

AN ABSTRACT OF THE THESIS OF

Michael Bryan Ek for the degree of Master of Science in Atmospheric Sciences presented on December 10, 1982. Title: The Influence of the Diurnal Variation of Stability on Potential Evaporation.

Abstract approved: _____ **Redacted for Privacy** _____

A method of calculating surface evapotranspiration by separately including the effects of vegetation and atmospheric evaporative demand under the condition of nonlimiting soil moisture is presented. A literature survey is conducted to determine the effects of plants on evapotranspiration.

To represent the atmospheric evaporative demand, the original potential evaporation equation of Penman (1948) is utilized and then modified to include the effect of atmospheric stability using turbulent exchange coefficients formulated by Louis et al. (1982). The original and modified Penman expressions are compared for different asymptotic cases. Using boundary layer data from the Wangara experiment (Clarke et al., 1971), the diurnal variations of the original and modified Penman equations are compared. The daily total potential evaporation using linearized and integrated forms of the original and modified expressions are also compared. Finally, the nonlinear effects of averaging both the original and modified expressions are examined. It is found that including the diurnal variations of stability in the modified expression causes large hourly differences with

the original expression under non-neutral conditions, while daily averages of the two compared fairly well. The diurnal variation of the surface moisture flux appears to be much larger than predicted by the original Penman expression. However, the original Penman expression remains a reasonable estimate of the 24-hour total potential evaporation.

The Influence of the Diurnal Variation
of Stability on Potential Evaporation

by

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A THESIS

Submitted to

Oregon State University

in partial fulfillment of
the requirements for the
degree of

Master of Science

Completed December 10, 1982

Commencement June 1983

APPROVED:

Redacted for Privacy

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Dean of Graduate School

Date thesis is presented December 10, 1982

Typed by Cindy Beck for Michael Bryan Ek

ACKNOWLEDGEMENTS

I am very grateful for all the help Dr. Larry J. Marht gave while I was preparing this thesis. As my major professor, he provided valuable inspiration and guidance. I also appreciate the useful suggestions given by the members of my committee, Drs. Lyle D. Calvin, Hua-Lu Pan, and William McDougal.

Thanks also to Chris Corley, Brenda Mobley, and Paula Jean Ladd for accurately entering the copious amounts of boundary layer data into the computer. In addition, I must commend Robert C. Heald of Willamette Digital for the help he gave through his excellent computer programming skills.

Thanks also must be extended to Cindy Beck, whose remarkable typing skills made the final copy of this thesis possible.

Finally, many warm thanks go to all my family and friends, whose patience and support proved invaluable.

Support in the form of a graduate assistantship was sponsored by the Air Systems Command, United States Air Force Contract #F19628-81-K-0046, a project entitled "A Boundary Layer Parameterization for a General Circulation Model".

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The Influence of the Diurnal Variation
of Stability on Potential Evaporation

1. INTRODUCTION

Evapotranspiration is the flux of water vapor from soil and vegetation to the atmosphere and is of major importance on both the micro and macro scales. In the surface boundary layer it is important in the maintenance of the surface energy balance; in the troposphere it is a principle component of the hydrologic cycle.

Evapotranspiration, ET, is dependent on several factors, namely, the resistance of the soil and plant surfaces to water vapor flux, plant leaf density, water content of the plant canopy, soil moisture content, and atmospheric evaporative demand also called potential evaporation. An expression for ET is

$$ET = E_s + E_t + E_i \quad (1.1)$$

where E_s is soil evaporation, E_t is plant transpiration, and E_i is the reevaporation of water intercepted by the plant canopy. For the case of no interception, (1.1) becomes

$$ET = E_s + E_t \quad (1.2)$$

Although the soil evaporation and plant transpiration are difficult to determine separately, it is useful to segregate the two in order to better describe the physical processes behind them. The soil evaporation may be defined as

$$E_s = C_s F_1(L) F_2(M) E_p \quad (1.3)$$

where C_s is the soil coefficient which accounts for the soil's resistance to water vapor flux, a function of soil type, and is independent

of the soil moisture content and plant leaf density. $F_1(L)$ is a function of plant leaf density and accounts for the reduction of soil evaporation due to shading and wind speed decrease resulting from an overlying plant canopy. $F_2(M)$ accounts for the reduction in soil evaporation due to soil moisture deficit. E_p is the potential evaporation, a reference value equal to the evaporation from a free water surface.

Similarly, the plant transpiration may be defined as

$$E_t = C_p G_1(L) G_2(M) E_p \quad (1.4)$$

where C_p is a plant coefficient which accounts for the plant properties which influence root uptake and stomatal response to water deficit and like C_s is defined to be independent of the soil moisture content and plant leaf density. $G_1(L)$ is a function of the plant leaf density and accounts for the effect of increasing plant transpiration with increasing surface area of the transpiring surface. $G_2(M)$ is analogous to $F_2(M)$; it accounts for the reduction in plant transpiration due to decreasing soil moisture.

All of the terms in (1.3) and (1.4) in one form or another have been researched extensively, although C_s is rarely mentioned in the literature because the resistance of the soil to water vapor flux is generally considered to be a function of soil moisture ($F_2(M)$) and not soil type. Even so, some authors have made references to its existence (Penman, 1948; and Godard, 1964). In nearly all studies of evapotranspiration C_p and $G_1(L)$ are combined as a single term, PC, a plant coefficient in which both the effects of stomatal resistance and plant leaf density are included. Eagleson (1978) chose to keep the two terms apart although he stated that C_p was believed by many to have a value of unity for all species, suggesting that $PC = G_1(L)$. Implicitly assuming that $C_p = 1$, Al-Khafaf *et al.* (1978) determined that $G_1(L)$ ($F_1(L)$) is an exponentially increasing (decreasing) function of increasing plant leaf density. That is to say, increasing plant density causes increased transpiration and decreased direct soil evaporation. Similarly, Monteith (1975), summarizing the work of several

authors, observed that soil evaporation is greatly reduced under a developed plant canopy, implying that $F_1(L)$ is small. Many studies have been completed concerning the reduction of soil evaporation and plant transpiration due to limiting soil moisture (e.g. Ritchie et al., 1972; Hanson, 1976; Al-Khafaf et al., 1978; Meyer, 1980).

The focus of this thesis is on the PC plant coefficient and the potential evaporation, the two components necessary for the calculation of the actual surface evapotranspiration under the condition of non-limiting soil moisture. For the purposes of atmospheric boundary layer research under this condition, it is sometimes convenient to model the surface evapotranspiration by calculating the potential evaporation and then multiplying it by the appropriate plant coefficient for that surface.

. Primarily a literature survey, the section on PC outlines the methods of PC determination and typical seasonal values for different plant species. The primary emphasis of this thesis will be upon the analysis, modification, and testing of the potential evaporation equation first developed by Penman (1948).

2. PLANT COEFFICIENTS

In our terminology, the plant coefficient PC is defined as the ratio of plant transpiration E_t to potential evaporation E_p for the case of non-limiting soil moisture. However, since measurements of evapotranspiration ET generally do not distinguish between E_t and soil evaporation E_s , the PC cannot be determined in this way. But for the case of vegetation with high plant densities, as is the situation found in many agricultural evapotranspiration studies, the plant transpiration is much greater than the soil evaporation. Plant transpiration may then be approximated by evapotranspiration; e.g. Norero (1972) suggested that soil evaporation is nearly zero under a dense stand of alfalfa. In this case the PC may then be approximated as the ratio of ET to E_p . For the case of vegetation with low plant densities, taking the ratio of ET to E_p as the PC would include the undesirable effects of soil evaporation.

2.1 PC DETERMINATION

The PC for a high leaf density plant is obtained by measuring ET and dividing by free water E_p . The methods of measuring or calculating E_p are discussed in Section 3. The ET may be measured directly or indirectly. An example of a direct measurement is the eddy correlation method where the actual vertical flux of water vapor is measured (Hicks, 1973). Using an instrument such as a Lyman-alpha hygrometer, the actual vertical flux may be obtained on an extremely short time scale (Buck, 1976).

Evapotranspiration may be estimated indirectly by means of a budget. The surface energy budget represents one method in which the flux of latent heat LE, and thus ET, is roughly approximated as the net radiation R_n at the surface less the soil S and sensible H heat fluxes. R_n , S, and H are measured; the residual term LE can then be transformed into an equivalent ET (Fritschen, 1965). Unfortunately,

using this method allows all the errors to be collected in the residual term, LE.

The soil moisture budget method involves a hydrological equation which balances ET with precipitation, soil moisture storage change, and runoff at the surface or by infiltration to the ground water beneath (Strahler, 1975). Good estimates of ET using this method can be obtained over a season or month, or even a week, but not for a day or less. Instrumentation used in measuring the change in soil moisture include lysimeters and other gravimetric devices, and the neutron probe. A lysimeter is an isolated block of soil in which soil moisture can be closely monitored. In general, gravimetric methods involve a "before" and "after" soil moisture measurement which is then related to ET. The neutron probe is an instrument that measures neutrons in the soil and relates their number to water content, the change being attributed to ET.

In some cases, analytical expressions of PC are created by using existing data on plant coefficients, plant leaf densities, ET, and E_p (Ritchie and Burnett, 1971; Kristensen, 1974; Hanson, 1976; Al-Khafaf *et al.*, 1978).

The PC may be implicitly included in an equation for the actual ET. Using the work of Penman (1948), Monteith (1965) included a stomatal resistance term in an expression for ET (see (3.9)). Another example is a special form of the Blaney-Criddle method in which mean monthly air temperatures and percent of daylight hours, plus empirical climate and crop growth stage coefficients are combined to give actual ET for a growing season (USDA, 1967).

2.2 PC VALUES

Figure 2.1 is an idealized example of the annual variation of potential evaporation, evapotranspiration, soil evaporation, and plant transpiration. Note that the transpiration curve peaks more sharply in summer than the soil evaporation or potential evaporation curves. This is due to the large increase in the plant coefficient largely associated with plant leaf density. The sum of the E_s and E_t curves

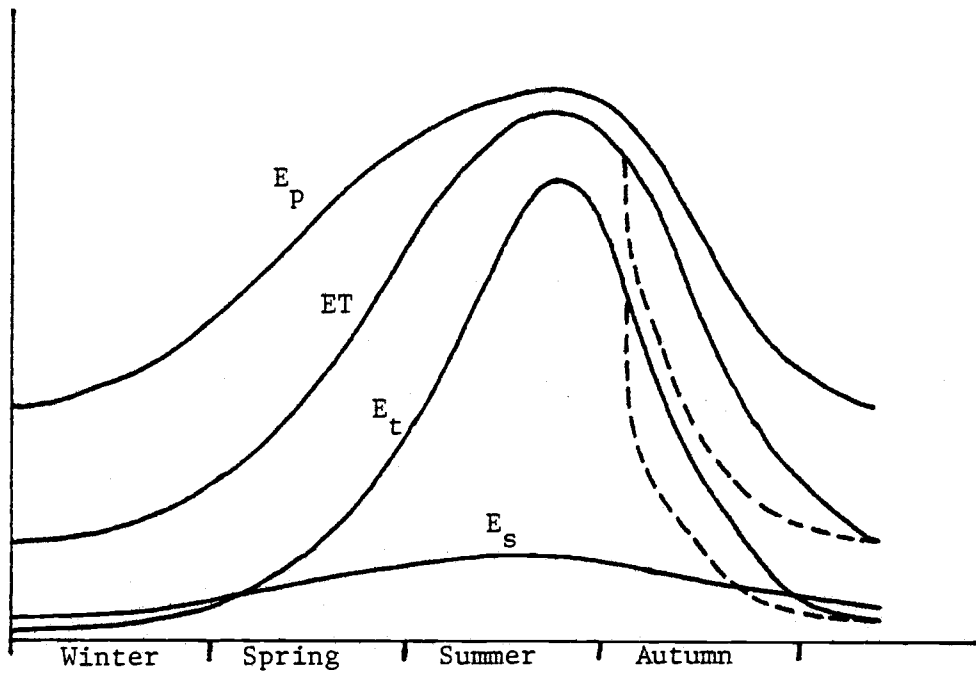


Figure 2.1 Typical examples of annual curves of potential evaporation (E_p), evapotranspiration (ET), plant transpiration (E_t), and soil evaporation (E_s). Curves generalized from Al-Khafaf (1978), Bates (1982), Cuenca (1981), Fritschen (1981), Kanemasu (1976), Nelson (1976), and Russell (1980). Dashed line represents a crop harvest.

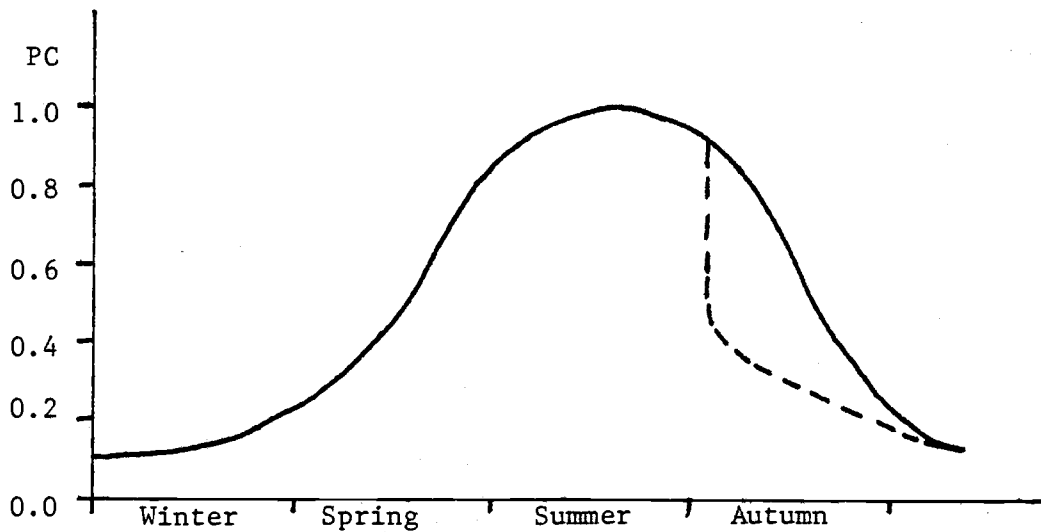


Figure 2.2 Typical example of an annual plant coefficient (PC) curve. Curve generalized from Bates (1982) and Wright (1978).

gives an ET curve which also peaks sharply. In Fig. 2.2 the value of the PC curve peaks in summer when the ratio of E_t to E_p is greatest. Note the decrease of the plant coefficient after the summer peak as the plant approaches maturity. Note also that with a crop harvest (at maturity, dashed line) the PC curve falls sharply.

The plant coefficients listed in Appendix A represent typical values from numerous studies. Listed are plant coefficients for vegetation types for different times of the year at various geographic locations for the condition of non-limiting soil moisture. Unfortunately, most of our knowledge of PC is based primarily on agricultural crops. Plant coefficients are not generally available for natural vegetation because of a lack of studies or appropriate studies.

The diurnal variations of PC are also not generally available because of a lack of study, though it is commonly known that the stomatal resistance of some types of vegetation changes in response to the diurnal variations in temperature and other meteorological variables.

3. POTENTIAL EVAPORATION

In the literature potential evaporation E_p , sometimes called potential evapotranspiration ET_p , is defined and calculated in different ways. Generally it may be defined as the evaporation that would be obtained from a surface of any configuration under a given set of meteorological conditions if there was no moisture deficit at the surface, i.e. a condition of non-limiting water supply. Thus, E_p is largely determined by climatic conditions (Tanner and Fuchs, 1968).

In some studies E_p , or ET_p , is defined as the ET over a short green crop, usually grass, which completely shades the ground, is of uniform height, and is never short of water (Penman, 1956). In other cases E_p is defined as the evaporation from an exposed water surface such as an evaporating pan or an atmometer (a porous ceramic or porous paper evaporating surface connected to a water reservoir) (Rosenberg, 1974). Evaporation from an exposed water surface and the ET over a short green crop do not always agree, especially in dry conditions when pan evaporation exceeds that of short green crop ET due to limiting soil moisture (Dale, 1977). For this reason the potential chosen here is with respect to evaporation from free water surfaces.

3.1 POTENTIAL EVAPORATION EQUATIONS

Potential evaporation E_p , as defined by the pan evaporation, is usually estimated by a variety of models. Most of these depend on empirically derived implicit or explicit constants having to do with climate and occasionally geography. Most are based on aerodynamic or mass transfer relations, energy balance or other thermal considerations, or a combination of these approaches.

Aerodynamic or mass transfer models incorporate the vertical profile of humidity and some function of the wind. This is one of the oldest and most widely used methods of determining E_p . Dalton (1834) used the equation,

$$E_p = C(e_o - e_a) \quad (3.1)$$

where C is an empirically derived function of wind speed, e_o is the vapor pressure at the surface (taken to be the saturation value when calculating E_p), and e_a is the vapor pressure at some standard height above the surface. Dalton's equation, as well as many other aerodynamic equations, is a special case of the more general bulk aerodynamic equation

$$\begin{aligned} E_p &= \rho L_v (\overline{w'q'})_{sfc} = -\rho L_v K_q \frac{\partial \bar{q}}{\partial z} \\ &= \rho L_v C U_z (\bar{q}_{sfc} - \bar{q}_a) \end{aligned} \quad (3.2)$$

where ρ is the mean atmospheric density, K_q and C_q are exchange coefficients for water vapor, normally considered to be a function of stability, U_z is the wind speed at some reference height z , \bar{q} is the mean specific humidity, \bar{q}_a is the mean specific humidity at the same reference height above the surface, and \bar{q}_{sfc} is the mean specific humidity at the surface. \bar{q}_{sfc} is taken to be the saturation value when computing E_p .

Thermally-based equations depend on some function of air temperature for determining E_p . Thornthwaite (1948) assumed an empirical relationship between the mean monthly temperature and E_p using the equation

$$E_p = -1.6 \left(\frac{10\bar{T}}{I} \right)^a \quad (3.3a)$$

$$I = \sum_{i=1}^n \left(\frac{\bar{T}_i}{5} \right)^{1.514} \quad (3.3b)$$

$$\begin{aligned} a &= 6.75 \times 10^{-7} I^3 - 7.71 \times 10^{-5} I^2 + \\ &1.792 \times 10^{-2} I + 0.49239 \end{aligned} \quad (3.3c)$$

where \bar{T} is the mean monthly temperature and I is the sum of monthly heat indices ($i = 1$ to n) (Taylor, 1972). In this case, E_p is

evaluated for a growing season of n months. However, shorter time scale errors will exist because there is no physical basis for this method. For example, the evaporation during a cool, sunny, dry and windy day will be greater than a warm, humid, and calm day even though the Thornthwaite equation would indicate the opposite.

Energy balance models of E_p require measurement of net radiation. The Priestly-Taylor (1972) equation is

$$E_p = \frac{\Delta}{\Delta + \gamma} R_n \quad (3.4)$$

where Δ is the slope of the saturated vapor pressure curve, γ is the psychrometric constant, and R_n is the net radiation. The Jensen-Haise (1963) regression equation has an added function of temperature:

$$E_p = (0.14\bar{T} - 0.37)R_n \quad (3.5)$$

where \bar{T} is the mean air temperature and R_n is net radiation.

Finally, combination equations are derived by combining Dalton's law with the surface energy budget. The most widely used is the Penman or Penman-type equation. The traditional Penman equation (1948) is:

$$E_p = E_r + E_a \quad (3.6)$$

where E_r is the radiation term and E_a is the aerodynamic term defined as

$$E_r = \frac{\Delta}{\Delta + \gamma} R_n \quad (3.7)$$

$$E_a = \frac{\gamma}{\Delta + \gamma} f(u) (e_s - e_a) \quad (3.8)$$

where Δ , γ , and R_n are as defined above and e_s and e_a are the saturation and actual vapor pressures, respectively, of the air at a reference height of 2 meters, and $f(u)$ is an empirically derived

function of the wind speed. E_a is called the aerodynamic term for convenience even though it represents only a part of the bulk aerodynamic water vapor transport. Penman's original equation used R_n expressed in equivalent evaporation of water in millimeters per day, $e_s - e_a$ in mm(Hg), and $f(u)=0.35(1+0.54U_2)$ where U_2 is the average daily wind speed in meters per second at a reference height of two meters. E_p is then the potential evaporation in mm/day.

An actual evapotranspiration equation that may be transformed into potential evaporation is a modified form of the Penman equation given by Montieth (1965),

$$ET = \frac{\Delta(R_n - S) + \rho C_p (e_s - e_a)/r_a}{\Delta + \gamma/(1+r_s/r_a)} \quad (3.9)$$

where r_s is the stomatal resistance of the vegetation, r_a is the aerodynamic resistance and all other symbols are as defined above. When there is no stomatal resistance ($r_s=0$), (3.9) becomes

$$E_p = \frac{\Delta(R_n - S) + \rho C_p (e_s - e_a)/r_a}{\Delta + \gamma} \quad (3.10)$$

The methods of computing evaporation described above are each valuable in their own way. However, they all have drawbacks. The aerodynamic methods depend on the difficult measurement of surface moisture and do not include an energy budget. Thornthwaite's equation is dependent on monthly mean temperatures so that the evaporations calculated can only be useful on a monthly or seasonal basis. The Priestly-Taylor equation is the same as Penman's energy term, but excludes aerodynamic factors. The Jensen-Haise regression by its nature is empirical because it is site specific so that its use is limited.

Though theoretical in nature, the Penman equation is dependent on an empirical wind function in the aerodynamic term which is calibrated for evaporation for a daily total. But overall, because of the better results with combination equations, often those methods are favored over others when calculating E_p (Parmele, 1974). In other

words, the Penman relationship includes constraints due to both energy restrictions and atmospheric turbulent transport and thus includes influences due to radiation, wind, and humidity. Also, using Penman, surface temperature or surface saturation specific humidity is eliminated which in practice is difficult to define over land where the difference between vegetation and soil temperatures can exceed 5°C . Associated errors in the bulk aerodynamic relationship have been shown to be large (Yu, 1977). Nonetheless, to determine evaporation rates on time scales of less than a day, such as one hour, we must include the diurnal variation of the variables used in the usual Penman equation, as well as the diurnal variations in stability and soil heat flux. In particular, the influence of diurnal variations of stability on the surface moisture flux is often large.

3.2 PENMAN DERIVATION

It is useful to derive the Penman equation from first principles to show that the aerodynamic term includes a turbulent exchange coefficient for water vapor. Using Mahrt (1982), we start with the bulk aerodynamic equation for water vapor flux (3.2) using a reference height of 2 meters and that for temperature flux

$$\overline{(w'\theta')}_{\text{sfc}} = C_h U_2 (T_{\text{sfc}} - T_2) \quad (3.11)$$

where C_h is the heat exchange coefficient, T_{sfc} is the surface temperature, and T_2 is the air temperature at a reference height of 2 meters.

Next we introduce the surface energy balance

$$R_n - S = H + LE \quad (3.12a)$$

where R_n is the net radiation and S is the soil heat flux, and

$$H = \rho C_p \overline{(w'\theta')}_{\text{sfc}} \quad (3.12b)$$

$$LE = \rho L_v \overline{(w'q')}_{sfc}. \quad (3.12c)$$

It is important here to note that the soil heat flux S should be included in the surface energy balance. In the original Penman equation evaporation is calculated for an entire day, a time scale over which the sum of S is approximately zero. However, S must be included for hourly calculations of evaporation or the potential evaporation will be overestimated during the day and underestimated at night.

Continuing our derivation, we first substitute (3.11) into (3.12b), then (3.12b) and (3.12c) into (3.12a) which yields

$$R_n - S = \rho C_p C_h U_2 (T_{sfc} - T_2) + \rho L_v \overline{(w'q')}_{sfc}. \quad (3.13)$$

Solving for $T_{sfc} - T_2$ we obtain

$$T_{sfc} - T_2 = \frac{-\rho L_v \overline{(w'q')}_{sfc} + R_n - S}{\rho C_p C_h U_2}. \quad (3.14)$$

Since

$$q = \epsilon e / p \quad (3.15)$$

where ϵ is the ratio of the molecular weight of water to the apparent molecular weight of dry air (≈ 0.622), e is the vapor pressure, and p is the atmospheric pressure, the specific humidity can be expressed as a function of temperature after some mathematical manipulation

$$\begin{aligned} q_{ssfc} - q_s &= \frac{\epsilon}{p} (e_{ssfc} - e_s) \\ &= \frac{\epsilon}{p} \frac{de_s}{dT} (T_{sfc} - T_2) \\ &= \frac{\epsilon}{p} \Delta (T_{sfc} - T_2) \end{aligned} \quad (3.16)$$

where $\Delta = \frac{de_s}{dT} = \frac{L_v e_s}{R_v T_2^2}$ and the subscripts $ssfc$ and s refer to saturation at the surface and 2 meters, respectively. Substituting (3.16) into (3.17) gives

$$\overline{(w'q')}_{sfc} = C_q U_2 \left\{ \frac{\epsilon}{p} \Delta (T_{sfc} - T_2) + (q_s - q_a) \right\}. \quad (3.18)$$

Solving for $T_{sfc} - T_2$ we then get

$$T_{sfc} - T_2 = \frac{-C_q U_2 (q_s - q_a) + \overline{(w'q')}_{sfc}}{C_q U_2 \epsilon \Delta / p}. \quad (3.19)$$

Combining (3.14) and (3.19) to eliminate surface temperature yields

$$\frac{-\rho L_v \overline{(w'q')}_{sfc} + (R_n - S)}{\rho C_p C_h U_2} = \frac{-C_q U_2 (q_s - q_a) + \overline{(w'q')}_{sfc}}{C_q U_2 \epsilon \Delta / p}. \quad (3.20)$$

Solving for the potential evaporation $\rho L_v \overline{(w'q')}_{sfc}$ we obtain

$$\rho L_v \overline{(w'q')}_{sfc} = \frac{\frac{C_q}{C_h} \Delta (R_n - S) + \gamma \rho L_v C_q U_2 (q_s - q_a)}{\Delta \frac{C_q}{C_h} + \gamma} \quad (3.21)$$

where $\gamma = p C_p / \epsilon L_v$ is the psychrometric constant.

According to Dyer and Hicks (1970), the exchange coefficients for heat and moisture are the same over a wide range of stabilities although Verma *et al.* (1978) determined that under advective conditions C_h may exceed C_q . But, for lack of a better method it is assumed that $C_h = C_q$. (3.21) then becomes

$$\rho L_v \overline{(w'q')}_{sfc} = \frac{\Delta (R_n - S) + \gamma \rho L_v C_q U_2 (q_s - q_a)}{\Delta + \gamma} \quad (3.22)$$

or compared with the original Penman equation

$$E_p = E_r + E_a \quad (3.23a)$$

where

$$E_p = \rho L_v \overline{(w'q')}_{sfc} \quad (3.23b)$$

$$E_r = \frac{\Delta}{\Delta + \gamma} (R_n - S) \quad (3.23c)$$

$$E_a = \frac{\gamma}{\Delta + \gamma} \rho L_v C_q U_2 (q_s - q_a). \quad (3.23d)$$

This is the modified Penman equation and shows that the aerodynamic term depends on a turbulent exchange coefficient for water vapor.

This new relationship is similar to other Penman relationships except that here the wind function $f(u)$ is replaced by a dependence on $C_q U_2$.

3.2.1 Original Penman Exchange Coefficient

Using Penman's original aerodynamic term (3.8) and the modified aerodynamic term derived above (3.23d) we can determine the turbulent exchange coefficient implicit in Penman's original equation. We must first convert the units of the original aerodynamic term to the same mks units as the modified aerodynamic term.

In the original aerodynamic expression $e_s - e_a$ is expressed in the units of pressure mm(Hg), the $1 + 0.54U_2$ term is in m/s, and E_a is in mm/day, implying that the constant 0.35 is in mm day⁻¹/(m s⁻¹ mm(Hg)). If the units of $e_s - e_a$ are chosen to be Pa (=kg m⁻¹ s⁻²), then the constant becomes 0.35 mm day⁻¹/(m s⁻¹ mm(Hg)) x 7.5x10⁻³ mm(Hg)/Pa = 2.63x10⁻³ mm day⁻¹/(m s⁻¹ Pa). To convert E_a from mm/day (depth of water evaporated) to kg m⁻² s⁻¹ (mass flux of water per unit area), the constant then becomes 2.63x10⁻³ mm day⁻¹/(m s⁻¹ Pa) x 1m/1000mm x 1day/86400s x 1000kg(H₂O)/1m³(H₂O) = 3.04x10⁻⁸ kg m⁻² s⁻¹/(m s⁻¹ Pa). The expression for E_a becomes

$$E_a = \frac{\gamma}{\Delta + \gamma} 3.04 \times 10^{-8} (1 + 0.54U_2) (e_s - e_a) \quad (3.24)$$

To convert E_a from a mass flux (kg/m²s) to an energy flux (J/m²s) the expression must be multiplied by L_v . And since the vapor

pressure $e=pq/\epsilon$, $e_s - e_a$ can be substituted by $p(q_s - q_a)/\epsilon$. The expression for E_a then becomes

$$E_a = \frac{\gamma}{\Delta + \gamma} 3.04 \times 10^{-8} L_v (1 + 0.54 U_2) \frac{p}{\epsilon} (q_s - q_a) \quad (3.25)$$

where the units of L_v are in J/kg, the constant 3.04×10^{-8} in $\text{kg m}^{-2} \text{s}^{-1}/(\text{m s}^{-1} \text{Pa})$, the U_2 term in m/s, and p in Pa, so E_a is in $\text{J/m}^2 \text{s}$.

Since the original aerodynamic equation is now in mks units with E_a as an energy flux, we may set (3.25) equal to (3.23d).

$$\frac{\gamma}{\Delta + \gamma} 3.04 \times 10^{-8} L_v (1 + 0.54 U_2) \frac{p}{\epsilon} (q_s - q_a) = \frac{\gamma}{\Delta + \gamma} \rho L_v C_{qp} U_2 (q_s - q_a) \quad (3.26)$$

where C_{qp} is the turbulent exchange coefficient implicit in the original Penman aerodynamic equation. Cancelling terms and solving for C_{qp} we obtain

$$C_{qp} = 3.04 \times 10^{-8} \frac{p}{\epsilon \rho} \frac{(1 + 0.54 U_2)}{U_2} \quad (3.27)$$

where the constant 3.04×10^{-8} is in $\text{kg m}^{-2} \text{s}^{-1}/(\text{m s}^{-1} \text{Pa})$, p is in Pa, and ρ is in kg/m^3 , so C_{qp} is dimensionless. Since pressure and density are approximately constant we substitute in their typical values: $p=101325$ Pa and $\rho=1.26$ kg/m^3 so that for a good approximation

$$\begin{aligned} C_{qp} &= 3.9 \times 10^{-3} \frac{(1 + 0.54 U_2)}{U_2} \\ &= \left(\frac{3.9}{U_2} + 2.1 \right) \times 10^{-3}. \end{aligned} \quad (3.28)$$

The apparent turbulent exchange coefficient in Penman's original equation is of course a function of wind speed only. Clearly, when (3.28) is substituted into the modified Penman aerodynamic term (3.23d) we obtain Penman's original aerodynamic term (3.8), after proper constant conversions. Henceforth, when we refer to Penman's

original aerodynamic equation we mean the transformed equation using the Penman apparent turbulent exchange coefficient C_{qp} .

3.2.2 Stability and Roughness Modifications to the Turbulent Exchange Coefficient for Water Vapor

While the Penman equation has proven useful for a wide variety of conditions, this relationship, through Dalton's law, implies a turbulent exchange coefficient based exclusively on wind speed as shown above. Using neutral log theory, Businger (1956) modified the Penman relationship to include a dependence on roughness length which was incorporated later by Tanner and Pelton (1960) and van Bavel (1966).

In addition to surface roughness, Monteith (1965) suggested that his aerodynamic resistance term (see (3.10)) had a dependence on stability through an empirically-derived function of the bulk Richardson number. The Monteith aerodynamic resistance term is inversely proportional to $C_q U^2$ in the modified Penman aerodynamic equation derived earlier. Monteith also noted that a failure to consider the diurnal changes in stability could lead to spurious correlations between wind speed and evaporation, as in Penman's equation where the exchange coefficient is based on wind speed only.

Using earlier work, Fuchs and Tanner (1967) evaluated an exchange coefficient for water vapor based on surface roughness and stability through a modified bulk Richardson number. For the unstable case (daytime) they found that the computed exchange coefficients agreed well with the experimental exchange coefficients based on sensible heat fluxes. Unfortunately, their results were based on data from only one day.

Federer (1970) and Stricker and Brutsaert (1978) used a stability adjustment in the Penman relationship which required the evaluation of an Obukhov length and found that the effect of non-neutral stability cannot be neglected in the calculation of evaporation. In many studies the Penman-Monteith relationship has been modified to include certain aspects of stability using Monin-Obukhov theory for the unstable case (Stewart and Thom, 1973; Thom and Oliver, 1977;

Deheer-Amissah et al., 1981). However, these stability functions based on the Obukhov length and Monin-Obukhov theory cannot be determined from the climatic data used in the Penman equation.

Dooenbos and Pruitt (1977) modified Penman's original equation by multiplying by locally derived empirical coefficients which presumably adjust for the diurnal variation of the mean daily climatic data used in Penman's original equation. These coefficients appear to unknowingly account for the diurnal stability changes.

As indicated above, in many cases the influence of the diurnal variation in stability is found to be large although Bailey and Davis (1981) reported that stability is not important below a height of one meter. However, their work was based on a summertime study done in Ontario, Canada where there was a relatively weak diurnal variation in stability.

Stigter (1980) determined that for the case of atmospheric instability, the original Penman wind equation compares well with the stability dependent wind function (exchange coefficient) derived by Thom and Oliver (1977). If the work by Thom and Oliver is indeed valid, then the original Penman equation is approximately valid for the unstable case.

Although many authors have stated that diurnal changes in stability are important to diurnal variations of surface evaporation, studies using the convenient bulk Richardson number have only been done on the unstable (daytime) case. The influence of stable and neutral, as well as unstable, conditions must be considered in the calculation of the diurnal variation of evaporation.

3.2.3 Dependence of Exchange Coefficient on Stability via Louis

The Penman equation can be modified to explicitly include the influence of atmospheric stability by using the dependence of an exchange coefficient (C_q) on the bulk Richardson number as presented in Louis (1979) and Louis et al. (1982) for both the stable and unstable cases.

The dependence of the exchange coefficient C_{q1} (1 for Louis) on atmospheric stability can be expressed in terms of a Richardson number of the form

$$Ri = \frac{gz(T_z - T_{sfc})}{T_{sfc}U_z^2} \quad (3.29)$$

where g is the acceleration of gravity and z is the reference height of the atmospheric observations taken as 2 meters. T_z is the air temperature at the reference height and T_{sfc} is the temperature corresponding to the temperature of the surface or air at the surface. U_z is the wind speed at the reference height z . Based on previous observations and certain asymptotic constraints, the development in Louis (1979) along with modifications in Louis et al. (1982) lead to the following dependence for the unstable case ($Ri < 0$)

$$C_{q1} = \left(\frac{k}{\ln \frac{z}{z_0}} \right)^2 \left(1 + \frac{15|Ri|}{1 + C|Ri|^{\frac{1}{2}}} \right) \quad (3.30)$$

where z_0 is the roughness length and

$$C = 75 \left(\frac{k}{\ln \frac{z}{z_0}} \right)^2 \left(\frac{z+z_0}{z_0} \right)^{\frac{1}{2}} \quad (3.30a)$$

$$k = 0.4 \quad (3.30b)$$

and for the stable case ($Ri > 0$)

$$C_{q1} = \left(\frac{k}{\ln \frac{z}{z_0}} \right)^2 \left(\frac{1}{1 + 15Ri(1+5Ri)^{\frac{1}{2}}} \right) \quad (3.31)$$

Note that the exchange coefficient increases with increasing instability (increasing negative Richardson number). In the limit of extreme instability ($Ri \rightarrow -\infty$) after substituting for the Richardson

number from (3.29), the wind function becomes

$$C_{q1} U_z = \frac{2}{15} \left(\frac{z_0}{z+z_0} \right)^{1/2} \left(\frac{g}{T_{sfc}} (T_{sfc} - T_z) z \right)^{1/2}. \quad (3.32)$$

Thus the evaporation rate becomes independent of wind speed (free convection) and depends on surface heating through a square root dependence on the surface temperature excess. Note that the moisture flux modelled by the Penman relationship also does not vanish with vanishing wind speed due to the inclusion of a constant term in the wind function.

The above exchange coefficient for heat and water vapor is different than that for momentum with respect to the stability corrections. However, the differences between the neutral values of the exchange coefficients have been neglected in the above formulation based on Louis et al. (1982) in contrast to formulations considered in Louis (1979) and also Stewart and Thom (1973).

In the original Penman equation, stability effects are included indirectly and somewhat awkwardly through the wind function $f(u)$ in the aerodynamic equation (see (3.8)). In Sec. 3.2.1 the effective exchange coefficient C_{qp} implicit in Penman's original aerodynamic equation was derived which is proportional to $f(u)/U_2$. In Fig. 3.1 C_{qp} (3.28) is compared with C_{q1} , the exchange coefficient based on stability created by Louis, (3.30) and (3.31). It is seen that for a wind speed of about 4.9 m s⁻¹ the Penman exchange coefficient is approximately neutral as defined by the Louis exchange coefficient; below (above) this wind speed C_{qp} is unstable (stable). Note that as the wind speed increases, C_{q1} approaches neutrality (2.9×10^{-3}), increasing in the stable region and decreasing in the unstable region. C_{qp} follows the same trend as C_{q1} in the unstable region but an opposite trend in the stable region where C_{qp} asymptotically approaches the value 2.1×10^{-3} .

The influence of stability may be included more directly by replacing C_{qp} with a function of the Richardson number through the use

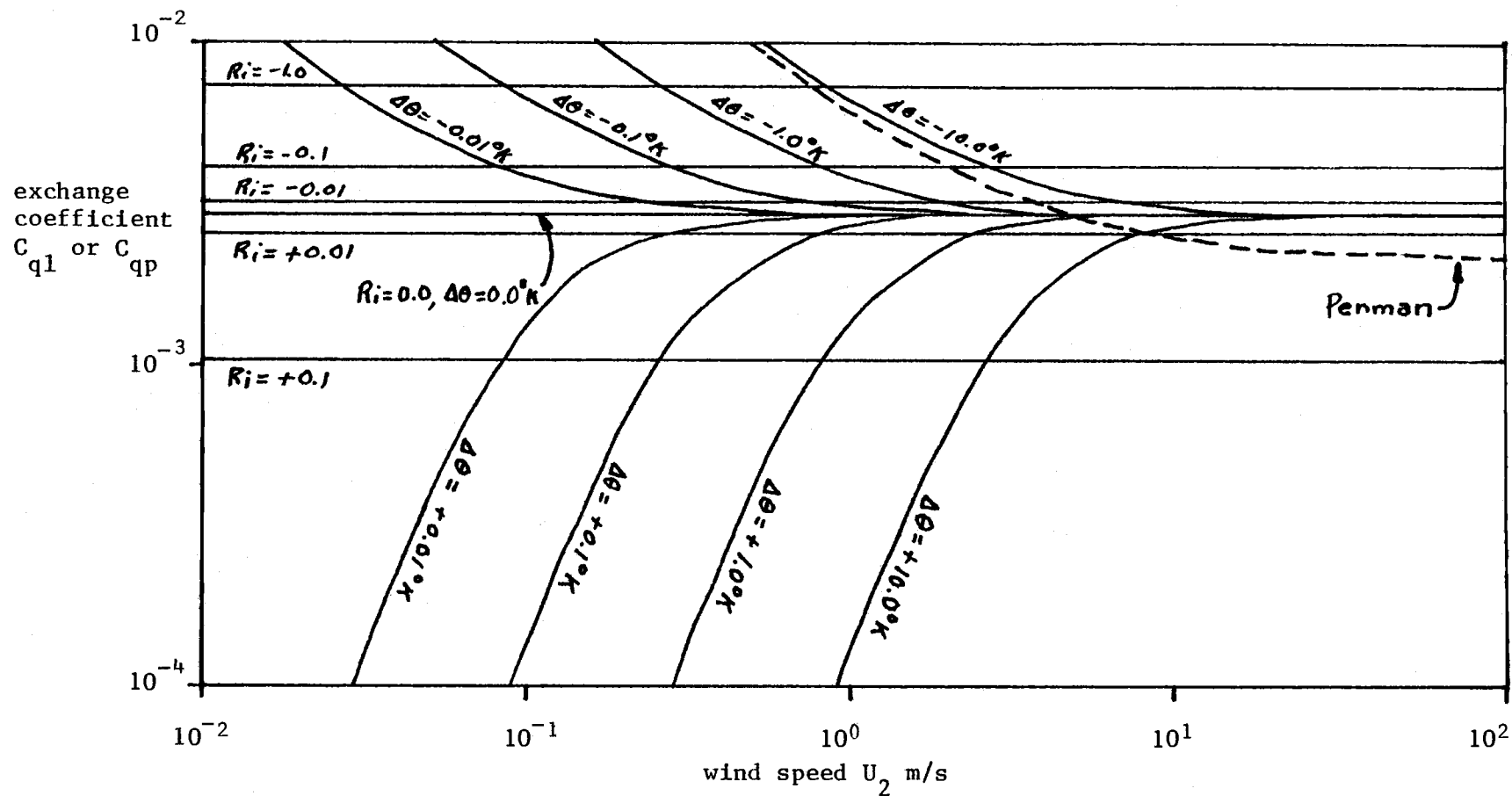


Figure 3.1 Turbulent exchange coefficients C_{q1} formulated by Louis et al. (1982) under different stability conditions and C_{qp} implicit in Penman's original relationship. Ri is the Richardson number from (3.29).

of C_{q1} . Unfortunately, the bulk Richardson number used in determining C_{q1} requires the evaluation of the temperature gradient which is not normally measured in evaporation experiments.

3.2.4 Limiting Cases of E_p

Now that an equation for potential evaporation that includes a dependence on atmospheric stability has been developed, it is interesting to look at the limiting cases for the original and modified Penman expressions. In each case the sign of the individual terms is specified in the expressions and the influence on the potential evaporation is determined.

Examining the individual terms in (3.23c) and 3.23d) it is seen that the terms $\Delta/\Delta+\gamma$, $\gamma/\Delta+\gamma$, ρ , and L_v are always positive so are included only implicitly in the discussion. Net radiation less soil heat flux is positive, zero, or negative; and specific humidity deficit is positive or zero. The wind speed is positive or zero and the exchange coefficients given by Louis and implied through Penman are always positive. The stability dependent wind function $C_{q1} U_2$ and the Penman wind function $C_{qp} U_2$ may also be utilized since when $U_2=0$, $C_{q1} U_2$ is positive for unstable conditions and zero otherwise, and $C_{qp} U_2$ is always positive.

From these individual terms the sign of the modified aerodynamic contribution (positive or zero) is determined. Then the sign (positive, zero, or negative) and the type of latent heat flux LE (evaporation) in each case is determined for the original and modified Penman relationships.

Solutions that do not satisfy the surface energy balance (3.12a) are not allowed. Net radiation less soil heat flux ($R_n - S$) must be balanced by the latent heat flux LE plus the contribution of the sensible heat flux H. Under stable, neutral, and unstable conditions the sensible heat flux is negative, zero, and positive, respectively.

Tables 3.1a and 3.1b show the different limiting cases of the original and modified Penman expressions allowed by the surface energy balance. It is seen that both the original and modified expressions

Table 3.1a Limiting cases of potential evaporation for the original Penman relationship.

Case	Surface Energy Balance $R_n - S = H + LE$						Type of Latent Heat Flux (evaporation)	
	Latent Heat Flux LE					Sensible Heat Flux H		
	Rad. term	aerodynamic term			rad+aero			
	$R_n - S = \text{rad. contrib.}$	U_2	$C_q U_2$	$q_s - q_a$	aero. contrib.			=LE
1a	0	0	>0	0	0	0	0	none
2a	<0	0	>0	0	0	<0	<0	dewfall
3a	>0	0	>0	0	0	>0	>0	free conv.*
4a	0	>0	>0	0	0	0	0	none
5a	<0	>0	>0	0	0	<0	<0	dewfall
6a	>0	>0	>0	0	0	>0	>0	mech. turb.**
7a	0	0	>0	>0	>0	>0	<0	(see text)
8a	<0	0	>0	>0	>0	(1)	<0	(see text)
9a	>0	0	>0	>0	>0	>0	>0	free conv.
10a	0	>0	>0	>0	>0	>0	<0	mech. turb.
11a	<0	>0	>0	>0	>0	(1)	<0	(2)
12a	>0	>0	>0	>0	>0	>0	<0	mech turb (3)
13a	>0	>0	>0	>0	>0	>0	0	mech turb (3)
14a	>0	>0	>0	>0	>0	>0	>0	mech turb (3)

Table 3.1b Limiting cases for the modified Penman relationship.

1b	0	0	0	0	0	0	0	0	none
2b	<0	0	0	0	0	<0	<0	<0	dewfall
3b	>0	0	>0	0	0	>0	>0	>0	free conv.
4b	0	>0	>0	0	0	0	0	0	none
5b	<0	>0	>0	0	0	<0	<0	<0	dewfall
6b	>0	>0	>0	0	0	>0	>0	>0	mech. turb.
7b	0	0	0	>0	0	0	0	0	none
8b	<0	0	0	>0	0	<0	<0	<0	dewfall
9b	>0	0	>0	>0	>0	>0	>0	>0	free conv.
10b	0	>0	>0	>0	>0	>0	<0	<0	mech turb.
11b	<0	>0	>0	>0	>0	(1)	<0	<0	(4)
12b	>0	>0	>0	>0	>0	>0	<0	<0	mech turb (5)
13b	>0	>0	>0	>0	>0	>0	0	0	mech turb (5)
14b	>0	>0	>0	>0	>0	>0	>0	>0	mech turb (5)

* upward water vapor flux due to free convection.

** upward water vapor flux due to mechanical turbulence.

(1) depends on magnitude of each contribution.

(2) dewfall, none, or mechanical turbulence depending on the size of each contribution.

(3) 12a=13a=14a because the original relationship is independent of stability.

(4) dewfall, none, or mech. turb. depending on the size of each contribution as dictated by the magnitude of the stability.

(5) 12b<13b<14b because evaporation under stable conditions is less than evaporation under neutral conditions which is less than evaporation under unstable conditions.

give the same results in most all cases. However, since the Penman wind function has no dependence on stability, for the cases where $U_2=0$ and $q_s - q_a > 0$ (Tab. 3.1a; 7a & 8a) the original Penman aerodynamic term incorrectly gives free convection of water vapor (a positive contribution) under stable and neutral conditions where there should be none. The stability dependent wind function correctly causes the modified aerodynamic equation to give no contribution under these conditions (Tab. 3.1b; 7b & 8b). Similar to the above situation, cases 1a and 2a exhibit the same inconsistency with 1b and 2b in the wind function except here $q_s - q_a = 0$. Then both the original and modified aerodynamic terms are zero.

It is seen that the limiting cases of potential evaporation are more accurately described by the modified Penman relationship because the original relationship fails to recognize the importance of atmospheric stability.

3.3 WANGARA POTENTIAL EVAPORATION ANALYSIS

In this section is the analysis of the diurnal variation of potential evaporation and micrometeorological data from the Wangara experiment conducted in the Southern Hemisphere winter of 1967 from 15 July to 27 August (Clark et al., 1971). Potential evaporation is calculated using the original Penman equation and the modified Penman equation using the exchange coefficient formulation developed by Louis (C_{q1}).

The 40 days of data provided by the Wangara study enable nearly 900 hourly evaporation calculations to be made for comparison. Unfortunately, the temperature and specific humidity at the reference height of two meters needed for the Penman calculation were not taken. These variables were approximated by temperature and humidity data available at a height of approximately 1.2 meters. In the daytime, use of the 1.2 meter temperatures would overestimate the saturation vapor pressure at 2m while the 1.2m specific humidity would underestimate the 2m specific humidity. Therefore, the net error in the estimation of the 2m humidity deficit is smaller than the above

individual errors.

Since the surface temperature was not measured, and cannot be defined over land surfaces, the temperature gradient in the lowest 2 meters could not be determined; consequently the surface-based bulk Richardson number could not be specified. Instead, the surface-based bulk Richardson number is estimated using observations at the one and four meter levels. Because the exchange coefficient is a slowly varying function of the Richardson number, errors in the estimation of the surface-based bulk Richardson number will cause much smaller errors in the exchange coefficient.

3.3.1 Diurnal Variations in the Wangara Experiment

Examined in this section are the diurnal variations of the micro-meteorological variables used to calculate potential evaporations using original or modified expressions, the potential evaporations, and the differences between them. The radiation expression (3.23c) which is common to both the original and modified Penman equations and the modified aerodynamic expression (3.23d) with $C_q = C_{qp}$ (3.28) for the original Penman equation and $C_q = C_{q1}$ (using (3.30) and (3.31)) for the modified Penman equation are used. The units of these equations are J/m^2s but it is of interest here to know the equivalent evaporation in units of mm/hr.

These three equations must be divided by L_v to obtain the mass of water evaporated per unit area per unit time (kg/m^2s). One $kg H_2O = 0.001 m^3 H_2O$ so that $1 kg/m^2s$ corresponds to an equivalent water depth change of $0.001m/s$. Changing to units of mm/hr requires multiplying the previous units by $1000 mm/m$ and by $3600 s/hr$. So, to convert the radiation and aerodynamic terms from J/m^2s to mm/hr they must be multiplied by $(0.001)(1000)(3600)/L_v$. The radiation and aerodynamic expressions become

$$E_r = \frac{3600}{L_v} \frac{\Delta}{\Delta + \gamma} (R_n - S) \quad (3.33)$$

$$E_a = 3600 \rho \frac{\gamma}{\Delta + \gamma} C_q U_2 (q_s - q_a) \quad (3.34)$$

with all units as they were previously except now the evaporations are expressed in mm/hr.

Since ρ and L_v vary by only a small percentage during the day they are considered constant and set equal to 1.275 kg/m^3 and $2.5 \times 10^6 \text{ J/kg}$, respectively. Of the other terms, $\Delta/\Delta+\gamma$, $\gamma/\Delta+\gamma$, and U_2 exhibit marked change while the terms $R_n - S$, C_q , and $q_s - q_a$ show substantial variation throughout the day. Fig. 3.2a shows the diurnal variations of these other terms averaged over 40 days. In Fig. 3.2b the diurnal variations have been scaled to show the relative magnitudes of these variations. Temperature is not a term found in the radiation or aerodynamic expressions but is included implicitly in the Δ and humidity deficit terms.

As expected, the net radiation less soil heat flux varies greatly, peaking at noon and dropping to an almost constant negative value at night. The averaged wind speed peaks in early afternoon and is small and almost constant during the night. The specific humidity deficit is smallest when the air temperature is lowest (just before radiation sunrise) and is highest when the air temperature is highest in the later afternoon. The term $\Delta/\Delta+\gamma$ is primarily dependent on temperature and thus exhibits a diurnal variation. The term $\gamma/\Delta+\gamma$ is merely $1 - \Delta/\Delta+\gamma$ so it is a mirror image of $\Delta/\Delta+\gamma$. Finally C_{q1} shows a very large diurnal variation with maximum (unstable) values occurring in the early afternoon, dropping off rapidly in later afternoon to an almost constant small (stable) value through the night, then increasing rapidly again in late morning. Also shown is the neutral value of the exchange coefficient, $C_{q1n} (=2.91 \times 10^{-3})$ as given by Louis.

The inferred Penman exchange coefficient C_{qp} is also shown in the same figure as C_{q1} . Note that the diurnal variation is just the opposite! The values are higher (unstable as defined by the Louis exchange coefficient) and more slowly varying at night dropping off rapidly to minimum (stable) values from late morning to late afternoon followed by an increase again at night.

The diurnal variation of wind functions $f(u) = C_q U_2$ contained in both the original and modified aerodynamic expressions are shown in Fig. 3.3. It is easy to see that the diurnal variation of the

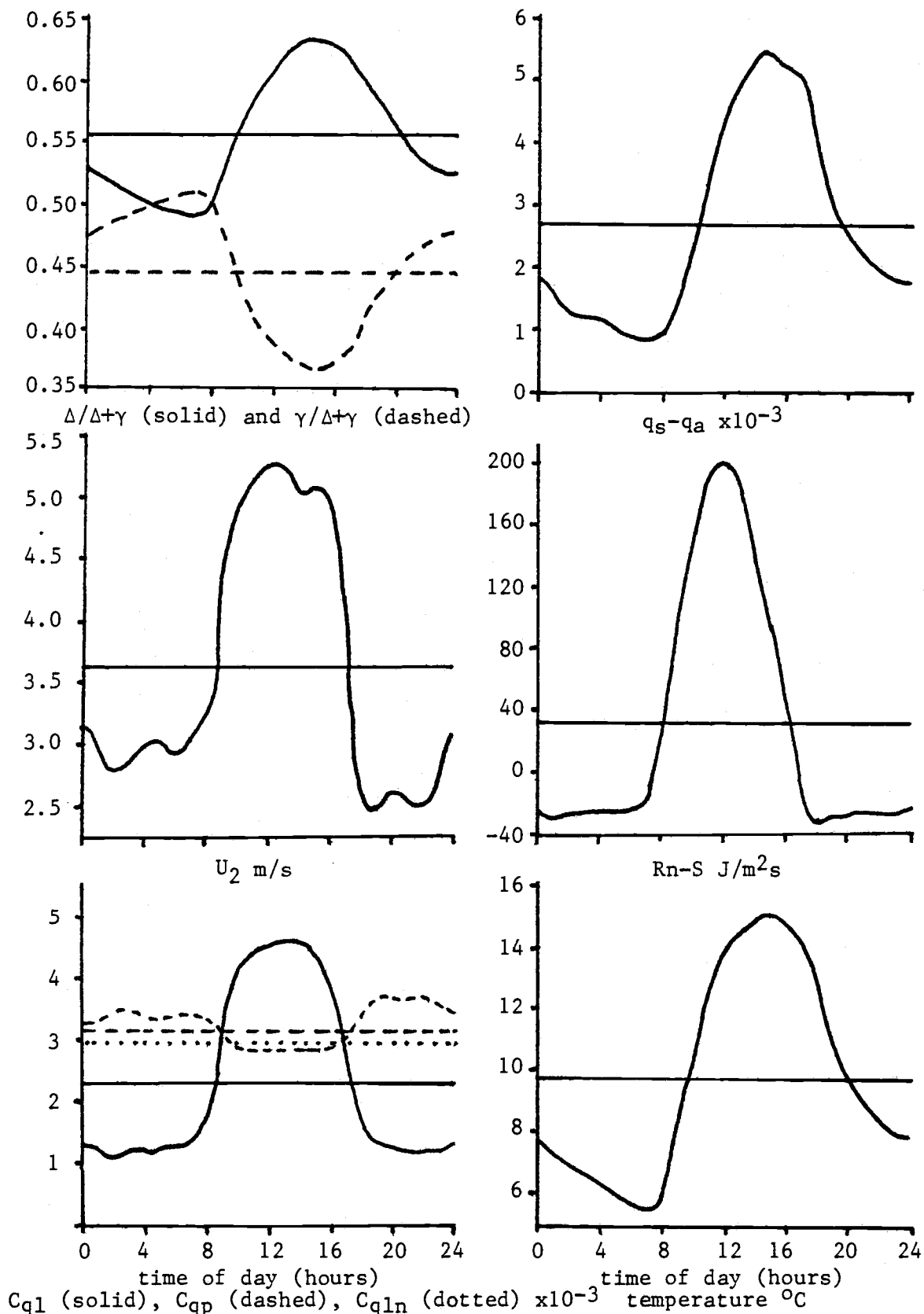


Figure 3.2a Diurnal variations and daily averages of micrometeorological variables used to calculate potential evaporation from Wangara data.

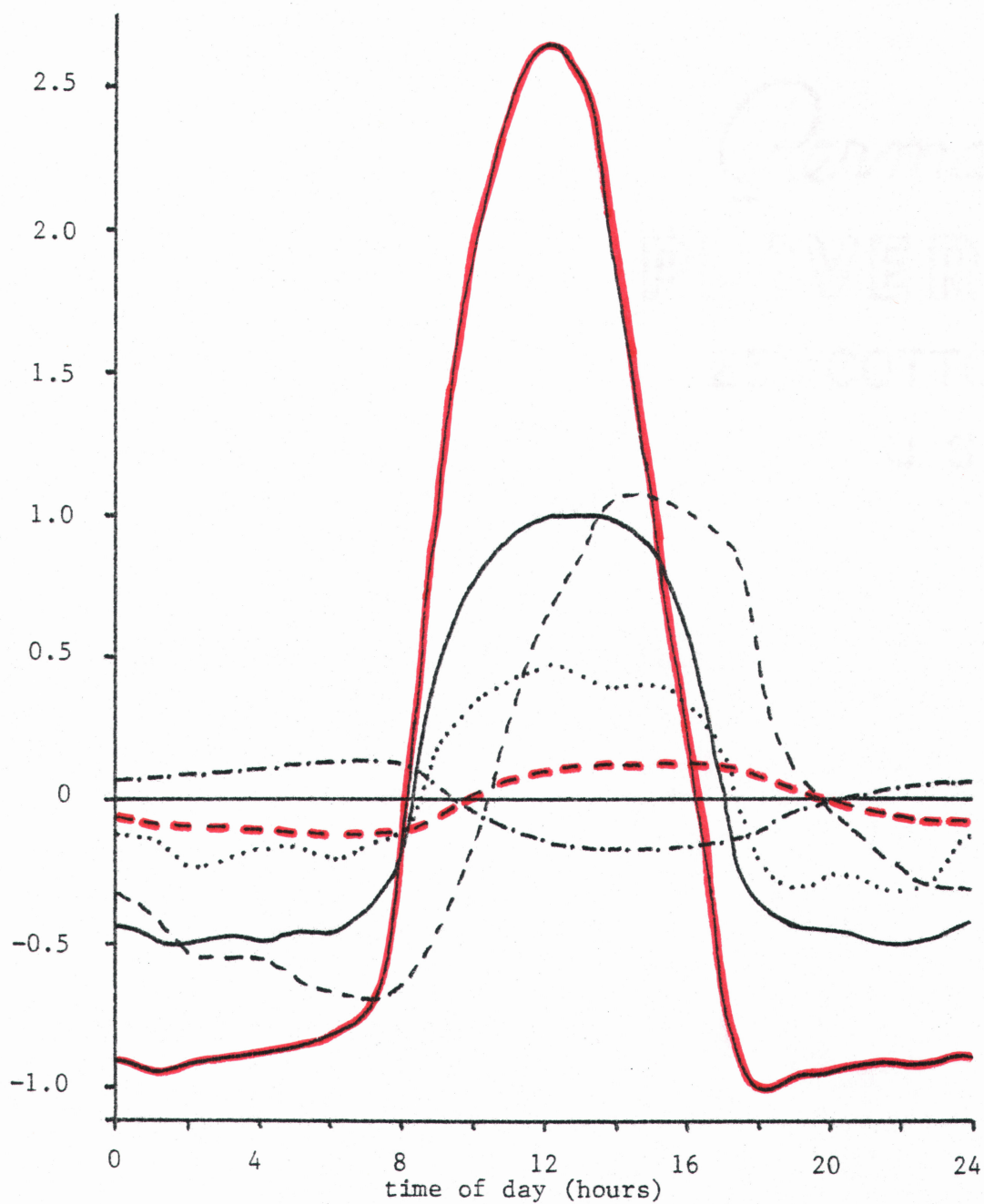


Figure 3.2b Scaled deviations of micrometeorological variables averaged over all the Wangara days. Variables from the radiation term are shown in red; $R_n - S$ (solid) and $\Delta / \Delta + \gamma$ (dashed). Variables from the aerodynamic term are shown in black; C_{q1} (solid), $q_s - q_a$ (dashed), U_2 (dotted), and $\gamma / \Delta + \gamma$ (dash-dot).

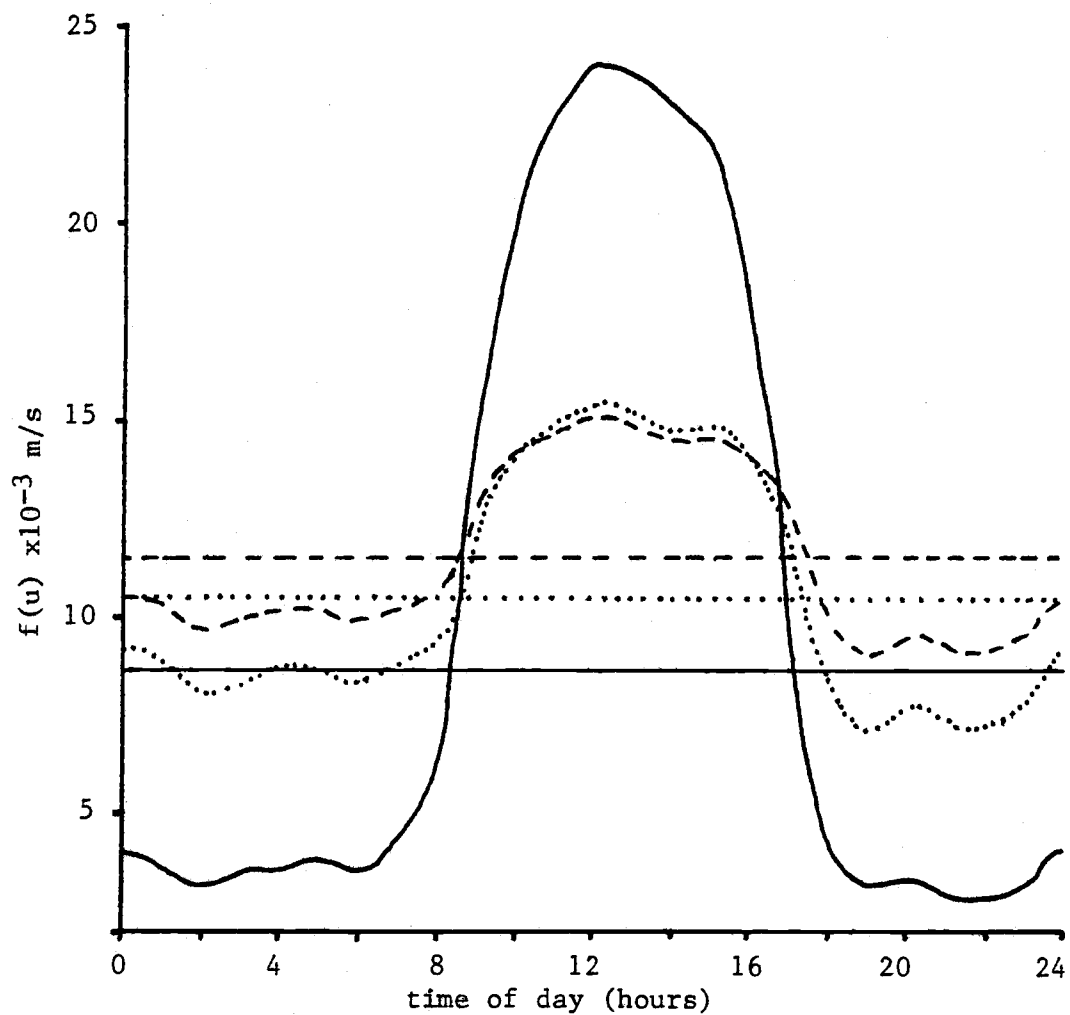


Figure 3.3 Diurnal variations of wind functions $f(u)=C_q U_2$ and their averages as first averaged over all Wangara days; $C_{q1} U_2$ (solid), $C_{qp} U_2$ (dashed), and $C_{q \ln} U_2$ (dotted).

Louis-stability wind function $C_{q1} U_2$ is much greater than the Penman wind functions $C_{qp} U_2$. Note also that the Penman wind function follows a diurnal pattern close to the neutral wind function $C_{q1n} U_2$ (also shown), but with a smaller decrease at night.

Fig. 3.4 shows the diurnal variation of the radiation expression (3.33) and the aerodynamic expressions using C_{qp} , C_{q1} , and C_{q1n} (from 3.34). The radiation expression peaks around noon, whereas the aerodynamic expressions peak in early afternoon. The aerodynamic terms are as large or nearly as large as the radiation term in contrast to the usual summertime case where the radiation term is largest. However, this was a winter-time experiment corresponding to low sun angles and thus smaller radiation values.

As expected, the aerodynamic term using the stability dependent exchange coefficient (C_{q1}) exhibits, on the average, considerably more diurnal variation than the aerodynamic term using the Penman exchange coefficient (C_{qp}) or a constant neutral exchange coefficient (C_{q1n}). In addition, it is noted that the aerodynamic terms using the Penman and constant neutral exchange coefficients are nearly the same. This indicates that the aerodynamic equation using the Penman exchange coefficient approximates the neutral case throughout the day. Finally, all the aerodynamic terms have nearly the same daily average which reflects the fact that on the average the 24-hour integrated potential evaporation may be predicted using the constant neutral exchange coefficient.

The differences between the original Penman aerodynamic equation (using C_{qp}) and the modified Penman aerodynamic equation (using C_{q1}), are shown in Table 3.2. Differences of about 30% of the modified term occur from late morning to late afternoon when the stability effect is greatest. Though small in absolute magnitude compared to daytime differences, nighttime differences are nearly the same as the modified term and accumulate for a longer period. On days where the diurnal variation in stability was large, early afternoon differences were found to be greater than 50% in some cases. The smallest differences occur when the atmosphere is in transition from stability to instability or visa versa, a neutral condition corresponding closely to the

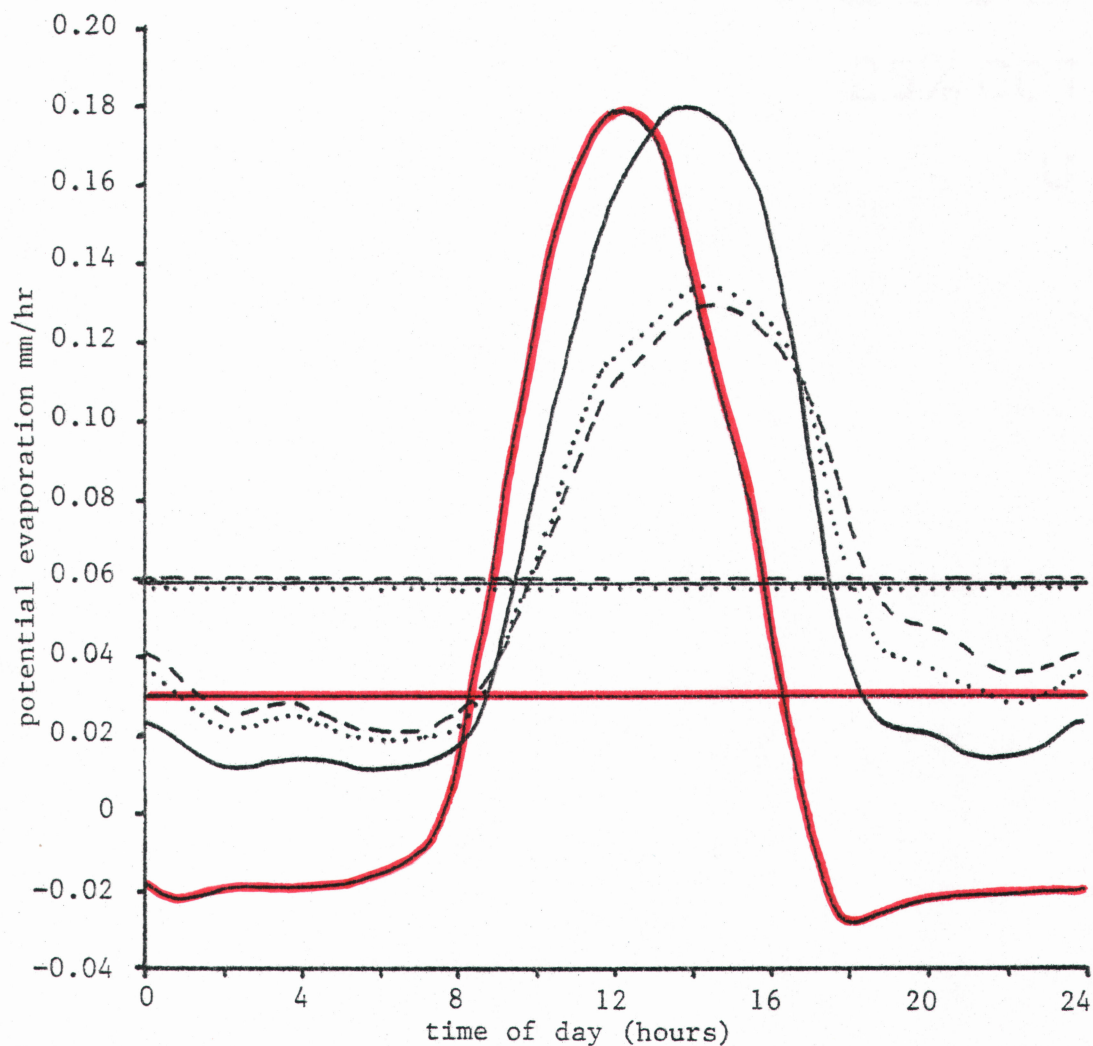


Figure 3.4 Diurnal variations and averages of the radiation term (red) and aerodynamic terms (black) of the Penman potential evaporation equation as averaged over all Wangara days. The three aerodynamic terms shown are the modified (solid), original (dashed), and modified with constant neutral exchange coefficient (dotted).

Table 3.2 Diurnal differences between the original Penman aerodynamic term and the modified term as averaged over all Wangara days, mm/hr. Number days averaged refers to the number of days used to average that particular hour.

Hour	#days avgd	Modified aerodynamic	Original aerodynamic	Absolute difference
1	37	0.01567	0.02847	0.01747
2	34	0.01180	0.02566	0.01386
3	38	0.01356	0.02695	0.01339
4	38	0.01522	0.02834	0.01312
5	39	0.01291	0.02374	0.01083
6	39	0.01074	0.02095	0.01021
7	36	0.01239	0.02102	0.00863
8	40	0.01479	0.02250	0.00771
9	39	0.04322	0.04015	0.00307
10	39	0.08134	0.06253	0.01881
11	39	0.12527	0.09003	0.03524
12	39	0.16101	0.11259	0.04843
13	35	0.17186	0.11926	0.05261
14	33	0.17969	0.12924	0.05045
15	36	0.17160	0.12834	0.04361
16	37	0.14504	0.12113	0.02391
17	37	0.09491	0.10824	0.01333
18	36	0.03365	0.07036	0.03671
19	38	0.01968	0.04862	0.02894
20	38	0.01950	0.04688	0.02738
21	36	0.01590	0.04059	0.02469
22	37	0.01415	0.03641	0.02226
23	37	0.01615	0.03540	0.01925
24	38	0.02282	0.04006	0.01724
Avg.	37.3	0.05846	0.05905	0.02309

original Penman aerodynamic term. Approximating the modified Penman aerodynamic evaporation by the original Penman aerodynamic evaporation on an hourly basis gives a daily average absolute difference of 40% of the modified term as averaged over all 40 days. On the days with a large diurnal variation in stability, it was found that this absolute error could exceed 60%. Summertime errors would be still larger because of the even greater diurnal variations in stability.

3.3.2 Influence of Nonlinearities on Daily Potential Evaporation

In most applications the daily total evaporation is estimated by using daily averaged variables. The original Penman equation is sometimes employed to estimate the 24-hour evaporation using daily averaged wind speed, net radiation, humidity deficit, and temperature with the neglect of the influence of atmospheric stability. Since the Penman relationship is nonlinear, significant errors could occur as is implied by the calculations of Pruitt and Doorenbos (1977). They found systematic errors of 50% or more in the Penman relationship based on daily averaged variables. Corrections must then be applied when using daily averaged variables (Pruitt and Doorenbos, 1977; Stigter, 1980).

These corrections are necessary because of the day to day variations of atmospheric stability (as shown in Sec. 3.3.1) and also because of the nonlinear interaction between diurnal variations of the exchange coefficient, wind, specific humidity deficit, and the $\gamma/\Delta+\gamma$ term in the aerodynamic equations and the diurnal variations of net radiation less soil heat flux and the $\Delta/\Delta+\gamma$ term in the radiation equation.

3.3.2a Daily Evaporation Comparisons

The differences associated with calculating daily potential evaporation from daily averaged (micrometeorological) input values are now considered and compared with the more correct procedure of summing hourly potential evaporation values for a daily total. The former case

is called the linearized potential evaporation and the latter daily integrated potential evaporation. Both the original and modified Penman equations are evaluated. In the original Penman equation (3.6), daily evaporation is usually calculated using the daily averaged input values of net radiation, temperature, wind speed, and specific humidity deficit. Soil heat flux and atmospheric stability have been added for hourly calculations and are kept in for the daily linearized calculations, although the soil heat flux is approximately zero when integrated over the day.

When comparing two quantities, the absolute difference rather than the average difference is more useful. The average difference includes sign, which, when averaging over many days could sum to near zero even though differences on individual days may be generally large.

Table 3.3 summarizes the linearized daily potential evaporation differences in mm/hr between different expressions averaged over the 40 days of the Wangara experiment. For individual daily differences see Appendix B. The linearized aerodynamic term of the original Penman is within 5% of the averaged value of the daily integrated original Penman. Also, the absolute difference between them is only about 6%. Even the maximum difference of the 40 individual days is only 11% of the integrated term. This would indicate that the linearization process by itself is not an important factor.

For the case of the modified Penman aerodynamic term a different story is found. The linearized term averages less than 60% of the integrated term with an absolute difference of 43% between the two! Differences for some individual days with large variations in stability exceeded 90% of the integrated term! It is concluded that taking the daily averaged stability causes large underestimations in the daily potential evaporation.

Comparing the integrated modified and integrated original Penman aerodynamic terms a very small difference between them is found averaging only 2%, but a larger absolute difference averaging 12%. An individual daily difference of about 24% of the integrated modified term is found on a day with a large diurnal variation in stability.

Table 3.3 Summary of the daily potential evaporation differences using integrated and linearized forms of the original and modified aerodynamic terms, and the radiation term as averaged over all Wangara days, mm/hr.

First term	Second term	Difference	<u>Difference</u> First	Absolute Difference	<u>Absolute</u> First	<u>Second</u> First
integrated original aerodynamic	linearized original aerodynamic	0.00286	0.04843	0.00376	0.06372	0.95416
0.05905	0.05619					
integrated modified aerodynamic	linearized modified aerodynamic	0.02132	0.36432	0.02132	0.36432	0.56662
0.05852	0.03720					
integrated modified aerodynamic	integrated original aerodynamic	-0.00060	-0.01024	0.00667	0.11401	1.00029
0.05852	0.05905					
integrated modified aerodynamic	linearized original aerodynamic	0.00227	0.03881	0.00816	0.13943	0.96018
0.05852	0.05619					
integrated radiation	linearized radiation	0.00385	0.13088	0.00385	0.13088	0.86912
0.02942	0.02557					

Similarly, the integrated modified and linearized original Penman aerodynamic terms also have a small difference between them, approximately 2%, and an absolute difference of about 15%. The maximum individual difference, about 28% of the integrated term, also occurs on a day with a large diurnal variation in stability. Because of the large differences involved in approximating the integrated modified term by the linearized modified aerodynamic term, it is concluded that on a daily basis the linearized original Penman is preferred for a good approximation of the integrated modified Penman aerodynamic term. This is not surprising since the Penman equation has been calibrated to perform well when daily values are summed over extensive periods.

Finally, the differences between the integrated and linearized radiation terms are small; the linearized radiation term is usually around 90% of the integrated term. The difference between them in this case equals the absolute difference, around 9% of the integrated term. A maximum individual daily difference of nearly 22% of the integrated term is found on a day with a large diurnal variation in temperature and radiation.

3.3.2b Explicit Nonlinear Effects

The nonlinear contribution to the integrated evaporation is determined by subtracting the 24-hour means from the hourly values for each variable, then multiplying products of resulting perturbations, summing these products for all hours of the day, then averaging. A nonlinear term will be a significant positive percentage of the linear (mean) term if both factors of the nonlinear term exhibit a similar diurnal variation (high positive correlation). A significant negative percentage will occur if the diurnal variation of both terms is 180 degrees out of phase (high negative correlation). Nonlinear terms involving more than two variables can be obtained in a similar manner.

Nonlinear terms must be determined for the radiation expression, and the original and modified aerodynamic expressions. From the radiation expression (3.33) it is seen that there are two terms that vary throughout the day, $\Delta/\Delta+\gamma$ and $R_n - S$. To determine their nonlinear

effect they are divided up into a 24-hour mean part (overbar) and an hourly deviation part (prime)

$$\frac{\Delta}{\Delta+\gamma} = \overline{\frac{\Delta}{\Delta+\gamma}} + \left(\frac{\Delta}{\Delta+\gamma}\right)' \quad (3.35)$$

$$R_n - S = \overline{(R_n - S)} + (R_n - S)' \quad (3.36)$$

Substituting back into the radiation expression yields

$$E_r = \frac{3600}{L_v} \left[\overline{\frac{\Delta}{\Delta+\gamma}} \overline{(R_n - S)} + \overline{\frac{\Delta}{\Delta+\gamma}} (R_n - S)' + \left(\frac{\Delta}{\Delta+\gamma}\right)' \overline{(R_n - S)} + \left(\frac{\Delta}{\Delta+\gamma}\right)' (R_n - S)' \right] \quad (3.37)$$

Taking the 24-hour average of E_r and noting that the 24-hour average of a single primed term equals zero

$$\overline{E_r} = \frac{3600}{L_v} \left[\overline{\left(\frac{\Delta}{\Delta+\gamma}\right)} \overline{(R_n - S)} + \overline{\left(\frac{\Delta}{\Delta+\gamma}\right)' (R_n - S)'} \right] \quad (3.38)$$

where there is one linear (mean) and one nonlinear term.

The original aerodynamic expression ((3.34) using C_{qp}) has 4 terms that vary throughout the day: $\gamma/\Delta+\gamma$, C_{qp} , U_2 , and $q_s - q_a$. However, C_{qp} can be expressed as a function of the wind speed: $C_{qp} = (a/U_2 + b)$ where $a = 3.9 \times 10^{-3}$ and $b = 2.1 \times 10^{-3}$ (see (3.28)). When multiplied by the wind speed this gives the wind function $C_{qp} U_2 = a + bU_2$. So, effectively, there are only 3 terms for which nonlinear effects must be considered. The terms are written as a mean plus deviation as before

$$\frac{\gamma}{\Delta+\gamma} = \overline{\frac{\gamma}{\Delta+\gamma}} + \left(\frac{\gamma}{\Delta+\gamma}\right)' \quad (3.39)$$

$$a + bU_2 = \overline{(a + bU_2)} + (a + bU_2)' \quad (3.40)$$

$$q_s - q_a = \overline{(q_s - q_a)} + (q_s - q_a)' \quad (3.41)$$

Substituting back into the aerodynamic expression and taking the 24-hour average of E_a yields

$$\begin{aligned} \bar{E}_a = 3600\rho & \left[\overline{\left[\frac{\gamma}{\Delta+\gamma} \overline{(q_s - q_a)} (a+b\bar{U}_2) + b \frac{\gamma}{\Delta+\gamma} \overline{U_2' (q_s - q_a)'} + \right.} \right. \\ & \overline{b \overline{(q_s - q_a)} \left(\frac{\gamma}{\Delta+\gamma} \right)' U_2'} + (a+b\bar{U}_2) \overline{\left(\frac{\gamma}{\Delta+\gamma} \right)' (q_s - q_a)'} + \\ & \left. \left. \overline{b \left(\frac{\gamma}{\Delta+\gamma} \right)' U_2' (q_s - q_a)'} \right] \right] \quad (3.42) \end{aligned}$$

where there are one linear and four nonlinear terms.

The modified aerodynamic expression ((3.34) using C_{q1}) also has 4 terms that vary throughout the day; all of these are independent. The mean plus prime terms are

$$U_2 = \bar{U}_2 + U_2' \quad (3.43)$$

$$C_{q1} = \bar{C}_{q1} + C_{q1}' \quad (3.44)$$

in addition to (3.39) and (3.41). As before substituting back into the aerodynamic expression and taking a daily average produces

$$\begin{aligned} \bar{E}_a = 3600\rho & \left[\overline{\left[\frac{\gamma}{\Delta+\gamma} \bar{C}_{q1} \bar{U}_2 \overline{(q_s - q_a)} + \frac{\gamma}{\Delta+\gamma} \bar{C}_{q1} \overline{U_2' (q_s - q_a)'} + \right.} \right. \\ & \overline{\left. \frac{\gamma}{\Delta+\gamma} \bar{U}_2 \overline{C_{q1}' (q_s - q_a)'} + \frac{\gamma}{\Delta+\gamma} \overline{(q_s - q_a)} \bar{C}_{q1}' \overline{U_2'} + \right.} \\ & \left. \left. \overline{C_{q1}' U_2' (q_s - q_a)'} \right] \right] \quad (3) \quad (4) \end{aligned}$$

$$\begin{aligned}
 & \overline{\frac{\gamma}{\Delta+\gamma} C_{q1}' U_2' (q_s - q_a)'} + \overline{C_{q1} \bar{U}_2 \left(\frac{\gamma}{\Delta+\gamma}\right)' (q_s - q_a)'} + & (5) & \quad (6) \\
 & \overline{C_{q1} (q_s - q_a) \left(\frac{\gamma}{\Delta+\gamma}\right)' U_2'} + \overline{C_{q1} \left(\frac{\gamma}{\Delta+\gamma}\right)' U_2' (q_s - q_a)'} + & (7) & \quad (8) \\
 & \overline{U_2 (q_s - q_a) \left(\frac{\gamma}{\Delta+\gamma}\right)' C_{q1}'} + \overline{U_2 \left(\frac{\gamma}{\Delta+\gamma}\right)' C_{q1}' (q_s - q_a)'} + & (9) & \quad (10) \\
 & \overline{(q_s - q_a) \left(\frac{\gamma}{\Delta+\gamma}\right)' C_{q1}' U_2'} + \overline{\left(\frac{\gamma}{\Delta+\gamma}\right)' C_{q1}' U_2' (q_s - q_a)'} \Big]. & (11) & \quad (12) \quad (3.45)
 \end{aligned}$$

Here there are one linear and 11 nonlinear terms.

From the equations derived above, it is seen that in the radiation expression (3.38) there is a nonlinear interaction between the diurnal variations of net radiation less soil heat flux and the term $\Delta/\Delta+\gamma$. In the aerodynamic expressions ((3.42) and (3.45)) there are nonlinear interactions between the diurnal variations of the exchange coefficient (only for the modified expression), wind, specific humidity deficit, and the term $\gamma/\Delta+\gamma$.

Since $\gamma = pC_p/\epsilon L_v$ is a function of pressure which has little diurnal variation, the variation of the $\Delta/\Delta+\gamma$ and $\gamma/\Delta+\gamma$ terms can be attributed to $\Delta = de_s/dT = L_v e_s/R_v T^2$ which is a function of the temperature through the saturation vapor pressure e_s and latent heat function. Since the daily variation of $\Delta/\Delta+\gamma$ is proportional to the diurnal variation of temperature, we call $\Delta/\Delta+\gamma$ a temperature function. Similarly, we call $\gamma/\Delta+\gamma$ an inverse temperature function because the daily variation of $\gamma/\Delta+\gamma$ is inversely proportional to the diurnal variation of temperature.

Table 3.4 is a summary of the linear and nonlinear terms of the radiation and the original and modified aerodynamic expressions averaged over all the Wangara days. For nonlinear terms of individual

Table 3.4 Summary of the linear and nonlinear terms of the radiation (3.38) and the original (3.42) and modified (3.45) aerodynamic expressions as averaged over all Wangara days, mm/hr. Total is the sum of the linear and nonlinear terms which is the daily average.

Radiation Term

total	linear term (1)	nonlinear term (2)	term (2)/(1)
0.02969	0.02574	0.00395	0.12998

Original Aerodynamic Term

total	linear term (1)	nonlinear terms+ (2)	(2)/(1)	(3)	(3)/(1)	(4)	(4)/(1)	(5)	(5)/(1)
0.06005	0.06048	0.00558	0.09103	-0.00118	-0.01755	-0.00453	-0.07964	-0.00030	-0.00541

Modified Aerodynamic Term

total	linear term (1)	nonlinear terms+ (2)	(2)/(1)	(3)	(3)/(1)	(4)	(4)/(1)	(5)	(5)/(1)
0.05917	0.04309	0.00591	0.14476	0.00966	0.28926	0.00731	0.19481	0.00139	0.04109
		(6)	(6)/(1)	(7)	(7)/(1)	(8)	(8)/(1)	(9)	(9)/(1)
		-0.00319	-0.07964	-0.00124	-0.00124	-0.00032	-0.00877	-0.00190	-0.05265
		(10)	(10)/(1)	(11)	(11)/(1)	(12)	(12)/(1)		
		-0.00058	-0.01906	-0.00028	-0.00777	-0.00069	-0.01910		

days see Appendix C. It is seen that the nonlinear term in the radiation expression is nearly 13% of the linear term averaged over all 40 days. On a day with a large diurnal variation of temperature and net radiation (less soil heat flux) the nonlinear term is 28% of the linear term. This is because the temperature function and net radiation (less soil heat flux) both reach maximum values in the afternoon. It is this effect that accounts for the differences between the linearized and integrated radiation terms (see Sec. 3.3.2a).

In the original Penman aerodynamic expression the nonlinear terms involving the wind and specific humidity deficit, and the specific humidity deficit and inverse temperature function are found to be the most significant of the nonlinear terms, approximately 9% and -8% of the linear term, respectively. On a day with large diurnal variations of atmospheric variables these terms account for about 19% and -20%, respectively. These two nonlinear terms are significant because the wind and specific humidity deficit have a high positive correlation dominated by high afternoon values; the inverse temperature function and specific humidity deficit have a high negative correlation because both have a dependence on the saturation vapor pressure, one inversely.

In similar evaporation studies, Jobson (1972) and Hage (1975) found analogous results; the wind and vapor pressure, and the vapor pressure and temperature (through a dependence on e_s) were found to be the two most significant nonlinear terms. However, both studies explicitly excluded any nonlinear terms with a dependence on exchange coefficients, i.e. they considered the exchange coefficient constant. In the study here, both of these nonlinear terms are of opposite sign and approximately equal, roughly cancelling each other. Since the other two nonlinear terms are quite small, the total effect of the nonlinear terms is roughly zero. It is concluded as in Sec. 3.3.2a that the daily linearized original Penman aerodynamic term is a good approximation of the daily integrated original Penman aerodynamic term.

Finally, of the eleven nonlinear terms in the modified Penman aerodynamic equation, seven are found to be unimportant, their sum amounting to less than about -9% of the linear term. As in the

original Penman aerodynamic expression, nonlinear interactions between the specific humidity deficit and inverse temperature function are found to be among those important, about 14% and -8% of the linear term, respectively. However, the correlation between the exchange coefficient (C_{q1}) and specific humidity deficit ($q_s - q_a$), and the correlation between the exchange coefficient (C_{q1}) and wind (U_2) lead to the most important nonlinear terms which are nearly 29% and 20% of the linear term, respectively. On a day with a particularly high diurnal variation in atmospheric variables, the former two nonlinear terms are found to be 33% and -14%, respectively. On this same day, the latter two nonlinear terms are found to be 57% and 26% of the linear term, respectively. The C_{q1} and U_2 interaction, and the C_{q1} and $q_s - q_a$ interaction are the largest nonlinear terms because of the very large variations in C_{q1} , U_2 , and $q_s - q_a$ throughout the day and the high positive correlations of C_{q1} with U_2 and $q_s - q_a$. It can now be seen why the linearized modified Penman greatly underestimates the integrated modified Penman aerodynamic term (see Sec. 3.2.2a); it is primarily due to the nonlinear interactions of the stability dependent exchange coefficient with the wind and humidity, and to a lesser extent to the nonlinear interactions of the humidity with the wind and inverse temperature function.

4. SUMMARY AND CONCLUSIONS

A method for calculating surface evapotranspiration under the condition of nonlimiting soil moisture has been presented. Vegetation has been taken into consideration through the use of plant coefficients, which account for vegetative density and stomatal resistance. The atmospheric influence is represented by the potential evaporation.

From a literature survey, it was determined that the plant coefficient for most vegetation types exhibits a seasonal variation because of a seasonal variation in plant transpiration. Unfortunately, most studies have been done on agricultural crops so little is known about plant coefficients of natural vegetation. Also, both potential evaporation and plant coefficients exhibit diurnal variations but little has been done to investigate this interaction.

The original Penman equation is favored over other potential evaporation equations because it explicitly includes constraints due to both energy restrictions and atmospheric turbulent transport. The Penman equation as modified here is a further improvement because of the inclusion of an atmospheric stability dependent turbulent exchange coefficient. The coefficient formulated by Louis et al. (1982) was used since it allows for both stable and unstable conditions which are necessary for diurnal calculations of evaporation.

In a comparison of different asymptotic cases it was found that the modified expression more accurately describes the free convection cases because of a failure of the original expression to recognize the importance of atmospheric stability.

Using boundary layer data from the Wangara experiment, the average diurnal variation of the modified Penman aerodynamic term was determined to be much greater than the original aerodynamic term because of the effect of atmospheric stability. As compared to the modified Penman relationship, the original aerodynamic term leads to an average daily absolute difference of 40%, more than 60% on days with large diurnal variations. Large underestimations (overestimations) of hourly evaporation occur in the afternoon (at night) under non-neutral conditions. The magnitude of the difference averages 30% in the early

afternoon, and was found to be more than 50% on days with large diurnal variations.

The difference between linearized and integrated daily estimates of evaporation is due mainly to nonlinear interactions of the exchange coefficient with the wind and the specific humidity deficit. On the average the linearized radiation term is a good approximation to the daily integrated because the contribution of the nonlinearity between the net radiation and temperature dependent coefficient is small. The use of the linearized original aerodynamic term to approximate the daily integrated term also leads to small differences because of an approximate cancelling of the nonlinear terms. This is quite reasonable since the original linearized Penman has been calibrated for time scales of one day or greater.

However, the use of the linearized modified aerodynamic term to approximate the daily integrated term results in gross underestimations because of the large nonlinear interactions between the diurnal variations of the stability-based exchange coefficient with specific humidity deficit and wind speed. When combined, these nonlinear terms average nearly 50% of the linearized term and may exceed 80% on days with large diurnal variations in stability.

Most studies of the Penman relationship are incomplete due to a failure to explicitly include the influence of the diurnal variation of atmospheric transport of water vapor. Such an influence was found to significantly contribute to the diurnal variation of surface evaporation. Thus, the modified Penman relationship, which accounts for the diurnal variations of stability, must be used in calculating evaporation for individual hours. The original Penman is of general use only for evaporation averaged over 24 hours or longer periods.

The calculations performed here should be applied to a summertime case when the diurnal variation of stability could be much larger in some locations leading to even larger diurnal variations of evaporation than found here. A comparison should be made between calculated values of potential evaporation as presented here and actual measured values of potential evaporation. There is a need to look more closely into the dependence of the Louis turbulent exchange coefficient on the

Richardson number and the way in which the Richardson number is calculated. The effects on the calculated value of potential evaporation when using wind, temperature, and humidity data from non-standard levels as well as incomplete radiation data (e.g. no upward longwave or soil heat flux) must also be examined. Also, the Penman relationships have not been tested for the case of nighttime dewfall.

BIBLIOGRAPHY

- Al-Khafaf, S., P. J. Wierenga and B. C. Williams, 1978: Evaporative flux from irrigated cotton as related to leaf area index, soil water, and evaporative demand. Agron. J., 70:912-917.
- Assenac, G., 1972: Etude de l'evapotranspiration reelle de quarte peuplements forest revs dans l'est de la France. Ann. Sci. Forest., 29(3), 369-389.
- Bailey, W. G. and J. A. Davies, 1981: The effect of uncertainty in aerodynamic resistance on evaporation estimates from the combination model. Boundary-Layer Met., 20:187-199.
- Bates, E. M., F. V. Pumphrey, D. C. Hane, and T. P. Davidson, 1982: Evapotranspiration relationships with pan evaporation of frequently irrigated wheat and potatoes. Oregon State University, Department of Crop Science. Manuscript to be published as a NOAA Technical Report.
- Black, T. A., C. B. Tanner, and W. R. Gardner, 1970: Evaporation from a snap bean crop. Agron. J., 62, 66-79.
- Buck, A. L., 1976: The variable-path Lyman-alpha hygrometer and its operating characteristics. Bull. Amer. Met. Soc., 57(9):1113-1118.
- Businger, J. A., 1956: Some remarks on Penman's equations for evapotranspiration. Neth. J. Agr. Sci., 4:77-80.
- Clarke, R. H., A. J. Dyer, R. R. Brook, D. G. Reid, and A. J. Troup, 1971: The Wangara Experiment: Boundary layer data. Tech. Paper No. 19, Div. Meteor. Phys., CSIRO, Australia.
- Cuenca, R. H., J. Erpenbeck, and W. O. Pruitt, 1981: Advances in computation of regional evapotranspiration. Proceedings of the ASCE, Water Forum '81, San Francisco, 73-80.
- Dale, R. F. and K. L. Scheeringa, 1977: The effect of soil moisture on pan evaporation. Agric. Met., 18, 463-474.
- Dalton, J., 1834: Meteorological observations and essays. Second edition, Manchester: printed by Harrison and Crosfield, for Baldwin and Cradock, London.
- DeHeer-Amissah, A., U. Hoegstroem, and A. S. Smedman-Hoegstroem, 1981: Calculation of sensible and latent heat fluxes, and surface resistance from profile data. Boundary-Layer Met., 20:35-49.
- Dooenbos, J. and W. O. Pruitt, 1975: Crop water requirements. In Irrigation and Drainage 24. FAO, Rome. 178 p.
- Dyer, A. J. and B. B. Hicks, 1970: Flux-gradient relationships in the constant flux layer. Quart. J. R. Met Soc., 96:715-721.
- Dylla, A. S., D. R. Timmons, and H. Shull, 1980: Estimating water use by irrigated corn in west central Minnesota. Soil Sci.

- Soc. Am. J., 44, 823-827.
- Eagleson, P. S., 1978: Climate, soil and vegetation, Parts 1-7. Water Resources Res., 14(5):705-776.
- Federer, C. A., 1970: Measuring forest evapotranspiration - theory and problems. U.S.D.A. Forest Service research paper NE-165.
- Fritschen, L. J., 1981: The vertical fluxes of heat and moisture at a vegetated land surface. GARP Study Conference on Land Processes in Atmospheric General Circulation models. NASA Goddard Space Flight Center, January 5-10.
- , 1965: Evapotranspiration rates of field crops determined by the Bowen ratio method. Agron. J., 58:339-342.
- Fuchs, M. and G. Stanhill, 1963: The use of class A evaporation pan data to estimate the irrigation water requirements of the cotton crop. Israel J. Agric. Res., 13:2, 63-78.
- , I. Hausenberg, and G. Stanhill, 1964: A field test of the control of cotton irrigation practice from class A pan data. Israel J. Agric. Res., 14:4, 237-239.
- and C. B. Tanner, 1967: Evaporation from a drying soil. J. Applied Met., 6:852-857.
- Godard, M., 1964: Transpiration et maturation du Ble'dans le Languedoc mediterraneen, p. 357-370. In L'eau et la production vegetale. Institute National de la Recherche Agronomique, Paris.
- Hage, K. D., 1975: Averaging errors in monthly evaporation estimates. Water Resources Res., 11(2):359-361.
- Hanson, C. L., 1976: Model for predicting evapotranspiration from native rangelands in the northern great plains. Trans. ASCE, 471-481.
- Hargreaves, G. H., 1974: Estimation of potential and crop evapotranspiration. Trans. ASCE, 701-704.
- Hicks, B. B., 1973: Eddy fluxes over a vineyard. Agric. Met., 12:203-215.
- Holmes, R. M. and G. W. Robertson, 1959: A modulated soil moisture budget. Mon. Wea. Rev., March: 101-105.
- Jensen, M. E. and H. R. Haise, 1963: Estimating evapotranspiration from solar radiation. J. Irrigation Drainage Div. Amer. Soc. Civil. Eng., 89:15-41.
- Jobson, H. E., 1972: Effect of using averaged data on the computed evaporation. Water Resources Res., 8(2):513-518.
- Kanemasu, E. T., L. R. Stone, and W. L. Powers, 1976: Evapotranspiration model tested for soybean and sorghum, Agron. J., 68:569-572.
- Kristensen, K. J., 1974: Actual evapotranspiration in relation to leaf area. Nordic Hydrology, 5:173-182.

- Lomas, J., E. Schlesinger, and J. Lewin, 1974: Effects of environmental and crop factors on the evapotranspiration rate and water-use efficiency of maize. Agric. Met., 13:239-251.
- Louis, J. F., 1979: A parametric model of vertical eddy fluxes in the atmosphere. Boundary-Layer Met., 17:187-202.
- , M. Tiedtke, and J. F. Geleyn, 1982: A short history of the operational PBL - Parameterization at ECMWF. Internal report, European Centre for Medium Range Weather Forecasts, Shinfield Park, Reading, Berks, U. K.
- Lourence, F. J. and W. O. Pruitt, 1971: Energy balance and water use of rice grown in the Central Valley of California. Agron. J., 63:827-832.
- Mahrt, L. J., 1982: Unpublished manuscript.
- Meyer, W. S. and G. C. Green, 1980: Water use by wheat and plant indicators of available soil water. Agron. J., 72:253-257.
- Monteith, J. L., 1965: Evaporation and environment. Sump. Soc. Exp. Biol., 19:205-234.
- , 1975: Vegetation and the Atmosphere, Volume 1 Principles. Academic Press, London, 278 pp.
- Nelson, S. H. and K. E. Hwang, 1976: Water usage by cabbage plants at different stages of growth. Can. J. Plant Sci., 56:563-566.
- Nkemdirim, L. C. and P. F. Haley, 1973: An evaluation of grassland evapotranspiration. Agric. Met., 11:373-383.
- Norero, A. L., J. Keller, and G. L. Ashcroft, 1972: Effect of irrigation frequency on the average evapotranspiration for various crop-climate-soil systems. Trans. ASCE, 662-666.
- O'Neill, P., L. Pochop, and J. Borrelli, 1979: Urban lawn evapotranspiration measurement and prediction. Trans. ASCE, 1050-1053.
- Parmele, L. H. and J. L. McGuinness, 1974: Comparisons of measured and estimated daily potential evapotranspiration in a humid region. J. Hydrol., 22:239-251.
- Penman, H. L., 1948: Natural evaporation from open water, bare soil, and grass. Proc. Roy. Soc. A., 193:120-195.
- , 1956: Evaporation - an inductive survey. Neth. J. Agric. Sci., 4:9-29.
- Priestly, C. H. B. and R. J. Taylor, 1972: On the assessment of surface heat flux and evaporation using large-scale parameters. Mon. Wea. Rev., 100:81-92.
- Ritchie, J. T. and E. Burnett, 1971: Dryland evaporative flux in a subhumid climate: II. Plant influences. Agron. J., 63:56-62.
- , and R. C. Henderson, 1972: Dryland Evaporative Flux in a Subhumid Climate: III. Soil Water Influence. Agron. J., 64:168-172.

- Rosenberg, N. J., 1974: Microclimate: The biological environment. John Wiley and Sons, Inc., New York, pp. 159-205.
- Russell, G., 1980: Crop evaporation, surface resistance and soil water status. Agric. Met., 21:213-226.
- Shepherd, W., 1972: Some evidence of stomatal restriction of evaporation from well-watered plant canopies. Water Resources Res., 8(4):1092-1095.
- Stanley, C. D. and R. H. Shaw, 1978: The relationship of evapotranspiration to open-pan evaporation throughout the growth cycle of soybeans. Iowa State J. Res., 53(2):129-136.
- Stewart, J. E. and A. S. Thom, 1973: Energy budgets in pine forest. Quart. J. R. Met. Soc., 99:154-170.
- Stigter, C. J., 1980: Assessment of the quality of generalized wind functions in Penman's equations. J. Hydrol., 45:321-331.
- Strahler, A. N., 1975: Physical Geography, John Wiley and Sons, Inc., New York, pp. 205-208.
- Stricker, H. and W. Brutsaert, 1978: Actual evapotranspiration over a summer period in the "Hupsel Catchment." J. Hydrol., 39:139-157.
- Tanner, C. E. and M. Fuchs, 1968: Evaporation from unsaturated surfaces: A generalized combination method. J. Geophys. Res., 73:1299-1304.
- Tanner, C. B. and W. L. Pelton, 1960: Potential evapotranspiration estimates by the approximate energy balance method of Penman. J. Geophys. Res., 65:3391-3413.
- Taylor, S. A., 1972: Physical Edaphology: The physics of irrigated and nonirrigated soils, W. H. Freeman and Co., San Francisco, pp. 45-84.
- Thom, A. S. and H. R. Oliver, 1977: On Penman's equation for estimating regional evaporation. Quart. J. R. Met. Soc., 103:345-357.
- Thorntwaite, C. W., 1948: An approach toward a rational classification of climate. Geog. Rev., 38:55-94.
- USDA (United States Department of Agriculture), Soil Conservation Service, Engineering Division, April 1967, Revised September 1970, Irrigation Water Requirements, Technical Release No. 21.
- van Bavel, C. H. M., 1966: Potential evaporation: The combination concept and its experimental verification. Water Resources Res., 2:455-467.
- , 1967: Changes in canopy resistance to water loss from alfalfa induced by soil water depletion. Agric. Met., 4:165-176.
- Vankatachari, A. and K. A. Reddy, 1978: Relationship of evapotranspiration with pan evaporation and evaluation of crop

- coefficient. *Acta Agronomica Scientiarum Hungaricae*, 27:107-110.
- Verma, S. B., N. J. Rosenberg, and S. L. Blad, 1978: Turbulent exchange coefficients for sensible heat and water vapor flux under advective conditions. *J. Applied Met.*, 17:330-338.
- Yu, T. W., 1977: Parameterization of surface evaporation rate for use in numerical modeling. *J. Applied Met.*, 16:393-400.
- Wright, J. L. and M. E. Jensen, 1978: Development and evaluation of evapotranspiration models for irrigation scheduling. *Trans. ASCE*, 88-91.

APPENDICES

APPENDIX A. Plant Coefficients

Plant coefficients (PC) for different seasons at different locations. Maximum refers to the maximum for the growing season; growing season is the average for the growing season. General refers to any location where modification by empirical coefficients of climate and geography are necessary for application of PC to a specific location. Necessary for the determination of PC is the way in which Ep and ET are obtained. Comb. refers to a combination equation, TB is a thermally based equation, pan is pan evaporation, ETB is an energy and thermally based equation. Lys is a lysimeter measurement and BC refers to an already existing Blaney-Criddle crop coefficient so that no Ep measurement was made.

Season	P.C.	Location where valid	Author, Date	Ep measurement method or equation	ET measurement method
<u>Agricultural plants:</u>					
<u>Crop: Alfalfa</u>					
1. Spring	.93	Arizona	van Bavel, 66	comb.	lys.
Summer	.99				
Fall	.96				
2. Maximum	1.00	Arizona	van Bavel, 67	comb.	lys.
3. Maximum	1.35	General	Hargreaves, 74	T.B.	lys.
Growing Season	1.00				
4. Maximum	.85	Semi-arid & Arid locations	Holmes, 59	----	BC
5. Maximum	1.10	General	USDA, 67	----	BC

Crop: Snap Beans

1. Maximum Growing Season	.90 .50	Kimberly, Idaho	Wright, 78	comb.	lys.
2. Summer	.48	Wisconsin	Black, 70	comb.	lys.
3. Maximum Growing Season	1.10 .85	General	USDA, 67	----	BC
4. Maximum Growing Season	1.15 .90	General	Hargreaves, 74	TB	lys.

Crop: Dry Beans

1. Maximum Growing Season	1.10 .85	General	USDA, 67	----	BC
2. Maximum Growing Season	1.15 .90	General	Hargreaves, 74	TB	lys.

Crop: Soy Beans

1. Maximum Growing Season	1.05 .65	General	USDA, 67	----	BC
2. Maximum Growing Season	.73 .57	India	Vankatachari, 78	----	BC
3. Maximum Growing Season	1.15 .90	General	Hargreaves, 74	TB	lys.

4. Maximum Growing Season	1.10 .70	Iowa	Stanley, 78	pan	neutron-probe/ gravimetric
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Crop: Corn

1. Summer Late Summer	1.15 1.01	Ohio	Parmele, 74	comb.	lys.
2. Maximum Growing Season	1.25 .80	Israel	Lomas, 74	comb.	lys.
3. Maximum Growing Season	1.15 .70	Minnesota	Dylla, 80	ETB	lys.
4. Maximum Growing Season	.89 .71	India	Vankatachari, 78	----	BC
5. Maximum Growing Season	1.15 .90	General	Hargreaves, 74	TB	lys.
6. Maximum Growing Season	1.08 .85	General	USDA, 67	----	BC

Crop: Cotton

1. Maximum	.70	Israel	Fuchs, 63 & 64	pan	gravimetric
2. Maximum Growing Season	.81 .60	India	Vankatachari, 78	----	BC
3. Maximum Growing Season	1.02 .60	General	USDA, 67	----	BC

4. Maximum Growing Season	1.15 .90	General	Hargreaves, 74	TB	lys.
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Crop: Grain (Wheat)

1. Maximum Growing Season	.81 .72	India	Vankatachari, 78	----	BC
2. Maximum Growing Season	.95 .70	Eastern Oregon	Bates, 82	pan	neutron probe
3. Maximum Growing Season	1.30 .75	General	USDA, 67	----	BC
4. Maximum Growing Season	1.15 .90	General	Hargreaves, 74	TB	lys.

Crop: Grain (Barley)

1. Growing Season	.76	Denmark	Kristensen, 74	Penman	neutron probe
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Crop: Grain (Sorghum)

1. Maximum Growing Season	.72 .56	India	Vankatachari, 78	----	BC
2. Maximum Growing Season	1.08 .70	General	USDA, 67	----	BC
3. Maximum Growing Season	1.15 .90	General	Hargreaves, 74	TB	lys.

Crop: Grass (short, green)

1. Summer	.80	SE England	Thom, 77	Penman	?
Winter	.60				
2. Growing Season	.73	Denmark	Kristensen, 74	Penman	neutron probe
3. Maximum	1.00	General	Hargreaves, 74	TB	lys.
Growing Season	1.00				
4. Spring	.75	Wyoming	O'Neill, 79	----	BC
Summer	.90				
Fall	.70				

Crop: Grass (long)

1. Growing Season	.74	Denmark	Kristensen, 74	Penman	neutron probe
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Crop: Grass (Pasture)

1. Summer	.90	Australia	Shepherd, 72	comb.	lys.
2. Maximum	1.50	Netherlands	Stricker, 78	bulk	energy
Growing Season	.85			aerodynamic	balance
3. Maximum	1.15	General	Hargreaves, 74	TB	lys.

Crop: Grass (Prairie)

1. Summer	.90	Canadian Great Plains	Nkemdirim, 73	Penman	lys/ neutron probe
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Crop: Potatoes

1. Maximum Growing Season	1.38 .90	General	USDA, 67	----	BC
2. Summer	.90	Australia	Shepherd, 72	comb.	lys.
3. Maximum Growing Season	.90 .50	Eastern Oregon	Bates, 82	pan	neutron probe
4. Maximum Growing Season	1.15 .90	General	Hargreaves, 74	TB	lys.

Crop: Rice

1. Maximum	1.20	Arid & Semi- Arid Conditions	Holmes, 59	----	BC
2. Summer	1.02	California/ Davis	Lourence, 71	reference crop	energy balance

Crop: Sugar Beets

1. Growing Season	.82	Denmark	Kristensen, 74	Penman	neutron probe
2. Maximum Growing Season	1.15 .90	General	Hargreaves, 74	TB	lys.

3.	Maximum Growing Season	1.25 .90	General	USDA, 67	----	BC
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Crop: Field and Oil Crops (Flax, peanuts, safflower, tomatoes)

1.	Maximum Growing Season	1.15 .90	General	Hargreaves, 74	TB	lys.
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Crop: Citrus Fruits (Orange, lemon, grapefruit)

1.	Maximum Growing Season	.75 .75	General	Hargreaves, 74	TB	lys.
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2.	Maximum Yearly Avg.	.72 .70	General	USDA, 67	----	BC
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Crop: Deciduous Fruits (peaches, plums, walnuts)

1.	Maximum Growing Season	1.10 .85	General	Hargreaves, 74	TB	lys.
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2.	Maximum Yearly Avg.	1.00 .50	General	USDA, 67	----	BC
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Crop: Grapes

1.	Maximum Growing Season	.75 .60	General	Hargreaves, 74	TB	lys.
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2.	Maximum Yearly Avg.	.82 .50	General	USDA, 67	----	BC
----	------------------------	------------	---------	----------	------	----

Crop: Sugar Cane

1. Maximum	1.25	General	Hargreaves, 74	TB	lys.
Growing Season	1.00				
2. Growing Season	.85	General	USDA, 67	----	BC

Crop: Summer Vegetables

1. Maximum	1.15	General	Hargreaves, 74	TB	lys.
Growing Season	.85				
2. Maximum	.85	General	USDA, 67	----	BC
Growing Season					

Non-agricultural plants:

Plant: Spruce, Pine Fir

Spring	1.00	France	Assenac, 72	?	?
Summer	1.00				
Fall	1.00				

Plant: Douglas Fir

Annual Range	.64 to	France	Assenac, 72	?	?
	1.00				

APPENDIX B. DAILY EVAPORATION COMPARISONS

The following pages contain the individual daily potential evaporation differences from Wangara data using integrated and linearized forms of the original and modified aerodynamic terms, and the radiation term, mm/hr.

1st term - integrated original aerodynamic term
 2nd term - linearized original aerodynamic term

day	#hours avgd	1st	2nd	diff	$\frac{\text{diff}}{\text{1st}}$	abs	$\frac{\text{abs}}{\text{1st}}$	$\frac{\text{2nd}}{\text{1st}}$
1	10	0.077	0.075	0.002	0.021	0.002	0.021	0.979
2	24	0.135	0.127	0.008	0.058	0.008	0.058	0.942
3	23	0.106	0.100	0.006	0.056	0.006	0.056	0.944
4	23	0.061	0.071	-0.011	-0.174	0.011	0.174	1.174
5	24	0.068	0.070	-0.003	-0.041	0.003	0.041	1.041
6	24	0.030	0.031	-0.001	-0.027	0.001	0.027	1.027
7	23	0.058	0.056	0.002	0.031	0.002	0.031	0.969
8	23	0.046	0.043	0.005	0.106	0.005	0.106	0.894
9	24	0.021	0.020	0.000	0.016	0.000	0.016	0.984
10	24	0.024	0.023	0.002	0.066	0.002	0.066	0.934
11	21	0.036	0.033	0.003	0.083	0.003	0.083	0.917
12	22	0.054	0.051	0.003	0.056	0.003	0.056	0.944
13	23	0.053	0.047	0.006	0.110	0.006	0.110	0.890
14	23	0.040	0.038	0.002	0.045	0.002	0.045	0.955
15	24	0.058	0.053	0.005	0.089	0.005	0.089	0.911
16	24	0.037	0.036	0.001	0.032	0.001	0.032	0.968
17	21	0.034	0.033	0.001	0.035	0.001	0.035	0.965
18	24	0.048	0.047	0.001	0.018	0.001	0.018	0.982
19	22	0.030	0.028	0.001	0.043	0.001	0.043	0.957
20	23	0.062	0.057	0.005	0.080	0.005	0.080	0.920
21	12	0.104	0.104	0.001	0.005	0.001	0.005	0.995
25	9	0.067	0.061	0.006	0.095	0.006	0.095	0.905
26	24	0.077	0.070	0.006	0.083	0.006	0.083	0.917
27	24	0.095	0.093	0.001	0.013	0.001	0.013	0.987
28	24	0.068	0.061	0.007	0.096	0.006	0.096	0.904
29	24	0.129	0.118	0.011	0.085	0.011	0.085	0.915
30	24	0.057	0.052	0.005	0.081	0.005	0.081	0.919
31	24	0.053	0.049	0.004	0.068	0.004	0.068	0.932
32	23	0.042	0.037	0.005	0.110	0.005	0.110	0.890
33	23	0.048	0.045	0.003	0.054	0.003	0.054	0.946
34	24	0.084	0.076	0.008	0.092	0.008	0.092	0.908
35	23	0.107	0.102	0.005	0.045	0.005	0.045	0.955
36	24	0.042	0.042	*	-0.005	*	0.005	1.005
37	24	0.026	0.024	0.002	0.077	0.002	0.077	0.923
38	24	0.049	0.045	0.009	0.081	0.004	0.081	0.919
39	24	0.024	0.027	-0.003	-0.115	0.003	0.115	1.115
40	23	0.036	0.033	0.003	0.089	0.003	0.089	0.911
41	24	0.071	0.065	0.005	0.076	0.005	0.076	0.924
42	12	0.036	0.032	0.005	0.128	0.005	0.128	0.872
43	21	0.086	0.086	*	0.006	*	0.006	0.994
44	12	0.066	0.062	0.003	0.051	0.003	0.051	0.949
all	895	0.059	0.056	0.003	0.046	0.004	0.065	0.954

* <0.001

1st term - integrated modified aerodynamic term
 2nd term - linearized modified aerodynamic term

day	#hours avgd	1st	2nd	diff	<u>diff</u> 1st	abs	<u>abs</u> 1st	<u>2nd</u> 1st
1	10	0.051	0.037	0.014	0.276	0.014	0.276	0.724
2	24	0.125	0.106	0.019	0.152	0.019	0.152	0.848
3	23	0.094	0.065	0.030	0.313	0.030	0.313	0.687
4	23	0.047	0.045	0.002	0.046	0.002	0.046	0.954
5	24	0.050	0.012	0.038	0.757	0.038	0.757	0.243
6	24	0.033	0.002	0.031	0.935	0.031	0.935	0.065
7	23	0.051	0.015	0.036	0.708	0.036	0.708	0.292
8	23	0.047	0.017	0.030	0.634	0.030	0.634	0.366
9	24	0.024	0.001	0.023	0.945	0.023	0.945	0.055
10	24	0.018	0.006	0.012	0.660	0.012	0.660	0.340
11	21	0.035	0.027	0.008	0.232	0.008	0.232	0.768
12	22	0.056	0.036	0.020	0.360	0.020	0.360	0.648
13	23	0.053	0.023	0.031	0.572	0.031	0.572	0.428
14	23	0.043	0.015	0.028	0.646	0.028	0.646	0.354
15	24	0.056	0.043	0.013	0.227	0.013	0.227	0.773
16	24	0.036	0.015	0.021	0.589	0.021	0.589	0.411
17	21	0.032	0.021	0.011	0.340	0.011	0.340	0.660
18	24	0.052	0.045	0.006	0.122	0.006	0.122	0.878
19	22	0.037	0.010	0.026	0.715	0.026	0.715	0.285
20	23	0.057	0.021	0.037	0.637	0.037	0.637	0.363
21	12	0.091	0.084	0.007	0.077	0.007	0.077	0.923
25	9	0.053	0.027	0.027	0.502	0.027	0.502	0.498
26	24	0.063	0.027	0.036	0.569	0.036	0.569	0.431
27	24	0.088	0.085	0.003	0.038	0.003	0.038	0.962
28	24	0.075	0.049	0.026	0.349	0.026	0.349	0.651
29	24	0.140	0.125	0.015	0.109	0.015	0.109	0.891
30	24	0.058	0.052	0.006	0.108	0.006	0.108	0.892
31	24	0.062	0.045	0.017	0.278	0.017	0.278	0.722
32	23	0.048	0.015	0.032	0.676	0.032	0.676	0.324
33	23	0.066	0.018	0.045	0.713	0.045	0.713	0.287
34	24	0.094	0.054	0.040	0.429	0.040	0.429	0.571
35	23	0.123	0.113	0.011	0.88	0.011	0.088	0.912
36	24	0.041	0.036	0.005	0.128	0.005	0.128	0.872
37	24	0.019	0.014	0.006	0.291	0.006	0.291	0.709
38	24	0.052	0.045	0.008	0.149	0.008	0.149	0.851
39	24	0.025	0.006	0.020	0.803	0.020	0.803	0.197
40	23	0.040	0.008	0.032	0.796	0.032	0.796	0.204
41	24	0.071	0.032	0.039	0.545	0.039	0.545	0.455
42	12	0.046	0.005	0.040	0.884	0.040	0.884	0.116
43	21	0.076	0.065	0.011	0.148	0.011	0.148	0.852
44	12	0.077	0.071	0.005	0.069	0.005	0.069	0.931
all	895	0.058	0.037	0.021	0.434	0.021	0.434	0.566

1st term - integrated modified aerodynamic term
 2nd term - integrated original aerodynamic term

day	#hours avgd	1st	2nd	diff	<u>diff</u> 1st	abs	<u>abs</u> 1st	<u>2nd</u> 1st
1	10	0.051	0.077	-0.026	-0.506	0.826	0.506	1.506
2	24	0.125	0.135	-0.009	-0.074	0.009	0.074	1.074
3	23	0.094	0.106	-0.012	-0.129	0.012	0.129	1.129
4	23	0.047	0.061	-0.014	-0.301	0.014	0.301	1.301
5	24	0.050	0.068	-0.018	-0.363	0.018	0.363	1.363
6	24	0.033	0.030	0.003	0.094	0.003	0.094	0.906
7	23	0.051	0.058	-0.007	-0.141	0.007	0.141	1.141
8	23	0.047	0.048	-0.001	-0.029	0.001	0.029	1.029
9	24	0.024	0.021	0.003	0.136	0.003	0.136	0.864
10	24	0.018	0.024	-0.007	-0.370	0.007	0.370	1.370
11	21	0.035	0.036	-0.001	-0.027	0.001	0.027	1.027
12	22	0.056	0.054	0.001	0.026	0.001	0.026	0.974
13	23	0.053	0.053	*	0.002	*	0.002	0.998
14	23	0.043	0.040	0.002	0.056	0.002	0.056	0.944
15	24	0.058	0.058	-0.002	-0.036	0.002	0.036	1.036
16	24	0.036	0.037	*	-0.018	*	0.018	1.018
17	21	0.032	0.034	-0.002	-0.060	0.002	0.060	1.060
18	24	0.052	0.048	0.003	0.067	0.003	0.067	0.933
19	22	0.037	0.030	0.007	0.187	0.007	0.187	0.813
20	23	0.057	0.062	-0.004	-0.077	0.004	0.077	1.077
21	12	0.091	0.104	-0.013	-0.149	0.013	0.149	1.149
25	9	0.053	0.067	-0.014	-0.260	0.014	0.260	1.260
26	24	0.063	0.077	-0.013	-0.218	0.013	0.210	1.210
27	24	0.088	0.095	-0.006	-0.072	0.006	0.072	1.072
28	24	0.075	0.068	0.007	0.097	0.007	0.097	0.903
29	24	0.140	0.129	0.011	0.076	0.011	0.076	0.924
30	24	0.058	0.057	0.001	0.023	0.001	0.023	0.977
31	24	0.062	0.053	0.009	0.142	0.009	0.142	0.858
32	23	0.048	0.042	0.006	0.120	0.006	0.120	0.880
33	23	0.064	0.048	0.016	0.244	0.016	0.244	0.756
34	24	0.094	0.084	0.011	0.114	0.011	0.114	0.886
35	23	0.123	0.107	0.016	0.133	0.016	0.133	0.867
36	24	0.041	0.042	-0.001	-0.021	0.001	0.021	1.021
37	24	0.019	0.026	-0.007	-0.367	0.007	0.367	1.367
38	24	0.052	0.049	0.003	0.061	0.003	0.061	0.939
39	24	0.025	0.024	0.001	0.050	0.001	0.050	0.950
40	23	0.040	0.036	0.004	0.102	0.004	0.102	0.898
41	24	0.071	0.071	*	*	*	*	1.000
42	12	0.046	0.036	0.009	0.204	0.009	0.204	0.796
43	21	0.076	0.086	-0.010	-0.127	0.010	0.127	1.127
44	12	0.077	0.066	0.011	0.140	0.011	0.140	0.860
all	895	0.058	0.059	-0.001	-0.024	0.007	0.124	1.024

1st term - integrated modified aerodynamic term
 2nd term - linearized original aerodynamic term

day	#hours avgd	1st	2nd	diff	$\frac{\text{diff}}{\text{1st}}$	abs	$\frac{\text{abs}}{\text{1st}}$	$\frac{\text{2nd}}{\text{1st}}$
1	10	0.051	0.075	-0.024	-0.474	0.024	0.474	1.474
2	24	0.125	0.127	-0.001	-0.011	0.001	0.011	1.011
3	23	0.094	0.100	-0.006	-0.066	0.006	0.066	1.066
4	23	0.047	0.071	-0.025	-0.527	0.025	0.527	1.527
5	24	0.050	0.070	-0.021	-0.419	0.021	0.419	1.419
6	24	0.033	0.031	0.002	0.070	0.002	0.070	0.930
7	23	0.051	0.056	-0.005	-0.105	0.005	0.105	1.105
8	23	0.047	0.043	0.004	0.030	0.004	0.080	0.920
9	24	0.024	0.020	0.004	0.151	0.004	0.151	0.849
10	24	0.018	0.023	-0.005	-0.280	0.005	0.280	1.280
11	21	0.035	0.033	0.002	0.058	0.002	0.058	0.942
12	22	0.056	0.051	0.004	0.081	0.004	0.081	0.919
13	23	0.053	0.047	0.006	0.112	0.006	0.112	0.888
14	23	0.043	0.038	0.004	0.098	0.004	0.098	0.902
15	24	0.056	0.053	0.003	0.056	0.003	0.056	0.944
16	24	0.036	0.036	0.001	0.015	0.001	0.015	0.985
17	21	0.032	0.033	-0.001	-0.023	0.001	0.023	1.023
18	24	0.052	0.047	0.004	0.084	0.004	0.084	0.916
19	22	0.037	0.028	0.008	0.222	0.008	0.222	0.778
20	23	0.057	0.057	*	0.008	*	0.008	0.992
21	12	0.091	0.104	-0.013	-0.143	0.013	0.143	1.143
25	9	0.053	0.061	-0.007	-0.140	0.007	0.140	1.140
26	24	0.063	0.070	-0.007	-0.109	0.087	0.109	1.109
27	24	0.088	0.093	-0.005	-0.059	0.005	0.059	1.059
28	24	0.075	0.061	0.014	0.184	0.014	0.184	0.816
29	24	0.140	0.118	0.022	0.155	0.022	0.155	0.845
30	24	0.058	0.052	0.006	0.102	0.006	0.102	0.898
31	24	0.067	0.049	0.012	0.200	0.012	0.200	0.800
32	23	0.048	0.037	0.010	0.216	0.010	0.216	0.784
33	23	0.066	0.045	0.016	0.285	0.018	0.285	0.715
34	24	0.094	0.076	0.018	0.196	0.018	0.196	0.804
35	23	0.123	0.102	0.021	0.172	0.021	0.172	0.828
36	24	0.041	0.041	-0.001	-0.026	0.001	0.026	1.026
37	24	0.019	0.024	-0.005	-0.261	0.005	0.261	1.261
38	24	0.052	0.045	0.007	0.137	0.007	0.137	0.863
39	24	0.025	0.027	-0.002	-0.060	0.002	0.060	1.060
40	23	0.040	0.033	0.007	0.182	0.007	0.132	0.818
41	24	0.071	0.065	0.005	0.075	0.005	0.075	0.925
42	12	0.046	0.032	0.014	0.306	0.014	0.306	0.694
43	21	0.076	0.086	-0.009	-0.120	0.009	0.120	1.120
44	12	0.077	0.063	0.014	0.184	0.014	0.184	0.816
all	895	0.058	0.056	0.002	0.020	0.008	0.146	0.980

1st term - integrated radiation term

2nd term - linearized radiation term

day	#hours avgd	1st	2nd	diff	<u>diff</u> 1st	abs	<u>abs</u> 1st	2nd 1st
1	10	-0.010	-0.012	0.002	-0.187	0.002	0.187	1.187
2	24	-0.001	-0.002	0.001	-1.275	0.001	1.275	2.275
3	23	0.021	0.013	0.003	0.141	0.003	0.141	0.859
4	23	0.023	0.021	0.002	0.087	0.002	0.007	0.913
5	24	0.031	0.028	0.003	0.069	0.003	0.089	0.911
6	24	0.029	0.024	0.005	0.179	0.005	0.179	0.821
7	23	0.022	0.017	0.005	0.219	0.005	0.219	0.781
8	23	0.031	0.027	0.003	0.107	0.003	0.107	0.893
9	24	0.024	0.020	0.003	0.143	0.003	0.143	0.857
10	24	0.009	0.008	0.001	0.087	0.001	0.087	0.913
11	21	0.041	0.038	0.003	0.067	0.003	0.067	0.933
12	22	0.040	0.034	0.005	0.138	0.003	0.138	0.862
13	23	0.033	0.026	0.006	0.192	0.006	0.192	0.808
14	23	0.033	0.027	0.006	0.186	0.006	0.186	0.814
15	24	0.039	0.036	0.003	0.079	0.003	0.079	0.921
16	24	0.018	0.015	0.004	0.202	0.004	0.202	0.798
17	21	0.022	0.019	0.003	0.128	0.003	0.128	0.872
18	24	0.038	0.034	0.004	0.102	0.004	0.102	0.898
19	22	0.035	0.030	0.005	0.130	0.005	0.130	0.870
20	23	0.030	0.027	0.003	0.096	0.003	0.096	0.904
21	12	0.014	0.012	0.001	0.105	0.001	0.105	0.895
25	9	-0.007	-0.008	0.001	-0.159	0.001	0.159	1.159
26	24	0.012	0.011	0.001	0.096	0.001	0.096	0.904
27	24	0.022	0.020	0.003	0.115	0.003	0.115	0.885
28	24	0.025	0.021	0.004	0.167	0.004	0.167	0.833
29	24	0.028	0.024	0.004	0.145	0.004	0.145	0.855
30	24	0.033	0.031	0.002	0.073	0.002	0.073	0.927
31	24	0.023	0.019	0.004	0.186	0.004	0.186	0.814
32	23	0.035	0.028	0.007	0.205	0.007	0.205	0.795
33	23	0.038	0.030	0.007	0.197	0.007	0.197	0.803
34	24	0.047	0.038	0.009	0.200	0.009	0.200	0.800
35	23	0.044	0.039	0.005	0.109	0.005	0.109	0.391
36	24	0.048	0.045	0.004	0.075	0.004	0.075	0.925
37	24	0.018	0.017	0.001	0.074	0.001	0.074	0.926
38	24	0.050	0.046	0.004	0.078	0.004	0.078	0.922
39	24	0.027	0.024	0.003	0.095	0.003	0.095	0.905
40	23	0.051	0.045	0.006	0.122	0.006	0.122	0.878
41	24	0.040	0.037	0.004	0.092	0.004	0.092	0.908
42	12	0.052	0.042	0.009	0.181	0.009	0.181	0.819
43	21	0.012	0.010	0.002	0.164	0.002	0.164	0.836
44	12	0.062	0.057	0.005	0.078	0.005	0.078	0.922
all	895	0.029	0.025	0.004	0.086	0.004	0.086	0.914

APPENDIX C. NONLINEAR TERMS

The following pages contain the daily linear and nonlinear terms of the radiation (3.38) and the original (3.42) and modified (3.45) aerodynamic expressions, mm/hr. Total is the sum of the linear and nonlinear terms which is the daily average.

Radiation Expression

day	#hours avgd	total	linear term (1)	nonlinear term (2)	$\frac{(2)}{(1)}$
1	10	-0.010	-0.012	0.002	-0.156
2	24	-0.001	-0.002	0.001	-0.559
3	23	0.022	0.018	0.003	0.167
4	23	0.023	0.021	0.002	0.096
5	24	0.031	0.029	0.003	0.100
6	24	0.030	0.024	0.005	0.223
7	23	0.022	0.017	0.005	0.285
8	23	0.031	0.027	0.003	0.124
9	24	0.024	0.020	0.003	0.170
10	24	0.009	0.008	0.001	0.096
11	21	0.041	0.039	0.003	0.073
12	22	0.040	0.034	0.006	0.164
13	23	0.033	0.027	0.006	0.242
14	23	0.033	0.027	0.006	0.234
15	24	0.039	0.036	0.003	0.087
16	24	0.018	0.015	0.004	0.257
17	21	0.022	0.019	0.003	0.148
18	24	0.038	0.034	0.004	0.115
19	22	0.035	0.030	0.005	0.151
20	23	0.030	0.027	0.003	0.108
21	12	0.014	0.012	0.001	0.118
25	9	-0.007	-0.008	0.001	-0.136
26	24	0.012	0.011	0.001	0.108
27	24	0.023	0.020	0.003	0.131
28	24	0.025	0.021	0.004	0.204
29	24	0.028	0.024	0.004	0.173
30	24	0.033	0.031	0.002	0.080
31	24	0.023	0.019	0.004	0.231
32	23	0.035	0.028	0.007	0.262
33	23	0.038	0.030	0.008	0.251
34	24	0.048	0.038	0.010	0.257
35	23	0.044	0.039	0.005	0.125
36	24	0.049	0.045	0.004	0.083
37	24	0.019	0.017	0.001	0.080
38	24	0.051	0.047	0.004	0.087
39	24	0.027	0.024	0.003	0.108
40	23	0.052	0.045	0.007	0.144
41	24	0.041	0.037	0.004	0.104
42	12	0.052	0.042	0.010	0.227
43	21	0.012	0.010	0.002	0.198
44	12	0.063	0.058	0.005	0.085
all	895	0.030	0.026	0.004	0.130

Original Aerodynamic Expression

day	#hours avgd	total	linear term (1)	nonlinear term (2)	(2) (1)	nonlinear term (3)	(3) (1)
1	10	0.078	0.079	0.004	0.049	-0.001	-0.011
2	24	0.137	0.136	0.014	0.104	-0.003	-0.024
3	23	0.109	0.107	0.011	0.101	-0.003	-0.029
4	23	0.061	0.072	-0.008	-0.107	-0.001	-0.008
5	24	0.068	0.072	0.001	0.014	-0.002	-0.024
6	24	0.030	0.034	*	0.006	*	-0.006
7	23	0.058	0.063	0.005	0.087	-0.002	-0.026
8	23	0.049	0.046	0.008	0.165	-0.002	-0.034
9	24	0.021	0.022	0.001	0.040	*	-0.006
10	24	0.025	0.024	0.002	0.090	*	-0.003
11	21	0.036	0.036	0.004	0.124	-0.001	-0.015
12	22	0.055	0.057	0.006	0.112	-0.001	-0.017
13	23	0.054	0.053	0.010	0.193	-0.002	-0.035
14	23	0.041	0.044	0.004	0.090	-0.001	-0.018
15	24	0.059	0.056	0.007	0.132	-0.001	-0.018
16	24	0.038	0.039	0.002	0.062	*	-0.011
17	21	0.035	0.035	0.002	0.074	-0.001	-0.015
18	24	0.049	0.051	0.002	0.042	*	-0.006
19	22	0.030	0.032	0.003	0.090	*	-0.005
20	23	0.062	0.061	0.007	0.118	-0.001	-0.023
21	12	0.106	0.106	0.001	0.012	*	-0.003
25	9	0.068	0.064	0.010	0.159	-0.002	-0.030
26	24	0.078	0.075	0.010	0.136	-0.002	-0.027
27	24	0.098	0.098	0.003	0.034	-0.001	-0.013
28	24	0.069	0.067	0.011	0.158	-0.002	-0.025
29	24	0.132	0.128	0.019	0.148	-0.005	-0.039
30	24	0.057	0.053	0.006	0.115	-0.001	-0.009
31	24	0.053	0.054	0.006	0.119	-0.001	-0.019
32	23	0.042	0.041	0.007	0.177	-0.001	-0.034
33	23	0.048	0.051	0.005	0.107	-0.001	-0.023
34	24	0.084	0.086	0.015	0.180	-0.004	-0.045
35	23	0.109	0.110	0.010	0.087	-0.001	-0.010
36	24	0.043	0.045	*	0.004	*	-0.002
37	24	0.027	0.026	0.003	0.112	*	-0.010
38	24	0.050	0.049	0.006	0.115	-0.001	-0.014
39	24	0.025	0.030	-0.003	-0.085	*	0.014
40	23	0.037	0.038	0.006	0.158	-0.001	-0.026
41	24	0.073	0.070	0.009	0.128	-0.002	-0.029
42	12	0.037	0.037	0.008	0.229	-0.001	-0.031
43	21	0.089	0.092	0.002	0.018	*	*
44	12	0.067	0.066	0.005	0.074	*	-0.008
all	895	0.060	0.061	0.006	0.091	-0.001	-0.018

* <0.001

Original Aerodynamic Expression (cont.)

day	nonlinear term (4)	(4) (1)	nonlinear term (5)	(5) (1)
1	-0.003	-0.043	-0.001	-0.006
2	-0.010	-0.074	*	0.001
3	-0.006	-0.052	*	-0.002
4	-0.003	-0.039	*	*
5	-0.003	-0.039	*	-0.003
6	-0.004	-0.115	*	0.004
7	-0.008	-0.126	*	-0.004
8	-0.003	-0.062	*	-0.009
9	-0.002	-0.092	*	*
10	-0.001	-0.036	*	-0.003
11	-0.003	-0.085	*	-0.004
12	-0.007	-0.124	-0.001	-0.012
13	-0.007	-0.124	-0.001	-0.022
14	-0.006	-0.129	*	-0.005
15	-0.003	-0.048	*	-0.006
16	-0.004	-0.092	*	-0.002
17	-0.002	-0.058	*	-0.006
18	-0.003	-0.068	*	-0.002
19	-0.004	-0.116	-0.001	-0.017
20	-0.004	-0.068	*	0.005
21	-0.002	-0.016	*	*
25	-0.003	-0.049	-0.001	-0.014
26	-0.005	-0.064	*	-0.002
27	-0.002	-0.022	*	-0.001
28	-0.006	-0.093	-0.001	-0.013
29	-0.009	-0.069	*	-0.003
30	-0.001	-0.024	*	-0.008
31	-0.006	-0.107	-0.001	-0.011
32	-0.005	-0.122	*	-0.007
33	-0.007	-0.128	*	-0.006
34	-0.012	-0.141	-0.001	-0.017
35	-0.009	-0.079	-0.001	-0.007
36	-0.003	-0.059	*	0.006
37	-0.001	-0.048	*	-0.003
38	-0.003	-0.070	*	-0.003
39	-0.003	-0.106	*	-0.002
40	-0.005	-0.138	*	-0.011
41	-0.004	-0.057	*	-0.006
42	-0.005	-0.146	-0.001	-0.037
43	-0.005	-0.050	*	0.001
44	-0.002	-0.035	*	-0.005
all	-0.005	-0.080	*	-0.005

Modified Aerodynamic Expression

day	#hours avgd	total	linear term (1)	nonlinear term (2)	(2) (1)	nonlinear term (3)	(3) (1)
1	10	0.052	0.043	0.003	0.076	0.005	0.107
2	24	0.127	0.108	0.015	0.136	0.010	0.090
3	23	0.095	0.074	0.010	0.140	0.009	0.117
4	23	0.047	0.047	-0.008	-0.169	*	-0.006
5	24	0.050	0.033	0.001	0.027	0.001	0.030
6	24	0.034	0.024	*	0.014	0.019	0.785
7	23	0.051	0.032	0.005	0.148	0.012	0.369
8	23	0.047	0.026	0.007	0.271	0.008	0.323
9	24	0.024	0.016	0.001	0.094	0.010	0.601
10	24	0.018	0.010	0.002	0.174	0.005	0.488
11	21	0.035	0.025	0.005	0.180	0.006	0.228
12	22	0.056	0.039	0.007	0.168	0.012	0.317
13	23	0.054	0.030	0.009	0.304	0.013	0.423
14	23	0.043	0.026	0.004	0.154	0.014	0.534
15	24	0.056	0.041	0.008	0.184	0.007	0.159
16	24	0.037	0.026	0.003	0.106	0.009	0.369
17	21	0.032	0.024	0.003	0.108	0.006	0.245
18	24	0.052	0.044	0.002	0.056	0.007	0.163
19	22	0.037	0.021	0.003	0.163	0.011	0.527
20	23	0.058	0.034	0.007	0.199	0.013	0.401
21	12	0.092	0.086	0.001	0.017	0.005	0.062
25	9	0.054	0.031	0.008	0.267	0.009	0.280
26	24	0.064	0.035	0.008	0.237	0.011	0.310
27	24	0.089	0.082	0.004	0.044	0.002	0.029
28	24	0.076	0.049	0.011	0.227	0.015	0.304
29	24	0.142	0.111	0.022	0.198	0.014	0.130
30	24	0.058	0.043	0.007	0.169	0.005	0.106
31	24	0.062	0.042	0.007	0.175	0.013	0.304
32	23	0.048	0.027	0.008	0.312	0.013	0.485
33	23	0.064	0.044	0.009	0.205	0.018	0.406
34	24	0.095	0.063	0.017	0.268	0.020	0.312
35	23	0.125	0.104	0.012	0.112	0.016	0.153
36	24	0.042	0.038	*	0.006	0.006	0.165
37	24	0.019	0.016	0.003	0.178	0.001	0.080
38	24	0.053	0.042	0.006	0.156	0.006	0.133
39	24	0.026	0.021	-0.003	-0.154	0.009	0.439
40	23	0.041	0.022	0.007	0.332	0.012	0.565
41	24	0.072	0.045	0.009	0.200	0.012	0.278
42	12	0.047	0.021	0.009	0.441	0.017	0.814
43	21	0.078	0.070	0.002	0.024	0.006	0.085
44	12	0.077	0.066	0.006	0.093	0.006	0.097
all	895	0.059	0.043	0.006	0.145	0.010	0.289

Modified Aerodynamic Expression (cont.)

day	nonlinear term (4)	(4) (1)	nonlinear term (5)	(5) (1)	nonlinear term (6)	(6) (1)
1	0.004	0.102	0.002	0.048	-0.002	-0.043
2	0.008	0.077	*	-0.001	-0.008	-0.074
3	0.011	0.154	0.002	0.028	-0.004	-0.052
4	0.010	0.209	0.003	0.064	-0.002	-0.039
5	0.018	0.537	0.005	0.148	-0.001	-0.039
6	0.002	0.063	-0.004	-0.156	-0.003	-0.115
7	0.012	0.364	0.004	0.127	-0.004	-0.126
8	0.010	0.396	0.002	0.087	-0.002	-0.062
9	0.002	0.115	*	-0.015	-0.001	-0.092
10	0.002	0.207	0.001	0.089	*	-0.036
11	0.004	0.161	*	-0.007	-0.002	-0.085
12	0.007	0.168	0.003	0.071	-0.005	-0.124
13	0.010	0.348	0.005	0.170	-0.004	-0.124
14	0.006	0.239	0.002	0.086	-0.003	-0.129
15	0.006	0.147	*	0.007	-0.002	-0.048
16	0.004	0.145	0.001	0.028	-0.002	-0.092
17	0.003	0.137	*	0.005	-0.001	-0.058
18	0.004	0.093	*	0.001	-0.003	-0.068
19	0.005	0.237	0.003	0.140	-0.002	-0.116
20	0.011	0.307	-0.001	-0.021	-0.002	-0.068
21	0.002	0.020	*	0.002	-0.001	-0.016
25	0.010	0.322	0.005	0.159	-0.002	-0.049
26	0.015	0.415	0.003	0.083	-0.002	-0.064
27	0.006	0.068	0.001	0.014	-0.002	-0.022
28	0.010	0.202	0.004	0.074	-0.005	-0.093
29	0.014	0.123	0.002	0.014	-0.008	-0.069
30	0.006	0.144	0.001	0.021	-0.001	-0.024
31	0.008	0.200	0.002	0.046	-0.004	-0.107
32	0.009	0.330	0.001	0.053	-0.003	-0.122
33	0.007	0.166	0.001	0.012	-0.006	-0.128
34	0.017	0.272	0.003	0.053	-0.009	-0.141
35	0.007	0.071	0.002	0.015	-0.008	-0.079
36	0.003	0.069	-0.001	-0.036	-0.002	-0.059
37	0.001	0.075	*	0.004	-0.001	-0.048
38	0.004	0.085	0.001	0.021	-0.003	-0.070
39	0.003	0.137	*	-0.012	-0.002	-0.106
40	0.006	0.262	0.002	0.107	-0.003	-0.138
41	0.014	0.320	0.001	0.026	-0.003	-0.057
42	0.007	0.313	0.007	0.329	-0.003	-0.146
43	0.005	0.076	-0.002	-0.023	-0.004	-0.050
44	0.002	0.032	0.001	0.015	-0.002	-0.035
<u>all</u>	0.007	0.195	0.001	0.041	-0.003	-0.080

Modified Aerodynamic Term (cont.)

day	nonlinear term (7)	(7) (1)	nonlinear term (8)	(8) (1)	nonlinear term (9)	(9) (1)
1	-0.001	-0.017	*	-0.010	-0.001	-0.025
2	-0.003	-0.032	*	0.001	-0.002	-0.022
3	-0.003	-0.041	*	-0.003	-0.002	-0.034
4	-0.001	-0.012	*	0.001	-0.001	-0.029
5	-0.001	-0.046	*	-0.006	-0.002	-0.065
6	*	-0.013	*	0.008	-0.004	-0.147
7	-0.001	-0.044	*	-0.007	-0.003	-0.088
8	-0.001	-0.056	*	-0.014	-0.002	-0.069
9	*	-0.014	*	*	-0.002	-0.101
10	*	-0.007	*	-0.007	*	-0.031
11	*	-0.021	*	-0.006	-0.001	-0.027
12	-0.001	-0.025	-0.001	-0.017	-0.002	-0.048
13	-0.002	-0.055	-0.001	-0.034	-0.002	-0.076
14	-0.001	-0.030	*	-0.009	-0.002	-0.089
15	-0.001	-0.025	*	-0.009	-0.001	-0.027
16	*	-0.018	*	-0.004	-0.002	-0.061
17	-0.001	-0.022	*	-0.009	-0.001	-0.033
18	*	-0.009	*	-0.003	-0.001	-0.020
19	*	-0.009	-0.001	-0.031	-0.001	-0.062
20	-0.001	-0.038	*	0.009	-0.003	-0.077
21	*	-0.004	*	*	-0.001	-0.013
25	-0.002	-0.051	-0.001	-0.024	-0.002	-0.055
26	-0.002	-0.046	*	-0.004	-0.002	-0.059
27	-0.001	-0.017	*	-0.001	-0.001	-0.014
28	-0.002	-0.036	-0.001	-0.018	-0.002	-0.050
29	-0.006	-0.052	*	-0.004	-0.004	-0.037
30	-0.001	-0.013	-0.001	-0.012	-0.001	-0.017
31	-0.001	-0.029	-0.001	-0.016	-0.002	-0.049
32	-0.002	-0.061	*	-0.013	-0.002	-0.090
33	-0.002	-0.044	*	-0.011	-0.004	-0.094
34	-0.004	-0.067	-0.002	-0.025	-0.005	-0.081
35	-0.001	-0.013	-0.001	-0.009	-0.002	-0.023
36	*	-0.003	*	0.009	-0.001	-0.026
37	*	-0.016	*	-0.004	*	-0.015
38	-0.001	-0.019	*	-0.004	-0.001	-0.017
39	0.001	0.025	*	-0.004	-0.001	-0.058
40	-0.001	-0.055	-0.001	-0.023	-0.002	-0.102
41	-0.002	-0.045	*	-0.009	-0.003	-0.064
42	-0.001	-0.061	-0.001	-0.071	-0.002	-0.118
43	*	0.001	*	0.002	-0.001	-0.011
44	-0.001	-0.009	*	-0.006	-0.001	-0.010
all	-0.001	-0.028	*	-0.009	-0.002	-0.053

Modified Aerodynamic Expression (cont.)

day	nonlinear term (10)	(10) (1)	nonlinear term (11)	(11) (1)	nonlinear term (12)	(12) (1)
1	*	-0.010	-0.001	-0.012	*	-0.008
2	*	0.003	*	*	-0.001	-0.006
3	*	-0.004	*	-0.006	-0.001	-0.013
4	*	0.006	*	-0.008	*	-0.005
5	*	0.001	-0.001	-0.034	-0.001	-0.017
6	-0.002	-0.086	0.001	0.021	*	0.011
7	-0.001	-0.045	-0.001	-0.036	-0.002	-0.048
8	*	-0.015	-0.001	-0.020	-0.001	-0.028
9	-0.001	-0.040	*	0.001	*	-0.016
10	*	-0.027	*	-0.006	*	-0.010
11	*	-0.010	*	-0.001	*	-0.012
12	-0.001	-0.025	*	-0.012	-0.001	-0.022
13	-0.001	-0.048	-0.001	-0.031	-0.002	-0.056
14	-0.002	-0.060	-0.001	-0.019	-0.001	-0.029
15	*	-0.007	*	-0.003	*	-0.009
16	*	-0.013	*	-0.006	*	-0.012
17	*	-0.016	*	0.004	*	-0.015
18	*	-0.010	*	-0.001	*	-0.005
19	-0.001	-0.042	*	-0.023	*	-0.022
20	*	*	*	0.006	-0.001	-0.030
21	*	-0.004	*	-0.001	*	*
25	-0.001	-0.022	-0.001	-0.029	-0.001	-0.035
26	*	-0.011	-0.001	-0.015	-0.001	-0.035
27	*	*	*	-0.003	*	-0.002
28	-0.001	-0.018	-0.001	-0.012	-0.001	-0.023
29	*	*	*	-0.004	-0.002	-0.014
30	*	-0.003	*	-0.004	*	-0.004
31	-0.001	-0.022	*	-0.009	-0.001	-0.018
32	-0.001	-0.034	*	-0.010	-0.001	-0.048
33	-0.001	-0.020	*	-0.002	-0.001	-0.026
34	-0.002	-0.025	-0.001	-0.013	-0.003	-0.044
35	-0.001	-0.013	*	-0.003	-0.001	-0.006
36	*	-0.010	*	0.005	*	-0.003
37	*	0.002	*	-0.001	*	-0.003
38	*	-0.003	*	-0.002	*	-0.007
39	-0.001	-0.027	*	-0.004	*	-0.001
40	-0.001	-0.026	*	-0.011	-0.001	-0.056
41	-0.001	-0.011	*	-0.002	-0.001	-0.024
42	-0.002	-0.104	-0.001	-0.042	-0.002	-0.113
43	*	-0.001	*	0.003	*	-0.003
44	*	-0.007	*	-0.001	*	-0.002
all	-0.001	-0.019	*	-0.008	-0.001	-0.019