RANGE OF USEFULNESS OF BETHE'S SEMIEMPIRICAL NUCLEAR MASS FORMULA

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RANGE OF USEFULNESS OF BETHE'S SEMIEMPIRICAL NUCLEAR MASS FORMULA

INTRODUCTION

The complicated experimental results on atomic nuclei have been defying definite interpretation of the structure of atomic nuclei for a long time. Even though various theoretical methods have been suggested, based upon the particular aspects of experimental results, it has been impossible to find a successful theory which suffices to explain the whole observed properties of atomic nuclei.

In 1936, Bohr (3, p. 344) proposed the liquid drop model of atomic nuclei to explain the resonance capture process of nuclear reactions. The experimental evidences which support the liquid drop model are as follows:

1. Substantially constant density of nuclei with radius

$$R = R_o A^{1/3} \qquad (1)$$

where A is the mass number of the nucleus and R_o is the constant of proportionality with the value of $(1.5 \pm 0.1) \times 10^{-13}$ cm.

2. Short range effect of nuclear force.
3. Fission by thermal neutrons of U^{235} and other odd-N nuclides.

4. Systematic variation of α decay energies with the numbers of neutrons and protons.

The liquid drop model assumes that the atomic nucleus behaves, in many respects, like a droplet of incompressible matter.

However, although the liquid drop model successfully explains many nuclear characteristics, an independent-particle model has been suggested by many people (14, p. 1969), (12, p. 1766), (8, p. 1275) to explain many nuclear properties such as nuclear angular momentum, magnetic dipole moments, electric quadrupole moments, islands of isomerism, relative parity of nuclear levels, and frequency of stable isotones and isotopes.

It has been suggested (6, p. 625) that special numbers of neutrons or protons in the nucleus form a particularly stable configuration, and it is especially prominent for the nuclei of 50 and 82 protons and for 50, 82, and 126 neutrons. (15, p. 235) For accounting for these properties of atomic nuclei, the liquid drop model is inherently insufficient.

Extensive investigations with regard to these aspects have led to the realization of the exceptional properties of atomic nuclei having 2, 8, 20, 28, 50, 82, and 126 protons or neutrons. These numbers are usually referred to as nuclear magic numbers.

We know that atoms with 2, 8, 18, etc., electrons are exceptionally stable, and we describe these configurations as closed electron shells. By analogy with this description, we deduce that nuclear shells are closed by magic numbers of protons or neutrons.

If we agree with the nuclear closed shell model, we shall arrive at the conclusion that when a new shell is begun, the binding energy of the newly added nucleons should be less than that of the preceding nucleons which served to complete the preceding shell. This fact was noticed early by Bethe in 1936 (2, p. 173). We should thus expect that the 3d, 9th, 21st, 29th, 51st, 83d, and 127th nucleon of a given kind is less strongly bound than the 2d, 8th, 20th, 28th, 50th, 82d, and 126th nucleon of the same kind.

The purpose of this paper is to investigate this aspect of atomic nuclei by means of the liquid drop model. The theoretical results from the liquid drop model will fail to predict accurately the binding energies of atomic nuclei in the neighborhoods of closed shells. The liquid drop model has been developed with the use of the so-called semiempirical nuclear mass formula, which was first proposed by Weizsäcker (19, p. 431) in 1935 and was simplified later by Bethe (2, p. 165). Therefore, we are going to investigate Bethe's form of the semiempirical

nuclear mass formula by comparing theoretical results with experimental data in terms of average binding energies and binding energies of last nucleons.

NUCLEAR BINDING ENERGIES AND SEMIEMPIRICAL MASS FORMULA

If we express the mass of a nucleus using atomic mass units, the mass is very close to an integer which is the mass number of the nucleus. However, the mass of a nucleus is always less than the arithmetical sum of the masses of constituent protons and neutrons. The difference between these two values,

$$M(A,Z) - (ZM_H + N M_n),$$

(where $M(A,Z)$ is the atomic mass, Z and N are the numbers of protons and neutrons, and M_H and M_n are the masses of hydrogen atom and neutron) is supposed to be the binding energy of the nucleus. Furthermore, dividing this mass defect by the mass number of the nucleus, we get an expression for average binding energy

$$B.E./A = \frac{M(A,Z) - (Z M_H + N M_n)}{A} \qquad (2)$$

which is supposed to be released by each nucleon to form the nucleus. The functional relationship between the average binding energy and the mass number is plotted in Figure 1. This curve tells us a very significant fact that the average binding energies are almost constant for nuclei with $A > 20$, with the value of about 8 Mev.

From the liquid drop model we can deduce the fact that those nucleons which are visualized as being at the nuclear surface have fewer near neighbors than nucleons

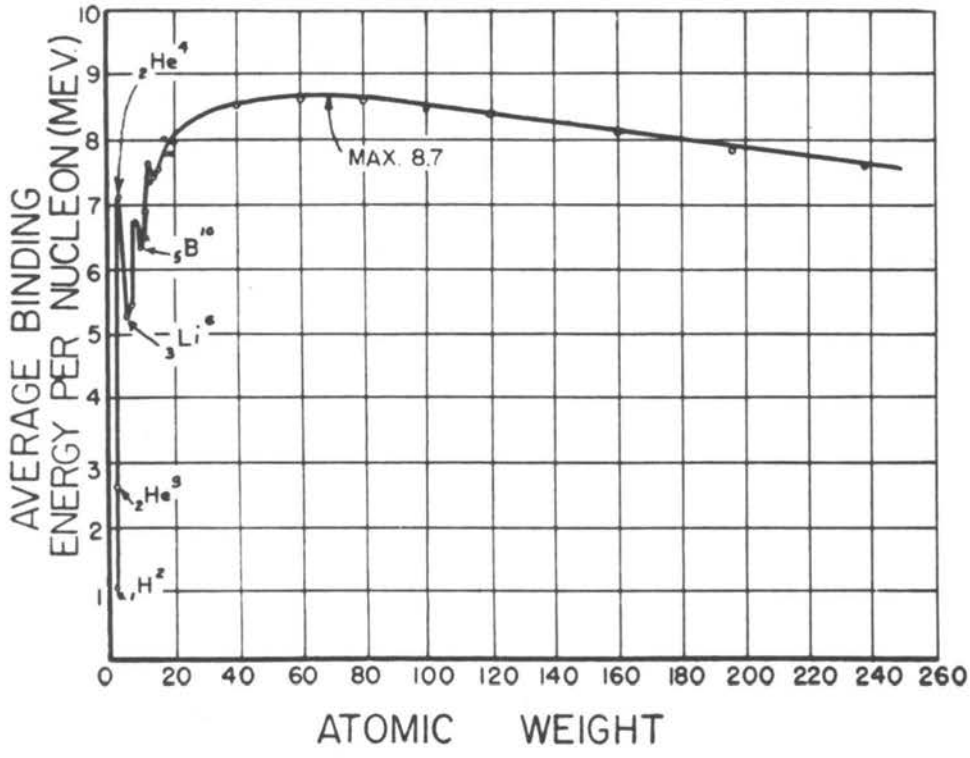


FIGURE 1.

AVERAGE BINDING ENERGY

FOR VARIOUS ELEMENTS

(5, p. 7)

which are deep within the nuclear volume. We can expect a deficit of binding energy for these surface nucleons, and the deficit of total binding energy will be proportional to the surface area of the nucleus. So, we can conclude that the total binding energy of the nucleus will partly decrease in proportion to the surface area which is also proportional to $A^{2/3}$ by equation (1).

We know from the experimental facts that the only known long-range force in nuclei is the coulomb repulsion between protons. So, we again assume that there is a deficit of binding energy due to the disruptive coulomb force. This deficit will be proportional to $Z^2/A^{1/3}$, where $A^{1/3}$ is proportional to the radius of the nucleus by equation (1).

Another deficit in binding energy depends on the isotopic number (N-Z) and is proportional to $(N-Z)^2/A$. Among the lightest elements there is a clear tendency for the number of neutrons and protons to be equal. This means that the nuclear binding becomes weak for the nucleus of large isotopic number. In rough approximation, we can set this deficit of binding energy due to symmetry effect to be proportional to $(N-Z)^2/A$, because the symmetry effect diminishes as A becomes large. This conclusion was also deduced from the study of nuclear forces. (2, p. 157)

With the above properties of nuclear binding energies

Weizsäcker (19, p. 454) derived an expression for the total mass (energy) of a nucleus as a function of Z and N; and following his idea, Bethe (2, p. 165) derived the following form for the total mass of a nucleus which is simpler than Weizsäcker's:

$$M(Z,N,A) = Z\,M_H + N\,M_n - \alpha\,A + \beta\,(N-Z)^2/A + \gamma\,A^{2/3} + \frac{3}{5}\,(e^2/R_o)\,Z^2/A^{1/3} \qquad (4)$$

where α, β, and γ are empirical constants having dimensions of energy. The first two terms in this formula are evidently the sum of the masses of the constituent nucleons, and the following terms respectively represent volume energy, asymmetry, surface energy, and coulomb repulsion energy effects.

This formula has been tested with experimental values of nuclear masses, and also the physical significance of the formula has been investigated extensively (4, p. 426), (1, p. 393), (9, p. 293). So, we have now the following modern version of Bethe's semiempirical nuclear mass formula:

$$M(Z,N,A) = M_H Z + M_n N - \alpha\,A + \beta\,(N-Z)^2/A + \gamma\,A^{2/3} + \varepsilon\,Z(Z-1)/A^{1/3} + \delta \qquad (5)$$

where $\delta = -36\,A^{-3/4}$ for even-A even-Z

$= 36\,A^{-3/4}$ for even-A odd-Z

$= 0$ for odd-A

in milli mass unit.

From the semiempirical mass formula (5) we can deduce the theoretical expression for average binding energy as follows:

$$B.E./A = \frac{M(Z,N,A) - (M_H Z + M_n N)}{A} =$$

$$- \alpha + \beta (N-Z)^2/A^2 + \gamma/A^{1/3} + \varepsilon Z(Z-1)/A^{4/3} + \delta/A. \quad (6)$$

A somewhat more detailed view of nuclear forces is given by the variations in the binding energy of the "last" proton or neutron in a group of nuclides (7, p. 302). The energy required to remove one neutron from the nucleus (Z,N) is called the neutron separation energy, S_n, and can be written

$$S_n(Z,N) = M(Z,N-1) + M_n - M(Z,N) \quad (7)$$

where M(Z,N) is the atomic mass of the nuclide, and M(Z,N-1) is the atomic mass of the lighter isotope which results when one neutron is removed from the nucleus (Z,N). In terms of binding energies, the neutron separation energy $S_n(Z,N)$ is the increment in total nuclear binding energy when one neutron is added to the lower isotope, (Z,N-1), thus

$$B_n(Z,N) = S_n(Z,N) = B.E.(Z,N) - B.E.(Z,N-1). \quad (8)$$

For this reason, the neutron separation energy is called the "binding energy of the last neutron." The binding

energy of the last proton is also defined by

$$B_p(Z,N) = S_p(Z,N) = M(Z-1,N) + M_H - M(Z,N)$$

$$= B.E.(Z,N) - B.E.(Z-1,N). \qquad (9)$$

In a similar say we may define the binding energy of any nuclear subgroup x(z,n) by (10, p. 57)

$$B_x(Z,N) = M(Z-z, N-n) + M_x(z,n) - M(Z,N). \qquad (10)$$

Nucleon separation energies are the nuclear analogues of the first ionization potentials of atoms.

RESEARCH PROCEDURE

To investigate the usefulness of the semiempirical nuclear mass formula, we proceeded with the following two methods:

1. Compare the variations of average binding energies from experimental results with the values from semiempirical mass formula.
2. Calculate the binding energies of the last neutrons and protons from the masses of nuclides and compare these values with average binding energies from semi-empirical mass formula.

For the first procedure, it was very important to obtain the exact values of the masses of as many nuclides as possible. For this purpose, we used the atomic mass table by Trigg (18) for most of the calculations. Other tables (17, p. 639), (5, p. 370) were also used. The method of getting experimental values of average binding energies was very simple. Equation (2) was simply used with

$$M_H = 1.008146 \text{ atomic mass unit}$$

$$M_n = 1.008986 \text{ atomic mass unit,}$$

and the experimental values of average binding energies were calculated for 171 nuclei. Most of these nuclei were stable.

The second step of the investigations was to calculate the theoretical average binding energies by using the semiempirical mass formula - equation (6). The main problem of this procedure was to determine the semiempirical constants which appeared in equation (6). The determination of the semiempirical constants has been made by many people (7, p. 383), (10, p. 287) in various ways. The following table collects the evaluations of the semiempirical constants in milli-mass units:

TABLE I

SEMIEMPIRICAL CONSTANTS

	α	4β	γ	ε
Bethe (1936)	14.885	83.770	14.176	0.623
Fermi (1945)	15.04	83.	14.0	0.627
Feenberg (1947)	15.035	77.755	14.069	0.627
Fowler (1947)	16.432	96.872	17.989	0.741
Metropolis-Reitwiesner (1950)	15.0825	82.970	14.0	0.627
Green (1954)	16.918	101.78	19.120	0.763
Evans (1955)	15.14	81.6	13.96	0.64
Halliday (1955)	15.04	83.	13.35	0.627

From the above table, the last four sets of constants were examined because we should have exact values of the masses of nuclides in determining some of the semiempirical constants and we could admit only recent data on the atomic

masses.

Among the last four sets, Green avoided the conventional practice of determining ε from the coulomb-energy difference of light mirror nuclei, and determined all four energy coefficients from a least-squares adjustment to the mass data for β -stable nuclides. (7, p. 383) This procedure leads to a larger value for ε . So, we omitted his values from our consideration in determining the constants. The semiempirical constants were determined from the values of Metropolis-Reitweisner, Evans, and Halliday as follows:

α . - - Since their three values were spread with almost equal widths, it seemed best to take mean value of those three. The mean value fell in the center of the total range.

4β . - - Two values were 83, and Evans had 81.6 ± 3.6. Since 83 was within the range of uncertainty of the Evans' value, we took 83.

γ . - - Since their distribution was not unique, we again took the mean of those three.

ε . - - Two values were 0.627 and Evans had 0.64 ± 0.02. Since the double value 0.627 was within the range of uncertainty of the Evans' value, we took 0.627.

In addition to this consideration, it was also noticed that they used the masses of proton and neutron

M_H = 1.008142

M_n = 1.008982.

However, their values differed from our new values by only four-thousandths in milli-mass units; therefore, these differences seemed to be negligible for the final values of the semiempirical constants.

From the above arguments, the following values were accepted for the calculation:

α = 15.088 m.m.u.

4β = 83 m.m.u.

γ = 13.77 m.m.u.

ε = 0.627 m.m.u.

To get accurate data, Standard Mathematical Tables, published by Chemical Rubber Publishing Company were used - particularly for the values of $A^{1/3}$ and $A^{4/3}$.

To calculate the binding energies of the last nucleon, equations (7) and (9) were used. Since we do not have so many data on the isotopic nuclear masses, it was very hard

to calculate enough data to study the binding energies of the last nucleon. In particular, a number of data for the nuclei in the neighborhood of magic numbers were badly needed. So, the unknown nuclear masses in these regions were calculated from decay energies, experimental data from (γ ,n) thresholds, (n, γ) reactions, and (d,p) reactions. (13, p. 481), (11, p. 362)

For instance, the binding energy of the last neutron for La^{139} was calculated from the value of Ba^{139}, the decay energy of Ba^{139} (3.5 Mev), and the decay energy of La^{138}. (11, p. 361) It has been reported (16, p. 303) that a 1-Mev γ -ray occurs in the K-capture of La^{138}; thus, this represents a lower limit to the decay energy. Then the binding energy of the last neutron in La^{139} is

$$B_n (57, 82) \geq 5.2 + (3.5 + 1.0) = 9.7 \text{ (Mev)}.$$

All the calculated data were converted from atomic mass unit to Mev by using

$$1 \text{ a.m.u.} = 931.16 \text{ Mev},$$

and the differences of average binding energies between experimental and theoretical values were evaluated for all nuclei. To see the variation of the binding energies of the last nucleon, the calculated values were compared with the theoretical average binding energies from the semiempirical formula.

The results of the first comparison between experimental and theoretical average binding energies were

plotted in Figures 2 and 3. In Figure 2, the differences between experimental and theoretical average binding energies were plotted as a function of the number of neutrons; and in Figure 3, the differences were plotted as a function of the number of protons.

The results of comparison between binding energies of the last neutron and average binding energies showed that there were three distinguishable parts in the neighborhood of the neutron magic numbers of 50, 82, and 126. So, each of these regions was plotted respectively in Figures 4, 5, and 6.

RESULTS

From Figure 2 we can see that differences between experimental and theoretical average binding energies take values from -0.7 Mev to 0.1 Mev, so that the maximum range of deviation is almost 0.8 Mev. The deviation is quite large for the nuclei with neutron number of less than 20, but the range of deviation becomes quite small for the nuclei with neutron numbers of more than 30. This means that the semiempirical nuclear mass formula is unusable for the nuclei in which the number of neutrons is less than 20.

It was a very interesting fact that we could find several peaks in Figure 2 for the neutron numbers of 20, 28, 50, 82, and roughly 126. It means that the experimental average binding energy becomes relatively large when the number of neutrons reaches a magic number. This is the direct proof of Elsasser's proposal in which he suggested that special numbers of nucleons in the nucleus form a particularly stable configuration (6, p. 635).

It was also found that the deviation curve drops down after the peaks around the magic numbers of neutrons. The semiempirical mass formula cannot predict this sort of periodical phenomenon on average binding energies.

Figure 3, in which the deviations between experimental

and theoretical average binding energies were plotted against the number of protons, shows a shape similar to Figure 2. It was again very irregular in the region of the number of protons less than 20, but peaks at the proton numbers of 20, 28, and 82 were very clear. This time the magic number of 50 protons did not appear clearly. This also showed peaks for the nuclei of 42, 48, and 54 protons.

In Figures 4, 5, and 6, the deviations of the binding energy of the last neutrons were plotted against the number of neutrons for the regions of 50, 82, and 126 neutrons. It was very noticeable that the binding energy of the last neutron usually takes on larger value than the theoretical average binding energy for the nuclei having even numbers of neutrons, and it is less than the theoretical average binding energy for the nuclei having odd numbers of neutrons. For 126 magic neutrons, the deviation was negative even for even-neutron nuclei; but the even-neutron nuclei gave larger values than odd-neutron nuclei through the whole region.

In Figure 4, we can see a very definite discontinuity of the even-neutron curve at the magic 50 neutrons. The rapid drop of more than 3 Mev was observed for the even-neutron curve at 52 neutrons, and it even becomes negative for 54 neutrons. The odd-neutron curve takes on relatively small values for the number of neutrons less

than 50, but the discontinuity was not so clear.

In Figure 5, both the even and odd curves showed very definite discontinuities at the magic number of 82. The even-neutron curve continued to decrease until N = 75, but it increased very rapidly in the region $75 < N < 82$. The sharp maximum takes place exactly at N = 82, and hereafter the curve decreases about 2 Mev while two more neutrons are added.

In this region the odd-neutron curve shows a very clear peak at N = 81; it decreases about 2 Mev by N = 83; and it goes down 2 more Mev by N = 85. After this, the curve increases again.

In Figure 6, where the deviations of the binding energy of the last neutron from the average binding energy were plotted for the region of $N > 110$, we can see that the even-neutron curve takes uniform values and finally it drops about 2.5 Mev at the neutron number of exactly 126. The odd-neutron curve also shows a very sharp peak in the neighborhood of 126 neutrons. The drop after the magic number of neutrons was of the order of 5 Mev.

From the above investigations, we can see in general that the appearances of magic numbers are so prominent that it is almost impossible to ignore the periodicity of the atomic nuclei. The liquid drop model cannot say anything about this periodicity, and it can be thoroughly understood from the above investigation that the theoretical prediction

of nuclear binding energies from the semiempirical nuclear mass formula is very poor in the neighborhood of magic numbers.

In Figure 7 the deviation of the binding energy of the last proton was plotted against the number of protons; however, the investigation of this graph shows that the magic properties for protons are uncertain from the viewpoint of nuclear binding energies. It would perhaps result from the fact that we have too many isotopes so that the detailed analysis for a particular number of protons is almost impossible.

CONCLUSIONS

The following conclusions have been reached from the preceding investigations of the semiempirical nuclear mass formula for nuclear binding energies:

1. It is unavoidable for one to conclude that the periodical magic properties of atomic nuclei are essential in interpreting the structure of atomic nuclei.

2. The semiempirical mass formula, together with the liquid drop model of atomic nuclei, is entirely incapable of explaining these magic properties. If the semiempirical mass formula is the final theoretical deduction in studying atomic nuclei, some additional factors must be included in the semiempirical mass formula so that it can predict accurately the magic properties of atomic nuclei.

3. The semiempirical mass formula is very incorrect for light nuclei.

4. The difference in the binding energy of the last neutron between odd and even neutron nuclides is very definite. Usually the binding energy of the last neutrons for the

even-neutron nuclei is larger than that of the odd-neutron nuclei. This is another evidence of the fact that nucleons form relatively stable configurations by pairing with each other.

5. The semiempirical mass formula must be developed further if we insist on the correctness of the liquid drop model of the atomic nucleus. Mathematically, the semiempirical mass formula has the form of a series expansion of a function of two variables. If we can find the explicit form of such a function, we should be able to determine a few more terms than the formula we now have. However, the physical significance of such procedures is obscure.

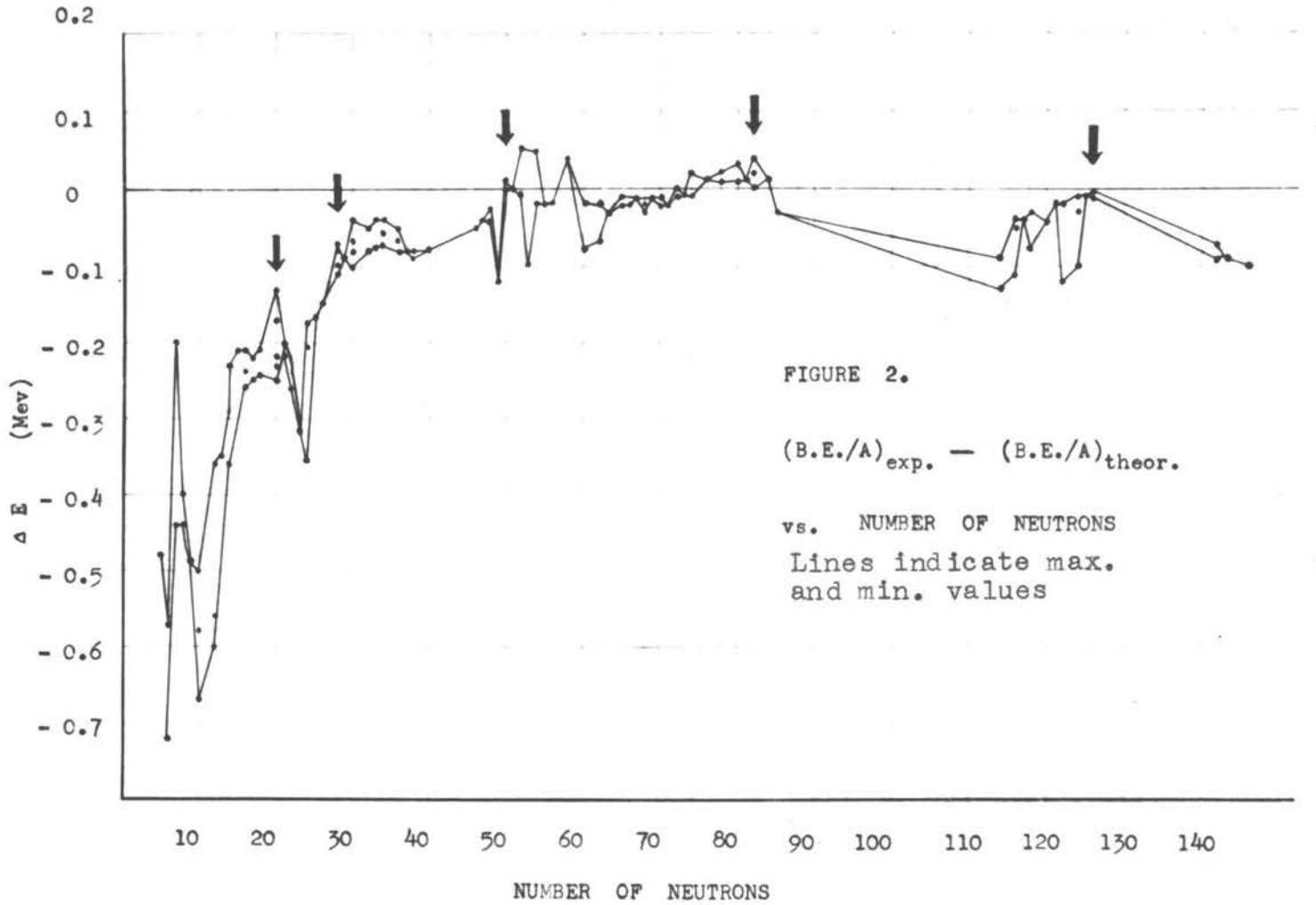


FIGURE 2.

$(B.E./A)_{exp.} - (B.E./A)_{theor.}$

vs. NUMBER OF NEUTRONS

Lines indicate max. and min. values

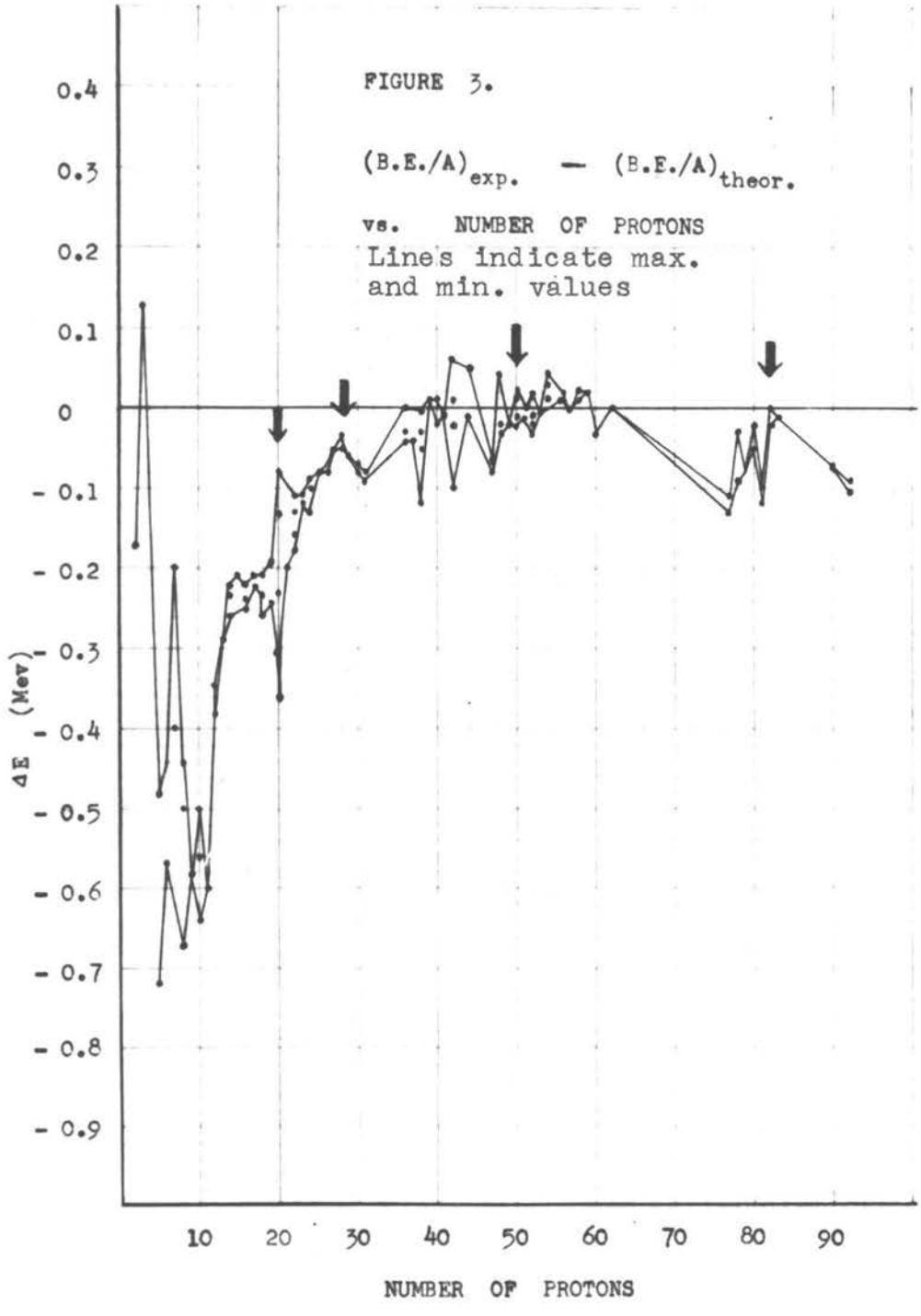


FIGURE 3.
(B.E./A)exp. — (B.E./A)theor.
vs. NUMBER OF PROTONS
Lines indicate max. and min. values
0.4
0.3
0.2
0.1
0
- 0.1
- 0.2
- 0.3
- 0.4
- 0.5
- 0.6
- 0.7
- 0.8
- 0.9
ΔE (Mev)
10
20
30
40
50
60
70
80
90
NUMBER OF PROTONS

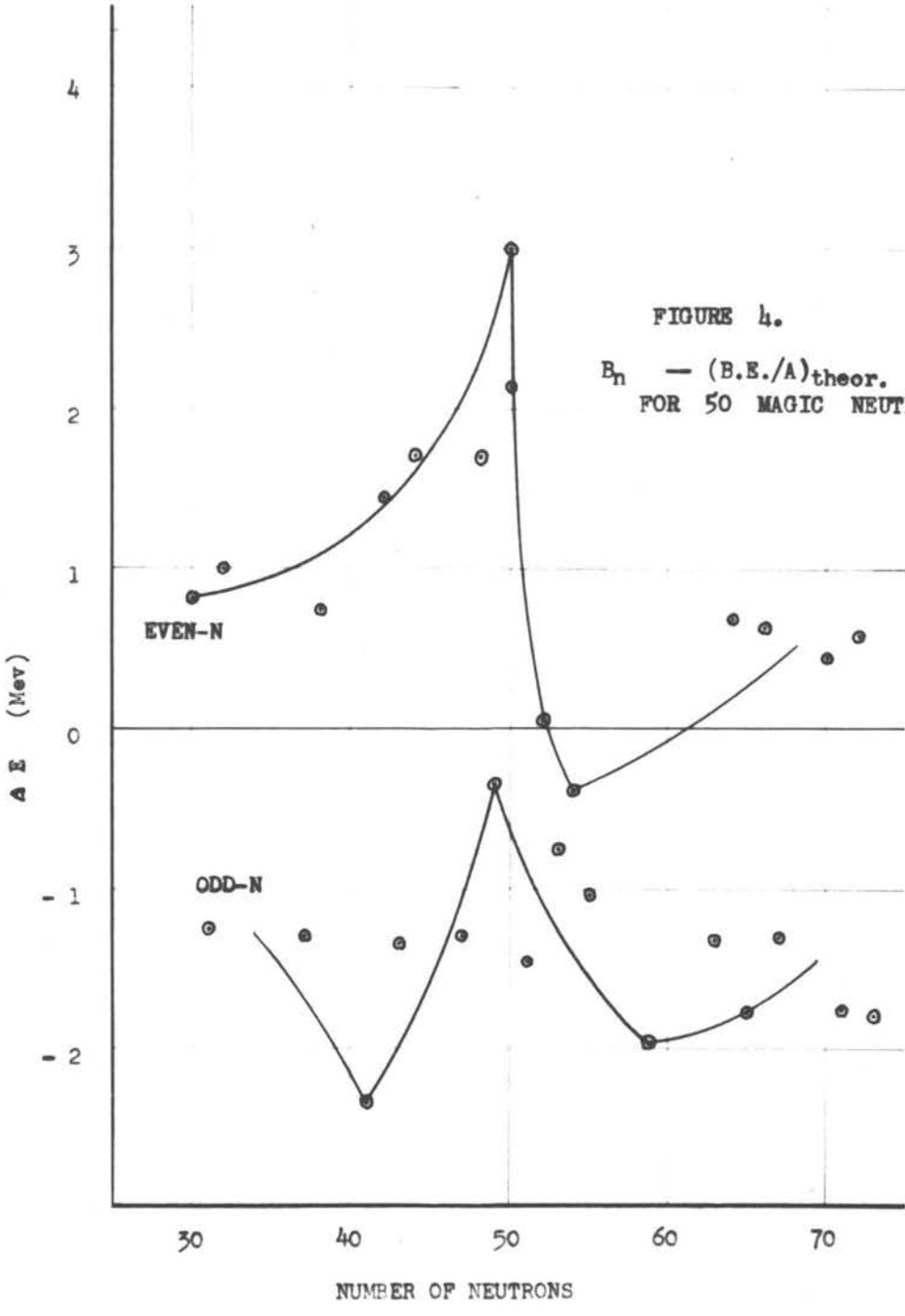


FIGURE 4.

B_n — $(B.E./A)_{theor.}$
FOR 50 MAGIC NEUTRONS

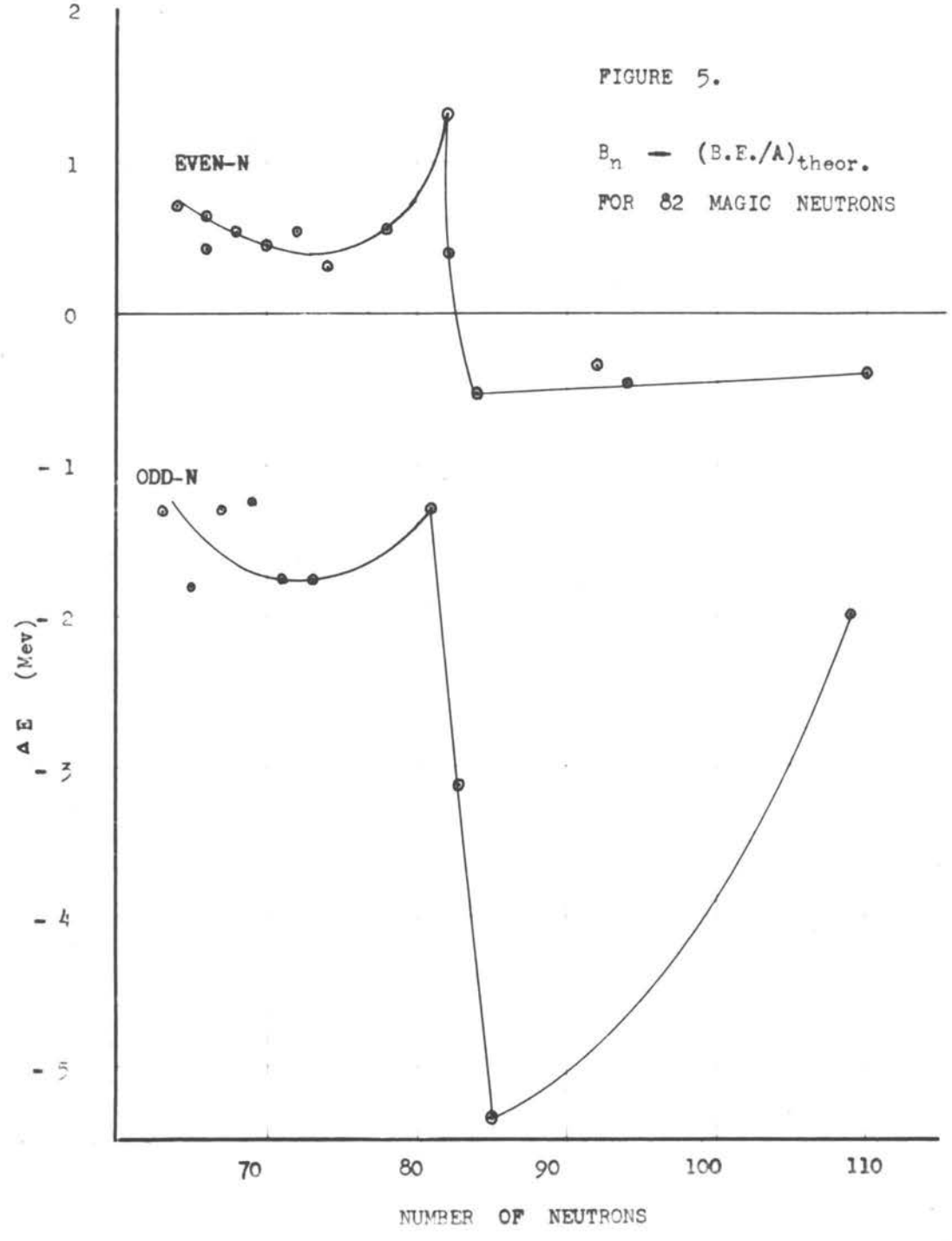


FIGURE 5.

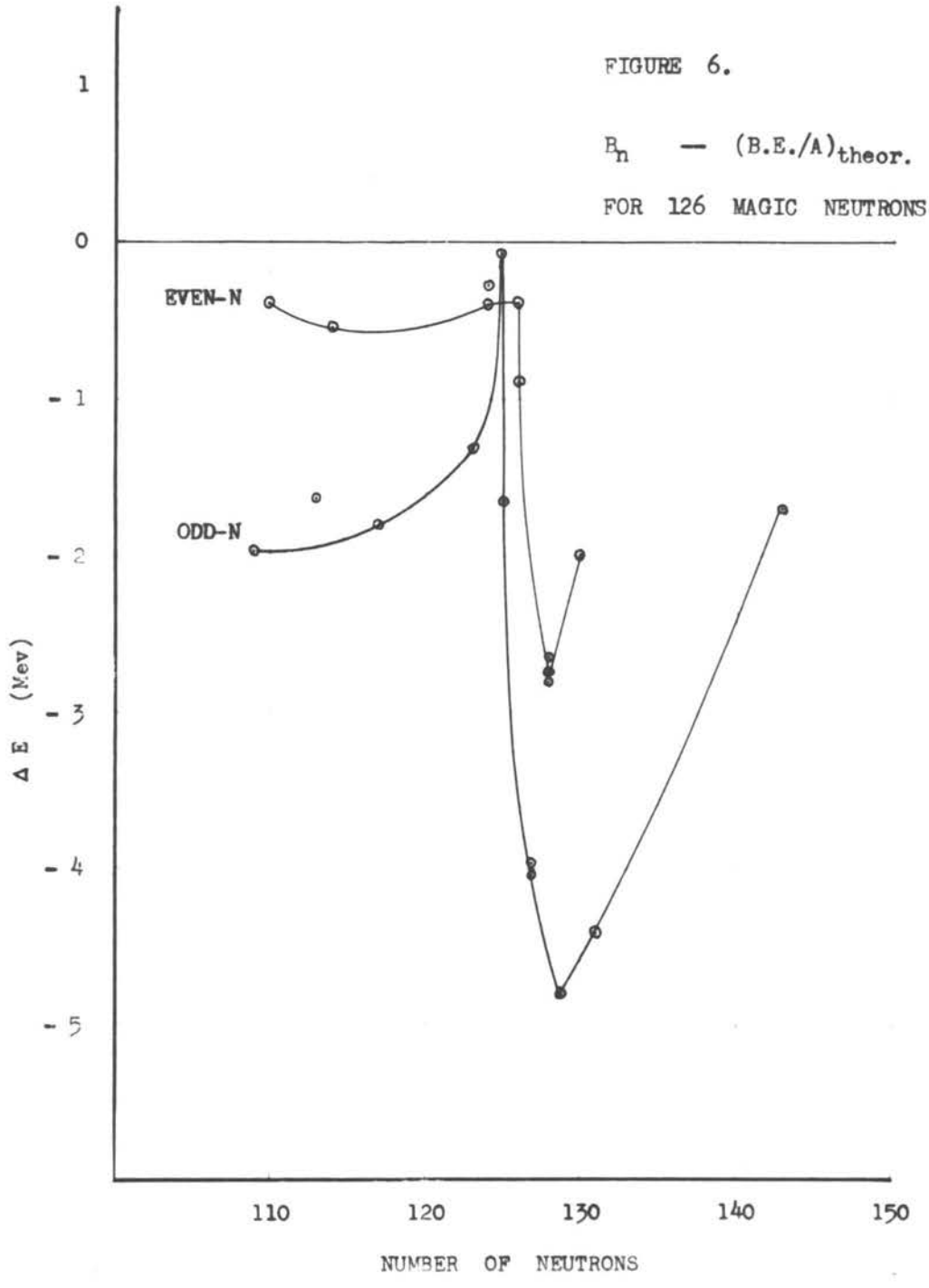
B_n — $(B.E./A)_{theor.}$

FOR 82 MAGIC NEUTRONS

FIGURE 6.

B_n — $(B.E./A)_{theor.}$

FOR 126 MAGIC NEUTRONS



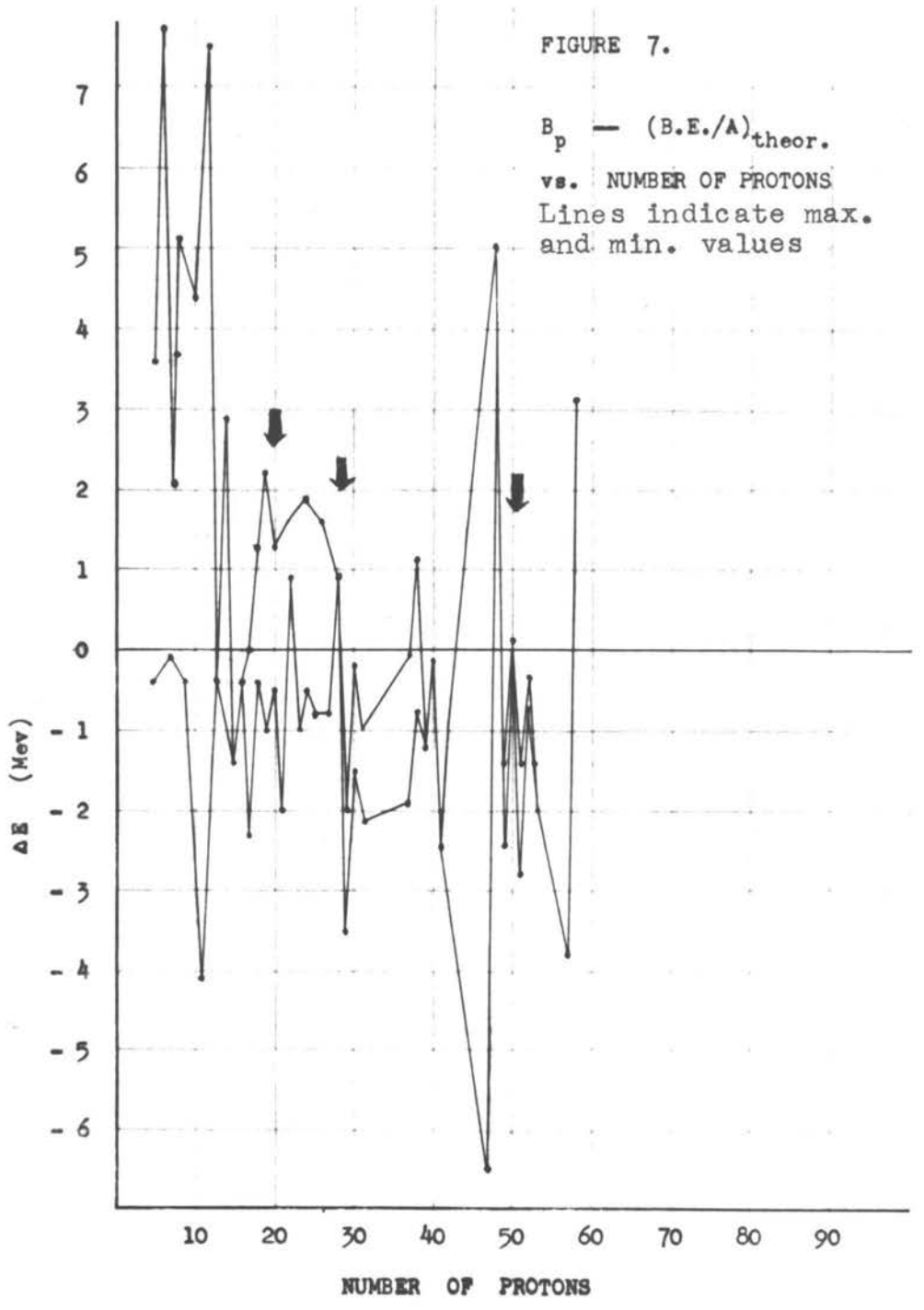


FIGURE 7.

B_p — $(B.E./A)_{theor.}$ vs. NUMBER OF PROTONS

Lines indicate max. and min. values

NUCLEAR BINDING ENERGIES IN MEV

Element	Z	N	A	(BE/A) exp.	(BE/A) theor.	B_p	B_n	(BE/A)exp. - (BE/A)theor.	B_p - (BE/A) theor.	B_n - (BE/A) theor.
H	1	1	2	1.13	-6.08	2.23	2.23	7.22	8.31	8.31
He	2	1	3	2.58	2.74	5.50		-0.17	2.76	
		2	4	7.07	8.76	19.80	20.56	-1.68	11.05	11.81
Li	3	3	6	5.33	5.21			0.13		
		4	7	5.61	6.69		7.25	-1.08		0.56
Be	4	5	9	6.46	7.27	16.87	1.67	-0.81	9.60	- 5.60
B	5	5	10	6.48	6.96	6.59	8.44	-0.48	- 0.37	1.48
		6	11	6.93	7.65	11.22	11.44	-0.72	3.57	3.79
C	6	6	12	7.68	8.25	15.94	18.68	-0.57	7.69	10.43
		7	13	7.47	7.91	17.51	4.96	-0.44	9.60	- 2.95
N	7	7	14	7.47	7.67	7.54	10.51	-0.20	- 0.13	2.84
		8	15	7.70	8.10	10.17	10.83	-0.40	2.07	2.73
O	8	8	16	7.98	8.41	12.15	15.60	-0.44	3.74	7.19
		9	17	7.75	8.25	13.37	4.15	-0.50	5.12	- 4.10
		10	18	7.77	8.44	16.11	8.04	-0.67	7.67	- 0.40
F	9	10	19	7.78	8.36	7.99	10.44	-0.58	- 0.37	2.08
Ne	10	10	20	8.03	8.54	12.88	16.88	-0.50	4.34	8.35
		11	21	7.97	8.61	13.03	6.76	-0.64	4.42	- 1.86
		12	22	8.08	8.64		10.34	-0.56		1.70
Na	11	12	23	7.92	8.52	4.43		-0.60	- 4.09	
Mg	12	12	24	8.26	8.62	16.10		-0.36	7.48	
		13	25	8.23	8.58		7.33	-0.35		- 1.25
		14	26	8.34	8.72		11.11	-0.38		2.39
Al	13	14	27	8.33	8.62	8.23		-0.29	- 0.39	
Si	14	14	28	8.45	8.68	11.57		-0.23	2.90	
		15	29	8.45	8.66		8.48	-0.21		- 0.18

(Continued)

Element	Z	N	A	(BE/A) exp.	(BE/A) theor.	B_p	B_n	(BE/A)exp. - (BE/A)theor.	B_p - (BE/A) theor.	B_n - (BE/A) theor.
		16	30	8.52	8.78		10.60	-0.26		1.82
P	15	16	31	8.48	8.69	7.30		-0.21	- 1.39	
S	16	16	32	8.49	8.71	8.87		-0.22	0.16	
		17	33	8.49	8.71		8.47	-0.22		- 0.24
		18	34	8.58	8.82		11.51	-0.24		2.68
		20	36	8.56	8.81			-0.25		
Cl	17	18	35	8.52	8.73	6.39		-0.21	- 2.33	
		20	37	8.57	8.79	8.77		-0.22	- 0.02	
A	18	18	36	8.52	8.73	8.55	15.35	-0.21	- 0.17	6.62
		20	38	8.61	8.84	10.15		-0.23	1.31	
		22	40	8.59	8.85			-0.26		
K	19	20	39	8.56	8.75	6.51		-0.19	2.23	
		21	40	8.54	8.74		7.92	-0.20		- 0.82
		22	41	8.58	8.82	7.80	9.90	-0.24	- 1.02	1.08
Ca	20	20	40	8.55	8.68	8.15		-0.13	- 0.53	
		22	42	8.61	8.85	10.14		-0.23	1.29	
		23	43	8.60	8.91		8.04	-0.31		- 0.87
		24	44	8.66	9.02		11.41	-0.36		2.39
		28	48	8.67	8.75			-0.08		
Sc	21	24	45	8.62	8.83	6.78		-0.21	- 2.04	
Ti	22	24	46	8.66	8.84	10.48		-0.18	1.64	
		25	47	8.66	8.83	9.72	8.53	-0.17	0.89	- 0.30
		26	48	8.72	8.88		11.75	-0.16		2.87
		27	49	8.71	8.84		7.99	-0.13		- 0.85
		28	50	8.75	8.86		10.98	-0.11		2.12
V	23	27	50	8.69	8.81	7.85		-0.12	- 0.96	
		28	51	8.74	8.84	7.82	10.96	-0.11	- 1.03	2.11

(Continued)

Element	Z	N	A	(BE/A) exp.	(BE/A) theor.	B_p	B_n	(BE/A)exp. - (BE/A)theor.	B_p - (BE/A) theor.	B_n - (BE/A) theor.
Cr	24	26	50	8.70	8.82			-0.13		
		28	52	8.78	8.87	10.80		-0.10	1.93	
		29	53	8.76	8.85	8.38	7.76	-0.09	- 0.47	- 1.08
		30	54	8.77	8.87		9.69	-0.10		0.82
Mn	25	30	55	8.76	8.84	8.04		-0.08	- 0.80	
Fe	26	28	54	8.73	8.80			-0.07		
		30	56	8.79	8.86	10.44		-0.07	1.59	
		31	57	8.77	8.84		7.58	-0.07		- 1.26
		32	58	8.79	8.87		9.85	-0.08		0.98
Co	27	32	59	8.77	8.82	7.99		-0.05	- 0.84	
Ni	28	30	58	8.74	8.78			-0.04		
		32	60	8.79	8.84	9.75		-0.05	0.91	
		33	61	8.79	8.82			-0.04		
		34	62	8.81	8.85			-0.04		
		36	64	8.79	8.84			-0.05		
Cu	29	34	63	8.76	8.81	5.30		-0.06	- 3.51	
		36	65	8.76	8.82	6.84		-0.06	- 1.98	
Zn	30	34	64	8.74	8.81	7.32		-0.07	- 1.49	
		36	66	8.76	8.83	8.64		-0.08	- 0.20	
		37	67	8.74	8.81		7.50	-0.08		- 1.31
		38	68	8.75	8.83		9.57	-0.08		0.74
		40	70	8.73	8.80			-0.08		
Ga	31	38	69	8.72	8.81	6.73		-0.09	- 2.08	
		40	71	8.71	8.80	7.84		-0.08	- 0.96	
Ge	32	41	73		8.79		6.44			- 2.35
		42	74		8.80		10.26			1.46
Se	34	43	77		8.77		7.42			- 1.35

(Continued)

Element	Z	N	A	(BE/A) exp.	(BE/A) theor.	B_p	B_n	(BE/A)exp. - (BE/A)theor.	B_p - (BE/A) theor.	B_n - (BE/A) theor.
		44	78		8.78		10.48			1.70
Kr	36	47	83	8.70	8.74		7.46	-0.04		- 1.28
		48	84	8.72	8.74		10.44	-0.03		1.70
		50	86	8.71	8.71			0		
Rb	37	48	85	8.69	8.73	6.80		-0.04	- 1.93	
		50	87	8.71	8.71	8.62		0	- 0.09	
Sr	38	46	84	8.68	8.73			-0.05		
		48	86	8.71	8.74	9.78		-0.03	1.05	
		49	87	8.70	8.82		8.43	-0.12		- 0.39
		50	88	8.73	8.72	7.88	10.87	0.01	- 0.84	2.15
Y	39	50	89	8.71	8.70	7.46	11.70	0.01	- 1.24	3.00
Zr	40	50	90	8.71	8.70	8.61		0.01	- 0.09	
		51	91	8.70	8.69		7.26	0.01		- 1.44
		52	92	8.70	8.70		8.74	0		0.04
		54	94	8.66	8.68			-0.02		
		56	96	8.63	8.65			-0.02		
Nb	41	52	93	8.67	8.68	6.19		-0.01	- 2.49	
Mo	42	50	92	8.67	8.65		13.10	0.02		
		52	94	8.66	8.60	7.68		0.06	- 0.93	
		53	95	8.65	8.75		8.00	-0.10		- 0.75
		54	96	8.65	8.67		8.27	-0.02		- 0.40
		55	97	8.64	8.66		7.62	-0.02		- 1.03
Ru	44	52	96	8.60	8.61			-0.01		
		54	98	8.62	8.57			0.05		
Pd	46	59	105		8.60		6.65			- 1.95
Ag	47	60	107	8.50	8.58			-0.08		
		62	109	8.51	8.58	2.01		-0.07	- 6.57	

(Continued)

Element	Z	N	A	(BE/A) exp.	(BE/A) theor.	B_p	B_n	(BE/A)exp. - (BE/A)theor.	B_p - (BE/A) theor.	B_n - (BE/A) theor.
Cd	48	58	106	8.54	8.50			0.04		
		60	108	8.55	8.57	13.55		-0.02	4.98	
		62	110	8.55	8.57	12.65		-0.02	4.08	
		63	111	8.53	8.56		7.23	-0.03		- 1.33
		64	112	8.54	8.56		9.23	-0.02		0.67
		65	113	8.52	8.55		6.73	-0.02		- 1.82
		66	114	8.53	8.55		8.96	-0.02		0.41
		68	116	8.51	8.53			-0.02		
In	49	64	113	8.52	8.54	6.10		-0.02	- 2.45	
		66	115	8.52	8.54	7.19		-0.02	- 1.35	
Sn	50	62	112	8.51	8.53			-0.02		
		64	114	8.52	8.53	8.56		-0.02	0.03	
		65	115	8.51	8.53			-0.01		
		66	116	8.52	8.53	8.64	9.18	-0.01	0.105	0.646
		67	117	8.51	8.52		7.20	-0.01		- 1.317
		68	118	8.51	8.52		9.06	-0.01		0.534
		69	119	8.50	8.51		7.03	-0.01		- 1.48
		70	120	8.50	8.50		8.95	0		0.45
		72	122	8.49	8.48			0.01		
		74	124	8.47	8.45			0.02		
Sb	51	70	121	8.48	8.49	5.71		-0.01	- 2.78	
		72	123	8.48	8.47	7.11		0	- 1.36	
Te	52	68	120	8.47	8.50			-0.02		
		70	122	8.48	8.49	8.21		-0.02	- 0.28	
		71	123	8.46	8.48		6.74	-0.02		- 1.74
		72	124	8.47	8.48	7.79	9.11	-0.01	- 0.69	0.63
		73	125	8.46	8.46		6.67	-0.01		- 1.79

(Continued)

Element	Z	N	A	(BE/A) exp.	(BE/A) theor.	B_p	B_n	(BE/A)exp. - (BE/A)theor.	B_p - (BE/A) theor.	B_n - (BE/A) theor.
		74	126	8.46	8.46		8.74	0		0.28
		76	128	8.44	8.43			0.01		
		78	130	8.42	8.40			0.02		
I	53	74	127	8.44	8.45	6.58		-0.01	- 1.87	
Xe	54	78	132	8.43	8.41		8.91	0.01		0.49
		80	132	8.41	8.39			0.03		
		82	136	8.40	8.35			0.04		
Ba	56	80	136	8.40	8.40			0.01		
		81	137	8.39	8.37		7.07	0.01		- 1.30
		82	138	8.39	8.37		8.74	0.02		0.37
La	57	82	139	8.36	8.36	4.61	9.7	0	- 3.76	1.34
Ce	58	82	140	8.38	8.36	11.50	5.28	0.02	3.13	
		83	141		8.35		5.28			- 3.07
		84	142	8.36	8.34			0.01		- 0.62
Pr	59	82	141	8.36	8.35			0.02		
Nd	60	85	145	8.29	8.32		2.97	-0.03		- 5.35
Sm	62	82	144	8.31	8.31			0		
Gd	64	92	156		8.25		7.91			- 0.35
		94	158		8.23		7.81			- 0.42
W	74	109	183		8.05		6.04			- 2.01
		110	184		8.05		7.62			- 0.42
Os	76	113	189		8.09		6.51			- 1.58
		114	190		8.08		8.65			0.56
Ir	77	114	191	7.87	7.99	-11.60		-0.12	-19.59	
		116	193	7.87	7.98	- 7.78		-0.11	-15.76	
Pt	78	114	192	7.90	7.99	14.10		-0.09	6.12	
		116	194	7.94	7.97	21.46		-0.04	13.49	

(Continued)

Element	Z	N	A	(BE/A) exp.	(BE/A) theor.	B_p	B_n	(BE/A)exp. - (BE/A)theor.	B_p - (BE/A) theor.	B_n - (BE/A) theor.
		117	195	7.93	7.96		6.13	-0.03		- 1.83
		118	196	7.93	7.96			-0.03		
		120	198	7.91	7.95			-0.04		
Au	79	118	197	7.87	7.95	- 3.87		-0.08	-11.82	
Hg	80	116	196	7.90	7.95			-0.05		
		121	201	7.90	7.92			-0.02		
		124	204	7.88	7.90			-0.02		
Tl	81	122	203	7.79	7.91			-0.12		
		123	204		7.90		6.54			- 1.36
		124	205	7.80	7.89	9.84	7.48	-0.10	1.94	- 0.41
		125	206		7.88		6.23			- 1.65
		126	207		7.88		6.97			- 0.91
		127	208		7.88		3.86			- 4.02
		128	209		7.86		5.08			- 2.78
		129	210		7.85		3.00			- 4.85
Pb	82	122	204	7.89	7.90	28.72		-0.02	20.82	
		124	206	7.88	7.89	26.40	8.15	-0.01	18.51	0.26
		125	207	7.88	7.88		6.78	-0.01		- 0.10
		126	208	7.87	7.88		7.34	-0.01		- 0.53
		127	209		7.87		3.87			- 4.00
		128	210		7.86		5.20			- 2.66
Bi	83	126	209	7.86	7.87	4.79	7.44	-0.01	- 3.08	- 0.43
		128	211		7.85		5.09			- 2.76
		131	214		7.83		3.36			- 4.47
Po	84	130	214		7.84		5.87			- 1.97
Th	90	142	232	7.63	7.70			-0.07		
U	92	142	234	7.61	7.70			-0.09		

(Continued)

Element	Z	N	A	(BE/A) exp.	(BE/A) theor.	B_p	B_n	(BE/A)exp. - (BE/A)theor.	B_p - (BE/A) theor.	B_n - (BE/A) theor.
		143	235	7.60	7.69		5.95	-0.09		- 1.75
		146	238	7.58	7.67			-0.09		

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