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Martin W. Ritchie for the degree of Master of Science in Forest Management presented on April 25, 1985.

Title: Development of Tree Height and Diameter Growth Equations for Mid-Willamette Valley Douglas-fir

Abstract approved: Signature redacted for privacy.
David W. Hann

Equations for predicting height growth, basal area growth and diameter inside bark are presented for Douglas-fir. Basal area growth equations for grand fir are also presented. The growth models were developed for use in an individual tree/distance independent growth simulator. Various model forms and measures of competitive stress were compared in both the height and basal area growth models. The height growth equation is a function of crown ratio and height divided by dominant stand height. Basal area growth is a function of tree diameter, crown ratio, crown competition at the plot level, stand basal area and either site index or predicted height growth. The diameter inside bark equation, used in backdating tree diameters, is a nonlinear function of diameter outside bark.

Development of Tree Height and Diameter Growth Equations for Mid-Willamette Valley Douglas-fir

by

Martin W. Ritchie

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APPROVED:

Signature redacted for privacy.

Associate Professor of Forest Management in charge of major

Signature redacted for privacy.

Head of the department of Forest Management

Signature redacted for privacy.

Dean of Graduate School

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Typed by Martin W. Ritchie

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CHAPTER 1

INTRODUCTION

The use of computer simulators of forest growth and yield has gained wide acceptance in forest management. Simulators are frequently used in stand management planning and in evaluation of alternative silvicultural treatments. Methods of projecting growth of stands, or trees, can also be useful in periodically updating existing forest inventory data.

Forest managers for Oregon State University's forest properties, in an effort to establish and maintain a forest wide data base, elected to develop an individual tree/distance independent growth model (Munro 1974). This modeling effort was conducted simultaneously with the installation of permanent inventory plots forest wide and the establishment of a data base management system described by Herzog (1984). The final goal of the overall project is to establish an integrated information system, to be available for use by forest managers, researchers, and students. The growth and yield simulator is but one element of this data management system.

None of the existing simulators are capable of providing the information needed on the University's Forest Properties. The only simulator available for western Oregon at this writing is DFSIM (Curtis et al. 1981). The DFSIM simulator has a number of limitations which make it unacceptable for many anticipated uses on OSU's Forest Properties. The model is not recommended for stands with less than 80% basal area in Douglas-fir or for multi-storied stands. Furthermore, DFSIM does not provide detailed descriptions of stand volumes by diameter classes or species groups. In addition, DFSIM is not recommended for updating forest inventory data.

Modeling Considerations

The individual tree/distance independent approach was chosen for this model. The required input data for this type of simulator will be readily available from the permanent inventory plots being established.

An individual tree/distance independent model is one which projects stand growth by simulating the growth of individual sample trees and then aggregating individual tree projections to provide stand growth and yield estimates. Distance independent refers to the nature of expressing competitive stress on individual trees without any consideration of actual distances between a subject tree and its competitors.

It is important that models developed for use in forest management make use of commonly measured stand and tree variables as predictors. The variables measured in forest inventory will vary with different ownerships and may change with time for any given forest depending on the needs of management. Because this model is being developed for a single ownership with a uniform inventory data base, the acceptable variables for defining the functional relationships of each component in the simulator are well defined.

The model components used to simulate stand dynamics include equations for predicting diameter increment, height increment, change in crown length and a model for predicting the occurrence of mortality. These equations are generally species specific, depending on the availability of data for a given species model development.

In this thesis, estimators for diameter increment for Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) and grand fir (*Abies grandis* (Dougl.) Lindl.), and height increment for Douglas-fir are presented. In addition, a model for predicting breast height diameter inside bark given outside bark diameter is presented.

The framework for integrating these components of tree growth is not an element of this study, neither is the development of crown change or mortality models.

The diameter inside bark (DIB) model was developed prior to modeling height or basal area increment. The model was used in backdating plot measurements. Backdating is necessary to estimate values for regression analysis at the start of the growth period. Inside bark diameter equations provide indirect estimates of change in bark thickness.

A height increment model is presented for Douglas-fir. The height increment model provides a means of predicting five year height growth given stand and tree variables. In addition, predicted values from this model can be considered as an indicator of site productivity in the basal area growth equations.

Basal area increment equations are presented for Douglas-fir and grand fir as a means of projecting future diameters of individual trees. The dependent variable in this model is five year basal area increment inside bark (expressed in square inches). Two models are presented for each species. One model includes predicted height growth as an index of site productivity. The other model includes site index instead of predicted height.

The primary advantage of including predicted height growth as an independent variable in the basal area increment equations is that it is a means of assuring that the two components are working harmoniously in the simulator. By linking the two components in this fashion, a stochastic element can be easily accommodated. The random error can be introduced to the height growth model and be carried through to the basal area increment model. A similar approach was used by Stage (1973), in introducing the error term in the basal area increment model.

Study Area

Oregon State University's College of Forestry maintains 14,062 acres of forest land, the majority of which is contained in McDonald and Dunn forests. McDonald forest and Dunn forest are adjacent tracts of land comprising approximately 11,500 acres located northwest of Corvallis in Benton County, Oregon. Most of the remaining acreage is in the Blodgett Tract in Columbia County, Oregon.

The ownership is dominated by second growth stands of Douglas-fir and grand fir with associated hardwoods, primarily bigleaf maple (Acer macrophyllum Pursh), red alder (Alnus rubra Bong.), madrone (Arbutus menziesii Pursh) and Oregon white oak (Quercus garryana Dougl.). These stands are primarily even-aged, although some two storied stands can be found. Site index (King 1966) ranges from 90 to 140 (high site IV to low site I) and stand ages range from 0 to 120 with a majority of the stands less than 50 years old.

The precipitation on McDonald and Dunn forests ranges from 40 to 60 inches per year. Elevations range from 300 to 2000 feet. The soils are predominantly well drained and of moderate depth. The three major soil associations on the forest are McAlpin-Abiqua, Dixonville-Philomath, and Price-Ritner.

Data

In a modeling effort such as this, one of the primary concerns is obtaining an adequate data base. The data requirements for growth can be quite costly. In order to adequately represent all the possible combinations of factors influencing tree growth, large numbers of observations are required. There are two approaches to measuring tree or stand growth. The first approach involves repeat measurements of plots over time. The alternative is to measure past growth on temporary plots, and then estimate stand and tree variables at the beginning of the growth period.

The growth measurements for this study were all taken on past growth in 136 stands. This established a five year growth period ending with the year of measurement. Growth data were collected from 1981 to 1983. A total of 20,143 standing trees were measured in this study. Of these, 723 Douglas-fir were felled for more detailed measurement. The standing tree measurements were meant to provide the data for mortality and basal area increment models. The felled trees provided the data for height increment and bark thickness models. A more detailed description of the data and the models developed are presented in the following chapters.

CHAPTER 2

NONLINEAR EQUATIONS FOR PREDICTING DIAMETER AND SQUARED DIAMETER INSIDE BARK AT BREAST HEIGHT FOR DOUGLAS-FIR

Abstract

Regression equations are presented for predicting diameter inside bark at breast height and squared diameter inside bark for Douglas-fir with diameter and squared diameter outside bark as the independent variables. Three types of equations were fitted to data collected from 724 Douglas-fir felled in western Oregon. A nonlinear model with a weight of $1.0/DOB^2$ provided a better fit according to Furnival's index of fit than did either a log-linear model or a weighted linear model fitted with ordinary least squares. Three applications of the equations are presented: estimating past diameters, estimating past basal area growth inside bark and converting predicted basal area growth inside bark to basal area growth outside bark.

Radial growth inside bark, as measured from increment cores, is often used in estimating past diameters. And, change in squared diameter inside bark has been used in the development of equations for predicting basal area growth (Cole and Stage 1972). Because diameter outside bark is the variable most commonly measured, such measurements must often be converted to dimensions inside bark. In this paper, regression equations are presented for predicting diameter inside bark at breast height and squared diameter inside bark for Douglas-fir on the basis of diameter outside bark. Application of these equations in studies of tree growth are discussed.

BACKGROUND

Diameter Inside Bark

Many of the previous studies of the relationship between diameters inside and outside bark have been directed toward obtaining estimates of past diameter. Typically, a linear relationship between diameters inside and outside bark has been assumed, simplifying both the estimation procedure and application of results. Johnson (1955, 1956) and Spada (1960) both used ordinary least squares regression to predict past diameter on the basis of double bark thickness:

$$BT = B_0' + B_1' \cdot DOB + e \qquad (1)$$

where:

BT = double bark thickness

B_0', B_1' = population parameter values

DOB = Diameter outside bark

e = a random error component with an expected value of zero and variance of σ^2.

Model (1) can be rearranged to provide the model used by Finch (1948) and Dolph (1981) for estimating past diameters:

$$DIB = B_0 + B_1 \cdot DOB + e \qquad (2)$$

where:

DIB = diameter inside bark

$B_0 = - B_0'$

$B_1 = 1 - B_1'$.

In working with model (2), Dolph (1981) also found that the residual variance exhibited heteroskedasticity. Consequently, he used weighted least squares regression to estimate the parameters.

Common to all of these studies was the inclusion of an intercept term. However, the presence of such a term can lead to questionable estimates of DIB for very small trees. These estimates are questionable because model (2) causes predicted DIB to exceed DOB for DOB's below $B_0/(1.0 - B_1)$. Because model (2) may be incorrectly specified for the full range of diameters, a more tenable form of the model might be

$$DIB = B_1 \cdot DOB + e \quad (3)$$

Both Monserud (1979) and Powers (1969) have applied weighted least squares regression to fit this model, using weights of 1.0/DOB and $1.0/DOB^2$, respectively.

Finally, a nonlinear relationship between DIB and DOB for some species has been reported by Loetsch et al. (1973). They found the nonlinearity to be particularly pronounced for trees under 8 inches in diameter.

Squared Diameter Inside Bark

Squared diameter inside bark (or basal area inside bark) has been predicted by Cole and Stage (1972) and Monserud (1979) according to the following model:

$$DIB^2/DOB^2 = \alpha_1 + e \ .$$

Because this model is the transformed version of a weighted least squares regression model, it can be restated as

$$DIB^2 = \alpha_1 \ DOB^2 + e_1$$

where:

$$Var(e_1) = \sigma^2 \cdot DOB^4 .$$

In this study, regression equations for DIB and DIB^2 will be used in developing estimators for past diameters and basal area growth inside bark, and in an equation for converting basal area growth inside bark to basal area growth outside bark.

THE STUDY

Source of Data

This study was conducted as part of a larger project to develop a stand growth model for Oregon State University's Forest Properties. One element of this project is the development of equations for predicting tree diameter growth. These equations first require estimations of DIB and DIB^2. Therefore, 724 Douglas-fir trees, ranging in DOB from 4.1 to 43.0 inches, were felled and their diameters, diameter growth and height growth were measured. The trees were selected to cover a range of stand conditions and site classes. On each tree, breast-height diameters inside and outside bark were measured to the nearest 0.1 inch according to both the long and the short axis of the cross section. At various times up to a year prior to felling, DOB of each tree had been measured with a diameter tape by a separate inventory crew.

Average diameters inside and outside bark were then calculated as the geometric means of their respective two measurements (Brickell 1976). This method provides unbiased estimates of basal area when the cross section is elliptical. However, the use of DOB calculated in this fashion does present a problem. If errors-in-variables problems are to be avoided, the method used to measure the independent variable should be the same as that which would be used in the application of the model (Monserud 1976). Thus, the diameter-tape measurement would have provided a more suitable independent variable for regression analysis, but because of the intervening diameter growth, the geometric mean diameter was substituted. As a result, the models developed will be slightly biased when applied to trees measured with a diameter tape.

Estimating DIB

Model (3) was fitted to the data by weighted least squares regression; weights of 1.0/DOB, $1.0/DOB^2$, and 1.0 (unweighted) were used. When the three weighting procedures were compared according to Furnival's (1961) index of fit, the index indicated that the weight of $1/DOB^2$ provided the best fit for the data. Residual plots from all three linear regressions revealed an unacceptable trend in the residuals as a result of forcing the regressions through the origin. If an intercept term is allowed in the equation, the trend in residuals is eliminated and the intercept is significant ($P < 0.0001$). A plot of the data shows an apparently linear relationship over the range of diameters sampled (Figure 1). Unfortunately, inclusion of the intercept term in model (2) results in unreasonable estimates for trees less than 2.0 inches in diameter.

In an effort to minimize this undesirable trend, two new models were tried. The first model:

$$\ln(DIB) = \ln(B_1) + B_2 \cdot \ln(DOB) + \ln(e) \qquad (4)$$

was fitted to the data by ordinary least squares regression. The second model:

$$DIB = B_1 \cdot DOB^{B_2} + e \qquad (5)$$

was fitted by nonlinear regression with a weight of $1.0/DOB^2$. Algebraic manipulation of model (5) showed that DIB will exceed DOB for trees under 0.5 inch DOB, a distinct improvement over the performance of model (2). Also, the asymptotic estimate of the 99 percent confidence interval about B_1 for model (5) does not include 1.0; thus, the nonlinear trend in the model may be significant. Parameter estimates (b_1 and b_2) and Furnival's index of fit for models (3), (4), and (5) are found in Table 1. A plot of model (5) is found in Figure 1.

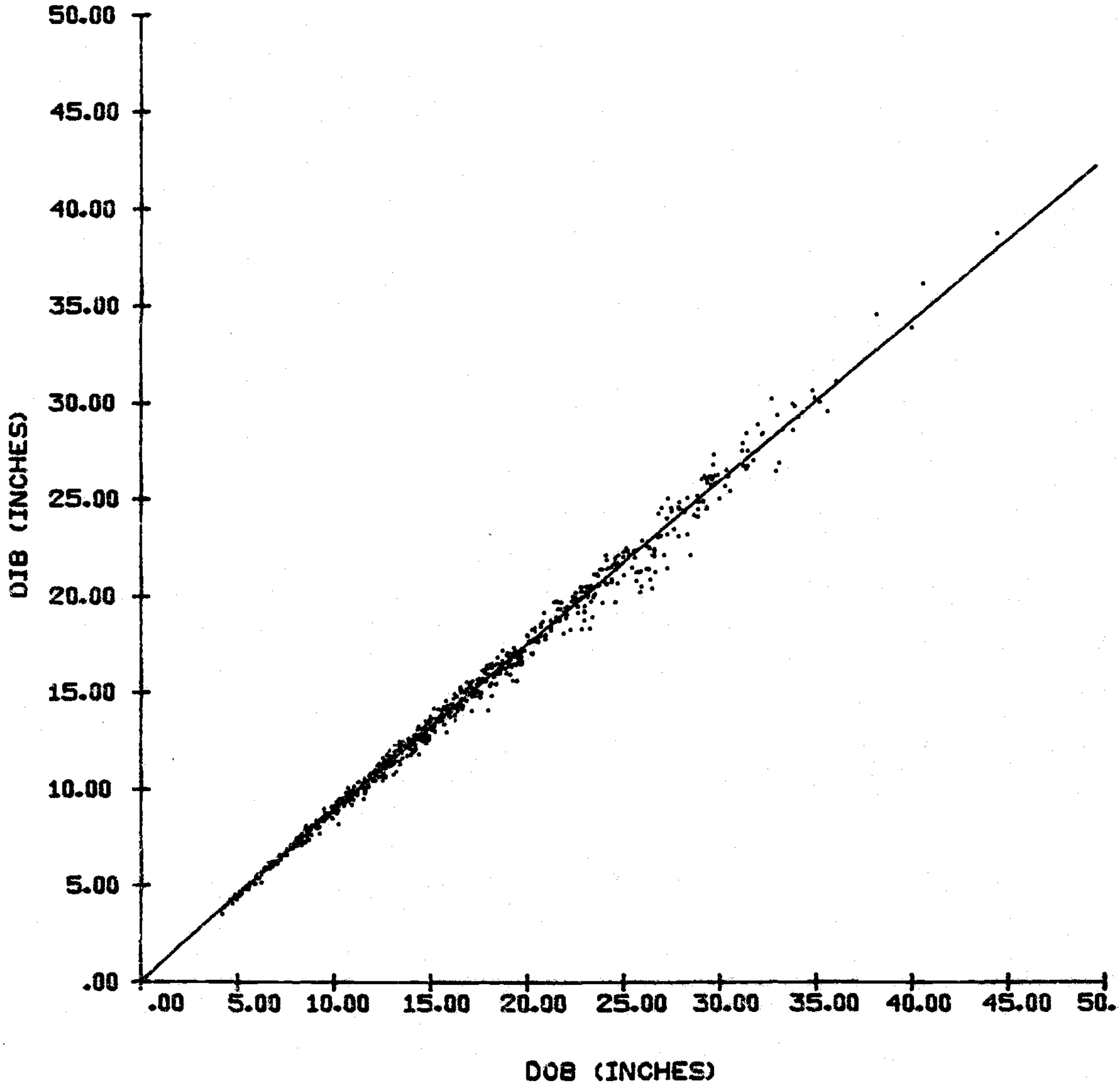

Figure 1. Regression of diameters inside bark at breast height on diameters outside bark for the 724 Douglas-firs. The nonlinear model (5) is plotted as a solid line.

Table 1. Parameter estimates and Furnival's index of fit for the linear, log-linear, and nonlinear models of diameter inside bark

Model	b_1	b_2	Index of fit
(3) Linear	0.887513	1.0	0.4507
(4) Log-linear[1]	0.972252	0.965836	0.4017
(5) Nonlinear	0.971330	0.966365	0.3952

[1] The parameter estimates for model (4) do not include a correction for bias introduced through the log transformation (Flewelling and Pienaar 1981).

Estimating DIB^2

Analysis of the DIB data suggested that a nonlinear model would give the best fit to the DIB^2 data. Therefore, parameters were estimated by nonlinear regression with a weight of $1.0/DOB^4$ for the following model:

$$DIB^2 = \alpha_1 (DOB^2)^{\alpha_2} + \Theta \quad (6)$$

where:

Θ = a random component with expected value of zero and variance of $\sigma^2 \cdot (DOB^4)$.

Estimates of final parameters were 0.941944 for α_1 and 0.966843 for α_2. Again, the asymptotic estimate for the 99 percent confidence interval about α_2 does not include 1.0.

APPLICATION OF RESULTS

DIB

While the applicability of these specific coefficients is limited by the range of the sample, the methodology employed in development and application is sufficiently general to be applied in other areas. The first application of the model for estimating DIB (5) is in the calculation of square-inch basal area increment inside bark (BAG) on the basis of current diameter and radial growth inside bark:

$$BAG = (\pi/4)[DIB^2 - (DIB - 2 \cdot RG)^2]$$

$$BAG = \pi(RG)(DIB - RG) \qquad (7)$$

where:

RG = radial growth (inside bark, in inches).

When model (5) is inserted into model (7) BAG is then estimated as

$$BAG = \pi(RG)[(0.971330 \cdot DOB^{0.966365}) - RG]$$

A second use of model (5) is in the estimation of past diameters. When the relationship between DIB and DOB is linear, the slope coefficient provides an indirect estimate of bark growth, as seen in the following expression for determining change in DOB:

$$DIB = b_0 + b_1 \cdot DOB$$

$$\Delta DIB = b_1 \cdot \Delta DOB$$

$$\Delta DOB = (1.0/b_1)\Delta DIB.$$

Then, in terms of radial growth:

$$\Delta DOB = (1.0/b_1)(2 \cdot RG).$$

Past diameter can then be expressed as

$$DOB_0 = DOB_1 - (2 \cdot RG)/b_1$$

where:

DOB_0 = diameter at the beginning of the growth period

DOB_1 = current diameter.

The nonlinear relationship complicates the estimation procedures somewhat, but the estimate for DOB_0 can be derived, in a similar fashion, as follows:

$$DIB = b_1 \ DOB^{b_2}$$

$$\Delta DIB = b_1(DOB_1^{b_2} - DOB_0^{b_2}).$$

Therefore,

$$DOB_0 = (DOB_1^{b_2} - (2 \cdot RG)/b_1)^{1/b_2}$$

Then, when b_1 and b_2 from model (5) are used:

$$DOB_0 = [DOB_1^{0.966365} - (2 \cdot RG/0.971330)]^{1.03481}$$

DIB^2

The relationship described by model (6) is the basis for deriving predicted basal area outside bark (BA_2) as a function of current diameter and predicted basal area growth inside bark (BAG):

$$DIB^2 = a_1(DOB^2)^{a_2},$$

then:

$$BAG = (\pi/4) \cdot \Delta DIB^2$$

$$= (\pi/4) \ a_1[(DOB_2^2)^{a_2} - (DOB_1^2)^{a_2}].$$

Therefore,

$$DOB_2^2 = [(4 \cdot BAG)/(\pi \cdot a_1) + (DOB_1^{2 \cdot a_2})]^{1/a_2}.$$

Inserting predicted basal area growth inside bark and converting the dependent variable to basal area gives the following solution:

$$BA_2 = (\pi/4)[(4 \cdot BAG)/(\pi \cdot a_1) + DOB_1^{2 \ a_2}]^{1/a_2}$$

$$= 0.785398 \ (1.35171 \cdot BAG + DOB_1^{1.93369})^{1.03429}$$

The Inverse Problem

The applications previously described imply inverse relationships of the form:

$$DOB = (DIB/b_1)^{1/b_2}$$

and

$$DOB^2 = (DIB^2/a_1)^{1/a_2}.$$

An alternative approach would be to derive separate regressions for DOB and DOB^2 in an approach analogous to that used by Myers and Alexander (1972). These two additional regressions were fitted to the data and found to provide estimates very close to those obtained by inverting models (5) and (6).

The closeness of these estimates was further checked by obtaining parameter estimates from models (5) and (6) and using them to compute the residual means and variances of the inverted forms. In both cases, the 95 percent confidence intervals about the means of the residuals included zero. Hence, there seems to be little practical difference between inverting the fitted equations and deriving two additional regressions. The method used in this paper does guarantee that estimated past diameters will be consistent with the current diameter measurement. That is, for trees with little or no radial growth, past diameter is constrained to be less than or equal to current diameter.

CONCLUSION

These data indicate that the ordinary least squares regression so frequently used in studies of this type may not always be applicable if one desires to characterize the full range of possible diameters. If estimates for small trees are desirable, then trees with diameters between 0 and 8 inches should be strongly represented in the sample. This is the range of data most likely to exhibit nonlinearity.

Although a nonlinear model is best fitted to the data of this study, such model forms must be fitted to data collected over a wider geographical area before conclusions can be drawn about their general applicability to Douglas-fir. The weighted nonlinear model does however, merit further consideration in studies of this type.

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CHAPTER 3

DEVELOPMENT OF A TREE HEIGHT GROWTH MODEL FOR DOUGLAS-FIR

Abstract

Douglas-fir height growth was modeled using linear and nonlinear regression analysis. Data for the study was provided by a sample of 866 trees for which five year height increment was measured. A predictive model is presented which expresses height growth as a function of potential height growth, crown ratio and height divided by dominant stand height. The adjusted coefficient of multiple determination ($\bar{R}^2$) for this model exceeded 0.70. In addition, four other models from previous height growth studies were compared to this model. The final model was found to predict well across crown classes. A number of different techniques for expressing competitive effects were considered. The most influential variables in the height growth analysis were related to tree position: crown competition factor in larger trees and height of the subject tree divided by the dominant stand height.

INTRODUCTION

Many previous studies of height growth in Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) have been devoted to development of site index equations (King 1966; Curtis et al. 1974; Cochran 1979). In such equations, development of tree height is expressed as a function of age and site productivity, where productivity is indexed by height attained by the dominant and/or codominant component of the stand at some reference age.

The utility of site curves in estimating height growth is limited since the equations are applicable only to the average growth of dominant or codominant trees in even-aged stands. A more general estimator would accurately predict an individual tree's height growth regardless of the tree's vigor or position within the stand.

Height growth prediction is frequently a weak link within the structure of stand growth simulators. This is largely due to the difficulty and expense involved in obtaining a data base which is of adequate size for parameter estimation and which is also free of measurement error. Consequently, the response of height growth to measures of competition and tree position within the stand has not been well documented in the literature.

The primary objective of this study was to develop a model for Douglas-fir height growth for application in an individual tree/distance independent growth model (Munro 1974). A secondary objective was to evaluate different model forms and measures of competition and position for their ability to characterize height growth of Douglas-fir.

Data

The study was conducted as one phase of a larger project to develop a stand simulator for Oregon State University's Forest Properties. To meet this need, a data base for growth model development was established consisting of measurements from a grid of variable radius plots in 136 stands.

The stands selected for this analysis covered the broadest possible range of site index and stand density. Most of the stands sampled were even-aged, however, some two-storied stands were also sampled.

At each variable radius plot within a stand, two nested, fixed area subplots were measured to provide a more representative sample in the smaller diameter classes. Each tree was measured for diameter, height, crown length, and distance to plot center. Five year radial growth was measured on all conifers, and five year height increment was measured with a collapsable pole on all conifers less than 25 feet tall. In addition, stand age and King's (1966) site index was estimated for each stand.

This growth data set contained a total of 20,143 trees. A subsample of 723 trees were felled and measured for last five years height increment. Trees with recent severe top damage or other severe damage were excluded from this subsample. No suppressed trees were felled. Of the standing trees with height growth measured directly with a 25 foot pole, 143 Douglas-fir met the criteria for the height growth data set. Many of these smaller trees were from the suppressed and intermediate crown classes. This provided a total of 866 Douglas-fir trees with measured height growth. The range of some of the key stand variables in the height growth data are presented in Table 2. A detailed

Table 2. Ranges and means for stand level variables represented in the height growth data.

Variable	Minimum	Maximum	Mean
Site index (ft)	90.0	142.0	113.2
Stand age (years)	18.0	137.0	52.6
Basal area (ft^2/ac)	11.7	247.0	138.0
CCF (%)	37.7	388.3	206.9

description of tree variables by the four crown classes is presented in Table 3.

A computer program was written to backdate all plots and compile a complex array of density measures. The backdating involved the estimation of tree heights, diameters, and expansion factors at the start of the five year growth period. Crown ratio was assumed constant over the growth period.

The density measures compiled by this routine include crown competition factor (CCF) (Krajicek et al. 1961), basal area per acre (BA), and foliage weight per acre (FW). Equations for predicting tree foliage weight were developed using Brown's (1978) data, local maximum crown width equations and the following model form:

$$FW = k_1(CL\ MCW\ CR^{k_2})^{k_3}$$

where:

CL = crown length

CR = crown ratio

MCW = estimated maximum crown width

k_1, k_2, k_3 = species specific coefficients.

All measures of density were computed at both the plot level and stand level.

Table 3. Number of trees (N) and minimum, mean and maximum for: five year height growth (ΔH), tree height (H), breast height diameter (DBH), crown ratio (CR) and height divided by dominant stand height (H/SH) by crown classes[1] for the height growth data.

	Crown Class			
	Dominant	Codominant	Intermediate	Suppressed
N	347	259	162	98
Min. ΔH	1.9	0.7	0.6	0.3
Mean ΔH	9.0	8.5	5.5	2.2
Max. ΔH	18.1	18.0	13.3	11.0
Min. H	1.5	0.2	0.3	0.2
Mean H	95.5	65.0	21.3	6.5
Max. H	187.2	160.6	94.4	22.5
Min. DBH	0.1	0.1	0.1	0.1
Mean DBH	18.0	10.2	2.6	0.7
Max. DBH	42.7	29.3	14.2	2.8
Min. CR	0.15	0.17	0.15	0.11
Mean CR	0.50	0.47	0.50	0.48
Max. CR	0.91	0.95	0.96	1.00
Min. H/SH	0.02	0.001	0.002	0.001
Mean H/SH	0.91	0.67	0.26	0.09
Max. H/SH	1.62	1.48	0.94	0.51

[1] In multi-storied stands, crown class for each tree was judged in the context of its immediate environment rather than solely upon its position in the stand. Therefore, an understory tree could be classed as a dominant if it recieved full sunlight from above and partly from the sides.

Competitive stress experienced by a given tree was characterized by subdividing these density variables into quantity by diameter intervals. Two methods were used to define these diameter intervals.

One method characterized density by three diameter intervals in a method similar to that used by Hann (1980). These intervals were defined such that the middle interval is composed of three one-inch diameter classes with the subject tree's diameter falling in the central class of the interval. The upper and lower intervals include all one-inch diameter classes above and below the middle interval. For each tree, the sum of the three variables equals total density per acre.

The second method used to express competitive stress was a two diameter interval subdivision of density. Wykoff et al. (1982) defined a variable expressing basal area in trees larger than the subject tree. Basal area in smaller trees can be calculated in a similar fashion, thus defining the two intervals. This two interval subdivision was also applied to the CCF and FW density measures.

Because we also wished to examine species effects, levels of both CCF and BA were further subdivided by three species groups: Douglas-fir, other conifers and hardwoods.

MODEL DEVELOPMENT

The height growth function was defined as a product of potential height growth and a modifier incorporating tree vigor and/or competition. This same general approach has been applied to other Douglas-fir (Arney 1972; Mitchell 1975; Krumland 1982), northern hardwood (Monserud 1975) and loblolly pine (*Pinus taeda* L.) height growth models (Daniels and Burkhart 1975).

This type of model is thought to be superior to models generated by strictly empirical means. It should provide more reliable estimates when the model is applied to trees not within the range of data used in model development. It is also a safeguard against developing a model which is peculiar to the specific data used in model development (Krumland 1982).

Potential Height Growth

From the height growth data, 264 site quality trees were chosen to test potential height growth estimators. Initially, Bruce's (1981) estimator using stand age and King's (1966) site index was considered:

$$SH = S \cdot EXP[a_1((SA + 13.25 - S/20)^{a_2} - (63.25 - S/20)^{a_2})]$$

SH = predicted stand height in feet

S = King's site index

SA = stand age

$$a_2 = -0.447762 - 0.894427(S/100) + 0.793548(S/100)^2 - 0.17166(S/100)^3$$

$$a_1 = \ln(4.5/S)/[(13.25 - S/20)^{a_2} - (63.25 - S/20)^{a_2}]$$

In this approach, SH is calculated for stand age at the start and end of the growth period. The difference provides an estimate of potential growth. However, this estimator was discarded because it predicts average height growth for only dominants and codominants in only even-aged, single-storied stands.

In order to characterize the growth of individual trees from multi-storied stands, potential height growth was modified to reflect individual tree height. This potential growth function makes use of growth effective age, defined as that age corresponding to the estimated stand site index and the subject tree's height. Similar estimators have been used in previous studies (Monserud 1975; Krumland 1982; Martin and Ek 1984). Growth effective age is determined by solving Bruce's (1981) height equation for age:

$$A = [\ln(H/S)/a_1 + (63.25 - S/20)^{a_2}]^{1/a_2} - 13.25 + S/20$$

where:

A = growth effective age estimated for the start of the growth period

H = total tree height in feet at the start of the growth period.

Potential height growth (ΔPH) is finally estimated using the growth effective age at the end of the growth period (A + 5), stand site index and the height of the tree at the start of the growth period:

$$\Delta PH = PH2 - H1$$

where:

PH2 = predicted height of the tree at the end of the growth period using site index and growth effective age of the tree at the end of the growth period in Bruce's equation.

H1 = total tree height at the start of the growth period.

Using the 264 site quality trees, a simple zero intercept linear regression was fit to this potential height growth estimator:

$$\Delta H_s = b(\Delta PH)$$

where:

ΔH_s = height growth of dominant and co-dominant site quality trees

The value of the slope coefficient in this regression (b = 1.14906) was significantly different than 1.0 ($P < 0.05$), indicating that the sample trees grew at a faster rate than indicated by the estimator of potential growth.

This result could be due to poor estimates of site index or to better than normal climatic conditions during the growth period. Because the felled trees were not sectioned, no check of the site estimator (King 1966) could be made. Precipitation records collected near the study area indicated that rainfall was 144 percent of normal during the growing seasons represented by the data (1976-1980).

Modifier Function

The first step in defining a modifier function for height growth was to screen over the various combinations of independent variables that were being considered for the height growth models. An all-combinations linear regression package was used to evaluate transformations of the predictor variables. Although several intrinsically linear models were considered, the most useful was a simple log-linear model.

$$\ln(\Delta H/\Delta H_s) = c_0 + c_1 \cdot x_1 + c_2 \cdot x_2 + \ldots + c_p \cdot x_p$$

where:

$x_1, x_2, \ldots, x_p$ = independent variables

$c_0, c_1, c_2, \ldots, c_p$ = parameter estimates

The index used for comparison in this analysis was the adjusted coefficient of multiple determination ($\bar{R}^2$) (Draper and Smith, 1981).

This analysis did help to identify the most influential variables in modeling height growth. The results can best be summarized in the following manner:

(1) Plot level measures of density or structure did not improve the fit when compared to the same variables computed over all plots in the stand.

(2) Among the different expressions of density, i.e., CCF, BA, and FW; CCF was generally the best independent variable.

(3) CCF in larger trees (CCFL) was the best single density related measure of competition ($\bar{R}^2$ = .4175). It was a great improvement over total stand CCF ($\bar{R}^2$ = .0203) and similar improvements in $\bar{R}^2$ were found when compared to BA and FW.

(4) Dividing density into two or three diameter intervals was not an improvement over simply CCFL. The addition of CCF in smaller trees (CCFS) produced a positive correlation with height growth. This is probably due to a strong negative correlation between CCFL and CCFS.

(5) Various transformations of a position variable defined as height divided by dominant stand height (H/SH) generally provided high $\bar{R}^2$ values in this analysis ($\bar{R}^2$ on the order of

.60). Addition of CCF did not result in a substantial improvement in fit for models already containing H/SH.

(6) Transformations involving crown ratio as an expression of tree vigor provided $\bar{R}^2$ values less than 0.10).

(7) Expressions of density subdivided by the three species groups did not improve the fits.

In the next step, nonlinear regression was used to examine and refine several candidate model forms. Selection of these candidate models was guided by the results of the linear regression variable screening runs. These models were evaluated on the basis of $\bar{R}^2$ and residual plots. From this step of the analysis, the following model was judged best:

$$\Delta H = [b \cdot \Delta PH][c_1(1 - EXP(c_2\ CR))(EXP(c_3((H/SH)^{c_4} - 1)))] \quad (1)$$

Four additional models were also fitted to the data set. These four represent the closest possible approximations to model forms used in previous height growth analyses:

$$\Delta H = [b \cdot \Delta PH][c_1(1 - EXP(c_2 \cdot CR^{c_3}))] \quad (2)$$

$$\Delta H = [b \cdot \Delta PH][c_1(EXP(c_2 \cdot BA))] \quad (3)$$

$$\Delta H = [b \cdot \Delta PH]\ c_1/[(1 + EXP(4 - c_2 \cdot CR))(1 + EXP(-3 + c_3 \cdot CCFL))] \quad (4)$$

$$H = [b\cdot\Delta PH][c_1(1 + c_2\cdot CR^{-.5} \cdot EXP(c_3\cdot CCFL + c_4\cdot CR))] \quad (5)$$

Model (2) was used by Arney (1972) for Douglas-fir. Model (3) is one of the height growth models considered by Martin and Ek (1984).

The models presented by Daniels and Burkhart (1975) and Krumland (1982) could only be approximated because the index of competition used in those studies was not calculated in this study. The approximation consisted of using CCFL as a measure of competitive stress. Model (4) is the approximation to Krumland's model. The Daniels and Burkhart approximation, model (5), was reduced to a four parameter model because the five parameter model would not converge in nonlinear analysis with this data set.

RESULTS AND DISCUSSION

The parameter estimates, mean squared error, and $\bar{R}^2$ values for all five models are presented in Table 3. Model (1) was far superior to any of the other models considered.

Table 4. Parameter estimates, adjusted coefficient of multiple determination ($\bar{R}^2$) and mean squared error (s^2) for five height growth models fit by nonlinear regression.

	Parameter Estimates					
Model	c_1	c_2	c_3	c_4	$\bar{R}^2$	s^2
(1)	1.1171	-4.26558	2.54119	0.25054	.7078	5.27
(2)	0.6614	-12.6726	1.57845	-------	.0810	16.57
(3)	0.6063	3.034E-4	-------	-------	.0577	16.99
(4)	1.0232	26.1344	0.18809	-------	.4885	9.22
(5)	1.7925	-0.18281	0.00285	1.02532	.4666	9.62

The height growth of sample trees was best characterized by model (1). The most influential variable in this model is H/SH. The effect of the H/SH modifier is shown in Figure 2. Increased height growth is predicted for trees in a superior competitive position (H/SH near 1). Height growth predictions for trees with H/SH in excess of 1.3 are suspect for this model. The range of H/SH did not extend much beyond this point. Trees in this position would most likely be predominant trees, for which the model should not be applied.

CR may be considered to be an index of tree vigor (Daniels and Burkhart 1975). A tree with a CR as low as 0.3 is growing at over 70 percent of its predicted maximum (Figure 3). This would indicate that, for most trees, vigor as indicated by crown ratio is not a seriously limiting factor (the average CR in this study was 0.5).

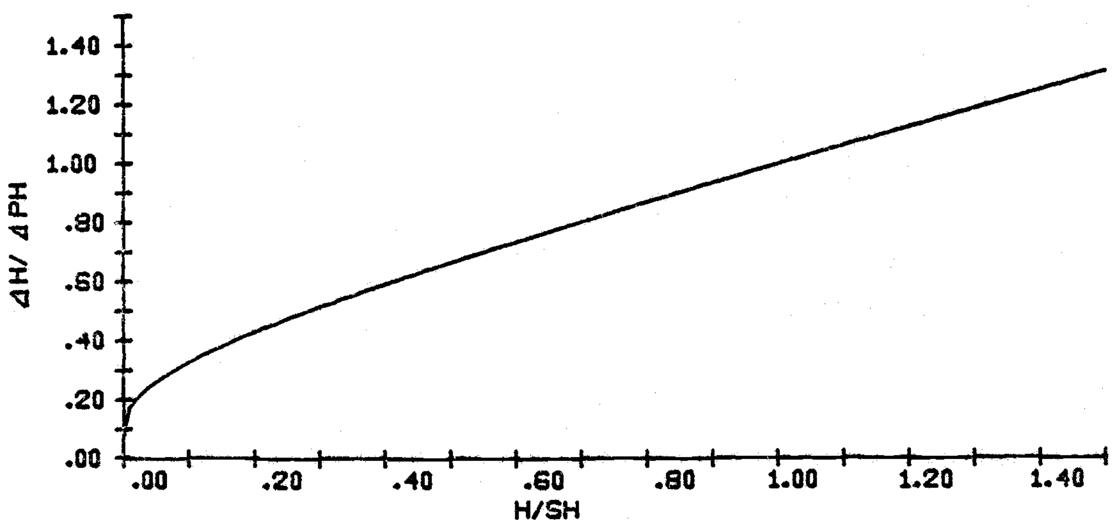

Figure 2. Modifier component for H/SH in model (1), assuming CR fixed at 1.0.

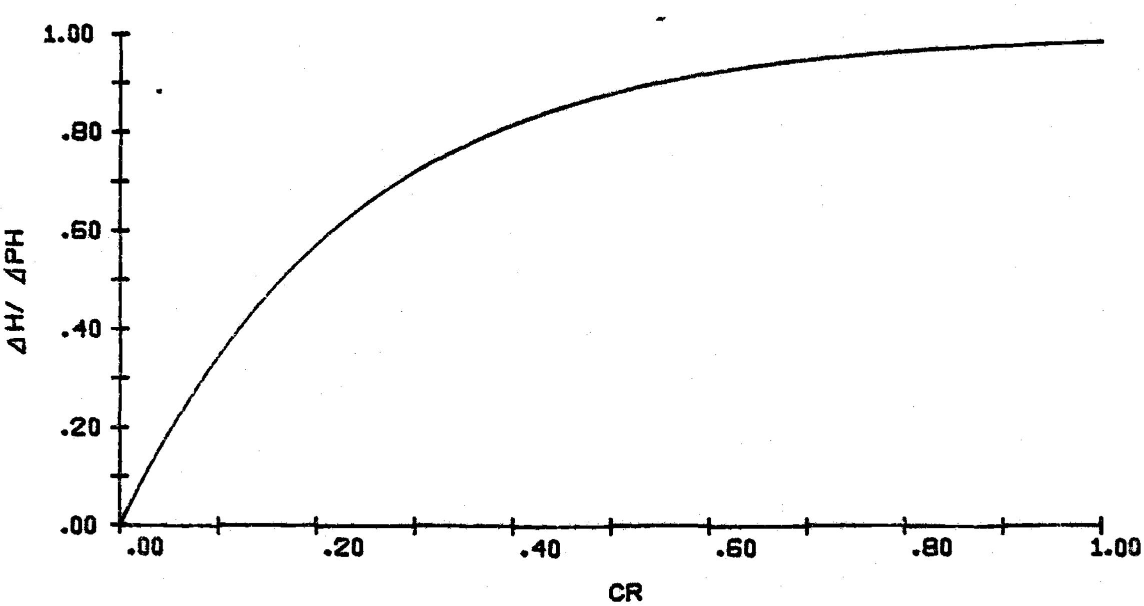

Figure 3. Modifier component for crown ratio in height growth model (1), assuming H/SH is fixed at 1.0.

Although model (1) explains more than 70 percent of the variation in height growth for all trees, a primary concern should be model performance for those crown classes that will form the major portion of intensively managed stands. A model which predicts well for intermediate and suppressed trees at the expense of those from the dominant and codominant crown classes would be poorly suited for management applications. To test the ability of model (1) to predict height growth in different components of a stand, $\bar{R}^2$ was calculated for each crown class. Model (1) does well in all crown classes, with the poorest fit being attributed to intermediate trees (Table 5).

Table 5. Adjusted coefficients of multiple determination ($\bar{R}^2$) for height growth model (1) by crown class.

Crown Class	$\bar{R}^2$
Dominant	0.7472
Codominant	0.7091
Intermediate	0.5056
Suppressed	0.8931

Height growth for dominants and codominants in this sample is predicted at least as well for the sample as a whole. It should be noted that the values in Table 5 were calculated using the crown class total sum of squares corrected for the overall mean rather than the crown class mean. This resulted in an artificially inflated index of fit for suppressed trees. The values for dominants and codominants were similar regardless of the sum of squares correction.

As the H/SH values in Table 3 indicate, the dominant and codominant trees came from a wide range of positions within the sampled stands. Therefore, the model should function well in both single-storied and multi-storied stands.

The approximation to Krumlands height growth model (4) provided the best fit among the remaining models. The CR function in this model differs from that used in (1), in that the modifier is not forced through zero and has a small positive value when CR approaches zero.

Of the remaining models, (2) and (3) had greatly reduced $\bar{R}^2$ values. This is consistent with the observation that, in log-linear regression analyses, both CR and BA were inferior to the position related variables.

Model (5) appeared to be over specified for this sample because its parameters could only be estimated by forcing a constant power on CR. Daniels and Burkhart (1975) included two CR terms in order to form a peaking function which would reduce height growth for trees with near full crowns. A sample that includes open grown trees is probably necessary to obtain convergence with this model.

All nonlinear models were fit using unweighted regression. Residual plots from model (1) did not show any trends over predicted values or over the independent variables. In addition, Furnival's index was used to compare unweighted regression with log transform and weighted (weight = $1/\Delta PH$) regressions. The unweighted regression provided the lowest index of fit. Thus, we did not find any evidence to suggest that weighted regression should have been used in this analysis.

In addition to those shown, a number of other nonlinear models were fitted to confirm the findings of the log-linear regression analysis. Stand level CCFL was replaced with a number of other expressions of density. In all cases $\bar{R}^2$ was reduced, anywhere from 0.2 to 0.4.

Because FW was, in general, an ineffective predictor variable in the height growth equations, no justification for the use of FW estimators as an index of competitive stress could be found in this analysis.

This result is not conclusive, however, due to the uncertainty associated with the particular estimation procedure used for FW.

The results of this analysis consistently supported the conclusion that CCF was superior to BA or FW as an index of competition. Furthermore, measures of tree position, with respect to its competitors, such as CCFL or H/SH, are superior to total density variables. This seems to indicate that the density of these stands has very little impact on individual tree height growth, as long as the tree is in an advantageous position with respect to its competitors.

For predicting height growth, tree position as defined by H/SH is far superior to any other density or position variables considered in this analysis. The strength of H/SH as an independent variable may also indicate that variables related to vertical position in the stand may be superior to diameter related variables such as BA or CCF.

Plot level expressions of density were also tested in this phase of the analysis. Again, residual variation was higher with plot level density variables than with a stand level estimate of the same variable. The $\bar{R}^2$ for a model with stand CCFL was reduced from 0.48 to 0.29 by inserting the plot level estimate of the same variable.

CONCLUSION

The model (1) developed in this analysis was very successful in explaining the variation of height growth due to tree vigor, tree position and site productivity. Included in the model is a slope correction on the potential component (b = 1.14906). In application, users may wish to force this correction to one to offset the effect of short term climatic fluctuations on the model. However, to the extent that this coefficient represents a true departure from the potential model, the slope may be a reasonable correction. Nevertheless, this slope correction can be varied in order to adapt the model to sites characterized by growth which does not appear to conform with the data used in this analysis.

High indices of fit were obtained when compared to previous modeling efforts. Krumland and Wensel (1981) for example, report an R^2 of 0.11 for a Douglas-fir height growth model. The improvement in fit is probably due, in part, to reduced measurement error. Height growth in this study was measured directly on all trees. Therefore, none of the error usually associated with repeat height measurements of standing trees was introduced to the error term about the regression surface. Without such precise measurements, a comparitive analysis of alternative model forms would likely have provided ambiguous results.

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CHAPTER 4

DEVELOPMENT OF EQUATIONS FOR PREDICTING FIVE YEAR BASAL AREA INCREMENT IN DOUGLAS-FIR AND GRAND FIR TREES

Abstract

Equations are presented for predicting individual tree basal area increment for Douglas-fir and grand fir in the east-central Coast Range of Oregon. Techniques for predicting future diameters from these equations are also presented. A number of methods of expressing stand density or structure are compared for the log-linear model of basal area growth. Final parameter estimates were obtained using weighted nonlinear regression analysis of a simple exponential model. Two equations are presented for each species, one with site index and the other with predicted height growth as independent variables in the model. The other variables used are diameter, crown ratio, crown competition factor in larger trees on the sample point, and stand basal area.

INTRODUCTION

Simulators of growth and yield can be a valuable tool in the management of forest stands. They provide resource managers with a means of predicting the growth response to various stand treatments, and they help guide long term planning by providing yield estimates over the length of a rotation.

Simulators can be classified on the basis of the primary modeling unit used in projecting growth (Munro 1974). In simulators for which the individual tree is the primary modeling unit, projections of stand growth and yield depend on estimates of the components of individual tree development that are then aggregated to produce stand level estimates. These individual tree components may include diameter growth, height growth, crown change, and mortality models. Individual tree models may be further classified by the presence or absence of intertree distances in measures of competitive stress. Models which incorporate some measure of distance between the subject tree and its competitors are referred to as distance dependent. Conversely, if competitive stress is quantified by some measure of overall density in a stand or plot, the model is referred to as distance independent.

This study was conducted as one part of a project to develop an individual-tree/distance-independent growth simulator for Oregon State University's Forest Properties. The objective is to develop distance independent type equations for projecting individual tree diameter growth for Douglas-fir (Pseudotsuga menziesii (Mirb.) Franco) and grand fir (Abies grandis (Dougl.) Lindl.). To meet these objectives, it was necessary to evaluate model forms, and to compare a number of expressions of density or structure on the basis of their explanatory power.

Data

The data base for model development was established simultaneously with the installation of inventory plots on the Forest Properties. A total of 136 stands were included in the growth modeling data base. In order for a stand to be selected for the modeling data set, there had to be a significant amount of stand basal area in Douglas-fir and grand fir and the stand must have been free from silvicultural treatment during the five years prior to measurement. This latter requirement is to assure that factors affecting growth are undisturbed during the growth period.

Plots were established on a systematic grid within each stand. Most stands contained one plot every two acres. However, some of the younger stands were sampled with one plot per acre to insure an adequate representation in these age classes, and some of the older stands were sampled at one plot every four acres.

Among the stands measured, basal area ranged from 10 square feet to over 270 square feet per acre. Site index (King 1966) ranged from 90 to 142 feet at a base age of 50. Stand ages varied between 20 and 120 years, with a heavy concentration of stands between 40 and 60 years old.

Each plot consisted of a variable radius point and two nested fixed radius sub-plots. The 20 BAF variable radius point was established to measure all trees greater than 8.0 inches in diameter at breast height (4.5 feet). The larger of the two fixed area sub-plots was a 15.56 foot radius plot to measure all trees between 4.1 and 8.0 inches in diameter. Finally, a 7.78 foot radius plot was established at the point to measure all trees 4.0 inches or smaller.

All trees, including hardwoods, were measured on each plot for total height, height to crown base and breast height diameter. Tree height and height to crown base were measured to the nearest 0.1 foot using the pole tangent method described by Curtis and Bruce (1968). Diameters were measured to the nearest 0.1 inch using a diameter tape. The previous five year radial growth was measured on increment cores taken from all live conifers greater than 3.0 inches in diameter. Radial growth was measured to the nearest 1/40 of an inch. All mortality trees estimated to have died in the last five years were included on the plot and so noted with a mortality code.

A total of 9526 Douglas-fir trees were measured for radial growth on the growth stands. An additional 595 grand fir and 152 other conifers were also measured. The other conifers were not considered in this analysis because they represented a wide variety of species, many of which are not native to the area. A number of different tree species have been introduced on the forest over the years, either individually or in small groups, but none of the species in this group are well enough represented to form a data base of any use in modeling.

A computer program was written to backdate stands. Backdating is necessary for data developed from temporary plots because predictor variables should represent measurements taken at the beginning of the growth period of interest. Bruce's (1981) height equation was used to backdate all trees for which height growth was not measured directly. Crown ratio was assumed constant over the five year growth period and diameters were backdated using the radial growth measurements.

Variables expressing stand and point density were then calculated. These variables include Crown Competition Factor (CCF)(Krajicek et al. 1961), basal area per acre (BA) and foliage weight per acre (FW).

CCF was calculated by summing individual tree (CCF_i) values over all trees in the stand or point:

$$PCCF = \sum_{i=1}^{np}(CCF_i \cdot EXPN_i)$$

$$SCCF = \sum_{i=1}^{ns}(CCF_i \cdot EXPN_i/M)$$

where:

PCCF = CCF at the point level

SCCF = CCF at the stand level

$CCF_i = 0.001803 \cdot MCW_i^2$

np = number of trees on a point

ns = number of trees in a stand

M = number of points in a given stand

$EXPN_i$ = expansion factor of the each tree

MCW_i = maximum crown width of each tree, estimated from crown width equations presented in Appendix A.

Since the foliage weight on a plot or stand should be indicative of demand for nutrients and water in the soil, as well as competition for light, it was felt that FW estimators may provide an improved means of estimating competitive stress.

Foliage weight was calculated for each tree using species specific equations developed using Brown's (1978) data and local maximum crown width equations:

$$FW = k_1 \; (CL \cdot MCW \cdot CR^{k_2})^{k_3}$$

where:

FW = foliage weight of a given tree

CL = crown length

CR = crown ratio

k_1, k_2, k_3 = species specific parameter estimates (Appendix B).

Foliage weight per acre was then calculated in the same manner as CCF. All measures of density were computed at both the point level and stand level.

Competitive stress experienced by a given tree was characterized by subdivisions of these three density variables based on diameter intervals. Two methods were used to define the diameter intervals.

One method characterized density by three diameter intervals (Hann 1980). These intervals were defined such that the middle interval is composed of three, one inch, diameter classes with the subject tree's diameter falling in the central class. The remaining two intervals are defined as comprising all diameters above and below this middle interval. For each tree the sum of the three variables equals total density per acre on the stand or point.

The second approach at indexing competition was a two diameter interval subdivision of density. Wykoff et al. (1982) defined a variable expressing basal area in trees larger in diameter than the subject tree. Basal area in smaller trees can be calculated in a similar fashion. This two interval breakdown was also applied to the CCF and FW density measures.

Because we also wished to examine species effects, levels of both CCF and BA were further subdivided by three species groups: Douglas-fir, other conifers, and hardwoods.

Due to the large size of the Douglas-fir data set and the cost involved with repeat regression analyses on a data file of such magnitude, a random sub-sample of approximately 20% (1910 trees) was selected for the model selection phase of the analysis. The remaining 80% (7616 trees) were reserved for final parameter estimation. The grand fir data set was left intact because there was not a sufficient number of observations for a subsample.

The primary advantage to subdividing a large data set is that parameter estimation can be done independently of variable selection. This allows for more reliable significance testing because there is no loss of degrees of freedom due to variable selection. This reduction in sample size for variable screening also resulted in a significant savings in computing costs.

MODEL DEVELOPMENT

Model Selection

The first step in the modeling effort was to define those factors which would characterize the "best" model. For this study, the best model was defined as that which both minimized the residual mean squared error, came closest to meeting the assumptions of regression analysis and characterized the relationship between basal area growth and the independent variables in a biologically meaningful fashion.

Diameter growth can be estimated by the use of either basal area growth equations or diameter growth equations. Both methods have been used in previous studies. Cole and Stage (1972) compared basal area and diameter increment, as well as logarithmic transformations of each. They concluded that the log of basal area growth best met the regression assumptions of normally distributed residuals and constant variance. However, in a study comparing basal area increment and diameter increment, West (1980) concluded that there was no *a priori* justification for choosing one over the other in projecting tree diameters.

In this analysis, basal area increment was used for the parameter estimation because it is more easily extrapolated to alternative growth period lengths (Cole and Stage 1972). The natural logarithm of basal area increment was used for variable screenings. The log transform makes it possible to linearize some nonlinear models. Such models are called intrinsically linear. Through the use of intrinsically linear models it is possible to apply linear least squares variable selection procedures to nonlinear models.

Variable Selection

The Douglas-fir data were used for all variable screening and selection and for model comparisons. A least squares all combinations variable selection routine was used to screen and select independent variables. The index used for comparison in this routine was the adjusted coefficient of multiple determination ($\bar{R}^2$)(Draper and Smith 1981). This index can be thought of as a relative mean squared error, such that a value of one indicates a perfect fit to the data and a value of zero indicates that the regression is no better than a simple mean.

The basic model used in variable selection is expressed as:

$$\ln(bag) = b_0 + b_1 \cdot X_1 + b_2 \cdot X_2 + \ldots + b_p \cdot X_p$$

where:

bag = tree inside bark basal area growth in square inches

$X_1, X_2, \ldots, X_p$ = independent variables

$b_0, b_1, b_2, \ldots, b_p$ = parameter estimates.

The variables influencing basal area growth can be divided into three groups: site productivity, competition, and tree size or vigor. Site productivity was characterized by site index (King 1966), transformations of slope and aspect (Stage 1976), potential height growth and predicted height growth. For a more thorough description of the height growth prediction, refer to Appendix C.

One of the most influential factors in basal area growth of individual trees is the level of competition for light, water and nutrients. Density variables can be employed in basal area growth models as indicators of the competitive stress being experienced by a given tree. In such models, high levels of density will generally indicate increased competition, and thus reduced growth rates.

By subdividing density variables with respect to the subject tree's size, it may be possible to more accurately model the response of basal area growth to various levels of competition. For example, stand basal area in larger trees and basal area in smaller trees, rather than total basal area, may be a better means of describing the stress on a given tree. If a stand is quite dense but the subject tree is larger than all of its competitors, one may expect less of a reduction of growth than if the same level of density was in trees larger than the subject tree. It is possible that density in smaller trees will show no significant effect on growth whatsoever. A similar analogy can be made with the three diameter class subdivision of density described earlier.

Another possibility is that the growth response to competition is, to some degree, dependent on the species of the competition. If a given tree is competing with hardwoods, that tree's growth response may be different than if the competition is primarily other conifer species. However, because species composition is not independent of site index and other productivity measures, it may be difficult to assess these types of relationships.

Finally, one might expect that density on a given plot is a more precise expression of the competitive stress experienced by a given tree than the stand level estimate of the same density variable. This effect should be more pronounced where density varies greatly within a given stand. If, however, a stand is perfectly homogeneous (tree diameters and spacing constant as in some very young plantations), any difference between point and stand estimates of density are due solely to sampling error and are not indicative of any real differences in competitive stress.

Tree variables include breast height diameter (DBH) squared and the natural logarithm of DBH. In addition, crown ratio and foliage weight were considered as indicators of tree vigor.

The results of the screening portion of the analysis can best be summarized in the following manner:

(1) The combination of DBH squared and logarithm of DBH form a peaking function in basal area growth over DBH. These two variables alone account for over 38% of the variability in the logarithm of basal area growth.

(2) Addition of crown ratio as an independent variable resulted in a significant improvement in fit ($\bar{R}^2 = 0.5974$). Crown ratio provided a slightly higher $\bar{R}^2$ (improvements less than 1%) than did various transformations of the subject tree's foliage weight or crown length.

(3) When DBH squared, logarithm of DBH, and CR were forced into the model, the addition of

productivity variables failed to improve $\bar{R}^2$ by more than 2%. Transformations of site index generally performed better than predicted height growth. Stage's (1976) slope and aspect transformations were not significant either with or without site index in the model.

(4) Measures of density compiled by point level summations were generally superior to their corresponding stand level variables.

(5) Various transformations of CCF generally resulted in better fits than basal area per acre. BA and CCF both performed better than foliage weight in this model.

(6) The three interval subdivision of density did not improve fits over use of the two interval subdivision. In either case, only the larger trees had a significant effect on basal area growth. Density in smaller trees or in the middle diameter interval of the three diameter class subdivision was insignificant ($P > .10$). As expected, increasing values of basal area and CCF in larger trees (CCFL) indicated a reduction in growth rates.

(7) Density variables summed over all diameter classes provided poorer fits than did the same

variables expressed in larger and smaller trees.

(8) Subdivisions of density variables by species did not provide any improvement over diameter interval subdivisions.

(9) The best combination of density variables was CCFL on the plot and stand BA.

From these analyses, it was determined that either predicted height growth or site index could be used as an indicator of site productivity. Furthermore, transformations of DBH and crown ratio constituted the main variables in the model.

The density/position variable which best expressed the effects of competing vegetation was CCFL. However, BA was also significant in this analysis, indicating that the effect of larger trees on basal area growth is not independent of stand density.

Final Basal Area Increment Models

The log transformation, as used in the variable selection phase of the analysis, will result in some degree of bias being introduced in predictions of the untransformed dependent variable (Flewelling and Pienaar 1981). Although the bias can be adjusted for, corrections for bias are generally dependent on meeting the assumption of normality of the residuals with respect to the logarithm of basal area growth. The log transform was rejected for final parameter estimation because of the severe non-normality of the residuals about the log-linear model.

An alternate approach to parameter estimation is to apply nonlinear regression to fit the exponential of the log-linear model, thus eliminating the need for the log transform of the dependent variable. In this regression, untransformed basal area growth is the dependent variable. An additive error is assumed with nonlinear regression. Furthermore, the resulting model is asymptotically unbiased, regardless of the distribution of the residuals.

The log-linear and weighted nonlinear regressions were compared on the same model using Furnival's (1961) index of fit. Furnival's index indicated that the log-linear fit was only slightly better than nonlinear with a weight of $1.0/DBH^2$. Test contours for skewness and kurtosis (Bowman and Shenton 1975) indicated that both models had non-normally distributed residuals. From this, it was concluded that weighted nonlinear regression was preferable to the log-linear estimation procedure. A weight of $1.0/DBH^2$ was used for final parameter estimation.

The two models chosen to fit to the remaining 80%

(7616 observations) of the Douglas-fir data and the total grand fir data set are:

$$\text{bag} = \text{EXP}[c_0 + c_1 \cdot \ln(\text{DBH}) + c_2(\text{DBH}^2) + c_3 \cdot \text{CR} + c_4 \cdot \Delta\text{H} + c_5 \cdot \text{PCCFL} + c_6 \cdot \text{SBA}] \quad (1)$$

and

$$\text{bag} = \text{EXP}[c_0 + c_1 \cdot \ln(\text{DBH}) + c_2(\text{DBH}^2) + c_3 \cdot \text{CR} + c_4 \cdot \ln(\text{S}) + c_5 \cdot \text{PCCFL} + c_6 \cdot \text{SBA}] \quad (2)$$

where:

CR = crown ratio

S = King's (1966) site index, in feet

PCCFL = point CCF in trees larger than the subject tree

SBA = stand basal area, in square feet.

The parameter estimates, $\bar{R}^2$, and mean squared errors (s^2) for both Douglas-fir and grand fir are presented in Table 1.

The appropriate model for predicting growth depends on, among other things, how the different components of growth are incorporated in the framework of a stand simulator. Basal area and height increment models may be linked in a two-stage fashion similar to that used by Stage (1973). If such an approach is used then predicted height growth would be used in the basal area increment model as a means of compensating for correlated errors between the two models. A random variable may then be introduced to the height growth

Table 6. Parameter estimates, mean squared error (s^2) and adjusted R^2 for Douglas-fir and grand fir basal area increment models.

	---Douglas-fir---		---Grand Fir---	
	Models			
	1	2	1	2
c_0	0.275735	-4.51337	-0.313843	-2.23906
c_1	1.0060822	1.0190520	1.318159	1.370403
c_2	-0.0002754	-0.0003326	-0.0008066	-0.0009438
c_3	1.13501	1.33031	1.09101	1.21110
c_4	0.0310146	1.07106	0.0241041	0.448166
c_5	-0.0027791	-0.0028096	-0.0017633	-0.0018079
c_6	-0.0013096	-0.0019739	-0.0022058	-0.0030200
s^2	0.586661	0.580577	0.821505	0.829858
$\bar{R}^2$	0.5571	0.5616	0.4713	0.4659

predictions and carried through to the diameter growth predictions.

An alternative approach is to assume that the equations are seemingly unrelated. The use of site index alone as the productivity variable in both height and diameter growth equations is an application of this approach.

In seemingly unrelated regressions, some correlation is assumed to exist between the errors of the two models (Kmenta 1971). If this correlation exists, and these two regressions are developed separately, the parameter estimates are unbiased and consistent. However, in order to obtain efficient estimates, the error correlation can be accounted for through generalized least squares. Although generalized least squares can be applied in nonlinear regression, the procedure is quite complex, and was rejected for this analysis.

PREDICTING FUTURE DIAMETERS

The primary function of these equations is in estimating future outside bark diameters. The method by which this can be accomplished was described by Ritchie and Hann (1984). The first step is to calculate projected tree basal area as a function of current diameter squared and estimated basal area growth. This is done by assuming the following relationship between inside and outside bark squared diameters:

$$DIB^2 = a_1(DOB^2)^{a_2}$$

where:

DIB = breast height diameter inside bark

DOB = breast height diameter outside bark

a_1, a_2 = species specific coefficients.

Then:

$$ba_2 = (\pi/4)[(4 \cdot \widehat{bag})/(\pi \cdot a_1) + DBH^{2 \cdot a_2}]^{1/a_2}$$

where:

ba_2 = projected outside bark basal area in square inches

$\widehat{bag}$ = predicted inside bark basal area growth in square inches from equation (1) or (2).

DBH = current outside bark breast height diameter in inches.

The equation for projected diameter (DBH_2) then, can be derived from the square root of ba_2. For Douglas-fir:

$$DBH_2 = [1.35171 \cdot \widehat{bag} + DBH^{1.93369}]^{0.517145}$$

and, for grand fir:

$$DBH_2 = [1.34747 \cdot \widehat{bag} + DBH^{1.95478}]^{0.511565}$$

CONCLUSION

These data indicate that basal area growth can be adequately modeled by a simple exponential function. A nonlinear least squares solution procedure was applied in parameter estimation because of problems with non-normality in the residuals of the log-linear fit of the same model.

Competitive stress is expressed in the model by crown competition factor in larger trees on the plot and total stand basal area. More complex expressions of stand or plot density, including species subdivisions, did not improve the explanatory power of the model.

These models should function well over a wide range of stand conditions. The stands sampled covered a wide range of site index and density, and tree size varied from very small saplings to overmature trees.

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APPENDICES

APPENDIX A

Equations for maximum crown width are necessary for calculating CCF and for estimating foliage weight. CCF is defined as the sum of the maximum crown areas for all trees in a stand or at a given point (Krajicek et al. 1961). The species specific MCW equations used to calculate CCF and FW in this study are all quadratic functions of diameter:

$$MCW = d_0 + d_1 \cdot DBH + d_2 \cdot DBH^2.$$

However, for most of these equations, the value of d_2 is zero. In these cases the equation reduces to a linear function over diameter. The MCW parameter estimates for each species, as well as their source in the literature, are presented in Table 7.

Table 7. Coefficients for maximum crown width equations by species, and the literature source for each.

Species	d_0	d_1	d_2	Source
Douglas-fir	4.7071	2.0168	-.0186	Arney (1973)
Yew/hemlock	4.20	1.42	0.00	Smith (1966)
Western red-cedar	4.0	1.6	0.00	Smith (1966)
Grand fir	5.0	1.5	0.00	Smith (1966)
Sitka spruce	6.5	1.8	0.00	Smith (1966)
Oregon white oak	3.0785	1.9242	0.00	Paine and Hann (1982)
Madrone	3.4299	1.3532	0.00	Paine and Hann (1982)
Alder	8.0	1.53	0.00	Smith (1966)
Ash/cotton-wood	0.5	1.62	0.00	Smith (1966)

APPENDIX B

Foliage weight estimates were developed in a two-stage process using the data from Brown (1978). The first stage involved the development of crown width equations. The crown width model was assumed to be:

$$CW/MCW = (CL/H)^{k_2}$$

where:

CW = crown width

k_2 = species specific parameter estimate.

Then, predicted crown width ($\widehat{CW}$) is simply:

$$\widehat{CW} = CR^{k_2} \cdot MCW$$

This predicted crown width was then used as an independent variable in fitting a model of the form:

$$FW = k_1 \, (\widehat{CW} \cdot CL)^{k_3},$$

where:

FW = foliage weight of an individual tree

CL = crown length

k_1, k_3 = species specific parameter estimates.

The complete model for predicting tree foliage weight is then:

$$FW = k_1 \, (CR^{k_2} \cdot MCW \cdot CL)^{k_3}.$$

The parameter estimates of k_1, k_2, and k_3 can be found in Table 3.

Table 8. Species specific parameter estimates for the tree foliage weight model.

Species	k_1	k_2	k_3
Douglas-fir	0.0261580	0.7652813	1.1889078
Ponderosa pine	0.0200009	0.6974895	1.2528906
Western redcedar	0.0267653	0.3550908	1.1418090
Spruce	0.0170113	1.1700041	1.4016785
Grand fir	0.0033270	0.6843938	1.5192013

APPENDIX C

Height growth was predicted for each tree, in order to be used as an independent variable in the basal area increment models. The equation was developed for Douglas-fir trees as a separate phase of the overall modeling project[1]. The height growth estimator is a product of a potential height growth function (PHG) and a modifier (MHG) which adjusts the potential according to tree position and vigor. Potential height growth is expressed as a function of site index and tree height. The modifier of height growth is a function of crown ratio and tree position. Predicted height growth (ΔH) is then estimated with the equation:

$$\Delta H = (PHG)\ (MHG)$$

where:

$$PHG = 1.14906\ (PH2 - H)$$

$$PH2 = S \cdot EXP[t_1((A + 5 + 13.25 - S/20)^{t_2} - (63.25 - S/20)^{t_2})]$$

S = King's (1966) site index, in feet

H = tree height, in feet

[1] Ritchie, M.W. and D.W. Hann. Development of a tree height growth model for Douglas-fir. Manuscript submitted for review, Journal of Forest Ecology and Management, 1985.

$$t_2 = -0.447762 - 0.894427(S/100) + 0.793548(S/100)^2 - 0.17166(S/100)^3$$

$$t_1 = \ln(4.5/S) / (13.25 - (S/20))^{t_2} - (63.25 - (S/20))^{t_2}$$

$$A = [\ln(H/S)/t_1 + (63.25 - S/20)^{t_2}]^{1/t_2} - 13.25 + S/20$$

and the modifier is defined as:

$$MHG = q_1(1 - EXP(q_2 \cdot CR)) (EXP(q_3((H/SH)^{q_4} - 1)))$$

where:

CR = crown ratio

H/SH = tree height divided by dominant stand height

q_1 = 1.117148

q_2 = -4.26558

q_3 = 2.54119

q_4 = 0.250537