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Title AN APPLICATION AND GENERATING METHODS FOR MELLIN TRANSFORMS

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Properties of Mellin transforms are applied to the summation of infinite series. A specific example of this application leads to the establishment of certain properties of a generalization of Lerch's zeta function.

Nine powerful methods of generating new tables of Mellin transforms from existing tables of Mellin and other integral transforms are described and illustrated. A table of over 200 examples, which were derived from one of the above mentioned methods, is included.

AN APPLICATION AND GENERATING METHODS FOR MELLIN TRANSFORMS

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AN APPLICATION AND GENERATING METHODS FOR MELLIN TRANSFORMS

1. INTRODUCTION

The pair of inversion formulae

(1) $$g(s) = \int_0^{\infty} f(x)\, x^{s-1} dx = \mathcal{M}\{f(x), s\}$$

(2) $$f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} g(s)\, x^{-s}\, ds$$

(c real) was first established by Mellin [17]. The function $g(s)$ is called the Mellin transform of $f(x)$ with respect to the parameter s. The theory of this transform is well established (see [5, vol. 1, ch. 2], [20, p. 46]). A sufficient condition for the existence of (1) and (2) is the absolute convergence of the integral (1). The main feature of $g(s)$ as given by (1) is that it represents an analytic function in an infinite strip, $\sigma_1 < \text{Re } s < \sigma_2$, of the complex s-plane. σ_1 and σ_2 are the limits of ordinary convergence of the integral in (1) and depend, of course, on the function $f(x)$. The strip of analyticity may extend into a left half-plane $-\infty < \text{Re } s < \sigma_2$, a right half-plane $\sigma_1 < \text{Re } s < \infty$, or the whole s-plane $-\infty < \text{Re } s < \infty$. In the latter case $g(s)$ represents an entire function. If $\sigma_1 = \sigma_2$, then $g(s)$ is defined only on the line $\text{Re } s = \sigma_1$.

A sufficient (but not necessary) condition for the existence of the inversion integral (2) is that the complex integration along the line parallel to the imaginary s-axis in the distance c from the origin is such that $\overline{\sigma}_1 < c < \overline{\sigma}_2$, where $\overline{\sigma}_1$ and $\overline{\sigma}_2$ are the limits of absolute convergence of (1).

The formula pair has proved to be of great use in the solution of numerous problems in analysis. We are concerned here with extensions of results given by MacFarlane [14], who used (1) and (2) to express infinite series of the form

$$\sum_{n=o}^{\infty} f(n)$$

in the form of a Mellin inversion integral, and by the use of the residue theorem transformed this integral back into another series which often converged faster or even terminated. This is, in a way, similar to the application of Poisson's summation formula [20, p. 60]. This application of (1) and (2) is by no means new, but was developed by Mellin himself [16]. Nevertheless, some interesting results have been given by MacFarlane. Also, he gave a short table of examples of pairs of the form (1) and (2). More extensive tables were published later [7, vol. 1].

In this thesis, two objectives are pursued: generalization of the results by MacFarlane; development of new tables of pairs of the form (1) and (2). In the first, a specific example of the general formula given by MacFarlane leads to the establishment of certain properties of a generalization of the Lerch zeta function. For theory on the Lerch zeta function see [2], [3], [4], [6, vol. 1, p. 27], [8], [12], and [18]. Although one particular result (transformation of Lerch's series into another series) is known [6, vol. 1, p. 29], it is produced here directly and without the use of Lerch's transformation formula and Hurwitz's series for the Hurwitz zeta function.

In pursuit of the second objective, nine methods of generating new tables of Mellin transforms are described and illustrated. The application of these methods proved to be extremely successful. Using only one of the nine methods, more than 200 new examples were obtained. At present the largest published table of Mellin transforms [7, vol. 1] contains only 245 examples.

2. MELLIN TRANSFORMS APPLIED TO INFINITE SERIES

Consider an infinite series of the form

(3) $$S = \sum_{n=0}^{\infty} f(n+a)$$

where $f(x)$ is given. We use the pair (1) and (2). We have by (2)

$$f(n+a) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} g(s)\,(n+a)^{-s}\,ds.$$

Now we sum on n, and provided it is permissible to interchange the order of summation and integration, we get

$$S = \sum_{n=0}^{\infty} f(n+a) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} g(s) \left(\sum_{n=0}^{\infty} (n+a)^{-s} \right) ds.$$

But

(4) $$\sum_{n=0}^{\infty} (n+a)^{-s} = \zeta(s,a), \quad \text{Re } s > 1$$

is Hurwitz's zeta function [6, vol. 1, p. 24]. The series (4) is ordinarily and absolutely convergent provided Re $s > 1$. Therefore,

(5) $$\sum_{n=0}^{\infty} f(n+a) = \frac{1}{2\pi i} \int_{\sigma_0 - i\infty}^{\sigma_0 + i\infty} \zeta(s,a)\, g(s)\, ds$$

where $\max[1, \overline{\sigma}_1] < \sigma_0 < \overline{\sigma}_2$. This formula represents the transformation of the l. h. s. infinite series into the r. h. s. Mellin inversion integral. The evaluation of this integral is usually performed by the aid of the residue theorem. This means that the integral in (5) is at first taken along a simple closed contour C which starts at

$s = \sigma_o + id$, progresses in some manner other than straight down to $s = \sigma_o - id$ (call this part of the contour C'), and then goes in a straight line parallel to the imaginary s-axis back to $s = \sigma_o + id$, $(d > 0)$. The contour is chosen such that the function

$$h(s) = \zeta(s, a)\, g(s)$$

has only singular points of one-valued character inside C. Then the residue theorem gives

$$\frac{1}{2\pi i} \int_C \zeta(s, a)\, g(s)\, ds = \sum \text{Res}\,[g(s)\zeta(s, a)]$$

where the summation is taken over all the singularities inside C. Now we let d and the minimum distance from the point $s = \sigma_o$ to C' tend simultaneously to infinity. When in this limiting case

$$\int_{C'} h(s)\, ds \rightarrow 0$$

we get

$$\frac{1}{2\pi i} \int_{\sigma_o - i\infty}^{\sigma_o + i\infty} \zeta(s, a)\, g(s)\, ds = \sum \text{Res}\,[\zeta(s, a)\, g(s)] .$$

Hence, in this case

$$(6) \qquad \sum_{n=o}^{\infty} f(n+a) = \sum \text{Res}\,[\zeta(s, a)\, g(s)] .$$

Two cases are now possible.

(i) $h(s) = \zeta(s, a)\, g(s)$ has an infinite number of singularities inside C. In this case, using equation (6), the original series is transformed into another infinite series. But the two series are usually of entirely different character. It might happen that the new series can be summed or that its convergence is faster than the original series.

(ii) $h(s) = \zeta(s,a)\, g(s)$ has a finite number of singularities inside C. In this case equation (6) gives the sum of the series

$$\sum_{n=o}^{\infty} f(n+a).$$

It is now necessary to have some information about the analytic character of $h(s) = \zeta(s,a)\, g(s)$. The Mellin transform of the summation function $f(x)$

$$g(s) = \int_{o}^{\infty} f(x)\, x^{s-1}\, dx$$

involves the function $\Gamma(s)$ in most cases. It is known that $\Gamma(s)$ has poles of the first order at $s = -\ell$, $(\ell = 0, 1, 2, \ldots)$ with residues

$$\frac{(-1)^{\ell}}{\ell!}\,.$$

The behavior of $\Gamma(s)$ at $s = \infty$ is given by Stirling's formula [6, vol. 1, p. 47] .

(7) $$\Gamma(s) = \left(\frac{2\pi}{s}\right)^{\frac{1}{2}} e^{-s}\, e^{s \log s}\left[1 + 0\left(\frac{1}{s}\right)\right] \text{ as } |s| \to \infty \text{ in } -\pi < \arg s < \pi.$$

(8) $$|\Gamma(\sigma + it)| = 0\left(|t|^{\sigma - \frac{1}{2}}\, e^{-\frac{\pi}{2}|t|}\right) \text{ as } |t| \to \infty \text{ with fixed } \sigma, (\sigma, t \text{ real}).$$

(9) $$\frac{\Gamma(\alpha + z)}{\Gamma(\beta + z)} = z^{\alpha - \beta}\left[1 + 0\left(\frac{1}{z}\right)\right] \quad \text{as} \quad |z| \to \infty.$$

The Hurwitz zeta function $\zeta(s,a)$ which occurs in $h(s)$ is a one-valued function analytic in every finite part of the s-plane except at $s = 1$, where $\zeta(s,a)$ has a first order pole with residue 1. The following properties of $\zeta(s,a)$ are given in [6, vol. 1, p. 26]:

(10) $$\zeta(0,a) = \frac{1}{2} - a$$

(11) $$\zeta(s,a) - \frac{1}{s-1} = -\psi(a) + \sum_{\ell=1}^{\infty} b_\ell (s-1)^\ell$$

(Laurent expansion of $\zeta(s,a)$ about $s = 1$)

(12) $$\frac{d}{ds}[\zeta(s,a)]_{s=0} = \log \Gamma(a) - \frac{1}{2}\log(2\pi)$$

(13) $$\zeta(-m,a) = \frac{-B_{m+1}(a)}{m+1}, \quad m = 0, 1, 2, \ldots$$

(The $B_m(x)$ are Bernoulli polynomials, [6, vol. 1, p. 35])

(14) $$\zeta(s,a) = 0\{|s|^{-\frac{1}{2}} e^{\frac{\pi}{2}|s||\sin\phi|} e^{-|s|\cos\phi[\log|s| - \log(2\pi)]}\}$$

as $|s| \to \infty$ in $-\pi < \arg s = \phi < -\frac{\pi}{2}$, $\frac{\pi}{2} < \arg s = \phi < \pi$.

3. GENERALIZATION OF LERCH'S ZETA FUNCTION

By applying (6) to a specific function $f(x)$, we will establish properties of a generalization of Lerch's zeta function.

Choose

$$f(x) = x^{-\nu} e^{-\beta x^{\delta}}, \quad \delta > 0, \ \mathrm{Re}\,\beta > 0.$$

Then

$$g(s) = \int_0^{\infty} f(x)\, x^{s-1}\, dx = \int_0^{\infty} x^{s-\nu-1} e^{-\beta x^{\delta}}\, dx.$$

Let $x^{\delta} = t$. Then

$$g(s) = \frac{1}{\delta} \int_0^{\infty} t^{\left(\frac{s-\nu}{\delta}\right)-1} e^{-\beta t}\, dt.$$

From [6, vol. 1, p. 1]

$$\Gamma(z) u^{-z} = \int_0^{\infty} e^{-ut} t^{z-1}\, dt, \quad \mathrm{Re}\, z > 0, \ \mathrm{Re}\, u > 0.$$

Therefore

$$g(s) = \frac{1}{\delta} \beta^{\left(\frac{\nu-s}{\delta}\right)} \Gamma\left(\frac{s-\nu}{\delta}\right), \quad \mathrm{Re}\, s > \mathrm{Re}\,\nu, \ \mathrm{Re}\,\beta > 0, \ \delta > 0.$$

Hence $\sigma_1 = \mathrm{Re}\,\nu$, $\sigma_2 = +\infty$.

From (5) we obtain

$$(15) \quad \sum_{n=0}^{\infty} (n+a)^{-\nu} e^{-\beta(n+a)^{\delta}} = \frac{\beta^{\frac{\nu}{\delta}}}{2\pi i \delta} \int_{\sigma_0 - i\infty}^{\sigma_0 + i\infty} \beta^{-\frac{s}{\delta}} \zeta(s,a) \Gamma\left(\frac{s-\nu}{\delta}\right) ds, \quad \sigma_0 > \max[1, \mathrm{Re}\,\nu].$$

If we denote

$$h_1(s) = \beta^{-\frac{s}{\delta}} \zeta(s,a) \Gamma\left(\frac{s-\nu}{\delta}\right),$$

then $h_1(s)$ has a first order pole at $s = 1$ [due to $\zeta(s,a)$], and an infinite set of poles of first order at $s = \nu - \delta\ell$, $(\ell = 0, 1, 2, \ldots)$

[due to $\Gamma\left(\frac{s-\nu}{\delta}\right)$]. The respective residues are:

(i) Res $[h_1(s)]$ at $s = 1$ is $\beta^{-\frac{1}{\delta}}\,\Gamma\left(\frac{1-\nu}{\delta}\right)$.

(ii) Res $[h_1(s)]$ at $s = \nu-\delta\ell$ is $\frac{(-1)^{\ell}}{\ell!}\,\delta\beta^{\ell-\frac{\nu}{\delta}}\,\zeta(\nu-\delta\ell, a)$.

We choose the path of integration C as indicated in Figure 1 below, such that half-circle C' of radius d separates the poles $s=\nu-N\delta$ and $s=\nu-(N+1)\delta$.

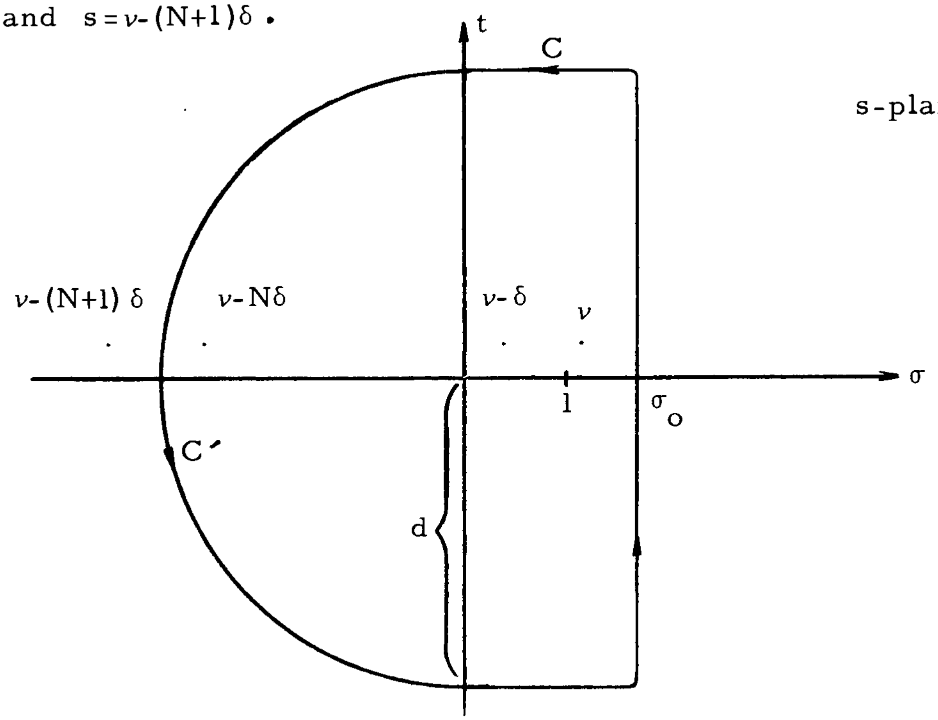

Figure 1.

Then $h_1(s)$ is one-valued and analytic inside and on C except at the points $s=1$, $s=\nu-\ell\delta$, $(\ell=0,1,\ldots,N)$. We now let N tend to infinity through positive integers.

Consider the contour integral

$$\int_C h_1(s)\,ds.$$

It is clear that the contributions along the horizontal lines of length σ_o vanish as $d \to \infty$ because of (8) and the fact that $\zeta(s,a)$ is bounded for $\operatorname{Re} s > 1$.

In order to investigate the contribution along the half-circle C', it is sufficient to investigate the behavior of $h_1(s)$ on the quarter-circle of radius d for $\frac{\pi}{2} < \phi < \pi$, since the modulus of $h_1(s)$ on the quarter-circle for $-\pi < \phi < -\frac{\pi}{2}$ is the same by Schwartz's reflection principle. From (7) and (14) we get

$$(16)\ h_1(s) = \beta^{-\frac{s}{\delta}} \zeta(s,a) \Gamma\left(\frac{s-\nu}{\delta}\right) = O\{|s|^{-2} e^{|s| \cos\phi[\log(2\pi) - \frac{1}{\delta}\log\beta]} \times$$

$$\times\, e^{|s| \sin\phi \left(\frac{\pi}{2} - \frac{\phi}{\delta}\right)}\, e^{|s| \cos\phi[\frac{1}{\delta}\log\left|\frac{s}{\delta}\right| - \log|s|]}\},$$

$$\text{as } |s| \to \infty \text{ in } \frac{\pi}{2} < \phi < \pi,\ s = |s| e^{i\phi}.$$

In formula (16) we have assumed β to be real and positive. The analytic continuation to complex β will be obtained later.

The following three cases are possible:

(i) $\delta > 1$: Then (16) is dominated by the last exponential function and $h_1(s)$ tends to infinity when $d \to \infty$. Thus, the contribution over the semi-circle tends to infinity as $d \to \infty$ and the formula (15) is not applicable, although the series in (15) converges for $\operatorname{Re} \beta > 0$.

(ii) $\delta = 1$: It is clear from (16) that the integral over the semi-circle C' vanishes as $d \to \infty$, provided $\beta < 2\pi$.

(iii) $0 < \delta < 1$: The integral over the semi-circle C' vanishes as $d \to \infty$, regardless of β.

Hence, in the cases (ii) and (iii) we obtain by the residue theorem and (15)

$$(17)\quad \sum_{n=0}^{\infty} (n+a)^{-\nu} e^{-\beta(n+a)^{\delta}} = \frac{1}{\delta}\Gamma\left(\frac{1-\nu}{\delta}\right) \beta^{\frac{(\nu-1)}{\delta}} + \sum_{\ell=0}^{\infty} \frac{(-1)^{\ell}}{\ell!} \beta^{\ell} \zeta(\nu-\delta\ell, a)$$

or

$$(18)\quad \sum_{n=0}^{\infty} (n+a)^{-\nu} e^{-\beta(n+a)^{\delta}} - \frac{1}{\delta} \beta^{(\nu-1)/\delta} \Gamma\left(\frac{1-\nu}{\delta}\right) = \sum_{\ell=0}^{\infty} \frac{(-1)^{\ell}}{\ell!} \beta^{\ell} \zeta(\nu-\delta\ell, a).$$

The r.h.s. series in (18) is a Taylor series around the origin and is therefore an analytic function of β in its circle of convergence, while the l.h.s. expression is valid only when $\operatorname{Re}\beta > 0$ and ν arbitrary; or $\operatorname{Re}\beta = 0$, $\operatorname{Im}\beta \neq 0$ and $\operatorname{Re}\nu > 0$; or $\beta = 0$ and $\operatorname{Re}\nu > 1$. Therefore, (18) represents the analytic continuation with respect to β of the l.h.s. of (18) valid for $\operatorname{Re}\beta > 0$ into the r.h.s. which is valid for unrestricted β when $0<\delta<1$, or for $|\beta| < 2\pi$ when $\delta = 1$. If $\delta > 1$, (18) is not valid.

In (18) put

$$e^{-\beta} = z, \quad \beta = \log\left(\frac{1}{z}\right).$$

Let

$$(19)\quad Y(z, \nu, a, \delta) = \sum_{n=0}^{\infty} (n+a)^{-\nu} z^{(n+a)^{\delta}}.$$

Then, from (18) we get

$$(20)\quad Y(z,\nu,a,\delta)-\frac{1}{\delta}\Gamma\left(\frac{1-\nu}{\delta}\right)\left(\log\frac{1}{z}\right)^{\frac{\nu-1}{\delta}}=\sum_{\ell=0}^{\infty}\frac{(\log z)^{\ell}}{\ell!}\zeta(\nu-\delta\ell,a).$$

Equation (20) is valid for unrestricted z if $0 < \delta < 1$. For $\delta = 1$, it follows from (20) that

$$(21)\quad Y(z,\nu,a,1)=z^{a}\Phi(z,\nu,a)=\Gamma(1-\nu)\left(\log\frac{1}{z}\right)^{\nu-1}+\sum_{\ell=0}^{\infty}\frac{(\log z)^{\ell}}{\ell!}\zeta(\nu-\ell,a),$$

$$\nu \neq 1,2,\ldots,|\log z| < 2\pi .$$

The function $\Phi(z,\nu,a)$ in (21) is defined by

$$(22)\quad \Phi(z,\nu,a)=\sum_{n=0}^{\infty}z^{n}(n+a)^{-\nu},\quad |z|<1.$$

$\Phi(z,\nu,a)$ is called Lerch's zeta function and is defined by the above power series for $|z| < 1$ and arbitrary ν. For $z=1$, (22) is Hurwitz's zeta function.

$$(23)\quad \Phi(1,\nu,a)=\zeta(\nu,a)=\sum_{n=0}^{\infty}(n+a)^{-\nu},\ \operatorname{Re}\nu > 1.$$

Lerch's function has been extensively investigated in [2], [3], [4], [6, vol. 1, p. 27], [8], [12], and [18].

Formula (21) was known previously [6, vol. 1, p. 29]. However, its derivation depended on the knowledge of Lerch's functional equation and Hurwitz's series for the Hurwitz zeta function. In this paper a generalization of formula (21), formula (20), was derived directly, without the use of either Lerch's functional equation or

Hurwitz's series.

Furthermore, Hardy's relation, see [8] and [16], also follows from (21).

$$(24)\quad \lim_{z\to 1}\left[\Phi(z,\nu,a) - \Gamma(1-\nu)\left(\log\frac{1}{z}\right)^{\nu-1}\right] = \zeta(\nu,a).$$

Therefore, (24) can be generalized by (20), and we get

$$(25)\quad \lim_{z\to 1}\left[Y(z,\nu,a,\delta) - \frac{1}{\delta}\Gamma\left(\frac{1-\nu}{\delta}\right)\left(\log\frac{1}{z}\right)^{\frac{\nu-1}{\delta}}\right] = \zeta(\nu,a),\quad 0 < \delta \le 1.$$

We might point out here that special cases of (19) have become increasingly important in Quantum Physics, cavity resonator theory and wave guide theory. For example, the Fermi-Dirac function, see [15] and [19], defined by

$$F_k(\eta) = \int_0^\infty x^k\,(e^{x-\eta}+1)^{-1}\,dx$$

is nothing but

$$F_k(\eta) = \Gamma(k+1)\,e^{\eta}\,\Phi(-e^{\eta}, k+1, 1)$$

where Φ is given by (22).

A vast number of special cases of

$$(26)\quad \Phi(z,\nu,1) = \sum_{n=1}^{\infty}\frac{z^n}{n^\nu}$$

are discussed in reference [13]. The function (26), called Jonquière's function, has been discussed and to some extent tabulated by Truesdell [21].

4. SOME METHODS FOR THE GENERATION OF MELLIN TRANSFORMS

It is obvious from the previous example that in order to treat other cases it is desirable to have extensive integral tables of the Mellin transform type. A short list of such integrals was given in MacFarlane's paper [14]. Later a much larger list of such integrals was given [7, vol. 1, p. 307-339]. In this chapter we shall present nine methods by which this last list can be substantially increased.

Consider

$$(27) \quad g(s) = \mathfrak{M}\{f(x); s\} = \int_0^\infty f(x)\, x^{s-1}\, dx, \quad \sigma_1 < \text{Re } s < \sigma_2.$$

Now express the kernel of this transform by another integral.

$$(28) \quad x^{s-1} = \frac{1}{\Gamma(1-s)} \int_0^\infty e^{-xt}\, t^{-s}\, dt, \quad \text{Re } x > 0, \ \text{Re } s < 1.$$

This is the well known definition of the Gamma function by Euler's integral. Substituting (28) into (27) we have

$$\mathfrak{M}\{f(x); s\} = \frac{1}{\Gamma(1-s)} \int_0^\infty f(x) \left(\int_0^\infty e^{-xt}\, t^{-s}\, dt \right) dx, \quad \sigma_1 < \text{Re } s < \min[1, \sigma_2].$$

If $f(x)$ is such that inversion of the order of integration is permissible, then

$$(29) \quad \mathfrak{M}\{f(x); s\} = \frac{1}{\Gamma(1-s)} \int_0^\infty t^{-s} \left(\int_0^\infty f(x)\, e^{-xt}\, dx \right) dt, \quad \sigma_1 < \text{Re } s < \min[1, \sigma_2].$$

But the inner integral is now the Laplace transform of $f(x)$ with respect to the parameter t. (For tables of this kind see [7, vol. 1, p. 127-226]) If we denote

(30) $$\mathcal{L}\{f(x);t\} = \int_0^\infty f(x)\, e^{-xt}\, dx,$$

then from (29) and (30) we get

(31) $$\mathcal{M}\{\mathcal{L}\{f(x);t\};1-s\} = \Gamma(1-s)\,\mathcal{M}\{f(x);s\}\,,\ \sigma_1 < \mathrm{Re}\ s < \min[1,\sigma_2]\,.$$

Thus, if we have a function $f(x)$ such that we know both its Mellin and its Laplace transform, then by (31) we have the Mellin transform of the Laplace transform of $f(x)$.

A specific example will illustrate the method. Consider

$$f(x) = \frac{x^{\nu-1}\, e^{-ax}}{1-e^{-x}}\,,\ \mathrm{Re}\ a > 0,\ \mathrm{Re}\ \nu > 1.$$

From [7, vol. 1, p. 313]

$$\mathcal{M}\{f(x);s\} = \Gamma(s+\nu-1)\,\zeta(s+\nu-1,a),\ \mathrm{Re}\ a > 0,\ \mathrm{Re}\ (s+\nu) > 2.$$

From [7, vol. 1, p. 144]

$$\mathcal{L}\{f(x);\ t\} = \Gamma(\nu)\,\zeta(\nu,t+a),\ \mathrm{Re}\ \nu > 1,\ \mathrm{Re}\ (t+a) > 0.$$

Therefore, by (31)

(32) $$\mathcal{M}\{\zeta(\nu,t+a);s\} = \int_0^\infty \zeta(\nu,t+a)\,t^{s-1}\,dt = \frac{\Gamma(s)\,\Gamma(\nu-s)}{\Gamma(\nu)}\,\zeta(\nu-s,a),\ \mathrm{Re}\ a > 0,\ \mathrm{Re}\ \nu > 1,$$

$$0 < \mathrm{Re}\ s < \mathrm{Re}\ \nu-1.$$

Formula (32) is not listed in any of the published tables of Mellin transforms.

It can easily be shown that for this particular example inversion of the order of integration, which led to (29) and (31), is permissible. In most cases a well known theorem about the sufficiency (not necessity) of interchanging the order of integration of repeated infinite integrals can be applied [1, p. 445, 449-50], [11, p. 536].

Theorem: If $f(x, y)$ is a non-negative continuous function in $x \geq a$, $y \geq b$, and

$$\phi(y) = \int_a^\infty f(x, y)dx, \quad \psi(x) = \int_b^\infty f(x, y)\, dy$$

are continuous for $y \geq b$ and $x \geq a$, and at least one of

$$\int_b^\infty \phi(y)dy \quad \text{and} \quad \int_a^\infty \psi(x)dx$$

is convergent, then the other is also convergent and the two are equal; i.e.

$$\int_a^\infty \int_b^\infty f(x, y)\, dy\, dx = \int_b^\infty \int_a^\infty f(x, y)\, dx\, dy.$$

For cases where this theorem does not apply, individual investigations will have to be made. (See also [9], [10])

Other useful formulas similar to (31) can be obtained if certain different integral representations for the kernel x^{s-1} are substituted into (27) and the order of integration is interchanged. Below is given a list of eight different integral representations for x^{s-1}, and the resulting formulas obtained when they are substituted for x^{s-1} in (27) and the order of integration is interchanged.

From [7, vol. 1, p. 317]

$$(33)\quad x^{s-1} = \frac{\sec\left(\frac{\pi s}{2}\right)}{\Gamma(1-s)} \int_0^{\infty} \sin(xt)\, t^{-s}\, dt,\ x>0,\ 0<\operatorname{Re} s<2.$$

$$(34)\quad \mathfrak{M}\{\mathfrak{F}_s\{f(x);t\};1-s\} = \frac{\Gamma(1-s)}{\sec\left(\frac{\pi s}{2}\right)} \mathfrak{M}\{f(x);s\},\ \max[0,\sigma_1]<\operatorname{Re} s<\min[2,\sigma_2],$$

where $\mathfrak{F}_s\{f(x);t\} = \int_0^{\infty} f(x)\sin(xt)\,dx$ is the Fourier sine transform of $f(x)$ with respect to the parameter t. When the Mellin and Fourier sine transforms of $f(x)$ are known, we have from (34) the Mellin transform of the Fourier sine transform of $f(x)$.

From [7, vol. 1, p. 319]

$$(35)\quad x^{s-1} = \frac{\csc\left(\frac{\pi s}{2}\right)}{\Gamma(1-s)} \int_0^{\infty} \cos(xt)\, t^{-s}\, dt,\ x>0,\ 0<\operatorname{Re} s<1.$$

$$(36)\quad \mathfrak{M}\{\mathfrak{F}_c\{f(x);t\};1-s\} = \frac{\Gamma(1-s)}{\csc\left(\frac{\pi s}{2}\right)} \mathfrak{M}\{f(x);s\},\ \max[0,\sigma_1]<\operatorname{Re} s<\min[1,\sigma_2],$$

where $\mathfrak{F}_c\{f(x);t\} = \int_0^{\infty} f(x)\cos(xt)\,dx$ is the Fourier cosine transform of $f(x)$. When both the Mellin and Fourier cosine transforms of $f(x)$ are known, we have from (36) the Mellin transform of the Fourier cosine transform of $f(x)$.

From [7, vol. 1, p. 308]

$$(37)\quad x^{s-1} = \pi^{-1}\sin(\pi s) \int_0^{\infty} (x+t)^{-1} t^{s-1}\, dt,\quad |\arg x|<\pi,\ 0<\operatorname{Re} s<1.$$

$$(38)\quad \mathfrak{M}\{\mathfrak{S}\{f(x);t\};s\} = \pi\csc(\pi s)\,\mathfrak{M}\{f(x);s\},\ \max[0,\sigma_1]<\operatorname{Re} s<\min[1,\sigma_2],$$

where $\mathcal{S}\{f(x);t\} = \int_0^\infty f(x)(x+t)^{-1}dx$ is the Stieltjes transform of f(x). When both the Mellin and Stieltjes transforms of f(x) are known, we have from (38) the Mellin transform of the Stieltjes transform of f(x).

From [7, vol. 1, p. 310]

(39) $$x^{s-1} = \frac{1}{B(s+\rho-1,1-s)}\int_0^\infty (x+t)^{-\rho}t^{s+\rho-2}dt, \quad \left|\arg\frac{1}{x}\right| < \pi, \ 1-\mathrm{Re}\ \rho < \mathrm{Re}\ s < 1.$$

(40) $$\mathfrak{M}\{\mathcal{S}_\rho\{f(x);t\}; s+\rho-1\} = B(s+\rho-1,1-s)\,\mathfrak{M}\{f(x);s\}, \quad \max[1-\mathrm{Re}\ \rho, \sigma_1] < \mathrm{Re}\ s < \min[1,\sigma_2],$$

where $\mathcal{S}_\rho\{f(x);t\} = \int_0^\infty f(x)(x+t)^{-\rho}dx$ is the generalized Stieltjes transform of f(x). When both the Mellin and generalized Stieltjes transforms of f(x) are known, we have from (40) the Mellin transform of the generalized Stieltjes transform of f(x).

From [7, vol. 1, p. 326]

(41) $$x^{s-1} = \frac{\Gamma\left(\frac{1}{4}+\frac{s+\nu}{2}\right)}{2^{\frac{1}{2}-s}\,\Gamma\left(\frac{3}{4}+\frac{\nu-s}{2}\right)}\int_0^\infty J_\nu(xt)(xt)^{\frac{1}{2}}t^{-s}dt, \quad x>0, \ 0<\mathrm{Re}\ s<\frac{3}{2}+\mathrm{Re}\ \nu.$$

(42) $$\mathfrak{M}\{\mathcal{H}_\nu\{f(x);t\};1-s\} = \frac{2^{\frac{1}{2}-s}\,\Gamma\left(\frac{3}{4}+\frac{\nu-s}{2}\right)}{\Gamma\left(\frac{1}{4}+\frac{s+\nu}{2}\right)}\,\mathfrak{M}\{f(x);s\},$$

$$\max[0,\sigma_1] < \mathrm{Re}\ s < \min\left[\sigma_2, \frac{3}{2}+\mathrm{Re}\ \nu\right],$$

where $\mathcal{H}_\nu\{f(x);t\} = \int_0^\infty f(x)\,J_\nu(xt)(xt)^{\frac{1}{2}}dx$ is the Hankel transform of order ν of f(x). When both the Mellin and Hankel transforms of f(x) are known, we have from (42) the Mellin transform of the Hankel transform of f(x).

From [7, vol. 1, p. 329]

(43) $$x^{s-1} = \frac{-2^{s-\frac{1}{2}}\pi \sec\left[\frac{\pi}{2}\left(\frac{3}{2}-s-\nu\right)\right]}{\Gamma\left(\frac{3}{4}+\frac{\nu-s}{2}\right)\Gamma\left(\frac{3}{4}-\frac{\nu+s}{2}\right)} \int_0^\infty Y_\nu(xt)(xt)^{\frac{1}{2}}\, t^{-s}\, dt,$$

$$x > 0, \quad 0 < \operatorname{Re} s < \frac{3}{2} - |\operatorname{Re} \nu|.$$

(44) $$\mathfrak{M}\{\mathcal{Y}_\nu\{f(x); t\}; 1-s\} = \frac{-\Gamma\left(\frac{3}{4}+\frac{\nu-s}{2}\right)\Gamma\left(\frac{3}{4}-\frac{\nu+s}{2}\right)}{2^{s-\frac{1}{2}}\pi \sec\left[\frac{\pi}{2}\left(\frac{3}{2}-s-\nu\right)\right]}\mathfrak{M}\{f(x); s\},$$

$$\max[0, \sigma_1] < \operatorname{Re} s < \min\left[\sigma_2, \frac{3}{2} - |\operatorname{Re} \nu|\right],$$

where $\mathcal{Y}_\nu\{f(x); t\} = \int_0^\infty Y_\nu(xt)(xt)^{\frac{1}{2}} f(x)\, dx$ is the Y-transform of order ν of $f(x)$. When both the Mellin and Y-transforms of $f(x)$ are known, we have from (44) the Mellin transform of the Y-transform of $f(x)$.

From [7, vol. 1, p. 331]

(45) $$x^{s-1} = \frac{2^{s+\frac{1}{2}}}{\Gamma\left(\frac{3}{4}-\frac{s+\nu}{2}\right)\Gamma\left(\frac{3}{4}+\frac{\nu-s}{2}\right)} \int_0^\infty K_\nu(xt)(xt)^{\frac{1}{2}}\, t^{-s}\, dt, \quad \operatorname{Re} x > 0,$$

$$\operatorname{Re} s < \frac{3}{2} - |\operatorname{Re} \nu|.$$

(46) $$\mathfrak{M}\{\mathcal{K}_\nu\{f(x); t\}; 1-s\} = \frac{\Gamma\left(\frac{3}{4}-\frac{s+\nu}{2}\right)\Gamma\left(\frac{3}{4}+\frac{\nu-s}{2}\right)}{2^{s+\frac{1}{2}}}\mathfrak{M}\{f(x); s\},$$

$$\sigma_1 < \operatorname{Re} s < \min\left[\sigma_2, \frac{3}{2} - |\operatorname{Re} \nu|\right],$$

where $\mathcal{K}_\nu\{f(x); t\} = \int_0^\infty K_\nu(xt)(xt)^{\frac{1}{2}} f(x)\, dx$ is the K-transform of order ν of $f(x)$. When both the Mellin and K-transforms of $f(x)$ are known, we have from (46) the Mellin transform of the K-transform of f(x).

From [7, vol. 1, p. 335]

(47) $$x^{s-1} = \frac{\Gamma\left(\frac{1}{4}+\frac{s+\nu}{2}\right)}{\Gamma\left(\frac{3}{4}+\frac{\nu-s}{2}\right)} 2^{s-\frac{1}{2}} \operatorname{ctn}\left[\frac{\pi}{2}\left(\frac{3}{2}-s+\nu\right)\right] \int_0^\infty t^{-s}\, \mathbf{H}_\nu(xt)(xt)^{\frac{1}{2}}\, dt,$$

$$x>0, \quad \max\left[0, \frac{1}{2}+\operatorname{Re}\nu\right] < \operatorname{Re}s < \operatorname{Re}\nu + \frac{5}{2}.$$

(48) $$\mathfrak{M}\{\mathbf{H}_\nu\{f(x); t\}; 1-s\} = \frac{\Gamma\left(\frac{3}{4}+\frac{\nu-s}{2}\right) \tan\left[\frac{\pi}{2}\left(\frac{3}{2}-s+\nu\right)\right]}{2^{s-\frac{1}{2}}\Gamma\left(\frac{1}{4}+\frac{s+\nu}{2}\right)} \mathfrak{M}\{f(x); s\},$$

$$\max\left[\sigma_1, 0, \frac{1}{2}+\operatorname{Re}\nu\right] < \operatorname{Re}s < \min\left[\sigma_2, \frac{5}{2}+\operatorname{Re}\nu\right],$$

where $\mathbf{H}_\nu\{f(x); t\} = \int_0^\infty f(x)\, \mathbf{H}_\nu(xt)(xt)^{\frac{1}{2}}\, dx$ is the $\mathbf{H}$-transform of order ν of f(x). When both the Mellin and $\mathbf{H}$-transforms of f(x) are known, we have from (48) the Mellin transform of the $\mathbf{H}$-transform of f(x).

For tables of integral transforms of the types mentioned in this chapter, see [7].

In conclusion we give a table of Mellin transforms. We do not attempt to be exhaustive, but include only those examples which were derived using the first of the methods described in the last chapter and which are not included in [7, vol. 1, p. 307-339].

In this table notations for many special functions are used. For the definitions of these notations see [7, vol. 1, p. 367-388].

Algebraic functions

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
1.	$(t+a)^{-2}$, $\operatorname{Re} a > 0$	$\Gamma(s)\,\Gamma(2-s)a^{s-2}$, $0 < \operatorname{Re} s < 2$
2.	$(t^{\frac{1}{2}}+a^{\frac{1}{2}})^{-1}$, $\lvert\arg a\rvert < \pi$	$\Gamma(\frac{1}{2}+s)\Gamma(\frac{1}{2}-s)\,a^{s-\frac{1}{2}}\csc(\pi s)$, $0 < \operatorname{Re} s < \frac{1}{2}$
3.	$(t+a)^{-\nu}$, $\operatorname{Re} a > 0, \operatorname{Re} \nu > 0$	$\frac{\Gamma(s)}{\Gamma(\nu)}\, a^{s-\nu}\, \Gamma(\nu-s)$, $0 < \operatorname{Re} s < \operatorname{Re} \nu$
4.	$(t^2+a^2)^{-\nu-\frac{1}{2}}$, $a > 0, \operatorname{Re} \nu - \frac{1}{2}$	$\dfrac{\Gamma(s)\, 2^{-s}\,\Gamma\left(\frac{2\nu-s+1}{2}\right)\pi^{\frac{1}{2}}}{a^{2\nu+1-s}\,\Gamma\left(\frac{1+s}{2}\right)\Gamma\left(\nu+\frac{1}{2}\right)}$ $(\max[0, \operatorname{Re} \nu - \frac{1}{2}] < \operatorname{Re} s < 1 + \operatorname{Re} 2\nu)$
5.	$[(t+a)^2 - a^2]^{-\nu-\frac{1}{2}}$, $\operatorname{Re} a > 0$ $\operatorname{Re} \nu > \frac{-1}{2}$	$\dfrac{\Gamma(s-\nu-\frac{1}{2})\,\Gamma(1+2\nu-s)}{(2a)^{2\nu+1-s}\,\Gamma(\nu+\frac{1}{2})}$ $\frac{1}{2} + \operatorname{Re} \nu < \operatorname{Re} s < 1 + \operatorname{Re} 2\nu$
6.	$[(t+a)^2-\beta^2]^{-\nu-\frac{1}{2}}$, $\operatorname{Re} a > \lvert\operatorname{Re} \beta\rvert$ $\operatorname{Re} \nu > -\frac{1}{2}$	$\dfrac{\Gamma(s)\,\Gamma(\nu+\frac{1-s}{2})\,\Gamma(1+\nu-\frac{s}{2})}{2^s\,\Gamma(\nu+\frac{1}{2})\,\Gamma(\nu+1)a^{2\nu+1-s}} \times$ $\times\ {}_2F_1(\nu+1-\frac{s}{2}, \nu+\frac{1-s}{2}; \nu+1; \frac{\beta^2}{a^2})$ $0 < \operatorname{Re} s < \operatorname{Re} 2\nu+1$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
7.	$([t+a]^2+\beta^2)^{-1}$, $\mathrm{Re}\, a > \lvert \mathrm{Im}\beta \rvert$	$\Gamma(s)\,\Gamma(1-s)\,\beta^{-1}(a^2+\beta^2)^{(s-1)/2}\sin[(1-s)\tan^{-1}(\frac{\beta}{a})]$ $0 < \mathrm{Re}\, s < 2$
8.	$[t+(t^2+a^2)^{\frac{1}{2}}]^{-\nu}$, $a > 0$, $\mathrm{Re}\,\nu > 0$	$\dfrac{\nu\Gamma(s)\, 2^{-s-1}\,\Gamma(\frac{\nu-s}{2})}{a^{\nu-s}\,\Gamma(\frac{\nu+s+2}{2})}$, $0 < \mathrm{Re}\, s < \mathrm{Re}\,\nu$
9.	$[t+a+\{(t+a)^2-a^2\}^{\frac{1}{2}}]^{-\nu}$, $\mathrm{Re}\, a > 0$, $\mathrm{Re}\,\nu > 0$	$\dfrac{\Gamma(s)\,\Gamma(s+\frac{1}{2})\,\Gamma(\nu-s)}{(2a)^{-s}\,a^{\nu}\,\pi^{\frac{1}{2}}\,\nu^{-1}\,\Gamma(1+\nu+s)}$, $0 < \mathrm{Re}\, s < \mathrm{Re}\,\nu$
10.	$(t+a+[(t+a)^2-\beta^2]\ )^{-\nu}$ $\mathrm{Re}\,\nu > 0$, $\mathrm{Re}\, a > \lvert\mathrm{Re}\,\beta\rvert$	$\dfrac{\nu\,\Gamma(s)\,\Gamma(\frac{\nu-s}{2})\,\Gamma(\frac{1-s+\nu}{2})}{\pi^{\frac{1}{2}}\,2^{s+1}\,a^{\nu-s}\,\Gamma(\nu+1)}\ \times$ $\times\ {}_2F_1\left(\frac{1+\nu-s}{2},\ \frac{\nu-s}{2};\ \nu+1;\ \frac{\beta^2}{a^2}\right)$, $0 < \mathrm{Re}\, s < \mathrm{Re}\,\nu$
11.	$(3t^2+a^2)(t^2+a^2)^{-2}$, $a > 0$	$\dfrac{2^{-2-s}\,\pi^{\frac{1}{2}}\,\Gamma(s+3)\,\Gamma(\frac{1-s}{2})}{a^{2-s}\ \Gamma(2+\frac{s}{2})}\mathrm{ctn}\,(\frac{\pi s}{2})$ $0 < \mathrm{Re}\, s < 2$
12.	$(t+\beta)^{-1}(t+\beta+1)^{-a}$ $\mathrm{Re}\,\beta > 0$, $\mathrm{Re}\, a > -1$	$\frac{\Gamma(s)}{\Gamma(a)}a^{-1}\ (1+\beta)^{s-a-1}\ \Gamma(1+a-s)\ \times$ $\times\ {}_2F_1(1,\ 1+a-s;\ a+1;\ \frac{1}{1+\beta})$ $0 < \mathrm{Re}\, s < 1+\mathrm{Re}\, a$
13.	$(t+\beta)^{-1}[1-(1+t+\beta)^{-a}]$ $\mathrm{Re}\, a, \beta > -1$	$\frac{\Gamma(s)}{\Gamma(a)}(1-s)^{-1}(1+\beta)^{s-a-1}\Gamma(1+a-s)\ \times$ $\times\ {}_2F_1(1,\ 1+a-s;\ 2-s;\ \frac{\beta}{1+\beta})$ $0 < \mathrm{Re}\, s < 1$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
14.	$(t+a-1)^n (t+a)^{-n-1}$, Re $a>0$	$\frac{\Gamma(s)}{n!} \Gamma(1-s+n)(a-1)^n a^{s-1-n} \times$ $\times\ {}_2F_1(-n, s; s-n; \frac{a}{a+1})$, $0<\text{Re } s<1$
15.	$(t+a-\frac{1}{2})^n (t+a+\frac{1}{2})^{-n-\frac{1}{2}}$ Re $a > -\frac{1}{2}$	$\frac{(-1)^n \Gamma(s)\, \pi^{\frac{1}{2}}\, 2^{2s}\, \Gamma(1-2s)}{\Gamma(n+\frac{1}{2})\, \Gamma(1-s-n)(a+\frac{1}{2})^{1-s}} \times$ $\times\ {}_2F_1(1-s, \frac{1}{2}-n;\ 1-s-n;\ \frac{a-\frac{1}{2}}{a+\frac{1}{2}})$ $0 < \text{Re } s < \frac{1}{2}$
16.	$(t+a-\frac{1}{2})^n (t+a+\frac{1}{2})^{-n-\frac{3}{2}}$ Re $a > -\frac{1}{2}$	$\frac{(-1)^n \Gamma(s)\, \pi^{\frac{1}{2}}\, 2^{2s-\frac{1}{2}}\, \Gamma(2-2s)}{\Gamma(n+\frac{3}{2})\, \Gamma(1-s-n)(a+\frac{1}{2})^{\frac{3}{2}-s}} \times$ $\times\ {}_2F_1(\frac{3}{2}-s, -n; 1-s-n;\ \frac{a-\frac{1}{2}}{a+\frac{1}{2}})$ $0 < \text{Re } s < 1$
17.	$(t+a)[(t+a)^2+\beta^2]^{-1}$ Re $a > \lvert \text{Im}\beta \rvert$	$\Gamma(s)(a^2+\beta^2)^{(s-1)/2}\Gamma(1-s)\cos[(1-s)\tan^{-1}(\frac{\beta}{a})]$ $0 < \text{Re } s < 1$
18.	$(t+a)[(t+a)^2-a^2]^{-\nu-\frac{3}{2}}$ Re $a>0$, Re $\nu>-1$	$\frac{\Gamma(s)\,\Gamma(s-\nu-\frac{3}{2})\,\Gamma(2+2\nu-s)}{\Gamma(s-1)\, 2^{2\nu+3-s}\, a^{2\nu+2-s}\, \Gamma(\nu+\frac{3}{2})}$ $\frac{3}{2}+\text{Re }\nu < \text{Re } s < 2 + \text{Re } 2\nu$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
19.	$(t+a)[(t+a)^2-\beta^2]^{-\nu-\frac{3}{2}}$ $\mathrm{Re}\, a > \lvert \mathrm{Re}\, \beta \rvert$, $\mathrm{Re}\, \nu > -1$	$\frac{\Gamma(s)\Gamma(1+\nu-\frac{s}{2})\Gamma(\frac{3}{2}+\nu-\frac{s}{2})}{\Gamma(\nu+\frac{3}{2})2^s a^{2+2\nu-s}\Gamma(\nu+1)} \times$ $\times\ {}_2F_1(\frac{3}{2}+\nu-\frac{s}{2}, 1+\nu-\frac{s}{2}; \nu+1; \frac{\beta^2}{a^2})$ $0 < \mathrm{Re}\, s < 2 + \mathrm{Re}\, 2\nu$
20.	$(t+a)^{\beta-\gamma}(t+a-\lambda)^{-\beta}\Gamma(\rho)$ $\mathrm{Re}\, a, \lambda, \rho > 0$	$\Gamma(s)\, a^{s-\gamma}\Gamma(\gamma-s)\, {}_2F_1(\beta, \gamma-s; \rho; \frac{\lambda}{a})$ $0 < \mathrm{Re}\, s < \mathrm{Re}\, \gamma$
21.	$[t^2+(a+b)^2]^{-1}[t^2+(a-b)^2]^{-1}$ $a, b > 0$, $a \neq b$	$\frac{\Gamma(s)}{4\,ab}\Gamma(1-s)\sin(\frac{\pi s}{2})[\lvert b-a \rvert^{s-1} - (b+a)^{s-1}]$ $1 < \mathrm{Re}\, s < 4$
22.	$\left[\frac{(t^2+1)^{\frac{1}{2}} - t}{t^2+1}\right]^{\frac{1}{2}}$	$\left(\frac{2}{\pi}\right)^{\frac{1}{2}} \Gamma(s)\sin\left(\frac{\pi}{4}-\frac{\pi s}{2}\right)\Gamma\left(\frac{1}{2}-s\right)$ $0 < \mathrm{Re}\, s < \frac{3}{2}$
23.	$\frac{[t+(t^2+a^2)\ ]^{-n-\frac{1}{2}}}{(t^2+a^2)^{\frac{1}{2}}}$, $a > 0$	$\frac{(-1)^n\Gamma(s)2^{-s}\Gamma\left(\frac{\frac{1}{2}-s-n}{2}\right)}{a^{\frac{3}{2}+n-s}\ \Gamma\left(\frac{\frac{1}{2}+s-n}{2}\right)}\tan\left[\frac{\pi}{2}(\frac{1}{2}-s)\right]$ $\max(0, -n-\frac{1}{2}) < \mathrm{Re}\, s < \frac{3}{2} - n$
24.	$\frac{(t-\beta+a)^n}{(t+a)^{\nu+n+1}}$, $\mathrm{Re}\, a > 0$ $\mathrm{Re}\, \nu > -1$	$\frac{\Gamma(s)\,\Gamma(1+\nu+n-s)}{\Gamma(\nu+n+1)}(a-\beta)^n a^{s-\nu-n-1} \times$ $\times\ {}_2F_1(-n, s; s-\nu-n; \frac{a}{a-\beta})$ $0 < \mathrm{Re}\, s < 1 + \mathrm{Re}\, \nu$

	f(t)	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
25.	$\dfrac{(t+a-\frac{\beta}{2})^{k-\mu-\frac{1}{2}}}{(t+a+\frac{\beta}{2})^{k+\mu+\frac{1}{2}}}$, $\mathrm{Re}\, a > \frac{1}{2}\lvert\mathrm{Re}\,\beta\rvert$	$\dfrac{\Gamma(s)\,\Gamma(2\mu+1-s)(a+\frac{\beta}{2})^{s-2\mu-1}}{\Gamma(2\mu+1)} \times$ $\times\, {}_2F_1(1+2\mu-s, \mu-k+\frac{1}{2}; 2\mu+1; \dfrac{\beta}{a+\frac{\beta}{2}})$ $0<\mathrm{Re}\, s<1+\mathrm{Re}\, 2\mu$
26.	$\dfrac{(t+2a)^{\frac{1}{2}}-t^{\frac{1}{2}}}{(t+2a)^{\frac{1}{2}}+t^{\frac{1}{2}}}$, $\mathrm{Re}\, a>0$	$\dfrac{\Gamma(s)\,\Gamma(1-s)\,\Gamma(s-\frac{1}{2})}{2^{-s}a^{-s}\pi^{\frac{1}{2}}\,\Gamma(2+s)}$, $0<\mathrm{Re}\, s<1$
27.	$\dfrac{(t^2+a^2+\beta^2)}{[t^2+(a+\beta)^2]\,[t^2+(a-\beta)^2]}$, $a,\beta>0$	$-\frac{1}{2}\Gamma(s-1)\Gamma(2-s)\cos(\frac{\pi s}{2}) \times$ $\times[\lvert\beta-a\rvert^{s-2}+(\beta+a)^{s-2}]$, $1<\mathrm{Re}\, s<2$
28.	$\dfrac{(t^2-\beta^2+a^2)}{[t^2+(a+\beta)^2][t^2+(\beta-a)^2]}$, $a,\beta>0$	$\frac{1}{2a}\Gamma(s)\Gamma(1-s)\cos(\frac{\pi s}{2}) \times$ $\times[(a+\beta)^{s-1}+\mathrm{sgn}(a-\beta)\lvert a-\beta\rvert^{s-1}]$ $0<\mathrm{Re}\, s<2$
29.	$\dfrac{(t+a+\beta)^{\frac{1}{2}}-(t+a-\beta)^{\frac{1}{2}}}{(t+a+\beta)^{\frac{1}{2}}+(t+a-\beta)^{\frac{1}{2}}}$ $\mathrm{Re}\, a>\lvert\mathrm{Re}\,\beta\rvert$	$\dfrac{\Gamma(s)\, 2^{-s-1}\beta\, \Gamma(\frac{1-s}{2})\,\Gamma(\frac{3-s}{2})}{\pi^{\frac{1}{2}}\, a^{1-s}} \times$ $\times\, {}_2F_1(\frac{2-s}{2}, \frac{1-s}{2}; 2; \frac{\beta^2}{a^2})$ $0<\mathrm{Re}\, s<\mathrm{Re}\,\nu$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
30.	$\left(\frac{t}{2}\right)^{-\frac{1}{2}} \frac{[t+(t^2+a^2)^{\frac{1}{2}}]^{\frac{1}{2}}}{[t^2+a^2]^{\frac{1}{2}}}$ $a>0$	$\frac{2^{-s}\Gamma(s)\Gamma(\frac{3}{4}-\frac{s}{2})}{a^{\frac{1}{2}-s}\Gamma(\frac{3}{4}+\frac{s}{2})} \operatorname{ctn}[\frac{\pi}{2}(s-\frac{1}{2})]$ $\frac{1}{2} < \operatorname{Re} s < \frac{5}{2}$
31.	$\frac{[t+a+([t+a]^2-a^2)^{\frac{1}{2}}]^{-\nu}}{([t+a]^2-a^2)^{\frac{1}{2}}}$ $\operatorname{Re} a>0,\ \operatorname{Re}\nu>-1$	$\frac{\Gamma(s)\, a^{s-\nu-1}\,\Gamma(s-\frac{1}{2})\,\Gamma(1+\nu-s)}{2^{1-s}\pi^{\frac{1}{2}}\,\Gamma(s+\nu)}$ $\frac{1}{2} < \operatorname{Re} s < 1+\operatorname{Re}\nu$
32.	$(t+a+\beta)^{2\nu}\{[(2\beta)^{\frac{1}{2}}+(\beta-a-t)^{\frac{1}{2}}]^{1-2\nu}+$ $+[(2\beta)^{\frac{1}{2}}-(\beta-a-t)^{\frac{1}{2}}]^{-2\nu}\}$ $\operatorname{Re}(a+\beta)>0,\ \operatorname{Re}\nu<0$	$\frac{\Gamma(s)\,\pi^{\frac{1}{2}}\,2^{\frac{3}{2}+2s}\,\beta^{\frac{1}{2}}\,\Gamma(-2s-2\nu)}{\Gamma(-2\nu)\Gamma(\frac{1-s}{2})(a+\beta)^{\frac{1}{2}-s-\nu}} \times$ $\times\ {}_2F_1(\frac{1}{2}-s-\nu,\nu+\frac{1}{2},\frac{1}{2}-s;\ \frac{a-\beta}{a+\beta})$ $0 < \operatorname{Re} s < -\operatorname{Re}\nu$
33.	$\frac{(t+\nu[(t+a)^2-a^2]^{\frac{1}{2}})}{[(t+a)^2-a^2]^{\frac{3}{2}}} \times$ $\times\{(t+a)+[(t+a)^2-a^2]^{\frac{1}{2}}\}^{-\nu}$ $\operatorname{Re} a>0,\ \operatorname{Re}\nu>-2$	$\frac{\Gamma(s)\,\Gamma(s-\frac{3}{2})\,\Gamma(2+\nu-s)}{2^{2-s}a^{2+\nu-s}\pi^{\frac{1}{2}}\Gamma(\nu+s-1)}$ $\frac{3}{2} < \operatorname{Re} s < 2+\operatorname{Re}\nu$
34.	$(t+a+\nu u_2)\, u_2^{-3}\, U_2^{-\nu}$ $\operatorname{Re} a > \lvert\operatorname{Re}\beta\rvert,\ \operatorname{Re}\nu>-2$	$\frac{\Gamma(s)\,2^{1-s}\,\Gamma(1+\frac{\nu-s}{2})\,\Gamma(\frac{3-s+\nu}{2})}{\pi^{\frac{1}{2}}a^{2+\nu-s}\,\Gamma(\nu+1)} \times$ $\times\ {}_2F_1(\frac{3-s+\nu}{2},\frac{2-s+\nu}{2};\nu+1;\ \frac{\beta^2}{a^2})$ $0 < \operatorname{Re} s < 2+\operatorname{Re}\nu$

$u_2=[(t+a)^2-\beta^2]^{\frac{1}{2}}$, $U_2=t+a+u_2$

	f(t)	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
35.	$r^{-3}(t+\nu r)(\frac{a}{R})^{\nu}$, $a>0$, $\text{Re}\,\nu>-2$	$\dfrac{\Gamma(s)\, 2^{1-s}\, \Gamma(\frac{2+\nu-s}{2})}{a^{2-s}\, \Gamma(\frac{\nu+s}{2})}$ $\frac{1}{2} < \text{Re } s < 2 + \text{Re } \nu$
36.	$(\frac{\nu^2-1}{r^3} + 3t\, \frac{t+\nu r}{r^5})(\frac{a}{R})^{\nu}$ $a>0$, $\text{Re } \nu > -3$	$\dfrac{\Gamma(s)\, 2^{-s}\, \Gamma(\frac{3-s+\nu}{2})}{a^{3-s}\, \Gamma(\frac{\nu+s-1}{2})}$, $\frac{3}{2} < \text{Re } s < 3 + \text{Re } \nu$
37.	$\frac{1}{\nu-1}(\frac{a}{R})^{\nu-1} + \frac{1}{\nu+1}(\frac{a}{R})^{\nu+1}$ $a>0$, $\text{Re } \nu > 1$	$\dfrac{2^{-s-1}\, \Gamma(s)\, \Gamma(\frac{\nu-s-1}{2})}{\nu^{-1}\, a^{-s}\, \Gamma(\frac{\nu+s+3}{2})}$, $0 < \text{Re } s < \text{Re } \nu - 1$
38.	$u_2^{-1}[\beta^{-\nu} U_2^{\nu} + \beta^{\nu} U_2^{-\nu}]$ $\text{Re}(a+\beta)>0$, $\lvert\text{Re } \nu\rvert < 1$	$\dfrac{\Gamma(s)\sin(\pi\nu)\, 2^{s}\beta^{\nu}\Gamma(1-s+\nu)\,\Gamma(1-s-\nu)}{a^{1-s+\nu}\, \pi\, \Gamma(\frac{3}{2}-s)} \times$ $\times\, {}_2F_1(1+\frac{\nu-s}{2}, \frac{1-s+\nu}{2}; \frac{3}{2}-s; 1-\frac{\beta^2}{a^2})$ $0 < \text{Re } s < 1 - \lvert\text{Re } \nu\rvert$
39.	$(t^2+a^2)^{-\frac{1}{2}}(e^{-i\pi\nu} a^{\nu} R^{-\nu} -$ $- a^{-\nu} R^{\nu})$, $a>0$, $\lvert\text{Re } \nu\rvert < 1$	$\dfrac{-\Gamma(s)(1+i)\, \sin(\pi\nu)\, 2^{-s}\, \Gamma(\frac{1-s+\nu}{2})}{a^{1-s}\, \Gamma(\frac{\nu+s+1}{2})}$, $0 < \text{Re } s < 1 - \lvert\text{Re } \nu\rvert$
40.	$(1\cdot 3\cdot 5\cdots[2n-1]\, a^{n}\, r^{-2n-1})$ $a>0$, $n \geq 1$	$\dfrac{\Gamma(s)\, 2^{n-s}\, \Gamma(\frac{2n+1-s}{2})}{a^{1-s+n}\, \Gamma(\frac{1+s}{2})}$ $n - \frac{1}{2} < \text{Re } s < 2n+1$

$u_2 = [(t+a)^2 - \beta^2]^{\frac{1}{2}}$, $U_2 = t + a + u_2$, $r = (t^2+a^2)^{\frac{1}{2}}$, $R = t + r$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
41.	$\sum_{m=0}^{n} \binom{\nu+m-1}{m} \frac{(t+a-1)^{n-m}}{(t+a)^{n-m+1}}$ Re $a > 0$	$\frac{\Gamma(s)}{(n!)^2} \Gamma(1+n-s)(a-1)^n a^{s-1-n} \times$ $\times\ {}_2F_1(-n, s+\nu; s-n; \frac{a}{a-1})$ $0 < \text{Re } s < 1$
42.	$(t^2+a^2)^{-n-1} \times$ $\times \sum_{0 \le 2m \le n+1} (-1)^m \binom{n+1}{2m} (\frac{a}{t})^{2m}$	$\frac{\Gamma(n-s-1)\, a^{s-2n-2}\, \Gamma(2n+2-s)}{n!\, (-1)^{n+1} \sec(\frac{\pi s}{2})}$ $n+1 < \text{Re } s < n+2$
43.	$(t^2+a^2)^{-n-1} \times$ $\times \sum_{0 \le 2m \le n} (-1)^m \binom{n+1}{2m+1} \left(\frac{a}{t}\right)^{2m+1}$ $a > 0$	$\frac{\Gamma(s-n-1)\, a^{s-2n-2}\, \Gamma(2+2n-s)}{n!\, (-1)^n \csc(\frac{\pi s}{2})}$ $1+2n < \text{Re } s < 3+2n$
	Exponential functions	
1.	$[1+\sqrt{at}]\, e^{-\sqrt{at}}$, Re $a > 0$	$2\Gamma(\frac{3}{2}+s)\, \Gamma(s)\, \pi^{-\frac{1}{2}} (\frac{a}{4})^{-s}$, Re $s > 0$
2.	$(t^2+a^2)^{-\frac{1}{2}} e^{-\nu \sinh^{-1}(\frac{t}{a})}$ $a > 0$, Re $\nu > -1$	$\frac{\Gamma(s)\, 2^{-s} a^{s-1}\, \Gamma(\frac{1-s+\nu}{2})}{\Gamma(\frac{1+\nu+s}{2})}$ $0 < \text{Re } s < 1 + \text{Re } \nu$
3.	$\frac{e^{-\frac{\pi t}{2}}}{(1+t^2)^2}[\frac{t\pi}{2}(1+t^2)+t^2-1]$	$\frac{\Gamma(s)}{2}[e^{\frac{1}{2}\pi i(1-s)}\Gamma(2-s, \frac{-i\pi}{2}) +$ $+ e^{\frac{1}{2}\pi i(s-1)}\Gamma(2-s, i\frac{\pi}{2})]$, Re $s > -1$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
4.	$\frac{e^{-\frac{t\pi}{2}}}{(1+t^2)^2}\left[\frac{\pi}{2}(1+t^2)+2t\right]$	$\frac{-\Gamma(s)}{2}\left[e^{\frac{i\pi}{2}(2-s)}\,\Gamma(2-s, \frac{-i\pi}{2}) + \frac{1}{2} e^{-\frac{i\pi}{2}(2-s)}\,\Gamma(2-s, i\frac{\pi}{2})\right]$, $\mathrm{Re}\, s>0$
5.	$(1+t^2)^{-2}\left\{2t - e^{-\frac{\pi t}{2}}\left[\frac{t\pi}{2}(1+t^2)+t^2-1\right]\right\}$	$\frac{\Gamma(s)}{2}\left[(i)^{s-1}\gamma(2-s, \frac{i\pi}{2}) + (-i)^{s-1}\gamma(2-s, -\frac{i\pi}{2})\right]$, $0<\mathrm{Re}\, s<3$
6.	$(1+t^2)^{-2}\left[t^2-1 + e^{-\frac{t\pi}{2}}\left\{\frac{\pi}{2}(1+t^2)+2t\right\}\right]$	$\frac{\Gamma(s)}{2}\left[(i)^{s-2}\gamma(2-s, \frac{i\pi}{2}) + (-i)^{s-2}\gamma(2-s, \frac{-i\pi}{2})\right]$, $0<\mathrm{Re}\, s<1$
	Logarithmic functions	
1.	$(t+1)^{-1}\log(t)$	$\Gamma(s)\,\Gamma(1-s)\,\pi\,\mathrm{ctn}(\pi s)$, $0<\mathrm{Re}\, s<1$
2.	$\log(t^2+1)$	$2\Gamma(s)\,\Gamma(-s)\cos(\frac{\pi s}{2})$, $-1<\mathrm{Re}\, s<0$
3.	$\log(1+4(\frac{a}{t})^2)$, $a>0$	$\frac{\pi s^{-1} a^s 2^{s+1}\cos(\frac{\pi s}{2})}{\sin(\pi s)}$, $0<\mathrm{Re}\, s<2$
4.	$\log\left[t^{\frac{1}{2}} + (t+1)^{\frac{1}{2}}\right]$	$\frac{-\Gamma(\frac{1}{2}+s)}{2\sqrt{\pi}\, s}\,\Gamma(-s)$, $-\frac{1}{2}<\mathrm{Re}\, s<0$
5.	$\log\left[\frac{t^2+(a+\beta)^2}{t^2+(a-\beta)^2}\right]$, $a, \beta>0, a\neq\beta$	$2\Gamma(s)\,\Gamma(-s)\cos(\frac{\pi s}{2}) \times [\lvert\beta-a\rvert^s - (\beta+a)^s]$, $0<\mathrm{Re}\, s<2$

	f(t)	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
6.	$(t+a)^{-\nu-1}[\psi(\nu+1)-\log(t+a)]$ $\mathrm{Re}\,\nu > -1,\ \mathrm{Re}\,a > 0$	$\frac{\Gamma(s)\,\Gamma(\nu+1-s)}{\Gamma(\nu+1)a^{\nu+1-s}}[\psi(1-s+\nu)-\log a]$ $0 < \mathrm{Re}\, s < 1 + \mathrm{Re}\,\nu$
7.	$\log \frac{(t^2+a^2)^{\frac{1}{2}}}{t}$, $a > 0$	$\frac{\Gamma(s)\,\pi^{\frac{1}{2}}\,2^{-s-1}\,\Gamma(\frac{1-s}{2})}{a^{-s}\,\Gamma(1+\frac{s}{2})}\,\mathrm{ctn}\,(\frac{\pi s}{2})$ $0 < \mathrm{Re}\, s < 2$
8.	$\frac{a}{2}t^{-1} - a^{-1}t\log\left(\frac{(t^2+a^2)^{\frac{1}{2}}}{t}\right)$	$\frac{-\Gamma(s)\,\pi^{\frac{1}{2}}\,\Gamma(1-\frac{s}{2})}{2^{s+1}\,a^{-s}\,\Gamma(\frac{3+s}{2})}\tan(\frac{\pi s}{2})$, $1 < \mathrm{Re}\, s < 3$
9.	$\frac{t^{-2}}{2} - r^{-2} + a^{-2}\log(\frac{r}{t})$ $a > 0$	$\frac{\Gamma(s)\,\pi^{\frac{1}{2}}\,\Gamma(\frac{3-s}{2})}{2^s\,a^{2-s}\,\Gamma(1+\frac{s}{2})}\,\mathrm{ctn}\,(\frac{\pi s}{2})$ $2 < \mathrm{Re}\, s < 4$
10.	$r\log(\frac{r+a}{t})$, $a > 0$	$\frac{\Gamma(s)\,2^{-s-1}\,\pi\,\Gamma(\frac{1-s}{2})}{a^{1-s}\,\Gamma(\frac{1+s}{2})}\,\mathrm{ctn}(\frac{\pi s}{2})$, $0 < \mathrm{Re}\, s < 2$
11.	$-1 + \frac{r}{a}\log(\frac{r+a}{t})$, $a > 0$	$\frac{\pi\,\Gamma(s)\,\Gamma(\frac{1-s}{2})}{2^{s+2}\,a^{-s}\,\Gamma(\frac{3}{2}+\frac{s}{2})}\,\mathrm{ctn}\,(\frac{\pi s}{2})$, $0 < \mathrm{Re}\, s < 2$
12.	$\frac{2}{\pi t} - \frac{2t}{\pi a r}\log(\frac{r+a}{t})$, $a > 0$	$\frac{\Gamma(s)\,2^{-s}\,\Gamma(1-\frac{s}{2})}{a^{1-s}\,\Gamma(\frac{s+\nu+1}{2})}\,\mathrm{ctn}\,[\frac{\pi}{2}(s-\nu)]$ $1 < \mathrm{Re}\, s < 3$
13.	$\frac{t}{a} + \frac{a}{3t} - \frac{r}{a}\log(\frac{r+a}{t})$, $a > 0$	$\frac{-\Gamma(s)\,\pi\,\Gamma(1-\frac{s}{2})}{2^{s+2}\,a^{-s}\,\Gamma(2+\frac{s}{2})}\tan(\frac{\pi s}{2})$, $1 < \mathrm{Re}\, s < 3$

$r = [t^2+a^2]^{\frac{1}{2}}$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
14.	$-\frac{2}{a} + \frac{a}{3t^2} + \frac{a^2 + 2t^2}{a^2 r} \log\left(\frac{r+a}{t}\right)$ $a > 0$	$\frac{\pi\, \Gamma(s)\, \Gamma(\frac{3}{2} - s)}{2^{s+1}\, a^{1-s}\, \Gamma(\frac{3}{2} + s)} \operatorname{ctn}\left(\frac{\pi s}{2}\right), 2 < \operatorname{Re} s < 4$
15.	$\frac{1}{3t} + \frac{4t}{a^2} + \frac{2a^2}{15t^3} - \frac{6a^2 t + 8t^3}{\pi a^3 r} \log\left(\frac{r+a}{t}\right)$ $a > 0$	$\frac{-\pi\, \Gamma(s)\, \Gamma(2 - \frac{s}{2})}{2^{s+1}\, a^{1-s}\, \Gamma(2 + \frac{s}{2})} \tan\left(\frac{\pi s}{2}\right), 3 < \operatorname{Re} s < 5$
16.	$\frac{a^2}{15t^2} - \frac{4t^2}{3a^2} - \frac{7}{9} + \frac{4t^2 r + a^2 r}{3a^3} \times \log\left(\frac{r+a}{t}\right), a > 0$	$\frac{\Gamma(s)\, \pi\, \Gamma(\frac{3}{2} - \frac{s}{2})}{2^{s+2}\, a^{-s}\, \Gamma(\frac{5}{2} + \frac{s}{2})} \operatorname{ctn}\left(\frac{\pi s}{2}\right), 2 < \operatorname{Re} s < 4$
17.	$\frac{2i}{\pi r^2} + \frac{t}{r^3}\left(1 - \frac{2i}{\pi} \log\left[\frac{R}{a}\right]\right)$ $a > 0$	$\frac{\Gamma(s)\,(1-i)\, 2^{1-s}\, \Gamma(\frac{2-s}{2})}{a^{2-s}\, \Gamma(\frac{s}{2})}, \frac{1}{2} < \operatorname{Re} s < 2$
18.	$\frac{a}{r^3}\left(1 - \frac{2i}{\pi} \log\left(\frac{R}{a}\right)\right) - \frac{2it}{\pi a r^2}$ $a > 0$	$\frac{\Gamma(s)(1-i)\, 2^{1-s}\, \Gamma(\frac{3-s}{2})}{a^{2-s}\, \Gamma(\frac{1+s}{2})}, \frac{1}{2} < \operatorname{Re} s < 1$
19.	$r^{-2}\left[t a^{-1} + a r^{-1} \log\left(\frac{R}{a}\right)\right]$ $a > 0$	$\Gamma(s)\, a^{s-2}\, 2^{-s}\, \Gamma\left(\frac{3}{2} - \frac{s}{2}\right) \times$ $\times\, \Gamma\left(\frac{1-s}{2}\right) \cos\left(\frac{\pi s}{2}\right), \frac{1}{2} < \operatorname{Re} s < 1$
20.	$r^{-2}\left[1 - t r^{-1} \log\left(\frac{R}{a}\right)\right], a > 0$	$-\Gamma(s)\, 2^{-s}\, a^{s-2} \left[\Gamma\left(1 - \frac{s}{2}\right)\right]^2 \cos\left(\frac{\pi s}{2}\right)$ $\frac{1}{2} < \operatorname{Re} s < 2$

$r = [t^2 + a^2]^{\frac{1}{2}}, \quad R = t + r$

	f(t)	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
21.	$\beta^{-1}(t+\alpha)u_2^{-2} - \beta u_2^{-3} \log(\frac{U_2}{\beta})$ $\mathrm{Re}\,(\alpha+\beta) > 0$	$\frac{\Gamma(s)\, \pi^{\frac{1}{2}}\, \beta\, \Gamma(3-s)\, \Gamma(1-s)}{2^{2-s} \alpha^{3-s} \Gamma(\frac{5}{2} - s)} \times$ $\times\, {}_2F_1(2-\frac{s}{2}, \frac{3-s}{2}; \frac{5}{2}-s; 1-\frac{\beta^2}{\alpha^2})$ $0 < \mathrm{Re}\, s < 1$
22.	$(t+\alpha)^{-1} \log(\gamma[t+\alpha])$, $\mathrm{Re}\,\alpha > 0$	$-\pi\alpha^{s-1} \csc(\pi s)[\psi(1-s) - \log\alpha]$ $0 < \mathrm{Re}\, s < 1$
23.	$(t+1)^{-1}\{\frac{\pi^2}{6} + \log^2(\gamma[t+1])\}$	$\Gamma(s) \frac{d^2}{ds^2} \Gamma(1-s)$, $0 < \mathrm{Re}\, s < 1$
24.	$\log(\gamma[r]) \tan^{-1}(\frac{\alpha}{t})$, $\alpha > 0$	$\Gamma(s)\, \Gamma(-s) \sin(\frac{\pi s}{2})\, \alpha^s \times$ $\times [\psi(-s) - \log\alpha - \mathrm{ctn}(\frac{\pi s}{2})]$ $0 < \mathrm{Re}\, s < 1$
25.	$\log[\gamma([(t+\alpha)^2 + \beta^2]^{\frac{1}{2}})] \times$ $\times \tan(\frac{\beta}{t+\alpha})$, $\mathrm{Re}\,\alpha > \lvert \mathrm{Im}\,\beta \rvert$	$\Gamma(s)\Gamma(-s)(\alpha^2+\beta^2)^{s/2} \sin[s \tan^{-1}(\frac{\beta}{\alpha})] \times$ $\times\{\psi(-s) - \frac{1}{2}\log(\alpha^2+\beta^2) - \tan^{-1}(\frac{\beta}{\alpha})\mathrm{ctn}[s \tan^{-1}(\frac{\beta}{\alpha})]$ $0 < \mathrm{Re}\, s < 1$
26.	$(t+\alpha)^{-n-\frac{1}{2}}[2(1+\frac{1}{3}+\cdots+\frac{1}{2n+1}) -$ $- \log(4\gamma(t+\alpha))]$ $\mathrm{Re}\,\alpha > 0$, $n \geq 1$	$\frac{\alpha^{s-n-\frac{1}{2}}\, \Gamma(s) 2^n\, \Gamma(\frac{1}{2}+n-s)}{\pi^{\frac{1}{2}} \cdot 1 \cdot 3 \cdot 5 \cdots (2n-1)} \times$ $\times [\psi(\frac{1}{2}+n-s) - \log\alpha]$, $0 < \mathrm{Re}\, s < n + \frac{1}{2}$

$u_2 = [(t+\alpha)^2 - \beta^2]^{\frac{1}{2}}$, $U_2 = t+\alpha+u_2$, $r + (t^2+\alpha^2)^{\frac{1}{2}}$

	f(t)	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
27.	$(t+\alpha)^{-n-1}[1+\frac{1}{2}+\cdots+\frac{1}{n}$ - $-\log(\gamma[t+\alpha])]$, $n \geq 1$, $\mathrm{Re}\,\alpha > 0$	$\frac{\Gamma(s)}{n!}\alpha^{+s-n-1}\Gamma(n+1-s)[\psi(n+1-s)$ - $-\log\alpha]$, $0 > \mathrm{Re}\ s < n+1$
	Trigonometric functions	
1.	$(t^2+\alpha^2)^{-\frac{\nu}{2}}\sin[\nu\tan^{-1}(\frac{\alpha}{t})]$ $\alpha > 0$, $\mathrm{Re}\,\nu > -1$	$\frac{\Gamma(s)\,\Gamma(\nu-s)}{\Gamma(\nu)}\alpha^{s-\nu}\sin(\frac{\pi\ (\nu-s)}{2})$ $\max[0, \mathrm{Re}\,\nu-1] < \mathrm{Re}\ s < 1+\mathrm{Re}\,\nu$
2.	$([t+\alpha]^2+\beta^2)^{-\frac{\nu}{2}}\sin[\nu\tan^{-1}(\frac{\beta}{\alpha+t})]$ $\mathrm{Re}\,\nu > -1$, $\mathrm{Re}\,\alpha > \lvert\mathrm{Im}\beta\rvert$	$\frac{\Gamma(s)}{\Gamma(\nu)}(\alpha^2+\beta^2)^{(s-\nu)/2}\Gamma(\nu-s)\ \times$ $\times\sin[(\nu-s)\tan^{-1}(\frac{\beta}{\alpha})]$, $0 < \mathrm{Re}\ s < \mathrm{Re}\,\nu+1$
3.	$(t^2+a^2)^{-\frac{1}{2}\nu}\sin[\nu\tan^{-1}(\frac{a}{t})]\ \times$ $\times\{\psi(\nu)-\log(t^2+a^2)^{\frac{1}{2}}+$ $+\tan^{-1}(\frac{a}{t})\mathrm{ctn}[\nu\tan^{-1}(\frac{a}{t})]\}$, $a>0$ $\mathrm{Re}\,\nu>-1$	$\frac{\Gamma(s)}{\Gamma(\nu)}a^{s-\nu}\Gamma(\nu-s)\sin[\frac{\pi}{2}(s-\nu)]\ \times$ $\times\{\log a-\psi(\nu-s)+\frac{\pi}{2}\mathrm{ctn}[\frac{\pi}{2}(s-\nu)]\}$ $\max[0, \mathrm{Re}\,\nu-1] < \mathrm{Re}\ s < 1+\mathrm{Re}\,\nu$
4.	$[(t+\alpha)^2+\beta^2]^{-\frac{\nu}{2}}\sin[\nu\tan^{-1}(\frac{\beta}{t+\alpha})]\ \times$ $\times\{\psi(\nu)-\log[(t+\alpha)^2+\beta^2]^{\frac{1}{2}}\ +$ $+\tan^{-1}(\frac{\beta}{\alpha+t})\mathrm{ctn}[\nu\tan^{-1}(\frac{\beta}{\alpha+t})]\}$ $\mathrm{Re}\,\alpha > \lvert\mathrm{Im}\beta\rvert$, $\mathrm{Re}\,\nu > -1$	$\frac{\Gamma(s)}{\Gamma(\nu)}\Gamma(\nu-s)(a^2+\beta^2)^{\frac{s-\nu}{2}}\sin[(\nu-s)\tan^{-1}(\frac{\beta}{\alpha})]\ \times$ $\times\{\psi(\nu-s)-\frac{1}{2}\log(a^2+\beta^2)+\tan^{-1}(\frac{\beta}{\alpha})\ \times$ $\times\ \mathrm{ctn}[(\nu-s)\tan^{-1}(\frac{\beta}{\alpha})]\}$ $0 < \mathrm{Re}\ s < \mathrm{Re}\,\nu+1$

	f(t)	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
5.	$[t^2+a^2]^{-\frac{\nu}{2}} \cos[\nu \tan^{-1}(\frac{a}{t})]$ $a > 0$, $\mathrm{Re}\, \nu > 0$	$\frac{\Gamma(s)}{\Gamma(\nu)} \Gamma(\nu-s)\, a^{s-\nu} \cos(\frac{\pi}{2}[s-\nu])$ $\max[0, \mathrm{Re}\, \nu-1] < \mathrm{Re}\, s < \mathrm{Re}\, \nu$
6.	$([t+a]^2+\beta^2)^{-\frac{\nu}{2}} \cos[\nu \tan^{-1}(\frac{\beta}{t+a})]$ $\mathrm{Re}\, a > \|\mathrm{Im}\beta\|$, $\mathrm{Re}\, \nu > 0$	$\frac{\Gamma(s)}{\Gamma(\nu)} (a^2+\beta^2)^{\frac{s-\nu}{2}} \Gamma(\nu-s) \cos[(\nu-s)\tan^{-1}(\frac{\beta}{a})]$ $0 < \mathrm{Re}\, s < \mathrm{Re}\, \nu$
7.	$(t^2+a^2)^{-\frac{\nu}{2}} \cos[\nu \tan^{-1}(\frac{a}{t})]\{\psi(\nu) -$ $- \log(t^2+a^2)^{\frac{1}{2}} - \tan^{-1}(\frac{a}{t}) \times$ $\times \tan[\nu \tan^{-1}(\frac{a}{t})]\}$ $a > 0$, $\mathrm{Re}\, \nu > 0$	$\frac{\Gamma(s)}{\Gamma(\nu)} a^{s-\nu} \Gamma(\nu-s) \cos(\frac{\pi}{2}[s-\nu]) \times$ $\times[\psi(\nu-s) - \log(a) + \frac{\pi}{2} \tan[\frac{\pi}{2}(s-\nu)]]$ $\max[0, \mathrm{Re}\, \nu-1] < \mathrm{Re}\, s < \mathrm{Re}\, \nu$
8.	$[(t+a)^2+\beta^2]^{-\frac{\nu}{2}} \cos[\nu \tan^{-1}(\frac{\beta}{t+a})] \times$ $\times\{\psi(\nu) - \log([t+a]^2+\beta^2)^{\frac{1}{2}} -$ $- \tan^{-1}(\frac{\beta}{t+a}) \tan[\nu \tan^{-1}(\frac{\beta}{t+a})]$ $\mathrm{Re}\, a > \|\mathrm{Im}\beta\|$, $\mathrm{Re}\, \nu > 0$	$\frac{\Gamma(s)}{\Gamma(\nu)} (a^2+\beta^2)^{\frac{s-\nu}{2}} \Gamma(\nu-s)\{[\psi(\nu-s) -$ $- \log(a^2+\beta^2)^{\frac{1}{2}}] \cos[(\nu-s)\tan^{-1}(\frac{\beta}{a})] -$ $- \tan^{-1}(\frac{\beta}{a}) \sin[(\nu-s)\tan^{-1}(\frac{\beta}{a})]$ $0 < \mathrm{Re}\, s < \mathrm{Re}\, \nu$
9.	$\int_0^{\frac{\pi}{2}} \frac{(\cos\theta)^{\nu+\frac{1}{2}} \cos[(\nu-\frac{1}{2})\theta]}{(\frac{1}{4}t^2+a^2\cos^2\theta)^{\nu+1}} d\theta$ $a > 0$, $\mathrm{Re}\, \nu > -1$	$\frac{2^{\nu-\frac{1}{2}} \Gamma(s-\frac{1}{2}-\nu)\, \Gamma(1+\nu-\frac{s}{2})}{a^{2\nu-s+2} \Gamma(\nu+1)\, \Gamma(\frac{1}{2}+\frac{s}{2}-\nu)}$ $\max[0, \frac{1}{2}+\mathrm{Re}\, \nu] < \mathrm{Re}\, s < 1$
10.	$\int_0^{\frac{\pi}{2}} \frac{(\cos\theta)^{\nu-\frac{1}{2}} \cos[(\nu+\frac{1}{2})\theta]}{(\frac{t^2}{4}+a^2\cos^2\theta)^{\nu}} d\theta$ $a > 0$, $\mathrm{Re}\, \nu > 0$	$\frac{\pi\, 2^{\nu-\frac{1}{2}} \Gamma(s)\, \Gamma(\frac{1}{2}+s-\nu)\, \Gamma(\nu-\frac{s}{2})}{a^{2\nu-s} \Gamma(\nu)\, \Gamma(1+s)\, \Gamma(\frac{1}{2}-\nu+\frac{s}{2})}$ $\max[0, \mathrm{Re}\, \nu-\frac{1}{2}] < \mathrm{Re}\, s < \mathrm{Re}\,(2\nu)$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
11.	$\int_0^\pi [(t^2+2-2\cos\phi)^{\frac{1}{2}}-t] \times (1+\cos\phi)\, d\phi$	$\frac{\Gamma(s)\, 2^{-1-s}\, B(s+2,\, 1-s)\, \pi}{[\Gamma(\frac{3+s}{2})]^2}$, $0 < \mathrm{Re}\, s < 1$
12.	$\tan(\frac{a}{t})$, $a>0$	$a^s\, s^{-1}\, \pi\, \sin(\frac{\pi s}{2})\, \csc(\pi s)$, $0 < \mathrm{Re}\, s < 1$
13.	$\tan^{-1}(t)$	$\Gamma(s)\, \Gamma(-s)\, 4\, \sin(\frac{\pi s}{2})$, $0 < \mathrm{Re}\, s < 1$
14.	$\tan^{-1}(\frac{a}{t})$, $a>0$	$\frac{-\Gamma(s)\, 2^{-s-1}\, \pi^{\frac{1}{2}}\, \Gamma(\frac{-s}{2})}{a^{-s}\, \Gamma(\frac{1+s}{2})} \tan(\frac{\pi s}{2})$, $0 < \mathrm{Re}\, s < 1$
15.	$t\, r^{-2} - a^{-1} \tan^{-1}(\frac{a}{t})$, $a>0$	$\frac{-\Gamma(s)\, \pi^{\frac{1}{2}}\, \Gamma(\frac{-s}{2})}{2^s\, a^{1-s}\, \Gamma(\frac{s-1}{2})} \tan(\frac{\pi s}{2})$, $0 < \mathrm{Re}\, s < 1$
16.	$a \tan^{-1}(\frac{2a}{t}) - \frac{t}{4} \log(1+4(\frac{a}{t})^2)$, $a>0$	$\Gamma(s)\, \Gamma(-s-1)\, \sin(\frac{\pi s}{2})\, 2^s\, a^{1+s}$, $0 < \mathrm{Re}\, s < 1$
17.	$\frac{a \tan^{-1}(\frac{a}{t}) + t \log(\gamma(r))}{r^2}$, $a>0$	$-\Gamma(s)\, a^{s-1}\, \Gamma(1-s) \sin(\frac{\pi s}{2})[\psi(1-s) - \log a - \frac{\pi}{2} \mathrm{ctn}(\frac{\pi s}{2})]$, $0 < \mathrm{Re}\, s < 1$
18.	$\frac{t \tan^{-1}(\frac{a}{t}) - a \log[\gamma(r)]}{r^2}$, $a>0$	$\frac{\pi}{\sin(\pi s)}\, a^{s-1} \cos(\frac{\pi s}{2})[\Psi(1-s) - \log a + \frac{\pi}{2} \tan(\frac{\pi s}{2})]$, $0 < \mathrm{Re}\, s < 2$

$r = (t^2 + a^2)^{\frac{1}{2}}$

	f(t)	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
19.	$\tan^{-1}\left(\frac{\beta}{t+\alpha}\right)$, $\mathrm{Re}\,\alpha > \lvert \mathrm{Im}\beta \rvert$	$\pi s^{-1} \csc(\pi s)(\alpha^2+\beta^2)^{\frac{s}{2}} \sin[s \tan^{-1}(\frac{\beta}{\alpha})]$ $0 < \mathrm{Re}\, s < 1$
20.	$\tan^{-1}\left(\frac{2\alpha t}{t^2-\alpha^2+\beta^2}\right)$, $\alpha, \beta > 0$	$-\Gamma(s)\,\Gamma(-s)\, \sin(\frac{\pi s}{2})[(\alpha+\beta)^s +$ $+ \mathrm{sgn}(\alpha-\beta)\lvert \alpha-\beta \rvert^s]$, $0 < \mathrm{Re}\, s < 1$
21.	$\frac{\beta \tan^{-1}(\frac{\beta}{t+\alpha}) + (t+\alpha)\log[\gamma(u)]}{u^2}$ $\mathrm{Re}\,\alpha > \lvert \mathrm{Im}\beta \rvert$	$-\Gamma(s)\,\Gamma(1-s)(\alpha^2+\beta^2)^{\frac{s-1}{2}}\{[\psi(1-s) -$ $- \log(\alpha^2+\beta^2)^{\frac{1}{2}}] \cos[(1-s)\tan^{-1}(\frac{\beta}{\alpha})]\} -$ $- \tan^{-1}(\frac{\beta}{\alpha}) \sin[(1-s)\tan^{-1}(\frac{\beta}{\alpha})]\}$, $0 < \mathrm{Re}\, s < 1$
22.	$\frac{(t+\alpha)\tan^{-1}(\frac{\beta}{t+\alpha}) - \beta \log[\gamma(u)]}{u^2}$ $\mathrm{Re}\,\alpha > \lvert \mathrm{Im}\beta \rvert$	$\Gamma(s)\Gamma(1-s)(\alpha^2+\beta^2)^{\frac{s-1}{2}} \sin[(1-s)\tan^{-1}(\frac{\beta}{\alpha})] \times$ $\times\{\psi(1-s) - \frac{1}{2}\log(\alpha^2+\beta^2) + \tan^{-1}(\frac{\beta}{\alpha})\,\mathrm{ctn}[(1-s)\tan^{-1}(\frac{\beta}{\alpha})]\}$ $0 < \mathrm{Re}\, s < 2$
23.	$\alpha \tan^{-1}(\frac{2\beta t}{t^2+\alpha^2-\beta^2}) + \beta\tan^{-1}(\frac{2\alpha t}{t^2+\beta^2-\alpha^2}) +$ $+\frac{t}{2}\log[\frac{t^2+(\alpha-\beta)^2}{t^2+(\alpha+\beta)^2}]$ $\alpha, \beta > 0$, $\alpha \neq \beta$	$-\Gamma(s)\Gamma(-s-1)\sin(\frac{\pi s}{2})[\lvert \beta-\alpha \rvert^{s+1} -$ $- (\beta+\alpha)^{s+1}]$, $0 < \mathrm{Re}\, s < 1$
24.	$\alpha^{\nu}\,\mathrm{ctn}(\pi\nu)\, r^{-1} R^{-\nu} - \alpha^{-\nu} \csc(\pi\nu) \times$ $\times\, r^{-1} R^{\nu}$, $\alpha > 0$, $\lvert \mathrm{Re}\,\nu \rvert < 1$	$-\Gamma(s)\, 2^{-s} \alpha^{s-1} \pi^{-1}\, \Gamma(\frac{1-s+\nu}{2}) \times$ $\times\, \Gamma(\frac{1-s-\nu}{2}) \cos(\frac{\pi}{2}[s+\nu])$ $0 < \mathrm{Re}\, s < 1 - \lvert \mathrm{Re}\,\nu \rvert$

$u = [(t+\alpha)^2 + \beta^2]^{\frac{1}{2}}$, $r = (t^2+\alpha^2)^{\frac{1}{2}}$, $R = t + r$

Hyperbolic functions

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
1.	$(t^2+a^2)^{-\frac{1}{2}} \sinh^{-1}(\frac{t}{a}), a>0$	$\Gamma(s)\, 2^{-s-1}\, a^{s-1} [\Gamma(\frac{1-s}{2})]^2 \sin(\frac{\pi s}{2})$ $0 < Re\ s < 1$
2.	$r^{-1} - \frac{2i}{\pi r} \sinh^{-1}(\frac{t}{2}), a>0$	$\frac{\Gamma(s)(1-i)\, 2^{-s}\, \Gamma(\frac{1-s}{2})}{a^{1-s}\, \Gamma(\frac{1+s}{2})}$, $0 < Re\ s < 1$
3.	$u_2^{-1} \sinh^{-1}(\frac{u_2}{\beta})$ $Re\ a>0,\ Re\ (a+\beta)>0$	$\frac{\Gamma(s)\, \pi^{\frac{1}{2}} [\Gamma(1-s)]^2}{(2a)^{1-s}\, \Gamma(\frac{3}{2}-s)} \times$ $\times\ {}_2F_1(1-\frac{s}{2}, \frac{1-s}{2}; \frac{3}{2}-s; 1-\frac{\beta^2}{a^2})$ $0 < Re\ s < 1$

Orthogonal polynomials; Gamma, Legendre and related functions

1.	$(t+a)^{-n-1}\, P_n(1-\frac{2}{t+a})$ $Re\ a > 0$	$\frac{\Gamma(s)}{(n!)^2}\, \Gamma(1-s+2n)(a-1)^n\, a^{s-1-2n} \times$ $\times\ {}_2F_1(-n, s-n; s-2n; \frac{a}{a+1})$ $0 < Re\ s < 1+n$
2.	$(t+1)^{-\beta}\, L_n^a(\frac{\lambda}{t+1})$, $Re\ \beta > 0$	$\frac{\Gamma(s)(a+1)\, \Gamma(\beta-s)}{n!\, \Gamma(\beta)}\, {}_2F_2(\beta-s, -n; a+1, \beta; \lambda)$ $0 < Re\ s < Re\ \beta$

$r = [t^2+a^2]^{\frac{1}{2}}$, $u_2 = ([t+a]^2 - \beta^2)^{\frac{1}{2}}$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
3.	$\psi(\frac{t+3}{4}) - \psi(\frac{t+1}{4})$	$2^{s+1}\,\Gamma(s)\,\Gamma(1-s)\,\Phi(-1,\ 1-s,\ \frac{1}{2})$ $0 < \operatorname{Re} s < 1$
4.	$\psi(\frac{t}{4}+1) - \psi(\frac{t}{4}+\frac{1}{2})$	$2^{s+1}\,\Gamma(s)\,\Gamma(1-s)(1-2^s)\zeta(1-s)$ $0 < \operatorname{Re} s < 1$
5.	$\psi(\frac{t+a+1}{2}) - \psi(\frac{t+a}{2})$, $\operatorname{Re} a > 0$	$2\pi\csc(\pi s)\,\Phi(-1,\ 1-s,\ a)$, $0 < \operatorname{Re} s < 1$
6.	$\psi(t-ia+1) - \psi(t+ia+1)$ $\lvert \operatorname{Im} a \rvert < 1$	$\Gamma(s)\,\Gamma(1-s)\,[\zeta(1-s,\ 1+ia) -$ $-\zeta(1-s,\ 1-ia)]$, $0 < \operatorname{Re} s < 1$
7.	$\frac{t}{2}[\psi(\frac{1}{4}t+\frac{1}{2}) - \psi(\frac{1}{4}t)] - 1$	$\Gamma(s)\,2^{s+1}\,\Gamma(1-s)(1-2^{s+1})\zeta(-s)$ $0 < \operatorname{Re} s < 1$
8.	$\psi'(\frac{1}{2} - \frac{t}{2a})$, $\operatorname{Re} a > 0$	$a^s\, 4\,\Gamma(s)\Gamma(2-s)\zeta(2-s)(1-2^{s-2})$ $0 < \operatorname{Re} s < 1$
9.	$\psi^{(n)}(t+a)$, $\operatorname{Re} a > 0$	$\Gamma(s)\,\Gamma(1-s+n)\,\zeta(1-s+n,\ a)(-1)^{n+1}$ $0 < \operatorname{Re} s < n$
10.	$1 - e^{t^2/4}\operatorname{Erfc}(\frac{t}{2})$	$-\Gamma(s+1)\Gamma(\frac{1-s}{2})(s^{-1})\,\pi^{-\frac{1}{2}}$, $-1 < \operatorname{Re} s < 0$
11.	$e^{at}\operatorname{Erfc}(\sqrt{at})$, $\lvert \arg a \rvert < \pi$	$\Gamma(s)\,a^{-s}\sec(\pi s)$, $0 < \operatorname{Re} s < \frac{1}{2}$
12.	$t^{-\frac{1}{2}} - (\pi a)^{\frac{1}{2}}\, e^{at}\operatorname{Erfc}(\sqrt{at})$ $\lvert \arg a \rvert < \pi$	$-\Gamma(s)\,\pi^{\frac{1}{2}}\,a^{\frac{1}{2}-s}\csc(\pi s)$, $\frac{1}{2} < \operatorname{Re} s < \frac{3}{2}$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
13.	$e^{\frac{a}{t}}$ Erfc $(\sqrt{\frac{a}{t}})$, Re $a>0$	$2\Gamma(\frac{1}{2}+s)\,\Gamma(-2s)\,\pi^{-\frac{1}{2}}\,4^s a^s$, $-\frac{1}{2}<\mathrm{Re}\, s<0$
14.	$1-\pi^{\frac{1}{2}}\, t^{-\frac{1}{2}}\, e^{\frac{a}{t}}$ Erfc $(\sqrt{\frac{a}{t}})$ Re $a>0$	$\Gamma(1+s)\,\Gamma(-2s)\,2^{2s+1}\,a^s$, $-1<\mathrm{Re}\, s<0$
15.	$-a^{\frac{1}{2}}+\pi^{\frac{1}{2}}t^{-\frac{1}{2}}(a+\frac{t}{2})\, e^{(\frac{a}{t})}\,\times$ $\times$ Erfc $(\sqrt{\frac{a}{t}})$, Re $a>0$	$\Gamma(2+s)\,\Gamma(-1-2s)\,2^{2s+2}\,a^{s+\frac{1}{2}}$ $-2<\mathrm{Re}\, s<-\frac{1}{2}$
16.	$\exp[\frac{(\beta+t)^2}{4a}]$ Erfc $[\frac{\beta+t}{\sqrt{a4}}]$ Re $a>0$, Re $\beta\geq 0$	$2^{\frac{1}{2}}\,(2a)^{\frac{s}{2}}\,\frac{\pi^{\frac{1}{2}}}{\sin(\pi s)}\exp(\frac{\beta^2}{8a})D_{s-1}[\frac{\beta}{\sqrt{2a}}]$ Re $s>0$
17.	$a^{\cdot}-\pi^{\frac{1}{2}}\, a^{\frac{3}{2}}\,(t+\beta)\, e^{a(t+\beta)^2}\,\times$ $\times$ Erfc$(a^{\frac{1}{2}}[t+\beta])$, Re $a>0$	$\frac{1}{2}\Gamma(s)(2a)^{1-\frac{s}{2}}\,\Gamma(2-s)\, e^{a\beta^2/2}$ $D_{s-2}[\beta\sqrt{2a}]$, $0<\mathrm{Re}\, s<2$
18.	$e^{\beta t}$ Erfc $(\sqrt{\beta(t+a)})$ Re $a>0$, $\lvert\arg\beta\rvert<\pi$	$\Gamma(s)\,\pi^{-1}\,\beta^{-s}\,e^{a\beta}\,\Gamma(\frac{1}{2}+s,\, a\beta)$ $0<\mathrm{Re}\, s<\frac{1}{2}$
19.	$(\frac{\pi}{t+a})^{\frac{1}{2}}-\pi\,\beta^{\frac{1}{2}}\,e^{\beta t}$Erfc$[\sqrt{\beta(t+a)}]$ Re $a>0$, $\lvert\arg\beta\rvert<\pi$	$\Gamma(s)\,\Gamma(\frac{3}{2}-s)\beta^{\frac{1}{2}-s}\,e^{a\beta}\,\Gamma(s-\frac{1}{2},\, a\beta)$ $0<\mathrm{Re}\, s<\frac{3}{2}$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
20.	$e^{-\frac{a^2}{4t}} \operatorname{Erf}\left[\frac{a}{2\sqrt{t}}\, e^{-\frac{i\pi}{2}}\right]$ $a > 0$	$\dfrac{-i\, 2^{\frac{\nu}{2}-s}\, a^{s-\frac{\nu}{2}}\, \Gamma(1+s+\nu)\, \Gamma(\frac{\nu}{4}-\frac{s}{2})}{\Gamma(\frac{3\nu}{4}+\frac{s}{2}+1)} \times$ $\times \tan\left[\frac{\pi}{2}\left(\frac{\nu}{2}-s\right)\right]$ $\max\left[0, -\frac{3}{2}-\operatorname{Re}\frac{\nu}{2}, \operatorname{Re}\frac{\nu}{2}-1\right] < \operatorname{Re} s < 1 + \operatorname{Re}\frac{\nu}{2}$
21.	$e^{\frac{t}{2}} \operatorname{Ei}\left(\frac{-t}{\alpha}\right)$, $\lvert\arg \alpha\rvert < \pi$	$\dfrac{-\Gamma(1+s)\,\pi}{\sin(\pi s)}\, \alpha^s\, s^{-1} = \dfrac{-\Gamma(s)\,\pi\,\alpha^s}{\sin(\pi s)}$ $0 < \operatorname{Re} s < 1$
22.	$e^{\frac{\alpha}{t}} \operatorname{Ei}\left(\frac{-\alpha}{t}\right)$, $\operatorname{Re} \alpha > 0$	$-\Gamma(1+s)\left[\Gamma(s)\right]^2 \alpha^s$, $-1 < \operatorname{Re} s < 0$
23.	$(-1)^{n-1}\, \alpha^n\, e^{\alpha t} \operatorname{Ei}(-\alpha t) +$ $+ \sum_{m=1}^{n} (m-1)!\, (-\alpha)^{n-m}\, t^{-m}$ $n > 1$, $\lvert\arg \alpha\rvert < \pi$	$\Gamma(s)\, \pi\, \alpha^{n-s} \csc\left[\pi(s-n)\right]$ $n < \operatorname{Re} s < n+1$
24.	$\sum_{m=1}^{n-1} \frac{(m-1)!}{(n-1)!} (-t)^{n-m-1} - \frac{(-t)^{n-1}}{(n-1)!} \times$ $\times\, e^{t} \operatorname{Ei}(-t)$, $n \geq 2$	$\Gamma(s)\, B(1-s, n+s-1)$, $0 < \operatorname{Re} s < 1$

	f(t)	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
25.	$e^t\,[\mathrm{Ei}\,(-2t) - \mathrm{Ei}\,(-t)]$	$\frac{\Gamma(s)}{2}[\psi(1-\frac{s}{2}) - \psi(\frac{1-s}{2})]$, $0 < \mathrm{Re}\ s < 1$
26.	$e^{-t}\,\overline{\mathrm{Ei}}(t)$	$-\Gamma(s)\,\pi\ \mathrm{ctn}\,(\pi s)$, $0 < \mathrm{Re}\ s < 1$
27.	$[\mathrm{ci}(t)]^2 + [\mathrm{si}(t)]^2$	$-2[\Gamma(s)]^2\,\Gamma(1-s)\,s^{-1}\cos(\frac{\pi s}{2})$ $0 < \mathrm{Re}\ s < 1$
28.	$\mathrm{ci}(t)\sin(t) + \mathrm{si}(t)\cos(t)$	$-\frac{\pi}{2}\Gamma(s)\sec(\frac{\pi s}{2})$, $0 < \mathrm{Re}\ s < 1$
29.	$\mathrm{ci}(t)\cos(t) + \mathrm{si}(t)\sin(t)$	$\Gamma(1-s)\,[\Gamma(s)]^2\cos(\frac{\pi s}{2})$, $0 < \mathrm{Re}\ s < 1$
30.	$\frac{\pi}{2} + \mathrm{ci}(t)\sin(t) + \mathrm{si}(t)\cos(t)$	$-\frac{\pi}{2}\Gamma(s)\sec(\frac{\pi s}{2})$, $-1 < \mathrm{Re}\ s < 0$
31.	$t^{-1}[-\mathrm{ci}(t)\sin(t) - \mathrm{si}(t)\cos(t)] +$ $+ [\mathrm{ci}(t)\cos(t) - \mathrm{si}(t)\cos(t)]$	$-\frac{\pi}{2}\Gamma(1+s)\,(1-s)^{-1}\csc(\frac{\pi s}{2})$ $1 < \mathrm{Re}\ s < 2$
32.	$\frac{\pi}{2}t^{-1} + t^{-1}[\mathrm{ci}(t)\sin(t) + \mathrm{si}(t)\cos(t)] +$ $+ \mathrm{si}(t)\sin(t) - \mathrm{ci}(t)\cos(t)$	$\frac{1}{2}\Gamma(s+1)\,\pi\,(1-s)^{-1}\csc(\frac{\pi s}{2})$ $0 < \mathrm{Re}\ s < 1$
33.	$\sin(2\sqrt{at})\,\mathrm{Ci}\,(2\sqrt{at})$ - $-\cos(2\sqrt{at})\,\mathrm{Si}\,(2\sqrt{at})$ $\mathrm{Re}\ a > 0$	$\Gamma(s+\frac{1}{2})\,\Gamma(\frac{1}{2}-s)\Gamma(2s)\,4^{-s}\,a^{-s}$ $0 < \mathrm{Re}\ s < \frac{1}{2}$
34.	$e^{at}\,\Gamma(-\nu, at)$, $\lvert\arg a\rvert < \pi$ $\mathrm{Re}\ \nu > -1$	$\frac{\pi\,\Gamma(s)a^{-s}}{\Gamma(\nu+1)}\csc[\pi(s-\nu)]$ $\max[0, \mathrm{Re}\,\nu] < \mathrm{Re}\ s < \mathrm{Re}\ \nu + 1$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
35.	$e^{\frac{a}{t}}\ \Gamma(-\nu, \frac{a}{t})$, Re $a>0$ Re $\nu > -1$	$\frac{a^s\ \Gamma(s+\nu+1)\ \Gamma(-s)\ \Gamma(-s-\nu)}{\Gamma(\nu+1)}$ $\max[-1-\mathrm{Re}\,\nu, -\mathrm{Re}\,\frac{\nu}{2}] < \mathrm{Re}\, s < -\mathrm{Re}\,\frac{\nu}{2} - \lvert \mathrm{Re}\,\frac{\nu}{2}\rvert$
36.	$e^{\beta(t+a)}\ \Gamma(-\nu, \beta(t+a))$ $\lvert\arg\beta\rvert < \pi$, Re $a>0$, Re $\nu>-1$	$\frac{\Gamma(s)}{\Gamma(\nu+1)}\ \Gamma(1-s+\nu)\ \beta^{-s}\ e^{a\beta}\Gamma(s-\nu, a\beta)$ $0 < \mathrm{Re}\, s < 1 + \mathrm{Re}\,\nu$
37.	$\gamma(\nu+1, t)$, Re $\nu > -1$	$-s^{-1}\ \Gamma(1+s+\nu)$, $-1-\mathrm{Re}\,\nu < \mathrm{Re}\, s < 0$
38.	$\{P^{-\nu}_{-\frac{1}{4}}\ [\frac{(t^2+a^2)^{\frac{1}{2}}}{t}]\}^2$, $a>0$ Re $\nu > -\frac{1}{4}$	$\frac{2^{\nu-2s-\frac{1}{2}}\ a^s\ \Gamma(\frac{1}{2}+s)}{\Gamma(2\nu+\frac{1}{2})[\Gamma(1+\frac{s}{2})]^2}\ \ B\ (1+s,\ \nu-\frac{s}{2})$ $-\frac{1}{2} < \mathrm{Re}\, s < \mathrm{Re}\, 2\nu$
39.	$[(t+a)^2-\beta^2]^{-\frac{\mu}{2}}\ P^{-\mu}_{\nu}(\frac{t+a}{\beta})$ Re $(a+\beta)>0$, Re $(\mu+\nu)>-1$ Re $(\mu-\nu)>0$	$\frac{\Gamma(s)\,\beta^{\nu+1}\ \Gamma(1-s+\mu+\nu)\ \Gamma(\mu-s-\nu)}{2^{\mu-s}\,a^{1-s+\mu+\nu}\Gamma(1-s+\mu)\Gamma(\mu-\nu)\Gamma(\mu+\nu+1)}\ \times$ $\times\ {}_2F_1(1+\frac{\mu+\nu-s}{2}, \frac{1-s+\mu+\nu}{2}; 1-s+\mu; 1-\frac{\beta^2}{a^2})$ $0 < \mathrm{Re}\, s < \mathrm{Re}\,\mu - \lvert\mathrm{Re}\,\nu\rvert$
40.	$r^{-\mu-1}\ P^{-\nu}_{\mu}\ (\frac{t}{r})$, $a>0$ Re $(\mu+\nu)>-1$	$\frac{\Gamma(s)\ 2^{\mu-s}\Gamma(\frac{\mu+\nu+1-s}{2})}{a^{1+\mu-s}\Gamma(\frac{\nu+s-\mu+1}{2})\Gamma(\mu+\nu+1)}$ $\max[0, \mathrm{Re}\,\mu-\frac{1}{2}] < \mathrm{Re}\, s < 1+\mathrm{Re}\,(\mu+\nu)$

$r = [t^2+a^2]^{\frac{1}{2}}$

	f(t)	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
41.	$u_1^{-\mu-1} P_\mu^{-\nu}\left(\frac{t+a}{u_1}\right)$ $\mathrm{Re}\, a > 0,\ \mathrm{Re}\,(\mu+\nu) > -1$	$\frac{\Gamma(s)\,\Gamma(s-\frac{1}{2}-\mu)\,\Gamma(1-s+\mu+\nu)}{(2a)^{1-s+\mu}\,\pi^{\frac{1}{2}}\,\Gamma(s+\nu-\mu)\Gamma(\mu+\nu+1)}$ $-\mathrm{Re}\,(\nu+\mu) < \mathrm{Re}\,(1-s) < \min[1, \frac{1}{2} - \mathrm{Re}\,\mu]$
42.	$u_2^{-\mu-1} P_\mu^{-\nu}\left(\frac{t+a}{u_2}\right)$ $\mathrm{Re}\, a > \lvert \mathrm{Re}\, \beta \rvert, \mathrm{Re}(\mu+\nu) > -1$	$\frac{\Gamma(s)\, 2^{\mu-s} \beta^\nu\, \Gamma(\frac{1-s+\mu+\nu}{2})\,\Gamma(\frac{2-s+\mu+\nu}{2})}{\Gamma(\mu+\nu+1)\,\pi^{\frac{1}{2}} a^{1-s+\mu+\nu}\,\Gamma(\nu+1)} \times$ $\times\, {}_2F_1\left(\frac{2-s+\mu+\nu}{2}, \frac{1-s+\mu+\nu}{2}; \nu+1; \frac{\beta^2}{a^2}\right)$ $0 < \mathrm{Re}\, s < 1 + \mathrm{Re}(\mu+\nu)$
43.	$u^{-\mu-1} P_\mu^{-\nu}\left(\frac{t+a}{u}\right)$ $\mathrm{Re}\, a > \lvert \mathrm{Im}\beta \rvert, \mathrm{Re}(\mu+\nu) > -1$	$\frac{\Gamma(s)\,\beta^\nu\,\Gamma(1-s+\mu+\nu)}{2^\nu a^{\nu+\mu+1-s}\,\Gamma(\nu+1)\Gamma(1+\nu+\mu)} \times$ $\times\, {}_2F_1\left(\frac{1-s+\nu+\mu}{2}, \frac{\mu+\nu-s}{2}; \nu+1; \frac{-\beta^2}{a^2}\right)$ $0 < \mathrm{Re}\ s < 1 + \mathrm{Re}\,(\mu+\nu)$
44.	$P_{-\frac{1}{4}}^{\nu}\left(\frac{r}{t}\right) P_{-\frac{1}{4}}^{-\nu}\left(\frac{r}{t}\right)$, $a > 0$	$\frac{2^{-s-\frac{1}{2}} a^s\, \pi^{-\frac{1}{2}}\, \Gamma(\frac{1+s}{2})\ B\,(1+s, \frac{-s}{2})}{\Gamma(\nu+\frac{s}{2}+1)\,\Gamma(\frac{s}{2}+1-\nu)}$ $-\frac{1}{2} < \mathrm{Re}\ s < 0$

$r = (t^2 + a^2)^{\frac{1}{2}}$, $u_1 = ([t+a]^2 - a^2)^{\frac{1}{2}}$, $u_2 = ([t+a]^2 - \beta^2)^{\frac{1}{2}}$, $u = ([t+a]^2 + \beta^2)^{\frac{1}{2}}$

	f(t)	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
45.	$r^{-1} P_{\frac{1}{4}}^{-\nu}(\frac{r}{t}) P_{\frac{1}{4}}^{-\nu}(\frac{r}{t})$ $\operatorname{Re}\nu > -\frac{3}{4},\ a > 0$	$\dfrac{2^{\frac{1}{2}-2s-\nu} a^{s-2} \Gamma(\frac{1}{2}+s)\, B(s, \nu+\frac{1-s}{2})}{\Gamma(2\nu+\frac{3}{2})[\Gamma(\frac{1+s}{2})]^2}$ $0 < \operatorname{Re} s < 1+2\operatorname{Re}\nu$
46.	$r^{-1} P_{\frac{1}{4}}^{-\nu}(\frac{r}{t}) P_{-\frac{1}{4}}^{-\nu-1}(\frac{r}{t})$ $a > 0,\ \operatorname{Re}\nu > -\frac{5}{4}$	$\dfrac{2^{\frac{7}{2}-2s+\nu} a^{s-1} \Gamma(\frac{1}{2}+s)\, B(s, \nu+\frac{3}{4}-\frac{s}{2})}{\Gamma(2\nu+\frac{5}{2})\, \Gamma(\frac{s-1}{2})\, \Gamma(\frac{s+3}{2})}$ $0 < \operatorname{Re} s < 2+\operatorname{Re} 2\nu$
47.	$r^{-\mu-1}[\Gamma(\mu+\nu+1)\operatorname{ctn}(\pi\nu)\, P_\mu^{-\nu}(\frac{t}{r})$ $-\Gamma(\mu-\nu+1)\csc(\pi\nu)\, P_\mu^{\nu}(\frac{t}{r})]$ $a > 0,\ \operatorname{Re}(\mu \pm \nu) > -1$	$-\Gamma(s)\, 2^{\mu-s} a^{s-\mu-1} \pi^{-1} \Gamma(\frac{1-s+\mu+\nu}{2}) \times$ $\times \Gamma(\frac{1-s+\mu-\nu}{2}) \sin(\frac{\pi}{2}[s+\nu-\mu])$ $\lvert\operatorname{Re}\nu\rvert - \operatorname{Re}\mu < \operatorname{Re}(1-s) < \min[1, \frac{3}{2} - \operatorname{Re}\mu]$
48.	$r^{-1}[(\nu+\frac{1}{4}) P_{-\frac{1}{4}}^{\nu}(\frac{r}{t}) P_{\frac{1}{4}}^{-\nu}(\frac{r}{t})$ - $-(\nu-\frac{1}{4}) P_{\frac{1}{4}}^{\nu}(\frac{r}{t}) P_{-\frac{1}{4}}^{-\nu}(\frac{r}{t})],\ a > 0$	$\dfrac{\Gamma(\frac{1}{2}+s)\, a^{s-2}\, 4^{1-s} \pi^{-\frac{1}{2}} B(s, \frac{1-s}{2})}{\Gamma(\frac{1}{4}+\nu+\frac{s}{2})\, \Gamma(\frac{1}{4}-\nu+\frac{s}{2})}$ $\frac{1}{2} < \operatorname{Re} s < \frac{3}{2}$
49.	$Q_{\nu-\frac{1}{2}}(t+1),\ \operatorname{Re}\nu > -\frac{1}{2}$	$\dfrac{[\Gamma(s)]^2\, \Gamma(\frac{1}{2}-s+\nu)}{2^{1-s}\Gamma(\frac{1}{2}+s+\nu)},\quad 0 < \operatorname{Re} s < \frac{1}{2} + \operatorname{Re}\nu$
50.	$Q_{\nu-\frac{1}{2}}(t+a),\ \operatorname{Re} a > 1, \operatorname{Re}\nu > -\frac{1}{2}$	$\dfrac{\Gamma(s)\, \Gamma(\frac{\nu-s}{2}+\frac{1}{4})\, \Gamma(\frac{\nu-s}{2}+\frac{3}{4})}{2^{s+1} a^{\nu-s+\frac{1}{2}} \Gamma(\nu+1)} \times$ $\times {}_2F_1(\frac{\nu-s}{2}+\frac{3}{4}, \frac{\nu-s}{2}+\frac{1}{4};\ \nu+1;\ a^{-2})$ $0 < \operatorname{Re} s < \frac{1}{2} + \operatorname{Re}\nu$

$r = (t^2 + a^2)^{\frac{1}{2}}$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
51.	$Q_{\nu-\frac{1}{2}}\left(\frac{t^2}{2a^2}+1\right), a>0, \mathrm{Re}\,\nu > -\frac{1}{2}$	$\frac{\Gamma(s)\, 2^{-s}\, a^s\, \pi\, B(s, \nu+\frac{1-s}{2})}{[\Gamma(\frac{1+s}{2})]^2}$ $0 < \mathrm{Re}\, s < 1 + \mathrm{Re}(2\nu)$
52.	$u_1^{-\mu}\, Q^{\mu}_{\nu-\frac{1}{2}}\left(\frac{t+a}{a}\right)$, $\mathrm{Re}\, a > 0$ $\mathrm{Re}\,(\mu+\nu) > -\frac{1}{2}$	$\frac{\Gamma(s)\sin[\pi(\mu+\nu)]\,\Gamma(s-\mu)\Gamma(\frac{1}{2}-s+\mu+\nu)}{\sin(\pi\nu)\,\Gamma(\frac{1}{2}+s-\nu-\mu)\, 2^{1+\mu-s}\, a^{\mu-s}}$ $\max[0, \mathrm{Re}\,\mu] < \mathrm{Re}\, s < \frac{1}{2} + \mathrm{Re}(\nu+\mu)$
53.	$u_2^{-\mu}\, Q^{\mu}_{\nu}\left(\frac{t+a}{\beta}\right)$, $\mathrm{Re}\, a > \lvert\mathrm{Re}\,\beta\rvert$ $\mathrm{Re}\,(\mu+\nu) > -1$	$\frac{2^{\mu-s-1}\beta^{\nu+1}\,\Gamma(s)\sin[\pi(\nu+\mu)]\,\Gamma(\frac{\mu+\nu-s}{2})}{\sin(\pi\nu)a^{\mu+\nu+1-s}\,\Gamma(\nu+\frac{3}{2})[\Gamma\frac{1-s+\mu+\nu}{2}]^{-1}} \times$ $\times\, {}_2F_1\left(\frac{\mu+\nu-s}{2};\ \frac{1-s+\mu+\nu}{2};\ \nu+\frac{3}{2};\ \frac{\beta^2}{a^2}\right)$ $0 < \mathrm{Re}\, s < 1 + \mathrm{Re}(\nu+\mu)$
54.	$u_2^{-\mu-1}\, Q^{\nu}_{\mu}\left(\frac{t+a}{u_2}\right)$, $\mathrm{Re}\, a > 0$ $\mathrm{Re}\,(a+\beta) > 0, \mathrm{Re}(\mu \pm \nu) > -1$	$\frac{\Gamma(s)\sin[\pi(\nu+\mu)]\,\pi^{\frac{1}{2}}\,\beta^{\nu}\,\Gamma(1-s+\mu+\nu)}{\sin(\pi\mu)\,\Gamma(\mu-\nu+1)\, 2^{1-s+\mu}\, a^{1-s+\mu+\nu}} \times$ $\times\, \frac{\Gamma(1-s-\nu+\mu)}{\Gamma(\frac{3}{2}-s+\mu)}\ {}_2F_1\left(\frac{\mu+\nu-s}{2}, \frac{1-s+\mu+\nu}{2}; \frac{3}{2}-s+\mu;\ ;\ 1-\frac{\beta^2}{a^2}\right)$ $0 < \mathrm{Re}\, s < 1 + \mathrm{Re}\,\mu - \lvert\mathrm{Re}\,\nu\rvert$

$u_1 = [(t+a)^2 - a^2]^{\frac{1}{2}}$, $u_2 = [(t+a)^2 - \beta^2]^{\frac{1}{2}}$

	$f(t)$	$g(s) = \int_0^{\infty} f(t)\, t^{s-1}\, dt$
	Bessel and related functions	
1.	$\mathbf{J}_{-\nu}(t) - J_{-\nu}(t)$	$\dfrac{\Gamma(s)\sin(\pi\nu)}{\pi 2^{s}}\, B\left(1-s, \dfrac{s-\nu}{2}\right)$ $\max[0, \operatorname{Re}\nu] < \operatorname{Re} s < 1$
2.	$H^{(1)}_{2\nu}(\sqrt{at})\, H^{(2)}_{2\nu}(\sqrt{at})$, $\operatorname{Re} a > 0$ $\lvert \operatorname{Re}\nu \rvert < \frac{1}{4}$	$\dfrac{2a^{-s}\Gamma(s+2\nu)\Gamma(\frac{1}{2}-s)\Gamma(s)\Gamma(s-2\nu)}{\pi^{\frac{1}{2}}\,\Gamma(2\nu+\frac{1}{2})\,\Gamma(\frac{1}{2}-2\nu)}$ $\max[-\operatorname{Re} 2\nu, \operatorname{Re}\nu + \lvert\operatorname{Re}\nu\rvert] < \operatorname{Re} s < \frac{1}{2}$
3.	$H^{(1)}_{\nu+\frac{1}{2}}(a^{\frac{1}{2}}t^{\frac{1}{2}})\, H^{(2)}_{\nu-\frac{1}{2}}(a^{\frac{1}{2}}t^{\frac{1}{2}}) +$ $+ H^{(1)}_{\nu-\frac{1}{2}}(a^{\frac{1}{2}}t^{\frac{1}{2}})\, H^{(2)}_{\nu+\frac{1}{2}}(a^{\frac{1}{2}}t^{\frac{1}{2}})$ $\operatorname{Re} a > 0$, $\lvert\operatorname{Re}\nu\rvert < 1$	$4\Gamma(s-\frac{1}{2})\nu\, \pi^{-\frac{3}{2}}\, a^{-s}\,\Gamma(1-s) \times$ $\times\, \Gamma(s+\nu)\Gamma(s-\nu)$, $\max[\frac{1}{2}, \lvert\operatorname{Re}\nu\rvert] < \operatorname{Re} s < 1$
4.	$I_{\nu}(t) - \mathbf{L}_{\nu}(t)$, $\operatorname{Re}\nu > -\frac{1}{2}$	$\dfrac{\Gamma(s+\nu)\, B(\nu+\frac{1}{2}, \frac{1-s-\nu}{2})}{\pi^{\frac{1}{2}}\,\Gamma(\nu+\frac{1}{2})\, 2^{\nu}}$ $-\operatorname{Re}\nu < \operatorname{Re} s < 1 - \operatorname{Re}\nu$
5.	$K_{\nu}(\sqrt{at})$, $\operatorname{Re} a > 0$	$\frac{1}{2}\Gamma(s+\frac{\nu}{2})\left(\frac{a}{4}\right)^{-s}\Gamma(s-\frac{\nu}{2})$ $\operatorname{Re} s > \lvert \operatorname{Re}\frac{\nu}{2} \rvert$
6.	$e^{\frac{t}{2}} K_0(\frac{t}{2})$	$[\Gamma(s)]^2\, \pi^{-\frac{1}{2}}\,\Gamma(\frac{1}{2}-s)$, $0 < \operatorname{Re} s < \frac{1}{2}$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
7.	$e^{\frac{a}{t}} K_{\frac{\nu}{2}}(\frac{a}{t})$, Re $a > 0$, $\lvert \text{Re}\, \nu \rvert < 1$	$\Gamma(s+\frac{1}{2})\pi^{-\frac{1}{2}}\cos(\frac{\pi\nu}{2})(2a)^s \times$ $\times \Gamma(\frac{\nu}{2}-s)\Gamma(-s-\frac{\nu}{2})$ $-\frac{1}{2} < \text{Re } s < -\lvert \text{Re } \frac{\nu}{2} \rvert$
8.	$(t+a)^{-\frac{\nu}{2}} K_\nu(2\sqrt{\beta(a+t)})$ Re $a>0$, Re $\beta>0$	$\Gamma(s)\beta^{-\frac{\nu}{2}}(\frac{\beta}{a})^{\frac{\nu-s}{2}} K_{\nu-s}(2\sqrt{a\beta})$ Re $s>0$
9.	$(t+\beta)^{\frac{1}{2}} e^{a(t+\beta)^2/2} K_{\frac{1}{4}}(\frac{a(t+\beta)^2}{2})$ Re $a>0$	$\Gamma(s)(2a)^{\frac{1}{4}-\frac{s}{2}} a^{-\frac{1}{2}} \Gamma(\frac{1}{2}-s) \times$ $\times \exp(\frac{a\beta^2}{2}) D_{s-\frac{1}{2}}[\beta\sqrt{2a}]$ $0 < \text{Re } s < \frac{1}{2}$
10.	$e^{\frac{a}{t}}[K_1(\frac{a}{t}) - K_o(\frac{a}{t})]$ Re $a>0$	$\Gamma(\frac{3}{2}+s)\Gamma(-s-1)\Gamma(-s)a^s \pi^{-\frac{1}{2}} 2^{s+1}$ $-\frac{3}{2} < \text{Re } s < -1$
11.	$\mathbf{H}_1(at) - Y_1(at) - \frac{2}{\pi}$, $a>0$	$\dfrac{\Gamma(s)2^{s-1}\Gamma(2-s)\Gamma(\frac{s-1}{2})}{\pi a^s \Gamma(\frac{3}{2}-\frac{s}{2})}$ $1 < \text{Re } s < 2$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
12.	$\mathbf{H}_\nu(t) - Y_\nu(t)$	$\dfrac{\Gamma(s+\nu)\, B\left(\frac{1-s-\nu}{2}, \frac{s-\nu}{2}\right)}{2^\nu \pi^{\frac{1}{2}} \Gamma(\nu+\frac{1}{2})}$ $\lvert \operatorname{Re} \nu \rvert < \operatorname{Re} s < 1 - \operatorname{Re} \nu$
13.	$\mathbf{H}_o(\sqrt{8at}) - Y_o(\sqrt{8at})$ $\operatorname{Re} a > 0$	$\Gamma(\frac{1}{2}+s)\Gamma(\frac{1}{2}-s)[\Gamma(s)]^2 (2a)^{-s}$ $0 < \operatorname{Re} s < \frac{1}{2}$
14.	$\dfrac{\pi(\mathbf{H}_o(t) - Y_o(t))}{2t} +$ $+ \dfrac{\pi(\mathbf{H}_1(t) - Y_1(t))}{2} - 1$	$\Gamma(s)\Gamma(1-s)\, 2^{s-1} \Phi(-1, 2-s, \frac{1}{2})$ $-1 < \operatorname{Re} s < 1$
	Generalized hypergeometric series	
1.	${}_2F_1(\nu+\frac{1}{2}, 2\nu+\frac{1}{2}; \nu+1; \frac{-4a^2}{t^2})$ $a > 0$, $\operatorname{Re} \nu > -\frac{1}{4}$	$\dfrac{2^{-s-6\nu-1} a^s \pi \Gamma(\nu+1)\Gamma(1+s+4\nu)}{\Gamma(\nu+\frac{1}{2})\, \Gamma(2\nu+\frac{1}{2})\, [\Gamma(1+\nu+\frac{s}{2})]^2} \times$ $\times B(1+s+2\nu, \frac{-s}{2})$, $\max[-1-\operatorname{Re} 4\nu, -1-\operatorname{Re} 2\nu] < \operatorname{Re} s < 0$
2.	${}_2F_1(\nu+\frac{1}{2}, \nu+\frac{3}{2}; 2\nu+1; \frac{-4a^2}{t^2})$ $a>0$, $\operatorname{Re} \nu > -1$	$\dfrac{2^{-4\nu-s-1} \pi a^s B(s+2\nu+1, \frac{-s}{2})}{(\nu+\frac{1}{2})[\Gamma(\frac{s}{2}+\nu+1)]^2 B(\nu+\frac{1}{2}, \nu+\frac{1}{2})}$ $-1-\operatorname{Re}(2\nu) < \operatorname{Re} s < 0$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
3.	${}_2F_1(\frac{1}{2}, \frac{1}{2}-\nu; 1+\nu; \frac{-4a^2}{t^2})$ $a>0, \mathrm{Re}\,\nu<\frac{1}{2}$	$\frac{2^{2\nu-s-1} a^s \pi^{\frac{1}{2}} \Gamma(1+s-2\nu)\Gamma(1+\nu)}{\Gamma(\frac{1}{2}-\nu)\Gamma(\nu+1+\frac{s}{2})\Gamma(1-\nu+\frac{s}{2})} \times$ $\times B(1+s, \frac{-s}{2})$, $\max[\mathrm{Re}(2\nu-1), -1] < \mathrm{Re}\, s < 0$
4.	${}_2F_1(\mu+\nu+\frac{1}{2}, \mu-k+\frac{1}{2}; \nu-k+\frac{1}{2};$ $; \frac{t-1}{t})$ $\mathrm{Re}(\nu \pm \mu) > -\frac{1}{2}$	$\frac{\Gamma(s)\Gamma(\nu-k+1)\Gamma(-s-1)}{\Gamma(\mu+\nu+\frac{1}{2})\Gamma(\nu-\mu+\frac{1}{2})\Gamma(\mu-k+\frac{1}{2})} \times$ $\times B(-s-2\mu, s+\mu-k)$ $-\mathrm{Re}\mu - \lvert\mathrm{Re}\mu\rvert > \mathrm{Re}\, s > \max[-\frac{1}{2}+\mathrm{Re}(k-\mu),$ $, -\frac{1}{2} - \mathrm{Re}(\mu+\nu)]$
5.	$(t+1)^{-\mu-\nu-\frac{1}{2}} \times$ $\times {}_2F_1(\mu+\nu+\frac{1}{2}, \mu-k+\frac{1}{2}; 2\mu+1; \frac{1}{t+1})$ $\mathrm{Re}(\mu+\nu) > -\frac{1}{2}$	$\frac{\Gamma(s)\Gamma(2\mu+1)B(\mu+\nu+\frac{1}{2}-s, k+s-\nu)}{a^{\mu+\frac{1}{2}}\Gamma(\mu+\nu+\frac{1}{2})\Gamma(\mu+s+\frac{1}{2}-\nu)}$ $\max[0, \mathrm{Re}(\nu-k)] < \mathrm{Re}\, s < \frac{1}{2} + \mathrm{Re}(\mu+\nu)$
6.	$(t+a+\frac{\beta}{2})^{-\mu-\nu-\frac{1}{2}} \times$ $\times {}_2F_1(\mu+\nu+\frac{1}{2}, \mu-k+\frac{1}{2}; 2\mu+1; \frac{\beta}{t+a+\frac{\beta}{2}})$ $\mathrm{Re}\, a > \frac{1}{2}\lvert\mathrm{Re}\,\beta\rvert, \mathrm{Re}(\mu+\nu) > -\frac{1}{2}$	$\frac{\Gamma(s)\Gamma(\frac{1}{2}+\mu+\nu-s)(a+\frac{\beta}{2})^{s-\mu-\nu-\frac{1}{2}}}{\Gamma(\mu+\nu+\frac{1}{2})} \times$ $\times {}_2F_1(\frac{1}{2}+\mu+\nu-s, \mu-k+\frac{1}{2}; 2\mu+1; \frac{\beta}{a+\frac{\beta}{2}})$ $0 < \mathrm{Re}\, s < \frac{1}{2} + \mathrm{Re}(\mu+\nu)$

	f(t)	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
7.	$(t+a-1)^n {}_2F_1(-n, \nu-\beta; -\beta-n; \frac{t+a}{t+a-1})$ $\mathrm{Re}\, a>0,\ \mathrm{Re}\,\beta>-1$	$\frac{\Gamma(1+s+\beta+n)\Gamma(-s)(a-1)^n a^s}{\Gamma(\beta+n+1)} \times$ $\times {}_2F_1(-n, \nu+s+n+1; s+1; \frac{a}{a-\beta})$ $-n-1-\mathrm{Re}\,\beta<\mathrm{Re}\, s<-n$
8.	$(t+a+\frac{\beta}{2})^{-\mu-\nu-\frac{1}{2}} \times$ ${}_2F_1(\mu+\nu+\frac{1}{2}, \mu-k+\frac{1}{2}; \nu-k+1; \frac{t+a-\frac{\beta}{2}}{t+a+\frac{\beta}{2}})$ $\mathrm{Re}(a+\frac{\beta}{2})>0,\ \mathrm{Re}(\nu\pm\mu)>-\frac{1}{2}$	$\frac{\Gamma(s)\Gamma(\nu-k+1)\Gamma(\frac{1}{2}+\mu+\nu-s)\Gamma(\frac{1}{2}+\nu-s-\mu)}{\Gamma(\nu+\mu+\frac{1}{2})\Gamma(\nu-\mu+\frac{1}{2})\Gamma(1-s-k+\nu)(a+\frac{\beta}{2})^{\mu+s+\nu-\frac{1}{2}}}$ $\times {}_2F_1(\frac{1}{2}+\mu+\nu-s, \mu-k+\frac{1}{2}; 1-s-k+\nu; \frac{2a-\beta}{2a+\beta})$ $0<\mathrm{Re}\, s<\frac{1}{2}+\mathrm{Re}\,\nu-\lvert\mathrm{Re}\,\mu\rvert$
9.	$(t+1)^{-\sigma} {}_{p+1}F_q(a_1, \ldots, a_p, \sigma, b_1, \ldots, b_q, \frac{a}{t})$ $\mathrm{Re}\,\sigma>0,\ p<q$	$\frac{\Gamma(s)\Gamma(\sigma-s)}{\Gamma(\sigma)} \times$ $\times {}_{p+1}F_q(\sigma-s, a_1, \ldots, a_p, b_1, \ldots, b_q, a)$ $0<\mathrm{Re}\, s<\mathrm{Re}\,\sigma$
	Other higher transcendental functions	
1.	$V_\nu(2t, 0)$, $\mathrm{Re}\,\nu>0$	$\frac{-\Gamma(s)}{2} \sin(\pi\nu)\csc(\frac{\pi}{2}[s-\nu])$ $\max[0, \mathrm{Re}\,\nu-2]<\mathrm{Re}\, s<\mathrm{Re}\,\nu$
2.	$r^{-1}K(\frac{a}{r})$, $a>0$	$\frac{\Gamma(s)\pi 4^{-s} a^{s-1} B(s, \frac{1-s}{2})}{[\Gamma(\frac{1+s}{2})]^2}$, $0<\mathrm{Re}\, s<1$

$r = [t^2+a^2]^{\frac{1}{2}}$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
3.	$r^{-1}\,[K\,(\frac{a}{r}) - E\,(\frac{a}{r})],\ a>0$	$\dfrac{\pi\Gamma(s)\, 4^{1-s} a^{s-1} B(s-1, \frac{3}{2}-\frac{s}{2})}{\Gamma(\frac{1+s}{2})\,\Gamma(\frac{s-1}{2})}, 1<\mathrm{Re}\, s<3$
4.	$r^{-2}[\,(2t^2+a^2) K\,(\frac{a}{r}) -$ $-2(t^2+a^2) E\,(\frac{a}{r})],\ a>0$	$\dfrac{\Gamma(s)\pi\, 4^{-s} a^{s+1} B(s, \frac{3}{2}-\frac{s}{2})}{[\Gamma(\frac{1+s}{2})]^2}, 0<\mathrm{Re}\, s<3$
5.	$\zeta(\nu, t+a)$, $\mathrm{Re}\, a>0$, $\mathrm{Re}\, \nu>1$	$\dfrac{\Gamma(s)\Gamma(\nu-s)}{\Gamma(\nu)}\, \zeta(\nu-s, a), 0<\mathrm{Re}\, s<\mathrm{Re}\, \nu-1$
6.	$\zeta(\nu, \frac{1}{2}t+1)$, $\mathrm{Re}\, \nu>1$	$\dfrac{\Gamma(s)}{\Gamma(\nu)}\, 2^s \Gamma(\nu-s) \zeta(\nu-s), 0<\mathrm{Re}\, s<\mathrm{Re}\, \nu-1$
7.	$\zeta(\nu, \frac{t+1}{2})$, $\mathrm{Re}\, \nu>1$	$\dfrac{2^\nu \Gamma(s)}{\Gamma(\nu)}\, (1-2^{s-\nu}) \Gamma(\nu-s) \zeta(\nu-s)$ $0<\mathrm{Re}\, s<\mathrm{Re}\, \nu-1$
8.	$S_{0,\nu}(t)$	$\Gamma(s)\pi^{-1} 2^{s-1} \sin(\frac{\pi s}{2}) \cos(\frac{\pi\nu}{2}) \times$ $\times\, \Gamma(\frac{s-\nu}{2}) \Gamma(\frac{s+\nu}{2}), \lvert\mathrm{Re}\, \nu\rvert<\mathrm{Re}\, s<1$
9.	$S_{-1,0}(t)$	$\Gamma(s)\Gamma(1-s) 2^s [\Gamma(\frac{s}{2})]^2 \cos(\frac{\pi s}{2})$ $0<\mathrm{Re}\, s<1$

$r = (t^2+a^2)^{\frac{1}{2}}$

	f(t)	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
10.	$S_{-1,\nu}(t)$, $\nu \neq 0$	$\Gamma(s)\pi^{-1}\nu^{-1}2^{s-1}\Gamma(1-s)\cos(\frac{\pi s}{2}) \times$ $\times \sin(\frac{\pi\nu}{2})\Gamma(\frac{s-\nu}{2})\Gamma(\frac{s+\nu}{2})$, $\|Re\,\nu\| < Re\, s < 2$
11.	$S_{2k,2\mu}(2\sqrt{at})$, $Re\, a > 0$ $Re(\pm k \pm \mu) > -\frac{1}{2}$	$\frac{2^{2k-1} a^{-s} \Gamma(\frac{1}{2}+s+k)\Gamma(\frac{1}{2}-s-k)}{\Gamma(\mu-k+\frac{1}{2})\Gamma(-k-\mu+\frac{1}{2})} \times$ $\times\ \Gamma(\mu+s)\Gamma(s-\mu)$ $\max[\|Re\,\mu\|, -\frac{1}{2}-Re\,k] < Re\, s < Re\,\mu + \frac{1}{2}$
12.	$S_{0,\frac{1}{2}}(t) - \frac{1}{2} S_{-1,\frac{1}{2}}(t)$	$\Gamma(s)(\frac{2}{\pi})^{\frac{1}{2}}(1-s)^{-1}\sin(\frac{\pi s}{2}-\frac{\pi}{4}) \times$ $\times \Gamma(s-\frac{1}{2})\Gamma(2-s)$, $0 < Re\, s < 1$
13.	$S_{1,\nu}(t) + \nu S_{0,\nu}(t)$, $Re\,\nu < 0$	$\frac{-\Gamma(1+s)\Gamma(-s)\Gamma(\frac{s-\nu}{2})\nu 2^{s-1}}{\Gamma(\frac{2-\nu-s}{2})}$ $\max[-1, Re\,\nu] < Re\, s < 0$
14.	$e^{\frac{a}{2t}} D_{-2\nu}(\sqrt{\frac{2a}{t}})$, $Re\, a > 0$, $Re\,\nu > 0$	$\frac{\Gamma(s+\nu)}{\Gamma(2\nu)} 2^{\nu+2s} a^{s} \Gamma(-2s)$, $Re\, -\nu < Re\, s < 0$
15.	$e^{\frac{(t+\beta)^2}{8a}} D_{-\nu}(\frac{t+\beta}{2a})$, $Re\, a > 0$, $Re\,\nu > 0$	$(2a)^{\frac{s}{2}} \frac{\Gamma(s)}{\Gamma(\nu)} \Gamma(s+\nu-1)\exp(\frac{\beta^2}{8a}) \times$ $\times D_{s-\nu}(\frac{\beta}{\sqrt{2a}})$, $0 < Re\, s < Re\,\nu$

	$f(t)$	$g(s) = \int_0^\infty f(t)\, t^{s-1}\, dt$
16.	$\exp\left(\frac{t^2-\beta^2}{8a}\right)\left[e^{\frac{-i\beta t}{4a}} D_{-\nu}\left(\frac{t-i\beta}{\sqrt{2a}}\right) + e^{\frac{i\beta t}{4a}} D_{-\nu}\left(\frac{t+i\beta}{\sqrt{2a}}\right)\right]$, $\operatorname{Re} a>0, \operatorname{Re}\nu>0$	$\frac{\Gamma(s)}{\Gamma(\nu)}\, 2^{\frac{\nu}{2}} a^{\frac{s}{2}}\, \Gamma\left(\frac{\nu-s}{2}\right)\exp\left(\frac{\beta^2}{4a}\right) \times$ $\times\, {}_1F_1\left(\frac{1+s-\nu}{2};\ \frac{1}{2};\ \frac{\beta^2}{4a}\right)$, $0<\operatorname{Re} s<\operatorname{Re}\nu$
17.	$e^{\left(\frac{t^2-\beta^2}{8a}\right)}\left[e^{\frac{i\beta t}{4a}} D_{-\nu}\left(\frac{t-i\beta}{\sqrt{2a}}\right) - e^{\frac{-it\beta}{4a}} D_{-\nu}\left(\frac{t-i\beta}{\sqrt{2a}}\right)\right]$, $\operatorname{Re} a>0, \operatorname{Re}\nu>1$	$-i\beta\frac{\Gamma(s)}{\Gamma(\nu)}\Gamma\left(\frac{1-s+\nu}{2}\right)2^{\frac{\nu}{2}} a^{\frac{s-1}{2}} \times$ $e^{-\frac{\beta^2}{4a}}\, {}_1F_1\left(\frac{s-\nu}{2};\ \frac{3}{2};\ \frac{\beta^2}{4a}\right)$, $0<\operatorname{Re} s<\operatorname{Re}\nu+1$
18.	$e^{\frac{a}{2t}}\, W_{-\mu,\nu}\left(\frac{a}{t}\right)$ $\operatorname{Re} a>0, \operatorname{Re}(\mu\pm\nu)>-\frac{1}{2}$	$\frac{a^s\,\Gamma(s+\mu)\Gamma(\frac{1}{2}+\nu-s)\Gamma(\frac{1}{2}-\nu-s)}{\Gamma(\mu+\nu+\frac{1}{2})\,\Gamma(\mu-\nu+\frac{1}{2})}$ $-\operatorname{Re}\mu<\operatorname{Re} s<\frac{1}{2}-\lvert\operatorname{Re}\nu\rvert$
19.	$E(a,\beta,\nu:\zeta:t)$, $\operatorname{Re}\nu>0$	$\frac{\Gamma(a)\,\Gamma(\beta)\Gamma(s+\nu)\, B(-s,a+s)B(-s,\beta+s)}{\Gamma(\zeta)\ B(-s,\zeta+s)}$ $\max[-\operatorname{Re}\nu, -\operatorname{Re} a, -\operatorname{Re}\beta]<\operatorname{Re} s<0$
20.	$(t+1)^{a-1}\times$ $\times\, G^{m,n+1}_{p+1,q}\left(\frac{1}{t+1}\,\middle\vert\, \begin{matrix} a, a_1,\dots,a_p \\ b_1,\dots,b_q\end{matrix}\right)$ $p+q<2(m+n)$, $\operatorname{Re} a>b_j+1, j=1,\dots,m$	$\Gamma(s)G^{m,n+1}_{p+1,q}\left(1\,\middle\vert\, \begin{matrix} s+a, a_1,\dots,a_p \\ b_1,\dots,b_q\end{matrix}\right)$ $0<\operatorname{Re} s<1-\operatorname{Re} a+\min_{1\le k\le m}\operatorname{Re} b_k$

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