

AN ABSTRACT OF THE THESIS OF

Hee Jung Kang for the degree of Doctor of Philosophy in
Industrial and Manufacturing Engineering presented on
May 2, 1989.

Title: Multiobjective Decision Making in Industrial
Energy and Environmental Planning

Abstract approved: **Redacted for privacy**

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Multiobjective Decision Making (MODM) has been suggested for the solution of complicated decision problems. Decision analysis in numerous areas, including industrial energy and environmental planning, necessarily requires consideration of multiple conflicting objectives. MODM has been successfully applied to a number of these problems of this type. Moreover, it has the ability to deal with both quantitative and qualitative factors, each which involve different units of measurement.

The objective of this study is to introduce a MODM process for energy and environmental planning problems in forest products manufacturing industries. Throughout the analytic process, the posteriori articulation of decision maker's (DM) preferences is assumed. This

mandates development of two procedures: (1) the generation of nondominated solutions and (2) evaluation of the solutions by DM judgement to determine the final, best-compromised solution.

For the first procedure, a Multiobjective Linear Programming (MOLP) model is introduced, formulated as a prototype example through the examination of fuel-mix options. Three objectives are observed in the MOLP model, including: (1) total energy costs, (2) environmental impacts, and (3) business and performance risks. In order to overcome the complexities caused by the use of different qualitative units of measurement, factors (2) and (3) are quantified in numerical values. The constraint method is then applied for the generation of nondominated solutions. As the second procedure, an evaluation procedure which includes multiple screening methods is proposed for ease of problem application for consideration of a large number of alternatives. This methodology is based on rating and pairwise comparison methods. Special emphasis is placed on the achievement of a higher DM level of confidence when the final solution is selected. The methodology can be divided into two regions: (1) step-by-step reduction of alternatives, and (2) judgmental options for upgrading DM confidence. This methodology provides a useful and flexible tool for problems as characterized above and for large-scale problems.

Multiobjective Decision Making in Industrial
Energy and Environmental Planning

by

Hee Jung Kang

A THESIS

submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Doctor of Philosophy

Completed May 2, 1989

Commencement June 1989

APPROVED:

Redacted for privacy

Professor of Industrial and Manufacturing Engineering
in charge of major

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Date thesis is presented May 2, 1989

Typed by B. McMechan for Hee Jung Kang

Acknowledgements

I would like to express my appreciation to my major professor, Dr. Thomas M. West, for his advice and assistance throughout the course of preparing this thesis. My special regards are also extended to several of my professors, including Dale McFarlane, Robert Safford, David Thomas, and Robert Wilson, among others I have had the pleasure of working with at Oregon State University.

I would also like to thank my lovely son, Sanggyu, for his well-being throughout the years and my appreciation to my wife, Mee Ran, for her unfailing patience and assistance. I and Mee Ran extend our sincere gratitude to our parents for their encouragement. Finally, my special thanks to Bill McMechan for his editorial and typing assistance during the preparation of this study.

Table of Contents

	<u>Page</u>
1. INTRODUCTION	1
1.1 Background	1
1.2 MOP and Energy and Environmental Planning .	4
1.3 Objectives of the Study	7
1.4 Organization of the Study	9
2. MOLP MODEL AND SOLUTION SYSTEMS	10
2.1 MOLP Backgrounds	10
2.1.1 Mathematical Presentation	10
2.1.2 MOLP Assumptions	11
2.2 Nondominated Solutions	12
2.2.1 Definition	12
2.2.2 Kuhn-Tucker Conditions for Nondominated Solutions	16
2.3 Overview of Generating Methods	17
2.4 Constraint Solution Method	20
2.4.1 Solution Procedure	20
2.4.2 Motivation for Use of the Constraint Method	24
3. MOLP MODELING FOR INDUSTRIAL ENERGY AND ENVIRONMENTAL PLANNING	26
3.1 Introduction	26
3.2 Industrial Background	28
3.3 Formulation of the Problem	31
3.3.1 Observed Objectives	31
3.3.2 MOLP Model	33
3.3.3 Assumptions and Limitations of the Model	38
3.3.4 Case Studies	39
3.4 Development of Input Data	39
3.5 Results	40
3.5.1 Payoff Table	41
3.5.2 Results of Constraint Problem	43
3.5.3 Listed of Generated Nondominated Solutions	45
3.6 Conclusion	51
4. CLASSICAL EVALUATION METHODS	53
4.1 Categorization of Methods	55
4.2 Simple Descriptive Methods	56
4.2.1 Satisfying Level	56
4.2.2 Lexicographic Ordering	57
4.3 Value Function Method	59
4.3.1 Theoretical Background	60
4.3.2 Development of Objective Weight	61

Table of Contents (continued)

	<u>Page</u>
4.3.3 Development of Value Functions	64
4.3.4 Results	69
4.4 ELECTRE Method	73
4.5 Conclusion	75
5. ANALYTIC HIERARCHY PROCESS	77
5.1 AHP Procedures	78
5.1.1 Decomposition of the Problem	78
5.1.2 Pairwise Comparison	79
5.1.3 Estimation of the Relative Weight ..	81
5.1.4 Aggregation of the Relative Weight ..	84
5.2 Extensions of AHP	85
5.3 Methodological Comparisons	86
6. PROPOSED METHODOLOGY: MULTIPLE SCREENING METHOD	91
6.1 Description of the Methodology	93
6.1.1 Development of the Objective Weight	93
6.1.2 Determination of Rating Intensity ..	93
6.1.3 Reduction of Alternatives	95
6.1.4 Pairwise Comparison of Rating Intensity	96
6.1.5 Sensitivity Analysis	97
6.1.6 Pairwise Comparison of Solution Outcomes	98
6.2 Industrial Energy and Environmental Planning Problem Applications	99
6.2.1 Development of the Objective Weights	99
6.2.2 Rating Intensity	102
6.2.3 Pairwise Comparison of Rating Intensities	103
6.2.4 Sensitivity Analysis	108
6.2.5 Pairwise Comparison of Solution Outcomes	111
6.2.6 Final Solution Alternative Implications	116
6.3 Potential Benefits and Problems of Evaluation Procedures	118
6.4 Conclusion	120

Table of Contents (continued)

	<u>Page</u>
7. CONCLUSIONS AND RECOMMENDATIONS	122
7.1 Conclusions	122
7.2 Recommendations for Further Research	125
7.2.1 MOLP Model	125
7.2.2 Evaluation Method	126
 BIBLIOGRAPHY	 128
 Appendix A: LP Formulation	 137
Appendix B: Example of LP Solution: Case A, Minimization of Total Energy Costs	 141
Appendix C: Example of Constraint Problem and Solution: Case A, Minimization of Total Energy Costs	 146
Appendix D: Comparison of Composite Weights	.153

List of Figures

<u>Figure</u>		<u>Page</u>
2.1	Nondominated solutions	14
2.2	Interpretation of the constraint method ..	21
4.1	Value functions	66
5.1	AHP hierarchical diagram	80
5.2	MOV hierarchical diagram	88
6.1	Sequence of Steps for multiple screening method	94
6.2	Hierarchical diagram with rating intensities	104
6.3	Sensitivity analysis with rating intensity: Case 2	112
6.4	Sensitivity analysis with rating intensity: Case 3	113
6.5	Sensitivity analysis for the top-ranked alternatives: Case 3	117

List of Tables

<u>Table</u>		<u>Page</u>
2.1	An example of nondominated and dominated solutions	15
2.2	Payoff table for the constraint method ...	23
3.1	Industrial structures and energy consumption	28
3.2	Energy consumption by end use (1983)	29
3.3	Typical particle emission levels of wood treatment industries	30
3.4	Pollutant emissions level of energy sources	31
3.5	Energy price and facility conversion cost	40
3.6	Subjective numerical values	40
3.7	Energy Intensity	41
3.8	Individual LP solutions, Cases A and B ...	42
3.9	Payoff table, Case A	43
3.10	Payoff table, Case B	43
3.11	Value of ξ_k , Case A	44
3.12	Value of ξ_k , Case B	45
3.13	Solutions of constraint problem Case A ...	46
3.14	Solutions of constraint problem Case B ...	47
3.15	Generated nondominated solutions	48
3.16	Ranges of solution outcomes	50
4.1	Objective weights by ranking method	62
4.2	Rating intensities	67
4.3	Scaling values to solution outcomes	68

List of Tables (continued)

<u>Table</u>		<u>Page</u>
4.4	Scaling values to rating intensities	68
4.5	Objective weights	70
4.6	Results of value function method	71
4.7	Results of value function method (rating intensities)	72
5.1	Scale values for pairwise comparisons	82
6.1	Pairwise Comparison Matrix for Objectives with respect to Goals	100
6.2	Pairwise Comparisons in Decimal Equivalents and with Columns Summed	101
6.3	Normalized Columns and Averaged Rows for Pairwise Comparisons	101
6.4	Pairwise Comparison Matrix for Rating Intensities	106
6.5	Summary of Composite Weights	107
6.6	Pairwise Comparison Matrix for Sensitivity Analysis	109
6.7	Summary of Sensitivity Analysis By Preference	109
6.8	Pairwise Comparison Matrix for Top-Ranked Alternatives	115
6.9	Fuel-Mix Planning of the Best Compromised Solution Alternative	118

MULTIOBJECTIVE DECISION MAKING IN INDUSTRIAL ENERGY AND ENVIRONMENTAL PLANNING

1. INTRODUCTION

1.1 Background

Contemporary procedures for decision analysis in the context of managerial problem solving have become more complex, requiring careful consideration of multiple objectives¹ for the decision making process. There is always a potential for conflict between differing objectives. Additionally, it is difficult to define the measurements of each objective. In the area of industrial energy and environmental planning, for example, supplying clean energy resources to production processes minimizes the impact of air pollution, but with an increase of total energy costs. Energy costs are usually measured quantitatively in terms of monetary values. However, the impact of pollution may not be sufficient to define a measurement solely confined to quantitative terms. Therefore, proper analysis of the problem mandates inclusion of qualitative factors.

¹The term "multiple objectives" is often referred to as either "multiple criteria" or "multiple attributes."

The energy/pollution problem described above cannot be analyzed adequately using the conventional methodologies of single objective programming (SOP). SOP procedures do not have the capacity to deal with decision analysis problems which contain possibly conflicting objectives. Moreover, given the goal of a single objective, its implications for modeling outputs are often too narrow.

To overcome the problems inherent in the SOP process, a modified and refined methodology is required which satisfies the need for simultaneous evaluation of conflicting objectives. This methodology must be based on evaluation techniques which can accommodate both the monetary and non-monetary effects of a decision problem (Freeman & Haveman, 1970; Goecoechea et al., 1982).

This approach is known as multiobjective decision making (MODM) or multiobjective programming (MOP).

Explicit needs for MODM have been pointed out in a number of studies, particularly when applying its benefits to planning decision problems. Cohon (1978) noted that MODM offers three major problem solving improvements when compared to the SOP procedure:

- 1) Promotion of an appropriate role for the decision maker in the decision making process;
- 2) Identification of a wide range of alternatives;
and
- 3) Viewing the problem in a realistic focus.

These characteristics are appropriate to the requirements of energy and environmental planning processes. The continued decrease in the real delivery price of energy resources since the mid-1980s has somewhat lessened the energy cost problem throughout modern industrial economies. Energy conservation activities and investments have been drastically diminished, leading to increases in energy consumption. In contrast, environmental regulation (i.e., the regulation of allowable pollutant emission levels) has been strengthened, offsetting some of the economic savings resulting from reduced energy prices. However, industrial energy planning decision making is still considered one of the strategic planning issues which can effect the future of individual firms and entire industries. Fuel-mix planning decisions with respect to energy supply and conversion facilities cannot be changed easily once the equipment has been installed.

Environmental planning (i.e., air pollution planning) is closely related to the issues of energy resource planning. Quantities of pollutant emissions are strongly influenced by the form and facilities of the energy resource in use. Therefore, it is recommended that an integral MOP approach, considering both energy and environmental problems be adopted for industrial planning procedures. The purpose of this paper is to

present a methodology in which these issues can be simultaneously analyzed.

1.2 MOP and Energy and Environmental Planning

In spite of efforts devoted to MOP over the past decade, a general model still does not exist (Kavrakoglu & Kiziltan, 1983). The methodologies developed to date reflect only a potential and present serious application limitations. The appropriate development of a methodology for a given problem and situation would be a noteworthy contribution to the process of utilizing the advantages of MOP procedures.

A number of factors should be considered in determining the characteristics of a successful methodology. These include the issues of: (1) when the decision maker's preference information is available; (2) what type of decision variables should be included, i.e., discrete vs. continuous or quantitative vs. qualitative; and (3) how the problem should be formulated, i.e., through linear or nonlinear objective functions and constraint sets. The most important factor, however, is the timing in the elicitation of the decision maker's preferences (Evans, 1984). Three technique groups have been proposed, including:

- 1) priori articulation of preferences,
- 2) progressive articulation of preferences, and
- 3) posteriori articulation of preferences.

The essential state-of-the-art has been summarized by Hwang et al. (1980) and Evans (1984). When the analyst has achieved priori articulation of preferences, then the multi-attribute utility function (MAUT) (Keeney & Raiffa, 1976) and goal programming (Charnes & Cooper, 1977) techniques can be used. Methodologies which rely on the progressive articulation of preferences are referred to as "interactive methods." When partial preference information is available, this method may be usefully applied. Moreover, two subcategories of interactive methods have been defined: either implicit or explicit tradeoffs among objectives. Implicit tradeoff information methods include STEM (Benayoun et al., 1971), modified STEM (Dinkelbach & Isermann, 1980), interactive multiple objective linear programming (IMOLP) (Quaddus & Holzman, 1986), and the method of the displaced ideal (Zeleny, 1974a). For explicit tradeoffs, the methods of Zionts-Wallenius (1976) and Geoffrion et al. (1972) may be the choices.

Methods for the posteriori articulation of preferences are also known as "generating methods" insofar as they defer the employment of the decision maker's preferences until entire sets of nondominated solutions are generated. The decision maker need only react to the results of these generated solution sets (Cohon, 1978), a factor implying the possible superiority of this method to other in the resolution of planning decision

problems. Modeling, usually represented in the form of multiobjective linear programming (MOLP), is utilized as a tool for generating the multiple alternatives, which are each composed of unique nondominated solutions. One additional step is required in choosing the best compromise problem solution: defining the trade-off values among objectives is widely used, and is desirable for problems in which the objective functions are identified quantitatively. If difficulties exist in deriving tradeoff values, adoption of other methodologies should be considered.

A number of MOP applications for energy and environmental planning have been developed since the first oil crisis in 1973. Most of them may be categorized as either "macroscopic" or "microscopic." The former category includes applications intended to optimize national strategic planning. The sizes of the models required are so large that huge amounts of data, time, and effort are required for their operation (Cherniavsky, 1974; Haddock, 1984; Kok, 1986, 1987). The latter category is intended for the in-depth study of specific application areas. The techniques include a number of MOP methodologies: the interactive method (Malakooti, 1986), the generating method (Kavarkoglu, 1982; Cohon et al., 1980; Hsu et al., 1987), MAUT (Anandalingam, 1987; Buehring et al., 1978), and goal programming (Spronk & Veeneklaas, 1983). However, the issue of

evaluating and choosing the best compromise solution has been only superficially studied in the generating method application.

The objective of this study is to introduce a MODM process for energy and environmental planning through an examination of fuel mix options in the forest products manufacturing sector. The MOLP model is used for describing the problem formulation and the resulting approach is based on the posteriori articulation of decision maker's preference information. An evaluation procedure for arriving at a final decision (i.e., the best compromise solution) will also be presented. This study will be conducted in a sequence of six steps:

- 1) Identification of the goal,
- 2) determination of decision variables,
- 3) model formulation,
- 4) generation of nondominated solutions,
- 5) choosing the best compromise solution through the evaluation of generated sets of nondominated solutions, and
- 6) post-optimality analysis (sensitivity analysis).

1.3 Objectives of the Study

This study is presented in two distinct parts. First, a MOLP model is introduced which involves consideration of nonmonetary valued objective functions

toward energy and environmental planning in the forest products manufacturing sector. Second, a methodology for conducting an evaluation of the nondominated solutions generated in the MOLP model is proposed. The model used in this study is of relatively restricted size, requiring a minimum volume of input data. The object is to develop this model as a prototype example which other analysts may use to conduct similar research. It is presumed that the model may also be applied to other manufacturing sectors or to the planning policies of individual firms. When a specific sector or firm requires inclusion of additional decision variables or other information, it should be possible to add supplements to the model.

The proposed evaluation procedures are based on a multi-screening method that narrows down nondominated solutions until a final solution is obtained. The procedure relies principally on the rating method and pairwise comparison suggested by Saaty (1980). It is a generally appropriate approach for situations in which the tradeoff values cannot be defined and for which a large number of alternatives exist. The employment of the methodology and the sequential approach outlined in the previous section will provide a valuable tool for the analysis of related planning decision problems. However, it should be borne in mind that there are limits to the solution capacity of this model, and a fur-

ther discussion of the strengths and weaknesses of the proposed model is included in Chapter 6.

1.4 Organization of the Study

The state-of-the-art of the generating method is described in Chapter 2. Solution procedures and topics related to the method applied in this study are also presented. The MOLP model is introduced in Chapter 3, along with the model assumptions, the input data, and the computational results. Chapter 4 is an examination of existing evaluation methods, presented with the intention of providing an elementary framework for considering evaluation methods and comparative study of the proposed and existing methodologies. The basic tools for the proposed methodology, discussed in Chapter 5, are the pairwise comparison procedures of the Analytic Hierarchy Process (AHP). The proposed procedure, accompanied by the final results, sensitivity analysis, and a discussion of potential applications, are presented in Chapter 6. The study is summarized, conclusions are reached and recommendations for future research are included in Chapter 7.

2. MOLP MODEL AND SOLUTION SYSTEMS

In this chapter, the MOLP model for the formulation of industrial energy and environmental planning problems, related issues, and model solution systems are discussed. The constraint method, adopted as a solution method for generating nondominated solutions, is also discussed.

2.1 MOLP Backgrounds

2.1.1 Mathematical Presentation

First, consider the general form of conventional single objective linear programming (SOLP), which can be formulated as:

Maximize $Z(\mathbf{x})$

subject to

$$g_i(\mathbf{x}) \leq 0, \quad i=1,2,\dots,m$$

$$x_j \geq 0, \quad j=1,2,\dots,n$$

where the objective function $Z(\mathbf{x})$ and the constraint $g_i(\mathbf{x})$ are linear and defined by the decision variable, \mathbf{x} . For this formulation, feasible solutions \mathbf{X} are defined as:

$$\mathbf{X} = \{ \mathbf{x} : \mathbf{x} \in \mathbb{R}^n, g_i(\mathbf{x}) \leq 0, x_j \geq 0, \forall_{ij} \},$$

where R^n is a set of Euclidean space. The objective of this problem is to find the optimal solution \mathbf{x}^* , $\mathbf{x}^* \in X$ in order to maximize the objective function $Z(\mathbf{x})$.

MOLP in turn pursues solutions for two or more objective functions and the principal difference between MOLP and SOLP lies in the presentation of the objective function(s). The MOLP model, with p -dimensional objective functions ($p \geq 2$), may be expressed as:

$$\text{Maximize } Z(\mathbf{x}) = [Z_1(\mathbf{x}), Z_2(\mathbf{x}), \dots, Z_p(\mathbf{x})]$$

subject to

$$g_i(\mathbf{x}) \leq 0, \quad i=1,2,\dots,m$$

$$x_j \geq 0, \quad j=1,2,\dots,n$$

or

$$\text{Maximize } Z(\mathbf{x}) = [Z_1(\mathbf{x}), Z_2(\mathbf{x}), \dots, Z_p(\mathbf{x})]$$

subject to

$$\mathbf{x} \in X,$$

where $Z(\mathbf{x})$ is the vector of the maximized objective function. A problem which seeks to minimize the objective function can be converted to a maximization problem by a multiplication factor of (-1) .

2.1.2 MOLP Assumptions

SOLP embodies assumptions relative to the problem being modeled, including the appropriateness of the formulation and such mathematical relationships as (McCarl & Spreen, 1988):

- 1) objective function appropriateness,

- 2) decision variable appropriateness,
- 3) constraint appropriateness,
- 4) proportionality,
- 5) additivity,
- 6) divisibility, and
- 7) certainty.

Among these assumptions, objective function appropriateness is relaxed in the use of MOLP since any one objective function cannot be the sole criteria for optimization. Therefore, the concept of the optimal solution, as applied in SOLP, can no longer be meaningful. Rather, MOLP generates "nondominated"¹ and "dominated" solutions as two mutually exclusive subsets of feasible solutions. A finite number of generated sets of unique nondominated solutions are comprised of alternatives provided for the decision maker's evaluation and the most preferable solution is termed the "best compromise" solution.

2.2 Nondominated Solutions

2.2.1 Definition

Zadeh (1963) defined nondominated solutions in non-scalar valued performance criteria (i.e., as vector optimization problems). This approach has provided a fundamental solution concept for MOP problems, as well

¹"Nondominated" solutions are also referred to as "noninferior," "efficiency," or "Pareto optimal" solutions.

as decision problems in economics, statistical decision theory, or in any decision problem with non-comparable criteria (Geoffrion, 1968). A recent study by Lowe et al. (1984) has suggested a more slightly restricted definition, but refinement of this definition is still in the development process. The classical definition of a nondominated solution for the maximization of the objective functions may be expressed as follows:

A feasible solution \mathbf{x}' , $\mathbf{x}' \in X$ is said to be a non-dominated solution if there exists no other feasible solution \mathbf{x} , $\mathbf{x} \in X$ such that for $k = 1, 2, \dots, p$, $Z_k(\mathbf{x}) \geq Z_k(\mathbf{x}')$ and for at least one value of k , $Z_k(\mathbf{x}) > Z_k(\mathbf{x}')$.

To illustrate this definition, consider the simplified example below:

Maximize

$$Z_1(\mathbf{x}) = 4x_1 + x_2$$

$$Z_2(\mathbf{x}) = x_2$$

subject to

$$x_1 + x_2 \geq 1$$

$$3x_1 + 2x_2 \leq 12$$

$$3x_1 + 6x_2 \leq 18$$

$$x_1, x_2 \geq 0.$$

This problem is displayed graphically in Figure 2.1.

Optimal solution of a problem with the objective function $Z_1(\mathbf{x})$ should be point C (4,0). In the same manner, point A (0,3) directs the optimal solution for

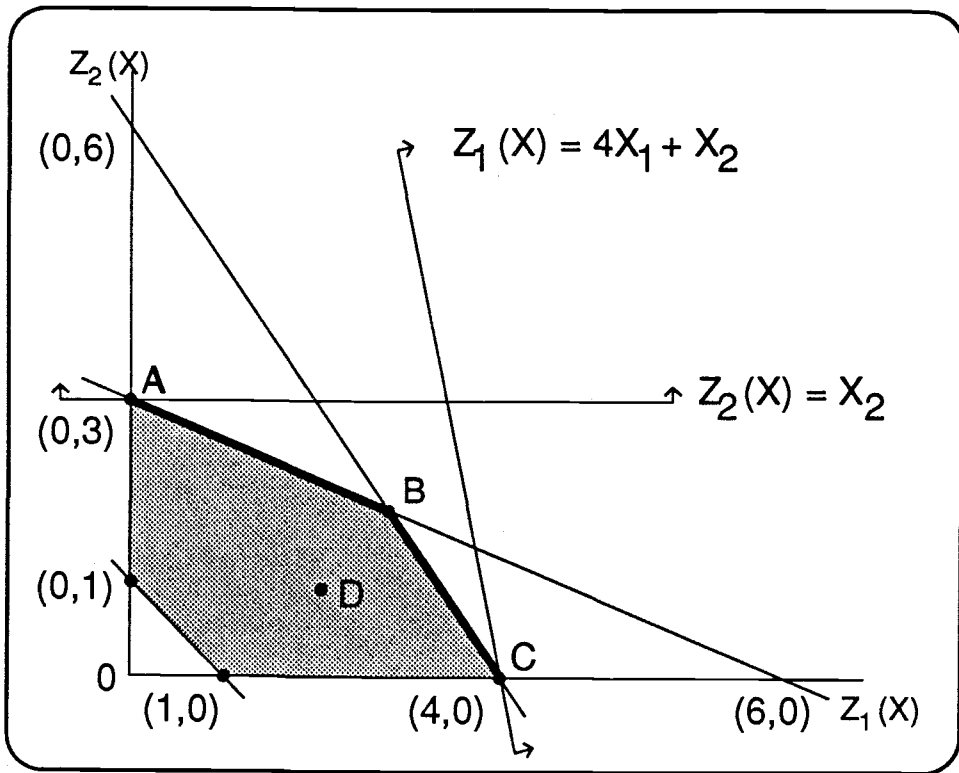


Figure 2.1 Nondominated solutions.

the objective function, $Z_2(x)$. Points A, B, and C are nondominated (feasible) corner solutions and segments AB and BC are the regions of the nondominated solutions (i.e., in this example, an infinite number of nondominated solutions). Table 2.1 indicates some of the feasible solutions and the values of the objective functions. Solution set A is dominated by solution set B for the objective function $Z_1(x)$, but solution set A dominates solution set B for $Z_2(x)$. The relationship between solution sets B and C remains the same. Thus, solutions sets A, B, and C are all nondominated solutions, while solution set D should be dominated since the value of both of its objective functions is less than those for solution set B. Therefore, a nondominated solution is a feasible solution for which increasing the values of any one objective function should be accompanied by a decrease in the values of one or more other objective functions (Zeleny, 1982). This definition thus directs attention to the concept of trade-off values among objective functions (see Chapter 4).

Table 2.1 An example of nondominated and dominated solutions.

Solution	$Z_1(x)$	$Z_2(x)$	Type
A (0,3)	3.0	3.0	Nondominated
B (3,1.5)	13.5	1.5	Nondominated
C (4,0)	16.0	0.0	Nondominated
D (2,1)	10.0	1.0	Dominated

2.2.2 Kuhn-Tucker Conditions for Nondominated Solutions

Kuhn and Tucker (1951) derived the necessary and sufficient conditions (KTC) for the optimality of the single objection programming problem (i.e., the scalar optimization problem) and extended the conditions for nondominated solutions for multiobjective problems (i.e., vector optimization). These conditions for nondominated solutions may be defined as follows:

A feasible solution \mathbf{x}^* is said to satisfy the necessary conditions for vector optimization if (Chankong & Haimes, 1983; Goicoechea et al., 1982):

- i) $Z(\mathbf{x})$ and $g_i(\mathbf{x})$ are differentiable, and
- ii) there exists $W_k \geq 0$, $k=1,2,\dots,p$ with multipliers $U_i \geq 0$, $i=1,2,\dots,m$, such that

$$\mathbf{x}^* \in X$$

$$U_i g_i(\mathbf{x}^*) = 0 \quad i=1,2,\dots,m$$

$$\sum_{k=1}^p W_k \nabla Z_k(\mathbf{x}^*) - \sum_{i=1}^m U_i \nabla g_i(\mathbf{x}^*) = 0 .$$

The conditions so defined are nearly equivalent to those defined for optimality in the scalar optimization problem. The single difference lies in the final conditions, which are replaced by a linear combination of the gradient of the p -objective function. These neces-

sary conditions are also sufficient conditions if $g_i(\mathbf{x}^*)$ is convex and $Z(\mathbf{x})$ is concave for maximization, or convex for minimization. Derivation procedures and interpretation of the final condition can be found in Zadeh (1963) and Cohon (1978).

2.3 Overview of Generating Methods

Several generating methods have been developed for decision problems based on the MOLP model and a posteriori articulation of the decision maker's preference information. Moreover, these methods may be applied to problems with a priori articulation of the decision maker's preferences, i.e., goal programming. Willis and Perlack (1980) have noted that generating methods fare relatively better than goal programming from the criteria quantification of trade-offs, quantity of information, and the validity of decision maker and analyst interactions. Goal programming is superior to generating methods only when computational burden is the principal criterion. Additionally, Zeleny (1974a) argued that there are limitations on the decision maker's ability to assess prior weights for goal programming. Therefore, a priori articulation does not always precisely reflect the decision maker's preference structures and generating methods have substantial potential for a wide range of applications.

To date a number of highly respected solution methods have been introduced, including: (1) noninferior set estimation (NISE), (2) the multiobjective simplex method, and (3) the constraint (ϵ -constraint) method.

The NISE method was developed by Cohon et al. (1979) and generally applied only to dual objective or bicriteria decision problems, including recent studies of regional forest planning (Allen, 1986) and energy economic planning (Hsu et al., 1987). This method involves approximation of the sets of nondominated solutions for problems with two objectives. Weights and trade-offs for the objective functions are applied to obtain NISE solutions. Problems with more than two objective functions or difficulties in the definition of the trade-off values between the objectives reflect limitations in the application of this method. However, a recent study (Appino, 1984) has proposed a new approach extending the effective range of the method to three objectives.

Application of the multiobjective simplex method has been subjected to a number of investigations (Ecker et al., 1980; Evans & Steuer, 1973; Philip, 1972; Yu & Zeleny, 1975; Zeleny, 1974b). Each method proposed included specific simplex-based algorithms for the generation of nondominated solutions. Cohon (1978) pointed out that these methods provide the most elaborate gener-

ating technique, but noted that they cannot be applied to large scale problems because of requirements for hundreds of decision variables and constraint sets, as well as extensive computer programming efforts.

The constraint method, used as the solution technique in the present study, has become a common approach to the generation of nondominated solutions. The concept underlying the method was first suggested by Marglin (1967), followed by development of the theoretical background (Haines et. al, 1971) and systematic computational procedures (Cohon, 1978). Subsequently, the constraint method has been applied to water resource planning problems (Cohn & Marks, 1973), power plant siting problems (Cohon et al., 1980), power systems planning (Kvrakoglu & Kiziltan, 1983), and design and evaluation of large-scale automation systems (Bard, 1986a). Bard, in particular, introduced a procedure for the inclusion of integer variables within the MOLP model. Basic operations involve the arbitrary selection of one objective function for optimization, while the others are converted to constraint sets. By application of this procedure, the simplex methods used for single objective optimization problems may be applied, and the following section includes a more detailed consideration of this approach.

2.4 Constraint Solution Method

2.4.1 Solution Procedure

As stated in section 2.3, the constraint method transforms the problem to a single objective programming base. Three transformation variations may be considered: (1) inequality constraint, (2) equality constraint, and (3) hybrid (weighting-constraint) approaches (Lin, 1976, 1977). For the analytical solution of relatively small-scale problems, the second variation appears to be the most efficient method (Chankong & Haimes, 1983). However, for the current study, the inequality variation is proposed in the following forms:

Maximize $Z_1(\mathbf{x})$

subject to

$$\mathbf{x} \in X$$

$$Z_k(\mathbf{x}) \geq \xi_k, \quad k=1,2,\dots,l-1,l+1,\dots,p$$

where $Z_1(\mathbf{x})$ is an arbitrarily chosen objective function for maximization. The lower bound, ξ_k , is interpreted as the satisfaction level of the k^{th} ($k \neq 1$) objective. Graphical representation of ξ_k for two objective problems, $Z_1(\mathbf{x})$ and $Z_2(\mathbf{x})$, is shown in Figure 2.2, where $Z_1(\mathbf{x})$ is selected as an objective function for maximization, $Z_2(\mathbf{x})$ is converted to a constraint set, S is

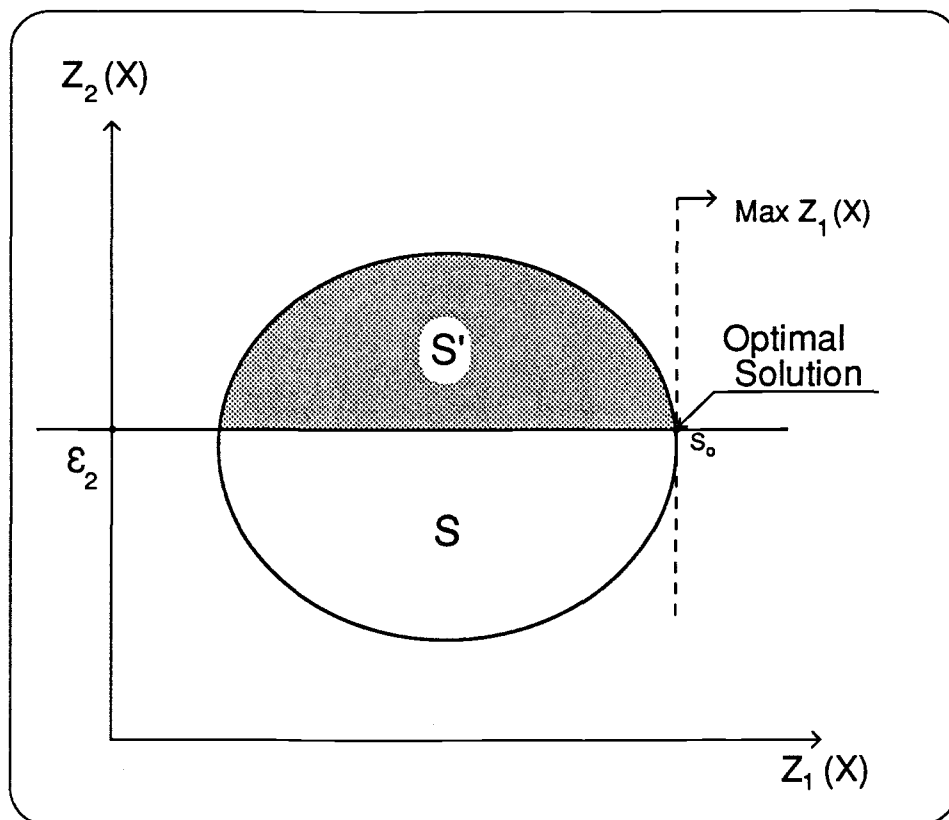


Figure 2.2 Interpretation of the constraint method.

the feasible set² of the original problem, and S' becomes the new feasible region which is restricted by the constraint $Z_2(\mathbf{x}) \geq \xi_2$.

The optimal solution, which will also be comprised of the nondominated solutions to the problem, should be the point S_0 . If the value of the lower bound, ξ_k , is too high, then no feasible solution exists; conversely, extremely low values of ξ_k generate too many solutions. Therefore, a well-defined algorithm for determining ξ_k values is recommended in order to capture maximum benefits from application of this method. Fortunately, Co-hon (1978) has suggested a sequential algorithm for the parametric variation of the ξ_k values:

- 1) Step 1, construct a payoff table as follows:
 - a) Solve the individual LP problem to find the optimal solution for p-objective functions. Let $\mathbf{x}^k = (x_1^k, x_2^k, \dots, x_n^k)$;
 - b) Compute each objective function value for each p-optimal solution, $Z_1(\mathbf{x}^k), Z_2(\mathbf{x}^k), \dots, Z_p(\mathbf{x}^k)$ for $k=1, 2, \dots, p$;
 - c) Construct a payoff table as indicated in Table 2.2; and

²In analysis of a MOP problem, the term "feasible set" is called the "payoff set" (Szidarovszky et al., 1986).

Table 2.2 Payoff table for the constraint method.

	$Z_1(\mathbf{x}^k)$	$Z_2(\mathbf{x}^k)$. . . $Z_p(\mathbf{x}^k)$
\mathbf{x}^1	$Z_1(\mathbf{x}^1)$	$Z_2(\mathbf{x}^1)$. . . $Z_p(\mathbf{x}^1)$
\mathbf{x}^2	$Z_1(\mathbf{x}^2)$	$Z_2(\mathbf{x}^2)$. . . $Z_p(\mathbf{x}^2)$
\vdots	\vdots	\vdots
\mathbf{x}^p	$Z_1(\mathbf{x}^p)$	$Z_2(\mathbf{x}^p)$. . . $Z_p(\mathbf{x}^p)$

- d) From the payoff table, let the largest number in the k^{th} column be M_k and the smallest number in the k^{th} column be n_k for $k=1,2,\dots,p$.
- 2) Step 2, change the MOLP problem to a constraint form. Hence, $Z_k(\mathbf{x})$ is defined by $n_k \leq Z_k \leq M_k$.
 - 3) Step 3, choose the number of different values of ξ_k that will be used for generation of non-dominated solutions. Let this number be r .
 - 4) Step 4, Determine the ξ_k value for $k=1,2,\dots,l-1,l+1,\dots,p$ where $\xi_k = n_k + [t/(r-1)](M_k - n_k)$. $t=0,1,2,\dots,(r-1)$
 - 5) Step 5, solve the constrained problem for every combination of ξ_k values.
 - 6) Step 6, if all objective constraints are binding, check for the feasibility of each solution that yields a nondominated solution.

The total number of individual problems is dependent on the number of objective functions and the values of r ,

defined in Step 3, which yields combinations of r^{p-1} , including feasible and infeasible solutions.

The Kuhn-Tucker conditions (KTC) for the constraint method are not confirmed in the present study since it is concerned only with the generation of numerical nondominated solutions. However, KTC may be derived from the third (last) condition presented in section 2.2.2. These procedures have been adequately explained by Chankong and Haimes (1983) and by Cohon (1978). The sequence of steps described in this section will subsequently be applied to solution of the MOLP model presented in Chapter 3.

2.4.2 Motivation for Use of the Constraint Method

Computational problems from application of the constraint method may be anticipated, particularly when a large value of r is selected. This negative aspect of the method is mitigated by the fact that this problem is common to all generating methods. The NISE method accepts problems limited to two objectives and the multiobjective simplex method has limitations when it is applied to real-time problems. Moreover, when a larger number of alternatives for decision maker evaluation are considered, confidence in the final choice may be increased (Zeleny, 1982). Generating every possible solution combination is also an advantage when problems are designed for decision making in the

planning area. Furthermore, the use of LP software with the capability of sensitivity analysis and parametric programming will lessen the difficulties inherent in the computational burden of large-scale problems.

3. MOLP MODELING FOR INDUSTRIAL ENERGY AND ENVIRONMENTAL PLANNING

3.1 Introduction

A number of conventional LP applications for industrial energy planning have been examined. Most of those investigated were limited to fuel-mix planning or inter-fuel competition analysis. For consideration of multiple objectives in the area of industrial energy planning, the complicated processes of decision analysis require the application of multiobjective linear programming (MOLP) methods. The continuing development of acceptable solution techniques and algorithms has also expanded the potentiality and the flexibility of this approach. MOLP has become a widely used multiobjective programming (MOP) tool.

For industrial environmental planning, use of the MOLP approach has been primarily confined to the area of air pollution problems which are closely related to issues of energy planning. However, this approach places a number of restrictions on problem formulation, including: (1) the number of different units of measurement of emitted pollutant effects and (2) considerations of variable influencing factors, such as at-

atmospheric stability and diffusion. Therefore, MOLP applications have to this point been focused principally on specific areas, e.g., acid rain control (Ellis, 1988), evaluation of air pollution abatement options with given maximum allowable emission levels (Kohn, 1978), and industrial location and fuel-mix planning for air pollution abatement (Guldman & Sheffer, 1980). A variety of methods have been adopted for the treatment of measurement units, including the index approach (Bretschneider & Kurfürst, 1987) and subjective numerical values (Kavrakoglu & Kiziltan, 1983).

As stated above, energy planning problems are closely related to issues of environmental planning. This is especially true when dealing with industrial contribution to air pollution problems. Therefore, it is recommended that an integrated approach for the simultaneous consideration of both energy and environmental problems be considered. This chapter presents an MOLP model for the analysis of these issues through examination of fuel-mix options in forest manufacturing industries. The model is aggregated for the entire industry and the input data for the real applications of the model are derived from Korean examples.

3.2 Industrial Background

The forest manufacturing industries (SIC¹ 33 and 34 in Korea) encompass a diversity of products: pulp production, the manufacture of paper and paperboard products from pulp, and other refined wood products, including veneers, plywood, fibreboard, and particle board. The total energy consumption by the industry in 1983 was 42 Peta joule (PJ = 10¹⁵ Joule),² accounting for 5.9 percent of the energy consumption of the entire industrial sector, while total value added accounted for only a net 5.0 percent (Table 3.1). This classifies Korean forest manufacturing industries as high energy intensive industries.

Table 3.1 Industrial structures and energy consumption.

Industry	Value Added (%)	Energy Consumption (%)
Food, tobacco	20.4	6.9
Textiles	17.7	10.3
Paper, wood	5.0	5.9
Chemicals	15.1	23.1
Non-metallic minerals	5.7	19.9
Iron and steel	6.6	27.9
Non ferrous metals	1.6	1.6
Ferrous metals	2.7	1.1
Machinery	23.0	4.9
Other manufacturing	2.2	0.4
Total:	100.0	100.0

Source: KIER (1985); BOK (1984).

¹Standard Industry Classification.

²1 PJ is equivalent to 9.479×10^{11} BTU or 2.389×10^{11} Kcal.

Energy use patterns by fuel type and end use device for the same year are shown in Table 3.2. Almost all fuels were consumed by indirect use devices (steam generating boilers) for the supply of medium temperature (250-350°F) process heat for production facilities and electricity for motive power sources. Oil products constitute the main energy source, accounting for 50 percent of the total energy use. Consumption of other petroleum substitute sources, such as natural gas and coal, were negligible. The residues from wood and paper processing industries can also serve as an energy source for either steam generation or auto electricity generation, but data for these applications is not available. A recent survey shows that the portion of residual use is still quite low (KIER, 1985). It is evident that the increase in use of substitutable energy sources at a lower cost than petroleum products is a primary alternative as a means to lessen energy costs problems in the Korean industrial sector.

Table 3.2 Energy consumption by end use (1983).

	Medium Temp. Indir. (≥250°F)	Low Temp. Direct (≤250°F)	Motive Power	Space Heat	Lighting	Other	Total
Bunker-C	19.50						19.50
Diesel oil		0.40	0.02 ^a				0.42
Coal				0.04			0.04
Electricity			21.47	0.13	0.42	0.02	22.04
Total:	19.50	0.40	21.49	0.17	0.42	0.02	42.00

Source: KIER (1984). Note ^a = electricity generation.

The major air pollution problem from forest product manufacturing industries is the emission of wood particles and dust. The amount and the particle size distribution varies, dependent on the type of production process in use. Detailed data on industrial pollution emissions in Korea is not currently available. For purposes of this study, comparable data for the U.S. wood treatment industries is provided in Table 3.3

Table 3.3 Typical particle emission levels of wood treatment industries.

Operation	Particle Loadings Ahead Of Cleaning Equipment (g/m^3)	Part. Size Distributed (% by mass $\leq 60 \mu\text{m}$)
Sawing	< 10	12-28
Grinding	< 10	12-28
Sawdust & fiber drying	0.3-2.0	97-100

Source: Suess et al. (1985).

The emissions of the pollutants, sulfur dioxide (SO_x), nitrogen oxide (NO_x) and others, are related to the type of fuel used and chemical materials added in the production process. The normalized pollutant emissions from the combustion of Bunker-C oil (B-C), anthracite coal, diesel fuel, and liquified natural gas (LNG) are listed in Table 3.4. The relatively high levels of SO_2 produced from the use of B-C oil and the high levels of carbon monoxide (CO_2) produced from

burning anthracite coal are of particular note. Diesel oil and LNG generate lesser amounts of air pollutants.

Table 3.4 Pollutant emissions level of energy sources.^a

	B-C Oil	Diesel Oil	Anth. Coal	LNG
SO _x (g)	74.8	19.1	27.5	0.4
NO _x (g)	7.8	5.2	3.5	2.3
CO (g)	0.6	0.6	103.0	0.3
Dust (g)	3.0	2.0	n/a	0.3

Note ^a = Normalized to 104 kcal fuel. Percent of sulfur contained (weight %) in each energy source is: B-C oil, 3.9%; diesel oil, 1.0%; anthracite coal, 0.6-0.8%; and LNG, less than 0.003%.

Source: DOE (1981).

In this paper, forest manufacturing industries products are classified in six major categories for purposes of model simplification: (1) newsprint paper, (2) printing paper, (3) craft paper, (4) paperboard, (5) other paper products, and (6) plywood.

3.3 Formulation of the Problem

3.3.1 Observed Objectives

Complicated real world decision problems often include numerous factors to be considered and evaluated to reach an optimal decision. Even though MOP has the ability to accept several objectives at the same time, it is almost impossible to facilitate all of the factors required for modeling and analytical procedures.

Subobjectives within each objective are often in conflict, qualitative in nature, or even incommensurable. Within these constraints, the observed and selected objectives should reflect the goal of the given decision problem as much as possible. This constraint bears a strong relationship to arrival at an acceptable confidence level when the decision maker must formulate the final decision.

The three objectives selected for the MOLP model proposed in this study are: (1) minimization of total energy costs, (2) minimization of environmental impacts, and (3) minimization of related business and performance risks. A list of the primary factors within each of the three objectives includes:

1) Relevant energy costs:

- Delivery price of each fuel,
- Replacement or conversion cost of energy supply facility due to substitution of fuel,
- Changes in operation and maintenance costs of energy supply facility,
- Fuel storage cost,
- Financial availability for replacement costs, and
- Price changes in the future;

2) Environmental impacts:

- Pollutant emission coefficient of fuel,
- Environmental conservation costs,

- Influences on existing environmental conservation devices, and
 - Potential effects on environmental quality standards;
- 3) Business and performance risks:
- Effects on product quality and production process,
 - The ability to change existing processes and facilities,
 - Transportation mode of fuel,
 - Fuel storage capacity and availability,
 - Maintenance and operation schedules, and
 - Risks on stable supply of substituted fuel.

While all the energy related cost factors can be measured quantitatively in monetary values, factors within the other two objectives must be measured in different units. To overcome this obstacle, these objectives will be measured by subjective numerical values. Moreover, only two energy related cost factors, the delivery price of fuel and the conversion costs of the energy supply, are considered in the model, with others transformed into constraints.

3.3.2 MOLP Model

In this section, an MOLP model for the energy and environmental planning problem is developed. The notation used in the model is as follows:

i = product categories, including a) newsprint paper, b) printing paper, c) craft paper, d) paperboard, e) other paper products, and f) plywood,

j = energy source, including a) oil products, b) coal, and c) gas (LNG),

EI_i = energy intensity of product i in Calorific value,

PV_i = production volume of product i per year,

TES_i = total energy consumption of product i ,

E_{ij} = consumption of j energy source for product i ,

TE_j = total consumption of j energy source,

S_j = total available supply of j energy source,

U_i = process capacity of product i ,

L_i = minimum market demand volume of product i ,

α = allowable fuel substitution rate of coal and gas,

β = existing supply rate of coal and gas,

P_j = delivery price of energy source j ,

C_j = coefficients of environmental impacts (numerical values),

R_j = coefficients of business and performance risks (numerical values),

FC_j = conversion costs of energy supply facility; initial cost, $j=2,3$ (coal and gas, respectively),

AE_j = additional fuel requirements for coal and gas due to reduction of oil product consumption,

$\frac{i(1+i)^n}{(1+i)^n - 1}$ = capital recovery factor of FC_j , and

FI_j = financial availability of facility conversion cost.

A mathematical formulation for the model is shown in the following equations.

Objective functions

1) Minimize total energy costs:

$$Z_1 = \sum_j P_j TE_j + \sum_j \left(\frac{i(1+i)^n}{(1+i)^n - 1} \right) FC_j AE_j$$

2) Minimize environmental impacts:

$$Z_2 = \sum_j C_j TE_j$$

3) Minimize related business and performance risks:

$$Z_3 = \sum_j R_j TE_j$$

Constraints

The following constraints are included:

$$1) EI_i PV_i - TES_i = 0, \quad \forall_i$$

$$2) TES_i - \sum_j E_{ij} = 0, \quad \forall_i$$

$$3) TE_j - \sum_i E_{ij} = 0, \quad \forall_j$$

$$4) TE_j \leq S_j, \quad \forall_j$$

$$5) PV_i \leq U_i, \quad \forall_i$$

$$\begin{aligned}
6) \quad PV_i &\geq L_i, & \forall_i \\
7) \quad E_{ij} - \alpha_j TES_i &\leq 0, & \forall_i \text{ \& } j=2,3 \\
8) \quad E_{ij} - \beta_j TES_i &\geq 0, & \forall_i \text{ \& } j=2,3 \\
9) \quad TE_j - \sum_i (\beta_j TES_i) - AE_j &= 0, & j=2,3
\end{aligned}$$

and

$$10) \quad FC_j AE_j \leq FI_j, \quad j=2,3$$

Critical decision variables denote the total consumption of energy source j (TE_j) and additional coal and gas requirements (AE_j). The first objective function, Z_1 , the minimization of total energy costs, is composed of the two cost items a) annual energy costs and b) initial facility conversion costs. The latter is changed to an annual basis. The objective value unit is 10^3 US\$. The other objective functions, Z_2 , minimization of environmental impacts, and Z_3 , related business and performance risks, are measured by numerical values, as stated in section 3.3.1

There are five main categories of constraint equations used in the model. These may be interpreted as follows:

1) Energy intensity and balance equation:

Constraint (1), identifies the relationship between energy intensity and total energy consumption for each product. Energy intensity represents the net amount of total energy directly consumed to obtain one final product

unit. At a given production facility, energy intensity is a useful tool for the estimation of total energy requirements. This may also be obtained by a different method, i.e., using the value added or monetary value of the total production volume in place of units of output product. Both methods not only require consideration of monetary values at the time of evaluation, but also have limitations when comparison of different units of economic activity is required. Constraints (2) and (3) represent the balance equations for total energy consumption.

2) Energy supply equation:

Constraint (4), limits the amount of each energy source.

3) Demand/supply equations:

The capacity of the production process and the minimum market demand for products are ensured by constraints (5) and (6), respectively.

4) Technical limitations:

The substitution of energy sources causes the conversion of energy supply facilities and even entire production processes. If critical difficulties ensue, the substitution of energy sources may not be possible. Constraint (7) limits the potential substitution rate for coal and gas resources, unless the entire production

process is changed. Constraint (8) denotes the existing consumption rate for coal and gas, and constraint (9) is used for the calculation of quantities of substituted energy sources.

5) Financial availability:

Constraint (10) limits financial availability for the installation and related conversion costs of facilities for substituted energy sources (AE_j).

3.3.3 Assumptions and Limitations of the Model

The MOLP model presented in section 3.3.2 has a number of limitations and certain assumptions are stated to enable its operation:

- 1) Decision makers have no priori articulation of preference information. This means that information on solution payoff (target values or satisfaction level) will not be available until initial solution sets are generated.
- 2) Electrical energy is not included in the considerations.
- 3) Energy intensity is assumed to be constant over time, i.e., considerations of energy conservation or new energy conversion technology options are not included.
- 4) The model is restricted to material conditions and data from the year 1983.

3.3.4 Case Studies

The present MOLP model is examined through the fuel-mix options. This is done by introducing two cases, A and B, according to the penetration rate of substitutable energy sources. In Case A, the maximum rate of coal and gas substitution (α_j) for oil products is limited to 25 percent of the total energy source. In Case B coal substitution is further limited to 15 percent of the total energy source, while gas substitution is increased to a maximum of 35 percent. The supply of coal and gas energy sources is limited in Korea. Both observed cases can be considered as the maximum substitutable ratio for each energy source, i.e., coal (25%) for Case A and gas (35%) for Case B.

3.4 Development of Input Data

The delivery price for each energy source and related facility conversion costs are shown in Table 3.5. The initial conversion costs are distributed on an annual basis at an interest rate of 12 percent, assuming a facility life of 10 years. The subjective numerical values use for objective functions (2) and (3), respectively, minimization of environmental impacts and business and performance risks, are given in Table 3.6. The data provided in Tables 3.5 and 3.6 represents the coefficients for each objective function.

Table 3.5 Energy price and facility conversion cost
(unit = 10^3 \$/PJ).

	Oil (B-C)	Coal	Gas
Delivery price	7.237	2.585	7.423
Conversion cost	-	0.346	0.307

Table 3.6 Subjective numerical values.

	Oil (B-C)	Coal	Gas
Environmental impacts (Z_2)	5.5	7.4	2.1
Business and performance risks (Z_3)	4.7	8.6	6.3

The energy intensity of all products is shown in Table 3.7, which has been calculated from the survey data of KIER (1985). Electricity consumption is not reflected in the calculation of energy intensity. It is assumed that no restrictions are applied to the total available supply of energy sources (S_j) and financial availability (FI_j) for model operations (nonbinding constraints). The other parameters used in the model for both Cases A and B are listed in Appendix A, which also includes all of the constraint set equations.

3.5 Results

The constraint method described in section 2.4 was applied to obtain an initial nondominated solution.

Table 3.7 Energy Intensity.

Product	Energy Intensity (TJ/10 ³ ton)*
Newsprint paper	8.5792
Printing paper	6.2097
Craft paper	6.2000
Paperboard	4.4427
Other products	13.0233
Plywood	1.4502

* TJ = Tera Joules (10¹² Joule)

The procedures and corresponding results are presented in the following sections.

3.5.1 Payoff Table

Constructing the payoff table is initiated by solving each individual LP model. Cases A and B thus required the solution of six LP models. The summary of results is shown in Table 3.8. One solution output objective function for Case A total energy costs, solved via LINDO/PC software, is given in Appendix B.

From Table 3.8, each optimum solution can be rearranged as follows:

Case A:

$$X^1 = (12481.46, 4160.49, 0, 3055.99, 0),$$

$$X^2 = (11376.96, 1104.50, 4160.49, 0, 4160.49),$$

$$X^3 = (15537.45, 1104.50, 0, 0, 0),$$

Table 3.8 Individual LP solutions, Cases A and B.

Objectives	Decision Variables (T_j)					Objective Values ^a
	TE_1	TE_2	TE_3	AE_2	AE_3	
<u>CASE A</u>						
Energy Costs (Z_1)	12481.46	4160.49	0	3055.99	0	102140.60
Environmental Impacts (Z_2)	11376.96	1104.50	4160.49	0	4160.49	79483.63
Performance Risks (Z_3)	15537.45	1104.50	0	0	0	82524.72
<u>CASE B</u>						
Energy Costs (Z_1)	14145.66	2496.29	0	1391.79	0	109306.60
Environmental Impacts (Z_2)	9712.77	1104.50	5824.68	0	5824.68	73825.36
Performance Risks (Z_3)	15537.45	1104.50	0	0	0	82524.72

^a = 10^3 \$US for Z_1 and numerical values for Z_2 and Z_3

Case B:

$$X^1 = (14145.66, 2496.29, 0, 1391.79, 0),$$

$$X^2 = (9712.77, 1104.50, 5824.68, 0, 5824.68),$$

$$X^3 = (15537.45, 1104.50, 0, 0, 0),$$

where X^k = optimal solutions for each objective function, and

$$X^k = (TE_1, TE_2, TE_3, AE_2, AE_3).$$

Using the above results, the payoff table and the largest and smallest number in each column (M_k, n_k) are identified in Tables 3.9 and 3.10 for Cases A and B, respectively.

Table 3.9 Payoff table, Case A.

	$Z_1(X^k)$	$Z_2(X^k)$	$Z_3(X^k)$
X^1	102140.60	99435.66	94443.08
X^2	117350.78	79483.63	89181.50
X^3	115299.66	93629.28	82524.72
M_k	117350.78	99435.66	94443.08
n_k	102140.60	79483.63	82524.72

Table 3.10 Payoff table, Case B.

	$Z_1(X^k)$	$Z_2(X^k)$	$Z_3(X^k)$
X^1	109306.60	96273.68	87952.70
X^2	118171.23	73825.36	91844.20
X^3	115299.66	93629.28	82524.72
M_k	118171.23	96273.68	91844.20
n_k	109306.60	73825.36	82524.72

3.5.2 Results of Constraint Problem

In order to apply the ξ_k constraint method, the MOLP model was changed to a constraint problem type. If objective Z_1 is arbitrarily chosen first, then the constraint problem becomes:

Minimize $Z_1(x)$

subject to

$$x \in X$$

$$Z_2(x) \leq \xi_2$$

$$Z_3(x) \leq \xi_3$$

The range of ξ_2 and ξ_3 is defined as $n_2 \leq \xi_2 \leq M_2$ and $n_3 \leq \xi_3 \leq M_3$. The number of different values for ξ_2

and ξ_3 are determined by the value of r . To generate a reasonable number of combinations, $r = 4$ is applied, then

$$\xi_2 = n_2 + (\%T)(M_2 - n_2) , \quad t=0,1,2,3$$

and

$$\xi_3 = n_3 + (\%T)(M_3 - n_3) . \quad t=0,1,2,3$$

The constraint problem shown above is exactly the same as the single LP problem with two more constraint sets. Solutions for every combination of ξ_k value can be easily obtained through the sensitivity analysis (i.e., changing the value of the right hand side) used in the LP problem. All values of ξ_k for Cases A and B are computed in Tables 3.11 and 3.12, respectively.

Table 3.11 Value of ξ_k , Case A.

T	ξ_1	ξ_2	ξ_3
0	102140.60	79483.63	82524.72
1	107210.66	86134.31	86497.51
2	112280.72	92784.98	90470.29
3	117350.78	99435.66	94443.08

$$\begin{aligned} \xi_1 &= n_1 + T/3(M_1 - n_1) \\ &= \underline{102140.60 + (T/3)(15210.18)} \end{aligned}$$

$$\begin{aligned} \xi_2 &= n_2 + T/3(M_2 - n_2) \\ &= \underline{79483.63 + (T/3)(19952.03)} \end{aligned}$$

$$\begin{aligned} \xi_3 &= n_3 + T/3(M_3 - n_3) \\ &= \underline{82524.72 + (T/3)(11918.36)} \end{aligned}$$

Table 3.12 Value of ξ_k , Case B.

T	ξ_1	ξ_2	ξ_3
0	109306.60	73825.36	82524.72
1	112261.48	81308.13	85631.21
2	115216.35	88790.91	88737.71
3	118171.23	96273.68	91844.20

	$\xi_1 = 109306.60 + (T/3)(8864.63)$		
	$\xi_2 = 73825.36 + (T/3)(22448.32)$		
	$\xi_3 = 82524.72 + (T/3)(9319.48)$		

The results of constraint problems for every feasible solution combination of ξ_k value are shown in Table 3.13 for Case A and Table 3.14 for Case B. The deleted combinations of ξ_k are nonfeasible solutions. In both tables, the value of decision variable AE3 has the same value as TE3. An example of combinations of the ξ_k value is given in Appendix C. From a total of 94 combinations, a total of 59 feasible solutions were generated. These include 31 for Case A and 28 for Case B.

3.5.3 Listed of Generated Nondominated Solutions

Some of the solution sets (Tables 3.13 and 3.14) show the same values for decision variables and objectives. These solutions are referred to as nonunique, nondominated solutions. It is obvious that only one of each set can be selected for further consideration in

Table 3.13 Solutions of constraint problem Case A.

	T Values			Decision Variables				Objective Values		
	ξ_1	ξ_2	ξ_3	TE1	TE2	TE3	AE2			
Z ₁	0	2		11376.96	1104.51	4160.49	0	117350.78		
		3		11376.96	1104.51	4160.49	0	117350.78		
	1	1		13188.11	1197.48	2256.36	92.98	116011.67		
		2		11896.35	2026.16	2719.45	921.66	112671.70		
		3		10604.58	2854.84	3182.53	1750.33	109331.70		
	2	1		14126.55	1850.30	665.10	745.80	112416.14		
		2		12834.79	2678.98	1128.18	1574.48	109076.10		
		3		11543.02	3507.66	1591.27	2403.16	105736.17		
	3	1		14518.79	2123.17	0	1018.66	110913.30		
		2		13500.12	3141.83	0	2037.32	106526.90		
		3		12481.46	4160.49	0	3055.99	102140.57		
	Z ₂	0	3		12481.46	4160.49	0	3055.99	99435.58	
1			2		13321.68	3017.69	302.59	1913.19	96235.54	
			3		11158.18	3239.94	2243.83	2135.44	90057.53	
2		1		14161.88	1874.89	605.18	770.38	93035.40		
		2		11998.39	2097.14	2546.41	992.64	86857.50		
		3		10199.52	2281.94	4160.49	1174.44	81720.75		
3		1		13054.45	1104.50	2483.00	0	85187.10		
		2		11376.96	1104.50	4160.49	0	79483.62		
		3		11376.96	1104.50	4160.49	0	79483.62		
Z ₃		0	3		12481.46	4160.49	0	3055.99	94443.04	
			1	1		9784.26	3381.09	3476.61	2276.58	96965.97
				2		12113.30	3141.82	1386.83	2037.32	92689.19
	3				13658.91	2983.04	0	1878.54	89851.03	
	2	1		11745.13	2123.16	2773.65	1018.66	90935.36		
		2		14074.18	1883.90	683.87	779.40	86658.57		
		3		14836.35	1805.60	0	701.10	85259.01		
	3	0		11376.96	1104.50	4160.49	0	89181.50		
		1		13333.05	1104.50	2204.40	0	86051.77		
		2		15289.12	1104.50	248.32	0	82922.03		
		3		15537.45	1104.50	0	0	82524.72		

Table 3.14 Solutions of constraint problem Case B.

	T Values			Decision Variables				Objective Values
	ξ_1	ξ_2	ξ_3	TE1	TE2	TE3	AE2	
Z ₁	1	2		11778.71	1191.03	3672.22	86.52	116737.53
		3		10768.62	1839.01	4034.32	734.50	114125.77
	2	1		13844.65	1227.55	1519.75	173.05	115303.80
		2		12834.56	1925.53	1881.86	821.03	112692.06
		3		11944.84	2496.29	2200.82	1391.79	110391.60
	3	0		15537.45	1104.50	0	0	115299.60
		1		14740.91	1901.04	0	796.54	111869.76
		2		14145.66	2496.29	0	1391.79	109306.60
		3		14145.66	2496.29	0	1391.79	109306.60
Z ₂	1	1		14638.68	1829.92	173.35	725.42	94418.19
		2		12946.95	2003.71	1691.30	899.21	89587.38
		3		11255.21	2177.50	3209.24	1073.00	84756.56
	2	1		13867.45	1293.42	1481.08	188.92	88952.54
		2		12175.72	1467.21	2999.02	362.71	84121.77
		3		10483.99	1641.00	4516.96	536.50	79290.95
	3	0		15537.45	1104.50	0	0	93629.28
		1		13595.89	1104.50	1941.56	0	87027.99
		2		11654.33	1104.50	3883.12	0	80426.67
		3		9712.77	1104.50	5824.68	0	73825.38
Z ₃	1	1		10047.58	2301.56	4292.81	1197.06	94061.76
		2		12668.03	2032.36	1941.56	927.86	89249.87
		3		14831.88	1810.07	0	705.57	85276.42
	2	1		11190.40	1568.43	3883.12	463.93	90547.05
		2		13810.84	1299.24	1531.87	194.74	85735.18
		3		15518.10	1123.85	0	19.35	82600.17
	3	1		11913.58	1104.50	3623.87	0	88322.91
		2		14114.40	1104.50	1423.05	0	84801.61
		3		15537.45	1104.50	0	0	82524.72

the evaluation process. The selection may be made arbitrarily. Then, all unique solutions will be comprised of sets of nondominated solutions, which also denote the alternatives to be evaluated by the decision maker's preferences. The list of generated sets of nondominated solutions is given in Table 3.15. The

Table 3.15 Generated nondominated solutions.

Alt.	Decision Variables					Objective Values		
	TE1	TE2	TE3	AE2	AE3	Z ₁	Z ₂	Z ₃
AE-1	11376.96	1104.50	4160.49	0	4160.49	117350.78	79483.62	89181.50
AE-2	13188.11	1197.48	2256.36	92.98	2256.36	116011.67	86134.31	86497.51
AE-3	11896.35	2026.16	2719.45	921.66	2719.45	112671.70	86134.31	90470.36
AE-4	10604.58	2854.84	3182.53	1750.33	3182.53	109331.70	86134.31	94443.04
AE-5	14126.55	1850.30	665.10	745.80	665.10	112416.14	92784.96	86497.51
AE-6	12834.79	2678.98	1128.18	1574.48	1128.18	109076.10	92784.96	90470.36
AE-7	11543.02	3507.66	1591.27	2403.16	1591.27	105736.17	92784.96	94443.04
AE-8	14518.79	2123.17	0	1018.66	0	110913.30	95564.80	86497.51
AE-9	13500.12	3141.83	0	2037.32	0	106526.90	97500.20	90470.36
AE-10	12481.46	4160.49	0	3055.99	0	102140.57	99435.58	94443.04
AV-1	13321.68	3017.69	302.59	1913.19	302.59	107210.67	96235.54	90470.36
AV-2	11158.18	3239.94	2243.83	2135.44	2243.83	107210.67	90057.53	94443.04
AV-3	14161.88	1874.89	605.18	770.38	605.18	112280.71	93035.40	86497.51
AV-4	11998.39	2097.14	2546.41	992.64	2546.41	112280.66	86857.50	90470.36
AV-5	10199.52	2281.94	4160.49	1174.44	4160.49	112279.69	81720.75	93773.52
AV-6	13054.45	1104.50	2483.00	0	2483.00	116523.78	85187.10	86497.51
AP-1	9784.26	3381.09	3476.61	2276.58	3476.61	107210.67	86134.38	96965.97
AP-2	12113.30	3141.82	1386.83	2037.32	1386.83	107210.67	92784.96	92689.19
AP-3	13658.91	2983.04	0	1878.54	0	107210.67	97198.50	89851.03
AP-4	11745.13	2123.16	2773.65	1018.66	2773.65	112280.65	86134.26	90935.36
AP-5	14074.18	1883.90	683.87	779.40	683.87	112280.65	92784.96	86658.57
AP-6	14836.35	1805.60	0	701.10	0	112280.65	94961.37	85259.01
AP-7	13333.05	1104.50	2204.40	0	2204.40	116386.43	86134.38	86051.77
AP-8	15289.12	1104.50	248.32	0	248.32	115422.01	92784.96	82922.03
AP-9	15537.45	1104.50	0	0	0	115299.66	93629.28	82524.72

(Table 3.15 continued on following page)

Table 3.15 (continued).

Alt.	Decision Variables					Objective Values		
	TE1	TE2	TE3	AE2	AE3	Z ₁	Z ₂	Z ₃
BE-1	11778.71	1191.03	3672.22	86.52	3672.22	116737.53	81308.19	88737.78
BE-2	10768.62	1839.01	4034.32	734.50	4034.32	114125.77	81308.19	91844.22
BE-3	13844.65	1227.55	1519.75	173.05	1519.75	115303.80	88420.92	85201.21
BE-4	12834.56	1925.53	1881.86	821.03	1881.86	112692.06	88790.91	88737.78
BE-5	11944.84	2496.29	2200.82	1391.79	2200.82	110391.60	88790.91	91474.01
BE-6	14740.91	1901.04	0	796.54	0	111869.76	95142.70	85631.22
BE-7	14145.66	2496.29	0	1391.79	0	109306.60	96273.68	87952.70
BV-1	14638.68	1829.92	173.35	725.42	173.35	112261.46	94418.19	85631.22
BV-2	12946.95	2003.71	1691.30	899.21	1691.30	112261.46	89587.38	88737.78
BV-3	11255.21	2177.50	3209.24	1073.00	3209.24	112261.46	84756.56	91844.22
BV-4	13867.45	1293.42	1481.08	188.92	1481.08	115216.34	88952.54	85631.22
BV-5	12175.72	1467.21	2999.02	362.71	2999.02	115216.34	84121.77	88737.78
BV-6	10483.99	1641.00	4516.96	536.50	4516.96	115216.34	79290.95	91844.22
BV-7	13595.89	1104.50	1941.56	0	1941.56	116256.85	87027.99	85631.22
BV-8	11654.33	1104.50	3883.12	0	3883.12	117214.04	80426.67	88737.78
BV-9	9712.77	1104.50	5824.68	0	5824.68	118171.23	73825.38	91844.22
BP-1	10047.58	2301.56	4292.81	1197.06	4292.81	112261.46	81308.19	94061.76
BP-2	12668.03	2032.36	1941.56	927.86	1941.56	112261.46	88790.91	89249.87
BP-3	14831.88	1810.07	0	705.57	0	112261.46	94969.86	85276.42
BP-4	11190.40	1568.43	3883.12	463.93	3883.12	115216.34	81308.19	90547.05
BP-5	13810.84	1299.24	1531.87	194.74	1531.87	115216.34	88790.91	85735.18
BP-6	15518.10	1123.85	0	19.35	0	115216.34	93666.04	82600.17
BP-7	11913.58	1104.50	3623.87	0	3623.87	117086.23	81308.19	88322.91
BP-8	14114.40	1104.50	1423.05	0	1423.05	116001.22	88790.91	84801.61

alternative code for each set is provided in the first column.

The ranges of solution outcomes for each objective is summarized in Table 3.16. Alternative BV-9 yielded the best outcome for the minimization of environmental impacts, but at the same time was the worst outcome for minimization of energy costs. The reverse was true for alternative AE-10. Despite the relatively narrow range of numerical values used in the objective function for environmental impacts, the range of solution outcomes was wider than those obtained for the business and performance risks objective function. The "best outcome" values for energy costs and environmental impacts were exactly the same as the values for each single objective problem (see Table 3.8). It is clear that the minimization of energy costs and minimization of environmental impacts, constituted conflicting objectives.

Table 3.16 Ranges of solution outcomes.

Outcome	Energy Cost (Z ₁)	Environmental Impacts (Z ₂)	Business & Perf. Risks (Z ₃)
Best	102140.57 (AE-10)	73825.38 (BV-9)	82524.72 (AP-9)
Worst	118171.23 (BV-9)	99435.58 (AE-10)	97965.97 (AP-1)
Range	16030.66	25610.20	14441.25
Remarks in parentheses indicate corresponding alternative.			

3.6 Conclusion

An MOLP model, with solution procedures and results for the problem under study, have been presented in this chapter. In most cases of environmental decision analysis, the decision maker does not have priori information on expected solution outcomes. The MOLP model of problem formulation with constraint methods is designed as a tool for providing information on alternative solutions for the decision maker.

The MOLP model presented here is relatively restricted in size, including only a small number of decision variables, constraint sets, and a minimum volume of input data. Despite this simplification, 49 sets of nondominated solutions were generated from use of the constraint method. This implies that a larger model may generate more numerous sets of solutions. However, this does not have to be the case. It is possible that under certain conditions one type of fuel may offer both economic and environmental advantages. In such cases, the analysts' work is greatly simplified.

Though the large number of solutions offers the decision maker a broad sense of the problem implications and a high level of confidence, restriction of

the problem to a manageable number of solution is desirable. This issue is discussed in the following two chapters.

4. CLASSICAL EVALUATION METHODS

In the preceding chapter all sets of nondominated solution alternatives and their performance values for each objective were evaluated in order to identify candidates for the best possible compromise alternative. In most cases, the tradeoff value method should be a valid approach, but only if it can be well-defined. Problems which reflect both quantitative and qualitative factors, such as those presented in the present study, may not lend themselves to this method. Moreover, solution outcomes for two of the objectives, environmental impacts and business and performance risks, can be represented only by the summation of numerical values, which would make it extremely difficult to examine the implications of these outcomes using tradeoff values. Therefore, it is desirable to consider the adoption of an alternative method of evaluation of the problems considered in this study.

The evaluation of solution alternatives is principally concerned with the formalization and systematic arrangement of the decision maker's preference structures. The application of intuitive judgement is not an adequate means and would even pose major difficul-

ties in the analysis of the present problem. Methods relying on intuitive judgements may ignore any number of relevant influencing variables in order to simplify the problem (Goicoechea, 1982). There is also the question of the ability of the analyst, as well as the problem of inconsistencies in the application of human intellect to any solution process. To overcome these limitations, systematic analytical methodology should be introduced and the choice of method to apply is dependent upon the characteristics of the problems faced. Hobbs (1979) suggested important criteria for consideration when choosing an analytical method for a given set of problem characteristics:

- 1) Theoretical validity: Preference structure and assumptions, risk and uncertainty, and appropriateness of the method to problems and data;
- 2) Flexibility: Maximum number of alternatives and objectives, characteristics of method, ease of sensitivity analysis, and extension to multiple decision makers;
- 3) Results compared with other methods: Extension to other methods which yield the same judgements and the significance of differences in the results due the choice of method; and
- 4) Ease of use: Time and costs, understandability, and decision maker confidence level.

4.1 Categorization of Methods

All of the analytical methods presented in this chapter for the evaluation of nondominated solution alternatives are confined to procedures based upon discrete finite numbers of alternatives. Methods may be classified in a number of ways, but two general classifications are more commonly used: (1) single and full dimensional methods, based upon the unit measurement dimensions for the objectives, and (2) descriptive and nondescriptive methods, based upon descriptive characteristics. Single dimensional methods convert the measurements for each objective into a single dimension unit, while the simultaneous consideration of all measurements is termed the full dimensional method. Several of the available methods, e.g., the satisfying level and the lexicographic ordering method, are descriptive insofar as they simply describe the behavior of the decision making process. However, these methods do not prescribe the most preferred course of action for problems requiring specific processes of decision making logic.

In the first category, it is preferable to consider a problem with the full dimensional method since the single dimensional method poses the chance of losing information and accuracy. However, more time and

effort are required for complex decision problems (West, 1976). From the second category, the descriptive method does not often guarantee arrival at a unique final solution. We are left with the full dimensional and nondescriptive methods, each of which allows use of individualized sets of assumptions and limitations. From this point of view, the tradeoff value method, with its strong theoretical background, can satisfy both of these conditions. The following sections present a comparison of the values of these methodological alternatives.

4.2 Simple Descriptive Methods

4.2.1 Satisfying Level

The satisfying level is also referred to as the "exclusionary screening method." Application of this method requires that the decision maker set a "satisfying" or acceptable level for all objectives. Alternatives are then compared with the given satisfying levels and selected with respect to their ability to meet the requirements, discarding those which prove less satisfactory. The upper levels of the objectives are set with minimization preferred and those for the lower levels seek maximization. Two undesirable results may arise from the use of this method.

First, none of the alternatives may be selected, an instance often found in problems in which the ob-

jective functions are in extreme conflict. The result, then, is that the objective with the superior value usually reflects lower performance. For example, consider a problem in which the acceptable upper limits of all objectives are set as follows:

- | | |
|------------------------------------|-----------|
| 1) Total energy costs: | 108552.83 |
| 2) Environmental impacts: | 84069.46 |
| 3) Performance and business risks: | 85412.97 |

No alternatives are selected on the basis of given acceptable levels. In this case an adjustment of the acceptable levels is required.

Second, consider the case when multiple solution alternatives exist. Tightening the levels would be a preferable option, but the principal difficulty would still remain, i.e., which objective acceptable level should be adjusted? The adjustment of a single objective level will have no effect upon the others, based on the assumption of objective independence.

4.2.2 Lexicographic Ordering

As noted in section 4.1, the satisfying level method considers all objectives as equal in importance. In reality, multiple objectives may be arranged in a hierarchical order in accordance with the decision maker's judgement of their values relative to each other. In contrast to the satisfying level, the lexicographic method assigns ordinal weights for evaluation

of the importance of each objective. The operational procedure, continued until a final solution is arrived at, is as follows:

- 1) The decision maker considers all objectives in a decreasing hierarchy of importance (i.e., establishes ranking);
- 2) The alternatives with the highest performance with respect to the highest ranked objective is selected; and
- 3) If more than two alternatives are selected, the solution alternative is chosen by consideration of the next higher ranked objective.

The advantage of this procedure lies in its simplicity. In most cases the decision maker reaches the final solution in relatively few steps. In the problem under consideration, if the environmental impacts objective is assumed to be the most important, the selected solution alternative is BV-9. This alternative reflects the poorest performance based for the total energy costs objective, a finding which implies that lexicographic ordering procedures, while requiring strongly independent assumptions, lack the ability to compensate poor performances from some objectives with superior performances on others (Holloway, 1979).

4.3 Value Function Method

The concept of the Multiobjective Value Function (MOV) method is a straightforward approach to the problem of multiobjective comparison (Troutt, 1988). For the evaluation of nondominated solution alternatives, the value function can be expressed as

$$V_j = \sum_{i=1}^n W_i V_i(X_{ij}) ,$$

where V_j = value of alternative j ,

W_i = weights assigned to objective i ,

$V_i(X_{ij})$ = value function or order-preserving utility function for X_{ij} , and

X_{ij} = solution outcome of objective i and alternative j .

This formula is expressed as a single additive weight value function. The total value of alternative j is obtained through the summation of weighted values. The weight assigned to objective i (W_i) reflects the marginal contribution to composite objectives, which is generally represented by a numerical value (cardinal value). In contrast, the lexicographic ordering procedure considers only the hierarchical ordering of objective importance (ordinal value) and this value is not even considered in satisfying level procedure.

Therefore, objective weight in the value function method is well-defined and unambiguous in its meaning.

Response to two principal queries are required in the application of the value function method: (1) how to assign the objective weight and (2) how to obtain value functions for solution outcomes. Step (1) may be resolved by the introduction of a reasonable method of evaluation. Step (2) is related to the scaling techniques that convert solution outcomes to a range of numerical scale values. This approach may encompass evaluation difficulties when both quantitative and qualitative factors are included in the problem.

4.3.1 Theoretical Background

In Chapter III, all sets of generated nondominated solutions were said to be subsets of three-dimensional Euclidian space. If subsets are finite and countable, value functions can exist. Proof of this theorem is subject to the binary relationship of the preference orders for a given subset of Euclidean space (Fishburn, 1970). It is certain that the given sets of nondominated solution alternatives satisfy these conditions. Detailed discussions of this theoretical background, including consideration of related issues, may be found in Keeney and Raiffa (1976), Einhorn and McCoach (1977), Fischer (1977), and Belton (1986).

4.3.2 Development of Objective Weight

Numerous methods for the assignment of objective weights have been proposed and potential applications for each method have been surveyed by Eckenrude (1965), Saaty et al. (1983), Hobbs (1980), and Mills (1988). Several of these alternative methods are considered below, including: (1) ranking, (2) the Churchman-Ackoff method, (3) rating, and (4) ratio method.

In the ranking method the decision maker orders objectives by decreasing preference, for instance assigning a numerical value of 1 for the most important objective, 2 for the next in importance, etc. "Importance," therefore, is ambiguously defined (Hobbs, 1980). The weight of each objective is calculated by the rank sum weight or the rank reciprocal weight. These methods are defined as follows (Eckenrude, 1965; Canada & Sullivan, 1988):

- 1) Rank sum method is

$$W_i = \frac{k - R_i + 1}{\sum_{i=1}^k (k - R_i + 1)}$$

and

- 2) Rank reciprocal weight is

$$W_i = \frac{1/R_i}{\sum_{i=1}^k (1/R_i)}$$

where W_i = gained weight of objective i ,

k = number of objectives, and

R_i = rank position of objective i .

An example of the results for these two methods with the ranking of each objective is shown in Table 4.1.

Table 4.1 Objective weights by ranking method.

Objective	Ranking (R_i)	Weight	
		Rank sum	Rank reciprocal
Total energy cost	2	0.333	0.273
Environmental impacts	1	0.500	0.545
Business and perform- ance risks	3	0.177	0.182

Totals:		1.000	1.000

Another (and similar) method, directed at provision of a consistency check for each assigned objective rank, was proposed by Churchman and Ackoff (1954).

This method, for reason of this check, is considered more defensible than ranking procedures (Hobbs, 1980).

It is composed of three steps:

- 1) The decision maker ranks objectives in decreasing preference order, assigning each a rating weight;
- 2) A consistency check is performed through the combination of the objective weights; and
- 3) Objective weights are adjusted in inconsistency occurs.

This procedure can be time-consuming when there are a large number of objectives and alternatives.

Rating is a commonly used method. The decision maker rates each objective on a numerical scale, e.g. from 1 to 10, with 1 as the least important and 10 as the most important. Though this method is attractive from the standpoint of ease of use, it cannot provide assurance that the weight is theoretically valid. Moreover, the definition of importance through a rating value does not reflect the relative value of unit changes in value functions (Hobbs, 1980).

The ratio method uses questioning procedures to develop a ratio of importance of any two objectives. This procedure is applied to all possible combinations of objectives until a comprehensive ratio of objectives is obtained. The total number of possible questions is $k(k-1)/2$, where k is the number of objectives.

For example, if a value of 2 is assigned in the comparison of two objectives, Z_1 and Z_2 , then objective Z_1 is twice as important as objective Z_2 . Equal importance is assigned by a value of 1. The range of importance values is not limited in the ratio method, but usually values of 1 to 9 are used (Saaty, 1980). Consistency checks are provided by the use of a $(k \times k)$ matrix format of paired comparisons or by eigenvector prioritization method, suggested by Saaty (1980). This procedure is analyzed in greater detail in Chapter 5.

4.3.3 Development of Value Functions

Following the assignment of a weight to each objective, value functions for solution outcomes must be developed. The most common method is to assign numerical values (cardinal scaling) with respect to the degree of solution outcome. An arbitrary scale may be used, but ten-point (0 to 10) or one hundred-point (0 to 100) scales are the logical and usual choices since they lend themselves to the intuitive ability of the decision maker (West, 1976). In use of the scaling method, certain critical assumptions should be pointed out. First, when the ten-point method is applied, a scale value of 4.0 is twice as favorable as 2.0. In like manner, a scale value of 2.0 is twice as favorable as 1.0. Second, when the assigned scaling values are identical for all objectives, the implication is that the difference between scale values of 4.0 and 2.0 for one objective is identical to the difference between the same scale values for other objectives.

The measurable value function of the solution outcome is a step function since sets of generated nondominated solutions are treated as a finite number of discrete random variables. A large number of solution outcomes enable a continuous function for the purpose of convenient calculation of scaling value. Figure 4.1 displays the continuous value function rather than the

discrete step function. As an alternative method for obtaining scaling values with relatively simple procedures, rating intensity may be defined. The purpose of rating intensity is to define the solution outcomes by several groups, according to their similarities. Five groups may be introduced, including: (1) excellent, (2) above average, (3) average, (4) below average, and (5) poor. The ranges of rating intensity for each objective are calculated from:

$$\begin{aligned} & (\text{best solution}) + (\text{worst solution} \\ & \quad - \text{best solution}) \cdot k/5 , \end{aligned}$$

where k is the rating intensity (i.e., 1 through 5 for, respectively, excellent through poor). These are also given in the horizontal axis of Figure 4.1 and rating intensities for each solution outcome are provided in Table 4.2. Ranges of scaling values are assigned from 1 to 9, rather than from 1 to 10. The reason for use of this range is to allow use of the consistency analysis for the Analytic Hierarchy Process (AHP) considered in Chapter 5. (Scaling value of 1 for worst solution outcome, with a rating intensity of "poor," to 9 for the best solution outcome, with a rating intensity of "excellent.") Tables 4.3 and 4.4 show the solution outcomes and rating intensities corresponding to the scaling values. These figures have been interpreted in accordance with the system described in Figure 4.1.

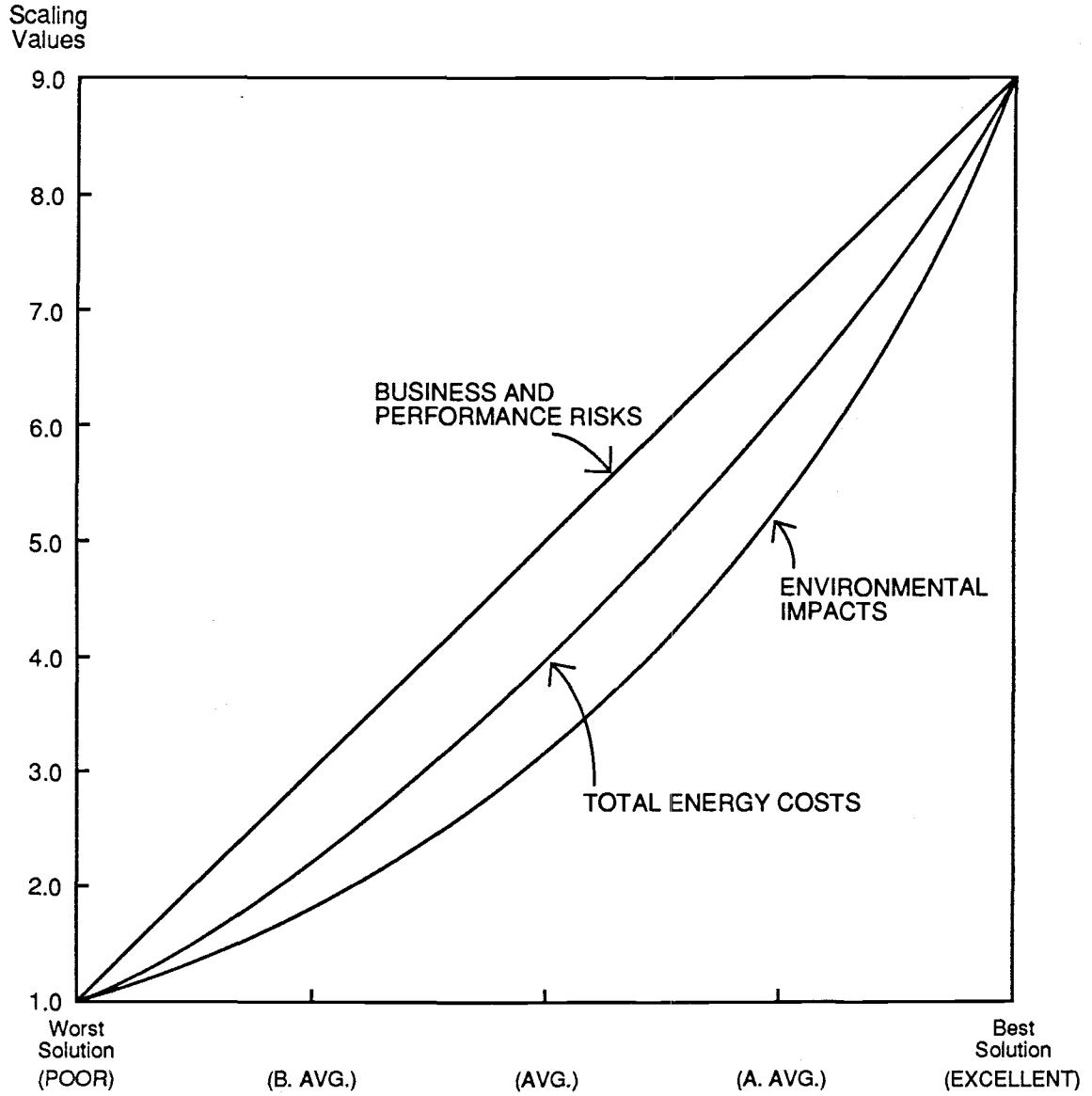


Figure 4.1 Value functions.

Table 4.2 Rating intensities.

Alt.	Energy Costs	Environmental Impacts	Business & Perf. Risks
AE-1	POOR	A AVG	AVG
AE-2	POOR	AVG	A AVG
AE-3	B AVG	AVG	AVG
AE-4	AVG	AVG	POOR
AE-5	B AVG	B AVG	A AVG
AE-6	AVG	B AVG	AVG
AE-7	A AVG	B AVG	POOR
AE-8	AVG	POOR	A AVG
AE-9	A AVG	POOR	AVG
AE-10	EXCELLNT	POOR	POOR
AV-1	A AVG	POOR	AVG
AV-2	A AVG	B AVG	POOR
AV-3	B AVG	B AVG	A AVG
AV-4	B AVG	AVG	AVG
AV-5	B AVG	A AVG	B AVG
AV-6	POOR	AVG	A AVG
AP-1	A AVG	AVG	POOR
AP-2	A AVG	B AVG	B AVG
AP-3	A AVG	POOR	AVG
AP-4	B AVG	AVG	AVG
AP-5	B AVG	B AVG	A AVG
AP-6	B AVG	POOR	EXCELLNT
AP-7	POOR	AVG	A AVG
AP-8	POOR	B AVG	EXCELLNT
AP-9	POOR	B AVG	EXCELLNT
BE-1	POOR	A AVG	AVG
BE-2	B AVG	A AVG	B AVG
BE-3	POOR	AVG	A AVG
BE-4	B AVG	AVG	AVG
BE-5	AVG	AVG	AVG
BE-6	B AVG	POOR	A AVG
BE-7	AVG	POOR	A AVG
BV-1	B AVG	POOR	A AVG
BV-2	B AVG	B AVG	AVG
BV-3	B AVG	AVG	B AVG
BV-4	POOR	AVG	A AVG
BV-5	POOR	AVG	AVG
BV-6	POOR	A AVG	B AVG
BV-7	POOR	AVG	A AVG
BV-8	POOR	A AVG	AVG
BV-9	POOR	EXCELLNT	B AVG
BP-1	B AVG	A AVG	B AVG
BP-2	B AVG	AVG	AVG
BP-3	B AVG	POOR	EXCELLNT
BP-4	POOR	A AVG	AVG
BP-5	POOR	AVG	A AVG
BP-6	POOR	B AVG	EXCELLNT
BP-7	POOR	A AVG	AVG
BP-8	POOR	AVG	EXCELLNT

Table 4.3 Scaling values to solution outcomes.

Scaling Values	Solution Outcomes		
	Energy Costs	Environ. Impacts	Perform. Risks
9	102140	73825	82524
8	103410	75126	84329
7	104841	76826	86135
6	106398	78730	87940
5	108152	81028	89745
4	110106	83829	91550
3	112292	87420	93355
2	114835	92033	95160
1	118171	99435	96965

Table 4.4 Scaling values to rating intensities.

Rating Intensity	Energy Costs	Environ. Impacts	Perform. Risks
EXCELLNT	9.0	9.0	9.0
A AVG	6.2	5.3	7.0
AVG	4.0	3.2	5.0
B AVG	2.2	1.8	3.0
POOR	1.0	1.0	1.0

In utility function the scaling values shown in Figure 4.1 could be "utility" values, which are usually given on a scale of 0 to 1. A utility function is a value function which satisfies specific theoretical properties. In the present study, the utility function approach has not been applied since the MOLP model is deterministic and generated nondominated solutions are already defined. This approach is suitable for decision problems with priori articulations of the decision

maker's preferences, especially when stochastic elements play an important role in the decision process (i.e., the scaling procedure). This problem can be accommodated by the introduction of probability distributions for given consequences, i.e., for the solution outcomes (Szidarovszky, 1986). However, from the viewpoint of the utility concept, the total energy costs and environmental impacts objectives represent risk-seeking attitudes. The linear function of the business and performance risks objective indicates a risk-neutral attitude.

4.3.4 Results

The development of objective weights may be derived through application of the methods discussed above. Each method generates a different set of weight values and the choice of a proper method is dependent upon the decision maker's judgement and understanding of the strengths and weaknesses of each procedure. The ratio method, with Saaty's (1980) pairwise comparison procedures, has been applied in the development of objective weights indicated in Table 4.5.

The matrix form representing the ratio of importance by pairwise comparison is developed and explained in Chapter 5. Summary results of the value function method are given in Table 4.6. Table 4.7 reflects the results obtained by application of rating intensity.

Table 4.5 Objective weights.

Objective	Weight
Total energy costs	0.3695
Environmental impacts	0.4067
Business & performance risks	0.2238

Total:	1.0000

While both of these results indicate that alternative BV-9 is preferred, no particular relationship between the two procedures exists. The results obtained by the use of rating intensity involve somewhat robust decision making. This may reduce the influence and dependency of changes in objective weights and scaling values. All solution alternatives generated from the MOLP model have been predefined in accordance with their solution outcomes. The consideration of other possible alternatives has not been encompassed for reason of the non-independence among objectives. For problems in which other alternatives can be considered, the rating intensity method provides a flexible decision making process. However, two important principles should be borne in mind for grouping solution outcomes: (1) maximization of inter-group variance and (2) minimization of intra-group variance (Tzeng & Shiau, 1987).

Table 4.6 Results of value function method.

Alts	Energy (0.3695)	Environ (0.4067)	Per. risk (0.2238)	Weights	Rank
AE-1	1.2	5.7	5.3	3.948	3
AE-2	1.6	3.3	6.8	3.455	27
AE-3	2.8	3.3	4.6	3.406	35
AE-4	4.4	3.3	2.4	3.505	19
AE-5	2.9	1.9	6.8	3.366	47
AE-6	4.5	1.9	4.6	3.465	24
AE-7	6.4	1.9	2.4	3.675	12
AE-8	3.6	1.4	6.8	3.421	30
AE-9	5.9	1.2	4.6	3.698	8
AE-10	9.0	1.0	2.4	4.269	2
AV-1	5.5	1.3	4.6	3.590	16
AV-2	5.5	2.4	2.4	3.545	17
AV-3	3.0	1.8	6.8	3.362	48
AV-4	3.0	3.1	4.6	3.399	36
AV-5	3.0	4.7	2.8	3.647	13
AV-6	1.4	3.6	6.8	3.503	21
AP-1	5.5	3.3	1.0	3.598	15
AP-2	5.5	1.9	3.3	3.543	18
AP-3	5.5	1.2	4.9	3.617	14
AP-4	3.0	3.3	4.3	3.413	31
AP-5	3.0	1.9	6.7	3.381	42
AP-6	3.0	1.5	7.5	3.397	38
AP-7	1.5	3.3	7.0	3.463	26
AP-8	1.8	1.9	8.8	3.407	34
AP-9	1.8	1.7	9.0	3.371	46
BE-1	1.4	4.9	5.6	3.763	7
BE-2	2.3	4.9	3.8	3.693	9
BE-3	1.8	2.8	7.5	3.482	23
BE-4	2.8	2.7	5.6	3.386	41
BE-5	3.8	2.7	4.0	3.397	37
BE-6	3.2	1.5	7.3	3.426	29
BE-7	4.4	1.3	6.0	3.497	22
BV-1	3.0	1.6	7.3	3.393	39
BV-2	3.0	2.5	5.6	3.378	43
BV-3	3.0	3.7	3.8	3.464	25
BV-4	1.8	2.6	7.3	3.356	49
BV-5	1.8	3.9	5.6	3.504	20
BV-6	1.8	5.7	3.8	3.834	4
BV-7	1.5	3.0	7.3	3.408	33
BV-8	1.2	5.2	5.6	3.811	5
BV-9	1.0	9.0	3.8	4.880	1
BP-1	3.0	4.9	2.6	3.683	11
BP-2	3.0	2.7	5.3	3.393	40
BP-3	3.0	1.6	7.5	3.438	28
BP-4	1.8	4.9	4.6	3.687	10
BP-5	1.8	2.7	7.2	3.374	44
BP-6	1.8	1.7	9.0	3.371	45
BP-7	1.3	4.9	5.8	3.771	6
BP-8	1.6	2.7	7.7	3.413	32

Table 4.7 Results of value function method
(rating intensities).

Alt.	Energy (0.3695)	Environ. (0.4067)	Per. risks (0.2238)	Weights	Rank
AE-1	1.0	5.3	5.0	3.644	10
AE-2	1.0	3.2	7.0	3.238	23
AE-3	2.2	3.2	5.0	3.233	32
AE-4	4.0	3.2	1.0	3.003	44
AE-5	2.2	1.8	7.0	3.112	41
AE-6	4.0	1.8	5.0	3.329	20
AE-7	6.2	1.8	1.0	3.247	21
AE-8	4.0	1.0	7.0	3.451	18
AE-9	6.2	1.0	5.0	3.817	4
AE-10	9.0	1.0	1.0	3.956	2
AV-1	6.2	1.0	5.0	3.817	4
AV-2	6.2	1.8	1.0	3.247	21
AV-3	2.2	1.8	7.0	3.112	41
AV-4	2.2	3.2	5.0	3.233	32
AV-5	2.2	5.3	3.0	3.640	15
AV-6	1.0	3.2	7.0	3.238	23
AP-1	6.2	3.2	1.0	3.816	7
AP-2	6.2	1.8	3.0	3.694	8
AP-3	6.2	1.0	5.0	3.817	4
AP-4	2.2	3.2	5.0	3.233	32
AP-5	2.2	1.8	7.0	3.112	41
AP-6	2.2	1.0	9.0	3.234	30
AP-7	1.0	3.2	7.0	3.238	23
AP-8	1.0	1.8	9.0	3.116	38
AP-9	1.0	1.8	9.0	3.116	38
BE-1	1.0	5.3	5.0	3.644	10
BE-2	2.2	5.3	3.0	3.640	15
BE-3	1.0	3.2	7.0	3.238	23
BE-4	2.2	3.2	5.0	3.233	32
BE-5	4.0	3.2	5.0	3.898	3
BE-6	2.2	1.0	7.0	2.786	46
BE-7	4.0	1.0	7.0	3.451	18
BV-1	2.2	1.0	7.0	2.786	46
BV-2	2.2	1.8	5.0	2.664	49
BV-3	2.2	3.2	3.0	2.786	48
BV-4	1.0	3.2	7.0	3.238	23
BV-5	1.0	3.2	5.0	2.790	45
BV-6	1.0	5.3	3.0	3.196	37
BV-7	1.0	3.2	7.0	3.238	23
BV-8	1.0	5.3	5.0	3.644	10
BV-9	1.0	9.0	3.0	4.701	1
BP-1	2.2	5.3	3.0	3.640	15
BP-2	2.2	3.2	5.0	3.233	32
BP-3	2.2	1.0	9.0	3.234	30
BP-4	1.0	5.3	5.0	3.644	10
BP-5	1.0	3.2	7.0	3.238	23
BP-6	1.0	1.8	9.0	3.116	38
BP-7	1.0	5.3	5.0	3.644	10
BP-8	1.0	3.2	9.0	3.685	9

4.4 ELECTRE Method

The ELECTRE (Elimination and Choice Translating Algorithm) is another evaluation method for ranking (ordering) the alternatives. This method has been presented in two models: ELECTRE I and ELECTRE II¹ (Goi-coechea, 1982). ELECTRE I yields only a partial ordering of the alternatives, whereas ELECTRE II proves a complete ordering. Thus, ELECTRE I acts as a screening technique for reduction of the number of alternatives. For practical purposes, both models are combined into a single overall method for ranking alternatives.

Several studies of the application of this method to the energy and environmental planning area have been conducted: the evaluation of waste disposal systems (Albrecht, 1980); energy conservation strategies in the urban transportation sector (Tzeng & Shiau, 1987), and an analysis of energy supply systems (Capros et al., 1988).

The basic concept of ELECTRE I is the outranking relationships which allow the ordering of alternatives. These are represented by a concordance index, $c(i,j)$, and a discordance index, $d(i,j)$. Alternative i is preferred to alternative j ($i > j$) if and only if $c(i,j) \geq p$ and $d(i,j) \leq q$, where p and q ($0 \leq p \leq 1$,

¹ELECTRE I and II were introduced, respectively, by Benayoun, Roy, and Sussman (1966) and Roy and Bertier (1971).

$0 \leq q \leq 1$) are threshold values determined by the decision maker. The indices $c(i, j)$ and $d(i, j)$ are defined as follows:

$$c(i, j) = \frac{\sum_{k \in i_k > j_k} W_k + \frac{1}{2} \sum_{k \in i_k = j_k} W_k}{\sum_k W_k}$$

and

$$d(i, j) = \text{Max}_{k \in i_k < j_k} \frac{i_k(f/l) - j_k(\bar{f}/l)}{k(l)},$$

where

W_k = the k^{th} objective weight,

$i_k > j_k$ = alternative i , which is superior to alternative j at the k^{th} objective,

$i_k = j_k$ = when alternatives i and j have no difference at the k^{th} objective,

$i_k < j_k$ = when alternative i is inferior to alternative j at the k^{th} objective,

$i_k(f/l) - j_k(\bar{f}/l)$ = scale interval at the k^{th} objective, and

$k(l)$ = total range of scale.

As can be seen from the above formulas, objective weights and interval scales should be developed prior to calculation of the concordance and discordance indices. Then, a preference graph of the results from ELECTRE I is constructed to determine the kernel representing the preferred alternatives. Complete ordering of the alternatives, which have already been partially

ordered, requires the use of the ELECTRE II model.

There are shortcomings to this method, including:

- 1) Unlike the value function method, decision maker's preferences are not fully defined (Tzeng & Shiau, 1987);
- 2) When the ELECTRE II model is used for the complete ordering of alternatives, the development of the concordance index is very complicated due to selection of a large number of parameters; and
- 3) ELECTRE II does not amplify small differences between evaluation of the decision maker's judgements (Capros et al., 1988).

4.5 Conclusion

The inclusion of qualitative factors expressed as numerical values simplified the MOLP model through the capture of different factor units. However, under other circumstances, this technique could produce difficulties when using the tradeoff value method, which is the most valid approach for evaluating alternatives.

The methods reviewed in this chapter were not originally intended for the evaluation of sets of non-dominated solutions, rather they were intended for problems with predefined and restricted numbers of alternatives. In other words, these methods were developed for problems for which the decision maker has

priori articulation of information. Therefore, strong assumptions and limitations (i.e., non-interdependence of objectives) are always required. Nonetheless, almost all of the methods reviewed are relatively easy to use and to understand. Descriptive methods, satisfying level, and lexicographic ordering procedures are all extremely easy to use, but they cannot often guarantee an optimal solution. While the ELECTRE methods can be more finely refined, they require complicated procedures, particularly for problems having a large number of objectives or alternatives. The value function method is a very powerful and flexible procedure, within which method rating intensity can be adopted as a primary filtering step for narrowing the choice of alternatives to a manageable quantity. This technique may provide a basis for the development of more elaborated evaluation methods with the potential of increasing the decision maker's confidence level.

5. ANALYTIC HIERARCHY PROCESS

The analytic hierarchy process (AHP) decomposes a decision problem into multiple levels of decision elements to which judgement value is provided. The procedures differ from classical methods of evaluation in their practical nature and theoretical backgrounds. For example, eliciting processes of developing objective weights and evaluating alternatives have been combined from different methods in the value function method in order to obtain the total weight of each alternative. In the AHP, both procedures are identical and input data are calculated using a pairwise comparison method.

The characteristics of AHP are deterministic. There is no ability to permit the assessment of risk attitudes, such as in the utility function approach (Arbel & Seidmann, 1984). Moreover, AHP assumes the "intransitivity"¹ of preference when decision elements are compared. AHP has been successfully applied to various problems since it can be used either as a normative or descriptive tool. In energy and environmental planning this includes industrial energy rationing

¹If $A \succ B$ and $B \succ C$, the "intransitivity" implies $A \succ C$. When "transitivity" is applied, then A must be $\succ C$.

(Saaty & Mariano, 1979), regional energy planning (Blair, 1980), and nuclear waste management (Saaty & Gholamnezhad, 1982). A number of similar applications have been adequately surveyed by Zahedi (1986), Saaty (1982), and Dyer et al. (1988).

The analytical procedures of AHP involve four major components (Zahedi, 1986), which are:

- 1) Decomposition of decision problems into a hierarchy of decision elements (objectives, alternatives);
- 2) Pairwise comparisons of decision elements for collecting the decision maker's judgement;
- 3) Estimation of the relative weights (priority weight) of the decision element, determined by the eigenvalue method; and
- 4) Aggregation of the relative weights for the final decision.

Discussion of these four components is based on Saaty's first publication (1980).

5.1 AHP Procedures

5.1.1 Decomposition of the Problem

The application of AHP for solving decision problems begins with the decomposition of a problem into a hierarchy of decision elements. Small-sized problems do not always require this step. However, structuring the decision hierarchy is a useful tool for identifying

and understanding the relationships between the decision elements. This is particularly true for complex problems. Saaty also noted that it yields a great deal of information on the structures and purposes of the problem.

General rules to structure a hierarchy do not exist, but the most specific elements of the problems should be included, e.g., the goals for the problem, objectives (criteria), and alternatives (Wind & Saaty, 1980). Figure 5.1 shows a typical hierarchical diagram for an AHP with m levels. Level 1 indicates the overall goal or the macro-objective of the problem; level 2 details the objectives; and level 3 includes objectives in much greater detail (i.e., subobjectives). The lowest level represents the course of actions or alternatives for the problem. The number of elements at each level, with the exception of the first level, are suggested to be limited to a maximum of seven to nine for convenient pairwise comparison (Saaty, 1980).

5.1.2 Pairwise Comparison

This step relates to the development of input data to obtain the objective weight and alternative scores. Collection of the input data is formatted through the matrix of pairwise comparisons by asking the decision maker to evaluate the elements of one level with respect to the next higher levels. The decision maker's

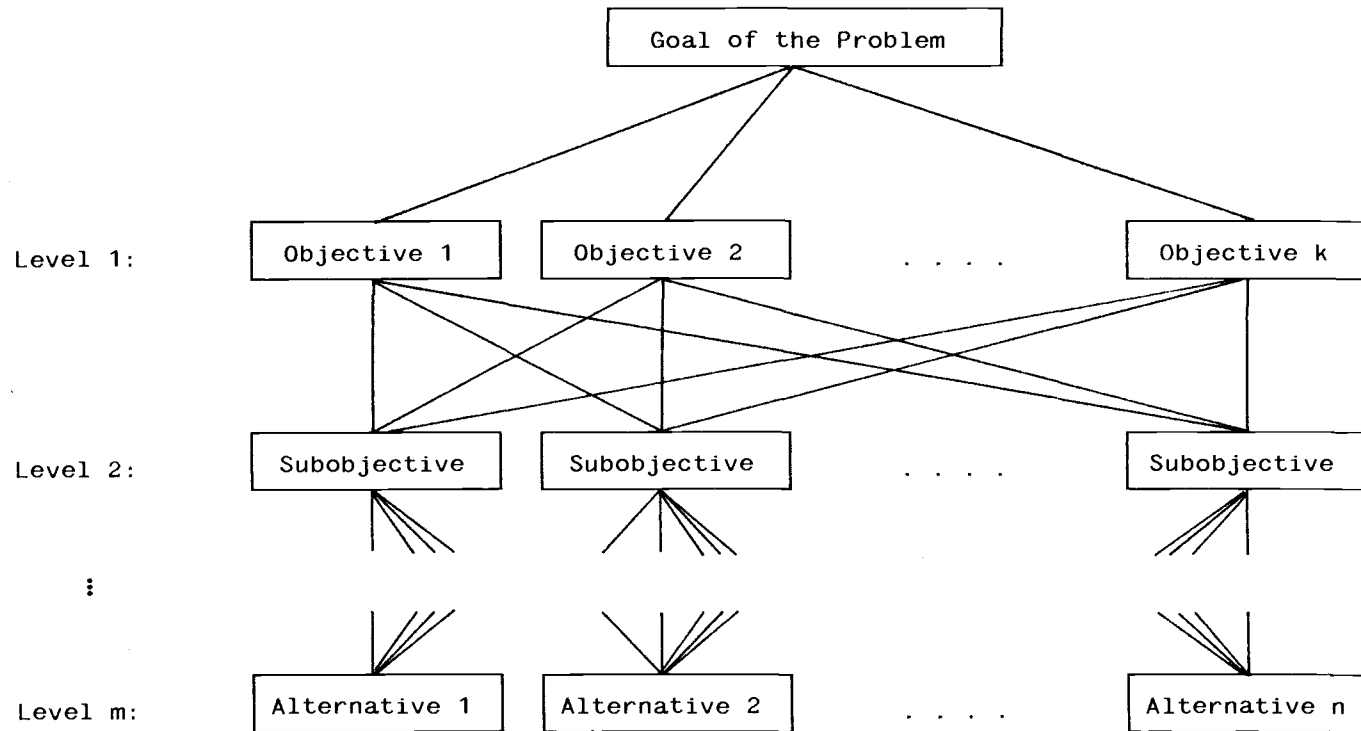


Figure 5.1 AHP hierarchical diagram.

judgement is represented by the "importance value" between two elements. When n elements are being compared, $n(n-1)/2$ questions are required. As an example, the matrix form of pairwise comparisons in the four elements at one level can be represented by:

	A	B	C	D
A	1	3	5	6
B	1/3	1	2	3
C	1/5	1/2	1	2
D	1/6	1/3	1/2	1

Judgement values in row 1 and column 2 indicate that element A is three times as important as element B. In transposed position (row 2, column 1), value 1/3 represents the reciprocal of value 3. The pairwise comparisons reflect diagonal relationships, meaning that once one-half of the matrix element has been collected, the remaining elements can be obtained by determining the corresponding reciprocal values of each. Judgement values are quantified by a nine-point scale, which is shown in Table 5.1.

5.1.3 Estimation of the Relative Weight

The relative weights of elements at each level can be estimated from the pairwise comparison matrix. This step is divided into two components: estimation of relative weights and consistency checks (consistency index (CI) and consistency ratio (CR)).

Table 5.1 Scale values for pairwise comparisons.

Value	Definition	Explanation
1	Equal importance	Both elements contribute equally
3	Weak importance	Experience and judgement slightly favor one element over another
5	Strong importance	An element is strongly favored
7	Very strong importance	An element is very strongly favored
9	Absolute importance	Favoring one factor over another is unquestionable
2,4 6,8	Intermediate values between two adjacent	Used for compromise between two judgements
Incr. of 0.1	Intermediate values in increments of 0.1 (example: 6.3)	Used for even finer gradations of judgements

Source: Saaty (1986), Bard (1986b).		

If we let A be the matrix of pairwise comparison with n elements at one level of the hierarchy, the relative weights may be depicted by:

$$A = [a_{ij}] = \begin{matrix} & \begin{matrix} 1 & 2 & \dots & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ \vdots \\ n \end{matrix} & \begin{bmatrix} W_1/W_1 & W_1/W_2 & \dots & W_1/W_n \\ W_2/W_1 & W_2/W_2 & \dots & W_2/W_n \\ \vdots & \vdots & \dots & \vdots \\ W_n/W_1 & W_n/W_2 & \dots & W_n/W_n \end{bmatrix} \end{matrix} .$$

Thus, the following equation holds:

$$\sum_{j=1}^n a_{ij} W_j = n W_i, \quad i=1, \dots, n$$

which is equivalent to

$$AW = nW ,$$

where $W =$ the relative weight, $W = (W_1, W_2, \dots, W_n)^T$,

and

$n =$ the number of elements and is called the largest eigenvalue.

In practical situations the decision maker does not know the ratio W , therefore accurate values of the relative weight W of matrix A are not available. Instead, use the largest eigenvalue λ_{\max} of matrix A to obtain an estimate of the weight W . Thus, matrix A always contains inconsistencies and the above formula becomes

$$AW = \lambda_{\max}W .$$

From this formula, the relative weight, W , can be obtained.

The eigenvalue λ_{\max} is always greater than or equal to n ($\lambda_{\max} \geq n$) since it is estimated from n . If λ_{\max} is equal to n , perfect consistency exists. From this point of view, the consistency index (CI) is defined by

$$CI = (\lambda_{\max} - n) / (n - 1)$$

and the consistency ratio (CR) is

$$CR = (CI / RI) ,$$

where RI is the average index of randomly generated weights.

Saaty obtained the RI values through the simulation shown below, suggesting that a CR value of 0.1 or less is considered an acceptable level for the results.

n	1	2	3	4	5	6	7	8	9
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45

5.1.4 Aggregation of the Relative Weight

The aggregation of relative weights at each level is calculated from step 3. Then, the composite relative weights of the elements of the lowest level can be obtained. In problems with three levels, the objective weight can be calculated directly from the pairwise comparison matrix. Alternative scores at level 3 are also calculated from the pairwise comparison matrix. Finally, the composite relative weights for each alternative with respect to level 1 can be calculated from:

Relative weight of alternative n

$$= \sum_{\text{all obj.}} (W_k \cdot S_{kn}) ,$$

where W_k : relative weights of objective k, and

S_{kn} : scores of alternative n with respect to objective k.

The detailed calculation procedures, as applied to the problem considered in the present study, are explained in Chapter 6.

5.2 Extensions of AHP

Since Saaty introduced the AHP procedure, various alternative methods and criticisms have been provided (Zahedi, 1986). Most of them have concentrated upon the method of estimating relative weights. For general AHP applications, the eigenvalue method has been used most often, largely for the reason that it provides a consistency index. Saaty (1980) also suggested the use of the geometric mean, the harmonic mean, and logarithmic least squares.

The geometric mean is a suitable method when multiple decision makers are involved. This method permits the use of several judgements through pairwise comparisons. However, the logarithmic least square method offers statistical optimality properties under specific practical models of pairwise comparison (Jong, 1984). Jensen (1984) proposed an alternative for this method, a least squares method which yields least squares optimal weights. Renormalization procedures have also been proposed to lessen the ambiguity of relative weights. Redefining questions to enable pairwise comparison of objectives was developed by Belton and Gear (1983, 1985). Though the strength of the AHP lies in its use of the eigenvalue method, the most

desirable method for the estimation is yet to be determined.

The assumptions of independence between decision elements provides simple procedures and minimal effort for the solution of multiple objective problems. Like most other methods, the AHP mandates this degree of independence. However, in some cases, relaxing the independence requirement plays an important role. A recent methodology has been suggested by Saaty and Takizawa (1986) which is directed at solving problems involving the functional dependence of objectives on alternatives, of objectives on objectives, and alternatives on alternatives. This method is based on the AHP feedback systems and, of course, its procedures are more complicated than those of standard AHP procedures.

5.3 Methodological Comparisons

AHP can be applied only to problems in which the conditions for the existence of multiobjective value functions (MOV) are satisfied (Kamenetzky, 1982). Therefore, in this section AHP is mainly compared with the MOV method with emphasis on practical points of view. When considering various methodologies, the principal difference between the two methods lies in the process of developing objective weights and alternative scores. Although these procedures can be elicited easily in AHP, they can be strengthened consider-

ably by the adoption of more conventional definitions (Belton, 1986). The most interesting considerations are included below:

1) Problem structures.

The structural (hierarchical) diagram of the MOV method differs from that of the AHP. This is shown in Figure 5.2, indicating an approach to a similar AHP problem given in Figure 5.1. Belton (1986) observed that even though these two diagrams have a different structure, it is worth noting that the decision elements at one level are connected, each contributing to the next higher level. In other words, no differences of any consequence exist between the two methods.

2) Quantitative data.

Each method has the ability to deal with both quantitative and qualitative data. However, the AHP is basically oriented to the use of qualitative data since it requires only qualitative inputs (e.g., importance value) (Bard, 1986). This is not necessarily a disadvantage. When the decision maker wants to differentiate between two outcomes, the application of importance value with AHP can amplify differences with greater facility than the MOV method.

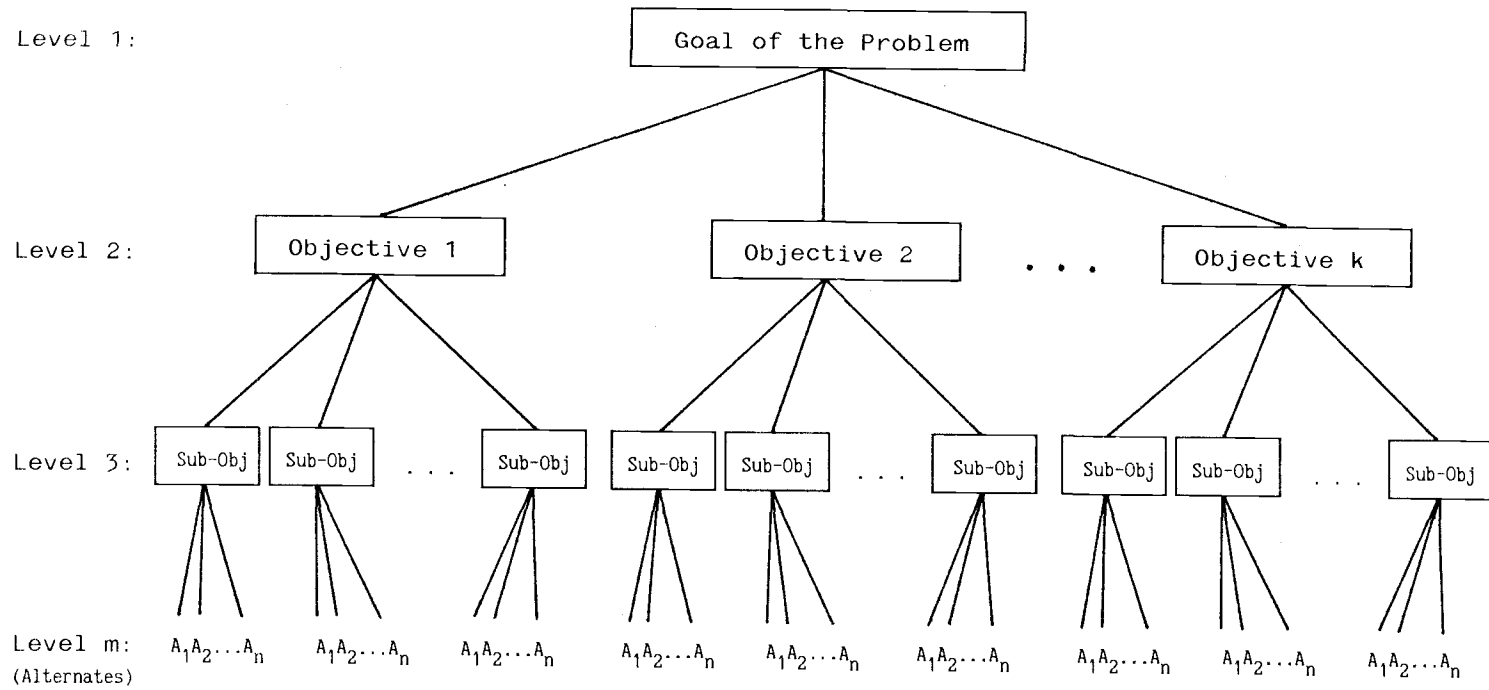


Figure 5.2 MOV hierarchical diagram.

3) Attitude toward risks.

As discussed above, both methods are deterministic and do not allow stochastic elements. Unlike the AHP, the MOV method may be changed to a multiple attribute utility process by the introduction of probability distributions. In this case, some restrictions and assumptions are applied. The AHP is not appropriate for the type of problem in which stochastic elements are the key factors of consideration.

4) Introduction of new alternatives.

For AHP, several studies related to the occurrence of rank reversal by the introduction of new alternatives have been reported (Troutt, 1988; Belton & Gear, 1983, 1985). This issue is still controversial and no real comparative judgements can be offered in this section.

5) Consistency.

Since the AHP assumes "intransitivity" in the pairwise comparisons, it requires a consistency check for accurate problem analysis. In the MAV method, "intransitivity" is not permitted. Therefore, it can provide no formal mechanism for checking reliability (Belton, 1986).

6) Number of alternatives.

Both methods encompass significant computational burdens for problems with large numbers

of alternatives. This is especially true for the AHP. This condition has led to the recommendation that the number of alternatives be reduced prior to applying AHP procedures for purposes of ease of analysis and the derivation of maximum benefits from use of the method.

From the above discussion, it may be concluded that each method has its own specific strengths and weaknesses. Ultimately, decisions with regard to the selection of an appropriate method are dependent on the characteristics of the problem under consideration and its requirements with respect to trustworthiness and performance or other factors discussed in Chapter 4.

6. PROPOSED METHODOLOGY: MULTIPLE SCREENING METHOD

The primary objective of this study has been to propose a methodology for the selection of a final alternative from listed nondominated solutions which are characterized by processing large numbers of solution alternatives, different units of measurements, and conflicting objectives. These development of this methodology is discussed in this chapter.

The final solution alternative implies the best compromise solution for the decision maker's acceptable confidence level. Cohon (1978) pointed out that the greatest degree of interaction between the decision maker and the decision problem will result in the best performance for a problem solution. Zeleny (1982) also noted that raising the decision maker's confidence level is accomplished by the inclusion of a large number of feasible alternatives at each step of the evaluation process and the extent to which there is correspondence between the decision maker's judgement and the analytical process. These two requirements create the demand for complicated evaluation procedures. Therefore, the methodology presented in this chapter,

the "multiple screening method," is intended to reconcile the conflicts within these requirements. The analytical tools in the proposed method are based on the pairwise comparison of the AHP approach, incorporated with other procedures.

The proposed method is comprised of multiple screening procedures that allow step-by-step pruning of the solution alternatives. The main components of the method are as follows:

- 1) Development of rating intensity measurement;
- 2) Reduction of alternatives by a disjunctive method;
- 3) Pairwise comparison of the rating intensities;
- 4) Pairwise comparison of the solution outcomes;
and
- 5) Post-optimality (sensitivity) analysis.

These components may be grouped in two categories. The first three are required in order to arrive at a final decision without complex procedures, while the solution alternatives are easily reduced through rating intensities and the disjunctive method. The last two components are intended to reinforce the decision maker's confidence level, which is selected through decision maker judgement. The procedures are based on the assumption that objectives and alternatives are independent considerations.

6.1 Description of the Methodology

The sequence of step for this method is illustrated in Figure 6.1. Detailed explanation for each step are listed in the following sections.

6.1.1 Development of the Objective Weight

While numerous methods for the development of objective weights are available, the AHP pairwise comparison method is applied to the proposed method. The motivation for this decision lies mainly in the ability of this method to accept "intransitivity" while providing the same procedures for evaluation of both the objectives and the alternatives.

6.1.2 Determination of Rating Intensity

As discussed in Chapter 4, the rating intensity for all solution outcomes for each objective identifies the solution group or cluster. In the proposed method, rating intensity provides two important factors. First, rating intensities are used as guidelines in the selection of the acceptable levels which are required in the disjunctive method. Second, rating intensities facilitate the AHP pairwise comparisons by replacement of solution alternatives. The number of rating intensities will match the number of alternatives, no matter how many alternatives are involved. The determination

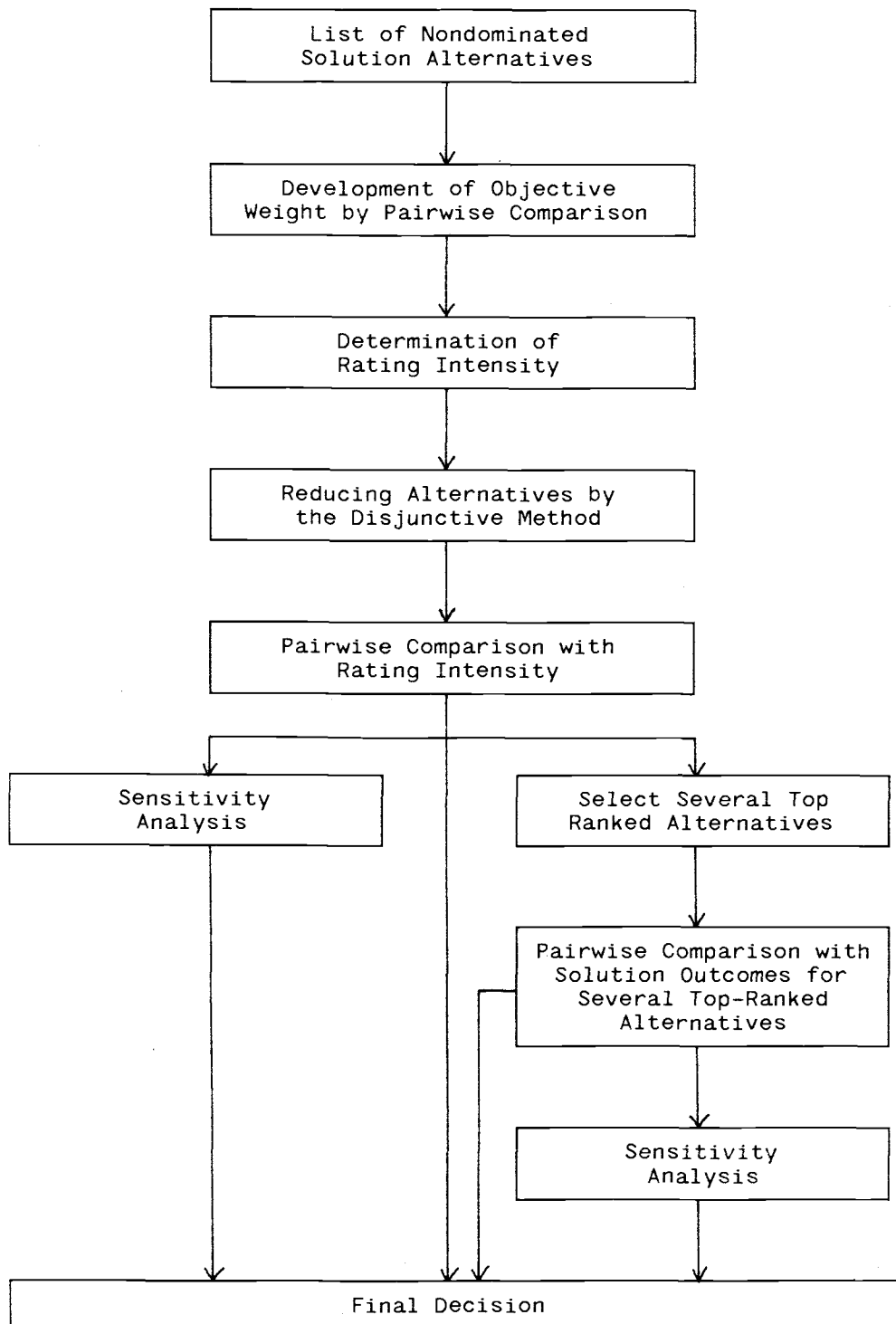


Figure 6.1 Sequence of Steps for Multiple Screening Method.

of the number of rating intensities is entirely dependent on the nature of the problem faced and the decision maker's judgement. When the objectives are in extreme conflict within the problem, the number of rating intensities will be increased. However, it is generally recommended that five to seven rating intensities seem to be a reasonable number. If too many are defined, then the benefit of compact procedures will be lost. If too few are used, then the procedure will be too robust for the small number expected.

6.1.3 Reduction of Alternatives

After assignment of rating intensities according to the number of solution outcomes, alternatives must be narrowed down to a manageable number. Two considerations are important in accomplishing this step: (1) Providing a reasonable number of alternatives and (2) not neglecting alternatives which offer the best performance. The meaning of "reasonable number" as applied to alternatives is ambiguous. However, the above two considerations are affected by the adapted method.

Although several methods for reducing alternatives have been suggested (Morse, 1980), the simple descriptive method is applied in this study. Two methods are considered: conjunctive and disjunctive. The conjunctive method is basically identical to the satisfying level procedure. As could be expected, this method is

not suitable for problems in which some of the objectives are in extreme conflict, as is the case in the present study. For the disjunctive method, solution alternatives are selected that meet the given acceptable level or the rating intensity for at least one objective. Therefore, this method has been chosen for the present study since it satisfies the two criteria noted above. However, the number of alternatives to be selected by the decision maker cannot be controlled unless the given acceptable level is adjusted.

6.1.4 Pairwise Comparison of Rating Intensity

Alternatives selected by the disjunctive method may not guarantee a reasonable number of alternatives. This condition is beyond control with AHP pairwise comparison procedures. Saaty (1980) recommended limiting to nine the number of elements at each hierarchical level. As discussed in the preceding section, rating intensities act as alternatives for the pairwise comparison procedures. Therefore, the number alternatives must equal the number of rating intensities. After obtaining the relative weight of the rating intensities, the composite weights for each of the solution alternatives can be calculated.

This procedure is valid only when the number of alternatives selected by the disjunctive method are much greater than those of the rating intensities. The

converse is very unusual for problems in which conflicting objectives are involved.

Upon completion of this step, the decision maker is faced with two options, based on stopping the analysis or reference to additional analytical procedures:

- 1) When the decision maker is confident in the results of the pairwise comparisons of rating intensities, choose the solution alternative with the highest composite weight. No further procedures are required.
- 2) When the decision maker lacks confidence, following the pairwise comparisons of solution outcomes for several of the top-ranked alternatives or the sensitivity analysis steps described in the following section, the former procedure is preferred since rating intensities involve the concept of robust decision making. This minimizes the implications of sensitivity analysis.

6.1.5 Sensitivity Analysis

When the decision maker has selected the top-ranked solution alternatives (i.e., the best compromised solution), it may be expected that the judgements arrived at through application of the procedures are sometimes uncertain. Basically, sensitivity analysis is used to determine how sensitive decision outcomes

are to changes in judgement, i.e., sensitivity analysis shows how changes in judgement input data affect the decision.

As observed from the pairwise comparison procedures, two groups of judgements are identified: judgements of relative weight for the objectives and the alternatives. The decision maker may change either or both. No specific and simple procedure exists which may be used to perform AHP sensitivity analysis. The relative weights of the objectives and alternatives must be recalculated. For large judgmental variations, the consistency ratio is also checked. Thus, to reduce the computational burden, it is recommended that the decision maker select a few of the top-ranked solution alternatives for performance of sensitivity analysis.

In general, when the following situations arise, apply pairwise comparison procedures of solution outcomes for several of the top-ranked alternatives:

- 1) In cases where there are large changes in judgement; and
- 2) When there are changes in the relative weight of both objectives and alternatives (rating intensities).

6.1.6 Pairwise Comparison of Solution Outcomes

This step relates solely to situations where the decision maker has decided upon the application of

pairwise comparisons of solution outcomes, rather than of rating intensities. The procedures are applied to only a few of the top-ranked solution alternatives. Sensitivity has the drawback of providing inexact results in certain cases, due to the mechanisms of the rating intensity. In this step, pairwise comparison may compensate for this drawback, as well as easily differentiate or amplify small judgement differences among the top-ranked solution alternatives. After completing this procedure, sensitivity analysis may be applied to lessen uncertainty in the decision maker's judgements.

6.2 Industrial Energy and Environmental Planning

Problem Applications

For purposes of illustration, the sequence of steps described in section 5.1 is applied to the industrial energy and environmental problem.

6.2.1 Development of the Objective Weights

The input data for all pairwise comparisons of the three objectives with respect to the problem goals are shown in Table 6.1. Energy costs are equally as important as environmental impacts and are 1.5 times as favored as business and performance risks. In the second row, environmental impacts are twice as important

Table 6.1 Pairwise Comparison Matrix for Objectives with Respect to Goal.

	Energy Costs	Environmental Impacts	Business & Perform. Risks
Energy costs (ENER)	1.0	1.0	1.5
Environmental impacts (ENVT)		1.0	2.0
Bus. & perform. risks (RISK)			1.0

as business and performance risks. From these relationships, the ranking of the objectives may be intuitively derived, i.e. environmental impacts \succ energy costs \succ business and performance risks.

After obtaining the pairwise comparison matrix, the next step is to calculate each relative weight on an objective and consistency index to check the consistency of the pairwise comparisons. Calculation tables may be used for ease of obtaining the approximate value of the objective weights (Canada et al., 1984). These are shown in Tables 6.2 and 6.3. The matrix in Table 6.3 was obtained by dividing each element in Table 5.2 by the sum of its corresponding column. The right hand side values in Table 6.3 represent each objective weight that resulted from the arithmetic mean of each row.

Table 6.2 Pairwise Comparisons in Decimal Equivalents and with Columns Summed.

	ENER	ENVT	RISK
ENER	1.000	1.000	1.500
ENVT	1.000	1.000	2.000
RISK	0.667	0.500	1.000
Sum	2.667	2.500	4.500

Table 6.3 Normalized Columns and Averaged Rows for Pairwise Comparisons.

	ENER	ENVT	RISK	Sum	Average
ENER	0.375	0.400	0.3333	1.1083	0.3695
ENVT	0.375	0.400	0.4444	1.2194	0.4067
RISK	0.250	0.200	0.2223	0.6723	0.2238
Sum	1.000	1.000	1.0000		1.0000

The value of the λ_{\max} and consistency index (CI) are obtained through the following procedures. From $[A][W]$, the new vector becomes:

$$\begin{bmatrix} 1.000 & 1.000 & 1.500 \\ 1.000 & 1.000 & 2.000 \\ 0.667 & 0.500 & 1.000 \end{bmatrix} \begin{bmatrix} 0.3695 \\ 0.4067 \\ 0.2238 \end{bmatrix} = \begin{bmatrix} 1.1119 \\ 1.2238 \\ 0.6736 \end{bmatrix} .$$

Then, the largest eigenvalue (λ_{\max}) can be found by

$$\begin{aligned} \lambda_{\max} &= [\sum(\text{new vector}/\text{weight})] / \text{no. of element} \\ &= \left[\frac{1.1119}{0.3695} + \frac{1.2238}{0.4067} + \frac{0.6736}{0.2238} \right] / 3 \\ &= 3.00937 . \end{aligned}$$

The above procedures are the approximation method of obtaining maximum eigenvalues.

Next, the consistency index (CI) and the consistency ratio (CR) can be obtained by the following formulas:

$$\begin{aligned} \text{CI} &= (\lambda_{\max} - n) / (n - 1) \\ &= 0.0047 \end{aligned}$$

and

$$\begin{aligned} \text{CR} &= (\text{CI} / \text{RI}) \\ &= 0.0081 . \end{aligned}$$

The CR, 0.0081, is much less than the suggested and empirically acceptable 0.1.

6.2.2 Rating Intensity

Five rating intensities are introduced for the purpose of defining groups of solution outcomes, including: excellent, above average, average, below average, and poor. The ranges of solution outcome for each respective rating intensity are also identical to those of the applied in the value function method described in Chapter 4. All rating intensities for the solution alternatives were given in Table 4.2. With this information, the decision maker is able to set up an acceptable level (i.e., of rating intensity) in order to reduce the number of solution alternatives. In the case under study, the upper level should be defined since all the objectives are presented in the preferred minimization. These are given below:

Total energy cost	: above average
Environmental impacts	: above average
Bus. & performance risks	: excellent .

The following step is to select solution alternatives by application of the disjunctive method to reduce the number of alternatives to the solution alternatives which meet the given acceptable rating intensity for at least one objective. In accordance with the given level of acceptance, as applied with the described methods, 24 solution alternatives were selected and 35 alternatives were eliminated. The selected alternatives are as follows:

AE-1, AE-7, AE-9, AE-10,
 AV-1, AV-2, AV-5,
 AP-1, AP-2, AP-3, AP-6, AP-8, AP-9,
 BE-1, BE-2,
 BV-6, BV-8, BV-9,
 BP-1, BP-3, BP-4, BP-6, BP-7, BP-8.

6.2.3 Pairwise Comparison of Rating Intensities

The defined rating intensities in the preceding section replace the solution alternatives and are given in Figure 6.2. The decision maker performs pairwise comparisons for these rating intensities with respect to each objective. The pairwise comparisons, along with the relative weights (scores) for the rating intensities, are summarized in Table 6.4. The composite

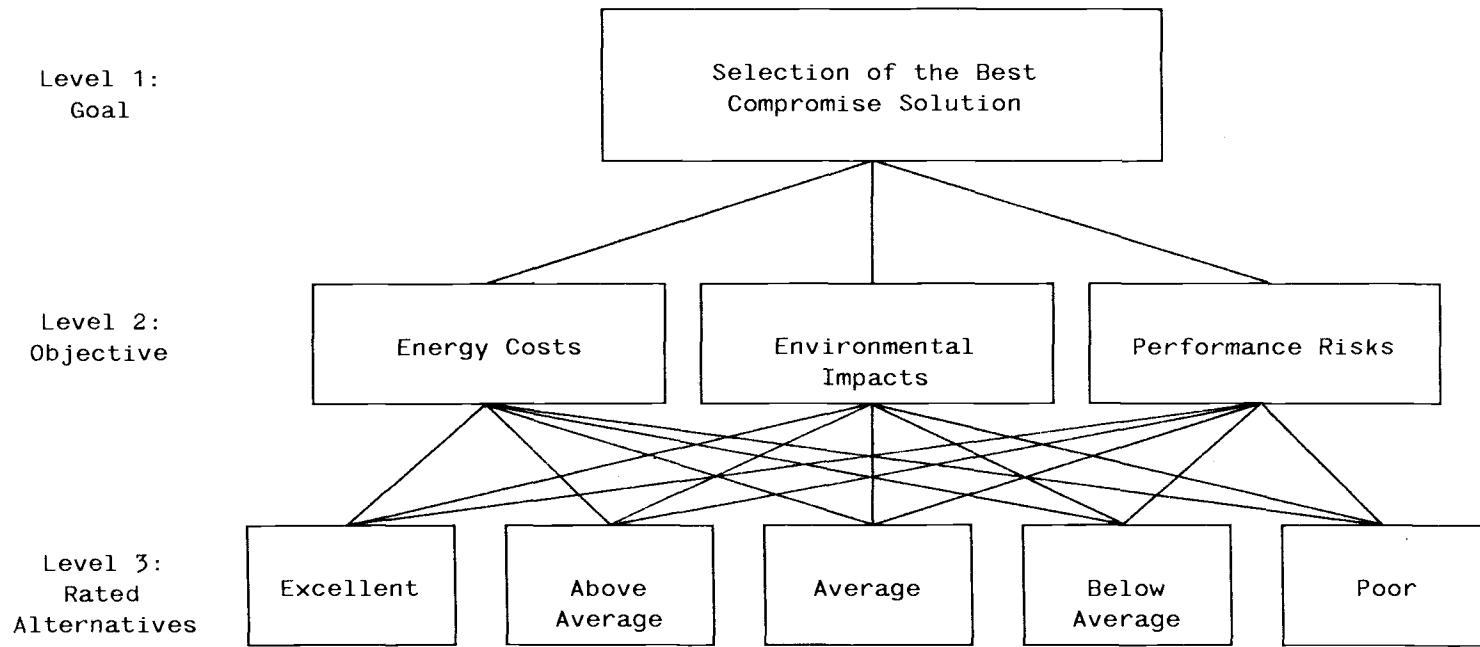


Fig. 6.2 Hierarchical Diagram with Rating Intensities.

weight of the solution alternatives can be obtained by multiplying the objective weight matrix by the rating intensity weights for the corresponding objectives. For example, the rating intensity for each objective and its relative weight for the objective BV-9 are:

<u>Objective (weight)</u>	<u>Rating Intensity (weight)</u>
ENER (0.3695)	Poor (0.07117)
ENVT (0.4067)	Excellent (0.38259)
RISK (0.2238)	Below average (0.13003)

Then, the composite weights are:

$$0.3695 (0.07117) + 0.4067 (0.38259) \\ + 0.2238 (0.13003) = 0.211 .$$

Similar calculations for the other solution alternatives, in ranked order, are presented in Table 6.5.

The results shown in Table 6.5 indicate that the solution alternative BV-9 is the best compromised solution, followed in relatively close order by the second ranked alternative, AP-1 (91.8% of the total weight of BV-9). The results of the composite weights for all solution alternatives is shown in Appendix D. As may be seen from this example, reducing alternatives by disjunctive methods works very well.

At this step, the question may occur to the decision maker, should BV-9 be selected? When the decision maker strongly confirms the judgmental data by pairwise comparisons and confidence in the results is satisfied, then the solution alternative may be selected as the

Table 6.4 Pairwise Comparison Matrix for Rating Intensities.

ENER	Excel.	A. Avg.	Avg.	B. Avg.	Poor	Relative Weights
Excellent	1.0	1.5	2.0	3.5	4.5	0.36725
Above Avg.		1.0	1.8	2.7	3.5	0.27605
Average			1.0	1.5	2.5	0.17077
B. Average				1.0	2.0	0.11502
Poor					1.0	0.07117

CR = 0.006						

ENVT	Excel.	A. Avg.	Avg.	B. Avg.	Poor	Relative Weights
Excellent	1.0	1.7	2.2	3.5	5.0	0.38259
Above Avg.		1.0	2.0	2.5	3.5	0.26654
Average			1.0	2.0	3.0	0.17777
B. Average				1.0	2.0	0.10770
Poor					1.0	0.06565

CR = 0.011						

RISK	Excel.	A. Avg.	Avg.	B. Avg.	Poor	Relative Weights
Excellent	1.0	1.3	1.7	2.5	3.5	0.32484
Above Avg.		1.0	1.5	2.0	3.0	0.26452
Average			1.0	1.5	2.5	0.19437
B. Average				1.0	1.5	0.13003
Poor					1.0	0.08624

CR = 0.001						

Table 6.5 Summary of Composite Weights.

Alt.	Energy Cost	Environ. Impacts	Business & Perf. Risks	Total Weight	% Max
BV-9	POOR	EXCELLNT	B. AVG	0.211	100.0
AP-1	A. AVG	AVG	POOR	0.194	91.8
AE-10	EXCELLNT	POOR	POOR	0.182	86.1
AV-5	B. AVG	A. AVG	B. AVG	0.180	85.3
BE-2	B. AVG	A. AVG	B. AVG	0.180	85.3
BP-1	B. AVG	A. AVG	B. AVG	0.180	85.3
AE-1	POOR	A. AVG	AVG	0.178	84.4
BE-1	POOR	A. AVG	AVG	0.178	84.4
BP-4	POOR	A. AVG	AVG	0.178	84.4
BP-7	POOR	A. AVG	AVG	0.178	84.4
BV-8	POOR	A. AVG	AVG	0.178	84.4
AP-2	A. AVG	B. AVG	B. AVG	0.175	82.9
AE-9	A. AVG	POOR	AVG	0.172	81.6
AP-3	A. AVG	POOR	AVG	0.172	81.6
AV-1	A. AVG	POOR	AVG	0.172	81.6
BP-8	POOR	AVG	EXCELLNT	0.171	81.2
AE-7	A. AVG	B. AVG	POOR	0.165	78.3
AV-2	A. AVG	B. AVG	POOR	0.165	78.3
BV-6	POOR	A. AVG	B. AVG	0.164	77.6
AP-8	POOR	B. AVG	EXCELLNT	0.143	67.7
AP-9	POOR	B. AVG	EXCELLNT	0.143	67.7
BP-6	POOR	B. AVG	EXCELLNT	0.143	67.7
AP-6	B. AVG	POOR	EXCELLNT	0.142	67.3
BP-3	B. AVG	POOR	EXCELLNT	0.142	67.3

best compromise solution. However, if this is not true, then the decision maker must take one further step to verify the final results. This includes sensitivity analysis or pairwise comparison of solution outcomes for the several top-ranked alternatives shown in Table 6.5, rather than the rating intensities.

6.2.4 Sensitivity Analysis

Examining the sensitivity of an objective weights judgement is useful when there is uncertainty regarding the pairwise comparisons or how they affect the final decision. In problems with more than three objectives, a number of ways to combine changes in objective weights are possible. In this section, two case examples are considered: (1) total energy costs, preferred (given greater importance) to environmental impacts, and (2) total energy cost, less preferred than environmental impacts. Pairwise comparisons and the relative weights for these two cases are given in Table 6.6. The results of the relative weights are slightly changed and the inconsistency ratio for both cases is acceptable.

A summary of the recalculations of the composite weights for the solution alternatives is shown in Table 6.7, which includes only the top-ranked solution alternatives. In both cases, solution alternative BV-9 is the top-ranked choice. For the ranking order, no

Table 6.6 Pairwise Comparison Matrix for Sensitivity Analysis.

More Pref.	ENER	ENVT	RISK	Relative Weight
ENER	1.0	1.2	1.5	0.3926
ENVT		1.0	2.0	0.3827
RISK			1.0	0.2247

CR = 0.021				

Less Pref.	ENER	ENVT	RISK	Relative Weight
ENER	1.0	0.8	1.5	0.3413
ENVT		1.0	2.0	0.4359
RISK			1.0	0.2228

CR = 0.004				

Table 6.7 Summary of Sensitivity Analysis: By Preference.

Alt.	Base Case	More Preferred	Less Preferred
BV-9	0.211	0.204	0.220
AP-1	0.194	0.196	0.191
AE-10	0.182	0.189	0.173
AV-5	0.180	0.176	0.184
BE-2	0.180	0.176	0.184
BP-1	0.180	0.176	0.184
AE-1	0.178	0.174	0.183

changes were found in the case of the "more preferred," but the ranking of solution alternative AE-10 was lowered in the "less preferred" case. This was attributed to the poor performance of alternative AE-10 for environmental impacts.

Frequently, the decision maker would like to know the break-even point for objective weights that have changed as the best solution alternatives. This can be useful and realistic information where combinations of objective weights are introduced in order to measure their effect upon a final decision. The typical sensitivity analysis shows only how sensitive relative weights for each alternative are to changes in one objective weight. This method fails to consider the other objective weights. For example, when the objective weight for total energy costs is increased, one of the other objective weights should be decreased. Sensitivity analysis without rearrangement of the remaining weights provides only a very narrow sense of output. The following procedures, for a problem with k objectives, are useful:

- 1) Select the objective that will be changed first, then vary (i.e., increase or decrease) its weight;
- 2) Select the next objective and change its weight, continuing this procedure until $(k-1)$ objective weights have been changed; and

- 3) Calculate the weight for the one remaining objective by:

$$1.0 - \sum (\text{changed objective weight}) .$$

The above formula states that the sum of the objective weights at any point should be equal to 1.0.

Three examples cases were observed, including:

- 1) Linearly increasing the weights for total energy costs and environmental impacts objectives at the same rate;
- 2) Decreasing the weights for both objectives in (1); and
- 3) Linearly increasing the weight for the total energy costs objective while decreasing the weight for the environmental impacts objective.

In case (1), no changes were found and alternative BV-9 remained the best choice. Rank reversal occurred for cases (2) and (3), as shown, respectively, in Figures 6.3 and 6.4. For both of these cases, alternative BV-9 remained the best choice in the region of small changes in objective weight.

6.2.5 Pairwise Comparison of Solution Outcomes

As previously discussed and as determined in the preceding section, sensitivity analysis imposes limitations and restrictions on computational procedures. If the decision maker has decided in favor of the regular pairwise comparison of several top-ranked solution

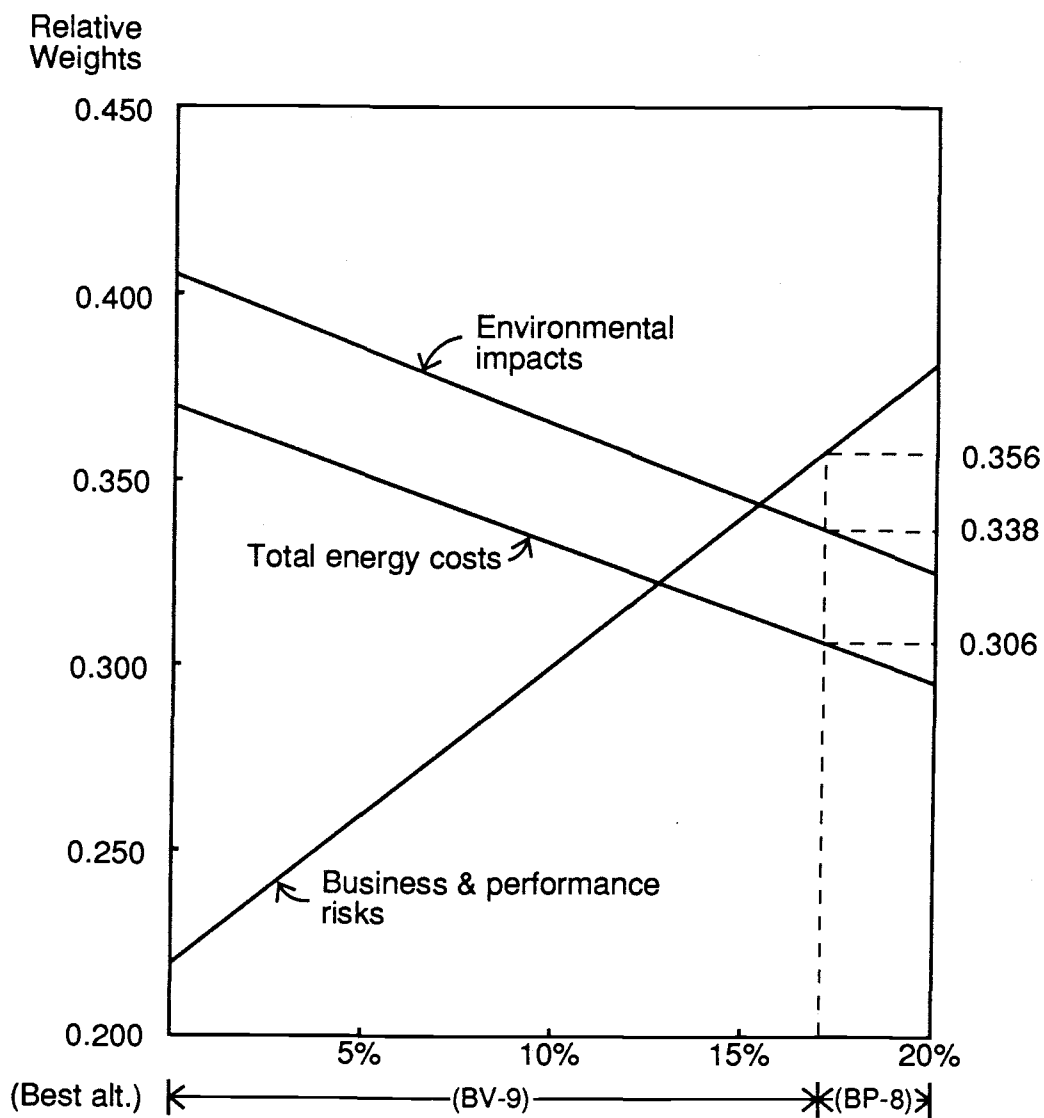


Figure 6.3 Sensitivity Analysis with Rating Intensity: Case 2.

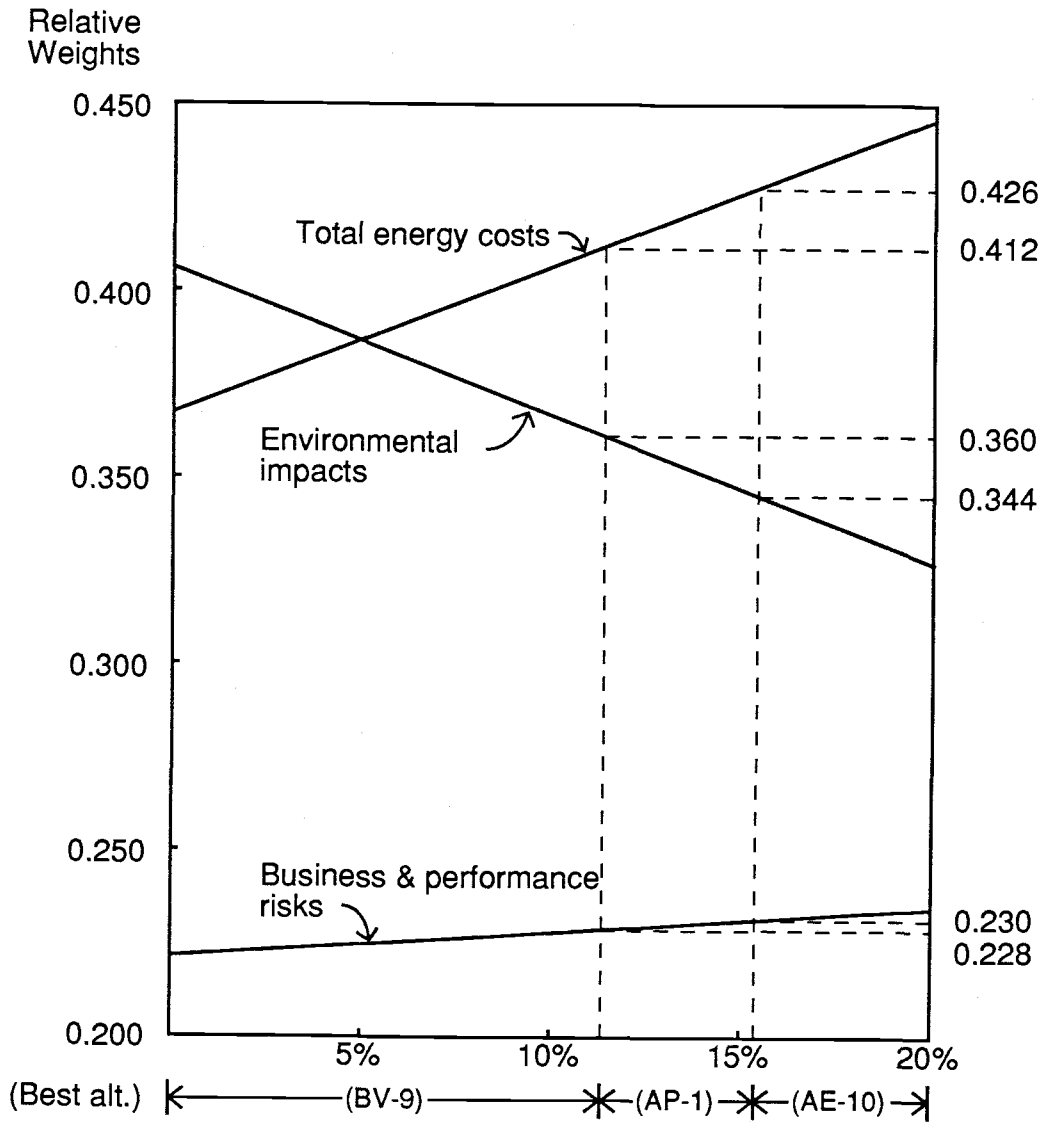


Figure 6.4 Sensitivity Analysis with Rating Intensity: Case 3.

alternatives rather than sensitivity analysis for rating intensities, pairwise comparison matrices must be made for each of the objectives with respect to the selected alternatives. Thus, with just a few selected alternatives, the problem is reduced to a small and manageable size.

For the decision maker's selection of the three top-ranked alternatives, BV-9, AP-1, and AE-10, pairwise comparison matrices for the three alternatives with respect to each objective are given in Table 6.8.

The composite weights for these alternatives was

BV-9 : 0.416 ,

AP-1 : 0.256 ,

AE-10 : 0.328 ,

and the order in ranking is $BV-9 > AE-10 > AP-1$. These results show that alternative BV-9 remains the top-ranked choice, which is consistent with the preceding procedures. However, rank reversal occurred between AP-1 and AE-10. At this step, alternative AE-10 was second ranked, which may be partially attributed to the reasons which follows. The performance of AE-10 for total energy costs and business and performance risks, with respect to alternative AP-1, was somewhat better and the use of rating intensity would not be expected to distinguish smaller variations in the solution outcome. The second factor is that the decision maker's judgement for the pairwise comparison procedures

Table 6.8 Pairwise Comparison Matrix for Top-Ranked Alternatives.

ENER	BV-9	AP-1	AE-10	Relative Weights
BV-9	1.0	1/3.0	1/5.5	0.104
AP-1		1.0	1/2.0	0.304
AE-10			1.0	0.591
CR = 0.001				

ENVT	BV-9	AP-1	AE-10	Relative Weights
BV-9	1.0	3.5	6.0	0.674
AP-1		1.0	3.0	0.232
AE-10			1.0	0.093
CR = 0.03				

RISK	BV-9	AP-1	AE-10	Relative Weights
BV-9	1.0	2.0	1.5	0.460
AP-1		1.0	1/1.5	0.221
AE-10			1.0	0.319
CR = 0.001				

between rating intensity and solution outcomes was inconsistent, i.e., it would have been impossible to achieve consistent results. Since direct comparison of the two results cannot be determined, sensitivity

analysis could in this case be used to help dissolve conflict issues for the decision maker. However, if rank reversal had occurred between the first and second positions, the decision maker would have to refine his judgements for the pairwise comparison procedures.

For sensitivity analysis, the three cases observed in this section resulted in the following:

- 1) Increasing the weights of energy costs and environmental impacts: no change;
- 2) Decreasing the weights of the objectives in (1): no change; and
- 3) Increasing the weight of energy costs and decreasing the weight of environmental impacts: see Figure 6.5.

From Figure 6.5, solution alternative BV-9 must be the best choice, based on the decision makers's given judgements.

6.2.6 Final Solution Alternative Implications

The final decision is the selection of solution alternative BV-9 as the best compromised solution. The fuel-mix planning for BV-9 is shown in Table 6.9.

Each of the objective function values from the given fuel-mix planning are:

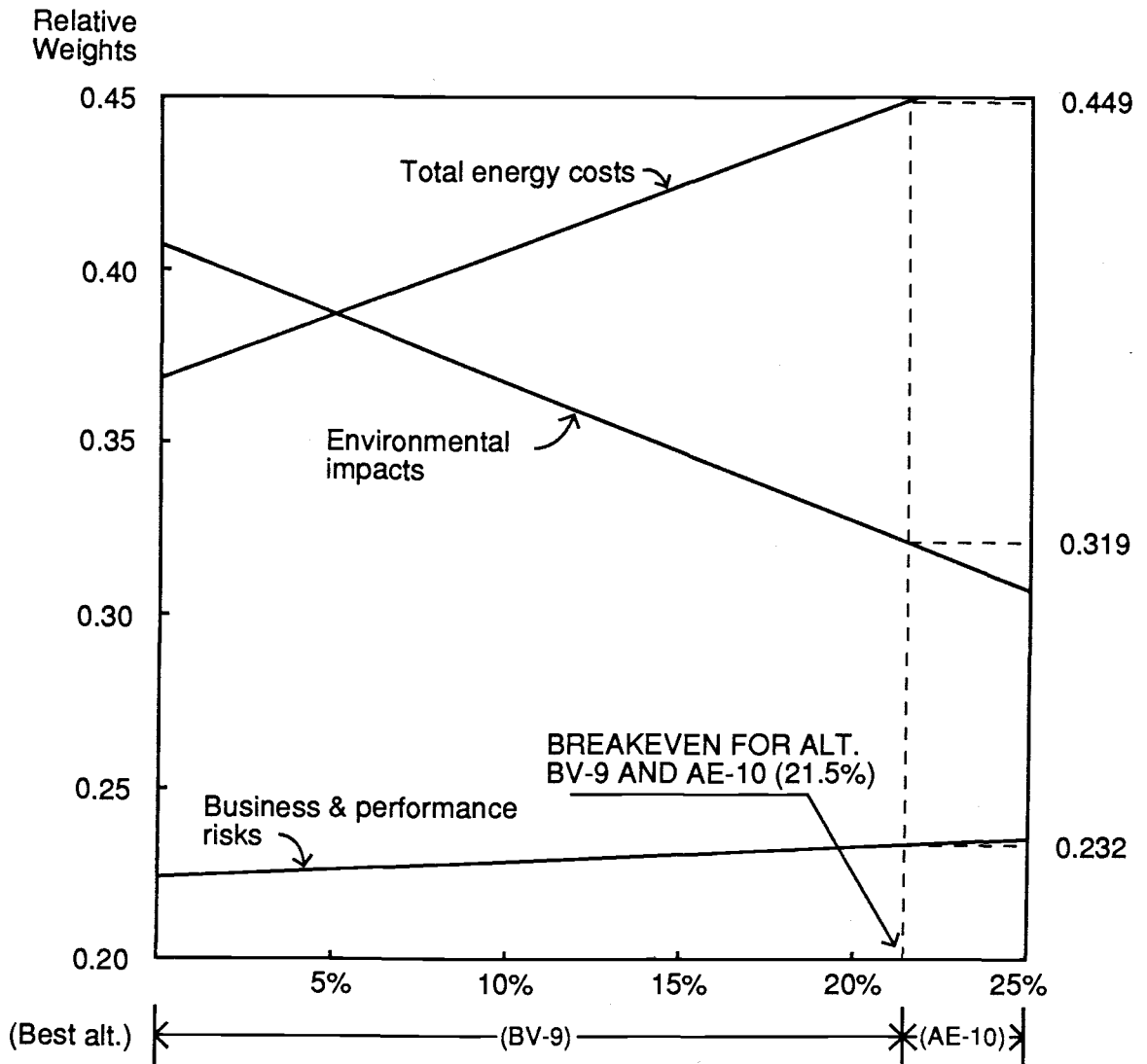


Figure 6.5 Sensitivity Analysis for the Top-Ranked Alternatives: Case 3.

Table 6.9 Fuel-Mix Planning of the Best Compromised Solution Alternative.

	Fuel Mix	
	TJ	Percent
Oil	9712.77	58.4
Coal	1104.50	6.6
Gas	5824.68	35.0
Total	16641.95	100.0

Total energy costs¹

= 118171.23 (10^3 US \$),

Environmental impacts

= 73825.38, and

Business and performance risks

= 91844.22.

These results imply that the maximum use of gas energy resources (up to 35% of total energy) is recommended, requiring 10111.64 (10^3 US \$, first costs) for the conversion of gas burning facilities.

6.3 Potential Benefits and Problems of Evaluation

Procedures

It is not a simple matter to determine the best compromised solution alternatives for decision problems with conflicting objectives and large numbers of alternatives. These issues may be inherent in the problem

¹Including facility conversion costs.

without the decision maker's priori articulation. No matter which evaluation methods are applied, the burden of a large number of alternatives cannot be lessened until the total number of alternatives is reduced to manageable proportions. The procedures proposed have been offered in response to these problems, and with the object of deriving benefits from the AHP procedure.

The potential advantages of using the proposed methodology include:

- 1) The ability to produce a higher level of decision maker confidence;
- 2) The ability to accept decision makers's multiple judgements by the adoption of the AHP procedure;
- 3) Greater flexibility of means, i.e., the ability to follow either an entire procedure or selected partial steps; and
- 4) Appropriateness of the methodology for consideration of both quantitative and qualitative factors.

Some of the weakness included in the use of the proposed procedure are:

- 1) Additional time and costs are required, in comparison to the use of existing methods (i.e., value function method), when all of the procedures are followed;

- 2) Less transparent and understandable than more simple methodologies, e.g., the value function method; and
- 3) The need to maintain consistency throughout all procedures. For example, decision maker's judgements for the pairwise comparison of rating intensities and solution alternatives may not be structured similarly. Even through the use of a consistency ratio provides guidelines for consistency checks, this measure provides consistency only within one procedure and is not comprehensive.

6.4 Conclusion

The proposed procedures are ideally suited for situations where (1) the decision problem does not require the decision maker's prior articulation of preferences and where (2) the decision maker's prior articulation of preferences is available for problems with a large number of pre-defined alternatives.

In both types of problems, procedures to reduce the number of alternatives will lessen the decision maker's computational burden. The multiple steps of screening or pruning procedures are helpful in order to maintain the decision maker's confidence level. For use of the AHP procedure in the proposed method, the

somewhat ambiguous nature of obtaining judgmental values for any two decision elements should be fully understood.

7. CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

The objectives for this study were presented in two distinct parts: (1) Presentation of an MOLP model for energy and environmental planning in the forest manufacturing industries; and (2) Proposal of a methodology for conducting evaluations of the nondominated solutions generated by the MOLP model.

MOLP models are formulated in relatively restricted sizes and, therefore, do not require large volumes of input data. The model also acts as a tool for generating solution alternatives, composed of courses of action for decision maker evaluation. The constraint method was applied in order to generate entire sets of nondominated solutions. Through the computational procedures, a considerable number of nonfeasible solutions were found. Even though only a small number of combinational iterations were used, a large number of solution alternatives was generated, resulting in burdensome computational procedures. However, this is inevitable for those decision problems in which the decision maker does not provide priori articulations of preference information. On the other hand,

the generation of a large number of solution alternatives provides the decision maker with a broad sense of the implications of the solution space, which is particularly important in planning problems.

In the model, three objectives were observed for which the preferences could be minimized: (1) total energy costs, (2) environmental impacts, and (3) business and performance risks. In real applications of the model, the last two objectives cannot be measured only in terms of quantitative values since they imply quantitative factors and the use of different units of measurement. This causes inconvenience and a certain amount of complexity for the process of model formulation. In order to overcome these problems, subjective numerical values were applied for the most simple quantifications. This precluded the application of a "tradeoff value" approach, while necessitating the assumption that procedural "independence" exists between the objectives and the alternatives for the evaluation of nondominated solutions.

Several existing evaluation methods for generated nondominated solutions were examined with the intention of providing an elementary idea and comparative study of the proposed methodology. The major findings from this examination were that the classical method (i.e., the value function method) may not be adequate for problems with a large number of alternatives and which

fail to satisfy the decision maker's confidence level upon selection of the final solution alternative. The proposed evaluation method was comprised of a multiple screening approach, based on rating and pairwise comparison methods. By this means, the solution alternatives were reduced, step-by-step, through application of the decision maker's judgement to the results of each individual procedural step, until a final solution alternative was selected.

The advantage of the proposed method in comparison to conventional single-step evaluation methods is that the former is capable of approaching the decision problem with flexibility and it strengthens higher confidence levels for the decision maker. Throughout the implementation of this method, the assumption of "independence" did not prove to be critical in the problem considered in this study since each of the solution alternatives were defined by the MOLP model. Moreover, the proposed method captures some of the strengths of AHP procedures by the use of pairwise comparisons, such as providing "intransitivity" and the ability to effect multiple decision maker's judgements. In the evaluation procedure, the solution alternative BV-9, implying the maximum use of gas energy resources, was selected as the best compromised solution for the given decision maker's judgements. This alternative was identical to the results obtained from a single LP problem when the

environmental impacts objective was selected (see Chapter 3), given that the decision maker's judgement could be characterized as displaying an extreme risk-taking attitude and that the relative weight of this objective was higher than the others. These facts could lead to the conclusion that extreme risk-seeking or a risk aversal attitude expressed in the decision maker's preferences could degrade the implications and benefits of the MODM procedures. Moreover, the higher objective weight reinforced the above result.

7.2 Recommendations for Further Research

The different issues for future research are discussed in the following sections.

7.2.1 MOLP Model

The MOLP model introduced in this study has been formulated as a prototype example for obtaining the best fuel-mix planning decisions. For refinement of the model, several additional factors should be considered. First, the relaxation of the assumptions should be considered. Namely, electrical energy resources were not included in the model simplification. Though electricity is not directly related to environmental impacts, it generally occupies a large portion of the total energy requirements. Introducing electrical energy could affect the obtained fuel-mix planning deci-

sion. Neither are energy conservation or new conversion technologies considered. These options could result in the decrease of total energy consumption and, therefore, should be included in the model for proper problem analysis and implementation.

Second, the model could be changed to reflect a dynamic structure in order to be applied as a forecasting tool for energy resources, complete with the ability to observe environmental impacts. The given model was fixed for a specific year's fuel-mix planning. Providing additional parameters, e.g., potential energy price changes in the future and additional decision variables, would make the MOLP model more realistic and powerful for forecasting energy resources.

7.2.2 Evaluation Method

The proposed evaluation methodology, based on multiple screening procedures, was applied and examined through predefined alternatives provided by the MOLP model for energy and environmental planning problems considered in this study. To obtain detailed comments and the widest implications of the procedural steps, the method should be applied to general types of problems, i.e., large-scale problems in which priori articulations of the decision maker's preferences are available. Moreover, experimental comparative studies with

other evaluation methods is also an area recommended for further study.

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APPENDICES

Appendix A

LP Formulation

Case A

MIN 7.237 TE1 + 2.585 TE2 + 7.423 TE3 + .346 AE2 + .307 AE3

SUBJECT TO

- 2) 8.5792 PV1 - TES1 = 0
- 3) 6.2097 PV2 - TES2 = 0
- 4) 6.2 PV3 - TES3 = 0
- 5) 4.4427 PV4 - TES4 = 0
- 6) 13.0233 PV5 - TES5 = 0
- 7) 1.4502 PV6 - TES6 = 0
- 8) TES1 - E11 - E12 - E13 = 0
- 9) TES2 - E21 - E22 - E23 = 0
- 10) TES3 - E31 - E32 - E33 = 0
- 11) TES4 - E41 - E42 - E43 = 0
- 12) TES5 - E51 - E52 - E53 = 0
- 13) TES6 - E61 - E62 - E63 = 0
- 14) TE1 - E11 - E21 - E31 - E41 - E51 - E61 = 0
- 15) TE2 - E12 - E22 - E32 - E42 - E52 - E62 = 0
- 16) TE3 - E13 - E23 - E33 - E43 - E53 - E63 = 0
- 17) TE1 <= 20000
- 18) TE2 <= 5000
- 19) TE3 <= 4350
- 20) PV1 <= 211
- 21) PV1 >= 152
- 22) PV2 <= 286
- 23) PV2 >= 195

- 24) PV3 <= 174
- 25) PV3 >= 128
- 26) PV4 <= 710
- 27) PV4 >= 553
- 28) PV5 <= 154
- 29) PV5 >= 98
- 30) PV6 <= 7950
- 31) PV6 >= 6620
- 32) - .25 TES1 + E12 <= 0
- 33) - .25 TES1 + E13 <= 0
- 34) - .25 TES2 + E22 <= 0
- 35) - .25 TES2 + E23 <= 0
- 36) - .25 TES3 + E32 <= 0
- 37) - .25 TES3 + E33 <= 0
- 38) - .25 TES4 + E42 <= 0
- 39) - .25 TES4 + E43 <= 0
- 40) - .25 TES5 + E52 <= 0

41) - .25 TES5 + E53 <= 0
42) - .25 TES6 + E62 <= 0
43) - .25 TES6 + E63 <= 0
44) - .06 TES1 + E12 >= 0
45) - .06 TES2 + E22 >= 0
46) - .02 TES3 + E32 >= 0
47) - .03 TES4 + E42 >= 0
48) - .09 TES6 + E62 >= 0
49) TE2 - .06 TES1 - .06 TES2 - .02 TES3 - .03 TES4 - .09 TES6 - AE2
= 0
50) TE3 - AE3 = 0
51) 1.957 AE2 <= 10000
52) 1.736 AE3 <= 10000

END

Case B

MIN 7.237 TE1 + 2.585 TE2 + 7.423 TE3 + .346 AE2 + .307 AE3

SUBJECT TO

- 2) 8.5792 PV1 - TES1 = 0
- 3) 6.2097 PV2 - TES2 = 0
- 4) 6.2 PV3 - TES3 = 0
- 5) 4.4427 PV4 - TES4 = 0
- 6) 13.0233 PV5 - TES5 = 0
- 7) 1.4502 PV6 - TES6 = 0
- 8) TES1 - E11 - E12 - E13 = 0
- 9) TES2 - E21 - E22 - E23 = 0
- 10) TES3 - E31 - E32 - E33 = 0
- 11) TES4 - E41 - E42 - E43 = 0
- 12) TES5 - E51 - E52 - E53 = 0
- 13) TES6 - E61 - E62 - E63 = 0
- 14) TE1 - E11 - E21 - E31 - E41 - E51 - E61 = 0
- 15) TE2 - E12 - E22 - E32 - E42 - E52 - E62 = 0
- 16) TE3 - E13 - E23 - E33 - E43 - E53 - E63 = 0
- 17) TE1 <= 20000
- 18) TE2 <= 5000
- 19) TE3 <= 6550
- 20) PV1 <= 211
- 21) PV1 >= 152
- 22) PV2 <= 286
- 23) PV2 >= 195

- 24) PV3 <= 174
- 25) PV3 >= 128
- 26) PV4 <= 710
- 27) PV4 >= 553
- 28) PV5 <= 154
- 29) PV5 >= 98
- 30) PV6 <= 7950
- 31) PV6 >= 6620
- 32) - .15 TES1 + E12 <= 0
- 33) - .35 TES1 + E13 <= 0
- 34) - .15 TES2 + E22 <= 0
- 35) - .35 TES2 + E23 <= 0
- 36) - .15 TES3 + E32 <= 0
- 37) - .35 TES3 + E33 <= 0
- 38) - .15 TES4 + E42 <= 0
- 39) - .35 TES4 + E43 <= 0
- 40) - .15 TES5 + E52 <= 0

41) - .35 TES5 + E53 <= 0
42) - .15 TES6 + E62 <= 0
43) - .35 TES6 + E63 <= 0
44) - .06 TES1 + E12 >= 0
45) - .06 TES2 + E22 >= 0
46) - .02 TES3 + E32 >= 0
47) - .03 TES4 + E42 >= 0
48) - .09 TES6 + E62 >= 0
49) TE2 - .06 TES1 - .06 TES2 - .02 TES3 - .03 TES4 - .09 TES6 - AE2
= 0
50) TE3 - AE3 = 0
51) 1.957 AE2 <= 20000
52) 1.736 AE3 <= 15000

END

Appendix B

Example of LP Solution: Case A,
Minimization of Total Energy Costs

LP OPTIMUM FOUND AT STEP 36

OBJECTIVE FUNCTION VALUE

1) 102140.600

VARIABLE	VALUE	REDUCED COST
TE1	12481.460000	.000000
TE2	4160.487000	.000000
TE3	.000000	.186000
PV1	152.000000	.000000
TES1	1304.038000	.000000
PV2	195.000000	.000000
TES2	1210.891000	.000000
PV3	128.000000	.000000
TES3	793.600000	.000000
PV4	553.000000	.000000
TES4	2456.813000	.000000
PV5	98.000000	.000000
TES5	1276.283000	.000000
PV6	6620.000000	.000000
TES6	9600.324000	.000000
E11	978.028700	.000000
E12	326.009600	.000000
E13	.000000	.000000
E21	908.168600	.000000
E22	302.722900	.000000
E23	.000000	.000000
E31	595.200000	.000000
E32	198.400000	.000000
E33	.000000	.000000
E41	1842.610000	.000000
E42	614.203200	.000000
E43	.000000	.000000
E51	957.212600	.000000
E52	319.070900	.000000
E53	.000000	.000000
E61	7200.243000	.000000
E62	2400.081000	.000000
E63	.000000	.000000
AE2	3055.986000	.000000
AE3	.000000	.307000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	6.139740
3)	.000000	6.139740
4)	.000000	6.153580
5)	.000000	6.150120
6)	.000000	6.160500
7)	.000000	6.129360
8)	.000000	7.237000
9)	.000000	7.237000
10)	.000000	7.237000
11)	.000000	7.237000
12)	.000000	7.237000
13)	.000000	7.237000
14)	.000000	-7.237000
15)	.000000	-2.931000
16)	.000000	-7.237000
17)	7518.538000	.000000
18)	839.512700	.000000
19)	4350.000000	.000000
20)	59.000000	.000000
21)	.000000	-52.674060
22)	91.000000	.000000
23)	.000000	-38.125940
24)	46.000000	.000000
25)	.000000	-38.152190
26)	157.000000	.000000
27)	.000000	-27.323140
28)	56.000000	.000000
29)	.000000	-80.230040
30)	1330.000000	.000000
31)	.000000	-8.888798
32)	.000000	4.306000
33)	326.009600	.000000
34)	.000000	4.306000
35)	302.722900	.000000
36)	.000000	4.306000
37)	198.400000	.000000
38)	.000000	4.306000
39)	614.203200	.000000
40)	.000000	4.306000
41)	319.070900	.000000
42)	.000000	4.306000
43)	2400.081000	.000000
44)	247.767300	.000000
45)	230.069400	.000000
46)	182.528000	.000000
47)	540.498800	.000000
48)	1536.052000	.000000
49)	.000000	.346000
50)	.000000	.000000
51)	4019.436000	.000000
52)	10000.000000	.000000

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE	OBJ COEFFICIENT RANGES		
	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
TE1	7.237000	.186000	4.306000
TE2	2.585000	4.306000	24.517440
TE3	7.423000	INFINITY	.186000
PV1	.000000	INFINITY	52.674060
TES1	.000000	INFINITY	6.139740
PV2	.000000	INFINITY	38.125940
TES2	.000000	INFINITY	6.139740
PV3	.000000	INFINITY	38.152190
TES3	.000000	INFINITY	6.153580
PV4	.000000	INFINITY	27.323140
TES4	.000000	INFINITY	6.150120
PV5	.000000	INFINITY	80.230040
TES5	.000000	INFINITY	6.160500
PV6	.000000	INFINITY	8.888798
TES6	.000000	INFINITY	6.129360
E11	.000000	.000000	4.306000
E12	.000000	4.306000	24.558960
E13	.000000	INFINITY	.000000
E21	.000000	.000000	4.306000
E22	.000000	4.306000	24.558960
E23	.000000	INFINITY	.000000
E31	.000000	.000000	4.306000
E32	.000000	4.306000	24.614320
E33	.000000	INFINITY	.000000
E41	.000000	.186000	.000000
E42	.000000	4.306000	24.600480
E43	.000000	.000000	.186000
E51	.000000	.000000	4.306000
E52	.000000	4.306000	24.642000
E53	.000000	INFINITY	.000000
E61	.000000	.000000	4.306000
E62	.000000	4.306000	24.517440
E63	.000000	INFINITY	.000000
AE2	.346000	4.306000	24.642000
AE3	.307000	INFINITY	.307000

ROW	RIGHTHAND SIDE RANGES		
	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	.000000	1304.038000	3358.051000
3	.000000	1210.891000	3358.051000
4	.000000	793.599900	3358.051000
5	.000000	2456.813000	3358.051000
6	.000000	1276.283000	3358.051000
7	.000000	9600.323000	3358.051000
8	.000000	978.028700	7518.538000
9	.000000	908.168600	7518.538000
10	.000000	595.200000	7518.538000
11	.000000	1842.610000	7518.538000
12	.000000	957.212600	7518.538000
13	.000000	7200.243000	7518.538000
14	.000000	7518.538000	12481.460000
15	.000000	839.512700	3055.986000
16	.000000	.000000	614.203200
17	20000.000000	INFINITY	7518.538000
18	5000.000000	INFINITY	839.512700
19	4350.000000	INFINITY	4350.000000
20	211.000000	INFINITY	59.000000
21	152.000000	59.000000	152.000000
22	286.000000	INFINITY	91.000000
23	195.000000	91.000000	195.000000
24	174.000000	INFINITY	46.000000
25	128.000000	46.000000	128.000000
26	710.000000	INFINITY	157.000000
27	553.000000	157.000000	552.999900
28	154.000000	INFINITY	56.000000
29	98.000000	56.000000	98.000000
30	7950.000000	INFINITY	1330.000000
31	6620.000000	1330.000000	6620.000000
32	.000000	839.512700	247.767300
33	.000000	INFINITY	326.009600
34	.000000	839.512700	230.069400
35	.000000	INFINITY	302.722900
36	.000000	595.200000	182.528000
37	.000000	INFINITY	198.400000
38	.000000	839.512700	540.498800
39	.000000	INFINITY	614.203200
40	.000000	839.512700	319.070900
41	.000000	INFINITY	319.070900
42	.000000	839.512700	1536.052000
43	.000000	INFINITY	2400.081000
44	.000000	247.767300	INFINITY
45	.000000	230.069400	INFINITY

46	.000000	182.528000	INFINITY
47	.000000	540.498800	INFINITY
48	.000000	1536.052000	INFINITY
49	.000000	3055.986000	2053.876000
50	.000000	.000000	.000000
51	10000.000000	INFINITY	4019.436000
52	10000.000000	INFINITY	10000.000000

Appendix C

Example of Constraint Problem and Solution:

Case A, Minimization of Total Energy Costs

$$(\xi_2 = 86134.31, \xi_3 = 90470.29)$$

Constraint Problem

MIN 7.237 TE1 + 2.585 TE2 + 7.423 TE3 + .346 AE2 + .307 AE3
SUBJECT TO

- 2) 8.5792 PV1 - TES1 = 0
- 3) 6.2097 PV2 - TES2 = 0
- 4) 6.2 PV3 - TES3 = 0
- 5) 4.4427 PV4 - TES4 = 0
- 6) 13.0233 PV5 - TES5 = 0
- 7) 1.4502 PV6 - TES6 = 0
- 8) TES1 - E11 - E12 - E13 = 0
- 9) TES2 - E21 - E22 - E23 = 0
- 10) TES3 - E31 - E32 - E33 = 0
- 11) TES4 - E41 - E42 - E43 = 0
- 12) TES5 - E51 - E52 - E53 = 0
- 13) TES6 - E61 - E62 - E63 = 0
- 14) TE1 - E11 - E21 - E31 - E41 - E51 - E61 = 0
- 15) TE2 - E12 - E22 - E32 - E42 - E52 - E62 = 0
- 16) TE3 - E13 - E23 - E33 - E43 - E53 - E63 = 0
- 17) TE1 <= 20000
- 18) TE2 <= 5000
- 19) TE3 <= 4350
- 20) PV1 <= 211
- 21) PV1 >= 152
- 22) PV2 <= 286
- 23) PV2 >= 195

- 24) PV3 <= 174
- 25) PV3 >= 128
- 26) PV4 <= 710
- 27) PV4 >= 553
- 28) PV5 <= 154
- 29) PV5 >= 98
- 30) PV6 <= 7950
- 31) PV6 >= 6620
- 32) - .25 TES1 + E12 <= 0
- 33) - .25 TES1 + E13 <= 0
- 34) - .25 TES2 + E22 <= 0
- 35) - .25 TES2 + E23 <= 0
- 36) - .25 TES3 + E32 <= 0

- 37) - .25 TES3 + E33 <= 0
- 38) - .25 TES4 + E42 <= 0
- 39) - .25 TES4 + E43 <= 0
- 40) - .25 TES5 + E52 <= 0
- 41) - .25 TES5 + E53 <= 0
- 42) - .25 TES6 + E62 <= 0
- 43) - .25 TES6 + E63 <= 0
- 44) - .06 TES1 + E12 >= 0
- 45) - .06 TES2 + E22 >= 0
- 46) - .02 TES3 + E32 >= 0
- 47) - .03 TES4 + E42 >= 0
- 48) - .09 TES6 + E62 >= 0
- 49) TE2 - .06 TES1 - .06 TES2 - .02 TES3 - .03 TES4 - .09 TES6 - AE2
= 0
- 50) TE3 - AE3 = 0
- 51) 1.957 AE2 <= 10000
- 52) 1.736 AE3 <= 10000
- 53) 5.5 TE1 + 7.4 TE2 + 2.1 TE3 <= 86134.31
- 54) 4.7 TE1 + 8.6 TE2 + 6.3 TE3 <= 90470.29

END

Solution

LP OPTIMUM FOUND AT STEP 40

OBJECTIVE FUNCTION VALUE

1) 112671.700

VARIABLE	VALUE	REDUCED COST
TE1	11896.350000	.000000
TE2	2026.156000	.000000
TE3	2719.446000	.000000
PV1	152.000000	.000000
TES1	1304.038000	.000000
PV2	195.000000	.000000
TES2	1210.891000	.000000
PV3	128.000000	.000000
TES3	793.600000	.000000
PV4	553.000000	.000000
TES4	2456.813000	.000000
PV5	98.000000	.000000
TES5	1276.283000	.000000
PV6	6620.000000	.000000
TES6	9600.324000	.000000
E11	652.019200	.000000
E12	326.009600	.000000
E13	326.009600	.000000
E21	835.515100	.000000
E22	72.653490	.000000
E23	302.722900	.000000
E31	595.200000	.000000
E32	198.400000	.000000
E33	.000000	.000000
E41	1768.905000	.000000
E42	73.704390	.000000
E43	614.203200	.000000
E51	957.212600	.000000
E52	.000000	.000000
E53	319.070900	.000000
E61	7087.498000	.000000
E62	1355.388000	.000000
E63	1157.439000	.000000
AE2	921.654200	.000000
AE3	2719.446000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	14.141090
3)	.000000	14.141090
4)	.000000	14.154930
5)	.000000	14.151470
6)	.000000	14.161850
7)	.000000	14.130710
8)	.000000	14.161850
9)	.000000	14.161850
10)	.000000	14.161850
11)	.000000	14.161850
12)	.000000	14.161850
13)	.000000	14.161850
14)	.000000	-14.161850
15)	.000000	-14.161850
16)	.000000	-14.161850
17)	8103.649000	.000000
18)	2973.844000	.000000
19)	1630.554000	.000000
20)	59.000000	.000000
21)	.000000	-121.319200
22)	91.000000	.000000
23)	.000000	-87.811930
24)	46.000000	.000000
25)	.000000	-87.760570
26)	157.000000	.000000
27)	.000000	-62.870740
28)	56.000000	.000000
29)	.000000	-184.434000
30)	1330.000000	.000000
31)	.000000	-20.492360
32)	.000000	.000000
33)	.000000	.000000
34)	230.069400	.000000
35)	.000000	.000000
36)	.000000	.000000
37)	198.400000	.000000
38)	540.498800	.000000
39)	.000000	.000000
40)	319.070900	.000000
41)	.000000	.000000
42)	1044.693000	.000000
43)	1242.642000	.000000
44)	247.767300	.000000
45)	.000000	.000000
46)	182.528000	.000000
47)	.000000	.000000
48)	491.358800	.000000

49)	.000000	.346000
50)	.000000	.307000
51)	8196.322000	.000000
52)	5279.042000	.000000
53)	.000000	.540632
54)	.000000	.840718

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE	OBJ COEFFICIENT RANGES		
	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
TE1	7.237000	3.831438	2.585604
TE2	2.585000	4.030500	INFINITY
TE3	7.423000	7.212476	2.259565
PV1	.000000	INFINITY	121.319200
TES1	.000000	INFINITY	14.141090
PV2	.000000	INFINITY	87.811930
TES2	.000000	INFINITY	14.141090
PV3	.000000	INFINITY	87.760570
TES3	.000000	INFINITY	14.154930
PV4	.000000	INFINITY	62.870740
TES4	.000000	INFINITY	14.151470
PV5	.000000	INFINITY	184.434000
TES5	.000000	INFINITY	14.161850
PV6	.000000	INFINITY	20.492360
TES6	.000000	INFINITY	14.130710
E11	.000000	INFINITY	.000000
E12	.000000	.000000	56.564360
E13	.000000	.000000	56.564360
E21	.000000	.000000	.000000
E22	.000000	INFINITY	.000000
E23	.000000	.000000	56.564360
E31	.000000	.000000	.000000
E32	.000000	.000000	56.619720
E33	.000000	INFINITY	.000000
E41	.000000	.000000	.000000
E42	.000000	INFINITY	.000000
E43	.000000	.000000	56.605880
E51	.000000	.000000	.000000
E52	.000000	INFINITY	.000000
E53	.000000	.000000	56.647400
E61	.000000	.000000	.000000
E62	.000000	.000000	.000000
E63	.000000	.000000	.000000
AE2	.346000	4.030500	INFINITY
AE3	.307000	7.212476	2.259565

ROW	RIGHTHAND SIDE RANGES		
	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	.000000	590.140200	277.565400
3	.000000	661.095400	310.938300
4	.000000	590.140200	277.565400
5	.000000	673.888800	316.955500
6	.000000	687.187100	323.210200
7	.000000	590.140200	305.145300
8	.000000	652.019200	323.210200
9	.000000	687.187100	323.210200
10	.000000	595.200000	323.210200
11	.000000	687.187100	323.210200
12	.000000	687.187100	323.210200
13	.000000	687.187100	323.210200
14	.000000	323.210200	687.187100
15	.000000	194.964700	414.520300
16	.000000	323.210200	687.187100
17	20000.000000	INFINITY	8103.649000
18	5000.000000	INFINITY	2973.844000
19	4350.000000	INFINITY	1630.554000
20	211.000000	INFINITY	59.000000
21	152.000000	32.353290	68.787320
22	286.000000	INFINITY	91.000000
23	195.000000	50.073000	106.461700
24	174.000000	INFINITY	46.000000
25	128.000000	44.768610	95.183900
26	710.000000	INFINITY	157.000000
27	553.000000	71.343000	151.684500
28	154.000000	INFINITY	56.000000
29	98.000000	24.817840	52.765970
30	7950.000000	INFINITY	1330.000000
31	6620.000000	210.416000	406.937100
32	.000000	491.358800	247.767300
33	.000000	652.019200	326.009600
34	.000000	INFINITY	230.069400
35	.000000	835.515100	302.722900
36	.000000	491.358800	182.528000
37	.000000	INFINITY	198.400000
38	.000000	INFINITY	540.498800
39	.000000	1157.439000	614.203200
40	.000000	INFINITY	319.070900
41	.000000	957.212600	319.070900
42	.000000	INFINITY	1044.693000
43	.000000	INFINITY	1242.642000
44	.000000	247.767300	INFINITY
45	.000000	230.069400	72.653490
46	.000000	182.528000	INFINITY
47	.000000	491.358800	73.704390

48	.000000	491.358800	INFINITY
49	.000000	921.654200	4188.208000
50	.000000	2719.446000	3040.923000
51	10000.000000	INFINITY	8196.322000
52	10000.000000	INFINITY	5279.042000
53	86134.310000	4837.502000	5005.715000
54	90470.290000	5008.381000	2355.632000

Appendix D

Comparison of Composite Weights

	<u>Non-Reduced</u>		<u>Reduced</u>	
	<u>Alt.</u>	<u>Total Weight</u>	<u>Alt.</u>	<u>Total Weight</u>
1	BV-9	0.211	BV-9	0.211
2	AP-1	0.194	AP-1	0.194
3	AE-10	0.182	AE-10	0.182
4	AV-5	0.180	AV-5	0.180
5	BE-2	0.180	BE-2	0.180
6	BP-1	0.180	BP-1	0.180
7	AE-1	0.178	AE-1	0.178
8	BE-1	0.178	BE-1	0.178
9	BP-4	0.178	BP-4	0.178
10	BP-7	0.178	BP-7	0.178
11	BV-8	0.178	BV-8	0.178
12	AP-2	0.175	AP-2	0.175
13	AE-9	0.172	AE-9	0.172
14	AP-3	0.172	AP-3	0.172
15	AV-1	0.172	AV-1	0.172
16	BP-8	0.171	BP-8	0.171
17	AE-7	0.165	AE-7	0.165
18	AV-2	0.165	AV-2	0.165
19	BE-5	0.164	BV-6	0.164
20	BV-6	0.164	AP-8	0.143
21	AE-3	0.158	AP-9	0.143
22	AV-4	0.158	BP-6	0.143
23	AP-4	0.158	AP-6	0.142
24	BE-4	0.158	BP-3	0.142
25	BP-2	0.158		
26	AE-2	0.158		
27	AV-6	0.158		
28	AP-7	0.158		
29	BE-3	0.158		
30	BV-4	0.158		
31	BV-7	0.158		
32	BP-5	0.158		
33	AE-4	0.155		
34	AE-6	0.150		
35	AE-8	0.149		
36	BE-7	0.149		
37	AE-5	0.146		
38	AV-3	0.146		
39	AP-5	0.146		
40	BV-3	0.144		
41	AP-8	0.143		
42	AP-9	0.143		
43	BP-6	0.143		
44	BV-5	0.142		
45	AP-6	0.142		
46	BP-3	0.142		
47	BV-2	0.130		
48	BE-6	0.128		
49	BV-1	0.128		