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Title: RESPONSE OF GUYED OFFSHORE TOWERS TO STOCHASTIC LOADS IN THE PRESENCE OF STEADY CURRENT.

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The validity of the widely used stochastic linearization method for analysing the response of guyed towers to stochastic loads is investigated. The governing equations of a guyed tower have two main sources of nonlinearities, fluid-structure interaction and the restoring force of the cables. In this study, two models are considered. First, a linearized single degree of freedom (SDOF) model of a guyed tower is developed using the stochastic linearization approach. It is solved in the frequency domain, giving the statistical response of the guyed tower to random waves and earthquake loading. The results are compared to the response statistics of a time simulation of a single degree of freedom system

that fully incorporates the nonlinearities of the cable system and the fluid-structure interaction.

Second, the dynamic response of a multiple degree of freedom (MDOF) model of a guyed offshore tower to stochastic earthquake loads and steady uniform current is investigated. The nonlinear cable stiffness and the fluid-structure interaction were again linearized using the stochastic linearization method. Numerical results for several load cases are presented and discussed. The displacement statistics from the MDOF linear analysis were compared to the statistics of the SDOF time simulation that fully incorporated the nonlinearities of the structure. The increased damping of the structure with increasing current was found to significantly reduce the stochastic forces and moments of the structure.

The SDOF model is adequate for predicting statistics of displacements, but a MDOF model is needed to accurately predict statistics of maximum moments in the tower.

It is found that the results from the linearized model agree reasonably well with the results from the time simulation of the fully nonlinear system. The computer time using the stochastic linearization approach is several order of magnitude less than for a conventional time simulation method. It is therefore justifiable to use this method in the early design stages of guyed towers.

Response of Guyed Offshore Towers to Stochastic Loads in the Presence of Steady Current.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

RESPONSE OF GUYED OFFSHORE TOWERS TO STOCHASTIC LOADS IN THE PRESENCE OF STEADY CURRENT

CHAPTER I

INTRODUCTION

The need for fossil fuel has forced oil production into deeper waters than ever before. Guyed offshore towers are structures that offer an economical way for oil production in such deep waters. Since they are compliant, they must be designed dynamically. The two principal methods to investigate random dynamic loads on structures are time simulations and frequency response methods. Frequency domain methods are only applicable to linear systems while nonlinearities can easily be included in time simulations. But time simulations are very time consuming and interpretation of the results is tedious.

In this work, which consists of two papers submitted for publication, guyed towers under stochastic loads due to wave action and earthquakes and a uniform steady current are investigated. In chapter II, the validity of the stochastic linearization is investigated by comparing the displacement statistics from a time simulation of a SDOF guyed tower to the displacement statistics of a linearized SDOF system due to various combination of stochastic wave and earthquake loadings. The analytical results are compared to wave tank model tests (Sekita and Maruyanna, 1986) and to a large scale model that was installed in the Gulf of Mexico (Finn and Young, 1978). No experimental data of earthquake loading of guyed tower is currently available in the literature, so only analytical results from those two methods is presented. The results compared fairly well between those two methods as discussed in chapter II.

In chapter III, a MDOF model is developed and its response to stochastic earthquake loads in the presence of a steady uniform current investigated. Guyed towers are fairly soft and are designed to rely upon their dynamic softness to reduce loadings. They will therefore have more modal frequencies in the excitation frequency range of earthquakes than conventional fixed offshore towers. The statistics of the MDOF tower displacement are compared to the displacements of the SDOF model developed in chapter II. The statistics of the forces and moments are then calculated and the influence of higher modes included in the MDOF towers are discussed.

CHAPTER II

RESPONSE OF GUYED OFFSHORE TOWERS TO STOCHASTIC LOADS: TIME DOMAIN VS. FREQUENCY DOMAIN

ABSTRACT

The validity of the widely used stochastic linearization method for analysing the response of guyed towers to stochastic loads is investigated. The governing equation of a guyed tower has two sources of nonlinearities, fluid-structure interaction and the restoring force of the cables. In this study, a linearized single degree of freedom model of a guyed tower is developed using stochastic linearization approach. It is solved in the frequency domain, giving the statistical response of the guyed tower. The results are compared to the response statistics of a time simulation that fully incorporates the nonlinearities of the cable system and the fluid-structure interaction.

It is found that the results from the linearized model agree reasonably well with the results from the time simulation of the fully nonlinear system. The computer time using the stochastic linearization approach is several order of magnitude less than for a conventional time simulation method. It is therefore justifiable to use this method in the early design stages of guyed towers.

INTRODUCTION

The guyed tower is one of the more promising structural concepts for offshore oil production in deep waters. Many of these structures will also be located in seismically active regions. It is therefore essential that their behavior under intense excitations be fully understood. Due to the random nature of the loads, probabilistic methods are more appropriate than deterministic methods for the analysis of guyed towers. Monte Carlo simulations are popular due to their simplicity in implementation. But, extensive computer time is required, huge amounts of data are generated and interpretation of the results is tedious.[1-4] On the other hand, analyses in the frequency domain are not as involved as in the time domain.

The primary aim of this paper is to investigate the validity of the widely used stochastic linearization method for analysing the response of guyed towers to stochastic loads. The governing equation of a guyed tower has two sources of nonlinearities, fluid-structure interaction and the restoring force of the cables. Therefore, it has both nonlinear stiffness and damping. Emphasis is placed upon the structural response under combined wave and earthquake loadings. First, the nonlinear equations of motion are solved in the time domain, fully incorporating the nonlinearities of the problem. Then the equations of motion are linearized to allow solution in the frequency domain using spectral techniques.

A conceptual design[5,6] is shown in Figure 2.1. The tower is long and slender and depends upon a group of guy lines for lateral stability. Each guy line consists of a lead line, a clump weight and a trailing line. The presence of the clump weights limits the maximum tensions in the cables. In survival sea states, the clump weights leave the bottom and the system becomes softer, decreasing the dynamic amplification factor.

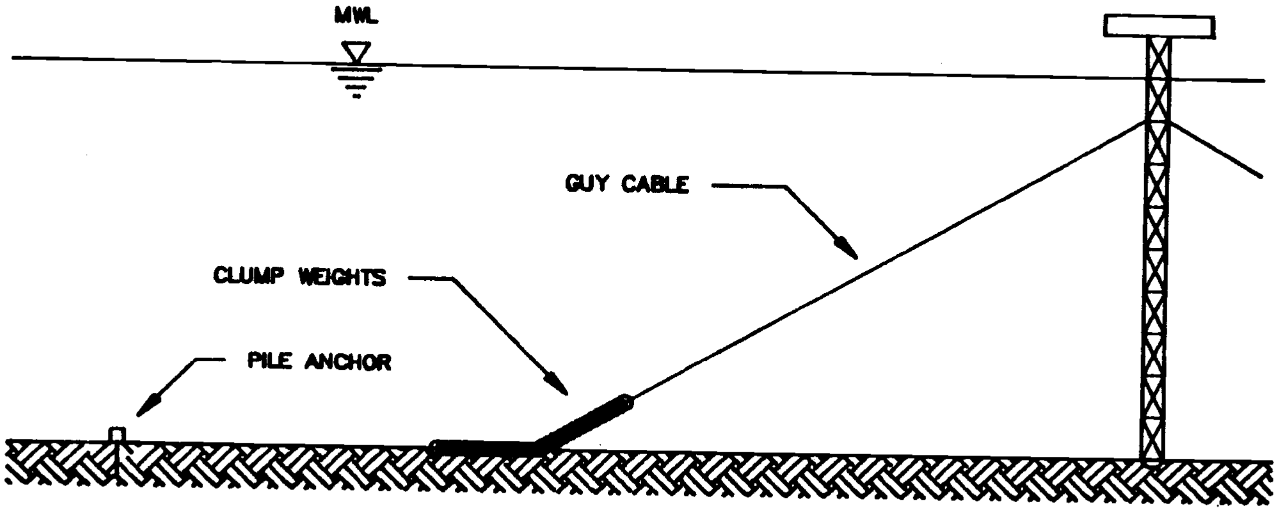

FIGURE 2.1 Compliant guyed tower

A guyed tower was first constructed as a one-fifth scale instrumented model.[7] It was located in 293 feet of water in the Gulf of Mexico in 1978. Results of the model test confirmed the adequacy of linearized analysis[5] of this complex structure as a means of obtaining peak loads. However, test data showed that simplified analysis would not adequately describe the kinematic and dynamic responses of the tower as a function of time.

Several authors have investigated the responses of guyed towers subject to wave and current loading. The well known Morison equation,[8] including relative velocities and accelerations, was used to calculate the wave forces on the structures. Hanna et al.[9] analysed the response of the guyed tower in the time domain, using a normal mode superposition approach. Anagnostopoulos[10] used a similar approach to investigate a fixed offshore structure.

A number of authors[11-14] have noted the importance of dynamic mooring properties in determining the overall (deterministic) tower response. Inclusion of the guyline dynamics prevents overestimation of the maximum deck motions and underestimation of the maximum guyline tension that are apparent in decoupled analysis. Leonard and Young[12] used a nonlinear finite element program with an implicit Newmark's integration method to analyse the coupled response of compliant structures. Wilson and Orgill[15] developed a model of a guyed tower that incorporated the nonlinear stiffness of the cable array and optimized the stiffness characteristics. The results indicated that the peak tower rotations for storm conditions are very sensitive to the damping ratio. Mo and Moan[2] compared various kinematic models for the fluid field in the splashing zone using a Monte Carlo approach. They found that the subharmonic components generated due to the nonlinearities in the drag force and sea elevation formulation could contribute significantly to the structural response. Dutta[16] investigated the the effect of flow dependent drag and inertia coefficients and found it to be marginal. Sekita and Maruyanna[17] did wave tank experiments with a 1/100 scale model. There measurements compared reasonably well with analytical results obtained using regular waves. They conclude that the effect of drag force coefficient is significant only in the area of resonant periods.

Due to the random nature of wave loading, it is appropriate to use a frequency-domain analysis to determine the response of a compliant offshore structure. But frequency-domain analysis is only applicable to linear systems. The method of stochastic linearization (equivalent linearization) is a widely used approximate method for probabilistic analysis of nonlinear structural dynamics problems.[18-23] In the method of stochastic linearization it is assumed that a solution can be obtained from linearized equations. The error in the linearization, i.e. the difference between the linear and the nonlinear equation, is then minimized in a suitable way. The usual choice is to minimize the mean-square error. This procedure gives implicit expressions for the equivalent linear coefficient. Smith and Sigbjornsson[6] and Arockiasamy, et al.[24] evaluated the stochastic structural response of a guyed tower subject to wave action. Smith and Sigbjornsson used two basically different approaches, stochastic linearization and an iterative approach and found them to give almost identical results. Eatock Taylor and Rajagopalan[25] investigated the influence of current on the wave induced dynamic response of offshore structures. They concluded that linearized analysis can lead to markedly different results from a more complete nonlinear analysis and that the method of stochastic linearization appeared to provide nonconservative results. They also concluded that it is important to retain the nonlinear dependence of drag force on wave and current velocity, in evaluating the load spectra of the equivalent linearized structural system. That agrees with results by Gudmestad and Conner[26] who also investigated the effects of current on the non-linear drag force and linearization methods.

The principal methods for analyses of earthquake responses of offshore platforms are time history and response spectrum techniques. A simple, acceptable way to incorporate the randomness of earthquake motions is to model the ground accelerations as a white noise.[27] Penzien, et al.[28] investigated the stochastic response of fixed offshore towers to random sea waves and strong motion earthquakes. They concluded that a stochastic analysis should be carried out to establish wave design loads and suggested that two earthquake intensities should be used for design: one for which the tower remains elastic and another allowing some inelastic deformations to occur. Penzien[29] outlined a method which allowed for the possibility of yielding in the tower structure, and accounts for foundation-structure interaction. Kirkley and Murtha[30] presented a simple method for adopting existing (land based) earthquake response techniques to the analysis of offshore structures.

Nair, et al.[31] investigated the seismic response of a fixed offshore platform, using spectrum and time history analyses. The accuracy of the SRSS (Square Root of the Sum of the Squares) method of modal combination was found to be less than satisfactory in estimating individual peak values of member forces. But the accuracy of the response spectrum technique as a design tool seems to be adequate.

This paper uses a time domain analysis to calculate the overall nonlinear global response to wave and earthquake loading of a guyed tower modeled as a single degree of freedom system. Time domain analysis has the disadvantage of being computationally expensive. An equivalent linear single degree of freedom system was developed. It was solved in the frequency domain, giving the statistical response of the guyed tower with less computer time than conventional time domain methods. A single degree of freedom system was adopted in this work, to minimize the computational effort involved with a multi degree of freedom system and to focus on the comparison of the two methods. Assuming the response of the guyed tower to be stationary and ergodic with respect to certain moments (weakly stationary), the statistics of the response can be calculated from a single time history.[32] The Monte Carlo method will therefore not be used, i.e. it is not necessary to calculate a set of time histories.

SYSTEM MODELING

The structure is assumed to be a rigid truss of length L with a pivot at the sea floor. The motion of the model is confined in a plane. Finn and Young[7] observed that the motion of a 1/5 scale test tower in the gulf of Mexico exhibited elliptical motion in the horizontal plane, but dominant in one direction.

The cables were modeled as nonlinear springs with a static force - displacement curve as shown in Figure 2.2. Several authors[11-14] have noted the importance of incorporating the dynamic mooring properties in determining the tower response and guyline tension. Inclusion of the guyline dynamics prevents overestimation of the maximum deck motion and underestimation of the maximum guyline tension. But since the focus of this work is on the comparison of results from the stochastic linearization method to a fully nonlinear time simulation, exclusion of the guyline dynamics is immaterial to that comparison.

With the deck mass, M_p, lumped at a point and the truss mass, m, distributed uniformly per unit length, the equation of the tower rotation, θ, about the base is derived from consideration of summation of moments on the free body shown in Figure 2.3:

$$I_{twr}\,\ddot{\theta} + C_{twr}\,\dot{\theta} + K_{twr}\,\theta = -\,I_g\,\ddot{X}_g + M_{nl} + M_D + M_I \qquad (1)$$

where

$$I_{twr} = M_p L^2 + M_c z_c^{\,2} + m\,L^3/3 + \rho\,A_{twr} C_a d^3/3 \qquad (2)$$

$$K_{twr} = K_1 z_k + F_b z_b + F_{bt} d^2/2 - M_p g\,L - M_c g\,z_c - mgL^2/2 - F_s z_k \qquad (3)$$

$$I_g = M_p L + M_c z_c + mL^2/2 + \rho\,A_{twr} C_a d^2/2 \qquad (4)$$

$$M_{nl} = K_{nl}(1 - e^{-c_1|\theta|}) \qquad (5)$$

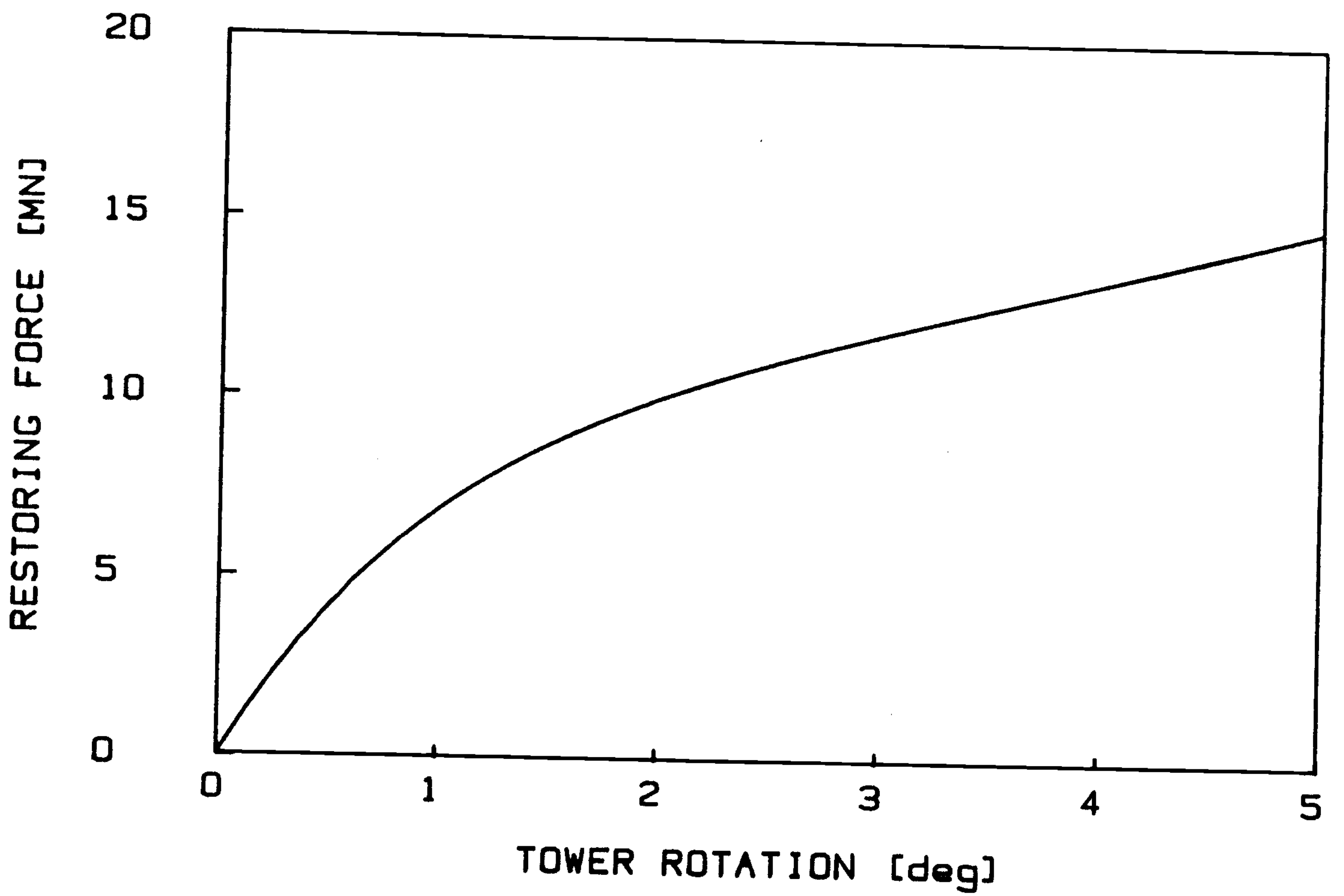

FIGURE 2.2 Restoring force of a cable system as a function of tower rotation for example 3.

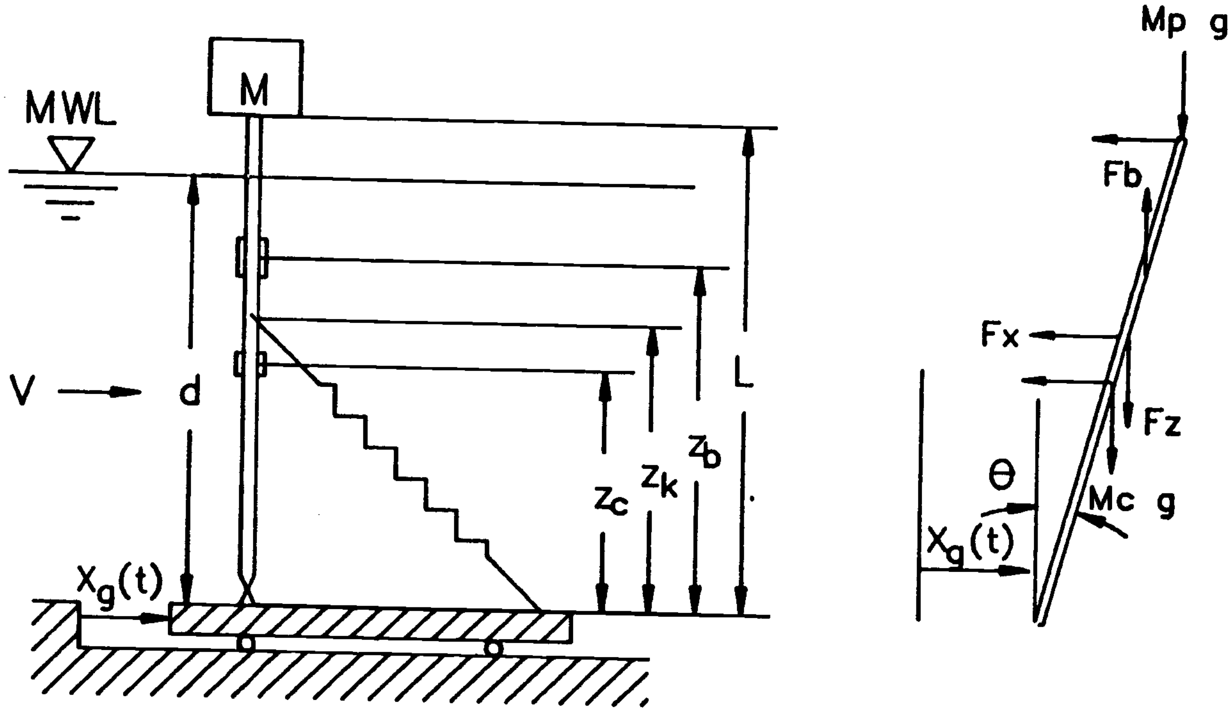

FIGURE 2.3 Guyed tower model and free body diagram

The terms I_{twr}, C_{twr} and K_{twr} are the linear inertia, damping and stiffness of the tower; I_g is an inertia term associated with the ground acceleration $\ddot{X}_g$; M_{nl} is the moment due to nonlinearities in the static cable stiffness; M_D and M_I are the moments due to the fluid velocity and acceleration respectively; M_c is a concentrated mass located at z_c above the sea floor; ρ is the density of sea water; A_{twr} is the equivalent inertial area of the tower; C_a the added mass coefficient; and d is the water depth. The terms K_l and K_{nl} are the horizontal linear and nonlinear stiffness contributions of the cable system; z_k is the location of the guyline attachment to the tower; F_s is the vertical force due to the cable system; and c_1 is a scaling factor for the cable nonlinearities. The term F_b is the vertical force due to buoyancy tanks located at z_b; F_{bt} is the buoyancy force per unit length of the tower; and g is the acceleration of gravity.

HYDRODYNAMIC LOADING

The wave force, dF, on an element of length ds at a location s above the sea floor is calculated using a generalized form of the Morison equation:[33]

$$dF(s) = 0.5\rho\, C_d D_{twr} \left| V(s) + \dot{\beta}(s,t) \right| (V(s) + \dot{\beta}(s,t))\, ds$$

$$+ \rho A_{twr}(1+C_a)\dot{u}(s,t)ds - \rho A_{twr} C_a(\ddot{\theta}(t)s + \ddot{X}_g(t))ds \qquad (6)$$

where

$$\dot{\beta}(s,t) = u(s,t) - \dot{\theta}(t)s - \dot{X}_g(t) \qquad (7)$$

C_d is the drag coefficient; D_{twr} is the equivalent drag diameter of the tower; V(s) is a steady current and u(s,t) is the horizontal particle velocity due to waves. The interaction of waves and current was neglected in this work and the current simply superimposed on the waves. Integrating over the water depth (to the mean water level) gives the moment due to drag and inertia of the fluid and the added mass (included in I_{twr} and I_g shown previously):

$$M_D = 0.5\rho\ C_d D_{twr} \int_0^d \left|V+\dot{\beta}\right| (V+\dot{\beta}) s\ ds \tag{8}$$

$$M_I = \rho\ A_{twr}(1+C_a) \int_0^d \dot{u}\ s\ ds \tag{9}$$

Here, the arguments of V(s), $\dot{\beta}(s,t)$ and $\dot{u}(s,t)$ have been dropped for convenience. The wave kinematics are evaluated using Airy's linear wave theory.[33] Therefore,

$$u(s,t) = \omega \frac{\cosh ks}{\sinh kd} \rho(t) \tag{10}$$

$$= \frac{\cosh ks}{\cosh kd} u(d,t) \tag{11}$$

where ω is the wave frequency and k is the wave number defined by the dispersion equation:[33]

$$\omega^2/g = k \tanh kd \tag{12}$$

Similarly,

$$\dot{u}(s,t) = \frac{\cosh ks}{\cosh kd} \dot{u}(d,t) \tag{13}$$

The moment due to fluid acceleration, M_I, can then be integrated in closed form to give:

$$M_I = \rho A_{twr}(1+C_a) \frac{\dot{u}(d,t)}{k^2 \cosh(kd)} (kd\sinh(kd)-\cosh(kd)+1) \tag{14}$$

The moment due to relative fluid velocity, M_D, in equation (8) can then be calculated numerically by evaluating quantities at discrete stations s_i $(i = 0,\ldots,M)$ in the interval $0 < s < d$ and then integrating numerically by the trapizoid rule using M intervals

$$M_D = 0.5\rho\, C_d\, D_{twr}\Big(\sum_{i=0}^{M-1} 0.5(s_{i+1}-s_{i-1})M'(s_i)$$

$$+0.5(s_M-s_{M-1})M'(s_M)\Big) \qquad (15)$$

where

$$M'(s_i) = \left|V(s_i)+\dot\beta(s_i,t)\right| (V(s_i)+\dot\beta(s_i,t))\, s_i \qquad (16)$$

For random waves the wave amplitude is assumed to be a superposition of simple harmonics. Using a method proposed by Borgman[1,34] and modified by Wilson,[35] one can simulate fluid accelerations and velocities for random waves from Pierson-Moskowitz wave spectra.

The wave height, $\eta(t)$, can be written as

$$\eta(t) = 2\sum_{n=1}^{N}(s(\omega'_n)\delta\omega'_n)^{0.5}\cos(k'_n x - \omega'_n t + \Phi_n) \qquad (17)$$

where $s(\omega')$ is the two sided amplitude spectrum, $\delta\omega'_n = \omega_n - \omega_{n-1}$ and $\omega'_n = (\omega_n - \omega_{n-1})/2$. Φ_n are independent random numbers uniformly distributed over the interval 0 to 2π and k'_n is defined by the dispersion equation:

$$\omega_n'^{2}/g = k'_n \tanh k'_n d \qquad (18)$$

If ω_n are equally spaced between 0 and ω_{max}, then $\eta(t)$ will repeat itself with period $2\pi/\omega_1$. One way of avoiding this periodicity is to select the set of ω_n using the cumulative spectrum:

$$S_c(\omega) = 2 \int_0^{\omega} s(t)dt \tag{19}$$

but, since

$$s(\omega'_n) = (S_c(\omega_n) - S_c(\omega_{n-1}))/2 \tag{20}$$

the periodicity is avoided if the ω_n is chosen such that $(S_c(\omega_n) - S_c(\omega_{n-1}))/2 = a^2$ is constant[1], and ω_n defined as the solution of

$$S_c(\omega_n) = \frac{n}{N} S_c(\omega_{max}) \tag{21}$$

The one sided amplitude spectra proposed by Pierson-Moskowitz is of the form:

$$S(\omega) = \frac{A}{\omega^5} \exp(\frac{-B}{\omega^4}) \tag{22}$$

where

$$A = \alpha g^2 \tag{23}$$

$$B = \beta \left(\frac{g}{U_{wind}}\right)^4 \tag{24}$$

and α=8.1 10^{-3} and β=0.74 are empirically determined constants, g is the acceleration of gravity and U_{wind} is the reference wind velocity. The cumulative spectrum is then:

$$S_c(\omega) = \int_0^{\omega} \frac{A}{\omega'^5} \exp\left(\frac{B}{\omega'^4}\right)d\omega' \tag{25}$$

$$= \frac{A}{4B} \exp\left(\frac{-B}{\omega^4}\right) \tag{26}$$

The amplitude and ω_n are then determined by

$$a^2 = S_c(\infty)/N = A/(4B) \tag{27}$$

$$\omega_n = \left(B/(\ln(N/n)+B/\omega^4_{max}) \right)^{0.25} \tag{28}$$

and

$$\eta(t) = H/2 \sum_{n=1}^{N} \cos(k'_n x - \omega'_n t + \Phi_n) \tag{29}$$

where $H/2 = a\sqrt{2}$. Using Airy's linear wave theory,[33] the horizontal partical velocity $u(x,t)$ and acceleration $\dot{u}(x,t)$ can be calculated from the simulated random surface profile:

$$u = H/2 \sum_{n=1}^{N} \omega'_n \, G(s) \cos(k'_n x - \omega'_n t + \Phi_n) \tag{30}$$

$$\dot{u} = H/2 \sum_{n=1}^{N} \omega'^{\,2}_n \, G(s) \sin(k'_n x - \omega'_n t + \Phi_n) \tag{31}$$

where

$$G(s) = \cosh k'_n s \,/\, \sinh k'_n d \tag{32}$$

Assuming that the free surface elevation is Gaussian, and that linear wave theory is valid, one finds the partical velocity and acceleration to be Gaussian also. Their power spectrum densities are given by:

$$S_{uu}(\omega) = (\omega\, G(s))^2 \, S_{\eta\eta}(\omega) \tag{33}$$

$$S_{\dot{u}\dot{u}}(\omega) = (\omega^2\, G(s))^2 \, S_{\eta\eta}(\omega) \tag{34}$$

Their cross spectral density is zero so the velocity and acceleration are statistically independent.

EARTHQUAKE LOADING

The time history of a ground acceleration during a strong motion earthquake is characterized by a noise-like function. Furthermore, no two earthquakes at the same location are identical. Therefore, ground motions induced by an earthquake may be modeled as a random process. Several methods have been proposed to simulate free surface accelerations.[36,37] The earliest attempts were to simulate the accelerations as a stationary white noise.[38] But an earthquake is nonwhite in that its spectral density is not uniform with respect to frequency. It is also nonstationary since its statistics are time dependent. Kanai[39] and Tajimi[40] initially proposed the widely used filter transfer function:

$$|H_{a_1}(\omega)|^2 = \frac{\omega_g^4 + 4\zeta_g^2\omega_g^2\omega^2}{(\omega_g^2 - \omega^2)^2 + 4\zeta_g^2\omega_g^2\omega^2} \tag{35}$$

were ω_g and ζ_g are the characteristic ground frequency and ground damping ratio. Kanai suggested 5π rad/s for ω_g and 0.6 for ζ_g as being representative for firm ground condition. The power spectral density function for the filtered process is then

$$S_{a_1}(\omega) = |H_{a_1}(\omega)|^2 \, S_o \tag{36}$$

where S_o is constant (PSD of white noise). Nonstationarity can be introduced by multiplying the stationary process by a deterministic envelope function.[41,42,37] The envelope function is used to represent the initial build-up phase and final die-down phase of a realistic earthquake acceleration. Philippacopoulos and Wang[43] presented some seismic inputs that may be used for nonlinear structures. They where generated from recorded earthquakes and produce maximum non-linear response.

A stationary Kanai-Tajimi earthquake spectra was used in this research. It was modified to make the variance of the ground velocity finite.[38] A high pass filter transfer function of the form:

$$|H_{a_2}(\omega)|^2 = \frac{\omega^4}{(\omega_1^2 - \omega^2)^2 + 4\zeta_1^2\omega_1^2\omega^2} \tag{37}$$

is used in this research. The constants ω_1 and ζ_1 are selected to obtain the required filter characteristics. The power spectral density of the final process is then:

$$S_{\ddot{X}g}(\omega) = |H_{a_2}(\omega)|^2 \; |H_{a_1}(\omega)|^2 \; S_o \tag{38}$$

A direct method to obtain the simulated stationary process, $\ddot{X}_g(t)$, is to lump the area under the PSD curve at equal frequency intervals $\delta\omega$ and let these areas equal one half the squared amplitude of a set of discrete harmonics:[38]

$$A_i^2/2 = (S_{\ddot{X}g}(-\omega_i) + S_{\ddot{X}g}(\omega_i))\;\delta\omega \tag{39}$$

so

$$\ddot{X}_g(t) = \Sigma\,(4S_{\ddot{X}g}(\omega_i)\delta\omega)^{0.5}\sin(\omega_i t + \phi_i) \tag{40}$$

where ϕ_i is a random number uniformly distributed between 0 and 2π. This method is similar to the one used to simulate the wave height.

LINEARIZATION

The governing equation, equation (1), contains two nonlinear terms, in the drag, M_D given by equation (15) and in the cable force M_{nl} given by equation (5). In order to use a spectral approach, these two terms need to be linearized. The stochastic linearization technique replaces the original nonlinear system with an equivalent linear system based on a certain optimization criterion. The nonlinear drag in equation (1) is asymmetric, i.e. $M_D(-\dot{\beta}) \neq M_D(\dot{\beta})$. It is assumed that $\theta(t)$ consists of a deterministic (constant) offset, θ_o, and a zero mean random process $\hat{\theta}(t)$, i.e.

$$\theta(t) = \theta_o + \hat{\theta}(t) \tag{41}$$

The way chosen here to linearize equations (5) and (15) is to separately linearize the stiffness and drag terms, such that the expected values of the square of the differences between the linear and nonlinear terms are minimum

$$\delta_{1i} = \left|V+\dot{\beta}_i\right|(V+\dot{\beta}_i) - a_i - b_i\,\dot{\beta}_i \qquad i=1,\ldots,M \tag{42}$$

$$\delta_2 = M_{nl} - c - e\,\hat{\theta} \tag{43}$$

where the arguments of $V(s)$ and $\dot{\beta}(s,t)$ have been dropped for convenience. The terms $\dot{\beta}_i$ and M_{nl} are defined by equations (7) and (5), respectively. The coefficients a_i, b_i, c and e are selected such that

$$\langle \delta_{1i}{}^2 \rangle = \text{minimum} \tag{44}$$

$$\langle \delta_2{}^2 \rangle = \text{minimum} \tag{45}$$

where <...> represents the expected value operator. For a zero mean Gaussian random process, the expected value can be calculated according to

$$\langle f(x)\rangle = \frac{1}{\sigma\sqrt{(2\pi)}} \int_{-\infty}^{\infty} f(x)\ \exp\left(-\frac{x^2}{2\sigma^2}\right)dx \tag{46}$$

were σ is the standard deviation of x.

Minimizing $\langle \delta_{1i}{}^2\rangle$ with respect to a_i and b_i, one obtains

$$\frac{\partial}{\partial a_i}\langle \delta_{1i}{}^2\rangle = 0 \tag{47}$$

$$\frac{\partial}{\partial b_i}\langle \delta_{1i}{}^2\rangle = 0 \tag{48}$$

Hence

$$a_i = \langle\ |V+\dot{\beta}_i|(V+\dot{\beta}_i)\ \rangle \tag{49}$$

$$b_i = \langle\ |V+\dot{\beta}_i|(V+\dot{\beta}_i)\dot{\beta}_i\rangle / \langle\ \dot{\beta}_i{}^2\ \rangle \tag{50}$$

The mathematical expectations can be evaluated analytically to yield, after some manipulations

$$a_i = (\sigma_{\dot{\beta}_i}{}^2 + V^2)\ \mathrm{erf}\left(\frac{V}{\sigma_{\dot{\beta}_i}\sqrt{2}}\right) + \sqrt{(2/\pi)}\quad V\ \sigma_{\dot{\beta}_i}\ \exp\left(\frac{V^2}{2\sigma_{\dot{\beta}_i}{}^2}\right) \tag{51}$$

$$b_i = V\ \mathrm{erf}\left(\frac{V}{\sigma_{\dot{\beta}_i}\sqrt{2}}\right) + \sqrt{(8/\pi)}\quad \sigma_{\dot{\beta}_i}\ \exp\left(\frac{V^2}{2\sigma_{\dot{\beta}_i}{}^2}\right) \tag{52}$$

where $\sigma_{\dot{\beta}_i}$ is the standard variation of $\dot{\beta}_i$. Similarly, for $\langle\delta_2{}^2\rangle$ to be minimum,

$$\frac{\partial}{\partial c} \langle \delta_2{}^2 \rangle = 0 \tag{53}$$

$$\frac{\partial}{\partial e} \langle \delta_2{}^2 \rangle = 0 \tag{54}$$

So

$$c = \langle M_{nl}(\theta) \rangle \tag{55}$$

$$e = \langle M_{nl}(\theta)\ \hat{\theta} \rangle / \langle \hat{\theta}^2 \rangle \tag{56}$$

Substituting the linearized expressions equations (51), (52), (55) and (56), into the governing equation, equation (1) and rearranging, one obtains

$$I_{twr}\ddot{\hat{\theta}} + C_{twr}\dot{\hat{\theta}} + K_{twr}\ \hat{\theta} + K_{twr}\theta_o = -\ I_g\ddot{X}_g + c + e\ \hat{\theta}$$

$$+ C_1\ \Sigma\alpha_i a_i + C_1\ \Sigma\alpha_i b_i \dot{\beta}_i + g(\omega)\ \dot{u}(d,t) \tag{57}$$

where

$$C_1 = 0.5\ \rho\ C_d\ D_{twr} \tag{58}$$

$$\alpha_i = \begin{cases} 0.5(s_{i+1} - s_{i-1})s_i & i < M \\ 0.5(s_m - s_{m-1})s_m & i = M \end{cases} \tag{59}$$

$$g(\omega) = \rho A_{twr}(1+C_a)\ \frac{(kd\sinh(kd)-\cosh(kd)+1)}{k^2\cosh(kd)} \tag{60}$$

Equation (57) has to be satisfied in an average sense. Taking the expected value of equation (57) results in:

$$K_{twr}\theta_o = c + C_l\ \Sigma\alpha_i a_i \tag{61}$$

The mean tower offset becomes

$$\theta_o = (c + C_l\ \Sigma\alpha_i a_i)/K_{twr} \tag{62}$$

Substituting back in the values for $\dot{\beta}_i$ from equation (7) and for θ_o from equation (62), one can write equation (57) as

$$I_{twr}\ddot{\hat{\theta}} + C_{eq}\dot{\hat{\theta}} + K_{eq}\hat{\theta} = - I_g\ddot{X}_g - C_g\dot{X}_g$$

$$+ f(\omega)\ u(d,t) + g(\omega)\ \dot{u}(d,t) \tag{63}$$

where

$$K_{eq} = K_{twr} - e \tag{64}$$

$$C_{eq} = C_l\ \Sigma\alpha_i b_i s_i + C_{twr} \tag{65}$$

$$C_g = C_l\ \Sigma\alpha_i b_i \tag{66}$$

$$f(\omega) = C_l\ \Sigma\alpha_i b_i \cosh(ks_i)/\cosh(kd) \tag{67}$$

The linearized coefficients, K_{eq}, C_{eq} and C_g depend upon θ_o and the variance of $\dot{\beta}_i$ and $\dot{\hat{\theta}}$. The problem is therefore implicit, i. e. values of $\sigma_{\dot{\beta}_i}$ have to be guessed initially, then calculated and compared to the guesses. The variance of $\dot{\beta}_i$ can be calculated several ways. The way chosen here is from the expected value operator

$$\sigma_{\dot{\beta}_i}^2 = \langle (u_i - \dot{\hat{\theta}}\, s_i - \dot{X}_g)^2 \rangle \tag{68}$$

$$= \sigma_{u_i}^2 + s_i \sigma_{\dot{\hat{\theta}}}^2 + \sigma_{\dot{X}g}^2 - 2\, s_i \langle u_i \dot{\hat{\theta}} \rangle - 2 \langle \dot{X}_g \dot{\hat{\theta}} \rangle \tag{69}$$

where $\langle u_i \dot{X}_g \rangle = 0$ since u_i and $\dot{X}_g$ have been assumed to be uncorrelated. The calculation of the first three terms in equation(69) is relatively straight forward.

The variances $\sigma_{u_i}^2$ and $\sigma_{\dot{X}g}^2$ can be calculated according to

$$\sigma_{u_i}^2 = \int_0^{\infty} \omega^2 \frac{\cosh(ks_i)^2}{\sinh(kd)^2} S_{\eta\eta}(\omega) d\omega \tag{70}$$

$$\sigma_{\dot{X}g}^2 = \int_{-\infty}^{\infty} S_{\dot{X}g}(\omega) d\omega = \int_{-\infty}^{\infty} \omega^{-2} S_{\ddot{X}g}(\omega) d\omega \tag{71}$$

where $S_{\eta\eta}(\omega)$ is the Pierson-Moskowitz spectrum and $S_{\ddot{X}g}(\omega)$ is the modified Kanai-Tajimi acceleration spectrum.

The variance of the angular velocity of the tower, $\sigma_{\dot{\hat{\theta}}}^2$ is:

$$\sigma_{\dot{\hat{\theta}}}^2 = \int_{-\infty}^{\infty} S_{\dot{\hat{\theta}}}(\omega) d\omega = \int_{-\infty}^{\infty} \omega^2 S_{\hat{\theta}}(\omega)\, d\omega \tag{72}$$

where the power spectral density of the tower displacement can be obtained using equation (63)

$$S_{\hat{\theta}}(\omega) = |H_1(\omega)|^2 S_{Xg}(\omega) + |H_2(\omega)|^2 S_u(\omega) \tag{73}$$

$$H_1(\omega) = \frac{I_g \omega^2 - j\, C_g \omega}{(K_{eq} - I_{twr}\omega^2)^2 + j C_{eq}\omega} \tag{74}$$

$$H_2(\omega) = \frac{f(\omega) - j\, \omega\, g(\omega)}{(K_{eq} - I_{twr}\omega^2)^2 + j C_{eq}\omega} \tag{75}$$

where $j = \sqrt{(-1)}$.

The two cross terms in equation (69) can be calculated using several approaches. The one chosen here is to use the unit impulse response function. First, consider $\langle u_i \dot{\hat{\theta}} \rangle$. For a stationary processes, it can be shown that

$$\langle u_i \dot{\hat{\theta}} \rangle = - \langle \dot{u}_i \hat{\theta} \rangle \tag{76}$$

From equation (63), the stationary response of $\hat{\theta}(t)$ is given by

$$\hat{\theta}(t) = \int_{-\infty}^{\infty} h(t-\tau)e(\tau)d\tau + \int_{-\infty}^{\infty} h(t-\tau)w(\tau)d\tau \tag{77}$$

where e(t) is the force due to earthquake and w(t) is the force due to wave loads

$$w(t) = f(\omega)\ u(d,t) + g(\omega)\ \dot{u}(d,t) \tag{78}$$

$$e(t) = -I_g \ddot{X}_g - C_g \dot{X}_g \tag{79}$$

Then since e(t) and u(t) were assumed to be independent, we have

$$\langle \dot{u}_i \hat{\theta} \rangle = \int_{-\infty}^{\infty} h(\tau)\langle w(t-\tau)\dot{u}_i(t)\rangle d\tau \tag{80}$$

Equation (80) can be evaluated by making use of the autocorrelation function and integral transforms as follows. The autocorrelation function is defined as:

$$R_{uu}(\tau) = \langle u(t)u(t+\tau)\rangle \tag{81}$$

For a stationary process, ensembled averages are independent of time, thus

$$\frac{dR_{uu}}{d\tau}(\tau) = \langle u(t-\tau)\dot{u}(t)\rangle \tag{82}$$

$$\frac{d^2R_{uu}}{d\tau^2}(\tau) = -\langle\dot{u}(t-\tau)\dot{u}(t)\rangle \tag{83}$$

The autocorrelation function can also be expressed as an inverse Fourier transform of the power spectral density

$$R_{uu}(\tau) = \int_{-\infty}^{\infty} S_u(\omega)e^{j\omega\tau}d\omega \tag{84}$$

$$\frac{dR_{uu}}{d\tau}(\tau) = \int_{-\infty}^{\infty} j\omega\ S_u(\omega)e^{j\omega\tau}d\omega \tag{85}$$

$$\frac{d^2R_{uu}}{d\tau^2}(\tau) = -\int_{-\infty}^{\infty} \omega^2\ S_u(\omega)e^{j\omega\tau}d\omega \tag{86}$$

Therefore

$$\langle u(t-\tau)\dot{u}(t)\rangle = \int_{-\infty}^{\infty} j\omega\ S_u(\omega)e^{j\omega\tau}d\omega \tag{87}$$

$$\langle\dot{u}(t-\tau)\dot{u}(t)\rangle = \int_{-\infty}^{\infty} \omega^2\ S_u(\omega)e^{j\omega\tau}d\omega \tag{88}$$

Substituting equation (78) into equation (80) and using the results from equations (87) and (88), one obtains

$$\langle\dot{u}_i\hat{\theta}\rangle = \int_{-\infty}^{\infty} f(\omega)\frac{\cosh(ks_i)}{\cosh(kd)}\ j\omega S_u(\omega)\int_{-\infty}^{\infty} h(\tau)e^{j\omega\tau}d\omega d\tau$$

$$+\int_{-\infty}^{\infty} g(\omega)\frac{\cosh(ks_i)}{\cosh(kd)}\ \omega^2 S_u(\omega)\int_{-\infty}^{\infty} h(\tau)e^{j\omega\tau}d\omega d\tau \tag{89}$$

The integral involving $h(\tau)$ can be evaluated using an integral transform

$$\int_{-\infty}^{\infty} h(\tau)e^{j\omega\tau}d\tau = \frac{(K_{eq}-I_{twr}\omega^2) + j\omega C_{eq}}{(K_{eq}-I_{twr}\omega^2)^2+(\omega C_{eq})^2} \tag{90}$$

Substituting this into the above equation, noting that $S_u(w)$, $f(\omega)$, $g(\omega)$ and $\cosh(ks_i)$ are even functions with respect to ω and using the relation from equation (76) one obtains

$$\langle u_i\dot{\hat{\theta}}\rangle = C_{eq}\int_{-\infty}^{\infty} f(\omega)\,\frac{\cosh(ks_i)}{\cosh(kd)}\,\frac{\omega^2 S_u(\omega)\,d\omega}{(K_{eq}-I_{twr}\omega^2)^2+(\omega C_{eq})^2}$$

$$- \int_{-\infty}^{\infty} g(\omega)\,\frac{\cosh(ks_i)}{\cosh(kd)}\,\omega^2\,\frac{(K_{eq}-I_{twr}\omega^2)\,S_u(\omega)\,d\omega}{(K_{eq}-I_{twr}\omega^2)^2+(\omega C_{eq})^2} \tag{91}$$

The last cross term in equation (69) can be evaluated similarly:

$$\langle \dot{X}_g\dot{\hat{\theta}}\rangle = -\langle \ddot{X}_g\hat{\theta}\rangle \tag{92}$$

$$\langle \ddot{X}_g\hat{\theta}\rangle = \int_{-\infty}^{\infty} h(\tau)\langle e(t-\tau)\ddot{X}_g(t)\rangle d\tau \tag{93}$$

since $w(t)$ and $X_g(t)$ are uncorrelated

$$\langle \ddot{X}_g\hat{\theta}\rangle = -I_g\int_{-\infty}^{\infty} h(\tau)\int_{-\infty}^{\infty} S_{Xg}(\omega)e^{j\omega\tau}d\omega d\tau$$

$$+ C_g\int_{-\infty}^{\infty} h(\tau)\int_{-\infty}^{\infty} \frac{j}{\omega}\, S_{Xg}(\omega)e^{j\omega x}d\omega d\tau \tag{94}$$

Since the power spectrum is an even function of ω, the cross term can be written as

$$\langle \dot{X}_g \dot{\hat{\theta}} \rangle = I_g \int_{-\infty}^{\infty} \frac{(K_{eq}-I_{twr}\omega^2)S_{Xg}(\omega)d\omega}{(K_{eq}-I_{twr}\omega^2)^2+(\omega C_{eq})^2}$$

$$- C_g C_q \int_{-\infty}^{\infty} \frac{S_{Xg}(\omega)d\omega}{(K_{eq}-I_{twr}\omega^2)^2+(\omega C_{eq})^2} \qquad (95)$$

Now we can evaluate $\sigma_{\dot{\beta}_i}$ in equations (51) and (52): first, assume $\sigma_{\dot{\beta}_i}^2$ and θ_o; next calculate the linearization coefficients, a_i, b_i, c and e and from them, an improved value of $\sigma_{\dot{\beta}_i}^2$ and θ_o; continue this iteration process until satisfactory convergence of $\sigma_{\dot{\beta}_i}$ and θ_o is obtained.

<u>Solution Procedure</u>

The time domain solution was done using a fourth order Runge-Kutta algorithm.[44] The forces along the tower were integrated using a Simpson rule. The tower was assumed to be at rest initially, i.e. $\theta(0) = \dot{\theta}(0) = 0$. The time step was varied according to maximum loading frequency. It was approximately $\delta t=0.1\ T_{min}$ were $T_{min} = 2\pi/\omega_{max}$ and ω_{max} is the maximum frequency of the simulated wave field. It was taken as three times the frequency where the PSD-spectrum is maximum. For a Pierson-Moskowitz power spectrum, that frequency is:

$$\omega_o = 0.877\ g/U_{wind} \qquad (96)$$

The mean and variance of the tower offset was then calculated and compared to the ones obtained by the frequency domain procedure.

Solving the equations in the frequency domain required an iteration on θ_o and $\sigma_{\dot{\beta}_i}$. Initially θ_o was assumed to be 0 and the variance of β_i was approximated by:

$$\sigma_{\dot{\beta}_i}^{2} = \sigma_{u_i}^{2} + \sigma\dot{x}_g^{2} \tag{97}$$

When solving for a range of wind velocities, the previous values of $\sigma_{\dot{\beta}_i}^{2}$ and θ_o were used as initial guesses. Knowing those, the a_i's and b_i's could be calculated from equations (51) and (52), giving C_{eq} and C_g and the steady force due to the current. Then the $\sigma_{\dot{\hat{\theta}}}^{2}$ and the cross terms $\langle u_i \dot{\hat{\theta}} \rangle$ and $\langle \theta \dot{\hat{X}}_g \rangle$ are evaluated by using equations (72), (89) and (95). Those integrations are done with Simpsons rule from 0 to ω_{max} and then the remainder, from ω_{max} to ∞ is integrated by a 15 point Gauss-Lagurre procedure.[44] The maximum frequency, ω_{max}, is three times the frequency of maximum spectral density for the waves.

RESULTS

To validate the solution procedures described in the previous sections the results from the time domain solution and the frequency domain solution were compared to each other and experimental results and examples reported in the literature for validation.

Example 1: Wave Tank Model Test

Sekita and Maruyanna[17] carried out a wave tank model test of a 1/100 scale guyed tower, pin-end connected to the sea bottom. The system properties are listed in Table 2.1. The cable used in the experiment behaved as a hardening spring rather than as a soft-stiff-soft spring

Table 2.1. Tower specifications.

		Example 1	Example 2	Example 3
M_{twr}	kg/m	$2.06\ 10^4$	$2.23\ 10^3$	$3.70\ 10^4$
M_{plf}	kg	0	$6.80\ 10^3$	$6.80\ 10^6$
F_{spg}	N	0	10^5	10^7
F_{btwr}	N/m	0	$1.61\ 10^4$	$2.92\ 10^5$
L_{twr}	m	200	100	480
L_{spg}	m	140	84	442
K_1	N/rad	$7.620\ 10^7$	$8.40\ 10^6$	$5.79\ 10^8$
K_{nl}	N/rad	$1.905\ 10^7$	$1.18\ 10^7$	$-4.34\ 10^8$
ζ_{twr}		0.05	0.03	0.01
D_{twr}	m	8.0	6.80	35.0
A_{twr}	m^2	12.6	1.41	26.3
C_a		1.0	1.0	1.0
C_d		1.0	0.6	0.7
ω_n	rad/s	0.302	0.576	0.234
T_n	s	20.8	10.9	26.9

characteristic of guyline cables. The results are presented as the ratio of the maximum tower offset at location 180 m above the bottom (full scale values used as in reference 17) to the wave height vs. wave period for regular waves (single wave). For random waves, the ratio of the standard variation of the tower offset to twice the standard variation of the wave amplitude is used:

$$X/H = 180 \frac{\sigma_\theta}{2\sigma_\eta} \tag{98}$$

where

$$\sigma_\eta^{\,2} = \int_0^\infty S_{\eta\eta}(\omega)\, d\omega = \alpha U_{wind}^{\;\;4}/(4\beta g^2) \tag{99}$$

$S_{\eta\eta}(\omega)$ is the Pierson-Moskowitz spectrum as defined by equation (22) and σ_θ is the standard deviation of the tower rotation. The period of the significant wave height can be estimated as suggested by Bretschneider:[45]

$$T_s = (4/5)^{0.25}\, 2\pi/\omega_o \tag{100}$$

were ω_o is the frequency where the PSD is maximum. The surge response of the tower to a 10 m high waves are plotted in Figure 2.4. As can be seen, there is close agreement between the analytical solutions and the experimental results for wave periods away from the natural period of the structure. But realistic guyed towers will be designed to have the first natural frequency lower than the main frequency of the waves. The results near the natural frequency of the tower are very sensitive to the drag coefficient used. A damping ratio of 5% of the critical value was used in addition to the hydro-

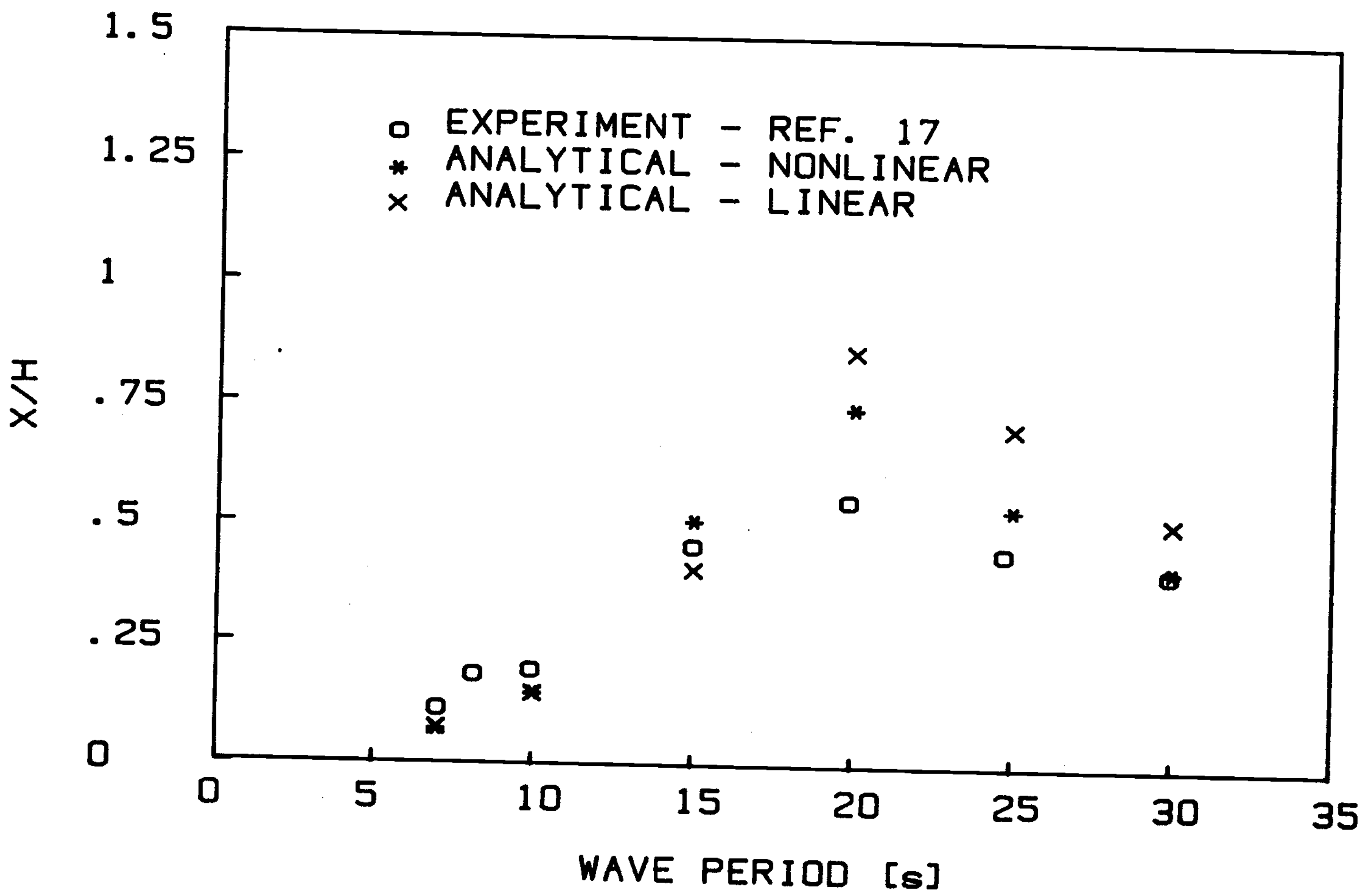

FIGURE 2.4 Surge response of example guyed tower # 1 to a single wave, H=10 m.

dynamic damping to account for the structural damping of tower and the damping due to the guylines. Figure 2.5 shows the results for random waves. The frequency domain solution compares very well with the experimental results.

Example 2: Large Scale Model

Finn and Young[7] discussed tests results of a large scale model of a guyed tower installed in the Gulf of Mexico. They reported measured tower motions from two major storms. The significant wave heights from those two storms were 2.7 m (8.9 ft) and 2.2 m (7.2 ft). Figure 2.6 shows a simulated wave profile with significant wave height equal to 2.2 m. The significant wave height is calculated as:

$$H_s = 4\ \sigma_\eta = 2\ (\alpha/\beta)^{0.5} U_{wind}{}^2/g \tag{101}$$

Significant wave height of 2.2 m corresponds to a 10.1 m/s wind. The specifications of the test tower are listed in Table 2.1. Time simulation of the tower response to that wave record is shown in Figure 2.7. The maximum displacement of the tower was measured[7] as 0.69 m which compares reasonably well with the 0.58 m response obtained with the time simulation. Note that the same wave profile was not used, only one with the same significant wave height. Wishahy and Arockiasamy[46] analysed this tower using the finite element method. By using the same wave profile as reported by Finn and Young[7] (H_s=2.2 m), they computed maximum deck displacements of 0.65 m and 0.56 m for beam and truss element models respectively. The standard deviation of the tower offset was calculated using the frequency domain method. The results are shown in Figure 2.8. The standard deviation

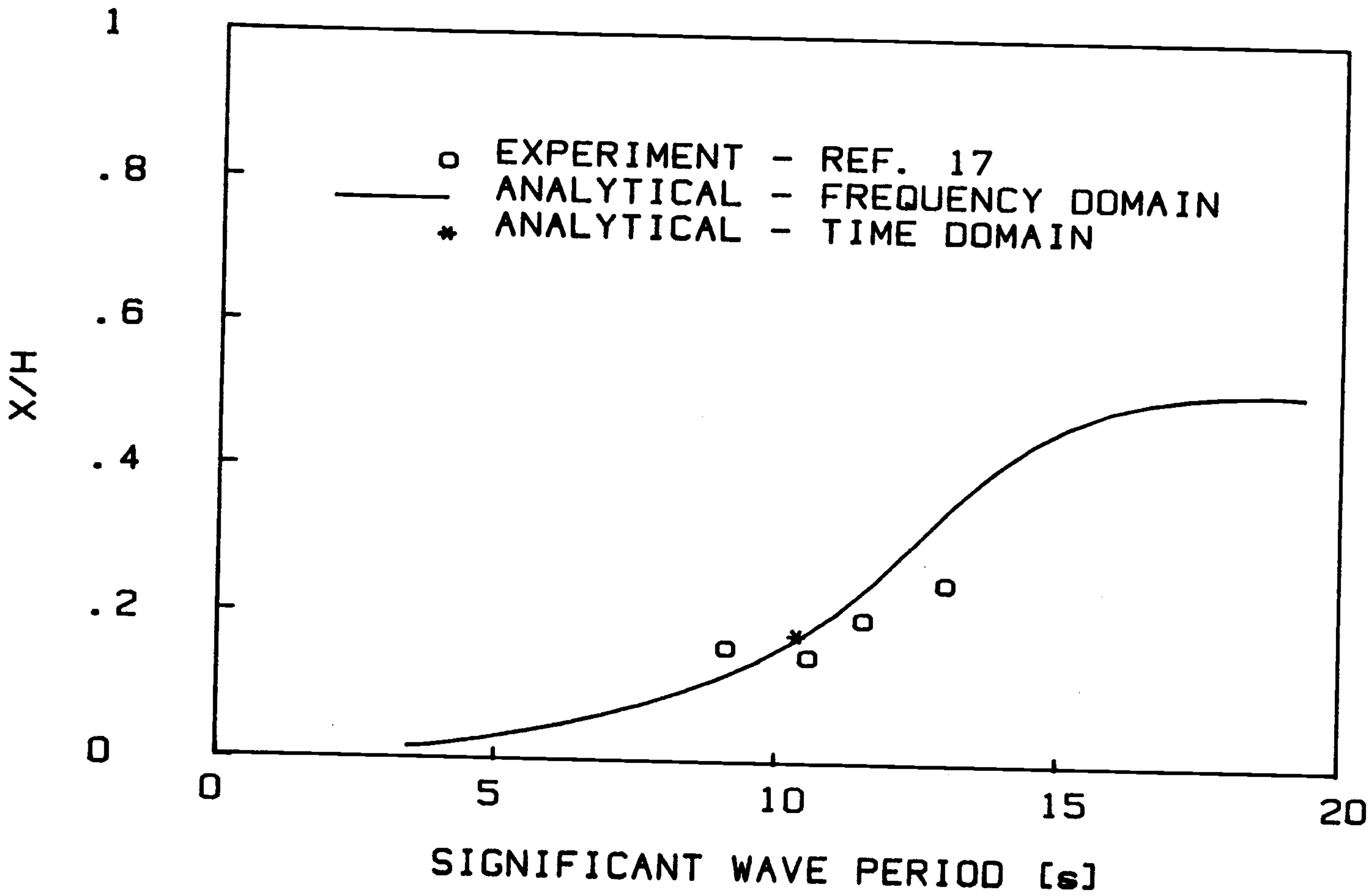

FIGURE 2.5 Surge response of example guyed tower # 1 to random waves.

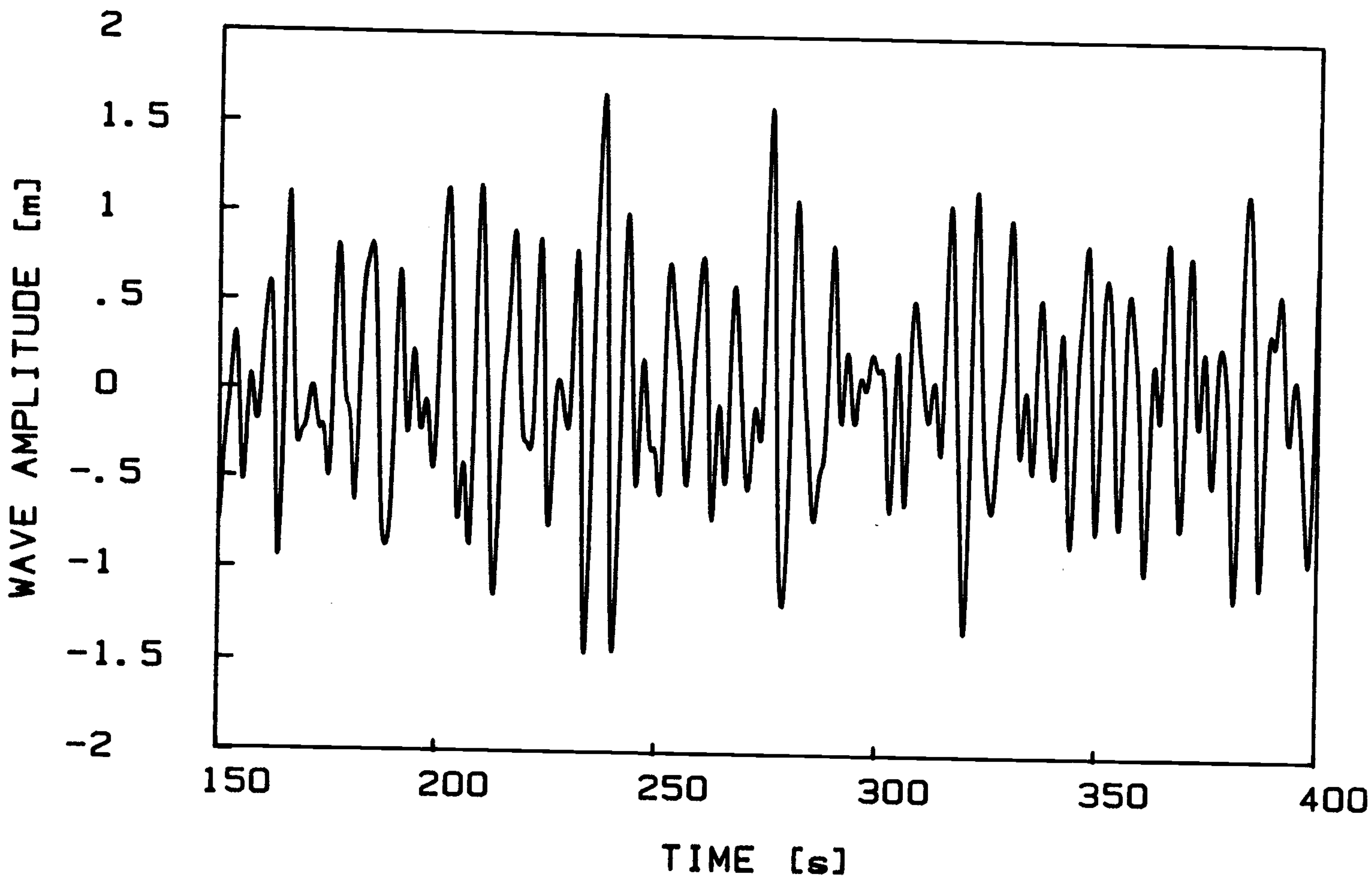

FIGURE 2.6 Simulated wave amplitude record, H_s = 2.2 m for example guyed tower # 2.

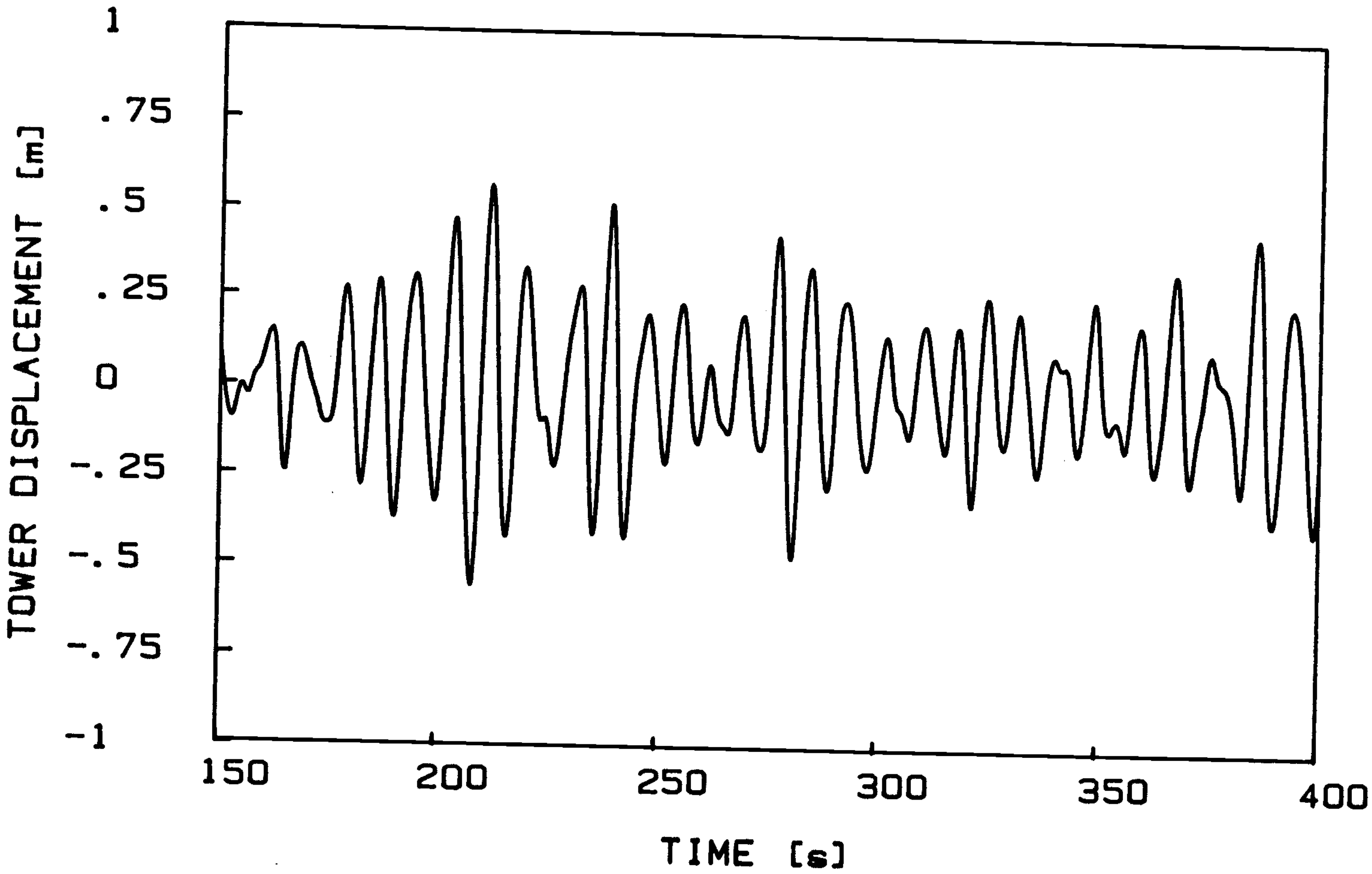

FIGURE 2.7 Response of example guyed tower # 2 to a simulated wave record with H_s=2.2 m.

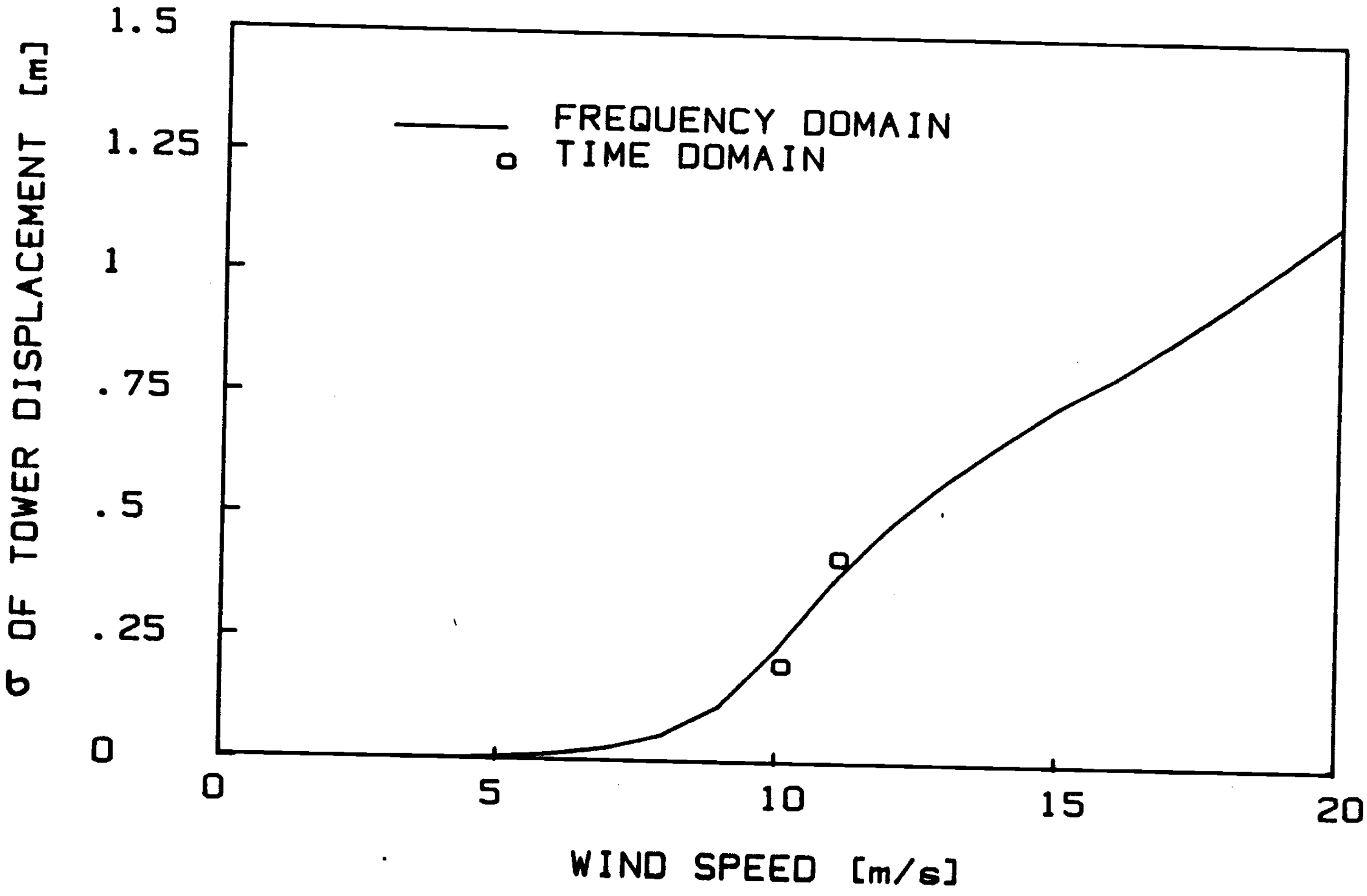

FIGURE 2.8 Standard deviation of deck displacment for example guyed tower # 2.

from time simulations with significant wave heights 2.2 m and 2.7 m (corresponding to wind velocity of 10.1 m/s and 11.3 m/s) are also shown in the figure. Estimating the maximum displacement as $3\sigma_x$, the maximum displacement calculated by the frequency domain method becomes 0.57 m and 1.2 m compared to 0.58 m and 1.1 m as obtained by the time simulation.

Example 3: Guyed Tower in 457 m Water Depth:

The two methods were also compared using a guyed tower model similar to the one used by Finn[5] and Dutta.[16] The response of this tower to waves, earthquakes, current and combination of these loads was investigated. The parameters used to specify the earthquake spectrum defined by equations (35), (37) and (38) where ω_g = 15.7 rad/s and ζ_g = 0.6 as suggested by Kanai[39] for firm ground condition; ω_1 = 0.4 rad/s and ζ_1 = 0.9; S_o = 0.004267 m^2/s^3 (0.04593 ft^2/s^3) which corresponds to a class of earthquakes having an average intensity similar to the N-S component of the 1940 El Centro earthquake.[47] The frequency, ω_1, and damping, ζ_1, were selected such that the standard deviation of the ground velocity would be approximately 0.14 m/s. Then the maximum ground velocity can be approximated as

$$\dot{X}_{g,max} = 3\,\sigma_{\dot{X}g} \tag{103}$$

$$= 0.42 \text{ m/s (17 in/s)} \tag{104}$$

which is the maximum ground velocity of the N-S component of the 1940 El Centro earthquake obtained by Berge and Housner[48] by integrating the acceleration record. The power spectral densities of the ground acceleration and velocity are shown in Figures 2.9 and 2.10, respectively.

Figures 2.11 and 2.12 show the standard deviation of the tower rotation with and without an earthquake loading. The frequency domain solution compares very well with the time simulation for both cases in the absence of current. When current is superimposed on the waves, the standard deviation of the response decreases in the frequency domain solution, but increases slightly in the time domain solution. This discrepancy may be due to overestimation of the drag coefficient in the frequency domain. Adding the current will significantly increase the damping ratio of the system, as can be seen in Figure 2.13. Also, for high wind speeds and current, the nonlinearities in the system increase. For wind speeds above 20 m/s, the wave forces are dominant and the effects of the earthquake and current on the standard deviation of the response are marginal. The mean tower offset has been plotted in Figure 2.14. It is clear that the results obtained by the two approaches are in close agreement. The softening effect of the nonlinear cable system can be seen in Figure 2.15. The natural frequency decreases for higher wind speeds since the amplitude of the tower increases. That is the advantage of guyed towers, since the frequency of the peak wave energy approaches the natural frequency of the tower for high wind speeds. The natural frequency of the tower moves therefore away from the peak energy in the waves. The earthquake lowers the natural frequency slightly, while the current has major effects.

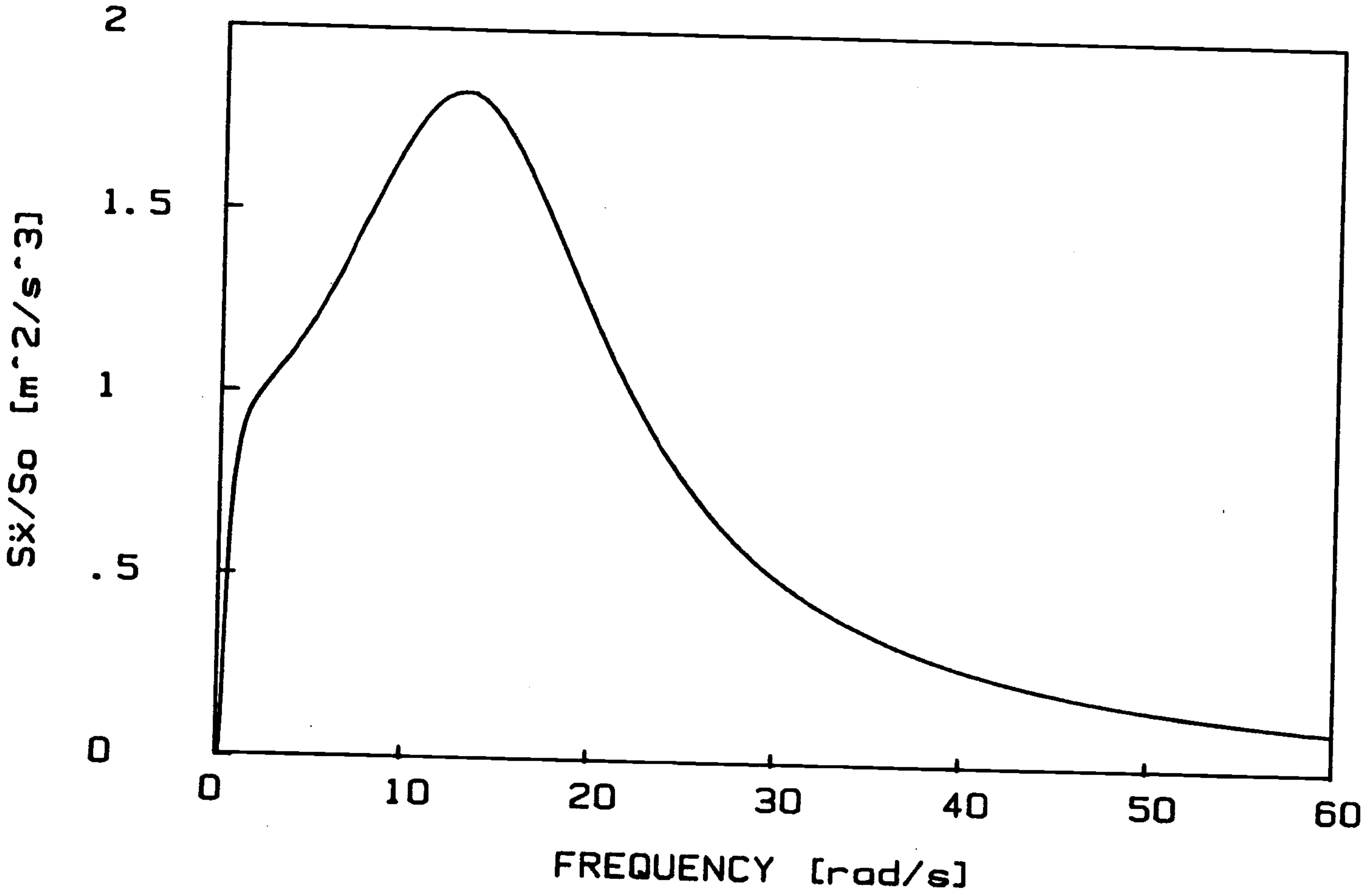

FIGURE 2.9 Power spectral density of ground acceleration, ω_g = 15.7 rad/s, ζ_g = 0.6, ω_1 = 0.4 rad/s and ζ_1 = 0.9. For example guyed tower # 3.

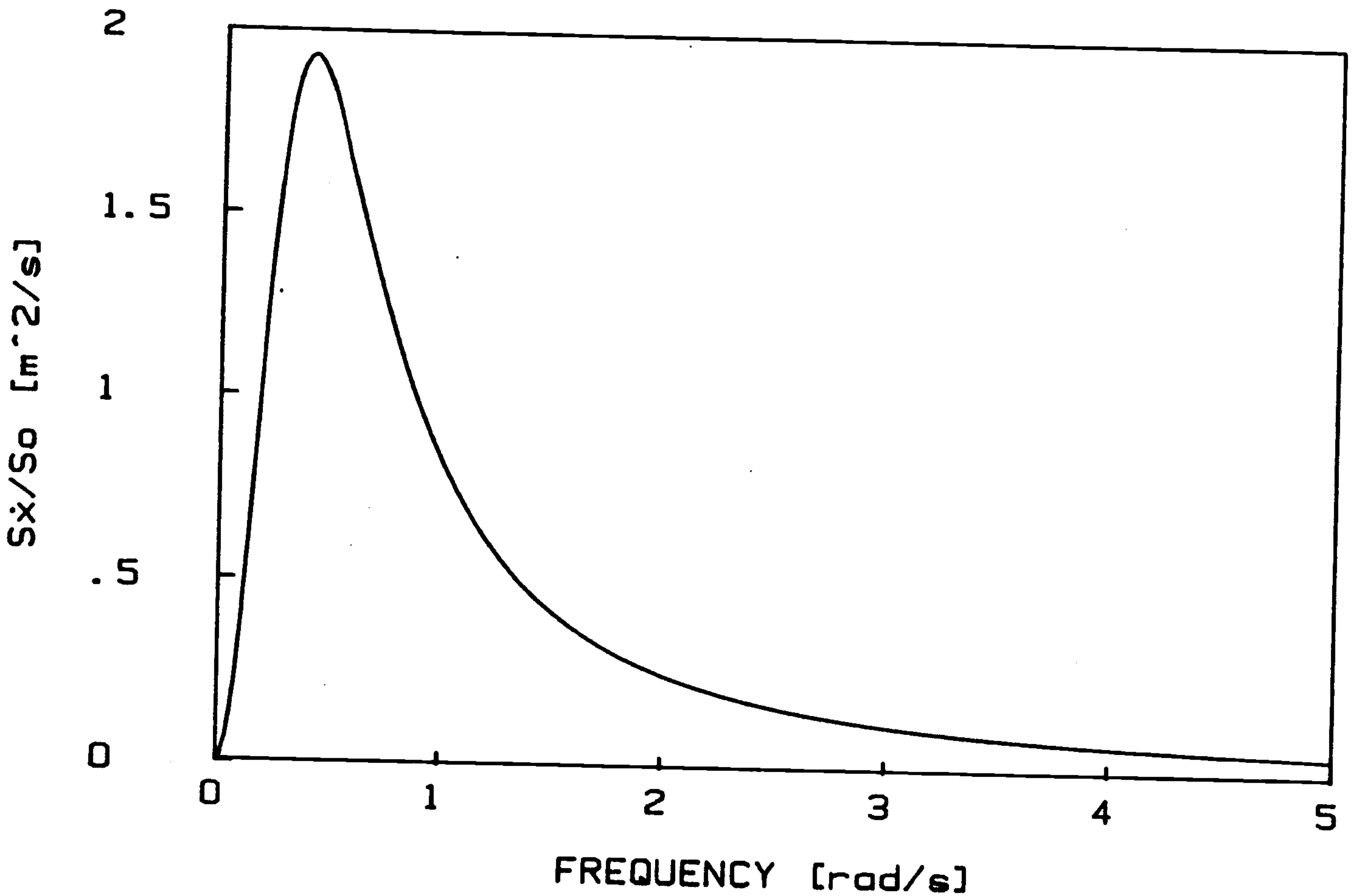

FIGURE 2.10 Power spectral density of ground velocity, $\omega_g = 15.7$ rad/s, $\zeta_g = 0.6$, $\omega_1 = 0.4$ rad/s and $\zeta_1 = 0.9$. For example guyed tower # 3.

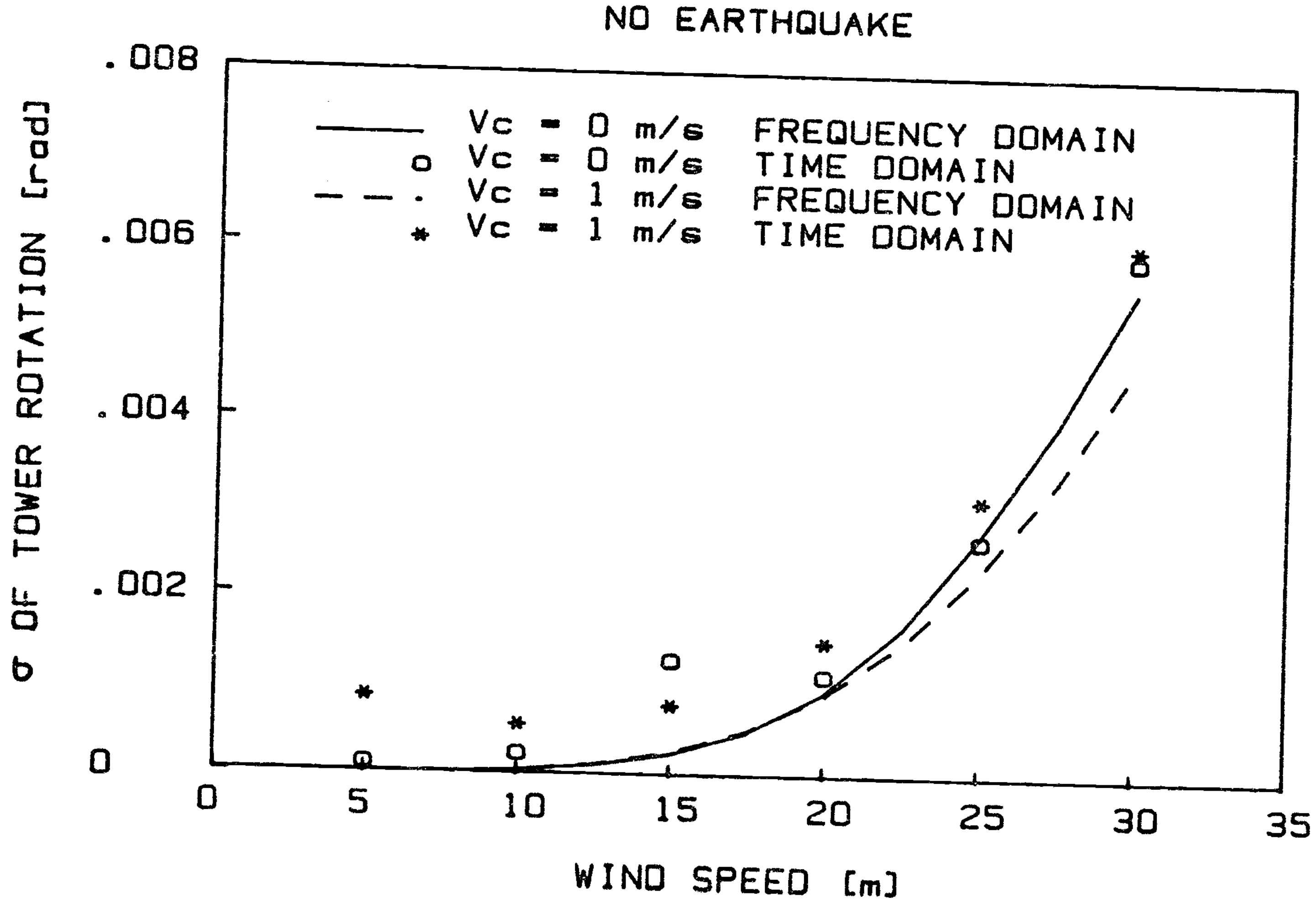

FIGURE 2.11 Standard deviation of rotation for example guyed tower # 3 without earthquake loads.

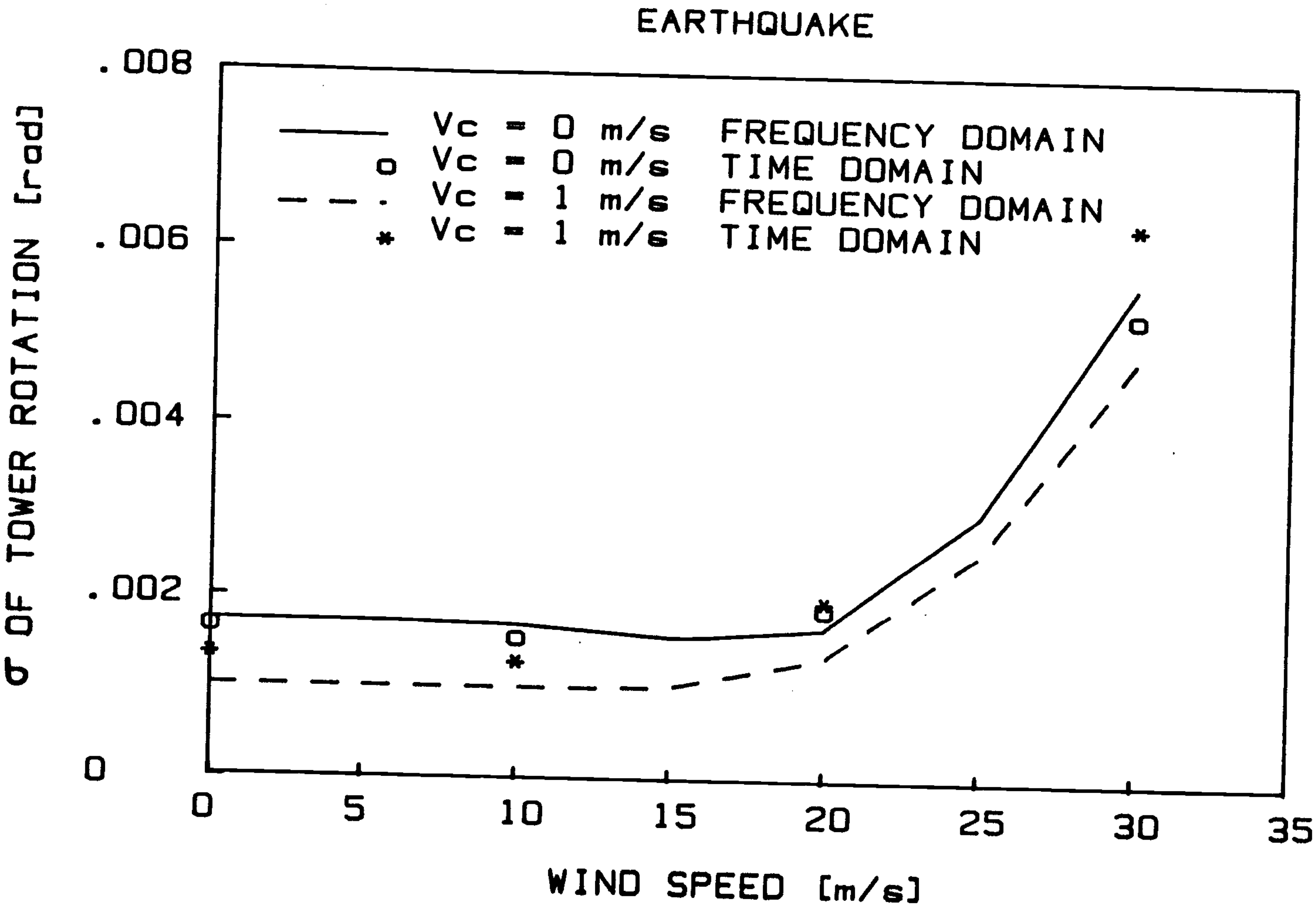

FIGURE 2.12 Standard deviation of rotation for example guyed tower # 3 with earthquake loads.

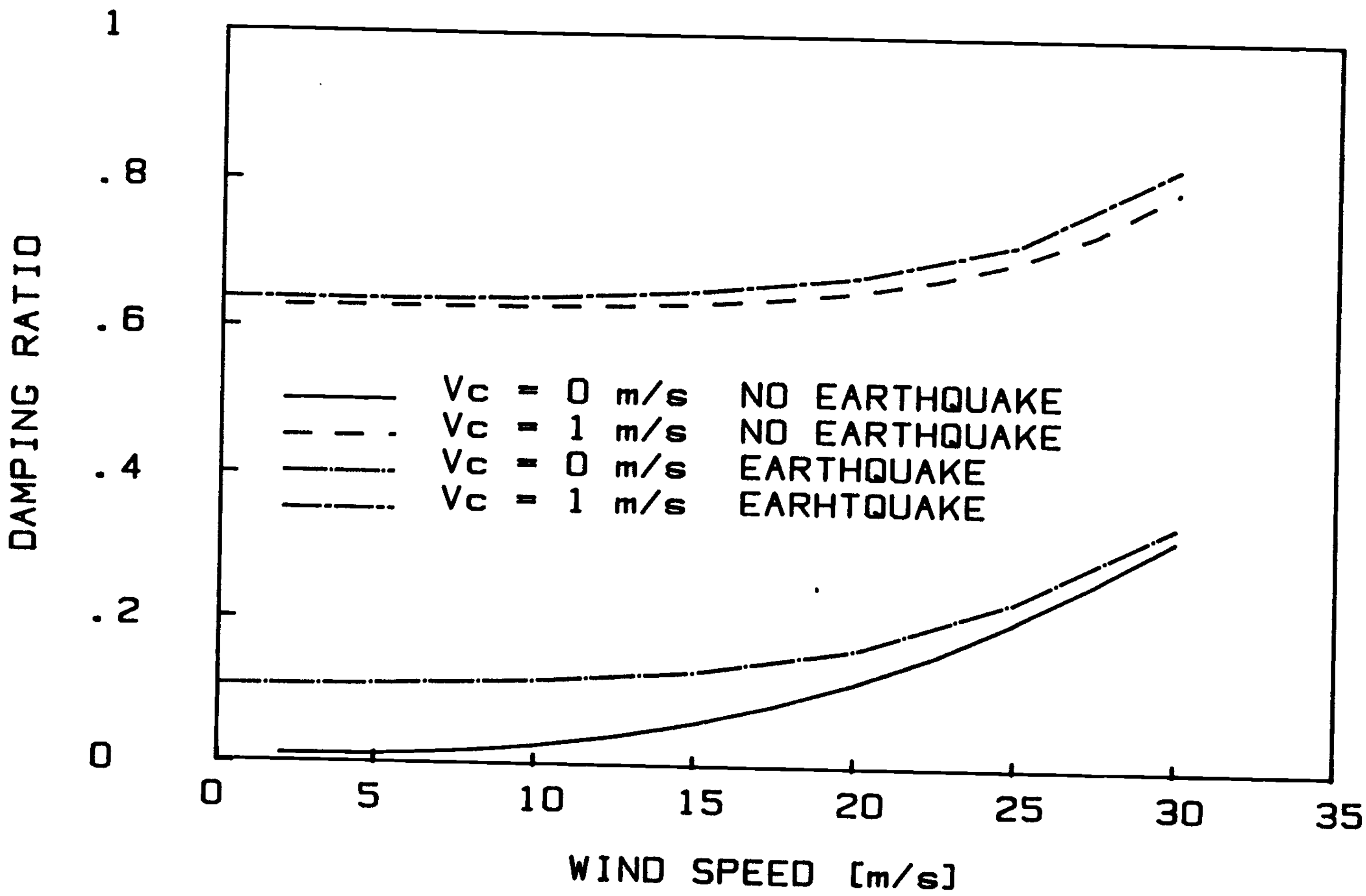

FIGURE 2.13 Damping ratio of linearized example guyed tower # 3, frequency domain

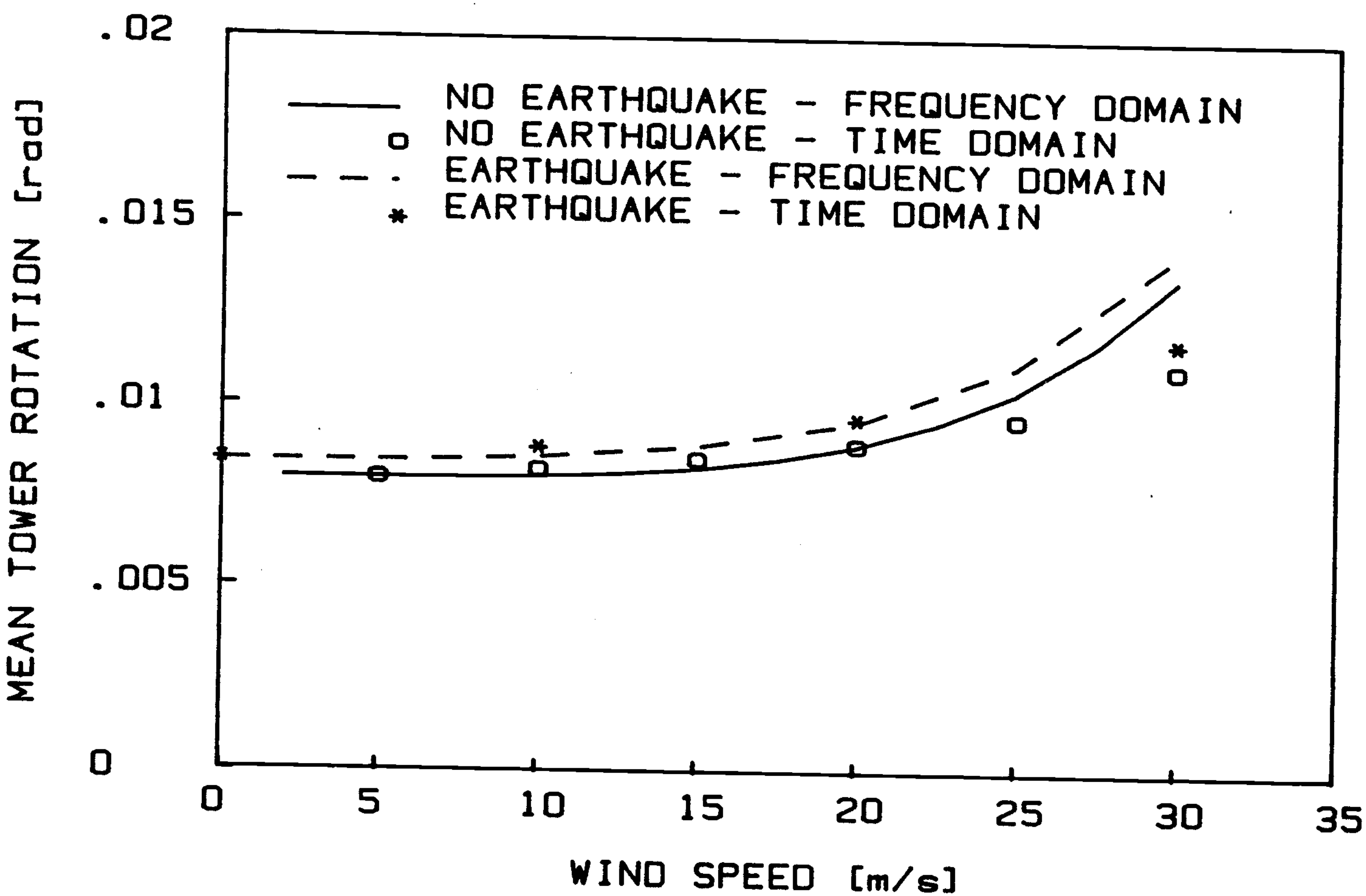

FIGURE 2.14 Mean offset of linearized example guyed tower # 3.

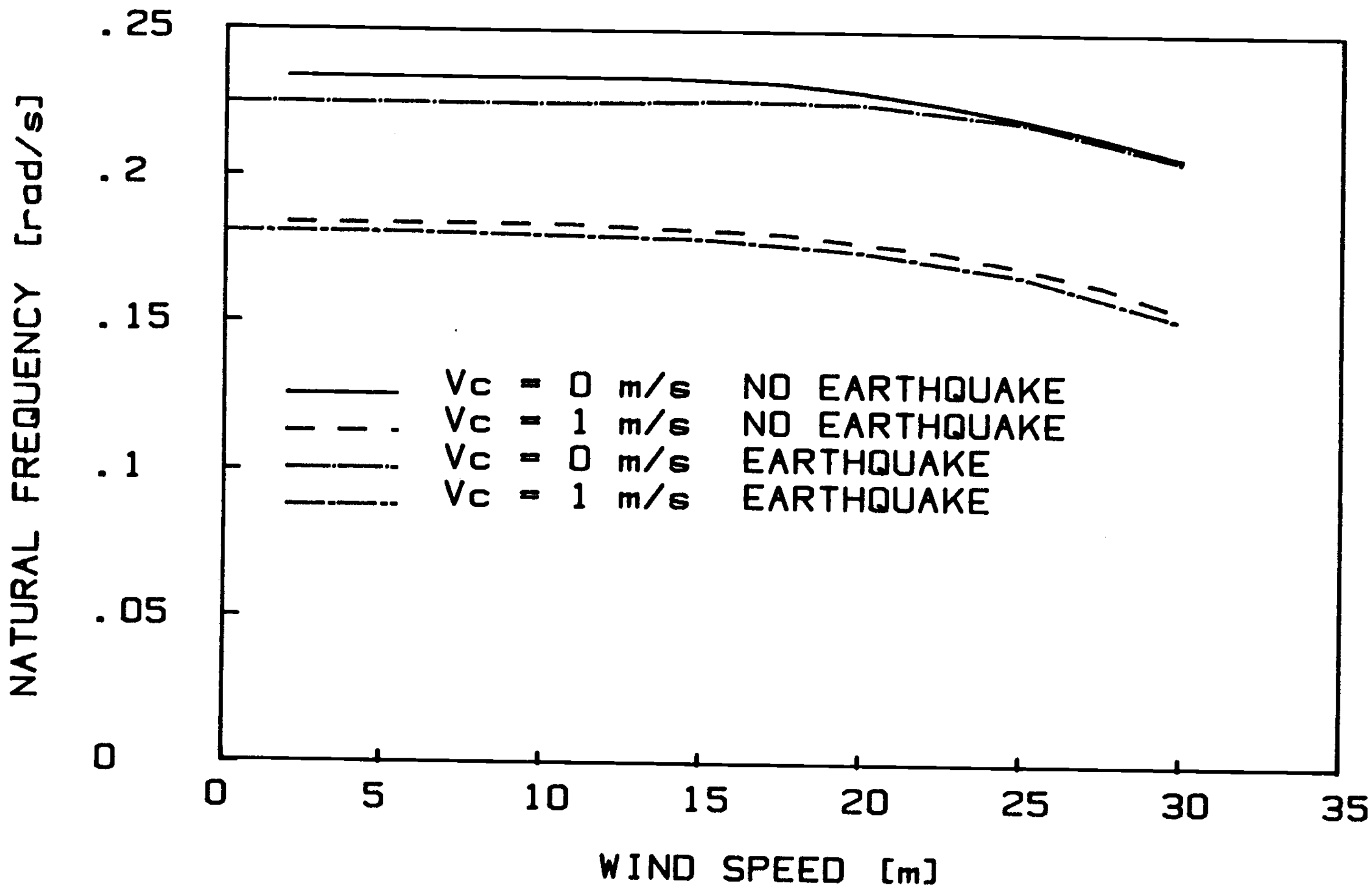

FIGURE 2.15 Natural frequency of linearized example guyed tower # 3, frequency domain.

CONCLUSION

In this study, a linearized single degree of freedom model of a guyed tower was developed using the stochastic linearization approach. It was solved in the frequency domain, giving the statistical response of the guyed tower. The results were compared to the response statistics of a time simulation that fully incorporated the nonlinearities of the cable system and the fluid-structure interaction.

It was found that the results from the linearized model agree reasonably well with the results from the time simulation of the fully nonlinear system. Both models were in good agreement with experimentally observed results. The computer time using the stochastic linearization approach is several order of magnitude less than for a conventional time simulation method. Considering other uncertainties in analysing guyed towers (selection of drag and inertia coefficients, specifying earthquake spectrum) it is justifiable to use this method in the early design stages of guyed towers.

ACKNOWLEDGEMENT

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REFERENCES

1. Borgman, L.E., "Ocean Wave Simulation for Engineering Design," Journal of Waterways and Harbors Division, vol. 95, no. WW4, pp. 557-583, November 1969.

2. Mo, M., and Moan, T., "Environmental Load Effect Analysis of Guyed Towers," Proceedings, 3rd International Offshore Mechanics and Arctic Engineering Symposium, Lousiana, February 1984.

3. Shinozuka, M., "Monte Carlo Solution of Structural Dynamics," Computers and Structures, vol. 2 pp. 855-876, 1972.

4. Spanos, P.D., "Monte Carlo Simulations of Responses of Non-Symmetric Dynamic System to Random Excitations," Computers & Structures, vol. 13, pp.371-376, 1981.

5. Finn, L.D., "A New Deepwater Offshore Platform - The Guyed Tower," Proceedings, Offshore Technology Conference, OTC 2688, May 1976.

6. Smith, E. and Sigbjornsson, R., "Nonlinear Stochastic Analysis of Compliant Platforms," Proceedings, Offshore Technology Conference, OTC 3801, May 1980.

7. Finn, L.D and Young, K.E., "Field Test of a Guyed Tower," Proceedings, Offshore Technology Conference, OTC 3131, pp. 681-688, May 1978.

8. Morison, J.R., O'Brian, M. P., Johnson, J.W. and Shaaf, S.A., "The Force Exerted by Surface Waves on Piles," Petrol. Trans., AIME, vol. 189, pp. 149-154, 1950.

9. Hanna, S.Y., Mangiavacchi, A. and Suhendra, R., "Nonlinear Dynamic Analysis of Guyed Tower Platforms," Journal of Energy Resources Technology, vol. 105, pp.205-211, June 1983.

10. Anagnostopoulos, S.A, "Dynamic Response of Offshore Platforms to Extreme Waves Including Fluid - Structure Interaction," Engineering Structures, vol. 4, pp. 179-185, July 1984.

11. Morrison, D.G., "Guyed Tower With Dynamic Mooring Properties," Journal of Structural Engineering, vol. 109, no. 11, pp. 2578-2590, November 1983.

12. Leonard, J.W. and Young, R.A., "Coupled Response of Compliant Offshore Platforms," Engineering Structures, vol. 7, no. 2, pp. 74-84, April 1985.

13. Brinkmann, C.R., "Dynamic Interaction of a Guyed Tower With Its Guying System," Transactions of the ASME, vol. 105, pp.290-295, September 1983.

14. Triantafyllou, M. S., Kardomates, G. and Bliek, A., "The Statics and Dynamics of the Mooring Lines of a Guyed Tower for Design Applications," Proceedings, BOSS 1982, vol. 1, pp. 546-564.

15. Wilson, J.F. and Orgill, G., "Optimal Cable Configurations for Passive Dynamic Control of Compliant Towers," Journal of Dynamic Systems, Measurement, and Control, vol. 106, no. 4, December 1984.

16. Dutta, A., "Some Fundamental Studies on Guyed Tower Response Under Wave Loadings," Proceedings, 5th International Offshore Mechanics and Arctic Engineering Symposium, Japan, April 1986

17. Sekita, K. and Maruyanna, M., "Model Tests on Guyed Towers with the Focus on their Nonlinear Characteristics," Proceedings, 5th International Offshore Mechanics and Arctic Engineering Symposium, Japan, April 1986

18. Spanos, P.D., "Random Response to Flow-Induced Forces," Journal of the Engineering Mechanics Division, Proceedings ASCE, vol. 107, No. EM6, December 1981.

19. Spanos, P.D., "Stochastic Linearization in Structural Dynamics," Applied Mechanics Review, vol. 34, no. 1, January 1981

20. Iwan, W.D, "On Defining Equivalent Systems for Certain Ordinary Non-Linear Differential Equations," International Journal of Nonlinear Mechanics, vol. 4, pp. 325-334., 1969

21. Iwan, W.D, "A Generalization of the Concept of Equivalent Linearization," International Journal of Nonlinear Mechanics, vol. 8, pp. 279- 287, 1973

22. Iwan, W.D. and Yang, I-M., "Application of Statistical Linearization Techniques to Nonlinear Multidegree-of-Freedom Systems," Transactions of the ASME, Journal of Applied Mechanics", June 1972.

23. Iwan, W.D. and Mason, A.B., "Equivalent Linearization for Systems Subjected to Non-Stationary Random Excitation," International Journal of Non-Linear Mechanics, vol. 15, pp. 71-82, 1980.

24. Arockiasamy, M., Reddy, D.V., Cheema, P.S. and El-Tahan, H., "Stochastic Response of Compliant Platforms to Irregular Waves," Ocean Engineering, vol. 10, no. 5, pp. 303-312, 1983.

25. Eatock Taylor, R. and Rajagopalan, A., "Load Spectra for Slender Offshore Structure in Waves and Currents," Earthquake Engineering and Structural Dynamics, vol. 11, pp. 831-842, 1983.

26. Gudmestad, O.T. and Connor, J.J., "Linearization Methods and the Influence of Current on the Nonlinear Hydrodynamic Drag Force," Applied Ocean Research, vol. 5, no. 4, pp. 184-194, 1983.

27. Newmark, N.M. and Rosenblueth, E., "Fundamentals of Earthquake Engineering," Prentice-Hall, 1971.

28. Penzien, J., Kaul, M.K. and Berge, B., "Stochastic Response of Offshore Towers to Random Sea Waves and Strong Motion Earthquakes," Computers and Structures, vol. 2, pp. 733-756, 1972.

29. Penzien, J., "Seismic Analysis of Platform Structure-Foundation Systems," Proceedings, Offshore Technology Conference, OTC 2352, May 1975.

30. Kirkley, O.M. and Murtha, J.P., "Earthquake Response Spectra for Offshore Structures," Proceedings, Offshore Technology Conference, OTC 2356 May 1977.

31. Nair, D., Valdivieso, J.B. and Johnson, C.M., "Comparison of Spectrum and Time History Techniques in Seismic Design of Platforms," Proceedings, Offshore Technology Conference, OTC 3823, May 1980.

32. Newland, D.E., "An Introduction to Random Vibration and Spectral Analysis," Longman, 1975.

33. Sarpkaya, T. and Isaacson, M., "Mechanics of Wave Forces on Offshore Structures," Van Nostrand Reinhold Company, 1981.

34. Borgman, L.E., "Techniques for Computer Simulations of Ocean Waves," Topics in Ocean Physics, Italian Physical Society, North-Holland Publishing Company, 1982.

35. Wilson, J.F., editor,"Dynamics of Offshore Structures," John Wiley & Sons, 1984.

36. Spanos, P.D. and Loli, V., "A Statistical Approach to Generation of Design Spectrum Compatible Earthquake Time Histories," Soil Dynamics and Earthquake Engineering, vol. 4, no. 1, 1985.

37. Doblere, M., Alarcon, E. and Benitz, G., "A Comparison Among Different Spectrum Compatible Earthquake Simulation Methods," Applied Mathematic Modelling, vol. 5, October 1981.

38. Clough, R.W. and Penzien, J., "Dynamics of Structures," McGraw-Hill, 1975.

39. Kanai, K., "Semi-empirical Formula for Seismic Characterisitcs of the Ground," University of Tokyo Bulletin Earthquake Research Institute, vol. 35, pp. 309-325, 1957.

40. Tajimi, H., "A Statistical Method of Determining the Maximum Response of a Building Structure during an Earthquake," Proceedings, 2nd World Conference on Earthquake Engineering, Tokyo, Japan 1960.

41. Ruitz, P. and Penzien, J., "Stochastic Seismic Response of Structures," Journal of the Engineering Mechanics Division ASCE, April 1971.

42. Spanos, P.D., "Probabilistic Earthquake Energy Spectra Equations," Journal of the Engineering Mechanics Division ASCE, vol. 106 pp. 147-159, February 1980.

43. Philippacopoulos, A.J. and Wang, P.C., "Seismic Inputs for Nonlinear Structures," Journal of Engineering Mechanics, vol. 110, no. 5, pp. 828-836, May 1984.

44. Carnahan, B., Luther, H.A. and Wilkes, J.O., "Applied Numerical Methods," Wiley, 1969.

45. Bretschneider, C.C., "On the Determination of the Design Ocean Wave Spectrum," Look Lab/Hawaii, vol. 7, no. 1, University of Hawaii, 1977.

46. Wishahy, M.A. and Arockiasamy, M., "Response of Guyed Towers for Linear and Nonlinear Behavior of Mooring Cables," Canadian Journal of Civil Engineering, vol. 12, no. 1, pp. 200-212, March 1985

47. Penzien, J. and Kaul, M.K., "Response of Offshore Towers to Strong Motion Earthquakes," Earthquake Engineering and Structural Dynamics, vol. 1, pp. 55-68, 1972.

48. Berg, G.V. and Housner, G.W., "Integrated Velocity and Displacement of Strong Earthquake Ground Motion," Bulletin of the Seismological Society of America, vol. 51, no. 2, pp.175-189, April 1961.

CHAPTER III

EFFECTS OF CURRENTS ON THE STOCHASTIC RESPONSE TO EARTHQUAKES OF MDOF MODELS OF GUYED OFFSHORE TOWERS

ABSTRACT

The dynamic response of a multiple degree of freedom (MDOF) guyed offshore towers to stochastic earthquake loads and a steady uniform current is investigated. The nonlinear cable stiffness and the fluid-structure interaction were linearized using the stochastic linearization method. To investigate the importance of higher modes on the stochastic forces and moments of the guyed tower, numerical results for several load cases are presented and discussed. The displacement statistics from the MDOF linear analysis were compared to the statistics of an equivalent single degree of freedom (SDOF) time simulation that fully incorporated the nonlinearities of the structure. The increased damping of the structure with increasing current was found to significantly reduce the stochastic forces and moments of the structure.

INTRODUCTION

The primary aim of this paper is to investigate the response of MDOF guyed towers to earthquake loads in the presence of a steady ocean current. Guyed offshore towers are generally softer than conventional fixed platforms, so they will have more frequencies in the excitation range of an earthquake than fixed platforms. It is therefore important to know the effect of higher modes on the forces and moments of a guyed tower.

In recent years, the guyed tower has become feasible for offshore oil production in water depths ranging from 200-400 m. A sketch of a guyed tower[1] is shown in Figure 3.1. The tower is long and slender and depends upon a group of guy lines for lateral stability. Each guy line consists of a lead line, a clump weight and a trailing line. The presence of the clump weights limits the maximum tensions in the cables.

Some guyed towers may be located in seismically active regions. Earthquakes are therefore a critical design consideration for guyed towers. Due to the random nature of the wave and earthquake loads, probabilistic methods are more appropriate than deterministic methods for the analysis of guyed towers. Monte Carlo simulations are popular due to their simplicity in implementation. But, extensive computer time is required, huge amounts of data are generated and interpretation of the results is tedious.[1-3] On the other hand, analyses in the frequency domain are not as involved as in the time domain, and are therefore preferable. However, they are only applicable to linear systems.

The governing equation of an offshore guyed tower has two main sources of nonlinearities, fluid-structure interaction and the restoring force of the cable system. Therefore, it has both nonlinear stiffness and damping. The equations of motion can be linearized to allow solution in the frequency domain using spectral techniques.

The present paper presents an equivalent linear multiple degree of freedom system that was developed using the stochastic lineariza-

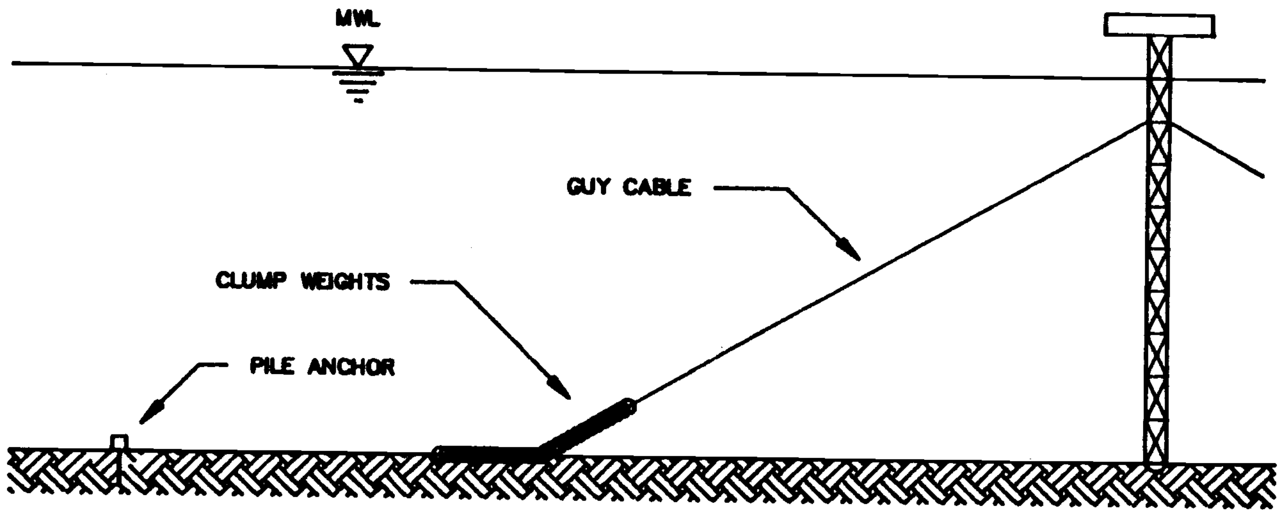

FIGURE 3.1 Compliant guyed tower.

tion method. It was solved in the frequency domain, giving the statistical response of the guyed tower with less computer time than conventional time domain methods. A MDOF system should be used to obtain accurate statistics of tower forces and moments. A SDOF system may give reasonable displacement statistics for the first mode, but it is not suitable to obtain forces and moments due to higher modes. The effects of current on the standard deviation of the response statistics are investigated by means of numerical examples.

PREVIOUS WORK

Several authors[4-12] have investigated and conducted parametric studies on the responses of guyed and fixed offshore towers subject to wave and current loading in the time domain. Some of them have retained the dynamic cable properties of the mooring system. Inclusion of the guyline dynamics prevents overestimation of the maximum deck motions and underestimation of the maximum guyline tension that are apparent in decoupled analysis. Sekita and Maruyanna[13] conducted wave tank experiments with a 1/100 scale model. Their measurements compared reasonably well with analytical results obtained using regular waves. They conclude that the effect of drag force coefficient is significant only in the area of resonant periods.

Since earthquakes are random in nature, it is appropriate to use a frequency-domain analysis to determine the dynamic response of structures due to earthquake loading. But frequency-domain analysis is only applicable to linear systems. The method of stochastic linearization is a widely used approximate method for probabilistic analysis of nonlinear structural dynamics problems.[14-17] The error in the linearization, i.e. the difference between the linear and the nonlinear equation, is minimized in a suitable way. The usual choice is to minimize the mean-square error. This procedure gives implicit expressions for the equivalent linear coefficients. Brynjolfsson and Leonard[18] used a SDOF system to investigate the validity of the stochastic linearization method by comparing the displacement statistics due to combined stochastic earthquake, wave and current loads from a nonlinear time simulation to the results of a linearized model obtained in the frequency domain. They conclude that the stochastic linearization gives a reasonable good agreement to the fully nonlinear time simulation. Smith and Sigbjornsson[19] evaluated the stochastic structural response of a guyed tower subject to wave action. They used two basically different approaches, stochastic linearization and an iterative approach and found them to give almost identical results. Some authors have investigated the effects of current on the hydrodynamic drag force in waves. Eatock Taylor and Rajagopalan[20] and Gudmestad and Connor[21] obtained different results than Wu[11] when they assumed that the velocity of the fluid particle is much greater than the structural velocity. That assumption may not be valid for compliant offshore structure.

The two most commonly used methods for analysing the earthquake response of structures are time history and response spectrum techniques. Nair, et al.[3] investigated the seismic response of a fixed offshore platform, using spectrum and time history analyses. They found the accuracy of the response spectrum technique as a design tool to be adequate. Kirkley and Murtha[22] presented a simple method for adopting existing (land based) earthquake response techniques to the analysis of offshore structures.

Penzien, et al.[23,24] investigated the stochastic response of fixed offshore towers to random sea waves and strong motion earthquakes using the stochastic linearization method and the response spectrum method. They found the two methods to give similar results for lower period towers, but that the difference between the results increase with the period of the structure.

SYSTEM MODELING

A SDOF system can be used to obtain fairly accurate statistics of the tower displacements,[18] but a MDOF system is necessary to obtain accurate tower forces and moments.[25] The motion of the guyed tower model used in this work is confined in a plane. The current is assumed to be in the same direction as the earthquake, which is a conservative assumption. A schematic diagram of an example tower used in this analysis can be seen in Figure 3.2. The example tower was discretized into nine beam elements with uniform properties and geometry along the tower. A representative commercially-available finite element program[26] was used to model the tower and generate the stiffness and mass matrices. The linearized geometric stiffness[27] was included in this analysis by utilizing a general stiffness element. The base of the tower is fixed against translation, but has a rotational stiffness. Other nodes have x-y translation and rotation about the z axis. In the dynamic analysis of the linearized MDOF system, the Guyan reduction procedure[28] was used to reduce the

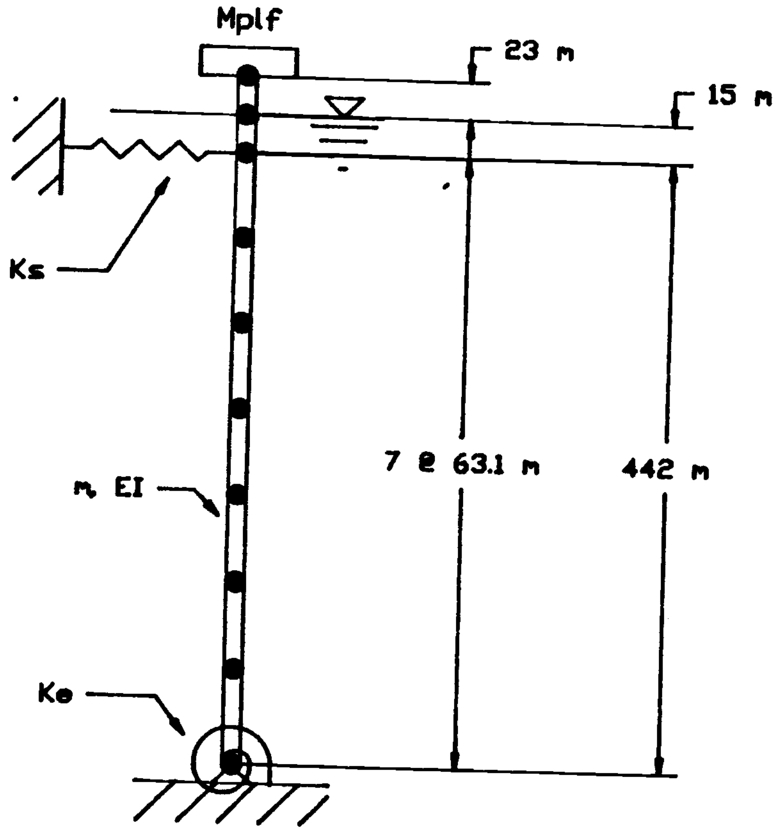

FIGURE 3.2 Idealized guyed tower model.

degrees of freedom to x translation only. The structural mass matrix is therefore fully populated.

The cables were modeled as nonlinear massless spring with a static force-displacement curve as shown in Figure 3.3. The linearized stiffness of the cable element was calculated using the stochastic linearization method. The dynamics of the cable system were neglected in this work.

With the deck mass, M_p, lumped at a point and the mass of the truss, m, uniformly distributed per unit length, the nonlinear equation of motion for the guyed tower can be written as:[24]

$$[M_v]\{\ddot{x}\} + [C_{twr}]\{\dot{x}\} + [K_{twr}]\{x\} = - [M_v]\{1\}\ddot{x}_g - [K_{nl}(x)]\{x\} + [C_h]\{|V-\dot{x}-\dot{x}_g|(V-\dot{x}-\dot{x}_g)\} \qquad (1)$$

where

$$[M_v] = [M_{twr} + M_a]$$

$$[M_a] = \rho\ C_a\ [V_e] \qquad (2)$$

$$[C_h] = 0.5\ \rho\ C_d\ [A_e] \qquad (3)$$

$$K_{nl,nn} = c_{2,nn}\ (1 - e^{c_{1,nn}|x_n|}) \qquad (4)$$

$\{x\}$, $\{\dot{x}\}$ and $\{\ddot{x}\}$ are the horizontal tower displacement, velocity and acceleration; $[M_{twr}]$, $[C_{twr}]$ and $[K_{twr}]$ are the linear portions of the structural mass, damping and stiffness matrices of the tower; $[M_a]$ is the diagonal added mass matrix from the Morison equation;[29] $\{1\}$ is a vector filled with one's; $\dot{x}_g$ and $\ddot{x}_g$ are the ground velocity

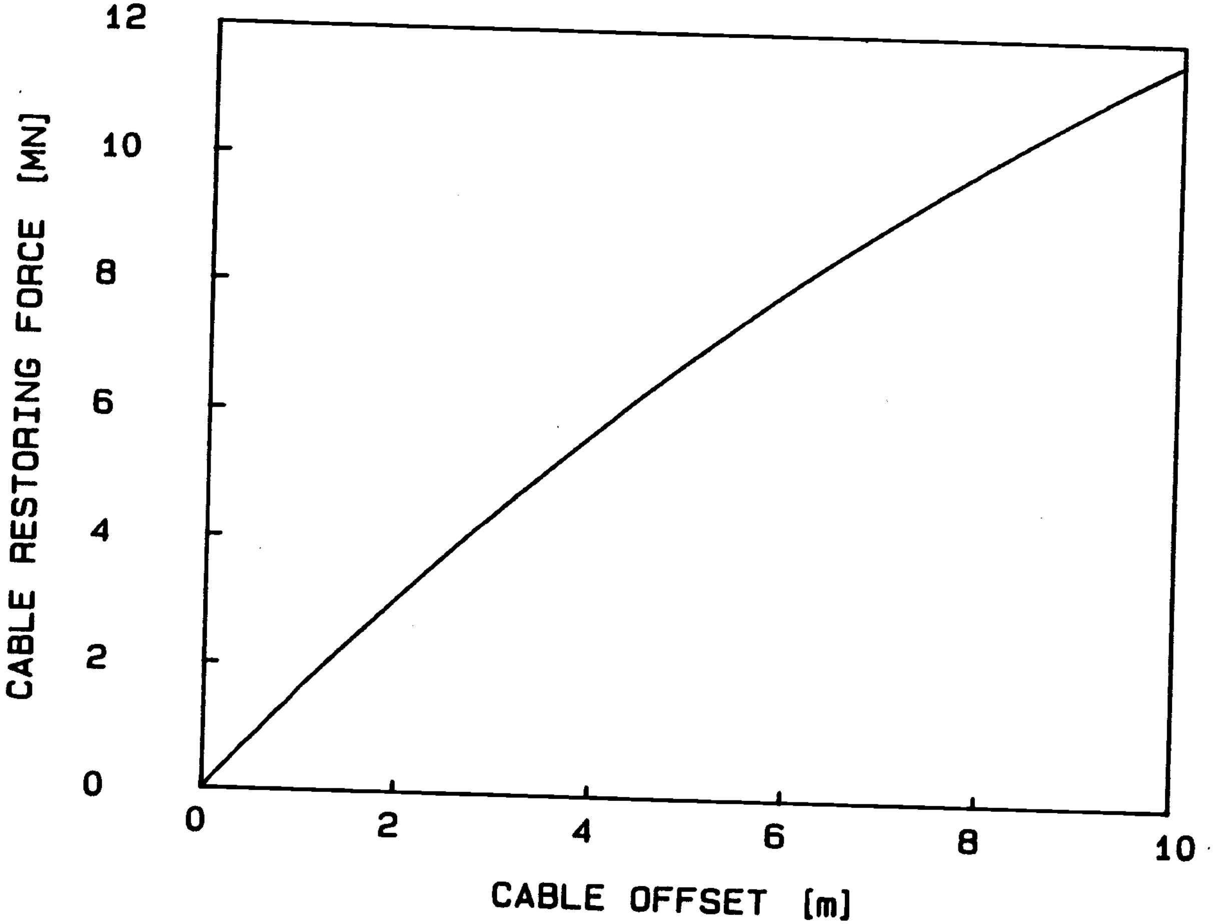

FIGURE 3.3 Force-displacement curve for the guying system

and acceleration; $[K_{nl}(x)]$ is the diagonal nonlinear cable stiffness (one element in present study); n is the node where the cable is connected to the tower; $[C_h]$ is the diagonal drag matrix and V is the steady current; ρ is the density of sea water; V_e and A_e are the equivalent inertial volume and projected area of the tower; C_a and C_d are the added mass and the drag coefficients, respectively; and c_1 and c_2 are constants that determine the cable nonlinearities. They were determined by fitting equation (4) to force-displacement curves for 20 cables obtained by catenary equations.[12]

EARTHQUAKE LOADING

The ground motions induced by an earthquake may be modeled as a random process. The earliest attempts were to simulate the accelerations as a stationary white noise.[25] But an earthquake is nonwhite since its spectral density is not uniform with respect to frequency. Kanai[30] and Tajimi[31] initially proposed the widely used filter transfer function:

$$|H_g(\omega)|^2 = \frac{\omega_g^{\,4} + 4\zeta_g^{\,2}\omega_g^{\,2}\omega^2}{(\omega_g^{\,2}-\omega^2)^2 + 4\zeta_g^{\,2}\omega_g^{\,2}\omega^2} \qquad (5)$$

were ω_g and ζ_g are the characteristic ground frequency and ground damping ratio. Kanai suggested 5π rad/s for ω_g and 0.6 for ζ_g as being representative for firm ground condition. The power spectral density (PSD) function for the filtered process is then

$$S_{\ddot{x}_g\ddot{x}_g}(w) = |H_g(\omega)|^2 \, S_0 \qquad (6)$$

where S_0 is constant (PSD of white noise). A zero mean ergodic Gaussian process is used in this research as the stochastic model for the ground acceleration, with a stationary Kanai-Tajimi earthquake spectra. The spectrum was modified to make the variance of the ground velocity finite.[25] A high pass filter transfer function of the form:

$$|H_1(\omega)|^2 = \frac{\omega^4}{(\omega_1^2-\omega^2)^2 + 4\zeta_1^2\omega_1^2\omega^2} \tag{7}$$

is used in this research. The constants ω_1 and ζ_1 are selected to obtain the required filter characteristics. The power spectral density of the final process is then:

$$S_{\ddot{x}_g\ddot{x}_g}(\omega) = |H_1(\omega)|^2 \; |H_g(\omega)|^2 S_0 \tag{8}$$

<u>LINEARIZATION</u>

The governing equation, equation (1), contains two nonlinear terms, in the drag force and in the cable stiffness $[K_{nl}]$ given by equation (4). In order to use a spectral approach, these two terms need to be linearized. The stochastic linearization technique replaces the original nonlinear system with an equivalent linear system based on a certain optimization criterion. The way chosen here to linearize equation (1) is to separately linearize the stiffness and drag terms, such that the expected values of the square of the differences between the linear and nonlinear terms are minimum.

It is assumed that $\{x(t)\}$ consists of a deterministic (constant) offset, $\{x_0\}$, and a zero mean random process $\{\hat{x}(t)\}$, i.e.

$$\{x(t)\} = \{x_o\} + \{\hat{x}(t)\} \tag{9}$$

Then the difference between the linear and nonlinear terms can be expressed as

$$\{\delta_1\} = \{|V-\dot{\beta}|(V-\dot{\beta})\} - \{a\} + [b]\{\dot{\beta}\} \qquad (10)$$

$$\{\delta_2\} = [K_{nl}(x)]\{x\} - \{c\} - [e]\{\hat{x}\} \qquad (11)$$

where

$$\{\dot{\beta}\} = \{\dot{x}\} + \{\dot{x}_g\} \qquad (12)$$

is the absolute velocity of the structure. The coefficients {a}, [b], {c} and [e] are selected such that

$$\langle\{\delta_1\}^T\{\delta_1\}\rangle = \text{minimum} \qquad (13)$$

$$\langle\{\delta_2\}^T\{\delta_2\}\rangle = \text{minimum} \qquad (14)$$

where <...> represents the expected value operator. For a zero mean Gaussian random process, the expected value can be calculated according to

$$\langle f(x)\rangle = \frac{1}{\sigma\sqrt{(2\pi)}} \int_{-\infty}^{\infty} f(x)\ \exp(-0.5x^2/\sigma^2)\ dx \qquad (15)$$

where σ is the standard deviation of x.

Minimizing $\langle\{\delta_1\}^T\{\delta_1\}\rangle$ with respect to {a} and [b], one obtains

$$\frac{\partial}{\partial a_k} \langle\{\delta_1\}^T\{\delta_1\}\rangle = 0 \qquad (16)$$

$$\frac{\partial}{\partial b_{km}} \langle\{\delta_1\}^T\{\delta_1\}\rangle = 0 \qquad (17)$$

Since δ_{1k} only depends upon $\dot{\beta}_k$, [b] becomes diagonal and

$$a_k = < |V_k - \dot{\beta}_k| \, (V_k - \dot{\beta}_k) > \quad (18)$$

$$b_{kk} = < |V_k - \dot{\beta}_k| \, (V_k - \dot{\beta}_k)\dot{\beta}_k > / < \dot{\beta}_k^{\,2} > \quad (19)$$

The mathematical expectations can be evaluated analytically to yield, after some manipulations

$$a_k = (\sigma_{\dot{\beta}_k}^{\,2} + V_k^{\,2}) \text{ erf } (V_k/(\sigma_{\dot{\beta}_k}\sqrt{2})$$
$$+ (2/\pi)^{0.5} \, V_k \, \sigma_{\dot{\beta}_k} \exp \, (-V_k^{\,2}/(2\sigma_{\dot{\beta}_k}^{\,2})) \quad (20)$$

$$b_{kk} = V_k \text{ erf } (V_k/(\sigma_{\dot{\beta}_k}\sqrt{2}) + (8/\pi)^{0.5}\sigma_{\dot{\beta}_k} \exp \, (-V_k^{\,2}/(2\sigma_{\dot{\beta}_k}^{\,2})) \quad (21)$$

where $\sigma_{\dot{\beta}_k}$ is the standard variation of $\dot{\beta}_k$. Similarly, for $<\{\delta_2\}^T\{\delta_2\}>$ to be minimum,

$$\frac{\partial}{\partial c_k} <\{\delta_2\}^T\{\delta_2\}> = 0 \quad (22)$$

$$\frac{\partial}{\partial e_{km}} <\{\delta_2\}^T\{\delta_2\}> = 0 \quad (23)$$

Hence, after some manipulations

$$\{c\} = <[K_{n1}(x)]\{x\}> \quad (24)$$

$$<\{\hat{x}\}\{\hat{x}\}^T>[e] = <\{[K_{n1}(x)]\{x\} \, \{\hat{x}\}^T> \quad (25)$$

In this work, $[K_{n1}(x)]$ has only one element, so equations (24) and (25) can be rewritten as

$$c_n = <K_{n1,nn}(x_n)x_n> \quad (26)$$

$$e_{nn} = \langle K_{nl,nn}(x_n) x_n \hat{x}_n \rangle / \langle \hat{x}_n^2 \rangle \tag{27}$$

where n is the node of cable attachment to the tower. More generally, Atalik and Utku[32] have shown that under certain general conditions on $[K_{nl}(x)]$, that

$$e_{km} = \langle \frac{\partial}{\partial x_m} F_k \rangle \tag{28}$$

where

$$\{F(x)\} = [K_{nl}(x)]\{x\} \tag{29}$$

Substituting the linearized expressions, equations (20), (21), (26) and (27), into the governing equation, equation (1), and rearranging, one obtains

$$[M_v]\{\ddot{x}\} + [C_{twr}]\{\dot{x}\} + [K_{twr}]\{x\} = -[M_v]\{1\}\ddot{x}_g - \{c\} - [e]\{x\}$$

$$+ [C_h]\{a\} - [C_h][b](\{\dot{x}\} + \{1\}\dot{x}_g) \tag{30}$$

Equation (30) has to be satisfied in an average sense: Taking its expected value, one obtains

$$[K_{twr}]\{x_0\} = -\{c\} + [C_h]\{a\} \tag{31}$$

The mean tower offset becomes

$$\{x_0\} = [K_{twr}]^{-1}(-\{c\} + [C_h]\{a\}) \tag{32}$$

Substituting back in the values for $\{x_0\}$ from equation (32), one can write equation (30) as

$$[M_v]\{\ddot{\hat{x}}\}+[C_{eq}]\{\dot{\hat{x}}\}+[K_{eq}]\{\hat{x}\} = -[M_v]\{\ddot{x}_g\}-[C_g]\{\dot{x}_g\}$$

where

$$[K_{eq}] = [K_{twr}] + [e] \tag{34}$$

$$[C_{eq}] = [C_{twr}] + [C_h][b] \tag{35}$$

$$[C_g] = [C_h][b] \tag{36}$$

The linearized matrices, $[K_{eq}]$, $[C_{eq}]$ and $[C_g]$ depend upon $\{x_0\}$ and the variance of $\{\dot{\beta}\}$ and $\{\hat{x}\}$. The problem is therefore implicit, i. e. values of $\{\sigma_{\dot{\beta}}\}$ and $\{x_0\}$ have to be guessed initially, then calculated and compared to the guesses.

UNCOUPLED EQUATIONS OF MOTION

The classical normal mode superposition is used to calculate the response statistics of the structure

$$\{\hat{x}\} = [\Phi]\{Y\} \tag{37}$$

where $[\Phi]$ is the modal matrix determined from the undamped homogeneous form of equation (33) and $\{Y\}$ is the generalized modal coordinate vector. Substituting equation (37) into equation (33) and premultiplying by $[\Phi]^T$, one obtains

$$[M^*]\{\ddot{Y}\} + [C_0]\{\dot{Y}\} + [K^*]\{Y\} = \{P^*\} \tag{38}$$

where

$$[M^*] = [\Phi]^T[M_v][\Phi] \tag{39}$$

$$[K^*] = [\Phi]^T[K_{eq}][\Phi] \tag{40}$$

$$[C_0] = [\Phi]^T[C_{eq}][\Phi] \tag{41}$$

$$\{P^*\} = -[\Phi]^T[M_V]\{1\}\ddot{x}_g - [\Phi]^T[C_g]\{1\}\dot{x}_g \tag{42}$$

The generalized mass and stiffness matrices, [M*] and [K*], will be diagonal due to orthogonality of normal modes, but the generalized damping matrix, $[C_0]$, will not satisfy the orthogonality condition, since it includes the damping from the fluid structure interaction. The coupling can be removed by a procedure similar to equivalent linearization as outlined by Penzien and Kaul[24]. Define a generalized diagonal damping matrix, [C*], such that

$$\langle\{\delta_3\}^T\{\delta_3\}\rangle = \text{minimum} \tag{43}$$

where

$$\{\delta_3\} = [C_0]\{\dot{Y}\} - [C^*]\{\dot{Y}\} \tag{44}$$

Then

$$\frac{\partial}{\partial c_k^*}\langle\{\delta_3\}^T\{\delta_3\}\rangle = 0 \tag{45}$$

$$\langle\,(\sum_{j=1}^{N} c_{0kj}\dot{Y}_j - c_k^*\,\dot{Y}_k)\dot{Y}_k\rangle = 0 \tag{46}$$

$$c_k^* = \sum_{j=1}^{N} \frac{c_{0kj}\langle\dot{Y}_j\dot{Y}_k\rangle}{\langle\dot{Y}_k^{\,2}\rangle} \tag{47}$$

This gives a set of uncoupled equations

$$[M^*]\{\ddot{Y}\} + [C^*]\{\dot{Y}\} + [K^*]\{Y\} = \{P^*\} \tag{44}$$

since [M*], [C*] and [K*] are diagonal. To fully describe the linearized system, $\{\sigma_{\dot{\beta}}\}$, $\{\sigma_x\}$, $\{x_0\}$ and $\langle\dot{Y}_k\dot{Y}_j\rangle$ need to be evaluated.

RESPONSE STATISTICS

To evaluate the statistics of the various response quantities, it is convenient to define an arbitrary response quantity, z(t), which is related to the normal coordinates, {Y}, through a set of known coefficients, {Z}

$$z(t) = \sum_{k=1}^{N} Z_k Y_k \tag{49}$$

Then the autocorrelation function for z(t) is

$$R_{zz}(\tau) = \sum_{k=1}^{N} \sum_{j=1}^{N} Z_k Z_j \langle Y_k(t) Y_j(t+\tau)\rangle \tag{50}$$

but $Y_k(t)$ can be written as

$$Y_k(t) = \int_{-\infty}^{t} P_k{}^*(\tau)\ h_k(t-\tau)d\tau \tag{51}$$

where

$$h_k(t) = \frac{\exp(\zeta_k\ \omega_k\ t)\ \sin(\omega_{dk} t)}{\omega_{dk}\ m_k{}^*} \tag{52}$$

$$\omega_k = \sqrt{(k_k{}^*/m_k{}^*} \tag{53}$$

$$\omega_{dk} = w_k\sqrt{(1-\zeta_k{}^2)} \tag{54}$$

$$\zeta_k = c_k{}^*/(2\ m_k{}^*\ \omega_k) \tag{55}$$

Substituting equation (51) into equation (50) one obtains after some manipulations

$$R_{zz}(\tau) = \sum_{k=1}^{N} \sum_{j=1}^{N} Z_k Z_j \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_k(\theta_1) h_j(\theta_2) R_{P_k^* P_j^*}(\tau+\theta_1-\theta_2) d\theta_1 d\theta_2 \tag{56}$$

The cross correlation function for the generalized forces can be obtained from equation (42)

$$P_k^*(t) = -(p_k \ddot{x}_g(t) + q_k \dot{x}_g(t)) \tag{57}$$

where

$$p_k = \{\Phi_k\}^T [M_v] \{1\} \tag{58}$$

$$q_k = \{\Phi_k\}^T [C_g] \{1\} \tag{59}$$

The cross correlation function for the loads can then be written as

$$R_{P_k^* P_j^*}(\tau) = p_k p_j R_{\ddot{x}_g \ddot{x}_g}(\tau) + p_k q_j R_{\ddot{x}_g \dot{x}_g}(\tau) + p_j q_k R_{\dot{x}_g \ddot{x}_g}(\tau) + q_k q_j R_{\dot{x}_g \dot{x}_g}(\tau) \tag{60}$$

Substituting equation (60) into equation (56) and taking the Fourier transform of the resulting equation , one obtains

$$S_{zz}(\omega) = \sum_{k=1}^{N} \sum_{j=1}^{N} Z_k Z_j \bar{H}_k(\omega) H_j(\omega) \left(p_k p_j S_{\ddot{x}_g \ddot{x}_g}(\omega) + p_k q_j S_{\ddot{x}_g \dot{x}_g}(\omega) + p_j q_k S_{\dot{x}_g \ddot{x}_g}(\omega) + q_k q_j S_{\dot{x}_g \dot{x}_g}(\omega)\right) \tag{61}$$

The cross spectral densities can be evaluated from the ground acceleration spectrum, equation (8)

$$S_{\dot{x}_g \dot{x}_g}(\omega) = \frac{1}{\omega^2} S_{\ddot{x}_g \ddot{x}_g}(\omega) \tag{62}$$

$$S_{\dot{x}_g \ddot{x}_g}(\omega) = i\omega\, S_{\dot{x}_g \dot{x}_g}(\omega) \tag{63}$$

$$S_{\ddot{x}_g \dot{x}_g}(\omega) = -i\omega\, S_{\dot{x}_g \dot{x}_g}(\omega) \tag{64}$$

where $i = \sqrt{(-1)}$. The power spectrum of the process z(t) is therefore fully defined in terms of the ground acceleration spectrum. The variance of z(t) is obtained by integrating equation (61):

$$\sigma_{zz}^{\;2} = \int_{-\infty}^{\infty} S_{zz}(\omega)\, d\omega \tag{65}$$

Equation (61) and subsequently equation (65) can be used to evaluate the parameters $\sigma_{\dot{\beta}_k}$ and $\sigma_{\hat{x}_k}$. According to equation (37), one can write $\hat{x}_k$ as

$$\hat{x}_k(t) = \sum_{j=1}^{N} \Phi_{kj}\; Y_j \tag{66}$$

The variance of the relative velocity, $\sigma_{\dot{\beta}_k}$, can be evaluated similarly. From equation (12)

$$\dot{\beta}_k = \dot{\hat{x}}_k + \dot{x}_g \tag{67}$$

so

$$S_{\dot{\beta}_k \dot{\beta}_k}(\omega) = S_{\dot{\hat{x}}_k \dot{\hat{x}}_k}(\omega) + S_{\dot{x}_g \dot{x}_g}(\omega) + S_{\dot{\hat{x}}_k \dot{x}_g}(\omega) + S_{\dot{x}_g \dot{\hat{x}}_k}(\omega) \tag{68}$$

and

$$\sigma^2_{\dot{\beta}_k \dot{\beta}_k} = \int_{-\infty}^{\infty} S_{\dot{\beta}_k \dot{\beta}_k}(\omega)\, d\omega \tag{69}$$

The first two terms in equation (68) can be determined by using equations (65) and (62). The cross spectral density, $S_{\dot{\hat{x}}_k \dot{x}_g}(\omega)$ can be evaluated using a method similar to that used to evaluate $S_{zz}(\omega)$.

$$R_{\hat{x}_k \dot{x}_g}(\tau) = \sum_{j=1}^{N} \Phi_{kj} \langle Y_j(t)\dot{x}_g(t+\tau)\rangle \tag{70}$$

Using equation (51) for $Y_j(t)$, one obtains

$$R_{\hat{x}_k \dot{x}_g}(\tau) = \sum_{j=1}^{N} \Phi_{kj} \langle \int_{-\infty}^{\infty} h_j(\theta) P_j{}^*(t-\theta)\dot{x}_g(t+\tau)d\theta\rangle \tag{71}$$

$$R_{\dot{\hat{x}}_k \dot{x}_g}(\tau) = \sum_{j=1}^{N} \Phi_{kj} \langle \int_{-\infty}^{\infty} h_j(\theta)(p_j R_{\ddot{x}_g \ddot{x}_g}(\tau+\theta)+q_j R_{\dot{x}_g \ddot{x}_g}(t+\tau))d\theta \tag{72}$$

So

$$S_{\dot{\hat{x}}_k \dot{x}_g}(\omega) = \sum_{j=1}^{N} \Phi_{kj}\ \bar{H}_j(\omega)\ S_{\ddot{x}_g \ddot{x}_g}(\omega)\ \left(p_j + \frac{i}{\omega} q_j\right) \tag{73}$$

and since

$$S_{\dot{\hat{x}}_k \dot{x}_g}(\omega) = \bar{S}_{\dot{x}_g \hat{x}_k}(\omega) \tag{74}$$

one obtains

$$S_{\dot{x}_g \dot{\hat{x}}_k}(\omega) = \sum_{j=1}^{N} \Phi_{kj}\ H_j(\omega)\ S_{\ddot{x}_g \ddot{x}_g}(\omega)\ \left(p_j - \frac{i}{\omega} q_j\right) \tag{75}$$

The generalized damping coefficient is evaluated according to equation (47). The expected value of $\langle \dot{Y}_j \dot{Y}_k \rangle$ can be calculated by same methods as described previously. The cross correlation of the time derivatives of a stationary random processes satisfies

$$R_{\dot{Y}_j \dot{Y}_k}(\tau) = \langle \dot{Y}_k(t) \dot{Y}_j(t+\tau) \rangle \quad (76)$$

$$= -\frac{d^2}{d\tau^2} R_{Y_j Y_k}(\tau) \quad (77)$$

But the autocorrelation function is the inverse Fourier transform of the power spectral density, so

$$R_{Y_j Y_k}(\tau) = \int_{-\infty}^{\infty} S_{Y_j Y_k}(\omega)\, e^{i\omega\tau}\, d\omega \quad (78)$$

$$\frac{d^2}{d\tau^2} R_{Y_j Y_k}(\tau) = -\int_{-\infty}^{\infty} \omega^2\, S_{Y_j Y_k}(\omega)\, e^{i\omega\tau}\, d\omega \quad (79)$$

Combining equations (77) and (79), one obtains

$$R_{\dot{Y}_j \dot{Y}_k}(\tau) = \int_{-\infty}^{\infty} \omega^2\, S_{Y_j Y_k}(\omega)\, e^{i\omega\tau}\, d\omega \quad (80)$$

$$\langle \dot{Y}_j(t) \dot{Y}_k(t) \rangle = \int_{-\infty}^{\infty} \omega^2\, S_{Y_j Y_k}(\omega)\, d\omega \quad (81)$$

But $S_{Y_j Y_k}(\omega)$ can be evaluated by equation (61)

$$S_{Y_j Y_k}(\omega) = \bar{H}_j(\omega)\, H_k(\omega)\, \big(p_j p_k S_{\ddot{x}_g \ddot{x}_g}(\omega) + p_j q_k\, S_{\ddot{x}_g \dot{x}_g}(\omega)$$

$$+ p_k q_j S_{\dot{x}_g \ddot{x}_g}(\omega) + q_j q_k S_{\dot{x}_g \dot{x}_g}(\omega)\big) \quad (82)$$

Using the procedures described above for a general response quantity, z(t), one develops all the equations needed to evaluate the tower response statistics. The elastic forces due to the random response are[25]

$$\{f\} = [K_{eq}]\{x\} \tag{83}$$

$$= [K_{eq}][\Phi]\{Y\} \tag{84}$$

Using the orthogonality relations of modal coordinates, one can write equation (84) as

$$\{f\} = [Mv][\Phi][\omega_n^2]\{Y\} \tag{85}$$

$$= [B]\{Y\} \tag{86}$$

where $[\omega_n^2]$ is a diagonal matrix with the natural frequency of mode n squared in column n. The shear force is then the summation of the elastic forces minus the reaction force due to the cable system

$$\{f_s\} = [L_1]\ \{f\} - [K_{cable}][\Phi]\{Y\} \tag{87}$$

$$= [\bar{B}]\{Y\} \tag{88}$$

where $[L_1]$ is a lower triangular matrix filled with 1's and $[K_{cable}]$ is the equivalent stiffness of the cable system (one element in the present study). The moments can be calculated similarly by integrating the shear forces along the tower. That gives

$$\{Mo\} = [\bar{\bar{B}}]\{Y\} \tag{89}$$

Equation (65) can then be used to calculate the statistics of the shear forces and moments.

SOLUTION PROCEDURE

The procedure described so far is implicit, since the linearized matrices depend upon the statistics of the response. The standard deviation of the absolute velocity, $\{\sigma_{\dot{\beta}}\}$, the steady tower offset, $\{x_0\}$, and the cable stiffness have to be estimated initially and then compared to calculated values. This is repeated until convergence (relative error less than 0.0001 in the present study), usually in less than 5 iterations. The variance of the absolute velocity was assumed to be very small (10^{-5}) at the start of the iteration, $\{x_0\}$ was assumed to be zero and the cable system was assumed to be linear ($[K_{nl}(x)] = [0]$). In examples where results were available from considerations of other currents, previous values were used as initial guesses for different currents. The mass and stiffness matrices, natural frequencies, mode shapes and the static displacements where calculated using a commercially available finite element program,[26] which is representative of the class of linear finite element programs. Then the steady forces, the statistics of the response and the nonlinear contribution of the cable system where calculated with another program and the stiffness of the cable system modified accordingly. In each iteration the stiffness changes and it is therefore necessary to re-calculate new mass and stiffness matrices, natural frequencies, mode shapes and static displacements in each iteration. The integration of the power spectral density is performed with Simpsons rule from 0 to 10 rad/s and then the remainder, from 10 rad/s to ∞ is integrated by a 15 point Gauss-Lagurre procedure.[33]

EXAMPLE PROBLEMS

The responses of the previously described guyed tower to a strong motion earthquake in the presence of steady uniform currents were investigated. The values of the current used in this work was 0, 0.3, 0.6 and 1.0 m/s. The parameters used to specify the earthquake spectrum defined by equations (5), (7) and (8) were ω_g = 15.7 rad/s and ζ_g = 0.6 as suggested by Kanai[30] for a firm ground condition; ω_1 = 0.4 rad/s and ζ_1 = 0.9; S_0 = 0.004267 m^2/s^3 (0.04593 ft^2/s^3) which corresponds to a class of earthquakes having an average intensity similar to the N-S component of the 1940 El Centro earthquake.[24] These values of ω_1 and ζ_1 give the standard deviation of the ground velocity approximately as 0.14 m/s. Then the maximum ground velocity can be approximated as

$$\dot{x}_{g,max} = 3\ \sigma_{\dot{x}_g} \tag{90}$$

$$= 0.42 \text{ m/s (17 in/s)} \tag{91}$$

which is the maximum ground velocity of the N-S component of the 1940 El Centro earthquake obtained by Berge and Housner[34] by integrating the acceleration record. The specifications of the tower used in this analysis are similar to the ones used by Dutta[12] and Mo and $Moan_1$ and are listed in Table 3.1. The first four natural frequencies, in the absence of current, are 0.273, 1.48, 5.07 and 10.8 rad/s

Table 3.1. Tower specifications.

M_p	$6.80\ 10^6$	kg
m	$3.70\ 10^4$	kg/m
K_1	$1.648\ 10^6$	N/m
c_1	0.045	1/m
c_2	$-1.324\ 10^6$	N/m
EI	$4.905\ 10^{13}$	N m^2
ζ_n	0.05	
A_e	35.1	m^2/m
V_e	26.3	m^3/m
C_a	1.0	
C_d	0.7	

(periods: 23, 4.3, 1.2 and 0.58 s). The corresponding mode shapes can be seen in Figure 3.4.

The steady tower displacement due to the current and the standard deviation of the tower displacements due to an earthquake can be seen in Figures 3.5 and 3.6, respectively. Results from a time simulations of a nonlinear SDOF model[18] are also plotted on the same graphs. The linearized MDOF model compares fairly well with the nonlinear SDOF model.

The effect of current is to reduce the standard deviation of the displacements, but increase the steady offset. The steady offset increases approximately as the current velocity squared, as expected from the Morison's equation. When a current of 1 m/s is superimposed on the structure, then the standard deviation of the tower displacement is reduced about 40% compared to the standard deviation in the absence of current. The standard deviation of the displacement decreases because of increased hydrodynamic damping of the system with increasing current, as can be seen in Figure 3.7. There are shown the total modal damping of the first four modes. The total modal damping consists of the structural damping in addition to the hydrodynamic damping. Structural damping of 5% was assumed for each mode. The effect of current on the damping of the second mode is quit significant. The hydrodynamic damping of the second mode increases from 1.4% of critical in the absence of current to 12.5% for 1 m/s current.

The effect of stochastic earthquake loads and current on the equivalent cable stiffness are shown in Figure 3.8. The earthquake

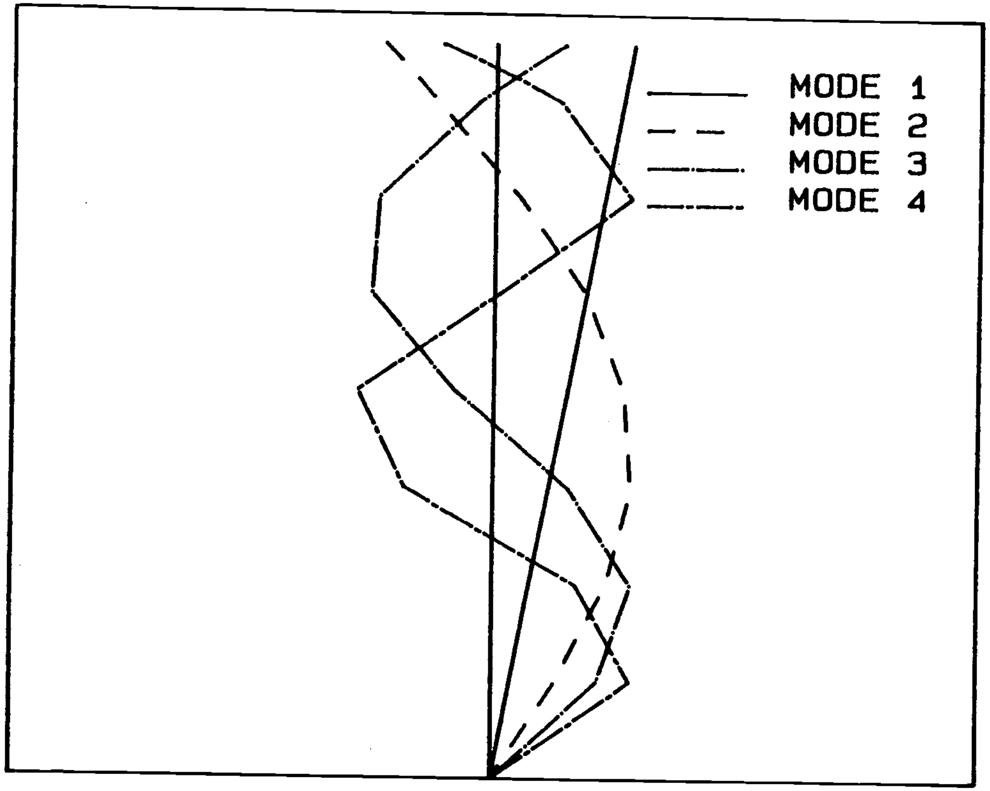

FIGURE 3.4 First four mode shapes for idealized guyed tower.

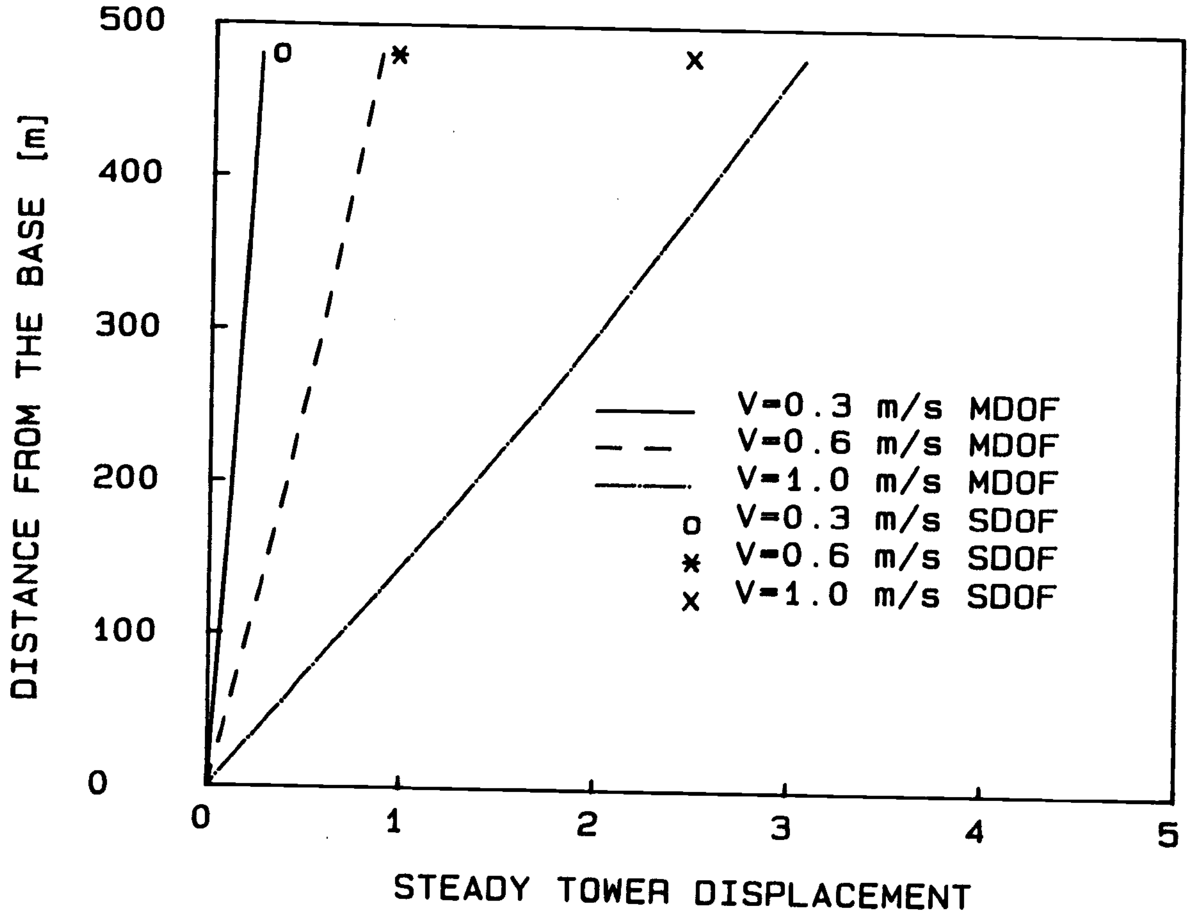

FIGURE 3.5 Steady tower offset due to current.

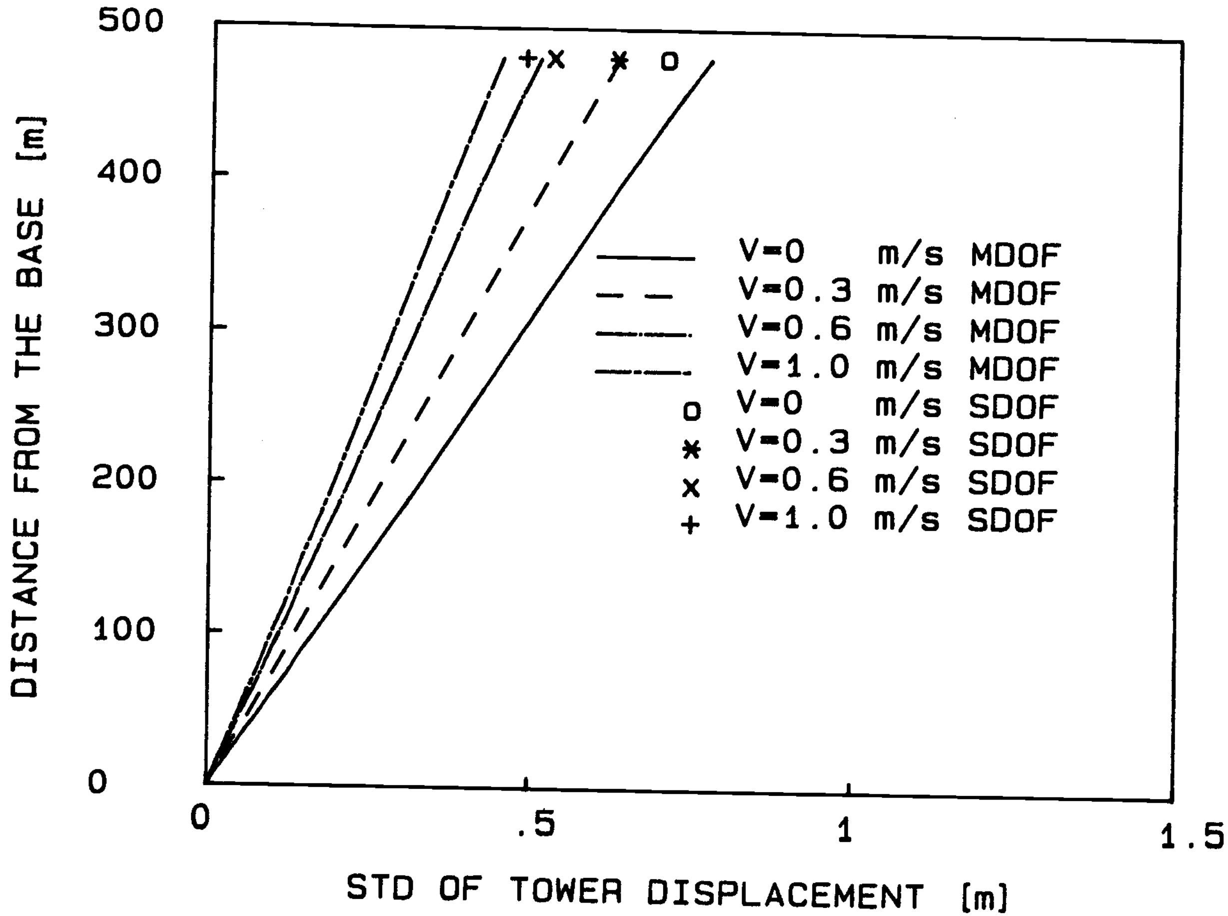

FIGURE 3.6 Standard deviation of the tower displacement due to an earthquake.

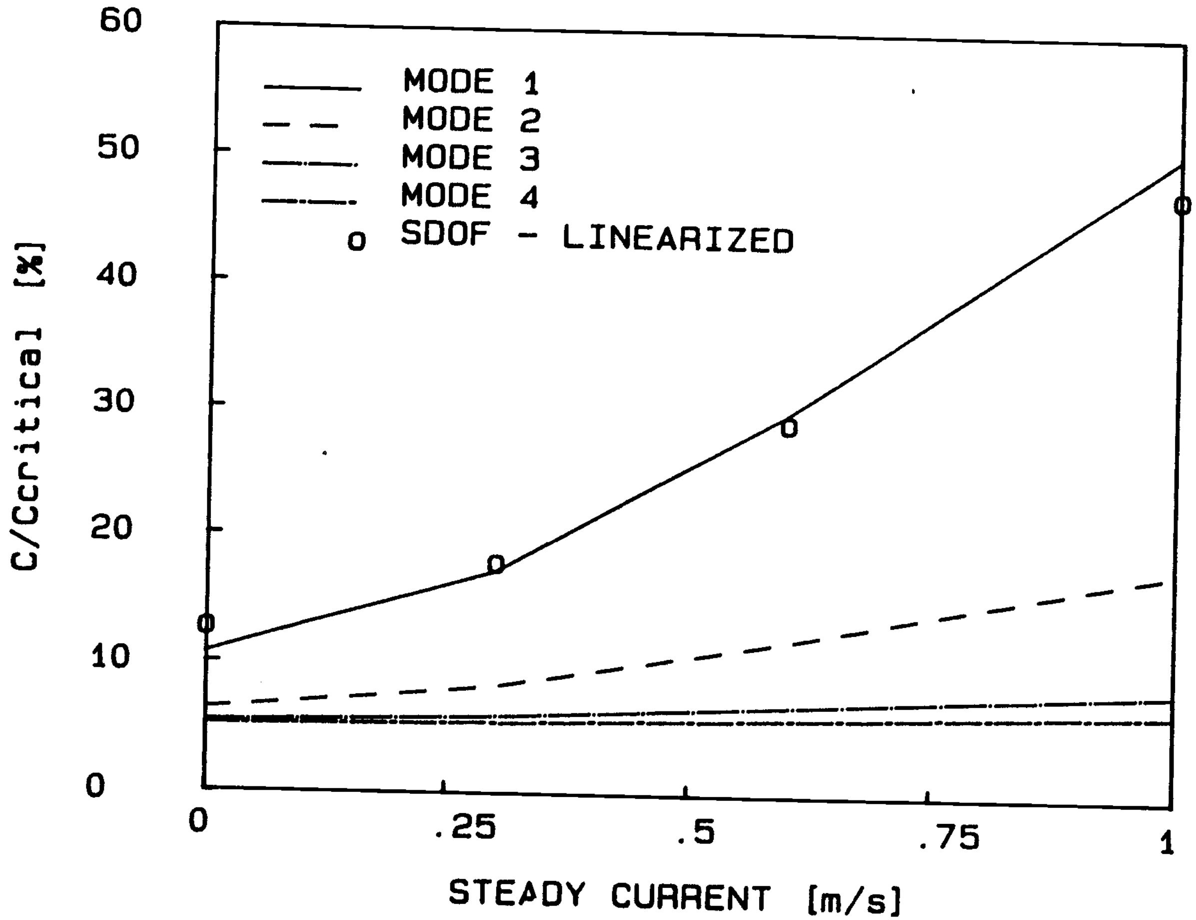

FIGURE 3.7 Total modal damping of first four modes of the guyed tower, $\zeta_T = \zeta_s + \zeta_h$.

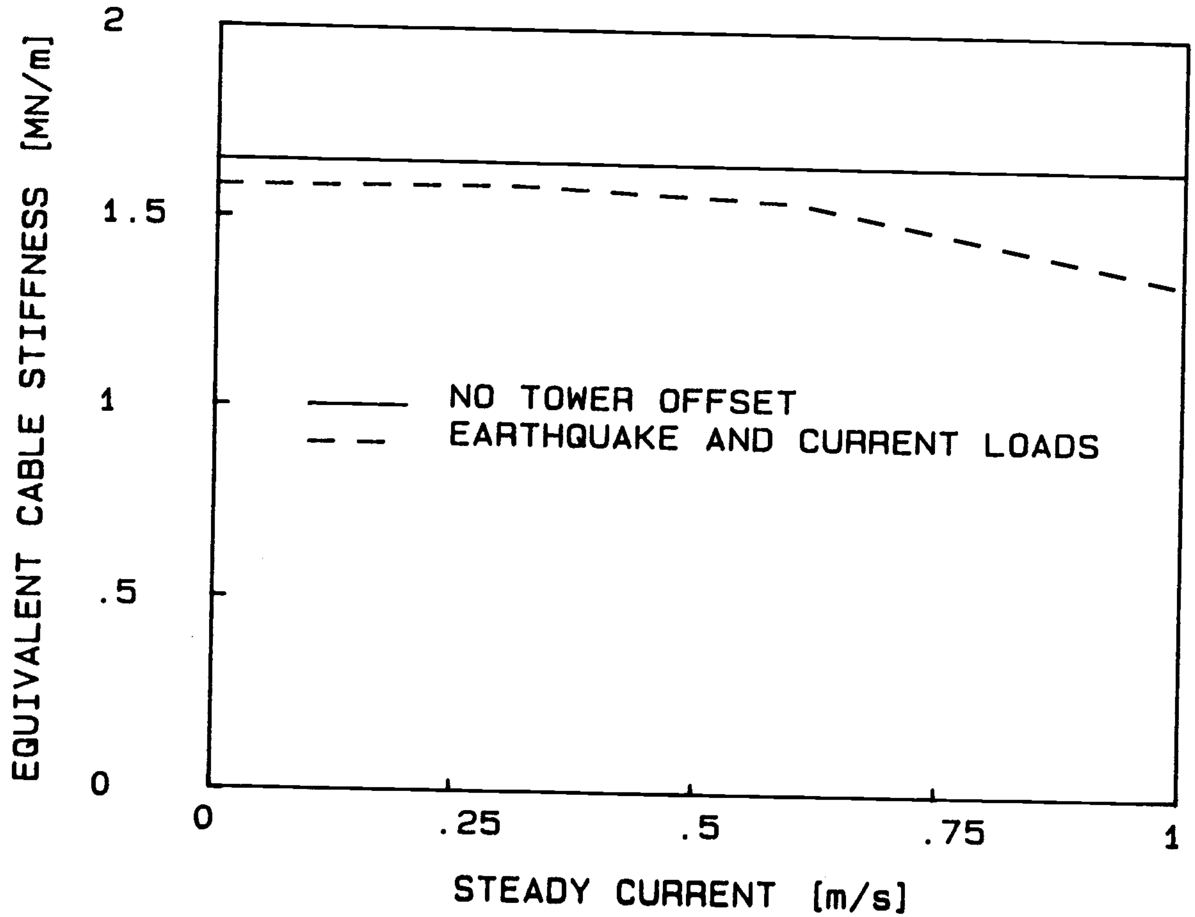

FIGURE 3.8 Linearized guyline stiffness.

reduces the linear cable stiffness by 4% in the absence of current, but when a 1 m/s current is added, the linear stiffness drops about 20%. That affects the natural frequency of the first mode which drops 9% compared to the linear frequency (nonlinear effects ignored). The higher modes do not change significantly with this change in the cable stiffness.

The moments throughout the tower due to the steady current are shown in Figure 3.9. The maximum moment increases with steady current since the steady drag forces are approximately proportional to the current velocity squared and the cable stiffness become softer with increasing tower offset.

The standard deviations of the shear forces are plotted in Figure 3.10. The maximum values occur at the base of the tower and at the cable attachment to the tower. The distribution of standard deviation of the moments obtained from these shear forces are shown in Figure 3.11. The presence of current reduces the standard deviations of the moments since damping of the structure is increased. If the maximum tower moment due to the earthquake is estimated as the steady moment plus or minus three standard deviations, then it is apparent that the steady moment is also important since they are of the same order of magnitude. So even though the standard deviation of the moment drops from 249.0 MN m with no current to 171.5 MN m with 1 m/s current, the absolute maximum moment remains about the same, 747 MN m and 771 MN m respectively.

The moment at the tower base was also compared to a SDOF nonlinear time simulation. The frequency domain solution gave consis-

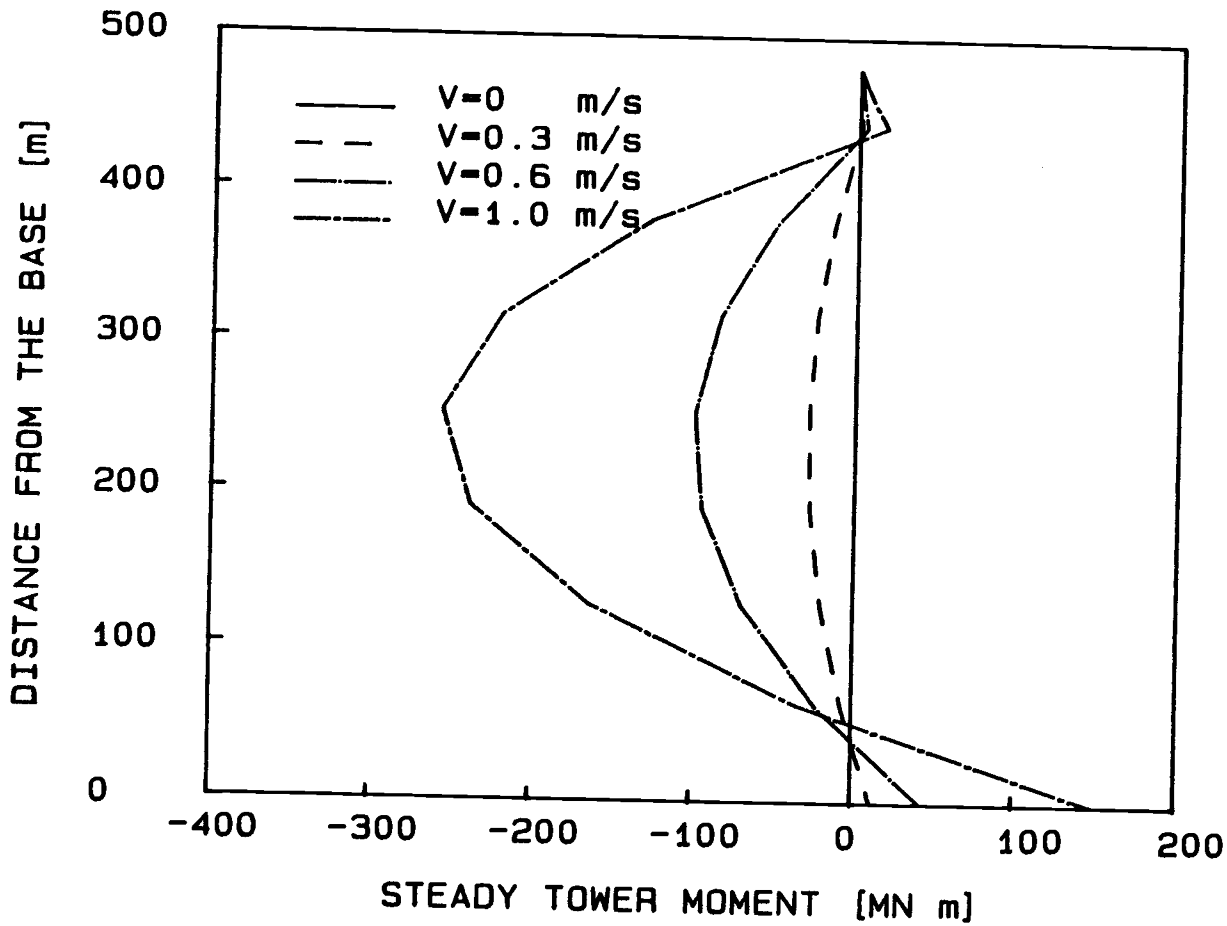

FIGURE 3.9 Tower momnets due to steady current.

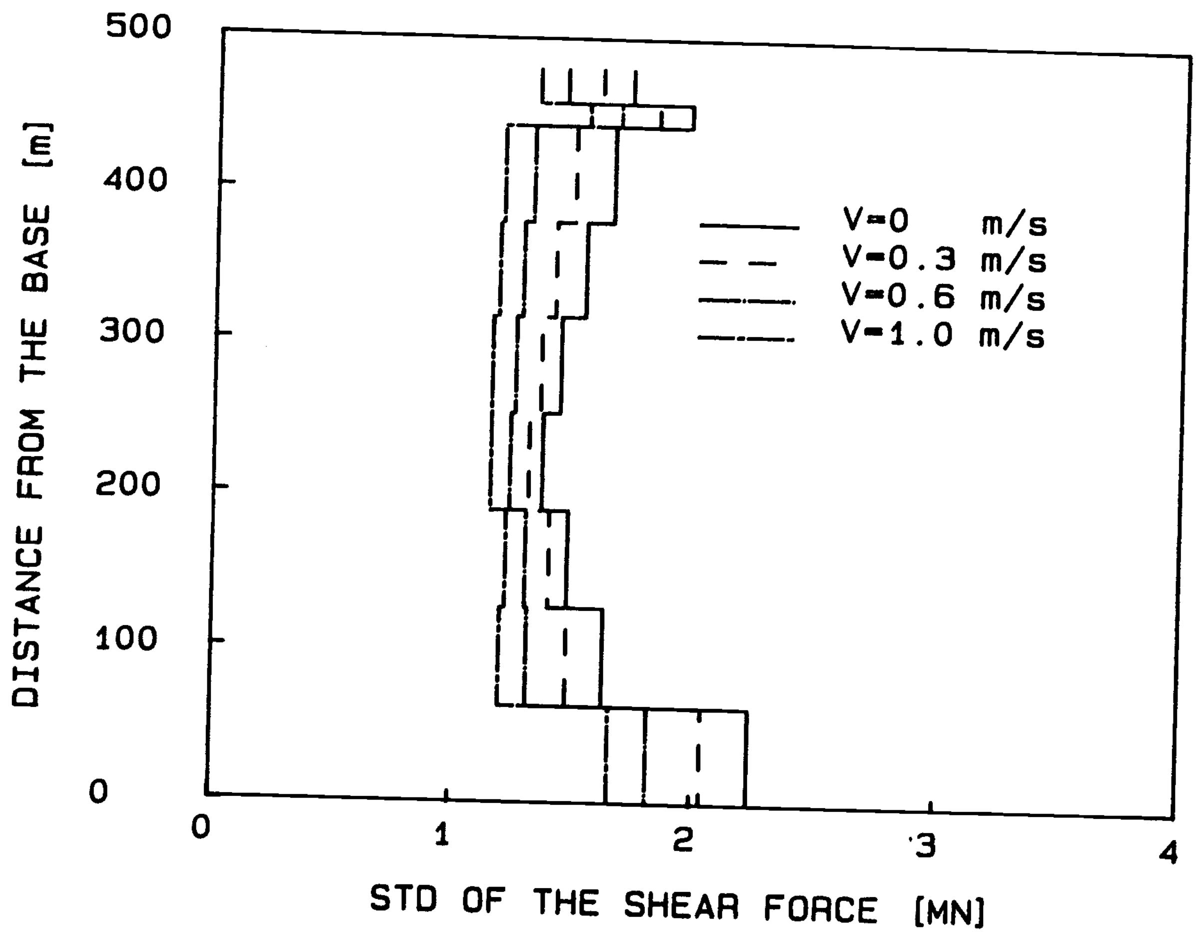

FIGURE 3.10 Standard deviation of the shear force along the tower due to an earthquake.

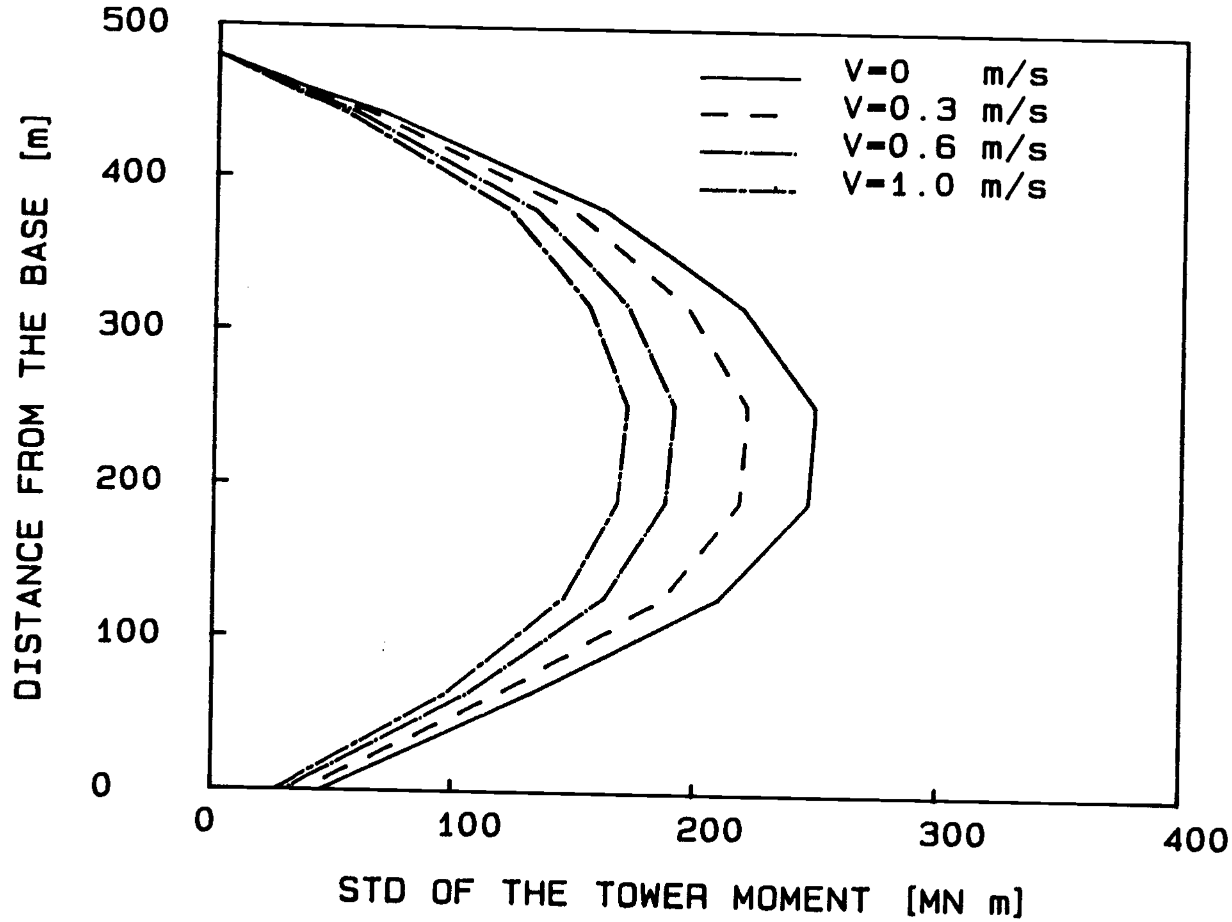

FIGURE 3.11 Standard deviation of the moment distribution along the tower.

tently higher moment at the base than the SDOF system. That is probably due to higher mode contributions to the rotation at the base.

Figures 3.12, 3.13 and 3.14 show the power spectral density of the tower displacement and the tower moments at the middle and the base of the tower. The displacement is mainly due to the first mode, the pendulum mode, but there is also a contribution from the second mode. Higher modes did not contribute significantly to the tower displacement. The moment at the center is governed by the second mode, the bending mode, with some contribution from the first mode. The peak of the power spectral density function for the first mode is an order of magnitude less than for the second mode as can be seen from Figure 3.13. The peaks for the third and fourth mode are then two order of magnitudes less than for the second mode. The moment at the base is mainly due to the first mode, with some contributions from the second mode as mentioned previously.

CONCLUSION

In this study, a linearized multiple degree of freedom model of a guyed tower was developed using the stochastic linearization approach. It was solved in the frequency domain, giving the statistical response of the guyed tower due to a strong motion earthquake and a steady uniform current. The displacement statistics were compared to the statistics of a SDOF time simulation that fully incorporated the nonlinearities of the cable system and the fluid-structure interaction.

It was found that the displacement statistics from the MDOF linearized model agree reasonably well with the results from the time

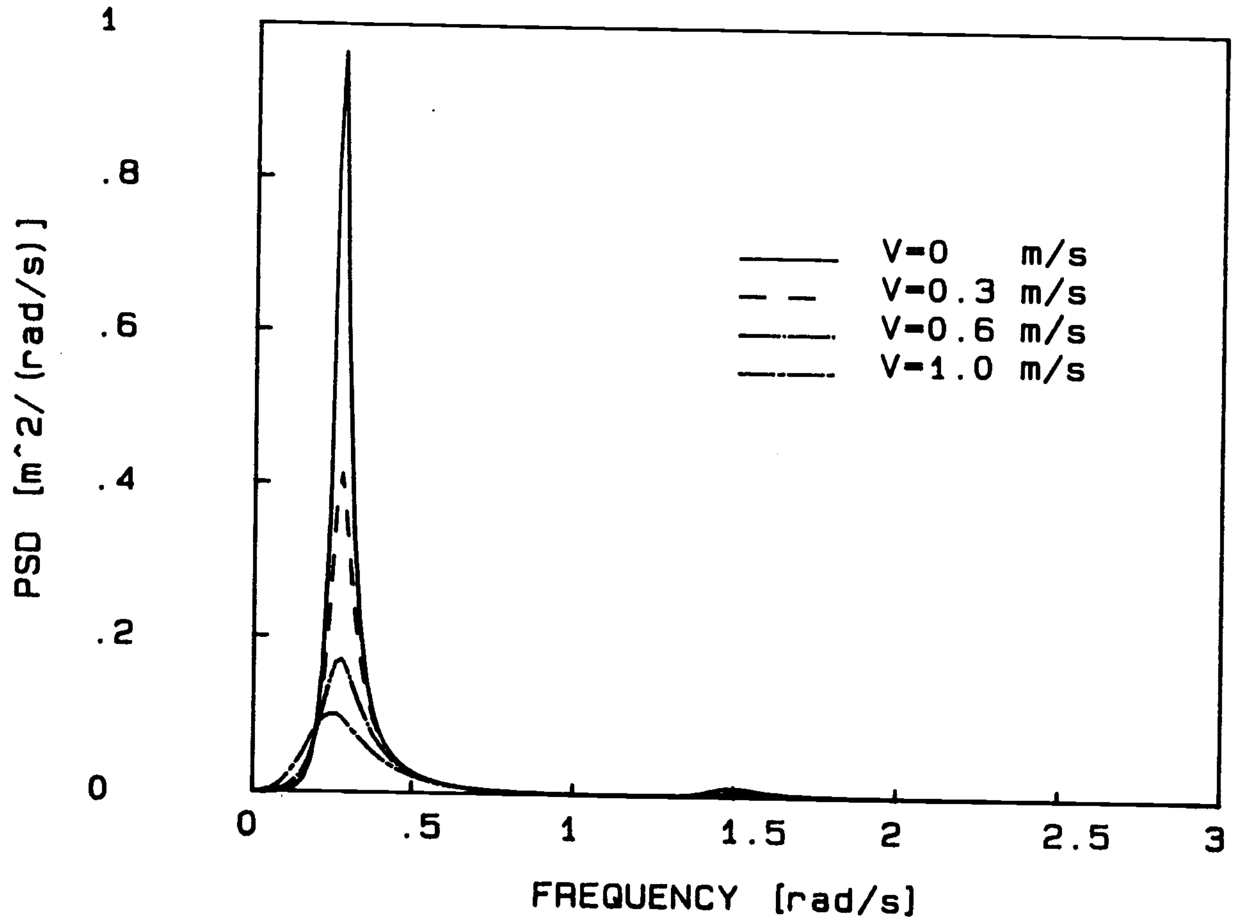

FIGURE 3.12 Power spectral density of the tower displacement 252 m above the sea bottom.

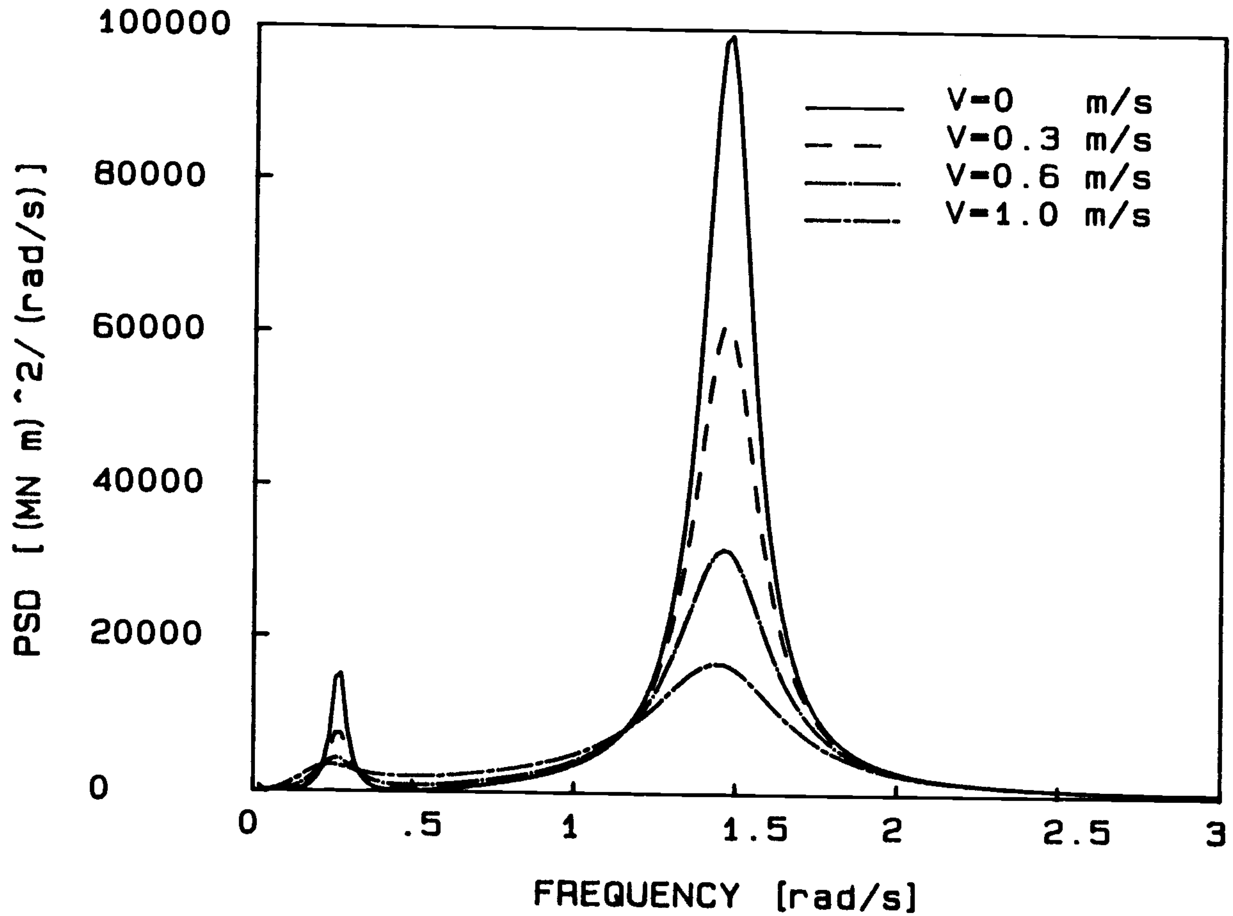

FIGURE 3.13 Power spectral density of the tower moment 252 m above the sea bottom.

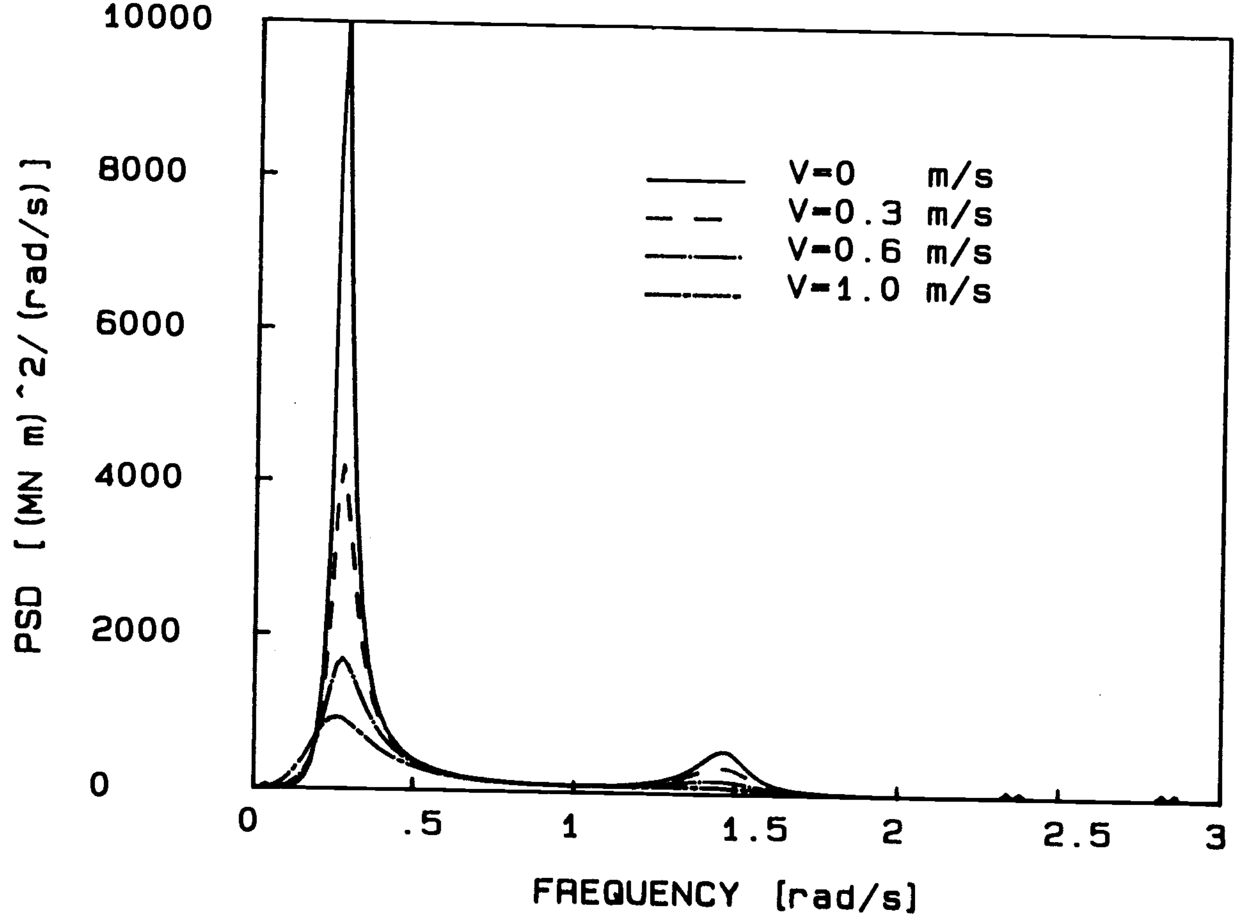

FIGURE 3.14 Power spectral density of the tower moment at the base of the tower.

simulation of the fully nonlinear SDOF system. The steady current was found to have a significant effect on the statistics of the forces and moments on the tower. It reduces the moments since the damping of the first two modes increases significantly with increasing current. However, the maximum moment on the tower was found to be similar with and without a steady current, since the steady moment increases with current but the standard deviation of the moment decreases with current. The first four modes of vibration in the MDOF model contributed to the force and moment statistics of the structure. It is therefore necessary to include at least four modes of vibration when calculating the forces and moments of a guyed tower due to stochastic earthquake loads. The computer time using the stochastic linearization approach is several order of magnitude less than for a conventional time simulation method. Considering other uncertainties in analysing guyed towers (selection of drag and inertia coefficients, specifying earthquake spectrum) it is economical to use this method in the early design stages of guyed towers.

Further research is needed to include the effects of the vertical component of the ground displacement. It might have significant effect on the structure and cause buckling in some members of the structure. Also, since ground velocity is an important parameter when calculating the loads on an offshore structure, further research should be devoted to investigate the power spectral density of the ground displacements, to accurately represent the PSD of the ground velocity. And, as always, there is a need for comparison of the analytical results with model tests to achieve further knowledge about the uncertainties involved in the mathematical formulation.

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REFERENCES

1. Mo, M., and Moan, T., "Environmental Load Effect Analysis of Guyed Towers," Proceedings, 3rd International Offshore Mechanics and Arctic Engineering Symposium, Lousiana, February 1984.

2. Spanos, P.D., "Monte Carlo Simulations of Responses of Non-Symmetric Dynamic System to Random Excitations," Computers & Structures, vol. 13, pp.371-376, 1981.

3. Nair, D., Valdivieso, J.B. and Johnson, C.M., "Comparison of Spectrum and Time History Techniques in Seismic Design of Platforms," Proceedings, Offshore Technology Conference, OTC 3823, May 1980.

4. Finn, L.D and Young, K.E., "Field Test of a Guyed Tower," Proceedings, Offshore Technology Conference, OTC 3131, pp. 681-688, May 1978.

5. Hanna, S.Y., Mangiavacchi, A. and Suhendra, R., "Nonlinear Dynamic Analysis of Guyed Tower Platforms," Journal of Energy Resources Technology, vol. 105, pp.205-211, June 1983.

6. Anagnostopoulos, S.A, "Dynamic Response of Offshore Platforms to Extreme Waves Including Fluid - Structure Interaction," Engineering Structures, vol. 4, pp. 179-185, July 1984.

7. Morrison, D.G., "Guyed Tower With Dynamic Mooring Properties," Journal of Structural Engineering, vol. 109, no. 11, pp. 2578-2590, November 1983.

8. Leonard, J.W. and Young, R.A., "Coupled Response of Compliant Offshore Platforms," Engineering Structures, vol. 7, no. 2, pp. 74-84, April 1985.

9. Brinkmann, C.R., "Dynamic Interaction of a Guyed Tower With Its Guying System," Transactions of the ASME, vol. 105, pp. 290-295, September 1983.

10. Wilson, J.F. and Orgill, G., "Optimal Cable Configurations for Passive Dynamic Control of Compliant Towers," Journal of Dynamic Systems, Measurement, and Control, vol. 106, no. 4, December 1984.

11. Wu, S.C., "The Effects of Current on Dynamic Response of Offshore Platforms," Proceedings, Offshore Technology Conference, OTC 2540, pp. 185-196, May 1976.

12. Dutta, A., "Some Fundamental Studies on Guyed Tower Response Under Wave Loadings," Proceedings, 5th International Offshore Mechanics and Arctic Engineering Symposium, Japan, April 1986

13. Sekita, K. and Maruyanna, M., "Model Tests on Guyed Towers with the Focus on their Nonlinear Characteristics," Proceedings, 5th International Offshore Mechanics and Arctic Engineering Symposium, Japan, April 1986

14. Spanos, P.D., "Random Response to Flow-Induced Forces," Journal of the Engineering Mechanics Division, Proceedings ASCE, vol. 107, No. EM6, December 1981.

15. Spanos, P.D., "Stochastic Linearization in Structural Dynamics," Applied Mechanics Review, vol. 34, no. 1, January 1981

16. Iwan, W.D. and Yang, I-M., "Application of Statistical Linearization Techniques to Nonlinear Multidegree-of-Freedom Systems," Transactions of the ASME, Journal of Applied Mechanics", June 1972.

17. Iwan, W.D. and Mason, A.B., "Equivalent Linearization for Systems Subjected to Non-Stationary Random Excitation," International Journal of Non-Linear Mechanics, vol. 15, pp. 71-82, 1980.

18. Brynjolfsson, S. and Leonard, J.W., "Response of Guyed Offshore Towers to Stochastic Loads, Time Domain vs. Frequency Domain," to be published.

19. Smith, E. and Sigbjornsson, R., "Nonlinear Stochastic Analysis of Compliant Platforms," Proceedings, Offshore Technology Conference, OTC 3801, May 1980.

20. Eatock Taylor, R. and Rajagopalan, A., "Load Spectra for Slender Offshore Structure in Waves and Currents," Earthquake Engineering and Structural Dynamics, vol. 11, pp. 831-842, 1983.

21. Gudmestad, O.T. and Connor, J.J., "Linearization Methods and the Influence of Current on the Nonlinear Hydrodynamic Drag Force," Applied Ocean Research, vol. 5, no. 4, pp. 184-194, 1983.

22. Kirkley, O.M. and Murtha, J.P., "Earthquake Response Spectra for Offshore Structures," Proceedings, Offshore Technology Conference, OTC 2356 May 1977.

23. Penzien, J., Kaul, M.K. and Berge, B., "Stochastic Response of Offshore Towers to Random Sea Waves and Strong Motion Earthquakes," Computers and Structures, vol. 2, pp. 733-756, 1972.

24. Penzien, J. and Kaul, M.K., "Response of Offshore Towers to Strong Motion Earthquakes," Earthquake Engineering and Structural Dynamics, vol. 1, pp. 55-68, 1972.

25. Clough, R.W. and Penzien, J., "Dynamics of Structures," McGraw-Hill, 1975.

26. ANSYS, users manual, Swanson Analysis Systems Inc., May 1986.

27. Paz, M., "Structural Dynamics, Theory and Computation," Van Nostrand Reinholdt Company, 1980.

28. Cook, R.D., "Concepts and Application of Finite Element Analysis," John Wiley & Sons, 1981.

29. Morison, J.R., O'Brian, M. P., Johnson, J.W. and Shaaf, S.A., "The Force Exerted by Surface Waves on Piles," Petrol. Trans., AIME, vol. 189, pp. 149-154, 1950.

30. Kanai, K., "Semi-empirical Formula for Seismic Characteristics of the Ground," University of Tokyo Bulletin Earthquake Research Institute, vol. 35, pp. 309-325, 1957.

31. Tajimi, H., "A Statistical Method of Determining the Maximum Response of a Building Structure during an Earthquake," Proceedings, 2nd World Conference on Earthquake Engineering, Tokyo, Japan 1960.

32. Atalik, T.S. and Utku, S., "Stochastic Linearization of Multi-Degree-of-Freedom Non-Linear Systems," Earthquake Engineering and Structural Dynamics, vol. 4, pp. 411-420, 1976.

33. Carnahan, B., Luther, H.A. and Wilkes, J.O., "Applied Numerical Methods," Wiley, 1969.

34. Berg, G.V. and Housner, G.W., "Integrated Velocity and Displacement of Strong Earthquake Ground Motion," Bulletin of the Seismological Society of America, vol. 51, no. 2, pp. 175-189, April 1961.

CHAPTER IV

CONCLUSION

In chapter II, the validity of the stochastic linearization method was investigated by comparing the displacement statistics due to stochastic wave and earthquake loads of a fully nonlinear time simulation of a single degree of freedom system to a linearized model of a guyed tower solved in the frequency domain. It was found that the results from the linearized model agree reasonably well with results from the time simulation of the fully nonlinear system. Both models were in good agreement with experimentally observed results.

After verifying that the linearized model gave good results for a SDOF guyed towers, a multiple degree of freedom model of a guyed tower was developed using the stochastic linearization approach. It was then solved in the frequency domain, giving the statistical response of the guyed tower due to a strong motion earthquake. The effect of a steady uniform current on the tower response was also investigated. To verify the MDOF model, the displacement statistics were compared to the statistics of a SDOF time simulation that fully incorporated the nonlinearities of the cable system and the fluid--structure interaction.

It was found that the results from the MDOF linearized model agreed reasonably well with the results from the time simulation of the fully nonlinear SDOF system. The steady current was found to have significant effect on the statistics of the forces and moments on the tower. It reduces the standard deviation of the moments since the damping of the system increases with increasing current. The hydrodynamic damping affected only the first three modes significantly. In this research, the first four modes of vibration contributed to the forces and moments of the tower to stochastic earthquake loads.

The SDOF model is adequate for predicting statistics of displacements, but a MDOF model is needed to accurately predict statistics of maximum moments in the tower.

The computer time using the stochastic linearization approach is several order of magnitude less than for a conventional time simulation method. Considering other uncertainties in analysing guyed towers (selection of drag and inertia coefficients, specifying earthquake spectrum) it is economical to use this this method in the early design stages of guyed towers.

Further work is needed to include the dynamics of the cables in the dynamic analysis, to accurately predict the maximum tension in each individual cable. The nonlinearities of each individual cable segment can be treated in a similar manner as the nonlinearities of the cable system described in chapter III. The vertical component of the ground motion has also been excluded from this analysis. It might have significant effect on the structure and cause buckling in some members of the structure.

The uncertainities involved in the analytical formulation would be, as always, best evaluated from experimental measurements of model towers.

BIBLIOGRAPHY

Anagnostopoulos, S.A, "Dynamic Response of Offshore Platforms to Extreme Waves Including Fluid - Structure Interaction," Engineering Structures, vol. 4, pp. 179-185, July 1984.

ANSYS, users manual, Swanson Analysis Systems Inc., May 1986
Arockiasamy, M., Reddy, D.V., Cheema, P.S. and El-Tahan, H., "Stochastic Response of Compliant Platforms to Irregular Waves," Ocean Engineering, vol. 10, no. 5, pp. 303-312, 1983.

Berg, G.V. and Housner, G.W., "Integrated Velocity and Displacement of Strong Earthquake Ground Motion," Bulletin of the Seismological Society of America, vol. 51, no. 2, pp. 175-189, April 1961.

Borgman, L.E., "Ocean Wave Simulation for Engineering Design," Journal of Waterways and Harbors Division, vol. 95, no. WW4, pp. 557-583, November 1969.

Borgman, L.E., "Techniques for Computer Simulations of Ocean Waves," Topics in Ocean Physics, Italian Physical Society, North-Holland Publishing Company, 1982.

Bretschneider, C.C., "On the Determination of the Design Ocean Wave Spectrum," Look Lab/Hawaii, vol. 7, no. 1, University of Hawaii, 1977.

Brinkmann, C.R., "Dynamic Interaction of a Guyed Tower With Its Guying System," Transactions of the ASME, vol. 105, pp. 290-295, September 1983.

Brynjolfsson, S. and Leonard, J.W., "Response of Guyed Offshore Towers to Stochastic Loads, Time Domain vs. Frequency Domain," to be published.

Carnahan, B., Luther, H.A. and Wilkes, J.O., "Applied Numerical Methods," Wiley, 1969.

Clough, R.W. and Penzien, J., "Dynamics of Structures," McGraw-Hill, 1975.

Doblere, M., Alarcon, E. and Benitz, G., "A Comparison Among Different Spectrum Compatible Earthquake Simulation Methods," Applied Mathematic Modelling, vol. 5, October 1981.

Dutta, A., "Some Fundamental Studies on Guyed Tower Response Under Wave Loadings," Proceedings, 5th International Offshore Mechanics and Arctic Engineering Symposium, Japan, April 1986

Eatock Taylor, R. and Rajagopalan, A., "Load Spectra for Slender Offshore Structure in Waves and Currents," Earthquake Engineering and Structural Dynamics, vol. 11, pp. 831-842, 1983.

Finn, L.D., "A New Deepwater Offshore Platform - The Guyed Tower," Proceedings, Offshore Technology Conference, OTC 2688, May 1976.

Finn, L.D and Young, K.E., "Field Test of a Guyed Tower," Proceedings, Offshore Technology Conference, OTC 3131, pp. 681-688, May 1978.

Gudmestad, O.T. and Connor, J.J., "Linearization Methods and the Influence of Current on the Nonlinear Hydrodynamic Drag Force," Applied Ocean Research, vol. 5, no. 4, pp. 184-194, 1983.

Hanna, S.Y., Mangiavacchi, A. and Suhendra, R., "Nonlinear Dynamic Analysis of Guyed Tower Platforms," Journal of Energy Resources Technology, vol. 105, pp.205-211, June 1983.

Iwan, W.D, "On Defining Equivalent Systems for Certain Ordinary Non-Linear Differential Equations," International Journal of Nonlinear Mechanics, vol. 4, pp. 325-334., 1969

Iwan, W.D, "A Generalization of the Concept of Equivalent Linearization," International Journal of Nonlinear Mechanics, vol. 8, pp. 279- 287, 1973

Iwan, W.D. and Yang, I-M., "Application of Statistical Linearization Techniques to Nonlinear Multidegree-of- Freedom Systems," Transactions of the ASME, Journal of Applied Mechanics", June 1972.

Iwan, W.D. and Mason, A.B., "Equivalent Linearization for Systems Subjected to Non-Stationary Random Excitation," International Journal of Non-Linear Mechanics, vol. 15, pp. 71-82, 1980.

Kanai, K., "Semi-empirical Formula for Seismic Characteristics of the Ground," University of Tokyo Bulletin Earthquake Research Institute, vol. 35, pp. 309-325, 1957.

Kirkley, O.M. and Murtha, J.P., "Earthquake Response Spectra for Offshore Structures," Proceedings, Offshore Technology Conference, OTC 2356 May 1977.

Leonard, J.W. and and Young, R.A., "Coupled Response of Compliant Offshore Platforms," Engineering Structures, vol. 7, no. 2, pp. 74-84, April 1985.

Mo, M., and Moan, T., "Environmental Load Effect Analysis of Guyed Towers," Proceedings, 3rd International Offshore Mechanics and Arctic Engineering Symposium, Lousiana, February 1984.

Morison, J.R., O'Brian, M. P., Johnson, J.W. and Shaaf, S.A., "The Force Exerted by Surface Waves on Piles," Petrol. Trans., AIME, vol. 189, pp. 149-154, 1950.

Morrison, D.G., "Guyed Tower With Dynamic Mooring Properties," Journal of Structural Engineering, vol. 109, no. 11, pp. 2578-2590, November 1983.

Nair, D., Valdivieso, J.B. and Johnson, C.M., "Comparison of Spectrum and Time History Techniques in Seismic Design of Platforms," Proceedings, Offshore Technology Conference, OTC 3823, May 1980.

Newland, D.E., "An Introduction to Random Vibration and Spectral Analysis," Longman, 1975.

Newmark, N.M. and Rosenblueth, E., "Fundamentals of Earthquake Engineering," Prentice-Hall, 1971.

Paz, M., "Structural Dynamics, Theory and Computation," Van Nostrand Reinholdt Company, 1980.

Penzien, J., Kaul, M.K. and Berge, B., "Stochastic Response of Offshore Towers to Random Sea Waves and Strong Motion Earthquakes," Computers and Structures, vol. 2, pp. 733-756, 1972.

Penzien, J. and Kaul, M.K., "Response of Offshore Towers to Strong Motion Earthquakes," Earthquake Engineering and Structural Dynamics, vol. 1, pp. 55-68, 1972.

Penzien, J., "Seismic Analysis of Platform Structure-Foundation Systems," Proceedings, Offshore Technology Conference, OTC 2352, May 1975.

Philippacopoulos, A.J. and Wang, P.C., "Seismic Inputs for Nonlinear Structures," Journal of Engineering Mechanics, vol. 110, no. 5, pp. 828-836, May 1984.

Ruitz, P. and Penzien, J., "Stochastic Seismic Response of Structures," Journal of the Engineering Mechanics Division ASCE, April 1971.

Sarpkaya, T. and Isaacson, M., "Mechanics of Wave Forces on Offshore Structures," Van Nostrand Reinhold Company, 1981.

Sekita, K. and Maruyanna, M., "Model Tests on Guyed Towers with the Focus on their Nonlinear Characteristics," Proceedings, 5th International Offshore Mechanics and Arctic Engineering Symposium, Japan, April 1986

Shinozuka, M., "Monte Carlo Solution of Structural Dynamics," Computers and Structures, vol. 2 pp. 855-876, 1972.

Smith, E. and Sigbjornsson, R., "Nonlinear Stochastic Analysis of Compliant Platforms," Proceedings, Offshore Technology Conference, OTC 3801, May 1980.

Spanos, P.D., "Probabilistic Earthquake Energy Spectra Equations," Journal of the Engineering Mechanics Division ASCE, vol. 106 pp. 147-159, February 1980.

Spanos, P.D., "Monte Carlo Simulations of Responses of Non-Symmetric Dynamic System to Random Excitations," Computers & Structures, vol. 13, pp.371-376, 1981.

Spanos, P.D., "Random Response to Flow-Induced Forces," Journal of the Engineering Mechanics Division, Proceedings ASCE, vol. 107, No. EM6, December 1981.

Spanos, P.D., "Stochastic Linearization in Structural Dynamics," Applied Mechanics Review, vol. 34, no. 1, January 1981

Spanos, P.D. and Loli, V., "A Statistical Approach to Generation of Design Spectrum Compatible Earthquake Time Histories," Soil Dynamics and Earthquake Engineering, vol. 4, no. 1, 1985.

Tajimi, H., "A Statistical Method of Determining the Maximum Response of a Building Structure during an Earthquake," Proceedings, 2nd World Conference on Earthquake Engineering, Tokyo, Japan 1960.

Triantafyllou, M. S., Kardomates, G. and Bliek, A., "The Statics and Dynamics of the Mooring Lines of a Guyed Tower for Design Applications," Proceedings, BOSS 1982, vol. 1, pp. 546-564.

Wilson, J.F. and Orgill, G., "Optimal Cable Configurations for Passive Dynamic Control of Compliant Towers," Journal of Dynamic Systems, Measurement, and Control, vol. 106, no. 4, December 1984.

Wilson, J.F., editor,"Dynamics of Offshore Structures," John Wiley & Sons, 1984.

Wishahy, M.A. and Arockiasamy, M., "Response of Guyed Towers for Linear and Nonlinear Behaviour of Mooring Cables," Canadian Journal of Civil Engineering, vol. 12, no. 1, pp. 200-212, March 1985

Wu, S.C., "The Effects of Current on Dynamic Response of Offshore Platforms," Proceedings, Offshore Technology Conference, OTC 2540, pp. 185-196, May 1976.