)) = 2 = 11.7 + 0.01 \times 7000 = 60.3$, the next compensating income is:

$$\begin{aligned}
 c(1) &= c^0 - \frac{1}{2} (q(p^1, c^0) + q(p^0, c^0)) (p^0 - p^1) \\
 &= 7000 + 63.65 \times 6.7 = 7426.4550
 \end{aligned}$$

This is a new start and the next, now is generated similarly. The iteration for $c^{(m)}$ converges after $m = 5$. We omit the figures referring to the iterations steps after $k = 1$ and the converged values only are tabulated. From 3-38 compensating variation of move from p^1 to p^0 is $CV = c^0 - c^1 = 3549.39$, thus the equivalent variation measure due to the reduction in prices (from p^0 to p^1) is $EC = 3549.39$. The compensating variation measure can be calculated starting from the initial situation. We estimate $CV = c^0 - c^1 = 1819.06$ (the table is not shown here). Since from this simple case the expenditure function can be calculated directly, the welfare measures are evaluated from 3-51 and 3-52 for the same price change.

Thus the true measures are $CV = 1817.0$ and $EV = 3551.4$. We observe that the approximation errors are quite small, the compensating variation error is 0.11% and compensating variation error is 0.05%. Unlike the errors associated to consumer surplus approximations are quite high, 37% for EV and 24% for CV.

Table C-2
Equivalent Variation for $q = 2 - 1 p + .01 y$

K	m	pk	q(m)	c(m)
0		5	67.000	7000.0000
1	1	11.7	60.3000	7426.4550
	2		64.5646	7440.7410
	3		64.7074	7441.2197
	4		64.7122	7441.2358
	5		64.7123	7441.2360
2	5	18.4	62.2661	7866.6137
3	5	25.1	59.6502	8275.0332
4	6	31.8	56.8532	8665.3190
5	5	38.5	53.8621	9036.2153
6	5	45.2	50.6638	9836.3769
7	5	51.9	47.2437	9714.3660
8	5	58.6	43.5864	10,018.6477
9	5	65.3	39.6758	10,297.5762
10	4	72.0	35.4939	10,549.3940

APPENDIX D

A FORMULATION OF THE HOUSEHOLD PRODUCTION MODEL

Consider the maximization problem faced by the household.

$$1. \quad \underset{z}{\text{Max}} \quad [u(z) : px = m, \quad F(x, z, k) = 0]$$

where:

x : is the vector of inputs

z : the vector of commodities

k : is a vector of fix production factors as household characteristics.

A way to illustrate the household's decision problem is by a two-stage optimization procedure. In the first stage the household determines the cost function which is defined as:

$$2. \quad m(p, z, k) = \underset{x}{\text{min}} \quad [px : F(x, z, k) = 0]$$

The solution value for x which minimize costs for a given z corresponds to the conditional demand.

$$3. \quad x^* = x^*(p, z, k)$$

The second stage optimization problem defines the indirect utility function as:

$$4. \quad v(p, m, k) = \max_z [u(z) : m = m(p, z, k)]$$

The function $v(p, m, k)$ can be interpreted as a generalized indirect utility function in the Epstein (1981) generalized duality context.

The expenditure function also can be defined, with a single constraint, for a nonlinear constraint. It take the form:

$$5. \quad c(p, u, k) = \min_z [m(p, z, k) : u = u(z)]$$

The generalization of the Shephard's Lemma given by Epstein (1981) is:

$$6. \quad \partial m(p, z, k) / \partial p_j = \partial c(p, u, k) / \partial p_j$$

and solving by z^* we get the Hicksian demand equations for commodities.

$$7. \quad z_i^*(p, u, k) = \partial m / \partial p_j [\partial c(p, u, k) / \partial p_j, p, k]$$

Similarly to standard duality, substituting equation 4 in equation 7 the Marshallian demand system is given by

$$8. \quad z^* = z^*(p, m, k)$$

This system must satisfy the same properties that the linear budget constraint case. However, the Slutsky matrix must be defined differently. The element a_{ij} of the "generalized Slutsky matrix" becomes:

$$9. \quad a_{ij} = (\partial m / \partial p_i \partial z_j) [(\partial z_i / \partial p_i) + (\partial m / \partial p_j) (\partial z_i / \partial m)]$$

Thus, like the standard case this matrix must be symmetric and negative semi-definite. It can be shown that these properties are necessary conditions for integrability. (See Epstein, 1981).

A special form of $m(p, k, z)$, which seems to be useful in empirical work, is considered in the following maximization problem:

$$10. \quad v(\pi(p, k), m) = \max_z [u(z) : m = \pi(p, k) \cdot \psi(z)]$$

Where $\pi(p, k)$ and $\psi(z)$ are row and column vector function respectively. Since $\pi(p, k)$ is only a function of exogenous variables, can be considered as a vector of exogenous parameters in equation (6), we get.

$$11. \quad \psi_i(z) = \partial c(\pi(p, k), u) / \partial \pi_i$$

However, if the elements of $\psi(z)$ can be identified we can redefine a new vector of endogenous variables $z^* = \psi(z)$ and a new utility function $u^*(z^*)$ such that:

$$12. \quad u^*(z^*) = u^*(\psi(z)) = u(z)$$

Then the maximization problem given by equation (10) takes the form:

$$13. \quad v(\pi(p,k),m) = \max_z [u^*(z^*) : m = \pi(p,k) z^*]$$

Since equation (10) and (13) are equivalent and the respective indirect utility functions are identical, we can treat our problem under the standard duality framework using a linear constraint.

Note that unlike the shadow price approach (Pollack and Wachter, 1975 and Barnett, 1977), we redefine the commodity quantities instead of commodity prices. In the case of constant return to scale and nonjoint technology the cost function takes the form:

$$14. \quad m(p,k,z) = \pi(p,k) z$$

Where the vector $\pi(p,k)$ is interpreted as the implicit commodity prices. Obviously in this case both methods become identical.

Unlike the Barnett (1977) method, we can deal with non homogenous cost function. For example, consider the nonjoint technology for the two commodity case, $z_1 = z_1(x)$, $z_2 = z_2(x)$. Also, assume that $z_1(x)$ is homogenous of degree t and $z_2(x)$ is homogenous of degree ℓ , then the cost function is $m(p,k,z) = z_1^{1/t} \pi_1(p,k) + z_2^{1/\ell} \pi_2(p,k)$, a non homogenous function.