

AN ABSTRACT OF THE THESIS OF

Arents, Chester Abbo for the M.E. in Mech. Engr.
(Name) (Degree) (Major)

Date Thesis presented May 16, 1953

Title A Dynamic Procedure for Evaluating Resilient
Mountings

Abstract Approved Redacted for privacy
(Major Professor)

The high frequency transmission characteristics of machinery isolators has been neglected for many years. However, the development of high speed machinery and the importance of sound transmission through structures make it desirable to evaluate the transmissibility of machinery mountings above their natural frequency. The purpose of this thesis is to suggest a successful method for comparing steady-state response of resilient mountings in a frequency range from approximately 10 cycles per second to 2000 cycles per second at maximum static loads of approximately 100 pounds. These limits vary depending upon the load, capacity of the driver, and components of the measuring system.

The dynamic method of testing resilient mountings, particularly rubber mountings, which is evolved should aid one in selecting the apparatus for research and study in this particular field. The results that one might expect from the test setup are presented. Also, the limitations of this method of testing are discussed.

A DYNAMIC PROCEDURE FOR EVALUATING
RESILIENT MOUNTINGS

by

CHESTER ABBO ARENTS

A THESIS

submitted to

OREGON STATE COLLEGE

in partial fulfillment of
the requirements for the
degree of

MECHANICAL ENGINEER

June 1953

APPROVED:

Redacted for privacy

Professor of Mechanical Engineering
In Charge of Major

Redacted for privacy

Head of Department of Mechanical Engineering

Redacted for privacy

Chairman of School Graduate Committee

Redacted for privacy

Dean of Graduate School

Date thesis is presented May 16, 1953

Typed by Miriam Balog

ACKNOWLEDGMENT

The author wishes to express his grateful appreciation to those who have made possible his research activities in the field of vibration and sound. He is indebted to Oregon State College for the education opportunities presented both as an undergraduate and as a graduate student. He is particularly indebted to Professor S. H. Graf, Oregon State College, who encouraged the writer to enter the field of mechanical vibrations. Also, the author wishes to acknowledge the support given to him by Illinois Institute of Technology and the Office of Naval Research.

Ariston
Valley Paper Co.
100 X 140
U.S.A.

TABLE OF CONTENTS

	Page
I. Introduction	1
Scope	1
Vibration Isolation	1
Frequency Range	2
II. Simple Theory	4
Transmissibility Defined	4
Transmissibility Referred to Decibels	7
Wave Phenomena	9
III. Equipment	10
Components	10
Wave Form with Lissajou Figures	10
Importance of Shielding	12
IV. Procedure of Testing	13
Static Load-Deflection Test	13
Dynamic Test	13
Limitations and Refinements	15
V. Discussion	18
Typical Commercial Mounting	18
Dynamic and Static Stiffness	20
Effect of Wave Length on Transmissibility	21
Sound Isolation	22
VI. Summary	23
Bibliography	24

LIST OF FIGURES

	Page
Figure 1. Schematic Representation of Test Arrangement	5
Figure 2. Block Diagram of Complete Test Setup . .	11
Figure 3. Static Testing Machine	14
Figure 4. Typical Rubber Mount	19

NOMENCLATURE

In the tabulation below are the letter symbols used throughout this report. Wherever feasible, the letter symbols proposed in the American Standard ASA Z10.3-1948 have been used. These standard symbols are indicated by an asterisk.

c	*Viscous damping constant (lb sec/in.)
E	Voltage
F	*Force (lb)
F _o	Force, maximum (ft/sec ²)
f	*Frequency (cps)
g	*Acceleration, gravitational (ft/sec ²)
k	*Stiffness (lb/in.)
k _{dy}	Stiffness, dynamic (lb/in.) *
k _{st}	Stiffness, static (lb/in.)
m	*Mass (slugs)
n	Attenuation of a mounting (decibels)
n _o	Attenuation of a mounting referred to a single-mass, single elastic element system
r	Frequency ratio ($=\omega/\omega_n$)
t	Time (seconds)
x	Displacement (in.)
ε	Transmissibility of a mounting
ε _o	Transmissibility referred to a single-mass, single elastic element system
ρ	Damping ratio ($=c/c_c$)

NOMENCLATURE--Continued

ω *Circular frequency (radians/sec)

ω_n Natural frequency ($= \sqrt{k/m}$)

A DYNAMIC PROCEDURE FOR EVALUATING RESILIENT MOUNTINGS

I. INTRODUCTION

Scope. In recent years there has been an increasing interest in rubber resilient mountings, particularly in their response to some form of dynamic loading. These loads can be classified either as impulsive or as steady-state vibratory loads. It is the measurement of the dynamic response of rubber resilient mountings to the latter with which this paper is concerned.

Vibration Isolation. Most engineering design problems in the field of vibration isolation are concerned primarily with minimizing the transmission of harmonic forces either from a machine to its foundation, or vice versa, at a frequency for which ordinary vibration theory is valid. For a limited group of problems this is not sufficient. For example, in installations aboard a ship the so-called vibration isolator must serve also as a noise attenuator, where noise is broadly interpreted here to include air-borne and structure-borne vibrations in the audio-frequency range. The lumped-constant model fails over this wide frequency range and it must be assumed that the resilient element and other parts of the mounting behave as essentially elastic continua.

It is well-known that even at frequencies where elastic

elements, such as metal springs, behave in a manner compatible with the ordinary vibration theory, such media as rubber or rubberlike materials, cork, and felt display dynamic characteristics which preclude schematic representation by the Voigt model [1, p.538] [10] [13].¹ Further, irrespective of the type of resilient element in a mounting, any theory involving lumped constants and rigid-body loads will be valid for the isolation of structure-borne vibrations only if the wave length of the vibration transmitted is much larger than any physical dimension of the mounting or the load upon it. (It will be assumed here that the load on any mounting to be tested will be sufficiently small in size to offer only a mass impedance.) If the forcing frequency is such that the wave length of the vibration transmitted is of, or less than the order of the dimensions of the resilient element, then the mounting behaves as continuous media and wave phenomena appear.

Frequency Range. Previous methods of dynamic testing described in the literature [4] [5] [12] [13] are based on the assumption that the mounting to be tested responds as a lumped-constant (Voigt) model. Thus, the response of the mounting in the frequency where wave phenomena occur in the resilient element is not accounted for or measured. The

¹Numbers in brackets refer to the bibliography at the end of this paper.

experimental method described here furnishes a means of measuring the response of a loaded resilient mounting to dynamic, steady-state loads at frequencies of approximately 10 cps to 1500-2000 cps.

II. SIMPLE THEORY

Transmissibility Defined. In considering the effectiveness of resilient mountings, it is well known that the transmissibility ϵ (defined as the ratio of the force transmitted by the mounting to the exciting force) depends not only upon the transmission characteristics of the mounting but also upon the properties of the support. On the other hand, if the mounting is fastened to a fixed base, the transmissibility ϵ_0 is a function of the mounting characteristics only.

It is not possible to devise a laboratory test setup that would simulate accurately the mass, damping, and stiffness characteristics of all the structures it would be possible to encounter in design problems. As a consequence, the empirical determination of ϵ in the laboratory is rather meaningless. However, the transmissibility ϵ_0 of the single mass-mounting-rigid foundation system permits comparison of various mountings and establishes a criterion for evaluating their probable effectiveness in the field.

A schematic representation of the proposed test setup is shown in Fig. 1. A distinct advantage of this particular arrangement driven in the manner indicated is that the transmissibility ϵ_0 , as defined previously, can be shown to be equal to the ratio of the acceleration amplitudes of the masses m_1 and m_2 . Since the proposed method uses

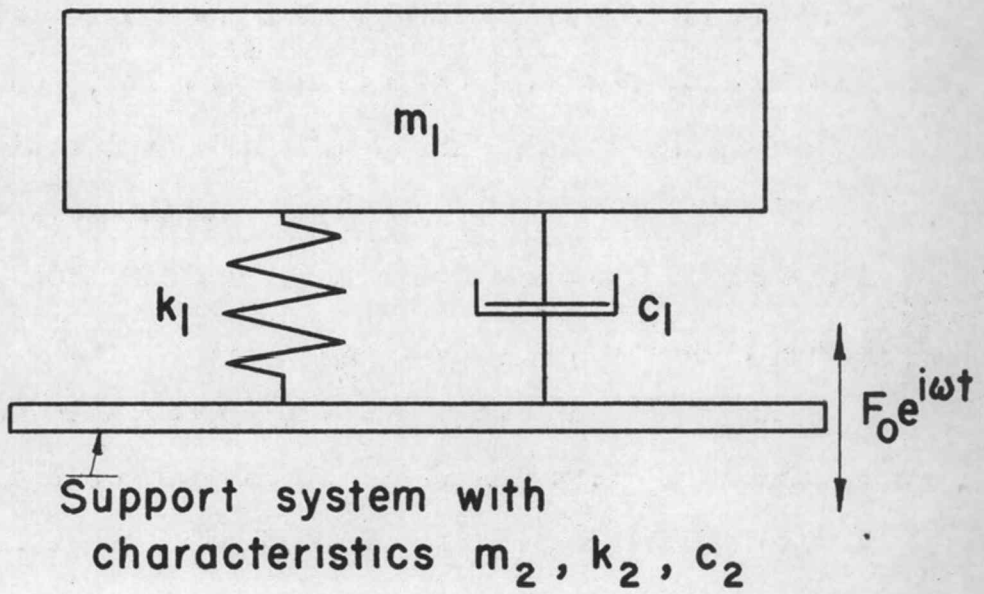


Figure 1
Schematic Representation of
Test Arrangement

this relation, it may not be out of place to recall that for harmonic motion, the transmissibility ϵ_0 of an idealized system consisting of a mass m_1 mounted on a rigid foundation by means of a resilient medium with stiffness k_1 and equivalent damping c_1 and can be expressed in complex notation as

$$\epsilon_0 = \frac{k_1 + ic_1\omega}{k_1 - m_1\omega^2 + ic_1\omega} \quad (1)$$

In the same complex notation, with reference to the system shown in Fig. 1, the equation which characterizes the motion of the mass m_1 is

$$x_1(-m\omega^2) + (x_1 - x_2)(k_1 + ic_1\omega) = 0 \quad (2)$$

from which

$$\frac{x_1}{x_2} = \frac{k_1 + ic_1\omega}{k_1 - m\omega^2 + ic_1\omega} \quad (3)$$

Thus, it is seen that the transmissibility ϵ_0 of the first idealized system is equal to the ratio of the displacements of the masses m_1 and m_2 in the system shown in

Fig. 1, and from (1) the following well-known equation is obtained:

$$\epsilon_o = \left[\frac{1 + 4\rho^2 r^2}{(1 - r^2)^2 + 4\rho^2 r^2} \right]^{\frac{1}{2}} \quad (4)$$

where ρ is the damping ratio (c_1/c_c) and r is the frequency ratio (ω/ω_n). Thus, if the displacements are sinusoidal, the transmissibility ϵ_o , which is a measure of the effectiveness of a mounting, can be determined readily in a setup similar to that of Fig. 1 by direct measurement of the accelerations of the masses m_1 and m_2 . The ratio of these accelerations does not depend directly upon the characteristics of the support system. Thus, m_2 , k_2 , and c_2 can be chosen so as to permit the entire assembly to be driven over a considerable frequency range. However, the motion of all portions of the support plate under the base of the mounting must be directed in a direction coincident with the main axis of the mounting, and must be sinusoidal and in phase.

Transmissibility Referred to Decibels. For the case of electrical circuits with the same impedance levels there is a decibel scale analogous to that used in the field of acoustics, namely:

$$n = 20 \log_{10} \frac{E_1}{E_2} \quad (5)$$

where n is the number of decibels and E_1 and E_2 are the voltages of the two energy levels to be compared.

The method requires that the two accelerometers used to measure the accelerations of m_1 and m_2 have approximately equal calibration constants over the desired range of frequencies. If this can be assumed, then the ratio of accelerations \ddot{x}_1 and \ddot{x}_2 will be equal to the corresponding ratio of voltages E_1 and E_2 and

$$n_o = 20 \log_{10} \epsilon_o \quad (6)$$

or

$$\epsilon_o = 10^{n_o/20} \quad (7)$$

Equation (6) is used as the fundamental criterion of comparison of resilient mountings. In the frequency range where the lumped-constant model is useful, the response (that is, the transmissibility ϵ_o) computed from (4) or empirically determined by using (7) is found to be very nearly the same. In the frequency range above, (6) provides an empirical criterion of comparison which is very useful.

Wave Phenomena. An obvious question arises when it is found that the lumped-constant model is not adequate for a large portion of the audio-frequency spectrum, namely, is there a mathematical model of a rubber resilient mounting which can account for the wave phenomena always measured in an actual test. A discussion of this model is not within the intended scope of this paper, but it might be well to point out that certain work has been done to account for these phenomena analytically.

Although the wave effects would be expected by anyone interested in the audio-frequency response of rubber resilient mountings, the first systematic study of the phenomena was made at Illinois Institute of Technology [17]. Later, Harrison and Sykes [11], Orlacchio [18], and others investigated the problem both analytically and empirically. In a more recent paper, MacDowell and Muster [16] discussed a continuum model which both accounts for the wave phenomenon and is valid for the low frequency vibration resonances.

III. EQUIPMENT

Components. For the sake of clarity the testing arrangement is shown in Fig. 2 as a block diagram which includes an electro-magnetic driving system and the electronic measuring system. The low energy levels being measured require that the electro-magnetic driver be removed from both the power amplifier and audio-oscillator, which are the source of oscillating power, and the components of the measuring system. Considerable electronic noise results from grouping these units in close proximity to each other.

The accelerometers (Massa, Type 117) are fastened as rigidly as possible to the metal support plate directly below the mounting and to a metal plug directly above it. This is done primarily to insure that any phase lag attributable to the distance between the points of measurement will be negligible. Oiled, mating mechanical joints at each of these points minimizes the losses across these interfaces.

Wave Form with Lissajou Figures. In the frequency region where wave effects occur, the phase difference in the accelerometer signals can be attributed to the response of the mounting alone.

The dual-beam oscilloscope furnishes a constant check on the wave form of the input and output accelerometer signals. The Lissajou figures also serve to indicate the phase

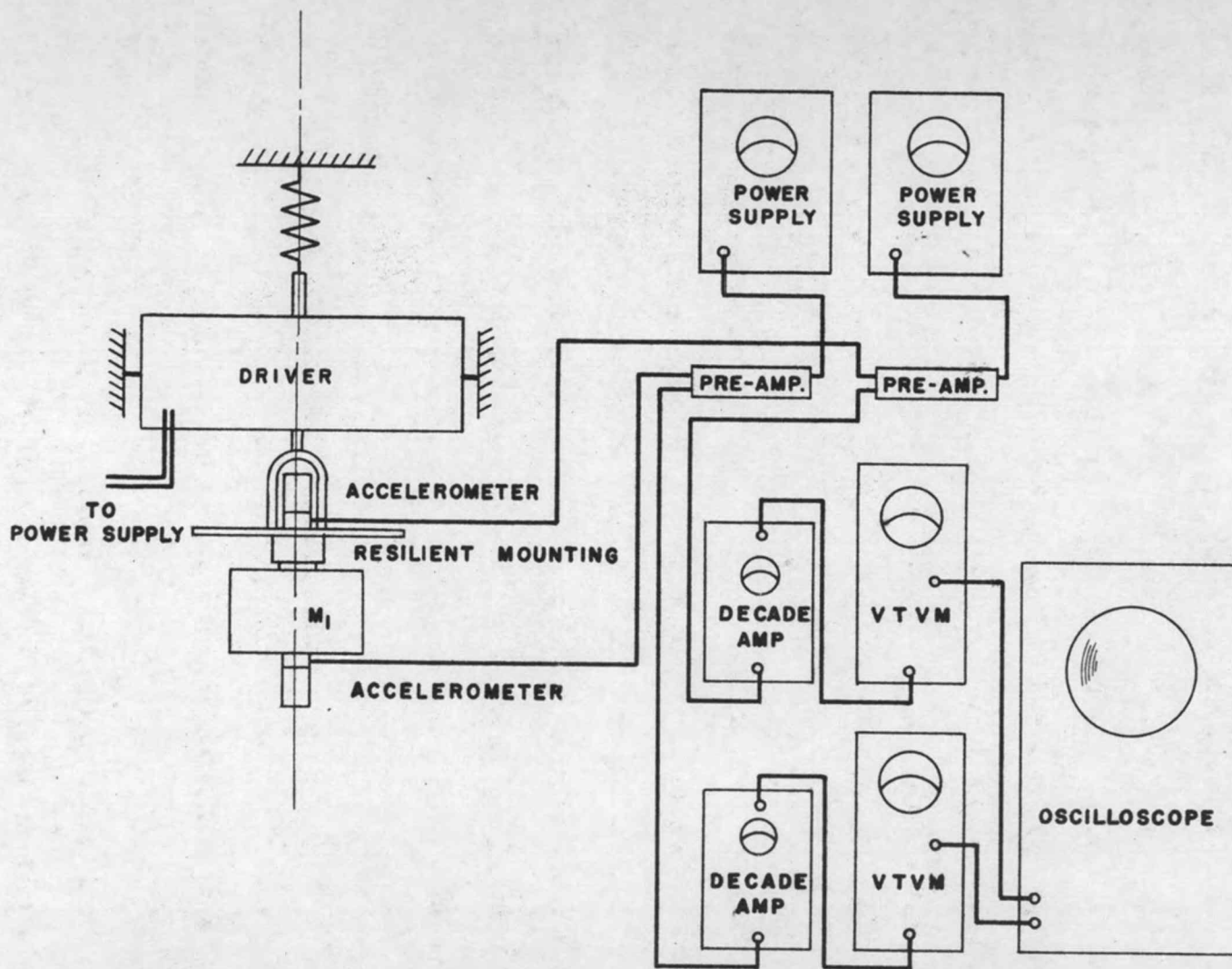


Figure 2 — Block Diagram of Complete Test Setup

relationship existing between the two signals. No data are taken unless the wave form of the measured accelerations is sinusoidal, although considerable harmonic distortion introduces only a small variation in the decibel level of an undistorted signal.

Importance of Shielding. Shielded cables are used throughout the driving and measuring systems and all the components, such as electromagnetic driver, oscillator, voltmeters, cables, etc., are grounded to minimize the electronic noise level in the instruments. Usually, it is possible to reduce this level to approximately -30 db,² which permits uncorrected readings to be made when the measured accelerometer signal is not less than -20 db.³

²Zero decibel level, 1 millivolt across 500,000 ohms.

³The addition of a signal 10 db above the background noise introduces an error of approximately 0.5 db [9, p.6], which is within the limits of error in the overall instrumentation.

IV. PROCEDURE OF TESTING

Static Load Deflection Test. Prior to any dynamic tests of the mounting, a static load-deflection test is made by means of the device shown in Fig. 3 or a similar testing machine. The value of the incremental static stiffness k_{st} , which is defined as the slope of the tangent to the load-deflection curve, is obtained from a plot of the data. After each change in the applied load, the corresponding deflection is not measured until the deflection gage indicates no appreciable motion [2]. Naturally, this method of "static" testing excludes all long term time effects, such as creep, but short term time effects are accounted for by permitting the material to relax.

The maximum static test load for a given mounting is set arbitrarily at approximately 150 per cent of the load at which the mounting is rated by the manufacturer. When this value of applied load is reached and the corresponding deflection measured, the entire load is removed quickly and elastic recovery-time data are taken. Initially, just after the load is removed, deflection measurements are made at very short time intervals. Usually, after two to five minutes, the unloaded mounting approaches an equilibrium position asymptotically which indicates the amount of permanent set present in the resilient element of the mounting.

Dynamic Test. For the dynamic phase of the test, the

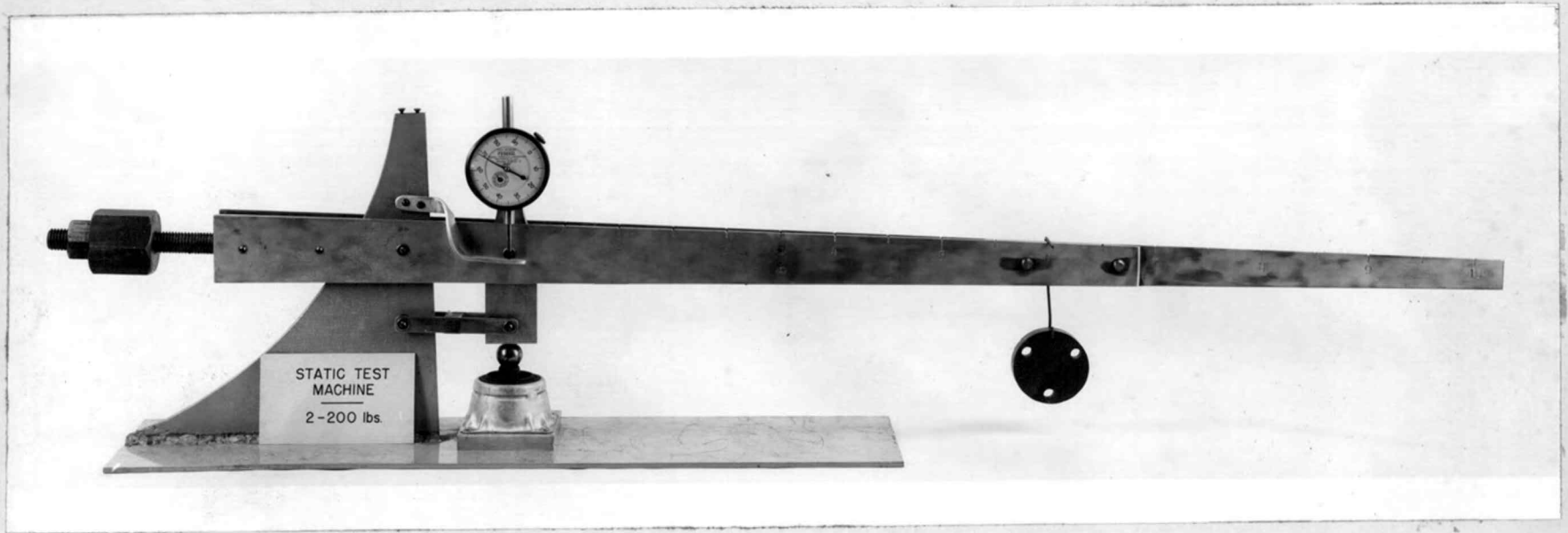


Figure 3
Static Testing Machine

mounting is placed in the test setup under a static load approximately equal to the manufacturer's rated load. The natural frequency of the mountings loaded in this way is usually about 10 to 20 cps.

Starting at a frequency value below the natural frequency of the mounting the following procedure is followed. At discrete frequencies, the acceleration signals from the upper (above the mounting) and lower (below the mounting) accelerometers are observed on the oscillograph to determine their wave form and, by means of a Lissajou figure, their phase relation. If the motion at both points is sinusoidal, the decibel levels are recorded and the phase relation noted.

The data from this test is plotted in with frequency as the abscissa and decibel attenuation (n_0) as the ordinate. This type of plot and the significance of the various critical regions are explained and discussed later.

Limitations and Refinements. In general, the method described here has limits of practicability which are governed by the physical size and shape of the components of the testing arrangement and by the effects of air-borne noise. It has been found that the former is the source of extraneous vibrations at the transverse and longitudinal resonant frequencies of the various components. However, these resonances are very different from the wave resonances that occur in the rubber resilient element. Any rubber wave resonance is relatively broad (usually, from 200 to 300 cps)

whereas the high-Q resonances of the metallic components are much narrower and peak very rapidly. Further, the extraneous vibrations are not normally accompanied by a 90° phase shift at their peaks such as accompanies the wave resonances in the rubber. Only one type of extraneous vibration appears as a true resonance and is accompanied by a phase shift. This is at the frequencies at which the weight W_1 is caused to resonate longitudinally. The available weights are cast steel parallelepipeds from 3" to 24" high and about one square foot in horizontal cross-section. As can be seen from these dimensions, the lowest frequency at which these longitudinal resonances could become important is about 4000 cps, well above the upper limit of practicability of the method for other reasons. Thus, although their presence is highly undesirable, the extraneous vibrations can be detected and identified rather readily and the test results corrected accordingly. Another means of minimizing their effect will be discussed later.

If weights larger than approximately 100 pounds are used, for most mountings it is not possible to drive the system at high frequencies. This manifests itself in the fact that sufficiently large sinusoidal forces can not be furnished at the input side of the mounting to cause a corresponding acceleration at the output side which is above the noise level of the instruments. No fixed limitation can be assigned where this will occur, but at some value of

frequency it always occurs.

The purpose of the test is to measure the dynamic response of a resilient mounting to structure-borne vibrations only. Air-borne noise from the driver or some radiating component in the system can be picked up very readily by the accelerometers. This can be minimized by placing caps over the accelerometers, but, at best, this serves only to raise the upper frequency limit of the method slightly and does not completely eliminate the trouble.

For small weight tests it is possible to enclose the entire driven assembly in a container from which the air can be removed. Thus, no air-borne noise ever reaches the upper accelerometer. However, this is not a convenient way to solve the problem for the larger weights.

V. DISCUSSION

Many resilient mountings have been tested by the described method. Unfortunately, the detailed results of these tests are available for only limited distribution.

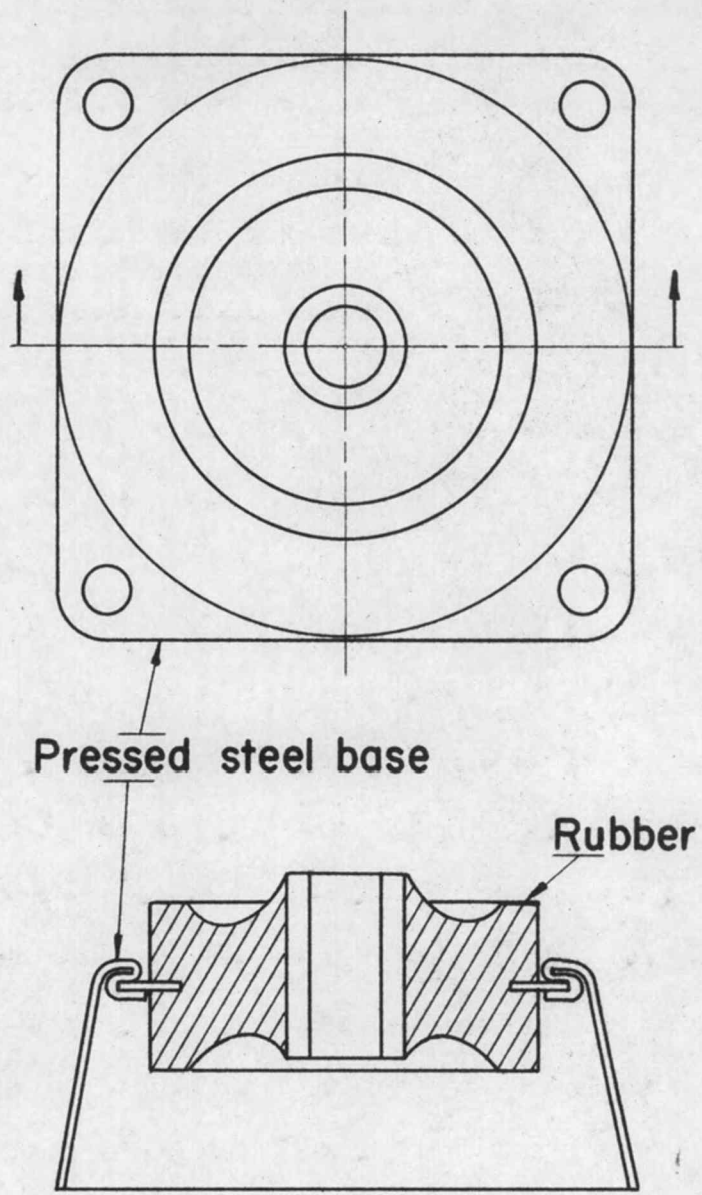
Typical Commercial Mounting. The mounting shown in Fig. 4 is typical of the largest class of commercial mountings, but is not representative of all resilient mountings. The main elastic element is rubber in shear and deflects almost linearly up to a load value in excess of its rated load. This linearity is typical of rubber in shear [14, p.29] and similar mountings are used very widely.

Although, as was mentioned, the resilient element of this mounting is rubber, other materials, such as, cork, felt, metal springs, wire mesh, and glass wool, in the shape of rods, tubes and plates are sometimes used. Additional damping resistance, other than the internal damping in the mounting materials is furnished by means of dry friction⁴ and air or oil damping devices.⁵

As can be seen in Fig. 4 the mounting consists of a pressed steel base which is bonded to a rubber resilient

⁴A current commercial mounting uses a wire mesh which furnishes a dry friction resistance caused by the rubbing of the wires on each other.

⁵There are several mountings with air or oil damping devices which operate by limiting the flow of air or oil through a small orifice.



Pressed steel base

Rubber

Figure 4
Typical Rubber Mount

element (used principally in shear) containing a bonded steel tube. When the mounting is in use, the base is usually fastened to the foundation or substructure and the load is fastened to the steel tube.

Dynamic and Static Stiffness. Ordinary vibration theory [6, p.14] [19, p.51] using lumped constants gives a good engineering approximation of the dynamic response of a mounting over the low frequency portion of the audio-frequency spectrum, provided that the dynamic effect on the stiffness of the rubber is accounted for in the design computation. For steel and most metal springs this effect is negligible, but for materials such as rubber or rubberlike materials, cork and felt, it has been established by many observers [1, p.93 ff] [7] [8] [10] that the stiffness of the latter materials is greater under a dynamic load than under a static load of the same magnitude. For rubber, this phenomenon has been shown to be dependent upon

- (1) the ambient temperature of testing [1, p.202] [15],
- (2) frequency [20],
- (3) the static strain in the specimen [3],
- (4) the amplitude of vibration [8], and
- (5) the type of rubberlike material of which the specimen is made.

The values of the ratio of dynamic to static stiffness of approximately 1.5 to 4.0 are not unusual; thus, this has

the effect of increasing the natural frequency to about 1.2 to 2.0 times the value computed from static constants.

Effect of Wave Length on Transmissibility. At sufficiently high forcing frequencies the wave length of the vibrations propagated in the resilient element are of the same order of magnitude as the dimensions of the element itself and wave resonances result in the mounting material. These resonances tend to effect the overall response of the mounting by decreasing the possible attenuation through it, and, in effect, increase the transmission of sound at these wave resonant frequencies.

In Fig. 4, it is shown how the resilient element is bonded to the pressed-steel base. When the relatively rigid base is caused to move in a vertical sinusoidal motion at an audio-frequency, due to the axisymmetrical construction of the mount the energy of the motion is imparted to the rubber in compressional acoustic waves, essentially toroidal in shape. It has been shown that these acoustic waves do resonate approximately as a function of the radial distance through the rubber which separates the base and the vertical steel tube (Fig. 4).

Dillon and Gehman [7] have demonstrated that the damping in a viscoelastic material, such as rubber, varies inversely with frequency, which would indicate that the observed transmissibilities should increase with frequency. The results of several investigators have corroborated this

[17, p.60 ff] and a mathematical model of a mounting has been discussed which includes this factor [16].

Witte, et al. [21] have shown that the velocity of wave propagation increases with frequency. Empirical results on mountings [17, p.86 ff] again corroborate these results in that the harmonics of the wave resonances do not occur at integral multiples of the fundamental, but rather at frequencies somewhat higher than the multiples.

Sound Isolation. The inference that can be drawn from the above discussion is that poor sound isolation by a mounting will result from a combination of light loads on⁶ and large radial rubber dimensions in a resilient mounting. Not all field installations require a high degree of sound isolation, but in those instances where it is desirable, the above criterion may be useful.

⁶From this it follows that the ratio of the mass of the resilient element to the mass of the load is small. See [16] for a discussion of the effects of small mass ratio.

VI. SUMMARY

By using the relationship given in equation (7) to define the transmissibility ϵ_0 , the test results obtained by this method can be used to compare the dynamic response of resilient mountings up to a practical frequency limit of about 2000 cps at static loads of 100 pounds or less. If air-borne noise effects and wave resonances in the components in the testing arrangement are accounted for, data can be taken at frequencies greater than 2000 cps. A vacuum chamber arrangement is one way this can be accomplished. This range of frequencies includes (a) the region where the simple viscous vibration theory is a good engineering approximation of the dynamic response of a mounting, and (b) a region of higher frequencies where wave resonances in the mounting material cause pronounced deviations in the response predicted by this theory.

BIBLIOGRAPHY

1. Alfrey, T. Mechanical behavior of high polymers. New York, Interscience Publishers, 1949. 58lp.
2. American society for testing materials. Tentative method of test for compressive strength of plastics. 1949 (ASTM Designation: D695-49T)
3. Baldwin, F.P. Determination of the dynamic properties of rubberlike materials by means of a modified Yerzely oscillograph. Rubber age 67:51. 1950
4. Chilton, E.G. Dynamic characteristics of rubber supports from vibration table data. Journal of applied physics 17:492-495. 1946
5. Cornell, D.H. and J.R. Beatty. Laboratory testing of rubber torsion springs. Rubber age 60:679-688. 1947
6. Crede, C.E. Shock and vibration isolation. New York, Wiley, 1951. 328p.
7. Dillon, J.H. and S.D. Gehman. Hysteresis and methods for its measurement in rubberlike materials. India rubber world 115:2-16. 1946
8. Gehman, S.D. Rubber in vibration. Journal of applied physics 13:402. 1942
9. General radio company. The noise primer. 3rd ed. Cambridge, 1948. 53p.
10. Hardy, H.C. and F.G. Tyzzer. Noise and vibration reduction in felt mountings. Journal of acoustical science 18:253. 1946
11. Harrison, M. and A.O. Sykes. Wave effects in isolation mounts. Journal of acoustical society of America 24:62-71. 1952
12. Howard, H.C. Comparative study of vibration absorbers. Industrial and engineering chemical journal 13:231-235. 1921
13. Hull, E.H. Influence of damping in the elastic mounting of vibrating machines. Transactions of the American society of mechanical engineers 53: 155-165. 1931

Bibliography--Continued

14. Imperial chemical industries, ltd. Rubber in engineering. Brooklyn, Chemical publishing company, inc., 1946. 267p.
15. Ivey, D.G., B.A. Mrowca and E. Guth. Propagation of vetrasonic bulk waves in high polymers. Journal of applied physics 20:486. 1949
16. MacDowell, E.L. and D. Muster. A mathematical model of a rubber resilient mounting. Proceedings, first midwestern conference on solid mechanics. To be published summer 1953
17. Mechanical engineering department of Illinois institute of technology. Resilient mountings for reciprocating and rotating machinery. Chicago, 1950. 413p. (Engineering report no. 2)
18. Orlacchio, A.W. Transmissibility of shock mounts. Washington, National bureau of standards, 1952. 2lp. (Report 1494)
19. Timoshenko, S. Vibration problems in engineering. 2nd ed. New York, Van Nostrand, 1937. 470p.
20. Witte, S.S., B.A. Mrowca and E. Guth. Propagation of audio frequency sound in high polymers. Journal of applied physics 20:481. 1949.