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Title: A MODEL FOR THE DETERMINATION OF OPTIMUM SETTING DIMENSIONS FOR TRACTOR YARD/SWING OPERATIONS

Abstract approved: ____ ________________

A problem of interest to forest managers is the optimum arrangement of truck roads and landings for economical logging operations. This problem becomes more complex if a combined yarding and swinging operation is considered. It is possible to formulate a mathematical model to express the cost per unit volume for a particular configuration of truck roads, landings, and swing roads arranged to accommodate this type of harvest operation. Then, any of several numerical methods may be employed to assess the sought-for optimum configuration. The parameters of interest are the truck road spacing, landing spacing, and swing road length that produce the smallest logging cost. This paper critically examines several assumptions made in formulation of the problem, including yarding cost computation, tractor movement patterns, and average yarding distance.

This paper uses a numerical method not frequently employed with constrained objective functions: Newton Multivariate Gradient Iteration. A computer program was developed to implement the iteration procedure, using a Hewlett-Packard 9830A Desktop Computer. The solution procedure reduced iterations required for convergence

from several thousand experienced with exhaustive enumeration techniques to less than thirteen. The use of this gradient method, observations on its behavior, and insights into the analytical approach to a complex problem are the subjects of this paper.

A MODEL FOR THE DETERMINATION OF OPTIMUM SETTING DIMENSIONS FOR TRACTOR YARD/SWING OPERATIONS

by

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PROBLEM STATEMENT

The density of truck roads and landings in timber harvest units is a matter of concern to the forest manager seeking to minimize logging costs. Several approaches to optimum road and landing spacing have been made in the literature of forest management. One special case involves the use of tractor swings to shorten yarding distance.

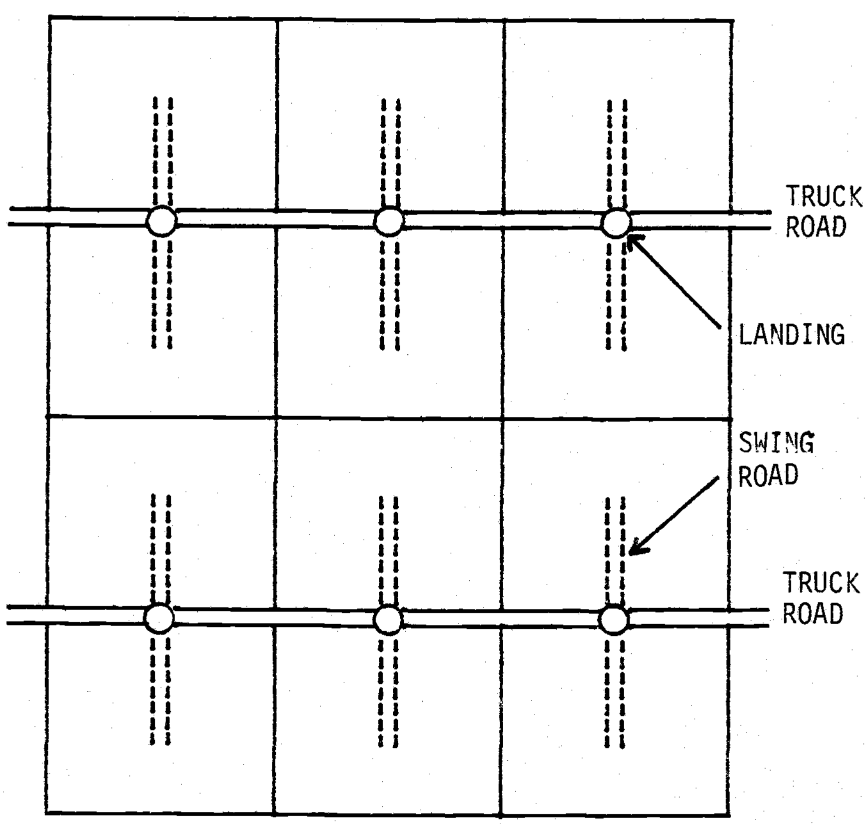

Figure 1. Tractor Yard/Swing Problem.

Although many other authors have studied optimal road and landing spacing for a single yarding machine (Matthews, 1942; Rowan, 1976; Weller, 1977), operations involving both yarding and swinging have not been investigated. Current consideration of this practice on timberlands in Borneo prompted the following examination of the problem.

Consider a block of forest land, large enough to be regarded as infinite, with a system of parallel, evenly spaced truck roads. Along each truck road, landings are spaced at regular intervals. From each landing a tractor swing road extends in both directions at right angles to the truck road. (This scheme is shown in Figure 1.)

OBJECTIVES

The objectives of this study are:

1) Prepare a mathematical formulation for the average logging cost associated with a combination yard/swing operation.
2) Set up a solution procedure using a gradient search method, to find optimum dimensions for:
    a) the spacing between truck roads,
    b) the spacing between landings, and
    c) the distance that the tractor swing road must extend into the setting.
3) Critically examine any assumptions which must be made in the formulation of the problem.
4) Write a computer routine to perform the optimization calculations.

5) Run through several examples to become familiar with the solution procedure and draw managerially applicable conclusions from the outcome.

6) Discuss the overall validity and practicality of the model, and make suggestions for its improvement.

LITERATURE REVIEW

The classic discussion of road/landing spacing problems is a forest economics text published in 1942 (Matthews, 1942). Matthews' treatment of yarding costs, breakeven analysis, and minimization is excellent, although perhaps limited by the nonexistence of advanced digital computers in 1942. Matthews' assumptions on average yarding distance are used in this paper, although other authors have developed more mathematically precise formulations (Peters, 1977; Suddarth and Herrick, 1964). More recently, other studies of road/landing spacing optimization have been published which sought to bring more detail into the costing-out of road construction, yarding, and landing construction. One of these was oriented to iterative solution by computer (Carter, Gardner, & Brown, 1973). Another took a practical approach to road/landing spacing optimizations, de-emphasizing the preciseness of the basic mathematical model (Rowan, 1976). Yet another author developed nomographs for finding optimum spacing (Weller, 1977). None of these writers have addressed the problem set forth in this paper: a combined yarding/swinging operation.

Many texts exist on the subject of optimization of non-linear multivariate functions. McMillan (1975) presents a concise matrix formulation for Newton multivariate gradient iteration. Himmelblau (1972) discusses a host of related topics, including other gradient methods, direct search methods, and penalty function methods. Simmons (1975) devotes considerable discussion to analytical approaches such as use of Lagrange multipliers. A chapter on constrained local optima was particularly useful in explaining some apparently contradictory results in the early phases of this study. Gottfried & Weismann (1973) discuss several direct search algorithms, including binary series, Fibonacci series, and golden section series. My first attempt at a solution algorithm used a direct-search scheme of my own design: exponential increment (Nickerson, 1976). An operations research text by Wagner (1969) discusses topics which suggested an alternate formulation of the problem, using the integral of a "density" function for logging cost. The literature is so rich in techniques for analysis of problems like the one in this paper, that it was an exercise in restraint to remain focused on one approach.

MATHEMATICAL FORMULATION

Throughout this paper, metric units will be used: volume in cubic meters, distance in meters, area in hectares. A "front-end" and "back-end" to translate between metric and non-metric units are simple additions to the solution technique discussed here.

We have assumed a simplified model for both yarding and swing cost:

Yarding Cost per Unit Volume

= Fixed Cost, Yarding + (Variable Cost per Unit Distance

x Yarding Distance)

Swing Cost per Unit Volume

= Fixed Cost, Swing + (Variable Cost per Unit Distance x Swing Distance)

This is a simple model that says yarding cost is a linear function of yarding distance. In reality, it may also be a function of ground slope, log size, soil condition, and other factors. Innumerable time studies, such as those upon which U.S. Forest Service yarding cost adjustment factors are based (USFS, 1976), support this observation. But on a given piece of ground, these other factors may be uniform enough to be regarded as constant; this is the assumption we will proceed upon in this study. Tables of yarding costs generated by the U.S. Forest Service and the Bureau of Land Management, based on cost studies, bear out the near-linear relation of yarding cost to yarding distance, assuming other factors are held constant (USFS, 1976; BLM, 1977).

Variable names used in this formulation are:

Variable or coefficient	Units
K = average logging cost	$/m^3
Y = one-half truck road spacing	meters
X = landing spacing	meters
L = swing road length	meters
F1 = swing machine movein cost	$
F2 = yarding machine movein cost	$
F3 = landing construction and setup cost	$

V = volume removed	m^3/hectare
A = yarding distance factor	meter/meter
C = swing road construction cost	\$/meter
R = truck road construction cost	\$/meter
V1 = swing variable cost	\$/m^3/meter
V2 = yarding variable cost	\$/m^3/meter
V3 = swing fixed cost	\$/m^3
V4 = yarding fixed cost	\$/m^3

One must investigate the movement of logs from stump to landing before formulating a cost function. Looking at Figure 1, the first question to resolve is, "May logs be yarded onto the truck road?" If this is permissible, let us assume the logs may be loaded at their point of arrival on the truck road; surely if a yarding tractor can operate onto the truck road surface, the loader can operate along the truck road, too. If yarding onto the truck road is allowed, we will call this condition "Case 1." Also, in order for us to have a clear understanding of average yarding distance, we must make an assumption about the path the yarding vehicle will travel. Let us assume it takes the shortest path: the yarding tractor moves directly to the road in a direction perpendicular to the road. The yarding distance factor (A), equal to or greater than 1, is used to allow for "weave" in the yarding tractor's route caused by obstacles such as stumps and terrain roughness.

Let us examine Case 1 in detail. If yarding is permitted onto the truck road, it obviously follows that there is a segment of the setting that will be yarded to the truck road (segment 1 in Figure 2), and another segment that will be yarded to the swing road, then swung to the landing (segment 2 in Figure 2). We are interested in defining the boundary, or breakeven line, between these two segments.

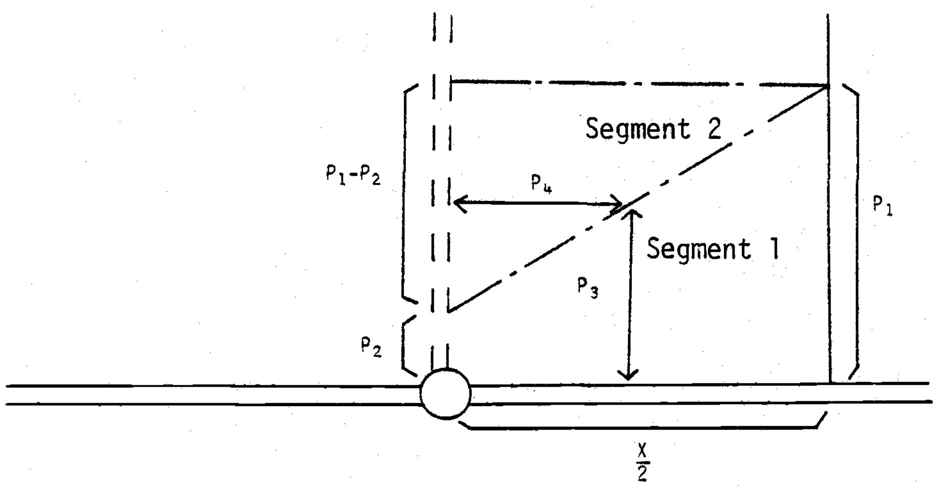

Figure 2. Breakeven Point if Yarding Allowed to Truck Road

Consider a log lying along the boundary of the unit, on the breakeven line we are seeking to define. Let the distance from the truck road be P_1. The cost to yard the log to the truck road is:

$$A\ V_2\ P_1 + V_4 \qquad \text{(eq. 1)}$$

The cost to yard the log to the swing road, and then swing it to the landing is:

$$\overset{\text{(Yarding)}}{\left(\frac{A\ V_2\ X}{2} + V_4\right)} + \overset{\text{(Swinging)}}{\left(V_1\ P_1 + V_3\right)} \qquad \text{(eq. 2)}$$

Since the two costs are by definition equal at the breakeven point, we have:

$$(A\ V_2\ P_1) + V_4 = ((1/2)A\ V_2\ X) + (V_4) + (V_1\ P_1) + (V_3) \qquad \text{(eq. 3)}$$

or:

$$P_1 = \frac{((1/2)A\ V_2\ X) + V_3}{A\ V_2 - V_1} \qquad \text{(eq. 4)}$$

Now consider a log along the swing road, a distance from the truck road. More specifically, the log lies just off the swing road, so that the yarding tractor will have to deliver it to the swing road, thereby experiencing only the fixed yarding cost. The associated costs are:

$$\text{Cost yarded to truck road} = (A\ V_2\ P_2) + V_4 \qquad \text{(eq. 5)}$$

$$\text{Cost yarded to swing road and swung to landing} = \underset{\text{(Yarding)}}{(V_4)} + \underset{\text{(Swinging)}}{(V_1\ P_2 + V_3)} \qquad \text{(eq. 6)}$$

Again equating the two costs to define the breakeven condition:

$$(A\ V_2\ P_2) + V_4 = V_4 + (V_1\ P_2) + V_3 \qquad \text{(eq. 7)}$$

or:

$$P_2 = \frac{V_3}{A\ V_2 - V_1} \qquad \text{(eq. 8)}$$

It can be demonstrated that the breakeven line is a straight line: consider a log somewhere along the breakeven line, located a distance P_3 from the truck road and P_4 from the swing road. The associated costs of moving this log are:

Cost yarded to truck road = $A\ V_2\ P_3 + V_4$ (eq. 9)

Cost yarded to swing road and swung to landing = $((A\ V_2\ P_4) + V_4) + ((V_1\ P_3) + V_3)$
(Yarding) (Swinging) (eq. 10)

Once again equating the two costs, we obtain:

$$(A\ V_2\ P_3) + V_4 = (A\ V_2\ P_4) + V_4 + (V_1\ P_3) + V_3 \qquad \text{(eq. 11)}$$

or:

$$P_3 = \frac{(A\ V_2\ P_4) + V_3}{A\ V_2 - V_1} \qquad \text{(eq. 12)}$$

Now, if the breakeven line is truly a straight line, it can be seen that similar triangles exist, described by the arbitrary position of the log at (P_3, P_4) and by the log at (P_1, boundary). This is described mathematically by equating the ratios:

$$\frac{P_3 - P_2}{P_4} = \frac{P_1 - P_2}{X/2}$$

Substituting expressions (4) (8) and (12) we obtain:

$$\frac{\dfrac{(A\ V_2\ P_4) + V_3}{A\ V_2 - V_1} - \dfrac{V_3}{A\ V_2 - V_1}}{P_4} \stackrel{?}{=} \frac{\dfrac{(1/2)(A\ V_2 X) + V_3}{A\ V_2 - V_1} - \dfrac{V_3}{A\ V_2 - V_1}}{X/2}$$

$$\frac{A\ V_2\ P_4}{A\ V_2 - V_1}\ \frac{1}{P_4} \stackrel{?}{=} \frac{X/2\ A\ V_2}{A\ V_2 - V_1}\ \frac{1}{X/2}$$

$$\frac{A\ V_2}{A\ V_2 - V_1} = \frac{A\ V_2}{A\ V_2 - V_1} \qquad \text{Q. E. D.}$$

An assumption that will greatly simplify the computation of average yarding distance for logs that are moved directly to the truck road is that $P_2 = 0$, i.e., that the breakeven line intercepts the landing. An examination of this assumption is made in Appendix C, concluding that the assumption is a safe one.

The above derivations, although somewhat lengthy and tedious, demonstrate the mechanics employed in deriving the cost functions for each case. Detailed examination for other segments and other cases will be omitted.

It develops that there are several distinct configurations of yarding patterns that may occur. Figure 3 shows Case 1a--yarding permitted to the truck road, $Y \geq P_1$. The configuration can be subdivided into five segments with distinct geometry. Similarly, Figure 4 shows Case 1b--yarding permitted to the truck road, $Y \leq P_1$--depicted with three segments. Case 1c, shown in Figure 5, is a degeneration to a condition of yarding directly to the truck road, which bears checking as a possible alternative. Figure 6 shows Case 2--yarding not permitted onto the truck road. Note that each "setting" is symmetric about both the truck road and the swing road.

Total cost for each setting is the sum of yarding cost for each segment, plus the sum of swing cost for each segment, plus road, landing, and movein cost. Average cost--the quantity we wish to minimize--is the total cost divided by the volume on the setting. The average cost functions for Cases 1a, 1b, 1c, and 2 are shown in Equations 13-16.

By carefully keeping the cost equations in the form of a summation of terms, the obtaining of the partial derivatives is made to be an easier task. Considering Case 1a, we must obtain partial derivative expressions with respect to L, Y, and X, and second

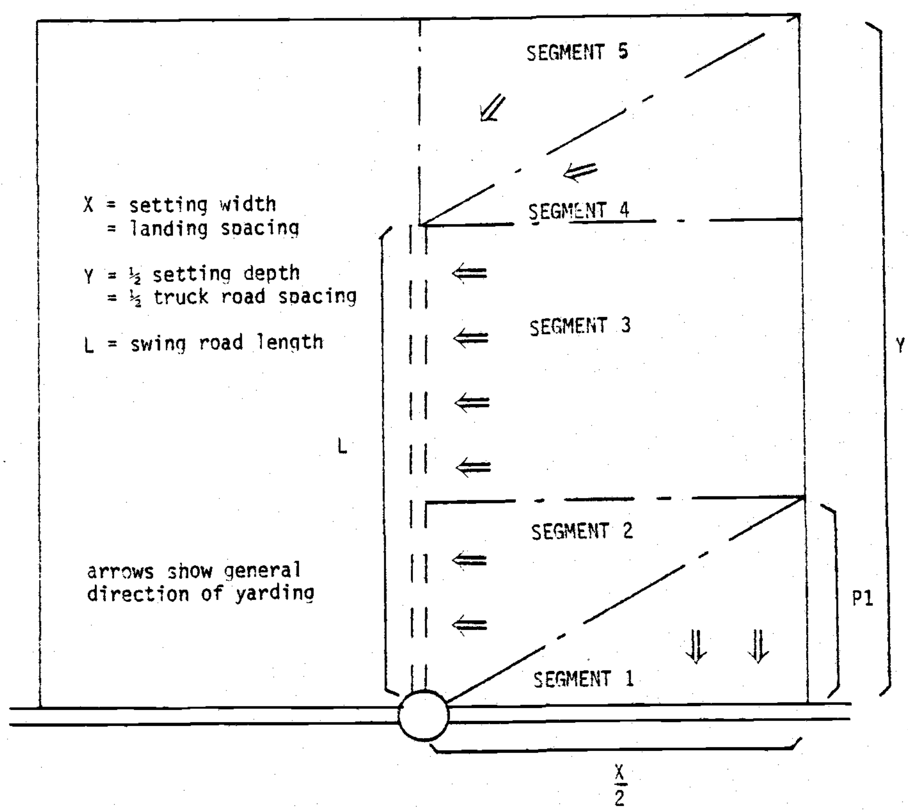

Figure 3. Case 1a: Yarding Permitted onto Truck Road, $P_1 \leq Y$

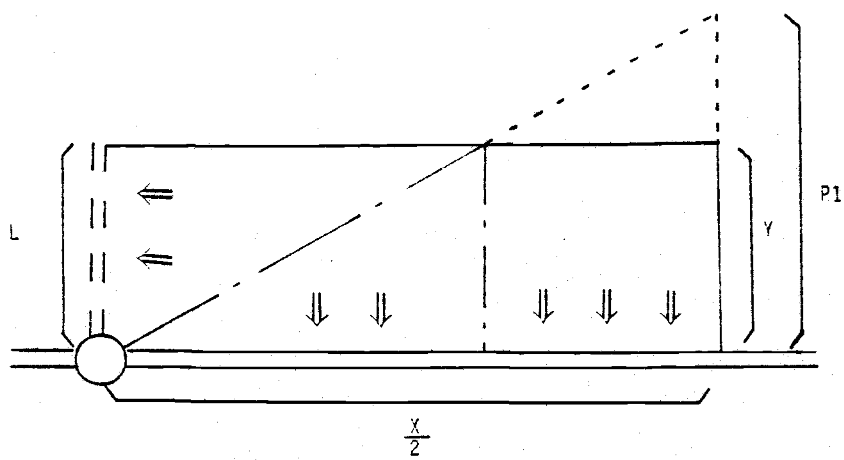

Figure 4. Case 1b: Yarding Permitted onto Truck Road, $P_1 > Y$

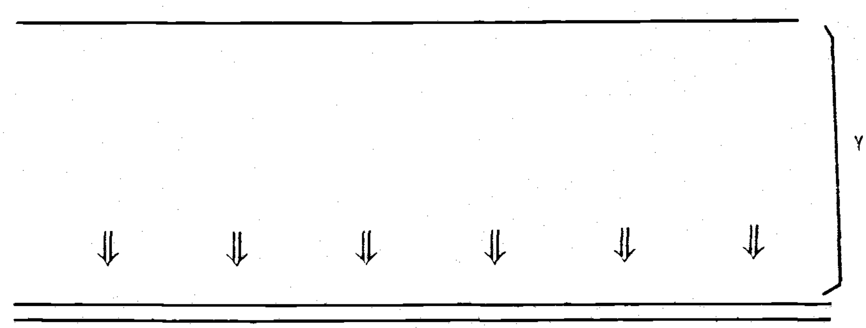

Figure 5. Case 1c: Yarding Direct to Truck Road

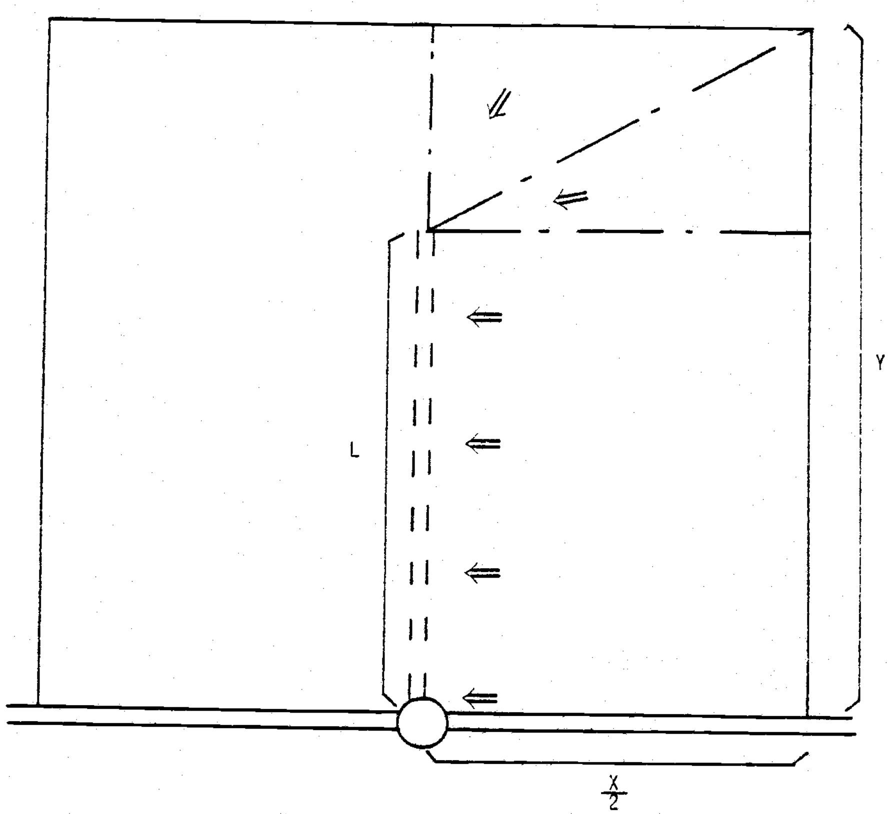

Figure 6. Case 2: Yarding not Permitted onto Truck Road

Equation 13: Cost Function for Case 1a

COST/UNIT VOLUME = K =

$$\frac{2}{Y}\left[\left(\frac{\frac{1}{6}A^2V_2X+\frac{1}{3}AV_3}{AV_2-V_1}\right)(V_2) + V_4\right]\left[\frac{\frac{1}{8}AV_2X+\frac{1}{4}V_3}{AV_2-V_1}\right]$$

$$+ \frac{2}{Y}\left[\left(\frac{AX}{6}\right)(V_2) + V_4\right]\left[\frac{\frac{1}{8}AV_2X+\frac{1}{4}V_3}{AV_2-V_1}\right]$$

$$+ \frac{2}{Y}\left[\left(\frac{AX}{4}\right)(V_2) + V_4\right]\left[\frac{L}{2} - \frac{\frac{1}{4}AV_2X+\frac{1}{2}V_3}{AV_2-V_1}\right]$$

$$+ \frac{2}{Y}\left[\left(\frac{1}{3}A\sqrt{Y^2-2YL+L^2+X^2}\right)(V_2) + V_4\right]\left[\frac{Y}{4} - \frac{L}{4}\right]$$

$$+ \frac{2}{Y}\left[\left(\frac{2}{3}A\sqrt{Y^2-2YL+L^2+\frac{1}{16}X^2}\right)(V_2) + V_4\right]\left[\frac{Y}{4} - \frac{L}{4}\right]$$

$$+ \frac{2}{Y}\left[\left(\frac{\frac{1}{3}AV_2X+\frac{2}{3}V_3}{AV_2-V_1}\right)(V_1) + V_3\right]\left[\frac{\frac{1}{8}AV_2X+\frac{1}{4}V_3}{AV_2-V_1}\right]$$

$$+ \frac{2}{Y}\left[\left(\frac{L}{2} + \frac{\frac{1}{4}AV_2X+\frac{1}{2}V_3}{AV_2-V_1}\right)(V_1) + V_3\right]\left[\frac{L}{2} - \frac{\frac{1}{4}AV_2X+\frac{1}{2}V_3}{AV_2-V_1}\right]$$

$$+ \frac{4}{Y}\left(LV_1 + V_3\right)\left(\frac{Y}{4} - \frac{L}{4}\right)$$

$$+ \frac{R}{2YV}$$

$$+ \frac{2CL + F_1 + F_2 + F_3}{2YXV}$$

Equation 14: Cost Function for Case 1b

COST/UNIT VOLUME = K =

$$(2)\left(\frac{AYV_2}{2} + V_4\right)\left(\frac{1}{2} - \frac{AV_2 - V_1}{AV_2X + 2V_3}\ Y\right)$$

$$+ \quad (2Y)\left(\frac{AYV_2}{3} + V_4\right)\left(\frac{AV_2 - V_1}{2AV_2X + 4V_3}\right)$$

$$+ \quad (2Y)\left(\frac{A^2V_2{}^2 - AV_1V_2}{3AV_2X + 6V_3}\ XY + V_4\right)\left(\frac{AV_2 - V_1}{2AV_2X + 4V_3}\right)$$

$$+ \quad (2Y)\left(\frac{2YV_1}{3} + V_3\right)\left(\frac{AV_2 - V_1}{2AV_2X + 4V_3}\right)$$

$$+ \quad \frac{R}{2YV}$$

$$+ \quad \frac{F_1 + F_2 + F_3 + 2CL}{2YXV}$$

Equation 15: Cost Function for Case 1c

COST/UNIT VOLUME = K =

$$\frac{AV_2Y}{2} + V_4 + \frac{R}{2YV}$$

Equation 16: Cost Function for Case 2

COST/UNIT VOLUME = K =

$$\frac{AV2LX}{4Y} + \frac{V4L}{Y}$$

$$+ \frac{2}{Y} \left[1/3A\sqrt{Y^2 - 2YL + L^2 + X^2}\ (V2) + V4 \right] \left[\frac{Y}{4} - \frac{L}{4} \right]$$

$$+ \frac{2}{Y} \left[2/3A\sqrt{Y^2 - 2YL + L^2 + 1/16X^2}\ (V2) + V4 \right] \left[\frac{Y}{4} - \frac{L}{4} \right]$$

$$+ \frac{V1L^2}{2Y} + \frac{V3L}{Y}$$

$$+ \frac{4}{Y} \left[LV1 + V3 \right] \left[\frac{Y}{4} - \frac{L}{4} \right]$$

$$+ \frac{R}{2YV}$$

$$+ \frac{2CL + F1 + F2 + F3}{2YXV}$$

partials with respect to L^2, Y^2, X^2, LY, LX, and YX. We can do this on a term-by-term basis for the ten terms comprising Equation 1. Liberal employment of the basic rules of calculus leads us to the desired expressions shown in Equations 1.1a-c (Appendix A) and 1.2a-f (Appendix B). A systematic procedure for checking our derivations is to calculate a derivative artificially, by making a differential increment in one variable while holding the others constant, and comparing this value on a term-by-term basis with the value obtained by our expressions for the derivative.

DISCUSSION OF ASSUMPTIONS

It is necessary to make some assumptions to clarify or simplify the problem at hand. For this formulation, some of the most significant assumptions are:

1) We are seeking to design an optimal layout on a block of land without existing transportation systems, with uniform timber distribution, uniform topography, and sufficient size that we may consider a set of regular, rectangular settings without concern about "leftover" land.

2) Yarding or swinging costs vary directly with yarding or swinging distance, in the form:

   Yarding Cost = Fixed Cost + (Variable Cost x Yarding Distance)

3) The average yarding distance for a triangular segment of a setting is the distance from the centroid of the segment to the landing Matthews, 1942).

4) Yarding along the length of the swing road will be perpendicularly into the swing road, and yarding along the truck road will be perpendicularly into the truck road.

Let us examine several of these assumptions in some detail.

Linear Yarding Distance/Yarding Cost Relationship

Yarding cost curves developed by the U.S. Forest Service and the Bureau of Land Management show a linear or near-linear relation between yarding distance and yarding cost. These curves were derived empirically, from a limited pool of cost studies. Similar cost curves might be obtained, however, by a deterministic approach. Were we to know the expected velocity of the yarding tractor, the expected time associated with hookup, unhook, etc., the necessary labor, fuel, operating, depreciation, overhead, and maintenance costs, we could assemble a yarding-cost model. In doing this, it would become clear that certain phases of the yarding operation would be linearly dependent on yarding distance (move unloaded, move loaded), and other phases would be fixed amounts (hook, unhook, turnaround). Of course, this presupposes that other factors such as slope, soil conditions, and stump spacing are at a constant level; variation in these factors could be expected to affect yarding cost.

Average Yarding Distance for Triangular Segment

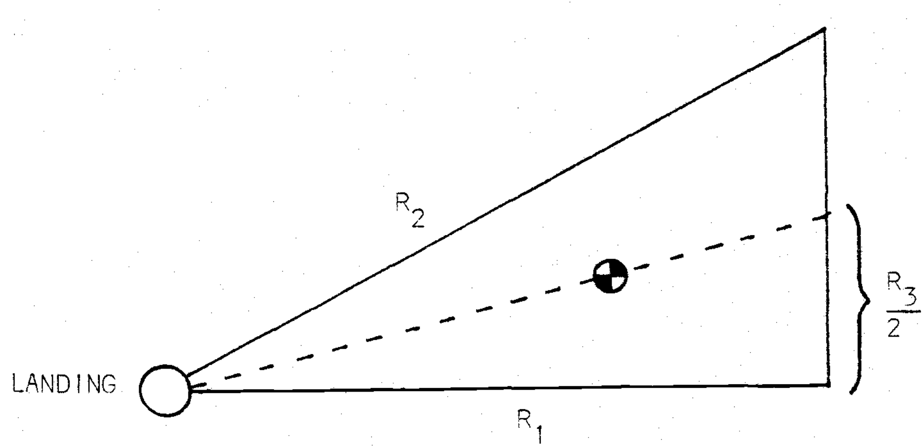

Figure 7. Centroid of Triangular Setting for Matthews' Average Yarding Distance

Matthews (1942) assumes that the average yarding distance of a right triangle segment of ground equals the distance from the centroid of the segment to the landing (Figure 7). Although this is nearly correct, a more mathematically precise result is obtained by integrating over the area:

$$\text{AVERAGE YARDING DISTANCE} = \int^{\text{area}} \frac{\text{distance}}{\text{area}} \, d\,\text{AREA}$$

Fortunately, this integration was performed by Suddarth and Herrick (1964) for a right triangular segment, and generalized by Peters (1977) for any triangular segment, leading to the following formulation for averaging yarding distance:

$$AYD = \left[\frac{R_1 + R_2}{6\,R_3^{\,2}}\right]\left[R_3^{\,2} + (R_1 - R_2)^2\right] + \left[\frac{(R_3^{\,2} - (R_1-R_2)^2)((R_1+R_2)^2 - R_3^{\,2})}{12R_3^{\,3}}\right] \ln\left(\frac{R_1+R_2+R_3}{R_1+R_2-R_3}\right) \quad \text{(eq. 17)}$$

Where R_1, R_2, R_3 can be dimensions of the sides of the right triangular segment, as shown in Figure 7. Using this same convention, Matthews' formulation for average yarding distance is:

$$AYD = 2/3\sqrt{R_1^{\,2} + \left[\frac{R_3}{2}\right]^2} \quad \text{(eq. 18)}$$

It should be apparent that the less-correct Matthews' formulation is more computationally attractive, particularly when faced with the necessity of differentiating twice. If Matthews' formulation is not too far off, we may be safe in using it.

We can reduce both Matthews' and Paters' formulations to ones involving a single variable. Let $R=R_1/R_3$. This is tantamount to scaling the right triangular setting down until the sides take on the dimensions:

$$R_3 = 1$$
$$R_1 = R$$
$$R_2 = \sqrt{R^2 + 1}$$

We can then compare the two methods on the basis of the scaled-down setting, where:

$$AYD^*_{matthews} = 2/3 \sqrt{R^2 + (1/2)^2} \qquad \text{(eq. 19)}$$

$$AYD^*_{peters} = 1/3\left(\sqrt{R^2 + 1} + R^2 \ln \frac{R + \sqrt{R^2+1} + 1}{R + \sqrt{R^2+1} - 1}\right) \qquad \text{(eq. 20)}$$

This is equivalent to the formulation by Suddarth and Herrick for right triangles. A plot of the percent difference between the two methods (see Figure 8) shows the interesting result that a peak difference of about 4 1/2% is experienced, when the ratio of R_1/R_3 is about 0.42, and that the Matthews formulation is strictly less than the Peters formulation. The difference is a magnitude that can be accepted, in view of the computational advantages.

<u>Yarding Perpendicular to the Swing Road</u>

The assumption that yarding to a road is accomplished by a yarding tractor moving into the road at right angles is implicit in formulations by Matthews, by Carter, Gardner, & Brown, and by Rowan. The same assumption is made in this formulation (see Figure 9). There may be physical or procedural restrictions that effectively result in perpendicular yarding into the swing road, but the options should be investigated.

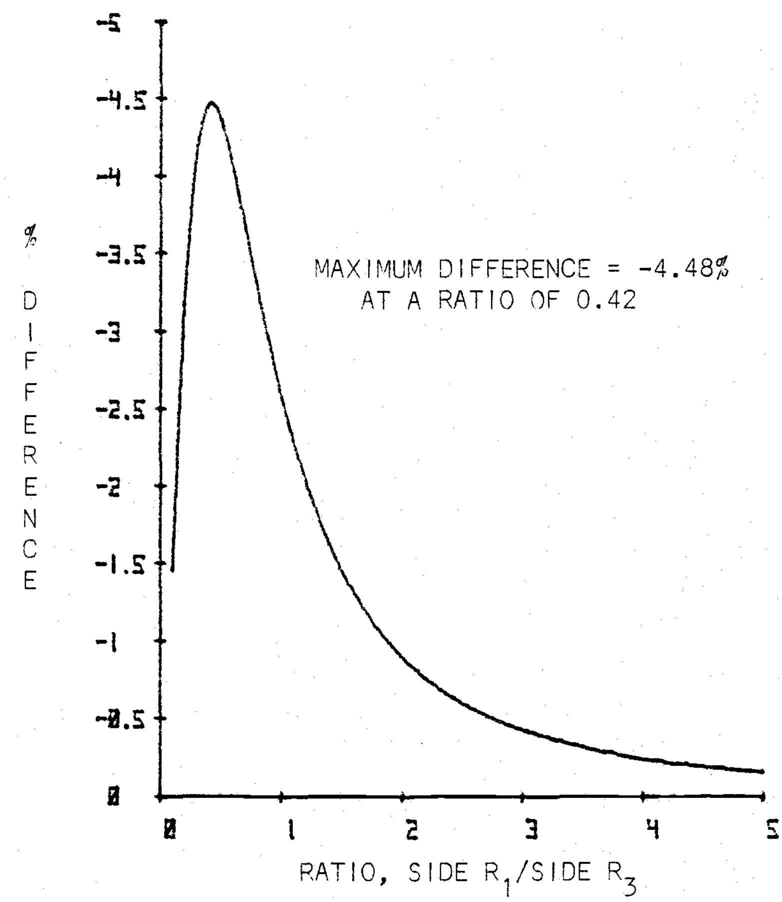

Figure 8. Difference between Peters and Matthews Average Yarding Distance

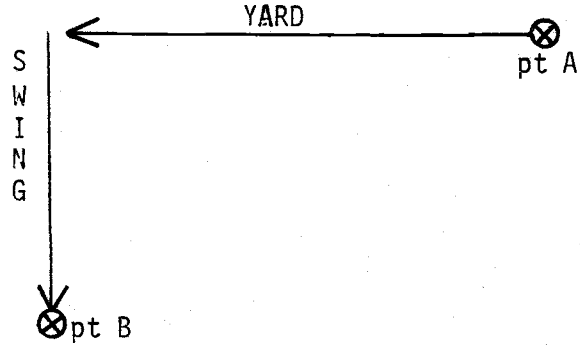

Figure 9. Yarding Perpendicular to Road.

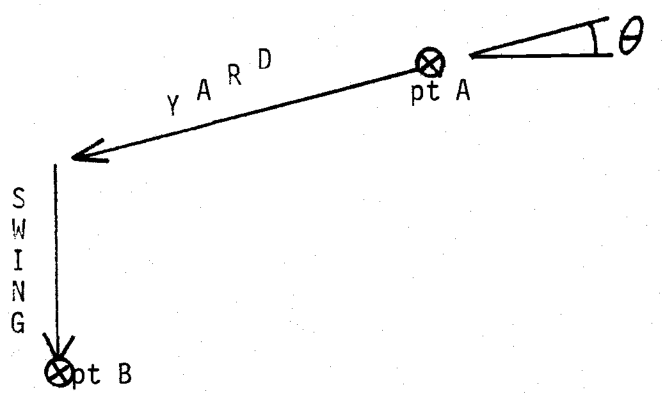

Figure 10. Yarding Not Perpendicular to Road.

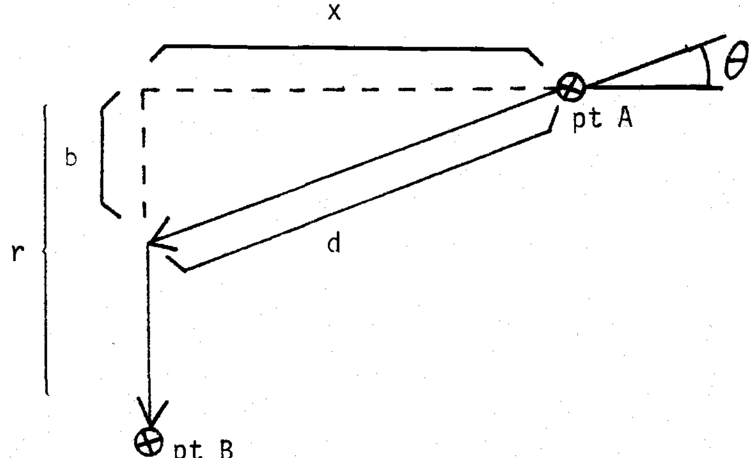

Figure 11. Tradeoffs when Yarding Not Perpendicular to Road.

Consider the incremental cost of yarding from point A to point B as shown in Figure 11:

$$\text{Cost} = AV_2 d + V_1 \ (r - b)$$

$$= AV_2 \sqrt{b^2 + x^2} + V_1 \ (r - b) \qquad \text{(eq. 21)}$$

now, let:

$$k = \frac{\text{Cost}}{xAV_2}$$

$$= \sqrt{\frac{b^2}{x^2} + \frac{x^2}{x^2}} + \frac{V_1}{AV_2} \left(\frac{r}{x} - \frac{b}{x}\right) \qquad \text{(eq. 22)}$$

and let:

$$k_1 = k - \frac{V_1}{AV_2} \frac{r}{x}$$

$$= \sqrt{(b/x)^2 + 1} - \frac{V_1}{AV_2} (b/x) \qquad \text{(eq. 23)}$$

This in effect isolates on the right side of the equation the terms that describe the angle of inclination from the perpendicular, labelled θ in Figure 10. From Figure 11 it can be seen that b/x = tan θ. Therefore, the economically optimum angle of inclination can be inferred from the minimum k_1, i.e.:

$$\frac{d\ k_1}{d\ (b/x)} = 0 \Longrightarrow b/x_{optimal} \Longrightarrow \theta_{optimal}$$

$$\frac{d\ k_1}{d\ (b/x)} = \frac{b/x}{\sqrt{1 + (b/x)^2}} - \frac{V_1}{AV_2} = 0$$

$$b/x = \frac{V_1}{AV_2} \sqrt{1 + (b/x)^2}$$

squaring both sides:

$$(b/x)^2 = (\frac{V_1}{AV_2})^2 \quad (1 + 2b/x + (b/x)^2)$$

$$0 = ((\frac{V_1}{AV_2})^2 - 1) \; (b/x)^2 + 2(\frac{V_1}{AV_2})^2 \; b/x + (\frac{V_1}{AV_2})^2 \qquad \text{(eq. 24)}$$

Now, let $(\frac{V_1}{AV_2}) = N$. Equation 24 becomes:

$$(N^2-1) \; (b/x)^2 + 2N^2 \; (b/x) + N^2 = 0$$

which can be solved by the quadratic equation:

$$b/x = \frac{-2N^2 + \sqrt{4N^4 - (4)\;(N^2-1)\;(N^2)}}{2\;(N^2-1)}$$

$$= \frac{-2N^2 + \sqrt{4N^2}}{2(N^2-1)}$$

$$= \frac{-N\;(N+1)}{(N+1)(N-1)}$$

$$= \frac{N}{1-N}$$

$$b/x_{optimal} = \frac{\frac{V_1}{AV_2}}{(1 - \frac{V_1}{AV_2})} \qquad \text{(eq. 25)}$$

and,

$$\theta_{optimal} = \arctan \left\{ \frac{\frac{V_1}{AV_2}}{1 - \frac{V_1}{AV_2}} \right\} \qquad \text{(eq. 26)}$$

Figure 12 is a graphical display of optimal angle of inclination from the perpendicular over a range of $\frac{V_1}{V_2}$ for various values of A. It appears that the perpendicular yarding assumption is not too good: $\theta_{optimal}$ is significantly different from zero for most of the range of values we can imagine. However, if we investigate the difference in variable yarding cost between perpendicular yarding and yarding at $\theta_{optimal}$, we see the assumption in a much more favorable light.

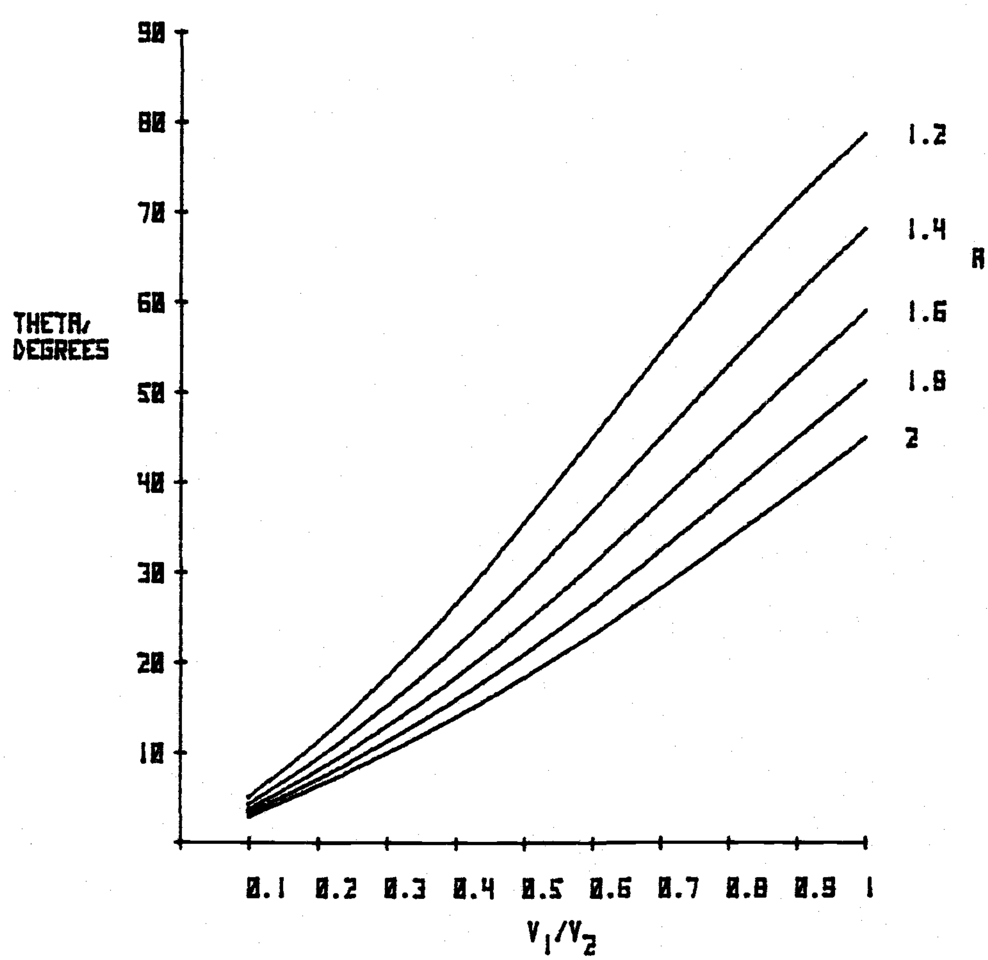

Figure 12. Optimum Yarding Angle for Various Values of A

Assume that x = 50 and r = 300:

A	V_1	V_2	$\theta_{optimal}$	cost, yarding perpendicular	cost, yarding at $\theta_{optimal}$	% difference
2.0	0.03	0.30	3.0	39.00	38.96	0.1
1.2	0.10	0.10	78.7	36.00	35.59	1.2
2.0	0.10	0.10	45.0	40.00	39.14	2.2
1.2	0.03	0.30	5.2	27.00	26.94	0.2

The percent differences shown are of a magnitude that we can readily accept. In view of the computational advantages of the yarding-perpendicular assumption, we will proceed on this basis. Nonetheless, our findings on optimum angle of inclination as shown nondimensionally in Figure 12, can stand alone as a significant observation with applicability in other circumstances.

SOLUTION PROCEDURE

The recursion equation for the Newton multivariate nonlinear gradient iteration, in matrix notation (McMillan, 1975), is:

$$\hat{X}_{n+1} = \hat{X}_n - \hat{H}^{-1}\,\hat{Y}^T \qquad \text{(eq. 27)}$$

where, $\hat{X}_n$ = mx1 column vector of variable estimates at the nth iteration

$\hat{Y}^T$ = transpose of the 1xm row vector of the gradient of the function--the mx1 vector of partial derivatives

$\hat{H}^{-1}$ = inverse of the Hessian matrix--the mxm matrix of second partial derivatives

For this problem:

$$m = 3$$

$$\hat{X}_n = \begin{bmatrix} L_n \\ Y_n \\ \lambda_n \end{bmatrix}$$

$$\hat{Y}^T = \begin{bmatrix} \partial K/\partial L \\ \partial K/\partial Y \\ \partial K/\partial X \end{bmatrix}$$

$$\hat{H} = \begin{bmatrix} \partial^2 K/\partial L^2 & \partial^2 K/\partial L \partial Y & \partial^2 K/\partial L \partial X \\ \partial^2 K/\partial Y \partial L & \partial^2 K/\partial Y^2 & \partial^2 K/\partial Y \partial X \\ \partial^2 K/\partial X \partial L & \partial^2 K/\partial X \partial Y & \partial^2 K/\partial X^2 \end{bmatrix}$$

By determining the first and second partial derivatives for the cost functions in each case, it is possible to make an initial guess at the optimum values for L, Y, and X, and by means of equation 27, improve upon these guesses in an iterative fashion until the optimal solution is reached.

Separation of Cases

Each case was examined separately. The objective was to arrive at the optimum point for each case, and compute the associated cost. The permissible case with the lowest cost would be the preferred yarding configuration.

Initial Guess

As is frequently the situation with Newton-Raphson iteration, the value of the initial guess is important in determining the behavior of the iteration. It became apparent that a generally low initial guess produced the least problem. Difficulties encountered with bad initial guesses included non-convergent oscillation and extreme fluctuation. A "good" initial guess produced a well-behaved monotonic convergence that was beautiful to behold.

Constrained Function

In all cases the function we are seeking to minimize is not unconstrained. Moreover, the constraints operating are a little extraordinary. To begin with, Y and X must be strictly greater than zero, and L must be less than Y. In Case 1a, L and Y must both be greater than P_1 (which is a function of X). In Case 1b, Y must be less than P_1. The iteration function described earlier will not recognize these constraints, of course. Consequently, if the iteration function takes us into a non-feasible region, the computer algorithm must be prepared to nudge us back into the feasible region. At each iteration, if the infeasible region is entered, the variable values are readjusted in the direction of the feasible region. Obviously, iteration must commence in a region of feasibility. This procedure worked nicely in practice.

Stopping Criterion

The existence of constraints means that at the sought-for minimum-cost point, the partial derivative of one or more independent variables may not be zero. Inspection of the partial derivative is not sufficient to act as a stopping criterion. Similarly, the increment of each independent variable cannot serve as a stopping criterion, since at a minimum point where the cost hypersurface intersects a constraint hypersurface, the gradient may be far from flat. A practical stopping criterion is the change in the cost function from iteration to iteration: if this change drops below some small amount, we are not making a significant gain by continuing to iterate.

Convexity/Non-convexity

It is of importance to know whether a minimum point determined by iteration is a global minimum or a local minimum. In an unconstrained problem, examination of the sign of the determinants of the principal minor matrices of the Hessian matrix can be used to assess convexity or non-convexity. In an unstrained function, a global minimum would have to lie in a zone of convexity, and if the function were convex over its entire range of interest, a minimum found by an iterative search would be certainly a global minimum. In a constrained situation, however, this need not be true. A simplified example in one independent variable will illustrate. Consider F(x) in Figure 13, where x is constrained to lie between a and b. Note that the global minimum lies in the convex zone, but that, were our gradient search to commence in part of the non-convex zone, we would converge to an incorrect local minimum at x = b. In Figure 14, an incorrect local minimum exists within the convex zone, which we would converge on if we started in the convex (or part of the non-convex) zone, but the global minimum at x = b will be reached only if we start looking somewhere within the zone of non-convexity. In Figure 15, again the global minimum lies within the zone of non-convexity where the constraint intersects the function, at x = b, but an initial guess anywhere will reach this point, providing our stopping criterion is sufficiently flexible to carry us past the inflection point. In this case, the convex zone is actually a "quasi-convex" zone; it is indistinguishable from a genuinely convex zone on the basis of the Hessian matrix test. The upshot of all this is that one must have some feel for the nature of the cost function if one is to feel confident that the global minimum has been found. Therefore, the solution algorithm for this problem includes a mechanism for forcing the iteration into or out of a zone of convexity, by readjusting the variable values at any iteration on the basis of the Hessian matrix convexity test. Specifically, it was observed that a non-convexity "hangup" sometimes occurred when swing road length (L) equalled 1/2 setting length (Y). If this condition occurred in the absence of a convexity force, subsequent iterations continued to demonstrate the condition: L remained equal to Y, and the function remained non-convex. By readjusting L to halfway between the current iteration values for P_1 (minimum possible L) and Y (maximum possible L), a zone of convexity was almost always encountered in subsequent iterations.

Problems with zones of non-convexity and constraints making up non-convex sets are certainly not unique to the Newton Multivariate Gradient Method. Appendix D shows similar anomalies occurring with several other well-known gradient algorithms.

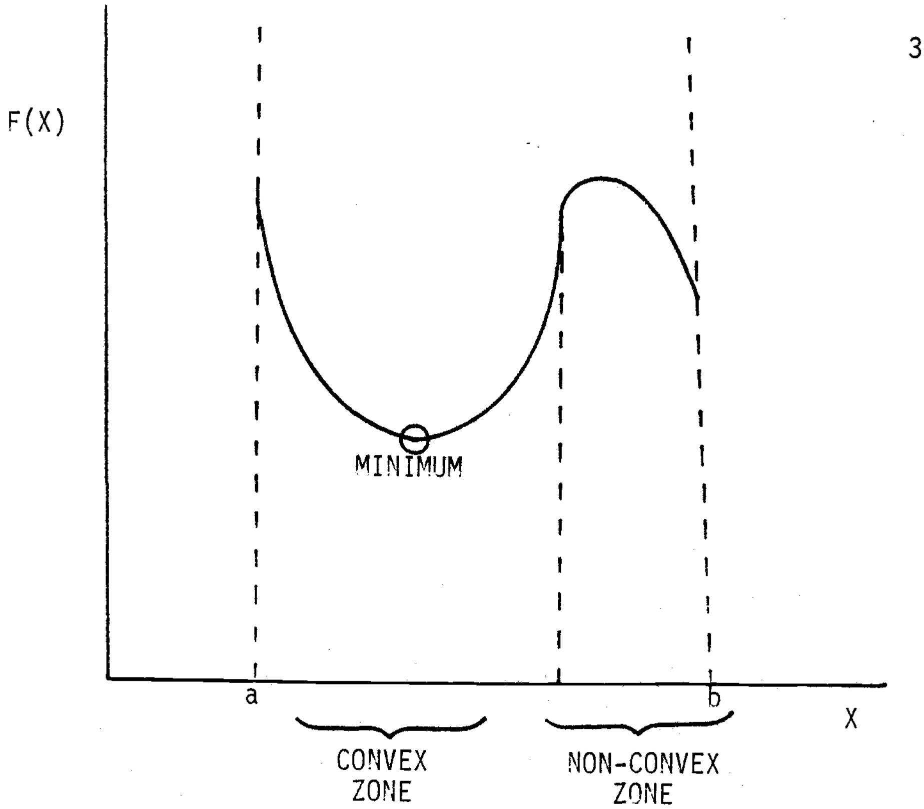

Figure 13. Global Minimum Within Convex Zone

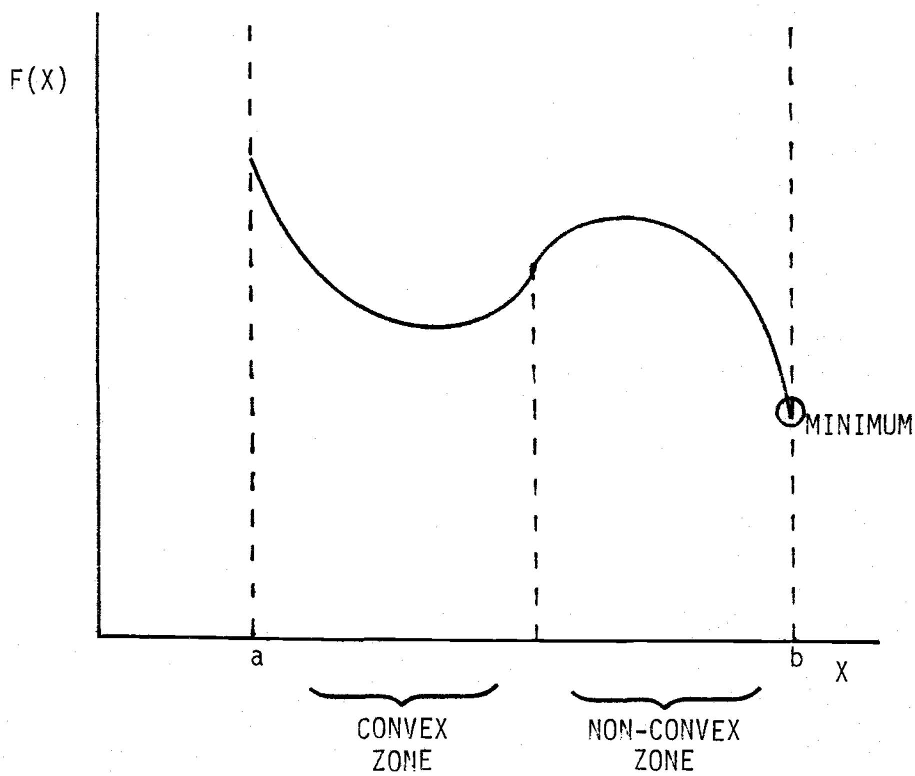

Figure 14. Global Minimum Not Within Convex Zone

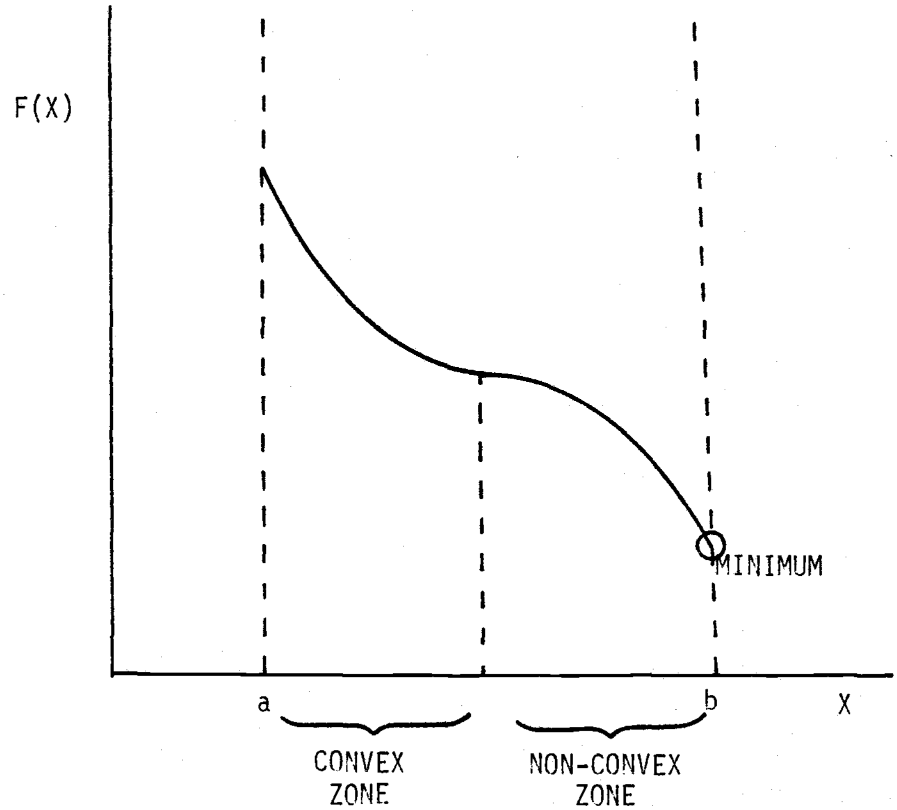

Figure 15. Global Minimum Not Within Quasi-convex Zone.

A computer program for performing the minimum search was written in BASIC for the Hewlett-Packard 9830 computer. Results for several example problems were obtained. A listing of this program is included in Appendix E.

Case 1c Direct Solution

Case 1c involves a single variable (see eq. 15). A direct solution is easily determined. Optimum Y occurs when $\frac{dK}{dY} = 0$:

$$\frac{dK}{dY} = \frac{AV_2}{2} - \frac{R}{2Y^2V} = 0$$

$$\Longrightarrow Y = \sqrt{\frac{R}{AV_2V}}$$

PROBLEM STATEMENT SUMMARIZED

Case 1a: Minimize $K = f(L,Y,X)$

s.t. $L \geq P_1$

$L \leq Y$

$Y \geq P_1$

$Y,X,L > 0$

Case 1b: Minimize $K = f(L,Y,X)$

s.t. $L = Y$

$Y \leq P_1$

$Y,X,L > 0$

Case 1c: Minimize $K = f(Y)$

s.t. $Y > 0$

Case 2: Minimize $K = f(L,Y,X)$

s.t. if $L = 0$ then swing movein = 0

$L \leq Y$

$Y,X > 0$

$L \geq 0$

RESULTS

Three examples were run. For Examples 1 and 2, a run was made without forcing the iteration into a zone of convexity, and another was made applying this correction. Example 3 was based on actual costs as currently listed in the Bureau of Land Management's Schedule 20 of logging costs in the Pacific Northwest (BLM, 1977).

Inputs were:

	Example 1	Example 2	Example 3
A	1.2	1.2	1.2
V	300.00	300.00	300.00
V_1	0.06	0.08	0.0023
V_2	0.11	0.10	0.0035
V_3	0.25	0.25	0.38
V_4	0.55	0.40	0.59
C	0.60	1.60	0.60
R	33.00	33.00	33.00
F_1	180.00	180.00	180.00
F_2	325.00	325.00	320.00
F_3	200.00	200.00	200.00
Initial L	20	20	250
Initial Y	20	20	250
Initial X	20	20	250

Table 1. Inputs for Example Problems.

Output summaries can be seen in Figures 16-20. Note that for Example 1, Case 1a and 2, an improvement (i.e., a lower minimum) was made by forcing the iteration into the zone of convexity. The function must be analogous in appearance to the two-dimensional function in Figure 14. In Example 2, Case 1a, attempting to force the iteration into the convex zone fails, suggesting that the convex zone is actually quasi-convex, as diagrammed in Figure 15.

Note that in all three examples, Case 1b degenerates in the direction of Case 1c. A very high truck road cost biases this outcome somewhat.

The interpretation of the results for each example is as follows:

		Example 1	Example 2	Example 3
If yarding is permitted onto truck road:	Optimum configuration	1a	1c	1c
	Optimum L =	121.7	--	--
	Optimum Y =	146.8	95.7	511.8
	Optimum X =	66.5	--	--
	Cost/m^3 =	\$12.23	\$11.89	\$2.74
If yarding not permitted onto truck road:	Optimum configuration	2	2	2
	Optimum L =	133.5	58.8	557.0
	Optimum Y =	156.4	126.1	703.2
	Optimum X =	53.7	71.4	183.1
	Cost/m^3 =	\$12.50	\$13.80	\$2.93

Table 2. Results for Example Problems

CONCLUSIONS

This study was an attempt to build a deterministic model of cost associated with a tractor yard/swing operation. If a model such as this is employed with the caution deserved by its simplifying assumptions, the forest manager may make some valuable inferences.

Case 1a--$P_1<Y$

Iteration	L	Y	X	Condition	Cost/M^3	Convexity
1	20.0	20.0	17.1	$P_1<Y$	64.63	Not convex
2	28.2	28.2	22.4	$P_1<Y$	41.19	Not convex
3	39.7	39.7	29.0	$P_1<Y$	27.37	Not convex
4	55.8	55.8	37.4	$P_1<Y$	19.39	Not convex
5	77.2	77.2	47.5	$P_1<Y$	15.06	Not convex
6	103.1	103.1	57.8	$P_1<Y$	13.06	Not convex
7	128.3	128.3	64.9	$P_1<Y$	12.41	Not convex
8	144.0	144.0	66.3	$P_1<Y$	12.30	Not convex
9	147.8	147.8	65.9	$P_1<Y$	12.29	Not convex

Case 1b--$P_1>Y$

Iteration	L	Y	X	Condition	Cost/M^3	Convexity
1	20.0	20.0	20.0	$P_1>Y$	59.60	Convex
2	29.7	29.7	30.3	$P_1>Y$	34.46	Convex
3	43.7	43.7	46.0	$P_1>Y$	21.83	Convex
4	62.2	62.2	73.0	$P_1>Y$	15.78	Convex
5	80.5	80.5	124.6	$P_1>Y$	13.43	Convex
6	83.1	83.1	245.0	$P_1>Y$	12.99	Convex
7	85.3	85.3	483.7	$P_1>Y$	12.78	Convex
8	87.6	87.6	954.7	$P_1>Y$	12.68	Convex
9	89.1	89.1	1892.5	$P_1>Y$	12.64	Convex
10	90.1	90.1	3764.0	$P_1>Y$	12.62	Convex
11	90.7	90.7	7504.3	$P_1>Y$	12.61	Convex

Case 1c--Yarding Perpendicular to Truck Road

Optimum Y = 91.3

Cost/M^3 = 12.60

Case 2--Yarding not Permitted onto Truck Road

Iteration	L	Y	X	Cost/M^3	Convexity
1	20.0	20.0	20.0	59.94	Not convex
2	28.3	28.3	25.8	38.73	Not convex
3	40.3	40.3	33.0	26.20	Not convex
4	57.1	57.1	41.2	18.97	Not convex
5	80.0	80.0	49.3	15.08	Not convex
6	108.3	108.3	54.6	13.28	Not convex
7	136.0	136.0	55.3	12.67	Not convex
8	152.9	152.9	54.1	12.56	Not convex
9	157.0	157.0	53.8	12.55	Not convex

Figure 16. Example 1 Not Forced into Convex Zone

Case 1a--$P_1 < Y$

Iteration	L	Y	X	Condition	Cost/M^3	Convexity
1	20.0	20.0	17.1	$P_1 < Y$	64.63	Not convex
2	26.1	28.2	22.4	$P_1 < Y$	41.12	Not convex
3	35.2	39.9	29.4	$P_1 < Y$	27.02	Convex
4	37.9	55.9	37.5	$P_1 < Y$	19.19	Convex
5	57.2	77.4	47.5	$P_1 < Y$	14.93	Convex
6	80.7	103.3	57.9	$P_1 < Y$	12.96	Convex
7	103.8	128.3	65.2	$P_1 < Y$	12.33	Convex
8	118.3	143.4	66.8	$P_1 < Y$	12.24	Convex
9	121.7	146.8	66.5	$P_1 < Y$	12.23	Convex

Case 1b--P1>Y

Iteration	L	Y	X	Condition	Cost/M^3	Convexity
1	20.0	20.0	20.0	$P_1 > Y$	59.60	Convex
2	29.7	29.7	30.3	$P_1 > Y$	34.46	Convex
3	43.7	43.7	46.3	$P_1 > Y$	21.83	Convex
4	62.2	62.2	73.0	$P_1 > Y$	15.78	Convex
5	80.5	80.5	124.6	$P_1 > Y$	13.43	Convex
6	83.1	83.1	245.0	$P_1 > Y$	12.99	Convex
7	85.3	85.3	483.7	$P_1 > Y$	12.78	Convex
8	87.6	87.6	954.7	$P_1 > Y$	12.68	Convex
9	89.1	89.1	1892.5	$P_1 > Y$	12.64	Convex
10	90.1	90.1	3764.0	$P_1 > Y$	12.62	Convex
11	90.7	90.7	7504.3	$P_1 > Y$	12.61	Convex

Case 1c--Yarding Perpendicular to Truck Road

Optimum Y = 91.3

Cost/M^3 = 12.60

Case 2--Yarding not Permitted onto Truck Road

Iteration	L	Y	X	Cost/M^3	Convexity
1	20.0	20.0	20.0	59.94	Not convex
2	14.2	28.3	25.8	38.37	Convex
3	23.4	40.3	32.9	25.96	Convex
4	38.2	57.3	41.0	18.81	Convex
5	59.3	80.3	49.0	14.96	Convex
6	86.1	108.5	54.3	13.20	Convex
7	113.2	136.1	55.1	12.61	Convex
8	129.7	152.6	54.0	12.50	Convex
9	133.5	156.4	53.7	12.50	Convex

Figure 17. Example 1 Forced into Convex Zone.

Case 1a--$P_1<Y$

Iteration	L	Y	X	Condition	Cost/M^3	Convexity
1	20.0	20.0	8.8	$P_1<Y$	101.85	Not convex
2	27.6	27.6	12.0	$P_1<Y$	61.70	Not convex
3	38.0	38.0	16.4	$P_1<Y$	39.12	Not convex
4	51.8	51.8	22.3	$P_1<Y$	26.36	Not convex
5	69.2	69.2	30.5	$P_1<Y$	19.31	Not convex
6	89.0	89.0	42.0	$P_1<Y$	15.68	Not convex
7	105.9	105.9	58.4	$P_1<Y$	14.05	Not convex
8	110.2	110.2	68.9	$P_1<Y$	13.67	Not convex
9	102.8	102.8	61.9	$P_1<Y$	14.00	Not convex
10	107.4	107.4	66.9	$P_1<Y$	13.76	Not convex
11	104.7	104.7	64.9	$P_1<Y$	13.87	Not convex
12	105.9	105.9	64.8	$P_1<Y$	13.84	Not convex
13	106.2	106.2	64.8	$P_1<Y$	13.84	Not convex

Case 1b--$P_1>Y$

Iteration	L	Y	X	Condition	Cost/M^3	Convexity
1	20.0	20.0	20.0	$P_1>Y$	61.12	Convex
2	29.6	29.6	30.4	$P_1>Y$	35.52	Convex
3	43.2	43.2	46.8	$P_1>Y$	22.55	Convex
4	61.4	61.4	73.9	$P_1>Y$	16.19	Convex
5	81.7	81.7	123.4	$P_1>Y$	13.45	Convex
6	94.8	94.8	227.0	$P_1>Y$	12.53	Convex
7	95.6	95.6	452.2	$P_1>Y$	12.21	Convex
8	95.4	95.4	905.1	$P_1>Y$	12.05	Convex
9	95.6	95.6	1809.1	$P_1>Y$	11.97	Convex
10	95.6	95.6	3616.7	$P_1>Y$	11.93	Convex
11	95.7	95.7	7231.4	$P_1>Y$	11.91	Convex
12	95.7	95.7	14460.7	$P_1>Y$	11.90	Convex

Case 1c--Yarding Perpendicular to Truck Road

Optimum Y = 95.7

Cost/M^3 = 11.89

Case 2--Yarding not Permitted onto Truck Road

Iteration	L	Y	X	Condition	Cost/M^3	Convexity
1	20.0	20.0	20.0	$P_1<Y$	61.59	Not convex
2	28.2	28.2	26.4	$P_1<Y$	39.88	Not convex
3	39.6	39.6	34.7	$P_1<Y$	27.23	Not convex
4	55.4	55.4	44.8	$P_1<Y$	20.07	Not convex
5	75.8	75.8	56.0	$P_1<Y$	16.34	Not convex
6	98.9	98.9	65.6	$P_1<Y$	14.76	Not convex
7	118.7	118.7	70.0	$P_1<Y$	14.31	Not convex
8	128.2	128.2	70.3	$P_1<Y$	14.24	Not convex
9	129.9	129.9	70.2	$P_1<Y$	14.23	Not convex

Figure 18. Example 2 Not Forced into Convex Zone

Case 1a--$P_1 < Y$

Iteration	L	Y	X	Condition	Cost/M^3	Convexity
1	20.0	20.0	8.8	$P_1<Y$	101.85	Not convex
2	25.9	27.6	12.0	$P_1<Y$	61.43	Not convex
3	35.3	38.9	17.0	$P_1<Y$	37.27	Not convex
4	51.3	57.3	26.1	$P_1<Y$	22.72	Convex
5	93.9	93.9	48.6	$P_1<Y$	14.91	Not convex
6	108.1	108.1	64.7	$P_1<Y$	13.81	Not convex
7	106.7	106.7	64.7	$P_1<Y$	13.83	Not convex
8	106.5	106.5	64.6	$P_1<Y$	13.84	Not convex

Case 1b--$P_1 > Y$

Iteration	L	Y	X	Condition	Cost/M^3	Convexity
1	20.0	20.0	20.0	$P_1>Y$	61.12	Convex
2	29.6	29.6	30.4	$P_1>Y$	35.52	Convex
3	43.2	43.2	46.8	$P_1>Y$	22.55	Convex
4	61.4	61.4	73.9	$P_1>Y$	16.19	Convex
5	81.7	81.7	123.4	$P_1>Y$	13.45	Convex
6	94.8	94.8	227.0	$P_1>Y$	12.53	Convex
7	95.6	95.6	452.2	$P_1>Y$	12.21	Convex
8	95.4	95.4	905.1	$P_1>Y$	12.05	Convex
9	95.6	95.6	1809.1	$P_1>Y$	11.97	Convex
10	95.6	95.6	3616.7	$P_1>Y$	11.93	Convex
11	95.7	95.7	7231.4	$P_1>Y$	11.91	Convex
12	95.7	95.7	14460.7	$P_1>Y$	11.90	Convex

Case 1c--Yarding Perpendicular to Truck Road

Optimum Y = 95.7

Cost/M^3 = 11.89

Case 2--Yarding not Permitted onto Truck Road

Iteration	L	Y	X	Cost/M^3	Convexity
1	20.0	20.0	20.0	61.59	Not convex
2	14.1	28.2	26.4	38.81	Convex
3	0.0	39.0	33.0	26.49	Convex
4	6.1	54.9	42.2	19.41	Convex
5	19.8	75.5	53.1	15.75	Convex
6	36.6	98.5	63.6	14.24	Convex
7	51.1	117.1	70.0	13.85	Convex
8	57.8	125.1	71.4	13.81	Convex
9	58.8	126.1	71.4	13.80	Convex

Figure 19. Example 2 Forced into Convex Zone

Case 1a--$P_1 < Y$

Iteration	L	Y	X	Condition	Cost/M^3	Convexity
1	250.0	250.0	43.7	$P_1 < Y$	4.89	Not convex
2	304.2	340.9	61.1	$P_1 < Y$	3.78	Convex
3	330.1	443.8	82.8	$P_1 < Y$	3.21	Convex
4	429.3	551.2	116.3	$P_1 < Y$	2.95	Convex
5	488.6	619.5	163.3	$P_1 < Y$	2.84	Convex
6	486.0	629.9	225.5	$P_1 < Y$	2.81	Convex
7	546.0	604.1	313.0	$P_1 < Y$	2.81	Convex

Case 1b--$P_1 > Y$

Iteration	L	Y	X	Condition	Cost/M^3	Convexity
1	250.0	250.0	250.0	$P_1 > Y$	3.62	Convex
2	348.4	348.4	401.6	$P_1 > Y$	3.06	Convex
3	449.0	449.0	687.2	$P_1 > Y$	2.83	Convex
4	496.6	496.6	1301.6	$P_1 > Y$	2.78	Convex
5	489.5	489.5	2621.9	$P_1 > Y$	2.76	Convex
6	495.7	495.7	5210.5	$P_1 > Y$	2.75	Convex
7	501.5	501.5	10359.4	$P_1 > Y$	2.75	Convex

Case 1c--Yarding Perpendicular to Truck Road

Optimum Y = 511.8

Cost/M^3 = 2.74

Case 2--Yarding not Permitted onto Truck Road

Iteration	L	Y	X	Cost/M^3	Convexity
1	250.0	250.0	250.0	3.99	Not convex
2	179.8	359.5	217.2	3.35	Convex
3	335.9	493.1	193.1	3.05	Convex
4	468.9	617.8	186.3	2.95	Convex
5	541.4	688.0	183.6	2.94	Convex
6	557.0	703.2	183.1	2.93	Convex

Figure 20. Example 3 Forced into Convex Zone.

For instance, based on Example 3, a manager might conclude:

> 1. If I permit logging onto the truck road, there is no advantage to moving in a swing machine or building landings. My truck spacing should be about 1000 meters.
>
> 2. If I do not permit logging onto the truck road, I will spend about 6 1/2% more on logging costs. If I stand to save an equal amount on reduced road maintenance or repair, this is an alternative to consider.
>
> 3. If I do not permit logging onto the truck road, my landings should be about 180 meters apart, my truck roads should be about 1400 meters apart, and my swing roads should extend about 550 meters into the setting from the landing.

The efficiency of the Newton Gradient search is quite remarkable. An early approach to this problem involved an exhaustive enumeration technique, requiring about 18,000 iterations. By contrast, the Newton algorithm closed on a solution in at most thirteen iterations. Execution time for these two approaches was 85 minutes and 1 1/2 minutes, respectively; we could expect a proportionate reduction in execution time if we were working with a computer system with a generally faster operating speed.

There are several considerations that could improve the validity of the model set forth in this paper. Any of these considerations would be fertile ground for further investigations. Several topics are:

Yard/Swing Synchronization

The ability of the swing machine to remove logs at a rate matching the arrival of the yarding machine is an interesting aspect of the problem. Unaccounted costs that could arise are those resulting from idle time for either machine, due to unmatched service rates or inadequate storage at the swing road. Queueing theory offers tools for analyzing this aspect. Perhaps the model developed here could infer costs assuming a theoretical 100% matching efficiency between the two machines. Then, variation in experienced cost could be attributed to inefficiencies in the match of service rates.

Sensitivity and Risk

The sensitivity of the cost function at the optimum point to variations in the variable values can be investigated as part of a solution. This should be an easy task, since the Newton optimization search necessitates calculation of the gradient at each iteration. A simple investigation of the range of costs accompanying a variation of ±10% of the optimum values, for example, may reveal that the cost function is not very sensitive.

There may be a greater element of risk associated with one case than with another. For example, if the swing tractor is moved in, the risk of equipment breakdown, and consequent increase in logging cost, is likely to change. This can be reflected, or rather hidden, in the associated operating cost of the equipment. One should be aware of its existence when making inferences from the model.

Time Value of Money and Changes in Technology

An optimization scheme such as the one presented here is a tool that could be considered when planning the layout of long-term management facilities for a large block of land. However, the time effects--time value of money and significant changes in technology--are not usually dealt with. Consider the effect on optimum spacing made by a major change in skidder technology. A company that operates with crawler tractors could find itself managing its land with suboptimal road and landing spacing if it were to convert to rubber-tired skidders. Similarly, if expenditures for road construction are made in Year 0, an optimal solution based on a five-year management framework may look quite different than an optimal solution based on a 20-year management framework, due to time value of money.

Verification

An attempt to verify a deterministic model by empirical cost collection would be an interesting undertaking. It may be difficult, however, to find many examples of operations similar to the configurations discussed here. Even a small sample of such operations could be compared with logging cost predictions from this model, offering some measure of non-rigorous verification of the model's validity.

Further investigation of this problem in several areas would be interesting:

1. Allow for variations in logging cost due to ground slope.
2. Investigate the effect of yarding into the truck road or swing road in a non-perpendicular direction.

3. Allow for setting shapes constrained by ownership lines, topography, or existing roads or landings.
4. Generalize the model to provide insights for other types of operations, such as pre-bunching under skyline corridors or skyline operation with lateral yarding.

The application of a sophisticated mathematical algorithm to a collection of constrained equations such as those generated in this study has been shown to give reasonable, understandable results; this alone was a major objective of the study.

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APPENDICES

APPENDIX A

APPENDIX A

First Derivatives of Case 1a Objective Function

EQUATION 1.1a SUMMARY OF COST FUNCTION PARTIAL DERIVATIVE - CASE 1a

$$\frac{\partial K}{\partial L} = 0$$

$$+ 0$$

$$+ \frac{AV_2X}{4Y} + \frac{V_4}{Y}$$

$$+ \frac{1}{4Y}\left[\left(\frac{(\frac{2}{3}AV_2Y)(L-Y)}{(Y^2-2YL+L^2+X^2)^{1/2}}\right) - \left(\frac{(\frac{2}{3}AV_2L)(L-Y)}{(Y^2-2YL+L^2+X^2)^{1/2}}\right) - \left((\tfrac{2}{3}AV_2)(Y^2-2YL+L^2+X^2)^{1/2}\right) - 2V_4\right]$$

$$+ \frac{1}{4Y}\left[\left(\frac{(\frac{4}{3}AV_2Y)(L-Y)}{(Y^2-2YL+L^2+\frac{1}{16}X^2)^{1/2}}\right) - \left(\frac{(\frac{4}{3}AV_2L)(L-Y)}{(Y^2-2YL+L^2+\frac{1}{16}X^2)^{1/2}}\right) - \left((\tfrac{4}{3}AV_2)(Y^2-2YL+L^2+\tfrac{1}{16}X^2)^{1/2}\right) - 2V_4\right]$$

$$+ 0$$

$$+ \frac{V_1}{Y} L + \frac{V_3}{Y}$$

$$+ \frac{-2V_1}{Y} L + V_1 - \frac{V_3}{Y}$$

$$+ 0$$

$$+ \frac{C}{YXV}$$

APPENDIX A

EQUATION 1.1b SUMMARY OF COST FUNCTION PARTIAL DERIVATIVE - CASE 1a

$$
\begin{aligned}
\frac{\partial K}{\partial Y} = {} & \frac{-2}{Y^2}\left[\left(\frac{\frac{1}{6}A^2V_2X + \frac{1}{3}AV_3}{AV_2 - V_1}\right) V_2 + V_4\right]\left[\frac{\frac{1}{8}AV_2X + \frac{1}{4}V_3}{AV_2 - V_1}\right] \\
& + \frac{-2}{Y^2}\left[\frac{AX}{6} V_2 + V_4\right]\left[\frac{\frac{1}{8}AV_2X + \frac{1}{4}V_3}{AV_2 - V_1}\right] \\
& + \frac{-2}{Y^2}\left[\frac{AX}{4} V_2 + V_4\right]\left[\frac{L}{2} - \frac{\frac{1}{4}AV_2X + \frac{1}{2}V_3}{AV_2 - V_1}\right] \\
& + \frac{1}{2}\left[\frac{(\frac{1}{3}AV_2)(Y-L)}{(Y^2-2YL+L^2+X^2)^{\frac{1}{2}}} - \frac{(\frac{1}{3}AV_2L)(Y-L)}{Y(Y^2-2YL+L^2+X^2)^{\frac{1}{2}}} + \frac{\frac{1}{3}AV_2L}{Y^2}(Y^2-2YL+L^2+X^2)^{\frac{1}{2}} + \frac{V_4L}{Y^2}\right] \\
& + \frac{1}{2}\left[\frac{2}{3}AV_2 \frac{Y-L}{(Y^2-2YL+L^2+\frac{1}{16}X^2)^{\frac{1}{2}}} - \frac{\frac{2}{3}AV_2L}{Y}\frac{Y-L}{(Y^2-2YL+L^2+\frac{1}{16}X^2)^{\frac{1}{2}}} - \frac{-\frac{2}{3}AV_2L}{Y^2}\, Y^2-2YL+L^2+\frac{1}{16}X^2 {}^{\frac{1}{2}} - \frac{-V_4L}{Y^2}\right] \\
& + \frac{-2}{Y^2}\left[\frac{\frac{1}{3}AV_2X + \frac{2}{3}V_3}{AV_2 - V_1} V_1 + V_3\right]\left[\frac{\frac{1}{8}AV_2X + \frac{1}{4}V_3}{AV_2 - V_1}\right] \\
& + \frac{-2}{Y^2}\left[\left(\frac{L}{2} + \frac{\frac{1}{4}AV_2X + \frac{1}{2}V_3}{AV_2 - V_1}\right) V_1 + V_3\right]\left[\frac{L}{2} - \frac{\frac{1}{4}AV_2X + \frac{1}{2}V_3}{AV_2 - V_1}\right] \\
& + \frac{L^2V_1 + LV_3}{Y^2} \\
& + \frac{-R}{2Y^2V} \\
& + \frac{-2CL - F_1 - F_2 - F_3}{2Y^2XV}
\end{aligned}
$$

APPENDIX A

EQUATION 1.1c SUMMARY OF COST FUNCTION PARTIAL DERIVATIVE - CASE 1a

$$\frac{\partial K}{\partial X} = \left(\frac{\frac{1}{12} A_3 V_2{}^3}{Y(AV_2 - V_1)^2}\right) X + \frac{\frac{1}{6} A^2 V_2{}^2 V_3}{Y(AV_2 - V_1)^2} + \frac{\frac{1}{4} AV_2 V_4}{Y(AV_2 - V_1)}$$

$$+ \left(\frac{\frac{1}{12} A^2 V_2{}^2}{Y(AV_2 - V_1)}\right) X + \frac{\frac{1}{4} AV_2 V^4 + \frac{1}{12} AV_2 V_3}{Y(AV_2 - V_1)}$$

$$+ \left(\frac{-\frac{1}{4} A^2 V_2{}^2}{AV_2 Y - V_1 Y}\right) X + \frac{AV_2 L}{4Y} - \frac{\frac{1}{4} AV_2 V_3 + \frac{1}{2} AV_2 V_4}{AV_2 Y - V_1 Y}$$

$$+ \frac{AV_2 X (Y - L)}{6Y \left(Y^2 - 2YL + L^2 + \frac{1}{16} X^2\right)^{1/2}}$$

$$+ \left(\frac{\frac{1}{6} A^2 V_2{}^2 V_1}{Y(AV_2 - V_1)^2}\right) X + \frac{\frac{1}{3} AV_1 V_2 V_3}{Y(AV_2 - V_1)^2} + \frac{\frac{1}{4} AV_2 V_3}{Y(AV_2 - V_1)}$$

$$+ \left(\frac{\frac{1}{4} A^2 V_1 V_2{}^2}{Y(AV_2 - V_1)^2}\right) X - \frac{\frac{1}{2} AV_1 V_2 V_3}{Y(AV_2 - V_1)}$$

$$+ 0$$

$$+ 0$$

$$+ \frac{-2CL - F_1 - F_2 - F_3}{2YX^2 V}$$

APPENDIX B

APPENDIX B

Second Derivatives of Case 1a Objective Function

EQUATION 1.2a

$$\frac{\partial^2 K}{\partial L^2} = 0$$

$$+ \; 0$$

$$+ \; 0$$

$$+ \; \frac{AV_2}{6Y} \quad \frac{2Y^3-6Y^2L + 6YL^2 + 3YX^2 - 2L^3 - 3LX^2}{(Y^2-2YL+L^2+X^2)^{3/2}}$$

$$+ \; \frac{AV_2}{3Y} \quad \frac{2Y^3-6Y^2L+6YL^2+\frac{3}{16} YX^2 - 2L^3-\frac{3}{16}LX^2}{(Y^2-2YL+L^2+\frac{1}{16}X^2)^{3/2}}$$

$$+ \; 0$$

$$+ \; \frac{V_1}{Y}$$

$$+ \; \frac{-2V_1}{Y}$$

$$+ \; 0$$

$$+ \; 0$$

EQUATION 1.2b

$$\frac{\partial^2 K}{\partial Y^2}$$

$$= \frac{4}{Y^3}\left[\frac{\frac{1}{6}A^2V_2X+\frac{1}{3}AV_3}{AV_2-V_1}\ V_2 + V_4\right]\left[\frac{\frac{1}{8}AV_2X + \frac{1}{4}V_3}{AV_2-V_1}\right]$$

$$+ \frac{4}{Y^3}\left[\frac{AX}{6}\ V_2 + V_4\right]\left[\frac{\frac{1}{8}AV_2X + \frac{1}{4}V_3}{AV_2-V_1}\right]$$

$$+ \frac{4}{Y^3}\left[\frac{AX}{4}\ V_2 + V_4\right]\left[\frac{L}{2} - \frac{\frac{1}{4}AV_2X + \frac{1}{2}V_3}{AV_2 - V_1}\right]$$

$$+ \frac{AV_2}{6}\left[\frac{(Y^2-2YL+L^2+X^2)^{\frac{1}{2}}}{(Y^2-2YL+L^2+X^2)} - \frac{Y\ (\ Y - L\ )}{(Y^2-2YL+L^2+X^2)^{3/2}}\right] - \frac{AV_2L}{3}\left[\frac{L - Y}{Y^2-2YL+L^2+X^2)^{3/2}}\right] +$$

$$\frac{AV_2L^2}{6}\left[\frac{L - Y}{Y(Y^2-2YL+L^2+X^2)^{3/2}} - \frac{1}{Y^2(Y^2-2YL+L^2+X^2)^{1/2}}\right] +$$

$$\frac{AV_2L}{6}\left[\frac{Y^2(Y - L)}{Y^4(Y^2-2YL+L^2+X^2)^{1/2}} - \frac{2Y(Y^2-2YL+L^2+X^2)^{1/2}}{Y^4}\right] + \frac{-V_4L}{Y^3}$$

$$+ \frac{AV_2}{3}\left[\frac{(3Y^2-2YL-L^2)}{Y^2(Y^2-2YL+L^2+X^2)^{1/2}} - \frac{(Y^3-Y^2L-YL^2+L^3+\frac{1}{16}X^2L)(Y-L)}{Y^2(Y^2-2YL+L^2+\frac{1}{16}X^2)^{3/2}} - \right.$$

$$\left.\frac{(Y^3-Y^2L-YL^2+L^3+\frac{1}{16}X^2L)(2)}{Y^3(Y^2-2YL+L^2+\frac{1}{16}X^2)^{1/2}}\right] - \frac{V_4L}{Y^3}$$

$$+ \frac{4}{Y^3}\left[\frac{\frac{1}{3}AV_2X+\frac{2}{3}V_3}{AV_2-V_1}\ V_1 + V_3\right]\left[\frac{\frac{1}{8}AV_2X+\frac{1}{4}V_3}{AV_2-V_1}\right]$$

$$+ \frac{4}{Y^3}\left[\left(\frac{L}{2} + \frac{\frac{1}{4}AV_2X+\frac{1}{2}V_3}{AV_2-V_1}\right) V_1 + V_3\right]\left[\frac{L}{2} - \frac{\frac{1}{4}AV_2X + \frac{1}{2}V_3}{AV_2-V_1}\right]$$

$$+ \frac{-2L^2V_1 - 2LV_3}{Y^3}$$

$$+ \frac{R}{Y^3V}$$

$$+ \frac{2CL + F_1 + F_2 + F_3}{Y^3XV}$$

APPENDIX B

EQUATION 1.2c

$$\frac{\partial^2 K}{\partial X^2}$$

$$= \frac{\frac{1}{12}A^3 V_2^{\,3}}{Y(AV_2-V_1)^2}$$

$$+ \frac{\frac{1}{12}A^2 V_2^{\,2}}{Y(AV_2-V_1)}$$

$$+ \frac{-\frac{1}{4}A^2 V_2^{\,2}}{Y(AV_2-V_1)}$$

$$+ \frac{6Y\sqrt{Y^2-2YL+L^2+X^2}\;AV_2(Y-L) - \dfrac{6X^2YAV_2(Y-L)}{\sqrt{Y^2-2YL+L^2+X^2}}}{36\,Y^2\,(Y^2-2YL+L^2+X^2)}$$

$$+ \frac{48Y\sqrt{Y^2-2YL+L^2+\frac{1}{16}X^2}\;AV_2(Y-L) - \dfrac{3X^2\,YAV_2\,(Y-L)}{\sqrt{Y^2-2YL+L^2+\frac{1}{16}X^2}}}{2304\,Y^2\,(Y^2-2YL+L^2+\frac{1}{16}X^2)}$$

$$+ \frac{\frac{1}{6}A^2 V_2^{\,2} V_1}{Y(AV_2-V_1)^2}$$

$$+ \frac{-\frac{1}{4}A^2 V_1 V_2^{\,2}}{Y(AV_2-V_1)^2}$$

$$+ \; 0$$

$$+ \; 0$$

$$+ \frac{2CL + F_1 + F_2 + F_3}{YX^3V}$$

APPENDIX B

EQUATION 1.2d

$$\frac{\partial^2 K}{\partial L \partial Y}$$

$$= \ 0$$

$$+ \ 0$$

$$+ \ \frac{-AV_2X}{4Y^2} - \frac{V_4}{Y^2}$$

$$+ \ \frac{AV_2}{6} \ \frac{(Y^2)(Y^2-2YL+L^2+X^2)^{1/2}\ (3L^2-2YL-Y^2+X^2)}{Y^4\ (Y^2-2YL+L^2+X^2)}$$

$$- \ \frac{AV_2}{6} \ \frac{(Y^3-Y^2L-YL^2+L^3+LX^2)\ \dfrac{(L-Y)\ Y^2}{(Y^2-2YL+L^2+X^2)^{1/2}}}{Y^4\ (Y^2-2YL+L^2+X^2)} + \frac{V_4}{2Y^2}$$

$$+ \ \frac{AV_2}{3} \ \frac{(Y^2)(Y^2-2YL+L^2+1/16X^2)^{1/2}\ (3L^2-Y^2-2YL+1/16X^2)}{Y^4\ (Y^2-2YL+L^2+1/16X^2)}$$

$$- \ \frac{AV_2}{3} \ \frac{(Y^3-Y^2L=YL^2+L^3+1/16X^2L)\ \dfrac{(L\ -\ Y)\ Y^2}{(Y^2-2YL+L^2+1/16X^2)^{1/2}}}{} + \frac{V_4}{2Y^2}$$

$$+ \ 0$$

$$+ \ \frac{-V_1L}{Y^2} - \frac{V_3}{Y^2}$$

$$+ \ \frac{2V_1L}{Y^2} + \frac{V_3}{Y^2}$$

$$+ \ 0$$

$$+ \ \frac{-C}{Y^2XV}$$

APPENDIX B

EQUATION 1.2e

$$\frac{\partial^2 K}{\partial L \partial X}$$

$$= \quad 0$$

$$+ \quad 0$$

$$+ \quad \frac{AV_2}{4Y}$$

$$+ \quad \frac{AV_2X}{6} \; \frac{Y - L}{Y^2-2YL+L^2+X^2)^{3/2}} - \frac{AV_2XL}{6Y} \; \frac{Y - L}{Y^2-2YL+L^2+X^2)^{3/2}} - \frac{AV_2X}{6Y(Y^2-2YL+L^2+X^2)^{1/2}}$$

$$+ \frac{AV_2X}{48} \; \frac{Y - L}{Y^2-2YL+L^2+\frac{1}{16}X^2)^{3/2}} - \frac{AV_2XL}{48Y} \; \frac{Y - L}{(Y^2-2YL+L^2+\frac{1}{16}X^2)^{3/2}}$$

$$- \quad \frac{AV_2Y}{48Y(Y^2-2YL+L^2+1/16X^2)^{1/2}}$$

$$+ \quad 0$$

$$+ \quad 0$$

$$+ \quad 0$$

$$+ \quad 0$$

$$+ \quad \frac{-C}{YX^2V}$$

APPENDIX B

EQUATION 1.2f

$$\frac{\partial^2 K}{\partial Y \partial X}$$

$$= \frac{\frac{1}{12}A^3V_2{}^3X}{Y^2(AV_2-V_1)^2} - \frac{\frac{1}{6}A^2V_2{}^2V_3}{Y^2(AV_2-V_1)^2} - \frac{\frac{1}{4}AV_2V_4}{Y^2(AV_2-V_1)}$$

$$+ \frac{-\frac{1}{12}A_2V_2{}^2X}{Y^2(AV_2-V_1)} - \frac{\frac{1}{4}AV_2V_4}{Y^2(AV_2-V_1)} - \frac{\frac{1}{12}\,AV_2V_3}{Y^2(AV_2-V_1)}$$

$$+ \frac{\frac{1}{4}A^2V_2{}^2X}{Y^2(AV_2-V_1)} - \frac{AV_2L}{4Y^2} + \frac{\frac{1}{4}AV_2V_3}{Y^2(AV_2-V_1)} + \frac{\frac{1}{2}AV_2V_4}{Y^2(AV_2-V_1)}$$

$$+ \frac{-(AV_2)(Y-L)(1-\frac{L}{Y})(X)}{6(Y2-2YL+L^2+X^2)^{3/2}} + \frac{(AV_2L)(X)}{6Y^2(Y^2-2YL+L^2+\frac{1}{16}X^2)^{1/2}}$$

$$+ \frac{-(AV_2)(Y-L)(1-\frac{L}{Y})(X)}{48(Y^2-2YL+L^2+\frac{1}{16}X^2)^{3/2}} + \frac{(AV_2L)(X)}{48Y^2(Y^2-2YL+L^2+\frac{1}{16}X^2)^{1/2}}$$

$$+ \frac{-\frac{1}{6}A^2V_2{}^2V_1X}{Y^2(AV_2-V_1)^2} - \frac{\frac{1}{3}AV_1V_2V_3}{Y^2(AV_2-V_1)^2} - \frac{\frac{1}{4}AV_2V_3}{Y^2(AV_2-V_1)}$$

$$+ \frac{\frac{1}{4}A^2V_1V_2{}^2X}{Y^2(AV_2-V_1)^2} + \frac{\frac{1}{2}AV_1V_2V_3}{Y^2(AV_2-V_1)^2} + \frac{\frac{1}{2}AV_2V_3}{Y^2(AV_2-V_1)}$$

$$+ \ 0$$

$$+ \ 0$$

$$+ \frac{2CL + F_1 + F_2 + F_3}{2Y^2X^2V}$$

APPENDIX C

APPENDIX C

Examination of $P_2 = 0$ Assumption

In Case 1a we must compute the average yarding distance for segment 1. This segment is trapezoidal in shape if we assume some positive value for the dimension P_2, but is triangular in shape if $P_2 = 0$ (see Figure 2). Let us investigate the merits of assuming that $P_2 = 0$.

The unadjusted average yarding distance from segment 1 can be determined by considering a strip of width dZ located a distance Z from the swing road (see Figure 21). The average yarding distance for the segment is:

$$AYD_1 = \int_0^{X/2} \frac{d\ AREA*AYD_i}{TOTAL\ AREA}$$

$$= \int_0^{X/2} \frac{\left(P_2 + \frac{Z(P_1-P_2)}{X/2}\right)dZ \quad \left(P_2 + \frac{Z(P_1-P_2)}{X/2}\right)(1/2)}{\frac{P_1+P_2}{2} \quad \frac{X}{2}}$$

$$= \frac{2}{X(P_1+P_2)} \int_0^{X/2} \left[P_2^{\ 2} + \frac{2P_2(P_1-P_2)}{X/2} \ Z + \frac{(P_1-P_2)^2}{(X/2)^2} \ Z^2\right] dZ$$

$$= \frac{2}{X(P_1+P_2)} \left[P_2^{\ 2}Z + \frac{P_2(P_1-P_2)}{X/2} \ Z^2 + \frac{(P_1-P_2)^2}{3(X/2)^2} Z^3\right] \Bigg|_0^{X/2}$$

$$= \frac{2}{X(P_1+P_2)} \left[P_2^{\ 2}+P_2 \ (P_1-P_2)^2 + \frac{(P_1-P_2)^2}{3}\right] \quad (X/2)$$

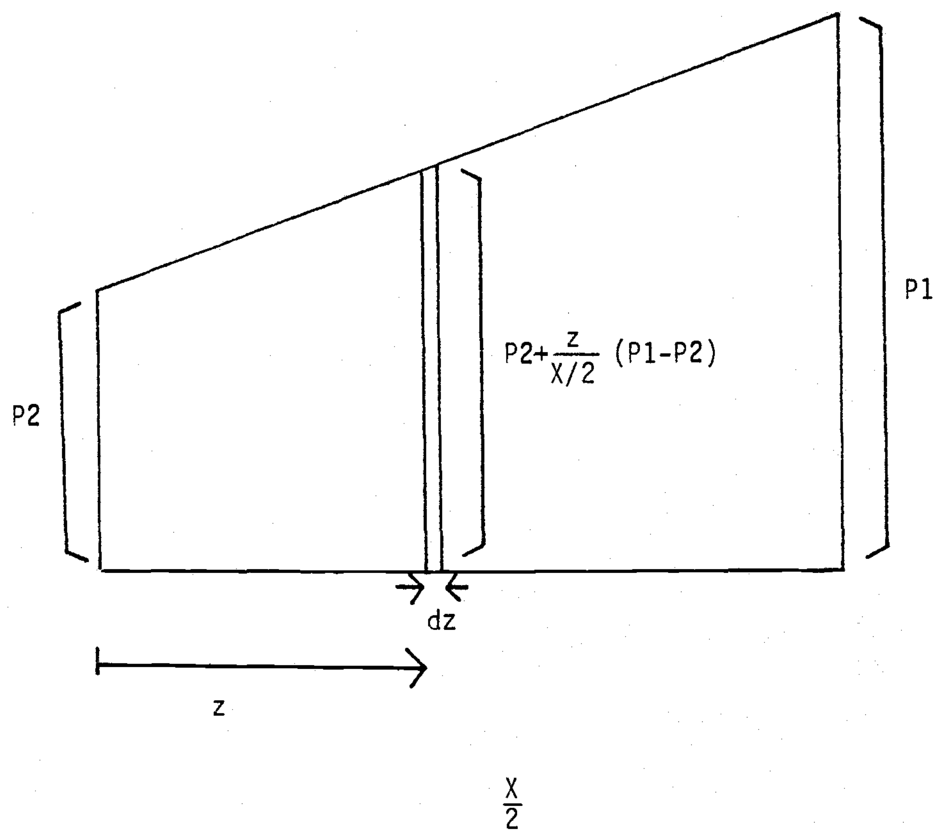

Figure 21. Integrating to Find Average Yarding Distance.

$$AYD_1 = \frac{P_2^{\,2} + P_2(P_1 - P_2) + \frac{(P_1 - P_2)^2}{3}}{} \qquad \text{(eq. 28)}$$

This expression can be simplified a little by expanding terms:

$$AYD_1 = 1/3 \; \frac{P_1^{\,2} + P_1P_2 + P_2^{\,2}}{P_1 + P_2}$$

Substituting expressions (4) and (8) we obtain:

$$AYD_1 = \frac{1}{3(AV_2 - V_1)} \left[1/2AV_2X + 2V_3 - \frac{1/2AV_2V_3 + V_3^{\,3}}{1/2AV_2X + 2V_3} \right] \qquad \text{(eq. 29)}$$

This is a cumbersome expression! However, if we choose some values for A, V_1; V_2, V_3, and X, it appears that it is difficult to concoct a combination resulting in a value for P_2 that is significant in relation to P_1 (see Table 3). What is the effect of assuming that $P_2 = 0$, i.e., that the breakeven line intercepts the landing? In this case the yarding distance for segment 1 is:

$$AYD_1\text{(SIMPLIFIED)} = \frac{P_1}{3} = \frac{1/2AV_2X + V_3}{3(AV_2 - V_1)} \qquad \text{(eq. 30)}$$

The last two columns in Table 3 show the average yarding distances corresponding to the full (eq. 29) and simplified (eq. 30) models. Note that there is little significant difference in any of the cases investigated, unless X is quite small. We will make the assumption that the simplified model--breakeven line intercepts the landing--is adequate.

A	V_1	V_2	V_3	X	P_1	FULL AYD	SIMPLE AYD	DIFFERENCE %
1.0	.03	.06	.85	500	528.3	176.6	176.1	-0.27
1.0	.05	.06	.25	500	1525.0	508.5	508.3	-0.03
1.0	.03	.10	.85	500	369.3	123.2	123.1	-0.10
1.0	.03	.12	.25	500	336.1	112.0	112.0	-0.01
1.5	.03	.06	.85	500	389.2	129.9	129.7	-0.13
1.5	.05	.06	.25	500	568.8	189.6	189.6	-0.01
1.5	.03	.10	.85	500	319.6	106.6	106.5	-0.05
1.5	.03	.12	.25	500	301.7	100.6	100.6	-0.003
1.0	.03	.04	1.20	500	1120.0	377.2	373.3	-1.03
1.5	.03	.06	.85	50	51.67	18.2	17.2	-5.58

Table 3. Difference Between Full and Simplified AYD Models

APPENDIX D

APPENDIX D

Anomalies Occurring in Several Gradient Methods

Several classic gradient methods that can result in suboptimal solutions are diagrammed in Figures 22-24. "Contours" of a hypothetical constrained objective function in two variables, X_1 and X_2, are shown, with a global optimum at Δ. In Figure 22, the conjugate gradient method (Gottfried & Weisman, 1973) moves from an unfortunate choice of starting point, along a path of steepest gradient to a suboptimal point. One can see that the same point would have been reached even without the constraint. Cauchy's Method (Simmons, 1975) (Figure 23), in which iteration is in cardinal directions (i.e., optimizing one variable at a time), also moves to a suboptimal point, due to an unlucky choice of starting point. Rosen's Gradient Projection Method (Gottfried & Weisman, 1973) (Figure 24) moves along a path of steepest gradient until encountering a constraint, then along a path of steepest gradient projected onto the constraint to an optimum--or in this example, to a suboptimum. The conclusion is that none of these methods are necessarily foolproof--the danger of closing on a non-global optimal value is always present.

The well-known Kuhn-Tucker Conditions can be used to test that a solution is an optimum for a constrained objective function. But the conditions are necessary and sufficient to indicate a global optimum only if the constraints define a convex set and the objective function being minimized is convex throughout the feasible region. We have seen in our examples that the feasible region contains zones of non-convexity. Therefore, we cannot rely on the Kuhn-Tucker Conditions to verify that we have a global optimum.

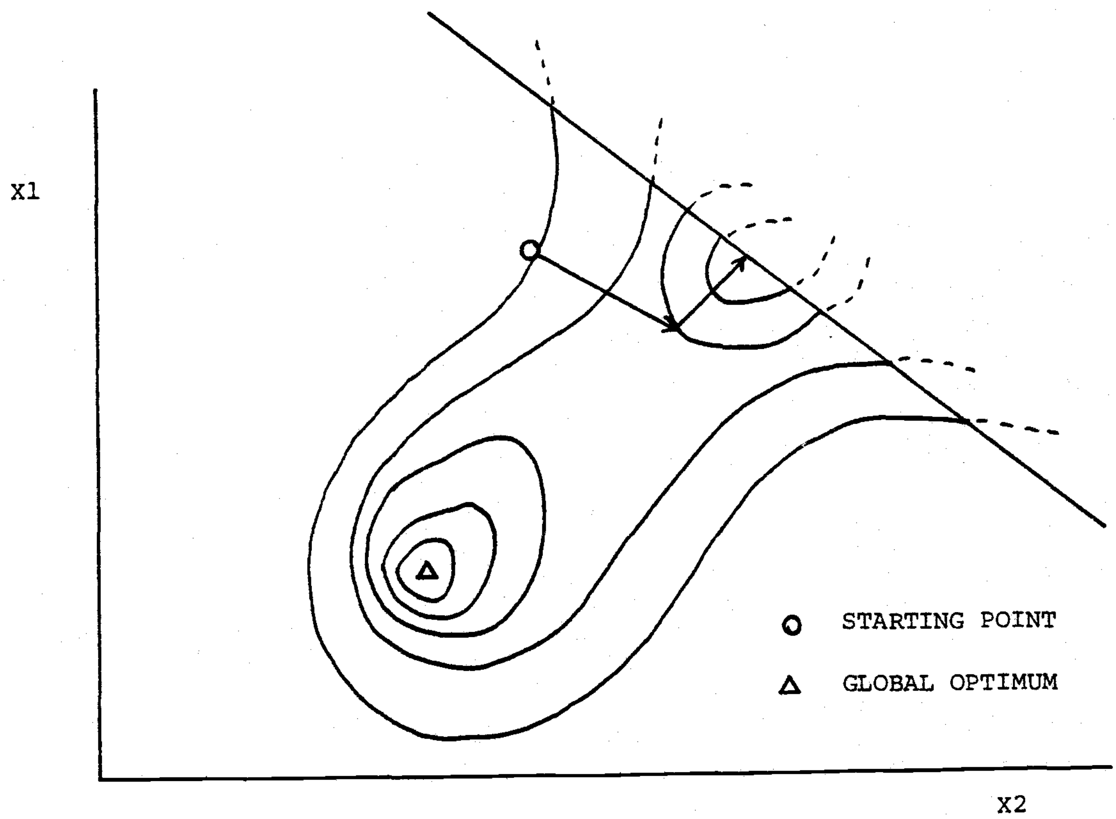

Figure 22. Conjugate Gradient Iteration Method.

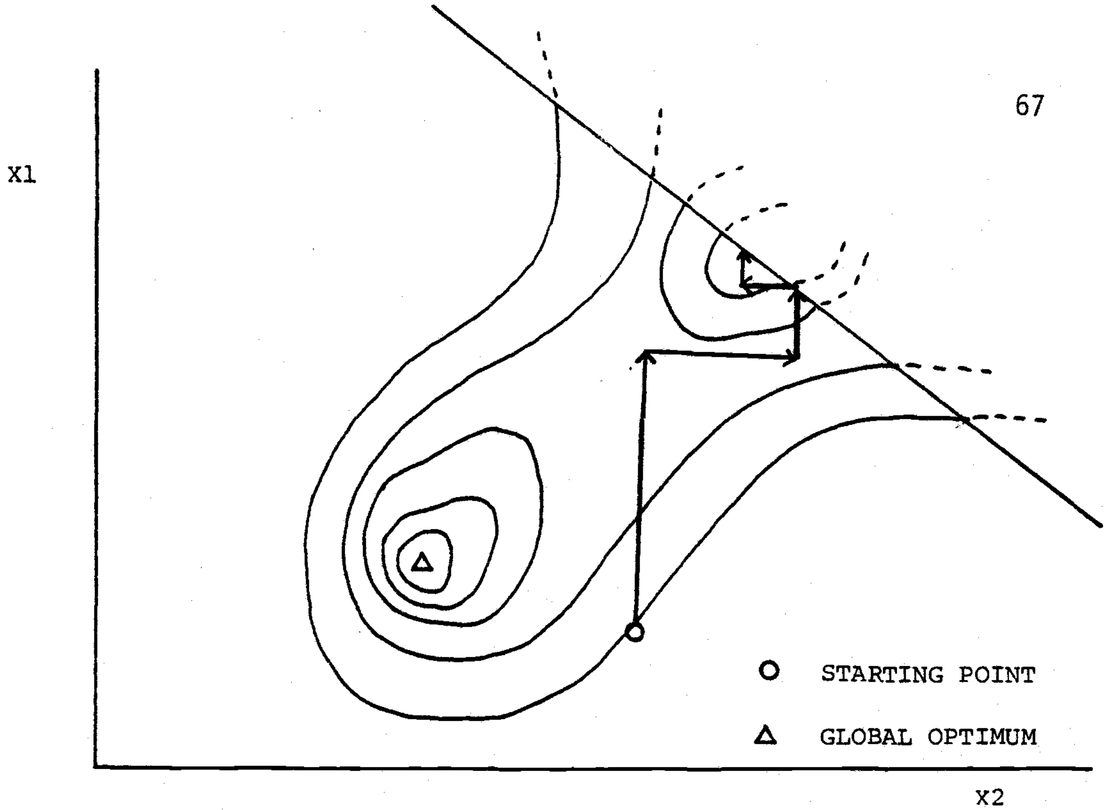

Figure 23. Cauchy's Gradient Iteration Method.

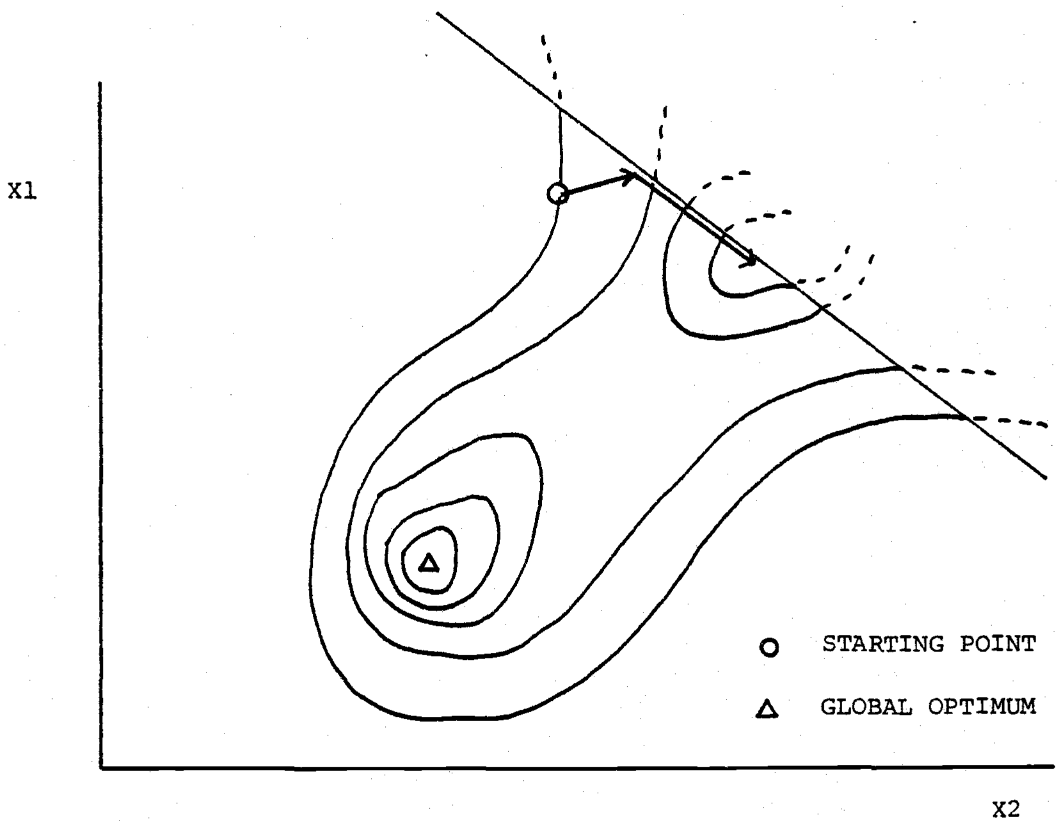

Figure 24. Rosen's Gradient Projection Method.

APPENDIX E

```
5 DIM C[10],I[3,10],J[10,1],L[3,1],K[20],S[6,10],H[3,3],T[3,3],X[3,1],V[3,1]
6 DIM W[3,1],Z[3,1]
8 PRINT
9 PRINT
10 DISP "TRACTOR YD DIST ADJ FCTR,A";
15 INPUT A
16 PRINT "TRACTOR YARDING DISTANCE ADJUSTMENT FACTOR (A) ="A
20 DISP "VOLUME,M3 PER HECTARE,V";
25 INPUT V
26 PRINT "VOLUME,CUBIC METERS PER HECTARE (V) ="V
30 DISP "SWING VRBL COST,V1";
35 INPUT V1
36 PRINT "SWING VARIABLE COST $ PER CUBIC METER PER METER (V1) ="V1
40 DISP "YARD VRBL COST,V2";
45 INPUT V2
46 PRINT "YARD VARIABLE COST $ PER CUBIC METER PER METER (V2) ="V2
50 DISP "OTHER SWING COST,V3";
55 INPUT V3
56 PRINT "OTHER SWING COST $ PER CUBIC METER (V3) ="V3
60 DISP "OTHER YARD COST,V4";
65 INPUT V4
66 PRINT "OTHER YARD COST $ PER CUBIC METER (V4) ="V4
70 V=V/10000
75 DISP "SWING RD CONST COST,C";
80 INPUT C
81 PRINT "SWING ROAD CONSTRUCTION COST $ PER METER (C) ="C
85 DISP "TRUCK RD CONST COST,R";
90 INPUT R
91 PRINT "TRUCK ROAD CONSTRUCTION COST $ PER METER (R) ="R
95 DISP "SWING MACHINE MOVEIN,F1";
100 INPUT F1
101 PRINT "SWING MACHINE MOVEIN COST $ (F1) ="F1
105 DISP "YARD MACHINE MOVEIN,F2";
110 INPUT F2
111 PRINT "YARD MACHINE MOVEIN COST $ (F2) ="F2
115 DISP "LANDING CONST COST,F3";
116 INPUT F3
117 PRINT "LANDING CONSTRUCTION COST $ (F3) ="F3
```

```
118 PRINT
119 PRINT
120 PRINT
121 PRINT "                              CASE 1A--P1<Y"
122 PRINT
123 PRINT "ITERATION    L          Y          X    CONDITION  COST/M3   CONVEXITY
135 C9=0
136 PRINT "-----------------------------------------------------------------------
137 PRINT
138 DISP "INITIAL X,Y,L";
139 INPUT X1,Y1,L1
140 L=L1
145 Y=Y1
150 X=X1
155 P1=(A*V2*X/2+V3)/(A*V2-V1)
156 IF P1 <= Y THEN 159
157 X=0.95*X
158 GOTO 155
159 L=L+(Y-L)*(L>Y)+(P1-L)*(L<P1)
185 C9=C9+1
190 WRITE (15,195)C9,L,Y,X;
195 FORMAT F6.0,4F10.1
200 GOSUB 5000
480 WRITE (15,481)K;
481 FORMAT 2F12.2
490 GOSUB 1000
491 GOSUB 3000
492 X[1,1]=L
493 X[2,1]=Y
494 X[3,1]=X
495 V[1,1]=L[1,1]
496 V[2,1]=L[2,1]
497 V[3,1]=L[3,1]
498 MAT T=INV(H)
499 MAT W=T*V
500 MAT Z=X-W
501 L=Z[1,1]
502 Y=Z[2,1]
503 X=Z[3,1]
```

```
505 GOSUB 6000
506 IF C9#1 THEN 509
507 K1=K
508 GOTO 512
509 K2=ABS(K-K1)
510 K1=K
511 IF K2<0.01 THEN 515
512 P1=(A*V2*X/2+V3)/(A*V2-V1)
513 L=L+((Y+P1)/2-L)*(U1<0 AND U2<0 AND U3<0)
514 GOTO 155
515 PRINT
516 PRINT
517 PRINT
518 LINK 2,5,5
900 END
1000 REM PARTIAL DERIVATIVE SUBROUTINE
1030 K[1]=-2/Y^2
1040 K[2]=(A*V2*X+2*V3)/(A*V2-V1)
1050 K[3]=SQR(Y^2-2*Y*L+L^2+X^2)
1060 K[4]=SQR(Y^2-2*Y*L+L^2+(X^2/16))
1070 K[5]=(2/3)*A*V2
1080 K[6]=Y*(A*V2-V1)
1090 K[7]=A^2*V2^2
1100 K[8]=Y*(A*V2-V1)^2
1200 REM DK/DL
1210 I[1,1]=I[1,2]=I[1,6]=I[1,9]=0
1215 I[1,3]=((A*V2*X)/(4*Y)+(V4/Y))
1220 I[1,4]=(((K[5]*Y)*(L-Y)/K[3])-((K[5]*L)*(L-Y)/K[3])-(K[5]*K[3])-(2*V4))
1225 I[1,4]=I[1,4]/(4*Y)
1230 I[1,5]=((2*K[5]*Y)*(L-Y)/K[4])-((2*K[5]*L)*(L-Y)/K[4])-(2*K[5]*K[4])-(2*V4)
1235 I[1,5]=I[1,5]/(4*Y)
1240 I[1,7]=((V1*L)/Y+(V3/Y))
1250 I[1,8]=(-V1*L*2)/Y+V1-(V3/Y)
1260 I[1,10]=C/(Y*X*V)
1300 REM DK/DY
1310 I[2,1]=K[1]*(((K[2]*A/6)*V2)+V4)*(K[2]/8)
1320 I[2,2]=K[1]*((A*V2*X/6)+V4)*(K[2]/8)
1330 I[2,3]=K[1]*((A*V2*X/4)+V4)*(L/2-(K[2]/4))
1340 I[2,4]=((K[5]/4)*(Y-L)/K[3])-((K[5]/4*L)*(Y-L)/(Y*K[3]))
```

```
1345 I[ 2, 4 ]=I[ 2, 4 ]+((K[ 5 ]*L*K[ 3 ])/(4*Y^2))+(V4*L/(2*Y^2))
1350 I[ 2, 5 ]=((K[ 5 ]*(Y-L))/(2*K[ 4 ]))-((K[ 5 ]*L*(Y-L))/(2*Y*K[ 4 ]))
1355 I[ 2, 5 ]=I[ 2, 5 ]+((K[ 5 ]*L*K[ 4 ])/(2*Y^2))+((V4*L)/(2*Y^2))
1360 I[ 2, 6 ]=K[ 1 ]*((K[ 2 ]*V1/3)+V3)*(K[ 2 ]/8)
1370 I[ 2, 7 ]=K[ 1 ]*(L*V1/2+((K[ 2 ]*V1/4)+V3))*(L/2-K[ 2 ]/4)
1380 I[ 2, 8 ]=((L^2*V1)+(L*V3))/(Y^2)
1390 I[ 2, 9 ]=-R/(2*Y^2*V)
1400 I[ 2, 10 ]=(-C*L-F1-F2-F3)/(2*Y^2*X*V)
1500 REM DK/DX
1510 I[ 3, 1 ]=((K[ 7 ]*A*V2*X)/(12*K[ 8 ]))+((A*V2^2*V3)/(6*K[ 8 ]))+((A*V2*V4)/(4*K[ 6 ])
1520 I[ 3, 2 ]=((K[ 7 ]*X)/(12*K[ 6 ]))+(((3*A*V2*V4)+(A*V2*V3))/(12*K[ 6 ]))
1530 I[ 3, 3 ]=((-K[ 7 ]*X)/(4*K[ 6 ]))+((A*V2*L)/(4*Y))
1535 I[ 3, 3 ]=I[ 3, 3 ]-(((A*V2*V3)+(2*A*V2*V4))/(4*K[ 6 ]))
1540 I[ 3, 4 ]=(A*V2*X*(Y-L))/(6*Y*K[ 3 ])
1550 I[ 3, 5 ]=(A*V2*X*(Y-L))/(48*Y*K[ 4 ])
1560 I[ 3, 6 ]=((K[ 7 ]*V1*X)/(6*K[ 8 ]))+((A*V1*V2*V3)/(3*K[ 8 ]))+((A*V2*V3)/(4*K[ 6 ]))
1570 I[ 3, 7 ]=((-K[ 7 ]*V1*X)/(4*K[ 8 ]))-((A*V1*V2*V3)/(2*K[ 8 ]))-((A*V2*V3)/(2*K[ 6 ]))
1580 I[ 3, 8 ]=I[ 3, 9 ]=0
1590 I[ 3, 10 ]=((-C*L*2)-F1-F2-F3)/(2*Y*X^2*V)
2000 MAT J=CON
2020 MAT L=I*J
2120 RETURN
3000 REMHESSIANMATRIX
3010 S[ 1, 1 ]=S[ 1, 2 ]=S[ 1, 3 ]=S[ 1, 6 ]=S[ 1, 9 ]=S[ 1, 10 ]=0
3020 S[ 1, 4 ]=(A*V2)/(6*Y*K[ 3 ]^3)
3030 S[ 1, 4 ]=S[ 1, 4 ]*(2*Y^3-6*Y^2*L+6*Y*L^2+3*Y*X^2-2*L^3-3*L*X^2)
3040 S[ 1, 5 ]=2*Y^3-6*Y^2*L+6*Y*L^2+3*Y*X^2/16-2*L^3-3*L*X^2/16
3050 S[ 1, 5 ]=(S[ 1, 5 ]*A*V2)/(3*Y*K[ 4 ]^3)
3060 S[ 1, 7 ]=V1/Y
3070 S[ 1, 8 ]=-2*V1/Y
3100 K[ 9 ]=4/Y^3
3110 S[ 2, 1 ]=((K[ 2 ]/6*V2)+V4)*K[ 9 ]*K[ 2 ]/8
3120 S[ 2, 2 ]=(A*X/6*V2+V4)*K[ 9 ]*K[ 2 ]/8
3130 S[ 2, 3 ]=(A*X/4*V2+V4)*K[ 9 ]*(L/2-K[ 2 ]/4)
3140 S[ 2, 4 ]=((A*V2/6)*((1/K[ 3 ])-((Y*(Y-L))/K[ 3 ]^3)))-((A*V2*L/3)*((L-Y)/K[ 3 ]^3))
3145 S[ 2, 4 ]=S[ 2, 4 ]+(A*V2*L^2/6)*(((L-Y)/(Y*K[ 3 ]^3))-(1/(Y^2*K[ 3 ])))
3150 S[ 2, 4 ]=S[ 2, 4 ]+(A*V2*L/6)*(((Y-L)/(Y^2*K[ 3 ]))-(2*K[ 3 ]/Y^3))
3155 S[ 2, 4 ]=S[ 2, 4 ]-V4*L/Y^3
```

```
3160 K[17]=Y^3-Y^2*L-Y*L^2+L^3+X^2*L/16
3165 S[2,5]=(3*Y^2-2*Y*L-L^2)/(Y^2*K[4])
3170 S[2,5]=S[2,5]-K[17]*(Y-L)/(Y^2*K[4]^3)
3175 S[2,5]=S[2,5]-2*K[17]/(Y^3*K[4])
3180 S[2,5]=S[2,5]*A*V2/3-(V4*L/Y^3)
3200 S[2,6]=(K[2]/3*V1+V3)*K[9]*K[2]/8
3210 S[2,7]=((L/2+K[2]/4)*V1+V3)*(L/2-K[2]/4)*K[9]
3220 S[2,8]=(-2*L^2*V1-2*L*V3)/Y^3
3230 S[2,9]=R/(Y^3*V)
3240 S[2,10]=(2*C*L+F1+F2+F3)/(Y^3*X*V)
3300 S[3,1]=(K[7]*V2*A)/(12*K[8])
3310 S[3,2]=K[7]/(12*K[6])
3320 S[3,3]=K[7]/(-4*K[6])
3325 K[10]=A*V2*Y*(Y-L)
3330 S[3,4]=(6*K[3]*K[10])-(6*X^2*K[10]/K[3])
3340 S[3,4]=S[3,4]/(36*Y^2*K[3]^2)
3350 S[3,5]=(48*K[4]*K[10])-(3*X^2*K[10]/K[4])
3360 S[3,5]=S[3,5]/(2304*Y^2*K[4]^2)
3370 S[3,6]=K[7]*V1/(K[8]*6)
3380 S[3,7]=K[7]*V1/(K[8]*-4)
3390 S[3,8]=S[3,9]=0
3400 S[3,10]=(2*C*L+F1+F2+F3)/(Y*X^3*V)
3500 S[4,1]=S[4,2]=S[4,6]=S[4,9]=0
3510 S[4,3]=(-A*V2*X/(4*Y^2)-V4/(Y^2))
3520 K[11]=Y^3-Y^2*L-Y*L^2+L^3+L*X^2
3530 K[12]=3*L^2-2*Y*L-Y^2+X^2
3540 K[13]=(L-Y)*Y^2/K[3]
3550 S[4,4]=(Y^2*K[3]*K[12]-K[11]*K[13])/(Y^4*K[3]^2)
3560 S[4,4]=S[4,4]*A*V2/6+V4/(2*Y^2)
3570 K[11]=K[11]-(15/16*X^2*L)
3580 K[12]=K[12]-(15/16*X^2)
3590 K[13]=(L-Y)*Y^2/K[4]
3600 S[4,5]=(Y^2*K[4]*K[12]-K[11]*K[13])/(Y^4*K[4]^2)
3610 S[4,5]=S[4,5]*A*V2/3+V4/(2*Y^2)
3620 S[4,7]=(-V1*L-V3)/(Y^2)
3630 S[4,8]=(2*V1*L+V3)/(Y^2)
3640 S[4,10]=-C/(Y^2*X*V)
3700 S[5,1]=S[5,2]=S[5,6]=S[5,7]=S[5,8]=S[5,9]=0
```

```
3710 S[5,3]=A*V2/(4*Y)
3720 S[5,4]=(Y-L)/K[3]^3
3725 S[5,4]=S[5,4]*(A*V2*X/6)*(1-L/Y)-(A*V2*X/(6*Y*K[3]))
3730 S[5,5]=(Y-L)/K[4]^3
3735 S[5,5]=S[5,5]*(A*V2*X/48)*(1-L/Y)-(A*V2*X/(48*Y*K[4]))
3760 S[5,10]=-C/(Y*X^2*V)
3800 S[6,1]=I[3,1]/-Y
3810 S[6,2]=I[3,2]/-Y
3820 S[6,3]=I[3,3]/-Y
3830 S[6,6]=I[3,6]/-Y
3840 S[6,7]=I[3,7]/-Y
3850 S[6,8]=S[6,9]=0
3870 K[15]=-A*V2*(Y-L)*(1-L/Y)*X
3880 K[16]=A*V2*L*X
3890 S[6,4]=K[15]/(6*K[3]^3)+K[16]/(6*Y^2*K[3])
3900 S[6,5]=K[15]/(48*K[4]^3)+K[16]/(48*Y^2*K[4])
4000 S[6,10]=I[3,10]/-Y
4010 MAT H=ZER
4020 FOR I=1 TO 10
4040 H[1,1]=H[1,1]+S[1,I]
4050 H[2,2]=H[2,2]+S[2,I]
4060 H[3,3]=H[3,3]+S[3,I]
4070 NEXT I
4110 FOR I=1 TO 10
4130 H[1,2]=H[2,1]=H[1,2]+S[4,I]
4140 H[1,3]=H[3,1]=H[1,3]+S[5,I]
4150 H[2,3]=H[3,2]=H[2,3]+S[6,I]
4160 NEXT I
4220 RETURN
5000 REM COST SUBROUTINE
5012 PRINT "     P1<Y ";
5300 C[1]=((A^2*V2*X/6+V3*A/3)/(A*V2-V1)*V2)+V4
5310 C[1]=((C[1]*2/Y)*((A*V2*X/8+V3/4)/(A*V2-V1)))
5320 C[2]=(A*X*V2/6+V4)*(2/Y)
5330 C[2]=C[2]*((A*V2*X/8+V3/4)/(A*V2-V1))
5340 C[3]=(A*X*V2/4+V4)*(2/Y)
5350 C[3]=C[3]*(L/2-((A*V2*X/4+V3/2)/(A*V2-V1)))
5360 C[4]=(2/Y)*((A*V2/3*SQR(Y^2-2*Y*L+L^2+X^2))+V4)
5370 C[4]=C[4]*(Y/4-L/4)
```

```
5 REM
7 PRINT "                          CASE 1B--P1>Y"
8 PRINT
9 PRINT
122 PRINT
123 PRINT "ITERATION    L         Y          X    CONDITION  COST/M3   CONVEXITY
124 M1=A*V2*V3-V1*V3
125 M3=4*A*V1*V2-2*A^2*V2^2-2*V1^2
126 M5=A^3*V2^3-2*A^2*V1*V2^2+A*V1^2*V2
135 C9=0
136 PRINT "------------------------------------------------------------------
137 PRINT
140 L=L1
145 Y=Y1
150 X=X1
155 P1=(A*V2*X/2+V3)/(A*V2-V1)
156 IF P1 >= Y THEN 159
157 X=1.05*X
158 GOTO 155
159 L=Y
185 C9=C9+1
190 WRITE (15,195)C9,L,Y,X;
195 FORMAT F6.0,4F10.1
200 GOSUB 5000
480 WRITE (15,481)K;
481 FORMAT 2F12.2
490 GOSUB 1000
491 GOSUB 3000
492 X[1,1]=L
493 X[2,1]=Y
494 X[3,1]=X
495 V[1,1]=L[1,1]
496 V[2,1]=L[2,1]
497 V[3,1]=L[3,1]
498 MAT T=INV(H)
```

```
5380 C[5]=(2/Y)*((2*A*V2/3*SQR(Y^2-2*Y*L+L^2+X^2/16))+V4)
5390 C[5]=C[5]*(Y/4-L/4)
5400 C[6]=(2/Y)*((A*V2*X/3+2*V3/3)*V1/(A*V2-V1)+V3)
5410 C[6]=C[6]*(A*V2*X/8+V3/4)/(A*V2-V1)
5420 C[7]=(2/Y)*((L*V1/2+(A*V1*V2*X/4+V1*V3/2)/(A*V2-V1))+V3)
5430 C[7]=C[7]*(L/2-((A*V2*X/4+V3/2)/(A*V2-V1)))
5435 C[8]=(4/Y)*(L*V1+V3)*(Y/4-L/4)
5450 C[9]=R/(2*Y*V)
5460 C[10]=(2*C*L+F1+F2+F3)/(2*Y*X*V)
5470 K=C[1]+C[2]+C[3]+C[4]+C[5]+C[6]+C[7]+C[8]+C[9]+C[10]
5480 RETURN
6000 REM CONVEXITY CHECK
6010 U1=H[1,1]
6020 U2=H[1,1]*H[2,2]-H[2,1]*H[1,2]
6030 U3=DET(H)
6040 IF (U1 >= 0 AND U2 >= 0 AND U3 >= 0) THEN 6070
6050 PRINT "  NOT CONVEX"
6060 GOTO 6080
6070 PRINT "  CONVEX"
6080 RETURN
```

```
499 MAT W=T*V
500 MAT Z=X-W
501 L=Z[1,1]
502 Y=Z[2,1]
503 X=Z[3,1]
505 GOSUB 6000
506 IF C9#1 THEN 509
507 K1=K
508 GOTO 155
509 K2=ABS(K-K1)
510 K1=K
511 IF K2<0.01 THEN 514
512 GOTO 155
514 PRINT
515 PRINT
516 PRINT
517 PRINT
518 PRINT "                              CASE 1C--YARDING PERPENDICULAR TO TRUCK ROAD"
519 Y=SQR(R/(A*V2*V))
521 K=A*V2*Y/2+V4+R/(2*Y*V)
522 WRITE (15,524)"OPTIMUM Y = "Y
523 WRITE (15,525)"COST/M3 = "K
524 FORMAT F10.1
525 FORMAT F8.2,/,/,/
526 LINK 3,5,5
900 END
1000 REM PARTIAL DERIVATIVE SUBROUTINE
1252 FOR J=1 TO 9
1253 I[1,J]=0
1254 NEXT J
1260 I[1,10]=C/(Y*X*V)
1300 REM DK/DY
1382 I[2,1]=M1/M2+A*V2/2
1383 I[2,2]=2*Y*M3/M4
1384 I[2,3]=2*Y*X*M5/M6
1385 I[2,4]=I[2,5]=I[2,6]=I[2,7]=I[2,8]=0
1390 I[2,9]=-R/(2*Y^2*V)
1400 I[2,10]=(-C*L-F1-F2-F3)/(2*Y^2*X*V)
```

```
1500 REM DK/DX
1572 I[3,1]=-Y*A*V2*M1/(M2^2)
1573 I[3,2]=-3*Y^2*A*V2*M3/(M4^2)
1574 I[3,3]=(Y^2*M5/M6)-(Y^2*X*M7*M5/(M6^2))
1575 I[3,4]=I[3,5]=I[3,6]=I[3,7]=0
1580 I[3,8]=I[3,9]=0
1590 I[3,10]=((-C*L*2)-F1-F2-F3)/(2*Y*X^2*V)
2000 MAT J=CON
2020 MAT L=I*J
2120 RETURN
3000 REMHESSIANMATRIX
3072 FOR J=1 TO 10
3073 S[1,J]=0
3074 NEXT J
3222 S[2,1]=S[2,4]=S[2,5]=S[2,6]=S[2,7]=S[2,8]=0
3223 S[2,2]=2*M3/M4
3224 S[2,3]=2*X*M5/M6
3230 S[2,9]=R/(Y^3*V)
3240 S[2,10]=(2*C*L+F1+F2+F3)/(Y^3*X*V)
3392 S[3,1]=2*Y*A^2*V2^2*M1/(M2^3)
3393 S[3,2]=6*Y^2*A^2*V2^2*M3/(M4^3)
3394 S[3,3]=-(Y^2*M5)*(M7/(M6^2)+M8/(M6^2)-2*M7/(M6^3))
3395 S[3,4]=S[3,5]=S[3,6]=S[3,7]=S[3,8]=S[3,9]=0
3400 S[3,10]=(2*C*L+F1+F2+F3)/(Y*X^3*V)
3632 FOR J=1 TO 9
3633 S[4,J]=0
3634 NEXT J
3640 S[4,10]=-C/(Y^2*X*V)
3737 FOR J=1 TO 9
3738 S[5,J]=0
3739 NEXT J
3760 S[5,10]=-C/(Y*X^2*V)
3902 S[6,1]=S[3,1]/Y
3903 S[6,2]=S[3,2]/(Y/2)
3904 S[6,3]=S[3,3]/(Y/2)
3905 S[6,4]=S[6,5]=S[6,6]=S[6,7]=S[6,8]=S[6,9]=0
4000 S[6,10]=I[3,10]/-Y
4010 MAT H=ZER
```

```
4020 FOR I=1 TO 10
4040 H[1,1]=H[1,1]+S[1,I]
4050 H[2,2]=H[2,2]+S[2,I]
4060 H[3,3]=H[3,3]+S[3,I]
4070 NEXT I
4110 FOR I=1 TO 10
4130 H[1,2]=H[2,1]=H[1,2]+S[4,I]
4140 H[1,3]=H[3,1]=H[1,3]+S[5,I]
4150 H[2,3]=H[3,2]=H[2,3]+S[6,I]
4160 NEXT I
4220 RETURN
5000 REM COST SUBROUTINE
5002 M2=A*V2*X+2*V3
5003 M4=3*M2
5004 M6=M4*M2
5005 M7=6*A^2*V2^2*X+12*A*V2*V3
5006 M8=12*A^2*V2^2*X+12*A*V2*V3
5441 C[1]=Y*((M1/M2)+(A*V2/2))
5442 C[2]=Y^2*(M3/M4)
5443 C[3]=Y^2*X*(M5/M6)
5444 C[4]=V4
5445 C[5]=C[6]=C[7]=C[8]=0
5446 PRINT "    P1>Y ";
5450 C[9]=R/(2*Y*V)
5460 C[10]=(2*C*L+F1+F2+F3)/(2*Y*X*V)
5470 K=C[1]+C[2]+C[3]+C[4]+C[5]+C[6]+C[7]+C[8]+C[9]+C[10]
5480 RETURN
6000 REM CONVEXITY CHECK
6010 U1=H[1,1]
6020 U2=H[1,1]*H[2,2]-H[2,1]*H[1,2]
6030 U3=DET(H)
6040 IF (U1 >= 0 AND U2 >= 0 AND U3 >= 0) THEN 6070
6050 PRINT "  NOT CONVEX"
6060 GOTO 6080
6070 PRINT "  CONVEX"
6080 RETURN
```

```
5 DIM C[10],I[3,10],J[10,1],L[3,1],K[20],S[6,10],H[3,3],T[3,3],X[3,1],V[3,1]
6 DIM W[3,1],Z[3,1]
8 PRINT
9 PRINT
121 PRINT "                         CASE 2--YARDING NOT PERMITTED ONTO TRUCK ROAD"
122 PRINT
123 PRINT "ITERATION    L         Y          X    CONDITION  COST/M3   CONVEXITY
135 C9=0
136 PRINT "-------------------------------------------------------------------
137 PRINT
140 L=L1
145 Y=Y1
150 X=X1
151 F4=F1
155 L=L*(L>0)+(Y-L)*(L>Y)
185 C9=C9+1
190 WRITE (15,195)C9,L,Y,X;
195 FORMAT F6.0,4F10.1
200 GOSUB 5000
480 WRITE (15,481)K;
481 FORMAT 2F12.2
490 GOSUB 1000
491 GOSUB 3000
492 X[1,1]=L
493 X[2,1]=Y
494 X[3,1]=X
495 V[1,1]=L[1,1]
496 V[2,1]=L[2,1]
497 V[3,1]=L[3,1]
498 MAT T=INV(H)
499 MAT W=T*V
500 MAT Z=X-W
501 L=Z[1,1]
502 Y=Z[2,1]
503 X=Z[3,1]
505 GOSUB 6000
506 IF C9#1 THEN 509
507 K1=K
508 GOTO 513
```

```
509 K2=ABS(K-K1)
510 K1=K
511 IF K2<0.01 THEN 515
513 L=L+(Y/2-L)*(U1<0 AND U2<0 AND U3<0)
514 GOTO 155
515 PRINT
516 PRINT
517 PRINT
900 END
1000 REM PARTIAL DERIVATIVE SUBROUTINE
1030 K[1]=-2/Y^2
1040 K[2]=(A*V2*X+2*V3)/(A*V2-V1)
1050 K[3]=SQR(Y^2-2*Y*L+L^2+X^2)
1060 K[4]=SQR(Y^2-2*Y*L+L^2+(X^2/16))
1070 K[5]=(2/3)*A*V2
1080 K[6]=Y*(A*V2-V1)
1090 K[7]=A^2*V2^2
1100 K[8]=Y*(A*V2-V1)^2
1200 REM DK/DL
1210 I[1,2]=I[1,3]=I[1,7]=I[1,9]=0
1215 I[1,1]=A*V2*X/(4*Y)+V4/Y
1220 I[1,4]=(((K[5]*Y)*(L-Y)/K[3])-((K[5]*L)*(L-Y)/K[3])-(K[5]*K[3])-(2*V4))
1225 I[1,4]=I[1,4]/(4*Y)
1230 I[1,5]=((2*K[5]*Y)*(L-Y)/K[4])-((2*K[5]*L)*(L-Y)/K[4])-(2*K[5]*K[4])-(2*V4)
1235 I[1,5]=I[1,5]/(4*Y)
1240 I[1,6]=V1*L/Y+V3/Y
1250 I[1,8]=(-V1*L*2)/Y+V1-(V3/Y)
1260 I[1,10]=C/(Y*X*V)
1300 REM DK/DY
1310 I[2,1]=-A*V2*L*X/(4*Y^2)-V4*L/Y^2
1320 I[2,2]=I[2,3]=I[2,7]=0
1340 I[2,4]=((K[5]/4)*(Y-L)/K[3])-((K[5]/4*L)*(Y-L)/(Y*K[3]))
1345 I[2,4]=I[2,4]+((K[5]*L*K[3])/(4*Y^2))+(V4*L/(2*Y^2))
1350 I[2,5]=((K[5]*(Y-L))/(2*K[4]))-((K[5]*L*(Y-L))/(2*Y*K[4]))
1355 I[2,5]=I[2,5]+((K[5]*L*K[4])/(2*Y^2))+((V4*L)/(2*Y^2))
1360 I[2,6]=-V1*L^2/(2*Y^2)-V3*L/Y^2
1380 I[2,8]=((L^2*V1)+(L*V3))/(Y^2)
1390 I[2,9]=-R/(2*Y^2*V)
1400 I[2,10]=(-C*L-F1-F2-F3)/(2*Y^2*X*V)
```

```
1500 REM DK/DX
1510 I[3,1]=A*V2*L/(4*Y)
1540 I[3,4]=(A*V2*X*(Y-L))/(6*Y*K[3])
1550 I[3,5]=(A*V2*X*(Y-L))/(48*Y*K[4])
1580 I[3,8]=I[3,9]=I[3,2]=I[3,3]=I[3,6]=I[3,7]=0
1590 I[3,10]=((-C*L*2)-F1-F2-F3)/(2*Y*X^2*V)
2000 MAT J=CON
2020 MAT L=I*J
2120 RETURN
3000 REMHESSIANMATRIX
3010 S[1,1]=S[1,2]=S[1,3]=S[1,7]=S[1,9]=S[1,10]=0
3020 S[1,4]=(A*V2)/(6*Y*K[3]^3)
3030 S[1,4]=S[1,4]*(2*Y^3-6*Y^2*L+6*Y*L^2+3*Y*X^2-2*L^3-3*L*X^2)
3040 S[1,5]=2*Y^3-6*Y^2*L+6*Y*L^2+3*Y*X^2/16-2*L^3-3*L*X^2/16
3050 S[1,5]=(S[1,5]*A*V2)/(3*Y*K[4]^3)
3060 S[1,6]=V1/Y
3070 S[1,8]=-2*V1/Y
3100 K[9]=4/Y^3
3105 S[2,1]=A*V2*L*X/(2*Y^3)+2*V4*L/(Y^3)
3110 S[2,2]=S[2,3]=S[2,7]=0
3140 S[2,4]=((A*V2/6)*((1/K[3])-((Y*(Y-L))/K[3]^3)))-((A*V2*L/3)*((L-Y)/K[3]^3))
3145 S[2,4]=S[2,4]+(A*V2*L^2/6)*(((L-Y)/(Y*K[3]^3))-(1/(Y^2*K[3])))
3150 S[2,4]=S[2,4]+(A*V2*L/6)*(((Y-L)/(Y^2*K[3]))-(2*K[3]/Y^3))
3155 S[2,4]=S[2,4]-V4*L/Y^3
3160 K[17]=Y^3-Y^2*L-Y*L^2+L^3+X^2*L/16
3165 S[2,5]=(3*Y^2-2*Y*L-L^2)/(Y^2*K[4])
3170 S[2,5]=S[2,5]-K[17]*(Y-L)/(Y^2*K[4]^3)
3175 S[2,5]=S[2,5]-2*K[17]/(Y^3*K[4])
3180 S[2,5]=S[2,5]*A*V2/3-(V4*L/Y^3)
3200 S[2,6]=V1*L^2/Y^3+2*V3*L/Y^3
3220 S[2,8]=(-2*L^2*V1-2*L*V3)/Y^3
3230 S[2,9]=R/(Y^3*V)
3240 S[2,10]=(2*C*L+F1+F2+F3)/(Y^3*X*V)
3325 K[10]=A*V2*Y*(Y-L)
3330 S[3,4]=(6*K[3]*K[10])-(6*X^2*K[10]/K[3])
3340 S[3,4]=S[3,4]/(36*Y^2*K[3]^2)
3350 S[3,5]=(48*K[4]*K[10])-(3*X^2*K[10]/K[4])
3360 S[3,5]=S[3,5]/(2304*Y^2*K[4]^2)
```

```
3390 S[3,8]=S[3,9]=S[3,1]=S[3,2]=S[3,3]=S[3,6]=S[3,7]=0
3400 S[3,10]=(2*C*L+F1+F2+F3)/(Y*X^3*V)
3500 S[4,1]=-A*V2*X/(4*Y^2)-V4/Y^2
3510 S[4,2]=S[4,3]=S[4,7]=S[4,9]=0
3520 K[11]=Y^3-Y^2*L-Y*L^2+L^3+L*X^2
3530 K[12]=3*L^2-2*Y*L-Y^2+X^2
3540 K[13]=(L-Y)*Y^2/K[3]
3550 S[4,4]=(Y^2*K[3]*K[12]-K[11]*K[13])/(Y^4*K[3]^2)
3560 S[4,4]=S[4,4]*A*V2/6+V4/(2*Y^2)
3570 K[11]=K[11]-(15/16*X^2*L)
3580 K[12]=K[12]-(15/16*X^2)
3590 K[13]=(L-Y)*Y^2/K[4]
3600 S[4,5]=(Y^2*K[4]*K[12]-K[11]*K[13])/(Y^4*K[4]^2)
3610 S[4,5]=S[4,5]*A*V2/3+V4/(2*Y^2)
3620 S[4,6]=-V1*L/Y^2-V3/Y^2
3630 S[4,8]=(2*V1*L+V3)/(Y^2)
3640 S[4,10]=-C/(Y^2*X*V)
3700 S[5,2]=S[5,6]=S[5,7]=S[5,8]=S[5,9]=S[5,3]=0
3710 S[5,1]=A*V2/(4*Y)
3720 S[5,4]=(Y-L)/K[3]^3
3725 S[5,4]=S[5,4]*(A*V2*X/6)*(1-L/Y)-(A*V2*X/(6*Y*K[3]))
3730 S[5,5]=(Y-L)/K[4]^3
3735 S[5,5]=S[5,5]*(A*V2*X/48)*(1-L/Y)-(A*V2*X/(48*Y*K[4]))
3760 S[5,10]=-C/(Y*X^2*V)
3800 S[6,1]=-A*V2*L/(4*Y^2)
3850 S[6,8]=S[6,9]=S[6,2]=S[6,3]=S[6,6]=S[6,7]=0
3870 K[15]=-A*V2*(Y-L)*(1-L/Y)*X
3880 K[16]=A*V2*L*X
3890 S[6,4]=K[15]/(6*K[3]^3)+K[16]/(6*Y^2*K[3])
3900 S[6,5]=K[15]/(48*K[4]^3)+K[16]/(48*Y^2*K[4])
4000 S[6,10]=I[3,10]/-Y
4010 MAT H=ZER
4020 FOR I=1 TO 10
4040 H[1,1]=H[1,1]+S[1,I]
4050 H[2,2]=H[2,2]+S[2,I]
4060 H[3,3]=H[3,3]+S[3,I]
4070 NEXT I
```

```
4110 FOR I=1 TO 10
4130 H[1,2]=H[2,1]=H[1,2]+S[4,I]
4140 H[1,3]=H[3,1]=H[1,3]+S[5,I]
4150 H[2,3]=H[3,2]=H[2,3]+S[6,I]
4160 NEXT I
4220 RETURN
5000 REM COST SUBROUTINE
5012 PRINT "          ";
5300 C[1]=A*V2*L*X/(4*Y)+V4*L/Y
5310 C[2]=C[3]=C[7]=0
5360 C[4]=(2/Y)*((A*V2/3*SQR(Y^2-2*Y*L+L^2+X^2))+V4)
5370 C[4]=C[4]*(Y/4-L/4)
5380 C[5]=(2/Y)*((2*A*V2/3*SQR(Y^2-2*Y*L+L^2+X^2/16))+V4)
5390 C[5]=C[5]*(Y/4-L/4)
5400 C[6]=V1*L^2/(2*Y)+V3*L/Y
5435 C[8]=(4/Y)*(L*V1+V3)*(Y/4-L/4)
5450 C[9]=R/(2*Y*V)
5460 C[10]=(2*C*L+F1+F2+F3)/(2*Y*X*V)
5470 K=C[1]+C[2]+C[3]+C[4]+C[5]+C[6]+C[7]+C[8]+C[9]+C[10]
5480 RETURN
6000 REM CONVEXITY CHECK
6010 U1=H[1,1]
6020 U2=H[1,1]*H[2,2]-H[2,1]*H[1,2]
6030 U3=DET(H)
6040 IF (U1 >= 0 AND U2 >= 0 AND U3 >= 0) THEN 6070
6050 PRINT "  NOT CONVEX"
6060 GOTO 6080
6070 PRINT "  CONVEX"
6080 RETURN
```