AN ABSTRACT OF THE DISSERTATION OF

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Title: The Language of Mathematics: A Functional Definition and the Development of an Instrument to Measure Teacher Perceived Self-Efficacy

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Mathematics is permeated with language; it appears in the form of new words and some old words with new meanings. There are new symbols to be able to read and consume; much information is presented in tabular or graphic form, and finally the language in a mathematics class has its own semantics, syntax and traditions of argumentation and expression. It is this language, used in the mathematics classroom, which students must absorb and develop fluency with—all while learning the mathematics expressed by this language. Traditionally, the language of mathematics has been overlooked in the classroom, as if students could learn it by just being exposed, rather than having explicit instruction. Numerous professional organizations have called for a focus on language in mathematics education, yet it appears that this important topic is overlooked in the classroom. This research project concentrated on developing a working definition of the language of mathematics and then, speculating that the reason teachers avoid teaching the language of mathematics, it developed the Language of Mathematics Teacher Self-Efficacy Scale (LoMTES), a measurement instrument to measure teacher perceived self-efficacy regarding the teaching of the language of mathematics. Bandura's socio-cognitive theory was the guiding force in developing this

instrument. Bandura indicates that self-efficacy is predictive—that teachers with high perceived self-efficacy on a topic are generally capable of teaching it, while teachers with low perceived self-efficacy on a topic tend to skip over the topic or teach it in a minimal way. Self-efficacy, however, is a changeable construct; thus, this instrument could be used to identify teachers with low perceived self-efficacy regarding the teaching of the language of mathematics, which would enable the mathematics education community to explore possible interventions designed to improve student learning by improving teacher perceived self-efficacy.

The Language of Mathematics: A Functional Definition and the Development of an
Instrument to Measure Teacher Perceived Self-Efficacy

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Joyce Elizabeth

Martha Ida

John Gilbert

Brenda Denise

and Kenneth Russell

my most significant teachers.

You built my heart.

Where would I be without you?

The Language of Mathematics: A Functional Definition and the Development of an Instrument to Measure Teacher Perceived Self-Efficacy

Chapter 1: General Introduction

Language presents a number of difficulties for students in mathematics. It is easy to get confused when reading about mathematics or when hearing someone talk about mathematics. It is also difficult to devise good ways to explain one's own mathematical perspective clearly. As students are learning mathematics they must contend not only with new vocabulary, but also with vocabulary of the same words, with new, more precise meanings. There are also symbols that are useful in mathematics but are difficult to absorb as meaningful representations. Additionally, mathematics students are expected to develop skills to diagram and graph information so that meaning can be shared and extracted visually. Finally, there are a number of syntactic and cultural conventions for students to contend with as they decode language from others and as they develop their own communication style in mathematics classes.

The Language of Mathematics has been a veiled topic within the mathematics classroom. Teachers naturally focus on mathematics instead of language and students are traditionally expected to learn the new language skills needed for mathematics by exposure rather than explicit instruction—in much the same way they learned their original, natural language. But the language of mathematics is not a natural language. First of all, there are no native speakers;

secondly, mathematics requires an additional natural language to supplement communication; and finally, one cannot be fluent in this language without being literate. While students are learning mathematics they are also learning the new forms of expression containing new rituals, many of which are unlike anything they have learned while learning their original, natural language.

A number of professional organizations have recognized the need for a focus on language in mathematics education, including the National Council of Teachers of Mathematics, The American Mathematical Association, and The American Mathematical Association of Two Year Colleges. And yet, in investigating mathematical language the author found little focused research on the topic. It also was apparent that even though language is the way in which mathematics is transmitted, teachers rarely focus on language when teaching mathematics. The question of why this is the case was at the heart of this dissertation project which focused on speculations that teachers are either unaware of how to teach the language of mathematics or that they may not believe that they are capable of successfully implementing language instruction within their mathematics classrooms. This study developed an instrument to measure teacher confidence in the ability to teach the language of mathematics.

Bandura's self-efficacy theory is one possible explanation of why teachers rarely focus on mathematical language. Bandura (1997) posited that behavior is influenced by an individual's self-appraisal among other factors. It is this self-appraisal that may be shaping teacher choices regarding language. A teacher's

perceived self-efficacy is a powerful influence on what she chooses to teach. If she does not believe she can succeed at teaching a particular topic, she will probably not be able to. Bandura indicated, however, that self-efficacy is not a static state, that it can be altered, thus creating a concordant change in a teacher's confidence and the attached ability to teach language.

In order to view teacher perceived self-efficacy toward the teaching of the language of mathematics it was necessary to develop an instrument to measure the construct. This dissertation project was two-fold: (1) to create a functional definition of the language of mathematics and (2) to create a measurement instrument to determine teacher perceived self-efficacy regarding the language of mathematics.

The first article in this collection focuses on a functional definition of the language of mathematics and reviews the literature to establish the importance of the topic and to identify the language difficulties that students face as they learn mathematics. The second article tackles the question of language being a rare focus in the mathematics classroom and details a research project of developing a measurement instrument to assess teacher perceived self-efficacy regarding the language of mathematics by creating the Language of Mathematics Teacher Self-Efficacy Scale (LoMTES).

The two articles together detail an investigation into language of mathematics and the development of an instrument that would enable mathematics educators to determine teacher confidence in teaching language. It is hoped that

the instrument will facilitate the identification of teachers who would benefit from helpful intervention. Successful intervention of increasing teacher self-efficacy should result in increased student learning of and facility with the language of mathematics.

Chapter 2: Functionally Defining the Language of Mathematics: A Literature Review

*All words begin as servants, eager to oblige and assume whatever
function may be assigned them, but, that accomplished, they
become masters, imposing the will of their predefined intention
and dominating the essence of human discourse. It is for this
reason that articulate conversation must demand not only clarity
of thought and expression but also preciseness of word choice and
meaning.* (Pajares, 1992, pp. 308-309)

The eloquent statement by Pajares focuses on the general need for clarity
of language, but the statement is even more relevant in mathematics because the
use of precise language and expression is part of the very spirit of the subject. As a
demonstration of the difficulty students face with language, Smith (2002) related
the following quote from an unidentified biology text: "Cells divide in order to
multiply" (p. vi). When thinking in biology or English mode, the statement makes
perfect sense. Our brains interpret the terms *divide* and *multiply* in an English
sense as *separate* and *increase*. But in a mathematical sense, the statement is
humorous, because it is pointless. Division and multiplication are inverse
operations; in mathematics the poor cell is unable to reproduce, leaving nature to
develop only one-celled organisms. English speaking school students must
somehow learn to differentiate between the English and mathematical meanings of
a word. This simple example exposes only one of the several language issues faced
in mathematics. Students must learn to negotiate through considerable confusion
created by language as they are simultaneously trying to follow the abstractions
that make up the bulk of the curriculum in the mathematics classroom.

A number of mathematics educators and educational establishments have focused on the idea of mathematical literacy (Cooper, 2003; NCTM, 1989, 2000; OCED, 2003; Romberg, 2000; South African Government, 2003). In 1986 the National Council of Teachers of Mathematics (NCTM) began its journey toward a standards document when the Board of Directors created the Commission on Standards for School Mathematics. When the group was created, it was given two charges:

1. Create a coherent vision of what it means to be mathematically literate both in a world that relies on calculators and computers to carry out mathematical procedures and in a world where mathematics is rapidly growing and is extensively being applied in diverse fields.
2. Create a set of standards to guide the revision of the school mathematics curriculum and its associated evaluation toward this vision. (NCTM, 1989, p. 1)

In personal communication, Romberg (September 16, 2003) asserted that mathematical literacy is the key to this mathematics education reform. He defined literacy more broadly than merely reading and writing. In fact, it might more generally be called fluency. Students need fluency in mathematics, which includes an ability to communicate their understanding and to comprehend mathematical statements. Language is important in developing mathematical fluency, because mathematics demands precision of language, rejecting ambiguity and vagueness and valuing brevity and elegance of expression. In order for students to attain the vision of being mathematically literate, they need a language of mathematics as a significant tool to develop that literacy (Cooper, 2003; NCTM, 1989; 2000, OCED, 2003; Romberg, 2000).

Identifying the Problem

The term *language of mathematics* is a common yet unfortunately vague term. A conscientious teacher faces confusion when reading the NCTM communication standard, which states in part: "Instructional programs from pre-kindergarten through grade 12 should enable all students to use the language of mathematics to express mathematical ideas precisely" (2000, p. 60). There is no indication of what is meant by the term *language of mathematics*. There is apparently an inherent assumption that educators have some sort of tacit agreement of meaning on this important term.

What is the Language of Mathematics?

This idea of a language of mathematics is not new. Galileo spoke of the great book of nature being written in the language of mathematics. More recent theoreticians have also focused on the topic of mathematics as a language. Esty (1999) and Devlin (1998) have both written books entitled *The Language of Mathematics*. Kline (1953) indicated that the language is so rich it suggests new ideas and is, " . . . cleverer than the people who invented it" (pp. 176-177). Kline also maintained that Leibniz sought to broaden the scope of mathematical language and create a universal, technical language. Lampert (1998) spoke of the process of refining language in mathematics. While some mathematics educators have focused on applications of language (Austin & Howson, 1979; Bakhurst, 1988; Berenson, 1997; Bradley, 1990; Kaput, 1989; Trafton & Bloom, 1990), other theorists have argued that mathematics should be viewed as a foreign

language and that students should be instructed accordingly (Borasi & Agor, 1990; Esty, 1992; Rotman, 1990; Sharma, 1985a, 1985b; Usiskin, 1996).

Not only have some theoreticians indicated a view of mathematics as a language, but others have focused on more specific details regarding language of mathematics. Morgan (1996) took issue with the use of the definite article in *The Language of Mathematics*, indicating the use of the word *the* indicates " . . . an assumption of uniqueness, suggesting that *one* description of this language will be sufficient to characterize any text that arises within the practice of mathematics" (p. 2). She also argued that it is a mistake to focus on vocabulary and symbolism without including linguistics. Miura (2001) separated language into two distinctions, divided according to its purpose: (a) instructional representations (teacher language, external to the student), and (b) cognitive representations (student language, constructed by the student).

In considering a definition of the language of mathematics Kane, Byrne, and Hater (1974) described human language as, " . . . a system of vocal behavior used to communicate among people" (p. 1). In describing the particular language of mathematics, they depicted it as,

> . . . the written language found in mathematics instructional materials; in journals, books and papers about mathematics; on chalkboards and overhead projector screens in classrooms; in bank statements and supermarket advertisements and in the writings of a host of fields related to mathematics. (p. 3)

Although it covers the places where the language of mathematics can be found, this definition generally defines the language of mathematics as the language used

in mathematics. Defining a term with itself offers inadequate insight. Also this definition narrows to a written language and is in direct contrast to the more global definition of language stated first which focused on vocalization. The written language of mathematics can, however, be vocalized, which is implied by the NCTM (1980, 1989, 2000) and American Mathematical Association of Teachers of Two-Year Colleges (AMATYC, 1995) documents. An earlier focus on language by Kane (1968) gets closer to the heart of the matter:

> Mathematical English (ME) is a hybrid language. It is composed of ordinary English (OE) commingled with various brands of highly stylized formal symbol systems. The mix of these two kinds of language varies greatly from elementary school texts to books written for graduate students. The OE component often is laced with residue from formal symbol systems. . . . ME and OE exhibit different characteristics and consequently may require different skills on the part of readers. (p. 296)

Thus Kane addressed the difficulties in the confusion between English and mathematical language. Daniels (1995) also indicated that there are a number of differences between the ordinary language used in most American classrooms and the language used in mathematics although it is all said to be English.

Although not offering a definition, Pimm (1987) addressed the language of mathematics as involving (a) meaning, (b) symbols and the things symbolized, and (c) syntax. Similarly, Curcio (2001) identified (a) vocabulary, (b) semantics, (c) syntax, and (d) symbols. Because mathematicians communicate in more ways than in vocabulary and sentence structure, it is difficult to pin down the language aspects of mathematics. Any definition of language of mathematics would need to employ a broader mode than merely vocabulary, semantics, and syntax. A major

component of communication for mathematicians is also lodged in diagrams and graphs.

Mathematics educators are left without a sufficient definition of the language of mathematics. Because *language* can be defined only by experience, it is impossible to be specific; but the vagueness of the use of the term language of mathematics needs more precision in order to be useful to mathematics educators.

The Problem

A literature search revealed only the above, unsatisfying definitions of the language of mathematics. This term appears to be common, yet there is no convincing definition of this ubiquitous term. In order to narrow the search for meaning it was necessary to develop a functional definition. In so doing this review will first look at what is meant by the term *language*, and then more narrowly, *the language of mathematics.*

Defining the Language of Mathematics

What is Language?

Darwin (1871) spoke of language ability as ". . . an instinctive tendency to acquire an art." Pinker (1995) also embraced language as an instinct. He stated: "The workings of language are as far from our awareness as the rationale for egg laying is from the fly's" (p. 21). He spoke of the effortlessness, transparency, and automaticity of language. Although he included a significant glossary in both his 1995 and his 1999 texts, a definition of the term *language* does not appear in either of them. Chomsky (1972b) also avoided defining the term.

Language is generally an undefined term. Almost every person on earth acquires a language whether it is spoken or signed. Because of the commonality of the term *language*, deliberations tend to rely on the idea that everyone knows what is meant by the term. Jackendoff (2002) for instance assumed no imperative to define language in his book containing over 400 pages of text entitled *Foundations of Language: Brain, Meaning, Grammar, Evolution.*

Fauconnier and Turner (2002) alluded to language as a form and posited the following argument, " . . . what is behind form is not a thing at all but rather the human power to construct meanings. It, too, no matter the circumstances, can be unleashed dynamically and imaginatively to make sense" (p. 6). They maintained, "Form carries meaning with no loss" (p. 4). In addition to language, Fauconnier and Turner argued that humans have three additional forms: math, music, and art. From their descriptions, language can be defined as an instrument to convey meaning. Note, however, that these theorists see language and mathematics as distinct, not overlapping, forms.

Chomsky (2000) indicated that every speaker of a language speaks his own personal language—that our languages intersect enough that we can communicate. This idea is explained by Calvin and Bickerton in *Lingua ex Machina* (2000) as follows:

> . . . however little a word evokes in my mind, that little will be a subset, however weird or limited, of the set of things that the same word evokes in the mind of those who are expert in the relevant field. If it isn't—if the word 'orange' evoked in me some of the properties of bananas—we're in real trouble. But this seldom

happens, and if it does, we conclude there's something wrong in
the brain of the person concerned. (pp. 16-17)

As a child interconnects with those people within her sphere, she develops

an ability to understand language, which is easier than the later developed skill of

producing messages (Calvin & Bickerton, 2000). These researchers indicate that

this is because producing messages requires syntax, whereas comprehending a

message can involve a number of other modes of information, such as eyes, body

language, tone, and context. They assert that it is syntax, not symbols, that

separates language skills of our species from communication systems of other

species.

The major mystery of language is that it is possible for a speaker to create

an infinite number of sentences and that most sentences have never been uttered

before (Chomsky 2000, 1972; Lasnik, 1990). This literature review is placed in the

vision of Chomsky that language is an intrinsic, human, creative ability, and that

humans understand what is meant by the term *language*.

How is Language Acquired?

Brown (1994) identified two diametrically opposed positions on

acquisition of language. The first position he discussed was a behaviorist

perspective that assumed children are blank slates who are then shaped through

their environment and conditioned by reinforcement. Brown's second perspective,

in opposition to the behaviorist perspective, was the instinctive perspective—that

children have specific innate knowledge of the nature of language and a general

sense of semantic structure. Chomsky (2000) posited an imaginary Language

Acquisition Device (LAD) that children are genetically encoded with, a Universal

Grammar (UG) sense that allows them to disentangle syntax and choose the

appropriate syntax for their native language. Children naturally develop language

skills just as they naturally learn to walk (Chomsky, 1972a, 2000; Calvin &

Bickerton, 2000). This is not to imply that children are never taught certain aspects

of language; it does, however, imply that certain parts of language are developed

by the combination of exposure and an internal, genetic sense of how language

works. "A conscious knowledge of formal grammar is not a necessary condition

for the acquisition of fluent language skills" (Allen & VanBuren, 1972, p. 149).

Chomsky spoke of an "underlying language competence" allowing the learner to

create never before uttered sentences. He believed:

> Language is not a 'habit structure.' Ordinary linguistic behavior
> characteristically involves innovation, formation of new sentences
> and new patterns in accordance with rules of great abstractness and
> intricacy. This is true both of the speaker, who constructs new
> utterances appropriate to the occasion, and of the hearer who must
> analyze and interpret these novel structures. There are no known
> principles of association or reinforcement, and no known sense of
> 'generalization' that can begin to account for this characteristic
> 'creative' aspect of normal language use. (pp. 153-154).

Chomsky (1972b) also argued that he knows no "technology" of language

teaching for second language learning. He further indicated: ". . . it should serve as

a warning to teachers that suggestions from the 'fundamental disciplines' must be

viewed with caution and skepticism" (p. 153). Thus Chomsky implied that,

although linguists work on the macrostructure of language, they may not be

helpful to the teacher who is passing on information to those new to a language.

Chomsky's revelation means that students have an instinctive, general sense of grammar and they learn language both from exposure and direct, explicit instruction. There is comfort in this perspective, because it is impossible to identify all the rules of a formal grammar system in a simple way (Allen & VanBuren, 1972). Educators can teach a practical grammar, however, and give some of the rules of structure. It is the learner's task to generalize and apply the structure to develop an internal, more complete set of rules of the particular grammar. This work embraces the perspective of some linguists and neurophysiologists—that language acquisition is instinctive (Chomsky, 2000; Pinker, 1995, 1999; Calvin & Bickerton, 2000).

How are Language and Learning Related?

Vygotsky (2002) revealed important connections between language and thought. "Thought development is determined by language, i.e., by the linguistic tolls of thought and by the sociocultural experience of the child" (p. 94). Bakhurst (1988) also asserted that language is an essentially social phenomenon and that a set of shared social meanings in a language represents a culture. Devlin (1998, 2000), Sfard (1991, 1995), Sfard and Linchevski (1994), Vygotsky and Davydov (as cited in Confrey, 1991), used language as a metaphor for thinking. As Vygotsky (2002) indicated, "The child's intellectual growth is contingent on his mastering the social means of thought, that is, language" (p. 94).

Ellis (1993) also theorized a social focus in language: "A language is a unique, highly complex, ordered conceptual system. It is the most central factor in

the social life of those who share it, and it is the most crucial thing that differentiates one community from another" (p. 199). It is impossible to separate language from learning in that much of our thinking is in language (Vygotsky, 2002). This review embraces the sociocultural perspective that language and thought are so intricately intertwined that it is impossible to separate them.

This project explores the teaching of the language of mathematics. The Chomskian school of linguistics is embraced in envisioning language as an instinctive act, learned both from exposure and direct explanation. It is easier to receive messages than to produce them. In the Vygotskian tradition, language is inseparable from thought.

In applying the concepts of language to mathematics and developing a functional definition, the questions arises: Is *language* a metaphorical or a literal term? This is an important distinction. As the review continues each of these perspectives is considered.

The Language of Mathematics—Literally or Metaphorically?

In promoting a literal meaning of the language of mathematics, Esty (1992, 1999), Esty and Teppo (1994), Rin (2001), Sharma (1985a, 1985b), and Usiskin (1996) used the adjectives *foreign* or *second*. Esty (1999) advocated the capitalization of Mathematics to indicate its standing as a proper language. Usiskin (1996) made a strong case for the literal definition of language in mathematics, identifying that it has a grammar, sentences, verbs, and a well-constructed syntax. He speculated that the reason the language of mathematics is:

. . . not treated by scholars as a language in the same sense that English or Japanese or French is a language is that, except for the small whole number, mathematics does not tend to originate as a spoken language." (p. 233).

Perhaps some of the difficulty students have with the language of mathematics is indeed related to a second language nature of the subject. In traditional foreign language study, the student is focused on learning the meaning of words and the structure of the language. The foreign language curriculum re-exposes students frequently as it recycles words and syntax; students in foreign language classes have many opportunities to learn the language specifics. Mathematics teaching, however, tends to focus on concepts and to treat the language portion of the subject minimally. Are there any mathematics students who have not had a symbol or word quickly defined once and then been expected it use it thereafter with fluency?

Perhaps the pinnacle of viewing the literal meaning of the language of mathematics occurs at Montana State University. Warren Esty (1999) has taught a course called "The Language of Mathematics" for over 10 years. His goal is " . . . to have the students assimilate the basic concepts and language skills which are fundamental to mathematics" (p. *iv*). The prerequisites are an ability to read English at the college level and completion of beginning algebra. The course focuses on language and is primarily a reading course.

Although mathematics has syntax and a semantic structure and functions like other languages in that it borrows words (i.e., *similar*) and gives words (i.e., *irrational*) to other languages, three major differences between mathematics and

natural languages make a literal view impossible: (a) No one is a native speaker of this language; (b) the language doesn't stand alone in that it uses an additional, natural language in tandem with the mathematical symbolism; and (c) it is impossible to be fluent in this language without being literate.

Because of the symbolic density in mathematical writing, it is tempting to argue that mathematical language is an international language. Mathematicians, however, also use words from their native languages within their mathematical conversations; these words become part of their personal lexicon within mathematics and thus mathematics commonly uses a natural language as part of its communication contrivance. Thus the language of mathematics is also used with massive amounts of an additional, natural language. Also, unlike natural languages, the language of mathematics is primarily a written language. The complications of symbols and graphical representations makes fluency in mathematical language quite another matter from learning to read. The literal perspective of language in mathematics is not a useful construct for this study, because it uses a broader perspective than mechanics and translation. Having rejected a literal perspective, one is led to a metaphorical perspective to consider as a possible viewpoint.

Some theorists approach language as a metaphorical term (Kaput, 1989; Goodson-Espy, 1998; Sfard, 1991; Sfard & Linchevski, 1994) Metaphorically, the phrase language of mathematics addresses the dichotomy of our mathematical experience: reasoning and communication. Differentiating between mathematics as

reasoning (another NCTM Standard) and mathematics as communication appears to be an issue of semantics and hearkens back to the earlier discussion that language and thought are intricately intertwined. Kaput (1989) remarked, "Mathematics is, among other things, a collection of languages, and languages have dual, interlocking roles: They are instruments of communication and instruments of thought" (p. 167). Can humans differentiate between their reasoning process and their communication process? Vygotsky would say, "no." Language allows us not only to share, but also to process our reasoning. It appears that the two different standards of reasoning and communication are hierarchically related rather than horizontally related.

Mathematical reasoning has been spoken of as metaphor (Goodson-Espy, 1998; Sfard, 1991). In studying the cognitive structure of mathematics learning. Sfard (1991) and Sfard and Linchevski (1994) considered cognitive processes in mathematics. They indicated a reification process whereby a student would experience a concept as an abstraction and, in applying it to later concepts, reify the abstraction into a concrete object. Devlin (1998 & 2000) used the metaphor of language in expressing the nature of mathematics. Kaput (1989) addressed the semantics of mathematics and indicated his belief that it is the study of symbols and syntax that causes alienation in algebra. He posited a "relational semantics" in considering students' thinking processes and cautioned against a literal perspective on the phrase language of mathematics as too focused on syntax, exacerbating student problems.

This project uses a metaphorical perspective in dealing with the term language of mathematics. Mathematics is not a natural language as it is not the first language of anyone. It does not stand on its own in that mathematicians require another, natural language to make the connections among different symbols in used in written mathematics. Mathematical language is also primarily a written language because if there is very much complication in what is being communicated, the written form is crucial so that the sender and the recipient of the message concur on the information shared.

If the language of mathematics is not literally a language, then what is it? In developing a functional definition, this article first considers other distinctions.

A Functional Definition

Some argue that rather than being called a language, mathematics is better addressed as a register (Forman, 1996; Halliday, 1978; Kang & Pham, 1995; Pimm, 1991; Winslow, 1998), but the linguistic term *register* refers to a contextual style that a speaker or writer uses, dependent upon degree of formality (Chomsky, 1972b; Liles, 1972; Shorter Oxford English Dictionary, 2002). Discussion among teenagers thus uses a register different from that used when those same teenagers give formal speeches in English class. Accordingly, *register* is not precise in describing language aspects of mathematics.

It can be argued that instead of a register, one could consider mathematics as a genre (Wertsch, 1991; Winslow, 1998) of language, meaning kind or type. Wertsch (1991) contended that language in different disciplines should be looked

at as specific curriculum *genres*, produced by teachers and students. These language genres change for students throughout their school day as they move from subject to subject (Winslow, 1998). Interaction patterns between teacher and student are demonstrated in the written genres produced by students. These written genres are context specific and each subject has a specialized discourse. This term *genre* is a more nearly accurate term and probably the appropriate one, but the term language of mathematics is robust. So many people use it that it is unlikely to be altered to genre, thus this review continues to use the term language and apply it as a metaphor rather than accept the term literally.

For the purposes of this project, language of mathematics refers to a metaphorical rather than literal meaning of language and specifies a *genre of* English as clarified above, *genre* meaning *kind* or *type*, characterized by a particular form or purpose (Shorter Oxford English Dictionary, 2002). Because language is an undefined term, we can accept the term language of mathematics only as a generalized construct.

The term language of mathematics needs to be defined and there is apparently no satisfying definition; thus the author offers a functional definition to drive the literature review. For this review, the language of mathematics is viewed as a metaphorical term referring to a genre constituted by the union of a subset of English and a set of symbolic forms that enable the communication process in English speaking mathematics classrooms. This study focuses on English speaking classrooms and uses the following functional definition:

In English speaking classrooms, the Language of Mathematics is used in combination with English and a set of symbolic forms. This language refers to the specific words, symbols, graphs, and conventions used when discussing or writing about mathematics.

The definition is depicted graphically in figure 1.
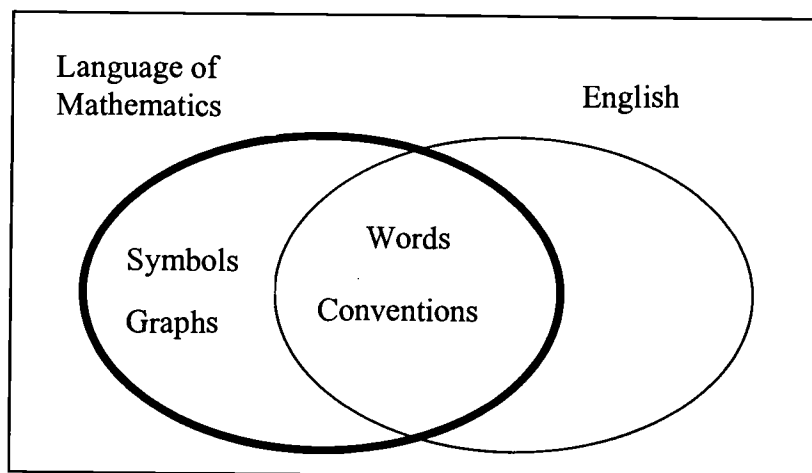
Figure 1: Graphical representation of language of mathematics used in English speaking classrooms as the union of subsets of English and symbolic forms.

The four factors of the definition need to be clarified. *Words* indicates not only specific vocabulary used in mathematics such as rhombus, polynomial, and square root, but also terms that have different meanings in English and mathematics such as similar, continuous, divide, rational, and random. The term *symbols* refers to icons that have encoded meaning that can be read, but are not normally used in mathematics in an alphabetic form. The term *graphs* refers to visual representations, such as coordinate graphs, diagrams, and any visual representations of data. Finally, the term *conventions* refers to the grammatical and cultural traditions of expression used in mathematics including phrasing such as *let x =* , or *is decreased by*, or *is a function of x,* along with things such as the

accepted forms of argument, the expectation of precision of language, the respect for brevity, and the admiration of elegance.

A number of researchers were contacted regarding this definiton. Pajares (personal communication, October 13, 2003) responded with , "I see no problem with this." Kilpatrick (personal communication, October 14, 2003) said, "I'm not aware of any definition of the term, and yours seems reasonable to me." Yackel (personal communication, October 12, 2003) suggested it be identified as a personal definition and also suggested considering the use of the term *register* instead of language. Cobb made a suggestion, "I guess I would broaden the definition a little to include forms of (mathematical) argumentation as these distinguish mathematical langauge from everyday language." The author concurs with Cobb's concern and feels that argumentation would be subsumed in the category of "conventions" within the definition. Romberg said,

> "First the major point in NCTM's standards is not about language, but about literacy. We viewed mathematics as a language (its signs, symbols, and rules for use all invented to help make sense of some phenomena), and were concerned that students should not just learn the language, but how it evolved and how it can be used. Your working definition is fine except the intersection is with other languages (and those of other disciplines such as physics, or archeology).

Although agreeing with Romberg, the author finds it useful to narrow the definition to a smaller intersection of language (English) in order to pinpoint a smaller area of study for now. Cooper cautioned that the definition is limited to the language of American (later changed to English speaking) school mathematics and also stated, "I find your definition intriguing, connecting the notions of English

language usage with standard mathematical usage." Lampert opposed the functional definition as follows: "This seems like a very limited definition given the volumes of research that have been written about the use of mathematical language in teaching and learning." The author respectfully continues to embrace this definition. Lampert studies communication, which is important in mathematics. This study is designed to look at the narrower topic of language and needs a narrow, more limited definition than would be used in a communication study in order to avoid being overly broad.

Literature on the Language of Mathematics

This review continues by focusing on literature related to the language of mathematics. It uses the proffered functional definition of language of mathematics, focusing on English speaking classrooms and therefore addresses the language of mathematics in the context of English. Language issues pertinent to the mathematics classroom are addressed, recognizing specific difficulties present for students as they learn this language of mathematics. The review first considers how the language is learned and then contemplates specific problems students face as they hear teacher language and develop their own language, while assimilating features of the words, symbols, graphs, and conventions of mathematics.

Because this mathematical language is the form in which mathematics is expressed, students must develop a certain amount of fluency or communicative competence in order to learn the subject. Pimm (1987) identified the importance of addressing language in mathematics teaching when he stated:

> If we are to view mathematics as a language, communicative competence becomes an important consideration, and meaningful communication an overwhelming concern . . . . Being fluent in a language, then, involves the ability to tap into the resources implicit in it and to use these potentialities for one's own ends. (p. 6)

Traditionally mathematics language skills have been learned mostly by "osmosis" rather than by explicit instruction (Daniels, 1995; Durkin & Shire, 1991). One of the questions for mathematics educators regarding language and mathematics is how is this language learned?

How do Teachers Learn the Language of Mathematics?

There seems to be consensus that mathematics educators should address language when teaching mathematics, yet a review of three secondary mathematics methods textbooks (Huetnick & Munshin, 2000; Posamentier & Stepelman, 1999; Sobel & Malaetsky, 1999) revealed a total of 27 pages listed in the indices under *language, communication, speaking, reading, writing,* or any form of the word *verbal.* With a total of 1,371 pages in the texts, this yields less than two percent of the pages that at least touch on language. The books range from having zero pages on these topics to less than four percent. Three texts of mathematics for elementary teachers (Billstein, Libeskind, & Lott, 2001; Bennett & Nelson, 2004; Troutman & Lichtenberg, 2003) yielded only 3 out of a total of 2,153 pages (only one-tenth of one percent) that addressed any of the search words.

The *Handbook of Research on Mathematics Teaching and Learning* (Grouws, 1992) has 732 text pages. Its index identifies nine pages related to the term *language.* On closer inspection, the pages are focused on students who are

learning mathematics in a language other than the one spoken in the home. While this is an important focus, it does not address the general need for guidance on language issues in mathematics education. Further inspection of the index reveals no entries for *communication, discussion, speaking, reading, discourse,* or any word with the same root as *verbal.* There are a total of nine pages referring to *language,* seven pages referring to *written symbols,* and one page referring to *written composition,* making a total of approximately two percent of the pages that are identified with any of the search words, and only about 1% if the second language pages are not included in the count.

A teacher, faced with the 2000 *Standards,* is given a formidable task—to teach students to use the language of mathematics, yet the mathematics education community offers little help. Many educators and documents cited above call for increased emphasis on communication in the language of mathematics. Unfortunately there is no apparent consensus regarding the pedagogical implications of how students learn this language and no information on how teachers can approach the dictated task.

Some studies indicate that discussion is a powerful way for students to develop language and social norms of discussion. (Hiebert, et al., 1997; Yackel, 2001). A number of researchers have studied the impact of discussion in developing understanding (Cobb, Wood, & Yackel, 1993; Fennema, Franke, Carpenter, & Carey, 1993; Wearne & Hiebert, 1988); however, there seems to be a paucity of literature focused on how students acquire mathematical language.

Student Difficulties with Language in Mathematics

Pimm (1991) contended that, "Part of learning mathematics is gaining control over the mathematics register so as to be able to talk like, and more subtly to mean like, a mathematician" (p. 18). As students struggle with the semantics and syntax of mathematical language, their language eventually makes a shift to include more mathematical terms and structure (Daniels, 1995). This portion of the review addresses difficulties students experience with learning language in mathematics. It is subdivided into four sections: (a) teacher language and student language, (b) vocabulary, (c) symbols and graphs, and (d) conventions. These categories are not totally distinct and some of the research discussed could fit into more than one category. The choices of category are thus sometimes arbitrary.

Teacher language and student language. Gregory indicated that the language of teaching must be common to both the teacher and the student and identified the importance of understanding for both teacher and student in his century old text:

> "In all true teaching thought passes in both directions—from pupil to teacher as well as from teacher to pupil. It is as needful that the man shall clearly understand the child as it is that the child shall understand the man" (Gregory, 1886, p. 53).

In this book of advice to teachers, *The Seven Laws of Teaching*, he included a section entitled *The Law of Language*, with the following recommendations:

> No one has more language than he has learned, and acquisition of a large vocabulary is the work of a lifetime. A teacher may know ten thousand words; the child will scarcely know as many hundreds, but these few hundreds of words represent the child's ideas, and

within this narrow circuit of signs and thoughts the teacher must
come if he would be understood. Outside of these the teacher's
language is as unmeaning to the child as if it were mere drum-taps.
His language may sometimes be partially and vaguely understood
by reason of the known words scattered through it but may as
frequently mislead as lead aright. (Gregory, 1886, pp. 49-51).

This admonition to teachers from over 100 years ago is applicable today.
As teachers enculturate students into the esoteric mathematical language structure,
they must address language issues, whether implicitly or explicitly. This idea
seems obvious, but it is particularly significant in mathematics education. Teachers
must model mathematical language every day, and Gregory's caution implores
them to be aware of which portions of their language will not be understood by
students.

Adler (1998 & 1999) looked at uses of language in the mathematics
classroom as she studied schools in South Africa and considered language from
the perspective of multiple language speakers. All of the schools she chose taught
classes primarily in English, but some of the teachers and students did not have
English as their main, personal language. Adler's focus was on the nature of
language within the mathematics classroom while exploring the "benefits and
constraints of *explicit mathematics language teaching*" (1999, p. 47). She looked
at the concept of code-switching which is a term in foreign language study and
refers to the shifting between two distinct languages. This qualitative study
considered the use of language within the mathematics classroom, focusing on
how mathematics teachers in a multilingual setting managed the complicated
balance between: (a) the use of formal mathematical language and informal

language, and (b) a language of instruction which is not the main language of pupils.

The six teachers were evenly divided among the three main multilingual educational contexts in South Africa. Adler found that teachers in multilingual settings moved between talk used for thinking and talk used as a display of knowledge. She argued that teaching and learning mathematics entails this moving back and forth. Adler also found that explicit language teaching is a struggle, particularly for teachers and students who use English for teaching/learning mathematics and another language as a first language.

She put forth the idea of transparency of talk in terms of its visibility and invisibility. At times the teachers wanted invisible talk, comfortable to learners. At other times the teachers employed visible talk, particularly in teaching mathematical language. The invisible talk was used to clarify concepts. The visible talk was used for specific vocabulary and for formal statements of mathematics. Teachers struggled with the balance between the two types of language codes and which forms of language were of best use in a classroom situation. The researcher reported that teachers developed a complex practice of shifting explicitly between everyday and mathematical discourses and between verbal and symbolic forms as they attempted to enculturate students into mathematics.

While Adler's research on code-switching is between English and other South African languages, a parallel can be made with possible code-switching employed by mathematics teachers, interchanging mathematical English and other

classroom English. If mathematics educators embrace the metaphor of mathematics as a language, it may reasonable to apply this code-switching construct of visible and invisible talk to English speaking mathematics classrooms. Although instruction is in English, teachers might want some of their language to be visible (as they teach about the formal language and constructions of mathematics), and they might want some of their language to be invisible (as they teach the concepts of mathematics).

Similarly to Adler, Miura (2001) separated language into two distinctions and saw the mathematics classroom language as divided according to its purpose: (1) instructional representations (teacher language, external to the student), and (2) cognitive representations (student language, constructed by the student). When teachers are in control of classroom discourse, the pattern is frequently teacher initiation, student response, then teacher evaluation (Cazden, 2001; Fullerton, 1995). Fullerton indicated that students used different language when working separately rather than when the class was in a whole group discussion. Working separately from the teacher, the student language changed to self-directing speech, particularly when a pupil worked through the complex thinking needed to solve a problem. Fullerton (1995)was interested in the relationship between oral language and the learning of mathematics and investigated the degree to which students were able to hear and practice language patterns of mathematics. Her qualitative study consisted of nearly fifty sessions as a participant-observer in classrooms as fourth to sixth grade students worked on geometry.

Fullerton found that students tended to give short answers to the teachers'

whole group questions. She also found that students often ended those responses

with a rising inflection, as if asking for confirmation of their answers. (An

octahedron? Maybe a hexahedron?) When students worked in groups, however,

Fullerton found that they asked more questions as they worked through a task

together. Instead of being directed to the teacher, most of these questions were

directed to each other. The researcher also found a difference in student language

in general. "In contrast to the language used by children when the teacher was in

control, children's language flowed more naturally in small group settings" (p. 14).

Fullerton concluded that students need verbal interaction with others as

they learn the mathematical language. She found that both teachers and students

provided language models, and students used a mixture of the new mathematics

language and less formal language as they developed ways to explain their ideas.

("You slide a circle to make a cylinder" p. 12.) She urged, " . . . teachers must

maximize talk opportunities for children" (p. 16). She posited that this

maximization can best be done if teachers devise situations in which students are

encouraged to articulate their ideas within small group discussion. Similarly,

Lampert and Cobb referred to evidence (Brown, Stein & Forman, as cited in

Lampert & Cobb, 2003, p. 246) that ". . . with appropriate support and structures in

place, teachers can improve the quality of their mathematics instruction to build

the capacity of students to think, reason, solve complex problems, and

communicate mathematically" (p. 246).

Whether mathematics is viewed as a language, as a particular register within the language of English, or as a genre, there are certainly hazards with usage that can ensnare a student trying to develop facility in mathematics. Not only the words, but also the symbols are perilous if approached carelessly. In addition to these problems, students are faced with the specific syntax and semantics used in writing and speaking about mathematics, which can also confuse students. The subsequent portions of this section of the review address focused topics important to mathematical language: (1) vocabulary, (2) symbols and graphs, and (3) conventions.

Vocabulary. Students frequently are confused when learning the distinct vocabulary for mathematics and falter when confronted with a need for specific words. Rubenstein (2002) was concerned about difficulties of the language in her elementary students and demonstrated some of those difficulties through this list of actual student remarks recorded in her classroom:

> Is the diameter the short one or the long one?
> I forget, I think obtuse is the wide kind of angle.
> Eight is a multiple of 24, I think, . . . or is it a factor? (p. 243)

Rubenstein showed that in addition to the considerable conceptual difficulties of mathematics, students are faced with a large set of new terms and arguably a new language. Zazkis (1999) argued that mathematics students are faced with a significant number of vocabulary words, and some of them are old words with new, more precise meanings.

Pimm (1987) identified that some common English words undergo grammatical shifts as they are used in the mathematics register. As an example he indicated that the term, *diagonal* in English was originally an adjective (*diagonal line*), yet in mathematics it has undergone a syntactic category shift to the noun *a diagonal*. He also identified student remarks that indicated that some students thought of the term *diagonal* in respect to physical orientation, and that *diagonal* indicated a non-vertical, non-horizontal line. This example shows the kind of confusion that students experience in mathematical language, particularly with words that are ordinary English words, used differently in mathematics. Gregory (1886) cautioned teachers about the difficulty students have with words that have more than one meaning.

Between English and mathematics there are numerous examples of polysemous words (words with different, but related meanings) such as the terms *similar, irrational, opposite*, and *function*. Halliday (1978) noted the possible student confusion with the common term *multiply*, which means to increase except when it means to decrease as in multiplication by a half. Because of the subtleties and different shades of meaning, much of the vocabulary of mathematics is thorny for students to access, particularly if they are unfamiliar with the mathematical concepts behind the vocabulary.

As she studied polysemy with different meanings <u>within</u> the mathematics language, Zazkis (1999) focused on lexical ambiguity that arises in mathematics classes. She examined the terms *quotient* and *divisor* with pre-service elementary

teachers through classroom transcripts and student interviews (*n* unidentified). The quotient in the division of 11 by 4 for example can be interpreted as 2, or as some form of another answer: 2r3, 2.75, $2\frac{3}{4}$, $\frac{11}{4}$, or the number of rows in an array of 11 elements with 4 columns. The major issue in this example depends upon the definition of division, and the number set in which the student is working. In standard reading situations, one is often able to determine meaning from context, but Zazkis' point is that some mathematical situations lack contextual cues. Zazkis concluded, " . . . the regular 'tools' to determine meaning, such as context or grammatical form, are not always sufficient" (p. 8). Zazkis' project identified the confusion that some students experience because contextual cues are frequently not available. The study report is unfortunately short and lacks detail other than a discussion of her students' views of the two words.

Durkin and Shire (1991), in a theoretical argument, identified 80 ambiguous words used commonly in school mathematics. As students take on the language needed in mathematics classes, they encounter new meanings, complicated symbolism, and multiple interpretations. In addition to these difficulties, students are faced with syntactic and semantic issues that generate confusion for the student. Students struggle between meaning and structure as they learn mathematical concepts and acquire skills to communicate their mathematical thinking. The new language that mathematics students confront has complicated semantics and difficult syntax that is not the same as the structure of English, yet it is similar enough to be confusing.

Some sentences in mathematics are vocalized in the traditional manner taught in school English classes. Although there are many new, polysyllabic words, the word attack techniques students have assimilated elsewhere can work to sound out these new words. But mathematical language is more complicated than the language students are faced with elsewhere; in addition to the complicated but alphabetic words, students must contend with a mixture of signs. (Kane et al., 1974; Winslow, 1998).

Symbols and graphs. Cajori (1929) identified three types of mathematical symbols: (1) ones that originated as abbreviations of words such as f (function), cos (cosine), *ln* (natural log) and + (et); (2) symbols that are pictographic such as $\triangle$ (triangle), $\parallel$ (parallel), or $\bigcirc$ (circle); and (3) symbols that are ideographic or arbitrary such as $\therefore$ (therefore), $\times$ (multiply), and $\sim$ (similar). Some abbreviative symbols eventually lose the clarity of their abbreviation, functioning as ideographic symbols: π (periphery of a circle), and i (imaginary). One of the major stumbling blocks in mathematical language comes from ideographic symbols (Kane, Byrne, & Hater, 1974; Skypek, 1982; Winslow, 1998). The meanings of these symbols must be memorized in order to be pronounced because they are not inherently connected to their symbols the way the meanings of abbreviated or pictorial symbols are connected cognitively. The ideographic symbols are the most difficult to learn.

In addition to the problem of pronouncing the considerable number of symbols, there is the difficulty of the direction of reading (Marks & Mousley,

1990). Some mathematical symbolism is read in the standard left to right manner

of traditional English texts ($2x^2 + 7$), but then some symbolism is read vertically

($\frac{2}{3}$), some is read right to left ($13\overline{)124}$), and sometimes there is a mixture of

directions ($\sum_{j=2}^{7}\frac{3}{5}j^{-1}$). Even something as simple as $100 is read contrary to the

standard English direction. This must create significant confusion for students,

especially if the direction of reading for each type of symbol is only implied.

Franzblau and Warner (2001) demonstrated that not only does

mathematical notation require an adjustment in reading direction, but also the

notation poses a difficulty for students attempting to use symbols to record their

thinking. Additionally, as students are faced with problems with symbols and the

complication of reading direction, they are also faced with complicated

information compacted by notation. This information cannot be gleaned from

context. In their speculation on student learning, Kane et al. (1974) pointed out:

> . . . all the subject matter related to interpreting correctly a
> sequence such as $34_{five} = 19_{ten}$ is to be found in the mathematics
> curriculum with little likelihood that these concepts will arise
> naturally outside the classroom. In short, comprehending the
> language of mathematics is a task more closely related to a specific
> subject matter (mathematics) than is comprehending a story such
> as *The Emperor's New Clothes*. (pp. 9-10)

Not only is mathematics dense with polysemous vocabulary, but

mathematics also has symbolism that is polysemous, which is possibly confusing

to learners. For example: (2, 7) may mean the point located on the Cartesian plane

at $x = 2$ and $y = 7$ or it may mean the open set of real numbers that are larger than 2

but smaller than 7. The symbol ~ sometimes means *not* and sometimes means

similar. Students can also be confused about graphing something like $x = 1$ which

is interpreted in at least two ways as seen in Figure 2.
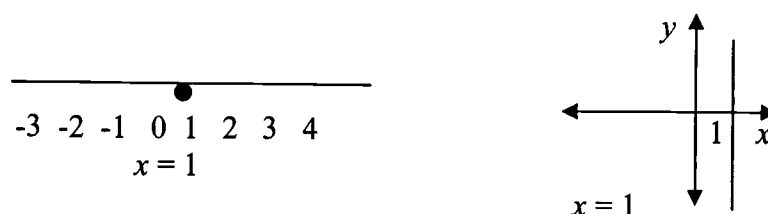
Figure 2: Two different graphs of $x = 1$.

Even the simple equal sign (=) can be interpreted as, *"equals, means, makes,*

leaves, the same as, gives, results in, any one of which is itself multi-meaning."

(Durkin & Shire, 1991, p. 73).

Conventions. Concerned about ambiguity in mathematics, Alro and

Skovsmose (1998) presented an exposition about semantic difficulties in teaching

as they took exception with a Danish textbook direction to students to answer the

question, "How much do newspapers fill?" This use of the term *fill* is not so

common in English speaking classrooms, but in the research study Alro and

Skovkomose showed their students' confusion between area and volume and made

an important point. In order to function mathematically, students need to make

sense of the mathematical language and develop the necessary language skills.

This point is supported by Kang & Pham (1995): "Such language includes specific

vocabulary, syntax, and other features of the mathematics register that represent

mathematical concepts as well as the language that is used to teach mathematics"

(p. 3).

Rowland (1999) focused on the semantic difficulty created by a confusing use of pronouns in mathematics. He studied transcripts of classroom interchanges between elementary school teachers and their pupils, and focused on the subtleties of pronoun use within the classroom. He also included information from a variety of unidentified transcripts, covering " . . . a number of one-off interviews and teaching episodes with students spanning the age-range 10 to 25" (p. 25). He indicated that he was the teacher in some of the situations, but not in all. The information he presented is an amalgamation of the impressions he developed from the transcripts.

Rowland took great exception to the widespread use of the pronoun "we" as teachers presented mathematics. He pointed out that, "It is improbable that the child is included in the 'we' in phrases like 'what we said you had to do.' The phrase could be intended to imply 'What I said in your presence'" (p. 19). He also pointed out that the pronoun "we" indicates an anonymous expert community the teacher invokes in order to impose a certain classroom practice.

Regarding the student use of the pronoun "I," Rowland included a set of student remarks, showing the use of "I" when the student was confused (I want to find . . .), and a shift to the generalized, "you" when understanding finally occurred and the student generalized the task verbally (Oh, hey, you just . . .). Rowland also argued against vague uses of the pronouns, "it" and "you." While his may seem a picayune perspective, Rowland contended that teachers who are sensitive to student pronoun use may be able to recognize cognitive shifts in student thinking.

However, with no mention of validity or reliability, the study is incomplete, and the conclusions must be viewed as tentative.

Students in mathematics are faced with learning to translate among a variety of ways to represent relations: verbal, graphic, tabular, and symbolic (Brophy, 1991; Fennema & Franke, 1992). The ability to connect these different representations of relationships gives students a broader understanding of the relationship and offers multiple ways to proceed with problems through various perspectives.

Student difficulty with semantics and syntax is sometimes unrecognized by teachers (Nathan & Koedinger, 2000). In a study of high school algebra and geometry students ($n = 76$) and a similar follow up study ($n = 171$) of the same level of students, the researchers chose 12 problems for the students to solve. Half of the problems were arithmetic in nature (defined as result unknown), and half were algebraic in nature (defined as start unknown). Both types were presented in three ways: (1) as a traditional word problem, (2) in equation form, but written in words, and (3) in equation form, and written in symbols. The students had the least difficulty with the traditional word problems and the most difficulty with the symbolic form; however, when teachers and mathematics educators ranked the difficulty of the problems, they indicated that the symbolic form would be the easiest for students and word problems would be the most challenging. A lack of information on validity and reliability calls the study into question so the results must be taken as conditional rather than as established.

DeCorte and Verschaffel (1991) maintained that there is robust research (unidentified) that

> ... shows psychological significance of the semantic classification of word problems. A major finding in this respect derives from children's performance on such tasks: word problems that can be solved by the same arithmetic operation but differ with respect to their underlying semantic structure have very different degrees of difficulty. (p. 119, italics theirs)

This assertion is reaffirmed by Esty and Teppo (1996) who studied university students who had taken enough algebra for precalculus but did not perform well at the prerequisite level. The research hypothesis was that the difficulties these students had with word problems were not because the students did not understand the mathematical relationships expressed in the words, but that they did not understand the underlying algebraic language and the use of symbolism required to express these concepts. Esty and Teppo considered this lack of ability as an indication of a deficiency in understanding of the language of algebra.

The sample was one of convenience. All 137 students who attended the first day of any of the five sections of pre-calculus during one particular term at Montana State University were included. The article does not indicate any differentiation of students based on mathematical background, age, recency of any mathematics class, socio-economic status, or any other indicator. Each student in Esty and Teppo's study completed one of two quizzes. There is no indication of whether the quizzes were assigned to students randomly. Each quiz contained two situational word problems, and included diagrams. One of the two problems on the

quiz had students recall how to find the area of a shape and then apply the formula directly. (Nathan and Koedinger, 2000, called this type of problem "result unknown.") The other problem had students recall an area formula again, but this time the desired variable was within the formula part of the relation and students could not solve for it without using an algebraic method. (Nathan and Koedinger, 2000, called this type of problem "start unknown.")

The research design was simple and direct. It purported to study the lack of algebraic thinking in pre-calculus students, but unfortunately failed to collect background information on the students. Because it was given on the first day of class, there could have been additional students who did not belong in that class. This could of course include students who were "shopping" for a class, and who were not ready for pre-calculus or perhaps students who were lacking confidence, but were actually ready for the subsequent course. It could also include students who just wanted to brush up on their skills. This situation confounded the demographics of the sample. A closer identification of the sample would make the inference clearer and more powerful. With the data listed as percentages and no statistical analysis, the results were unclear. Additionally, the quizzes were developed by the researchers and did not appear to have been subjected to tests of reliability or validity. This fact calls the entire study into question as it is not clear if it is suitably focused on the planned topics of algebraic thinking and language.

In studying sixth grade students, Swafford and Langrall (2000) found that pre-algebra students were able to generalize problematic situations and to solve

problems with their own, sometimes non-standard, equations. They had difficulty generalizing however, being unable to apply solution methods of one problem to a similar problem. This study demonstrated that before studying algebra, these students did not see equations as semantic objects, but rather as shorthand notation. It appeared, though, that students naturally developed a sense of the need for mathematical language because they tended to invent notation as code for their thinking.

Rosnick and Clement (1980) completed an often cited study of how students interface between mathematical symbols and verbal descriptions of real world problems. They studied nine students, most of whom had taken one semester of calculus, and all of whom had created a reversed equation ($P = 6S$, instead of $S = 6P$) when translating the following problem:

> Write an equation using the variables S and P to represent the following statement: There are six times as many students as professors at this university. Use S for the number of students and P for the number of professors. (p. 4)

Rosnick and Clement further studied six of the students who wrote incorrect equations for the problem. The format alternated between remediation and interviewing. The remediation techniques were as follows:

1. Simply telling the students that the reversal is incorrect.
2. Telling the student that the variable should be thought of as "number of students," not "students."
3. Pointing out (with pictures) that since "students" is a bigger group than "professors," one must multiply the professors by six to create an equality.
4. Asking the students to test the equations by "plugging in" numbers.
5. Specifically showing the students how to set up a proportion to solve the problem.

6. Demonstrating a correct solution to the students, using an analogous problem. (p. 6)

Rosnick and Clement concluded that the misconception they studied was resistant and that students' misconceptions were not quickly eliminated. They further concluded that the error they were studying could not be corrected just by demonstrating the correct solution or by explaining why it was wrong and that the mistakes were not casual or careless, merely from lack of concentration. They surmised that the errors were the result of, " . . . deeply ingrained and resilient misconceptions" (p. 16) and indicated that a subsequent study found similar results. The authors used flawed reasoning in reporting their results. They concluded that the misconceptions students had regarding variables and equations were deep-seated and resistant to change and that a student's ability to write a correct answer to a problem was not an indication of understanding. This idea may be accurate, but what the researchers actually showed instead was that six particular engineering students had deep-seated misconceptions that were not remediated by a quick-fix algorithmic technique, delivered in a traditional, telling format, meant to be completed in one sitting.

Rosnick and Clement's interventions were limited to telling students directions for writing equations rather than creating opportunities for students to build an internal framework of understanding. The researchers missed some important intervention possibilities. They could have used a number of other techniques: manipulatives, translation, or conceptualization—techniques used by many K-12 teachers and recommended by NCTM (1989, 2000). The remediation

could have begun with student conceptualization and continued with helping students create patterns that allowed them to see the contradictions in their thinking process while they formulated new conceptions. The research did not attempt a number of possible approaches and gave up after trying the six interventions listed, assuming all possibilities were exhausted.

Rosnick and Clement showed their own lack of pedagogical content knowledge rather than the students' inabilities to adjust their conceptual understanding. There is no connection of this research to any other research project. Merely four references were cited in this study and three of them were to the authors themselves. There is no evidence of validity or reliability. The researchers also made the error of assuming that six engineering students, chosen from a group of nine students from one class who answered a particular problem incorrectly, could represent all students. Although this study is often cited, its weaknesses make it of little value.

In summary. In learning mathematics students are faced with a number of difficulties as they try to develop fluency. In order to develop that fluency they need skills with language that will enable them to communicate their fluency and understanding of mathematical concepts. Unfortunately that language, which is ill defined and conceivably impossible to define, is not traditionally a focus of the teaching in mathematics. The subject of mathematics, however, requires a precision of expression and a strong mathematics student needs communication skills utilizing the language of mathematics.

Some theorists have seen the language of mathematics as a literal language and advocated the teaching of mathematics as a foreign or second language. Others view the term language of mathematics metaphorically, as is advocated in this review. The interplay between language and thought is so complex that it is impossible to separate them. The language of mathematics is also so different from a natural language, because (1) students are trying to embrace a genre of language in which there are no native speakers, (2) the language requires an additional, natural language, and (3) a student cannot be fluent in this language without being literate.

Finally, students who learn mathematics in English face numerous difficulties in learning this genre of language for a number of reasons. Many common words from English have different meanings in mathematics. There are words and symbols that have multiple meanings within mathematics. Encoding and decoding information in graphs is a significant skill that is difficult to learn. There are syntactic and cultural tenets that students need to assimilate that mathematical literacy so advocated by the *Standards*.

Implications for Research

The language of mathematics has been given lip service from the mathematics education community, but there is little focused support for teachers. There is sometimes a disconnect between teacher language and student language. Students are faced with complex vocabulary and symbolism. They have difficulty taking things they understand in English and translating them to mathematics. In

addition to difficulties with translation of word problem sentences into mathematics and concerns with symbols and polysemy, students in mathematics are faced with difficult syntax and semantics, confusing language constructions, and cultural language traditions. Students confront difficulties as they pursue understanding of mathematics, while filtering information through the complicated structures within mathematical language.

A number of researchers are addressing the issue of communication in mathematics by completing qualitative studies within classrooms. In addition to this research the author suggests research focused directly on language. Three major problems present themselves: (1) Language has been shown to be important in mathematics education, yet a focus on language issues is mathematics classes is rare. (2) It is teachers who are on the front lines of developing a language focus in mathematics classes, yet teachers are not given instruction in language concepts themselves and it appears there is no consensus on how teachers should employ language in order to teach effectively. (3) Students are being expected to assimilate language concepts in mathematics, yet there is not enough convincing research to indicate how students learn this language.

If language is important in mathematics education and yet there is rarely a focus on language in mathematics classes, it would be useful to determine why this situation exists and how it can be alleviated. It may be that there is not enough time in the curriculum. The teachers may feel that because they are not evaluated on language teaching, it is not important to address it. Perhaps the issue is situated

in the ubiquity of standardized testing which does not include items designed to probe the language of mathematics skills that students have acquired. Although professional organizations promote a focus on language concepts in mathematics classes, it may be that teachers do not support these ideas and thus do not feel compelled to teach language. An additional reason that teachers do not focus their mathematics classes on language issues may be that the teachers feel it is not their job to focus on language, that the English teachers have that in their purview. It may also be that teachers, not having had much schooling in language of mathematics, may not feel confident (self-efficacious) to teach the language of mathematics.

As students are expected to learn the language of mathematics, researchers can facilitate the process by identifying which language concepts may be unnecessary in mathematics (rationalizing the denominator, rarely used vocabulary, archaic notation $a : b = c : d$) that we continue to teach. It would also be useful to determine how students assimilate vocabulary. There may be effective techniques to help students develop rich visualizations of mathematical problems. Research on assessment techniques might determine ways to enrich student understanding of language concepts. A focus on the actual language acquisition process could look for how students learn the difference between an elegant presentation and a contrived one, and teachers could benefit from research to determine if it is necessary to address language explicitly or whether students can learn it through classes rich in communication activities.

In addition to needing information on student learning, there is a need for information on how teachers acquire and employ language in their teaching. Fruitful investigation similar to Adler's code-switching investigation could focus on whether teachers employ code-switching within the English language, tending to use informal language at particular times and more formal mathematical language at other particular times. The question of teacher preparation also could be addressed in two ways: whether teachers are adequately prepared or not and whether any preparation should be done in the in-service time rather than in preservice experiences. It may also be that methods classes are lacking in language concepts for mathematics teaching.

Clearly, a number of productive research projects related to the language of mathematics could enhance understanding of language concepts within the mathematics classroom. This area is a rich assortment of researchable problems whose answers could facilitate student learning and capacity to develop mathematical literacy.

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Chapter 3: Development and Initial Validation of the Language of Mathematics Teacher Self-Efficacy Instrument (LoMTES)

Language is at the heart of nearly every human endeavor, and yet somehow there is a cultural belief that there is no language in mathematics. How could one do mathematics without language? How could mathematics be taught without language? When a problem is particularly significant, even the strongest mathematics teacher will talk himself through a solution to the answer. The ability to phrase a mathematical thinking process in language is necessary in becoming skilled in mathematics. The importance of language in teaching mathematics was indicated by Durkin (1991):

> . . . mathematics education begins and proceeds in language, it
> advances and stumbles because of language, and its outcomes are
> often assessed in language. Such observations could be made of
> most school curricula, but the interweaving of mathematics and
> language is particularly intricate and intriguing. (p. 3)

Mathematics insists upon precision of language. It actively discourages ambiguity and vagueness, and even honors and values brevity and elegance of expression. Although language is major tool for teaching mathematics, its importance and impact may have been overlooked because of the possibility that the tool is so transparent that it is invisible. But this transparent language is used incessantly. As teachers pass on mathematical knowledge they need language as a vehicle to transfer it through. Students need communication skills as they recognize meanings and connections, as they demonstrate and share their mathematical understanding, and as they employ mathematics while working with

others. Unfortunately mathematics education has long sidestepped the Language of Mathematics.

Language and Thought

As students weigh complex relations in mathematics, their language facilitates their thinking. Vygotsky (2002) argued that word meaning encompasses both thought and speech and that the two cannot be separated when studying the meaning of words. He referred to an unspecified study showing that speech movements facilitate reasoning and that inner speech is helpful for imprinting and organizing content. Vygotsky also depicted inner speech as a mechanism to facilitate the selection of essential material from the nonessential and as a significant factor in making the transition from thought to external speech; this is not to imply that all thought uses language. It is only to say that much of our thinking is processed through language.

Smith (2002) argued that " . . . language is inseparable from the way we perceive objects, categories, and relationships in the world" (p. 36). The neurophysiologist Calvin (Calvin and Bickerton, 2000) discussed how instantaneously our language is produced in the brain. "If the brain is working in language mode, words are put together in whole phrases and clauses and even sentences before they're sent to the speech organs to be pronounced" (p. 43). Language and thought are clearly inseparable (Coulter, 2001; Vygotsky, 2002), and students of mathematics use mathematical language as they think through complex problems and create an internal framework of understanding.

Communication is Essential in Learning

Vygotsky's (2002) zone of proximal development is built on the concept of communication being essential to learning. Bruner (1984) recounted the results of a study (1931-1932) by Vygotsky and Luria which was suppressed by the Soviet government as being a criticism of peasants. This account shows a very basic example of learning developing through interaction:

> The principal finding of their study—suppressed for years, and finally appearing not in the form in which Vygotsky wrote it, but only in Luria's book (1979) of many years later—was that participation in an agricultural collective had the effect of promoting growth in the thinking of the peasants involved, which took them from childlike, primitive forms of thinking to adult forms of thought. Collective activity, in a word, led peasants along the way to adult thinking. (p. 94)

Language and thinking appear to be inseparable. Von Glasserfeld (1987) theorized that mathematical learning is developed for students as they actively create knowledge while interacting with other students and the teacher. He also maintained that this interaction is developed through language. Students are thus scaffolded to higher understanding. Habermas (as cited in Coulter, 2001) indicated that the only major use of language is to develop understanding. Sfard (1991) and Sfard and Linchevski (1994) indicated the importance of communication as students reify concepts from an abstract understanding to a concrete tool. Additionally, a number of researchers (Ball, 1993; Cobb, Wood, & Yackel, 1992, 1993; Cazden, 2001) showed uses of discussion in developing understanding in mathematics. The culture often views mathematics as separate from the humanistic

action of communication and language, yet clearly this perspective is directly contrary to student needs.

Literacy and Language

In 1987 the National Council of Teachers of Mathematics appointed a board and gave them two charges: (1) to "Create a coherent vision of what it means to be mathematically literate," (NCTM, 1989, P. 1); and (2) to develop a set of standards of how that vision would be reflected in the classroom. From this charge of only two tasks, the NCTM Principles and Standards were developed. The term mathematically literacy was used in a very broad sense, incorporating not only reading and writing, but also the understandings and applications of mathematics. It could be termed fluency. Looking at mathematical literacy as a major goal in mathematics education, one notes that in order to develop literacy in mathematics, it is paramount that a student develop facility with the language of mathematics. This language is a major tool that students need for mathematical literacy (Cooper, 2003; NCTM, 1989; OCED, 2003; Romberg, 2000).

This research project focused on the language of mathematics as a sub-topic of mathematical literacy within English speaking classrooms and thus viewed the language of mathematics from that perspective. The language of mathematics has been viewed in a metaphorical rather than literal meaning of language, specifying a *genre* of English, meaning *kind* or *type*, characterized by a particular form or purpose (Shorter Oxford English Dictionary, 2002). Because the term *language of mathematics* is so common, it remains in this work, but it is

however taken to mean language genre. The following generalization of what is

meant by the language of mathematics was the functional definition in this study:

> In English speaking classrooms the *language of mathematics* is
> used in combination with English and is the union of a subset of
> English and a set of symbolic forms. This language refers to the
> specific words, symbols, graphs, and conventions used when
> discussing or writing about mathematics.

In the definition *words* indicates the specific vocabulary used in mathematics such

as ratio, polynomial, and square root, along with terms that are common in English

but have a different meaning in mathematics—words such as similar, radical,

power, and function. The term *symbols* refers to icons used to encode meaning that

is not usually written in alphabetic form. The term *graphs* refers to diagrams,

graphs, and any visual representation of mathematics or data. The term

conventions refers to the grammatical and cultural traditions of expression

including how mathematical work is represented, accepted forms of argument, and

the expectation of precision. The functional definition is depicted graphically in

figure 3.
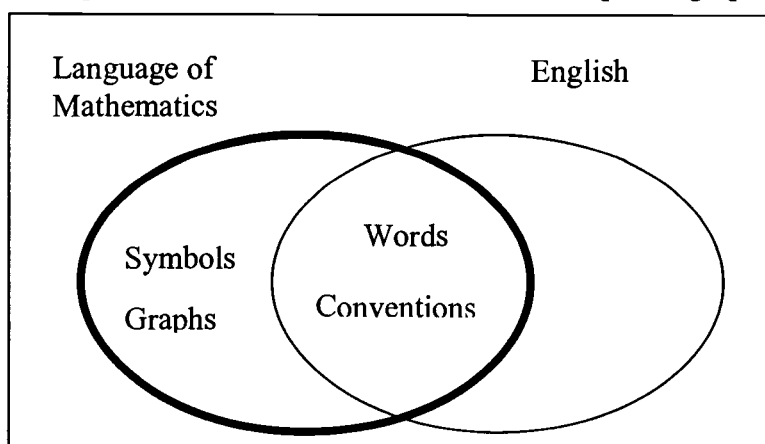
Figure 3: Graphical representation of language of mathematics used in English
speaking classrooms as the union of subsets of English and symbolic forms.

Language of Mathematics Varies from Standard English

Smith (2002) indicated that mathematics requires a language different from our natural language. It is important that students acquire this mathematical language to expand their thinking in mathematics. Think of the inherent confusion presented by the concept of *similar* in mathematics. An English sense of the word *similar* allows a conclusion that all triangles are similar. A mathematical use of the term *similar* is significantly different, allowing only particular combinations of triangles to be labeled similar.

In addition to the concepts veiled in vocabulary, the copious symbols in mathematics can be interpreted as part of mathematical language. Neophytes to mathematics sometimes consider the symbols to be a major block to understanding, but of course the symbols facilitate quicker thinking. An example that might convince those neophytes would be the following problem: *fifty-four minus thirty-nine.* The standard symbols are so ingrained in our thinking process that most adults will automatically convert to the symbolic representation of 54 – 39 before completing a solution. This same simplification of thinking occurs for mathematicians using more complicated symbols, freeing the mathematician to think beyond the boundaries of wordy explanations of relations. Facility with the language of mathematics is indispensable in being able to progress in mathematics and that language, while using many English words, is not the same as English.

Mathematical language is also different from Standard English is its attention to careful precision. A student in mathematics is subjected to a cultural

constraint requiring a more demanding meticulousness than is found in standard communication. The situation was summarized by Dr. Art Clemons at a mathematics department meeting (Southern Oregon University, November 7, 2000): "English is a natural, sloppy, mushy language and our real purpose is to teach students to use a precise language—math . . . . Translating and clarifying language is a beneficial learning experience." This aspect of the language of mathematics may be surprising to novices.

In addition to the specific concerns listed above, students need to learn to translate among a variety of ways to represent relations: verbal, graphic, tabular, and symbolic (Brophy, 1991; Fennema & Franke, 1992). The ability to view relations from a variety of perspectives increases student mathematical power by offering multiple modes of attack. This translation process is part of the language of mathematics that students learn in our classrooms. In learning the language and in learning mathematics, communication is an essential ingredient.

Professional Support for a Focus on Language

We have raised the standards in mathematics education, and out of 10 of them there are four: Reasoning and Proof, Communication, Connections, and Representation that clearly require the use of language—if not traditional, verbal language, then at least the encoding of information into a form from which it can be retrieved. Although they have not clearly defined the term language of mathematics, the National Council of Teachers of Mathematics (NCTM) communication standard explicitly refers to it: NCTM placed an emphasis on

mathematics as communication as early as 1980 in its *Agenda for Action* and in

both of its standards documents (1989, 2000). The communication standard from

the more recent NCTM document is as follows:

> Instructional programs from prekindergarten through grade 12
> should enable all students to—
>
> - Organize and consolidate their mathematical thinking
>   through communication;
> - Communicate their mathematical thinking coherently and
>   clearly to peers, teachers, and others;
> - Analyze and evaluate the mathematical thinking and
>   strategies of others;
> - Use the language of mathematics to express mathematical
>   ideas precisely. (2000, p. 60)

The American Mathematical Association of Two-Year Colleges

(AMATYC) has also developed a standards document (1995). Although their

standards do not use the terms *communication* or *language*, the importance of

language is indicated in the following standard: "Students will acquire the ability

to read, write, listen to, and speak mathematics" (p. 11). This standard implies the

vision of mathematics as a language. The American Mathematical Association's

(MAA) Committee on Undergraduate Program in Mathematics (CUPM, 2003) has

developed the recommendation that students should, "Develop mathematical

thinking and communication skills" (p. 11). The National Research Council in its

1989 report on American mathematics education asserted that students need to be

able to read technical language. It also alluded to language issues when saying,

"Doing mathematics is much like writing. In each, the final product must express

good ideas clearly and correctly" (p. 44). The South African Government (2003)

indicated the following outcome for their mathematics students, "Use

mathematical language to communicate mathematical ideas, concepts,

generalizations and thought processes" (p. 3). Australia's mathematical literacy

program has students participate in the Programme for International Student

Assessment, which assesses students on "the use of mathematical language"

(OECD, 2003, p. 1).The above citations of various organizations and documents

clearly point to a need for students to learn a language of mathematics so that they

can communicate with each other. Communication activities would help students

learn mathematics and to share their thinking in mathematics as they meet the

requirements of a good mathematics education.

The standards are a statement of values, and they indicate a vision of

mathematics (Friel, Bally, Cooney, & Lappan, 1990; Hiebert, 1999; Martinez &

Martinez, 1998; Romberg, 1990). One might argue that mathematics is an isolated

endeavor and a focus on communication merely creates confusion, pulling the

individual away from the necessary, focused, isolated attention that it takes to do

mathematics. Durkin (1991) addressed this concern in the following argument on

mathematical language:

> Why focus on language? The mathematician, after all, works in an
> abstract and highly symbolic subject where precision and
> formalism are critical . . . surely the mathematician transcends the
> vagaries and pitfalls of everyday discourse. (p. 3)

He countered that argument by referring to Pimm (1991, as cited by Durkin) " . . .

such a view disregards two of the essential ingredients of mathematics: people and

communication" (p. 3). Mathematics is not isolated from the humanistic side of

thinking; in fact, it requires the use of language and communication in order to establish concepts and to further significant arguments. Smist and Barkman (1996) completed a study to indicate that forcing the brain to extract patterns is more effective than having teachers identify patterns for students. As students identify these patterns they need mathematical language to describe and justify their choices.

Burton (1998) found that mathematicians indicated an increased need for communication skills and an increased need to work collaboratively. Although her study was focused on a non-randomized subset of mathematicians from only the British Isles, the information is tentative evidence of language needs of individual mathematicians. Burton admitted to surprise in her study of 70 mathematicians when she found merely four (5.7%) of the mathematicians interviewed claimed to do only individual work. Their collection of reasons for working together reads like a support document for cooperative education:

- Talking is a good way to get a problem done,
- It shares the work, you benefit from the experience of others,
- It increases the quantity and quality of ideas,
- You have someone off whom to bounce idea,
- It enhances the range of skills,
- You get into areas that you might not have thought of going into,
- You learn a lot from more senior colleagues,
- Under the pressure of writing up, you mustn't let the others down,
- There is someone to take over when you reach a deadend [sic],
- You share 'the euphoria' with someone,
- You feel less isolated,
- You can benefit from a novice/expert combination. (p. 128)

Note the need for communication indicated by the mathematicians' list. This attitude of mathematicians certainly supports curricular reform encouraging a focus on verbal skills and other mathematical communication methods. It also identifies a perception among mathematicians of the need for skills in mathematical communication, which is affirmed by Burton and Morgan (2000). The mathematics education reform movement calls for greater focus on communication; professional mathematicians concur.

In addition to the needs of practicing mathematicians, there is a need for students to use language as they develop arguments about mathematical relationships. Lampert and Cobb (2003) identified these student needs:

> If students are to engage in mathematical argumentation and produce mathematical evidence, they will need to talk or write in ways that expose their reasoning to one another and to their teacher. These activities are about communication and the use of language. (p. 237)

David Pimm (1991) and Paul Cobb (2000) discussed the value of language in developing understanding in mathematics. Arzarello (1998) indicated "The role of the verbal code . . . is crucial for developing an algebraic way of thinking in solving problems" (p. 249).

Reform efforts identify the need for and value of communication and language skills in mathematics education. Mathematicians need communication and language skills as they work together. Students require language and communication proficiency as they develop the ability to make cogent arguments within the mathematics class.

A Focus on Language is Rare

Although the need for a focus on language in mathematics education has been established, it appears that teachers are not concentrating on language in their teaching (Lampert and Cobb, 2003). In her report comparing traditional and reform middle school level mathematics classrooms, Forman (1996) demonstrated that reform classrooms offered more opportunity for students to develop fluency in mathematical language than traditional classrooms did. She indicated that this was because participation was more broad while in traditional classrooms students were more likely restricted to reading text and listening to the teacher rather than participating actively. Forman further indicated that Stodolsky (as cited in Forman, 1996) found fifth grade students had opportunities to discuss with other students only 1% of the time in mathematics classes as compared to 34% of the time in social studies classes. This imbalance in traditional classrooms makes learning the language of mathematics difficult for students, giving them little opportunity to practice using the complex vocabulary and constructions necessary in mathematics.

A pilot study by this author asked Oregon high school teachers ($n = 116$, randomly chosen from all Oregon high school mathematics teachers) about mathematical language. The three statements "Learning to speak mathematics is an important skill," "Learning to read mathematics is an important skill," and "Learning to write mathematics is an important skill" were included (among other items) and reported on a five-point Likert scale. Each of the items was scored as

strongly agreeing at more than 24 standard deviations above the mean, and reliability established at 95.9% after a Bonferroni adjustment. The project indicated that Oregon high school teachers are likely to view language issues as important. Additional questions asking whether teachers actually help students to learn to speak, read, and write mathematics had mixed results with moderate reliability. Weaknesses in the study prohibit inferences, but it is reasonable to consider that further study could be insightful.

Having established that the language of mathematics is an important topic and yet it is not a consistent focus in mathematics classrooms, one is left with discomfort of the disconnection between theory and practice. If we know that students should learn to use the language of mathematics, yet teachers are not teaching it, then the mathematics education community is faced with a contradiction. This contradiction merits further study in order to determine how to resolve the inconsistency and how to support teachers effectively as they try to implement reform.

The Problem

In summary, the problem is one of contradiction because theory and policy stand in opposition to action. Language is inherent in thinking; improving language can have considerable effect in expanding thinking; and so educational theory posits communication as essential to learning. Argument and proof are at the heart of mathematics and require facility with language, including the language of mathematics. Thus the language of mathematics is an integral part of

mathematics, and of practical importance for communicating within the mathematics classroom.

Accordingly, mathematics education reform calls for a focus on the language of mathematics. Yet, there is no particular consensus on what is meant by language of mathematics, and American mathematics classrooms seldom have communication and language as an important focus. This conflict between what is determined to be valuable for students and what is actually happening in the classroom triggers speculation on the possible causes.

Speculations on Why the Problem Exists

In speculating on the reasons that we have conflicting conditions, there are a number of possibilities. Initial conversations with three veteran teachers indicate that the reason some teachers do not focus on the language of mathematics in their teaching may be:

- There is not enough time in the curriculum.
- Teachers are not evaluated on language.
- Standardized tests capture the focus.

Additional reasons may be:

- Teachers are inattentive to reform specifics.
- Teachers are unsupportive of the Communication Standard.
- Language is taught by other teachers in the school.
- Teachers lack pedagogical content knowledge.
- Teachers believe mathematical language is taught by "osmosis."
- Teachers do not feel confident (self-efficacious) to teach the language of mathematics.

While all of the speculations above merit study, this research project focused on the final speculation, that of mathematics teachers' confidence in teaching the

language of mathematics. This an important problem and merits study in order to resolve a basic contradiction between theory and practice.

Significance of the Problem

This problem is important because of the contradiction between practice and conviction at a basic level of mathematics instruction. If mathematical literacy is a (the?) primary goal in mathematics education, then language of mathematics is principal. The teacher is the foremost designer of day-to-day instruction. If the teacher has personal beliefs or concerns that influence the quality and quantity of classroom efforts to help students to ". . . use the language of mathematics to express mathematical ideas precisely" (NCTM, 2000, p. 60), then those beliefs may be the stumbling block to implementation of reform. If teachers have beliefs that keep them from addressing language, then they will probably not implement the reform topics of communication and language. Determining a way to establish teacher self-efficacy on this topic may make it possible to facilitate the teaching of language within the mathematics classroom. If teachers are not confident that they can initiate students' learning of the language of mathematics, they probably will not attempt to train students in that subject matter. If teachers do not focus on language, then students will have less opportunity to expand their mathematical thinking and their argumentation skills. They will be less able to think/talk their way through a problem, and less able to communicate problematic situations and solutions to others in their chosen vocations. Students need language that enables them to think and communicate mathematically.

In order to assess teacher confidence in their ability to teach the language of mathematics, it is necessary to have a measurement instrument. Because no instrument was available, this research project was developed to create and initially validate such an instrument. The vision of confidence used was that of self-efficacy as developed by Bandura.

Bandura's Social Cognitive Theory

Bandura (1986) posited his theory of triadic reciprocal determination, which asserts that human activity is caused by a complex interaction among behavior, personal factors, and environmental factors. He argues that, ". . . people are both producers and products of social systems" (1997, p. 6) and argued that in studying self-influence we must recognize a reciprocal relationship among three factors. Interpersonal factors influence and are influenced by both behavior and the external environment. Similarly, behavior influences and is influenced by the external environment. It is these three elements of interpersonal factors, behavior, and the external environment that make up Bandura's triadic reciprocal causation. This relationship is graphically depicted in Figure 4.

P $\Rightarrow$ Interpersonal Factors
(cognitive, affective, and biological events)

P

B $\longleftrightarrow$ **E**

B $\Rightarrow$ Behavior   E $\Rightarrow$ External Environment
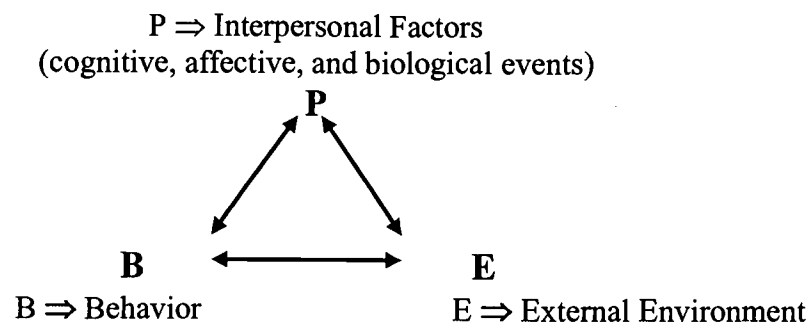
Figure 4. Adaptation of Bandura's triadic reciprocal causation. (1997, p. 6).

As an example of triadic reciprocal causation, consider a teacher instructing students on randomization. He plans his lesson and decides upon a behavior based upon his interpersonal perspective of randomness and previous interpersonal experience in teaching the same and similar topics and also chooses his behavior based upon the external environment such as the text and the given group of students. His behavior is determined both by his interpersonal factors and by external factors.

As this teacher begins to teach randomization, the classroom environment changes and he may choose to change his approach (behavior) or his interpersonal thinking based on his perceptions of the environment and his interpersonal thinking. Bandura's theory shows the three determinants of (a) personal factors, (b) the environment, and (c) behavior are interconnected because each influences the other two. It is this triadic relationship that is at the heart of Bandura's social cognitive theory on which this study is based.

Behavior is influenced then by an individual's self-appraisal among other factors. As Bandura (1997) explained:

> The choice of actions from among alternatives is not completely and involuntarily determined by environmental events. Rather, the making of choices is aided by reflective thought, through which self-influence is largely exercised. People exert some influence over what they do by the alternatives they consider; how they foresee and weigh the visualized outcomes, including their own self-evaluative reactions; and how they appraise their abilities to execute the options they consider. . . . thus, for example, an individual will behave differently in an efficacious frame of mind than in an inefficacious one. But the individual remains the agent of the thought, the effort, and the actions. (p. 7)

Henson (2001) summarized the triadic relationship: "We are products of the dynamic interplay between the external, the internal, and our current and past behavior" (p. 3). Bandura indicated that people both produce and are products of their environment and social systems. They are self-reflective, self-organizing, and self-regulating based on experiences and interactions with their environment. The three aspects of triadic reciprocal causation together account for choices people make regarding their actions.

Pajares (2002b) asserted, "Social cognitive theory is rooted in a view of human agency in which individuals are agents proactively engaged in their own development and can make things happen by their actions" (p. 2). He further explained that it is individual beliefs that modify thoughts, feelings, and actions. In saying: "What people think, believe, and feel affects how they behave" (1986, p. 25), Bandura exemplifies the essence of his theory of perceived self-efficacy.

This perceived self-efficacy is a powerful construct that predicts human behavior. Bandura defines it as " . . . a judgment of one's ability to organize and execute given types of performances" (1997, p. 21). He also asserted "Peoples level of motivation, affective states, and actions are based more on what they believe than on what is objectively true" (p. 2). Pajares elaborated as follows: "People's accomplishments are generally better predicted by their self-efficacy beliefs than by their previous attainments, knowledge, or skills" (2002b, p. 4). The predictive nature of self-efficacy makes knowledge of a teacher's perceived self-efficacy an important measure in predicting their accomplishments in teaching the

language of mathematics. Woolfolk and Hoy (1990) strengthened Pajares' stance by noting, "Teachers' sense of efficacy is a consistent relationship between characteristics of teachers and the behavior or learning of students" (p. 81).

Self-efficacy is task specific. A person might feel particularly efficacious about being able to build a bookcase, but less self-efficacious about being able to sail a boat. Because of the task-specificity of self-efficacy, it cannot be measured in general. It must be measured in reference to a particular activity. It may be that teacher perceived self-efficacy is a factor in the teaching of language in mathematics. If teachers do not feel confident about being able to teach the language of mathematics, they will not likely be able to help students learn it. In order to determine teacher perceived self-efficacy, it is necessary to measure it. Although there are numerous instruments to measure self-efficacy, there is none designed to measure the perceived self-efficacy of mathematics teachers as regards the teaching of the language of mathematics. This research project, which developed an instrument to measure teacher perceived self-efficacy for teaching the language of mathematics, was situated in Bandura's theory.

Teachers' perceived self-efficacy is significant for student learning. Ashton and Webb (1986) studied seasoned teachers teaching students in basic skills classes who were placed there because of severe academic difficulties. The teachers' efficacy beliefs predicted their students' levels of mathematics and language achievement over the course of an academic year. If teachers do not believe that they can teach the language of mathematics, they are probably right.

The Research Project

This research project focused on the fact that there is not an instrument to assess teacher confidence to teach the language of mathematics. An instrument to measure perceived teacher self-efficacy on this topic could allow the mathematics education community to determine teacher confidence, which shapes and predicts teacher ability and student success. Interventions could then be developed that would help low-efficacious teachers develop stronger skills for teaching the language of mathematics.

As Bandura (1997) noted, "Beliefs of personal efficacy constitute the key factor of human agency. If people believe they have no power to produce results they will not attempt to make things happen" (p. 3). If teachers have low perceived self-efficacy regarding the teaching of the language of mathematics, then they would be less likely to teach it and students would thus have less opportunity to learn that portion of the curriculum. Conversely, teachers who have high perceived self-efficacy regarding the teaching of the language of mathematics, then they would be more likely to teach it.

In order to address the question of teacher perceived self-efficacy on this construct of teaching the language of mathematics, a measurement instrument was developed and partially validated. It was designed to measure teacher perceived self-efficacy as regards teaching the language of mathematics. This Language of Mathematics Teacher Efficacy Scale (LoMTES) was designed with the hope that it could eventually enable educators to determine teachers' efficacy levels for

teaching language concepts in mathematics. Because perceived self-efficacy is predictive, the instrument should be helpful in determining which teachers would be successful in teaching this construct and which may be able to benefit from additional support.

Method

The purpose of this study was to make the initial steps in developing and partially validating the Language of Mathematics Teacher Efficacy Scale (LoMTES) to measure elementary teachers' perceived self-efficacy for teaching the language of mathematics. This instrument was developed in five steps (Bandura, 2001; Gall, Borg, & Gall, 1996):

(1) Defining the Factors: open-ended interviews of subjects of interest to determine the difficulties of teaching the language of mathematics;
(2) Building a Prototype: using factors developed in the open-ended interviews to create efficacy measurement items and an original prototype;
(3) Evaluating and Adjusting the Prototype: critical review, field testing, and statistical analysis;
(4) Field Testing the Instrument: instrument testing on a group from the target population;
(5) Establishing Initial Validity: collecting data on reliability and validity.

Target Population

The Language of Mathematics Teacher Efficacy Scale was developed for inservice elementary teachers (grades 1-6). This level was chosen because these teachers set the groundwork for students of mathematics. The elementary teachers' approach to language issues could make a huge difference in the development of mathematical literacy.

Step 1: Defining the Factors

Four university mathematics education professors were asked to suggest elementary teachers who were exemplary mathematics teachers. A pool of eight mathematics educators who were willing to be interviewed were acquired from that list, teachers who were identified most frequently being chosen first. There were four females and four males; all of them had teaching credentials to teach elementary students. They taught in levels from first grade to seventh grade at a total of five different schools.

Each teacher participated in an hour-long one-on-one interview with the researcher. In the open-ended interviews, the researcher attempted to elicit the more difficult aspects of teaching the language of mathematics (Bandura, 2001; Gall, Borg, & Gall, 1996). The interview protocol (Appendix A) was designed to encourage each interviewee to discuss language of mathematics, teaching the language of mathematics, and each of the four sub-topics of the language of mathematics: words, symbols, graphs, and conventions.

At the end of the interview, each participant was given contact information and strongly encouraged to communicate with the researcher later if other thoughts came to mind. Written notes of each interview were formatted and organized. The notes of each of the eight interviews were sent to every participant with two requests: (1) for the participant to indicate approval or alteration of the notes of each personal interview, and (2) for the interviewees to read the comments of other participants and identify any reactions based on the opportunity to see the

comments of the others. All eight participants responded to both of the two requests.

The final collected information from the interviews was color-coded by topic and sorted and synthesized in order to determine significant factors which were used to develop items for the efficacy scale. Bandura emphasized the importance of finding factors that "regulate functioning in the selected domain" (2001, p. 3), and thus particular care was given to determine the factors that the educators believed could make a difference in successfully teaching the language of mathematics.

Step 2: Building a Prototype

Item development. The comments from the open-ended interviews were used to develop an item pool. As suggested by Bandura (2001) an attempt was made to avoid the following problems as much as possible: ambiguous or poorly worded items, technical jargon, and multiple questions within one item. The items were short and explicit (Bandura, 2001). Items were devised to determine judgment of capacity and capacity to create particular outcomes in order to reflect self-efficacy. Because efficacy beliefs vary in generality, strength, and level (Bandura, 2001), the items were varied to reflect those issues.

Each of the four factors (words, symbols, graphs, and conventions) of the functional definition was represented by ten items in the prototype survey (see Appendix B). In addition to the language of mathematics items, the prototype instrument also included demographic items such as age, gender, years of

teaching, levels of teaching, type of teaching endorsement, and number of college

credit hours in mathematics. One item from each of the four factor groups is

shown below:

> Words: I am not able to find lots of places to use mathematical
> words when I teach other subjects.

> Symbols: I can help students understand that mathematical
> symbols are just code—a short cut for writing.

> Graphs: I am able to get students to discuss what a graphs shows
> after it is completed.

> Conventions: I cannot help students to learn to translate between
> words and equations and vice versa.

Each of the items was investigated for readability and clarity and then built

into an initial instrument of 40 items. The prototype instrument included 10 self-

efficacy items for each of the four sub-topics (words, symbols, graphs, and

conventions). Items were ordered on the instrument by random selection. Half of

the items were negatively worded, and these were chosen to be negatively worded

by random selection.

Response Scales. Pajares, Hartley, & Valiante (2001) administered two

versions of a writing self-efficacy scale ($n = 497$) in which the only difference was

in response possibilities. One instrument consisted of 10 items, each measured on

a six point Likert scale. The second contained the same 10 items, but on a 100

point scale separated into 10 intervals. The second approach developed greater

reliability. Items in the instrument being developed in this project were initially

also to be measured on a 100 point scale with responses separated into 10 intervals

from 0 to 100. This scale was deemed more sensitive and reliable than using only a

few response points (Bandura, 2001; Pajares, Hartley, & Valiante, 2001). When

preservice and inservice elementary teachers checked the initial document for face

validity, the general response was confusion because of the 100 point scale. The

teachers were concerned that it was like a 100 point grading scale, where 65 was

failing. Many indicated a reluctance to score themselves below 70 on the

positively worded items. For this reason and as an attempt to maintain the

advantages of Pajares' 10 interval scale, the response values were altered to be 0 to

10 in intervals of size one. Respondents were asked to determine ratings as to how

they viewed themselves at that particular point in time rather than at any future

vision or in any sense of possible eventuality (Bandura, 2001).

Step 3: Evaluating the Prototype

The prototype was evaluated in two steps. First a critical review to assess

the prototype for face and content validity. Second the prototype was field tested

with 43 elementary teachers. Statistical analysis on the results of this

administration led to alterations in the instrument items.

Critical review for face and content validity. Cates (1985)

defined face validity as referring to " . . . whether the instrument appears to be all

right. That is, does it *look right?*" (p. 122). He defined content validity as the " . . .

extent to which an instrument covers content which is appropriate to the research

study, to the samples and population to be studied" (p. 122). Face and content

validity of the prototype were assessed by five professional mathematics educators

and four self-efficacy specialists.. Content validity was determined to see if the test

content paralleled the objectives contained in the factors from the open-ended

interviews and in the four factors of the functional definition (Hopkins, Stanley, &

Hopkins, 1990). Adjustments to the instrument were made based on comments

from the panel of experts. Eighteen individuals from the target group also

inspected the initial instrument, paying careful attention to readability and clarity

(Bandura, 2001).

Field testing. The prototype instrument was administered to a sample (n

$= 43$) from the target population in order to determine if the efficacy items held

sufficient gradations of difficulty and that they avoided ceiling effects (Bandura,

2001). All respondents were certified elementary teachers; they worked in 12

different schools. Procedures outlined by Bandura in his 2001 set of guidelines for

constructing self-efficacy scales were followed In order to minimize response bias:

- The self-efficacy judgments were recorded privately rather than given publicly.
- The questionnaire was identified by code number rather than by name.
- Respondents were informed that their responses would remain confidential and be used only with number codes by the researcher.
- The scale is labeled, with the nondescript title of "Mathematics Teaching Survey" rather than "Self-Efficacy."

Analysis. The prototype was completed by 43 certified teachers (29

females and 14 males) in 12 schools There were 12 prototype surveys which were

not used due to missing data, leaving a pool of 31 prototype surveys to analyze.

An iterative reliability analysis was performed to determine the subset of questions which best independently correlated to each of the four factor groups of words, symbols, graphs, and conventions. Item Total Item Correlations > 0.3 were used to determine the difficulty, validity, and reliability of each item in the prototype (Enochs, Smith, & Huniker, 2000; Robinson, Shaver, & Wrightsman, 1991). In the iterative reliability analysis each factor group was analyzed independently. Test instrument items were iteratively deleted if their total item correlation was less than 0.3. This level was identified as exemplary by Robinson, Shaver, & Wrightsman, (1991).

Table 1: Item-Total Correlations After Prototype Analysis, iteratively deleting items < 0.30. A * indicates a negatively worded item.

Scale	Item	Item-Total Correlation	Scale	Item	Item-Total Correlation
Words	1	0.67	Graphs	9	0.57
	6*	0.44		18	0.50
$\alpha = 0.75$	10*	0.42	$\alpha = 0.82$	22	0.45
	11	0.45		23	0.70
	16*	0.40		26*	0.62
	19*	0.33		29	0.74
	21	0.63		31*	0.58
	30	0.60		35*	0.40
Symbols	2	0.34	Conventions	5	0.43
	8	0.36		7*	0.51
$\alpha = 0.79$	12	0.67	$\alpha = 0.80$	20*	0.59
	13*	0.60		25	0.50
	14	0.50		27*	0.67
	15	0.43		28	0.50
	32*	0.55		39	0.55
	34*	0.53			

The language teaching items were hypothesized to fit in four factor groups: words, symbols, graphs, and conventions. Reliability was determined by

Cronbach's alpha within each of these four constructs (Bandura, 2001;

Charalambos& Philippou, 2003; Fouad, Smith, & Enochs, 1997; Hagedorn &

Enochs, L, & Smith, P. & Huinker, D., 2000; Robinson, Shaver & Wrightsman,

1991), the acceptance level was chosen at 0.70. Adjustments in the items were

made based on the results of the reliability tests.

The statistical analysis deleted nine of the initial items, leaving an

instrument of 31 items (see Appendix B) to be tested in the field on a larger

sample. The 31 remaining items in the survey instrument distributed nearly evenly

among the four factor groups, each group containing eight items except for the

conventions group which maintained seven items. The reliability estimates of each

factor group had an alpha value ranging from 0.75 to 0.82 with a mean of 0.79 as

can be seen in Table 1.

Step 4: Field Testing the Instrument

In order to begin validation so that the scores were meaningful, the final

instrument developed from the successful pilot test items was administered to a

sample of the target subjects. The instrument was formatted two ways: (1) as a

traditional paper document and also (2) in an electronic format which sent results

to the researcher's email account. The paper version was completed by 22

elementary teachers chosen by convenience. The electronic version was emailed to

a stratified random sample of 6000 teachers of grades 1-6. The sample involved

50% suburban, 28% urban, 22% rural teachers from across the United States. The

electronic instrument was returned by 223 respondents in 39 states, yielding a total

of 245 respondents. After 14 surveys were removed due to missing data, the pool of respondents was 231.

Step 5: Establishing Initial Validity

Construct validity. Construct validity was measured by an accumulation of evidence (Brown, 1996). The validation will be an ongoing process, but the initial attempt to begin construct validation originated after the final instrument was administered to the group of the target subjects ($n = 231$). The final instrument was a subset of the prototype instrument; because of this fact, the seven prototype samples that arrived after the prototype analysis was completed were included in the 231 documents that were analyzed as the final instrument. This is reasonable as all items contained the same wording as in the prototype and no additional questions were added to the instrument. The only change was to delete nine of the original items.

All negative items were reversed coded and the reliability results of the data without the original prototype scores was compared with the reliability results of the data with the original prototype scores was compared. An item analysis was conducted on each of the 31 items. The Item Total Item Correlations were used to determine homogeneity (Enochs, L, & Smith, P. & Huinker, D. 2000; Fouad, Smith, & Enochs, 1997). Reliability was established by Cronbach's alpha. These values are summarized in Table 2. Because the prototype results were so similar with the final results in terms of reliability and alpha, it was decided to include the original prototype study results in the final pool of results used for analysis. This

method of including data from the earlier study and using it in the later is sometimes referred to as bootstrapping a sample, which is a statistical iterative sampling technique and could be implemented because the second document was a reduction of the first. This technique increased n from 231 to $n = 266$.

Demographics of the population sample. Of the 266 respondents used in the final analysis, their self-reports indicated that 17% were male and 83% were female. They represented 39 states, and all claimed to be certified to teach at least one grade level in the range of 1-6. The current teaching levels identified were 2.7% teaching pre first grade, 13.6% teaching first grade, 20.4% teaching second grade, 15.8% teaching third grade, 11.3% teaching fourth grade, 19.0% teaching fifth grade, 12.7% teaching sixth grade, and 4.5% teaching above grade 6. The teachers reported teaching experience had a mean of 15 and median of 12.5 years.

Data analysis. Because the original prototype data was so consistent, only one item was deleted from the field tested instrument through Item-Total Item-Correlations, removed for an item-total item-correlation below 0.30, leaving 30 items in the instrument at that time. The remaining 30 items were subjected to frequency analyzed for discrimination by a frequency analysis, and seven of them were removed because they failed to discriminate between respondents as the responses tended to be clustered at the high end of the scale, indicating that respondents tended to answer alike. The removal for non-discrimination resulted in 23 items remaining in the instrument.

Table 2: Reliability established through Item-Total Item Analysis and Cronbach's alpha for the data collected. A * indicates a negatively worded item.

Scale	Final Data with Prototype			Final Data without Prototype		
	Item number new⇒old	Item Total Correlation	Alpha	Item number new⇒old	Item Total Correlation	Alpha
Words	1⇒1	0.34	0.71	1⇒1	0.32	0.73
	2⇒6*	0.40		2⇒6*	0.44	
	3⇒10*	0.32		3⇒10*	0.34	
	4⇒11	0.47		4⇒11	0.47	
	5⇒16*	0.46		5⇒16*	0.46	
	6⇒19*	0.51		6⇒19*	0.56	
	7⇒21	0.38		7⇒21	0.38	
	8⇒30	0.52		8⇒30	0.53	
Symbols	9⇒2	0.44	0.76	9⇒2	0.46	0.77
	11⇒12	0.51		11⇒12	0.61	
	12⇒13*	0.53		12⇒13*	0.54	
	13⇒14	0.47		13⇒14	0.46	
	14⇒15	0.52		14⇒15	0.53	
	15⇒32*	0.54		15⇒32*	0.56	
	16⇒34*	0.39		16⇒34*	0.39	
Graphs	24⇒9	0.53	0.81	24⇒9	0.55	0.81
	25⇒18	0.41		25⇒18	0.42	
	26⇒22	0.57		26⇒22	0.57	
	27⇒23	0.62		27⇒23	0.64	
	28⇒26*	0.48		28⇒26*	0.51	
	29⇒29	0.57		29⇒29	0.60	
	30⇒31*	0.48		30⇒31*	0.46	
	31⇒35*	0.59		31⇒35*	0.59	
Conventions	17⇒5	0.37	0.76	17⇒5	0.49	0.78
	18⇒7*	0.49		18⇒7*	0.49	
	19⇒20*	0.62		19⇒20*	0.65	
	20⇒25	0.45		20⇒25	0.45	
	21⇒27*	0.52		21⇒27*	0.53	
	22⇒28	0.52		22⇒28	0.55	
	23⇒39	0.39		23⇒39	0.39	

Confirmatory factor analysis. In order to determine the strength of each of the four hypothesized factors of words, symbols, graphs, and conventions, the items characterizing each of the factors were subjected to confirmatory factor analysis (Enochs, L, & Smith, P. & Huinker, D., 2000; Krathwohl, 1998). Freed, Ryan, and Hess (1991) have asserted,

> Factor analysis is used to reduce the data from a large set of measures to a smaller set of factors that retain all the basic information of the measures, but does not reflect redundancies found in the original measures. Factor analysis can be used as a confirmatory procedure to verify theoretically or empirically derived constructs.

This view of factor analysis can be coupled with that of Kim and Mueller (1978) who stated,

> Most confirmatory factor analysis can provide self-validating information. If a given factorial hypothesis is supported by the data, we will in general also have greater confidence in the appropriateness of the factor analytic model for the given data. (p. 46).

This analysis was used to determine items that correlated highly outside of their factor group. Seven items that correlated highly outside of their group were deleted from the final instrument, leaving a total of 16 items. The 16 items were balanced with respect to scale as exactly four of the items loaded on each of the four factors of words, symbols, graphs, and conventions. It had been hypothesized that the confirmatory factor analysis would identify that the survey items would load on the four factors from the functional definition: words, symbols, graphs, and conventions. Factor Analysis indicated a general grouping of items in the four categories, but there were some relatively high correlations outside of the groups.

The remaining 16 items were subjected to a confirmatory path analysis in order to confirm the model hypothesized by the functional definition.

Path analysis. LISREL was used to confirm a path analysis model, identifying the strength between the observed and the latent variables. Two items were deleted to strengthen the model, leaving a total of 14 items (see Appendix B) in the instrument, The Chi Square Goodness of Fit (ratio of Chi Square to its degrees of freedom) of 2.7 indicated the model was not rejected, (Mueller, 1996). The LISREL™ model also yielded a Goodness of Fit Index of 0.91, being greater than 0.90. Thus the model can be tentatively accepted. The model diagram is shown in figure 2. the initial model is shown in figure 3, and the final model is shown in figure 4.

Latent and observed variables. The LoMTES instrument measures items as observed variables in order to assess the latent variables which cannot be easily measured. The four latent variables of the LoMTES are words, symbols, graphs, and conventions. Observable variable examples are shown below:

> Words: I am able to get my students to use mathematical vocabulary appropriately.
>
> Symbols: I can teach students to read and to write mathematical symbols.
>
> Graphs: I can get students to recognize a need to create diagrams to model mathematical situations.
>
> Conventions: I am not able to teach appropriate ways for students to justify their thinking in mathematics.

Further examples of observable items designed to measure the latent variables can be found in Appendix B. Figure 5 below shows the initial LISREL™ model which included 16 items. Following that, Figure 6 shows the final LISREL™ model comprising a total of 14 items.
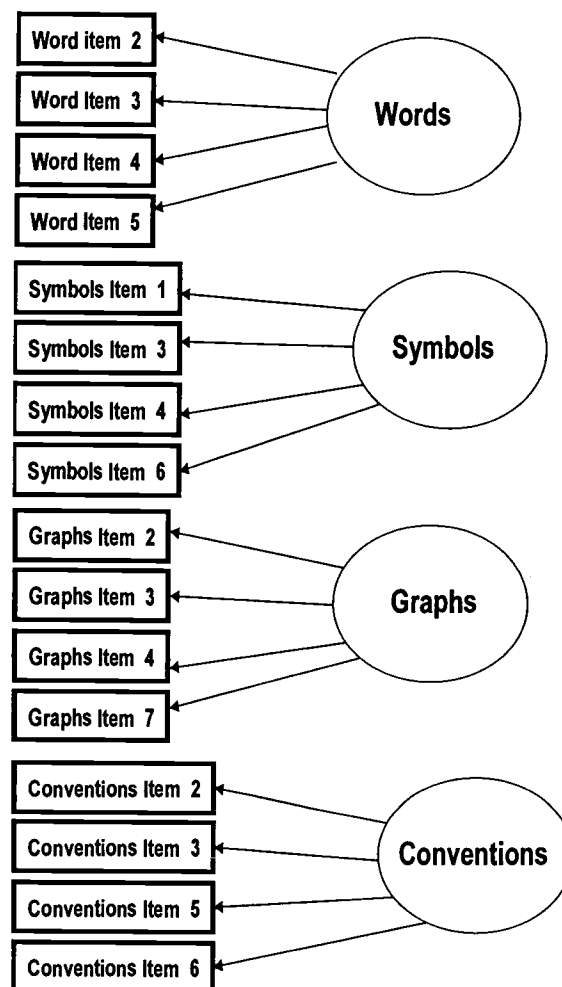
Figure 5: Initial LISREL™ model showing factor loading on four latent variables using 16 observed variables
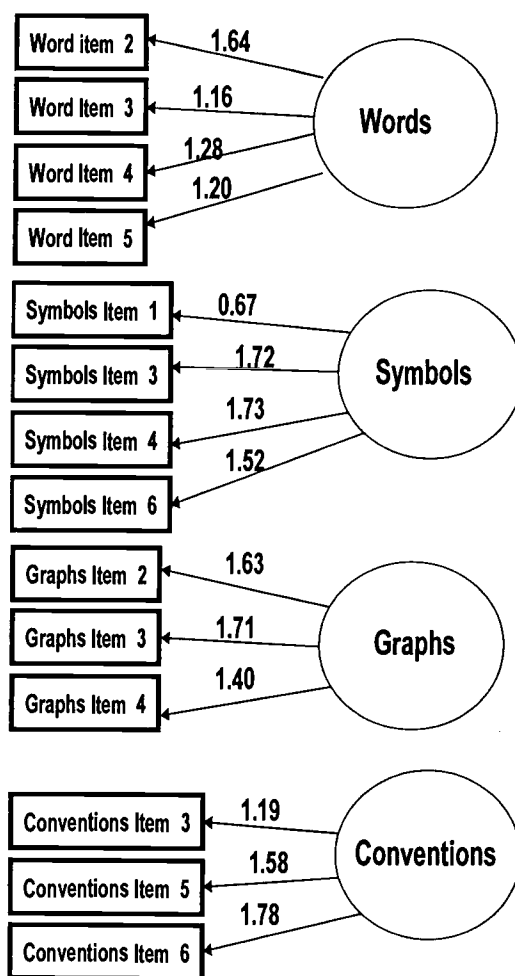
Figure 6: Final LISREL™ model showing factor loading on four latent variable using 14 observed variables

Convergent validity. In the final form of the survey, teachers were

asked questions designed to illicit information that would help to determine

convergent validity, by ascertaining if teachers scoring high on the LoMTES were

likely to be successful in teaching the language of mathematics. To this end, the

responses on the 14 items in the final instrument were analyzed along with the

answers to the convergent validity questions included in the final administration of the instrument. It was assumed that teachers who were more confident about teaching mathematics should score high on the LoMTES.

The first item focused on perceived self-efficacy regarding the teaching of mathematics in general. The first was: *I am one of the most outstanding teachers in my school.* Teachers answered in integer values between 0 and 10 inclusive, where 10 indicated total agreement and 0 stood for no agreement at all. The ANOVA shows that the teachers who scored high on the LoMTES were more likely to consider themselves one of the most outstanding teachers in the school, indicating they are probably more self-efficacious. This was significant at the $p < 0.001$ level.

Figure 3: ANOVA table comparing teachers answers on *I am one of the most outstanding teachers in my school* with teachers' scores on the LoMTES.
Anova: Single Factor

SUMMARY

Groups	Count	Sum	Average	Variance
1	7	582	83.14	114.14
2	12	1150	95.83	302.15
3	9	833	92.56	362.03
4	10	1075	107.50	224.94
5	46	4630	100.65	538.72
6	22	2380	108.18	294.82
7	31	3386	109.23	247.11
8	34	3863	113.62	224.12
9	13	1554	119.54	64.77
10	19	2293	120.68	153.56

ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	16486.18	9	1831.80	6.13	1.43E-07	1.93
Within Groups	57713.74	193	299.03			
Total	74199.92	202				

The second convergent validity item was: *If you had your choice, would you choose to be the one to teach mathematics to your students?* Answers were on a five point scale from definitely no to definitely yes. It was originally hypothesized that because these questions focus on perceived self-efficacy, teachers scoring high on the LoMTES should be more likely to score high on these items and teachers scoring low on the LoMTES should be more likely to score low on these two items. The means of the five groups shows an increasing relationship. The more confidence a teacher had, the more likely the teacher was to have a high score on the LoMTES. The relaionship was significant beyond the $p = 0.001$ level.

Table 4: ANOVA showing the relationship between teacher scores on the LoMTES and their answers to *If you had your choice, would you choose to be the one to teach mathematics to your students?*

Anova: Single Factor

SUMMARY

Groups	Count	Sum	Average	Variance
1	13	1178	90.62	360.92
2	15	1458	97.20	163.17
3	19	1800	94.74	356.76
4	58	6049	104.29	407.75
5	120	13435	111.96	294.88

ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	11056.25	4	2764.06	8.52	2.07E-06	2.41
Within Groups	71369.97	220	324.41			
Total	82426.22	224				

Finally, teachers were asked to complete the Subject Preference Inventory (SPI) developed by Markle (1978). In this instrument teachers are given pairs of

subjects commonly taught in grades 1-6—all 28 pairs that can be made from

language arts, science, health, social studies, art, music, reading, and mathematics.

The teachers choose which subject in the pair they would prefer to teach. In

translating teacher answers to useful information, each respondent was given a

score from 0 to 7 that indicated the number of times they chose mathematics over

another subject. An ANOVA compared the groups designated by the number of

times the teachers chose mathematics. The analysis was based on the teachers'

scores on the LoMTES and showed the groups were significantly different at the p

$= 0.01$ level. Teacher who chose to teach mathematics more frequently were more

likely to score high on the LoMTES, indicating a likely higher sense of self-

efficacy. Bandura (1997) indicated that teachers are more likely to choose to teach

a topic they are successful with than another so it was hypothesized that teachers

who have a higher score would be more effective mathematics teachers and thus

more likely to be effective at teaching the language of mathematics.

Table 5: ANOVA table showing teachers preferring to teach mathematics more often were different from teachers choosing less often as compared by LoMTES scores.

Anova: Single Factor

SUMMARY

Groups	Count	Sum	Average	Variance
7	58	6409	110.5	257.34
6	45	5196	115.47	330.62
5	51	5459	107.04	254.32
4	26	2721	104.65	425.20
3	18	1787	99.28	355.51
0-2	10	963	96.30	162.46

Table 5: ANOVA Table (continued)

ANOVA

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	5923.55	5	1184.71	3.98	0.001812	2.26
Within Groups	60067.22	202	297.36			
Total	65990.76	207				

Teachers who scored high on perceived self-efficacy on the LoMTES differed in distinct ways from teachers who scored low on perceived self-efficacy. The overall results indicated that the instrument identifies teachers high in perceived self-efficacy.

Limitations of the Study

The limitations in the development of the LoMTES are centered around three issues: (1) the sample, (2) the nature of self-reported responses, and (3) the electronic format of the instrument. Each of the three foci will be addressed in turn.

Although an attempt was made to collect a representative sample, the survey was completed voluntarily. It may be that the volunteers are significantly different from the general target population. Also, the study is focused on English, and the language issues may change when a different natural language is the vehicle used. Additionally, the survey was entitled Mathematics Teaching Survey, and it may be that relatively strong math teachers were more likely to respond, with weaker teachers less inclined to complete the instrument.

Because the responses are self-reported, bias may have been introduced. The assumption was made that the respondents told the truth. Without external validation such as student test scores, student interviews, or observations, the responses may be questionable.

Although the use of the electronic format of collecting survey responses facilitated sampling from a geographically large pool of elementary teachers, the collection of data had some unforeseen problems. In the electronic instrument, the data were collected through drop-down menus. If the respondent skipped a question or stopped responding before sending the results, the data were confounded by default responses appearing as actual teacher responses. Significant effort was needed to determine and remove survey responses that were likely in this category. The electronic format introduced another issue. This format required that the teacher have email access. It was initially believed that the electronic format would bias the responses toward younger teachers, but the pool of respondents had a mean of 15 years of teaching experience and a median of 12.5 years.

There are concerns about bias in the sample. Electronic sampling is a new technique and possible bias has not been adequately researched. There is bias in traditional mailed surveys also. It was a matter of concern that the sample could tend to bias toward younger, more computer savvy teachers, but results indicate the average teacher had taught 12-15 years.

Implications for Future Research

This study used the functional definition of language of mathematics put forward in this paper to develop an instrument for measuring teacher perceived self-efficacy of inservice elementary mathematics teachers as regards teaching the language of mathematics. The instrument was developed in five stages by: (1) Defining the Factors, (2) Building a Prototype, (3) Evaluating the Prototype, (4) Field Testing the Instrument, and (5) Establishing Initial Validity

The implications for future research offer a number of possibilities. The research could be focused in three areas: (1) determining why teachers tend not to teach language concepts in mathematics, (2) subjecting the LoMTES to additional validation procedures in order to strengthen the measure, and (3) investigating possible interventions for teachers of mathematics in order to increase student learning of the language of mathematics.

This research project focused on the speculation that the reason a focus on language is rare in mathematics is because teachers are not confident about teaching it. Additional research could be focused on other possible speculations in this area. The problem may be rooted in the lack of class time teachers have. It may be that teachers are unaware or even unsupportive of the NCTM Communication Standard. It could be that teachers do not see language as part of their responsibility in teaching. It also may be that teachers are lacking in the necessary pedagogical content knowledge that would enable them to teach the language of mathematics.

Validation of an instrument is a continual process, and the LoMTES would be strengthened by additional investigation in that area. This study was an initial effort and should be extended. This could be accomplished by many further administrations of the instrument. The groundwork is also laid to examine the differences in responses based on paper or electronic forms of data collection. Explorations to establish a baseline of convergent, divergent, and predictive validity are paramount

The application of the LoMTES could be used to determine teacher self-efficacy as regards the teaching of the language of mathematics and a number of possible interventions could be evaluated. It may be that teachers would improve their perceived self-efficacy by vicarious experience, seeing other teachers as they successfully teach language concepts in mathematics. It might be possible to improve teacher perceived self-efficacy also by social persuasion, having peers encourage each other to success. The key may also be in developing teacher activities that would help them to have success in teaching language which would increase their belief in the possibility of succeeding again.

As Bandura (1997) indicated, "Teachers sense of instructional efficacy partly determines how much their students learn." (p. 248). Self-efficacy is predictive of success in a task and self-efficacy can be influenced by intervention; thus if teachers' efficacy levels for teaching language concepts in mathematics can be determined, it may be possible to increase teacher skill and thus student learning. The Language of Mathematics Teacher Efficacy Scale developed in this

project should be helpful in improving the quality of teaching in the language of

mathematics. This improved teaching should result in students who are more fluent

in mathematical language, creating students with improved mathematical literacy

who are thus more adept at mathematics.

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Chapter 4: General Conclusion

This research project analyzed literature related to the language of mathematics and how it is taught in the classroom. It found that there is no consensus on the meaning of the term *language of mathematics* and developed a functional definition of the term involving four factors of words, symbols, graphs, and conventions. The literature review determined that although there is professional consensus that students need to learn the language of mathematics, it appears that teachers are not teaching it.

Speculating on the contradiction of a need for students to learn language of mathematics and yet a lack of teaching focus, the project explored the possibility of the problem being rooted in teacher perceived self-efficacy. If teachers are not efficacious about being able to teach the language of mathematics, then they will be unlikely to succeed at teaching it. In order to study this construct, a measurement instrument was needed, yet there was none. This project developed an instrument to measure teacher perceived self-efficacy regarding the teaching of the language of mathematics and was termed the Language of Mathematics Teacher Efficacy Scale (LoMTES). The scale was developed in five steps: (1) defining the factors by interviewing exemplary professionals in the field, (2) building a prototype by devising items to reflect not only the factors determined in those interviews but also to reflect the four factors of the functional definition of language of mathematics, (3) evaluating and adjusting the prototype by applying various statistical techniques, (4) field testing the instrument by surveying teachers

certified to teach at the 1-6 levels, and (5) establishing initial validity through statistical analysis.

The result of the survey is an instrument to measure teacher perceived self-efficacy regarding the teaching of the language of mathematics. Further validation would indicate the strength of the instrument. The instrument could be used to determine which teachers could benefit from intervention to increase teacher perceived self-efficacy and ultimately student learning in the language of mathematics.

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APPENDICES

APPENDIX A: Interview Protocol

1. Shaping the Topic

   a. I am interested in language issues in mathematics; so first of all, I would like to know how you look at language in the mathematics class.

   b. How do you address those issues?

   c. Are there specific student problems you see when you teach that?

   d. Additional probing/follow-up questions based on the direction of the teacher's response.

2. Focusing on Words

   a. How do you teach vocabulary?

   b. What if I think of vocabulary in three ways: (1) concrete words, meaning words that have some visual component such as trapezoid, sphere, and denominator; (2) abstract words, such as ratio, prime number, and congruent; and (3) multiple meaning words, meaning words that mean one thing in English and something else in mathematics, such as similar, plane, mean, divide, power, or random. Am I leaving out any particular type if I group them by being concrete, abstract, and of multiple meaning?

   c. Do you approach the teaching of words differently for different types as we identified?

d. How do you approach teaching concrete words?

e. How do you approach teaching abstract words?

f. How do you approach teaching words with multiple meanings?

g. What are the difficulties students face in learning vocabulary?

h. What advice would you give a beginning teacher about teaching words?

i. Do you have any other thoughts or things to add that focus particularly on words?

j. Additional probing/follow-up questions based on the direction of the teacher's response.

3. Focusing on Symbols

a. What sort of symbols do you teach in your curriculum?

b. Can we categorize symbols in any way such as we did with words?

c. Do you have a specific way you usually approach the teaching of new symbols?

d. What are the difficulties students face in learning to use symbols?

e. What advice would you give a beginning teacher about teaching symbols?

f. Do you have any other thoughts or things to add that focus particularly on symbols?

g. Additional probing/follow-up questions based on the direction of the teacher's response.

4. Focusing on Graphs

   a. I don't know if you think of graphs as part of the language issues in mathematics, but I would like to talk about them. What kind of graphing do you teach?

   b. Do you think of graphs in categories?

   c. What are the difficulties students face in learning to create graphs?

   d. What are the important issues in teaching graphing?

   e. What advice would you give a beginning teacher about teaching graphing?

   f. What about diagrams? How do you teach those?

   g. What are the difficulties students face in learning to create diagrams?

   h. What advice would you give a beginning teacher about helping students learn to create diagrams?

   i. Do you have any other thoughts or things to add that focus particularly on diagrams or graphs?

   j. Additional probing/follow-up questions based on the direction of the teacher's response.

5. Focusing on Conventions

   a. In mathematics we have certain conventions in communication such as justifying your reasoning, writing your work in such a way that the reader can follow your path of thinking, identifying what

each variable means, and being very precise. How do you teach those concepts?

b. How do you teach communication concepts to your students?

c. How do you get students to develop precision in their explanations?

d. What do you do to teach students how to combine reasoning and methods in mathematics?

e. What are the difficulties students face in learning these language conventions in mathematics?

f. What advice would you give a beginning teacher about helping students learn the conventions of language in mathematics?

g. Do you have any other thoughts or things to add that focus particularly on conventions?

h. Additional probing/follow-up questions based on the direction of the teacher's response.

6. In Closing

a. We began by talking about language in mathematics. Are there any additional thoughts you have on the topic, either specifically on the four topics we discussed or in general?

b. How important is it to thing about language in teaching mathematics?

7. If you think of anything else, please contact me: _____

8. When I finish the interviews in _____ days, I will put all this information together and send it to you; ;then you can check that I recorded it correctly and add or comment on anything.

Mathematics Teaching Survey (Original Prototype—40 Items)

1) Gender:Male _____ Female _____ 2) Age: _____ 3) Name of School: _____ in _____ (city)
4) Prof. Status: Preservice Teacher _____ Certified Teacher _____ 5) Years of Experience _____ 6) Current grade you teach:_____
7) Grade level(s) you are (will be) certified to teach _____ 8) Subject area(s) you are (will be) certified to teach _____
9) Number of years of high school math_____ 10) Approx. number of math credit hours in college ____ [quarters or sem.?] ____
11) How many terms of calculus have you completed? 0 1 2 3 4 5 6 [quarters or semesters?] _____

	Directions: Circle the appropriate level to which you agree with each of the statements as regards teaching mathematics.
	
	0 1 2 3 4 5 6 7 8 9 10
	do not
	agree
	at all moderately
	        agree agree
	       totally
		NOTE:
		Answer about your <u>abilities</u>, not what you would do or what you actually do in practice.

1.	If I come across a word (such as *similar*) that has one meaning in English and a different one in mathematics, I am able to compare and contrast the English and mathematical meanings of the word as I teach it to my students. 0 1 2 3 4 5 6 7 8 9 10	
2.	I can help students understand that mathematical symbols are just code—a short cut for writing. 0 1 2 3 4 5 6 7 8 9 10	
3.	I am not able to help students use the language of mathematics to develop precise arguments. 0 1 2 3 4 5 6 7 8 9 10	
4.	I am unable to teach my students to write word problems to match computational problems that I give them. 0 1 2 3 4 5 6 7 8 9 10	
5.	In helping students with word problems, I am able to use words that show action in the problem and to act it out as we discuss it (such as "Fred put five figs in a basket" rather than "Fred had five figs"). 0 1 2 3 4 5 6 7 8 9 10	

Mathematics Teaching Survey (Original Prototype—40 Items) (Continued)

	Directions: Circle the appropriate level to which you agree with each of the statements as regards teaching mathematics.	
	0 1 2 3 4 5 6 7 8 9 10 do not moderately agree agree agree totally at all	**NOTE:** **Answer about your <u>abilities</u>, not what you would do or what you actually do in practice.**
6.	I am not able to mix formal and informal language in teaching mathematical terms. (Using informal language when teaching a concept and more formal language after students understand the ideas.) 0 1 2 3 4 5 6 7 8 9 10	
7.	I am not able to get students to appreciate that there are many good ways to solve a mathematical problem. 0 1 2 3 4 5 6 7 8 9 10	
8.	I can teach students to read and to write mathematical symbols. 0 1 2 3 4 5 6 7 8 9 10	
9.	I am able to get students to discuss what a graph shows after it is completed. 0 1 2 3 4 5 6 7 8 9 10	
10	I am unable to use multiple perspectives when I teach students the meaning and use of words in mathematics. 0 1 2 3 4 5 6 7 8 9 10	
11.	I am able to get my students to use mathematical vocabulary appropriately. 0 1 2 3 4 5 6 7 8 9 10	
12.	I can teach my students to connect equations to something concrete instead of just letting equations be abstract. 0 1 2 3 4 5 6 7 8 9 10	
13.	I cannot help students learn to translate between words and equations and vice versa. 0 1 2 3 4 5 6 7 8 9 10	

Mathematics Teaching Survey (Original Prototype—40 Items) (Continued)

	Directions: Circle the appropriate level to which you agree with each of the statements as regards teaching mathematics.

	0 1 2 3 4 5 6 7 8 9 10	NOTE:
	do not agree at all moderately agree agree totally	Answer about your <u>abilities</u>, not what you would do or what you actually do in practice.

14.	I am able to explain mathematical symbols in many ways as I teach them. 0 1 2 3 4 5 6 7 8 9 10
15.	I can teach mathematical symbols in much the same way as I teach mathematical words. 0 1 2 3 4 5 6 7 8 9 10
16.	When I teach mathematical words, I cannot seem to give students lots of examples. 0 1 2 3 4 5 6 7 8 9 10
17.	I am not able to model diagrams such that students can make them too. 0 1 2 3 4 5 6 7 8 9 10
18.	I can get students to recognize a need to create diagrams to model mathematical situations. 0 1 2 3 4 5 6 7 8 9 10
19.	I am not able to check in with students to see if they understand mathematical words when I teach them. 0 1 2 3 4 5 6 7 8 9 10
20.	I am not able to get students to verbally explain and connect ideas in mathematics. 0 1 2 3 4 5 6 7 8 9 10
21.	I can help students as they use a mix of formal and informal language while learning the vocabulary used in mathematics. 0 1 2 3 4 5 6 7 8 9 10
22.	I am able to get my students to do lots of graphs. 0 1 2 3 4 5 6 7 8 9 10

Mathematics Teaching Survey (Original Prototype—40 Items) (Continued)

	Directions: Circle the appropriate level to which you agree with each of the statements as regards teaching mathematics.	NOTE: Answer about your <u>abilities</u>, not what you would do or what you actually do in practice.

	0 1 2 3 4 5 6 7 8 9 10
	do not
	agree
	at all moderately
	                        agree agree
	                                                                    totally	

23.	I can get students to graph very carefully. 0 1 2 3 4 5 6 7 8 9 10	
24.	I am not able to find lots of places to use mathematical words when I teach other subjects. 0 1 2 3 4 5 6 7 8 9 10	
25.	I am able to get students to explain their mathematical thinking. 0 1 2 3 4 5 6 7 8 9 10	
26.	I cannot get students to recognize the value of graphs to display information visually. 0 1 2 3 4 5 6 7 8 9 10	
27.	I am not able to teach appropriate ways for students to justify their thinking in mathematics. 0 1 2 3 4 5 6 7 8 9 10	
28.	I can get my students to discuss mathematics as they work in collaborative groups. 0 1 2 3 4 5 6 7 8 9 10	
29.	I am able to teach students to read graphs after they make them. 0 1 2 3 4 5 6 7 8 9 10	
30.	I am able to explain the words used in mathematics so that they really make sense to students. 0 1 2 3 4 5 6 7 8 9 10	
31.	I am not able to find ideas for data that is interesting for students to graph. 0 1 2 3 4 5 6 7 8 9 10	
32.	I am not able to get my students to translate the equations they write into English. 0 1 2 3 4 5 6 7 8 9 10	

Mathematics Teaching Survey (Original Prototype—40 Items) (Continued)

	Directions: Circle the appropriate level to which you agree with each of the statements as regards teaching mathematics.	NOTE: Answer about your **abilities**, not what you would do or what you actually do in practice.
	0 1 2 3 4 5 6 7 8 9 10 do not agree at all moderately agree agree totally	

33.	I am unable to help students see the similarities and differences between types of graphs. 0 1 2 3 4 5 6 7 8 9 10
34.	I am unable to teach students the understanding of a concept before I introduce the abstract symbols for it. 0 1 2 3 4 5 6 7 8 9 10
35.	I am not able to get students to create diagrams for their word problems. 0 1 2 3 4 5 6 7 8 9 10
36.	I can help students understand mathematics informally before expecting them to have a formal way of looking at mathematics. 0 1 2 3 4 5 6 7 8 9 10
37.	I am not able to teach students to approach a problem in more than one way. 0 1 2 3 4 5 6 7 8 9 10
38.	I am able to reinforce abstract vocabulary frequently so that students can learn the words. 0 1 2 3 4 5 6 7 8 9 10
39.	I am able to listen to students and understand their misconceptions from what they say. 0 1 2 3 4 5 6 7 8 9 10
40.	I am unable to teach students to write equations to represent their thinking. 0 1 2 3 4 5 6 7 8 9 10

Mathematics Teaching Survey (Field Tested Instrument—31 Items)

1) Gender:Male ____ Female ____   2) Age: ____   3) Name of School: _____ in _____ (city)
4) Prof. Status: Preservice Teacher ____ Certified Teacher ____ 5) Years of Experience ____6) Current grade you teach:____
7) Grade level(s) you are (will be) certified to teach _____ 8) Subject area(s) you are (will be) certified to teach _____
9) Number of years of high school math___ 10) Approx. number of math credit hours in college ___ [quarters or sem.?] ___
11) How many terms of calculus have you completed? 0 1 2 3 4 5 6   [quarters or semesters?] _____

	Directions: Circle the appropriate level to which you agree with each of the statements as regards teaching mathematics. 0 1 2 3 4 5 6 7 8 9 10 do not / moderately / agree agree / agree / totally at all	NOTE: Answer about your abilities, not what you would do or what you actually do in practice.
1.	If I come across a word (such as *similar*) that has one meaning in English and a different one in mathematics, I am able to compare and contrast the English and mathematical meanings of the word as I teach it to my students. 0 1 2 3 4 5 6 7 8 9 10	
2.	I can help students understand that mathematical symbols are just code—a short cut for writing. 0 1 2 3 4 5 6 7 8 9 10	
3.	In helping students with word problems, I am able to use words that show action in the problem and to act it out as we discuss it (such as "Fred put five figs in a basket" rather than "Fred had five figs"]. 0 1 2 3 4 5 6 7 8 9 10	
4.	I am not able to mix formal and informal language in teaching mathematical terms. (Using informal language when teaching a concept and more formal language after students understand the ideas.) 0 1 2 3 4 5 6 7 8 9 10	
5.	I am not able to get students to appreciate that there are many good ways to solve a mathematical problem. 0 1 2 3 4 5 6 7 8 9 10	

Mathematics Teaching Survey (Field Tested Instrument—31 Items) (Continued)

	Directions: Circle the appropriate level to which you agree with each of the statements as regards teaching mathematics.	
	0 **1** **2** **3** **4** **5** **6** **7** **8** **9** **10** do not moderately agree agree agree totally at all	**NOTE:** Answer about your <u>abilities</u>, not what you would do or what you actually do in practice.
6.	I can teach students to read and to write mathematical symbols. 0 1 2 3 4 5 6 7 8 9 10	
7.	I am able to get students to discuss what a graph shows after it is completed. 0 1 2 3 4 5 6 7 8 9 10	
8.	I am unable to use multiple perspectives when I teach students the meaning and use of words in mathematics. 0 1 2 3 4 5 6 7 8 9 10	
9.	I am able to get my students to use mathematical vocabulary appropriately. 0 1 2 3 4 5 6 7 8 9 10	
10.	I can teach my students to connect equations to something concrete instead of just letting equations be abstract. 0 1 2 3 4 5 6 7 8 9 10	
11.	I cannot help students learn to translate between words and equations and vice versa. 0 1 2 3 4 5 6 7 8 9 10	
12.	I am able to explain mathematical symbols in many ways as I teach them. 0 1 2 3 4 5 6 7 8 9 10	
13.	I can teach mathematical symbols in much the same way as I teach mathematical words. 0 1 2 3 4 5 6 7 8 9 10	
14.	When I teach mathematical words, I cannot seem to give students lots of examples. 0 1 2 3 4 5 6 7 8 9 10	
15.	I can get students to recognize a need to create diagrams to model mathematical situations. 0 1 2 3 4 5 6 7 8 9 10	

Mathematics Teaching Survey (Field Tested Instrument—31 Items) (Continued)

	Directions: Circle the appropriate level to which you agree with each of the statements as regards teaching mathematics.
	0 1 **2** **3** **4** **5** 6 7 **8** 9 **10** do not agree at all moderately agree agree totally **NOTE:** Answer about your <u>abilities</u>, not what you would do or what you actually do in practice.
16.	I am not able to check in with students to see if they understand mathematical words when I teach them. 0 1 2 3 4 5 6 7 8 9 10
17.	I am not able to get students to verbally explain and connect ideas in mathematics. 0 1 2 3 4 5 6 7 8 9 10
18.	I can help students as they use a mix of formal and informal language while learning the vocabulary used in mathematics. 0 1 2 3 4 5 6 7 8 9 10
19.	I am able to get my students to do lots of graphs. 0 1 2 3 4 5 6 7 8 9 10
20.	I can get students to graph very carefully. 0 1 2 3 4 5 6 7 8 9 10
21.	I am able to get students to explain their mathematical thinking. 0 1 2 3 4 5 6 7 8 9 10
22.	I cannot get students to recognize the value of graphs to display information visually. 0 1 2 3 4 5 6 7 8 9 10
23.	I am not able to teach appropriate ways for students to justify their thinking in mathematics. 0 1 2 3 4 5 6 7 8 9 10
24.	I can get my students to discuss mathematics as they work in collaborative groups. 0 1 2 3 4 5 6 7 8 9 10
25.	I am able to teach students to read graphs after they make them. 0 1 2 3 4 5 6 7 8 9 10

Mathematics Teaching Survey (Field Tested Instrument—31 Items) (Continued)

	Directions: Circle the appropriate level to which you agree with each of the statements as regards teaching mathematics.	NOTE: Answer about your abilities, not what you would do or what you actually do in practice.

	0 1 2 3 4 5 6 7 8 9 10
	do not
	agree
	at all moderately
	                agree agree
	                                                                    totally	

26.	I am able to explain the words used in mathematics so that they really make sense to students. 0 1 2 3 4 5 6 7 8 9 10	
27.	I am not able to find ideas for data that is interesting for students to graph. 0 1 2 3 4 5 6 7 8 9 10	
28.	I am not able to get my students to translate the equations they write into English. 0 1 2 3 4 5 6 7 8 9 10	
29.	I am unable to teach students the understanding of a concept before I introduce the abstract symbols for it. 0 1 2 3 4 5 6 7 8 9 10	
30.	I am not able to get students to create diagrams for their word problems. 0 1 2 3 4 5 6 7 8 9 10	
31.	I am able to listen to students and understand their misconceptions from what they say. 0 1 2 3 4 5 6 7 8 9 10	

Mathematics Teaching Survey (Final Instrument—14 Items)

	Directions: Circle the appropriate level to which you agree with each of the statements as regards teaching mathematics.
	0 1 2 3 4 5 6 7 8 9 10 do not moderately agree agree agree totally at all **NOTE:** Answer about your <u>abilities</u>, not what you would do or what you actually do in practice.

1.	I can help students understand that mathematical symbols are just code—a short cut for writing. 0 1 2 3 4 5 6 7 8 9 10
2.	I am not able to mix formal and informal language in teaching mathematical terms. (Using informal language when teaching a concept and more formal language after students understand the ideas.) 0 1 2 3 4 5 6 7 8 9 10
3.	I am not able to get students to appreciate that there are many good ways to solve a mathematical problem. 0 1 2 3 4 5 6 7 8 9 10
4.	I can teach students to read and to write mathematical symbols. 0 1 2 3 4 5 6 7 8 9 10
5.	I am unable to use multiple perspectives when I teach students the meaning and use of words in mathematics. 0 1 2 3 4 5 6 7 8 9 10
6.	I am able to get my students to use mathematical vocabulary appropriately. 0 1 2 3 4 5 6 7 8 9 10
7.	I can teach my students to connect equations to something concrete instead of just letting equations be abstract. 0 1 2 3· 4 5 6 7 8 9 10
8.	I am able to explain mathematical symbols in many ways as I teach them. 0 1 2 3 4 5 6 7 8 9 10
9.	When I teach mathematical words, I cannot seem to give students lots of examples. 0 1 2 3 4 5 6 7 8 9 10

Mathematics Teaching Survey (Final Instrument—14 Items)

	Directions: Circle the appropriate level to which you agree with each of the statements as regards teaching mathematics.
	0 1 2 3 4 5 6 7 8 9 10 do not moderately agree agree agree totally at all **NOTE:** Answer about your **abilities,** not what you would do or what you actually do in practice.
10.	I am not able to model diagrams such that students can make them too. 0 1 2 3 4 5 6 7 8 9 10
11.	I can get students to recognize a need to create diagrams to model mathematical situations. 0 1 2 3 4 5 6 7 8 9 10
12.	I am able to get my students to do lots of graphs. 0 1 2 3 4 5 6 7 8 9 10
13.	I am able to get students to explain their mathematical thinking. 0 1 2 3 4 5 6 7 8 9 10
14.	I am not able to teach appropriate ways for students to justify their thinking in mathematics. 0 1 2 3 4 5 6 7 8 9 10