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Robin M. Cross for the degree of Doctor of Philosophy in Agricultural and Resource Economics presented on December 7, 2004.

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Agricultural revenues, the product of stochastic prices and yields, lead to markets which are incomplete, thereby entreating and complicating economic inquiry. The following three essays explore the incomplete nature of agricultural markets and consider the implications of incompleteness for a range of policy questions and economic tools.

The first essay, "*Cooperative Pricing Policy Under Stress: The Case of Tri Valley Growers*," explores the incomplete contract markets that arise from unobservable yield processes. I formalize a common class of forward contracts, decompose them into a convex combination of yield derivatives, then derive the arbitrage-free forward price bounds. These bounds are used to show how the Board of Directors of a large agricultural cooperative, Tri Valley Growers, overstated earnings in order to liquidate financial equity.

The second essay, "*Adapting Cooperative Structure for the New Global Environment*," follows up on the first by showing that the liquidating strategy Tri Valley's Board pursued was rational in terms of maximizing expected net present value of future cash flows. I derive a condition under which optimal equity retention is strictly greater for investor-owned than for cooperatively owned firms. Finally, I use ruin probabilities associated with the standard first-crossing-time problem, together with numerical integration methods, to verify that this condition held under the market conditions in which Tri Valley and its investor-owned rivals operated.

The third essay, "*DEA and The Law of One Price*," explores the effect of variable prices on technical efficiency estimation. Data commonly are furnished in value, rather than factor terms. This raises the question of how value-based DEA models coincide with factor-based models. A sufficient condition for the two models to coincide is that all firms face the same set of prices. In practice, however, prices commonly vary across firms. I show that, unless an unreasonable restriction holds, the two models do not coincide. I decompose the resulting estimation error into its technology and firm-related components. Using Farrell's original 1957 data set to illustrate, the resulting estimation error is found to be both systematic and one-sided.

Essays in Incomplete Agricultural Markets

by

Robin M. Cross

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I understand that my dissertation will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my dissertation to any reader upon request.

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Robin M. Cross, Author

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CONTRIBUTION OF AUTHORS

Professor Steven T. Buccola co-authored essays one and two, providing guidance on all aspects of the research and writing process. Professor Rolf Färe co-authored essay three, guiding the motivation of claims and writing process. Professor Enrique Thomann provided guidance on contracting and pricing issues for essay one and the ruin probability specification for essay two. Professors Carlos Martins provided much of the initial motivation for essay three and important suggestions on modeling issues in essays one and three. Mister Benjamin Rashford provided valuable comments on audience and research objectives throughout a number of revisions.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

LIST OF FIGURES

LIST OF TABLES

1. General Introduction

Agricultural revenues, the product of stochastic prices and yields, lead to markets which are incomplete, thereby entreating and complicating economic inquiry. The following three essays explore the incomplete nature of agricultural markets and consider the implications of incompleteness for a range of policy questions and economic tools.

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2. Cooperative Pricing Policy Under Stress: The Case of Tri-Valley Growers

Recent economic forces, such as reduced trade barriers, improved manufacturing efficiency overseas, and food retail consolidation, have combined to put U.S. food manufacturers under intense competitive pressure. The latter have responded through re-tooling, consolidation, partnering, and diversification. Many large investor-owned food manufacturers have endured, enhancing profitability while maintaining financial liquidity. In contrast, a troubling wave of bankruptcies has swept through the cooperative sector, including such long-standing industry giants as Tri Valley Growers, the Rice Growers Association, Farmland Industries, and many regional cooperatives.

It is understandable that cooperative success should be counter-cyclical. A cooperative's patronage-based equity structure permits the cooperative to raise equity capital from its own members and, often, to exercise significant discretion in reporting net earnings. Thus, the cooperative can overcome thin capital markets to challenge monopoly power and manage cash flow despite volatile prices. Not surprisingly, many recently bankrupt cooperatives were formed amid the turbulent economic environment of the Great Depression.

The cooperative has discretion in reporting net earnings for two reasons. First, the member-supplied input values are declared *ex post*, that is, after processing and marketing activities have been completed and prevailing market conditions have been assessed. Second, there is often little with which to compare and validate these values. Local cash price information tends to be thin on account of variable yields, seasonal production, high transportation costs, and geographic dispersion of producers. Further, cooperative raw-product contracts often differ from those of investor-owned firms (IOF's) in both delivery and pricing policies, precluding direct comparison of forward prices.

Cooperative innovations add potential stability and flexibility to ever-changing agricultural markets and communities. For this reason, government legislation provides incentives for cooperative formation through favorable tax treatment and a secure source of borrowing capital.

However, the turbulent markets that once fostered cooperative formation have changed. First, raw-product markets have matured. Cooperative members can choose from a wider number of processors than before, eroding the value of the cooperative's promise to process and market member raw-product. Second, capital markets have matured, forcing lenders to reduce risk premia and narrowing the cooperative's borrowing advantage. Third, healthy equity market returns have increased the perceived opportunity cost that members face when holding non-tradable cooperative equity. For instance, from 1986 to 1995, the S&P index of IOF food manufacturers (#250) posted a total return of 12.4% to investor equity through capital gains and cash dividends. Over the same period, non-interest-bearing cooperative member equity returned nothing, except of course the increasingly strained benefits from continued cooperative patronage.

As the cooperative's advantages erode, pressure to liquidate grows. In this new environment, discretionary power becomes a double-edged sword. Under-reporting or over-reporting net processing returns provides a short-run mechanism for the cooperative board to distribute wealth to active members, undermining lenders' collateral and retired members' equity.

This equity liquidation strategy is possible because lenders typically secure cooperative loans with cooperative, rather than member, assets. Lenders then use financial covenants to restricting cash payments to members to, say, some portion of reported raw-product value. Herein lies the catch, since lenders often must rely on the cooperative's own valuation of raw-products in reporting net earnings. The cooperative board has the power to overstate the raw-product economic value, increasing allowable cash payments to members and undermining the lender's security position.

Such a strategy is not, of course, sustainable in the long-run and would be attractive only in an environment in which the short-run gains from liquidation outweigh the long-run benefits from continued cooperative patronage. Further, the strategy would benefit *active* members, those currently supplying raw-product, at the expense of retired members and lenders. It is just such an environment, however, that has faced a number of cooperative boards over the past decade and a half.

A thorough exploration of these phenomena would draw upon a wide range of economic tools. What incentive structures and economic conditions trigger the cooperative board to turn its discretionary power from equity-building to liquidation? What exactly is the principal-agent problem between lender and cooperative board? And finally, what is the impact on agricultural markets and communities if we lose the flexibility inherent in the traditional cooperative equity structure?

Unfortunately, one simple problem complicates each of these questions, namely, unobservable raw-product values. If we are to measure the discretionary power available to cooperative boards, we must have an unambiguous economic value with which to compare reported values. Many studies have addressed such values in the autarkic environment of the cooperative itself, along with the member supply incentives and fairness issues associated with them (Buccola, 1985, Buccola, 1994). Our challenge is to define a raw-product contract valuation method consistent with the external market environment, that is, with the presence of competing firms and competing, though dissimilar acreage-based raw-product contracts.

In this paper, we introduce such a method. We then apply it to the case of Tri Valley Growers, formerly the largest fruit and vegetable cooperative in the United States. We show how the Tri Valley board used its discretionary power to materially overstate economic values and circumvent lending restrictions in the years leading up to the processing giant's financial ruin. While clearly not the sole cause of its financial difficulty, the impact on financial liquidity was material. The pricing strategy succeeded in transferring wealth to active members at the expense of lenders, retired members, and the agricultural communities in which Tri Valley operated.

We begin by formalizing a class of forward contracts known as acreage-based contracts which were used by Tri Valley and are common to agricultural markets. We then consider the fruit and vegetable markets in which Tri Valley operated. We find these markets to be incomplete and hence inconsistent with assumptions critical to popular pricing models. Given incomplete markets, we derive an empirical expression for an interval within which Tri Valley's declared forward contract prices would have been consistent with arbitrage. While interesting in its own right, the interval also demonstrates that Tri Valley's board set prices that were unambiguously too high and thus both possessed and exercised discretionary power over reported profits. We conclude by illustrating how the board distributed cash to members in excess of lender restrictions, jeopardizing lender security, retired member equity, and the cooperative's long-term financial health.

The Acreage-Based Forward Contract and its Decomposition

Contracting was prevalent in the commodity markets in which Tri Valley participated. For instance, because of the high weight-to-value and perishable nature of tomatoes, 99% of California's processing tomatoes were planted under contract between 1991 and 2000 according to the California Tomato Growers Association (CTGA). As a result, there were no organized cash markets and hence no cash price information available to either growers or processors. Similar conditions existed in other fresh fruit and vegetable markets.

In order to evaluate Tri Valley's pricing strategy, we must first formalize the class of contracts used by the cooperative and its IOF rivals. Common in many agricultural markets, acreage-based contracts differ substantially from the standard forward contracts described in most financial textbooks. For this reason, we review briefly the standard forward contract and its decomposition into a balanced combination of European put and call options. We then introduce the acreage-based contract and its decomposition. Readers familiar with the agricultural contract literature will recognize the *broker contract* (Dimitri) and the *market premium contract* (Hamilton) as special cases of the acreage-based contract. See Hueth and Ligon for an discussion of these contract structures.

Standard Forward Contract

Consider a forward contract between two parties, which, for simplicity, we refer to as the farmer and the processor, acting in their respective roles of growing and processing raw agricultural produce. The standard agricultural forward contract obliges the farmer to deliver a target amount of agricultural produce, say y^{targ} units, at a specified time T for a predetermined *strike price* p^{strike}. The *payoff* at expiration, $\pi_T^{fwd}\left(p_T^{spot}; p^{strike}, y^{targ}\right)$, is also a function of the random *spot price* p_T^{spot}, sometimes referred to as the *random source*. The payoff and decomposition of the standard forward contract are

$$\pi_T^{fwd}\left(p_T^{spot};p^{strike},y^{targ}\right) = y^{targ}\left(p^{strike}-p_T^{spot}\right)$$

$$(1)\qquad = y^{targ}\left(p^{strike}-p_T^{spot}\right)^- + y^{targ}\left(p^{strike}-p_T^{spot}\right)^+$$

$$= \pi_T^{put}\left(p_T^{spot};p^{strike},y^{targ}\right)+\pi_T^{call}\left(p_T^{spot};p^{strike},y^{targ}\right)$$

where $\pi_T^{put}\left(p_T^{spot};p^{strike},y^{targ}\right)$ and $\pi_T^{call}\left(p_T^{spot};p^{strike},y^{targ}\right)$ are the payoff functions associated with European put and call options, respectively. The payoff function in (1) has the straightforward graphical interpretation shown in figure 2.1:

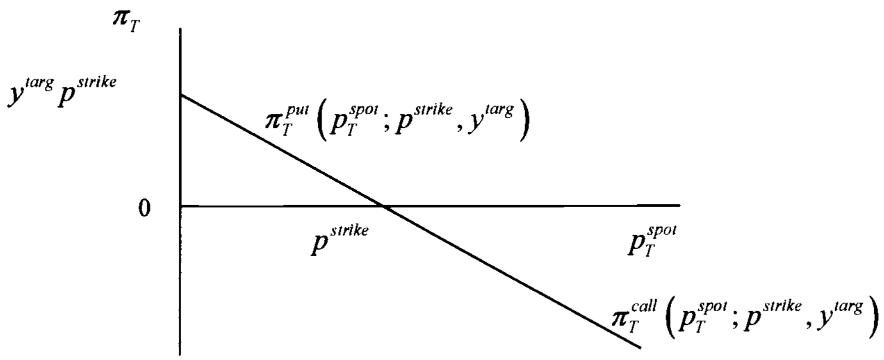

Figure 2.1. Payoff Functions Associated with y^{targ} European Put and Call Options

Here, the portion of the payoff associated with the put option lies above the x-axis, and the portion associated with the call option lies below. The payoff is greatest along the y-axis, where the spot price is zero.

Acreage-Based Forward Contract

As can be seen from equation (1), the payoff function associated with the standard forward contract depends on a spot price that is well defined. However, as we have noted for tomatoes, spot prices are often poorly defined on account of seasonal production, perishable goods, high transportation or storage costs, geographic dispersion, and variable yields. In response to these conditions, industry participants in these markets have innovated forward contracting to achieve a contract whose payoff is well defined without spot prices. The acreage-based contract specifies a particular tract of land and set of required agronomic practices. Generally, the contract specifies a *base price* p^{base} for deliveries up to some target quantity y^{targ}. Excess produce from a particular tract may be accepted, but at a predetermined *surplus price* p^{surpl}.

Thus, the farmer's payoff at expiration from the acreage-based contract can be defined and decomposed as

(2)

$$\begin{aligned}\pi_T^{ac}\left(y_T;\, p^{base},\, p^{surpl},\, y^{targ},\, C\right) &= p^{base}\min\left(y_T,\, y^{targ}\right) + p^{surpl}\max\left(y_T - y^{targ},\, 0\right)\\ &= p^{base}\min\left(y_T - y^{targ}\right)^{-} + p^{surpl}\max\left(y_T - y^{targ}\right)^{+} + M\\ &= \pi_T^{put}\left(y_T;\, p^{base},\, y^{targ}\right) + \pi_T^{call}\left(y_T;\, p^{base},\, y^{targ}\right) + M\end{aligned}$$

where C is the cost of the agronomic practices specified and M is a positive constant based on target quantity, base price, and agronomic cost. Equation (2) states that the payoff for an acreage-based contract can be decomposed into a constant and a convex combination of yield derivatives, namely, a number p^{base} of yield-based put options and p^{surpl} yield-based call options.

The payoff of the acreage-based forward contract is symmetric to farmer and processor if we assume both parties have an equal opportunity to cultivate the crop at agronomic cost C. Under the acreage-based contract, farmers lose the ability to separate planting decisions from contracting decisions. They do not, however, lose the ability to contract with qualified third parties, that is, to subcontract.

The payoff function in (2) also has a straightforward graphical interpretation, as shown in figure 2.2.

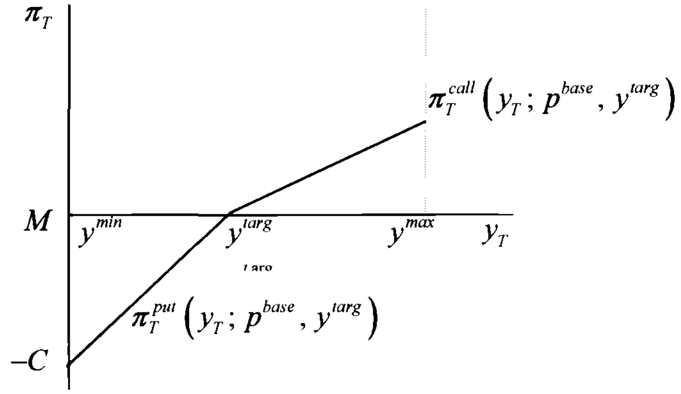

Figure 2.2. Payoff Functions for a Number of Yield-Based European Put and Call Options

In figure 2.2, the upper portion of the payoff line is associated with the call option and the lower portion with the put, and y^{min} and y^{max} are the theoretical minimum and maximum yield, respectively[1]PT.

Tri Valley Growers used a special case of the acreage-based contract in which the price was the same for all produce delivered ($p^{base} = p^{surpl}$). In contrast, most investor-owned firms do not accept delivery over some base amount ($p^{surpl} = 0$). We now turn our attention to the exact nature of the market in which Tri Valley operated and the difficulty of contract pricing in this market.

Contract Pricing

Arbitrage pricing models are often used to estimate contract prices that are unique and independent of both individual preferences and firm cost structure. However, characteristics of the market in which Tri Valley operated violate conditions critical to arbitrage models. In

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1. Compact support for yield is discussed in the appendix.

this section, we begin with a brief definition of arbitrage and note two classic arbitrage pricing models. We then consider the completeness our market of interest. Finally, we derive a set of bounds for the acreage-based contract's forward prices that are arbitrage-free in incomplete markets and independent of both individual preferences and firm cost structure.

Arbitrage

The concept of arbitrage, namely, risk-free profit with zero initial investment, is a highly intuitive economic concept and a necessary condition for many fundamental economic results. It can be shown, for example, that freedom from arbitrage is both a necessary and sufficient condition for the existence of a broad class of general price equilibria (Kreps).

Adapting Harrison and Pliska's (1981) notation, we define an arbitrage opportunity as the existence of an investment strategy h, that is, some portfolio of assets and/or contracts with payoff π_t^h at time t fulfilling the following weak arbitrage conditions,

(3) $\pi_0^h = 0, \quad \pi_T^h \geq 0, \quad \mathrm{E}\left[\pi_T^h\right] > 0$.

Here, subscripts 0 and T, denote the initial and terminal time periods, respectively. In words, (3) describes a portfolio with a zero initial investment and a payoff at expiration that is non-negative with probability one and has a strictly positive expected value. In addition, it is assumed that no funds will be withdrawn or injected over the trading period, that is, changes in the portfolio composition during the trading period will be financed only by gains or losses in the portfolio. Such a condition leads to a portfolio that is referred to as *self-financing.*

Classical Arbitrage Pricing Models and Completeness

Celebrated arbitrage pricing models, such as the Black-Scholes (Black and Scholes, Merton) and Term-Structure of Interest models (Cox, Dothan, Vasicek), provide prices for derivative contracts newly introduced to the market, such as the premium for a European call option, or the strike price for a standard forward contract, which are unique, free from arbitrage opportunities, and independent of individual preferences and firm cost structures. These models make use of the ingenious fact that, under certain conditions, it is possible to construct a self-financing portfolio consisting solely of assets or contracts already traded in the market, whose value is identical to the payoff of the contract of interest. Such a portfolio is called a *replicating* portfolio.

Unique prices from such arbitrage models depend critically on the ability to construct a self-financing replicating portfolio, which in turn requires that markets be frictionless and contain a sufficient number of traded assets, i.e., that markets be complete. The question of market completeness is essentially one of sufficient dimension, similar to the concept of identification in solving a system of equations. Many definitions of complete markets exist (Ethridge). Bjork offers a heuristic meta-theorem for considering whether or not a market is complete:

> Let M denote the number of underlying traded assets in the model excluding the risk free asset, and let R denote the number of random sources. Generically we then have the following (relation). The model is complete if and only if $M \geq R$.

In this context, an asset is said to be *traded* if it is continuously traded in a liquid market.

To illustrate this rule-of-thumb for our particular example, consider first the situation in which a farmer, at planting time, plans to plant, grow, harvest and sell tomatoes at a local well attended cash market. If growing practices and input costs are known at planting time, then the farmer faces two random sources, the tonnage of tomatoes that will be harvested and the price they will fetch at the market, i.e., yield and price. The number of traded assets is, for our purposes, zero, since there are no contracts in this market, and the tomato crop is not traded continuously throughout the year, but only just after harvest. Thus, the number of assets M is less than the number of random sources R, and the market is incomplete. In fact, this particular market is *perfectly* incomplete, since there is no hedging opportunity whatsoever.

Next, consider the situation in which a farmer contracts a portion of the tomato crop under the standard forward contract. Again, at planting time, the farmer plans to plant, grow, and harvest tomatoes. If the harvest is unexpectedly large, the farmer will fulfill the contract and sell the excess at the local cash market. If the harvest is unexpectedly small, the farmer will purchase just enough tomatoes at the local cash market to fulfill the contract. Again, the farmer faces two random sources, yield and market price. And again, the number of (continuously) traded assets is zero, since forward contracts are traded only at planting, and the not contracted (open) tomato harvest is traded only at harvest time. Thus, the market remains incomplete.

Finally, consider our particular market of interest. The local cash market is not well attended due to perishability, geographic dispersion, and the prevalence of contracting itself. Without the local cash market, forward contracts are difficult to fill when yields are low and there are few buyers when yields are high. So, instead of the standard forward contract, the processor agrees to purchase the entire harvest from the particular acreage for a predetermined price, i.e., the acreage-based rather than tonnage-based forward contract. With the price set, the number of random sources is reduced to one, namely yield. Like the standard forward contract, the acreage-based contract is traded only at planting time, and thus the number of continuously traded assets remains zero, and our market remains incomplete.

The Arbitrage Bounds

The inability to construct a perfectly replicating portfolio for the acreage-based forward contract might tempt one to assume that any forward price would be free from arbitrage opportunities. This would be a common though incorrect assumption. As we will show, it is possible to construct a portfolio with payoff π^h that always performs as well or better than the contract of interest, i.e., a *superreplicating* portfolio.

The superreplicating portfolio arises naturally from the concept of the dominating portfolio (Merton) and the existence of an interval of arbitrage-free prices (Harrison and Kreps). Superreplication has been used to derive price bounds in a variety of incomplete market frameworks, including the presence of transaction costs and restricted short selling (Bensaid *et al;* Constantinides; Rouge; Zariphopoulou). Here, we use a simple buy-and-hold strategy, like that used by Bensaid *et al.* to derive an analytical expression for the arbitrage bounds associated with the acreage-based contract. To our knowledge, we are the first to do so. In contrast to Bensaid et al., whose random source evolved according to a multi-period discrete-time binomial tree model with transaction costs, we consider a single period discrete-time frictionless model, but allow the random source to take on values in a continuous, closed interval along the non-negative real line.

For convenience we let the surplus price in the acreage-based contract be fixed and focus instead on the upper and lower arbitrage bounds on the base price, denoted p^{min} and p^{max}, respectively. We define these bounds in terms of a superreplicating portfolio h and the acreage-based contract payoff

$$(4) \qquad p^{min} = \inf\left\{p^{base} \in \Re_+ : \pi_0^h = 0, \quad \pi_T^h \geq \pi_T^{ac}\left(y_T; p^{base}, p^{surpl}, y^{targ}, C\right)\right\}.$$

Here, the lower arbitrage bound is the greatest lower bound on the base price such that, using only proceeds from the forward contract sale, a portfolio can be constructed whose value at expiration is no less than, i.e., *dominates*, the payoff associated with the resulting acreage-based forward (Merton). The upper bound is defined similarly. Prices outside the interval $\left(p^{min}, \quad p^{max}\right)$ lead to arbitrage opportunities, while prices inside the interval do not (Karatzas and Kou; Kreps).

Deriving the Interval for the Acreage-based Contract

As is standard for pricing purposes, we will treat Tri Valley's acreage-based contract as a new offering and denote its payoff at expiration $\pi_T^{tvg}\left(y_T; p^{tvg,\,base}, p^{tvg,\,surpl}, y^{targ}, C\right)$. In addition to this new offering, we will refer to a reference contract, already traded in our market of interest. Denote the payoff associated with this reference contract as $\pi_T^{ref}\left(y_T; p^{ref,\,base}, p^{ref,\,surpl}, y^{targ}, C\right)$. For simplicity, we will assume that the surplus price does not exceed the base price. Assume further, that Tri Valley's target yield y^{targ} is identical to that of the reference and lies between the upper and lower yield limits y^{min} and y^{max}.

If Tri Valley's base price is too high, we will conduct arbitrage by purchasing one Tri Valley contract and selling one reference contract. Thus, the superreplicating portfolio for Tri Valley's acreage-based contract is simply one reference contract. An opposite strategy can be used if the base price is too low. Because forward contracts require no initial investment, off-setting positions require no initial investment, and hence fulfill condition $\pi_0^h = 0$ in (3).

Given these assumptions, the base price for the Tri Valley contract must, to avoid an arbitrage opportunity, lie within the open interval (see Appendix)

$$(5) \qquad \left(p^{ref,\,base} + B^{-},\quad p^{ref,\,base} + B^{+}\right)$$

where B is given by,

$$(6) \qquad B = \left(p^{ref,\,surpl} - p^{ref,\,surpl}\right)\left(\frac{y^{min} - y^{targ}}{y^{targ} - y^{max}}\right)$$

a function of surplus prices and yield parameters.

In Tri Valley's case, the base and surplus prices are equal ($p^{base} = p^{surpl}$), which results in the arbitrage interval

$$(7) \qquad \left(p^{ref,\,base}\left(\frac{y^{targ} - y^{min}}{y^{max} - y^{min}}\right) + p^{ref,\,surpl}\left(\frac{y^{max} - y^{targ}}{y^{max} - y^{min}}\right),\ p^{ref,\,base}\right).$$

Here, the upper bound is simply the reference contract's base price, and the lower bound is a weighted average of the reference contract's base and surplus price. Derivation of this interval is similar to that of (5) and is left to the reader.

To illustrate, consider the case in which the Tri Valley's price for surplus production is greater than that of the reference contract ($p^{ref,\,surpl} < p^{tvg,\,surpl}$). A base price can then be identified for Tri Valley's contract which leads to payoffs that are greater than or equal to those associated with the reference contract. This result is illustrated in figure 2.3.

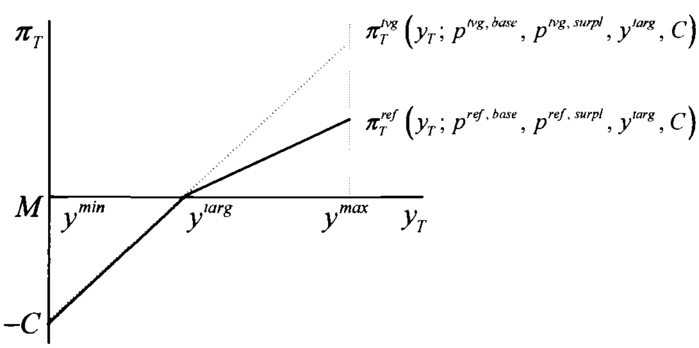

Figure 2.3. A Tri Valley Contract Payoff that Dominates the Reference Contract Payoff

For yields up to and including the target yield, payoffs are identical for the two contracts. At higher yields, however, the Tri Valley contract payoff is strictly greater than the reference contract payoff.

Alternatively, a base price can be identified for the Tri Valley contract which leads to payoffs that are less than or equal to those associated with the reference contract, as illustrated in figure 2.4.

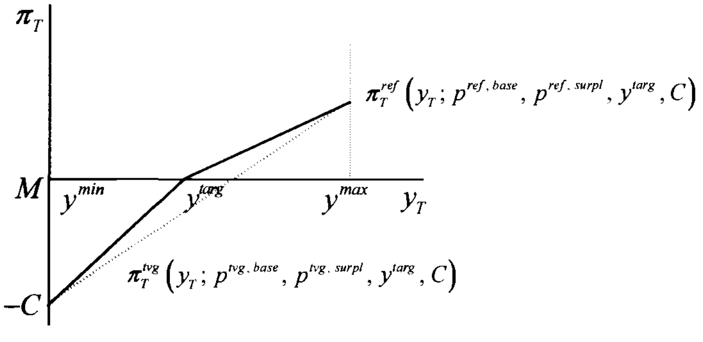

Figure 2.4. Tri Valley Payoff that is Dominated by the Reference Contract Payoff

At the upper and lower yield extremes, the two contract's payoffs are identical. Between these two points, the reference contract payoff is strictly greater than Tri Valley's.

Chambers shows that introducing an additional asset narrows, or at least does not widen, the arbitrage-free interval. Our intervals in (5) and (7) are consistent with this result, since, without the reference contract, no opportunity for arbitrage exists, and therefore, the arbitrage-free interval would be the entire price domain $\Re_+$.

Empirical Illustration of the Bounds

In this section, we illustrate the arbitrage-free bounds on Tri Valley's processing tomato forward contract prices from 1977 to 1996. The upper bound, which lies at the heart of our policy question is a widely published statistic, and is thus free from estimation error. Though not relevant to our specific policy question, we estimate the lower price bounds over the same period. The purpose of this second exercise is two-fold: First, we wish to illustrate a potential bid and ask price spread. Second, we wish to explore a contemporary set of empirical methods popular for estimating agricultural yield-related parameters in the insurance and derivative pricing literature. Toward this second purpose, we systematically identify a number of possibly fatal flaws inherent in the model specification and type of agricultural data commonly used. The empirical questions raised constitute a compelling set of future research questions.

Background

Tri Valley contracted raw-product with its members via acreage-based contracts, using a single stated price, $p^{tvg,\, base} = p^{tvg,\, surpl}$, for all produce harvested from a specified acreage. The arbitrage-free interval for Tri Valley's forward price, therefore, corresponds to equation (7). We denote this forward price p^{tvg}.

From 1977 to 1996, processing tomatoes represented approximately 40% of Tri Valley's total reported raw-product value. Investor-owned rivals, including Hunts, Heinz, Campbell, Ragu, and Del Monte used another special case of acreage-based contract in which surplus yield was not accepted, thus the surplus price was zero. We refer to this as the reference contract and denote the base price p^{ref}. This base price was set each year across the industry through collective bargaining with the growers' association CTGA.

Target delivery y^{targ} for the reference contract was set according to the average of the grower's best three yields over the past five years. This rule generated targets slightly higher than growers' historical average yields. In light of the upward yield trend in the tomato industry, this simple rule-of-thumb served as an innovative proxy for next year's expected yield. Contracts included quality premiums and penalties that varied somewhat according to processor needs. We do not model these quality differences, but instead assume that quality terms are the same across contracts.

We assume that agricultural input costs C are fixed. This is reasonable, since input prices are known at time of contracting as is contracted acreage. In addition, size and ripeness of processing tomatoes are determined rather late in the season. Accordingly, opportunity to vary input levels in response to weather conditions is limited. Finally, we assume the processor's opportunity to produce or source tomatoes is the same as the grower's. The payoff associated with these contracts is therefore symmetric between the two parties. This is reasonable, given the experienced operations staff at most processors, the mature and well capitalized processing tomato industry, and the availability of modern production technologies and equipment.

Based on these considerations and the assumption that the lower bound on yield is zero, the arbitrage-free interval to be estimated can be simplified as

$$(8) \qquad \left(p^{ref}\left(\frac{y^{targ}}{y^{max}}\right),\ p^{ref} \right).$$

As noted in the introduction, the upper arbitrage bound p^{ref} is of greatest interest to our question, because it is above this bound that the Tri Valley Board regularly set its forward price. As can be seen from (8), the upper bound is independent of yield-related terms, most notably, the support of the yield density, as well as the target yield specified in both the reference contract and the Tri Valley contract. This upper bound is a widely published industry statistic and is thus free from estimation error. In contrast, the lower bound retains two yield-related terms, and thus depends critically on the exact nature of the yield distribution.

Yield Distribution

In keeping with the approach proposed by Nelson and Preckel (1989) and recently examined by Ker and Coble (2003), yield is modeled as a 4-parameter Beta or Pearson Type I distribution, with a lower bound on yield y^{min} of zero. This distribution is desirable because it allows for consistent estimation of the theoretical upper bound on yield y^{max}. While other distributions could be considered, such as the truncated normal, the Beta family of distributions has flexible higher moments, of which symmetry is a special case of interest. Adapting the notation of Spanos (1999), the densities associated with the 4-parameter Beta family of distributions F is the set

(9)

$$F = \left\{ f\left(yield_t;\alpha,\beta,y^{min},y^{max}\right) = \frac{1}{B[\alpha,\beta]}\frac{\left(yield_t - y^{min}\right)^{\alpha-1}\left(y^{max} - yield_t\right)^{\beta-1}}{\left(y^{max} - y^{min}\right)^{\alpha+\beta-1}} \;;\alpha,\beta > 0 \right\}.$$

where B represents the Beta function (Abramowitz). The density is symmetric when α is equal to β, uni-modal for values of α and β greater than one, and non-negative for values of y^{min} greater than or equal to zero.

Because of the importance of the upper bound on yield, inference around this estimate is of particular interest. Affinity-based specification tests between parametric and non-parametric densities by Fan (1994) and Chen (1999) are now well suited to densities with compact support and perform well in much smaller samples than earlier specification test methods (White). However, such tests rely critically on *a priori* knowledge of the supports themselves, for example, efficiency scores in the [0, 1] interval, or test scores in the [0, 100] interval. Since the upper support for the density is our parameter of interest, merely raise the issue here and leave specification testing for yield distributions to future work.

Data

To estimate the yield-related terms, yield data was used from five counties in which Tri Valley operated is available for various years from each county's commissioner's office. A total of 83 observations are available for various years from 1965 through 2001, the majority from 1991 through 2000. To illustrate, processing tomato yields from Kern County, along with a quadratic time-trend, are plotted in figure 2.5.

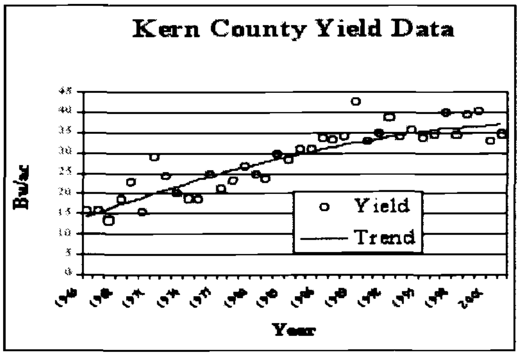

Figure 2.5. Kern County Yields with Quadratic Trend

The use of county-level yield data to explore farm-level issues is a common practice in contemporary work. For an overview, see Just and Weninger and Ker and Coble. This practice raises some serious concerns, since county-level yield averages are not sufficiently disaggregated to convey all farm-level information. In our sample, 42,000 acres of tomatoes were harvested on average per county in the five-county area studied, representing about 15% of California acreage in each county. A typical farm unit would instead include 300 to 900 acres. Because farm-level yield variations are not perfectly correlated with one another, over-aggregation should depress variance estimates and potentially biases density parameter estimates. Unfortunately, aggregation's effect on estimates of the Beta distribution's upper and lower bound has not, to our knowledge, been explored. Some attempts were made by the authors to explore this issue by further aggregating the data and re-estimating parameter values. However, convergence could not be achieved over a sufficient range of aggregation to provide useful information. Clearly, future work is warranted. Inference tests and estimator properties are disclosed and discussed hereafter under this strong qualification.

Fortunately, price data is less problematic. Reference contract prices are p^{ref} from 1966 through the present are published by the CTGA. Tri Valley contract prices p^{rg} from 1964 to 2001 are found in the company's audited *Additions to Financial Statements*. In our empirical example below, we consider the 20-year period from 1977 to 1996, the year before Tri Valley's financial difficulties were widely identified.

Estimation Methods

A two-stage model was used to estimate the four parameters of the yield distribution in equation (9). First, a quadratic time trend was fitted to the county-level yield data using Ordinary Least Squares (OLS). Yield parameters were estimated from the de-trended data by maximizing a Beta likelihood function.

The time trend was fitted with OLS, rather than Maximum Likelihood Estimation (MLE), because resulting gradient vectors are indeterminate when maximum yield parameter y^{max} is expressed as a function of time. As proposed by Just and Weninger, the first stage quadratic trend model was estimated after higher-order terms were found insignificantly different from zero:

$$\text{(10)} \quad \begin{aligned} \hat{y}_t = 37.37 &- 0.18\,t - 0.01\,t^2 \\ (1.01) &\quad (0.15) \quad (0.004) \end{aligned}$$

where, standard errors are shown in parentheses. The adjusted RP^{2}P was 0.77. Second stage maximum likelihood estimates were obtained from the de-trended data by solving

$$\text{(11)} \quad \max_{\alpha,\beta,y^{min},y^{max}} \left\{ \sum_{t=1}^{T} \ln\left(f\left(\hat{y}_t; \alpha, \beta, y^{min}, y^{max} \right) \right) \right\},$$

where f is the four-parameter Beta density defined in (9).

When observations are independently and identically distributed (IID), and when the choice of F is correct, MLE is consistent, asymptotically efficient, and tends to perform better in small samples than GMM. However, the variance of MLE estimates of the Beta distribution parameters begin to approach the Kramer-Rao lower bound only as sample sizes approach 1,000 or more (Carnahan). Two-stage MLE model variance regularly exceeds that of single-stage models (Greene). In this light, our sample size, n = 83, is quite small.

When the true density, say *g*, does not come from the assumed family of densities in *F*, as may often be the case, parameter solutions to the likelihood problem are known as Quasi-Maximum Likelihood estimates (QMLE). QMLE parameter estimates are consistent under the certain restrictive regularity conditions, but are less efficient in finite samples than when density specification is correct (White).

Parameter estimates from two-stage MLE models, given certain regularity conditions, are also consistent (Murphy and Topel). However, three characteristics of the specific model employed here cast profound doubt on the properties of our estimates. First, the consistency of our likelihood estimates from (11) depends on IID observations. It is straightforward to show that estimates obtained from our quadratic time trend in (10) are dependent[2]. Second, the asymptotic normality of our OLS estimates from (10) does not hold, since the time trend at powers above 1 does not converge (Greene). Third, the OLS model in (10) is correctly specified when the dependent and independent variables are from a jointly normal distribution. In contrast, our likelihood function (11) is correctly specified only when these same OLS residuals are distributed 4-parameter Beta. To resolve this contradiction, we would either need to establish the consistency of this particular two-stage QMLE model, or we would need to further explore the use of the Beta regression model suggested by Spanos (1999). Again, we raise these questions for future inquiry. Estimates and inference around these estimates are disclosed for the interested reader under these strong qualifications.

Results

Model estimates converged normally, using the BFGS, DFP, and Newton algorithms in the GAUSS programming environment. Convergence was not achieved under the BHHH algorithm. Parameter values and estimated significance levels were nearly identical under the

---

[2] Compare the off-diagonal elements of the classical error covariance matrix $\sigma^2 I$ to those of the quadratic time trend $\sigma^2 M$ where $M = X(X'X)^{-1}X'$.

three successful algorithms. Parameters obtained under BFGS are used below. Consistent with Just and Weninger's findings, a likelihood ratio test failed to reject the joint null hypothesis that the lower bound was zero ($y^{min} = 0$), and that the distribution was symmetric ($\alpha = \beta$), at the 95% confidence level. Incidentally, symmetry was rejected when the data were de-trended with a linear OLS model rather than the quadratic (10). The resulting estimates with standard errors in parenthesis are

$$\alpha = \beta = \underset{(10.68)}{68.31}$$

(12)

$$y^{max} = \underset{(0.70)}{74.77}$$

where shape parameters α and β from equation (9) are equal under symmetry, and the yield upper bound y^{max} is measured in bushels per acre.

Target Yield

Monte Carlo integration is used to estimate expected target yields y_t^{targ} over the 20-year period considered. Monte Carlo is a useful tool for estimating expected values of functions of random variables, when analytical solutions are not possible. See the appendix for a more detailed discussion of the Monte Carlo methods used.

Monte Carlo estimates of the target yields are shown in figure 2.5 along with the quadratic yield trend from (10).

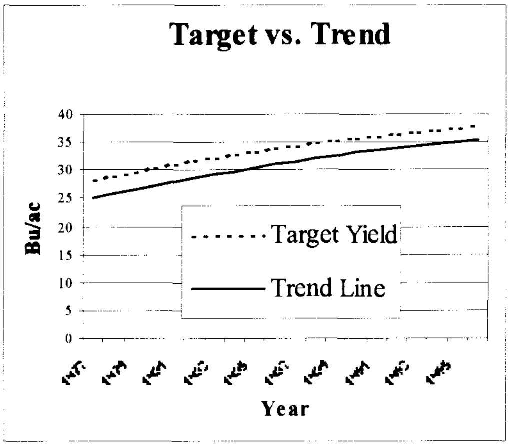

Figure 2.6. Target Yields with Trend

As can be seen from figure 2.6, the target yield lies consistently above the trend, suggesting that the IOF processor allows, on average, slightly greater deliveries than the growers' expected yield.

Example Arbitrage-Free Interval

With requisite parameters in hand, it is now straight-forward to calculate the arbitrage-free interval (8). For example, consider Tri Valley's interval in 1996. The upper price bound p_{1996}^{ref}, published by the CTGA, was \$60.33. The lower price bound is simply the ratio of the estimated target yield $\hat{y}_{1996}^{target}$ (37.95) over the estimated maximum yield $\hat{y}_{1996}^{max}$ (72.95), multiplied by the reference contract price p_{1996}^{ref}. This would indicate a lower price bound of \$31.38 and a spread between the upper and lower bound of \$28.95.

In words, a Tri Valley forward price equal to that of the reference contract price of \$60.33 means a tomato grower can achieve a positive profit with no initial investment, i.e., conduct arbitrage, by simply delivering tomatoes to Tri Valley and buying tomatoes under the reference contract. To verify this, consider first a yield below the target yield. In this case, the grower pays \$60.33 for the sub-contracted tomatoes and sells to Tri Valley at the same price, thus breaking even. Alternatively, when the yield is above the target yield, the grower pays nothing for tomatoes produced in excess of the target, but sells the entire yield to Tri

Valley at $60.33 per ton. This leads to a strictly positive profit. Thus, riskless profits are achieved.

A Tri Valley forward price equal to the lower bound of $31.38 means a tomato grower can conduct arbitrage by buying tomatoes under the Tri Valley contract and delivering tomatoes to the IOF processor for the $60.33 reference price. In the case of a yield below the target, the grower pays $31.38 for all sub-contracted tomatoes and sells them to the IOF for $60.33. Thus, yield shortfall results in a positive profit. In the case of a high yield, say the maximum theoretical yield of 72.95 bushels per acre, the grower again pays just under $31.38 for all 72.95 bushels, but sells only the target yield of 37.95 bushels per acre to the IOF. In this case, the higher reference contract price is just enough to off-set the disparity in delivery amounts, and the grower breaks even.

Comparison to Tri Valley's Reported Prices

Given the example above, reporting a forward price for the Tri Valley contract above the reference price of $60.33 per ton might seem preposterous. However, in 1996, Tri Valley paid members based on a "fair market" forward price of $60.88, $0.55 above the upper arbitrage bound. This price was reported to Tri Valley's lenders as well as the Internal Revenue Service. From 1977 through 1996, Tri Valley reported forward prices above the upper arbitrage bound in 16 of the 20 years, averaging, in 2001 dollars, $5.10 per ton above the bound. Figure 2.7 compares Tri Valley's nominal published prices to the corresponding nominal arbitrage-free upper bound.

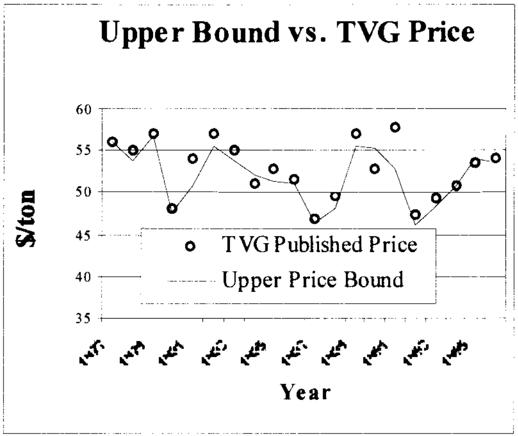

Figure 2.7. Nominal Tri Valley Published Prices and Upper Arbitrage-Free Bounds

By setting prices at or above the upper arbitrage bound, Tri Valley accepted considerably more yield risk than did its IOF competitors, but with no off-setting raw-product cost concession. Such a strategy is neither intuitively reasonable nor consistent with the notion of arbitrage.

From 1977 through 1996, Tri Valley's processing tomato costs were inflated by a total of $17 million, in 2001 dollars. If this policy were simultaneously pursued in all of its raw-product categories, Tri Valley's raw-product costs were over-reported, and thus, net earnings were under-reported, by $41 million or 15% during the same period. This amount represents 37% of Tri Valley's average working capital during the 1977 through 1996 period.

Circumventing Loan Restrictions

We have established that Tri Valley reported forward prices that were above the upper arbitrage bound and thus were unambiguously "too high." But, the question remains, "How can overstated raw-product prices impact liquidity?" In this section, we answer this question by examining the interplay between lender financial covenants and reported profits. We find that by overstating raw-product costs, Tri Valley's board-of-directors circumvented member payout restrictions, distributing more cash to members that would have been possible under more reasonable raw-product prices.

Lenders use financial covenants to encourage sound financial practices among borrowers and, failing that, to expedite the collection process. Violation of a covenant gives the lender the option to collect the loan, often triggering bankruptcy and asset liquidation under duress. Lenders employ a wide variety of financial covenants, including capital covenants, such as a minimum debt-to-equity ratio, and dividend covenants, such as a maximum dividend-to-earnings ratio. While many of Tri Valley's internal pricing and loan documents have been destroyed, an example of such a covenant survives in a 1996 loan agreement between Tri Valley and a primary lender. The lender restricts cash payments to members to no more than the declared raw-product value, plus the lesser of cooperative profits or 8% of the declared raw-product value.

The net effect of this restriction in profitable years was to cap total member payouts at 108% of member raw-product value. To circumvent the restriction, directors simply overstated forward contract prices, lowering reported profits and raising member raw-product values upon which the payouts were based.

Consider, for example, the profitable 1983 fiscal year. Had Tri Valley used the upper arbitrage bound to value member raw-product, it would have reported a profit of 22% and a total cash payment to members of 109.4% of economic value, a violation of the payout cap. By inflating raw-product value, however, the Board reported a profit of only 18% and a total cash payment to members of 106.8% of economic value, well under the 108% cap. The additional payout under this strategy was $1.6 million. Between 1977 and 1996, a total of $14.4 million was distributed to member growers in this manner.

Conclusion

We have defined a common class of acreage-based forward contracts and decomposed them into a convex combination of yield-based derivatives. We have explored the market characteristics that give rise to such contracts and found them to be incomplete. We then derived an analytical expression for bounds on acreage-based forward prices which are independent of agent preference or firm cost structure. Forward prices within these bounds do not permit arbitrage opportunities. Finally, we estimated these bounds for a set of acreage-based forward contracts used by Tri Valley Growers. We found published forward prices to be outside the arbitrage bounds, and thus to lead to grower arbitrage opportunities, that is, positive profits with no initial investment. The forward pricing strategy's financial impact was material.

Our analysis considered only the extent to which published prices exceeded the upper arbitrage bound. By definition the "fair market price" lies below the upper arbitrage bound. Tri Valley's published prices may have exceeded the actual fair market prices by a considerably wider margin than the $5.10 per ton that they exceeded the upper bound. Thus, the strategy impact disclosed in this analysis is a minimum impact. A fair market price well below the upper arbitrage bound would increase the impact of the Board's pricing policy many fold.

Our Tri Valley example illustrates how discretionary pricing allowed Tri Valley's Board, consisting almost entirely of grower-members, to pursue a member payout policy that appeared to comply with, but in truth violated certain lending restrictions. The Board's payout policy, at best, did not enhance Tri Valley's cash reserves or liquidity levels, and at worst, left the cooperative vulnerable to the future financial stresses ubiquitous to the competitive food processing sector.

We have left unexplored the economic triggers which lead a cooperative to favor strategies of liquidation over preservation. Identifying these triggers and the source of investor-owned firm's endurance could provide insight for future cooperative policy. Nor have we explored the discretionary pricing policy as a cooperative survival strategy when preservation remains optimal, as might have been the case in earlier years when so many cooperative's were forming. Such resilience amid economic hardship could have substantial value to local agricultural markets and communities.

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Appendix

Compact Support for Yield

The existence of an upper and lower bound on yield is relevant to our analysis. The compactness of the production set is one of the fundamental axioms underlying production theory. Compactness is consistent with the notion of scarcity and the law of diminishing returns, and given a fixed input and technology set provides the basis for the existence for the production possibilities frontier (Fare). Plant physiological models have provided further support for an upper limit on yield, dating back to the von Liebig's law of the minimum (Paris) and the Mitscherlich function (Dillon). Empirical work has provided further evidence for an upper limit on productive capacity, particularly in the context of fertilizer and weather considerations (Gallagher, Kaufmann, Spillman). Even Just and Weninger in their successful methodological critique of certain flexible skew models, acknowledge the "biological potential of the plant" (Kaufmann) as a motivation for compact support and offer no argument to the contrary.

Deriving the Arbitrage-free Interval

The farmer's payoff at expiration from the acreage-based contract can be defined and decomposed as

(2)

$$\begin{aligned}
\pi_T^{ac}\left(y_T;\, p^{base},\, p^{surpl},\, y^{targ},\, C\right) &= p^{base}\min\left(y_T,\, y^{targ}\right) + p^{surpl}\max\left(y_T - y^{targ},\, 0\right) \\
&= p^{base}\min\left(y_T - y^{targ}\right)^{-} + p^{surpl}\max\left(y_T - y^{targ}\right)^{+} + M \\
&= \pi_T^{put}\left(y_T;\, p^{base},\, y^{targ}\right) + \pi_T^{call}\left(y_T;\, p^{base},\, y^{targ}\right) + M
\end{aligned}$$

where C represents the cost of the agronomic practices specified and M is a positive constant based on target quantity, base price, and agronomic cost.

Given some fixed surplus price, we wish to derive the upper and lower bound for a base price associated with the Tri Valley contract. Consider first, the case in which $p^{tvg,\,surpl} \geq p^{ref,\,surpl}$. Then, the greatest lower bound, such that the payoff at expiration of the Tri Valley contract is dominated by the payoff associated with the reference contract for all yield levels is given by

$$\text{(A.1)} \quad p^{min} = \inf\left\{p^{tvg,base} \in \Re_+ : \ \pi_T^{ref} \geq \pi_T^{tvg}(y_T;\, p^{base}, p^{surpl}, y^{targ}, C)\right\}.$$

Exploiting the piecewise linear structure of the payoff function, we can re-write the problem as

$$\text{(A.2)} \quad p^{min} = \sup\left\{p^{tvg,base} \in \Re_+ : \left(p^{ref,\,base} - p^{tvg,\,base}\right)\left(y^{targ} - y^{min}\right) \geq \left(p^{tvg,\,surpl} - p^{ref,\,surpl}\right)\left(y^{max} - y^{targ}\right)\right\}.$$

Solving the inequality for the base price,

$$\text{(A.3)} \quad p^{min} = \sup\left\{p \in \Re_+ : \ p^{tvg,\,base} \leq p^{ref,\,base} + \left(p^{ref,\,surpl} - p^{tvg,\,surpl}\right)\frac{\left(y^{max} - y^{targ}\right)}{\left(y^{targ} - y^{min}\right)}\right\}.$$

At the supremum, (A.3) holds with equality

$$\text{(A.4)} \quad p^{min} = p^{ref,\,base} + \left(p^{ref,\,surpl} - p^{tvg,\,surpl}\right)\frac{\left(y^{max} - y^{targ}\right)}{\left(y^{targ} - y^{min}\right)}.$$

Since $p^{tvg,\,surpl} \geq p^{ref,\,surpl}$, we can re-write (A.4) as

$$\text{(A.5)} \quad p^{min} = p^{ref,\,base} + \left(p^{ref,\,surpl} - p^{tvg,\,surpl}\right)^{-}\frac{\left(y^{max} - y^{targ}\right)}{\left(y^{targ} - y^{min}\right)}.$$

Defining B^{-} as the non-positive portion of (A.5), we obtain

$$\text{(A.6)} \quad p^{min} = p^{ref,\,base} + B^{-}.$$

The least upper bound, such that the new contract's payoff at expiration is greater than the payoff associated with the reference contract at all yield levels, is given by

(A.7) $p^{max} = \sup\left\{p^{base} \in \Re_+ : \ \pi_T^{ref} \le \pi_T^{tvg}(y_T; p^{base}, p^{surpl}, y^{targ}, C)\right\}$.

Again, exploiting the payoff function's simple piecewise linear structure and the fact that $p^{tvg,surpl} \ge p^{ref,surpl}$, (A.7) can be rewritten

(A.8) $p^{max} = \inf\left\{p^{tvg,base} \in \Re_+ : \ p^{tvg,base} \ge p^{ref,base}\right\}$.

At the infimum, (A.8) holds with equality, so that

(A.9) $p^{max} = p^{ref,base}$.

The arbitrage-free interval, for the case in which $p^{tvg,surpl} \ge p^{ref,surpl}$, is then

(A.10) $\left(p^{ref,base} + B^-, \quad p^{ref,base}\right)$

where B is given by the following function of price and yield parameters

(A.11) $B = \left(p^{ref,surpl} - p^{tvg,surpl}\right)\left(\dfrac{y^{max} - y^{targ}}{y^{targ} - y^{min}}\right)$.

Repeating steps (A.1) through (A.10) for the alternative case, in which $p^{tvg,surpl} < p^{ref,surpl}$, we obtain the following arbitrage-free interval for the new contract's base price

(A.12) $\left(p^{ref,base} + B^-, \quad p^{ref,base} + B^+\right)$.

Monte Carlo Methods

To generate the Monte Carlo estimate of the target yield $\hat{y}_t^{target}$ in year *t*, we begin by generating five pseudo-random yield draws, one for each of the five previous years. Draws are taken from the four-parameter Beta density given in equation (9). Associated parameter values are given by (12) and time trend by (10). Next, we select the top three yield draws and average them to obtain our first observation, denoted $y_{t,1}^{target}$. Finally, we repeat this process *m* times and average the results. Adapting Campbell's notation , our models is

$$\text{(A.13)} \quad \hat{y}_t^{target} = \frac{1}{m}\sum_{j=1}^{m} y_{t,j}^{target}.$$

Monte Carlo estimates are consistent, asymptotically normal, and accurate to an arbitrary level that depends on *m* (Campbell). Confidence intervals can be readily calculated, as

$$\text{(A.14)} \quad \text{Prob}\left[y_t^{target} \in \left(\hat{y}_t^{target} - z,\ \ \hat{y}_t^{target} + z\right)\right] = 95\%, \quad z = \frac{1.96\,\sigma\left(y_{t,j}^{target}\right)}{\sqrt{m}}.$$

Equation (A.14) states that the true target yield y_t^{target} lies within the given interval with 95% probability. Because the standard deviation of the draws $\sigma\left(y_{t,j}^{target}\right)$ is not known, we estimate it using the same Monte Carlo integration method:

$$\text{(A.15)} \quad \hat{\sigma}^2\left(y_{t,j}^{target}\right) = \frac{1}{m}\sum_{j=1}^{m}\left(y_{t,j}^{target} - \hat{y}_t^{target}\right)^2$$

where $\hat{y}_t^{target}$ is given by equation (A.13). Solving equation (A.14) for *m*, we determine that 1,000,000 draws are required to obtain an accuracy level *z* of 0.001 bushels per acre at the 95% confidence level.

3. Adapting Cooperative Structure to the New Global Environment

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Adapting Cooperative Structure to the New Global Environment

Cooperatives flourish when competition is weak and decline when it is robust. Many agricultural cooperatives were, for example, formed during the Depression, while many struggle in today's competitive environment. Factors contributing to the present cooperative stress are many. Maturing capital markets and falling risk premia have eroded cooperatives' historical borrowing advantage. Declining transportation costs and trade barriers have expanded market boundaries. Healthy returns in equity markets have facilitated the capitalization of investor-owned firms, increasing the opportunity cost of holding non-tradable and non-dividend-bearing cooperative equity.

The notion that cooperatives should prosper when markets are poorly competitive has long been maintained as the competitive yardstick hypothesis. If cooperative formation is rational when market imperfections arise, we might wonder if dissolution is rational when the imperfections fade. In the present paper we show that the answer is yes. More generally, we show that cooperatives would wish to choose from a continuum of cooperative, new-generation, and investor-owned firm (IOF) models as market conditions and interest groups vary. For this purpose, we develop a general organizational framework of the agricultural processing firm, paying attention to the mechanisms distinguishing the cooperative from the IOF structure, the tension between active and retired members, and the influence of competition in raw and finished product markets. Optimality is considered first from the perspective of the cooperative member, in terms of maximizing the expected present value of net cash flows, then from the perspective of the policy maker, in terms of minimizing the probability of financial ruin.

We find, as markets become increasingly competitive, that a traditional cooperative encourages lower investment and higher probability of bankruptcy than does the IOF. However, member willingness to capitalize the cooperative improves abruptly as inactive members gain influence on the cooperative board or as the firm's financial structure merges toward the IOF model. Our findings are illustrated, using data from the internal records of Tri Valley Growers, which until its bankruptcy in 2000 was on of the nation's leading agricultural processing cooperative.

The Cooperative Problem

The traditional patronage-based agricultural processing cooperative raises equity capital from its grower members in return for the promise to process and market their raw agricultural products. At planting, the cooperative contracts with members to supply produce y_t from specified acreages. At delivery, the cooperative is obligated to pay some portion (1-*a*) of the "value" of the raw product in cash, the balance retained as cooperative equity. We denote the member's resulting net proceeds from farming as *f*. After processing and marketing activities are completed, the cooperative must return to its grower-members a portion (1-*b*) of processing returns π_t, the balance again retained as cooperative equity. We denote the member's resulting net proceeds from processing as *g*. Retained amounts are returned to members over a period of time *N*, known as the *revolve period*, provided the cooperative remains in business. We denote the member's proceeds from equity revolve as *h*.

The stream of cash flows faced by an *active* cooperative member, that is one currently supplying raw product, is then the sum of these three components *f*, *g*, and *h*. The member-director seeks the retain rates *a* and *b* that will maximize the sum of the expected net present values (ENPV) of the three cash flow components in the current year *t*; that is,

(1)

$$\max_{a,b}\left\{E\left[f\left(a;y_t,\mathbf{w}_t,r\right)+g\left(b;\pi_t,\mathbf{p}_t,r,\tau\right)+\sum_{k=t+1}^{N}h\left(a,b;y_t,\pi_t,\mathbf{w}_t,\mathbf{p}_t,r,\tau,N,1\left[s_{k-1}\right]\right)\right]\right\}$$

where vectors $\mathbf{w}_t$ and $\mathbf{p}_t$ are farming and processing prices, respectively, *r* is the discount rate, τ is the tax rate, and $1[s_{k-1}]$ is unity if the cooperative remains in business and zero otherwise.

Bankruptcy occurs when equity falls below zero, namely when the cooperative becomes *insolvent*, and we assume no further cash payments are made after that event. The cooperative declares bankruptcy just after its current cash payments to members. Problem (1) assumes current-year *t* is the active member's first and only farm production year and the entire crop is contracted through the cooperative.

Cooperative boards tend to be dominated by active members. Yet extending problem (1) to *inactive* members, namely to those who have retired from farming but continue on the board, is straightforward. Assume, again for the moment, that the inactive member's first and only farm production year was t-1. In the current year, then, f and g are zero and the inactive member's problem is

$$(2) \qquad \max_{a,b}\left\{E\left[\sum_{k=t+1}^{N} h\left(a,b;\, y_{t-1},\pi_{t-1},\mathbf{w}_{t-1},\mathbf{p}_{t-1},r,\tau,N,1\left[s_{k-1}\right]\right)\right]\right\}.$$

The Investor-owned Problem

The IOF raises equity capital by promising stockholders a perpetual annual share of net processing returns. This share, determined *ex post*, is paid to the stockholder as current dividend, the balance retained in the firm's equity account to preserve or enhance future earnings. Unlike in the cooperative model, the portion of earnings retained is not identified by stockholder name but intended rather to contribute to average unit stock value.

IOF investors need not be farm producers. For comparison with cooperatives, however, we begin by considering an active investor, that is, one who both farms under contract with and invests in the IOF processor. We will assume, also for comparison, that the cash portion a of raw-product value held back to purchase IOF shares is the same as in cooperative problem (1). The investor's problem is then to find the retain proportion b of processing returns that maximizes the ENPV of the three cash-flow components:

$$(3) \qquad \max_{b}\left\{E\left[f\left(a;\, y_t,\mathbf{w}_t,r\right)+g\left(b;\,\pi_t,\mathbf{p}_t,r,\tau\right)+\sum_{k=t+1}^{\infty} h\left(b;\,\pi_k,\mathbf{p}_k,r,\tau,1\left[s_{k-1}\right]\right)\right]\right\}.$$

Two features of problem (3) suggest, other things equal, that the IOF investor has a greater stake in the future well-being of the firm than does a cooperative member. First, the time horizon in (3) has risen from that in (1) and (2), namely from N to infinity. Second, future returns h no longer represent a return of retains as they did in (1) and (2) but instead a share of, and thus a stake in, future processing profits. Extending (3) to the inactive IOF investor is also straightforward. If t-1 were the inactive investor's first and only farm production year, current-year return f would be zero. Returns g and h would remain, affected only by the quantity of stock purchased the previous year. The inactive-investor problem is,

$$(4) \qquad \max_{b}\left\{E\left[g\left(b;\,\pi_t,\mathbf{p}_t,r,\tau\right)+\sum_{k=t+1}^{\infty} h\left(b;\,\pi_k,\mathbf{p}_k,r,\tau,1\left[s_{k-1}\right]\right)\right]\right\}.$$

Cooperative Survival: Choosing Between Two Processors

The survival of the cooperative structure in the face of IOF competition is best examined by considering an economy with two processors, a cooperative and an IOF, supplied by a number of farm producers. Farmers contract a fixed percentage v of their acreage with the cooperative and the remaining percentage $(1- v)$ with the IOF. The IOF has a stipulated degree of monopoly power in its finished-goods market and potential monopsony power in its raw-product market. Its economic return to processing may therefore be positive. The cooperative firm has, in contrast, no monopoly power in finished goods. But by threatening to withdraw deliveries to the IOF, farmers may bargain raw-product prices up until all the IOF's monopoly rents have been extracted.

Let us assume the investor-owned firm measures the credibility of the cooperative's boycott threat by the amount of capital the farmers invest in the cooperative. Farmers must then weigh the potential cost of foregoing cash payments from their cooperative against the benefit of higher raw-product prices from their IOF customer. Because the likelihood of cooperative insolvency declines as member equity rises, the probability of cooperative ruin therefore becomes a simple but effective measure of the farmer's bargaining power. To formalize this, let raw-product input price p_j range between monopsony price $p_j^{\min}$ and the price p_j^{max} at which the farmer extracts all the processor's monopsony rent. The raw-product price paid by the IOF is then determined as

(5) $$p_j = P[s_{k-1}]\,p_j^{min} + (1 - P[s_{k-1}])\,p_j^{max},$$

where $P[s_{k-1}]$ is the probability of cooperative ruin in year k. Because that probability lies between zero and one, (5) is a convex combination of the minimum and maximum price. If the cooperative is poorly capitalized, its chances of ruin will be comparatively high and the IOF's raw-product price offer comparatively low.

Farmers supplying both the cooperative and IOF want their cooperative to adopt the retain policy maximizing the present value of combined cash flows from both companies. For active growers, for example, the problem is to choose cooperative policy (a, b) maximizing the sum of percentage v of equation (1) and percentage $(1 - v)$ of equation (3), where raw-product prices, determined as in equation (5), are influenced by the extent of cooperative capitalization and thus by policy (a, b) itself.

In specifying (1) - (4), we explicitly represent the differences between the cooperative's and IOF's forward contract terms and prices. Processing returns vary randomly. Firm equity is specified as a Brownian motion with drift (Hull), which evolves according to processing revenues and costs as well as member payouts and retains. The probability of ruin is then a standard first-crossing-time problem (Klugman), in which bankruptcy occurs when the firm's equity falls below zero.

In addition to Tri Valley's internal financial records, we use data from the S&P Index of eleven investor-owned food processors (Standard and Poor's Corporation), four of which competed directly with Tri Valley in both the input and output markets. Agronomic data are taken from the five California counties in which these firms competed (Agricultural Commissioners). Payment parameters are taken from firm-specific forward contract policy, loan agreements, and bylaws. Because (1) and (3) involve expectations of functions of random variables, and no closed-form solutions exist for the resulting integrals, we solve these problems using both quadrature and Monte Carlo numerical integration methods in the GAUSS programming environment (Abramowitz and Stegun; Campbell). Estimation techniques, tests, and data summary statistics are available on request.

Influence of Market Structure

We first examine the welfare implications of alternative cooperative capitalization policies (a, b) under alternative assumptions about the IOF's finished-goods market power. For this purpose we consider active farmers only, and assume they contract one-half their produce with the cooperative and one-half with the IOF ($v = 0.5$). Active farmers are assumed to have no influence over the IOF's pricing policy. Numerical results are shown in figure 3.1.

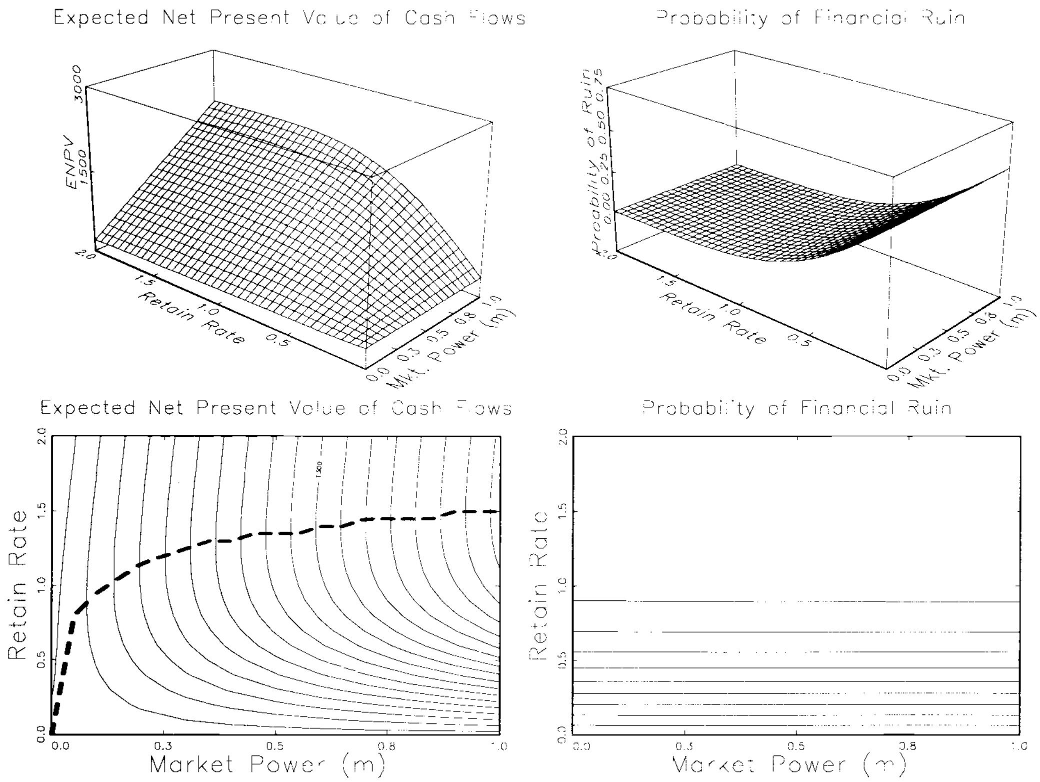

Figure 3.1. Member Welfare and Ruin Probability over a Range of Competitive Environments

In the upper-left panel of the figure, ENPV of cash flow is mapped over a continuum of finished-goods monopoly power, from perfect competition ($m = 0.0$) to pure monopoly ($m = 1.0$), and over a continuum of cooperative retain levels, from zero to twice the average 1977 - 1996 net Tri Valley Grower processing return. At a given degree of IOF monopoly power, the optimal retain rate is simply that associated with the highest ENPV elevation. In the upper-right panel of figure 3.1, the probability of cooperative ruin on or before the 15th year of operation is mapped over the same parameter values. Contour maps are provided below the corresponding surface maps.

The dashed line in the lower-left panel of figure 1 indicates the optimal retain rate, which rises as the IOF's monopoly power increases. The farm opportunity cost of another dollar the cooperative retains is always more than offset, that is, by the even greater raw-product-price bargaining power afforded with the IOF. When the IOF is a pure monopolist (*m*

= 1.0), the optimal retain is 150% of average annual processing return, implying members should contribute a portion of their own farm equity in addition to 100% of cooperative processing returns. Such would be necessary in the initial phases of cooperative organization, when the IOF's monopoly strength would presumably be highest. In more recent years at Tri Valley, when markets had become more competitive, retains averaged 43% of net return.

If, on the other hand, the IOF faces perfectly competitive finished-good prices ($m = 0.0$), the optimal retain is zero in figure 3.1 and the probability of ruin correspondingly high. The intuition is that, when finished-good competition is robust, the IOF's raw-product prices are similar to those paid by the cooperative. Without the reward of inducing higher IOF raw-product prices, the only reason to sustain cooperative investment is to preserve previously retained member equity. Herein lies the catch, for if retains really are zero, active members have no investment in the cooperative and its failure is inconsequential to them. Thus, the competitive yardstick hypothesis is verified.

Importantly, we find that the cooperative retains no equity when the IOF is a competitor in finished goods ($m = 0.0$) and the coop can manipulate both retain a (charged to raw product) and retain b (charged to processing return). The advantages of a dual retain policy are intriguing. By setting b at unity, for example, and a at the appropriate *negative* value, the cooperative can absorb an entire processing loss while paying members the full market value of their raw product, i.e., a negative net retain. Thus, cooperative ruin actually can relieve members of financial obligations provided that lenders are sufficiently unaware of true raw-product market value. Such an event contributed to the Tri Valley Grower bankruptcy. In the wake of its unexpectedly high 1997 processing loss, cash payments to members exceeded member raw-product value net of the loss, so the net retain was negative. That amount became a receivable from members, but the firm's failure in 2000 occurred before the receivable could be fully collected.

Influence of Board Composition

Let us consider now the interests of inactive as well as active farmers. Because active farmers supply raw product only in the current period and inactive ones only in the previous period, a convex combination of the active- and inactive-farmer problems allows us to examine the tradeoff between current and future cash-flow objectives. Imagine a weighting parameter z with values between zero and unity, zero corresponding to a completely active farmer and unity to a completely inactive one. The farmer is, in general, partially inactive, and z may be used to represent the proportion of the cooperative board occupied by inactive-member interests. Farmers continue as before to ship v percent of their produce to the cooperative and $(1 - v)$ to the IOF. Fortunately, the choice problem of this weighted-average farmer can be characterized as convex combinations of (1) - (4), taking (5) into account.

The impact on the cooperative's retain policy of the share of its board occupied by inactive-member interests is, as we shall see, of great policy significance. In examining that impact, we may assume any degree of competition we wish in the IOF's finished-good markets. For illustration in figure 3.2, such markets are assumed to be perfectly competitive.

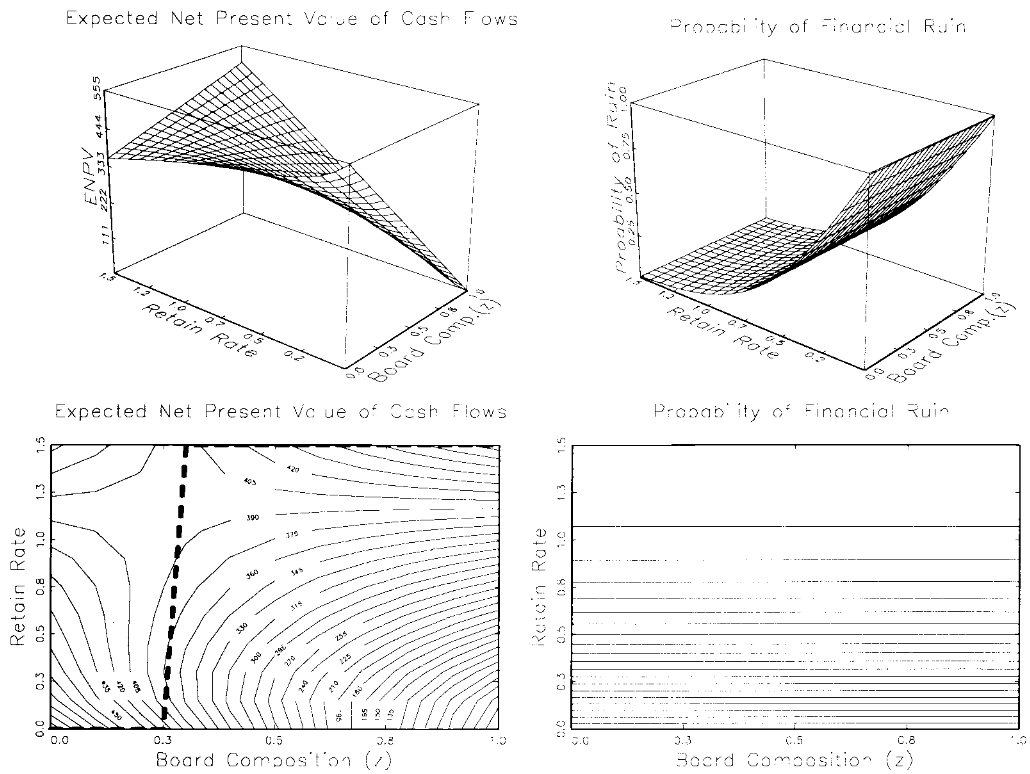

Figure 3.2. Member Welfare and Ruin Probability over a Range of Board Compositions

In the upper-left panel of figure 3.2, the farmer's expected net present value of the cash flow is mapped over a range of board compositions, from completely active ($z = 0.0$) to completely inactive ($z = 1.0$), and between a zero retain rate and one equaling Tri Valley's entire average annual net processing return (1.0). In the upper-right panel, the probability of cooperative ruin before the final year of the revolve period, in Tri Valley's case the 8th year of operation, is mapped over the same parameter values.

When few inactive members sit on the cooperative board (z is low), the expected NPV of future cash flows to our weighted-average farmer decreases monotonically as the cooperative's retain rate is boosted. That is, the board will vote for a zero-retain policy. When many inactive members sit on the board (z is high), however, expected returns instead rise monotonically with higher retains and the board will vote to hold the entire net processing profit on the company's books. At middling board compositions, ENPV first declines, then rises, as the retain rate is boosted; but it invariably reaches its peak at the point where the entire processing profit is retained. As the dashed line in the lower-left panel indicates, the switch from a zero to a 100% optimal retain is a bang-bang one, occurring when inactive board membership reaches approximately 28% ($z = 0.28$). Optimal bankruptcy probability, shown on the right side of figure 3.2, therefore is bang-bang as well and changes from nearly 100% to less than 1% when inactive board membership reaches the critical 28% level. The intuition is clear: active members have no incentive to keep their cooperative afloat because it extracts no additional raw-product income from the IOF processor. Inactive members do want to keep the cooperative afloat because its bankruptcy would eliminate their invested earnings.

The severe nature of the conflict between newer- and older-member interests would be predictive of a rather unstable financial policy. But the cooperative's patronage-based philosophy usually ensures active members a prominent if not exclusive presence in board decisions. Finite revolve periods N, whether fixed at, for example, eight years or varying with the member's recent patronage history, serve to reduce the member's financial interests in board affairs as time proceeds unless the member continues to patronize cooperative facilities. This is why ENPV falls in figure 2 as active members depart from the board, and suggests in a highly competitive environment that cooperatives normally will follow their active members' incentive to seek bankruptcy.

Influence of Cooperative Financial Structure: New-Generation Cooperatives

An equally important issue in modern cooperative policy is the extent of the cooperative's willingness to assume an IOF-style financial structure. The movement toward New-Generation cooperatives consists largely of such flexible adoption of IOF features, especially of the permanence and tradability of members' equity investment but also of the tonnage-based raw product contracts that investor-owned firms offer and the ordinary tax liabilities they typically face. Broadly speaking, we can represent any combination of cooperative and IOF structure we wish by considering a single firm and again forming the weighted sum of v times cooperative problems (1) and (2) and (1 - v) times respective IOF problems (3) and (4), so that $v = 0$ correspond to the traditional cooperative and $v = 1$ to the IOF. This requires explicitly modeling the transition from acreage- to tonnage-based forward contract terms and forward prices, as well as cooperative to IOF tax treatment, dividend, and equity revolve policy (Cross and Buccola). Parameter v was held at 0.5 in the above finished-goods market power discussion and at zero in the board composition discussion.

In permitting New-Generation parameter v to vary continuously, we arbitrarily hold inactive members to 15% of the cooperative board and again focus on a purely competitive finished-goods market ($m = 0.0$). Results are shown in figure 3.3.

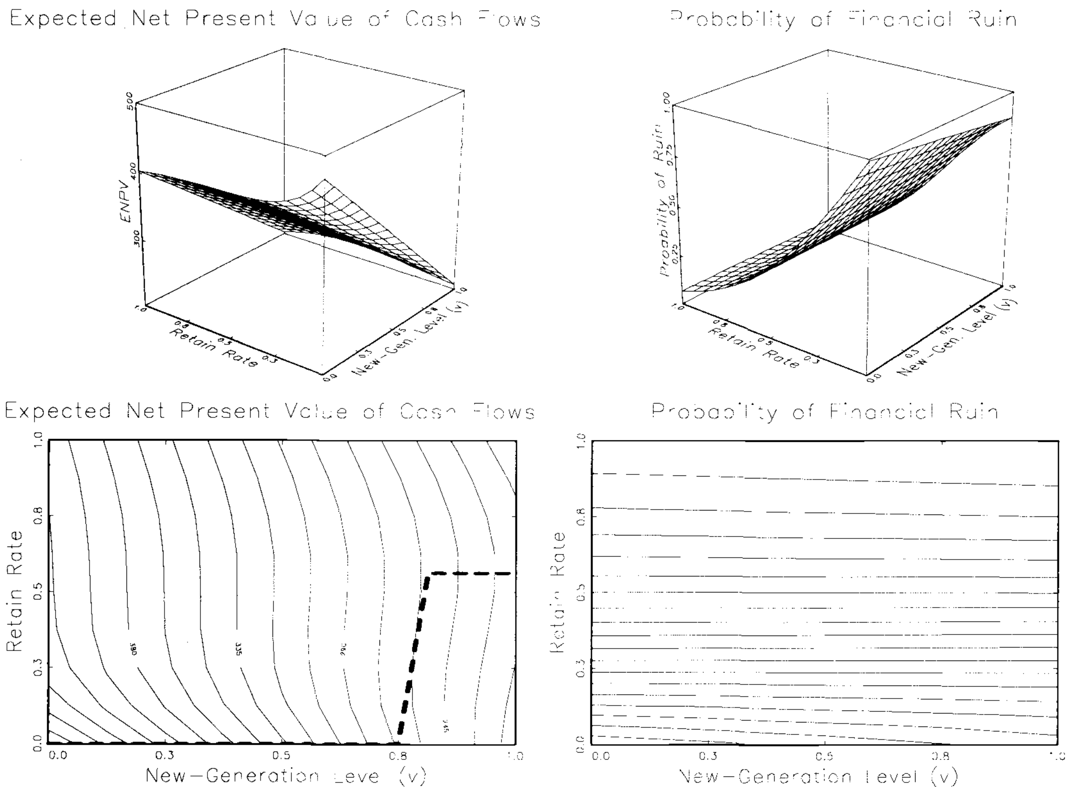

Figure 3.3. Member Welfare and Ruin Probability over a Range of New-Gen. Structures

In the upper-left panel of the figure, ENPV of farmer cash flow is mapped over the complete range of New-Generation possibilities and from a retain rate of zero to 100% of Tri Valley's average annual processing return. In the upper-right panel, the probability of cooperative ruin on or before the 15th year of operation is mapped over the same parameter values.

Just as in figure 3.2, member welfare in the traditional cooperative, represented by ENPV of member cash flows, monotonically falls as the retain rate rises, indicating a zero optimum retain under perfectly competitive conditions. But as the dashed line in the lower left hand panel reveals, the optimum abruptly rises to a 56% optimal equity retain when the New-Generation firm becomes approximately 80% IOF ($v = 0.80$). Interestingly, the indicated optimal retain level is close to investor-owned food processors' historical mean retain rate of 57% of annual earnings (Standard and Poor's Corporation). Because the probability of ruin falls monotonically as the retain rises (upper-right panel of figure 3), the pure IOF achieves a much lower chance of bankruptcy than does the traditional cooperative firm.

Why, then, in a competitive finished-goods environment do cooperatives not adopt a New Generation structure? In practice, capitalization shifts often meet with strong board resistance. In 1996, Tri Valley Growers' CEO tried introducing a number of New-Generation modifications to the firm's capital subscription and retirement plan. The proposals were popular with lenders but resisted by the largely active board of directors. The reason why lies in the slope depicted in the upper-left panel of figure 3. Member ENPV declines dramatically as the firm adopts New Generation policies because, in order to enhance members' interest in the cooperative's long-term financial health, more of their wealth must be invested in the cooperative and hence deferred to the future. In its limiting state as an IOF, the New Generation cooperative is governed entirely in the interests of inactive members, those with a stake only in the firm's permanent value and not in the welfare of its farmer suppliers. Put differently, members' interests in grower cash payments conflict with their long-run interests as equity shareholders. This conflict is the essential tension underlying New Generation cooperative design.

Conclusion and Extensions

As competitive yardstick theorists claim, farmers enhance future cash flows by capitalizing their cooperatives in response to imperfectly competitive markets. During the past three decades, however, the U.S. economy has moved decisively in the competitive direction. Trade restrictions have been reduced, freight markets deregulated, and local market contestability enhanced through improved preservation and transportation technology. It is important under these conditions to examine cooperative behavior in a competitive rather than a monopoloid environment.

When we do, and assume reasonably that active members continue to dominate board decisions, we find that the optimal cooperative capitalization rate is zero, with consequently high bankruptcy probabilities. Increasing the proportion of inactive members on cooperative boards, or moving toward more IOF financial structures under the aegis of New Generation cooperatives, will reduce these chances of ruin. But prospects for doing so remain questionable as long as boards remain in the hands of active members, for whom corrective policies require a direct sacrifice in personal wealth.

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Appendix

In this appendix, we specify the four underlying problems, motivate two key claims, and elaborate on the probability of ruin problem and the Monte Carlo integration methods used.

Active Cooperative Member Problem

The traditional patronage-based agricultural processing cooperative raises equity capital from its grower members in return for the promise to process and market their raw agricultural products. At planting, the cooperative contracts with members to supply produce from specified acreages. At delivery, the cooperative is obligated to pay some portion of the "value" of the raw product in cash. After processing and marketing activities are completed, that is *ex post*, the cooperative must return to its grower-members the remainder of the value of their raw product along with any processing profit or loss. These amounts may be returned as cash or scrip, the latter retained in the form of operating capital and hereafter referred to as *retains*. For convenience, we distinguish between the retain rates associated with raw product value and those associated with processing profit or loss, denoting them a and b respectively. Retains are paid to members over a period of time, known as the *revolve period*, provided the cooperative remains in business.

The stream of cash flows faced by an *active* cooperative member, that is one currently supplying raw product, may be separated into four components. The first consists of cash payments from her *farm production* activities in current year t, net of amounts retained as cooperative equity. The second consists of cash payments from *cooperative processing* activities in year t, again net of any retain. The third and fourth consist of the future revolve of the retained portions from previous farming and processing activities, respectively.

Let us assume that this active member would, on the cooperative board of directors, seek policies promoting her own interest. The active member-director would then seek the retain rates a^{coop} and b^{coop} that maximize the sum of the expected net present values (ENPV) of her four cash flow components; that is,

$$\max_{a^{coop},\, b^{coop}} \left\{ \mathrm{E} \left\{ e^{-r} \left[\left(1 - a^{coop}\right) \mathbf{p}_t^{fwd,coop} \mathbf{y}_t^{fwd} - \mathbf{w}_t^{ag} \mathbf{x}_t^{ag} \right] \right. \right.$$

$$+ e^{-r} \left(\frac{1}{A_t^{coop}} \right) \left(1 - b^{coop}\right) \left(1 - t^{coop}\right) \left(\mathbf{p}_t^{coop} \mathbf{y}_t^{coop} - \mathbf{w}_t^{coop} \mathbf{x}_t^{coop} \right)$$

(1)

$$+ \sum_{k=t+1}^{t+N} e^{-r(k-(t-1))} \frac{1}{N} 1_{\left[S_{k-1}^{coop}\right]} a^{coop} \mathbf{p}_t^{fwd,coop} \mathbf{y}_t^{fwd}$$

$$\left. \left. + \sum_{k=t+1}^{t+N} e^{-r(k-(t-1))} \frac{1}{N} 1_{\left[S_{k-1}^{coop}\right]} \left(\frac{1}{A_t^{coop}} \right) b^{coop} \left(1 - t^{coop}\right) \left(\mathbf{p}_t^{coop} \mathbf{y}_t^{coop} - \mathbf{w}_t^{coop} \mathbf{x}_t^{coop} \right) \right\} \right\}.$$

The problem in problem (1) assumes current-year t is the active member's first and only farm production year and that she contracts her entire crop through the cooperative. The first term in (1) is a function of the discount rate r, retain rate a^{coop} from farming proceeds, and farm input prices and quantities $\mathbf{w}_t^{ag}$ and $\mathbf{x}_t^{ag}$, respectively, and farm contracted output prices and quantities $\mathbf{p}_t^{fwd,coop}$ and $\mathbf{y}_t^{fwd}$, respectively. Specification of the output prices and quantities conforms to the definition of the acreage-based forward contract, discussed later. The second term in (1) incorporates total contracted acreage A_t^{coop}, retain rate b^{coop} from processing proceeds, net income tax rate t^{coop}, as well as processing input prices and quantities $\mathbf{w}_t^{coop}$ and $\mathbf{x}_t^{coop}$, respectively, and processing output prices and quantities $\mathbf{p}_t^{coop}$ and $\mathbf{y}_t^{coop}$, respectively. The third term is indexed over the revolve period of N years and depends on the indicator function $1_{\left[S_{k-1}^{coop}\right]}$, which is unity if the cooperative was still in business at the end of the previous period and zero otherwise.

In words, the first line in problem (1) represents the cash flow in the current-period from farming activities, less some amount retained as cooperative equity. The second line is the member's share of cash proceeds from processing activities, again, less some amount retained as cooperative equity. The third line represent the return of the cash retained from farming activities in line one over a finite number of future periods, so long as the cooperative remains in business. Similarly, line four is the future return of cash retained from processing activities.

Inactive Cooperative Member Problem

Cooperative boards tend to be dominated by active members. Yet extending problem (1) to *inactive* members, namely to those who have retired from farming but continue on the board, is straightforward. Assume, again for the moment, that the inactive member's first and only farm production year was t-1 and that she contracts her entire crop through the cooperative. In the current year, then, the first and second lines in (1) are zero because the inactive member does not farm and is thus not entitled to a share of processing returns. The inactive member's problem is

$$\max_{a^{coop},b^{coop}} \Bigg\{ \mathrm{E}\Bigg\{ e^{-r}\frac{1}{N}a^{coop}\mathbf{p}_{t-1}^{fwd,coop}\mathbf{y}_{t-1}^{fwd}$$

$$+e^{-r}\frac{1}{N}\left(\frac{1}{A_{t-1}^{coop}}\right)b^{coop}\left(1-t^{coop}\right)\left(\mathbf{p}_{t-1}^{coop}\mathbf{y}_{t-1}^{coop}-\mathbf{w}_{t-1}^{coop}\mathbf{x}_{t-1}^{coop}\right)$$

(2)

$$+\sum_{k=t+1}^{t+N} e^{-r(k-(t-1))}\frac{1}{N}1_{\left[S_{k-1}^{coop}\right]}1_{[k-t\le N]}a^{coop}\mathbf{p}_{t-1}^{fwd,coop}\mathbf{y}_{t-1}^{fwd}$$

$$+\sum_{k=t+1}^{t+N} e^{-r(k-(t-1))}\frac{1}{N}1_{\left[S_{k-1}^{coop}\right]}1_{[k-t\le N]}\left(\frac{1}{A_{t-1}^{coop}}\right)b^{coop}\left(1-t^{coop}\right)\left(\mathbf{p}_{t-1}^{coop}\mathbf{y}_{t-1}^{coop}-\mathbf{w}_{t-1}^{coop}\mathbf{x}_{t-1}^{coop}\right)\Bigg\}\Bigg\}.$$

The indicator function $1_{[k-t\le N]}$ truncates revolve payments when the member's particular revolve period has expired. For comparability to problem (1) the first two lines of problem (2) correspond to the member's current year proceeds, the second two lines to future period proceeds. In words, the first line of problem (2) represents the return of a portion of the cash retained from the previous year's farming activities. The second line of problem (2) represents the return of a portion of the cash retained from the previous year's processing activities. The third line represents the return of the remaining cash retained from the previous year's farming activities over a finite number of future periods, so long as the cooperative remains in business. Similarly, line four is the future return of cash retained from the previous year's processing activities.

Active Investor Problem

The investor-owned firm (IOF) raises equity capital by promising stockholders a perpetual annual share of net processing returns. This share, determined *ex post*, is paid to the stockholder as current dividend, the balance retained in the firm's equity account to preserve or enhance future earnings. Unlike in the cooperative model, the portion of earnings retained is not identified by stockholder name but intended rather to contribute to average unit stock value.

IOF investors need not be farm producers. For comparison with cooperatives, however, we begin by considering an active investor, i.e., one who farms under contract with the IOF she partly owns. We will assume again, for comparison, that the cash portion a^{coop} of raw-product value held back to purchase IOF shares is the same as in cooperative problem (1). The investor's problem is then to find the retained proportion b^{iof} of processing returns maximizing the ENPV of her three cash-flow components:

$$\max_{b^{iof}}\Big\{\mathrm{E}\Big\{e^{-r}\Big[\big(1-a^{coop}\big)\mathbf{p}_t^{fwd,iof}\mathbf{y}_t^{fwd}-\mathbf{w}_t^{ag}\mathbf{x}_t^{ag}\Big]$$

$$+e^{-r}\left(\frac{a^{coop}\mathbf{p}_t^{fwd,iof}\mathbf{y}_t^{fwd}}{p_t^{eq,iof}\,y_t^{eq,iof}}\right)\big(1-b^{iof}\big)d^{iof}\Big[\big(1-t^{iof}\big)\big(\mathbf{p}_t^{iof}\mathbf{y}_t^{iof}-\mathbf{w}_t^{iof}\mathbf{x}_t^{iof}\big)+\mathbf{w}_t^{fwd,iof}\mathbf{x}_t^{fwd,iof}\Big]$$

(3)

$$+\sum_{k=t+1}^{\infty}e^{-r(k-(t-1))}1_{\left[S_{k-1}^{iof}\right]}\left(\frac{a^{coop}\mathbf{p}_t^{fwd,iof}\mathbf{y}_t^{fwd}}{p_t^{eq,iof}\,y_k^{eq,iof}}\right)\big(1-b^{iof}\big)$$

$$d^{iof}\Big[\big(1-t^{iof}\big)\big(\mathbf{p}_k^{iof}\mathbf{y}_k^{iof}-\mathbf{w}_k^{iof}\mathbf{x}_k^{iof}\big)+\mathbf{w}_k^{fwd,iof}\mathbf{x}_k^{fwd,iof}\Big]\Big\}.$$

Here, the second term is a function of the market value of the firm $p_t^{eq,iof}\,y_t^{eq,iof}$, i.e., market price per share times the number of shares outstanding. Because cash dividends are no longer restricted to a portion of net processing returns alone, but rather to a portion of *free-cash-flow*, we allow the IOF to payout up to some portion d^{iof} of the combined after-tax net processing return plus the total value of raw product under contract $\mathbf{w}_t^{fwd,iof}\mathbf{x}_t^{fwd,iof}$. A number of relatively large variables, such as gross revenue or total equity capitalization would be reasonable approximation for this free-cash-flow proxy.

Two features of problem (3) suggest, other things equal, that the IOF investor has a greater stake in the future well-being of her firm than does a cooperative member. First, the time horizon in (3) has risen from that in (1) and (2), namely from N to infinity. Second, future returns no longer represent a return of retains as they did in (1) and (2) but instead a share of, and thus a stake in, future processing profits.

Inactive Investor Problem

Extending (3) to the inactive IOF investor is also straightforward. If t-1 were the inactive investor's first and only farm production year, and her entire crop were contracted through the firm, current-year net farming proceeds would be zero. The dividend terms would remain, affected only by the quantity of stock purchased the previous year. The inactive-investor problem is, then

(4)

$$\max_{b^{iof}}\left\{\mathrm{E}\left\{e^{-r}\left(\frac{a^{coop}\mathbf{p}_{t-1}^{fwd,iof}\mathbf{y}_{t-1}^{fwd}}{p_{t-1}^{eq,iof}y_t^{eq,iof}}\right)\left(1-b^{iof}\right)d^{iof}\left[\left(1-t^{iof}\right)\left(\mathbf{p}_t^{iof}\mathbf{y}_t^{iof}-\mathbf{w}_t^{iof}\mathbf{x}_t^{iof}\right)+\mathbf{w}_t^{fwd,iof}\mathbf{x}_t^{fwd,iof}\right]\right.\right.$$

$$+\sum_{k=t+1}^{\infty}e^{-r(k-(t-1))}1_{\left[s_{k-1}^{iof}\right]}\left(\frac{a^{coop}\mathbf{p}_{t-1}^{fwd,iof}\mathbf{y}_{t-1}^{fwd}}{p_{t-1}^{eq,iof}y_k^{eq,iof}}\right)\left(1-b^{iof}\right)$$

$$\left.\left.d^{iof}\left[\left(1-t^{iof}\right)\left(\mathbf{p}_k^{iof}\mathbf{y}_k^{iof}-\mathbf{w}_k^{iof}\mathbf{x}_k^{iof}\right)+\mathbf{w}_k^{fwd,iof}\mathbf{x}_k^{fwd,iof}\right]\right\}\right\}.$$

Acreage-Based Forward Contract Specifications

The acreage-based forward contract is an obligation to pay a base price $p_t^{base,i}$ for any produce harvested from a particular acreage, as specified by contracting firm *i*, up to a target level y_t^{targ}, and then some surplus price $p_t^{surpl,i}$ for any additional harvested produce. The price and delivery terms are then

$$(5)\quad \mathbf{p}_t^{fwd,i} = \begin{bmatrix} p_t^{base,i} & p_t^{surpl,i} \end{bmatrix}, \qquad \mathbf{y}_t^{fwd} = \begin{bmatrix} y_t^{targ} + \left(y_t - y_t^{targ}\right)^- \\ \left(y_t - y_t^{targ}\right)^+ \end{bmatrix}.$$

Claims

In this section we formalize two claims. First, under competition, the optimal traditional patronage-based cooperative equity retain rates a^{coop} and b^{coop} are zero. Second, the optimal investor-owned firm equity retain rate b^{iof} is strictly greater than that of the cooperative, under a first order condition, which we derive.

Claim 1: Given the traditional patronage-based equity retain and revolve policies, active directors, and competitive raw and processed goods markets,

$$(5)\qquad a^{coop*}, b^{coop*} = 0$$

where $\left\{a^{coop*}, b^{coop*}\right\}$ is the solution set to the active member-director's maximization problem from (1),

$$\arg\max_{a^{coop},b^{coop}} \mathrm{E}\Big\{e^{-r}\Big[\big(1-a^{coop}\big)\mathbf{p}_t^{fwd,coop}\mathbf{y}_t^{fwd} - \mathbf{w}_t^{ag}\mathbf{x}_t^{ag}\Big]$$

$$+e^{-r}\left(\frac{1}{A_t^{coop}}\right)\big(1-b^{coop}\big)\big(1-t^{coop}\big)\big(\mathbf{p}_t^{coop}\mathbf{y}_t^{coop} - \mathbf{w}_t^{coop}\mathbf{x}_t^{coop}\big)$$

(6)

$$+\sum_{k=t+1}^{t+N} e^{-r(k-(t-1))}\frac{1}{N}1_{\left[S_{k-1}^{coop}\right]}a^{coop}\mathbf{p}_t^{fwd,coop}\mathbf{y}_t^{fwd}$$

$$+\sum_{k=t+1}^{t+N} e^{-r(k-(t-1))}\frac{1}{N}1_{\left[S_{k-1}^{coop}\right]}\left(\frac{1}{A_t^{coop}}\right)b^{coop}\big(1-t^{coop}\big)\big(\mathbf{p}_t^{coop}\mathbf{y}_t^{coop} - \mathbf{w}_t^{coop}\mathbf{x}_t^{coop}\big)\Big\}.$$

Claim 2: Under the same conditions as in claim 1, the optimal investor-owned firm retain rate is strictly positive and thus strictly greater than the cooperative retain rate $b^{iof*} > b^{coop*}$ under the following condition,

(7) $$\sum_{k=t+1}^{t+N} e^{-r(k-(t-1))}\Big(\mathrm{P}'\big[S_{k-1}^{iof}\big] - \mathrm{P}\big[S_{k-1}^{iof}\big]\Big) > e^{-r}H$$

where H is the trade-off between the current dividend and expected future dividend

(8) $$H = \frac{\Big[\big(1-t^{iof}\big)\big(\mathbf{p}_t^{iof}\mathbf{y}_t^{iof} - \mathbf{w}_t^{iof}\mathbf{x}_t^{iof}\big) + \mathbf{w}_t^{fwd,iof}\mathbf{x}_t^{fwd,iof}\Big]G}{\mathrm{E}\Big[\big(1-t^{iof}\big)\big(\mathbf{p}_{t+1}^{iof}\mathbf{y}_{t+1}^{iof} - \mathbf{w}_{t+1}^{iof}\mathbf{x}_{t+1}^{iof}\big) + \mathbf{w}_{t+1}^{fwd,iof}\mathbf{x}_{t+1}^{fwd,iof}\Big]G}.$$

The first order condition in (7) has a familiar economic interpretation, that is an increase in the retain rate is preferred if the marginal benefit (LHS) is greater than the marginal cost (RHS). In our particular case, the benefit (LHS) is the present value gain in expected future cash dividends. The cost (RHS) is the loss in current-period cash dividends resulting from an increase in the retain rate, i.e., an increase in the amount of cash disbursed in the current-period.

We can see from (8) that the optimal equity retain rate is decreasing in current-period net processing returns. This suggests that an unexpected windfall in the current year induces the investor-director to take the cash today by lowering the retain rate (increasing the dividend).

Similarly, from the LHS of (7) we can see a higher retain rate becomes more desirable if additional equity has a greater effect on the chances that the firm remains solvent, i.e., additional retains have a higher marginal impact on the probability of continued operation. This suggests that if the firm already has high equity levels or stable earnings, additional retains may have little effect, and the director will be tempted to take cash today. Alternatively, for the thinly capitalized firm, a small increase in retained equity may dramatically increase the probability of future survival, thus inducing the director to retain a larger proportion of free cash flow.

Verification of Claim 1

To verify claim 1, recall the active director ENPV objective problem from (1),

$$
\begin{aligned}
&\mathrm{E}\Big\{e^{-r}\Big[\big(1-a^{coop}\big)\mathbf{p}_t^{fwd,coop}\mathbf{y}_t^{fwd}-\mathbf{w}_t^{ag}\mathbf{x}_t^{ag}\Big] \\
&+e^{-r}\left(\frac{1}{A_t^{coop}}\right)\big(1-b^{coop}\big)\big(1-t^{coop}\big)\big(\mathbf{p}_t^{coop}\mathbf{y}_t^{coop}-\mathbf{w}_t^{coop}\mathbf{x}_t^{coop}\big) \\
(9)\quad &+\sum_{k=t+1}^{t+N}e^{-r(k-(t-1))}\frac{1}{N}1_{\left[S_{k-1}^{coop}\right]}a^{coop}\mathbf{p}_t^{fwd,coop}\mathbf{y}_t^{fwd} \\
&+\sum_{k=t+1}^{t+N}e^{-r(k-(t-1))}\frac{1}{N}1_{\left[S_{k-1}^{coop}\right]}\left(\frac{1}{A_t^{coop}}\right)b^{coop}\big(1-t^{coop}\big)\big(\mathbf{p}_t^{coop}\mathbf{y}_t^{coop}-\mathbf{w}_t^{coop}\mathbf{x}_t^{coop}\big)\Big\}.
\end{aligned}
$$

Distributing the first two dividend terms, dropping the constants, and rearranging, we obtain

$$\mathrm{E}\Big\{-a^{coop}e^{-r}\mathbf{p}_t^{fwd,coop}\mathbf{y}_t^{fwd}$$

$$-b^{coop}e^{-r}\left(\frac{1}{A_t^{coop}}\right)\left(1-t^{coop}\right)\left(\mathbf{p}_t^{coop}\mathbf{y}_t^{coop}-\mathbf{w}_t^{coop}\mathbf{x}_t^{coop}\right)$$

(10)

$$+a^{coop}\sum_{k=t+1}^{t+N}e^{-r(k-(t-1))}\frac{1}{N}1_{\left[S_{k-1}^{coop}\right]}\mathbf{p}_t^{fwd,coop}\mathbf{y}_t^{fwd}$$

$$+b^{coop}\sum_{k=t+1}^{t+N}e^{-r(k-(t-1))}\frac{1}{N}1_{\left[S_{k-1}^{coop}\right]}\left(\frac{1}{A_t^{coop}}\right)\left(1-t^{coop}\right)\left(\mathbf{p}_t^{coop}\mathbf{y}_t^{coop}-\mathbf{w}_t^{coop}\mathbf{x}_t^{coop}\right)\Big\}.$$

Because bankruptcy is declared at the end of the operating period, immediately following cash payments to members, and because we are interested in the case in which the cooperative is still solvent in the current-period, we can expand the last two terms and pass expectations through as follows,

$$-a^{coop}e^{-r}\mathbf{p}_t^{fwd,coop}\mathbf{y}_t^{fwd}$$

$$-b^{coop}e^{-r}\left(\frac{1}{A_t^{coop}}\right)\left(1-t^{coop}\right)\left(\mathbf{p}_t^{coop}\mathbf{y}_t^{coop}-\mathbf{w}_t^{coop}\mathbf{x}_t^{coop}\right)$$

$$+a^{coop}e^{-2r}\frac{1}{N}\mathbf{p}_t^{fwd,coop}\mathbf{y}_t^{fwd}$$

$$+b^{coop}e^{-2r}\frac{1}{N}\left(\frac{1}{A_t^{coop}}\right)\left(1-t^{coop}\right)\left(\mathbf{p}_t^{coop}\mathbf{y}_t^{coop}-\mathbf{w}_t^{coop}\mathbf{x}_t^{coop}\right)$$

$$+a^{coop}\sum_{k=t+2}^{t+N}e^{-r(k-(t-1))}\frac{1}{N}\mathrm{P}\left[S_{k-1}^{coop}\right]\mathbf{p}_t^{fwd,coop}\mathbf{y}_t^{fwd}$$

(11)

$$+b^{coop}\sum_{k=t+2}^{t+N}e^{-r(k-(t-1))}\frac{1}{N}\mathrm{P}\left[S_{k-1}^{coop}\right]\left(\frac{1}{A_t^{coop}}\right)\left(1-t^{coop}\right)\left(\mathbf{p}_t^{coop}\mathbf{y}_t^{coop}-\mathbf{w}_t^{coop}\mathbf{x}_t^{coop}\right),$$

since

(12) $$\mathrm{E}\left\{1_{\left[S_{k-1}^{coop}\right]}\right\} = \mathrm{P}\left[S_{k-1}^{coop}\right].$$

Collecting terms, we obtain

$$-a^{coop}\mathbf{p}_t^{fwd,coop}\mathbf{y}_t^{fwd}\left[e^{-r} - e^{-2r}\frac{1}{N} - \sum_{k=t+2}^{t+N} e^{-r(k-(t-1))}\frac{1}{N}\mathrm{P}\left[S_{k-1}^{coop}\right]\right]$$

(13) $$-b^{coop}\left(\frac{1}{A_t^{coop}}\right)\left(1-t^{coop}\right)\left(\mathbf{p}_t^{coop}\mathbf{y}_t^{coop} - \mathbf{w}_t^{coop}\mathbf{x}_t^{coop}\right)$$

$$\left[e^{-r} - e^{-2r}\frac{1}{N} - \sum_{k=t+2}^{t+N} e^{-r(k-(t-1))}\frac{1}{N}\mathrm{P}\left[S_{k-1}^{coop}\right]\right].$$

Rather than take first and second order conditions at this point, it is sufficient to show that ENPV is decreasing monotonically in the retain rates. This is the case if the following strict inequality holds

(14) $$e^{-r} > e^{-2r}\frac{1}{N} - \sum_{k=t+2}^{t+N} e^{-r(k-(t-1))}\frac{1}{N}\mathrm{P}\left[S_{k-1}^{coop}\right].$$

The relation in (14) can be shown directly, starting from the fact that $1 < 2$ and the fact that $\mathrm{P}[\bullet] \in [0,1] \subseteq \Re_+$. Q.E.D.

Verification of Claim 2

As a result of claim 1, verifying claim 2 can be accomplished by verifying conditions under which

(17) $b^{iof*} > 0$,

where b^{iof*} refers to the optimal IOF retain rate. To see when this is true, consider the retain rate b^{iof*} that solves the active investor-director's objective function from problem (3),

$$
\begin{aligned}
(18)\quad & \mathrm{E}\Big\{ e^{-r}\Big[\big(1-a^{coop}\big)\mathbf{p}_t^{fwd,iof}\mathbf{y}_t^{fwd} - \mathbf{w}_t^{ag}\mathbf{x}_t^{ag}\Big] \\
& + e^{-r}\left(\frac{a^{coop}\mathbf{p}_t^{fwd,iof}\mathbf{y}_t^{fwd}}{p_t^{eq,iof}y_t^{eq,iof}}\right)\big(1-b^{iof}\big)d^{iof}\Big[\big(1-t^{iof}\big)\big(\mathbf{p}_t^{iof}\mathbf{y}_t^{iof} - \mathbf{w}_t^{iof}\mathbf{x}_t^{iof}\big) + \mathbf{w}_t^{fwd,iof}\mathbf{x}_t^{fwd,iof}\Big] \\
& + \sum_{k=t+1}^{\infty} e^{-r(k-(t-1))}1_{[S_{k-1}^{iof}]}\left(\frac{a^{coop}\mathbf{p}_t^{fwd,iof}\mathbf{y}_t^{fwd}}{p_t^{eq,iof}y_k^{eq,iof}}\right)\big(1-b^{iof}\big) \\
& \quad d^{iof}\Big[\big(1-t^{iof}\big)\big(\mathbf{p}_k^{iof}\mathbf{y}_k^{iof} - \mathbf{w}_k^{iof}\mathbf{x}_k^{iof}\big) + \mathbf{w}_k^{fwd,iof}\mathbf{x}_k^{fwd,iof}\Big]\Big\}.
\end{aligned}
$$

Because the portion a^{coop} of crop proceeds set aside to purchase equity shares in the firm is now an arbitrary and fixed variable, we omit the first term in (18). Factoring the dividend rate, we obtain

$$
\begin{aligned}
(19)\quad & \mathrm{E}\Big\{\big(1-b^{iof}\big)e^{-r}\left(\frac{a^{coop}\mathbf{p}_t^{fwd,iof}\mathbf{y}_t^{fwd}}{p_t^{eq,iof}y_t^{eq,iof}}\right)d^{iof}\Big[\big(1-t^{iof}\big)\big(\mathbf{p}_t^{iof}\mathbf{y}_t^{iof} - \mathbf{w}_t^{iof}\mathbf{x}_t^{iof}\big) + \mathbf{w}_t^{fwd,iof}\mathbf{x}_t^{fwd,iof}\Big] \\
& + \big(1-b^{iof}\big)\sum_{k=t+1}^{\infty} e^{-r(k-(t-1))}1_{[S_{k-1}^{iof}]}\left(\frac{a^{coop}\mathbf{p}_t^{fwd,iof}\mathbf{y}_t^{fwd}}{p_t^{eq,iof}y_k^{eq,iof}}\right) \\
& \quad d^{iof}\Big[\big(1-t^{iof}\big)\big(\mathbf{p}_k^{iof}\mathbf{y}_k^{iof} - \mathbf{w}_k^{iof}\mathbf{x}_k^{iof}\big) + \mathbf{w}_k^{fwd,iof}\mathbf{x}_k^{fwd,iof}\Big]\Big\}.
\end{aligned}
$$

For convenience, we hold the number of shares remains constant, i.e., no cash calls or new stock issuance. Then, because bankruptcy is a result of insolvency in the previous period, we may pass expectations through (Durrett, page128) to obtain

$$
\begin{aligned}
(20)\quad & \big(1-b^{iof}\big)e^{-r}\left(\frac{a^{coop}\mathbf{p}_t^{fwd,iof}\mathbf{y}_t^{fwd}}{p_t^{eq,iof}y_t^{eq,iof}}\right)d^{iof}\Big[\big(1-t^{iof}\big)\big(\mathbf{p}_t^{iof}\mathbf{y}_t^{iof} - \mathbf{w}_t^{iof}\mathbf{x}_t^{iof}\big) + \mathbf{w}_t^{fwd,iof}\mathbf{x}_t^{fwd,iof}\Big] \\
& + \big(1-b^{iof}\big)\sum_{k=t+2}^{\infty} e^{-r(k-(t-1))}\,\mathrm{P}\big[S_{k-1}^{iof}\big]\left(\frac{a^{coop}\mathbf{p}_t^{fwd,iof}\mathbf{y}_t^{fwd}}{p_t^{eq,iof}y_t^{eq,iof}}\right) \\
& \quad d^{iof}\,\mathrm{E}\Big\{\big(1-t^{iof}\big)\big(\mathbf{p}_k^{iof}\mathbf{y}_k^{iof} - \mathbf{w}_k^{iof}\mathbf{x}_k^{iof}\big) + \mathbf{w}_k^{fwd,iof}\mathbf{x}_k^{fwd,iof}\Big\}.
\end{aligned}
$$

Distributing and dropping the terms not related to the retain rate, we obtain

$$-b^{iof}\ e^{-r}\left(\frac{a^{coop}\mathbf{p}_t^{fwd,iof}\mathbf{y}_t^{fwd}}{p_t^{eq,iof}\ y_t^{eq,iof}}\right)d^{iof}\left[\left(1-t^{iof}\right)\left(\mathbf{p}_t^{iof}\mathbf{y}_t^{iof}-\mathbf{w}_t^{iof}\mathbf{x}_t^{iof}\right)+\mathbf{w}_t^{fwd,iof}\mathbf{x}_t^{fwd,iof}\right]$$

$$+\sum_{k=t+1}^{\infty}e^{-r(k-(t-1))}\,\mathrm{P}\left[S_{k-1}^{iof}\right]\left(\frac{a^{coop}\mathbf{p}_t^{fwd,iof}\mathbf{y}_t^{fwd}}{p_t^{eq,iof}\ y_t^{eq,iof}}\right)d^{iof}$$

$$\text{(21)}\qquad \mathrm{E}\left\{\left(1-t^{iof}\right)\left(\mathbf{p}_k^{iof}\mathbf{y}_k^{iof}-\mathbf{w}_k^{iof}\mathbf{x}_k^{iof}\right)+\mathbf{w}_k^{fwd,iof}\mathbf{x}_k^{fwd,iof}\right\}$$

$$-b^{iof}\sum_{k=t+1}^{\infty}e^{-r(k-(t-1))}\,\mathrm{P}\left[S_{k-1}^{iof}\right]\left(\frac{a^{coop}\mathbf{p}_t^{fwd,iof}\mathbf{y}_t^{fwd}}{p_t^{eq,iof}\ y_t^{eq,iof}}\right)d^{iof}$$

$$\mathrm{E}\left\{\left(1-t^{iof}\right)\left(\mathbf{p}_k^{iof}\mathbf{y}_k^{iof}-\mathbf{w}_k^{iof}\mathbf{x}_k^{iof}\right)+\mathbf{w}_k^{fwd,iof}\mathbf{x}_k^{fwd,iof}\right\}.$$

Simplifying, we obtain

$$G\sum_{k=t+2}^{\infty}e^{-r(k-(t-1))}\,\mathrm{P}\left[S_{k-1}^{iof}\right]\mathrm{E}\left\{\left(1-t^{iof}\right)\left(\mathbf{p}_k^{iof}\mathbf{y}_k^{iof}-\mathbf{w}_k^{iof}\mathbf{x}_k^{iof}\right)+\mathbf{w}_k^{fwd,iof}\mathbf{x}_k^{fwd,iof}\right\}$$

$$\text{(22)}\qquad -b^{iof}\ G\Big[e^{-r}\left[\left(1-t^{iof}\right)\left(\mathbf{p}_t^{iof}\mathbf{y}_t^{iof}-\mathbf{w}_t^{iof}\mathbf{x}_t^{iof}\right)+\mathbf{w}_t^{fwd,iof}\mathbf{x}_t^{fwd,iof}\right]$$

$$+\sum_{k=t+1}^{\infty}e^{-r(k-(t-1))}\,\mathrm{P}\left[S_{k-1}^{iof}\right]\mathrm{E}\left\{\left(1-t^{iof}\right)\left(\mathbf{p}_k^{iof}\mathbf{y}_k^{iof}-\mathbf{w}_k^{iof}\mathbf{x}_k^{iof}\right)+\mathbf{w}_k^{fwd,iof}\mathbf{x}_k^{fwd,iof}\right\}.$$

where G is the investor-director's share of available cash flow

$$\text{(23)}\qquad G=\left(\frac{a^{coop}\mathbf{p}_t^{fwd,iof}\mathbf{y}_t^{fwd}}{p_t^{eq,iof}\ y_t^{eq,iof}}\right)d^{iof}.$$

First order conditions, then, are positive if

$$G\sum_{k=t+1}^{\infty} e^{-r(k-(t-1))}\,\mathrm{P}'\left[S_{k-1}^{iof}\right]\mathrm{E}\left(1-t^{iof}\right)\left(\mathbf{p}_k^{iof}\mathbf{y}_k^{iof}-\mathbf{w}_k^{iof}\mathbf{x}_k^{iof}\right)+\mathbf{w}_k^{fwd,iof}\mathbf{x}_k^{fwd,iof} >$$

$$(24)\quad G\left[e^{-r}\left[\left(1-t^{iof}\right)\left(\mathbf{p}_t^{iof}\mathbf{y}_t^{iof}-\mathbf{w}_t^{iof}\mathbf{x}_t^{iof}\right)+\mathbf{w}_t^{fwd,iof}\mathbf{x}_t^{fwd,iof}\right]\right.$$

$$\left.+\sum_{k=t+1}^{\infty} e^{-r(k-(t-1))}\,\mathrm{P}\left[S_{k-1}^{iof}\right]\mathrm{E}\left\{\left(1-t^{iof}\right)\left(\mathbf{p}_k^{iof}\mathbf{y}_k^{iof}-\mathbf{w}_k^{iof}\mathbf{x}_k^{iof}\right)+\mathbf{w}_k^{fwd,iof}\mathbf{x}_k^{fwd,iof}\right\}\right.$$

Here, we keep G only for interpretation purposes. By our IID assumption, we can replace the future period index k with the previous period index $t-1$. Dividing through by the expected value term we obtain

$$(25)\quad \sum_{k=t+1}^{\infty} e^{-r(k-(t-1))}\,\mathrm{P}'\left[S_{k-1}^{iof}\right] > \left[e^{-r}H+\sum_{k=t+1}^{\infty} e^{-r(k-(t-1))}\,\mathrm{P}\left[S_{k-1}^{iof}\right]\right],$$

where H now represents the trade-off between the current dividend and expected future dividend

$$(26)\quad H=\frac{\left[\left(1-t^{iof}\right)\left(\mathbf{p}_t^{iof}\mathbf{y}_t^{iof}-\mathbf{w}_t^{iof}\mathbf{x}_t^{iof}\right)+\mathbf{w}_t^{fwd,iof}\mathbf{x}_t^{fwd,iof}\right]G}{\mathrm{E}\left\{\left(1-t^{iof}\right)\left(\mathbf{p}_{t+1}^{iof}\mathbf{y}_{t+1}^{iof}-\mathbf{w}_{t+1}^{iof}\mathbf{x}_{t+1}^{iof}\right)+\mathbf{w}_{t+1}^{fwd,iof}\mathbf{x}_{t+1}^{fwd,iof}\right\}G}.$$

Moving the probability term to the left hand side, we obtain the expression in (7). Q.E.D.

Probability of Ruin

In this section we define the event of financial ruin, the associated probability, and the first-crossing time problem (Bachelier). We then specify the relevant drift and noise parameters.

We define the event of bankruptcy or financial ruin to be triggered when firm equity falls below zero, namely when the firm becomes *insolvent*. The firm declares bankruptcy immediately following the current-period cash payments to members. Bankruptcy is assumed to be an absorbing state, that is, no further cash payments are made after the event.

We frame the equity level or *process* in terms of a standard Brownian motion with drift. The probability of bankruptcy is then a standard first-crossing-time problem, since bankruptcy occurs when the equity process first crosses the zero boundary.

Recall the standard Brownian motion X_t with drift term μ and noise term W_t. Adapting the notation of Bjork (1998), the *stochastic differential* of X and its initial state are given by

$$\begin{aligned} dX_t &= \mu dt + dW_t \\ X_0 &= m . \end{aligned} \tag{27}$$

To illustrate the associated first-crossing-time problem, consider a sample path corresponding to a zero drift with an initial state of unity and boundary of zero.

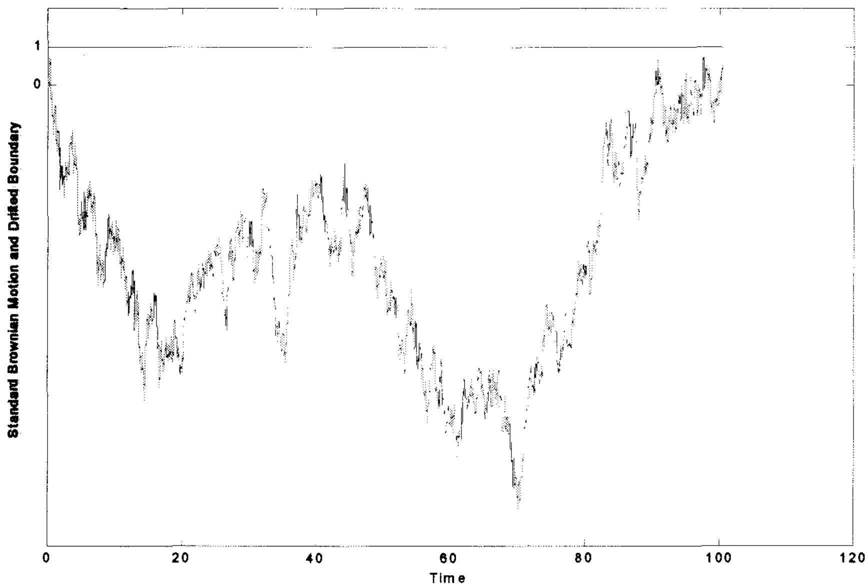

Figure 3.4: Sample First-Crossing-Time with Zero Drift

Here, the sample path of the standard Brownian motion without drift crosses the boundary, i.e., a bankruptcy event occurs, just after the 100th period.

To illustrate a path with drift, consider a negative drift of 0.1, other parameters the same.

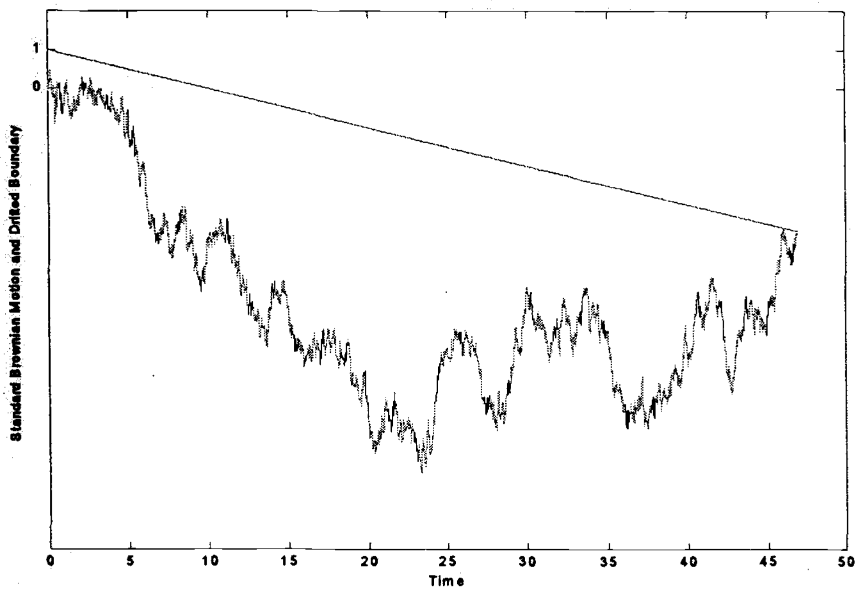

Figure 3.5: Sample First-Crossing-Time with Positive Drift

Here, the sample path first crosses just after the 45th period.

The first-crossing-time τ satisfies the following equation

(28) $\tau = \inf\{t : W_t = m + \mu t\}$.

The resulting probability of ruin on or before a specified time period t follows from Girsanov's Theorem, and adapting the notation of Klugman for our drifted boundary in figure 3.6, is given by

(29) $$\mathrm{P}[\tau \le t] = \int_0^t \frac{m}{s^{3/2}} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2s}(m + \mu s)^2\right\} ds.$$

It remains to specify the exact nature of the equity process for the cooperative and IOF firm of interest. Specifically, we must derive the drift terms and obtain the standard deviations in order to standardize the equity processes. To begin, consider our first claim.

Lemma 1: Given the traditional patronage-based equity retain and revolve policies, as specified in (1), the following is true of the cooperative equity process:

(30) $\mathrm{E}\{EQ_t \mid EQ_j\} = EQ_j, \quad t \ge j$.

In words, lemma 1 states that the equity expected in the future period t, given information about equity in, say, the current period j, is the equity level in the current period. The implication of this result is that the drift term (27) is zero for the cooperative equity process with an initial state at its equilibrium level.

To verify this, consider the equity at time t under the patronage-based policy

$$(31) \qquad \underset{t\geq j}{EQ_t} = EQ_j + \sum_{n=j}^{t} retain_n - \sum_{n=j}^{t} revolve_n .$$

Here, equity at time t is simply initial equity at time j, plus the sum of the retains, less the sum of the revolves.

Assume that membership is stable and the cooperative is already at its long-run equilibrium equity level at time j. Also, for accounting convenience, assume that yields and processing returns have been at average levels over the past N years. Then equity at time j is simply any accumulated non-member equity or non-revolving seed money EQ_0, plus the last N years of not-yet-revolved average retains. From the policy in (1) and (2), we can then expand (31) accordingly as

(32)

$$\begin{aligned}
\underset{t\geq j}{EQ_t} = \; & EQ_j \\
& + \sum_{n=j}^{t} a^{coop} \mathbf{p}_n^{fwd,coop} \mathbf{y}_n^{fwd} + \sum_{n=j}^{t} b^{coop} \left(1 - t^{coop}\right)\left(\mathbf{p}_n^{coop} \mathbf{y}_n^{coop} - \mathbf{w}_n^{coop} \mathbf{x}_n^{coop}\right) \\
& - \sum_{n=j}^{t} \frac{1}{N}\left(EQ_j - EQ_0\right)\left[\frac{N+j-n}{N}\right]^{+} \\
& - \sum_{n=j}^{t} \sum_{l=0}^{N-1} \frac{1}{N}\left[a^{coop} \mathbf{p}_{n-l}^{fwd,coop} \mathbf{y}_{n-l}^{fwd} + b^{coop}\left(1 - t^{coop}\right)\left(\mathbf{p}_{n-l}^{coop} \mathbf{y}_{n-l}^{coop} - \mathbf{w}_{n-l}^{coop} \mathbf{x}_{n-l}^{coop}\right)\right] 1_{[n-l>j]} .
\end{aligned}$$

Here the second from the last term allows a share of the existing member equity to revolve, until it has been depleted and the cooperative's equity reserves consists entirely of equity retains from periods after period j. Taking conditional expectations of both sides of (32) and factoring, we obtain

(33)

$$\mathrm{E}\left\{EQ_t \,\middle|\, EQ_j\right\} = EQ_j$$

$$+(t-j)\left(a^{coop}\overline{\mathbf{p}}^{fwd,coop}\overline{\mathbf{y}}^{fwd} + b^{coop}\left(1 - t^{coop}\right)\overline{\mathbf{p}}^{coop}\overline{\mathbf{y}}^{coop} - \overline{\mathbf{w}}^{coop}\overline{\mathbf{x}}^{coop}\right)$$

$$-\left(a^{coop}\overline{\mathbf{p}}^{fwd,coop}\overline{\mathbf{y}}^{fwd} + b^{coop}\left(1 - t^{coop}\right)\overline{\mathbf{p}}^{coop}\overline{\mathbf{y}}^{coop} - \overline{\mathbf{w}}^{coop}\overline{\mathbf{x}}^{coop}\right)\sum_{n=j}^{t}\left[\frac{N+j-n}{N}\right]^{+}$$

$$-\left(a^{coop}\overline{\mathbf{p}}^{fwd,coop}\overline{\mathbf{y}}^{fwd} + b^{coop}\left(1 - t^{coop}\right)\overline{\mathbf{p}}^{coop}\overline{\mathbf{y}}^{coop} - \overline{\mathbf{w}}^{coop}\overline{\mathbf{x}}^{coop}\right)\sum_{n=j}^{t}\sum_{l=0}^{N-1}\frac{1}{N}1_{[n-l>j]}$$

since, by our assumption above,

(34) $$EQ_j = EQ_0 + N\left(a^{coop}\overline{\mathbf{p}}^{fwd,coop}\overline{\mathbf{y}}^{fwd} + b^{coop}\left(1 - t^{coop}\right)\overline{\mathbf{p}}^{coop}\overline{\mathbf{y}}^{coop} - \overline{\mathbf{w}}^{coop}\overline{\mathbf{x}}^{coop}\right).$$

Here, the bar indicates expected value. This result depends, of course, on distributions with moments that are finite and constant over time, i.e., stationary, as well as the independence between prices and quantities, which is consistent with our assumption of no supply response to price in processing tomato production. Independence between processed prices and quantities, however, imposes fairly strong restrictions on demand elasticities, but is not unreasonable given the nature of the intermediate and finished good markets for such things and tomato paste or peeled tomatoes.

For our result to hold, we must show that

(35) $$\sum_{n=j}^{t}\left[\frac{N+j-n}{N}\right]^{+} + \sum_{n=j}^{t}\sum_{l=0}^{N-1}\frac{1}{N}1_{[n-l>j]} = (t-j).$$

To see why (35) holds, consider each summation term expanded out to its $(t\text{-}j)^{\text{th}}$ term

(36)

$$\sum_{n=j}^{t}\left[\frac{N+j-n}{N}\right]^{+} = \frac{N-0}{N}+\frac{N-1}{N}+\frac{N-2}{N}+\cdots+\frac{N-N}{N}, \text{ and}$$

$$\sum_{n=j}^{t}\sum_{l=0}^{N-1}\frac{1}{N}1_{[n-l>j]} = \frac{0}{N}+\frac{1}{N}+\frac{2}{N}+\cdots+\frac{N-0}{N}.$$

And, lemma 1 is verified.

Rewriting lemma 1, we obtain

(37) $\mathrm{E}\left\{EQ_t - EQ_j \,\middle|\, EQ_j\right\} = 0, \quad t \geq j$

Thus, the traditional patronage-based cooperative equity process drift is zero in its long-run equilibrium.

To obtain the drift of the investor-owned firm's equity process, it is convenient to consider net processing returns for firm i in period t as a single variable π_t^i, again from some stationary distribution with finite moments. Then, the investor owned firm equity level is given by

(38)

$$EQ_{t+1} = EQ_t + \left(1-t^{iof}\right)\pi_t^{iof} - \left(1-b^{iof}\right)d^{iof}\left[\left(1-t^{iof}\right)\pi_t^{iof} + \mathbf{w}_t^{fwd,iof}\mathbf{x}_t^{fwd,iof}\right].$$

Simplifying, we obtain

(39) $EQ_{t+1} - EQ_t = \left(1-t^{iof}\right)\left(1-\left(1-b^{iof}\right)d^{iof}\right)\pi_t^{iof} - \left(1-b^{iof}\right)d^{iof}\mathbf{w}_t^{fwd,iof}\mathbf{x}_t^{fwd,iof}.$

Passing expectations through gives us the following discrete time approximation for the drift term,

(40)

$$\mathrm{E}\left\{EQ_{t+1} - EQ_t \,|\, EQ_t\right\} = \left(1-t^{iof}\right)\left(1-\left(1-b^{iof}\right)d^{iof}\right)\overline{\pi}_t^{iof} - \left(1-b^{iof}\right)d^{iof}\overline{\mathbf{w}}_t^{fwd,iof}\overline{\mathbf{x}}_t^{fwd,iof}.$$

Note here that equity at time t is a function of equity and profit at time *t-1*.

In order to standardize these processes, we need discrete time approximations for variance. For the cooperative, for a one-year period, this term is given by

$$V\left\{EQ_{t+1} - EQ_t \mid EQ_t\right\} = \left(a^{coop}\right)^2 V\left\{\mathbf{p}_t^{fwd,coop}\mathbf{y}_t^{fwd} \mid \mathbf{p}_{t-1}^{fwd,coop}\mathbf{y}_{t-1}^{fwd}\right\}$$

(41)
$$+ \left[b^{coop}\left(1 - t^{coop}\right)\right]^2 V\left\{\pi_t^{coop} \mid \pi_{t-1}^{coop}\right\}$$

$$+ 2a^{coop}b^{coop}\left(1 - t^{coop}\right)COV\left\{\pi_t^{coop}, \mathbf{p}_t^{fwd,coop}\mathbf{y}_t^{fwd} \mid \bullet\right\}.$$

where • represents the appropriate conditioning variable or *filtration*.

Similarly, for the IOF, variance is given by

$$V\left\{EQ_{t+1} - EQ_t \mid EQ_t\right\} = \left[\left(1 - t^{iof}\right)\left(1 - \left(1 - b^{iof}\right)d^{iof}\right)\right]^2 V\left\{\pi_t^{iof} \mid \pi_{t-1}^{iof}\right\}$$

$$- \left[\left(1 - b^{iof}\right)d^{iof}\right]^2 V\left\{\mathbf{w}_t^{fwd,iof}\mathbf{x}_t^{fwd,iof} \mid \mathbf{w}_{t-1}^{fwd,iof}\mathbf{x}_{t-1}^{fwd,iof}\right\}$$

(42)
$$+2\left(1 - t^{iof}\right)\left(1 - \left(1 - b^{iof}\right)d^{iof}\right)\left(1 - b^{iof}\right)d^{iof}$$

$$COV\left\{\pi_t^{iof}, \mathbf{w}_t^{fwd,iof}\mathbf{x}_t^{fwd,iof} \mid \bullet\right\}$$

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4. DEA and The Law of One Price

In his classic paper, "The Measurement of Productive Efficiency," Farrell (1957) introduced a radial approach for estimating efficiency. His input oriented model is based on the Mahler (1939) inequality, which states that the normalized cost function is less than or equal to the input distance function. This duality between a support function, the cost function, and the corresponding distance function, Shephard's (1953) input distance function, is the basis for gauging and decomposing input efficiency. The primal, i.e., factor component, is a function of quantities. The dual, i.e., value component, is a function of prices. Thus, to estimate technical efficiency, quantity data are required.

However, it is common for data to be furnished in expenditure and revenue (value) terms, rather than physical input and output (factor) terms, e.g., labor expenditure rather than labor hours. Value data is commonly collected and reported by companies that are publicly traded or otherwise regulated and by companies and individuals seeking to borrow capital or pay taxes.

As a result, a number of DEA studies have utilized value-based data as a "proxy" for one or more physical inputs or outputs, an important example being Farrell himself (1957). This practice raises the two key questions addressed in this paper. First, how do value-based DEA models coincide with factor-based DEA models? Second, if they do not coincide, then what do value-based DEA models measure?

It is well known and we show that a sufficient condition for the two types of models to coincide is that all firms face the same set of prices, i.e., the Law of One Price holds[1]. However, prices are known to vary across firms in practice (Carsten, Engle, Rogoff). We show that if prices vary across firms, either the two models do not coincide, or a preposterous restriction on prices must hold.

Farrell initiates the discussion of what value-based models measure when the prices are assumed, incorrectly, to be the same across all firms. He states, "In this case, the (technical efficiency) index will reflect not only the technical efficiency of the firm but also the extent of its adaptation to a set of factor prices different from those facing it," (page 264). We formalize this claim, by introducing a multiplicative decomposition of this error into a technology and firm-related component.

[1] This fact was exploited by Färe and Grosskopf (1985).

Finally, we illustrate the potential magnitude and direction of these error terms. First, we re-consider a well-known data set in which prices varied across decision-making units, that is, Farrell's original 1957 empirical example. In Farrell's case, we find both the technology-related and firm-related error terms to be material, disrupting the relative efficiency ranking of the decision making units (DMU's) in question. Second, we use Monte Carlo integration methods to show that, for the Farrell case, the resulting error terms are one-sided and increasing in the level of price variation. Confidence intervals are included.

DEA estimates have been shown to be consistent (Kneip, Park, Simar) with a known asymptotic distribution (Gijbels, *et al*). In this paper we define prices as a schedule of positive linear transformations, and explore the magnitude and direction of errors in a small sample context. Other violations of the Law of One Price and their implications for the large sample properties of the DEA estimator are left to future work.

Formalities

In this section we introduce the concepts, notation, and definitions that will be used throughout the rest of the paper. We begin with a definition of the Law of One Price.

Consider the case of K firms and N inputs and let $w_{kn} \in \Re_+$ denote the price faced by firm k for input n. Then, the Law of One Price holds, provided

(1) $$\forall w_k \in \Re_+^N \quad w_{kn} = w_{k'n}, \quad k,k' = 1,\dots,K, \quad n = 1,\dots,N.$$

Thus, a violation of the Law of One Price exists whenever the following is true,

(2)

$$\exists n,k,k': \quad w_{kn} \neq w_{k'n} \quad w_k \in \Re_+^N, \quad n = 1,\dots N, \quad k,k' = 1,\dots,K, \quad k \neq k'.$$

Define $\alpha_{k'kn}$ for firms k and k' and good n as the following price ratio[2],

(3) $$\alpha_{k'kn} = \frac{w_{k'n}}{w_{kn}}.$$

Then, a violation of the Law of One Price is equivalent to

[2] This identity is used as a restriction on the DEA model proposed by Kuosmanen, Cherchye, and Simpilainen (2003).

(4) $$\exists k,k',n:\ \alpha_{k'kn} \neq 1,\quad n=1,\ldots N,\quad k,k'=1,\ldots,K,\quad k \neq k'.$$

Price indices are sometimes used to "recover" factor information from value data. For instance, to recover labor hours from labor cost data, a practitioner might divide labor costs by a suitable hourly wage-rate index. We define the *recovered* input vector $x_{vk} \in \Re_+^N$, where subscript v denotes the value-related component, as simply the input expenditure vector $e_k \in \Re_+^N$ element-wise divided by an input price vector $w_j \in \Re_+^N$:

(5) $$x_{vkn} = \frac{e_{kn}}{w_{jn}} \quad n=1,\ldots N,\quad k=1,\ldots,K$$

where the price vector may be associated with a particular firm $j=1,\ldots,K$, or with some average across firms, i.e., a price index. Note the trivial result that under the Law of One Price, as defined, the recovered input quantity x_{vkn} is equal to the actual quantity x_{kn},

(6)

$$x_{vkn} = \frac{e_{kn}}{w_{jn}} = \frac{x_{kn}w_{kn}}{w_{jn}} = \frac{x_{kn}w_{kn}}{w_{kn}} = x_{kn} \quad \forall n,j,k \quad n=1,\ldots N,\quad k=1,\ldots,K.$$

We now define the models that we will use throughout the paper. Consider, first, the Farrell input-oriented, factor-based measure of technical efficiency for firm k' with one output,

(7) $$\begin{aligned} TE_i(y_{k'},x_{k'}) \;=\; \min\{\lambda_{k'}:\ & \sum_{k=1}^{K} z_k y_k \geq y_{k'} \\ & \sum_{k=1}^{K} z_k x_{k1} \leq x_{k'1}\lambda_{k'} \\ & \vdots \\ & \sum_{k=1}^{K} z_k x_{kN} \leq x_{k'N}\lambda_{k'} \\ & z_k \geq 0, \quad k=1,\ldots,K\}, \end{aligned}$$

where z is the solution vector, if it exists, that satisfies the given constraints, and subscript i denotes the input orientation and the factor-based nature of the model . This technology satisfies free disposability of x and y and constant returns to scale (CRS) in the same variables.

Similarly, for the value-based technical efficiency measure, where again subscript v denotes the value-related component,

$$
(8) \quad TE_v(r_{k'}, e_{k'}) = \min\{\lambda_{vk'} : \sum_{k=1}^{K} z_{vk} r_k \geq r_{k'}, \; \sum_{k=1}^{K} z_{vk} e_{k1} \leq e_{k'1}\lambda_{vk'}, \; \vdots \; \sum_{k=1}^{K} z_{vk} e_{kN} \leq e_{k'N}\lambda_{vk'}, \; z_{vk} \geq 0, \quad k = 1,\ldots,K\}.
$$

Here, r_k is revenue for firm k and the model satisfies the same disposability and returns assumptions.

Claims

Claims regarding when value-based DEA models do and do not coincide with factor-based DEA models are not made lightly. In this section, we formalize our two key claims and then explore an implication of the second claim in a simple example.

Claim 1. If the Law of One Price holds, then the two models to coincide:

(9) $LOP \Rightarrow TE_v(r_{k'}, e_{k'}) = TE_i(y_{k'}, x_{k'})$.

Verification of this claim follows from the fact that inequality relations, as in (7), are preserved under division by a positive scalar, namely price.

The next claim makes use of our definition of the price ratio $\alpha_{k'kn}$ in (3).

Claim 2. If the Law of One Price does not hold, then either there is a firm for which the two models do not coincide or a restriction on prices and quantities must hold:

(10) $\neg LOP \Rightarrow$

either

(11) $\exists k': \; TE_v(r_{k'}, e_{k'}) \neq TE_i(y_{k'}, x_{k'})$

or

(12) $\forall k, k', \quad \alpha_{kk'l} = h(z, z_v, x, y, p, w), \quad k, k' = 1, \ldots, K$,

where h is a function of the intensity variables $z \in \Re_+^K$ and $z_v \in \Re_+^K$, and the input quantity and price matrices $x, w \in \Re_+^{K \times N}$ and output quantity and price matrices $y, p \in \Re_+^{K \times M}$ given by

$$(13) \qquad h(z, z_v, x, y, p, w) = \frac{z_k^* x_{k'l} x_{kn} p_k}{z_{vk}^* x_{k'n} x_{kl} p_{k'}} + \sum_{\substack{j=1 \\ j \neq k}}^{K} \frac{y_k p_k}{y_j p_{k'}} \left(\frac{z_j^* x_{k'l} x_{jn}}{z_{vk}^* x_{k'n} x_{kl}} - \frac{\alpha_{jk'l} z_{vj}^* x_{jl} p_{k'}}{z_{vk}^* x_{kl} p_j} \right).$$

Here, here the z^*'s denote the solution vectors satisfying the restrictions in (7) and (8), if solutions exists. Proof of this claim is provided in the appendix.

The restriction in (12) is a restriction on the exact nature of the violation of the LOP. It suggests that input-oriented technical efficiency scores may be identical for all firms when calculated using either value or factor data, but only for very specific sets of inputs, outputs, prices, and seemingly unrelated solution variables, as detailed in (13). We find no economic or probabilistic rationale for such a condition and would not expect it to hold for actual firms. Such a numerical coincidence can be manufactured, however, to illustrate the nature of the condition in (13).

We wish to illustrate a case in which firms do not face the same prices, but their technical efficiency estimates are identical for the factor-based DEA model and the value-based model. Consider a simple case of two firms, denoted k and k', respectively, where input vectors $x_k \in \mathfrak{R}_+^N$, price vectors $w_k \in \mathfrak{R}_+^N$, output vectors $y_k \in \mathfrak{R}_+^M$ for each firm are as follows:

Table 4.1. Actual Quantities and Prices for Firms *k* and *k'*

Firm	y	x_1	x_2	w_1	w_2
k	1	1	1	1	1
k'	1	2	1	½	2

Abusing notation somewhat, we will denote the input vector associated with firm *k'* as $x' \in \mathfrak{R}_+^N$, and similarly for the other vectors. Then, the inputs and the corresponding factor-based production possibilities frontier L(y) can be illustrated as follows:

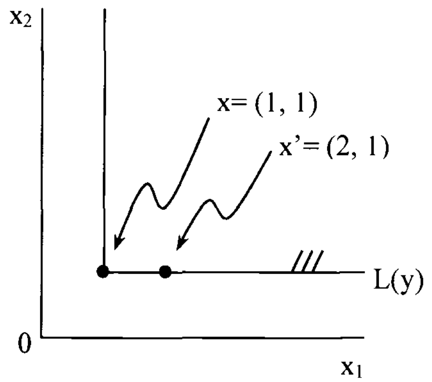

Figure 4.1. Factor-based Production Set

Note that the input-based DEA technical efficiency scores are 1 for both firms.

Now consider the situation in which the DEA practitioner does not observe inputs and prices for the two firms above, but instead is given expenditures $e_k \in \Re^N_+$ and a reference price index $w_l \in \Re^N_+$. Let us suppose the practitioner attempts to recover input information as in (5) using the reference price index vector. We will choose the prices faced by firm *k* as the reference price index, namely, the vector of 1's. Then the observed data set is given below.

Table 4.2. Observed (Recovered) Quantities for Firms *k* and *k'* and Reference Prices

Firm	y	x_{v1}	x_{v2}	w_{1l}	w_{2l}
k	1	1	1	1	1
k'	1	1	2	1	1

These recovered inputs and the resulting value-based production possibilities curve $L_v(y)$ are illustrated in figure 4.2 below.

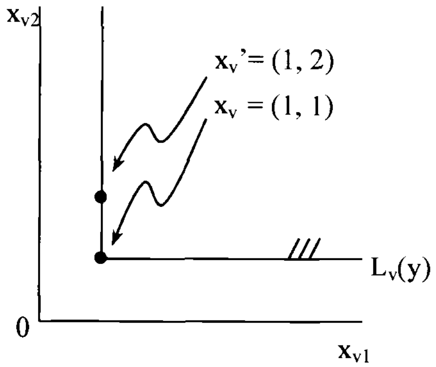

Figure 4.2. Value-based Production Set

Technical efficiency scores are unity for both firms under both the actual data and observed value data. Thus, the technical efficiency estimates from the two models coincide, despite the fact that prices vary across firms, i.e., the Law of One Price does not hold. The quantities and prices in this example were carefully constructed to lead to the "reflection" and resulting coincidence in efficiency scores. Such a coincidence in reality is, of course, preposterous. Thus, we turn our attention to measuring and illustrating the impacts on our efficiency estimates when the two models do not coincide.

Decomposition

In this section, we consider the case in which the LOP does not hold and the two models do not coincide. In this context, we ask the question: "If the value-based technical efficiency model does not, in fact, measure technical efficiency, then what does it measure?" In this section, we decompose the value-based model into three multiplicative components: the true technical efficiency score, a frontier-related error term, and a firm-specific error term. Later, we will use this decomposition to illustrate the magnitude and direction of the bias resulting from violations of the LOP.

To begin, let us consider a slightly more general case than that presented in the previous section. First, consider the location of a firm, labeled *a*, relative to the efficient frontier, labeled *b*, in the factor-based model.

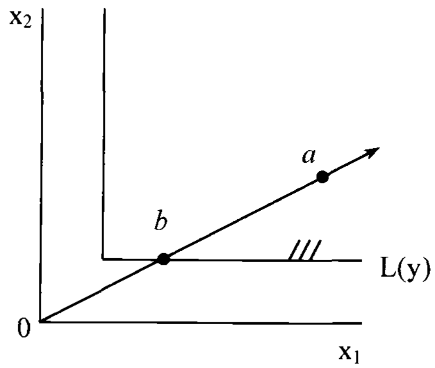

Figure 4.3. Factor-based Model

We denote the distance from the origin to the efficient frontier as 0*b*, and the distance from the origin to the firm as 0*a*. The input-oriented technical efficiency score in the factor model is then simply the ratio of these two distances 0*b*/0*a*.

Now, consider once more a case in which prices for the two inputs above and the DEA practitioner observes expenditures and a price index for each of the two goods, in lieu of the actual inputs and prices. Once more, let the practitioner recover factor data by dividing expenditures element-wise by the price index to obtain the following graph, with the location of the efficient frontier labeled *c*, and the location of the firm labeled *d*.

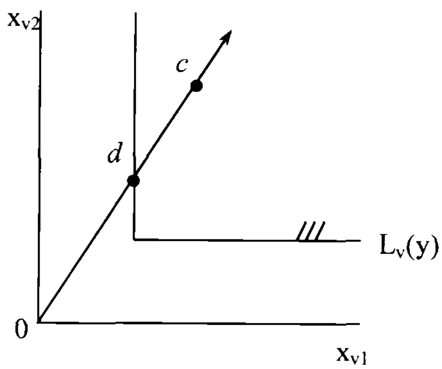

Figure 4.4. Value-based Model

Here, technical efficiency is calculated as 0*d*/0*c*.

Because prices vary across firms, there is no way that the practitioner can recover all of the factor information using a single index vector. As a result the two components of interest may differ between figures 4.3 and 4.4. First, the distances between the origin and the firms may differ. We will call this the *firm effect* and measure it as $0a/0c$. Second, the distances between the origin and the efficient frontiers may differ. We will call this the *technology effect* and measure it as $0d/0b$. We then have the following identity,

$$0d/0c = 0b/0a \bullet 0d/0b \bullet 0a/0c. \tag{14}$$

This identity states that the value-based technical efficiency score ($0d/0c$) can be decomposed, multiplicatively, into the true factor-based technical efficiency score ($0b/0a$), a technology effect ($0d/0b$), and a firm effect ($0a/0c$). We assign notation to this identity as follows,

$$TE_v(y_{vk'}, x_{vk'}) = TE_i(y_{k'}, x_{k'}) \bullet \gamma_{k'} \bullet \delta_{k'}, \tag{15}$$

where the technology and firm effects for firm k' are denoted $\gamma_{k'}$ and $\delta_{k'}$, respectively.

Empirical Example

In this section we use Farrell's original empirical example to show what may happen if the Law of One Price does not hold. In Farrell's example, output prices varied rather than input prices as examined in previous sections. Under the constant returns to scale assumption maintained throughout this paper, however, output price variation is exactly reciprocal to input price variation in the single output model, and Farrell's example is well suited to our point.

Farrell states that, due to scarcity of detailed price data and the variation in prices across firms, allocative efficiency, "…is a measure that is both unstable and dubious of interpretation; its virtue lies in leaving technical efficiency free of these faults," (page 261). Unfortunately, in his empirical illustration, Farrell utilizes value-based data, imparting to his technical efficiency estimates those properties he had reserved only for estimates of allocative efficiency. Farrell collected much of his data on state-level agricultural output from Table 700 on page 695 in a 1952 publication by the United States Department of Agriculture. Located a few pages from Table 700, we find a number of tables showing agricultural commodity prices by states. Without exception, these prices vary substantially.

Specifically, Farrell uses 1950 agricultural production data from the United States. He considers four inputs, including land and labor, measured in physical units, and materials and capital, measured in dollars. Farrell denotes these inputs as *b*, *c*, *d*, *e*, respectively, not to be confused with our labels in the previous section. Agricultural output, which he denotes *a*, includes cash receipts from farming plus the value of home consumption, measured in dollars.

We use the following DEA model to recreate Farrell's technical efficiency scores:

$$TE_v(1,\hat{x}_{k'}) = \min\{\lambda_{k'}: \sum_{k=1}^{K} z_k \geq 1$$

(16)
$$\sum_{k=1}^{K} z_k \hat{x}_{k1} \leq \hat{x}_{k'1}\lambda_{k'}$$
$$\vdots$$
$$\sum_{k=1}^{K} z_k \hat{x}_{k4} \leq \hat{x}_{k'4}\lambda_{k'}$$
$$z_k \geq 0, \quad k = 1,\ldots,48\}$$

where,

(17)
$$\hat{x}_1 = \frac{b}{a}, \quad \hat{x}_2 = \frac{c}{a}, \quad \hat{x}_3 = \frac{d}{a}, \quad \hat{x}_4 = \frac{e}{a}, \quad 1 = \frac{a}{a}$$

An example of the resulting data that appear in his paper is shown below for the first six states,

Table 4.3. Sample of Farrell's Original 1950 Data Set

	State	*Land*	*Labor*	*Materials*	*Capital*
1.	Maine	11.8	153.9	222.7	341.8
2.	New Hampshire	13.6	175.4	411.2	346.0
3.	Vermont	23.5	207.1	328.0	454.9
4.	Massachusetts	5.1	138.2	309.6	286.9
5.	Rhode Island	4.6	151.5	331.0	274.5
6.	Connecticut	5.3	125.7	317.3	258.3

From Table 445 on page 390 in the same source document, we see that average selling prices for cattle varied across states in 1950. An example of these average selling prices is given below for the same six states.

Table 4.4. Sample of 1950 Cattle Prices by State

	State	*Cents per lb.*
1.	Maine	23.3
2.	New Hampshire	25.0
3.	Vermont	23.4
4.	Massachusetts	32.6
5.	Rhode Island	39.1
6.	Connecticut	24.9

Similar variation is observed for milk and other crop and livestock commodity prices. In 1950, cattle and milk represented 18.9% and 14.1% of the total value US agricultural production, respectively, together about one third of total output. The average price for cattle across the 48 continental states was 25.9 cents per pound, and the standard deviation was 6.1 cents. Milk prices averaged 4.38 dollars per hundred-weight with a standard deviation of 1.13 dollars.

By using value-based data, Farrell imposed the assumption of the LOP when he claimed that his estimates represented "technical efficiency." Now, let us relax this assumption somewhat. We continue to assume that the Law of One Price holds across farms within each state, but acknowledge that cattle and milk prices vary across states.

Under this weaker set of assumptions, we wish to compare Farrell's estimates to efficiency scores estimated from data that is unaffected by milk and cattle price variation, but still consistent with Farrell's model specification in (17). To "correct" the data, we begin with the physical units of cattle and milk production by state. We multiply these quantities by a single national average price for cattle and milk in order to retain Farrell's use of revenue data, but without variation across states. We add these resulting cattle and milk revenues to the non-milk and cattle state revenues to obtain a single output value for each state. Finally, we divide the four inputs by this "corrected" output as in (17) to obtain a data set directly comparable to Farrell's, but free from price variation in approximately one third of its output.

Using the same model and normalization as given by (16) and (17) above, we estimate the "corrected" technical efficiency scores TE_i, the technology effect γ, and the firm effect δ. An example of these estimates, all multiplied by 100, is shown below for the same 6 states, starting with the recreated Farrell value-based technical efficiency scores TE_v. Statistical first moments and standard deviations for all 48 states are shown at the bottom.

Table 4.5. Resulting Technical Efficiency Estimates and Error Terms

State		*Farrell's* TE_v	*Corrected* TE_i	*Tech. Effect* γ	*Firm Effect* δ
1.	Maine	94	94	96	104
2.	New Hampshire	71	70	96	106
3.	Vermont	58	60	97	101
4.	Massachusetts	99	96	94	109
5.	Rhode Island	100	94	92	116
6.	Connecticut	100	100	93	107

Mean	**82.2**	**84.5**	**99.6**	**98.8**
Standard deviation	**12.6**	**12.2**	**2.6**	**5.9**

As you can see from table 4.5, value-based technical efficiency scores do not coincide with those of the partially corrected model. By correcting for price variation in just one third of total output and leaving all other expenditure and revenue terms unaltered, individual state efficiency scores differed between the two models by 3.8% up or down, on average and by as much as 22% in the case of Wisconsin, a state heavily dependant on milk production. On average, Farrell's value-based estimates are 2.7% lower than the corrected model, (82.2-84.5) / 84.5, a characteristic that we will see again later.

An important utility of DEA efficiency analysis is it ability to assign an objective rank to each DMU relative to its peers. The level of price variation described above was sufficient to disrupt the relative ranking of states by 2.7 rankings, up or down, on average. In particular, Wisconsin dropped 17 rankings from the 11th most efficient state under the corrected model to the 28th under Farrell's value-based model.

Expected Errors over a Range of Price Variation

The empirical illustration suggested that Farrell's value-based DEA model understated technical efficiency and that the error terms associated with each state varied. However, a number of critical questions remain unanswered. Is the expected magnitude of the error terms commensurate with the level of price variation or coincidental? If commensurate, are the expected magnitudes linear in price variation? Does the variation in the error terms behave similarly? What is the direction of these expected error terms? Are they one-sided?

In this section, we use Monte Carlo integration methods to illustrate the magnitude and direction of the expected technology effect γ and the firm effect δ over a range of price variation. We begin by introducing a model to illustrate the expected values of interest, along with the corresponding 95% confidence intervals. Then, we verify that the errors found in Farrell's empirical example lie within our 95% confidence interval, given the level of price variability in the markets for cattle and milk in 1950. Finally, we map the expected magnitudes of the error terms and confidence intervals over a range of price variation, and find that the expected technology effect introduces systematic and one-sided bias to technical efficiency scores.

Model and Assumptions

As in our empirical example, we will assume that the Law of One Price holds across farms within each of the 48 states, but will allow cattle and milk prices to vary across states. We will model the 1950 price vectors for milk and cattle in the 48 states $p_m \in \Re_+^{48}$, where m = milk, cattle, as vectors of real-valued, non-negative IID random variables, drawn from their respective 2-parameter Gamma distributions. While a number of distributions are appropriate to model price variation, the Gamma family is desirable because it is one-sided (Greene, 1980, 1990) with flexible higher moments, of which the Exponential and Chi-squared distributions are special cases. The Gamma family of distributions, expressed in terms of random variable p with density function f, is given by (Spanos) as the set F satisfying,

(18)
$$F = \left\{ f(p;\alpha,\beta) = \frac{\beta^{-1}}{\Gamma[\alpha]}\left(\frac{p}{\beta}\right)^{\alpha-1} \exp\left(-\frac{p}{\beta}\right), \quad \alpha,\beta > 0 \right\}$$

where $\Gamma[\alpha]$ is the Gamma function (Abramowitz). Special cases include the Exponential density when α equals 1 and the Chi-square density when β equals 2 (Wackerly). The mean and variance of the Gamma density are $\alpha\beta$ and $\alpha\beta^2$, respectively.

We will estimate the shape parameters using a Maximum Likelihood Estimation (MLE) model by solving the usual maximization problem

(19)
$$\max_{\alpha,\beta} \left\{ \sum_{k=1}^{48} \ln\left(f(p;\alpha,\beta)\right) \right\}$$

where f is the Gamma density defined in equation (18). MLE parameter estimates are consistent and asymptotically normal (Greene, 2003) under the null hypothesis, that is, when the true density g, is from the family of densities specified in (19). When the density is mispecified, i.e., when $g \notin F$, as may often be the case, solutions to (19) are Quasi-Maximum Likelihood estimates (QMLE). QMLE parameter estimates are consistent under certain restrictive conditions, but tend to be less efficient in finite samples than when density specification is correct (White). In either case, Farrell's sample size of K = 48 is quite small for inference purposes in this context.

We use Monte Carlo integration to calculate the expected value of γ and δ, the technology and firm effects, respectively. Monte Carlo is a useful tool for estimating expected values of functions of random variables when analytical solutions are not possible. In our case, the random variables of interest are cattle and milk prices. We will estimate the expected value of the technology-related and firm-related error terms which are generated by these random prices.

For instance, to estimate the expected value of technology-related error term $\gamma_{k'}$ for firm *k'*, we begin by generating 96 price observations, one for cattle and one for milk for each of the 48 states. Each price observation is a pseudo-random draw from the Gamma density given in (18) with parameter values α and β obtained by solving equation (19). Multiplying each state's physical production levels of cattle and milk by these prices, we are able to reconstruct one value-based data set. We then run the DEA model in (16). With these efficiency estimates and our "corrected" efficiency estimates from the previous section, we calculate the mean technology-related error using equation (15). We repeat this process *m* times and average the results. Adapting the notation of Campbell (1997), our model is,

$$\text{(20)} \qquad \hat{\gamma}_{k'} = \frac{1}{m}\sum_{j=1}^{m}\gamma_{k',j}$$

Monte Carlo estimates are consistent, asymptotically normal, and accurate to an arbitrary level, depending on *m* (Campbell, 1997). Confidence intervals can be readily calculated, as

$$\text{(21)} \qquad \text{Prob}\left[\gamma_{k'} \in \left(\hat{\gamma}_{k'} - z,\ \ \hat{\gamma}_{k'} + z\right)\right] = 95\%, \quad z = \frac{1.96\,\sigma\left(\gamma_{k',j}\right)}{\sqrt{m}}.$$

Equation (21) states that the true error $\gamma_{k'}$ lies within the given interval with 95% probability. Because the true standard deviation of the draws $\sigma\left(\gamma_{k',j}\right)$ is not known, we estimate it using the same Monte Carlo integration method:

$$\text{(22)} \qquad \hat{\sigma}\left(\gamma_{k',j}\right) = \frac{1}{m}\sum_{j=1}^{m}\left(\gamma_{k',j} - \hat{\gamma}_{k'}\right)^{2}$$

where $\hat{\gamma}_{k'}$ is given by equation (20). Solving equation (21) for *m*, we determine that 200,000 draws are required to obtain an accuracy level *z* of 0.0001 percent at the 95% confidence level.

MLE Results

MLE parameter estimates, normalized using the scalar average price, converged normally in the Matlab programming environment which uses the estimation procedure detailed in Hahn and Shapiro (1994). Parameters for normalized cattle prices were 18.7 and 0.054 for α and β, respectively, and 14.8 and 0.068 for milk. Parameter estimates were significantly different from zero, though inference with respect to such parameters and with so small a sample size is dubious.

Monte Carlo Results

Our first task is to verify that the value-based error terms from the Farrell data $TE_v(y_{vk'}, x_{vk'})$, $\gamma_{k'}$, and $\delta_{k'}$, listed in table 4.3 of the empirical section, lie within our 95% confidence intervals. We find that they do, as shown in figure 4.5.

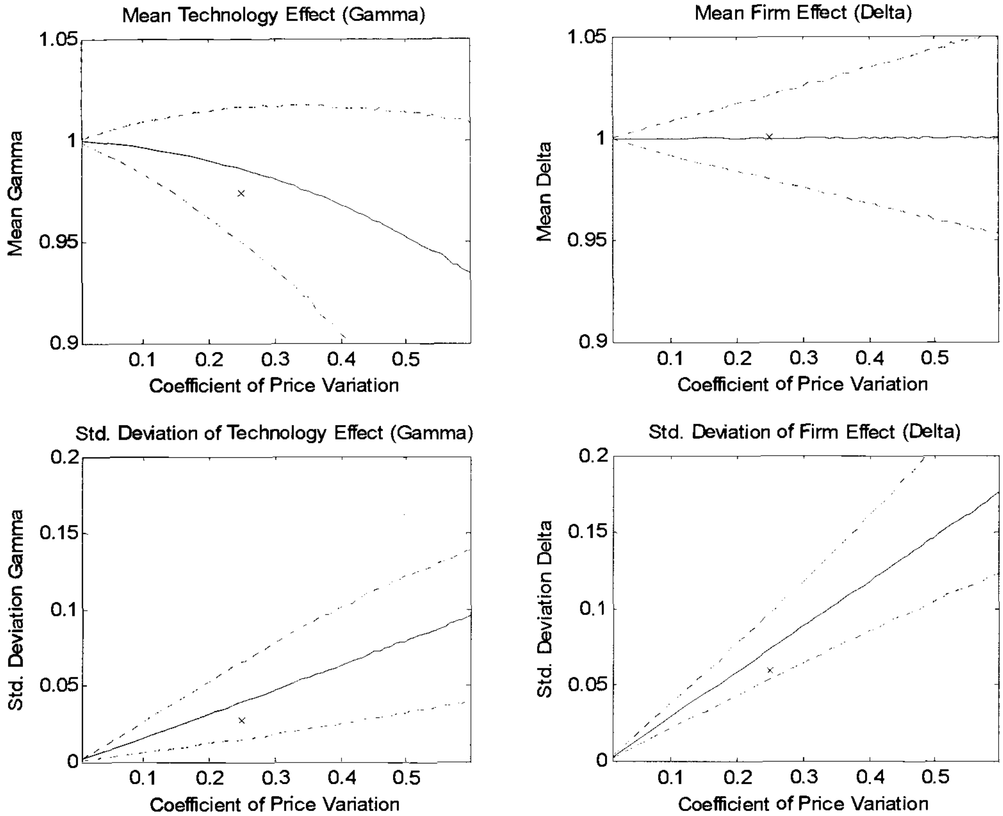

Figure 4.5. Expected Error Terms with 95% Confidence Intervals

Here, the solid lines represent the expected values with dotted lines indicating the 95% confidence intervals. Mean error levels are shown in the two upper panels and expected standard deviations in the two lower panels. Technology-related terms appear along the left side and firm-related terms along the right. In each panel the "x" corresponds to the level of error observed in the Farrell data from table 4.3. We can see that these error levels, along with their observed standard deviations lie within the 95% confidence intervals estimated by our model.

Beginning in the upper left hand panel of figure 4.4, the expected technology-related error term is decreasing at an increasing rate and ranges from zero error (gamma = 1) when prices do not vary, to a 5% downward bias (gamma = 0.95) when the coefficient of price variation reaches 0.5. This downward sloping line suggests that price volatility leads to systematic downward bias in value-based technical efficiency scores. Downward bias is not surprising, since we would expect volatility to disburse observations pushing the efficient input isoquant toward the origin. The width of the 95% confidence error grows from zero when prices do not vary, to 14% at a coefficient of price variation of 0.50. At this higher level of price variation then, technical efficiency scores may range from 1% overstated to 13% understated.

As we saw from the Farrell empirical example, a 2.7% downward bias was sufficient to cause considerable disruption in the relative efficiency rankings of states. It is also worth noting that we are considering the impact of variation in only one third of total output value. Clearly, price variation in other inputs and outputs could materially alter the magnitude of the bias demonstrated here.

Moving to the upper right panel of figure 4.4, we see that the expected firm effect is invariant to price volatility, that is, delta is unity over all levels of price variation. The width of the 95% confidence interval, however, increases from zero when prices do not vary, to 8% when the coefficient of price variation reaches 0.5. Again, average value-based technical efficiency scores may be understated or overstated by as much as 4% at this level of price variation. This result is again not surprising, since we would expect random price variation to disburse observations, but not in a particular direction.

Conclusion

We have shown that when the Law of One Price does not hold, value-based DEA models coincide with factor-based DEA models only under a preposterous restriction on prices and quantities. Finding no economic or probabilistic justification for such a restriction, we defined a multiplicative decomposition of the resulting estimation error into a technology effect and firm effect. To illustrate the possible magnitude and direction of this error we re-examined Farrell's original 1957 empirical data set and found evidence that the Law of One Price did not hold. Correcting for a portion of the price variation, we found the errors in Farrell's original technical efficiency scores to be material and the relative state rankings to be disrupted. Finally, mapping expected technology and firm effects over a range of price volatility levels, we found the bias resulting from the technology effect to be systematic and one-sided.

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Appendix

Claim 2. Under constant returns to scale (CRS), for one output (M=1), if the Law of One Price does not hold, then either there is a firm for which the value-based technical efficiency model does not coincide with the factor-based technical efficiency model or a restriction on prices and quantities must hold:

(10) $\neg LOP \Rightarrow$

either

(11) $\exists k': \quad TE_v(r_{k'}, e_{k'}) \neq TE_i(y_{k'}, x_{k'})$

or

(12) $\forall k, k', \quad \alpha_{kk'l} = h(z, x, y, p, w), \quad k, k' = 1, \ldots, K.$

The restriction in (12) is given by

(13) $$h(z, z_v, x, y, p, w) = \frac{z_k^* x_{k'l} x_{kn} p_k}{z_{vk}^* x_{k'n} x_{kl} p_{k'}} + \sum_{\substack{j=1 \\ j \neq k}}^{K} \frac{y_k p_k}{y_j p_{k'}} \left(\frac{z_j^* x_{k'l} x_{jn}}{z_{vk}^* x_{k'n} x_{kl}} - \frac{\alpha_{jk'l} z_{vj}^* x_{jl} p_{k'}}{z_{vk}^* x_{kl} p_j} \right).$$

To verify this, recall from (4)

(4) $$\neg LOP \Leftrightarrow \exists n, k, k': \quad \alpha_{k'kn} \neq 1 \quad n = 1, \ldots N, \quad k, k' = 1, \ldots, K.$$

If the Law of One Price is violated, then (4) holds and one of two possibilities exist, either the two models do coincide, or they do not. The restriction on alpha in (12) is a simple result of the prior.

To see that this is true, let us first derive alpha for firms k and k' and good j. Recall the definition of factor-based, input-oriented technical efficiency, under CRS and free-disposability for M=1, from (7):

(7) $$\begin{aligned} TE_i(y_{k'}, x_{k'}) \quad = \quad \min\{\lambda_{k'}: \quad & \sum_{k=1}^{K} z_k y_k \geq y_{k'} \\ & \sum_{k=1}^{K} z_k x_{k1} \leq x_{k'1} \lambda_{k'} \\ & \vdots \\ & \sum_{k=1}^{K} z_k x_{kN} \leq x_{k'N} \lambda_{k'} \\ & z_k \geq 0, \quad k = 1, \ldots, K\} \end{aligned}$$

First, we may divide each firm's input vector by its output scalar, under CRS and M=1.

$$TE_i(y_{k'},x_{k'}) = \min\{\lambda_{k'} : \sum_{k=1}^{K} z_k \geq 1$$

(1A)
$$\sum_{k=1}^{K} z_k \frac{x_{k1}}{y_k} \leq \frac{x_{k'1}}{y_{k'}}\lambda_{k'}$$
$$\vdots$$
$$\sum_{k=1}^{K} z_k \frac{x_{kN}}{y_k} \leq \frac{x_{k'N}}{y_{k'}}\lambda_{k'}$$
$$z_k \geq 0, \quad k=1,\ldots,K\}$$

Next, we may divide each of the N inequalities by the RHS input/output ratio, since the inequality relation is preserved under positive scalar division.

$$TE_i(y_{k'},x_{k'}) = \min\{\lambda_{k'} : \sum_{k=1}^{K} z_k \geq 1, \quad z_k \geq 0, \quad k=1,\ldots,K$$

(2A)
$$\sum_{k=1}^{K} z_k \frac{x_{k1}y_{k'}}{x_{k'1}y_k} \leq \lambda_{k'}$$
$$\vdots$$
$$\sum_{k=1}^{K} z_k \frac{x_{kN}y_{k'}}{x_{k'N}y_k} \leq \lambda_{k'}\}$$

If lambda is greater than or equal to each of the N LHS terms, then it is greater than or equal to the maximum of those terms.

(3A)
$$TE_i(y_{k'},x_{k'}) = \min\{\lambda_{k'} : \sum_{k=1}^{K} z_k \geq 1, \quad z_k \geq 0, \quad k=1,\ldots,K$$
$$\lambda_{k'} \geq \max\left\{\sum_{k=1}^{K} z_k \frac{x_{k1}y_{k'}}{x_{k'1}y_k},\ldots,\sum_{k=1}^{K} z_k \frac{x_{kN}y_{k'}}{x_{k'N}y_k}\right\}\}$$

Because the set of interest is compact, a maximal element exists. Denote the good associated with this maximal element as n and the solution vector of K weights as z^*. Then lambda can be expressed with equality.

(4A)
$$\lambda_{k'}(y_{k'},x_{k'}) = \sum_{k=1}^{K} z_k^* \frac{x_{kn}y_{k'}}{x_{k'n}y_k}$$
$$\text{where } \sum_{k=1}^{K} z_k^* \geq 1, \quad z_k^* \geq 0$$

Similarly, for the value-based measure, indicated by subscript v, and maximal element associated with good l not necessarily n.

(5A)
$$\lambda_{vk'}(r_{k'},e_{k'}) = \sum_{k=1}^{K} z_{vk}^{*} \frac{x_{kl} w_{kl} y_{k'} p_{k'}}{x_{k'l} w_{k'l} y_k p_k}$$
$$\text{where } \sum_{k=1}^{K} z_{vk}^{*} \geq 1, \quad z_{vk}^{*} \geq 0.$$

Then, if the two models coincide,

(6A)
$$\sum_{k=1}^{K} z_k^{*} \frac{x_{kn} y_{k'}}{x_{k'n} y_k} = \sum_{k=1}^{K} z_{vk}^{*} \frac{x_{kl} w_{kl} y_{k'} p_{k'}}{x_{k'l} w_{k'l} y_k p_k}$$
$$\text{where } \sum_{k=1}^{K} z_k^{*} \geq 1, \quad \sum_{k=1}^{K} z_{vk}^{*} \geq 1, \quad z_k^{*}, z_{vk}^{*} \geq 0.$$

Substituting in our definition of alpha,

(7A)
$$\sum_{k=1}^{K} z_k^{*} \frac{x_{kn} y_{k'}}{x_{k'n} y_k} = \sum_{k=1}^{K} z_{vk}^{*} \alpha_{kk'l} \frac{x_{kl} y_{k'} p_{k'}}{x_{k'l} y_k p_k}$$
$$\text{where } \sum_{k=1}^{K} z_k^{*} \geq 1, \quad \sum_{k=1}^{K} z_{vk}^{*} \geq 1, \quad z_k^{*}, z_{vk}^{*} \geq 0.$$

Simplifying, factoring out alpha associated with firms k and k' and good l, and changing our index accordingly,

(8A)
$$z_k^{*} \frac{x_{kn}}{x_{k'n} y_k} + \sum_{\substack{j=1 \\ j \neq k}}^{K} z_j^{*} \frac{x_{jn}}{x_{k'n} y_j} = z_{vk}^{*} \alpha_{kk'l} \frac{x_{kl} p_{k'}}{x_{k'l} y_k p_k} + \sum_{\substack{j=1 \\ j \neq k}}^{K} z_{vj}^{*} \alpha_{jk'l} \frac{x_{jl} p_{k'}}{x_{k'l} y_j p_j}$$
$$\text{where } j = 1,\ldots,K, \quad \sum_{k=1}^{K} z_k^{*} \geq 1, \quad \sum_{k=1}^{K} z_{vk}^{*} \geq 1, \quad z_k^{*}, z_{vk}^{*} \geq 0.$$

Solving for our alpha,

(9A)
$$\alpha_{kk'l} = \frac{\dfrac{z_k^{*} x_{kn}}{x_{k'n} y_k} + \displaystyle\sum_{\substack{j=1 \\ j \neq k}}^{K} \frac{1}{y_j} \left(\frac{z_j^{*} x_{jn}}{x_{k'n}} - \frac{z_{vj}^{*} \alpha_{jk'l} x_{jl} p_{k'}}{x_{k'l} p_j} \right)}{\dfrac{z_{vk}^{*} x_{kl} p_{k'}}{x_{k'l} y_k p_k}}$$
$$\text{where } \sum_{k=1}^{K} z_k^{*} \geq 1, \quad \sum_{k=1}^{K} z_{vk}^{*} \geq 1, \quad z_k^{*}, z_{vk}^{*} \geq 0.$$

Simplifying further, we obtain,

(10A)
$$\alpha_{kk'l} = \frac{z_k^{*} x_{k'l} x_{kn} p_k}{z_{vk}^{*} x_{k'n} x_{kl} p_{k'}} + \sum_{\substack{j=1 \\ j \neq k}}^{K} \frac{y_k p_k}{y_j p_{k'}} \left(\frac{z_j^{*} x_{k'l} x_{jn}}{z_{vk}^{*} x_{k'n} x_{kl}} - \frac{\alpha_{jk'l} z_{vj}^{*} x_{jl} p_{k'}}{z_{vk}^{*} x_{kl} p_j} \right)$$
$$\text{where } \sum_{k=1}^{K} z_k^{*} \geq 1, \quad \sum_{k=1}^{K} z_{vk}^{*} \geq 1, \quad z_k^{*}, z_{vk}^{*} \geq 0.$$

Q.E.D.

5. General Conclusion

In these three essays, we analyzed pricing policy, optimal capitalization levels, ruin probabilities, and efficiency measurement all within the context of agricultural market imperfection. First, incomplete agricultural contract markets result from unobservable yield processes. The resulting arbitrage-free forward price bounds are useful for analyzing firm-level pricing decisions and identifying potentially illicit pricing behavior. In competitive markets, cooperatively owned firms tend to retain less equity than investor-owned firms, and thus experience a higher probability of financial ruin. Finally, variable prices and imperfect price information lead to DEA technical efficiency estimates that are biased and one-sided.

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