INVESTIGATION OF A TWO-BAY FOLDED PLATE STRUCTURE

by

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INVESTIGATION OF A TWO-BAY FOLDED PLATE STRUCTURE

PART I INTRODUCTION

A folded plate structure consists of a series of flat plates connected together along their edges, and end diaphrams which transmit whole structure load to supports. The materials utilized for this type of structure are not limited, but reinforced concrete is favorably used over wood, steel and aluminum.

Folded plates which are sometimes called prismatic shells or hipped plates are extensively used at present time, mainly for roof construction. Other areas where this type of structure is able to be applied are bridge, airplane, and building designes. The popularity of the folded plate structures owe to the following advantages: (1) These structures are easier to form and fabricate; (2) design computations are generally simple and use of complicated mathematical equations are not needed in most analyses; (3) folded plate structures have interesting architectural appearances.

The first design theory of folded plate or hipped plate structure was published in Germany by G. Ehlers and H. Craemer in 1930. Since this publication, a great deal of roof and bunker structures have been designed in Europe by this theory.

In the United States, the most widely used design method was introduced by G. Winter and M. Pei (8, p. 505) in 1947. In their analysis, however, the effects of the joint displacement was not considered at all. This simplifying assumption sometimes caused erroneous results in the analysis and design of the structures. In 1954, therefore, the effect of joints displacements was included in the analysis proposed by I. Gaafer (2, p. 743). From then on, many technical papers were published in this country on the simplified treatment of the folded plates, including the publications by H. Simpson (6, p. 1), E. Traum (7, p. 103), and H. Nilson (3, p. 215).

To present the detailed discussions of each simplified design method was not intended by the writer. However, Simpson's and Traum's methods deserved some description. In 1958, H. Simpson simplified the theory such that the bending moments and stresses of a folded plate structure were determined by just assuming a unit rotation of each plate and superimposing the results to a no rotation case. E. Traum introduced another simplified method of analysis in 1959. His method was similar to the Simpson's in using the superimposing idea, but was different in dealing directly with deflections rather than rotations. The writer further simplified the theory in analyzing a steel model by utilizing merits of each theory by Simpson and Traum. This method of analysis will be called "simplified bending theory" in this paper.

In some cases, basic beam theory of strength of materials was engaged in computing folded plate stresses. This method will be defined as "beam theory". The detailed descriptions of these two theories are presented in the next part of the discussion.

As mentioned briefly above, for the last two decades, numerous papers have been written on the subject of analyzing folded plate structures in the United States as well as in European countries. On the other hand, there has been little information published concerning the experimental study of the same structures. In his paper I. Gaafer (2, p. 743) compared the analytical values with experimental results of an aluminum model with uniform loads applied symmetrically about the center line of the model cross section. In 1961, A. Scordelis, E. Croy and I. Stubbs (5, p. 139) investigated a folded plate model with unsymmetrical cross section, and the results were compared with stress and deflection values computed by a few simplified methods. In their experiment, uniform loads were applied on the entire model.

For the experimental investigation of this report, a steel folded plate model was loaded along the top ridges such that line loadings were created instead of uniformly distributed loads. Furthermore in this experiment, the model was loaded symmetrically first and then unsymmetrically about the center of cross section. Accordingly the purposes of this thesis are summarized to be:

(1) To compare the longitudinal stress and ridge deflection values of simplified bending theory with the experimental results for both symmetrical and unsymmetrical line loading cases;

(2) To compare the same values computed by beam theory with the same experimental results for both loading cases.

The discussions of this thesis will be presented in order, the theory, analysis of a model, experimental study, comparison of analytical and experiment results, and conclusion.

PART II THEORY

A. Beam Theory

1. Assumptions:

   (1). The structure is monolithic.

   (2). The material is elastic and homogeneous.

   (3). There is no distortion of cross section in transverse direction.

   (4). Twisting caused by unsymmetrical loading is negligible. (Since the plates are very thin, twisting moment is negligible comparing with flexure moment.) ?

   (5). Longitudinal strain varies linearly across the depth of structure.

2. Method of Analysis

In beam theory, the structure is treated as a box beam. Accordingly the general flexure formula and simple beam deflection formula are used to determine the longitudinal stresses and vertical deflections of the structure. A step by step outline of this analysis is shown below:

   (1). Find an equivalent rectangular cross section for each bay and find the moment of inertia of the equivalent section.

   (2). Determine the moment caused by the external loads.

(3). Applying the flexure formula, compute the longitudinal stesses of the structure.

(4). Find the vertical deflections of the structure by using the beam deflection formula for uniform loads.

B. Simplified Bending Theory

1. Assumptions:

(1). The material is homogeneous and elastic.

(2). The structure is monolithic.

(3). Longitudinal stresses vary linearly over the depth of each plate.

(4). Individual plates possess negligible torsional resistance.

(5). End diaphragms are infinitely stiff in vertical plane, but flexible in horizontal plane.

2. Method of Analysis

The following procedure will be used for analyzing a steel plate model in Part III.

(1). Assuming temporarily that the joints are held against translation, take one inch center strip and treat this as a continuous beam. Determine the ridge moments at each joint and the ridge reactions caused by the moments.

(2). Resolve the ridge reactions into plate loads and compute plate moments and longitudinal stresses caused by the moments. This longitudinal stress is called free edge stress.

(3). Because of continuity of the structure, stresses in two adjacent plates should be the same at their common edges. To satisfy this condition the free edge stresses are distributed in the same manner as suggested by E. Traum (6, p. 118). (See page 17)

(4). Deflections of plates parallel and perpendicular to their own planes due to the longitudinal stresses are computed.

(5). Assuming all the plates can rotate, above solutions should be corrected. This correction is made by assuming an arbitrary rotation of each plate and repeating the steps (1), (2), and (3) for the slab moment caused by these rotations. The rotation angle is assumed to vary as a half sine wave along length of the structure.

(6). The actual rotations of each plate are now equated to the arbitrary rotations of the plates multiplied by proportionality factors X and Y. These equations are solved for the proportionality factors.

(7). The final solution is obtained by adding the results of no rotation to each of the rotation solution multiplied by its respective proportionality factor.

PART III ANALYSIS OF A STEEL PLATE MODEL

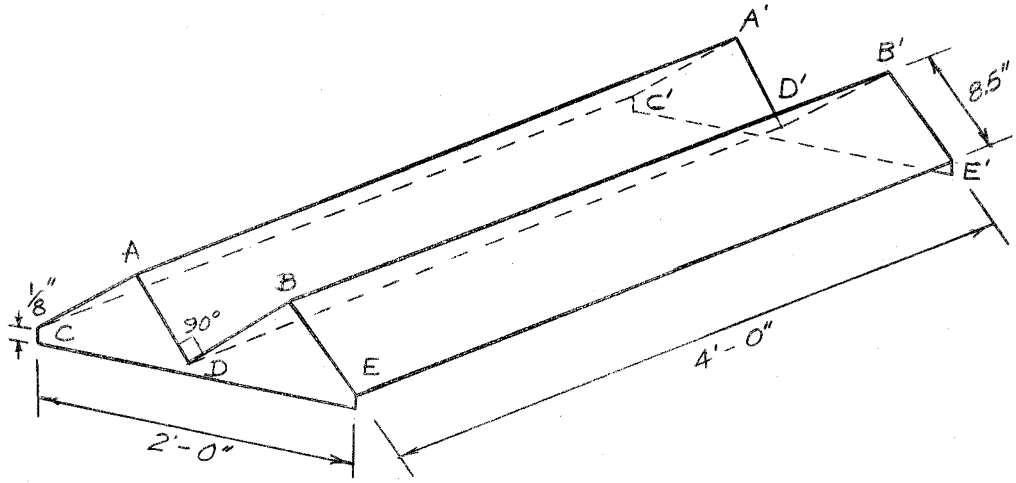
A steel plate model, four feet long and two feet wide as shown in Figure 1, will be analyzed by two methods; the first method is based on the beam theory and the second on the simplified bending theory. The analysis is composed of determining the longitudinal stresses and vertical deflections of points A, B, C, D and E (Figure 1.) at the half and quarter points of the model length.

The loading conditions for this analysis are:

(1) Total load of 1200 pounds uniformly distributed on ridges AA' and BB'.

(2) Total load of 600 pounds uniformly distributed along the ridge AA' only.

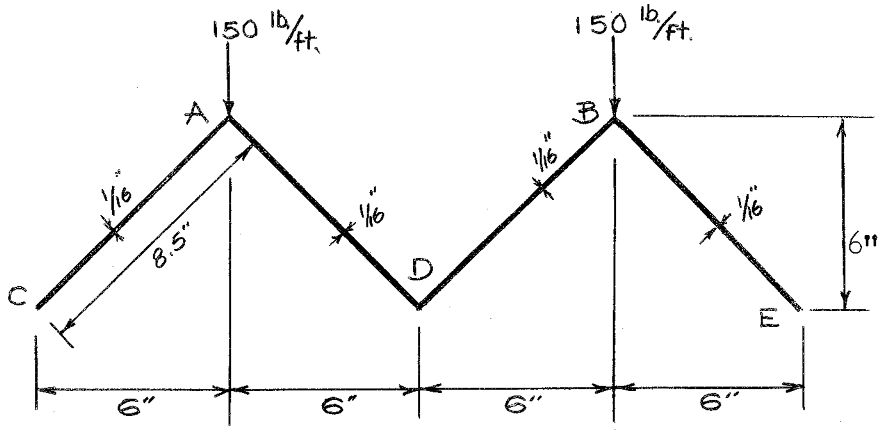
A. Analysis by Beam Theory



Two-Bay Folded Plate Model

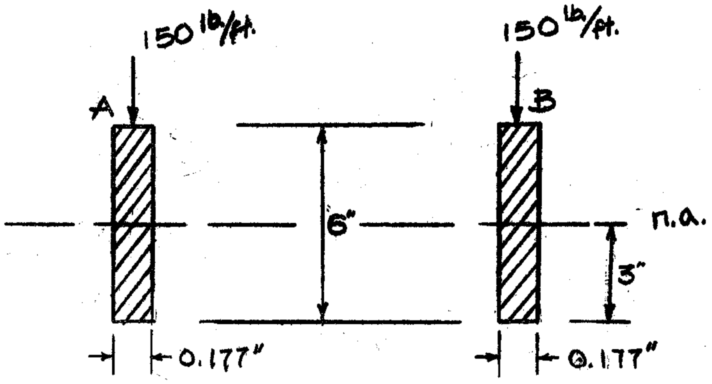
Figure 1

1. Symmetrical Loading Condition



Cross Section with Symmetrical Loading

Figure 2



Equivalent Rectangular Cross Section

Figure 3

Since Section A is identical to Section B, only Section A is analyzed.

Uniform line load = $\frac{\text{Total Load}}{L} = \frac{1200}{4(2)} = 150$ lbs per ft.

Width of equivalent section= $\frac{(1/16)\ 8.5(2)}{6} = 0.177$ in.

Neutral axis = 3 in. from top or bottom

$I_{xx} = \frac{b\ h^3}{12} = \frac{0.177(6)^3}{12} = 3.18\ \text{in.}^4$

$M_{(\frac{1}{2})} = \frac{wL^2}{8} = \frac{150\ (4)\ (4)\ (12)}{8} = 3600$ in.-lbs.

$M_{(\frac{1}{4})} = 2700$ in.-lbs.

Longitudinal stresses,

$S_{(\frac{1}{2})} = \frac{M\ c}{I} = \frac{3600\ (\pm 3)}{3.18} = \pm 3400$ psi.

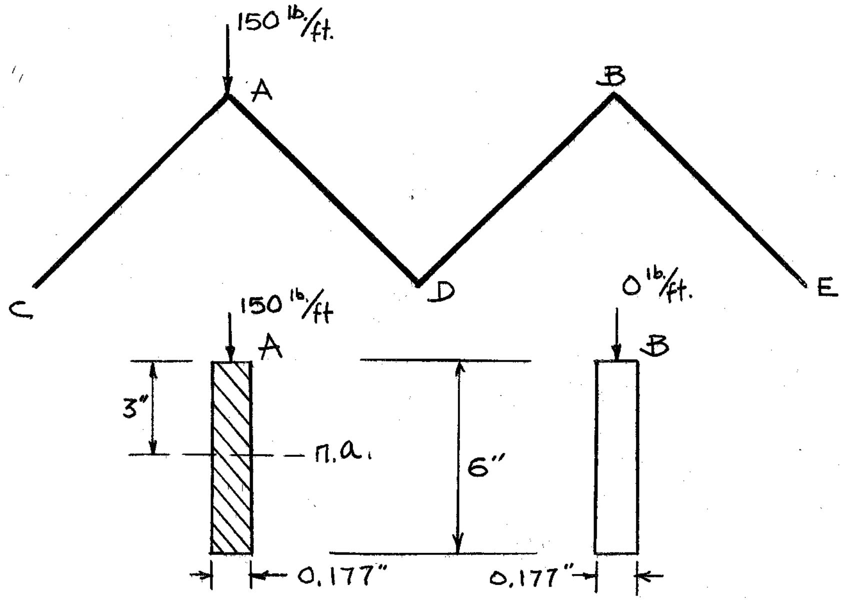
$S_{(\frac{1}{4})} = \frac{2700(\pm 3)}{3.18} = \pm 2550$ psi.

Vertical deflection,

$$V_{(\frac{1}{2})} = \frac{5\ WL^4}{384\ E\ I} = \frac{5(150)\ (16)^4(1728)}{384(30,000,000)(3.18)} = 0.00905 \text{ in. down}$$

$$V_{(\frac{1}{4})} = \frac{w\ x}{24\ E\ I}(L^3 - 2Lx^2 + x^3) = 0.00596 \text{ in. down}$$

2. Unsymmetrical Loading Condition



Unsymmetrical Loading and Equivalent Section

Figure 4

For Section A., the loading condition is exactly the same as the one in symmetrical case. Accordingly the results are:

$S_{(\frac{1}{2})} = \pm 3400$ psi.

$S_{(\frac{1}{4})} = \pm 2550$ psi.

$V_{(\frac{1}{2})} = 0.00905$ in. down

$V_{(\frac{1}{4})} = 0.00596$ in. down

For Section B., no vertical deflections and longitudinal stresses are introduced because there is no external load applied on the section.

B. Analysis by Simplified Bending Theory

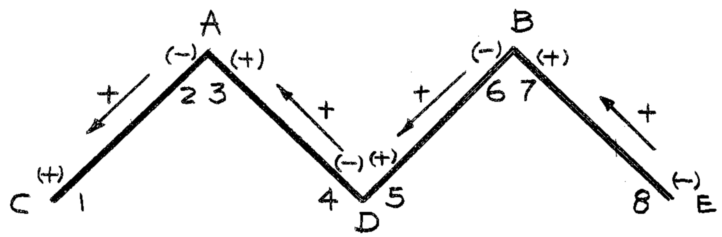
The methods of analyzing the structure by the simplified bending theory are almost identical in symmetrical and unsymmetrical loading conditions. In this analysis, therefore, only the unsymmetrical case is investigated in detail. The final results of symmetrical loading condition are tabulated at the end of Part V. (See tables 1 and 2).

(1) Taking one inch wide center strip of the cross section of the model, we treat this as an one-way slab on unyielding supports. Ridge moments do not exist since the external line load is acting on the ridge.

The external forces are resolved into the plate loads as shown in Figure 5.

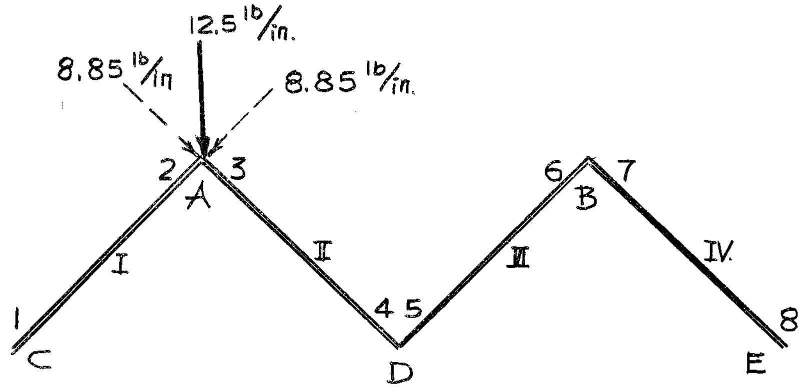
Total load on the model = 600 pounds

Sign convention for plate loads is shown below:



$w_{(1)}$ = Uniform line load = $\frac{600}{4(12)}$ = 12.5 lbs. per inch

(2)



One Inch Center Strip

Figure 5

$I_{(pl)}$ = Moment of Inertia of a plate

$$= \frac{0.00625(8.5)^3}{12} = 3.19 \text{ in.}^4$$

$Z_{(pl)}$ = Section modulus of a plate

$$= \frac{3.19}{4.25} = 0.75 \text{ in.}^3$$

$$M_{(\frac{1}{2})} \text{ of plate I or II} = \frac{w\,L^2}{8} = \frac{8.85(48)(48)}{8}$$

$$= 2545 \text{ in.-lbs.}$$

$$M_{(\frac{1}{4})} \text{ of plate I or II} = \frac{8.85(12)(36)}{2}$$

$$= 1910 \text{ in.-lbs.}$$

$S_{(f.e.)}$ = Free edge stress at center

$$= \frac{M}{\pm Z} = \frac{2545}{0.75} = \pm 3400 \text{ psi}$$

(3) Since the longitudinal stresses in two adjacent plates should be the same and there exist an unknown shearing force at each ridge, the free edge stresses are distributed in order to determine the final stresses. The distribution is performed in Figure 6.

(4) After final longitudinal stresses for the top and bottom of the plates are found, the deflections of plates in their own planes are determined by assuming the individual plate deflects just like a simply supported beam does. The deflection formula is derived as follows:

Beam length = L   Beam Depth = d

$$M = \frac{w\,L^2}{8} \qquad S = \frac{M\,d}{2\,I}$$

$$\text{Deflection at center} = D_{(\frac{1}{2})} = \frac{5\,wL^4}{384\,EI} = \frac{5\,L^2M(8)}{384\,EI}$$

$$= \frac{5\,ML^2}{48\,EI}$$

$$= \frac{5\,L^2}{48\,EI}\;\frac{2IS}{d}$$

Substituting the average of top and bottom stresses for S:

$$D_{(\frac{1}{2})} = \frac{5\,L^2}{48E}\;\frac{2}{d}\;\frac{(\,S_n - S_{n-1})}{2}$$

$$= \frac{L^2\;\;(\,S_n - S_{n-1})}{9.6\,d\,E}$$

Using the deflection formula presented on page 16 and substituting ($S_n - S_{n-1}$) values shown in Figure 6:

$$D_I = \frac{48(48)}{9.6(8.5)(3)(10)^7}\;(S_n - S_{n-1})$$

$$= 9.42\,(10)^{-7}\,(+6073.7) = +0.00572 \text{ in.}$$

Similarly,

D_{II} = -0.00435 in. D_{III} = +0.00206 in.

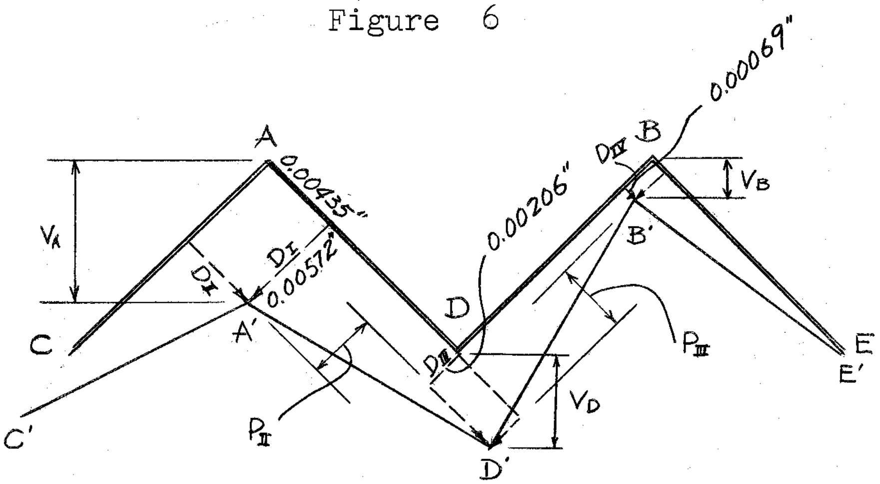
D_{IV} = -0.00069 in.

C	A	A	D	D	B	B	E	Ridges
0.53		0.53		0.53		0.53		Plate Area, in²
0.5		0.5		0.5		0.5		Distribution Factor
-0.5		-0.5		-0.5		-0.5		Carry Over Factor
+3400	-3400	-3400	+3400	0	0	0	0	Free Edge Stress
			-1700	+1700				
		+850			-850			
	+425	-425			+425	-425		
-212.5			+212.5	-212.5			+212.5	
			-212.5	+212.5				
		+106.25			-106.25			
	+53.12	-53.12			+53.12	-53.12		
-26.6			+26.6	-26.6			+26.6	
			-26.6	+26.6				
		+13.3			-13.3			
-3.37	+6.65	-6.65			+6.65	-6.65	+3.37	
+3157.5	-2916.2		+1700		-484.8		+242.5	Final Stress, Psi

1 2	3 4	5 6	7 8	Free edge
+6073.7	-4616.2	+2184.8	-727.3	$S_n - S_{n-1}$

Free Edge Stress Distrubution

Figure 6



Deflected Structure

Figure 7

Vertical deflections of ridges are easily found from Figure 7:

$$V_{(A)} = \frac{1}{1.414}(0.00572 + 0.00435)$$

$$= 0.00756 \text{ in. down}$$

$$V_{(D)} = 0.706(0.00435 + 0.00206)$$

$$= 0.00453 \text{ in. down}$$

$$V_{(B)} = 0.706(0.00206 + 0.00069)$$

$$= 0.00194 \text{ in. down}$$

$$V_{(C)} = 0.706(0.00572) = 0.00404 \text{ in. down}$$

$$V_{(E)} = 0.706(0.00069) = 0.00049 \text{ in. down}$$

From Figure 7 also, perpendicular deflections of plates are found as shown below:

$$P_{(II)} = 0.00572 - 0.00206 = 0.00366 \text{ in.}$$

$$P_{(III)} = 0.00206 - 0.00069 = 0.00137 \text{ in.}$$

(5) The longitudinal stresses and vertical deflections computed on previous pages should now be corrected for the stresses resulting from the rotation of plates II and III. The computation for the rotation of plate II is illustrated in Figure 8.

First the arbitrary rotation 'r' of plate II is shown in Figure 8(a). In Figure 8(b), the Fixed End Moment is determined by assuming the ridges A and D are fixed and assuming $\frac{EI}{d}$ equals 1000 ft-lbs.

Accordingly:

$$F.E.M. = \frac{6\ E\ I\ P}{d^2} \qquad 'r' = \frac{P}{d} \text{ and } \frac{E\ I\ 'r'}{d} = 1000''\#$$

$$= \frac{6\ EI}{d}('r')$$

$$= 6000 \text{ in.-lbs.}$$

The final moments in Figure 8(b) are applied to the plates and reactions are computed in (c). In Figure 8(d), the known reactions act on the structure and create plate loads. The longitudinal moments and the free edge stresses are found below:

$$M_I = M_{III} = \frac{w\ L^2}{8} = \frac{625(4)(4)(12)}{8} = 15{,}000 \text{ in.-lbs.}$$

$$M_{II} = M_{IV} = 375(2)(12) = 9{,}000 \text{ in.-lbs.}$$

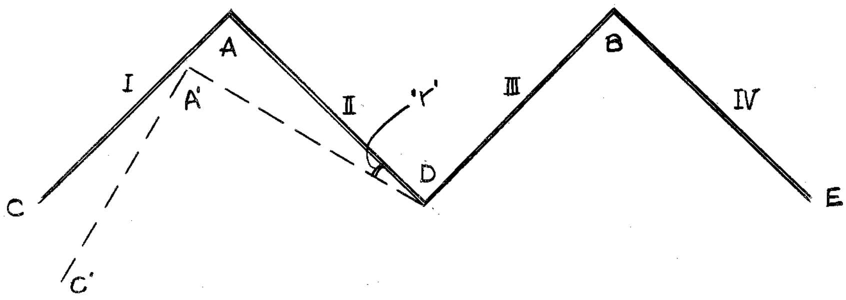
$$S(f.e.) = \frac{M}{\pm Z} = \pm 20{,}000 \text{ psi for plates I and III.}$$

$$= \pm 12{,}000 \text{ psi for plates II and IV.}$$

The free edge stresses are distributed in Figure 8(e) by the same method illustrated in Figure 6.

(6) Let $'R'_{II}$ and $'R'_{III}$ be the actual rotations of plates II and III which are easily computed from P_{II} and P_{III}.

(a)

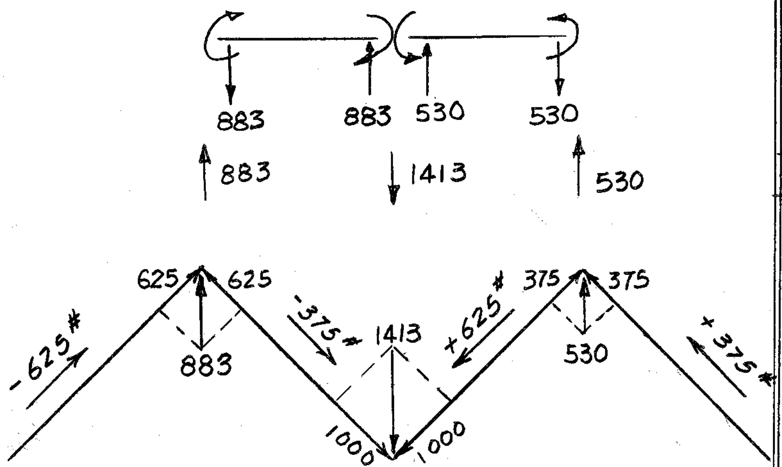


(b)

Distri. Factor	0	1	0.5	0.5	1	0
F. E. M.		-6000	-6000			
			+3000	+3000		
		+1500			+1500	
Final Moment		-4500	-3000	+3000	+1500	

(c)

(d)



(e)

Distrib. Factor	0.5		0.5		0.5		0.5	
Carry Over Fact.	-0.5		-0.5		-0.5		-0.5	
F. E. S. KSI	-20	+20	-12	+12	+20	-20	+12	-12
			-2	+4	-4	+2		
		-17	+17	- 8.5	-7.5	+15	-15	+7.5
	+8.5		-0.25	+0.5	+0.5	+0.25		
		-0.125	+0.125			-0.125	+0.125	-0.065
	+0.065							
Final Stress, psi	-11,425	+2875		+8,000		-2875		-12,563
$S_n - S_{n-1}$, Psi	-14,310		-5125		+10,875		+9,688	

Stresses due to the Arbitrary Rotation
of One inch Strip of Plate II

Figure 8

Assuming X and Y are constants:

$$'R'_{II} = X\ 'r' \,, \qquad (1)$$

$$'R'_{III} = Y\ 'r' \,. \qquad (2)$$

Using numbers:

$$'R'_{II} = \frac{P_{II}}{d} = \frac{0.00366''}{8.5''} = 0.000432$$

$$'R'_{III} = \frac{0.00137''}{8.5''} = 0.000161$$

From $\frac{E\ I\ 'r'}{d} = 1000$ in.-lbs. on page 19

$$'r' = \frac{d(1000)}{E\ I} = \frac{8.5(1000)}{30(2.45)(100)} = 1.16$$

Substituting values for $'R'_{II}$, $'R'_{III}$ and 'r' into Equation (1) and Equation (2):

$0.000432 = X\ (1.16)$,

$0.000161 = Y\ (1.16)$.

Solving for X and Y:

$X = 0.000372$, $\qquad Y = 0.0001$.

Since X and Y values are very small, the effect of the plate rotation on the model is negligible. In this analysis, therefore, the corrections of stress and deflection are not required. The final computed longitudinal stresses and vertical deflections at half and quarter points are summarized in Table 1 and Table 2 for both symmetrical and unsymmetrical cases.

PART IV EXPERIMENTAL STUDY

A. Description of Model

As shown in Figure 1 and Figure 9, the model was 4 feet long and 2 feet wide. The cross section was of a double triangular shape (Figure 10) and it was composed of two bays CAD and DBE, while each bay was composed of two plates. The material of the model was cold rolled steel plate (A 366- 58 T). The thickness of plate was 1/16 inch everywhere except for two 1/8 inch thick end diaphragms.

In constructing the model, two triangular bays CAD and DBE were made separately by bending a half of the model section 90 degrees along the ridges AA' and BB' (Figure 1) and then two bays were welded together along one edge of each bay. Finally the model was painted white.

To measure the longitudinal and transverse stresses, 4 SR-4 rosettes and 4 SR-4 type A-7 strain gages were attached to the surface of bay CAD as shown in Figure 11(a). Before these gages had been mounted on the specified spots (Figure 9(a) and (b)), the paint was carefully removed and plate surface was cleaned.

Six Ames dial gages were installed at different places for different loading conditions to measure the vertical deformations of inside ridges and vertical and horizontal deflections of two outer edges at half and quarter points of the model length. On the test data in the Appendix, the exact locations of the Ames dial gages for each test will be found.

B. Test Procedures

It was assumed under the symmetrical load that both sides (bay CAD and DBE) of the model would behave in the same way, and so the test data obtained from one bay was substituted for the other half of the section. In the unsymmetrical case, however, it was necessary to load the structure twice to get the data for both sides since the strain gages had been mounted on the surface of bay CAD only.

For this experimental study, two sets of tests were conducted in identical manner. Each set of tests was composed of three different loading conditions which are listed below:

(1) Symmetrical line loading along ridges AA' and BB'

(2) Unsymmetrical line loading along the ridge AA' only

(3) Unsymmetrical line loading along the ridge BB' only

The first loading condition was obtained by placing 23 inch long copper plates weighing 5.5 pounds each symmetrically between the ridges AA' and BB'. Second and third loading conditions were satisfied by placing the same copper weights symmetrically between just one ridge of the model and a 4 by 6 inch wood beam supported from outside. Figure 12(a) and (b) show the symmetrical and unsymmetrical loads respectively.

The maximum loads applied for this study were 1122 pounds for the first set and 1496 pounds for the second. The complete experimental data are tabulated in the Appendix A.

C. Results

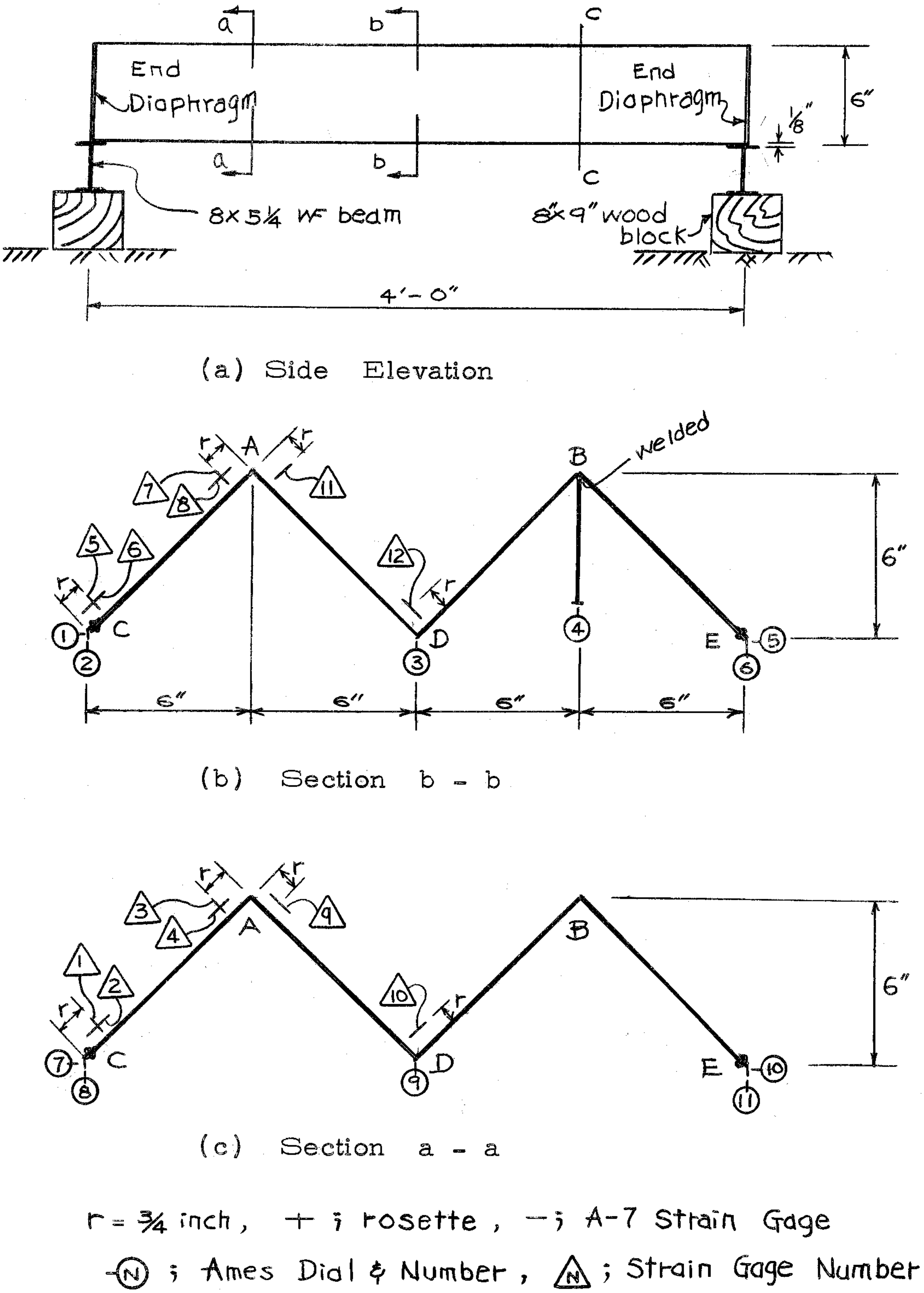
After the longitudinal and transverse strains were measured by SR-4 rosettes and A-7 strain gages, these strains were converted to longitudinal stresses and transverse moments by utilizing Hook's law and the standard flexure formula. Since the dial gauges were able to read as accurately as 1/2,000 inch of deflections, no conversion was needed.

The stress and deflection values obtained from the tests were plotted against loads. Because the stress (or deformation) should increase linearly, straight lines were drawn to represent the plotted values.

The results of the first set of tests did not agree in the straight-line relationship between the stress (or deflection) and total load. The reason for this disagreement was assumed to be that two bottom edges resting on supports were not straight, and this caused an initial distortion of structure.

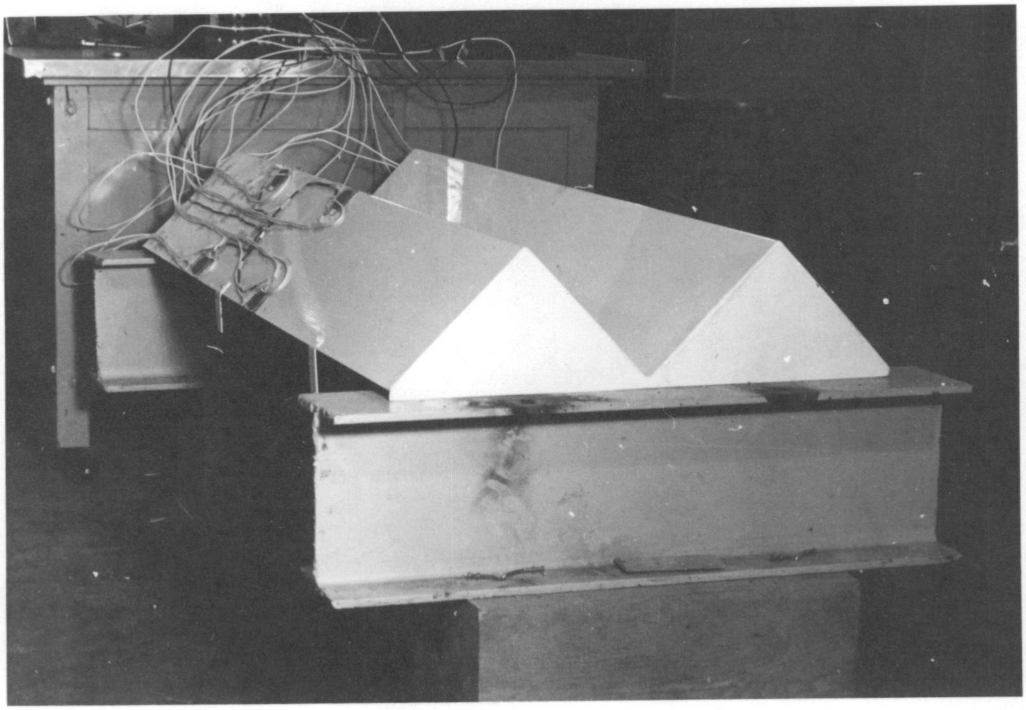
After the irregular bottom edges were straightened, the second set of tests were conducted, and the results satisfied the straight-line relationship. The results of second set, therefore, represented whole experimental values, and used for comparing with theoretical values.

Before the experimental values were compared, one correction was made for longitudinal stress. As shown in Figure 9, the longitudinal stresses were measured at 3/4 inch from the top or bottom of each plate, but the theoretical values were computed for extreme fibers. This correction was done by increasing the experimental values in proportion to the distance from neutral axis to extreme fiber and to the locations of the strain gages. The final experimental values of longitudinal stress and deflection are listed in Part V for the comparison with theoretical results.



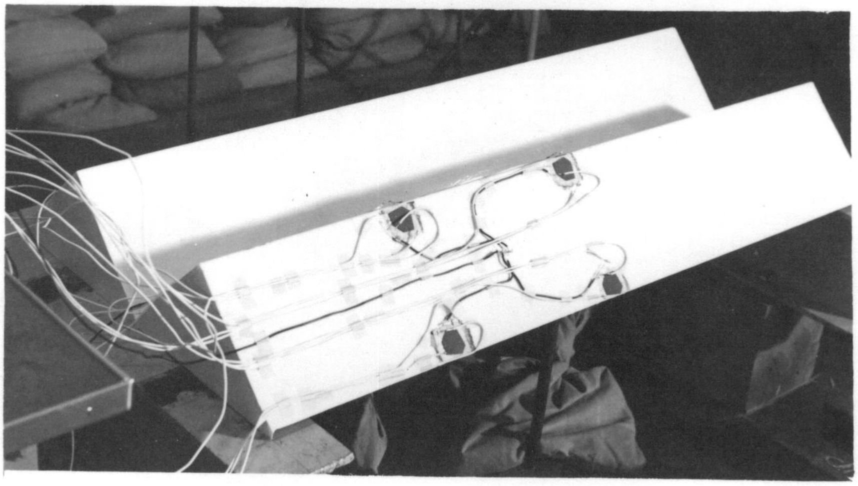
General View of Model and Locations of Gages

Figure 9

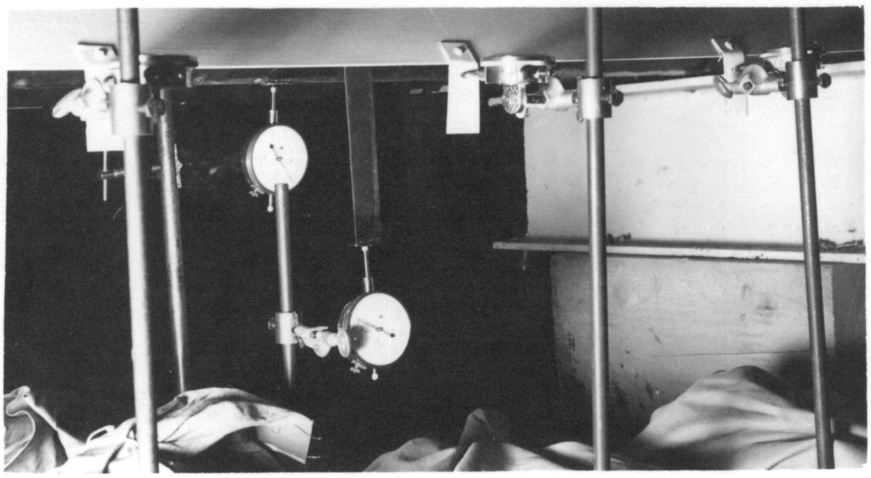


Two-bay Folded Plate Model without Load

Figure 10



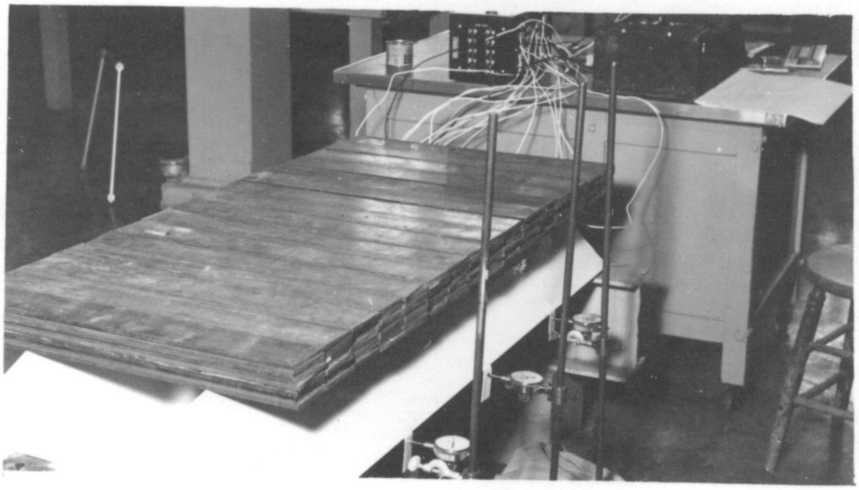
(a) SR-4 Strain Gages



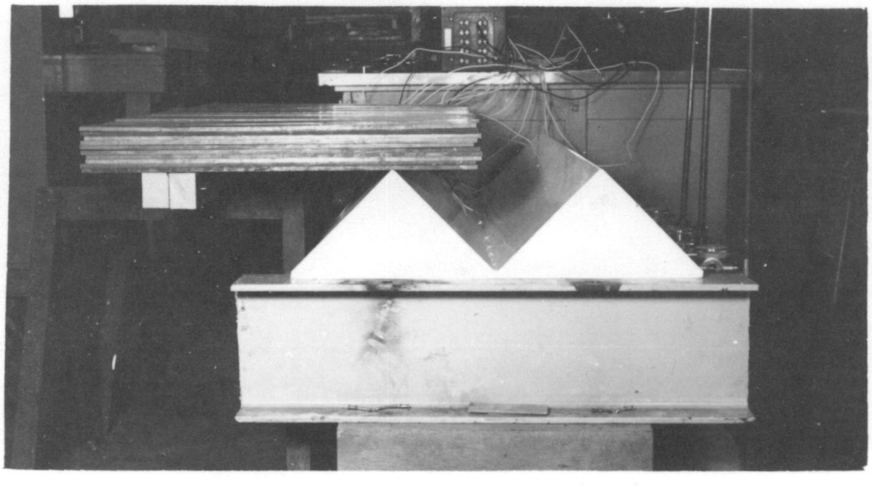
(b) Ames Dials

SR-4 Strain Gages and Ames Dials

Figure 11



(a) Symmetrically Loaded Model



(b) Unsymmetrically Loaded Model

Loaded Model

Figure 12

PART V COMPARISON OF EXPERIMENTAL RESULTS WITH ANALYTICAL INVESTIGATION

The final values of longitudinal stress and vertical deflection obtained by the three different methods (Beam Theory, Simplified Bending and Experimental Study) are summarized in Table 1 and Table 2. Table 1 lists all of the longitudinal stress values for the top and bottom fibers of each plate at half and quarter points of the structure length under both symmetrical and unsymmetrical loads. Table 2, on the other hand, lists the corresponding values of vertical deflections.

For the purpose of comparison, three curves representing the values of beam theory, simplified bending theory and experimental study are plotted using the same coordinates. Figure 13 shows the longitudinal stress variation along the transverse direction under the symmetrical load. The same variation but for the unsymmetrical load is shown in Figure 14. In Figures 15 and 16, the vertical stresses are plotted for the ridge points A, B, C, D, and E for the symmetrical and unsymmetrical loads respectively.

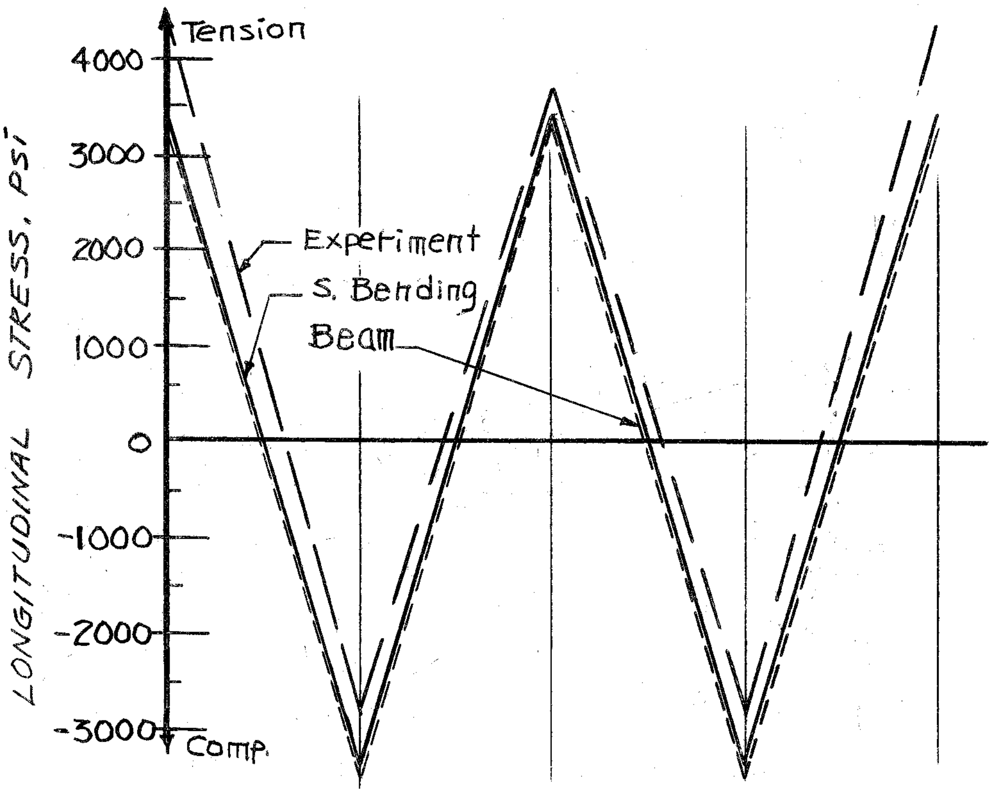
In plotting the experimental results, the average longitudinal stress values of two adjacent plates are used to represent the common edge stresses when these two values are not the same. In addition, it was necessary to compute the stresses and deflections of unmeasured points by using the known values and the proportionality between the half and quarter point values.

A comparison of the analytical results obtained by the simplified bending theory with the experimental values indicates a good agreement everywhere in the cross section of the beam except for the two outer edges. The percentage difference of the maximum longitudinal stresses of interior ridges is 4.4% in symmetrical loading and 18.2% in unsymmetrical loading. For the vertical deflections, the percentage difference is 4.45% for the symmetrical case and 11.1% for the unsymmetrical condition.

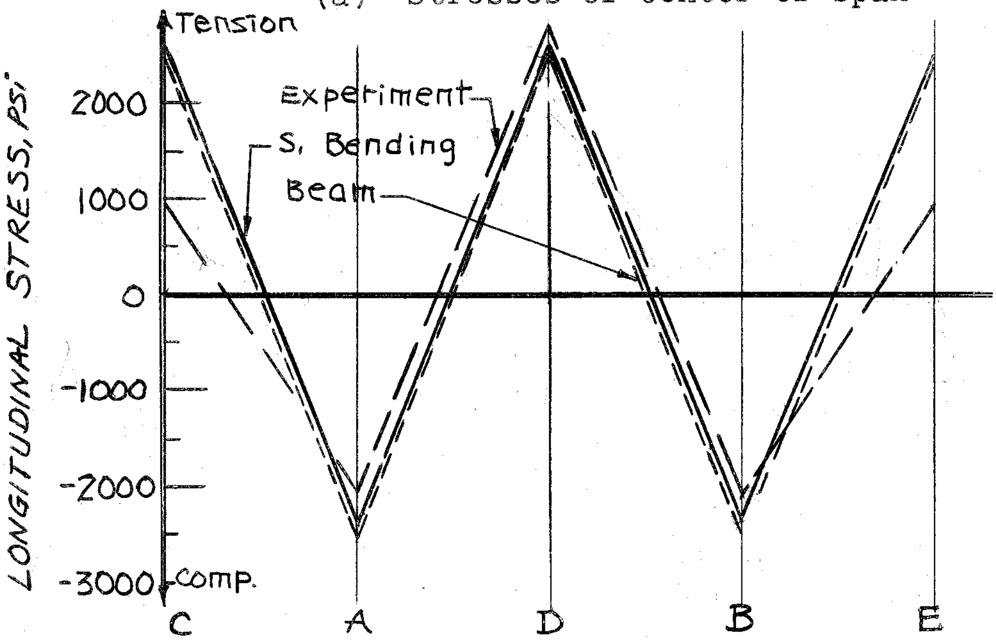
Under the symmetrical loading, the stresses and deflections of the interior ridges computed by the beam theory agree fairly well with the experimental results. However in unsymmetrical loading the beam theory yields somewhat high discrepancies. The percentage difference of the maximum longitudinal stress for the symmetrical load is 4.5%, but 52.5% for the unsymmetrical load. The same differences for the vertical deflections are 4.75% and 61.3%

As briefly mentioned above, the most noticeable disagreement appears at the outer edges. At these points, the maximum percent difference of longitudinal stress between the simplified bending theory and experimental data is 68%. This percent difference is higher in the case of deflection and even the directions of deformations of the two free edges are opposite. Both the simplified bending and beam theories predicted that the two outer free edges would deflect downward, but in the experiment, excessive upward deflections were measured.

This excessive upward deformations of the two outer edges are assumed to be resulted from the shear differences existing between two adjacent sections of the plates with a free end. It is not attempted here to correct or modify this disagreement by formulating a new analysis method for the free edges since this is not the purpose of this thesis.



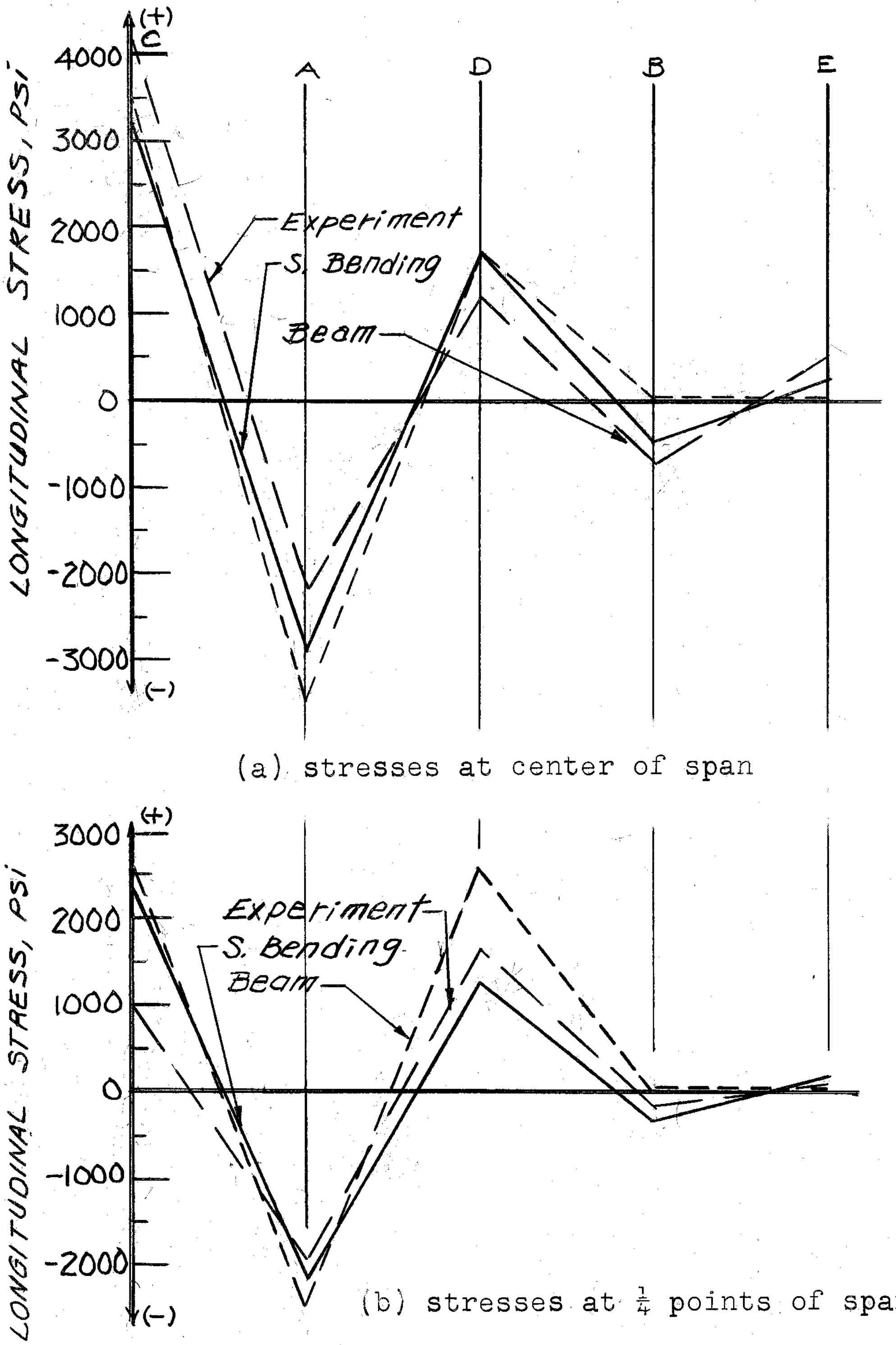
(a) Stresses of center of span



(b) Stresses at $\frac{1}{4}$ points of span

Longitudinal Stresses Under Symmetrical Loads

Figure 13

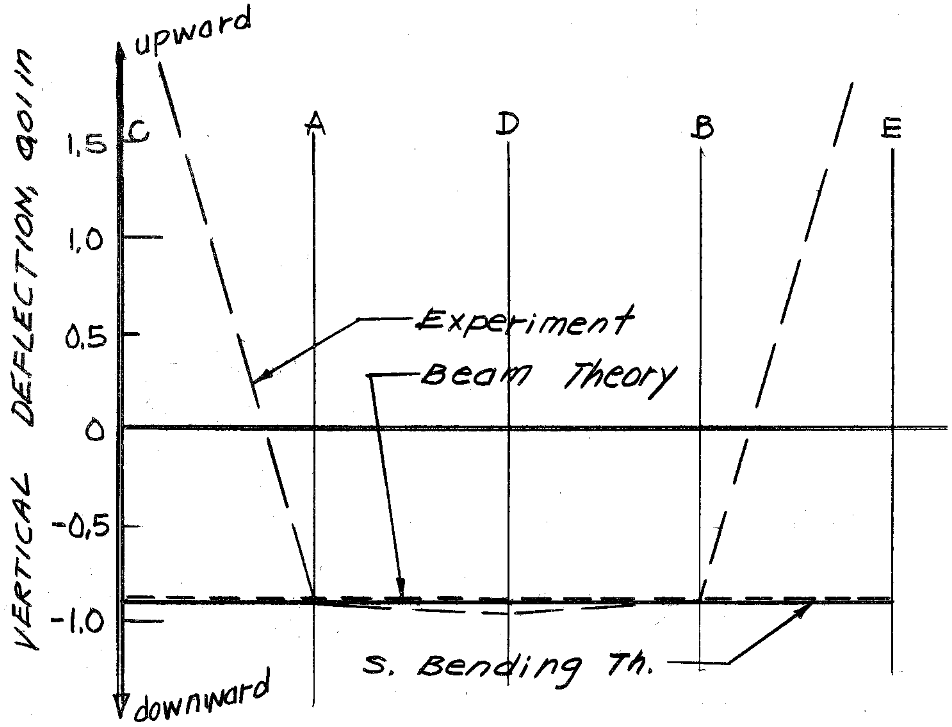


(a) stresses at center of span

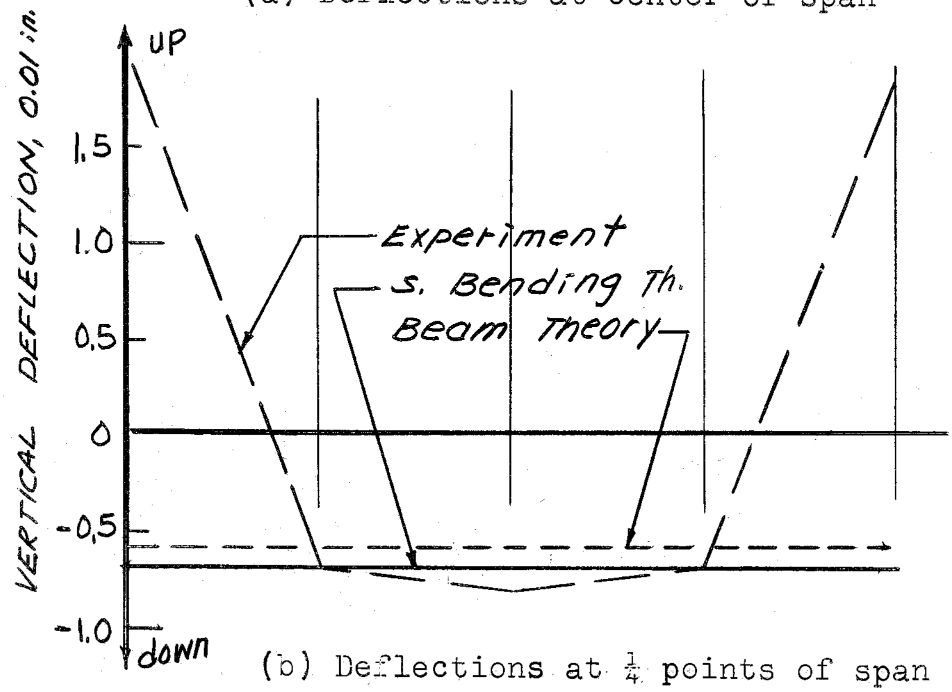
(b) stresses at $\frac{1}{4}$ points of span

Longitudinal Stresses Under Unsymmetrical Load

Figure 14



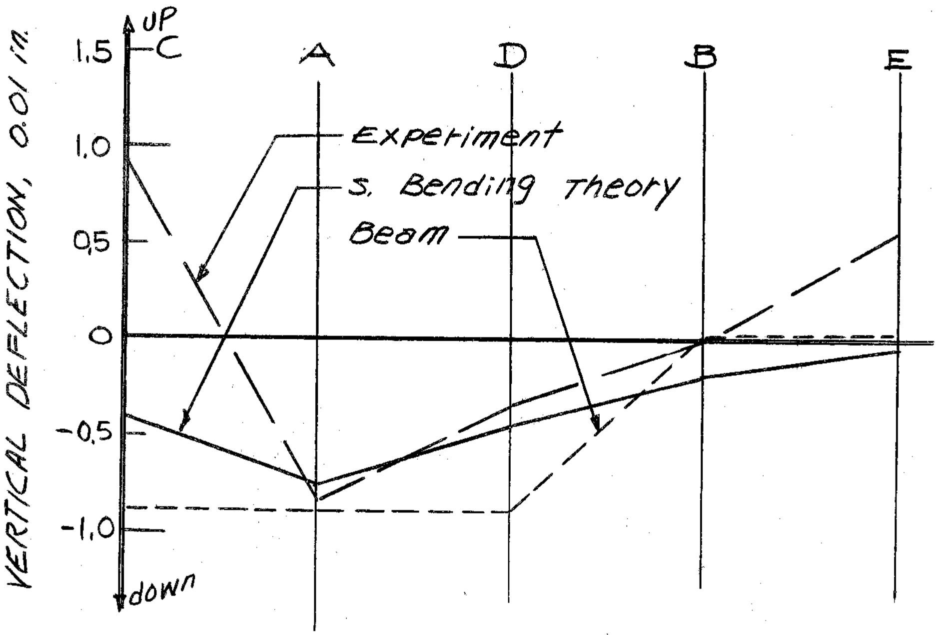
(a) Deflections at center of span



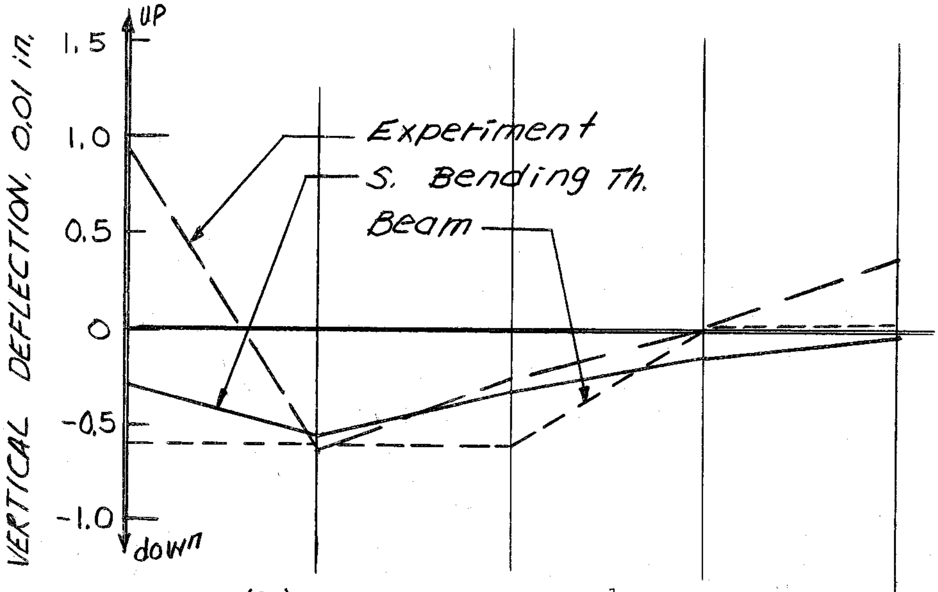
(b) Deflections at ¼ points of span

Vertical Deflections Under Symmetrical Load

Figure 15



(a) Deflections at center of span



(b) Deflections at $\frac{1}{4}$ points of span

Vertical Deflections Under Unsymmetrical Load

Figure 16

TABLE 1

Plate	Point	SYMMETRICAL LOAD Longitudinal Stress at ½ pt.	SYMMETRICAL LOAD Longitudinal Stress at ¼ pt.	UNSYMMETRICAL LOAD Longitudinal Stress at ½ pt.	UNSYMMETRICAL LOAD Longitudinal Stress at ¼ pt.	Result Type
I	1	+3400	+2550	+3400	+2550	Beam Theory
		+3400	+2545	+3157.5	+2375	S. Bending
		+4360	+ 950	+4130	+ 995	Experiment
	2	-3400	-2550	-3400	-2550	Beam Theory
		-3400	-2545	-2916	-2183	S. Bending
		-3340	-2160	-2490	-1940	Experiment
II	3	-3400	-2550	-3400	-2550	Beam Theory
		-3400	-2545	-2916	-2183	S. Bending
		-2310	*-----	-2100	*-----	Experiment
	4	+3400	+2550	+3400	+2550	Beam Theory
		+3400	+2545	+1700	+1272	S. Bending
		*-----	+2670	*-----	+ 910	Experiment
III	5	+3400	+2550	0.000	0.000	Beam Theory
		+3400	+2545	+1700	+1272	S. Bending
		*-----	*-----	*-----	+2330	
	6	-3400	-2550	0.000	0.000	Beam Theory
		-3400	-2545	- 484.8	- 363.0	S. Bending
		*-----	*-----	-1030.0	*-----	Experiment

Continued on next page . . . .

TABLE 1 (Continued)

Plate	Point	SYMMETRICAL LOAD Longitudinal Stress at ½ pt.	SYMMETRICAL LOAD Longitudinal Stress at ¼ pt.	UNSYMMETRICAL LOAD Longitudinal Stress at ½ pt.	UNSYMMETRICAL LOAD Longitudinal Stress at ¼ pt.	Result Type
IV	7	-3400	-2550	0.000	0.000	Beam Theory
		-3400	-2545	- 484.8	- 363.0	S. Bending
		*-----	*-----	- 376.0	- 182.0	Experiment
	8	+3400	+2550	0.000	0.000	Beam Theory
		+3400	+2545	+ 242.5	+ 182.0	S. Blending
		*-----	*-----	+ 515.0	+ 119.0	Experiment

Results of Longitudinal Stress in psi

* These values were not measured in experiment.

TABLE 2

Ridge	SYMMETRICAL LOADING — Vertical Deflect. at ½ pt.	SYMMETRICAL LOADING — Vertical Deflect. at ¼ pt.	UNSYMMETRICAL LOADING — Vertical Deflect. at ½ pt.	UNSYMMETRICAL LOADING — Vertical Deflect. at ¼ pt.	Result Type
A	-0.00905	-0.00596	-0.00905	-0.00596	Beam Theory
	-0.0091	-0.0069	-0.00756	-0.00566	S. Bending
	*-------	*-------	-0.0085	*-------	Experiment
B	-0.00905	-0.00596	0.0000	0.0000	Beam Theory
	-0.0091	-0.0069	-0.00194	-0.00185	S. Bending
	-0.0090	*-------	-0.0001	*-------	Experiment
C	-0.00905	-0.00596	-0.00905	-0.00596	Beam Theory
	-0.0091	-0.0069	-0.00404	-0.00302	S. Bending
	*-------	*-------	*-------	+0.0095	Experiment
D	-0.00905	-0.00596	-0.00905	-0.00596	Beam Theory
	-0.0091	-0.0069	-0.00453	-0.00339	S. Bending
	-0.0095	-0.0080	-0.0035	*-------	Experiment
E	-0.00905	-0.00590	-0.000	-0.000	Beam Theory
	-0.0091	-0.0069	-0.0004	-0.00039	S. Bending
	+0.026	*-------	+0.00505	*-------	Experiment

Results of Vertical Deflections in inch

- deflection down
+ deflection upward
* deflection not measured

Part VI CONCLUSIONS

On the basis of analytical and experimental investigations carried out in this report and the proceeding discussions of comparing results, following conclusions are made:

1. The longitudinal stresses and vertical deflections computed by the simplified bending theory will in general predict the actual behaviors of the folded plate structures under either symmetrical and unsymmetrical loads. Accordingly, it is safe to design these types of structures by simplified bending theory within comparable span-depth, span-bay and span-thickness ratios to the model tested.
2. Analysis by the beam theory will agree with the actual stresses and deformations resulted from the symmetrical load, but will yield values seriously in error if the structure is unsymmetrically loaded. It is not recommended, therefore, to use the beam theory in actual design of the folded plates unless symmetrical loading only is assured.

3. It appears to be unsafe to utilize either simplified bending or beam theory for analyzing and designing the outer plates with a free edge.

It is hoped that further study and research is to be continued along the line of the free edge investigations so that one simple method of analysis will satisfy not only the interior plates, but also exterior free plates.

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