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Simulated Annealing and Threshold Accepting are two stochastic search algorithms that have been successfully used on a variety of complex and difficult problem sets. Due to their stochastic nature they are not guaranteed to produce the same result for each run. This means that these techniques actually produce a distribution of solutions which are based on the input parameters and the problem instance. Most research in the area of Simulated Annealing and Threshold Accepting focuses on the single run performance of these algorithms and does not consider sampling multiple runs and taking the best result, known as multisampling. Previous work that has looked at multisampling did not explore a variety of settings or problem instances which has left gaps in the understanding of the multisampling performance of Simulated Annealing and Threshold Accepting.

This work examines the use of single runs and multisampling on four instances

of the Traveling Salesman Problem. A systematic exploration is done of the variables which affect the performance of these two heuristics. A pairwise analysis is then performed to identify if there are cases where it is advantageous to employ a multistart method instead of a single run. The conclusion is that in a majority of cases a single run outperforms the multistart method but there are cases in which the multistart method outperforms single runs.

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A Comparison of Single Runs Versus Multistart for Simulated
Annealing and Threshold Accepting

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Matthew Crews

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I understand that my thesis will become part of the permanent collection of Oregon State University libraries. My signature below authorizes release of my thesis to any reader upon request.

Matthew Crews, Author

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Chapter 1: Introduction

1.1 Stochastic Optimization

In the world of Optimization there is a field referred to as Stochastic Optimization. This field includes optimization heuristics which utilize random numbers to help guide them in finding solutions to complex objective functions[16]. The randomness of their search allows them to navigate around local optimum that deterministic algorithms may be susceptible to becoming stuck in. The downside to the use of random numbers is that the randomness of the search precludes the algorithm from necessarily finding the same solution each time it is run. This leads to the algorithms producing a distribution of solutions to a problem instead of deterministically arriving at the same solution every time. This raises the question of whether it is better to run a single instance of these Stochastic Optimization heuristic or to instead run multiple instances, known as multistart, and take the best result from the distribution of solutions that are generated.

Simulated Annealing is a stochastic optimization metaheuristic put forward by Kirkpatrick et al. [47] and later independently by Cerný [20]. It mimics the behavior of slowly cooling metal to achieve a low energy state and applies it to optimization problems. An initial solution is created and many small modifications to the solution are proposed. The acceptance of modifications to the solution are

accepted based on a Pseudo-Temperature which is analogous to the temperature of a heated metal. As the algorithm proceeds the Pseudo-Temperature is slowly decreased like the cooling of metal. As the Pseudo-Temperature decreases then the probability of accepting modifications to the solution decreases as well. Simulated Annealing found success when applied to the Traveling Salesman Problem [47]. Threshold Accepting is a modification of Simulated Annealing proposed by Dueck and Scheuer [31]. They made a simplification to the acceptance of the solution modifications which made the algorithm less computationally expensive. They claimed that this increased the efficiency and that Threshold Accepting had faster convergence to the optimum than Simulated Annealing.

A natural benchmark problem for these Stochastic Optimization Heuristics is the Traveling Salesman Problem. The Traveling Salesman Problem is one of the classic combinatorial optimization problems which is frequently used as a benchmark problem for optimization heuristics. The problem statement is: given a set of cities, choose a route which visits each city only once, returns to the origin city and is the minimal distance of travel. It was first described in a booklet for salesman in Germany in the 1800's [91]. Its first mathematical formulation was done by Menger [60]. Later it was shown to be an NP-Complete problem [72] which is a class of problems which have no polynomial time solution technique rendering large instances difficult, if not impossible, to solve.

1.2 Research Objective

The goal of this research is to identify what factors may impact whether it is more efficient and robust to run a single instance of a stochastic optimization heuristic for a long period of time versus a multistart method which runs several shorter instances taking the same total amount of time. To achieve this objective a systematic analysis of the input variables is performed across several problem instances. By performing this experiment with several problems instances it ensures that the results are not due to an interaction with the heuristic and a specific problem instance.

1.3 Contribution

While there have been two papers which have looked at a comparison of multistart methods, one for Simulated Annealing [29] and another for Threshold Accepting [69], they did not thoroughly cover the the possible variables which impact the distribution of solutions: problem instance, cooling coefficient, moves per temperature step, number of restarts etc. This thesis examines each of these factors in a systematic way to thoroughly understand whether there is a possible combination of factors which would make it advantageous to use multistart methods versus a singular, long run. Appropriate statistical tests are used to verify if there is a statistically significant difference between the results of these techniques.

1.4 Organization of the Document

This thesis begins with a literature review which is divided into three sections: The Traveling Salesman Problem, Simulated Annealing, and Threshold Accepting. The Traveling Salesman Problem section reviews some of the history of the problem and the techniques that have been developed to solve it. Both the Simulated Annealing and Threshold Accepting sections describe the original development of the respective techniques, the early refinements of them and subsections which go over some of the significant methods used to make them more efficient and robust.

The Methods section provides a thorough explanation of how this research was conducted, the rationale for the problems that were chosen and the means by which parameters were set. The Results sections examines the outcome of the experiments. The Conclusion reflects on what the experiments have made clear and provides thoughts for future work.

Chapter 2: Literature Review

The body of Traveling Salesman Problem (TSP), Simulated Annealing (SA) and Threshold Accepting (TA) research is vast and has already taken up many books. In this literature we review the history of each of these subjects and then discuss some of the latest research developments relevant to the topic of this thesis. Additionally Simulated Annealing and Threshold Accepting have several subsections which are devoted to separate subfields of study which attempt to increase the efficiency of these algorithms. In these sections we review the various refinements that have been made to these algorithms, and document their effectiveness in a myriad of use cases.

2.1 The Traveling Salesman Problem

The Traveling Salesman problem is a deceptively simple combinatorial problem which is easy to state but difficult to solve. In fact the Euclidian version of the problem has been shown to belong to the most complex set of problems, NP-complete [72]. The problem is thus: given a set of cities choose a route which visits each city only once, returns to the origin city and is the minimal distance for doing so. It is one of the classic combinatorial optimization problems and has remained difficult for large problem sets in spite of advances in computational

power and algorithmic complexity. To date the largest solved instance is 85,900 cities by the Concorde solver [12].

Müller-Merbach [65] were the first ones to recognize the TSP, although without any mathematical notation in a booklet [91] which describes how a salesman must travel around the countryside visiting the various cities in Germany. The intent was to identify the route which would take the salesperson through each of the cities only once while minimizing the amount of walking that would need to be done.

The first researcher to formulate the problem was K. Menger in [60]. Menger composed the problem as the identification of the minimum spanning tree or Hamiltonian path of the set of points X which contains all of the cities. Menger reported further findings on the properties of this problem in [61]. He noted that greedy algorithms which selected the next closest city would yield suboptimal results. He also looked into modifications of the TSP problem where the objective was to find a Steiner tree [62]. Here Menger also reformulates the problem as a messenger problem with the problem coded as permutations of cities.

In the 1930s Menger spent time as a guest lecturer at Harvard and it was at that time that Hassler Whitney was exposed to the problem [63] while he was finishing his PhD in graph theory. Whitney brought the problem to Princeton where he posed the problem of visiting each of the 48 states [27]. It was during this time that a great deal of thought went into the problem at Princeton [7]. Here it saw practical application and its connections to Hamiltonian games and Hamiltonian graphs were recognized [34]. In what was one of the first vehicle routing problems Flood [6] notes the application of the TSP problem to the bus routing problem

they were dealing with in 1940-1941.

Shortly after this the RAND Corporation became involved in the effort to solve TSP problems [52]. They recognized the practical importance and application of the TSP problem to transportation problems. Dantzig and Koopman [6] also had the insight that there was a connection between Linear Programming and the TSP which would later become integral in finding solutions to larger problems. Eventually the TSP found its way into a RAND Report by the effort of Julia Robinson [76] who tried to formulate a solution technique. While the effort was unsuccessful it was one of the first times that the problem was referred to as the Traveling Salesman Problem. It was this work that motivated Flood and Fulkerson to continue working on the problem [10]. During this time Birkhoff [17] had the insight that the TSP could be expressed as a convex hull of the permutations of cities where the matrix columns and rows would sum to 1. This paved the way for formulating the TSP as a linear programming problem.

The efforts to solve the TSP lead to the development of Mixed Integer Programming, Cutting Planes and the Branch-and-Bound Methods of combinatorial optimization. Mixed Integer Programming came out of the work of Dantzig, Fulkerson and Johnson [27]. Their insight was that the recently developed algorithm, the Simplex Method, could be used to help solve the TSP. By solving a LP relaxation of the TSP they could gain insight into what the optimal TSP route would be. In their groundbreaking work they calculated a solution to a 42 city problem which was remarkably large for the time. It was the beginning of the Cutting-Planes methods for solving TSPs using LP solvers. This breakthrough set

off a flurry of research in the development of MIP and spurred new research on techniques for solving the TSP.

After the development of the Cutting-Planes method came another technique which would be dubbed the Branch-and-Bound method. This method was first developed in the works of Bock [18], Croes [25], Eastman [32] and Rossman and Twery [79]. The work of Dantzig, Fulkerson and Johnson [27] was fundamental in beginning the development of this new method and the seeds of it can be seen in their earlier work. This new method would divide the solution space into separate subsets, called Branching, and would then compute the lower bound of those subsets, which is called Bounding. If one subset was found to have a higher lower bound than another it was set to the side and the more promising branch was further branched upon and bounded. This iterative branching and bounding of solutions eventually leads to a feasible optimum. If a branch was found to not yield a feasible solution then the next most promising branch was explored.

Soon after the developments of the Cutting-Planes and Branch-and-Bound methods for finding exact solutions to the TSP came another approach from the field of heuristics. The most impactful of these heuristics was the Lin-Kernighan method developed by Shen Lin and Brian Kernighan at Bell Labs in 1973 [56]. Their method took an existing tour, removed edges connecting cities and then repaired the tour based on a set of rules which would seek a superior solution. The number of edges that would be removed at each step is referred to as the numbers of "opts". A move which removed 2 edges would be considered a 2-Opt move. A move which removed 3 edges would be a 3-Opt move and so forth. The

general case is referred to as a k-Opt move where k can be any integer value. This idea of removing and replacing edges was first noted by Flood [34] who saw that by iteratively removing and replacing edges a superior solution could be achieved. The first formulation of this 2-Opt technique was by Croes [25] whose work was an extension of Flood. At the time he called the 2-Opt move an "inversion" since it would reverse the direction of a part of the solution. Lin and Kernighan's major contribution with their algorithm is that they formulated a method of considering many swaps simultaneously which before had been computationally too expensive.

Also occurring in the 1970s was the work of Saman Hong [39] who was the first person to actually code the method of Dantzig, Fulkerson and Johnson. The code integrated the ideas of Cutting-Planes and the Branch-and-Bound methods. This would later come to be known as branch-and-cut [71]. Other versions of this integration of techniques was done by Crowder and Padberg [26] and Grötchel and Holland [35].

Later on the Branch-and-Cut technique would lead to major successes in solving large instances of the TSP problem. Padberg and Rinaldi [71] solved a set of 42 instances in 1987 with problem sizes between 48 and 2,392 cities. The 2,392 city problem was the largest instance of the TSP which had been solved to date. Some of the most significant contributions that Padberg and Rinaldi made were techniques for reducing the amount of information that had to be stored as the algorithm progressed. This allowed their implementation to tackle problem instances that were far larger than those that had been attempted previously due to limitations in computer hardware. They also set themselves apart from previous

coding efforts by heavily employing the branching component of the Branch-and-Cut method. Their work is still the foundation for many of the modern TSP solvers used today.

Even with all of this continued effort on exact solvers the work on heuristic methods had not ceased. The work of Lin and Kernighan had set the stage for another leap forward in problem sizes. In 2000 Helsgaun put together several modifications to the Lin-Kernighan algorithm which would dramatically improve its effectiveness [37]. Helsgaun introduced the idea of 5-Opt moves where the intermediate steps were infeasible. This allowed the algorithm to navigate through complex exchanges of vertices to find better solutions. He also improved the pruning of edges to be considered in the swapping. At the time of its introduction it was shown to find every existing optimal solution for all solved TSP instances to date. While no algorithm has solved the 1.9 million-city World TSP the best solution found so far was found by the Helsgaun modified Lin-Kernighan algorithm.

While these exact and heuristic techniques were enjoying much success there was another class of heuristics that began to emerge in the 1980's which also showed success with the TSP. Instead of being deterministic methods they utilized a stochastic search process that integrated the use of random numbers into the search for good solutions to the TSP. One of the earliest of these techniques is called Simulated Annealing. It sought to mimic the physical phenomenon of slowly cooling metal. Simulated Annealing performs a slow cooling of the solution to achieve a minimum of the objective function which corresponds to slowly cooling metal so it finds a low energy state. After the success of the Simulated

Annealing algorithm came another stochastic search algorithm named Threshold Accepting. Threshold Accepting was a simplification of Simulated Annealing and was purported to converge faster. The next two sections detail the development of these algorithms and the state of the art in modifications to them for efficiency and robustness.

2.2 Simulated Annealing

One of the most common and heavily studied stochastic optimization techniques is Simulated Annealing first put forth by Kirkpatrick et al. [47] in 1983 and independently by Cerný in 1985 [20]. This technique seeks to utilize the idea of annealing metal and apply it to optimization. Simulated Annealing is an adaptation of the Metropolis-Hastings algorithm [64] which was originally intended to provide a robust and fast way to sample from a distribution which is difficult to directly sample from. Simulated Annealing performs a guided random search of the solution space to find the global optimum. It starts with an initial solution and will generate neighboring solutions and evaluate the change in the objective function. If the objective function improves, the neighbor becomes the new solution. If the change in the objective function worsens then the algorithm will test to see if this neighbor is accepted or not based on probability calculation. To guide the search the algorithm uses a Pseudo-Temperature as a parameter to determine the probability of accepting an inferior move. This Pseudo-Temperature (hereafter Temperature) is analogous to the process of heating and cooling metal to increase

its strength.

When a metal is hot there is an excess of energy in the system and it is easy for atoms to rearrange themselves into new formations. As the system cools the atoms will want to arrange themselves into the lowest possible energy conformation. If the cooling occurs slowly enough the atoms will form a regular crystalline pattern which is a low energy state for the system. If the metal is cooled too rapidly the atoms do not have sufficient time to arrange themselves in a low energy state and will become frozen in a less than minimal energy state. The Simulated Annealing algorithm seeks to mimic this behavior through allowing substantial change to the solution initially and over time reducing the acceptance of inferior states. Simulated Annealing starts with a high Initial Temperature so that most moves will be accepted. As time goes on the algorithm will "cool" the system and become less and less likely to accept inferior solutions. Eventually the algorithm will cease to accept any solutions which are inferior to the current solution. At this point the solution is essentially frozen and the stochastic search has come to an end. The randomness is in the selection of the next move and whether the algorithm will accept an inferior solution. This means that there is no guarantee that it will arrive at the same solution every time.

Kirkpatrick et. al. [47] used the acceptance probability that is found in the Metropolis-Hastings algorithm. The probability of acceptance of any move can be computed using Equation 2.1 where ΔE represents the change to the objective function and T the current temperature of the system.

$$P_{acceptance}(\Delta E, T) = \begin{cases} 1 & , if \Delta E < 0 \\ e^{-\Delta E/T} & , if \Delta E > 0 \end{cases} \quad (2.1)$$

From this we see that as T approaches 0 then the probability of accepting an inferior solution also approaches 0. Equation 2.1 is applicable when the objective is minimization, and is easily modified for maximization. The entire Simulated Annealing algorithm can be written only a few lines of pseudo-code.

There are several additional parameters which govern the behavior of the Simulated Annealing. There is also the number of moves per temperature step, which represents the number of neighbors that will be visited at each temperature step. This number needs to be large enough for the algorithm to have sufficient time to explore the solution space before moving on to the next temperature. The other important property in the cooling schedule, which is the set of temperature states that will be explored for the duration of the algorithm. Kirkpatrick et. al. proposed the exponential cooling schedule in their initial paper [47].

$$T(t) = T_o \times \alpha^t \quad (2.2)$$

After all of the moves for a given temperature have been explored then the current temperature of the system is updated using Equation 2.2. The α is the cooling coefficient and determines the rate at which the system cools. t is the number of temperatures that have already been visited. For values of α near 1 the system cools at a slow pace. It has been shown that the value of α should be

between 0.8 and 0.995.

The neighborhood of a solution is the set of solutions that can be reached from the current solution using the move the SA instance is employing. A move is a small transformation on the current solution which may or may not improve the objective function. An example would be to swap the order of items in a schedule or a small increase or decrease in a continuous variable. The change to the objective function the move makes is evaluated each time and is tested for acceptance. If the move is accepted then the current solution is updated with the move. If the move is not accepted then there is no change to the current solution.

Finite time implementations of simulated annealing are actually referred to as simulated quenching by Ingber [43] due to the fact that they do not provide a guarantee of reaching optimality. For each temperature there should be enough time given to the algorithm to reach a quasi-steady state where the value of the objective function is not substantially changing with time. Some papers which review the performance of finite-time temperature schedules are Van Laarhoven & Aarts [51], Collins, Eglese & Golden [24] and Romeo & Sangiovanni-Vincentelli [77].

The main focus of this work is to examine whether it is more efficient to run a single instance of the algorithm or several shorter instances, known as multistart. What sets this work apart from previous effort is that systematic examination of the possible contributing factors is performed. The answer to this question leads to the most efficient and robust means of employing these search algorithms. The following sections detail other research efforts that have examined methods for

increasing the efficiency of these algorithms including alternative cooling schedules, integration of local search algorithms, hybrid algorithms employing SA and parallelization strategies.

2.2.1 Cooling Schedules and Adaptive SA

Soon after the introduction of SA there was work which looked at different cooling schedules in order to increase the efficiency of the algorithm. The work of Szu and Hartley [82] looked at a cooling schedule which would converge much more rapidly than the geometric cooling schedule. They were working on continuous optimization with non-convex, non-linear objective functions. To overcome the potential drawbacks of a faster cooling schedule they adapted the move generation code to use a Cauchy distribution which has wide tails and would allow for large jumps in the solution space.

These early cooling schedules were all static in their control parameters and did not have mechanisms for adapting to the problems they were being applied to. The obvious weakness in this is that there was no flexibility in the heuristic to slow down or speed up the search based on the problem that was being explored. To address this shortcoming they developed the technique of Adaptive Simulated Annealing (ASA) or Dynamic Cooling Schedules which would modify the cooling parameters based on the progress of the heuristic. ASA measures the statistical properties of the solutions which are being generated to calculate what the next set of control parameters should be. One of the most common statistical measures

employed is provided in Aarts, Korst & Van Laarhoven [9] where a derivation is provided to estimate average cost of solutions and the standard deviation. The result of their derivations can be seen in Equation 2.3 which estimates the mean of the objective function for the current temperature and Equation 2.4 which estimates the variance of the objective function. The values of $\mathbb{E}_\infty(f)$ and $\sigma_\infty^2(f)$ can be approximated from the observations of the objective function that have been made for that temperature.

$$\mathbb{E}_c(f) = \mathbb{E}_\infty(f) - \frac{\sigma_\infty^2(f)}{c} \left(\frac{\gamma c}{\gamma c + 1} \right) \quad (2.3)$$

$$\sigma_c^2(f) = \sigma_\infty^2(f) \left(\frac{\gamma c}{\gamma c + 1} \right)^2 \quad (2.4)$$

where

$$\gamma = \frac{\mathbb{E}_\infty(f) - f^*}{\sigma_\infty^2(f)} \quad (2.5)$$

Huang, Romeo & Sangiovanni-Vincentelli [41] use a version of this derivation to make measurements of the quasi-equilibrium of the system. When it is determined that the system has reached a quasi-equilibrium then the temperature of the system is lowered. When the change in quasi-equilibrium is small then the algorithm terminates. This method of measuring the distribution of solutions being generated at a given temperature reduced the sensitivity of SA to the input parameters and provided a way for it to adapt to the problem instance being dealt with.

Ingber [42] also proposed the idea of reannealing or adapting the temperature schedule and time given to search at a given temperature to deal with the rapid

cooling schedule. This allowed SA to focus on searching the more sensitive areas of the solution space making it more efficient. He was working on finding cooling schedules which would converge much more rapidly than earlier proposals but the problem that can occur with too rapid of cooling is the heuristic getting caught in a local minima. His proposal was that the temperature would be increased or reheated, and the cooling coefficient modified for the region of the problem being explored. He called this process of reheating and slower cooling reannealing since it was restarting SA within SA. This concept was what later became known as Adaptive Simulated Annealing [43].

Ingber [44] followed up this work with a review of its use in real world applications where it proved to be highly effective in many optimization problems. Chen and Luk [22] showed it to be effective in the field of signal processing for the problems of maximum likelihood joint channel and data estimation, infinite-impulse-response filter design and evaluation of minimum symbol-error-rate decision feedback equalizer. It has also been used more recently on the minimum makespan problem and was found to compare favorably with Classic Simulated Annealing and Tabu Search [14]. Triki et al. took this idea of adaptive SA and showed that many of the earlier cooling schedules could be tuned to provide the same results [90]. They also put forth a new adaptive cooling schedule which tried to control the average decrease in the objective function over time and found that it had favorable results on several benchmark TSP instances.

2.2.2 SA with Local Search

One of the other approaches to increasing the efficiency of Simulated Annealing was to integrate a local search heuristic that would bolster SA's search when it was necessary to thoroughly examine a small subspace of the solution surface. SA is robust against not being caught in local optimum but may have difficulty in precisely identifying the global optimum. Adding problem specific local search heuristics alleviates the need for SA to iterate frequently around the global optimum. This iteration around the global optimum can be caused by the move SA is using not being flexible enough to land on the optimum. The difference between generating moves for neighboring solutions and local search is that the acceptance of moves is determined by an acceptance test and allows for inferior solutions while local search does not use the acceptance test and is exclusively looking for superior solutions. One of the first papers to explore this for the TSP came from Martin et al. [58]. They took the concept of the local search from the Lin-Kernighan heuristic and integrated it into Simulated Annealing. This made SA more efficient in choosing neighbors which were likely to generate superior solutions. Later Osman [70] took the idea of using Tabu search as the local search mechanism. He was examining vehicle routing problems and found that the combined power of SA with Tabu Search was able to yield significantly improved solutions.

In 1996 Yamada and Nakano [94] applied SA with a deterministic local search heuristic to the Job-Shop Scheduling Problem. Here they used SA to accept or reject new schedules which were then passed to a shifting bottleneck scheduling

optimizer which is a heuristic for job-shop scheduling problem with finite machines and jobs. This coupling of techniques proved to reliably find near optimal schedules for difficult benchmark problems and performed better than several other techniques which had been previously proposed.

Martin and Otto took the idea of SA with Local Search and explicitly subdivided the responsibilities of the two search techniques [58]. Their methodology was to have SA find areas of potential local optimum and then use a deterministic local search to thoroughly examine that space. In this case SA was not used as a means of honing in on the optimum but rather for identifying areas of potential interest. This meant that the SA component could accept dramatic changes to the objective function throughout the heuristic run. They found that this improved on the performance of the 3-Opt heuristic by 1.6% and Lin-Kernighan method by 1.3% for randomly generated TSPs.

Another application that benefits from the hybridization of SA and a local search heuristic is unconstrained nonlinear optimization. Hedar and Fukushima [36] took SA and married it with the Nelder-Mead local search technique. The advantage of this approach is that it leveraged the strength of SA in exploring a broad solution space but then used the Nelder-Mead [67] algorithm for honing in on the minimum to achieve accuracy which can be difficult for SA alone. This combination proved to be effective for their various unconstrained, non-linear test problems.

A different approach to integrating SA with a local search heuristic was put forward by Purushothama and Jenkins [74]. Instead of using SA and then switching

to local search as previous efforts had done they inserted local search into the SA at the end of each temperature step. The best solution that SA generated at that temperature step was taken as the input for the local search heuristic which would search for a solution. The solution the local search method found was then passed back to the SA algorithm which would then decrease the temperature and then perform another series of neighborhood searches at the new temperature. Purushothama and Jenkins found that this accelerated the convergence of SA and provided good solutions for the unit commitment problem.

An evolution of the previous thought was applied to Constrained Optimization Problems. Pedomallu and Ozdamar developed a Hybrid SA for Constrained Optimization Problems which used Feasible Sequential Quadratic Optimization as the local search heuristic [73]. The key difference in their work was that the local search was not triggered at each temperature step but probabilistically whenever SA found a superior solution. Near the end of the algorithm the local search was also triggered whenever an inferior solution was accepted.

2.2.3 Hybrid Genetic Algorithms and SA

While Simulated Annealing proved to be innovative at the time of its introduction it was not the first stochastic search method that had been developed. Genetic Algorithms (GA) are a set of techniques to evolve a population of solutions in order find the optimum. Holland was the inventor of Genetic Algorithms [38] and developed much of the formal framework for them in his time at University of

Michigan in the 1960s and 1970s. This popular optimization technique defined a set of rules which governed a population which would slowly evolve over time. The survival of solutions depended on their fitness which was determined by an objective function. Inferior solutions died off and surviving solutions were mutated into new solutions.

Lin, Kao and Hsu [55] developed a hybrid of SA and GA which embedded the SA inside of the GA search. A population of solutions would be generated and their fitness evaluated. The best of these would then be used as a seed for the SA search. The SA search would then generate a population of solutions which would then be evaluated. The new population was then mutated and the best handed back to the SA algorithm which then decreases its temperature. The authors proposed that this strategy utilized what best components of both GA and SA which provided a robust optimization method. One of their key assertions was that their algorithm was less computationally expensive than other algorithms which had been proposed to date.

A different approach for integrating the techniques was to pass the solution of one method to the other. Thangiah et al. [89] proposed a series of optimization steps called the GenSAT system which integrated Simulated Annealing, Tabu Search and Genetic Algorithms. In this case they would pass the solution found by one algorithm to the other as the seed solution in order to take advantage of the strength of each technique. They found that it produced new records for the Vehicle Routing Problem with Time Windows.

A refinement of this transition strategy was developed by Esbensen and Mazumder

[33]. They developed an algorithm they referred to as SAGA which would begin as a pure GA, over time begin to integrate SA and at the end transition to pure SA. They accomplish this by giving every solution in the population its own temperature. Mutations to the solutions are governed by SA which has little impact initially but as time progresses it begins to dominate the search process. Their conclusion was that the performance was superior to pure GA for macro-cell placement problems. Wong and Wong [93] used a similar integration strategy when hybridizing the techniques. They were concerned with optimizing the dispatching of power generators. They found that the use of SA as the acceptance strategy for mutations of the GA effective in preventing premature convergence. Their conclusion was that this hybrid outperformed pure SA. Jeong and Lee [45] continued this idea of using SA as the acceptance rule for mutations in GA but also added in the concept of Adaptive SA. This allowed the temperature governing the SA acceptance probability to increase should the heuristic deem it necessary to avoid premature convergence.

Mantawy, Abdel-Magid and Selim [57] resurfaced the idea of integrating GA, SA and Tabu Search in 1999. Instead of passing the solution of one technique to another as the seed they instead integrated components of each technique into one algorithm. The overarching heuristic was a GA where the generation of the population and mutation was done through Tabu Search and the acceptance of candidate solutions and mutations was governed by SA. They found it achieved superior solutions to GA, Tabu Search or SA alone on the Unit Commitment Problem.

Another integration of SA into a GA was done by Yu, Fang, Yao and Yuan [95]. They used SA as the means to mutate the populations generated by the GA. What was significant about this paper is that it included a proof of convergence that this combined approach could in theory achieve the global optimum.

2.2.4 Parallel SA

Although the original incarnation of SA was a serial algorithm the significant computational time required for some problem instances to arrive at a good solution quickly motivated researchers to look for ways to parallelize the algorithm. There are several different methodologies that have been employed to divide the SA algorithm up so that it can be accelerated with multiple processors. One of the most straightforward techniques is the parallelization of the objective function. This technique is problem dependent but can lead to significant speed gains if the problem instance allows. Early examples of this technique are found here [48] [49] [?].

One of the most simple and straightforward approaches is simply to run n independent trials on n separate processors. While this is infinitely scalable Bertocchi and Sergi showed that the speedup was not that significant [15].

Another approach has been to evaluate multiple moves at the same time. Aarts et al. [8] proposed several different methods for this approach. The unfortunate drawback to this approach though is that if there is no overhead put into synchronizing solutions different processors can get out of sync causing erroneous solutions.

One method to prevent this type of divergence in solutions from occurring is the acceptance of only one move of the many that were generated [49] [?] [48]. The drawback to this method is that significant computational effort may have gone into several good moves but they were not accepting.

Another possibility that has been explored in the multiple move generations strategy is the generation of many moves but the acceptance of several non-interacting moves. Darema et al. [28] successfully employed this strategy on a parallel gate array problem. The algorithm had to actively monitor which sections of the solutions were being modified to ensure that no conflicts occurred in the move generation process. In contrast to this Rose et al. [78] proposed a technique where multiple moves were generated and accepted and the inter-process communication would then synchronize the solution afterward. While this allows for far more parallelism it adds complexity to the resolution of conflicting moves.

A diverge and converge strategy was also proposed by Aarts and Laarhoven [50], and Azencott and Graffigne [13]. This method employs giving each processor its own copy of the solution and then having each of them independently work on their own solution. After a given amount of time those processors will then exchange information on which one has achieved the best solution, synchronize with the best solution and then carry on their own local search. This process does not violate the statistical rules which govern how much time a solution should be given to anneal.

Speculative computing has also been used as a parallization strategy where processors would work on new moves while the previous move was under evaluation.

This technique is most effective when the evaluation of the objective function of the generation of moves is expensive. One processor would begin the generation and evaluation of a move. At that same time another processor would be assigned to work on the next move assuming that the first move was not accepted. A third processor is asked to evaluate a new move assuming that the original move was accepted. When the first move is actually completed and the acceptance is actually determined the next step is already under evaluation. Which ever processor was working on the branch that was not accepted cancels its evaluation and gets reassigned a new branch. Witte et al. [92] showed that this speculative computing model could lead to substantial speed gains.

2.3 Threshold Accepting

Threshold Accepting (TA) is a technique that was proposed by Dueck and Scheuer [31] in 1989 which is similar in structure to Simulated Annealing except with a significant modification to the acceptance of inferior moves. Threshold Accepting does away with the generation of random numbers to determine if an inferior move is accepted and simply compares the ΔE of the objective function to the Current Threshold, T . This Threshold takes the place of Temperature in Simulated Annealing in determining which inferior moves are accepted. Therefore the acceptance probability of Simulated Annealing can be rewritten for Threshold Accepting as Equation 2.6.

$$P_{acceptance}(\Delta E, T) = \begin{cases} 1 & , if \Delta E < T \\ 0 & , if \Delta E > T \end{cases} \quad (2.6)$$

As the algorithm continues to run the value of the Threshold continues to decrease and therefore the number of inferior moves that are accepted is reduced. Dueck and Gunter claimed that this allowed Threshold Accepting to converge faster than Simulated Annealing. In their initial description of the Threshold Accepting algorithm they used a fixed set of Thresholds which had been precomputed. This was a significant departure from the cooling schedule that was used in Kirkpatrick et al. [47] for Simulated Annealing.

In 1991 Althöfer and Koschnick [11] showed that the long term performance of Threshold Accepting is similar to that of Simulated Annealing but does not provide the same guarantees that Simulated Annealing does. It was determined that TA will converge to toward the global optimum but will not necessarily arrive at it. This is due to the fact that it lacks the ergodicity of Simulated Annealing. The deterministic rule for determining whether an inferior solution will be accepted means that depending on the initial solution of the Threshold Accepting heuristic it may not be possible for the search to make it to the global optimum. Simulated Annealings stochastic acceptance of inferior moves means that it is always possible, though in some cases improbable, for the search to reach the global optimum from any initial starting point. Later on Dueck [30] proposed a further modification to the Threshold Accepting algorithm called the Great Deluge which changed the acceptance of inferior solutions from testing the change in the objective function

to testing acceptance based on the absolute value of the objective function. He claimed that this further reduction in complexity did not impede the search for high quality solutions.

2.3.1 Alternative Cooling Strategies

In 1995 Nissen and Paul [69] examined a modification of the Threshold Accepting algorithm where they integrated the geometric cooling schedule that Simulated Annealing used at first. Here they repeated the suggestion of Kirkpatrick et al. [47] that the value of the cooling coefficient should be between 0.8 and 0.995. They also proposed a method of evaluating what the initial Threshold should be based on random sampling of neighbor solutions. This provided additional assurance that the initial threshold would be sufficiently large and therefore accept a large percentage of the initial moves. Their proposal can be seen in Equation 2.7.

$$T_o = \beta \cdot \frac{1}{m} \sum_{i=1}^m f_i \quad (2.7)$$

where

f_i : objective function value of trial i

β : constant scalar

One of the other unique contributions of this work was the integration of a dynamic number of moves to be performed at each Threshold. The justification

for this was that as the heuristic progressed the difficulty of leaving suboptima increased and the probability of finding a superior objective function decreased. They proposed Equation 2.8 as a method of adjusting the number of moves at each temperature step. As the heuristic progressed more time is allocated to the local search eventually reaching a predetermined maximum T_{max}

$$I_t = \begin{cases} \frac{c \cdot T_0 \cdot I_{max}}{T_t} & c \cdot T_0 \leq T_t \leq T_0 \\ I_{max} & \text{else} \end{cases} \quad (2.8)$$

where

I_t : iterations to be performed at level t

I_{max} : maximum number of iterations

c : weighting factor

For their local search method they used the 2-Opt method which had shown earlier success. They examined three separate levels of search run time with a long run time, a short run time and a medium run time. One of the comparisons they made was the performance of the long search settings versus several iterations of the short run time. They refer to the multiple iterations of the short heuristic as a multistart method. They found that the improved reliability of the longer search method yielded superior results to multiple iterations of the shorter search a higher percentage of the time. In this case they used the CPU run time as a measure of computational effort and stated the the multistart method spent roughly the same

amount of time searching as the long search heuristic.

Another paper that came out in 1995 that discussed non-monotonic cooling schedules was the work of Hu, Khahng and Tsao [40] where they proposed a dynamic cooling schedule which could decrease or increase the threshold based on the path the heuristic was taking. They termed their Threshold Accepting variation Old Bachelor Acceptance (OBA). The key element of their algorithm was the temperature update strategy which is shown in Equation 2.9.

$$T_{t+1} = \left(\frac{age}{a} \right)^b - 1 \cdot (\Delta) \cdot \left(1 - \frac{i}{M} \right)^c \quad (2.9)$$

Here the *age* is the length of time since the last improvement in the objective function. When *age* = 0 then the Threshold is set to the lowest number possible making the acceptance of the next solution strict. Each time a move is calculated and not accepted the *age* increases. As *age* increases then so does the probability of accepting one of the new moves. For this equation *a* is the factor which governs the growth of the Threshold, *b* provides for power-law growth and *c* is used to scale the magnitude of T_t for the problem being optimized. They tested this non-monotone cooling schedule on several instances of the Traveling Salesman Problem with several different values of *a*, *b*, and *c* which corresponded to various search strategies. They found that the self-tuning properties of OBA provided for reliable solutions. The best performing strategy was "steepest descent, mildest ascent".

2.3.2 Backtracking and List Based TA

Another effort to improve the efficiency and robustness of TA was the idea of Backtracking proposed by Tarantilis, Kiranoudis and Vassiliadis [86]. Here they looked at the Vehicle Routing Problem (VRP) and the idea of allowing the Threshold to return to a previous value, or Backtrack, when no neighbors are found with a given threshold. They named this approach Backtracking Threshold Accepting (BATA). This thought of returning the threshold to a previous value is similar to the idea put forth by Ingber [42] for SA where the temperature in SA was reheated when it appeared the solution was converging prematurely. Tarantilis et al. [86] also proposed a method for initializing the value of the Threshold which removed the need for the practitioner to perform any tuning of the algorithm. For their neighborhood selection scheme they used a combination of 2-Opt, 1-1 Exchange and 1-0 Exchange. The proportion of each move that is generated is a controllable parameter in this instance. The algorithm terminates when there appears to be no more improvement in the objective function or no neighbors are accepted after a backtracking of the threshold has occurred. Tarantilis et al. [88] followed up this work in 2004 with an application of the BATA to an Open Vehicle Routing Problem (OVRP) and a Heterogeneous fixed fleet VRP [85] where they provided a thorough examination of the parameters which influence the performance of BATA. Again they found BATA to be highly effective and competitive with other available techniques.

At this same time Tarantilis and Kiranoudis proposed another technique for

controlling the acceptance of inferior moves termed List Based Threshold Accepting (LBTA) [87]. The modification that was proposed to the TA algorithm was a change in the method that thresholds are generated. The first difference is the calculation of the threshold values. Instead of simply taking the difference in the objective function they normalize the value as shown in Equation 2.10. Here s represents the current state of the solution, s' is the neighboring solution and $c()$ is a function to evaluate the objective.

$$T_{Norm} = \frac{c(s') - c(s)}{c(s)} \quad (2.10)$$

When the heuristic begins it generates an initial, feasible solution. The series of neighborhood solutions are generated and the normalized threshold values are recorded in a Threshold List. This list is then used as an input to the main optimization phase. As neighboring solutions are generated their normalized threshold is compared to the largest threshold contained in the list. If T_{Norm} is positive but lower than the T_{Max} contained in the list then the move is accepted, T_{Max} is removed from the list and T_{Norm} is added to the list. This method of removing the max and inserting new values into the list allows the algorithm to slowly converge while taking the variability of the solution space into account. This technique also proved to be effective on the job shop scheduling problem [83][81], heterogeneous fixed fleet vehicle routing problem [84], and zero-wait scheduling of multiproduct batch plants [54].

2.3.3 Hybridizing TA with other Optimization Methods

Since TA is such a simple algorithm it naturally lends itself to being integrated into other optimization strategies. One way that TA can be used is as a finishing heuristic when an initial heuristic is responsible for finding the region in which the optimum exists and then passing the responsibility of closing in on the optimum to TA. Bräyes et al. used TA as a means to improve upon the solution found by a parallel GA [19]. They showed that this hybrid approach worked well on benchmark problems for Vehicle Routing Problem with Time Windows. They particularly emphasized the robustness and speed of the TA heuristic.

Taking the inverse approach Nikolakopoulos and Sarimveis used TA as the overall search framework and then integrated a intensive local search algorithm for thorough neighborhood analysis [68]. They termed this use of Intensive Search with Threshold Accepting InTA. They studied the use of this approach on three variations of the Traveling Salesman Problem (TSP): the Asymmetric Traveling Salesman Problem (ATSP), Sequential Ordering Problem (SOP), and the Asymmetric Traveling Salesman Problem with Time Windows (ATSP-TW). All of these problems are generalizations of the TSP which are more complex and difficult to find solutions for. They found that in spite of the difficulty and complexity of these problems InTA required little tuning and was robust in finding good solutions to the benchmark problems.

Another integration of Threshold Accepting into another Heuristic was put forward by Chauhan and Ravi [21]. Their initial research focused on the use of

Differential Evolution (DE) for unconstrained optimization. DE is a genetic algorithm specifically formulated to deal with continuous, real number optimization problems where the objective function is noisy and possibly stochastic. It was formulated by Storn and Price [80] to deal with multidimensional, non-linear, and non-differentiable objective functions. What they found was that while DE was a powerful method for identifying the location of the optimal solution it was not necessarily able to identify it accurately. They proposed the use of Threshold Accepting as a finishing algorithm which would take the result of the DE optimization and then hone in on the optimal solution. They found that this two phase approach improved the overall performance of the optimization.

2.3.4 Other Variations of TA

One of the earliest attempts to increase the performance of TA while reducing its run time was to allow for adaptive moves and for adaptive temperature reduction [23]. In this paper Lin et al. examined the use of adapting the neighborhood of moves that both SA and TA focused on and also adapting the rate at which the Temperature/Threshold decreased. The test cases were scheduling problems and the ordering of the jobs was what determined the value of the objective function. They surmised that only a few of the jobs in the sequence had large impacts on the objective function while many of the remaining jobs would have little impact. They reasoned that the SA/TA algorithm should therefore focus its search on these high impact jobs. The algorithm would partition the jobs into separate

groups and would purposefully more heavily sample moves involving the jobs which had a higher impact on the objective function. As the problem progressed these groups could be reevaluated. They also allowed for variable decreases in temperature/threshold. Initially the temperature/threshold would cool at a normal rate but as the problem progressed a new rule would take over which would examine the results of the previous temperature/threshold step to determine what the next temperature/threshold should be. They found that Threshold Accepting beat or was comparable to Simulated Annealing for their benchmark cases.

Another modification of TA was used in the generation of fuzzy rule generation for classification by Ravi and Zimmermann [75]. Here they modified TA in two significant ways: the generation of the neighborhoods was deterministic and the threshold loop could terminate early. For their problem of identifying rules for a classification problem they found that a deterministic method of evaluating neighborhood solutions was an effective means of searching the solution space. They also allowed for early termination of a threshold loop. If too many solutions were being accepted the loop would terminate early and the threshold decreased. They found that the solution converged faster and provided higher classification power than previous methods.

There has also been work at automatically generating the control parameters for TA. Pérez, Pazos, Velez and Guillermo [46] provided a technique for generating the initial threshold regardless of the problem instance. They suggested an initial sampling of the neighborhood of solutions be done to form a distribution of the change in the objective function. The initial threshold was such that it would reject

40% of the moves that were generated. They also propose a test for whether the problem has reached equilibrium which is used to determine whether a threshold run is terminated or not. They found that this combination of automatic tests provided for good solutions without requiring any tuning by the practitioner.

2.4 Previous Multistart Research

There have been two previous papers which have dealt with the subject of multistart performance of Simulated Annealing and Threshold Accepting. The first was a short communication published in 1990 in *Parallel Computing* by Dodd [29]. In this paper Dodd examined the use of Simulated Annealing on the graph labelling problem. He used a fixed initial and final temperature for each of the experiments. Instead of having multiple moves evaluated at each temperature he had the algorithm only perform a single move at each temperature and only varied the value of the cooling coefficient to adjust the length of time the algorithm took.

For his analysis he wanted to observe the relationship between the number of moves the algorithm took and the probability of achieving the global optimum. He had a single instance take I moves, 2 instances take $\frac{I}{2}$ moves each, 4 instances take $\frac{I}{4}$ moves each and so forth. Each time he had multiple instances run he would take the best result of the values that was returned. He found that once a minimum number of moves were performed there was a linear relationship between number of moves a single run performed and the probability of finding the global optimum. This result suggested that there is no advantage in performing multiple

runs instead of a long single run. This analysis was only performed on one test problem so it was unclear if this behavior would be observed with other problem instances.

The second paper was done by Nissen and Paul [69] and looked at the performance of Threshold Accepting for different multistart strategies on the Quadratic Assignment Problem. They used three different sets of parameters corresponding to different search strategies: a long and intensive search, a fast search, and a moderate search. They used the geometric cooling schedule that Kirkpatrick et al. [47] proposed for the Simulated Annealing algorithm. Unlike Dodd [29] they had multiple moves performed at each temperature step and would vary the number based on the benchmark problem. What was held constant across the experiments was the value of the cooling coefficient. This is the parameter which determines the number of temperatures that will be explored by the algorithm. For the long search instance the cooling coefficient was 0.995 and for the short search instance the value was 0.8. When the number of temperature steps for each one is calculated it shows that the long search instance will visit 44 times more temperatures than the fast search instance which is a significant difference in the search time. They found that multiple instances of the shorter algorithm were unable to match the performance of the long search across all of the benchmark problems which is in alignment with the results of Dodd [29]. It is important to note that this paper only looked at the difference between long and short runs but did not explore settings in between these values.

A key difference between these previous examinations of multistart methods

and this work was the care that was taken in controlling the computational effort given to each heuristic run. These previous works used time as a measure of computational effort which is prone to error. The true amount of computational effort that is expended in an individual run can be obfuscated by other processes utilizing the CPU or inefficient algorithms and data structures. This work does not use time but the actual number of steps performed for each run as a measure of computational effort. This provides a more accurate measure of the computational effort that has gone into a run.

Chapter 3: Methods

The purpose of this research was to answer the question of whether, given a limited amount of time, it is better to run a single instance of a stochastic search algorithm or to instead run multiple shorter instances of the search algorithm and take the best result of that set. This idea stems from the fact that stochastic search methods do not guarantee the same result every time that they are run. This means what is really occurring when a run of a stochastic optimization algorithm is performed is that a sample is being taken from a random distribution. The distribution is formed by the relationship between the stochastic optimization technique and the problem instance that it is being used on. It is conceivable that a point exists where it is better to take the time that is allotted for a single run and run two or more searches and take the best result rather than spend the entire time on a single search. This springs from the fact that many large scale optimization problems have many local optimum where a stochastic search can become stuck and unable to make it out of the local minimum in order to find the global optimum. This behavior of getting stuck in local optimum may make it better to run multiple instances of a stochastic search to reduce the risk of this occurring with a single run.

To answer the question that has been put forth it is important to consider different stochastic optimization techniques. The two techniques used in this research

are Simulated Annealing and Threshold Accepting. They both work by successive iterative improvement and they both randomly generate the next possible solution. Where they differ is in their technique for accepting an inferior solution or not. Simulated Annealing will randomly accept an inferior solution but as the algorithm progress this probability decreases. With Threshold Accepting the acceptance of the inferior solution is deterministic and is determined by a threshold of acceptance which slowly becomes stricter.

At the end of the experiment the goal is to be able to say whether it is better to run a single instance of the stochastic optimization algorithm or to instead run two or more instances which individually have less time. This means the total quantity of computation is the same. To do this it is important to ensure that the amount of computational effort that goes into each of the runs is highly controlled. The computational effort of a stochastic search is determined by the number of moves that are performed. The following sections details how the computational effort is controlled to provide for valid comparisons.

3.1 Measuring Computational Effort

In order to compare the computational effort involved in each run of the stochastic optimization algorithm it is necessary to have a method of measuring which is not affected by the implementation of the algorithm or the hardware that it is run on. Time is a poor measure of computational effort because it is highly sensitive to factors outside of the algorithm. Most computers have an operating system

running which is managing the resources of the computer and will swap programs in and out of the CPU on the fly in order to provide a quality experience for the user. When this happens it will cause a long running program, such as an optimization run, to appear to take longer since it had to pause and resume when the operating system forced it to do so. This makes time an ineffective measure for comparison because the actual amount of time spent on computation and the amount spent idle is sensitive to factors outside of the control of the algorithm.

There is also the problem of the architecture that the algorithm is running on. Modern CPUs are incredibly complex and come in a wide variety of performance levels and capabilities. The exact same code could run dramatically faster or slower just by switching the architecture it was run on. Again, this defeats the purpose of having a reliable and repeatable method of comparing the computational effort that went into a single instance of a stochastic optimization run.

For this research the number of moves that the Simulated Annealing or Threshold Accepting algorithm performs is considered to be the measure of computational effort. The computational effort is calculated ahead of time by calculating the number of temperature/threshold steps that will occur and multiplying it by the number of moves performed at each temperature/threshold.

$$\text{Number of Temp Steps} \times \text{Moves per Temp Step} = \text{Computational Effort} \quad (3.1)$$

This method of measuring effort is not affected by how the algorithm is im-

plemented or what architecture the program is run on. It is not affected by any interruption in execution caused by the computers operating system.

3.2 Selecting Search Parameters

Simulated Annealing and Threshold Accepting both require a set of input parameters to determine how they will run. It is important that these parameters be selected so that most of the initial moves are accepted and that later on poor moves are only allowed infrequently. Simulated Annealing uses the Current Temperature of the system as the input to the function which determines if a poor move is accepted or not. The Current Temperature starts at the predefined Initial Temperature (T_i) and slowly decreases until it reaches the Final Temperature (T_f) at which point the algorithm ends. As the Current Temperature moves from the high Initial Temperature to the lower Final Temperature the number of inferior moves that the algorithm accepts decreases. In Threshold Accepting there is a Threshold Parameter which is analogous to the Temperature Parameter in Simulated Annealing and is what determines if a inferior move is accepted or not. The Current Threshold begins as the Initial Threshold (T_o) and slowly decreases until it reaches the Final Threshold (T_f).

The selection of the Initial Temperature/Threshold and the Final Temperature/Threshold is critical for the performance of these two algorithms. The span between the Initial and Final Temperature/Threshold needs to be several orders of magnitude. For these problems it was found that 4 orders of magnitude between

the Initial and Final values was sufficient for many of the initial moves to be accepted and essentially none of the inferior moves to be accepted at the end of the optimization run.

3.2.1 Number of Temperature Steps

Both of these algorithms use the geometric cooling schedule proposed by Kirkpatrick et al.[47] shown in Equation 3.2.

$$T(t) = T_o \times \alpha^t \quad (3.2)$$

Where $T(t)$ is the Current Temperature/Threshold for temperature step t , T_o is the Initial Temperature of the system and α is the Cooling Coefficient. After the algorithm has completed a full set of moves for a temperature step the value of t is incremented by 1 and the Current Temperature/Threshold is set equal to $T(t)$. The algorithm will then complete another set of moves using the new current temperature. This will continue until the value of $T(t)$ is less than the Final Temperature, T_f . , Once this has occurred the algorithm is complete and the final value of the objective function is recorded. This means that the Number of Temperature Steps (NTS) that either algorithm will go through can be evaluated using Equation 3.3.

$$NumberTemperatureSteps = \left\lceil \frac{\ln\left(\frac{T_f}{T_o}\right)}{\ln(\alpha)} \right\rceil \quad (3.3)$$

Since the Initial and Final Temperature will be fixed across the different algorithmic runs we can then evaluate the number of temperature steps that the algorithm will go through for various values of α . The set of values that α takes on must generate values of NTS which are divisible by one another. This allows for direct comparisons of the computational effort that went into the runs. To calculate the values of α the Minimum NTS is determined and then the corresponding α is calculated using a rearrangement of equation 3.4.

$$\alpha_i = \frac{T_f}{T_o}^{\frac{1}{MinNTS \times 2^i}} \quad (3.4)$$

Once values of α have been computed from this formula it is important to check that they will actually generate the number of temperature steps that are required once the rounding is taken into account in Equation 3.4. It may be necessary to hand tune the values of α and check the number of steps they will generate with Equation 3.4. This is due to the lack of infinite precision in floating point math.

3.2.2 Moves Per Temperature Step

Now that there is a way to evaluate the number of temperature steps that will occur for a given α , the last factor which must be selected is the Moves per Temperature Step (MPT). This determines how long the algorithm will search the solution space at a given temperature/threshold. These values need to be selected so that the algorithm has enough time to search and move from a possible local minimum. Several different settings of these parameters are evaluated to explore the impact

it has the solution distribution. What is also critical is that the settings that are used generate a number of moves which are all divisible by each other so that computational effort can be directly compared. To do this the values used are multiples of 2^n . Once the minimum MPT has been found for a problem the other settings are evaluated:

$$MPT_n = MinMPT \times 2^n \text{ for } n = \{0, 1, 2, 3\} \quad (3.5)$$

This method of determining the set of MPTs to use guarantee that the number of moves per temperature step will always be a multiple of one another when comparing algorithmic runs.

3.3 Calculating Computational Effort

Now that the method for finding each of the search parameters has been laid out it is possible to calculate the computational effort for each set of parameters. The full equation for Computational Effort (CE) is shown in Equation 3.6.

$$CE_{in} = \left[\frac{\ln \frac{T_f}{T_o}}{\ln \frac{T_f}{T_o} \frac{1}{MinNTS \times 2^i}} \right] \times (MinimumMPT \times 2^n) \quad (3.6)$$

With Equation 3.6 it is possible to calculate the total number of moves that will be performed by an experiment and therefore the number of runs that will need to be performed by one experiment to equal the computational effort of another

experiment. Table 3.1 gives an example of how many runs of Experiment 1 would need to be performed to equal the computational effort of Experiment 2 with both using an Initial Temperature = 100 and Final Temperature = 0.58.

Table 3.1: Example of the number of samples from Experiment 1 to match the computational effort of Experiment 2 with both using an Initial Temperature = 100 and Final Temperature = 0.58

			Experiment 2				
			$\alpha = 0.95$				
		MPT	10	20	30	40	
α	MPT	Total Moves	1000	2000	4000	8000	
Experiment 1	0.95	10	1000	1	2	4	8
		20	2000		1	2	4
		30	4000			1	2
		40	8000				1

To read this table, look at the parameters for Experiment 1 on the left side. There is a column for the values of α and for the values of Move Per Temperature (MPT). The resultant number of moves that the algorithm will perform for these settings is shown in the third column under Total Moves. The algorithm settings for Experiment 2 run across the top. The first row holds the values of α and the second row hold the values of MPT for Experiment 2. The intercept of the Experiment 1 settings and the Experiment 2 settings shows the number of runs that will need to be taken from Experiment 1 to equal the computational effort of Experiment 2. The intercept of Experiment 1, $\alpha=0.95$, MPT=10 and Experiment 2, $\alpha=0.95$, MPT=20 is 2. This means that 2 runs would need to be taken from Experiment 1 to equal the computational effort of Experiment 2. Therefore 2 runs would be taken from Experiment 1 and the best of those 2 would be what would

compete with the value from Experiment 2.

3.4 The Test Problems

For a thorough examination of the behavior of these parameters several different instances of the traveling salesman problem ranging from small cases of 100 and 127 cities to medium sized problems of 423 and 442 were examined. The 100 city problem shown in Figure 3.1, named rand100, was a randomly generated set of points arranged in distinct clusters where the distance between the clusters was significantly larger than the distance between cities in the clusters. This was to challenge the algorithms with a problem which had city clustering since the other three instances have a more uniform spread of cities across the space. The other three problem instances are taken from literature: bier127, pbn423, and pcb442.

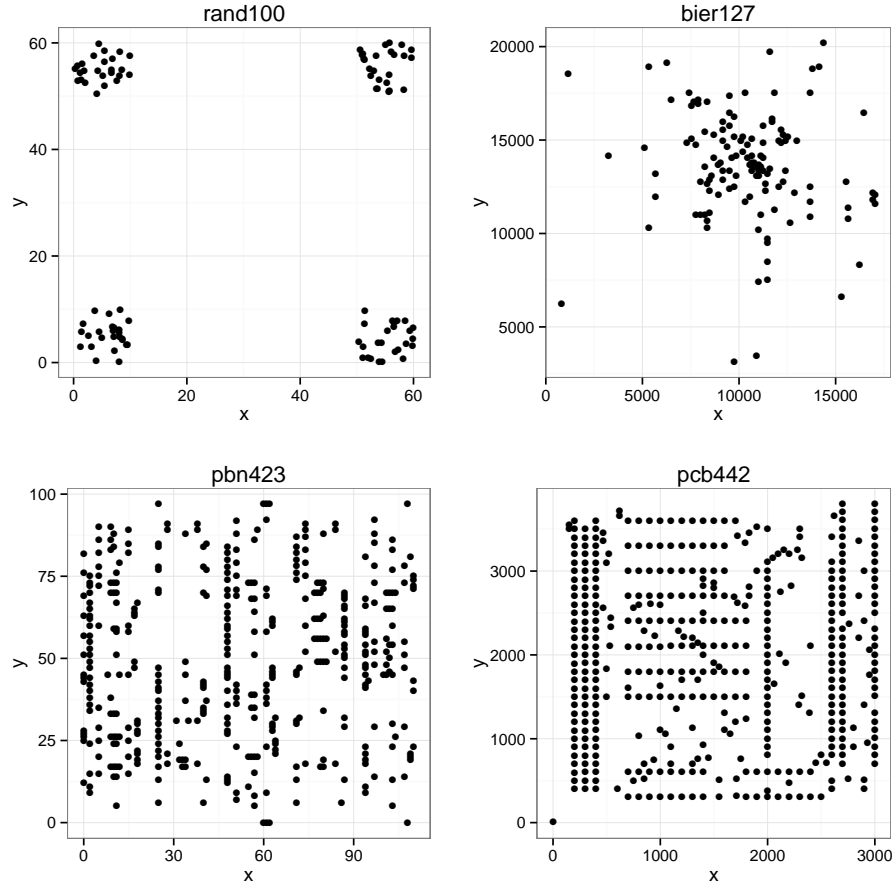


Figure 3.1: TSP test cases

3.5 Iterative Improvement Methods

There were two different methods used for the iterative improvement method utilized by Simulated Annealing and Threshold Accepting: 2-Opt and 3 city-swap. The 2-Opt method shown in Figure 3.2 was first proposed by Croes [25] and at the time he referred to it as inversions or subtour reversal since a part of the solution was reversed. It is a special case of the k -opt method described by Lin and

Kernighan [56] and is the most simple of the k-opt methods. Lin and Kernighan show that the 3-opt method provides better solutions than the 2-opt but the point of this examination is the comparison of the distribution of solutions rather than actually pursuing optimality.

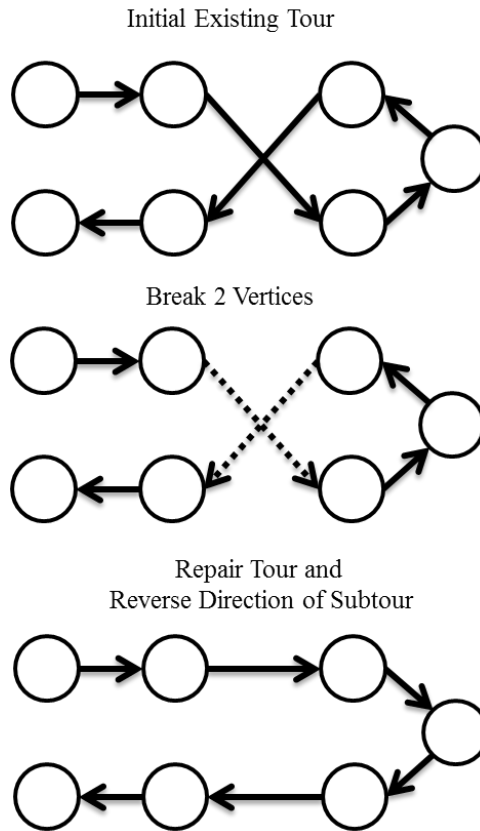


Figure 3.2: Example of 2-Opt move

In practice city-swap methods have not proven to be effective at finding high quality solutions for the traveling salesman problem. In spite of this a 3 city-city swap shown in Figure 3.3 is a meaningful iterative improvement method to study because its behavior is far different than the k-opt family of methods and could

lead to different conclusions at the analysis stage. In the 3 city-swap improvement a city and an adjacent city are selected at random. A third city, not necessary contiguous with the first two, is also selected. The algorithm then calculates all the permutations of swapping the 3 cities and selects the one which provides the greatest improvement as the proposed move.

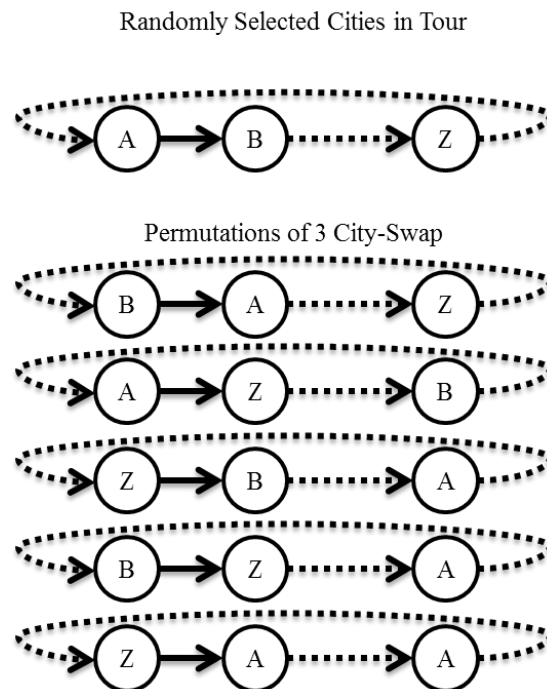


Figure 3.3: Example of 3-CitySwap move

3.6 Generating Distribution of Solutions

Once the settings for the Simulated Annealing and Threshold Accepting algorithms have been decided upon it was then necessary to perform many runs for each of

the settings to create an empirical distribution. In this case 40,000 individual optimization runs were performed for every combination of factor levels on each of the test problems. These 40,000 data points were then used as the basis for the empirical distribution when comparing the performance between different factor levels.

In order to generate the required volume of data the two algorithms were implemented in C++ using the C++AMP library created by Microsoft to perform massively parallel computations on Graphical Processing Units. The calculations were carried out on an AMD Radeon HD 7970 GPU. TinyMT is the pseudo-random number generator library that was used. TinyMT is a smaller version of the Mersenne-Twister algorithm created by Saito and Matsumoto [59]. The Mersenne-Twister is the standard PRNG used in many programming languages and statistical packages [2][1][4][5]. It has a long period of $2^{1937} - 1$ and generates a high quality random number stream. The TinyMT library was created by the same authors and has the same performance for randomness but requires far less memory and is better suited for scenarios which require a lower memory footprint [66]. The downside to TinyMT is that it has a significantly shorter period of $2^{127} - 1$. In spite of this shorter period, TinyMT is a high quality PRNG which passes the BigCrush test in the TestU01 library [66] provided by L'Ecuyer for testing the quality of PRNG [53]. Due to the limited cache memory in a GPU it is advantageous to have algorithms with smaller memory footprints. The implementation of TinyMT was provided by Microsoft via their CodePlex website [3].

3.7 Multisampling

Multisampling is a process of performing multiple runs of a heuristic and taking the best value from the results. This creates a solutions distribution that represents either the minimum or maximum value obtained from multiple runs of the heuristic. The distribution of solutions for a single run of the heuristic, S is determined by the combination of a problem instance, an optimization heuristic and a set of parameters for the heuristic. To multisample from this distribution several observations of the distribution S are taken and then the best of the values is chosen as the result. This is a new distribution referred to as $S_{n,best}$ where n is the number of observations of S that are taken. In the case of a minimization problem the formulation can be seen in Equation 3.7.

$$S_{n,min} = \min\{S_1, S_2, \dots, S_n\} \quad (3.7)$$

The impact of multisampling can be seen in Figure 3.4. What becomes apparent is that as the number of runs increase the solution distribution is shifting toward the left (minimization). Another observation is that the benefit of more samples decreases as the number of samples increases. This is due to the diminishing returns in continually resampling from the same distribution. This is as expected since additional samples are observations from S , and as n increases and $S_{n,best}$ shifts left the probability of an individual added run will improve the solution decreases.

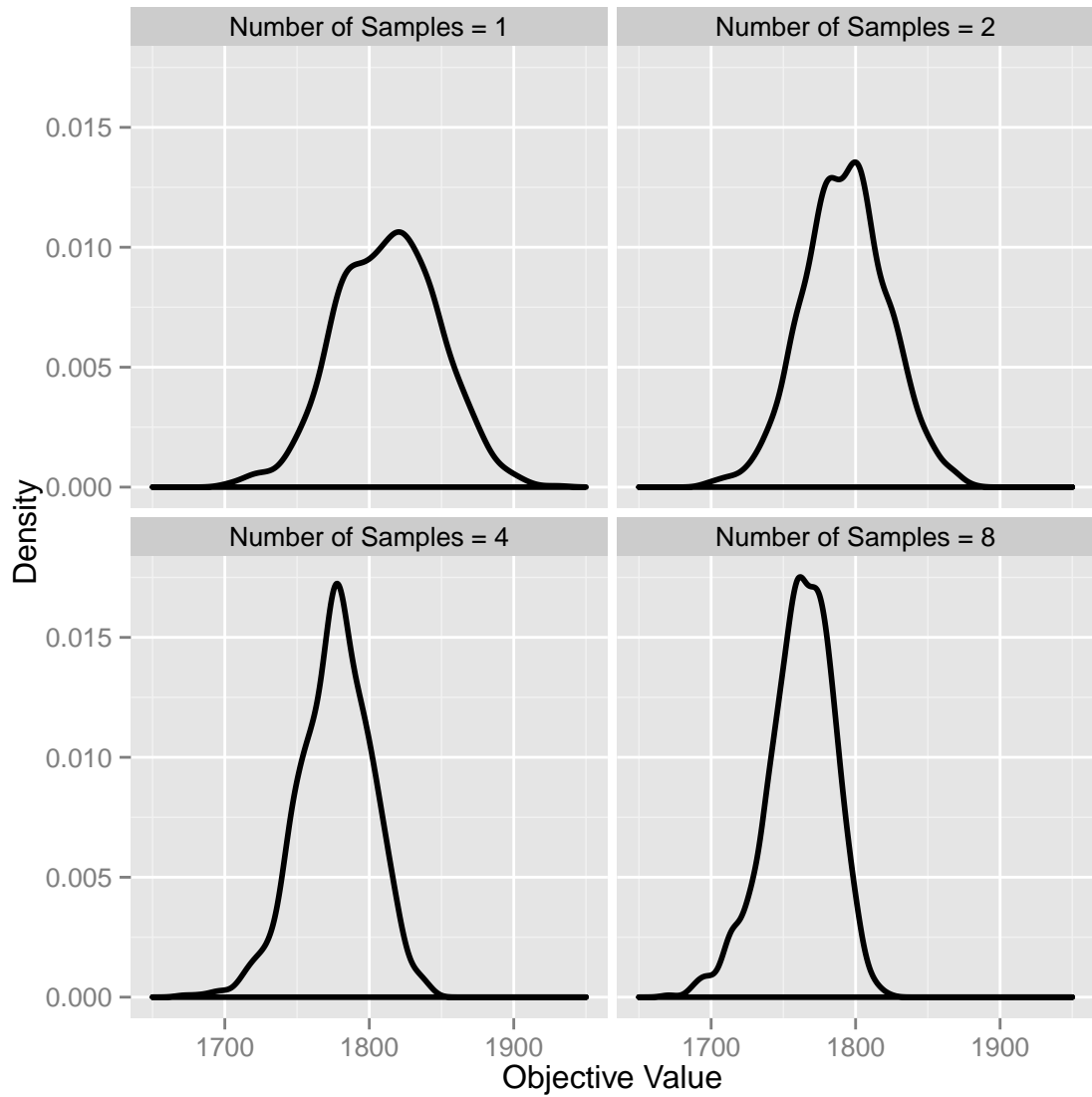


Figure 3.4: Example of the result of multisampling with Number of Samples = 1,2,4,8

Chapter 4: Results

The analysis done was systematic test of the input variables in question to cover a broad range of settings for the heuristics. Paired testing of the resulting distribution means was done for each of the experiment settings to see which heuristic settings outperformed one another when multisampling was employed to equate computational effort. This paired testing was done using Welch's t-test to compare the means of the distributions to see which settings outperformed the other. A thousand replications for each pairing of heuristic settings was performed. Welch's t-test was used since it does not make the assumption that the variability of the two distributions have equal variance.

4.1 Case 1: rand100

4.1.1 Simulated Annealing

For the first problem instance we can see the results of the raw distributions in Table 4.1. These are the results of taking 40,000 runs of SA using the parameters given in the table for the rand100 problem instance. We see that for the 2-Opt moves that the mean of the distributions and the standard deviation quickly begin to converge toward the optimum. This is a property of the flexibility of the 2-Opt move and the simple structure of the rand100 problem instance. Since there

are fewer cities in this problem instance the search space is significantly smaller than larger TSP instances. In the table the Cost is the number of moves that are performed for a single run. We also observe that the search heuristic begins to find the global optimum with as few as 8,000 moves with several settings of α and MPT. Graphs of a subset of the distributions are provided in Figure 4.1 as an example of what the distributions look like.

With the 3-City Swap move we do not observe as good of performance. In none of the settings did we observe an instance of the global optimum being achieved. There is also higher variability across all of the settings as well. This is to be expected since the 3-City Swap move does not perform as well as the 2-Opt move.

The t-test results for the 2-Opt move data can be seen in Table 4.2. The settings for Experiment 1 are the multistart settings and can be seen on the left side of the table. The settings for Experiment 2 are for single runs and be seen along the top of the table. The intersection of Experiment 1 and Experiment 2 in the table shows the result of the t-test that compared these settings. The 'Exper 1 Starts' field shows the number of runs that were performed with Experiment 1 settings to match the computational effort of the Experiment 2 settings. A negative value in the 't value' field indicates that the multistart of Experiment 1 outperformed a single instance of the Experiment 2 settings. In most instances we see that Experiment 1 settings with multistart do not perform as well as a single instance of the Experiment 2 settings. There are exceptions though which can be seen in Table 4.3. Here we see that in each case where the multistart method was superior to the single instance there were 2 runs of the multistart

Table 4.1: SA results for rand100 problem instance based on 40,000 observations, Optimum = 311.981

Move Type	α	MPT	Cost	BestObs.	mean	sd
2-Opt	0.82425	100	5000	365.312	527.83	63.85
		200	10000	335.815	402.38	38.68
		400	20000	316.51	345.5	19.19
		800	40000	312.567	326.28	9.78
	0.90788	100	10000	331.493	403.67	39.38
		200	20000	319.411	346.07	19.34
		400	40000	312.603	326.34	9.91
		800	80000	311.981	320.87	8.23
	0.95283	100	20000	316.665	346.25	19.4
		200	40000	312.91	326.39	9.63
		400	80000	311.981	320.85	8.07
		800	160000	311.981	318.39	7.4
	0.97613	100	40000	312.649	326.44	9.9
		200	80000	311.981	320.84	8.26
		400	160000	311.981	318.39	7.38
		800	320000	311.981	316.69	6.46
	0.988	100	80000	311.981	320.88	8.24
		200	160000	311.981	318.39	7.29
		400	320000	311.981	316.7	6.65
		800	640000	311.981	315.46	5.66
3-CitySwap	0.82425	1000	50000	333.366	636.41	80.39
		2000	100000	325.339	568.95	71.97
		4000	200000	322.751	511.98	65.02
		8000	400000	317.64	463.89	60.37
	0.90788	1000	100000	328.079	568.31	71.96
		2000	200000	321.816	511.93	64.79
		4000	400000	319.544	463.89	60.06
		8000	800000	317.791	421.96	55.92
	0.95283	1000	200000	321.204	511.41	65.14
		2000	400000	320.368	463.8	60.12
		4000	800000	316.847	421.43	56
		8000	1600000	316.395	385.29	47.74
	0.97613	1000	400000	319.835	464.12	60.38
		2000	800000	317.879	421.74	56.09
		4000	1600000	316.828	385.01	47.49
		8000	3200000	316.256	357.93	35.12
	0.988	1000	800000	317.79	421.72	56.13
		2000	1600000	316.494	384.85	47.61
		4000	3200000	316.281	357.67	35.09
		8000	6400000	316.077	341.52	24.22

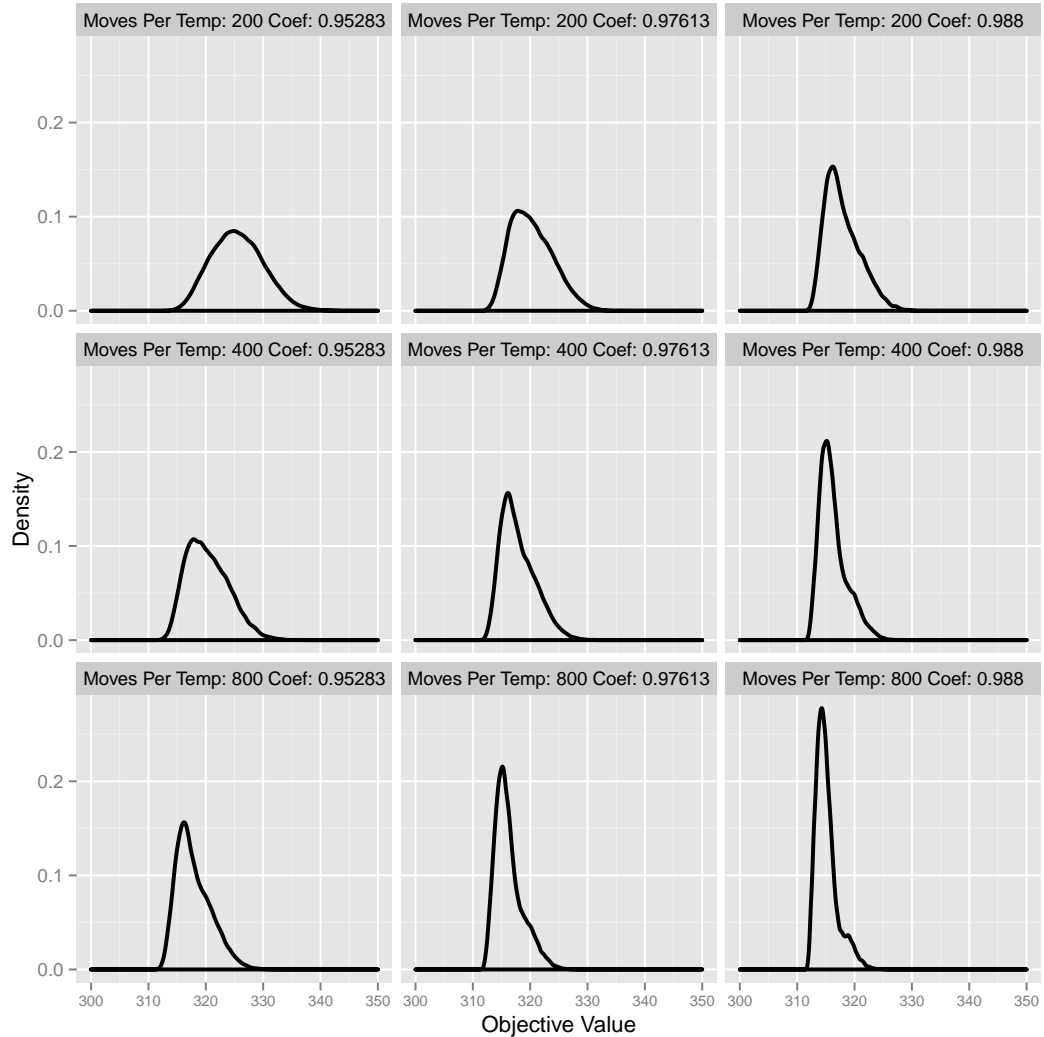


Figure 4.1: Empirical probability distributions for SA on the rand100 problem instance with Moves Per Temp: 200, 400, 800 and Cooling Coefficient: 0.95283, 0.97613, 0.98800. Optimum = 311.981

method. Apart from 2 runs of Experiment 1 there does not appear to be a pattern in the value of MPT of the Cooling Coefficients. The only other thing to note is that the multistart method is only superior when the source distribution is already

generating solutions near the global optimum.

In the case of the 3-City Swap method we do not observe any instances in Table 4.4 where the multistart method outperformed the single instance method. This is likely due to the 3-City Swap method not converging to the optimum as quickly as the 2-Opt method. This means that there is a more significant difference in performance between settings which are allowed to evaluate more moves and those which perform fewer moves. This difference means that the multistart method will have more difficulty finding equivalent solutions to the single search method since the underlying distribution it is sampling from is farther from the single search distribution. This difference is seen in Table 4.1 where we saw that the 2-Opt experiments were converging faster than the 2-City Swap moves and with lower spread. This means that there is more overlap in distributions for the 2-Opt distributions than for the 3-City Swap move distributions.

Table 4.3: SA 2-Opt rand100 instances where multistart outperformed single run methods

t value	df	p value	Coef 1	MPT 1	Cost 1	Exper 1 Samples	Coef 2	MPT 2	Cost 2
-3.27	1101.57	0.0011	0.98800	200	160000	2	0.98800	400	320000
-2.77	1092.83	0.0057	0.97613	400	160000	2	0.98800	400	320000
-2.73	1093.27	0.0063	0.95283	800	160000	2	0.98800	400	320000
-3.12	1204.20	0.0018	0.98800	200	160000	2	0.97613	800	320000
-2.43	1186.96	0.0154	0.97613	400	160000	2	0.97613	800	320000
-2.38	1187.82	0.0176	0.95283	800	160000	2	0.97613	800	320000
-2.58	1174.97	0.0099	0.98800	400	320000	2	0.98800	800	640000
-2.65	1165.24	0.0082	0.97613	800	320000	2	0.98800	800	640000

Table 4.4: rand100 SA 3-City Swap Welch's t-test results for 1,000 experiments comparing row settings to column settings

Coef 1	Moves per Temp	Coef 2																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																						
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0.82125	1000	1.0e+00	2.0e+00	4.0e+00	8.0e+00	1.6e+01	3.2e+01	6.4e+01	1.2e+02	2.4e+02	4.8e+02	9.6e+02	1.9e+03	3.8e+03	7.6e+03	1.5e+04	3.0e+04	6.0e+04	1.2e+05	2.4e+05	4.8e+05	9.6e+05	1.9e+06	3.8e+06	7.6e+06	1.5e+07	3.0e+07	6.0e+07	1.2e+08	2.4e+08	4.8e+08	9.6e+08	1.9e+09	3.8e+09	7.6e+09	1.5e+10	3.0e+10	6.0e+10	1.2e+11	2.4e+11	4.8e+11	9.6e+11	1.9e+12	3.8e+12	7.6e+12	1.5e+13	3.0e+13	6.0e+13	1.2e+14	2.4e+14	4.8e+14	9.6e+14	1.9e+15	3.8e+15	7.6e+15	1.5e+16	3.0e+16	6.0e+16	1.2e+17	2.4e+17	4.8e+17	9.6e+17	1.9e+18	3.8e+18	7.6e+18	1.5e+19	3.0e+19	6.0e+19	1.2e+20	2.4e+20	4.8e+20	9.6e+20	1.9e+21	3.8e+21	7.6e+21	1.5e+22	3.0e+22	6.0e+22	1.2e+23	2.4e+23	4.8e+23	9.6e+23	1.9e+24	3.8e+24	7.6e+24	1.5e+25	3.0e+25	6.0e+25	1.2e+26	2.4e+26	4.8e+26	9.6e+26	1.9e+27	3.8e+27	7.6e+27	1.5e+28	3.0e+28	6.0e+28	1.2e+29	2.4e+29	4.8e+29	9.6e+29	1.9e+30	3.8e+30	7.6e+30	1.5e+31	3.0e+31	6.0e+31	1.2e+32	2.4e+32	4.8e+32	9.6e+32	1.9e+33	3.8e+33	7.6e+33	1.5e+34	3.0e+34	6.0e+34	1.2e+35	2.4e+35	4.8e+35	9.6e+35	1.9e+36	3.8e+36	7.6e+36	1.5e+37	3.0e+37	6.0e+37	1.2e+38	2.4e+38	4.8e+38	9.6e+38	1.9e+39	3.8e+39	7.6e+39	1.5e+40	3.0e+40	6.0e+40	1.2e+41	2.4e+41	4.8e+41	9.6e+41	1.9e+42	3.8e+42	7.6e+42	1.5e+43	3.0e+43	6.0e+43	1.2e+44	2.4e+44	4.8e+44	9.6e+44	1.9e+45	3.8e+45	7.6e+45	1.5e+46	3.0e+46	6.0e+46	1.2e+47	2.4e+47	4.8e+47	9.6e+47	1.9e+48	3.8e+48	7.6e+48	1.5e+49	3.0e+49	6.0e+49	1.2e+50	2.4e+50	4.8e+50	9.6e+50	1.9e+51	3.8e+51	7.6e+51	1.5e+52	3.0e+52	6.0e+52	1.2e+53	2.4e+53	4.8e+53	9.6e+53	1.9e+54	3.8e+54	7.6e+54	1.5e+55	3.0e+55	6.0e+55	1.2e+56	2.4e+56	4.8e+56	9.6e+56	1.9e+57	3.8e+57	7.6e+57	1.5e+58	3.0e+58	6.0e+58	1.2e+59	2.4e+59	4.8e+59	9.6e+59	1.9e+60	3.8e+60	7.6e+60	1.5e+61	3.0e+61	6.0e+61	1.2e+62	2.4e+62	4.8e+62	9.6e+62	1.9e+63	3.8e+63	7.6e+63	1.5e+64	3.0e+64	6.0e+64	1.2e+65	2.4e+65	4.8e+65	9.6e+65	1.9e+66	3.8e+66	7.6e+66	1.5e+67	3.0e+67	6.0e+67	1.2e+68	2.4e+68	4.8e+68	9.6e+68	1.9e+69	3.8e+69	7.6e+69	1.5e+70	3.0e+70	6.0e+70	1.2e+71	2.4e+71	4.8e+71	9.6e+71	1.9e+72	3.8e+72	7.6e+72	1.5e+73	3.0e+73	6.0e+73	1.2e+74	2.4e+74	4.8e+74	9.6e+74	1.9e+75	3.8e+75	7.6e+75	1.5e+76	3.0e+76	6.0e+76	1.2e+77	2.4e+77	4.8e+77	9.6e+77	1.9e+78	3.8e+78	7.6e+78	1.5e+79	3.0e+79	6.0e+79	1.2e+80	2.4e+80	4.8e+80	9.6e+80	1.9e+81	3.8e+81	7.6e+81	1.5e+82	3.0e+82	6.0e+82	1.2e+83	2.4e+83	4.8e+83	9.6e+83	1.9e+84	3.8e+84	7.6e+84	1.5e+85	3.0e+85	6.0e+85	1.2e+86	2.4e+86	4.8e+86	9.6e+86	1.9e+87	3.8e+87	7.6e+87	1.5e+88	3.0e+88	6.0e+88	1.2e+89	2.4e+89	4.8e+89	9.6e+89	1.9e+90	3.8e+90	7.6e+90	1.5e+91	3.0e+91	6.0e+91	1.2e+92	2.4e+92	4.8e+92	9.6e+92	1.9e+93	3.8e+93	7.6e+93	1.5e+94	3.0e+94	6.0e+94	1.2e+95	2.4e+95	4.8e+95	9.6e+95	1.9e+96	3.8e+96	7.6e+96	1.5e+97	3.0e+97	6.0e+97	1.2e+98	2.4e+98	4.8e+98	9.6e+98	1.9e+99	3.8e+99	7.6e+99	1.5e+100	3.0e+100	6.0e+100	1.2e+101	2.4e+101	4.8e+101	9.6e+101	1.9e+102	3.8e+102	7.6e+102	1.5e+103	3.0e+103	6.0e+103	1.2e+104	2.4e+104	4.8e+104	9.6e+104	1.9e+105	3.8e+105	7.6e+105	1.5e+106	3.0e+106	6.0e+106	1.2e+107	2.4e+107	4.8e+107	9.6e+107	1.9e+108	3.8e+108	7.6e+108	1.5e+109	3.0e+109	6.0e+109	1.2e+110	2.4e+110	4.8e+110	9.6e+110	1.9e+111	3.8e+111	7.6e+111	1.5e+112	3.0e+112	6.0e+112	1.2e+113	2.4e+113	4.8e+113	9.6e+113	1.9e+114	3.8e+114	7.6e+114	1.5e+115	3.0e+115	6.0e+115	1.2e+116	2.4e+116	4.8e+116	9.6e+116	1.9e+117	3.8e+117	7.6e+117	1.5e+118	3.0e+118	6.0e+118	1.2e+119	2.4e+119	4.8e+119	9.6e+119	1.9e+120	3.8e+120	7.6e+120	1.5e+121	3.0e+121	6.0e+121	1.2e+122	2.4e+122	4.8e+122	9.6e+122	1.9e+123	3.8e+123	7.6e+123	1.5e+124	3.0e+124	6.0e+124	1.2e+125	2.4e+125	4.8e+125	9.6e+125	1.9e+126	3.8e+126	7.6e+126	1.5e+127	3.0e+127	6.0e+127	1.2e+128	2.4e+128	4.8e+128	9.6e+128	1.9e+129	3.8e+129	7.6e+129	1.5e+130	3.0e+130	6.0e+130	1.2e+131	2.4e+131	4.8e+131	9.6e+131	1.9e+132	3.8e+132	7.6e+132	1.5e+133	3.0e+133	6.0e+133	1.2e+134	2.4e+134	4.8e+134	9.6e+134	1.9e+135	3.8e+135	7.6e+135	1.5e+136	3.0e+136	6.0e+136	1.2e+137	2.4e+137	4.8e+137	9.6e+137	1.9e+138	3.8e+138	7.6e+138	1.5e+139	3.0e+139	6.0e+139	1.2e+140	2.4e+140	4.8e+140	9.6e+140	1.9e+141	3.8e+141	7.6e+141	1.5e+142	3.0e+142	6.0e+142	1.2e+143	2.4e+143	4.8e+143	9.6e+143	1.9e+144	3.8e+144	7.6e+144	1.5e+145	3.0e+145	6.0e+145	1.2e+146	2.4e+146	4.8e+146	9.6e+146	1.9e+147	3.8e+147	7.6e+147	1.5e+148	3.0e+148	6.0e+148	1.2e+149	2.4e+149	4.8e+149	9.6e+149	1.9e+150	3.8e+150	7.6e+150	1.5e+151	3.0e+151	6.0e+151	1.2e+152	2.4e+152	4.8e+152	9.6e+152	1.9e+153	3.8e+153	7.6e+153	1.5e+154	3.0e+154	6.0e+154	1.2e+155	2.4e+155	4.8e+155	9.6e+155	1.9e+156	3.8e+156	7.6e+156	1.5e+157	3.0e+157	6.0e+157	1.2e+158	2.4e+158	4.8e+158	9.6e+158	1.9e+159	3.8e+159	7.6e+159	1.5e+160	3.0e+160	6.0e+160	1.2e+161	2.4e+161	4.8e+161	9.6e+161	1.9e+162	3.8e+162	7.6e+162	1.5e+163	3.0e+163	6.0e+163	1.2e+164	2.4e+164	4.8e+164	9.6e+164	1.9e+165	3.8e+165	7.6e+165	1.5e+166	3.0e+166	6.0e+166	1.2e+167	2.4e+167	4.8e+167	9.6e+167	1.9e+168	3.8e+168	7.6e+168	1.5e+169	3.0e+169	6.0e+169	1.2e+170	2.4e+170	4.8e+170	9.6e+170	1.9e+171	3.8e+171	7.6e+171	1.5e+172	3.0e+172	6.0e+172	1.2e+173	2.4e+173	4.8e+173	9.6e+173	1.9e+174	3.8e+174	7.6e+174	1.5e+175	3.0e+175	6.0e+175	1.2e+176	2.4e+176	4.8e+176	9.6e+176	1.9e+177	3.8e+177	7.6e+177	1.5e+178	3.0e+178	6.0e+178	1.2e+179	2.4e+179	4.8e+179	9.6e+179	1.9e+180	3.8e+180	7.6e+180	1.5e+181	3.0e+181	6.0e+181	1.2e+182	2.4e+182	4.8e+182	9.6e+182	1.9e+183	3.8e+183	7.6e+183	1.5e+184	3.0e+184	6.0e+184	1.2e+185	2.4e+185	4.8e+185	9.6e+185	1.9e+186	3.8e+186	7.6e+186	1.5e+187	3.0e+187	6.0e+187	1.2e+188	2.4e+188	4.8e+188	9.6e+188	1.9e+189	3.8e+189	7.6e+189	1.5e+190	3.0e+190	6.0e+190	1.2e+191	2.4e+191	4.8e+191	9.6e+191	1.9e+192	3.8e+192	7.6e+192	1.5e+193	3.0e+193	6.0e+193	1.2e+194	2.4e+194	4.8e+194	9.6e+194	1.9e+195	3.8e+195	7.6e+195	1.5e+196	3.0e+196	6.0e+196	1.2e+197	2.4e+197	4.8e+197	9.6e+197	1.9e+198	3.8e+198	7.6e+198	1.5e+199	3.0e+199	6.0e+199	1.2e+200	2.4e+200	4.8e+200	9.6e+200	1.9e+201	3.8e+201	7.6e+201	1.5e+202	3.0e+202	6.0e+202	1.2e+203	2.4e+203	4.8e+203	9.6e+203	1.9e+204	3.8e+204	7.6e+204	1.5e+205	3.0e+205	6.0e+205	1.2e+206	2.4e+206	4.8e+206	9.6e+206	1.9e+207	3.8e+207	7.6e+207	1.5e+208	3.0e+208	6.0e+208	1.2e+209	2.4e+209	4.8e+209	9.6e+209	1.9e+210	3.8e+210	7.6e+210	1.5e+211	3.0e+211	6.0e+211	1.2e+212	2.4e+212	4.8e+212	9.6e+212	1.9e+213	3.8e+213	7.6e+213	1.5e+214	3.0e+214	6.0e+214	1.2e+215	2.4e+215	4.8e+215	9.6e+215	1.9e+216	3.8e+216	7.6e+216	1.5e+217	3.0e+217	6.0e+217	1.2e+218	2.4e+218	4.8e+218	9.6e+218	1.9e+219	3.8e+219	7.6e+219	1.5e+220	3.0e+220	6.0e+220	1.2e+221	2.4e+221	4.8e+221	9.6e+221	1.9e+222	3.8e+222	7.6e+222	1.5e+223	3.0e+223	6.0e+223	1.2e+224	2.4e+224	4.8e+224	9.6e+224	1.9e+225	3.8e+225	7.6e+225	1.5e+226	3.0e+226	6.0e+226	1.2e+227	2.4e+227	4.8e+227	9.6e+227	1.9e+228	3.8e+228	7.6e+228	1.5e+229	3.0e+229	6.0e+229	1.2e+230	2.4e+230	4.8e+230	9.6e+230	1.9e+231	3.8e+231	7.6e+231	1.5e+232	3.0e+232	6.0e+232	1.2e+233	2.4e+233	4.8e+233	9.6e+233	1.9e+234	3.8e+234	7.6e+234	1.5e+235	3.0e+235	6.0e+235	1.2e+236	2.4e+236	4.8e+236	9.6e+236	1.9e+237	3.8e+237	7.6e+237	1.5e+238	3.0e+238	6.0e+238	1.2e+239	2.4e+239	4.8e+239	9.6e+239	1.9e+240	3.8e+240	7.6e+240	1.5e+241	3.0e+241	6.0e+241	1.2e+242	2.4e+242	4.8e+242	9.6e+242	1.9e+243	3.8e+243	7.6e+243	1.5e+244	3.0e+244	6.0e+244	1.2e+245	2.4e+245	4.8e+245	9.6e+245	1.9e+246	3.8e+246	7.6e+246	1.5e+247	3.0e+247	6.0e+247	1.2e+248	2.4e+248	4.8e+248	9.6e+248	1.9e+249	3.8e+249	7.6e+249	1.5e+250	3.0e+250	6.0e+250	1.2e+251	2.4e+251	4.8e+251	9.6e+251	1.9e+252	3.8e+252	7.6e+252	1.5e+253	3.0e+253	6.0e+253	1.2e+254	2.4e+254	4.8e+254	9.6e+254	1.9e+255	3.8e+255	7.6e+255	1.5e+

4.1.2 Threshold Accepting

The raw distributions for Threshold Accepting runs on the rand100 problem can be seen in Table 4.5. Here again we see that the 2-Opt distributions are converging on the optimum more rapidly and with greater reliability. When we look at the t-test results for the 2-Opt distributions in Table C.3 in the appendix we see that there is only a single instance of the multistart method outperforming the single instance method. Both had an α value of 0.98800. The multistart method had 400 Moves Per Temperature while the single run instance ran 800. The t-test shows that the 2 runs with 400 Moves Per Temperature outperforms a single run with 800 Moves Per Temperature. This result is almost not significant though due to the p-value being 0.049 which is on the border of being considered not statistically significant.

When we look at the 3-City Swap data in Table C.4 in the appendix we do not see any instances where the multistart method outperformed a single run method. Again it appears that if the underlying distributions do not have substantial overlap then the multistart will not outperform a single run method.

Table 4.5: TA results for rand100 problem instance based on 40,000 observations, Optimum = 311.981

Move Type	α	MPT	Cost	BestObs.	mean	sd
2-Opt	0.82425	100	5000	408.26	620.43	72.65
		200	10000	354.53	453.96	46.59
		400	20000	327.44	369.61	23.18
		800	40000	314.286	334.75	8.35
	0.90788	100	10000	355.219	455.87	46.91
		200	20000	327.097	370.82	23.6
		400	40000	315.664	335.19	8.46
		800	80000	312.629	322.38	3.99
	0.95283	100	20000	332.126	371.16	23.64
		200	40000	316.537	335.39	8.53
		400	80000	312.474	322.42	3.97
		800	160000	311.981	317.81	2.87
	0.97613	100	40000	315.972	335.56	8.52
		200	80000	312.335	322.49	3.95
		400	160000	311.981	317.82	2.85
		800	320000	311.981	315.94	2.17
	0.98800	100	80000	312.521	322.51	4.02
		200	160000	311.981	317.83	2.87
		400	320000	311.981	315.92	2.17
		800	640000	311.981	314.79	1.67
3-City Swap	0.82425	1000	50000	349.806	636.47	77.71
		2000	100000	327.165	564.76	68.59
		4000	200000	321.135	505.17	61.09
		8000	400000	318.489	453.97	57.36
	0.90788	1000	100000	329.775	564.54	68.42
		2000	200000	321.866	503.82	61.05
		4000	400000	318.798	453.61	57.44
		8000	800000	316.618	407.7	52.74
	0.95283	1000	200000	319.812	504.42	61.79
		2000	400000	318.855	453.65	57.28
		4000	800000	316.522	407.98	53.1
		8000	1600000	316.473	369.81	42.23
	0.97613	1000	400000	318.535	453.78	57.25
		2000	800000	317.27	408.13	52.9
		4000	1600000	316.243	370.14	42.27
		8000	3200000	316.173	344.23	27.53
	0.98800	1000	800000	316.661	407.75	52.84
		2000	1600000	316.145	369.79	42.26
		4000	3200000	316.139	344.38	27.61
		8000	6400000	316.06	330.78	17.12

4.2 Case 2: bier127

When we look at the raw distribution data for the bier127 problem using Simulated Annealing in Table A.3 we find that there is significant overlap in the 2-Opt move distributions but not as significant overlap in the 3-City Swap distributions. We see the results of this in the t-test data for the 2-Opt distributions in Table C.5 where we find many instances of the multistart method outperforming single run methods. A summary of the instances where the multistart method was superior is shown in Table 4.6. In this case there are far more instances where the multistart method was superior when compared to the rand100 results. There is no trend of Cooling Coefficient, or MPT that explains why the multistart methods are superior. We do see that only multistarts with 2 or 4 starts outperform single run methods. We do not see multistarts with more than 4 starts outperforming single run methods. This suggests that the benefit of multistart is limited to cases with only a few starts and not in methods with many starts. What is also consistent is that the multistart underlying distribution requires a minimum number of moves, 40,000 in this case, to outperform single start runs. We do not see instances of the multistart method having less than 40,000 moves for a single iteration outperforming single, long runs. This suggests that there is a minimum amount of computational effort that must be put into generating solutions to achieve good results, no matter how many restarts are performed. For instances that did not spend at least 40,000 moves on a single run we see they all have substantially higher mean and standard deviation for their distribution of solutions. For the 3-City Swap distributions we

do not see any cases in Table C.6 where the multistart methods were superior.

For the Threshold Accepting experiments we do not see any instances where the multistart method outperformed a single run. Table C.7 and Table C.8 do not show any instances where the multistart method significantly outperformed the single run. This is a similar result to the rand100 problem which only had a single instances. This would suggest that the difference in the acceptance mechanism for Threshold Accepting has a significant effect on the distribution of solutions which reduces the overlap of solution distributions for different algorithm settings.

Table 4.6: SA bier127 instances where multistart outperformed single run methods

t value	df	p value	Coef 1	MPT 1	Cost 1	Exper 1	Samples	Coef 2	MPT 2	Cost 2
-2.19	1943.71	0.0289	0.95283	200	40000		2	0.98800	100	80000
-2.04	1892.42	0.0414	0.90788	400	40000		2	0.98800	100	80000
-2.86	1901.66	0.0043	0.97613	100	40000		2	0.97613	200	80000
-4.96	1904.14	0.0000	0.95283	200	40000		2	0.97613	200	80000
-4.89	1843.67	0.0000	0.90788	400	40000		2	0.97613	200	80000
-3.69	1902.87	0.0002	0.82425	800	40000		2	0.97613	200	80000
-2.54	1750.67	0.0111	0.97613	100	40000		4	0.98800	200	160000
-3.67	1900.44	0.0003	0.98800	100	80000		2	0.98800	200	160000
-3.30	1762.04	0.0010	0.95283	200	40000		4	0.98800	200	160000
-4.88	1926.87	0.0000	0.97613	200	80000		2	0.98800	200	160000
-2.65	1775.21	0.0081	0.90788	400	40000		4	0.98800	200	160000
-3.81	1924.84	0.0001	0.95283	400	80000		2	0.98800	200	160000
-3.70	1737.21	0.0002	0.82425	800	40000		4	0.98800	200	160000
-4.60	1917.02	0.0000	0.90788	800	80000		2	0.98800	200	160000
-2.84	1905.46	0.0045	0.95283	200	40000		2	0.95283	400	80000
-2.72	1845.23	0.0066	0.90788	400	40000		2	0.95283	400	80000
-3.91	1776.56	0.0001	0.97613	100	40000		4	0.97613	400	160000
-4.97	1920.07	0.0000	0.98800	100	80000		2	0.97613	400	160000
-4.68	1787.76	0.0000	0.95283	200	40000		4	0.97613	400	160000
-6.18	1944.00	0.0000	0.97613	200	80000		2	0.97613	400	160000
-4.01	1800.68	0.0001	0.90788	400	40000		4	0.97613	400	160000
-5.09	1942.19	0.0000	0.95283	400	80000		2	0.97613	400	160000
-5.09	1763.27	0.0000	0.82425	800	40000		4	0.97613	400	160000
-5.91	1935.17	0.0000	0.90788	800	80000		2	0.97613	400	160000
-2.68	1848.71	0.0074	0.98800	100	80000		4	0.98800	400	320000
-3.76	1831.56	0.0002	0.97613	200	80000		4	0.98800	400	320000
-3.08	1817.48	0.0021	0.95283	400	80000		4	0.98800	400	320000
-4.81	1771.29	0.0000	0.90788	800	80000		4	0.98800	400	320000
-3.77	1945.80	0.0002	0.95283	200	40000		2	0.90788	800	80000
-3.67	1895.14	0.0003	0.90788	400	40000		2	0.90788	800	80000
-2.45	1944.79	0.0143	0.82425	800	40000		2	0.90788	800	80000
-4.46	1781.23	0.0000	0.97613	100	40000		4	0.95283	800	160000
-5.49	1923.46	0.0000	0.98800	100	80000		2	0.95283	800	160000
-5.23	1792.39	0.0000	0.95283	200	40000		4	0.95283	800	160000
-6.70	1946.91	0.0000	0.97613	200	80000		2	0.95283	800	160000
-4.56	1805.25	0.0000	0.90788	400	40000		4	0.95283	800	160000
-5.61	1945.14	0.0000	0.95283	400	80000		2	0.95283	800	160000
-5.65	1767.98	0.0000	0.82425	800	40000		4	0.95283	800	160000
-6.43	1938.27	0.0000	0.90788	800	80000		2	0.95283	800	160000
-2.73	1775.72	0.0065	0.90788	800	80000		4	0.97613	800	320000

4.3 Case 3: pbn423

In the case of the pbn442 problem we do not observe any instances where the multistart method was able to outperform the single run method. Table A.5 and Table A.6 show that there is significant changes in the mean with each increase of computational effort. This suggests that the complexity of the TSP instance causes more spread in the solution distributions for the different levels of computational effort. Table C.9 and Table C.10 show the t-test result for SA and Table C.11 and Table C.8 hold the results for TA.

4.4 Case 4: pcb442

The pcb442 test problem had similar results to the pbn423 problem. Table C.13 and Table C.14 for the SA results show that there were no cases where the multistart method outperformed the single run method. The TA results in Table C.15 and Table C.16 show that there were no cases where TA performed better with multistart instead of single run. The similar result in pbn423 and pcb442 suggests that for larger problem instances of the TSP it will not be advantageous to employ a multistart search method regardless of the move type or acceptance method.

Chapter 5: Conclusion

The results from the four problems instances suggest there are situations where a multistart method performs better than a single start method for Simulated Annealing and Threshold Accepting for the TSP. Threshold Accepting only had a single instance where the multistart method outperformed the single run method but the phenomenon was observed. Simulated Annealing had many instances in the rand100 problem and the bier127 when employing the 2-Opt move to generate neighborhoods. This suggests that the type of move that is used has significant impact on whether a multistart method will be advantageous or not. The fact that Simulated Annealing and Threshold Accepting had significantly different results also shows that the criteria used to accept moves has a significant impact on whether multistart should be employed or not.

For the larger problem instances no combination of SA vs TA, Moves per Temperature, Cooling Coefficient, or Number of Samples had an impact whether a multistart method would outperform a single run method. This suggests that as TSP instances increase in size the need for a long search overwhelms the advantage that a multistart method may give from beginning at different initial solutions. Fundamentally the distribution of solutions is the result of the interaction between the problem instance, the heuristic being employed and the control parameters being used. This research showed that given the right combination of heuristic,

search parameters and problem instance the multistart method has a statistically significant advantage over a long single run. The point at which to begin using multistart instead of single runs is when the additional computational effort put into single run is no longer significantly improving the mean or standard deviation of the solution distribution. Until that point is reached it is best to continue employing a single long run.

5.1 Future Research

This work only examined 2 different move types for selecting neighboring solutions. It would be interesting to examine whether the 3-Opt method or v-opt methods would effect when it was best to use multistart. Lin and Kernighan [56] showed that there is significant improvement in using the 3-Opt method over the 2-Opt which may lead to additional interesting results. This work also only dealt with a single problem, the TSP. Both of these search heuristics have been used on a large variety of problem types. Seeing if these results would hold for other types of problem, both discrete and continuous, would provide further insight into their behavior. Meaningful test problems would be the Knapsack Packing Problem, Job Shop Scheduling and Multi-variable Distribution Fitting problems.

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APPENDICES

Appendix A: Raw Distribution Results

Table A.1: SA results for rand100 problem instance based on 40,000 observations, Optimum = 311.981

Move Type	α	MPT	Cost	BestObs.	mean	sd
2-Opt	0.82425	100	5000	365.312	527.83	63.85
		200	10000	335.815	402.38	38.68
		400	20000	316.51	345.5	19.19
		800	40000	312.567	326.28	9.78
	0.90788	100	10000	331.493	403.67	39.38
		200	20000	319.411	346.07	19.34
		400	40000	312.603	326.34	9.91
		800	80000	311.981	320.87	8.23
	0.95283	100	20000	316.665	346.25	19.4
		200	40000	312.91	326.39	9.63
		400	80000	311.981	320.85	8.07
		800	160000	311.981	318.39	7.4
	0.97613	100	40000	312.649	326.44	9.9
		200	80000	311.981	320.84	8.26
		400	160000	311.981	318.39	7.38
		800	320000	311.981	316.69	6.46
	0.988	100	80000	311.981	320.88	8.24
		200	160000	311.981	318.39	7.29
		400	320000	311.981	316.7	6.65
		800	640000	311.981	315.46	5.66
3-CitySwap	0.82425	1000	50000	333.366	636.41	80.39
		2000	100000	325.339	568.95	71.97
		4000	200000	322.751	511.98	65.02
		8000	400000	317.64	463.89	60.37
	0.90788	1000	100000	328.079	568.31	71.96
		2000	200000	321.816	511.93	64.79
		4000	400000	319.544	463.89	60.06
		8000	800000	317.791	421.96	55.92
	0.95283	1000	200000	321.204	511.41	65.14
		2000	400000	320.368	463.8	60.12
		4000	800000	316.847	421.43	56
		8000	1600000	316.395	385.29	47.74
	0.97613	1000	400000	319.835	464.12	60.38
		2000	800000	317.879	421.74	56.09
		4000	1600000	316.828	385.01	47.49
		8000	3200000	316.256	357.93	35.12
	0.988	1000	800000	317.79	421.72	56.13
		2000	1600000	316.494	384.85	47.61
		4000	3200000	316.281	357.67	35.09
		8000	6400000	316.077	341.52	24.22

Table A.2: TA results for rand100 problem instance based on 40,000 observations, Optimum = 311.981

Move Type	α	MPT	Cost	BestObs.	mean	sd
2-Opt	0.82425	100	5000	408.26	620.43	72.65
		200	10000	354.53	453.96	46.59
		400	20000	327.44	369.61	23.18
		800	40000	314.286	334.75	8.35
	0.90788	100	10000	355.219	455.87	46.91
		200	20000	327.097	370.82	23.6
		400	40000	315.664	335.19	8.46
		800	80000	312.629	322.38	3.99
	0.95283	100	20000	332.126	371.16	23.64
		200	40000	316.537	335.39	8.53
		400	80000	312.474	322.42	3.97
		800	160000	311.981	317.81	2.87
	0.97613	100	40000	315.972	335.56	8.52
		200	80000	312.335	322.49	3.95
		400	160000	311.981	317.82	2.85
		800	320000	311.981	315.94	2.17
	0.98800	100	80000	312.521	322.51	4.02
		200	160000	311.981	317.83	2.87
		400	320000	311.981	315.92	2.17
		800	640000	311.981	314.79	1.67
3-City Swap	0.82425	1000	50000	349.806	636.47	77.71
		2000	100000	327.165	564.76	68.59
		4000	200000	321.135	505.17	61.09
		8000	400000	318.489	453.97	57.36
	0.90788	1000	100000	329.775	564.54	68.42
		2000	200000	321.866	503.82	61.05
		4000	400000	318.798	453.61	57.44
		8000	800000	316.618	407.7	52.74
	0.95283	1000	200000	319.812	504.42	61.79
		2000	400000	318.855	453.65	57.28
		4000	800000	316.522	407.98	53.1
		8000	1600000	316.473	369.81	42.23
	0.97613	1000	400000	318.535	453.78	57.25
		2000	800000	317.27	408.13	52.9
		4000	1600000	316.243	370.14	42.27
		8000	3200000	316.173	344.23	27.53
	0.98800	1000	800000	316.661	407.75	52.84
		2000	1600000	316.145	369.79	42.26
		4000	3200000	316.139	344.38	27.61
		8000	6400000	316.06	330.78	17.12

Table A.3: SA results for bier127 problem instance based on 40,000 observations, Optimum = 118,282

Move Type	α	MPT	Cost	BestObs.	mean	sd
2-Opt	0.82425	100	5000	157356	185949.1	7903.82
		200	10000	134758	155092.9	5611.68
		400	20000	123475	137830.7	3929.6
		800	40000	119578	131213.9	3287.23
	0.90788	100	10000	136768	155460.8	5638.02
		200	20000	125019	137995.5	3942.67
		400	40000	120406	131233.9	3280.87
		800	80000	119702	129656.1	3179.37
	0.95283	100	20000	125406	138100.4	3955.34
		200	40000	120434	131281.4	3319.53
		400	80000	119545	129659	3202.17
		800	160000	119140	128493.3	3034.39
	0.97613	100	40000	120773	131287.1	3305.52
		200	80000	119100	129670.4	3220.98
		400	160000	119367	128497	3035.16
		800	320000	118673	126800.7	2804.17
	0.988	100	80000	119250	129708.9	3195.7
		200	160000	118915	128530.5	3032.32
		400	320000	118325	126838.3	2830.7
		800	640000	118367	125049.3	2571.68
3-CitySwap	0.82425	1000	50000	135926	160899.3	6503.24
		2000	100000	128015	152857.4	5755.1
		4000	200000	129143	146234	5091.53
		8000	400000	125003	140930	4510.93
	0.90788	1000	100000	132709	153023.4	5768.81
		2000	200000	127449	146261.5	5087.47
		4000	400000	125135	140949.7	4523.14
		8000	800000	122540	136796.2	4076.31
	0.95283	1000	200000	128225	146314.3	5094.22
		2000	400000	123796	140955.8	4526.45
		4000	800000	122468	136781.4	4079.11
		8000	1600000	121848	133544.9	3672.54
	0.97613	1000	400000	126191	140964.8	4517.64
		2000	800000	122903	136774.1	4084.74
		4000	1600000	121000	133536.2	3657.78
		8000	3200000	120436	130935.8	3289.12
	0.988	1000	800000	123037	136853.5	4035.97
		2000	1600000	121190	133538.1	3641.86
		4000	3200000	120936	130941.4	3301.9
		8000	6400000	119161	128883.9	2972.36

Table A.4: TA results for bier127 problem instance based on 40,000 observations, Optimum = 118,282

Move Type	α	MPT	Cost	BestObs.	mean	sd
2-Opt	0.82425	100	5000	154669	186439.3	7771
		200	10000	135700	155366.3	5511.6
		400	20000	125037	137594.5	3795.65
		800	40000	120239	129922.6	3037.78
	0.90788	100	10000	136054	155662.1	5523.07
		200	20000	124595	137713	3808.01
		400	40000	119394	129992.3	3036.47
		800	80000	118839	126925.1	2703.51
	0.95283	100	20000	124806	137810.3	3801.09
		200	40000	119997	130040.9	3063.17
		400	80000	118662	126897	2701.3
		800	160000	118339	124426.3	2209.68
	0.97613	100	40000	119686	130073	3037.36
		200	80000	118798	126923.9	2717.15
		400	160000	118572	124448	2224.77
		800	320000	118325	122492.9	1648.49
	0.98800	100	80000	118923	126928	2691.16
		200	160000	118601	124437.7	2224.86
		400	320000	118325	122442.5	1649.1
		800	640000	118294	121278.9	1220.92
3-City Swap	0.82425	1000	50000	139310	161552.7	6650.54
		2000	100000	130986	153305.2	5830.1
		4000	200000	129065	146258.8	5148.78
		8000	400000	125367	140656.4	4594.11
	0.90788	1000	100000	133812	153606.2	5842.03
		2000	200000	128933	146456.2	5182.63
		4000	400000	124264	140847.3	4630.71
		8000	800000	122186	136442.1	4102.03
	0.95283	1000	200000	127738	146652.8	5227.74
		2000	400000	125440	140962	4648.42
		4000	800000	122975	136510.9	4119.21
		8000	1600000	119929	133117.6	3683.27
	0.97613	1000	400000	125216	140960.9	4618.12
		2000	800000	123657	136544.5	4131.68
		4000	1600000	121500	133154.7	3694.32
		8000	3200000	120304	130561.5	3343.63
	0.98800	1000	800000	121949	136525.5	4133.8
		2000	1600000	122100	133164.6	3699.82
		4000	3200000	120314	130559.5	3338.76
		8000	6400000	119845	128569.7	3030.09

Table A.5: SA results for pbn423 problem instance based on 40,000 observations, Optimum = 1,365

Move Type	α	MPT	Cost	BestObs.	mean	sd
2-Opt	0.82425	1000	50000	2519.89	3151.11	196.63
		2000	100000	2036.28	2433.59	191.15
		4000	200000	1699.27	1965.98	181.46
		8000	400000	1558.24	1698.88	169.99
	0.90788	1000	100000	2005.93	2439.97	190.25
		2000	200000	1696.32	1970.35	180.74
		4000	400000	1551.03	1700.62	167.59
		8000	800000	1470.59	1579.34	154.24
	0.95283	1000	200000	1663.52	1971.92	181.09
		2000	400000	1541.21	1701.93	169.57
		4000	800000	1462.46	1579.5	154.69
		8000	1600000	1438.73	1530.43	138.12
	0.97613	1000	400000	1542.75	1702.75	169.54
		2000	800000	1472.66	1579.61	153.57
		4000	1600000	1421.35	1530.58	138.23
		8000	3200000	1397.92	1496.56	124.61
	0.988	1000	800000	1474.79	1579.78	155.31
		2000	1600000	1441.13	1530.77	138.33
		4000	3200000	1410.61	1496.81	124.61
		8000	6400000	1393.91	1467.98	112.39
3-CitySwap	0.82425	1000	50000	3909.61	4662.95	180.98
		2000	100000	3339.32	3983.12	163.46
		4000	200000	2858.79	3421.78	146.45
		8000	400000	2483.13	2969.77	129.24
	0.90788	1000	100000	3310.6	3988.13	163.7
		2000	200000	2863.17	3425.28	146.85
		4000	400000	2461.22	2973.04	129.78
		8000	800000	2199.66	2617.44	112.57
	0.95283	1000	200000	2859.7	3429.27	146.61
		2000	400000	2475.89	2972.97	129.77
		4000	800000	2184.44	2618.83	112.67
		8000	1600000	2010.39	2345.8	97.42
	0.97613	1000	400000	2458.87	2974.41	130.45
		2000	800000	2183.27	2620.08	112.51
		4000	1600000	1966.49	2347.2	96.99
		8000	3200000	1850.96	2135.2	82.53
	0.988	1000	800000	2145.21	2620.42	112.93
		2000	1600000	1990.79	2345.5	97.25
		4000	3200000	1828.33	2134.85	82.37
		8000	6400000	1722.14	1972.33	70.05

Table A.6: TA results for pbn423 problem instance based on 40,000 observations, Optimum = 1,365

Move Type	α	MPT	Cost	BestObs.	mean	sd
2-Opt	0.82425	1000	50000	3065.63	3643.59	221.52
		2000	100000	2373.71	2787.14	213.54
		4000	200000	1913.22	2201.22	200.76
		8000	400000	1640.99	1829.12	185.9
	0.90788	1000	100000	2368.64	2798.31	213.42
		2000	200000	1927.96	2209.06	200.09
		4000	400000	1618.4	1834.18	185.27
		8000	800000	1502.77	1627.07	168.6
	0.95283	1000	200000	1902.23	2214.07	201.52
		2000	400000	1650.13	1836.98	186.88
		4000	800000	1489.27	1628.3	167.22
		8000	1600000	1416.6	1534.42	149.23
	0.97613	1000	400000	1629.05	1838.4	186.01
		2000	800000	1502.32	1628.76	167.54
		4000	1600000	1436.24	1534.45	149.4
		8000	3200000	1405.04	1491.31	132.56
0.98800	1000	800000	1497.32	1629.42	167.97	
	2000	1600000	1431.68	1534.85	149.89	
	4000	3200000	1397.22	1491.19	131.48	
	8000	6400000	1388.12	1462.31	118.03	
3-City Swap	0.82425	1000	50000	3977.59	4753.08	181.21
		2000	100000	3450.04	4069.3	163.6
		4000	200000	2929.74	3498.99	146.81
		8000	400000	2562.34	3037.25	130.22
	0.90788	1000	100000	3429.56	4074.77	163.76
		2000	200000	2918.43	3502.63	148.02
		4000	400000	2510.52	3039.45	130.32
		8000	800000	2253.84	2680.01	113.68
	0.95283	1000	200000	2956.86	3504.72	147.6
		2000	400000	2568.21	3040.22	130.91
		4000	800000	2248.38	2680.17	114.54
		8000	1600000	2037.49	2401.79	99.38
	0.97613	1000	400000	2551.77	3041.77	130.98
		2000	800000	2219.33	2680.14	114.96
		4000	1600000	2055.52	2401.66	99.44
		8000	3200000	1841.38	2184.9	85.31
0.98800	1000	800000	2191.02	2681.03	114.44	
	2000	1600000	2026.6	2402.08	99.2	
	4000	3200000	1866.34	2184.29	84.95	
	8000	6400000	1708.2	2013.87	72.46	

Table A.7: SA results for pcb442 problem instance based on 40,000 observations, Optimum = 50,778

Move Type	α	MPT	Cost	BestObs.	mean	sd
2-Opt	0.82425	1000	50000	88216	103999.3	3679.67
		2000	100000	70181.1	81606.3	2910.39
		4000	200000	60362.5	67454.32	2328.45
		8000	400000	55896.7	59719.07	1934.43
	0.90788	1000	100000	71459.3	81764.94	2910.96
		2000	200000	60206.8	67554.54	2343.92
		4000	400000	55511.8	59762.51	1939
		8000	800000	53049.5	56508.08	1655.25
	0.95283	1000	200000	59690.5	67604.38	2329.94
		2000	400000	55597.6	59786.71	1935.52
		4000	800000	53430.7	56506.47	1644.72
		8000	1600000	52188.7	55158.22	1426.51
	0.97613	1000	400000	56181.3	59801.75	1937.44
		2000	800000	53331.8	56513.45	1623.35
		4000	1600000	51988.2	55153.42	1419.94
		8000	3200000	51789.2	54100.78	1240.82
0.988	1000	800000	53469.1	56512.81	1629.87	
	2000	1600000	52343.4	55157.8	1414.04	
	4000	3200000	51597.3	54091.4	1238.25	
	8000	6400000	50997.8	53128.98	1063.91	
3-CitySwap	0.82425	1000	50000	133496	157737.7	5977.19
		2000	100000	110976	132957	5345.85
		4000	200000	95058.6	113345.5	4743
		8000	400000	83780.4	98636.66	4109.74
	0.90788	1000	100000	110671	133351.5	5346.67
		2000	200000	94867.7	113614.7	4742.49
		4000	400000	84147.6	98768.99	4133.41
		8000	800000	75091.9	87924.25	3545.42
	0.95283	1000	200000	94563.2	113712.7	4758.81
		2000	400000	83101.3	98825.77	4125.33
		4000	800000	74995.8	87945.28	3547.69
		8000	1600000	68248.9	79637.66	3006.31
	0.97613	1000	400000	83658.5	98877.75	4150.53
		2000	800000	73704.5	87950.85	3556.78
		4000	1600000	69103.9	79623.38	2986.78
		8000	3200000	63737.4	73061.67	2483.24
0.988	1000	800000	74615.4	87919.79	3536.93	
	2000	1600000	68799.5	79643.04	2992.04	
	4000	3200000	63866.3	73069.61	2480.64	
	8000	6400000	60551.9	67943.77	2043.93	

Table A.8: TA results for pcb442 problem instance based on 40,000 observations, Optimum = 50,778

Move Type	α	MPT	Cost	BestObs.	mean	sd
2-Opt	0.82425	1000	50000	89586.5	102515.4	4134.88
		2000	100000	71388.5	80469.4	3358.66
		4000	200000	61337.8	66681	2699.67
		8000	400000	55162.2	59278.39	2127.06
	0.90788	1000	100000	72779.2	80631.85	3349.36
		2000	200000	60592.6	66781.83	2685.23
		4000	400000	55320.5	59320.95	2145.21
		8000	800000	53157.1	56292.7	1700.95
	0.95283	1000	200000	60955.3	66815.28	2696.84
		2000	400000	55122.1	59351.06	2138.69
		4000	800000	52735.6	56302.59	1691.77
		8000	1600000	52074.9	55005.68	1400.45
	0.97613	1000	400000	55535.6	59354.02	2139.47
		2000	800000	52967.6	56301.22	1704.72
		4000	1600000	52174	55001.96	1407.7
		8000	3200000	51623.9	53981.75	1204.76
0.98800	1000	800000	53208.8	56299.85	1703.49	
	2000	1600000	51961	54999.54	1396.86	
	4000	3200000	51535.1	53980.43	1218.22	
	8000	6400000	50965.8	53079.76	1044.52	
3-City Swap	0.82425	1000	50000	135583	158648.9	5895.5
		2000	100000	113201	134449.2	5345.03
		4000	200000	96064.1	115304	4773.26
		8000	400000	83887	100928.7	4196.46
	0.90788	1000	100000	114275	134778.9	5351.59
		2000	200000	96283.2	115500.1	4769.81
		4000	400000	85757.3	100961.8	4188.5
		8000	800000	76624.4	90070.63	3604.65
	0.95283	1000	200000	95248.4	115641	4765.28
		2000	400000	85232.8	100955.6	4184.41
		4000	800000	77238.4	90081.68	3611.74
		8000	1600000	70288.3	81522.37	3071.73
	0.97613	1000	400000	85718.5	101022.3	4187.82
		2000	800000	76427.4	90078.31	3627.23
		4000	1600000	69969.9	81514.23	3080.2
		8000	3200000	64732.6	74690.06	2575.11
0.98800	1000	800000	76528.2	90062.92	3614.63	
	2000	1600000	69851.5	81502.94	3067.37	
	4000	3200000	65141.6	74679.8	2577.25	
	8000	6400000	61642.3	69316.57	2119.65	

Appendix B: t-test Results

Table B.11: pbn423 TA 2-Opt Welch's t-test results for 1,000 experiments comparing row settings to column settings

Table with columns for Coef 1, Moves per Temp (1000, 2000, 4000, 8000), Coef 2 (0.82125, 0.90788, 0.92923, 0.97613, 0.98800), and various rows for p value, t value, and t value. The table contains multiple rows for different coefficients and their corresponding statistical results across various temperatures.

Table B.16: pcb442 TA 3-City Swap Welch's t-test results for 1,000 experiments comparing row settings to column settings

Coef 1	Moves per Temp	Coef 2																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																		
		100	200	400	800	1000	2000	4000	8000	10000	20000	40000	80000																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
0.82125	Exper 1 Starts	1.0e+00	2.0e+00	4.0e+00	8.0e+00	1.6e+01	4.0e+00	8.0e+00	1.6e+01	3.2e+01	8.0e+00	1.6e+01	3.2e+01	6.4e+01	1.3e+02	2.6e+02	5.1e+02	1.0e+03	2.0e+03	4.0e+03	8.0e+03	1.6e+04	3.2e+04	6.4e+04	1.3e+05	2.6e+05	5.1e+05	1.0e+06	2.0e+06	4.0e+06	8.0e+06	1.6e+07	3.2e+07	6.4e+07	1.3e+08	2.6e+08	5.1e+08	1.0e+09	2.0e+09	4.0e+09	8.0e+09	1.6e+10	3.2e+10	6.4e+10	1.3e+11	2.6e+11	5.1e+11	1.0e+12	2.0e+12	4.0e+12	8.0e+12	1.6e+13	3.2e+13	6.4e+13	1.3e+14	2.6e+14	5.1e+14	1.0e+15	2.0e+15	4.0e+15	8.0e+15	1.6e+16	3.2e+16	6.4e+16	1.3e+17	2.6e+17	5.1e+17	1.0e+18	2.0e+18	4.0e+18	8.0e+18	1.6e+19	3.2e+19	6.4e+19	1.3e+20	2.6e+20	5.1e+20	1.0e+21	2.0e+21	4.0e+21	8.0e+21	1.6e+22	3.2e+22	6.4e+22	1.3e+23	2.6e+23	5.1e+23	1.0e+24	2.0e+24	4.0e+24	8.0e+24	1.6e+25	3.2e+25	6.4e+25	1.3e+26	2.6e+26	5.1e+26	1.0e+27	2.0e+27	4.0e+27	8.0e+27	1.6e+28	3.2e+28	6.4e+28	1.3e+29	2.6e+29	5.1e+29	1.0e+30	2.0e+30	4.0e+30	8.0e+30	1.6e+31	3.2e+31	6.4e+31	1.3e+32	2.6e+32	5.1e+32	1.0e+33	2.0e+33	4.0e+33	8.0e+33	1.6e+34	3.2e+34	6.4e+34	1.3e+35	2.6e+35	5.1e+35	1.0e+36	2.0e+36	4.0e+36	8.0e+36	1.6e+37	3.2e+37	6.4e+37	1.3e+38	2.6e+38	5.1e+38	1.0e+39	2.0e+39	4.0e+39	8.0e+39	1.6e+40	3.2e+40	6.4e+40	1.3e+41	2.6e+41	5.1e+41	1.0e+42	2.0e+42	4.0e+42	8.0e+42	1.6e+43	3.2e+43	6.4e+43	1.3e+44	2.6e+44	5.1e+44	1.0e+45	2.0e+45	4.0e+45	8.0e+45	1.6e+46	3.2e+46	6.4e+46	1.3e+47	2.6e+47	5.1e+47	1.0e+48	2.0e+48	4.0e+48	8.0e+48	1.6e+49	3.2e+49	6.4e+49	1.3e+50	2.6e+50	5.1e+50	1.0e+51	2.0e+51	4.0e+51	8.0e+51	1.6e+52	3.2e+52	6.4e+52	1.3e+53	2.6e+53	5.1e+53	1.0e+54	2.0e+54	4.0e+54	8.0e+54	1.6e+55	3.2e+55	6.4e+55	1.3e+56	2.6e+56	5.1e+56	1.0e+57	2.0e+57	4.0e+57	8.0e+57	1.6e+58	3.2e+58	6.4e+58	1.3e+59	2.6e+59	5.1e+59	1.0e+60	2.0e+60	4.0e+60	8.0e+60	1.6e+61	3.2e+61	6.4e+61	1.3e+62	2.6e+62	5.1e+62	1.0e+63	2.0e+63	4.0e+63	8.0e+63	1.6e+64	3.2e+64	6.4e+64	1.3e+65	2.6e+65	5.1e+65	1.0e+66	2.0e+66	4.0e+66	8.0e+66	1.6e+67	3.2e+67	6.4e+67	1.3e+68	2.6e+68	5.1e+68	1.0e+69	2.0e+69	4.0e+69	8.0e+69	1.6e+70	3.2e+70	6.4e+70	1.3e+71	2.6e+71	5.1e+71	1.0e+72	2.0e+72	4.0e+72	8.0e+72	1.6e+73	3.2e+73	6.4e+73	1.3e+74	2.6e+74	5.1e+74	1.0e+75	2.0e+75	4.0e+75	8.0e+75	1.6e+76	3.2e+76	6.4e+76	1.3e+77	2.6e+77	5.1e+77	1.0e+78	2.0e+78	4.0e+78	8.0e+78	1.6e+79	3.2e+79	6.4e+79	1.3e+80	2.6e+80	5.1e+80	1.0e+81	2.0e+81	4.0e+81	8.0e+81	1.6e+82	3.2e+82	6.4e+82	1.3e+83	2.6e+83	5.1e+83	1.0e+84	2.0e+84	4.0e+84	8.0e+84	1.6e+85	3.2e+85	6.4e+85	1.3e+86	2.6e+86	5.1e+86	1.0e+87	2.0e+87	4.0e+87	8.0e+87	1.6e+88	3.2e+88	6.4e+88	1.3e+89	2.6e+89	5.1e+89	1.0e+90	2.0e+90	4.0e+90	8.0e+90	1.6e+91	3.2e+91	6.4e+91	1.3e+92	2.6e+92	5.1e+92	1.0e+93	2.0e+93	4.0e+93	8.0e+93	1.6e+94	3.2e+94	6.4e+94	1.3e+95	2.6e+95	5.1e+95	1.0e+96	2.0e+96	4.0e+96	8.0e+96	1.6e+97	3.2e+97	6.4e+97	1.3e+98	2.6e+98	5.1e+98	1.0e+99	2.0e+99	4.0e+99	8.0e+99	1.6e+100	3.2e+100	6.4e+100	1.3e+101	2.6e+101	5.1e+101	1.0e+102	2.0e+102	4.0e+102	8.0e+102	1.6e+103	3.2e+103	6.4e+103	1.3e+104	2.6e+104	5.1e+104	1.0e+105	2.0e+105	4.0e+105	8.0e+105	1.6e+106	3.2e+106	6.4e+106	1.3e+107	2.6e+107	5.1e+107	1.0e+108	2.0e+108	4.0e+108	8.0e+108	1.6e+109	3.2e+109	6.4e+109	1.3e+110	2.6e+110	5.1e+110	1.0e+111	2.0e+111	4.0e+111	8.0e+111	1.6e+112	3.2e+112	6.4e+112	1.3e+113	2.6e+113	5.1e+113	1.0e+114	2.0e+114	4.0e+114	8.0e+114	1.6e+115	3.2e+115	6.4e+115	1.3e+116	2.6e+116	5.1e+116	1.0e+117	2.0e+117	4.0e+117	8.0e+117	1.6e+118	3.2e+118	6.4e+118	1.3e+119	2.6e+119	5.1e+119	1.0e+120	2.0e+120	4.0e+120	8.0e+120	1.6e+121	3.2e+121	6.4e+121	1.3e+122	2.6e+122	5.1e+122	1.0e+123	2.0e+123	4.0e+123	8.0e+123	1.6e+124	3.2e+124	6.4e+124	1.3e+125	2.6e+125	5.1e+125	1.0e+126	2.0e+126	4.0e+126	8.0e+126	1.6e+127	3.2e+127	6.4e+127	1.3e+128	2.6e+128	5.1e+128	1.0e+129	2.0e+129	4.0e+129	8.0e+129	1.6e+130	3.2e+130	6.4e+130	1.3e+131	2.6e+131	5.1e+131	1.0e+132	2.0e+132	4.0e+132	8.0e+132	1.6e+133	3.2e+133	6.4e+133	1.3e+134	2.6e+134	5.1e+134	1.0e+135	2.0e+135	4.0e+135	8.0e+135	1.6e+136	3.2e+136	6.4e+136	1.3e+137	2.6e+137	5.1e+137	1.0e+138	2.0e+138	4.0e+138	8.0e+138	1.6e+139	3.2e+139	6.4e+139	1.3e+140	2.6e+140	5.1e+140	1.0e+141	2.0e+141	4.0e+141	8.0e+141	1.6e+142	3.2e+142	6.4e+142	1.3e+143	2.6e+143	5.1e+143	1.0e+144	2.0e+144	4.0e+144	8.0e+144	1.6e+145	3.2e+145	6.4e+145	1.3e+146	2.6e+146	5.1e+146	1.0e+147	2.0e+147	4.0e+147	8.0e+147	1.6e+148	3.2e+148	6.4e+148	1.3e+149	2.6e+149	5.1e+149	1.0e+150	2.0e+150	4.0e+150	8.0e+150	1.6e+151	3.2e+151	6.4e+151	1.3e+152	2.6e+152	5.1e+152	1.0e+153	2.0e+153	4.0e+153	8.0e+153	1.6e+154	3.2e+154	6.4e+154	1.3e+155	2.6e+155	5.1e+155	1.0e+156	2.0e+156	4.0e+156	8.0e+156	1.6e+157	3.2e+157	6.4e+157	1.3e+158	2.6e+158	5.1e+158	1.0e+159	2.0e+159	4.0e+159	8.0e+159	1.6e+160	3.2e+160	6.4e+160	1.3e+161	2.6e+161	5.1e+161	1.0e+162	2.0e+162	4.0e+162	8.0e+162	1.6e+163	3.2e+163	6.4e+163	1.3e+164	2.6e+164	5.1e+164	1.0e+165	2.0e+165	4.0e+165	8.0e+165	1.6e+166	3.2e+166	6.4e+166	1.3e+167	2.6e+167	5.1e+167	1.0e+168	2.0e+168	4.0e+168	8.0e+168	1.6e+169	3.2e+169	6.4e+169	1.3e+170	2.6e+170	5.1e+170	1.0e+171	2.0e+171	4.0e+171	8.0e+171	1.6e+172	3.2e+172	6.4e+172	1.3e+173	2.6e+173	5.1e+173	1.0e+174	2.0e+174	4.0e+174	8.0e+174	1.6e+175	3.2e+175	6.4e+175	1.3e+176	2.6e+176	5.1e+176	1.0e+177	2.0e+177	4.0e+177	8.0e+177	1.6e+178	3.2e+178	6.4e+178	1.3e+179	2.6e+179	5.1e+179	1.0e+180	2.0e+180	4.0e+180	8.0e+180	1.6e+181	3.2e+181	6.4e+181	1.3e+182	2.6e+182	5.1e+182	1.0e+183	2.0e+183	4.0e+183	8.0e+183	1.6e+184	3.2e+184	6.4e+184	1.3e+185	2.6e+185	5.1e+185	1.0e+186	2.0e+186	4.0e+186	8.0e+186	1.6e+187	3.2e+187	6.4e+187	1.3e+188	2.6e+188	5.1e+188	1.0e+189	2.0e+189	4.0e+189	8.0e+189	1.6e+190	3.2e+190	6.4e+190	1.3e+191	2.6e+191	5.1e+191	1.0e+192	2.0e+192	4.0e+192	8.0e+192	1.6e+193	3.2e+193	6.4e+193	1.3e+194	2.6e+194	5.1e+194	1.0e+195	2.0e+195	4.0e+195	8.0e+195	1.6e+196	3.2e+196	6.4e+196	1.3e+197	2.6e+197	5.1e+197	1.0e+198	2.0e+198	4.0e+198	8.0e+198	1.6e+199	3.2e+199	6.4e+199	1.3e+200	2.6e+200	5.1e+200	1.0e+201	2.0e+201	4.0e+201	8.0e+201	1.6e+202	3.2e+202	6.4e+202	1.3e+203	2.6e+203	5.1e+203	1.0e+204	2.0e+204	4.0e+204	8.0e+204	1.6e+205	3.2e+205	6.4e+205	1.3e+206	2.6e+206	5.1e+206	1.0e+207	2.0e+207	4.0e+207	8.0e+207	1.6e+208	3.2e+208	6.4e+208	1.3e+209	2.6e+209	5.1e+209	1.0e+210	2.0e+210	4.0e+210	8.0e+210	1.6e+211	3.2e+211	6.4e+211	1.3e+212	2.6e+212	5.1e+212	1.0e+213	2.0e+213	4.0e+213	8.0e+213	1.6e+214	3.2e+214	6.4e+214	1.3e+215	2.6e+215	5.1e+215	1.0e+216	2.0e+216	4.0e+216	8.0e+216	1.6e+217	3.2e+217	6.4e+217	1.3e+218	2.6e+218	5.1e+218	1.0e+219	2.0e+219	4.0e+219	8.0e+219	1.6e+220	3.2e+220	6.4e+220	1.3e+221	2.6e+221	5.1e+221	1.0e+222	2.0e+222	4.0e+222	8.0e+222	1.6e+223	3.2e+223	6.4e+223	1.3e+224	2.6e+224	5.1e+224	1.0e+225	2.0e+225	4.0e+225	8.0e+225	1.6e+226	3.2e+226	6.4e+226	1.3e+227	2.6e+227	5.1e+227	1.0e+228	2.0e+228	4.0e+228	8.0e+228	1.6e+229	3.2e+229	6.4e+229	1.3e+230	2.6e+230	5.1e+230	1.0e+231	2.0e+231	4.0e+231	8.0e+231	1.6e+232	3.2e+232	6.4e+232	1.3e+233	2.6e+233	5.1e+233	1.0e+234	2.0e+234	4.0e+234	8.0e+234	1.6e+235	3.2e+235	6.4e+235	1.3e+236	2.6e+236	5.1e+236	1.0e+237	2.0e+237	4.0e+237	8.0e+237	1.6e+238	3.2e+238	6.4e+238	1.3e+239	2.6e+239	5.1e+239	1.0e+240	2.0e+240	4.0e+240	8.0e+240	1.6e+241	3.2e+241	6.4e+241	1.3e+242	2.6e+242	5.1e+242	1.0e+243	2.0e+243	4.0e+243	8.0e+243	1.6e+244	3.2e+244	6.4e+244	1.3e+245	2.6e+245	5.1e+245	1.0e+246	2.0e+246	4.0e+246	8.0e+246	1.6e+247	3.2e+247	6.4e+247	1.3e+248	2.6e+248	5.1e+248	1.0e+249	2.0e+249	4.0e+249	8.0e+249	1.6e+250	3.2e+250	6.4e+250	1.3e+251	2.6e+251	5.1e+251	1.0e+252	2.0e+252	4.0e+252	8.0e+252	1.6e+253	3.2e+253	6.4e+253	1.3e+254	2.6e+254	5.1e+254	1.0e+255	2.0e+255	4.0e+255	8.0e+255	1.6e+256	3.2e+256	6.4e+256	1.3e+257	2.6e+257	5.1e+257	1.0e+258	2.0e+258	4.0e+258	8.0e+258	1.6e+259	3.2e+259	6.4e+259	1.3e+260	2.6e+260	5.1e+260	1.0e+261	2.0e+261	4.0e+261	8.0e+261	1.6e+262	3.2e+262	6.4e+262

Appendix C: Multistart Difference in Means

Table C.3: rand100 TA 2-Opt Multistart Difference in Means

Coef 1		Coef 2									
Moves per Temp		0.82425		0.90788		0.95283		0.97613		0.98800	
100	200	400	800	100	200	400	800	100	200	400	800
0.82425	100	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
	200	1.000	127.697	178.539	186.737	177.000	186.133	177.185	176.554	186.428	176.907
	400	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
	800	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
0.90788	100	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
	200	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
	400	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
	800	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
0.95283	100	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
	200	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
	400	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
	800	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
0.97613	100	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
	200	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
	400	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
	800	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
0.98800	100	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
	200	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
	400	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000
	800	1.000	2.000	4.000	8.000	16.000	32.000	16.000	32.000	64.000	128.000

Table C.6: **bier127 SA 3-City Swap Multistart Difference in Means**

Cofc1	Moves per Temp	Cofc2																			
		1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000
0.82425	Exper 1 Starts	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
	Difference in Means	0	-4189	7759	10775	4270	8032	10635	13312	8157	10941	13209	14715	10712	13288	14566	15562	12996	14727	15815	16330
	Exper 1 Starts	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
	Difference in Means	0	3467	6019	-219	3730	5879	8273	3855	6186	8170	9622	5956	8219	9473	10588	7957	9634	10841	11340	11340
	Difference in Means	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
0.90788	Exper 1 Starts	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
	Difference in Means	0	2362	233	2223	4248	358	2529	4145	5711	2299	4225	5592	6896	3932	5752	7149	7551	7551	7551	7551
	Exper 1 Starts	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
	Difference in Means	2	4	1	2	-139	2038	167	1935	2914	-63	2014	2765	3858	1722	2926	4111	4616	4616	4616	4616
	Difference in Means	3102	6272	0	3335	6133	8436	3460	6439	8333	9610	6210	8412	9461	10714	8120	9622	10967	11367	11367	11367
0.95283	Exper 1 Starts	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
	Difference in Means	2361	0	2221	4419	125	2528	4316	5652	2298	4395	5504	6838	4103	5664	7091	7645	7645	7645	7645	7645
	Exper 1 Starts	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
	Difference in Means	0	1676	0	1676	306	1573	2790	77	1652	2641	3736	1360	2801	3989	4625	4625	4625	4625	4625	4625
	Difference in Means	2	4	1	2	-103	796	1	2	1	2	1	2	1	2	1	2	1	2	1	2
0.97613	Exper 1 Starts	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
	Difference in Means	2431	0	2291	4450	0	2597	4347	5928	2368	4426	5779	6909	4134	5939	7162	7524	7524	7524	7524	7524
	Exper 1 Starts	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
	Difference in Means	1761	0	1638	2803	-230	1737	2654	3759	1445	2815	4012	4567	4567	4567	4567	4567	4567	4567	4567	4567
	Difference in Means	0	1163	0	1163	79	1015	1754	-213	1175	2007	2316	2316	2316	2316	2316	2316	2316	2316	2316	2316
0.98800	Exper 1 Starts	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
	Difference in Means	0	-149	380	12	633	991	991	991	991	991	991	991	991	991	991	991	991	991	991	991
	Exper 1 Starts	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
	Difference in Means	1665	2883	0	1744	2734	3815	1452	2895	4068	4536	4536	4536	4536	4536	4536	4536	4536	4536	4536	4536
	Difference in Means	886	0	737	1855	-292	897	2108	2318	2318	2318	2318	2318	2318	2318	2318	2318	2318	2318	2318	2318
0.98800	Exper 1 Starts	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
	Difference in Means	0	524	0	524	161	777	1153	1153	1153	1153	1153	1153	1153	1153	1153	1153	1153	1153	1153	1153
	Exper 1 Starts	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
	Difference in Means	2	4	1	2	2	4	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192
	Difference in Means	1131	982	1840	0	1143	2093	2459	2459	2459	2459	2459	2459	2459	2459	2459	2459	2459	2459	2459	2459
0.98800	Exper 1 Starts	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
	Difference in Means	0	588	0	588	0	841	1126	1126	1126	1126	1126	1126	1126	1126	1126	1126	1126	1126	1126	1126
	Exper 1 Starts	1	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
	Difference in Means	0	217	0	217	0	217	0	217	0	217	0	217	0	217	0	217	0	217	0	217
	Difference in Means	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table C.7: bier127 TA 2-Opt Multistart Difference in Means

Coef.1	Coef.2																			
	100	200	400	800	100	200	400	800	100	200	400	800	100	200	400	800				
0.82425	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64	16	32	64	128
0.90788	0	149.43	19778	-270	14602	19638	20733	14632	19626	20533	21946	19759	21897	21823	22136	21064	21851	22183	21872	21872
0.95283	0	5477	0	5477	-341	5386	6592	-311	5324	6732	8067	5457	6696	7944	8676	6833	7972	8723	9006	9006
0.97613	2	4	8	16	4	8	16	32	8	16	32	64	16	32	64	16	32	64	128	128
0.98800	0	267.40	407.80	4564.1	26470	40380	45501	43536	40120	45189	45976	46531	45622	45940	46407	46382	46077	46136	46129	45768
0.99283	0	5792	0	5652	6829	30	5640	6969	8216	5773	6933	8093	8824	7071	8121	8871	9061	9061	9061	9061
0.99788	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64	16	32	64	64
0.99800	2	4	8	16	4	8	16	32	8	16	32	64	16	32	64	16	32	64	128	128
0.99850	0	14744	20661	20907	14774	20049	21047	21955	20182	21011	21831	22295	21148	21860	22342	22187	21860	22342	22187	22187
0.99925	0	5792	0	5652	6829	30	5640	6969	8216	5773	6933	8093	8824	7071	8121	8871	9061	9061	9061	9061
0.99978	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64	16	32	64	64
0.99980	2	4	8	16	4	8	16	32	8	16	32	64	16	32	64	16	32	64	128	128
0.99985	0	5792	0	5652	6829	30	5640	6969	8216	5773	6933	8093	8824	7071	8121	8871	9061	9061	9061	9061
0.99992	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64	16	32	64	64
0.99995	2	4	8	16	4	8	16	32	8	16	32	64	16	32	64	16	32	64	128	128
0.99998	0	5792	0	5652	6829	30	5640	6969	8216	5773	6933	8093	8824	7071	8121	8871	9061	9061	9061	9061
0.99999	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64	16	32	64	64
1.00000	2	4	8	16	4	8	16	32	8	16	32	64	16	32	64	16	32	64	128	128

Table C.8: bier127 TA 3-City Swap Multistart Difference in Means

Coef 1	Moves per Temp	Coef 2																			
		0.82425				0.90788				0.95283				0.97613				0.98800			
		1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000
0.82425	2000	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64	16	32	64	128
	Difference in Means	0	4502	8285	11803	4058	8264	11227	13711	8235	11546	13376	15384	11260	13639	15120	16287	13617	15178	16047	16999
4000	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	3535	6913	-444	3513	6337	8773	3481	6655	8439	10485	6370	9001	10271	11444	8680	10329	11204	11982	
8000	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	2692	-22	2115	4522	-51	2434	4188	6225	2148	4750	6011	7022	4429	6069	6782	7724			
0.90788	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	1	2	-577	1493	-258	1158	3072	-543	1721	2858	3928	1399	2916	3689	4404				
4000	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	3740	7043	0	3719	6456	8931	3690	6775	8597	10628	6489	9159	10414	11688	8837	10472	11448	12251	
8000	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	2885	0	2309	4828	-29	2627	4483	6265	2342	5056	6051	7216	4734	6109	6976	7694			
0.95283	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	1643	0	1643	319	1309	3130	33	1871	2916	4018	1549	2974	3778	4586					
4000	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	-334	1226	0	0	-334	1226	0	228	1012	1873	-94	1070	1634	2277					
8000	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	2560	5059	0	2878	4725	6580	2593	5288	6366	7361	4966	6424	7122	7999					
0.97613	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	1740	0	1405	3215	-285	1968	3000	4040	1646	3058	3800	4583							
4000	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	1026	0	1026	563	812	1903	241	870	1664	2362									
8000	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	-214	435	0	0	-214	435	0	2	4	8	16	4	8	16	32	8	16	32	64
0.98800	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	1696	3447	0	2258	3233	4148	1936	3291	3908	4606									
4000	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	1208	0	994	1802	-322	1052	1562	2371											
8000	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	784	0	784	58	545	845													
0.98800	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	974	0	760	2071	0	818	1831	2358											
4000	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	591	0	591	0	352	783													
8000	2000	1	2	4	8	1	2	4	8	2	4	8	16	4	8	16	32	8	16	32	64
	Difference in Means	0	1	0	135	0	1	0	135	0	1	0	135	0	1	0	135	0	1	0	135

Table C.11: pbn423 TA 2-Opt Multistart Difference in Means

Coef1	Moves per Temp	Coef2																
		1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	
0.82425	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
	Difference in Means	0.00	780.55	1321.21	1653.79	763.47	1313.20	1656.09	1823.99	1298.13	1642.37	1813.66	1890.31	1646.90	1828.08	1878.82	1811.20	1823.35
	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
0.90788	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
	Difference in Means	0.00	521.90	863.82	-11.08	513.88	866.12	1039.63	-498.82	852.40	1029.30	1110.79	856.92	1043.72	1109.30	1128.28	1039.00	1106.32
	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
0.97613	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
	Difference in Means	0.00	323.21	-11.02	325.51	502.01	-26.08	311.79	491.68	581.87	316.31	506.10	580.39	608.88	501.37	576.40	603.66	613.01
	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
0.98800	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
	Difference in Means	0.00	535.24	875.69	0.00	524.22	877.39	1048.76	509.16	863.67	1038.43	1120.23	868.19	1052.86	1118.75	1142.72	1048.13	1114.76
	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
0.95283	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
	Difference in Means	0.00	331.77	510.26	-15.06	318.05	499.93	589.54	322.57	514.35	588.05	615.52	509.62	584.07	610.30	623.05	509.62	610.30
	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
0.97613	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
	Difference in Means	0.00	167.13	-13.72	166.80	251.54	-9.20	171.22	250.05	282.12	166.49	246.07	276.90	293.88	166.49	246.07	276.90	293.88
	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
0.98800	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
	Difference in Means	0.00	332.53	515.27	0.00	321.11	504.95	594.29	325.63	519.37	592.80	618.34	514.64	588.82	613.13	626.16	514.64	588.82
	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
0.97613	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
	Difference in Means	0.00	173.10	0.00	162.77	254.47	4.52	177.19	252.98	283.72	172.46	248.99	278.51	295.98	172.46	248.99	278.51	295.98
	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
0.98800	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
	Difference in Means	0.00	172.81	0.00	162.49	254.25	0.00	176.91	252.76	283.17	172.18	248.78	279.95	298.15	172.18	248.78	279.95	298.15
	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
0.98800	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
	Difference in Means	0.00	68.26	0.00	66.77	100.60	-4.73	62.78	95.38	115.42	68.26	100.60	-4.73	62.78	95.38	115.42	68.26	100.60
	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
0.98800	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
	Difference in Means	0.00	70.53	0.00	69.04	101.67	0.00	65.06	96.45	115.79	70.53	101.67	0.00	65.06	96.45	115.79	70.53	101.67
	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
0.98800	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
	Difference in Means	0.00	24.40	0.00	24.40	0.00	19.18	39.22	0.00	10.14	0.00	10.14	0.00	10.14	0.00	10.14	0.00	10.14
	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
0.98800	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00
	Difference in Means	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Exper 1 Starts	1.00	2.00	4.00	8.00	16.00	32.00	64.00	128.00	256.00	512.00	1024.00	2048.00	4096.00	8192.00	16384.00	32768.00	65536.00

Table C.13: pcb442 SA 2-Opt Multistart Difference in Means

Coef 1	Moves per Temp	Coef 2																																																																																																																																																																																																																																																																																																																																																																																																											
		0.82425	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000	1000	2000	4000	8000																																																																																																																																																																																																																																																																																																																																											
0.82425	Exper 1 Starts	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0	524288.0	1048576.0	2097152.0	4194304.0	8388608.0	16777216.0	33554432.0	67108864.0	134217728.0	268435456.0	536870912.0	1073741824.0	2147483648.0	4294967296.0	8589934592.0	17179869184.0	34359738368.0	68719476736.0	137438953472.0	274877906944.0	549755813888.0	1099511627776.0	2199023255552.0	4398046511104.0	8796093022208.0	17592186444416.0	35184372888832.0	70368745777664.0	140737491555328.0	281474983110656.0	562949966221312.0	1125899932442624.0	2251799864885248.0	4503599729770496.0	9007199459540992.0	18014398919081984.0	36028797838163968.0	72057595676327936.0	144115191352655872.0	288230382705311744.0	576460765410623488.0	1152921530821246976.0	2305843061642493952.0	4611686123284987904.0	9223372246569975808.0	18446744493139959616.0	36893488986279919232.0	73786977972559838464.0	147573955945119676928.0	295147911890393353856.0	590295823780786707712.0	1180591647561573415424.0	2361183295123146830848.0	4722366590246293661696.0	9444733180492587323392.0	18889466360985174646784.0	37778932721970349293568.0	75557865443940698587136.0	151115730887881391174272.0	30223146177576278234848.0	60446292355152556479696.0	120892584710305112959392.0	241785169420610225918784.0	483570338841220451837696.0	967140677682440903675392.0	1934281355364881807350784.0	38685627107297636147115616.0	77371254214595272294231332.0	154742508429190544588462624.0	309485016858381089176925248.0	618970033716760178353850496.0	123794006743352035670799392.0	247588013486704071347598784.0	495176026973408142719197568.0	990352053946816285438395136.0	1980704107893632570876790272.0	3961408215887265141753580544.0	7922816431774530283507160896.0	1584563283554906056701332192.0	3169126567109801113402264384.0	633825313421960222738066867808.0	12676506268439204446761337696.0	25353012536878408893522675392.0	50706025073756817787045350784.0	1014120501475136355740907056.0	20282410029502727114818111136.0	40564820059005454229636222272.0	81129640118010910847324444544.0	162259280236021820946648889088.0	324518560472043641893297778176.0	6490371209440872837875955553536.0	12980742418881745675751911073114645.0	25961484837763491351503822242289.0	519229696755269827026064454857.0	103845939351053975405212971115713.0	2076918787021079508104544233826.0	41538375740421590160908844804.0	8307675148084318112181716713.0	1661535029616636209103608342298.0	3323070059233272418236661509121012.0	6646140118544536661509121012.0	13292280237089066661509121012.0	26584560474178121012.0	53169120948366661509121012.0	106338241736332.0	21267648366661509121012.0	42535296736332.0	8507059347266661509121012.0	170141186945332.0	3402823728936661509121012.0	680564745787332.0	13611294915746661509121012.0	2722258983149332.0	54445179662986661509121012.0	10889035925997332.0	2177807185197332.0	4355614371595332.0	87112287431906661509121012.0	17422457483801332.0	348449149676026661509121012.0	69689829935205332.0	139379659870411332.0	2787593197408226661509121012.0	557518639481645332.0	11150372792632906661509121012.0	2230075588526581332.0	44601511775251626661509121012.0	8920302355051326661509121012.0	17840604710102526661509121012.0	3568120942020513326661509121012.0	713624188404076661509121012.0	1427248368008153326661509121012.0	285449673601626661509121012.0	570899347203253326661509121012.0	1141798694406506661509121012.0	2283597392812113326661509121012.0	4567194785624226661509121012.0	9134389571244453326661509121012.0	1826877914888906661509121012.0	3653755829777813326661509121012.0	7307511659555626661509121012.0	14615039111146661509121012.0	29230078222293326661509121012.0	5846015644456661509121012.0	116920311146661509121012.0	2338406288893326661509121012.0	4676812577786661509121012.0	9353625155573326661509121012.0	18707250311146661509121012.0	37414500622293326661509121012.0	7482900124576661509121012.0	14965802491153326661509121012.0	29931604982296661509121012.0	5986320996453326661509121012.0	11972641992906661509121012.0	23945283985813326661509121012.0	47890567971626661509121012.0	95781135943253326661509121012.0	19156227188506661509121012.0	38312453977013326661509121012.0	76624907954026661509121012.0	15324981590806661509121012.0	30649963181613326661509121012.0	61299926363226661509121012.0	122599846726453326661509121012.0	24519969345286661509121012.0	49039938690573326661509121012.0	98079877381146661509121012.0	196159754762293326661509121012.0	39231950952456661509121012.0	7846390190513326661509121012.0	1569278038106661509121012.0	31385560762113326661509121012.0	62771121524226661509121012.0	12554224304453326661509121012.0	25108448608906661509121012.0	50216897217813326661509121012.0	1004337944356661509121012.0	2008675888713326661509121012.0	4017351777426661509121012.0	8034703554853326661509121012.0	16069407109106661509121012.0	32138814218213326661509121012.0	64277628436426661509121012.0	1285552568728453326661509121012.0	25711051374568661509121012.0	514221075491373326661509121012.0	102844215093646661509121012.0	205688430187293326661509121012.0	411376860374586661509121012.0	822753720749173326661509121012.0	164550744148353326661509121012.0	329101488296706661509121012.0	65820297651346661509121012.0	131640595302686661509121012.0	263281190605373326661509121012.0	526562381210746661509121012.0	105312476242146661509121012.0	210624952484293326661509121012.0	421249904968586661509121012.0	842499809937173326661509121012.0	168499961987346661509121012.0	3369999239746661509121012.0	6739998479493326661509121012.0	13479996958986661509121012.0	2695999391796661509121012.0	5391998783593326661509121012.0	10783997567186661509121012.0	21567995135173326661509121012.0	43135990270346661509121012.0	86271980540693326661509121012.0	172543961081386661509121012.0	345087922162773326661509121012.0	690175844325546661509121012.0	138035168865113326661509121012.0	276070337730226661509121012.0	552140675460453326661509121012.0	11042813509206661509121012.0	22085627112013326661509121012.0	44171254214026661509121012.0	88342508428053326661509121012.0	17668501685606661509121012.0	35337003371213326661509121012.0	70674006742426661509121012.0	14134801348486661509121012.0	28269602696973326661509121012.0	56539205393946661509121012.0	11307840878786661509121012.0	22615681757573326661509121012.0	45231363515146661509121012.0	90462727030293326661509121012.0	180925454060586661509121012.0	361850908121173326661509121012.0	72370181624236661509121012.0	14474036244446661509121012.0	28948072488893326661509121012.0	57896144977786661509121012.0	115792289955573326661509121012.0	231584579911146661509121012.0	46316915982296661509121012.0	9263383175773326661509121012.0	18526763565546661509121012.0	3705352711146661509121012.0	74107151322893326661509121012.0	148214302557786661509121012.0	2964286051153326661509121012.0	5928572102296661509121012.0	1185714420456661509121012.0	2371428840913326661509121012.0	4742857681826661509121012.0	9485715363653326661509121012.0	1897143072736661509121012.0	379428614736661509121012.0	7588572294736661509121012.0	1517713116713.0	30354284804.0	60708569608.0	121417164804.0	242835329616.0	485670659232.0	97134131840.0	194268273664.0	388536547328.0	777073094656.0	155414618912.0	31082921792.0	62165843584.0	124331687168.0	248663374336.0	497326748672.0	994653497664.0	1989306995328.0	3978613990656.0	7957227981312.0	15914455962624.0	31828911925248.0	6365782384448.0	12731564768896.0	25463129537792.0	50926259075584.0	101852518151168.0	203705036302336.0	407410072604672.0	814820145209248.0	1629640310418496.0	3259280620836992.0	6518561241673984.0	13037124883351936.0	26074249766747872.0	52148499533495744.0	104296991071915488.0	20859398204389504.0	41718796408779008.0	83437592817578016.0	166875185635156032.0	333750371271572064.0	667500742543144128.0	1335001485086288256.0	2670002970172486528.0	5340005940344972992.0	10680011880689945984.0	21360023761379699968.0	42720047522759399936.0	85440095045518799872.0	17088019009107599744.0	34176038018215199488.0	68352076036430398976.0	136704152072870793952.0	273408304145741587904.0	546816608291483815808.0	1093633216582967637696.0	2187266433165735275392.0	4374532866327470554784.0	8749065732654941109568.0	17498134625309883119136.0	34996269250619766237312.0	69992538501239532474624.0	13998507700247107189472.0	2799701540049421439744.0	559940308009884287888.0	1119880616019767977776.0	2239761232039535955552.0	44795224640790719111104.0	89590449281580422222208.0	179180898563160844444416.0	358361777121681688888832.0	716723554243363377777664.0	1433447108486746755555328.0	2866894216973493511110656.0	5733788433946987022221312.0	11467576867893974044442624.0	22935153735787968088884448.0	45870307471575936177778896.0	91740614943171873555577792.0	183481229886343747111155584.0	36696245977268749422231168.0	7339249195453749844446336.0	14678493990907499688888672.0	29356987981814999377777344.0	5871397596362999875555488.0	11742795192739999515111776.0	2348559038757999903023552.0	4697118077515999806047104.0	9394236155031999607082208.0	187884723100719992141644416.0	37576944620143994228288896.0	75153889240287988457377792.0	150307782480575976914755584.0	300615564965151953891511168.0	6012311299303039077823144.0	12024622598606078154646288.0	24049245197212156311297696.0	480984903944243126225551392.0	961969807888486252451112704.0	192393961577692452451112704.0	38478792315538490490282304.0	769575846310769809805646048.0	15391516926215396191112704.0	307830338524379363823144.0	615660677048758727626288.0	12313213540975174554555776.0	24626427081950349111111552.0	492528541639006982222221056.0	985057083278013974444421112.0	1970114166556027888888422224.0	394022833215605577777644444.0	788045666431211155555588888.0	15760913328624222211111177776.0	315218266572444422222235552.0	6304365331448844444444711104.0	12608730662897688888881422208.0	252174613257953777776284444.0	50434922651595555555488888.0	100869853111911111111077776.0	201739706223822222222155552.0	403479412447644444443111104.0	80695884889528888888222208.

Table C.15: pcb442 TA 2-Opt Multistart Difference in Means

Coef 1	Moves per Temp	Coef 2																		
		0.82425	0.90788	0.95283	0.97613	0.98800														
0.82425	1000	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0					
	2000	0.0	20263.6	32991.2	39413.1	20335.4	32947.1	39470.8	41579.0	39654.7	39517.3	41701.1	42114.9	39455.4	41730.1	42367.3	41590.5	42094.8	42461.2	42509.6
	Difference in Means	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
0.90788	1000	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
	2000	0.0	12620.4	19161.6	71.8	12576.4	19219.4	21404.7	12683.9	19265.9	21526.9	22223.6	19204.0	21561.8	22239.4	22709.7	21166.2	22203.4	22703.6	23044.2
	Difference in Means	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
0.95283	1000	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
	2000	0.0	6522.2	-44.1	6409.9	8789.9	63.5	6456.4	8912.1	9766.2	6394.5	8950.0	9782.0	10372.6	8801.4	9746.0	10366.5	10873.2	11380.0	11886.7
	Difference in Means	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
0.97613	1000	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
	2000	0.0	12651.4	19266.6	0.0	12607.3	19324.3	21554.3	12714.9	19370.9	21676.5	22450.7	19308.9	21714.4	22466.4	22878.3	21565.8	22430.5	22872.2	23275.0
	Difference in Means	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
0.98800	1000	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
	2000	0.0	6478.7	0.0	6536.4	8943.0	107.6	6582.9	9065.2	9878.3	6521.0	9103.1	9894.1	10520.7	8954.5	9858.1	10514.6	10976.9	11439.6	11903.3
	Difference in Means	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
0.98800	1000	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
	2000	0.0	2213.3	0.0	2213.3	46.5	2335.4	3146.1	-15.4	2373.4	3161.9	3866.5	2224.8	3125.9	3860.4	4445.1	4445.1	4445.1	4445.1	4445.1
	Difference in Means	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
0.98800	1000	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
	2000	0.0	6507.4	6565.1	8909.0	0.0	6611.7	9031.1	9886.1	6549.7	9069.1	9901.9	10485.2	9850.5	9855.9	10479.1	11014.8	11595.0	12175.9	12756.9
	Difference in Means	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
0.98800	1000	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
	2000	0.0	2246.3	0.0	2368.5	3274.7	-61.9	2406.4	3290.5	3913.9	2257.8	3254.5	3907.8	4494.0	4494.0	4494.0	4494.0	4494.0	4494.0	4494.0
	Difference in Means	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
0.98800	1000	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
	2000	0.0	2274.9	2274.9	2274.9	2274.9	2274.9	2274.9	2274.9	2274.9	2274.9	2274.9	2274.9	2274.9	2274.9	2274.9	2274.9	2274.9	2274.9	2274.9
	Difference in Means	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
0.98800	1000	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
	2000	0.0	708.1	708.1	708.1	708.1	708.1	708.1	708.1	708.1	708.1	708.1	708.1	708.1	708.1	708.1	708.1	708.1	708.1	708.1
	Difference in Means	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
0.98800	1000	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
	2000	0.0	522.2	522.2	522.2	522.2	522.2	522.2	522.2	522.2	522.2	522.2	522.2	522.2	522.2	522.2	522.2	522.2	522.2	522.2
	Difference in Means	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
0.98800	1000	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0
	2000	0.0	439.3	439.3	439.3	439.3	439.3	439.3	439.3	439.3	439.3	439.3	439.3	439.3	439.3	439.3	439.3	439.3	439.3	439.3
	Difference in Means	1.0	2.0	4.0	8.0	16.0	32.0	64.0	128.0	256.0	512.0	1024.0	2048.0	4096.0	8192.0	16384.0	32768.0	65536.0	131072.0	262144.0

