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Title: PREDICTED ECONOMIC EFFECTS OF ENVIRONMENTAL QUALITY CONTROL POLICIES ON LINEAR FIRM MODELS AND AN APPLICATION TO AN IRRIGATED FARM MODEL

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Herbert H. Stoevener

Linear economic models were utilized to predict effects of various environmental control policies on individual firms. Four different linear models were specified and in some instances relatively minor changes in specification were made which resulted in additional sub-models. Models varied as to numbers and types of fixed factors, variable cost relationships, market products, and fixed factor requirements. Once each model or sub-model was described five or six policies were theoretically applied to that model. Policies used were: taxing market products, taxing variable factors, taxing a non-market externality (external diseconomy), a standard on the quality of the externality, subsidizing variable factors and subsidizing fixed factors.

It was assumed that the non-market externality would be produced in a fixed ratio with market products. Furthermore, the

assumption was made that alternative production techniques were available to the firm. The important aspect of the various techniques was that the proportion of the externality generated by a market product varied by production method. Consequently, strong emphasis in the analysis was placed on determining whether or not a given policy could induce the firm to switch to a lower externality generating production method.

In addition to the strictly theoretical analysis a linear irrigated farm model was described. The farm model produced irrigation return flows which were considered to be creating stream pollution. From the theoretical analysis likely policies for controlling return flows were ascertained. Some of these policies were then applied to the farm model. Specifically, a water tax (variable factor tax) and a constraint on delivered water were administered to the farm model.

Based on the theoretical analysis taxing market products did not appear to be a particularly desirable policy. For some models, the market product tax actually increased externality production. A tax on externality production (effluent tax) seemed to give the most consistent effects of all policies across all models. The externality tax either reduced or had no effect on externality production. The biggest shortcoming of the externality tax appeared to be administrative. Before the tax can be used the externality must be identifiable as to source. Consequently, a search was made for policies which generated results similar to the externality tax yet were not subject to the same administrative problem. It appeared that under specific conditions a variable factor tax, a tax on specialized fixed factors or a combination of a tax-subsidy scheme could be effective alternative policies. However, these latter policies, if improperly applied could result in increased externality production.

Taxes as high as 65 cents per acre inch of water were applied to the farm model. Depending on assumed conditions the water tax resulted in reduced irrigation return flows. When labor was constrained tax levels needed to be higher to reduce return flows compared to the case where labor was not constrained. Placing a restraint on delivered water also reduced return flows. Again, when labor was constrained this policy was not as effective as when labor was unconstrained. The water tax policy reduced net returns to the farm model considerably more than the constraint on delivered water. The main difference in net revenues was attributable to the total water tax bill rather than reductions from other added costs and/or enterprise changes.

Predicted Economic Effects of Environmental Quality Control Policies on Linear Firm Models and an Application to an Irrigated Farm Model

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Head of Department of Agricultural Economics

Dean of Graduate School

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I. INTRODUCTION

Increasing concern about environmental quality has brought with it intensified interest in the institutions and policies which could be used to affect such quality. Discussions concerning the environment have been wrought with emotionalism, charges and counter-charges. Although this rhetoric is at times quite interesting, it often contributes little towards solving or even identifying environmental problems.

Economists at least as far back as Pigou (29, Chapter 9) in 1920 have advocated policies such as taxing to alleviate problems associated with externalities.[1] Many of the writers have been mainly concerned with utilizing policies to achieve a social optimum. Kneese, for example, has written extensively in the area of water quality economics and has been quite concerned with the proper levels of pollution (non-pollution) (24).

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[1] Externalities are also known as external effects or spillovers. An externality exists "Any time provision of a good or service provides side effects whose value is not reflected in the price of the outputs sold or the resource used, . . ." (33, p. 18). This definition is also consistent with Baumol (5, p. 25).

Economists have also been interested in measuring the benefits that would flow from an "improvement" in some phase of environmental quality.[2] Such work has often involved the researchers in evaluating recreation benefits.

The Problem

Even though economists have long advocated taxation schemes to internalize externalities such policies have not been extensively used in the United States. Tax incentives have been used but mainly for installation of treatment equipment (24, p. 177). Unless the incentives more than pay for the cost of installing and operating the equipment it is difficult to see how the incentives alone can be effective. Industrial sewer charges have also been used by municipalities (24, p. 170). Industries responded to these charges in some instances by changing internal processes to reduce and alter effluent. Sewer charges are a form of an effluent tax advocated by many economists for internalizing externalities.

Kneese and Bower cite several examples of how industries respond to sewer use charges. They summarize their findings as follows:

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[2] See for example (20, 34 and 35).

> First, the imposition of a charge or surcharge tends to encourage plants to make changes that in many cases reduce not only the volume of effluents and the wastes in the effluents but also the water intake. Second, sewer charges tend to induce an examination of production processes that often uncovers relatively simple modifications which may result in net reductions in total production costs (24, p. 170).

They go on to discuss in-plant charges. From their research they conclude that responses to in-plant charges are very similar to responses to sewer charges. In addition to their discussion concerning responses to sewer use charges, Kneese and Bower also discuss the relative merits of various methods of pollution control. In comparing effluent charges to effluent standards they state,

> In a dynamic context, charges have the advantage of exerting a continuous pressure on the waste discharger to improve his waste handling technology. Under an effluent standards system, the waste discharger has no incentive to do more than meet the standard (24, p. 139).

They further point out that tax incentives such as rapid write-off on pollution control equipment and subsidies for the same type of equipment are not very effective. They conclude that such tax policies are potentially costly to the taxpayer and they are also likely to induce inefficient control. The inefficient control comes about since much of the tax legislation is specifically for the installation of treatment equipment whereas it may be cheaper if the firm were to make internal process changes rather than actually treat the effluent. Kneese and Bower arrive at the following conclusion concerning likely policy alternatives for controlling individual waste discharges:

> Our study leads us to the conclusion that the nation should give serious consideration to reorienting its policies toward effluent charges as a component of broader systems of regional water quality management and in turn as a component of over-all water resources management (24, p. 178).

It is necessary to point out that their discussion which has been summarized above related to the control and effects of such control for individual units, not entire river systems.

While studying the economics of water use in the beet sugar industry, Löf and Kneese (26) concluded that it would require only a small external stimulus in the form of either an effluent charge or standard to induce use of procedures that eliminate much of the biochemical oxygen demand (BOD) load in the effluent of these factories. They discovered from their study a clear and strong tendency for technical changes in this industry which reduced waste loads and water intake per unit of product. This is important from two standpoints. One is the reduced pollution load, and the other is the reduced intake of fresh water. Löf and Kneese also point out the dynamic aspects of internal technologies that need to be considered in policy formulation. They state,

> . . . a control method such as an effluent charge, which puts continuous pressure on the waste discharger to reduce the amount of waste emitted, can have desirable longer-run effects on the development of technology (26, p. 87).

Ruttan's work concerning induced innovation is particularly appropriate here. He argues that reallocation of resources due to price changes is not limited to movements between known technical alternatives as indicated by the neoclassical production function (31, p. 710). Rather, the firm can allocate resources to research for developing new techniques which may " . . . expand the scope for factor substitution along a perceived innovation possibility frontier . . ." (31, p. 711). If one defines a neoclassical production function as generating only one value for the output from a given combination of inputs, then " . . . the function must be so defined that it expresses the _maximum product_ obtainable from the combination at the existing state of technical knowledge" (12, p. 16). Using the latter definition for a production function leads to the implication that the innovation process alluded to by Ruttan may also just be a "rediscovery" of techniques already known but not generating the maximum output of the _marketable_ products. The latter approach is used in this study. That is, it is assumed that when the firm is permitted to ignore its externalities, it views the production function only with respect to its output of market products. When forced to consider the non-market externalities via taxes or other policies, the firm may switch to a technique already known but one that is less efficient in terms of the market products only.

Ruttan further suggests that the environmental movement ". . . is contributing to the creation of a social and political environment in which it may become feasible to more adequately institutionalize the redirection of technological effort. . ." (31, p. 713). In essence the approach taken in this study with respect to policies reflects attempts via institutions (including the market system) to redirect technical effort.

A recent study in North Carolina also indicates that firms do make internal adjustments to sewer use charges. Ethridge stated,

> Surcharges appear to induce poultry processing firms to use less water, and the pounds of wastes discharged by poultry processing firms also appear to be responsive to the cost of water (17, p. 78).

One implication of the above statement might be that charges on certain types of inputs may also be an effective means of controlling pollution.

Mills, in discussing incentives for controlling air pollution, makes a strong argument for the use of discharge or effluent fees as a means of controlling air pollution (28). He also points out that a fee system may be cheaper to administer than some other types of controls. The reason for the latter is that a charge is basically self-administering; however, there does not appear to be a general consensus in the literature concerning that point. In discussing various schemes, Mills states "For example, an excise tax on coal is less desirable than a tax on the discharge of pollutant resulting from burning coal because the former distorts resource use in favor of other fuels and against devices to remove pollutants from stack gases after burning coal"

(28, p. 103). This statement bears consideration, particularly where inputs may serve as substitutes for one another, yet the substitutes also have capabilities of producing pollution.

A conclusion which appears consistently in the literature reviewed is that effluent charges in some form or another are probably one of the better means of controlling pollution. This author has no basic argument with that concept; however, most of the articles, previously reviewed, fail to point out that before an effluent charge can be levied the effluent must be identifiable as to source. The particular type of pollution which will be considered later in this study, i.e., agricultural return flow pollution, does not readily lend itself to such identification. Consequently, a charges system of some type other than an effluent tax needs to be considered. For example, taxes on various types of inputs and output might produce results similar to effluent charges.

Another thing that the discussed studies do not do is to establish or at least point out theoretically how various types of controls will affect the individual firm. Ethridge (17) comes the closest to establishing a theoretical framework. He develops a theoretical demand equation for excess municipal treatment and total municipal treatment. Excess municipal treatment is the pounds of BOD per unit of time over and above the pounds of BOD allowed as "normal" waste. He then empirically fits these demand equations and derives elasticities with respect to the sewer surcharge. This then gives one an idea of the magnitude of response to changes in the surcharge rate, but it does not indicate how the firms might adjust internally. Whether or not proposed policies for controlling pollution from individual firms is going to be effective is quite dependent on how such firms react to the various policies.

Plott in an article in Economica points out that certain types of taxes placed for the purpose of internalizing externalities may in fact cause the opposite of the desired effect (30). Plott's argument depends on the concept of an inferior factor. Plott assumed the following model:

$$q_1 = f(X_1, X_2) \text{ and } q_2 = h(X_2)$$

where q_1 is a market product, q_2 is a non-market externality produced "jointly" and q_1,[3] $f(X_1, X_2)$ is the two factor production function for q_1, $h(X_2)$ is the single factor production function for q_2, and X_1 and X_2 are inputs. Since he assumed q_2 to be an external diseconomy, one suggestion for reducing q_2 would be to tax q_1. However, the following will show that such an action could increase the production of q_2.

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[3] This situation does not exactly fit a strict definition of joint production, e.g., found in Henderson and Quandt (21, p. 67-72) since q_2 could be produced without q_1 but not vice-versa. The same results can be demonstrated, however, using an implicit function, e.g., $H(q_1, q_2, X_1, X_2) = 0$.

The profit equation to be maximized is

$$\pi = p_1q_1 + p_2q_2 - r_1x_1 - r_2x_2 \tag{1}$$

where the p_i's are product prices and the r_i's are factor prices.

First order maximization conditions are

$$\frac{\partial \pi}{\partial X_1} = p_1 \frac{\partial f}{\partial X_1} - r_1 = 0 \tag{2}$$

$$\frac{\partial \pi}{\partial X_2} = p_1 \frac{\partial f}{\partial X_2} + p_2 \frac{\partial h}{\partial X_2} - r_2 = 0.$$

Second order conditions are

$$\frac{\partial^2 \pi}{\partial X_1^2} < 0; \quad \begin{vmatrix} \frac{\partial^2 \pi}{\partial X_1^2} & \frac{\partial^2 \pi}{\partial X_1 \partial X_2} \\ \frac{\partial^2 \pi}{\partial X_2 \partial X_1} & \frac{\partial^2 \pi}{\partial X_2^2} \end{vmatrix} > 0$$

$$\longrightarrow \quad f_{11} < 0; \quad \begin{vmatrix} p_1f_{11} & p_1f_{12} \\ p_1f_{12} & p_1f_{12} + p_2h_{22} \end{vmatrix} > 0 \tag{3}$$

where $f_{11} \equiv \frac{\partial^2 f}{\partial X_1^2}$ etc.

$$\longrightarrow \quad f_{11} < 0; \quad f_{11}(f_{22} + h_{22}\frac{p_2}{p_1}) - f_{12}^2 = |D| > 0.$$

The question of interest is what happens when the price of p_1 changes? The partials of the necessary conditions with respect to p_1 are

$$f_1 + p_1 \left(f_{11}\frac{\partial X_1}{\partial p_1} + f_{12}\frac{\partial X_2}{\partial p_1}\right) = 0$$

$$f_2 + p_1 \left(f_{12}\frac{\partial X_1}{\partial p_1} + f_{22}\frac{\partial X_2}{\partial p_1}\right) + p_2 h_{22}\frac{\partial X_2}{\partial p_1} = 0. \tag{4}$$

assuming that $\frac{\partial p_2}{\partial p_1} = 0$.

By subtracting f_1 from both sides of the first equation in (4), f_2 from both sides of the second equation and dividing through both with p_1, (4) can be written as

$$\begin{bmatrix} f_{11} & f_{12} \\ f_{12} & f_{22} + \frac{p_2}{p_1} h_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial X_1}{\partial p_1} \\ \frac{\partial X_2}{\partial p_1} \end{bmatrix} = \begin{bmatrix} -p_1^{-1} f_1 \\ -p_1^{-1} f_2 \end{bmatrix} \tag{5}$$

Then by Cramer's rule

$$\frac{\partial X_1}{\partial p_1} = \frac{-p_1^{-1} f_1 (f_{22} + \frac{p_2}{p_1} h_{22}) + p_1^{-1} f_2 f_{12}}{|D|}$$

$$= \frac{f_2 f_{12} - f_1 (f_{22} + \frac{p_2}{p_1} h_{22})}{p_1 \ |D|} . \tag{6}$$

Second order conditions, (3), imply that $(f_{22} + \frac{p_2}{p_1} h_{22}) < 0$. Assuming that f_1, $f_2 > 0$ which implies positive marginal physical productivities of q_1 with respect to both factors, one can reach the following conclusions concerning the sign of $\frac{\partial X_1}{\partial p_1}$. If $f_{12} > 0$ then

$\frac{\partial X_1}{\partial p_1} > 0$ since $|D| > 0$ and $p_1 > 0$ by (3). If $f_{12} < 0$ then $\frac{\partial X_1}{\partial p_1} \gtrless 0$ depending on the relative sizes of the terms in the numerator of (6). If it should be that $\frac{\partial X_1}{\partial p_1} < 0$ then by Plott's definition X_1 is an inferior factor, i.e., when the price of the product declines the use of the inferior factor, X_1, increases.

Suppose now that a production tax which is to be paid by the producer is levied against q_1. This action in effect lowers the price of q_1, inducing increased use of X_1. Since $q_2 = h(X_1)$ and it is assumed $h_1 > 0$ the increase in X_1 will increase production of q_2, the non-market externality. The result of the tax on q_1 then is just the opposite of the desired effect, i.e., reducing q_2. Using a similar approach as above it can be shown that $\frac{\partial X_2}{\partial p_2} > 0$ regardless of the sign of f_{12}. The latter implies a tax on X_2 would decrease the production of q_2. Plott summarizes the main implication of his argument thusly, ". . . the first problem, that of determining just exactly what variable should be taxed, becomes just as difficult as the second problem, that of determining what the optimum tax should be" (30, p. 87).

Even though the literature strongly supports the concept of effluent charges for controlling pollution, the fact still remains as stated earlier that such a policy has not been instituted extensively in the United States. The use of standards and the police power still seem to be the prevalent methods of enforcing environmental quality. The water quality act of 1965 is a prime example with the resulting sets of state standards (40). Wyoming's air quality act specifically makes reference to the use of cease and desist orders and fines (16, p. 7 and 10). One of the reasons for the lack of use of various charge schemes may well be the problems that could occur as pointed out by Plott, even though his argument definitely supports effluent charges.

It should be pointed out, however, that there is increasing interest in the use of some form of economic incentives for controlling pollution. Recent hearings before a subcommittee of the Joint Economic Committee of Congress reflect such interest (41). A statement by the Honorable Les Aspin, Representative from Wisconsin, gives a strong argument for economic incentives (41, p. 1247-1251). Other statements were offered against economic incentives; however, increased interest at least was apparent in those hearings.

The traditional policies have been directed against the offender, the one creating the externality. Little apparent consideration has been given to the effects of policies on the offender in terms of costs and reaction.[4] If policies are to be effective, it is imperative to

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[4] Wyoming's Air Quality Act, however, does make specific mention that in setting standards, consideration be given to "the social and economic value of the source of the air pollution . . ." (16, p. 7). See page 22 for a discussion of Coase's contribution toward this concept.

know how individual firms will react to and be affected by alternative control measures. The problem then is that there is a lack of knowledge concerning expected effects on firms of various policies advocated for controlling environmental quality. It is the purpose of this study to examine conceptually and empirically how alternative policies may affect specific types of firms. The types of firms to be considered are those that operate in competitive markets and whose decisions can be represented by linear economic theory.

Objectives

The specific objectives are to:

1. Deductively predict effects and economic consequences of alternative quality control policies on individual firms whose decision frameworks can be depicted by linear economic models.
2. Determine whether or not given policies produce consistent predictions over alternative linear economic models.
3. Ascertain whether any of the policies considered result in predictions similar to predictions based on effluent charges. Also determine policies that could be used when the source of the effluent is not identifiable.
4. Apply policies established in (3) to an empirical linear farm model and estimate magnitudes of charges (if charges are

deemed feasible) necessary to induce changes in return flows.

5. Utilize the analysis of the theoretical and empirical models to suggest hypotheses regarding predicted effects of the various policies.

Procedures

The order for the presentation of the results in this thesis follows chronologically the objectives. Chapters II through V each present linear economic models to which the various control policies are theoretically applied. The models differ mainly in their complexity as to numbers of variable and fixed factors and market products. The policies that are applied to the various models are: 1) taxing market products, 2) taxing variable factors, 3) taxing the non-market externality, 4) placing a quantitative restriction (standard)[5] on the quality of the externality, 5) subsidizing variable factors, and 6) subsidizing fixed factors. These four chapters achieve objective one and provide the basis for completing objectives two and three. The results of objectives two and three are summarized in Chapter VI. Also, the policies that seem relevant for controlling irrigation return flow pollution are determined in Chapter VI, based on the theoretical

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[5] See footnote 19, p. 49 for the specific definition of the type of standard used.

development to that point.

Chapter VII consists of the empirical results to complete objective four. A linear program model was developed for a representative farm in one area in Wyoming. The assumptions and specific characteristics of this linear programming model are discussed at the beginning of Chapter VII. The farm model is also related to theoretical models developed in Chapters II through V.

Chapter VIII presents the conclusions and suggests hypotheses that might be tested. Where possible, likely test procedures are pointed out.

Environmental Quality, Pollution and Governmental Intervention

Communication between people working on related problems within and between disciplines could be expedited by defining common words and phrases. Research concerning environmental problems is one area that is hindered by lack of understanding among scientists. In this section the terms "environmental quality" and "pollution" are defined.

The phrase "environmental quality" is used quite often (more specifically, water and air quality), yet frequently is left undefined. Absence of a definition is not critical, provided that there exists a common, widely accepted definition; however, review of studies in various disciplines indicates that such is not the case. Most authors who define water and/or air quality do so in a descriptive sense. That is, quality is defined to be the characteristics that describe the water. These authors then go on to associate the characteristics with particular uses.[6] Others seem to imply that quality is quite difficult to define and/or wish to make it a value loaded term.[7]

Part of the confusion arises since quality can be used as a noun or as an adjective. When quality is used as a noun, its meaning is essentially value free, whereas when used as an adjective, quality implies excellence or good "quality."[8]

The following definition of environmental quality seems consistent with several definitions that appear in the literature. As defined, quality is used as a noun. Environment quality is the characteristics or attributes of the environment which when taken together represent a physical, biological, and chemical description of the environment. The definition is hopefully value free.

A term that has a greater variety of interpretations than environmental quality is "pollution." Pollution can mean either the "act" of polluting or the state of being polluted. The most common pollution referred to both popularly and academically is water pollution. It

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[6] See for example (3 and 4).

[7] See (22, p. 99, and 45, p. 89).

[8] See (46).

seems that when water pollution is defined, the definition usually refers to the "act" of polluting, rather than to the "state" of being polluted.[9] Some definitions of pollution[10] refer to "natural quality" or before "man's activities" as reference points for judging pollution.

If pollution refers to the "state" of being polluted, then there does seem to be a need for a reference point. Even in this latter case, "before man" does not provide a very workable reference point. Reference points could be in terms of either time or location. If water quality measurements have been made over a period of years, one might conclude, for example, that a certain stream is polluted compared to what it was ten years ago. It appears, however, to make such a statement it would be necessary to indicate known damage that had occurred in those ten years. Reference could also be made to different locations. That is, water in a stream ten miles above a given point may have a given water quality which is quite different from the point of interest. If it could be demonstrated that the change in water quality over the ten mile stretch caused damage to at least one use, then it would seem logical to call the water at the point of interest polluted.

To be usable in a discipline a term must be capable of being

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[9] See for example (4, p. 14; 24, p. 9; and 42, p. ix).

[10] See for example (3, p. 23 and 7, p. 36).

related to concepts and theories within that discipline. Consequently, for economists to utilize the word "pollution," it must be defined in terms they can understand and in such a way so as not to conflict with definitions of the word in other disciplines.

For this study pollution is defined in the active sense, i.e., the act of polluting. Pollution then is a human alteration of environmental quality that presently or in the foreseeable future negatively affects someone's utility and/or cost function. For example, if a change in environmental quality diminishes someone's utility, then it can be termed pollution.

Conceptually, it seems easier to visualize how a change in water quality could negatively affect a firm's cost function. For example, an increase in dissolved salts upstream may cause a farmer downstream either to alter his cropping pattern and/or to apply more water at a higher cost.

The connection between such characteristics as dissolved oxygen (DO) and temperature of water and individual utility is not quite as direct. Lancaster (25), however, provides a conceptual framework within which the relationship can be reasoned. His basic argument is that goods themselves do not give utility to the consumer. The goods possess characteristics which give rise to utility.

Lancaster's framework involves goods, X_j, and consumption activities, y_k. An individual good or group of goods gives rise to activities, i.e.,

$$X_j = \sum_k a_{jk} y_k$$

which denotes a linear relationship between the good, X_j, and the associated activities, y_k. The coefficients, a_{jk}, are assumed to be determined by the intrinsic properties of the goods and possibly the context of technological knowledge in society; consequently, the a_{jk}'s are the same for all individuals.

The activities are linked with characteristics, z_i, by another linear relationship

$$z_i = \sum_k b_{ik} y_k.$$

Lancaster also assumes that the b_{ik}'s are the same for all consumers. The consumers' utility function, U, then is a function of the z's, i.e., U = U(Z). The link is now complete. Goods produce activities which in turn create characteristics which produce utility.

It is difficult to conceive of the characteristics in Lancaster's framework being those referred to in physical, chemical, and biological descriptions of environmental quality. It is doubtful that most consumers are even aware of the characteristics used to describe water quality, let alone are able to distinguish various levels. Consequently, it appears more reasonable to visualize other transformations being involved. For example, one might consider the arguments of U to be basic human wants (needs) as discussed by

Georgescu-Roegen (18). These wants, z_i's, are satisfied by activities, i.e., the activities in a sense produce "want satisfaction". The activities in turn depend on various characteristics of the environment. These environmental characteristics may be those that are recognizable by consumers.[11] For example, if the concern is with water then such things as color, odor, temperature, etc., may be recognizable by people. These water characteristics, C_i, then in turn imply certain levels of water quality as represented by dissolved oxygen, nitrogen, etc. The various water quality levels then would correspond with the goods in Lancaster's case. The association might go as follows:

$$U = U(Z); \; z_i = \sum_j d_{ij} \, y_j; \; y_j = \sum_k b_{jk} \, C_k \quad X_j = \sum_k a_{jk} \, C_k$$

That is, a given level of water quality, X_j, (e.g., 5ppm DO, 10ppm DS, etc.) gives rise to water characteristics distinguishable by the consumer, C_k, e.g., odor, color, etc. These characteristics, C_k, are then related to various levels of activities, y_j. The activities, y_j, in turn produce want satisfaction, z_i, which is the argument of the utility function. The process linking all these things together can undoubtedly be visualized in other ways. What has been presented is

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[11] There has been some interesting work concerning people's perception of air and water quality which indicates there may be a difference in perception or concern depending on what people are doing. See for example (8, 15 and 27).

only an example intended to show that the given definitions of environmental quality and pollution can be related to economic theory.

The definition of pollution unlike the one for "water quality" is not value free. Pollution as defined is definitely a "bad". Such an interpretation is consistent with popular use of the word. The definition is also dynamic in the sense that what is not pollution today may be tomorrow and vice-versa.

It is conceivable that a change in environmental quality may not have any affect on someone's utility and/or cost function. Such a change could be referred to as a neutral change in environmental quality. It is also possible that man induced changes may increase someone's utility and/or decrease firm costs. It seems logical to regard the latter change as an enhancement of environmental quality.

It is likely that a given change in environmental quality will be an enhancement to some and a degradation (pollution) to others. How then, can the effects to society be assessed? If only individual firm costs were affected by the change then the effects could be compared from an economic efficiency standpoint. That is, if firm A's costs were lowered $2, 000 by the change but firm B's costs were raised by $3, 000 then the external costs of the environmental change are effectively $1, 000. Does the presence of $1, 000 in external costs imply the need for governmental involvement to improve economic efficiency?

Many economists have argued that the presence of externalities is not a sufficient cause for governmental intervention. In Turvey's words, "When negotiation is possible, the case for government intervention is one of justice not economic efficiency; . . ." (39, p. 313). Furthermore, Pareto equilibrium can be achieved in the presence of external effects (11, p. 485).

Even if negotiation were not possible in the previous case, it is not clear that governmental intervention is desirable for economic efficiency. Still to be considered are the costs of intervention. One such cost is the direct cost such as the cost for bringing parties together or of instituting a policy which would lower the external costs to zero or some other level. Another cost and one that is often overlooked is the cost of intervention to the firm creating the external effect. Coase's seminal article in 1960 brings this point out quite clearly.

> The question is commonly thought of as one in which A inflicts harm on B and what has to be decided is: how should we restrain A? But this is wrong. We are dealing with a problem of a reciprocal nature. To avoid the harm to B would inflict harm on A. The real question that has to be decided is: should A be allowed to harm B or should B be allowed to harm A? The problem is to avoid the more serious harm. (14, p. 2)

If the cost to the firm creating the change in environmental quality and the direct governmental costs together are less than $1, 000 then it would improve economic efficiency if the change were not permitted. However, if it would cost over $1, 000 for these two items then society would be ahead (based on economic efficiency) to permit the environmental quality change.

In many instances a given environmental quality change will affect individuals as well as firms. If everyone involved is affected in the same direction then there is much less of a problem in reaching a solution. For example if all those affected are harmed by the given change then there may be a case for trying to get whoever caused the change to cease and desist. If negotiation is possible there does not appear any need for government intervention to achieve Pareto equilibrium. If negotiation is not possible there may be some need for government to intervene. Again, government intervention will depend on the benefits to be derived versus the costs. In the case of the firms, the benefits and costs could be ascertained directly and relatively accurately. If it is the recreational and aesthetic interests of consumers that are affected then the benefits of preventing the change will be much more difficult if not impossible to assess. The decision whether or not to intervene in this latter case then is likely to be a political decision with little if any reliance on economic efficiency.

If consumers are affected in different ways by a given quality change then only interpersonal utility comparison and/or a social welfare function could give an accurate picture of what society may desire. Due to the social scientists' inability to make interpersonal utility comparisons the public decision making process takes over to render a decision. Just how this process operates is not within the scope of this paper.[12]

Of course economic efficiency is not the only objective for decision making that may be used by the public. Such things as income distribution, enhancement[13] of environmental quality, regional development, etc., also enter into many public decisions. Consequently, it may not be economically efficient to prevent A from altering the environment but the public may still decide to intervene on other grounds.

It may be as Castle and Youmans state that ". . . economic efficiency becomes a means to other ends rather than competitive to those ends" (13, p. 1662). They are referring to national income as presently measured as the relevant indicator of economic efficiency. If we could measure all effects in terms of dollars, then one might argue that economic efficiency is an all encompassing criterion. With the present state of economics, the lack of a social welfare function and inability to objectively measure many intangibles, it is doubtful that an increase in national income is automatically

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[12] See (10 and 32) for a description and discussion of public decision making.

[13] Enhancement here refers to improving quality from an aesthetic viewpoint.

synonomous with the improved well being of society. We may presently be at or beyond the peak of the function between national income and individual fulfillment as shown by Castle and Youmans (13, p. 1663). That is, an increase in national income may cause a decline in individual fulfillment.

It still seems that economic efficiency can provide a valid benchmark in the form of the opportunity cost of achieving some goals. In any case it is imperative that the effects of governmental intervention on those who are directly involved not be ignored. That is, the benefits of doing something about a change in environmental quality that is external to A must be weighed against the costs to A of intervention.

II. POLICY EFFECTS ON MODEL 1 -- ONE NON-SPECIALIZED FIXED FACTOR PRODUCT, ONE MARKET PRODUCT, AND NO DOMINATING TECHNIQUE

The sequence to be followed in this and succeeding chapters which deal with the various models will be the same. First, the assumptions which describe the model will be set out. Once the models are specified the alternative policies will be applied. Different policies are expected to result in different effects on the models. Just how a firm as depicted by a given model will react to the policies will be explored and explained. The implications of the analysis will then be summarized.

Before describing Model I it is useful to note the assumptions that apply to all four models. First, it is assumed that the firms are operating in competitive markets for products and factors. Second, a non-market external diseconomy is produced in fixed proportions with each market product. The initial price of the externality is assumed to be zero. Third, the firms are assumed to have alternative production techniques available. The proportion of the non-market externality produced with a product is assumed to vary between techniques. Fourth, initial conditions are specified to be consistent with the general assumption that before policies are applied the firm is maximizing net revenues. The latter assumption is quite critical since it implies that prior to the policy the particular technique being used is the most efficient. It is also assumed that after a policy is applied, the firm will still seek to maximize net revenues or minimize loss.

One criterion for judging a proposed policy is whether or not it accomplishes the major goal of the policy making unit. It is assumed that that goal is to induce firms that produce the external diseconomy to reduce that production as much as is technically feasible without going out of business. The production point within a given model that is consistent with the above goal is referred to as the "optimum policy point". From the point of view of the policy making unit the outcome of a policy which induces the firm to produce at the "optimum policy point" will be referred to as the "best" outcome. If intermediate production points exist which, relative to the initial point, represent some reduction in the externality such points are referred to as "second best" outcomes or points. In other words, a "second best" outcome is one where externality production is lower than initially but not as low as at the "optimum policy point".

There may be a need to clarify the distinction between models and techniques. The "models" depict the particular way a firm is assumed to be organized. Such things as numbers of market products, fixed factors, etc., are the items that change between models. Production "techniques" on the other hand refer to particular methods of producing the same products within the models. In other words, different techniques have different resource requirements for producing the same items with the same resources within a given model. The words "technique" and "method" are used interchangeably.

The basic conjecture relevant to all models is that firms are ignoring the externality in their decision framework. As a result, they are also ignoring some production techniques, which when forced to consider the externality, may become the most efficient methods of production.

Specification[14] and Policy Applications for Model 1

The following assumptions along with those already discussed describe Model 1.

1. The firm has two techniques, A and B, available for producing one market product, designated X_m.
2. The non-market external diseconomy, X_n, is produced in fixed proportions with X_m. The amount of X_n produced per unit of X_m by techniques A and B is n_a and n_b, respectively.
3. Technique A generates relatively more X_n per unit of X_m

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[14] Many of the concepts utilized in this and succeeding model specifications are discussed in greater detail in (9).

than B, i.e., $n_a > n_b$.

4. The per unit variable costs for producing X_m are less for technique A than B, i.e., $VC_a < VC_b$.
5. There is some variable factor, V, that is, compared to other variable factors, relatively low priced to the firm and unlimited in quantity.
6. The amounts of the variable factor required per unit of X_m are designated v_a and v_b for techniques A and B, respectively. Method A requires more of V per unit of X_m than method B, i.e., $v_a > v_b$.
7. The firm faces one fixed factor constraint, K, which is nonspecialized as to technique and is perfectly divisible. The initial level of $K = K^o$. In other words both techniques can and do use K.
8. Technique A requires more units of K per unit of X_m than technique B, i.e., $k_a > k_b$.

These eight basic assumptions specify a linear economic model which can be depicted by two dimensional figures. Some of these assumptions will be relaxed, as the analysis proceeds; however, such relaxations are discussed at the appropriate time.

As noted in the chapter title no technique totally dominates another. Why the latter is so can be understood by referring again to assumptions four and eight. Technique A requires relatively more of the fixed factor, K, to produce one unit of X_m than Method B;

whereas, method B requires relatively more variable costs as denoted by VC to produce one unit of X_m than technique A.

It is assumed that initially the firm is producing X_m with technique B which is depicted in Figure 2. 1. Note that only variable costs are shown in Figure 2. 1. The values for X_m represented by $X_m^{a^o}$ and $X_m^{b^o}$ represent the maximum amounts of X_m that can be produced by techniques A and B, respectively. The maximums are determined by the level of the fixed factor constraint, K^o. Net revenue to the firm is represented by the rectangles $VC_aEAP^o_{x_m}$ for method A and $VC_bCBP^o_{x_m}$ for method B. Notice that $VC_bCBP^o_{x_m} > VC_aEAP^o_{x_m}$ since $ABCD > VC_b DEVC_a$. In other words at the market price for X_m, $P^o_{x_m}$, net revenue from using technique B is greater than net revenue from technique A. The price $P^o_{x_m}$ is the price of X_m expected by the firm at planning time.[15]

The initial firm position is also shown in Figure 2. 2. The rays OA and OB represent the two methods for producing X_m. Units of X_m are measured along the rays. Notice that the scales along OA and OB are different. The points $X_m^{a^o}$ and $X_m^{b^o}$ correspond with the same designations in Figure 2. 1.

Net revenue can be depicted in Figure 2. 2 by iso-net revenue lines such as nr^o. To be consistent with the initial firm position it must be that the net revenue at $X_m^{b^o}$ is greater than at $X_m^{a^o}$. The

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[15] Note that if P_{x_m} changes from $P^o_{x_m}$ the previous results will not necessarily hold. Implications of such an event will be discussed subsequently.

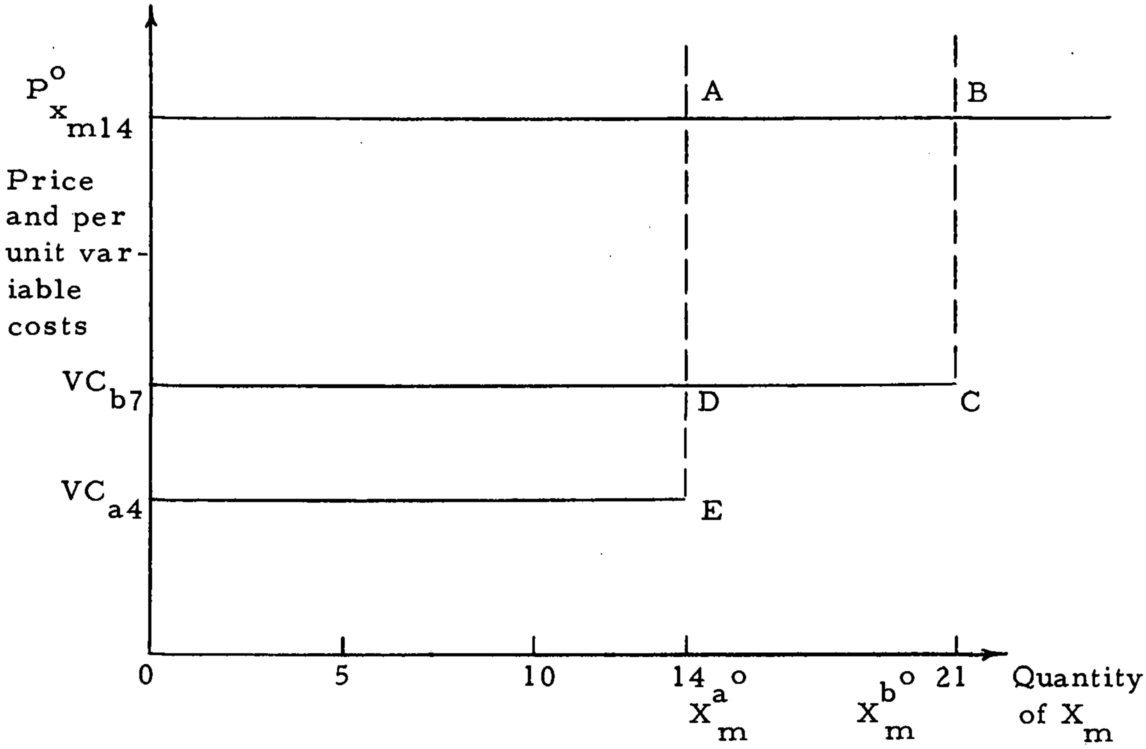

Figure 2. 1. Initial firm position--Model 1-- no dominating technique

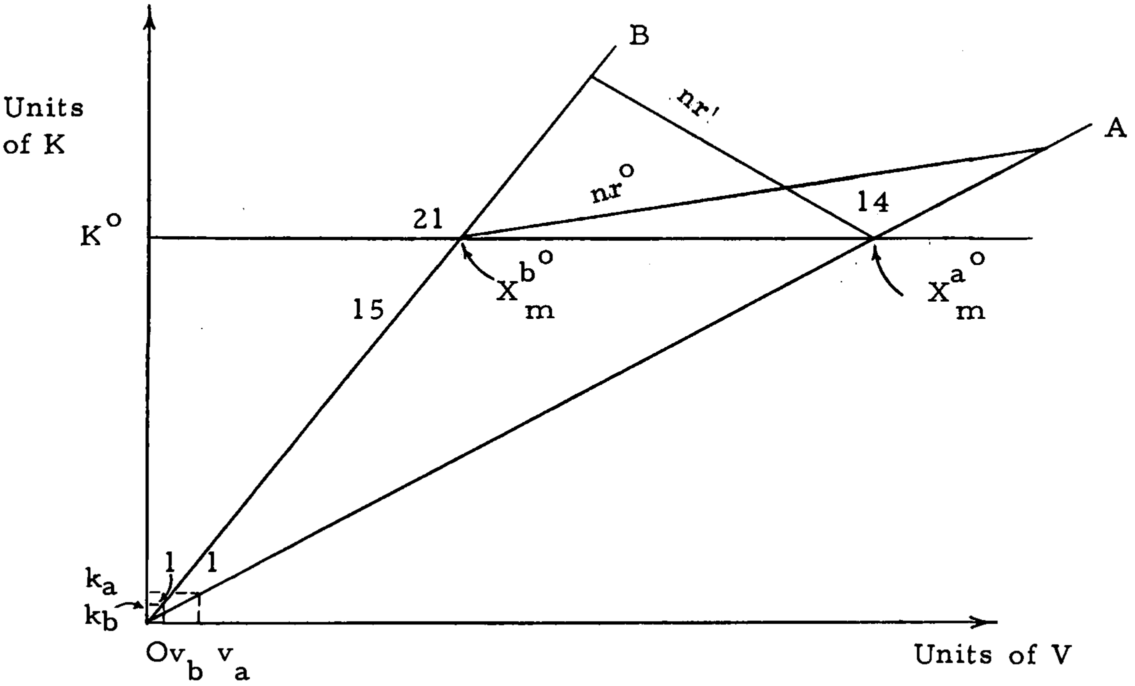

Figure 2. 2. Model 1--Relationships between techniques and fixed and one variable factors.

necessary conditions for a corner solution at $X_m^{b^o}$ are that the slope of nr be positive and that it be less than the slope of OB. It can be observed that nr^o meets these necessary conditions.

Mathematically an iso-net revenue line is of the form $nr_a = nr_b$ where $nr_i \equiv$ net revenue from technique i = a, b. The previous equality can be written as

$$(P^o_{x_m} - VC_a) \frac{K^i}{k_a} = (P^o_{x_m} - VC_b) \frac{K^j}{k_b} \qquad (1.1.1)$$ [16]

where i, j, refer to fixed factor levels and i need not equal j. For example $X_m^{b^o} = \frac{K^o}{k_b}$ and $X_m^{a^o} = \frac{K^o}{k_a}$. The per unit net revenues are represented by $(P^o_{x_m} - VC_a)$ for technique A and $(P^o_{x_m} - VC_b)$ for technique B. By previous assumption

$$(P^o_{x_m} - VC_a) > (P^o_{x_m} - VC_b) \qquad (1.1.2)$$

which implies that for (1.1.1) to hold it is necessary that $\frac{K^i}{k_a} < \frac{K^j}{k_b}$.

The condition necessary for the firm's initial point to be at $X_m^{b^o}$ is

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[16] Notice that the numbers of this equation designation refer to the model number, a sub-model (relevant in Chapters IV and V), and the equation number in that order.

$$(P^o_{x_m} - VC_a) \frac{K^o}{k_a} < (P^o_{x_m} - VC_b) \frac{K^o}{k_b} \qquad (1.1.3)$$

$$\Rightarrow (P^o_{x_m} - VC_a) \frac{K^o}{k_a} < (P^o_{x_m} - VC_a) \frac{K^i}{k_a} \qquad (1.1.4)$$

by substitution from (1.1.1) and setting $K^j = K^o$ in (1.1.1). Furthermore (1.1.4) implies that $K^i > K^o$. The conclusion then is that under the assumed conditions an iso-net revenue line will be associated with a higher fixed factor level for technique A than B. Line nr^o in Figure 2.2 is consistent with these conditions and represents the highest net revenue obtainable for given price and variable cost situations and the factor constraint K^o.

Taxing the Market Product

Suppose that a tax, T, were imposed on each unit of X_m produced with the hope of reducing the production of X_n by reducing X_m production. In this instance the size of the tax is crucial for predicting the outcome. Two things may result. One is that the firm will continue to produce X_m with technique B. In that case there will be no change in resource use or output level. The conditions necessary for the firm not to switch are that

$$(P^o_{x_m} - VC_a - T) \frac{K^o}{k_a} \leq (P^o_{x_m} - VC_b - T) \frac{K^o}{k_b}$$

$$\Rightarrow (P^o_{x_m} - VC_b)\frac{K^o}{k_b} - (P^o_{x_m} - VC_a)\frac{K^o}{k_a}$$

$$\geq T\frac{K^o}{k_b} - T\frac{K^o}{k_a} . \qquad (1.1.5)$$

That is, the amount of extra tax the firm must pay when using technique B versus A is less than or equal to the net revenue gained by using technique B instead of A. If the equality holds in (1.1.5) the implication is that the firm would be indifferent as to technique; however, for simplicity it is assumed that under such circumstances the firm will stay with the original technique, i.e., B in this instance.

The second possible outcome is that the firm will switch from B to A. The necessary condition for the switch is

$$(P^o_{x_m} - VC_a - T)\frac{K^o}{k_a} > (P^o_{x_m} - VC_b - T)\frac{K^o}{k_b} . \qquad (1.1.6)$$

Iso-net revenue lines will be

$$(P^o_{x_m} - VC_a - T)\frac{K^o}{k_a} = (P^o_{x_m} - VC_b - T)\frac{K^j}{k_b} . \qquad (1.1.7)$$

Together, (1.1.6) and 1.1.7) imply that

$$(P^o_{x_m} - VC_b - T)\frac{K^j}{k_b} > (P^o_{x_m} - VC_b - T)\frac{K^o}{k_b} ,$$

$$\Rightarrow K^j > K^o \text{ (assume } (P^o_{x_m} - VC_b - T) > 0). \qquad (1.1.8)$$

The interpretation is that the iso-net revenue line will be associated with a higher capital level for B than A. Such a line is shown in Figure 2.2 as nr'.

Another interpretation of (1.1.6) is as follows:

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m - TX^{a^o}_m > (P^o_{x_m} - VC_b)\, X^{b^o}_m - TX^{b^o}_m \quad \text{from}$$

$$(1.1.6) \text{ and } X^{i^o}_m = \frac{K^o}{k_i} \quad i=a,\ b,$$

$$\Longrightarrow (X^{b^o}_m - X^{a^o}_m)\ T > (P^o_{x_m} - VC_b)\, X^{b^o}_m - (P^o_{x_m} - VC_a)\, X^{a^o}_m$$

$$\Longrightarrow T > \frac{(P^o_{x_m} - VC_b)\, X^{b^o}_m - (P^o_{x_m} - VC_a)\, X^{a^o}_m}{X^{b^o}_m - X^{a^o}_m}. \qquad (1.1.9)$$

Inequality (1.1.9) states that the per unit tax, T, must be greater than the net revenue gained per unit of <u>added</u> product when method B is used instead of A. The right hand side of (1.1.9) is in a sense the marginal revenue of switching between techniques B and A.

A tax of the size represented by T^o in Figure 2.3 will be sufficient to cause the firm to switch from B to A. Notice now that the net revenue gained by switching to method A, VC_b DEVC_a, is greater than the net revenue lost from technique B, A'B'CD.

Has the optimum policy outcome been achieved? Without more information one cannot tell. The production of X_m has gone down

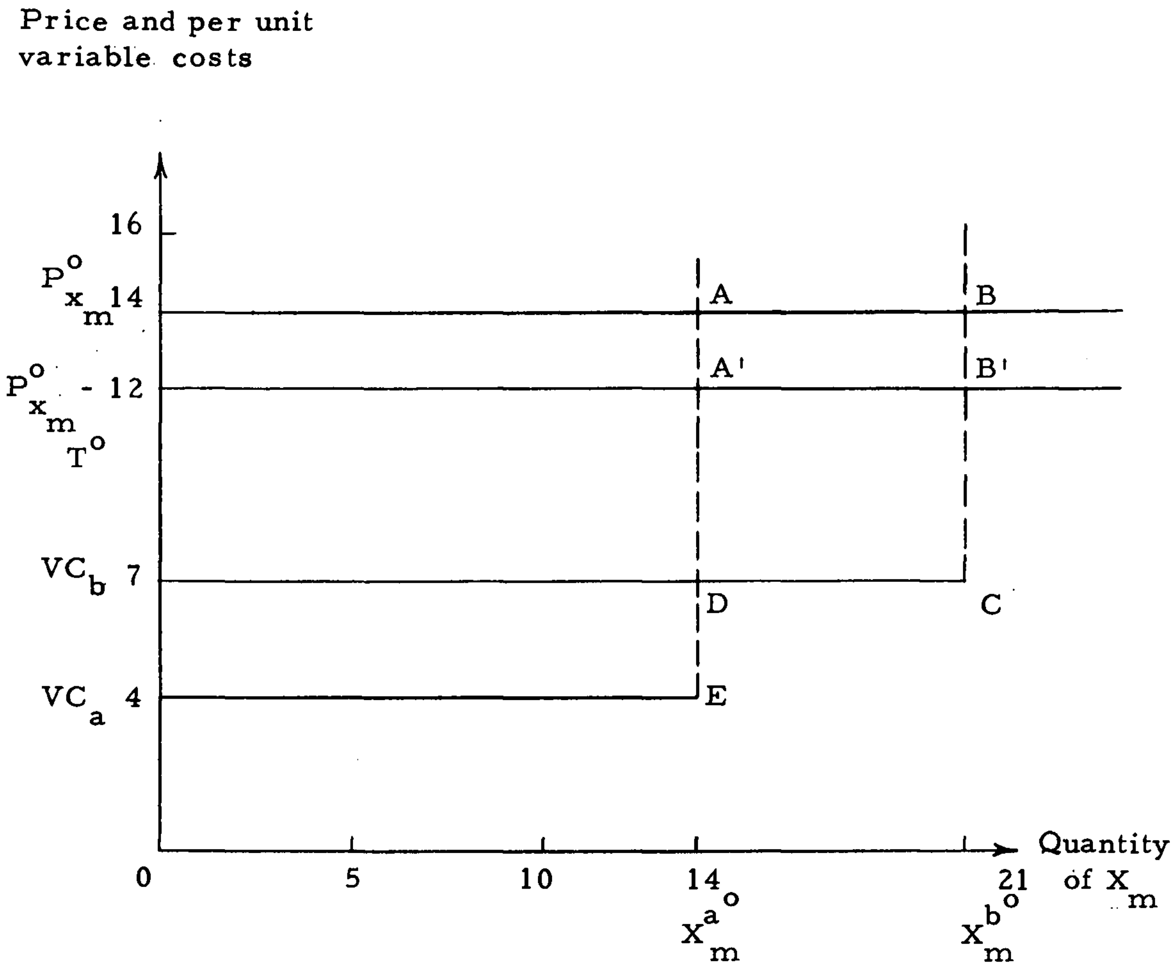

Figure 2. 3. Effect of a tax on X_m for Model 1

from $X^{b^o}_m$ to $X^{a^o}_m$. Assumption 3 does not say how much more X_n per unit of X_m is produced by A.

The necessary condition for a yes answer to the previous question is that

$$n_a(X^{a^o}_m) < n_b(X^{b^o}_m) \qquad \text{where } n_i \equiv \text{amount of } X_n \text{ produced}$$

per unit of X_m for method i, i = a, b. The previous inequality implies that at the maximum levels of X_m production as determined by the common constraint, K^o, more of the externality is produced by method B than A. If the necessary condition for the optimum policy outcome exists and the firm switches from method B to A then the policy has been successful.

Assuming that the firm actually switches from technique B to A, then the use of the variable factor V increases. The increase in the use of V as a result of an effective decrease in the price of X_m implies that V is an inferior factor as defined by Plott.[17]

Reference to Figure 2.3 should also make it clear that the results of the tax, T^o, are sensitive to increases in P_{x_m}. If the price of X_m at some later date should happen to rise above $P^o_{x_m}$ say to $P'_{x_m} = 16$ then the firm would be induced to switch back to technique B.

Instead of assumption 3 it might be that the amount of X_n (the externality) produced is some increasing function of V. If the other assumptions do not change then a tax level that would induce a switch from B to A would result in an increase in the production of X_n. Again, a result that is similar to a possibility pointed out by Plott as discussed in Chapter I.

If the firm initially started at point $X_m^{a^o}$, i.e., producing X_m

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[17] See Chapter I, pages 9-11.

with technique A then this policy would not bring about any switch. This result can be observed by examining Figure 2.3. If the original price were at, 12, then technique A would be used. Since $X_m^{a^o} < X_m^{b^o}$ the tax would reduce the returns to technique B more than to A so that A would still remain optimal.

Summary and Implications.

There is a possibility that a tax on the market product may not be effective. To predict a priori whether or not the "optimum policy point" will be achieved it seems necessary to be able to answer the following questions:

1. Does the firm have alternative production techniques available?
2. If the answer to (1) is yes, what are the relative proportions of the market good, the non-market externality, and the total amounts of these two under the constrained conditions that can be produced by alternative methods?
3. What are the variable costs of the alternative techniques and how do they compare relative to each other?

Given the assumptions of the model and if it is assumed that $X_n = f(V)$ in a positive sense then the tax cannot be effective without forcing the firm from business. This linear model along with the tax on X_m has demonstrated the possible perverse effects of taxing the market product as discussed by Plott. It raises the question of whether or not the inferior factor as discussed by Plott and others may not just be a result of changing production techniques. Since alternative production methods can be assumed into a theoretical continuous production model it is not obvious that internal production methods are changing. However, with the linear model as presented here it is more obvious what may be happening.

Taxing a Variable Factor

The firm's initial position is assumed to be as before, i. e.,

$$(P^o_{x_m} - VC_a) X^{a^o}_m < (P^o_{x_m} - VC_b) X^{b^o}_m$$

which implies $(P^o_{x_m} - VC_a) \frac{V^i}{v_a} < (P^o_{x_m} - VC_b) \frac{V^j}{v_b}$ (1. 1. 10)

Assume that a tax, T_v, is imposed on V. What if anything can be concluded about the following:

$$(P^o_{x_m} - VC_a - T_v v_a) \frac{V^i}{v_a} \overset{?}{\gtreqless} (P^o_{x_m} - VC_b - T_v v_a) \frac{V^j}{v_b}$$

$$\Longrightarrow (P^o_{x_m} - VC_a) \frac{V^i}{v_a} - T_v V^i \overset{?}{\gtreqless} (P^o_{x_m} - VC_b) \frac{V^j}{v_b} - T_v V^j.$$

(1. 1. 11)

Since $V^i > V^j$, $T_v V^i > T_v V^j$ which along with (1. 1. 10) permits the conclusion that

$$(P^o_{x_m} - VC_a)\frac{V^i}{v_a} - T_v V^i < (P^o_{x_m} - VC_b)\frac{V^j}{v_b} - T_v V^j .$$

(1.1.12)

Inequality (1.1.12) leads to the prediction that if the firm is utilizing technique B, a tax on V will <u>not</u> cause the firm to switch to technique A. In this instance the tax policy has not decreased the production of the externality; however, the possibility of it having the opposite effect has been eliminated.

Now suppose that the firm were originally operating with technique A, i.e.,

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m > (P^o_{x_m} - VC_b)\, x^{b^o}_m \qquad (1.1.13)$$

The iso-net revenue lines would appear similar to nr' in Figure 2.2. Again impose a tax, T_v, on V. Inequality (1.1.13) becomes

$$(P^o_{x_m} - VC_a)\frac{V^i}{v_a} - T_v V^i \overset{?}{\underset{>}{\leq}} (P^o_{x_m} - VC_b)\frac{V^j}{v_b} - T_v V^j.$$

(1.1.14)

It is known that $T_v V^i > T_v V^j$ which along with (1.1.13) does <u>not</u> permit any conclusion concerning (1.1.14). If the (>) holds in (1.1.14) the firm will not switch from A to B and (1.1.14) implies that

$$(P^o_{x_m} - VC_a)\frac{V^i}{v_a} - (P^o_{x_m} - VC_b)\frac{V^j}{v_b} > T_v V^i - T_v V^j.$$

(1.1.15)

In other words the difference in net revenue between A and B without the tax must be greater than the difference in tax between A and B.

Stated another way,

$$T_v < \frac{(P^o_{x_m} - VC_a)\frac{V^i}{v_a} - (P^o_{x_m} - VC_b)\frac{V^j}{v_b}}{(V^i - V^j)} . \qquad (1.1.16)$$

That is, to not induce a technique switch per unit tax must be less than the net-revenue gained per additional unit of V used when producing with method A instead of B. Inequality (1.1.16) can also be interpreted to say that the marginal unit cost (T_v) of V is less than the average net-marginal value product of V between the points $X^{b^o}_m$ and $X^{a^o}_m$ in Figure 2.2. The latter interpretation implies that the firm will produce at $X^{a^o}_m$, a situation quite similar to traditional marginal analysis.

If the equality holds in (1.1.14) the firm would supposedly be indifferent as to method A or B. If the (<) holds, (1.1.14) would imply that the firm will switch to method B. For such to be the case the per unit tax, (1.1.16), must be greater than the net-revenue advantage per unit of additional V used between A and B.

As with the previous policy whether or not the externality production has been decreased depends on at which point X_n is the greatest. If the firm were initially operating with technique B and $n_b X^{b^o}_m > n_a X^{a^o}_m$ then this policy would <u>not</u> be effective. If the initial point were $X^{a^o}_m$ then depending on the relative size of X_n produced the policy may be effective or it could result in the opposite of the desired effect. If it is assumed that X_n is a positive function of V, then this policy at least will not increase production of X_n.

A decrease in the price of X_m could induce the firm to switch to technique A if it were originally using method B. Such an event could happen either with or without the tax on V. The latter can be observed most readily by referring to Figure 2. 1. Without a tax a price for X_m below VC_b for example would result in positive net returns only for technique A. The only thing the tax does is raise the levels of both VC_a and VC_b. If after the tax on V, $VC_a + T_v V^i > VC_b + T_v V^j$ then a drop in P_{x_m} will not bring about a change in technique; otherwise the result could be the same as without the tax.

If the firm were initially utilizing method A and if the tax induced a technique switch to B then a falling P_{x_m} could induce a switch back to A. A switch back to A would not occur if the tax were large enough so that $VC_a + T_v V^I > VC_b + T_v V^j$.

Price increases will favor technique B. The only time an increase in P_{x_m} will by itself induce technique changes is when method A is being used. As implied previously, technique A will not be used unless it was the most profitable initially; consequently, the effect of price increases is not influenced by the present policy.

Summary and Implications.

The possibility still exists that this policy could result in the increased production of X_n; however, unless X_n decreases as V increases this result cannot happen. With the previous policy such a result could happen even if X_n were a positive function of V. The implication is that if the policy making unit is aware of a variable factor that is positively related to the externality then such a factor is a better candidate for taxing than is the market product. Experimentation with the size of tax could progress with only the knowledge that X_n was not a negative function of V.

If the policy unit knows that this firm <u>could</u> produce less X_n with a production technique other than the one the firm is using then it could proceed to look for a likely variable factor. A variable factor that is used in relatively large amounts by the high externality technique compared to a lower X_n producing technique would be a likely candidate.

It appears that this policy will be sensitive to decreases in P_{x_m}. If the firm produces a commodity with a highly variable price this policy may not be very effective. Of course, if the tax were high enough so that after the tax the desired technique totally dominated the other method then fluctuations in P_{x_m} would be immaterial.

To predict <u>a priori</u> how the firm will react to this policy in all respects appears to require as much information about the firm as predicting the reaction to taxing the market product. The main advantage then of this policy over the previous would depend on

X_n being positively related to V as discussed above.

Taxing the Non-market Externality

Assume the initial firm position to be at $X_m^{b^o}$ in Figure 2.1.

Suppose a tax, T_n, is levied on the externality X_n. Initially then,

$$(P^o_{x_m} - VC_a)\, X_m^{a^o} < (P^o_{x_m} - VC_b)\, X_m^{b^o} . \qquad (1.1.17)$$

The amount of X_n produced per unit of X_m is n_i where i = a, b depending on which technique is used.

Can a priori conclusions concerning the following statement be reached?

$$(P^o_{x_m} - VC_a)\, X_m^{a^o} - T_n\, n_a\, X_m^{a^o} \overset{?}{\substack{\geq \\ <}} (P^o_{x_m} - VC_b)\, X_m^{b^o}$$

$$- T_n\, n_b\, X_m^{b^o} . \qquad (1.1.18)$$

For assumption 3, $n_a > n_b \Rightarrow n_a T_n > n_b T_n$, but $X_m^{a^o} < X_m^{b^o}$ so that one cannot conclude without additional information anything about (1.1.18). To conclude that the less than prevails it is sufficient that $n_a X_m^{a^o} > n_b X_m^{b^o}$. If the less than holds the firm will continue to produce with technique B. The only effect of the tax will be to reduce the net revenue level. If the tax were large enough the firm could be forced out of business.

It is possible that $n_a X_m^{a^o} < n_b X_m^{b^o}$. If the latter is the case there will be a tax rate that is large enough to cause the greater than to hold in (1. 1. 18). In such a situation the firm would switch to technique A. In the short run at least the policy would have been effective. There is a possibility that the tax rate necessary to make the greater than hold will be so large that both sides of (1. 1. 18) will be negative. In the latter case the firm would cease production since variable costs would not be met by either technique.

With $n_a X_m^{a^o} < n_b X_m^{b^o}$ some tax rates will not cause the firm to switch techniques. These lower (relative to those that would bring about a switch) rates then would be ineffective in reducing the level of the externality.

Suppose that the original firm position were at $X_m^{a^o}$ in Figure 2. 1. That is,

$$(P_{x_m}^o - VC_a) X_m^{a^o} > (P_{x_m}^o - VC_b) X_m^{b^o} . \qquad (1. 1. 19)$$

Application of the per unit tax on X_n results in the following statement:

$$(P_{x_m}^o - VC_a) X_m^{a^o} - T_n n_a X_m^{a^o} \overset{?}{\gtrless} (P_{x_m}^o - VC_b) X_m^{b^o} - T_n n_b X_m^{b^o} \qquad (1. 1. 20)$$

Assumption 3 does not permit any conclusion concerning the relative size of $n_a X_m^{a^o}$ and $n_b X_m^{b^o}$; therefore, one cannot conclude anything about (1. 1. 20). If $n_a X_m^{a^o} < n_b X_m^{b^o}$ then the (>) will hold in (1. 1. 20) and the firm will either remain at $X_m^{a^o}$ or go out of business no matter how large the tax. If $n_a X_m^{a^o} > n_b X_m^{b^o}$ then there will be some tax that will cause the firm to switch from technique A to B. In other words $n_a X_m^{a^o} > n_b X_m^{b^o}$ is necessary for a switch in techniques but not sufficient. The sufficient conditions are that

$$(P_{x_m}^o - VC_a) X_m^{a^o} - T_n n_a X_m^{a^o} < (P_{x_m}^o - VC_b) X_m^{b^o} - T_n n_b X_m^{b^o} \qquad (1.1.21)$$

and $$(P_{x_m}^o - VC_b) X_m^{b^o} - T_n n_b X_m^{b^o} > 0$$

$$\Longrightarrow \quad T_n > \frac{(P_{x_m}^o - VC_a) X_m^{a^o} - (P_{x_m}^o - VC_b) X_m^{b^o}}{(n_a X_m^{a^o} - n_b X_m^{b^o})} . \qquad (1.1.22)$$

Statement (1.1.2) states that the per unit tax on the non-market externality must be greater than the net marginal revenue per unit of the additional non-market good produced when the firm stays at A versus switching to B. In marginal analysis terms the marginal cost (T_n) of $X_n^{a^o} - X_n^{b^o}$ is greater than its marginal revenue. In the latter situation the tax has achieved the optimum policy point. If $n_a X_m^{a^o} > n_b X_m^{b^o}$ but condition (1.1.22) is not met then the tax would not cause the firm to move to this "best" point.

In the first situation where a switch occurred (the (>) holding in (1.1.18)), a decrease in P_{x_m} will not do anything except maybe force the firm from business. However, if P_{x_m} should go up beyond its original level, $P^o_{x_m}$, there is the possibility that the firm will switch back to B. The latter two possibilities exist since $X^{a^o}_m < X^{b^o}_m$. A decreasing price will reduce the smallest side of (1.1.18) more than the larger side while an increasing price will increase the smaller (right) side more than the larger (left) side.

If the situation is such that the firm switches from method A to B (statement (1.1.21)) then an increasing price will <u>not</u> change the relative position of the two techniques. A decreasing price on the other hand could bring about a switch back to technique A. The reasoning is similar to above.

Summary and Implications.

The results of this policy as with the previous ones are sensitive to movements in the price of the market good. One advantage appears to be that the effluent tax will <u>not</u> result in an increase in the production of X_n.

It is quite important to know the relative total amounts of the externality that can be produced by various techniques. For example, if $n_a X^{a^o}_m > n_b X^{b^o}_m$ and the firm is already using technique B it would not do any good to tax X_n. If just n_a and n_b had been known the policy agency could apply a tax that would not change the situation (since $n_a > n_b$).[18]

If the policy unit knew the relative total proportions under constrained conditions of X_n produced by the two processes and the difference in net revenues then it could predict the outcome of a given tax. However, such information is not easily obtained.

One serious drawback to this policy is that the effluent must be identifiable. Identification could be quite difficult for certain types of pollution, e. g., pollution resulting from irrigation return flows. Monitoring effluent production may also be necessary if the effluent tax is to be effective.

It would probably be necessary for the policy agency to specify the characteristics of the effluent to be taxed. Such specification would add to the administrative costs of this policy. If the tax were set on particular effluent characteristics, the firm could react by changing the quality of its effluent by internal technique changes and/or treating the outflow. Although this possibility has not been specifically allowed for in the analysis of this policy, it is considered

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[18] Of course the firm could be driven from business but it has been assumed that such an effect is not desired. The revenue generated by the tax could still be useful but considering benefits from possible use of such revenues is not within the intent of this study.

more specifically under the next policy. The effects of treating the effluent are not unlike for the two policies. Internal process changes that alter the make-up of the discharge can be handled within the previous analysis. Suppose the firm can change internal techniques so that an effluent, say X'_n, is produced which will not be taxed. Such a possibility would be similar in effect to a situation where the firm's process did not emit any X_n. Both of these could be depicted in the previous analysis by either n_a or n_b being equal to zero.

A Standard on the Quality of the Externality

Conceptually, this policy seems more difficult to handle than the ones considered so far. One approach is to assume that as before X_n is produced jointly and in fixed proportions with X_m. Suppose that the policy is one of setting a standard on effluent quality.[19] Furthermore, assume that X_n does not meet that standard whether it is produced by

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[19] A standard may refer to proportions of specific characteristics, e.g., parts per million (ppm) of total dissolved solids and/or the total quantity of such characteristics permissible, e.g., so many tons of dissolved solids per day. As used in this study, the standard refers only to the proportions of characteristics. For example, if X_n represents water effluent, then the standard will constrain the proportion of characteristics contained in X_n, not the total amount of such characteristics. If the concentration of certain characteristics contained in X_n is higher than that permitted by the standard, the firm will need to alter X_n if it is to be released into the environment. Of course total volume of certain characteristics could be quite important, but this concern and likely effects on the analysis are treated in Chapter VI.

A or B. Under the assumptions of this case the only recourse available to the firm is to cease production or to "treat" X_n and change its form to X'_n which will meet the standard.

Treating X_n can be viewed in at least two ways. The simple case would be where only variable costs would be affected by the treatment process. Also, one could assume that each unit of X_n plus some constant amount of variable cost, say VC_n, would produce one unit of X'_n.

The initial position again is

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m < (P^o_{x_m} - VC_b)\, X^{b^o}_m .$$

The effluent standard would then impose the following situation on the firm:

$$\begin{aligned} &(P^o_{x_m} - VC_a)\, X^{a^o}_m - VC_n\, n_a\, X^{a^o}_m \\ &\overset{?}{\gtreqless} (P^o_{x_m} - VC_b)\quad X^{b^o}_m - VC_n\, n_b\, X^{b^o}_m . \end{aligned} \tag{1.1.23}$$

Statement (1.1.23) is practically identical to (1.1.18) except variable costs of treating X_n are used instead of a tax on X_n. As in (1.1.18), no conclusion can be reached regarding (1.1.23). As long as those making the policy can enforce it, one thing is certain. The certainty is that X'_n will be emitted instead of X_n. Such will be the case no matter which way statement (1.1.23) reads. If $n_a\, X^{a^o}_m > n_b\, X^{b^o}_m$, then no matter how high VC_n, the firm will not switch to technique A. If $n_a X^{a^o}_m < n_b X^{b^o}_m$, the firm might switch to A depending on the size of VC_n. In any case, the externality problem will have been reduced by this policy. This same conclusion can be reached even when the initial firm position is $X^{a^o}_m$. If VC_n is too high the firm could be forced out of business entirely.

Another way of visualizing the treatment of X_n is where not only variable costs are used, but also some of the fixed factor, K. The simplest assumption concerning the fixed factor is that X_n produced by either technique will require the same amount of fixed factor to alter it to X'_n. Then if the same assumption is made concerning the variable costs of treatment as previously, the situation will be practically identical to the one before. The only difference will be that X^b_m and X^a_m will be at lower levels, but the relative position of these two will not have changed. The fixed constraint could be K' where $K' < K^o$. From that point on the analysis and conclusions would be the same as where none of K is required to process X_n.

If the amount of the fixed factor required varies by method, the situation becomes more complex. Assume that the fixed factor required is proportionate to volume of X_n produced. An assumption must be made at this point regarding the production of X_n. Suppose $n_a X^{a^o}_m > n_b X^{b^o}_m$. With only K^o of K available the firm will not be able to produce at $X^{b^o}_m$. Suppose that the processing of X_n requires

k_n units of K per unit of X_n. Then, to operate at $X_m^{b^o}$ the firm would need $k_n n_b X_m^{b^o}$ additional units of K. Since in the short run it has been assumed that the latter is not possible, the firm must determine the maximum amount of X_m that can be produced while leaving enough K to process the related X_n.

It can be shown[20] that after allowances are made for enough K to process X_n, $K'' < K' < K^o$ where K^o is the total K available and K' and K'' are the amounts of K available to techniques B and A, respectively, for producing X_m. An example of the various levels of K is presented in Figure 2.4.

If the same assumption concerning variable costs is made as before (i.e., variable costs of processing X_n are at a constant rate per unit of X_n and the same between techniques) then the following statements describe the firm's initial and after the policy positions:

$$(P^o_{x_m} - VC_a) X_m^{a^o} < (P^o_{x_m} - VC_b) X_m^{b^o} \quad \text{(Initial)} \qquad (1.1.24)$$

$$(P^o_{x_m} - VC_a) X_m^{a''} - VC_n n_a X_m^{a''} \overset{?}{\gtrless} (P^o_{x_m} - VC_b) X_m^{b'}$$

$$- VC_n n_b X_m^{b'} . \qquad \text{(After)} \qquad (1.1.25)$$

Since $K' > K''$ and $k_a > k_b$ then $\frac{K'}{k_b} = X_m^{b'} > X_m^{a''} = \frac{K''}{k_a}$. To prove

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[20] See Appendix A, Part 1.

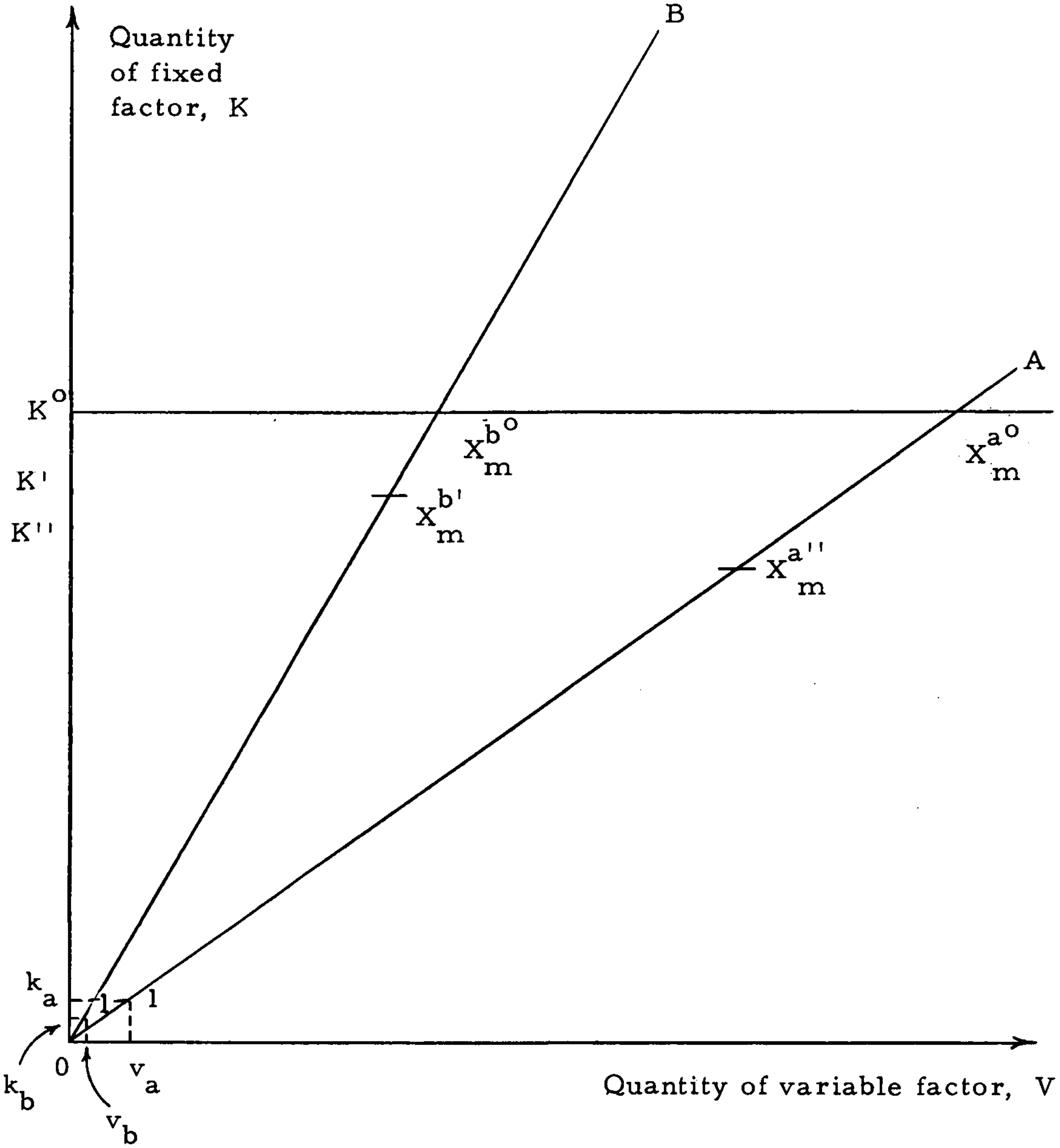

Figure 2.4. Effects of an effluent quality standard

that $K' > K''$ it was assumed that $n_a X_m^{a^o} > n_b X_m^{b^o}$ which also leads to the fact that $n_a X_m^{a''} > n_b X_m^{b'}$.[21] Since in (1.1.25) more will be subtracted from the smallest side (left) than the largest (right) the direction of the inquality as shown in (1.1.24) cannot change. Also due to the assumed conditions specifically $n_a > n_b$, $X_m^{a''}$ is likely to be reduced below $X_m^{a^o}$ by a larger percentage than $X_m^{b'}$ is reduced below $X_m^{b^o}$. Taking into account the relative sizes of $X_m^{a''}$ and $X_m^{b'}$ and the larger variable costs for processing X_n for method A it appears that the firm will <u>not</u> switch techniques. In other words only the (<) can hold in (1.1.25).

If initially the firm is utilizing method A what is likely to be the outcome?

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[21]

$$\frac{n_a X_m^{a''}}{n_b X_m^{b'}} = \frac{n_a \left(\frac{k_a K^o}{k_a + k_n n_a} \right) \Big/ k_a}{n_b \left(\frac{k_b K^o}{k_b + k_n n_b} \right) \Big/ k_b} = \frac{n_a (k_b + k_n n_b)}{n_b (k_a + k_n n_a)}$$

$$= \frac{n_a k_b + k_n n_a n_b}{n_b k_a + k_n n_a n_b} > 1$$

Since $n_a k_b > n_b k_a$ from (3) in Appendix A, Part 1.

$$(P^{o}_{x_m} - VC_a)\, X^{a^o}_{m} > (P^{o}_{x_m} - VC_b)\, X^{b^o}_{m}. \quad \text{(Initial)} \qquad (1.1.26)$$

After the policy the firm position will be as (1.1.25). For the firm to switch to method B it is required that the (<) hold in (1.1.25) which implies

$$(P^{o}_{x_m} - VC_a - VC_n\, n_a)\, X^{a''}_{m} < (P^{o}_{x_m} - VC_b - VC_n\, n_b)\, X^{b'}_{m}. \qquad (1.1.27)$$

Assume $VC_n = 0$, then

$$\frac{X^{b'}_{m}}{X^{a''}_{m}} > \frac{P^{o}_{x_m} - VC_a}{P^{o}_{x_m} - VC_b}. \qquad (1.1.28)$$

Inequality (1.1.28) implies that the more of the externality produced (the less of X_m) by method A with respect to B and/or the closer VC_a and VC_b are in size, the higher the probability that the firm will switch to technique B.

If $VC_n > 0$ then (1.1.27) implies

$$\frac{X^{b'}_{m}}{X^{a''}_{m}} > \frac{(P^{o}_{x_m} - VC_a - VC_n\, n_a)}{(P^{o}_{x_m} - VC_b - VC_n\, n_b)}. \qquad (1.1.29)$$

Now suppose that when VC_n is ignored a (<) appears in (1.1.28) which implies the firm would not switch to B due to the fixed factor requirements alone. When VC_n are considered and if $VC_n > 0$ then, the closer n_a and n_b are in magnitude the less likely is that (1.1.29) will hold and that the firm will change techniques. If n_a is quite large relative to $n_{b'}$, $X_m^{b'}$ will be larger relative to $X_m^{a''}$ and the right side of (1.1.29) is more likely to be smaller than the right side of (1.1.28); consequently, (1.1.29) has a higher possibility of holding which implies the firm will switch techniques. In other words, if a technique is being used that generates large amounts of X_n per unit compared to another method, this policy is more likely to result in internal method switches than if relative amounts of X_n are similar.

One definite advantage of the present policy is that fluctuations in the price of X_m will not affect the quality of the externality. If the standard is effectively enforced X_n' will be released and the policy agency should be satisfied. Price changes can affect what the firm does internally, however.

If the firm started with technique A and the standard brought about a switch to B the changes in P_{x_m} will have definite effects. That is, a price decrease could induce the firm to switch back to method A while a price increase will not change the firm's decision.

Summary and Implications.

Providing that enforcement is stringent predicting the effects of this policy on the externality requires much less information that other policies. Predicting the effects on the internal operations of the firm may be more difficult than such predictions for previous policies. To determine how the firm will be affected requires knowledge about the alternative techniques available, the cost structure of these techniques, the degree of treatment and/or alteration of X_n required, and the methods and costs of treating X_n.

The quality of the externality produced will not be affected by price changes; however, internal operations could be altered. It is also quite apparent with this policy (also true of previous policies) that the internal effects of price changes for X_m are quite dependent on the initial firm position.

From an administrative viewpoint effluent standards have many of the same problems as effluent charges. The effluent must be identifiable and measurable which could be expensive in terms of monitoring costs. The policy unit must also have effective enforcement practices available. In other words without the power to enforce, this policy is void.

Subsidizing a Variable Factor

The variable factor referred to so far, V, was assumed to be relatively low priced and closely related to the production of the externality. What might happen if the policy were to subsidize V? The answer depends on several things, first of which is the initial firm position, which will be assumed to be $X_m^{b^o}$ in Figure 2. 2.

After the policy of subsidizing V, the firm's net revenue situation would be

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m + S_v V^i \overset{?}{\substack{\geq \\ <}} (P^o_{x_m} - VC_b)\, X^{b^o}_m + S_v\, V^j \qquad (1.1.30)$$

where S_v is the per unit subsidy on V.

Without knowing something about the subsidy size and the relative sizes of V^i and V^j, the outcome of the policy cannot be predicted. For the firm to remain with method B it is necessary that

$$(P^o_{x_m} - VC_b)\, X^{b^o}_m - (P^o_{x_m} - VC_a)\, X^{a^o}_m > S_v V^i - S_v V^j. \qquad (1.1.31)$$

Statement (1.1.31) indicates that the net revenue advantage from using method B must be greater than the gain in subsidy if method A were used (note $V^i > V^j$). Whether or not (1.1.31) holds depends on the relative net revenue, relative amounts of V used and the size of the subsidy.

The (>) may hold in (1.1.30) given the proper size subsidy. In such a case the firm would switch to method A causing a reduction in X_m produced, an effect quite similar again to the inferior factor case of Plott's. It is similar since a decrease (subsidy) in the price of a factor also decreases output.[22] Whether or not the policy will

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[22] Using an argument similar to that developed in Chapter I, it can be shown that $\frac{\partial f}{\partial r_1} = - \frac{\partial X_1}{\partial P_{x_m}}$. If X_1 is an inferior factor then

cause a reduction in X_n depends on how much X_n is produced by each method. If $n_a X_m^{a^o} > n_b X_m^{b^o}$, there would not seem to be much point to subsidizing V. However, if $n_a X_m^{a^o} < n_b X_m^{b^o}$, then subsidizing V enough to cause the firm to switch techniques would reduce X_n. Naturally this appears to be the best policy for the <u>firm</u> since its net revenue after the action will be greater than before.

If the initial firm position were at $X_m^{a^o}$ then the subsidization policy would not cause the firm to switch methods under any circumstances, since

$$(P_{x_m}^o - VC_a) X_m^{a^o} + S_v V^i > (P_{x_m}^o - VC_b) X_m^{b^o} + S_v V^j. \tag{1.1.32}$$

Remember that $V^i > V^j$ and the greater than held initially so that the subsidy always adds a larger number to the side that was largest at the start. There would be no point to this policy no matter which method produces the most X_n.

Now suppose that there exists another variable factor (other than V) which is also utilized relatively more by one method than

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$$\frac{\partial X_1}{\partial P_{x_m}} < 0$$

$\Rightarrow \frac{\partial f}{\partial r_1} > 0$, which implies that output ($X_m = f(X_1, X_2)$) is positively related to the price of the inferior factor X_1.

another. Assume that the firm is initially producing with technique B. Since $VC_b > VC_2$ it seems to be a fair assumption that this other variable factor, V', is utilized more heavily by method B. Again, whether method B produces more or less X_n than A makes no difference as to the outcome of this policy since

$$(P^o_{x_m} - VC_b)\, X^{b^o}_m + S_{v'}\, (V')^j > (P^o_{x_m} - VC_a)\, X^{a^o}_m + S_{v'}\, (V')^i\,. \tag{1.1.33}$$

In (1.1.33) the subsidy adds the most to the side (left) that initially was the largest. The result is that the firm will continue to produce with method B. If B produces the least externality there is no need for the policy (at least as far as this situation is concerned). If A produces the least externality this policy will be ineffective.

If initially the firm were using method A, B utilized the most V' and B produced less X_n than A, then the subsidy policy would work. That is initially,

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m > (P^o_{x_m} - VC_b)\, X^{b^o}_m\,. \tag{1.1.34}$$

After the subsidy

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m + S_{v'}\, (V')^i \overset{?}{\underset{<}{\geq}} (P^o_{x_m} - VC_b) + S_{v'}\, (V')^j\,. \tag{1.1.35}$$

If $(V')^j$ is enough greater than $(V')^i$ and/or the subsidy is large enough the (<) could hold in (1.1.35). If such be the case the firm would switch to method B and reduce the level of X_n.

The effects of a price change for X_m are similar to previous policies. First, consider the case where $X_m^{b^o}$ was the initial firm position. The subsidy on V induced a switch from B to A as indicated by a (>) in (1.1.30). A decrease in P_{x_m} will not alter such a situation (other than maybe force the firm from business) since the larger side (left) of (1.1.30) will decrease less than the smaller side (right). A price increase for X_m would affect the situation in the opposite manner and could conceivably result in a switch back to technique B. If the initial firm position had been at $X_m^{a^o}$ the subsidy on V would not induce a change. Any effect of a price change would occur even without the policy.

If V' is the variable factor subsidized changes in P_{x_m} have opposite effects. If the initial position had been at $X_m^{b^o}$ then the subsidy on V' was shown not to have any effect on internal operations. Starting from $X_m^{a^o}$ there is the possibility that subsidizing V' could induce a switch to technique B. A rise in P_{x_m} would be favorable to technique B, resulting in no alteration after the subsidy. A decreasing price for X_m, however, could induce a switch back to technique A.

Summary and Implications

For the desired policy effects it is important that the policy unit know the total relative amounts of X_n produced by alternative techniques. Once the relative amounts are known the policy unit must also find a variable factor that is used relatively more by the least externality producing technique. If the wrong variable factor is chosen the end result would easily be an increase in the production of X_n.

The subsidy policy is likely to be popular with firms involved. It is also likely to be expensive to the public furnishing the subsidy unless it is used in conjunction with a tax on certain variables that would offset the subsidy cost.

Changes in the price of X_m could result in nullifying the effects of this policy. However, such a possibility is not different from most of the other policies.

Summary and Conclusions for Model 1

The policy of subsidizing a fixed factor was not considered for this model. The reason is simply that only one fixed factor is used and subsidizing it will not induce any changes in firm operations. If a given technique is the most efficient with K^o then it will be the most efficient with $K^o + \Delta K$. Obviously the only effect of subsidizing ΔK

is that the firm may increase production of X_m and X_n in the longer run assuming other things remain constant. If K^o (the amount already on hand) is subsidized also, economic rent to the firm will increase.

The effects of the various policies depended on several things. One important item was the initial firm position. Also important from a policy viewpoint was the relative total amounts of X_n produced by alternative techniques. To facilitate achieving Objectives 2 and 3 the following table (Table 2.1) presents the main effects of the various policies on Model 1.

For some combinations of starting points and externality positions two possible effects are shown. For example if the firm started at $X_m^{b^o}$ (producing X_m with technique B) and for either externality situation it was possible the tax on X_m could have induced a switch to method A. It was also possible the tax would not have induced a switch. The effects are shown for both possibilities which depended on the size of the tax among other things.

The columns concerning possible effects of price changes can be interpreted as follows: First of all price effects are considered relevant only in the situations where the policy being discussed induced some change in the initial technique being used. In some instances changes in P_{x_m} could induce the firm to change techniques whether the given policy had been applied or not. These latter effects due to price changes are ignored in Table 2.1. The intent of

Table 2. 1. Summary of Effects of Policies on Model 1.

Policy	Initial position		Possible effects of policies on[a]					Possible effects of P_{x_m} ↑ on[b]		Possible effects of P_{x_m} ↓ on[b]	
	Prod. point	Externality greater for tech.	Tech.	Net rev.	Prod. of X_m	Prod. of X_n	Use of V	Technique	X_n	Technique	X_n
Taxing the mkt. product X_m	$X_m^{b^o}$	B	0	-	0	0	0	0	0	0	0
			change	-	-	-	+	change back	+	0	0
		A	0	-	0	0	0	0	0	0	0
			change	-	-	+	-	change back	-	0	0
	$X_m^{a^o}$	B	0	-	0	0	0	0	0	0	0
		A	0	-	0	0	0	0	0	0	0
Taxing a variable factor, V	$X_m^{b^o}$	B	0	-	0	0	0	0	0	0	0
		A	0	-	0	0	0	0	0	0	0
	$X_m^{a^o}$	B	0	-	0	0	0	0	0	0	0
			change	-	+	+	-	0	0	change back	-
		A	0	-	0	0	0	0	0	0	0
			change	-	+	-	-	0	0	change back	+
Taxing the non-market externality, X_n	$X_m^{b^o}$	B	0	-	0	0	0	0	0	0	0
			change	-	-	-	+	change back	+	0	0
		A	0	-	0	0	0	0	0	0	0
	$X_m^{a^o}$	B	0	-	0	0	0	0	0	0	0
		A	0	-	0	0	0	0	0	0	0
			change	-	+	-	-	0	0	change back	+
A standard on the quality of the externality	$X_m^{b^o}$	B	0	-	-[c]	-[d]	0	0	0	0	0
		A	0	-	-[c]	-[d]	0	0	0	0	0
	$X_m^{a^o}$	B	0	-	-[c]	-[d]	0	0	0	0	0
			change	-	-	+[e]	-	0	0	change back	-
		A	0	-	-[c]	-[d]	0	0	0	0	0
			change	-	-	-	-	0	0	change back	+
Subsidizing a variable factor, V	$X_m^{b^o}$	B	0	+	0	0	0	0	0	0	0
			change	+	-	-	+	change back	+	0	0
		A	0	+	0	0	0	0	0	0	0
			change	+	-	+	+	change back	-	0	0

Table 2.1 (continued)

Policy	Initial position		Possible effects of policies on[a]					Possible effects of P_{x_m} ↑ on[b]		Possible effects of P_{x_m} ↓ on[b]	
	Prod. point	Externality greater for tech.	Tech.	Net rev.	Prod. of X_m	Prod. of X_n	Use of V	Technique	X_n	Technique	X_n
	X^{oa}_m	B	0	+	0	0	0	0	0	0	0
		A	0	+	0	0	0	0	0	0	0
Subsidizing a variable factor, V'	X^{ob}_m	B	0	+	0	0	0	0	0	0	0
		A	0	+	0	0	0	0	0	0	0
	X^{oa}_m	B	0	+	0	0	0	0	0	0	0
			change	+	+	+	+ (V')	0	0	change back	
		A	0	+	0	0	0	0	0	0	0
			change	+	+	-	+ (V')	0	0	change back	+

[a] A (+) indicates that the specific variable will increase, a (-) it will decrease, and (0) no change. If more than one sign is shown it means that both effects are possible.

[b] The only effects considered are on situations where the policy under consideration caused some change.

[c] Even though there is no change in basic technique some K must be used to process X_n; therefore, there is less available for producing X_m.

[d] The volume of X_n will be reduced since X_m production will be reduced; however, X_n will also be changed to the form X'_n which should meet the standard.

[e] Volume of X_n will go up but it will be released as X'_n.

considering the price changes was to see if any policies could cause effects that would be insensitive to P_{x_m}. As can be observed, none of the policies induced internal technique switches that were insensitive to some type of price movement. The effluent standard, however, would not be materially affected, since X'_n would be released in any event.

The policy of taxing the non-market externality was the only one for which the possibility of increasing X_n production did <u>not</u> exist. The effluent standard though could always be considered an improvement since at least X'_n would be released instead of X_n.

Provided that the proper type of variable factor is chosen for taxing (subsidizing), these policies could induce the proper type changes. If a variable factor is to be taxed (subsidized) successfully, then such factor should, under the constrained conditions, be used more (less) in total by the high externality producing technique. In other words, V meets the criterion for successful taxing and V' for successful subsidizing. By successful it is meant that the policy could at least induce a change under proper conditions (size of tax or subsidy for one) that would reduce X_n production.

The Remaining Theoretical Analysis

The next three chapters, III, IV, and V, present the theoretical analyses for three other linear models. The procedure and analysis for each model are similar to those presented in this chapter. The succeeding models, as mentioned earlier, are extensions of Model 1; consequently, the analysis becomes quite repetitive of that already discussed. The reader who is not interested in the detail may wish to skip to Chapter VI, where the results for all models are summarized. The major differences in the various models can be determined by reading only the first sections of each of the following three chapters.

III. POLICY EFFECTS ON MODEL 2--ONE NON-SPECIALIZED FIXED FACTOR, ONE MARKET PRODUCT AND ONE DOMINATING TECHNIQUE

Specification and Policy Applications for Model 2

Most of the assumptions for Model 1 are also relevant to this model. Notation established in Chapter II is used here also. The only difference between Models 1 and 2 is that it is assumed that technique A totally dominates technique B in Model 2. That assumption implies two things.

1. Technique A requires k_a units of the fixed factor K and method B requires k_b units of K to produce one unit of X_m, the market product. In this instance $k_a < k_b$.
2. The per unit variable costs for method A, VC_a, are assumed to be less than the per unit variable costs of B, VC_b.

It is also assumed that technique A, the dominating technique, produces more X_n per unit of X_m than does method B, i.e., $n_a > n_b$.

Taxing the Market Product

The initial net revenue maximizing technique is not dependent upon the level of P_{x_m} as it was in the previous model. If the firm produces at all it will use technique A due to the dominance. The latter can be observed in Figure 3.1. If $P_{x_m} > VC_a$ then the firm

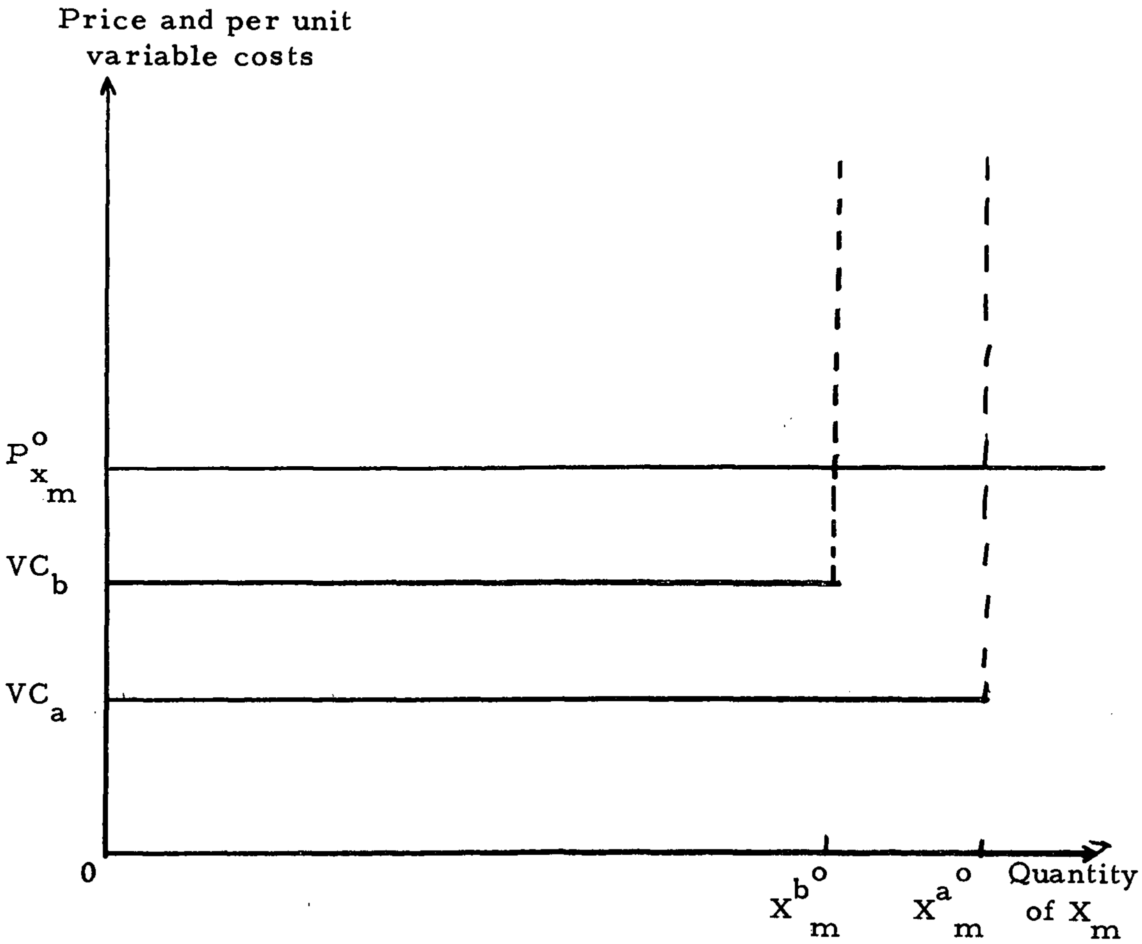

Figure 3.1. Initial firm position--Model 2-- one dominating technique

will make the most positive net revenue with technique A. If $P_{x_m} < VC_a$ then it would close down since it would sustain a loss by operating.

The initial firm position implies

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m > (P^o_{x_m} - VC_b)\, X^{b^o}_m . \qquad (2.1.1)$$

Since $VC_a < VC_b$ and $X^{a^o}_m > X^{b^o}_m$ it is clear that as long as

$(P_{x_m} - VC_a) > 0$ the left side of (2.1.1) will be larger than the right. Suppose a tax T is placed on each unit of X_m produced. The firm will view such a tax as a lowering of P_{x_m}. The situation after the tax will be

$$(P^o_{x_m} - VC_a - T)\, X^{a^o}_m \overset{?}{\underset{<}{\geq}} (P^o_{x_m} - VC_b - T)\, X^{b^o}_m . \qquad (2.1.2)$$

By assumption $(P^o_{x_m} - VC_a) > (P^o_{x_m} - VC_b)$ and subtracting a constant T, from both sides of an inequality will not change its direction, therefore $(P^o_{x_m} - VC_a - T) > (P^o_{x_m} - VC_b - T)$. It is also known that $X^{a^o}_m > X^{b^o}_m$. The only way that a (<) could possibly hold in (2.1.2) would be if $(P^o_{x_m} - VC_a - T) < 0$ which would imply that the firm would shut down. The conclusion is that only the (>) can hold (without the firm closing down) in (2.1.2); therefore, this policy will not achieve the "best" outcome.

Summary and Implications

Where a high externality producing technique dominates other lower externality producing techniques, taxing the market product will be ineffective. If such a policy is being considered the policy unit must be aware of whether or not dominating techniques exist. Production of X_n will be reduced only at the expense of forcing the firm from business. The price of X_m did not affect the initial firm position and that position cannot be changed by the tax. Consequently, fluctuations in P_{x_m} are unimportant to this policy and model.

Taxing a Variable Factor

The assumptions concerning the variable factor are the same as for Model 1 but are reiterated again here.

1. There is some variable factor, V, that is, compared to other variable factors, relatively low priced to the firm and readily available.
2. Technique A uses more V per unit of X_m than method B, i.e., $v_a > v_b$.

As before the initial situation is

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m > (P^o_{x_m} - VC_b)\, X^{b^o}_m \ . \qquad (2.1.3)$$

What can be concluded about the situation after the tax?

$$(P^o_{x_m} - VC_a - T_v v_a)\, X^{a^o}_m \overset{?}{\gtreqless} (P^o_{x_m} - VC_b - T_v v_b)\, X^{b^o}_m .$$

(2.1.4)

By assumption $v_a > v_b \Longrightarrow T_v v_a > T_v v_b$; therefore, one cannot conclude anything a priori about (2.1.4). To cause the firm to switch to technique B the following is necessary:

$$(P^o_{x_m} - VC_a - T_v v_a) X^{a^o}_m > (P^o_{x_m} - VC_b - T_v v_b) X^{b^o}_m$$

$$\Rightarrow (T_v (v_a X^{a^o}_m - v_b X^{b^o}_m) > (P^o_{x_m} - VC_a) X^{a^o}_m$$

$$- (P^o_{x_m} - VC_b) X^{b^o}_m \qquad (2.1.5)$$

$$\Rightarrow T_v (V^i - V^j) > (P^o_{x_m} - VC_a) X^{a^o}_m - (P^o_{x_m} - VC_b) X^{b^o}_m$$

since $V^i = v_a X^{a^o}_m$ and $V^j = v_b X^{b^o}_m$ or the total amount of the variable factor needed to produce $X^{a^o}_m$ and $X^{b^o}_m$, respectively.

In other words the tax savings (left side of (2.1.5)) associated with switching to method B must be greater than the net revenue advantage of A (right side of 2.1.5)). The left side is influenced by two things. One is the size of the tax. The other influence is the relative size of V^i and V^j. Consequently, a policy of taxing variable inputs to bring about technique changes is more likely to succeed if the input taxed is used to a much greater extent by the original technique.

From Figure 3.2 it is obvious that one implication of the condition necessary for a switch to occur is that $VC_a + T_v v_a > VC_b + T_v v_b$. In other words the tax on V has destroyed the domination of technique A over B. At this point this model is practically the same as Model 1.

If the price, $P^o_{x_m}$, should happen to be below VC_b in Figure 3.2 then the tax on V cannot bring about a technique switch. The firm

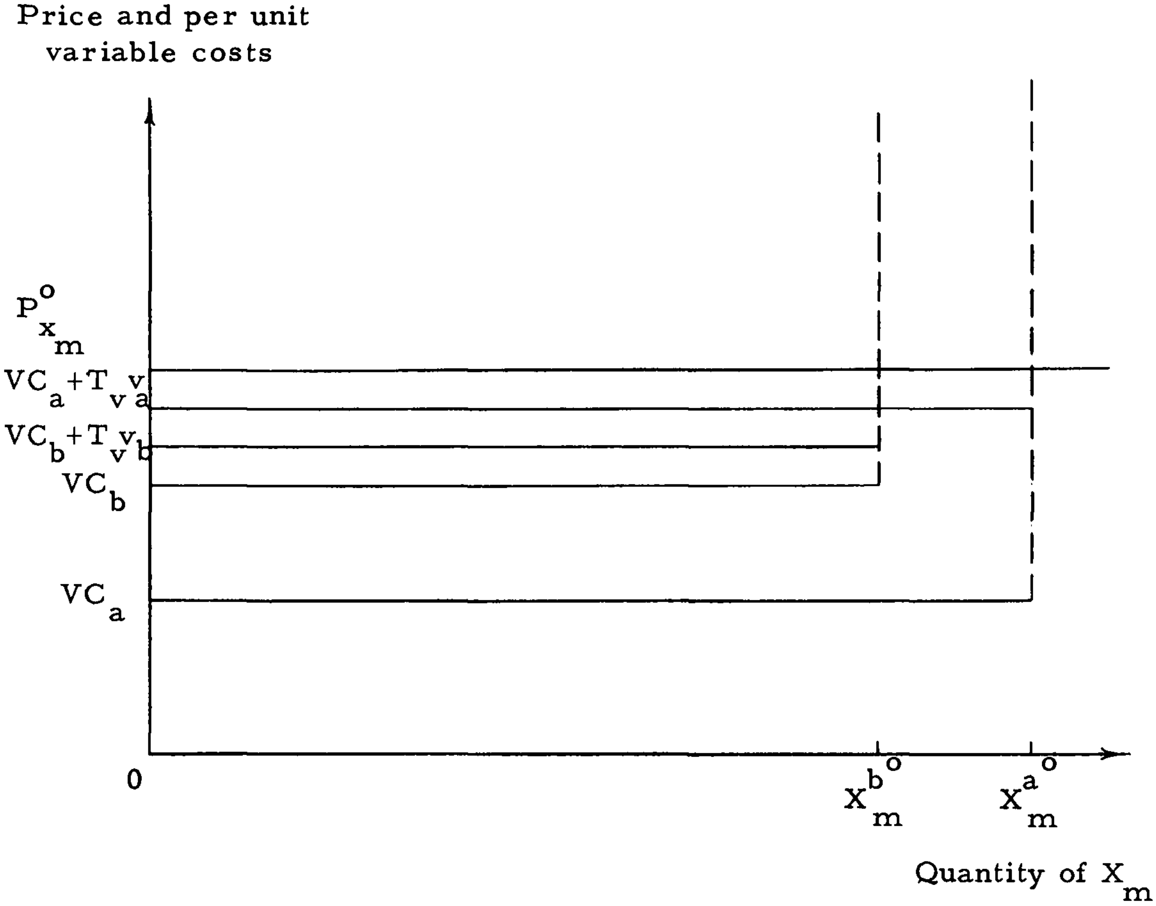

Figure 3. 2. Effects of taxing a variable factor--Model 2

would be forced from business before it switched to method B.

Given that the (<) holds in (2. 1. 4) which implies a switch to method B changes in P_{x_m} could alter the situation. An increase in P_{x_m} could induce the firm to switch back to method A while price decreases will do nothing more than maybe force the firm to shut down.

Summary and Implications

Given the proper size of the tax, T_v, this policy could achieve the "best outcome". For the latter to be so it must also be that P_{x_m} is not below the per unit variable costs of producing the product by the alternative technique. Increase in the price of X_m could induce a switch back to technique A.

The proper type of variable factor must be taxed. A variable factor that is used more heavily by the dominating technique A is the most likely prospect. Taxing a variable factor used heavily by method B would do nothing other than generate money for the policy unit. To determine the proper factor to tax it appears that knowledge about individual techniques is important.

Taxing the Non-market Externality

The firm initially is producing at $X_m^{a^o}$ utilizing method A. Further assume that the expected price is greater than either VC_a or VC_b as $P^o_{x_m}$ in Figure 3.1. Initially then

$$(P^o_{x_m} - VC_a)\, X_m^{a^o} > (P^o_{x_m} - VC_b)\, X_m^{b^o} \,. \tag{2.1.6}$$

Assume a tax, T_n, is placed on the non-market externality X_n. With the policy (2.1.6) becomes

$$(P^o_{x_m} - VC_a - n_a T_n) X^{a^o}_m \overset{?}{\underset{<}{\geq}} (P^o_{x_m} - VC_b - n_b T_n) X^{b^o}_m .$$

(2. 1.7)

Since $n_a > n_b$, $n_a T_n > n_b T_n$ and it is not possible to conclude anything about (2. 1. 7) without further information.

The necessary condition for the firm to switch methods is for the (<) to hold in (2. 1.7). From (2. 1. 7) then

$$(P^o_{x_m} - VC_a) X^{a^o}_m - (P^o_{x_m} - VC_b) X^{b^o}_m < (n_a X^{a^o}_m T_n - n_b X^{b^o}_m T_n)$$

(2. 1. 8)

where $n_a X^{a^o}_m = X^{a^o}_n > n_b X^{b^o}_m = X^{b^o}_n$. In words, the firm will switch techniques if the amount saved in taxes (right side of (2. 1. 8)) exceeds the additional revenue produced by utilizing technique A versus B. The cost to the firm of <u>switching</u> techniques then is the difference between the two sides of (2. 1. 8). However, the cost to the firm of the policy is $(P^o_{x_m} - VC_a) X^{a^o}_m - (P^o_{x_m} - VC_b - n_b T_n) X^{b^o}_m$ or $n_a X^{a^o}_m T_n$ depending on whether the firm switches techniques or not. The right side of (2. 1. 8) depends on the size of the tax and the relative amounts of X_n produced by each technique.

Notice the similarity between the effects on the original inequalities between this policy and the policy of taxing V. Statement (2. 1. 7) is quite similar to (2.1.4). The only difference is that v_a and v_b are used instead of n_a and n_b. It appears that if the proper variable factor were found, taxing it could be quite close to taxing the externality.

As expected the effects of changes in P_{x_m} are similar to the previous policy. If the switch from A to B occurred then a falling price for X_m will not change the situation; however, a price increase could induce the firm to switch back to A. Also note that if $P_{x_m} < VC_b$ initially, then this policy will not bring about any technique switch.

Summary and Implications

The form and the effects of taxing a variable factor are quite similar to this externality tax. The smaller the differences between VC_a and VC_b and $X_m^{a^o}$ and $X_m^{b^o}$ the better the chance that the tax on X_n will work. Also the greater the difference in externality production between the two techniques the more likely the tax will induce a switch from A to B. Again price increases for X_m could result in the firm switching back to technique A if it had, due to the policy, switched to B. The problems associated with administering an effluent or externality tax are the same as pointed out for Model 1.

A Standard on the Quality of the Externality

It may be useful to reiterate the assumptions of Model 1 that are also relevant at this point for Model 2. It is assumed that the standard is placed on the quality of the externality[23] and that X_n will not meet this standard. It is also assumed that both techniques produce X_n only in different quantities.

With the previous assumptions the only way the firm can continue to operate is if it changes the form of X_n say to X'_n which will meet the standard. Suppose first of all that X_n can be changed to X'_n by using variable costs only.

The initial position is again as shown in (2.1.6). After the effluent standard the relative situation is as follows:

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m - VC_n\, n_a\, X^{a^o}_m \overset{?}{\substack{\geq \\ <}} (P^o_{x_m} - VC_b)\, X^{b^o}_m - VC_n\, n_b\, X^{b^o}_m\, . \qquad (2.1.9)$$

Since $n_a X^{a^o}_m > n_b X^{b^o}_m$ no conclusion can be reached concerning whether or not the firm will switch techniques. The situation can be

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[23] See footnote 19, Chapter II, page 49.

analyzed identically as for the previous policy and statement (2.1.7). The only difference is that instead of a tax T_n, the variable cost of changing X_n to X'_n, VC_n, is involved. With this policy, providing there is some method of enforcement, the effluent will at least change form (to X'_n) if not quantity.

If fixed factors as well as variable are necessary for converting X_n to X'_n the situation can be somewhat different. If one assumes that no matter which technique is used the amount of fixed factor is the same the analysis will be about the same as the previous case. The only difference will be that X^b_m and X^a_m will be at lower levels but in the same relative positions. The case would be similar to one where the fixed constraint is K' (K' < K^o) instead of K^o. The subsequent analysis would then be identical to the case where there is no K required to process X_n.

Next, assume that the fixed factor required is proportionate to the volume of X_n. Then to produce at either $X^{a^o}_m$ or $X^{b^o}_m$ requires more K than is available in the short run. Relatively more K will need to be used for method A since $n_a > n_b$. The firm must determine the maximum amount of X_m that can be produced and still have enough K to process X_n.[24]

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[24] The procedure is the same as for Model 1. See Appendix A, Part 1, for the derivation of the relative sizes of K^o, K', and K''.

Under the assumed conditions then $K'' < K' < K^o$ where K' is the relevant constraint for method B and K'' is the relevant constraint for A.

Now the firm is faced with the following:

$$(P^o_{x_m} - VC_a) X^{a''}_m - VC_n n_a X^{a''}_m \overset{?}{\gtrless} (P^o_{x_m} - VC_b) X^{b'}_m - VC_n n_b X^{b'}_m . \qquad (2.1.10)$$

But $k_b > k_a$ (assumption 1) and $K' > K''$ so that the relative size now of $X^{a''}_m$ and $X^{b'}_m$ is not ascertainable (since $X^{a''}_m = \frac{K''}{k_a}$ and $X^{b'}_m = \frac{K'}{k_b}$). It is possible that now $X^{a''}_m < X^{b'}_m$. If that be the instance then the situation would appear similar to Figure 2.1 for Model 1. In effect, the dominance of method A over B has been broken by the policy if $X^{a''}_m < X^{b'}_m$. Breaking the dominance alone is not enough to assure that a switch of techniques will take place. If the (<) holds in (2.1.10) then a switch from A to B is implied. The possibility of a switch is enhanced by the total market output of method A being reduced below that for technique B. If VC_n is high enough it is possible that after the standard, technique B could dominate A. Dominance would occur if $X^{a''}_m < X^{b'}_m$ and $(VC_a + VC_n n_a) > (VC_b + VC_n n_b)$. If the latter result obtains then the switch to B will be totally insensitive to changes in P_{x_m}.

If after the standard is enforced $X_m^{b'} < X_m^{a''}$ the chances for an induced switch of techniques seems lower than as discussed above. It would still be possible for the (<) to hold in (2.1.10) but it is necessary now that $(VC_a + VC_n n_a) > (VC_b + VC_n n_b)$. When $X_m^{b'} > X_m^{a''}$ the latter cost condition was not necessary although it would have been sufficient. Even if the (<) in (2.1.10) holds it is not possible as it was above for method B to dominate A. Consequently, a switch back to method A could occur if the price of X_m went up. No change would occur (unless the firm shuts down) if the price of X_m falls.

Summary and Implications

Providing that enforcement is effective X'_n which meets the standard will be expelled. As far as the policy unit is concerned then the policy will appear to be effective.

What happens internally is not so easy to predict. If the variable costs of treating X_n and the fixed factor requirements for the same are relatively (to other costs and factor requirements) small then it appears that the firm will continue using method A. The disadvantage to such a procedure is that the volume of effluent may not change even though its quality has changed. The implication is that if water effluent standards are to be used then the standards must consider the fact that volume is not likely to be reduced. This would be quite important if the effluent quality standard was based on a stream standard. For example, suppose that to keep the stream dissolved oxygen at a certain level the firm being considered must not release over 1,000 pounds per day of BOD load. Here the total load is important so the effluent standard must consider that there is not likely to be a reduction in volume. The percent BOD removal would then need to be higher than if volume were expected to drop.

It is possible that the standard could cause the firm to switch techniques internally. Unless technique B dominates A (after the standard) price movements could cause the firm to switch back to A. If a switch back did occur due to changes in P_{x_m}, the volume of X_n could actually go up sometime after the standard had been set provided the specific standard was based on the effluent from technique B. The implication is that there is likely to be a need for monitoring and standards based on quantity as well as quality.

Subsidizing a Variable Factor

Initially the firm faces the following

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m > (P^o_{x_m} - VC_b)\, X^{b^o}_m \;. \qquad (2.1.11)$$

If V is subsidized the relationship in (2.1.11) will not be altered since method A uses more V than method B by assumption. That is

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m + v_a\, X^{a^o}_m\, S_v$$

$$> (P^o_{x_m} - VC_b)\, X^{b^o}_m + v_b\, X^{b^o}_m\, S_v\, . \qquad (2.1.12)$$

If the intent of the policy were to reduce production of X_n the preceding would not work.

Suppose that there exists another variable factor, V', which is used relatively more by method B than A. If V' is subsidized there is a chance that the firm will switch techniques. Statement (2.1.11) becomes

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m + v'_a\, X^{a^o}_m\, S_{v'} \overset{?}{\gtreqless} (P^o_{x_m} - VC_b)\, X^{b^o}_m$$

$$+ v'_b\, X^{b^o}_m\, S_{v'}\, . \qquad (2.1.13)$$

If the firm is going to switch to method B the (<) in (2.1.13) must hold, i.e.,

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m - (P^o_{x_m} - VC_b)\, X^{b^o}_m < S_{v'}(V')^j - S_{v'}\,(V')^i\, .$$

(2.1.14)

The subsidy (right side (2.1.14)) gained by switching to technique B must be greater than the net revenue lost (left side (2.1.14)). Graphically the situation in (2.1.14) would appear similar to Figure 3.3. Area D must be greater than area E for the firm to switch methods. Area D represents the additional subsidy received

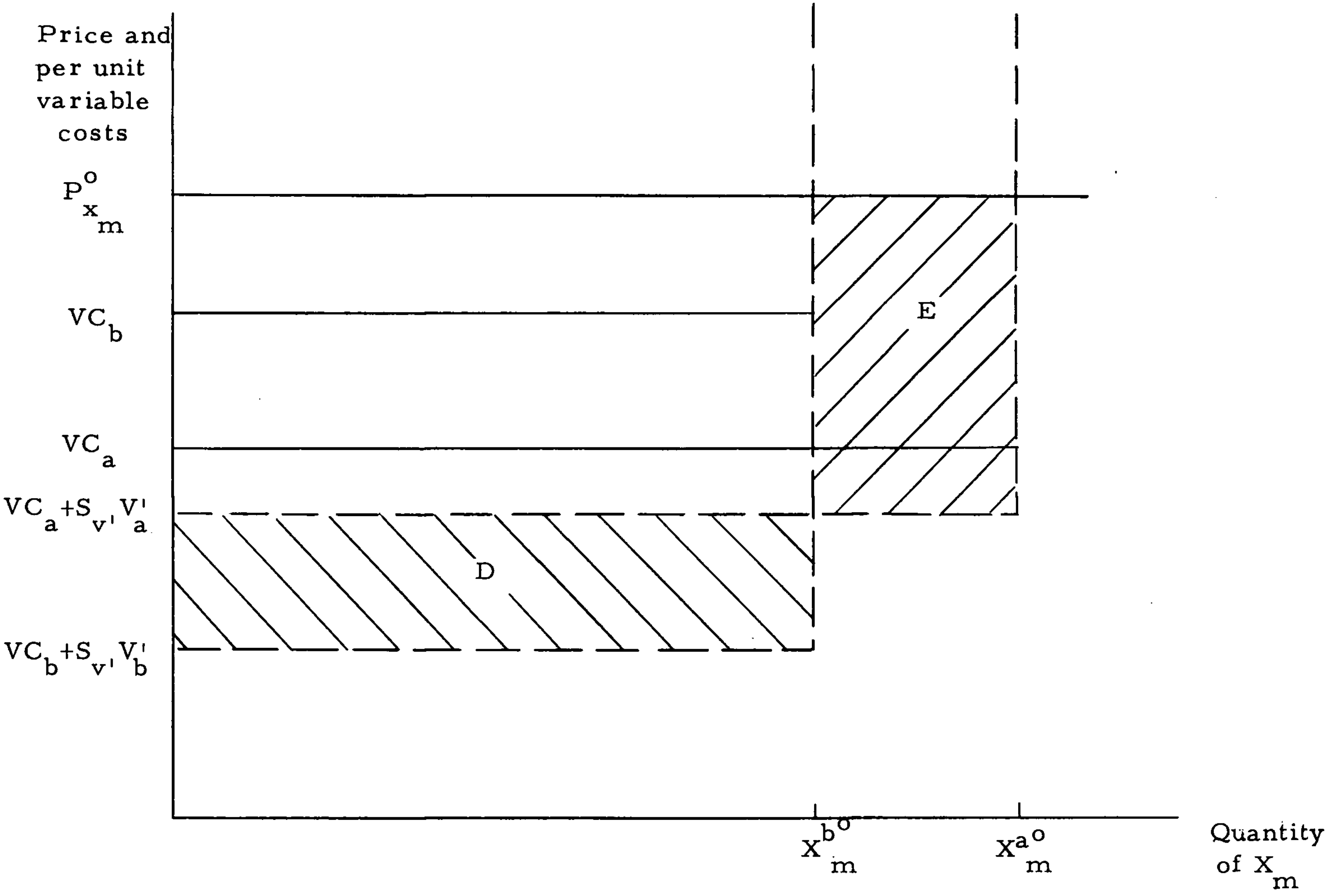

Figure 3.3. Effects of subsidizing a variable factor, V', on Model 2.

if $X_m^{b^o}$ is produced with method B while area E represents a combination of the net revenue gained by producing with method A and the subsidy realized on $X_m^{a^o} - X_m^{b^o}$.

Assuming that the firm did switch to method B a price rise for X_m could induce the firm to switch back to A. A decrease in $P_{x_m}^o$ implies the firm will keep producing with technique B.

Summary and Implications

This policy could induce the firm to switch to the "optimum policy point". For such a result the variable factor subsidized must be used relatively more by the high externality producing technique, A, than B. The level of the subsidy will be important. The less the difference in use of the subsidized factor between techniques the higher the subsidy will need to be. The implication of sensitivity to prices is that an effective subsidy may need to vary as price varies.

Summary and Conclusions for Model 2

The sixth policy, that of subsidizing a fixed factor, was not used on this model for the same reasons as discussed for Model 1. The rest of the policy results are less complicated for Model 2 than Model 1 due to the dominance assumption. The results are summarized in Table 3.1.

Table 3. 1. Summary of Effects of Policies on Model 2.

Policy	Possible effects of policies on					Possible effects of P_{x_m} ↑ on		Possible effects of P_{x_m} ↓ on	
	Technique[a]	Net revenue	Prod. of X_m	Prod. of X_n	Use of V	Technique	X_n	Technique	X_n
Taxing the market product	0	-	0	0	0	0	0	0	0
Taxing a variable factor, V,	0	-	0	0	0	0	0	0	0
	change	-	-	-	-	change back	+	0	0
Taxing the non-market externality, X_n	0	-	0	0	0	0	0	0	0
	change	-	-	-	-	change back	+	0	0
A standard on the quality of the externality	0	-	-[b]	-[c]	Indet.[d]	0	0	0	0
	change	-	-	-	Indet.[d]	0[e]	0[e]	0[e]	0[e]
	change[e]	-	-	-	Indet.[d]	change back[f]	+	0	0
Subsidizing a variable factor, V	0	+	0	0	0	0	0	0	0
Subsidizing a variable factor, V'	0	+	0	0	0 (V')	0	0	0	0
	change	+	-	-	+ (V')	change back	+	0	0

[a] Since technique A dominates B, the initial position is always $X_m^{a^o}$ and A always produces the most externality.

[b] This declines even though there is no switch since processing of X_n requires K which reduces the amount available for producing X_m.

[c] Volume of X_n is reduced since X_m is reduced; however, X'_n is released.

[d] Indeterminate because the use of specific variable factors, e. g. , V and V', for processing X_n was not assumed. Since an assumption about VC was made it could well be that V and/or V' would be used to convert X_n to X'_n.

[e] If after the standard $(VC_a+VC_{n_a}) > (VC_b+VC_{n_b})$ and $X_m^{a''} < X_m^{b'}$ then B dominates A and P_{x_m} will not affect the situation.

[f] This is if after the policy $X_m^{b'} < X_m^{a''}$.

All policies applied to this model were in a sense more effective than for the previous model since any induced changes did reduce X_n production. However, it must be pointed out that some of the policies did not induce any change at all. If the agency is looking for technique changes then it should not consider taxing X_m or subsidizing V.

The standard on the quality of the externality was insensitive to changes in P_{x_m} under one possible condition. That condition was that after the standard, technique B dominated A. Dominance of B over A depended on the relative amounts of K and VC_n required to convert X_n to X'_n. Other than this one case any changes of technique were susceptible to reversal by increasing prices for X_m. One implication is that the policy agency needs to consider flexible taxes or subsidies. More specifically if the firm to which the policy will be applied produces a product for which the price is not likely to go up then a tax (subsidy) rate that would induce a switch would be alright. However, if P_{x_m} might go up then the flexible level needs to be considered.

IV. POLICY EFFECTS ON MODEL 3--ONE NON-SPECIALIZED FIXED FACTOR AND TWO FIXED FACTORS SPECIALIZED BY TECHNIQUE

The difference between this and previous models is that it is assumed that each technique requires one specialized fixed resource. There is also assumed to be one non-specialized fixed resource that is needed and can be used by either method.

By making different assumptions about the relative levels of variable costs, use of the fixed factors, etc., a number of sub-models can be generated. Some of the assumptions are relevant to all the sub-models. These include the two mentioned above and the following:

1. Technique A which will be used initially requires k_{aa} units of K_a, the specialized fixed factor for method A, to produce one unit of X_m.
2. Only K_a is available at the outset.
3. To switch to method B the firm must acquire the specialized resource K_b at some per unit cost, P_{K_b}. Production of one unit of X_m by method B requires k_{bb} units of K_b.
4. There may be some salvage or resale value from disposing of the specialized resource for method A, K_a, and/or for disposing of some of the non-specialized resource K. Per unit resale values will be designated R_{K_a} and R_K for K_a and

K, respectively.

5. The per unit resource price and per unit resale values represent yearly prices and costs. For example, R_K represents a yearly return on money that could be received from disposing of one unit of K. Similarly, P_{K_b} represents the yearly cost of one unit of K_b including such things as yearly interest, depreciation, and maintenance over the appropriate time period.
6. Technique A produces more X_n per unit of X_m than technique B, again designated as n_a and n_b, respectively.
7. All fixed factors are perfectly divisible.
8. Method A uses more of the variable factor V per unit of X_m than method B, i. e., $v_a > v_b$.

One possible sub-model that will not be discussed is depicted in Figure 4. 1. If both K_a and K_b were initially available to the firm at the levels K_a^o and K_b^o then the only relevant constraint is K. Given such circumstances the analysis could proceed the same as the two previous models.

Specification and Policy Applications for Model 3. 1

It is assumed that the fixed factor K_a is available in quantities such that K is the only effective constraint, e. g., K_a^o in Figure 4. 1.

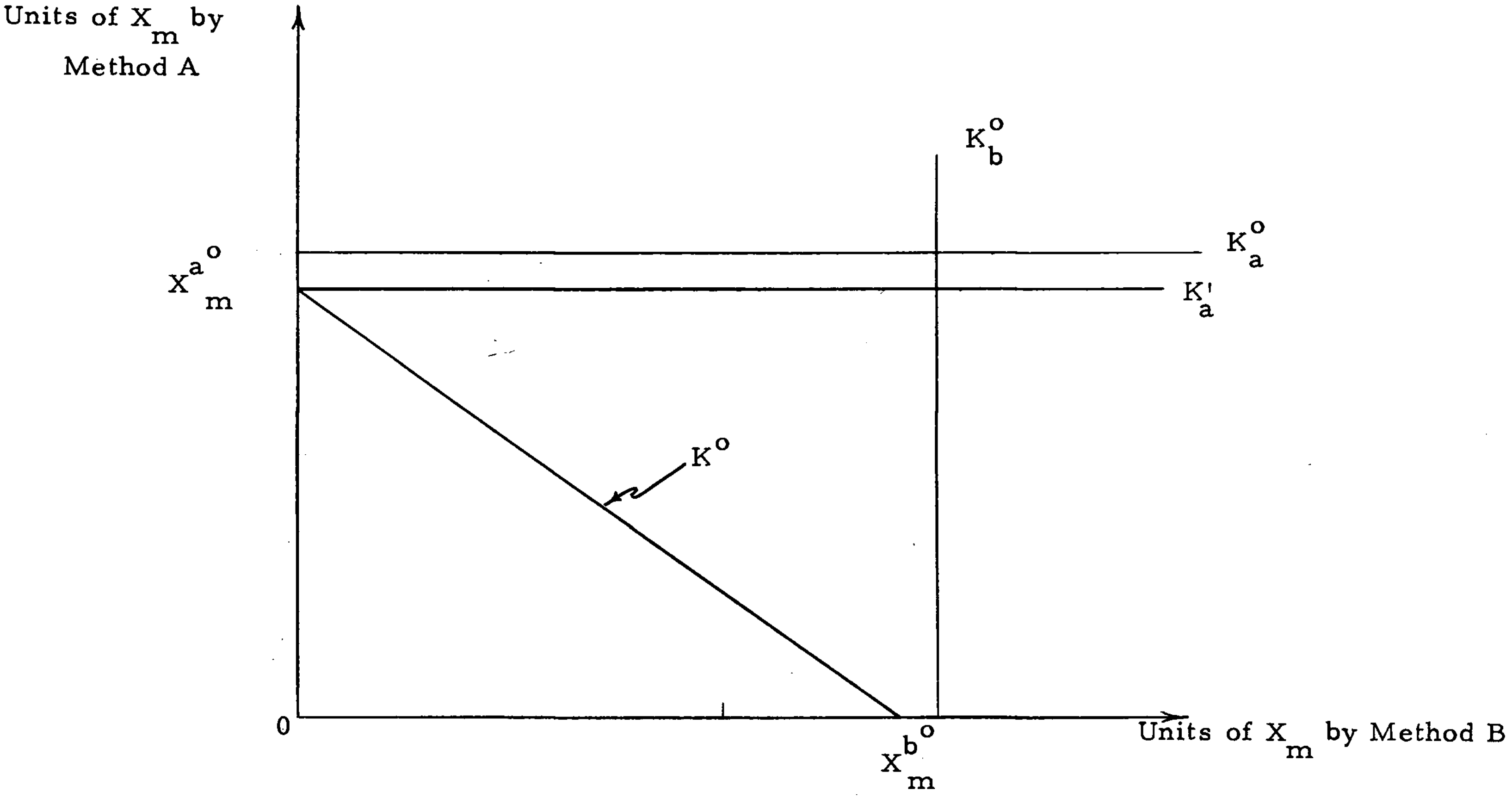

Figure 4. 1. Production possibilities, Model 3, with only K constraining.

The variable costs of producing X_m with technique A, VC_a, are less than the variable costs of producing with technique B, VC_b. Method A requires more K per unit of X_m than method B, i.e., $k_a > k_b$.

Also assume that the maximum X_m produced by A and B as constrained by only K is such that

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m > (P^o_{x_m} - VC_b)\, X^{b^o}_m \ . \qquad (3.1.1)$$

For this model and the ones to follow the time period considered is long enough for the firm to be able to obtain K_b, dispose of K_a and/or K, and maybe even obtain more K. The longer time requires that initial (before the policy is applied) conditions be specified so that such shifting of fixed resources is not economically feasible. These initial conditions will be similar for all sub-models but not identical.

The following expressions indicate the initial conditions which are assumed,

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m > (P^o_{x_m} - VC_b)\, X^{b^o}_m - P_{K_b}\, k_{bb}\, X^{b^o}_m$$
$$+ R_{K_a}\, k_{aa}\, X^{a^o}_m \qquad (3.1.2)$$

$$P_K\, k_a > (P^o_{x_m} - VC_a) + R_{K_a}\, (K^o_a - K''_a) \qquad (3.1.3)$$

where $K''_a = K'_a + k_{aa}$ (see Figure 4.1 for K'_a). Condition (3.1.3) makes it infeasible to sell the excess K_a and obtain enough K to produce one more unit of X_m with technique A.

$$P_{K_b} k_{bb} > R_{K_a} [K_a^o - K_a' + k_{aa}] + (VC_a - VC_b)$$
$$+ R_K (k_a - k_b). \tag{3.1.4}$$

Expression (3.1.4) implies that it costs more to obtain enough K_b to produce one unit of X_m by method B than could be gained in returns from salvaging excess K released by reducing production with technique A one unit and salvaging the excess K_a. Note that for submodel 3.1, $VC_a - VC_b < 0$ which implies a loss in basic net revenue if method B were used instead of A.

Taxing the Market Product

Will this policy bring about any change in the firm's production practices? The tax on the market product is similar to a reduction in price (tax incidence assumed to be to the producer). One consequence is that a large enough tax could force the firm out of business. The production of X_n would naturally be reduced; however, the cost to the firm would be at the extreme. With conditions as assumed this policy will not induce the firm to switch techniques. First of all the firm does not have any K_b and reducing its net revenue via a tax will make it even less feasible to obtain K_b. Even if K_b^o were available to the firm it still would not switch due to (3.1.1). The tax would reduce the right side (smaller already) of (3.1.1) by more than the left, since $X^{a^o}_m < X^{b^o}_m$. Conditions (3.1.2) and (3.1.3) will be affected similarly in that the right sides will be reduced the most by the tax. Inequality (3.1.4) will not be affected by the tax.

Summary and Implications

The policy of taxing X_m proved not to be effective under the assumed conditions. The effects of changes in P_{x_m} are not important then to this sub-model. Any change brought about in P_{x_m} would have happened without the tax.

Taxing a Variable Factor

Assume that there exists a variable factor V which is, relative to other variable factors, low priced. Furthermore, assume that technique A uses more V per unit of X_m than technique B ($v_a > v_b$). The initial conditions have been adequately described previously and cannot be different here. Initially it must be that (3.1.2) holds. After the tax (3.1.2) becomes

$$(P^o_{x_m} - VC_a) X^{a^o}_m - T_v v_a X^a_m \overset{?}{\substack{\geq \\ <}} (P^o_{x_m} - VC_b) X^{b^o}_m$$

$$- P_{K_b} k_{bb} X^{b^o}_m + R_{K_a} (k_{aa} X^{a^o}_m) - T_v v_b X^{b^o}_m. \qquad (3.1.5)$$

By assumption $v_a > v_b$ but $X_m^{a^o} < X_m^{b^o}$ so one cannot conclude anything about the relative size of $v_a X_m^{a^o}$ and $v_b X_m^{b^o}$. If $v_a X_m^{a^o} > v_b X_m^{b^o}$ then there is a possibility that this policy will induce the firm to switch techniques. Rewriting and assuming that the less than (<) holds in (3.1.5), which implies a switch, then

$$(P^o_{x_m} - VC_a) X_m^{a^o} - (P^o_{x_m} - VC_b) X_m^{b^o} + P_{K_b} K_b^o - R_{K_a} (k_{aa} X_m^{a^o})$$
$$< T_v v_a X_m^{a^o} - T_v v_b X_m^{b^o} . \qquad (3.1.6)$$

The main difference between the policy impact on this model and Model 1 (inequality (1.1.15))is the costs and salvage values of fixed factors. If the cost of K_b were greater than the salvage value of K_a then compared to (1.1.15) it would require a larger tax to induce a technique switch. If the salvage value of K_a exceeded the cost of K_b then the relative tax required to induce a technique switch would be smaller. Inequality (3.1.6) can be interpreted similarly as (1.1.16).

If $v_a X_m^{a^o} < v_b X_m^{b^o}$, taxing V will not induce a technique switch. If the policy agency attempted to bring about a technique switch by increasing T_v, the firm would eventually be forced from business.

The effects of changes in the price of X_m can be determined by observing (3.1.6). An increase in P_{x_m} will cause the left side of

(3.1.6) to decrease while the right side will not be affected (since $X^{b^o}_m > X^{a^o}_m$).. Consequently, the price rise will <u>not</u> induce the firm to change back to technique A. Conversely, a decrease in P_{x_m} will result in an increase in the left side of (3.1.6) which <u>could</u> bring about a change back to method A. If after the tax, T_v,

$$VC_a + T_v v_a + R_{K_a} k_{aa} > VC_b + T_v v_b + P_{K_b} k_{bb}$$

then technique B will in essence dominate A and the model will be insensitive to changes in P_{x_m}.

<u>Summary and Implications</u>

Whether or not this policy is effective depends on several things. First of all it must be that $n_a X^{a^o}_m > n_b X^{b^o}_m$ before the switch will reduce X_n production. Second, the variable factor to be taxed must be used more in total by method A than method B. Also the resale value of K_a as compared to the price for obtaining K_b is quite important as well as the relative amounts of each. The relative amounts of X_m that can be produced per unit of K are also important since these eventually determine the relative sizes of $X^{b^o}_m$ and $X^{a^o}_m$.

If a technique switch is induced by the policy only price decreases are likely to bring about a switch back. Under one set of circumstances the switch would not be influenced by any changes in P_{x_m}.

Taxing the Non-market Externality

From the previous assumptions it is not possible to determine which method produces the most X_n. Before it makes sense to attempt to induce the firm to switch methods with this policy, it must be known that $n_a X^{a^o}_m > n_b X^{b^o}_m$.

Assuming that $n_a X^{a^o}_m > n_b X^{b^o}_m$ after the tax (3. 1. 2) becomes

$$P^o_{x_m} - VC_a) X^{a^o}_m - T_n n_a X^{a^o}_m \overset{?}{\underset{<}{\geq}} (P^o_{x_m} - VC_b) X^{b^o}_m - T_n n_b X^{b^o}_m - P_{K_b} k_{bb} X^{b^o}_m + R_{K_a} k_{aa} X^{a^o}_m. \qquad (3.1.7)$$

For a technique switch to be indicated it is necessary that the (<) hold in (3. 1. 7). Since the tax will reduce the left side of (3. 1. 7) more than the right it is possible that the necessary condition for a switch could be obtained from this policy. This tax is similar to a relatively larger increase in the variable costs of technique A versus B. If the tax changes the total per unit cost relationships so that $(VC_a + R_{K_a} k_{aa} + T_n n_a) > (VC_b + P_{K_b} k_{bb} + T_n n_b)$ then the effects of the policy will not be sensitive to price of X_m. If the previous inequality goes the other direction, (<), it is still possible to meet the necessary condition for a method change but if the price should happen to go down conditions could imply a switch back to technique A.

The specialized fixed factors in this case do not alter analysis of the problem from, for example, Model 1. The only thing K_a and K_b do is influence the necessary size of the tax and extend the relevant time period.

A Standard on the Quality of the Externality

The assumptions concerning X_n and the standard are the same for previous models.[25] Providing that enforcement is effective, then, the firm must alter X_n if it is to produce at all. Whichever technique is used, this policy will at least change the form of the externality.

If X_n can be changed to X'_n by using only variable costs the analysis for this and the other sub-models can proceed as with the previous policy. For example suppose (3.1.2) is the initial condition. After the standard is enforced the firm position will appear as

$$(P^o_{x_m} - VC_a) X^{a^o}_m - VC_n n_a X^{a^o}_m \overset{?}{\gtreqless} (P^o_{x_m} - VC_b) X^{b^o}_m - VC_n n_b X^{b^o}_m - P_{K_b} k_{bb} X^{b^o}_m + R_{K_a} k_{aa} X^{a^o}_m , \qquad (3.1.8)$$

where VC_n is the variable cost associated with changing one unit of X_n to one unit of X'_n. Notice that (3.1.8) is quite similar to (3.1.7),

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[25] See page 49.

the only difference being that instead of T_n (3.1.8) has VC_n. Granted, one difference is that for the tax on X_n the policy agency could determine the size of T_n, whereas with this policy the agency will have little control over VC_n. Whether or not the firm switches techniques will be of little concern to the enforcement agency as long as X'_n is emitted instead of X_n. For determing whether the firm will switch techniques then, the analysis will be the same as for the previous policy.

As another approach, suppose that conversion of X_n to X'_n requires some use of the non-specialized fixed factor K as well as variable factors. Furthermore, assume that either technique will require the same amount of K. Again the analysis of all models can proceed as taxing X_n. The only difference is that instead of K^o being available for production of X_m only $K' < K^o$ will be available.

Suppose that the fixed factor required is proportional to the volume of X_n produced. Furthermore assume that more X_n is produced by technique A than technique B.

The time period for this case has been assumed to be long enough so that adjustments in fixed resources can be made. Consequently, the firm may either obtain the needed K for converting X_n to X'_n from its supply on hand, or obtain additional K.

Condition (3.1.3) implied that for technique A it would be less costly to obtain the K needed for altering X_n from production of X_m.

In other words, the net revenue lost by decreasing the production of X_m one unit is less than the cost of obtaining K outright.

Initially (3.1.2) must be considered. The first question to be answered is, what is the maximum X_m that can be produced by both methods while at the same time X_n' is produced instead of X_n? Initially K is constraining at level K^o. It can be demonstrated that[26]

$$K^o > K' > K''. \qquad (3.1.9)$$

By (3.1.9) and $k_a > k_b$ it can be concluded that $\frac{K''}{k_a} < \frac{K'}{k_b}$ which implies that $X_m^{a''} < X_m^{b'}$.

After the standard is effective,

$$(P^o_{x_m} - VC_a - VC_n n_a) \frac{K''}{k_a} \overset{?}{\gtrless} (P^o_{x_m} - VC_b - P_{K_b} k_{bb} - VC_n n_b)$$

$$\frac{K'}{k_b} + R_{K_a} k_{aa} X_m^{a^o} \qquad (3.1.10)$$

which also assumes that the cost of obtaining K exceeds net revenue that would be given up by method B if the K were taken from K^o.

From (3.1.2) it is known that

$$(P^o_{x_m} - VC_a - R_{K_a} k_{aa}) > (P^o_{x_m} - VC_b - P_{K_b} k_{bb}).$$

Since $VC_n n_a > VC_n n_b$, these terms will tend to decrease the per

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[26] See Appendix A, Part 1.

unit net revenue advantage of A over B. Furthermore, since method A is assumed to produce more X_n than method B, the maximum production of X_m by method A is reduced more than the maximum by B. Both of these factors (i.e., relative reduction of X_m and net revenues) tend to weaken (3.1.2) and make it more likely that the (<) will hold in (3.1.10) which would imply a technique switch. If the (<) does not hold in (3.1.10) it can be shown that

$$\frac{X_m^{b'}}{X_m^{a''}} > \frac{(P^o_{x_m} - VC_a - VC_n n_a) - R_{K_a} k_{aa} \frac{X_m^{a^o}}{X_m^{a''}}}{(P^o_{x_m} - VC_b - P_{K_b} k_{bb} - VC_n n_b)} \qquad (3.1.11)$$

The greater the relative difference between n_a and n_b, the larger the ratio $\frac{X_m^{b'}}{X_m^{a''}}$ will be. The latter is so since $n_a > n_b$, which implies that more K will be required to change the X_n produced by method A to X'_n than is required to change the X_n produced by method B. Large requirements of K for processing X_n by method A will result in a relatively larger reduction in X_m that can be produced by method A. The relationship between n_a and n_b will also tend to reduce the ratio on the right of (3.1.11). The implication is that the larger the relative difference in the externality produced by method A over B, the more likely that setting a standard will induce a technique switch. Other contributing factors to a switch are the relative net revenues and the amount of variable costs needed to convert

X_n to X'_n.

Depending on the situation after policy implementation the switch may or may not be sensitive to price of X_m. If after the policy the cost situation is such that B dominates A, then the switch will be insensitive to P_{x_m}. On the other hand, if a switch is implied without dominance, then the switch will be sensitive to a falling P_{x_m} but not to a rising P_{x_m}.

Summary and Implications

The problems of administering an externality standard are no different for this model than previous ones. Again, if it is enforceable, X'_n will be released into the environment instead of X_n and in a sense the policy will have been effective.

How the firm is affected internally is quite difficult to ascertain. If the amount of externality produced by the alternate technique is much less than that produced by A, then there is a strong likelihood the firm will switch techniques.

If after the standard is administered, technique B dominates A, then changes in P_{x_m} will not affect the situation. Lack of dominance by technique B could result in a switch back to A if P_{x_m} falls.

Subsidizing a Variable Factor

It could be assumed that the variable factor discussed here is the same, i.e., V, as discussed for the policy of taxing variable factors. Furthermore, it could also be assumed that $v_a > v_b$. For all the models there would be a chance that the subsidy could bring about a technique switch provided that $v_a X_m^{a^o} > v_b X_m^{b^o}$. However, it seems more realistic to assume that the subsidy will be placed on some variable factor for which the use coefficient is higher for the preferred technique B. Therefore, it will be assumed that for some variable factor, V', the use requirement for method B is greater than for method A, i.e., $v'_b > v'_a$.

After the subsidy the initial condition (3.1.2) will be transformed to

$$(P^o_{x_m} - VC_a - R_{K_a} k_{aa} + S_{v'} v'_a) X_m^{a^o} \overset{?}{\underset{<}{\geq}} (P^o_{x_m} - VC_b - P_{K_b} k_{bb} + S_{v'} v'_b) X_m^{b^o} \qquad (3.1.12)$$

where $S_{v'}$ is the subsidy on V' and v'_a and v'_b are the amounts of V' required per unit of X_m produced by methods A and B, respectively.

Depending on the relative size of v'_b and v'_a and the size of the subsidy it is possible that the (<) could hold in (3.1.12), which would imply a technique switch. Rewriting with the proper inequality for switching, (3.1.12) becomes

$$\frac{(P^o_{x_m} - VC_a - R_{K_a} k_{aa}) X^{a^o}_m - (P^o_{x_m} - VC_b - P_{K_b} k_{bb}) X^{b^o}_m}{v'_b X^{b^o}_m - v'_a X^{b^o}_m} < S_{v'}. \qquad (3.1.13)$$

In words, (3.1.13) states that for a switch to take place, the per unit subsidy on V' must be greater than the net revenue lost per unit of V' added when switching from A to B.

After a switch has taken place a change in P_{x_m} may or may not affect the situation. If the subsidy is such that after it is placed on V', $VC_a + R_{K_a} k_{aa} - S_{v'} v'_a \geq VC_b + P_{K_b} k_{bb} - S_{v'} v'_b$, then the switch will not be sensitive to P_{x_m}. If in the previous inequality a (<) holds, then a switch could still be implied but a drop in P_{x_m} could imply a switch back to A.

The subsidy brings up a possibility that was not viable with the other policies. A subsidy could increase the right side of (3.1.3) enough to reverse the direction of the inequality. Such an instance would then imply that it would be feasible for the firm to obtain additional K and produce more X_m and X_n by method A. It may be that a subsidy not large enough to bring about (3.1.13) could be large enough to reverse (3.1.3). Under those circumstances the subsidy would result in the opposite of the desired effect.

Summary and Implications

It is possible that the subsidy could induce a technique change. However, to bring about a switch it is important to subsidize a variable factor that is used more by the desired technique. In other words, if a variable factor existed which was specific to technique B, it would be the logical one to subsidize.

The size of the subsidy is also quite important. If the subsidy is too small, the firm may obtain more of its constraining factor, not switch techniques, and end up producing more X_n. If the subsidy is too large the firm might switch techniques, but also increase over-all size of operations to the point where more X_n is being produced than initially.

Decreases in P_{x_m} could possibly reverse the switch brought about by the policy. Provided that after the subsidy, technique B dominates A, changes in P_{x_m} will not reverse the original switch.

Subsidizing a Fixed Factor

Since it has been assumed that it is the objective of policy to induce firms to switch from technique A to B, it is logical to subsidize the specialized fixed factor for method B. After the policy, (3.1.2) will be changed to appear as follows:

$$(P^o_{x_m} - VC_a - R_{K_a} k_{aa}) X^{a^o}_m \overset{?}{\underset{<}{\geq}} (P^o_{x_m} - VC_b - P_{K_b} k_{bb}) X^{b^o}_m$$

$$+ S_{K_b} k_{bb} X^{b^o}_{m}, \qquad (3.1.14)$$

where S_{K_b} represents the subsidy on the fixed resource K_b. If the $(<)$ holds in (3.1.14), then a switch of techniques is implied and (3.1.14) becomes

$$(P^o_{x_m} - VC_a - R_{K_a} k_{aa}) X^{a^o}_m - (P^o_{x_m} - VC_b - P_{K_b} k_{bb}) X^{b^o}_m$$

$$< S_{K_b} k_{bb} X^{b^o}_m . \qquad (3.1.15)$$

Rewriting, (3.1.15) becomes

$$S_{K_b} > \frac{(P^o_{x_m} - VC_a - R_{K_a} k_{aa}) X^{a^o}_m - (P^o_{x_m} - VC_b - P_{K_b} k_{bb}) X^{b^o}_m}{k_{bb} X^{b^o}_m} \qquad (3.1.16)$$

which states that the subsidy per unit of K_b must be greater than the loss in net revenue per unit of K_b brought about by a switch from A to B.

Inequality (3.1.3) will not be affected by this policy; consequently, it cannot have the perverse effect of increasing production by technique A. If the subsidy were large enough so that

$$P^o_{x_m} - VC_b) + S_{K_b} k_{bb} > P_{K_b} k_{bb} + P_K k_b \qquad (3.1.17)$$

then the firm could continue producing with technique A at $X_m^{a^o}$ and also increase production with technique B ad infinitum, thus increasing the problem of the externality. It is difficult to compare a priori the relative sizes of S_{K_b} required to make (3.1.16) and (3.1.17) effective without specific coefficients. If a switch is indicated and if after the policy

$$(VC_a + R_{K_a} k_{aa}) < (VC_b + P_{K_b} k_{bb} - S_{K_b} k_{bb}) \qquad (3.1.18)$$

then the switch will be sensitive to downward movements in the price of X_m. If ($\geq$) holds in (3.1.18) then the firm will not switch back to technique A no matter what happens to P_{x_m}.

Summary and Implications

The fixed factor subsidized is important. There seems to be little logic in subsidizing some fixed factor that is used by technique A.

Subsidy size is also critical. It is possible for the subsidy to become too big. Firm size may become such that more of everything including X_n is produced.

Specification and Policy Applications for Model 3.1'

The only difference between this sub-model and (3.1) is that K_a is assumed to be the constraining factor instead of K (e.g., K'_a in Figure 4.2). As a consequence of this assumption and the fact that the firm is initially using method A, there will be excess K available. To prevent the possibility of the firm finding it feasible to just acquire some K_b and produce additional X_m, it is necessary to assume that

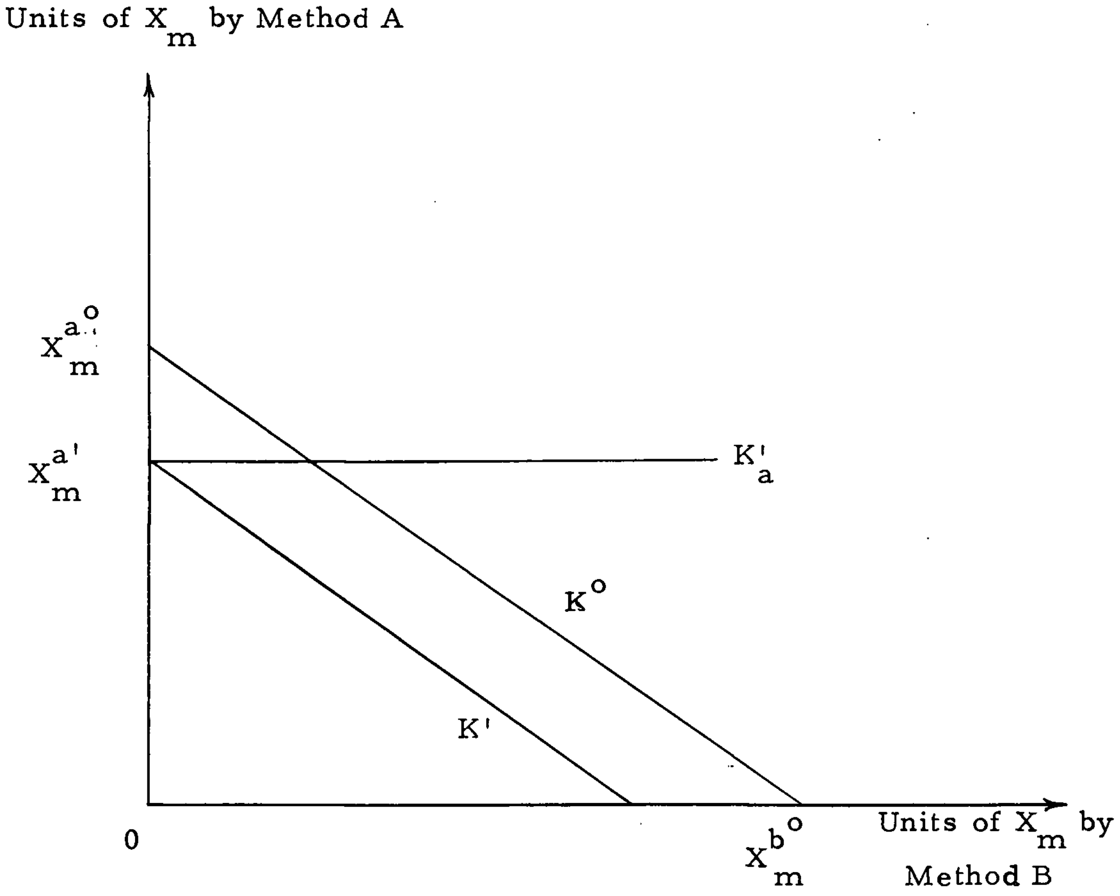

Figure 4.2. Production possibility--Model 3.1' when K_a is constraining.

$$P_{K_b} k_{bb} > (P^o_{x_m} - VC_b) + R_K (K^o - K''), \qquad (3.1'.1)$$

where $K'' = k_a X^{a'}_m + k_b$. That is, the cost of obtaining enough K_b to produce one unit of X_m with method B is greater than the net return from that unit plus salvage value of any K not used by $X^{a'}_m$ and the one unit of X_m produced with B. Inequality (3.1'.1) is a very limiting assumption as far as a technique switch is concerned. From (3.1'.1) it is readily (assuming $R_K (K^o - K'') \geq 0$) determined that $P_{K_b} k_{bb} > (P^o_{x_m} - VC_b)$ which further implies that any X_m produced by method B will result in negative net returns. Consequently, whenever (3.1'.1) is relevant a positive net revenue from using technique B is impossible unless income comes from other sources, e.g., a subsidy or salvage of fixed inputs.

Other initial conditions for Model 3.1' are similar to those for 3.1, such as

$$R_K (K^o - K') + (P^o_{x_m} - VC_a) X^{a'}_m > (P^o_{x_m} - VC_b) X^{b^o}_m$$
$$- P_{K_b} k_{bb} X^{b^o}_m + R_{K_a} k_{aa} X^{a'}_m \qquad (3.1'.2)$$

which is similar to (3.1.2). The following is similar to (3.1.3) in that the excess fixed factor (in this case K instead of K_a) cannot be traded for additional K_a:

$$P_{K_a} k_{aa} > (P^o_{x_m} - VC_a) + R_K (K^o - K'') \qquad (3.1'.3)$$

where $K'' = K' + k_a$ (see Figure 4.2 for k'),

Taxing the Market Product

The K'_a constraint will reduce the maximum X^a_m to some point less than $X^{a^o}_m$, such as $X^{a'}_m$ in Figure 4.2. To produce more X_m by technique A, the firms needs to obtain more K_a.

The tax on X_m will not induce the firm depicted by this model to change techniques, or increase production with method A. The tax will only make the smaller sides (right) of (3.1'.1) and (3.1'.3) even smaller. The tax will also reduce the right side of (3.1'.2) more than the left, since $X^{b^o}_m > X^{a'}_m$. Again, the only way the firm will reduce production of X_n is by being forced out of business. Consequently, taxing X_m can be considered to be ineffective for this model.

Taxing a Variable Factor

The same assumptions with respect to V as were made for Model 3.1 apply. Initially (3.1'.2) is relevant. If $v_a X^{a'}_m > v_b X^{b^o}_m$ then there is a chance that this policy could result in conditions that would imply a technique switch. After the tax the situation would be

$$(P^o_{x_m} - VC_a - T_v v_a) X^{a'}_m + R_K (K^o - K') \overset{?}{\substack{\geq \\ <}} (P^o_{x_m} - VC_b - T_b v_b) X^{b^o}_m$$

$$- P_{K_b} k_{bb} X_m^{b^o} + R_{K_a} k_{aa} X_m^{a'}. \qquad (3.1'.4)$$

The size of tax necessary for the (<) to hold in (3.1'.4) depends on the relative amounts of V used by the two techniques, the relative size of VC_a and VC_b, the resale value of K_a and the cost of K_b.

From (3.1'.1') it is apparent that $P_{K_b} k_{bb} > (P^o_{x_m} - VC_b)$. The latter implies that even if the conditions for a switch are met as in (3.1.6) the firm would lose money by switching unless the salvage value of K_a more than compensates for the loss in producing with B.

The final conclusion for this model is that a tax could bring about a switch in technique only if K_a has a positive salvage value. This salvage value must exceed any losses induced by producing with B. That is

$$RK_a k_{aa} X_m^{a'} > (P^o_{x_m} - VC_a - T_v v_a) X_m^{a'} + R_K (K^o - K')$$

$$- [(P^o_{x_m} - VC_b - T_v v_b) X_m^{b^o} - P_{K_b} k_{bb} X_m^{b^o}]. \qquad (3.1'.5)$$

If the resale value from K_a is not as stated then the firm will close down rather than switch methods. Certainly, then, the pollution would be reduced but the input demands, production of market products and net income would also be reduced.

Summary and Implications

Taxing the variable factor V will be successful only if the resale value of K_a is significant. It must also be that V is a factor used more in total by method A than B. For example, if $v_b = 0$ one of the terms that increases the right side of (3.1'.5) is eliminated. The larger v_a happens to be, the smaller the right side of (3.1'.5) will be. Both of the latter events would tend to reduce the right side of (3.1'.5) which implies a smaller salvage value necessary for a switch.

If a switch should occur, increases in P_{x_m} will help maintain the switch. It is difficult to tell what effect price decrease will have. If the firm had switched then it would need to obtain K_a (this has been sold) and sell K_b. Depending on the relative values of K_a and K_b and how permanent the price decrease may appear to be, it is possible that the firm could switch back.

Taxing the Non-market Externality

Inequalities (3.1'.1) and (3.1'.3) cannot be influenced (as to direction) by this policy. Statement (3.1'.2) can be influenced however, providing that $n_a X_m^{a'} > n_b X_m^{b^o}$. If the latter is so then (3.1'.2) could become

$$(P^{o}_{x_m} - VC_a - T_n n_a) X^{a'}_m + R_K (K^{o} - K')$$

$$< (P^{o}_{x_m} - VC_b - T_n n_b) X^{b^{o}}_m - P_{K_b} k_{bb} X^{b^{o}}_m + R_{K_a} k_{aa} X^{a'}_m \qquad (3.1'.6)$$

which implies a technique switch. The rest of the analysis and the qualifying statements concerning the price of X_m are the same for this policy as for Model 3.1 except (3.1'.1) makes it necessary to depend on resale of K_a to keep the firm from closing down if a switch is implied.

A Standard on the Quality of the Externality

It has been assumed that conversion of X_n to X'_n requires K. For this and the other "prime" models there is excess K initially available. If there is enough K for converting X_n to X'_n then the amount of X_m that can be produced by B does not change. After the policy, and assuming all K is used, condition (3.1'.2) will become

$$(P^{o}_{x_m} - VC_a - VC_n n_a) X^{a'}_m \overset{?}{\substack{\geq \\ <}} (P^{o}_{x_m} - VC_b - VC_n n_b) X^{b'}_m$$

$$- P_{K_b} k_{bb} X^{b'}_m + R_{K_a} k_{aa} X^{a'}_m \qquad (3.1'.7)$$

where $X_m^{b^o} > X_m^{b'} = \left(\frac{k_b K^o}{k_b + k_n n_b} \right) \Big/ k_b.$

Since $n_a > n_b$ it is not possible to determine the direction of the inequality in (3.1'.7). However, it does not seem likely that the (<) will hold (the (<) is necessary to imply a switch), since the left side is decreased by only one factor while the right side is decreased by two.

This model also implies a dependence on salvage value of K_a before a technique switch occurs. Furthermore, there will be less K_a available for salvaging under this model compared to 3.1, which also makes it less likely that a switch will be implied.

If there is not enough K at the outset to take care of converting X_n to X_n', then the X_m that can be produced by method A will decline after the policy is enforced. Statement (3.1'.7) will appear the same except that $X_m^{a''} < X_m^{a'}$ and will replace $X_m^{a'}$. Such a situation would increase the likelihood of a technique switch being implied; however, it still seems less likely to happen than with Model 3.1.

Summary and Implications

The main implication is that a standard on externality quality is less likely to induce an internal technique change when the initial technique is constrained by a specialized fixed factor, providing other assumptions are also met. If the policy agency can set its standard so that the quality of the effluent takes into account such factors as total stream load, then the previous implication will not be very important to the policy body. It is important in any event in trying to assess what happens to demand for factors of production.

If conditions are such that a technique switch occurs, the effects of changes in P_{x_m} are quite similar to the previous policy. In other words, price increases will have no effect, while a decrease may eventually imply a switch back.

The difficulties in administering this policy are the same for this model as already discussed for others.

Subsidizing a Variable Factor

The same assumptions concerning the type of variable factor to be subsidized apply here as did for Model 3.1. Specifically, V' is the factor to be subsidized. Technique B requires more V' per unit of X_m than method A, i.e., $v'_b > v'_a$.

This policy brings up a new possibility for this model. The subsidy will affect positively the right side of (3.1'.1); consequently, it may be that the inequality is reversed. The implication of the latter is that it may now be feasible to maintain $X_m^{a'}$ and also add some production of X_m by method B. Again the externality production would be increased instead of decreased. Furthermore, the subsidy may also be such that inequality (3.1'.3) is reversed in addition to

(3.1'1). Up to a point[27] then it may be feasible to increase production of X_m by both methods A and B. The issue could really be stretched by assuming that the subsidy was also large enough to make the following hold:

$$\left.\begin{array}{l} P_{K_b} k_{bb} + P_K k_b < (P^o_{x_m} - VC_b + S_{v'} v'_b) \\ \text{and} \\ P_{K_a} k_{aa} + P_K k_a < (P^o_{x_m} - VC_a + S_{v'} v'_a) \end{array}\right\} \qquad (3.1'.8)$$

which implies that the firm could expand indefinitely with both techniques.

It seems that for the subsidy to be effective it must permit the reversal of (3.1'.2), but not permit the existence of (3.1'.8). Whether or not such a situation can exist is difficult to tell without specific coefficient values. At best then, it seems that proper application of this policy by itself requires a high level of knowledge concerning the internal cost structures of the firm.

Summary and Implications

The important implication is that too large a subsidy could result in an increase in the production of X_m and X_n. To be able to

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[27] That point being where K^o is entirely used up, which could happen if $P_{K_b} k_{bb} < (P^o_{x_m} - VC_b + S_{v'} v'_b)$ and $P_{K_a} k_{aa} <$

determine a subsidy size that would bring about a technique switch without increasing X_n requires detailed information concerning the internal operations of the firm. The cost of obtaining accurate information may be quite high.

If a switch to technique B were precipitated by the subsidy, a rise in P_{x_m} would not affect the result. A fall in P_{x_m} could induce the firm to switch back to A although such a happening seems less likely with this model than with, say, Models 1 or 2.

Subsidizing a Fixed Factor

It is assumed for reasons already discussed that K_b is the appropriate factor to subsidize. Remember, K_b is the specialized fixed factor for technique B.

Since initially excess K exists, it may be that after the policy it is feasible to produce X_m with method A at $X_m^{a'}$ and some X_m by method B. That is, the subsidy could reverse (3. 1'. 1). A somewhat larger subsidy may even reverse (3. 1'. 1) without any salvage from K (the last term in (3. 1'. 1)). If either of the last two events occur, the externality situation will be aggravated.

A difficulty in determining the appropriate subsidy arises since

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$(P^o_{x_m} - VC_a + S_{v'} v'_a)$.

the minimum for reversing (3. 1'. 1) can be shown to be smaller than the minimum for reversing (3. 1'. 2).[28] If (3. 1'. 2) can be

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[28] To reverse (3. 1'. 1), the following is necessary:

$$P_{K_b} k_{bb} - S_{K_b} k_{bb} < (P^o_{x_m} - VC_b) + R_K (K^o - K''). \quad (1)$$

Rewritten, (1) becomes

$$S \equiv \frac{P_{K_b} k_{bb} - (P^o_{x_m} - VC_b) - R_K (K^o - K'')}{k_{bb}} < S_{K_b}. \quad (2)$$

To reverse (3. 1'. 2), it is necessary that

$$R_K (K^o - k') + (P^o_{x_m} - VC_a - R_{K_a} k_{aa}) X^{a'}_m$$

$$< (P^o_{x_m} - VC_b - P_{K_b} k_{bb}) X^{b^o}_m + S_{K_b} k_{bb} X^{b^o}_m. \quad (3)$$

Rewritten (3) becomes

$$\frac{R_K (K^o - K') + (P^o_{x_m} - VC_a - R_{K_a} k_{aa}) X^{a'}_m - (P^o_{x_m} - VC_b - P_{K_b} k_{bb}) X^{b^o}_m}{k_{bb} X^{b^o}_m}$$

$$< S_{K_b} \quad (4)$$

which implies that

$$S' \equiv \frac{P_{K_b} k_{bb} - (P^o_{x_m} - VC_b)}{k_{bb}} + \frac{R_K (K^o - K') + (P^o_{x_m} - VC_a - R_{K_a} k_{aa}) X^{a'}_m}{k_{bb} X^{b^o}_m}$$

$$< S_{K_b}. \quad (5)$$

Since the fraction to the right of the plus sign in (5) is greater than zero, then visual comparison will show that the minimum subsidy for reversing (3. 1'. 2), $S' + \epsilon$, is larger than the minimum for reversing (3. 1'. 1), $S + \epsilon$; i. e., $S' + \epsilon > S + \epsilon$.

reversed, then the firm would switch entirely to method B, producing $X_m^{b^o}$, which also generates less X_n than initially. The difficulty lies in choosing the right subsidy. One could go too far and choose a subsidy too large so that the firm could not only switch to B, but also obtain additional K, and end up producing more in total with technique B than was produced with A originally.

It can also be shown that a smaller subsidy is needed to induce a technique switch for Model 3.1' than for Model 3.1. The argument can be developed similarly as in Appendix A, Part 2.

Summary and Implications

Choosing the proper fixed factor to subsidize is quite important. It is logical to find one that is specific to the desirable technique. Subsidy size as with the previous policy is important also. One difference between this policy applied to Model 3.1 and to Model 3.1' should be emphasized. It is possible with Model 3.1' that too small a subsidy could increase production of X_m by technique A without inducing a switch. Such an event was not possible under Model 3.1, since K was constraining. Recall that K_a is the constraining factor for Model 3.1'.

Again, if a switch does occur, increases in P_{x_m} will not induce a reversal of techniques. Possibly, decreases in P_{x_m} could bring about a reversion to method A.

Specification and Policy Applications for Model 3.2

The assumptions are identical to those for Model 3.1 except

$$(P^o_{x_m} - VC_a) X^{a^o}_m < (P^o_{x_m} - VC_b) X^{b^o}_m , \qquad (3.2.1)$$

which represents the net revenue position if production were constrained by only K. Remember that the firm is assumed to have only K_a available (in addition to K) at the outset. The factor K is the constraining factor. Other initial conditions are also the same as those for Model 3.1 but they will be repeated here for reference. The explanation of the conditions can be obtained by referring to Model 3.1. These initial conditions are:

$$(P^o_{x_m} - VC_a) X^{a^o}_m > (P^o_{x_m} - VC_b) X^{b^o}_m - P_{K_b} k_{bb} X^{b^o}_m + R_{K_a} k_{aa} X^{a^o}_m , \qquad (3.2.2)$$

$$P_K k_a > (P^o_{x_m} - VC_a) + R_{K_a} (K^o_a - K''_a), \text{ and} \qquad (3.2.3)$$

$$P_{K_b} k_{bb} > R_{K_a} [K^o_a - K'_a + k_{aa}] + (VC_a - VC_b) + R_K (k_a - k_b). \qquad (3.2.4)$$

Taxing the Market Product

Even though short run net revenue would be greater if the firm utilized only method B (condition (3. 2. 1), which assumes no cost in switching to B) the initial conditions (3. 2. 2), (3. 2. 3) and (3. 2. 4) make it infeasible to switch (these latter conditions <u>do</u> consider switching costs). As with Model 3. 1 the tax on X_m will not induce the firm to change techniques. The tax will reduce the smaller sides of (3. 2. 2), (3. 2. 3) and (3. 2. 4) the most, thus not changing the direction of the inequalities, which is necessary for a technique switch.

Taxing a Variable Factor

The initial firm position is as in (3. 2. 2). Statement (3. 2. 1) along with (3. 2. 2) imply that

$$P_{K_b} k_{bb} X_m^{b^o} > R_{K_a} k_{aa} X_m^{a^o} .$$

A tax on V could possibly induce the firm to switch techniques, not change, or go out of business. If

$$v_a X_m^{a^o} > v_b X_m^{b^o} ,$$

then there should be a tax size that would induce a change without forcing the firm from business.

The tax will change (3. 2. 2) such that it will be the same as (3. 1. 5). If a switch is implied the (<) will hold in (3. 1. 5). The difference in the models is that the disadvantage of technique B in this model is due only to the cost of K_b. In other words, under similar prices for K_b, salvage values for K_a, variable costs and variable factor requirements and similar requirements of K, it should take a smaller tax on V to induce a technique switch for Model 3.2 than for Model 3.1.

Initial condition (3.2.3) is also relevant to this model but a tax on V would only reduce the right side, which is already the smallest. As a consequence, the tax on V will not make it feasible for the firm to obtain more K and increase production of X_m using method A. In other words, under the assumed conditions, $X^{a^o}_m$ is the maximum X_m that will be produced by method A before or after policy implementation.

Summary and Implications

If the costs of switching due to the fixed factors are ignored, technique B will generate more net revenue than A. However, due to the cost of obtaining K_b, the firm will use technique A. A tax on a variable factor that is used more in total by method A could reverse the situation. For the switch to occur the additional tax that would be paid if A were used instead of B must exceed the net revenue advantage (when all costs are considered) of A over B.[29] The implication is that it will require a smaller tax to induce a technique switch for a situation where the short run net revenues are larger for the preferred technique than if they were larger for the initial technique.

Once a switch is brought about, increases in P_{x_m} will not change the switch. Decreases in P_{x_m} might result in a switch back to technique A unless method B dominates A which is possible after the tax.

Taxing the Non-market Externality

Since initial conditions (3. 2. 2), (3. 2. 3) and (3. 2. 4) are the same as the initial conditions for Model 3. 1, the analysis is almost identical.. Condition (3. 2. 1) being different for this model (from (3. 1. 1)) affects only the magnitude of the tax required (assuming other conditions the same) and the necessary relationships between the cost of K_b and the salvage value of K_a. The basic analysis and conclusions are the same as with Model 3. 1.

A Standard on the Quality of the Externality

The analysis used and conclusions arrived at for this policy and

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[29] See statement (3. 1. 6), page 92.

Model 3. 1 do not change with the present model. The only difference that condition (3. 2. 1) makes is in the likelihood of a technique switch. It appears that the standard is more likely to result in a switch with this model than with Model 3. 1. To be able to predict the switch a priori, however, requires detailed information concerning the internal costs of the firm.

Subsidizing Fixed and Variable Factors

The change in this model from Model 3. 1 does not affect the analysis and conclusions regarding subsidizing a variable or fixed factor. Condition (3. 2. 1) will only affect subsidy sizes as compared to Model 3. 1. It will require a smaller subsidy to bring about a technique change under the present model.

Specifications and Policy Applications for Model 3. 2'

The only difference between Models 3. 2 and 3. 2' is that K_a is assumed to be the constraining factor for 3. 2'. The following initial conditions are relevant:

$$(P^o_{x_m} - VC_a)\, X^{a'}_m + R_K\,(K^o - K') > (P^o_{x_m} - VC_b)\, X^{b^o}_m$$

$$- P_{K_b}\, k_{bb}\, X^{b^o}_m + R_{K_a}\, k_{aa}\, X^{a'}_m, \qquad (3.2'.1)$$

$$P_{K_a} k_{aa} > (P^o_{x_m} - VC_a) + R_K (K^o - K''), \text{ where} \qquad (3.2'.2)$$

$$K'' = K' + k_a,$$

$$P_{K_b} k_{bb} > (P^o_{x_m} - VC_b) + R_K (K^o - K''), \text{ where} \qquad (3.2'.3)$$

$$K'' = k_a X^{a'}_m + k_b, \text{ and}$$

$$(P^o_{x_m} - VC_a) X^{a^o}_m < (P^o_{x_m} - VC_b) X^{b^o}_m . \qquad (3.2'.4)$$

The explanation of the above conditions is identical to that given for (3.1'.2), (3.1'.3) and (3.1'.1), respectively, for Model 3.1'. Figure 4.2 also describes the present model.

Taxing the Market Product

The tax on X_m will not bring about technique changes. If the tax were high enough a firm as depicted by this model could be forced out of business. This would reduce production of X_n providing those resources remained idle and were not usurped by other firms.

Taxing a Variable Factor

Application of this policy to Model 3.2' does not alter the analysis used or conclusions reached for the same policy applied to Model 3.1'. As discussed under Model 3.1', it is possible that the necessary condition for a switch can be met. Due to (3. 2'. 4), however, the firm will close down rather than switch unless the resale value of K_a is high. With the present model this resale value may not need to be as high compared to 3.1' for a switch to occur.

Taxing the Non-market Externality

The analysis is again the same for this model as for Model 3. 1'. The size of tax necessary to induce a technique switch is likely to be less under the present model as compared to 3. 1'.

A Standard on the Quality of the Externality

If all conditions are the same for Models 3. 1' and 3. 2' except (3. 2'. 4), it appears that a technique switch is more likely to occur with 3. 2'. Predicting the switch, however, requires much information about the internal cost structure of the firm.

Subsidizing Variable and Fixed Factors

Again the only difference the present model specification makes compared to 3. 1' is in the magnitude of the subsidies. Without specific coefficient values it is not possible to estimate what that magnitude might be.

Specification and Policy Applications for Model 3. 3

In this model method A dominates B with respect to variable costs and the fixed factor K. The model is depicted by Figure 4. 3. As can be observed the variable costs for method A are less than the variable costs for method B. Technique A also requires less of the common non-specialized fixed factor than B, i. e., $k_a < k_b$. The factor K is the effective constraint initially. Initial conditions

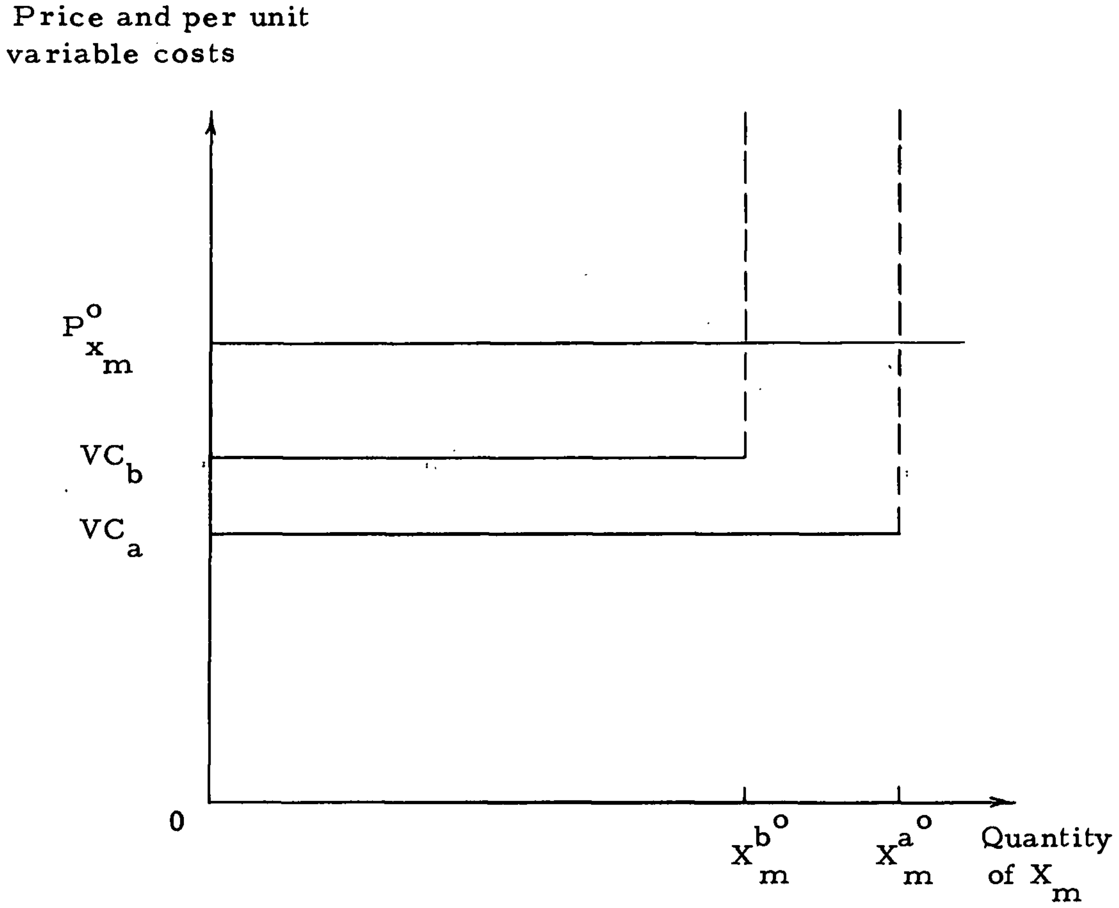

Figure 4. 3. Initial firm position--Model 3. 3--Technique A dominates B

are again similar to previous models but will be repeated below.

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m > (P^o_{x_m} - VC_a)\, X^{b^o}_m, \tag{3.3.1}$$

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m > (P^o_{x_m} - VC_b)\, X^{b^o}_m - P_{K_b}\, k_{bb}\, X^{b^o}_m$$

$$+ R_{K_a}\, k_{aa}\, X^{a^o}_m, \text{ and} \tag{3.3.2}$$

$$P_K\, k_a > (P^o_{x_m} - VC_a) + R_{K_a}\, (K^o_a - K''_a) \tag{3.3.3}$$

where $K''_a = K'_a + k_{aa}$.

Notice that no matter which method is used, there will be $K^o_a - K'_a$ amount of K_a available for disposal. These levels of K_a are the same as shown for Model 3.1, Figure 4.1. Since revenue from resale of the above amount of K_a is available to either technique it is ignored in the initial conditions.

Taxing the Market Product

This model is different than 3.1 through 3.2' since a switch to method B will reduce total taxes. The latter is so since $X^{a^o}_m > X^{b^o}_m$.

Can the tax on X_m induce any changes? After the tax, (3.3.2) becomes

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m - T\, X^{a^o}_m \overset{?}{\substack{\geq \\ <}} [(P^o_{x_m} - VC_b)\, X^{b^o}_m - T\, X^{b^o}_m]$$

$$- P_{K_b} k_{bb} X_m^{b^o} + R_{K_a} k_{aa} X_m^{a^o} . \qquad (3.3.4)$$

Since $T X_m^{a^o} > T X_m^{b^o}$, the tax could induce the firm to switch providing the tax savings are large enough, i.e.,

$$(T X_m^{a^o} - T X_m^{b^o}) > (P^o_{x_m} - VC_a) X_m^{a^o} - (P^o_{x_m} - VC_b) X_m^{b^o}$$

$$+ P_K K_b^o - R_{K_a} K_a^o . \qquad (3.3.5)$$

The tax savings (left of 3.3.5)) must be greater than the difference between the switching cost minus the resale value of K_a^o. An important item to note is that if the cost of K_b^o is greater than the salvage value of K_a^o, there is <u>no</u> chance the tax will bring about a change. The latter is so since method A dominates method B. It can be observed in Figure 4.3 that no matter what level the tax (reflected as a lower price for X_m), more net revenue can be obtained by using technique A. If $(P^o_{x_m} - T)$ should be coincidental with VC_b, then net revenue (excluding salvage value) with method B would be zero whereas it would still be positive with method A. The tax will not cause any change in the direction of the inequality in (3.3.3), which implies the firm cannot expand production with method A alone.

Summary and Implications

It is important for predicting the outcome of the tax to have good information concerning the elements in (3. 3. 5). The dangers in applying this policy are that it will be costly to the firms involved and it may be ineffective. If a switch is implied increases in P_{x_m} could result in a change back to method A. Decreases in P_{x_m} will not change the results.

Taxing a Variable Factor

A tax on the variable factor V has the same effect as increasing the variable costs of both methods; however, under the assumed conditions the variable costs of A will be raised relatively more than those of B. If the tax is going to induce a technique change it is necessary that

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m - T_v\, v_a\, X^{a^o}_m < (P^o_{x_m} - VC_b)\, X^{b^o}_m - P_{K_b}\, k_{bb}\, X^{b^o}_m$$

$$\pm R_{K_a}\, k_{aa}\, X^{a^o}_m - T_v\, v_b\, X^{b^o}_m . \qquad (3.3.6)$$

If (3. 3. 6) holds, Figure 4. 3 could be altered to appear as Figure 4. 4, although <u>not</u> <u>necessarily</u> due to the term $R_{K_a}\, k_{aa}\, X^{a^o}_m$. Notice that (3. 3. 6) can be rewritten as

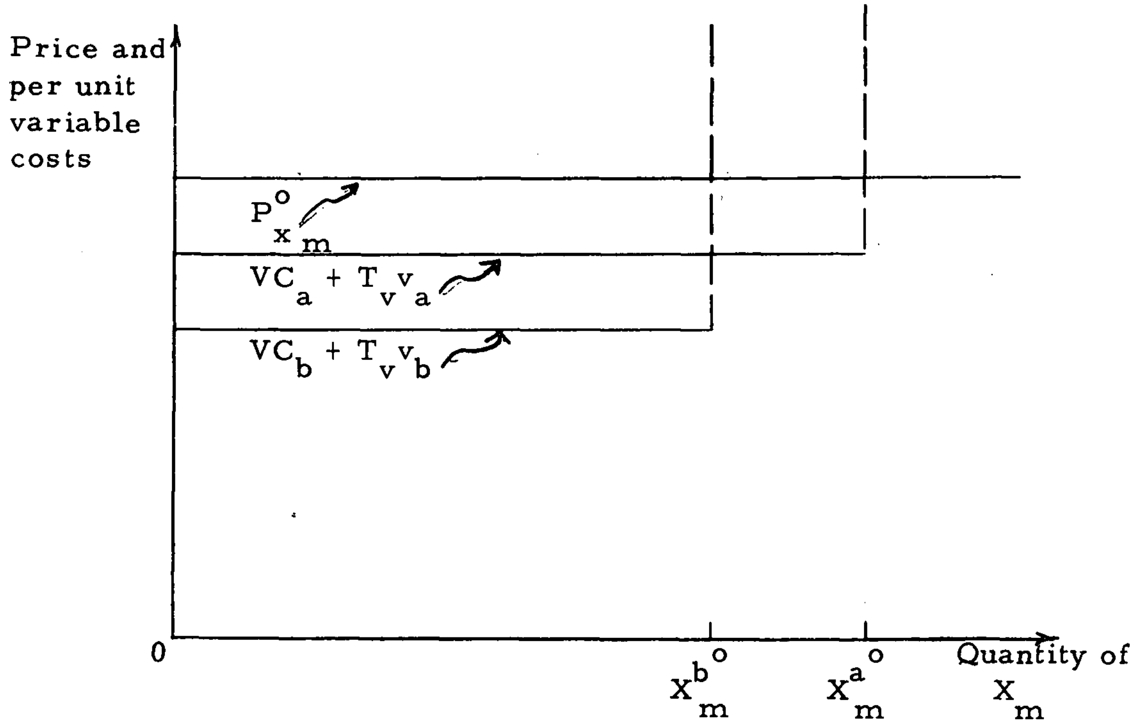

Figure 4.4. Possible effects of taxing V--Model 3.3.

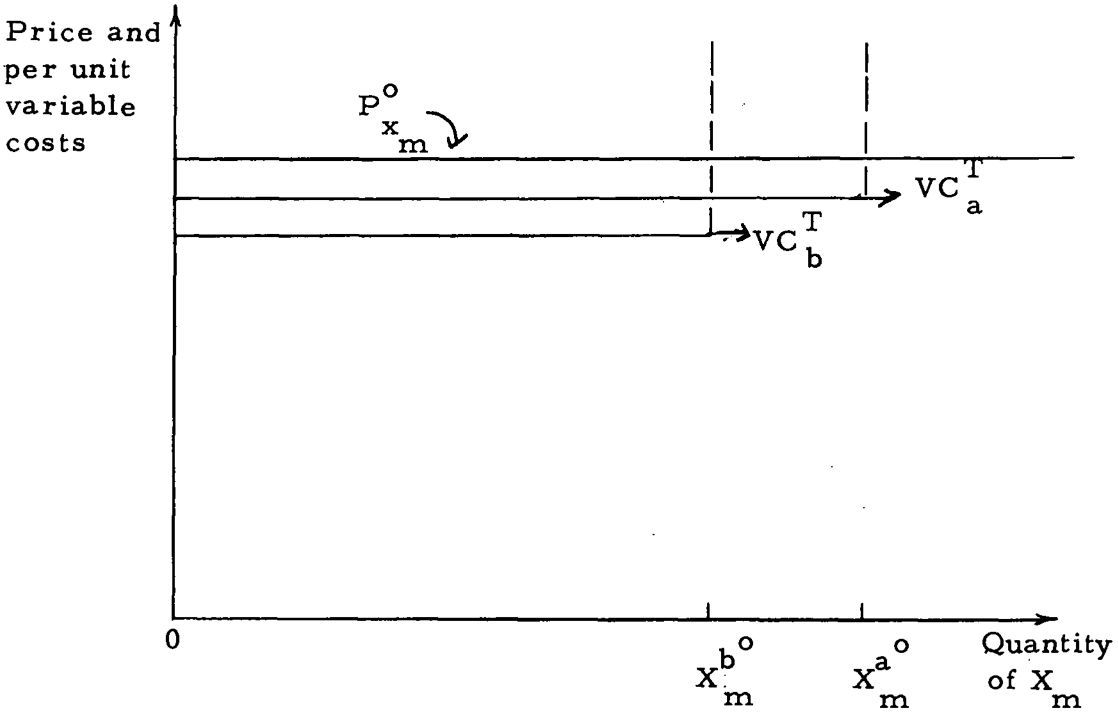

Figure 4.5. A case of implied method switch--Model 3.3

$$(P^o_{x_m} - VC_a - T_v v_a - R_{K_a} k_{aa}) X^{a^o}_m < (P^o_{x_m} - VC_b - T_v v_b - P_{K_b} k_{bb}) X^{b^o}_m . \qquad (3.3.7)$$

In a sense, all negative terms in both parentheses in (3.3.7) can be thought of as total per unit variable costs after tax and over a somewhat longer time period. Viewing the situation in the latter sense, then, it is necessary for a technique switch that the tax be high enough to result in a situation such as Figure 4.5, where

$VC^T_a \equiv VC_a + T_v v_a + R_{K_a} k_{aa}$ and $VC^T_b \equiv VC_b + T_v v_b + P_{K_b} k_{bb}$.

It is entirely conceivable, even under the assumed conditions, that before the tax $VC_a + R_{K_a} k_{aa} > VC_b + P_{K_b} k_{bb}$, in which case the dominance would be in essence destroyed. In either event (i.e., where dominance remains when consideration is given to resale and cost of fixed factors or where it is destroyed) the tax will be the impelling force for technique change. Depending on the price level, P_{x_m}, a tax could drive the firm from business rather than induce a method change. The latter could happen if P_{x_m} were between VC_a and VC_b in Figure 4.3 and if the salvage value of K_a were insignificant.

Summary and Implications

The magnitude of tax necessary to cause the firm to switch techniques will depend on the price of X_m, the price of K_b, salvage value of K_a, the relative amounts of V required by the techniques, and the relative size of VC_a and VC_b. The greater the ratio $\frac{v_a}{v_b}$ the more likely a tax on V will induce change. An increase in P_{x_m} after a switch would result in a switch back to technique A. Decreases in P_{x_m} will not change the results.

Taxing the Non-market Externality

Since it has been assumed that $n_a > n_b$ there is no question in this case as to whether or not X_n can be reduced by a technique change. As with Model 3.1 a tax on X_n could reverse the direction of the inequality in (3.3.2) thus implying a switch in techniques.

One difference between this model and 3.1 is that there is no situation which will make the technique switch irreversible if P_{x_m} varies. First of all before a technique switch is implied for this model it is necessary that

$$(VC_a + R_{K_a} k_{aa} + T_n n_a) > (VC_b + P_{K_b} k_{bb} + T_n n_b) \qquad (3.3.8)$$

whereas this condition was not necessary for Model 3.1. Such a situation as (3.3.8) could be depicted as Figure 4.5, where

$VC^T_a \equiv (VC_a + R_{K_a} k_{aa} + T_n n_a)$ and $VC^T_b \equiv (VC_b + P_{K_b} k_{bb} + T_n n_a)$. Once that situation exists there will be prices that will maximize net revenue for either technique.

Summary and Implications

Before this policy can work it must destroy the dominance of technique A over technique B. Once dominance is destroyed a technique change could be implied providing P_{x_m} were not so low that the firm were driven out of business first. If a switch is brought about, increases in P_{x_m} could result in a switch back to A.

A Standard on the Quality of the Externality

Since A dominates B the relative difference in n_a and n_b required to imply that $n_a X^{a^o}_m > n_b X^{b^o}_m$ is quite different than for the previous models. If one is trying to predict what will happen if this policy is applied this model points out that knowing that one technique will produce more of the externality in total than the other may not be enough. Total production of X_n by method A could be greater than X_n production for method B but n_a could be less than n_b. If $n_a < n_b$ then under certain conditions this policy cannot bring about a technique switch. The condition for which this statement is true is that resale value of K_a is insignificant or zero.

Suppose conversion of X_n to X'_n requires some K and some variable costs. Also assume zero resale value of K_a. After the policy the initial condition becomes

$$(P^o_{x_m} - VC_a - VC_n n_a) \frac{K''}{k_a} \overset{?}{\gtrless} (P^o_{x_m} - VC_b - P_{K_b} k_{bb} - VC_n n_b) \frac{K'}{k_b} .$$

(3.3.9)

It can be shown[30] that $\frac{K''}{k_a} > \frac{K'}{k_b}$ and if $n_a < n_b$ then $VC_n n_a < VC_n n_b$.

Due to the dominance of A over B the initial condition (3.3.2) implies that

$$(P^o_{x_m} - VC_a) > (P^o_{x_m} - VC_b - P_{K_b} k_{bb}) \qquad (3.3.10)$$

which along with $\frac{K''}{k_a} > \frac{K'}{k_b}$ implies that $(P^o_{x_m} - VC_a) \frac{K''}{k_a}$

$$> (P^o_{x_m} - VC_b - P_{K_b} k_{bb}) \frac{K'}{k_b} . \qquad (3.3.11)$$

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[30] It has been shown in Appendix A, Part 1, that $K'' = \frac{k_a K^o}{k_a + k_n n_a}$ and $K' = \frac{k_b K^o}{k_b + k_n n_b}$ so $X^{a''}_m = \frac{K''}{k_a} = \frac{K^o}{k_a + k_n n_a}$ and $X^{b'}_m = \frac{K'}{k_b} = \frac{K^o}{k_b + k_n n_b}$. But since $k_b > k_a > 0$ and $n_b > n_a > 0 \longrightarrow k_a + k_n n_a < k_b + k_n n_b \longrightarrow \frac{K''}{k_a} > \frac{K'}{k_b}$.

For the (<) to hold (which would imply a switch) in (3.3.9) it must be that

$$(P^{o}_{x_m} - VC_a - VC_n n_a) < (P^{o}_{x_m} - VC_b - VC_n n_b). \qquad (3.3.12)$$

If as discussed previously $n_b > n_a$ (3.3.12) is impossible.

It has been assumed in general for all the models in this chapter that $n_a > n_b$. Utilizing that assumption then (3.3.12) is possible and a switch could be implied. It is also possible even without this latter assumption that a switch could result if resale of K_a were quite significant.

Summary and Implications

Previously it was pointed out that it is important under some conditions to know the relative total X_n production by techniques rather than just the relative amounts per unit of X_m. Application of the present policy to Model 3.3 has shown that it may also be necessary to know the relative amounts of X_n produced per unit of X_m to make predictions about internal firm changes.

If application of the standard results in a switch of techniques, increases in P_{x_m} could induce a switch back. A falling P_{x_m} will not do anything except possibly force the firm to shut down. It is possible that after the policy B could dominate A if $X^{a''}_m < X^{b'}_m$ which would make price changes inconsequential.

Subsidizing a Variable Factor

It is possible that a subsidy on V' could lower the variable costs for technique B enough so that the dominance of method A could be broken. If a switch is implied then

$$(P^o_{x_m} - VC_a - R_{K_a} k_{aa} + S_{v'} v'_a) X^{a^o}_m < (P^o_{x_m} - VC_b - P_{K_b} k_{bb} + S_{v'} v'_b) X^{b^o}_m . \tag{3.3.13}$$

Since by assumption $X^{b^o}_m < X^{a^o}_m$ (3.3.13) implies that

$$(P^o_{x_m} - VC_a - R_{K_a} k_{aa} + S_{v'} v'_a) < (P^o_{x_m} - VC_b - P_{K_b} k_{bb} + S_{v'} v'_b). \tag{3.3.14}$$

The condition represented in (3.3.14) can be depicted as in Figure 4.5 where VC^T_a would be the same (excluding $P^o_{x_m}$) as the left of (3.3.14) and VC^T_b would be identical (excluding $P^o_{x_m}$) to the right.

As with Model 3.1 condition (3.3.3) could be reversed by the subsidy which would bring about more production of X_n instead of less. Furthermore, if the subsidy were such that (3.1'.8) held then the problem could become much more intense.

Summary and Implications

Subsidy size was quite important as it was with other models. The subsidy must destroy the dominance of technique A without permitting expansion in overall production.

Firm net revenues will be higher after the policy than before. Such a phenomena could have bearing on long run firm growth; however, such possibilities are ignored in this study.

Increases in the price of X_m could result in a switch back to technique A if a switch occurred due to the policy. A falling P_{x_m} will not change the policy result.

Subsidizing a Fixed Factor

There is little difference between the application of this policy to this model and to Model 3.1. The main difference is that the dominance of A over B must be broken, i.e.,

$$(VC_b + P_{K_b} k_{bb} - S_{K_b} k_{bb}) < (VC_a + R_{K_a} k_{aa}). \qquad (3.3.15)$$

It is possible that (3.3.15) can hold provided K_b is the factor subsidized.

The size of the subsidy is important to this model also. The effects of changes in the price of X_m are different for the present case compared to Model 3.1. After the policy and if a switch is indicated a rising P_{x_m} could induce a switch back with Model 3.3. A falling price for X_m will not affect the policy result. The price effects on Model 3.1 were just the opposite. Also it will not be possible after the subsidy for technique B to dominate A. Dominance of B was possible with Model 3.1. The conclusion is that under no circumstances will policy results be insensitive to changes in P_{x_m} with Model 3.3.

Specification and Policy Applications for Model 3.3'

As with previous "prime" models it is assumed that K_a is the initial constraining factor. Otherwise Model 3.3' is identical to Model 3.3. Initial conditions are

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m > (P^o_{x_m} - VC_b)\, X^{b^o}_m, \qquad (3.3'.1)$$

$$(P^o_{x_m} - VC_a)\, X^{a'}_m + R_K\,(K^o - K') > (P^o_{x_m} - VC_b)\, X^{b^o}_m$$

$$- P_{K_b}\, k_{bb}\, X^{b^o}_m + R_{K_a}\, k_{aa}\, X^{a'}_m, \qquad (3.3'.2)$$

$$P_{K_a}\, k_{aa} > (P^o_{x_m} - VC_a) + R_K\,(K^o - K'') \text{ where } K''$$

$$= K' + k_a, \text{ and} \qquad (3.3'.3$$

$$P_{K_b} k_{bb} > (P^o_{k_m} - VC_b) + R_K (K^o - K'') \text{ where}$$

$$K'' = k_a X^{a'}_m + k_b. \qquad (3.3'.4)$$

Explanation of the above conditions is identical to that given for the similar conditions under Model 3.1'.

Taxing the Market Product

It is obvious that the tax on X_m will not affect the direction of the inequality in (3.3'.3) so no change in this condition can be induced by the policy. It appears that there exists a possibility for changing the direction of the inequality in (3.3'.2) depending on the restrictive level of K_a. If K_a restricts maximum production by method A to some level to the left of $X^{b^o}_m$ in Figure 4.3, then there is no possibility that the tax can reverse (3.3'.2). The latter is so since the tax would reduce the right side (already the smallest) of (3.3'.2) the most.

Statement (3.3'.4) implies that even if the necessary condition for a switch exists, the firm will need to rely on income generated by salvaging K and K_a to obtain positive net revenue. This is so since (3.3'.4) implies that $P_{K_b} k_{bb} > (P^o_{x_m} - VC_b)$ (assuming $R_K (K^o - K'') \geq 0$).

Even without condition (3.3'.4) it can be demonstrated that the firm will operate with technique B instead of A only if it receives substantial income from salvaging the fixed resource, K_a.[31] The implication is that to justify not operating at $X_m^{a'}$ the firm must depend on revenues generated by disposal of fixed factors. In other words after the tax the firm would still choose to use technique A totally unless disposition of the fixed factor generated considerable income.

Suppose that the disposition of K_a generated just enough income to cover the acquisition costs of K_b. Then the necessary condition

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[31] After the tax the necessary condition for a switch is that

$$(P^o_{x_m} - VC_a - T)\, X^{a'}_m < (P^o_{x_m} - VC_b - T)\, X^{b^o}_m - P_{K_b}\, k_{bb}\, X^{b^o}_m + R_K\, k_{aa}\, X^{a'}_m - R_K\, (K^o - K') \qquad (1)$$

For (1) to prevail it must be that the tax reduced the left side of (3. 3'. 2) more than the right which implies that $X^{a'}_m > X^{b^o}_m$. It is also known that $(P^o_{x_m} - VC_a) > (P^o_{x_m} - VC_b)$; therefore, it must be that $(P^o_{x_m} - VC_a - T)\,(X^{a'}_m) > (P^o_{x_m} - VC_b - T)\,(X^{b^o}_m)$. (2)

(1) and (2) together imply that

$$R_{K_a}\, k_{aa}\, X^{a'}_m - P_{K_b}\, k_{bb}\, X^{b^o}_m - R_K\, (K^o - K') > 0. \qquad (3)$$

Notice that to produce $X^{b^o}_m$, K^o is required.

for a switch (see footnote 31) could be rewritten as

$$(P^o_{x_m} - VC_a - T) X^{a'}_m < (P^o_{x_m} - VC_b - T) X^{b^o}_m - R_K (K^o - K'). \tag{3.3'.5}$$

As shown by (2) in footnote 31 (3.3'.5) is impossible unless both sides are negative. If both sides are negative (3.3'.5) implies that $T > (P^o_{x_m} - VC_b)$ and $T > (P^o_{x_m} - VC_a)$; consequently, under these conditions the firm would shut down rather than switch or operate at the original position.

Summary and Implications

The necessary conditions can be met for the firm to switch techniques but the firm will not switch unless significant income can be generated from salvaging fixed resources. In addition this policy will be sensitive to changes in the price of X_m. If a switch is induced an increase in P_{x_m} could result in the firm switching back to method A. A falling P_{x_m} may eventually force the firm from business.

Taxing a Variable Factor

Depending on the restrictive level of K_a, the dominance of method A may be destroyed from the outset. For example if K_a restricts X_m production by method A to some point left of $X^{b^o}_m$

in Figure 4.3 then method B could with enough K_b produce more than method A. It is also possible that K_a will restrict X^a_m to some level between $X^{b^o}_m$ and $X^{a^o}_m$ in Figure 4.3 in which case the analysis would be quite similar to Model 3.3. As in the previous "prime" models, where (3.3'.4) was relevant, switching to B instead of closing down will be dependent upon salvage values of fixed resources. The latter can be observed by rewriting (3.3'.2) with the tax as

$$R_K (K^o - K') + (P^o_{x_m} - VC_a - T_v v_a) X^{a'}_m < (P^o_{x_m} - VC_b - T_v v_b$$

$$- P_{K_b} k_{bb}) X^{b^o}_m + R_{K_a} k_{aa} X^{a'}_m . \qquad (3.3'.6)$$

From (3.3'.4) it is known that $(P^o_{x_m} - VC_b - T_v v_b - P_{K_b} k_{bb}) < 0.$

For the firm to stay in business and switch techniques the right side of (3.3'.6) must be positive. For the latter to obtain, revenue must be forthcoming from resale of K_a.

Summary and Implications

If K_a restricts production of X_m by technique A so that it is less than $X^{b^o}_m$ then the model is about the same as Model 3.3. If K_a does not restrict X^a_m to a level less than $X^{b^o}_m$ it is still possible for a switch to be induced. It is necessary in either case that revenue be forthcoming from resale of K_a if the firm is to remain in business and switch.

If $X_m^{a'} > X_m^{b^o}$ an increase in P_{x_m} could induce the firm to switch back to method A. If $X_m^{a'} < X_m^{b^o}$ a price increase will not result in any changes. A decrease in P_{x_m} will not change matters if $X_m^{a'} > X_m^{b^o}$ but could induce a switch back if $X_m^{a'} < X_m^{b^o}$. It is also conceivable that after the tax B could dominate A and price changes would not alter the situation at all.

Taxing the Non-market Externality

If K_a restricts $X_m^{a'}$ to a point greater than $X_m^{b^o}$ this model becomes similar to 3.3. Condition (3.3'.4) however implies for Model 3.3' a need for substantial resale income from K_a if a switch is to be made.

If K_a restricts $X_m^{a'}$ such that it is less than $X_m^{b^o}$ (destroys dominance of A over B) then the analysis of this model is similar to Model 3.1'.

The implications are also the same as discussed for Models 3.3 and 3.1'.

A Standard on the Quality of the Externality

Analysis for this model and policy is quite similar to Model 3.1'. If after the policy $X_m^{a''} > X_m^{b'}$ (see Model 3.1') a switch can be implied only if $(VC_a + VC_n n_a + R_{K_a} k_{aa}) > (VC_b + VC_n n_b + P_{K_b} k_{bb})$.

If the previous conditions exist, increases in P_{x_m} could result in a switch back to method A. It is also possible that $X_m^{a''} < X_m^{b'}$ which means a falling price of X_m could induce a switch back unless after the policy technique B dominates A. For this model dominance will require resale value from K_a due to condition (3. 3'. 4). If dominance exists changes in P_{x_m} will not affect the situation.

Subsidizing a Variable Factor

Certainly the possibility exists that this policy can induce a technique switch given the present model. It is also possible that (3. 3'. 3) and (3. 3'. 4) could be reversed which would aggravate the problem with the externality.

If a switch were induced for this model it is possible that the switch could be insensitive to the price of X_m. If X_m^a is restricted by availability of K_a to some point less than $X_m^{b^o}$ and if after the policy (3. 3.14)holds then the switch will not be sensitive to P_{x_m}. Suppose $X_m^{a'} < X_m^{b^o}$ but (3. 3. 14) does not hold. Then a switch could be implied that could be reversed by a decline in P_{x_m}.

If X_m^a as restricted by K_a is greater than $X_m^{b^o}$ and a switch is implied then an increase in P_{x_m} might induce a switch back.

Subsidizing a Fixed Factor

The analysis for this model is quite similar to Model 3. 1'; consequently, only the major conclusions are presented.

It is possible that a subsidy on K_b could reverse (3. 3'. 4). Such a situation would increase externality production. It is also posisble that the subsidy could reverse (3. 3'. 2) which alone would reduce production of X_n. The implication is that subsidy size is quite crucial.

If K_a is restrictive enough so that $X_m^{a'} < X_m^{b^o}$ then if a switch is implied it is possible that method B could dominate A thus implying insensitivity to P_{x_m}. It is also possible that a switch could be induced with $X_m^{a'} < X_m^{b^o}$ that could be sensitive to a falling P_{x_m}. The former condition may require a larger subsidy than the latter. If $X_m^{a'} > X_m^{b^o}$ then the after policy price sensitivity will be identical to Model 3. 3.

Specification and Policy Applications for Model 3. 4

This model is almost the opposite of 3. 3. The variable costs of producing X_m by method A are greater than they are for method B. Method A also requires more K to produce one unit of X_m than B, i. e., $k_a > k_b$. The constraining factor is K. Method B then,

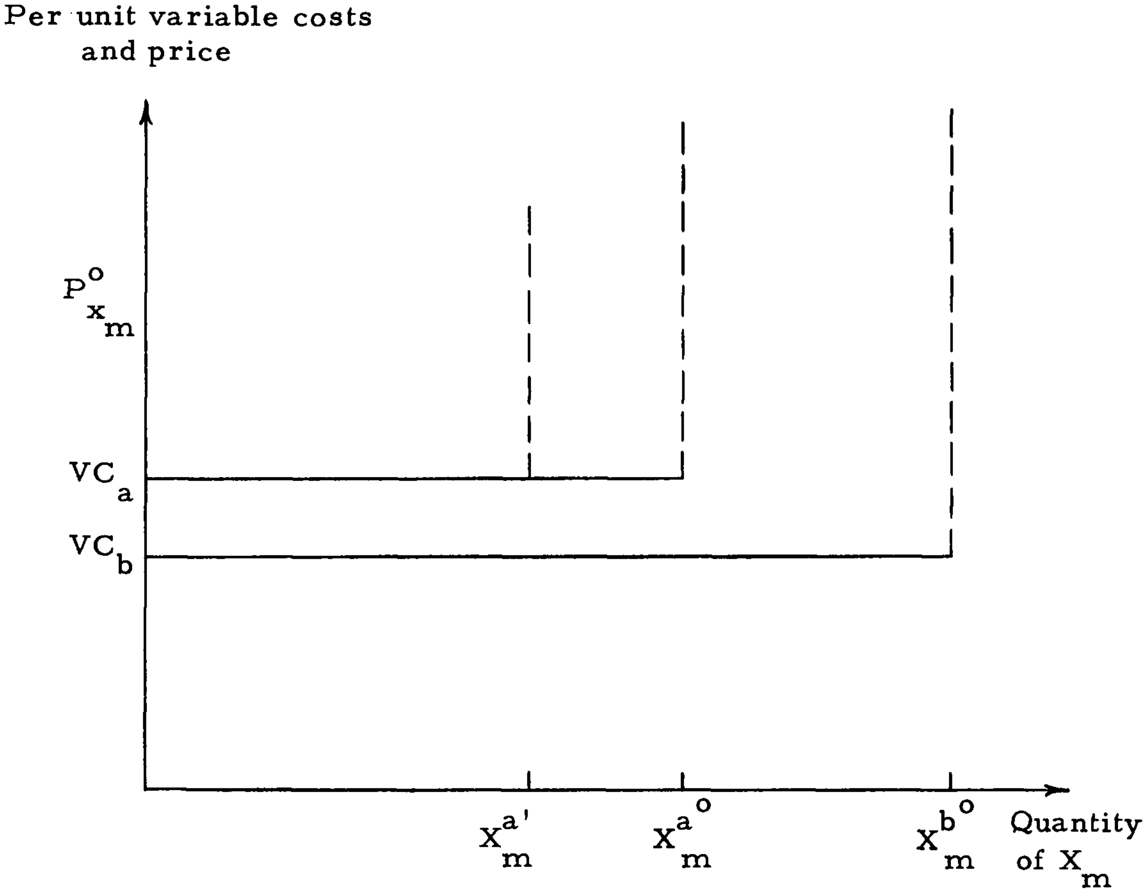

Figure 4.6. Initial firm position excluding costs and resale value of fixed factors--Model 3.4

dominates method A as far as the variable costs and the common fixed factor, K, are concerned (See Figure 4.6).

Without considering acquisition costs of K_b or salvage values of K_a more net revenue can be generated by technique B than A. Thus, the importance of considering longer run costs becomes apparent. Initial conditions are

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m < (P^o_{x_m} - VC_b)\, X^{b^o}_m \,, \qquad (3.4.1)$$

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m > (P^o_{x_m} - VC_b)\, X^{b^o}_m - P_{K_b}\, k_{bb}\, X^{b^o}_m$$

$$+ R_{K_a}\, k_{aa}\, X^{a^o}_m, \qquad (3.4.2)$$

$$P_K\, k_a > (P^o_{x_m} - VC_a) + R_{K_a}\, (K^o_a - K''_a) \text{ where } K''_a$$

$$= K'_a + k_{aa} \text{ and} \qquad (3.4.3)$$

$$P_{K_b}\, k_{bb} > R_{K_a}\, (K^o_a - K'_a + k_{aa}) + [VC_a - VC_b]$$

$$+ R_K\, (k_a - k_b). \qquad (3.4.4)$$

Taxing the Market Product

In this instance the firm has become locked into using a technique that is inferior in terms of net revenue to another available technique. One likely cause for such a situation is that when the original investment in K and K_a was made, method B was not available. If one assumes the firm had good information, then the reason for not switching techniques when B become available must have been due to the cost of obtaining K_b. That is (3.4.2), (3.4.3), and (3.4.4) hold. Since by assumption $X^{a^o}_m < X^{b^o}_m$ a tax on X_m will reduce the right side (the smallest) of (3.4.2) more than the left, leaving the inequality unchanged. The tax affects only the right (smallest) sides of (3.4.3) and (3.4.4) also. Consequently, the tax will not induce the firm to switch techniques. The policy may force the firm out of business and that is the only way it will reduce production of X_n.

Taxing a Variable Factor

Rewriting (3.4.2) the initial position becomes

$$(P^o_{x_m} - VC_a - R_{K_a} k_{aa}) X^{a^o}_m > (P^o_{x_m} - VC_b - P_{K_b} k_{bb}) X^{b^o}_m . \qquad (3.4.5)$$

Since by assumption $X^{b^o}_m > X^{a^o}_m$ it is necessary[32] for (3.4.5) that

$(P^o_{x_m} - VC_a - R_{K_a} k_{aa}) > (P^o_{x_m} - VC_b - P_{K_b} k_{bb})$ which when depicted appears in Figure 4.7. In other words when resale value of K_a and cost of K_b are considered the apparent dominance of B over A disappears. A tax on V can, under the assumed conditions, alter the net revenue relationship of A and B (B's will be decreased relatively less than A's). The firm can be induced to switch

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[32] Note, however, that this condition is not sufficient since $X^{b^o}_m > X^{a^o}_m$. That is, the necessary condition can exist and yet (3.4.5) can have a $(\nless)$ instead of $(\ngtr)$ sign.

techniques <u>with</u> $(P^o_{x_m} - VC_a - T_v v_a - R_{K_a} k_{aa}) > (P^o_{x_m} - VC_b - T_v v_b - P_{K_b} k_{bb})$; however if the net revenues after taxes are in this relative position the policy will be sensitive to movements in P_{x_m} in the negative direction. If the tax were large enough so that

$$(P^o_{x_m} - VC_a - T_v v_a - R_{K_a} k_{aa}) < (P^o_{x_m} - VC_b - T_v v_b - P_{K_b} k_{bb}) \qquad (3.4.6)$$

then once the firm switched to technique B it would remain with it or close down no matter what happens to P_{x_m}.

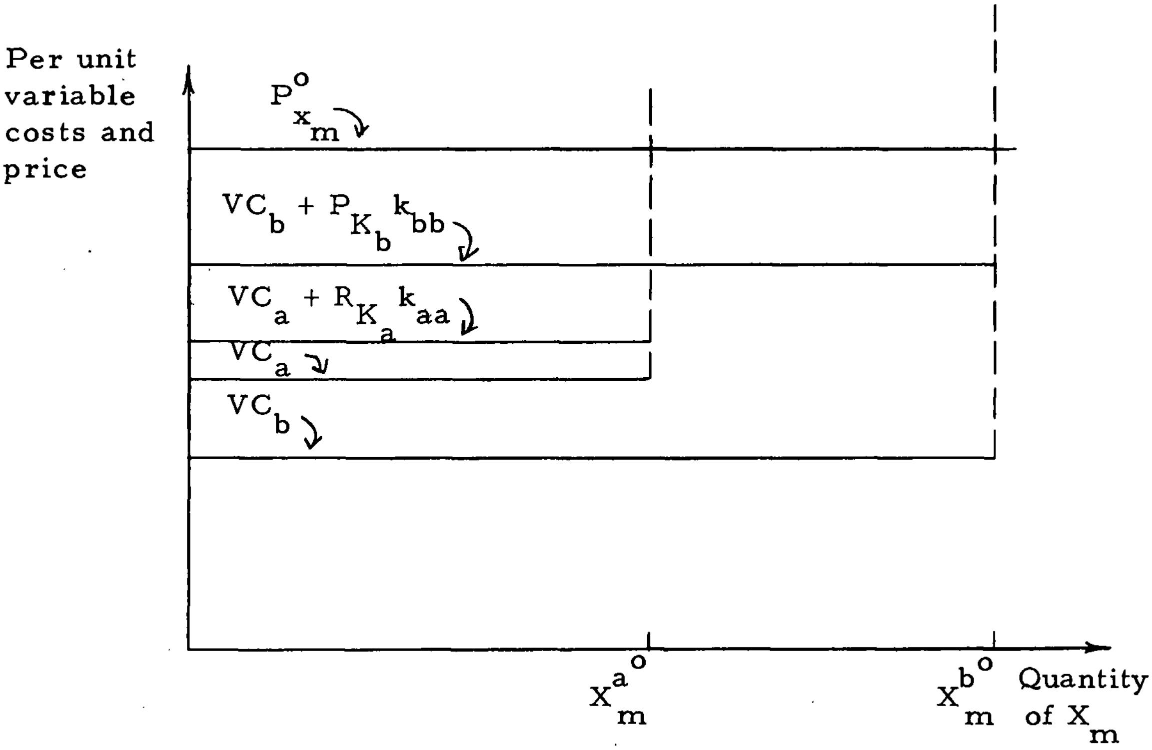

Figure 4.7. Initial firm position including costs and resale value of fixed factors--Model 3.4

Summary and Implications

Since a switch to technique B will increase production of X_m it is necessary that $n_a X_m^{a^o} < n_b X_m^{b^o}$ if the production of X_n is to be reduced. A successful tax (one large enough to induce a technique switch) will increase production of the market product, reduce production of the non-market externality, increase the use of some variable inputs, and decrease the use of V.

Given that a switch occurs due to the policy, only decreases in P_{x_m} can reverse the policy result. If after the tax technique B dominates A (i.e., (3.4.6) holds) the policy result will be insensitive to changes in P_{x_m}.

Taxing the Non-market Externality

Since $X_m^{a^o} < X_m^{b^o}$ it must be assumed that $n_a X_m^{a^o} > n_b X_m^{b^o}$ if a technique change is going to reduce X_n. Before the tax the existence of (3.4.2) implies that

$$(P^o_{x_m} - VC_a - R_{K_a} k_{bb}) > (P^o_{x_m} - VC_b - P_{K_b} k_{bb}. \qquad (3.4.7)$$

In other words the apparent dominance of B does not exist when salvage value of K_a and acquisition cost of K_b are considered.

For a switch of methods to be implied it is necessary that after the tax

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m - T_n n_a X^{a^o}_m < (P^o_{x_m} - VC_b)\, X^{b^o}_m - T_n n_b X^{b^o}_m$$

$$- P_{K_b} k_{bb} X^{b^o}_m + R_{K_a} k_{aa} X^{a^o}_m \,. \qquad (3.4.8)$$

It is sufficient but not necessary for (3.4.8) if

$$(VC_a + R_{K_a} k_{aa} + T_n n_a) > (VC_b + P_{K_b} k_{bb} + T_n n_b). \qquad (3.4.9)$$

If (3.4.8) holds then a switch to B is implied. The switch will be insensitive to changes in P_{x_m} if (3.4.9) also holds. If (3.4.9) does not hold but (3.4.8) does, then the switch will be sensitive to downward movements in P_{x_m} .

Summary and Implications

The policy can induce a technique switch. It is important to know the relative total amounts of the externality produced by both techniques.

The possibility for a switch which would be insensitive to changes in P_{x_m} exists with this model. Such a situation occurs if the tax results in technique B dominating A after the policy. Otherwise falling prices for X_m could induce the firm to switch back to method A.

A Standard on the Quality of the Externality

As discussed previously, even though there is apparent dominance of B over A, when costs of obtaining K_b and returns from salvaging K_a are considered this dominance breaks down.

After the policy it must be that $X^{a''}_{m} < X^{b'}_{m}$.[33] For a switch (3.4.2) must become

$$(P^o_{x_m} - VC_a - R_{K_a} k_{aa} - VC_n n_a) X^{a''}_m < (P^o_{x_m} - VC_b - P_{K_b} k_{bb} - VC_n n_b) X^{b'}_m . \qquad (3.4.10)$$

It is possible that the switch could be implied either with

$$(P^o_{x_m} - VC_a - R_{K_a} k_{aa} - VC_n n_a) > (P^o_{x_m} - VC_b - P_{K_b} k_{bb} - VC_n n_b) \qquad (3.4.11)$$

or with the $(>)$ being replaced by $(\leq)$.

As in previous models where this policy was applied X'_n will

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[33] See footnote 30, page 133 but notice for this model $k_a > k_b$ and $n_a > n_b$; consequently $k_a + k_n n_a > k_b + k_n n_b$ which implies $X^{b'}_m > X^{a''}_m$.

be produced instead of X_n. The policy agency, therefore, may not be too concerned about internal firm changes. If volume of X'_n is significant then understanding internal process changes could become important.

Summary and Implications

If (3.4.11) holds as shown the switch could be affected by decreases in P_{x_m}. If the ($\leq$) holds in (3.4.11) and (3.4.10) also holds then the switch will be insensitive to any change in P_{x_m}.

Again, the major implication is that enforcement must be very good if this policy is to be effective. The effluent must also be identifiable as discussed before.

Subsidizing a Variable Factor

The application of this policy to Model 3.4 results in effects that are little different from Model 3.1. As with Model 3.1 the possibility of B dominating A after policy implementation exists. There is also the chance that the subsidy could switch the direction of inequalities (3.4.3) and (3.4.4) and not (3.4.2) resulting in an aggravation of the problem.

Subsidizing a Fixed Factor

Firms which could be represented by this model are prime candidates for being influenced by subsidizing K_b. It is sufficient for a switch if the subsidy reduces the cost of K_b so that the resale value of K_a will pay for K_b. One way to view the situation is

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m - (P^o_{x_m} - VC_b)\, X^{b^o}_m \overset{?}{\substack{\geq \\ <}} R_{K_a}\, k_{aa}\, X^{a^o}_m - (P_{K_b} - S_{K_b})\, k_{bb}\, X^{b^o}_m \,. \tag{3.4.12}$$

If the salvage of K_a and subsidy to K_b cancel out the cost of K_b then the right side of (3.4.12) will be zero. By assumption, it is known that $(P^o_{x_m} - VC_b)\, X^{b^o}_m > (P^o_{x_m} - VC_b)\, X^{a^o}_m$ which if the right of (3.4.12) is zero implies that the ($<$) will hold in (3.4.12). The ($<$) holding in (3.4.12) implies a switch from A to B.

It is not necessary that the subsidy entirely offset the cost of K_b before a switch is implied. It is only necessary that the ($<$) hold in (3.4.12). Expressed another way (3.4.12) becomes

$$S_{K_b}\, k_{bb} > (P^o_{x_m} - VC_a)\, \frac{X^{a^o}_m}{X^{b^o}_m} - (P^o_{x_m} - VC_b) + P_{K_b}\, k_{bb} - R_{K_a}\, k_{aa}\, \frac{X^{a^o}_m}{X^{b^o}_m} \tag{3.4.13}$$

$$\longrightarrow S_{K_b}\, k_{bb} > (P^o_{x_m} - VC_a)\, \frac{k_b}{k_a} - (P^o_{x_m} - VC_b) + P_{K_b}\, k_{bb}$$

$$- R_{K_a} k_{aa} \frac{k_b}{k_a} \quad . \qquad (3.4.14)$$

Statement (3.4.14) stated that the subsidy necessary to induce a switch must be such that the return from producing one unit of X_m by method B ($S_{K_b} k_{bb}$) must be greater than the cost (right side of (3.4.14)). Since $k_a > k_b$, $\frac{k_b}{k_a} < 1$ and since $(P^o_{x_m} - VC_a) < (P^o_{x_m} - VC_b)$ the sum of the first two terms to the right of the (>) in (3.4.14) is negative. One can readily see that the larger $R_{K_a} k_{aa} \frac{k_b}{k_a}$ is relative to $P_{K_b} k_{bb}$ the smaller S_{K_b} needs to be.

If the subsidy becomes too large it might be feasible to not only switch to method B but also to obtain additional K and expand production so that more X_n would be produced with method B than originally with A. Again this points out the sensitiveness of the size of subsidy for bringing about the desired results.

Summary and Implications

It should require a smaller subsidy of K_b to induce a switch with this model compared to 3.1. This is so since shorter term net returns are higher for technique B than A under this model specification.

If the subsidy in effect cancels out the cost of K_b the policy results will *not* be affected by changes in P_{x_m}. If only the necessary condition for a switch is met a declining price for X_m could induce a switch back to A.

Specification and Policy Applications for Model 3.4'

Model 3.4' is identical to 3.4 except it is assumed that K_a is the initial constraining factor. The following are the beginning conditions:

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m < (P^o_{x_m} - VC_b)\, X^{b^o}_m \ , \qquad (3.4'.1)$$

$$(P^o_{x_m} - VC_a)\, X^{a'}_m + R_K\, (K^o - K') > (P^o_{x_m} - VC_b)\, X^{b^o}_m - P_{K_b}\, k_{bb}\, X^{b^o}_m + R_{K_a}\, k_{aa}\, X^{a'}_m \ , \qquad (3.4'.2)$$

$$P_{K_a}\, k_{aa} > (P^o_{x_m} - VC_a) + R_K\, (K^o - K'') \text{ where } K'' = K' + k_a \text{ and} \qquad (3.4'.3)$$

$$P_{K_b}\, k_{bb} > (P^o_{x_m} - VC_b) + R_K\, (K^o - K'') \text{ where } K'' = k_a\, X^{a'}_m + k_b\ . \qquad (3.4'.4)$$

Explanation of the above conditions is identical to that given for similar conditions under Model 3.1'.

Taxing the Market Product

A tax on X_m will not induce any changes in this model. It is possible the tax could force the firm from business if it were too high.

Taxing a Variable Factor

Clearly as before, the necessary conditions for a method switch can be specified but a switch will not take place unless the salvage value of K_a is significant. The main difference between the conditions necessary for switching for this model and 3.4 is that $X_m^{a'}$ (maximum X_m^a due to K_a constraint) is less than $X_m^{a^o}$. This difference implies that a lower tax may be needed (assuming everything else except $X_m^{a'}$ being the same and that $R_K(K^o - K') = 0$) to induce a method change for 3.4' compared to 3.4. However, if beginning net revenue is the same between models it may require a larger tax to induce a switch for 3.4' compared to 3.4. Once that tax level is determined it must be realized that it could force the firm from business unless the return from K_a is significant.

Taxing the Non-market Externality

Due to (3.4'.4) it will be necessary for the firm to receive income from the salvage of K_a before it will switch techniques instead of close down. The necessary condition for a switch is

$$(P^o_{x_m} - VC_a - T_n n_a) X^{a'}_m + R_K (K^o - K') < (P^o_{x_m} - VC_b - T_n n_b) X^{b^o}_m - P_{K_b} k_{bb} X^{b^o}_m + R_{K_a} k_{aa} X^{a'}_m. \qquad (3.4'.5)$$

In addition to (3.4'.5), it must also be that the right side of (3.4'.5) is greater than zero; otherwise, the firm will close down. It is difficult without specific coefficient values to compare the size of tax necessary to induce a switch between models 3.4 and 3.4'. If net revenue before the tax is equal between the models, then it will require a larger tax to induce a switch for Model 3.4' than for 3.4. However, if initial net revenue for 3.4' should be less than that for 3.4, it might require a smaller tax to induce a switch in 3.4' compared to 3.4.

Summary and Implications

This policy could induce a technique switch. Resale value from K_a must be forthcoming, however, before the switch will take place. It is possible that after the tax technique B could totally dominate A, i.e.,

$$(VC_a + T_n n_a + R_{K_a} k_{aa}) < (VC_b + T_n n_b + P_{K_b} k_{bb}). \qquad (3.4'.6)$$

If after the switch (3.4'.6) holds, the policy effects will be insensitive to changes in P_{x_m}. If the inequality in (3.4'.6) goes the other way, i.e., (<), then a falling P_{x_m} could result in a switch back to method A.

A Standard on the Quality of the Externality

It is possible that the standard could induce a technique switch. The necessary condition for a switch is

$$(P^o_{x_m} - VC_a - R_{K_a} k_{aa} - VC_n n_a) X^{a''}_m < (P^o_{x_m} - VC_b - P_{K_b} k_{bb} - VC_n n_b) X^{b'}_m. \qquad (3.4'.7)$$

If after the policy is applied, $X^{b'}_m > X^{a''}_m$[34] the analysis and implications will be the same as when this policy was applied to Model 3.4.

Since K_a is the constraining factor some K is not used to produce X_m with technique A. It is conceivable that this "extra" K could be sufficient to convert all X_n produced by method A to X'_m. If such should be the case the potentiality exists for $X^{b'}_m < X^{a''}_m$. Under the latter circumstance the per unit net revenue (after the policy) generated by method B must be greater than that generated by A for a switch to occur ((3.4'.7) holding).

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[34] Notice that $X^{a''}_m$ could be identical to $X^{a'}_m$ (the level of X^a_m before policy implementation).

Summary and Implications

Due to initial condition (3.4'.4) a switch to method B is dependent upon resale value from K_a. Without resale returns the firm will either continue production with method A or close down.

If after the standard is enforced $X_m^{b'} > X_m^{a''}$ and a switch occurs changes in P_{x_m} might not change the situation if B dominates A. Otherwise a falling P_{x_m} could cause reversion to A. However, if $X_m^{b'} <$ $X_m^{a''}$ a rise in P_{x_m} could result in a switch back to A.

Subsidizing a Variable Factor

The main difference between this model and 3.4 is that the price range for X_m that would indicate no change is smaller for 3.4'. The latter statement is made on the assumption that the only things different between the models is the maximum amount of X_m that can be produced by technique A and P_{x_m}. The argument can be more readily understood by looking at Figure 4.8. By definition $VC_a^T = VC_a +$ $R_{K_a} k_{aa} - S_{v'} v'_a$ and $VC_b^T = VC_b + P_{K_b} k_{bb} - S_{v'} v'_b$ so that Figure 4.8 represents the situation after policy application. Furthermore, if $VC_b^T \leq VC_a^T$ there would be no price situations under which the firm would stay with technique A. Consequently, Figure 4.8 represents the only situation which, if a switch were made that

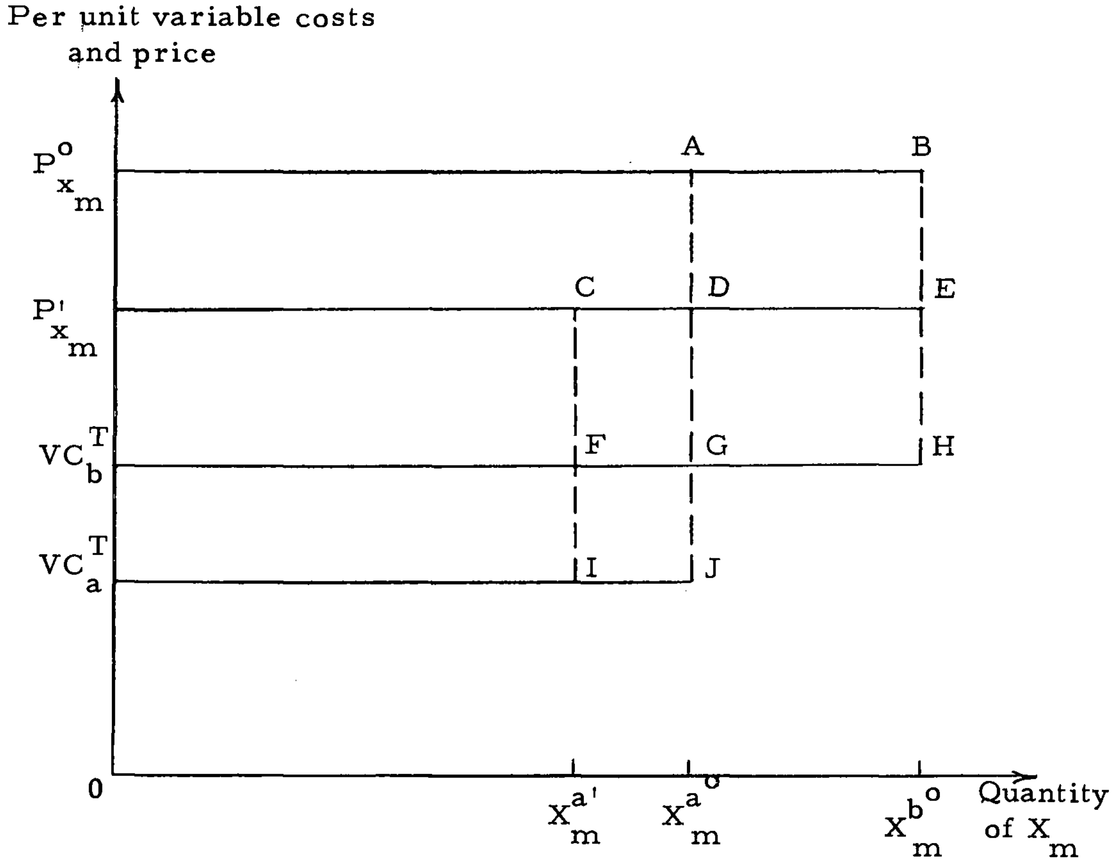

Figure 4.8. Comparison of price effects between Models 3.4 and 3.4'

would be sensitive to changes in P_{x_m}.

Model 3.4 is represented by the comparative rectangles VC^T_a JA $P^o_{x_m}$ and VC^T_b HB $P^o_{x_m}$ which are the areas of net revenue as earned by methods A and B, respectively, with price at $P^o_{x_m}$. Notice that the areas of these rectangles are equal which implies indifference as to technique. However, for prices lower than $P^o_{x_m}$ and greater than VC^T_a the net revenue for method A is the largest.[35] Model 3.4' is represented by rectangles VC_a^T IC P'_{x_m} and VC_b^T HE P'_{x_m} which represent the net revenues earned by methods A and B, respectively, at the price P'_{x_m}. Again the areas of these rectangles are equal indicating indifference as to method at P'_{x_m}. Any price lower than P'_{x_m} will generate higher net revenue for A than for B.[36] As can be observed $P^o_{x_m} > P'_{x_m}$ which indicates that under Model 3.4 the firm will stay with technique A over

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[35] The relevant comparison is for when $(P_{x_m} - VC_a^T)\, X^{a^o}_m \geq (P_{x_m} - VC_b^T)\, X^{b^o}_m$. (1)

In Figure 4.8 $X^{a^o}_m = 15$, $X^{b^o}_m = 21$ and $VC_b^T = VC_a^T + 3$ so (1) becomes $(P_{x_m} - VC_a^T)15 \geq (P_{x_m} - VC_a^T - 3)\, 21 \rightarrow 63 \geq 6(P_{x_m} - VC_a^T)$ $\rightarrow 10.5 \geq (P_{x_m} - VC_a^T)$ which implies for a positive net revenue that $0 \leq P_{x_m} - VC_a^T \leq 10.5 \rightarrow VC_a^T \leq P_{x_m} \leq 10.5 + VC_a^T$. Note that $P^o_{x_m} = VC_a^T + 10.5$.

[36] The argument here can be developed as footnote 35 making the proper substitutions.

a wider price range, $P^o_{x_m}$ to VC^T_a, versus P'_{x_m} to VC^T_a for Model 3.4'.

Another way of interpreting this difference between 3.4 and 3.4' concerns the size of the subsidy necessary to bring about a switch in techniques for a given price $P^o_{x_m}$. The argument can be developed to show that Model 3.4 requires a larger subsidy than Model 3.4' to bring about a switch,[37] if the same conditions are assumed other than $X^{a^o}_m > X^{a'}_m$. The previous development serves again to point out the complexity of knowing what will happen with this policy.

As with Model 3.1' it is possible that the subsidy could induce the firm to produce more X_m with technique A or maybe more with both techniques. In any event it appears that this is a sensitive policy to impose.

Summary and Implications

In most respects subsidizing V' produces results quite similar

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[37] A switch is implied if $(P^o_{x_m} - VC^t_a + S_{v'} v'_a) X^{a^o}_m < (P^o_{x_m} - VC^t_b + S_{v'} v'_b) X^{b^o}_m$ (1) where $VC^t_a = VC_a + R_{K_a} k_{aa}$ and $VC^t_b = VC_b + P_{K_b} k_{bb}$. Again using Figure 4.8 and footnote 35 (1) becomes $(P^o_{x_m} - VC^t_a) 15 + 15 S_{v'} v'_a < (P^o_{x_m} - VC^t_a) 21 - 63 + 21 S_{v'} v'_b$ (2)

to Model 3.4. The main difference in the two models is due to less X_m being produced initially under Model 3.4'.

One implication concerns the predictable sensitivity to P_{x_m}. For example suppose the policy agency felt that the minimum price to be expected for X_m was somewhere between $P^o_{x_m}$ and P'_{X_m}. It would then conclude if looking only at Model 3.4' that its policy would not likely be influenced by foreseeable price changes. However, for Model 3.4 a price below $P^o_{x_m}$ would imply a switch back to technique A (assuming the price at one time had been above $P^o_{x_m}$). In other words for 3.4' price could fall further without the firm switching back to B than it could fall for Model 3.4.

Another implication is that subsidy size is quite dependent upon the initial firm position. This position is in turn influenced by the

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since $VC^t_b = VC^t_a + 3$.

$$(2) \longrightarrow 63 - 6\,(P^o_{x_m} - VC^t_a) < 21\, S_{v'}\, v'_b - 15\, S_{v'}\, v'_a \qquad (3)$$

$$\longrightarrow \frac{21 - 2(P^o_{x_m} - VC^t_a)}{7\, v'_b - 5\, v'_a} < S_{v'} \equiv S^o_{v'} \qquad (4)$$

which is for Model 3.4. By a similar development it can be shown for 3.4' that

$S'_{v'} \equiv S_{v'} > \dfrac{21 - 3\,(P^o_{x_m} - VC^t_a)}{7\, v'_b - 4\, v'_a}$. Comparison of $S^o_{v'} + \epsilon$ and $S'_{v'} + \epsilon$ will reveal that $S^o_{v'} + \epsilon > S'_{v'} + \epsilon$ which implies $S^o_{v'} > S'_{v'}$.

relevant constraining factors.

Together with the possibility of increasing X_n production the two implications above point out the difficulties involved if the subsidy is to be a viable policy.

Subsidizing a Fixed Factor

Compared to Model 3.4 the potentiality for a subsidy resulting in added production by method B is stronger. The latter is so since under Model 3.4' excess K is available for use in producing some X_m^b and $X_m^{a'}$ at the same time. Comparisons as to subsidy size between Models 3.4 and 3.4' could also be made; however, the results and analysis would be quite similar to those discussed for taxing variable factors.

Subsidizing K_b could induce a technique switch. The only difference in effects between this "prime" model and others relates to subsidy size. Otherwise the results of applying the subsidy to this model adds nothing to what has already been discussed.

Specification and Policy Applications for Model 3.5

This model is distinguished from previous ones by the following assumptions. First, the variable costs of technique A are greater than those for method B, i.e., $VC_a > VC_b$. Secondly, it is assumed that B requires more K per unit of X_m than A, i.e., $k_a < k_b$.

Factor K is the initial constraint. Again for reference ease, initial conditions are stated.

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m \overset{>}{\underset{<}{=}} (P^o_{x_m} - VC_b)\, X^{b^o}_m \;. \tag{3.5.1}$$

Since per unit net revenue of A (ignoring K_a and K_b) is less than that for B and $X^{b^o}_m > X^{a^o}_m$ any of the three conditions in (3.5.1) may exist.

$$(P^o_{x_m} - VC_a)\, X^{a^o}_m > (P^o_{x_m} - VC_b - P_{K_b}\, k_{bb})\, X^{b^o}_m + R_{K_a}\, k_{aa}\, X^{a^o}_m \tag{3.5.2}$$

$$P_K\, k_a > (P^o_{x_m} - VC_a) + R_{K_a}\, (K^o_a - K''_a) \tag{3.5.3}$$

where $K''_a = K'_a + k_{aa}$.

In (3.5.3) K'_a is the amount of K_a used to produce $X^{a^o}_m$. The initial situation is depicted in Figure 4.9.

Notice that if either the (<) or (=) holds in (3.5.1) the cost of K_b must exceed resale value of K_a before (3.5.2) can hold. If the (>) holds in (3.5.1) there is no condition concerning K_a and K_b values implied for the validity of (3.5.2)

Taxing the Market Product

With the price of $P^o_{x_m}$ shown in Figure 4.9 the firm is maximizing net revenue by using method A. As long as price stays where

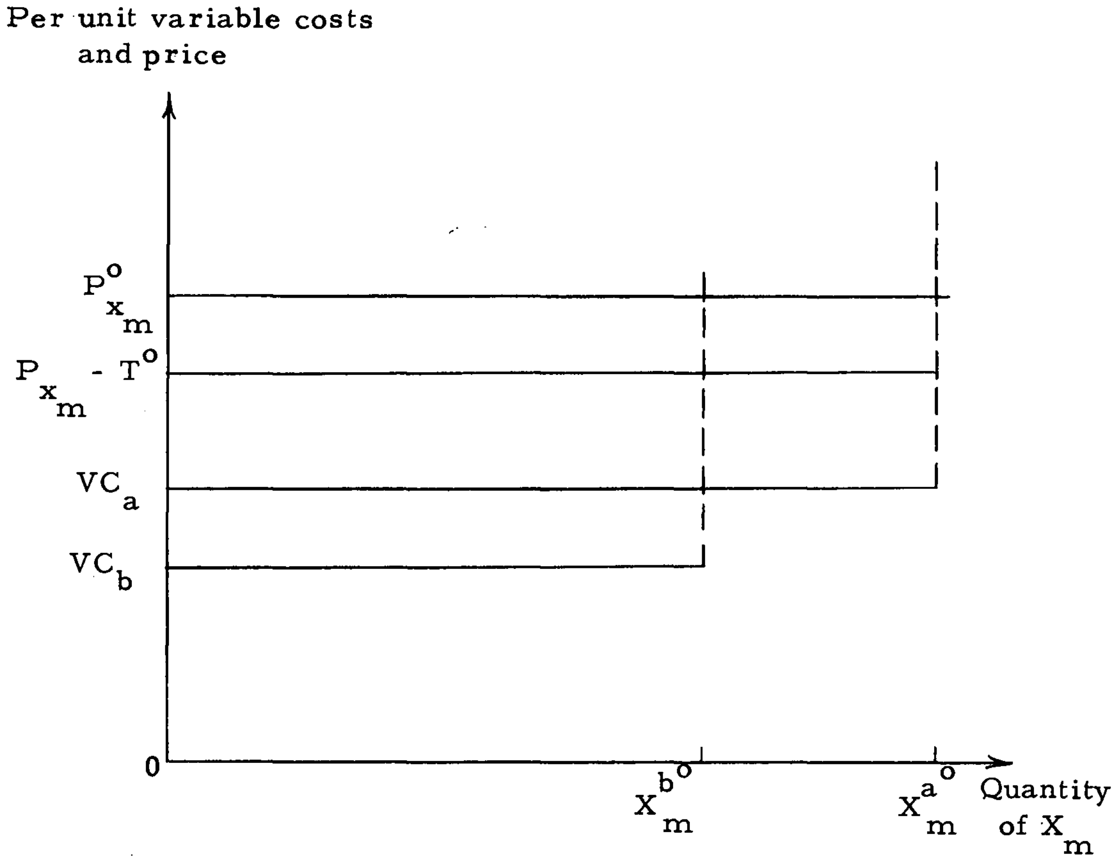

Figure 4.9. Initial and after tax on X_m positions--Model 3.5

it is the firm is effectively locked into technique A.

The tax on X_m will not affect the direction of the inquality in condition (3.5.3). However, there is a chance that the tax could induce a switch since (3.5.2) will be affected.

For the tax to result in a technique switch it is necessary that

$$(P^o_{x_m} - VC_a) X^{a^o}_m - T X^{a^o}_m < (P^o_{x_m} - VC_b) X^{b^o}_m - T X^{b^o}_m - P_{K_b} (k_{bb} X^{b^o}_m) + R_{K_a} (k_{aa} X^{a^o}_m) \qquad (3.5.4)$$

$$\longrightarrow (P^o_{x_m} - VC_a)\, X^{a^o}_m - (P^o_{x_m} - VC_b)\, X^{b^o}_m < TX^{a^o}_m - TX^{b^o}_m$$

$$- P_{K_b}\,(k_{bb}\, X^{b^o}_m) + R_{K_a}\,(k_{aa}\, X^{a^o}_m)\,. \qquad (3.5.5)$$

Even without the last two terms of (3. 5. 5) it is possible the tax could bring about a switch. All that is necessary is that the tax savings exceed the net revenue advantage of method A.[38] If the last two terms of (3. 5. 5) together are negative then the tax savings will need to be larger, implying a larger tax. Of course the tax could be too large and thus force the firm out of business. If this policy does work it will be very sensitive to upward changes in P_{x_m} (see Figure 4. 9).

Summary and Implications

The tax on X_m could induce a technique switch. The size of tax necessary will be affected by initial net revenue position before K_a and K_b are considered. Tax size will also be affected by the

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[38] Method A will have a basic (excluding cost of K_b and salvage of K_a)net revenue advantage if the (>) holds in (3. 5. 1). As noted earlier it is also possible that the (<) could hold in (3. 5. 1) in which case it must be that $P_{K_b}\,(k_{bb}\, X^{b^o}_m) > R_{K_a}\,(k_{aa}\, X^{a^o}_m)$ in (3. 5. 2). Assuming all other conditions the same including the relationship between cost of K_b and salvage of K_a the existence of the latter condition (i. e., (<) in (3. 5. 1)) would imply the need for a smaller tax to bring about a switch than if the former condition held.

relative cost of K_b and resale value of K_a. Upward movements of P_{x_m} could induce the firm to switch back to method A. A tax that goes up with price would need to be used to prevent a switch back to A.

Taxing a Variable Factor

The initial firm position after accounting for the cost of K_b and resale value of K_a is depicted in Figure 4.10.[39]

All that is necessary for the tax on V to be effective is for it to alter the net revenue relationships enough so that the net revenue after tax from B is larger than the net from A. The necessary situation is shown in Figure 4.10 by $VC_a + R_{K_a} k_{aa} + T_v v_a > VC_b + P_{K_b} k_{bb} + T_v v_b$. The area of net revenue for method B is larger than for A. Notice, however, that if P_{x_m} goes up enough technique A will again become the preferred method.

Taxing the Non-market Externality

A tax on X_n could persuade the firm to switch techniques. The

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[39] Notice that it is entirely conceivable that in Figure 4.10, $VC_b + P_{K_b} k_{bb} > VC_a + R_{K_a} k_{aa}$. Such a situation will not alter the analysis.

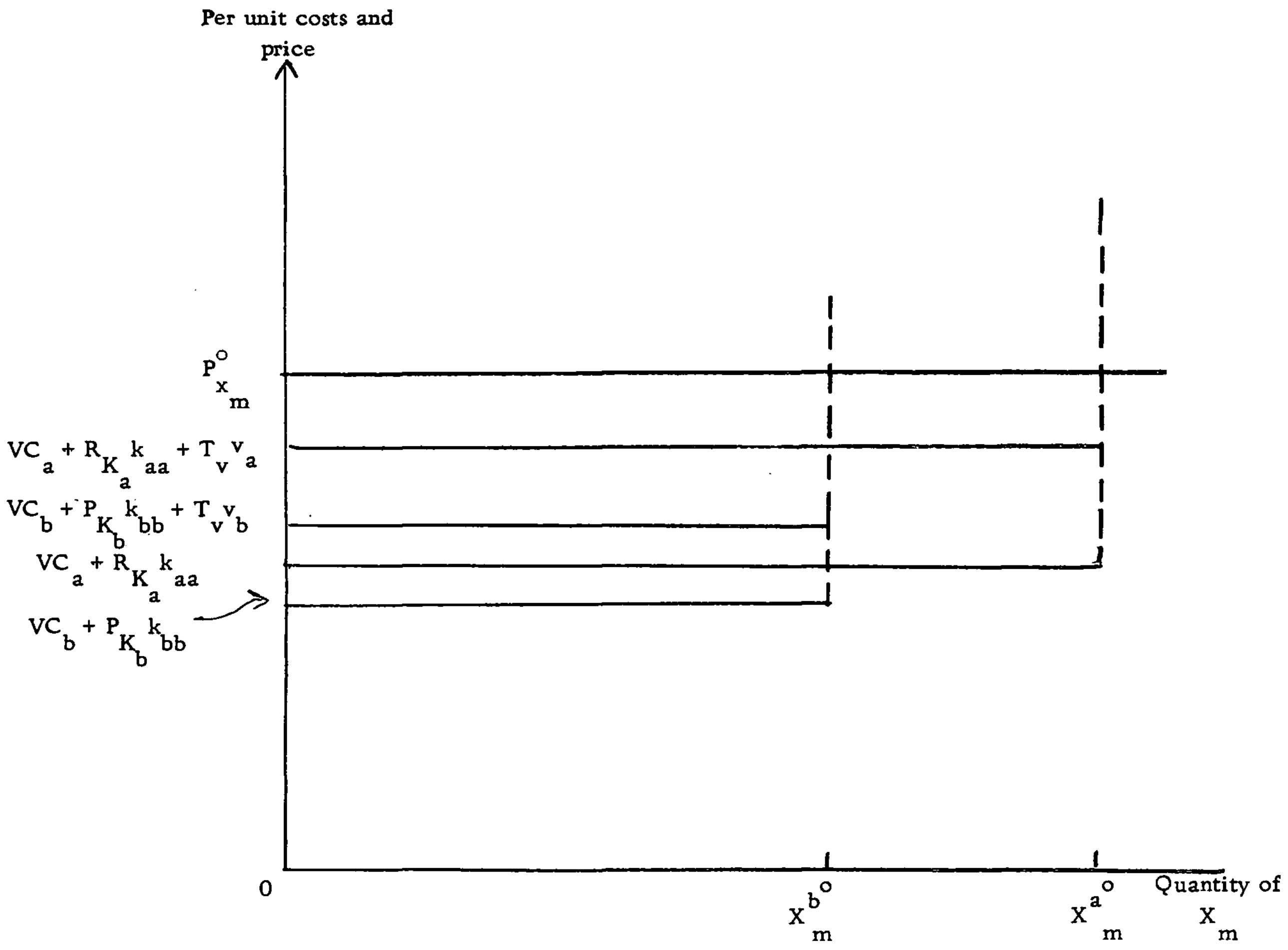

Figure 4. 10. Initial and after tax on V positions--Model 4. 5

necessary condition is

$$(P^{o}_{x_m} - VC_a - T_n n_a - R_{K_a} k_{aa}) X^{a^{o}}_m < (P^{o}_{x_m} - VC_b - T_n n_b$$

$$- P_{K_b} k_{bb}) X^{b^{o}}_m . \qquad (3.5.6)$$

Since $X^{a^{o}}_m > X^{b^{o}}_m$ (3.5.6) implies that

$$(VC_a + T_n n_a + R_{K_a} k_{aa}) > (VC_b + T_n n_b + P_{K_b} k_{bb}).$$

Even if (3.5.6) holds it can be observed that a rise in P_{x_m} could reverse the inequality which would imply a switch back to A.

The implication is that this policy could induce a technique switch for Model 3.5, however, the switch would be sensitive to rises in P_{x_m}.

A Standard on the Quality of the Externality

It is possible that $VC_a + R_{K_a} k_{aa} < VC_b + P_{K_b} k_{bb}$ or $VC_a + R_{K_a} k_{aa} \geq VC_b + P_{K_b} k_{bb}$. If the former condition holds it is conceivable that the effluent standard could bring about a technique change. Such a switch would be sensitive to upward changes in P_{x_m} if $X^{a''}_m > X^{b'}_m$. It must also be that

$$(P^{o}_{x_m} - VC_a - R_{K_a} k_{aa} - VC_n n_a) < (P^{o}_{x_m} - VC_b - P_{K_b} k_{bb}$$

$$- VC_n n_b). \quad (3.5.7)$$

When the previous conditions are placed together neither method dominates the other after the policy. Dominance is necessary if a switch is not to be price sensitive.

If $VC_a + R_{K_a} k_{aa} \geq VC_b + P_{K_b} k_{bb}$ then there is a possibility that a change could be implied that would not be P_{x_m} sensitive. To be insensitive to price it must be that after the policy, (3.5.7) holds and $X_m^{a''} < X_m^{b'}$ which implies dominance of B over A after the standard is set. It is conceivable that a switch could occur with $X_m^{a''} < X_m^{b'}$ and the inequality in (3.5.7) reversed. If these latter conditions exist, the change will then be sensitive to falling prices for X_m.

Subsidizing a Variable Factor

The subsidy on V' could bring about a switch from technique A to B. If the switch occurs it will be sensitive to changes in P_{x_m}.

The necessary and sufficient condition for a switch is

$$(P^o_{x_m} - VC_a - R_{K_a} k_{aa} + S_{v'} v'_a) X_m^{a^o} < (P^o_{x_m} - VC_b - P_{K_b} k_{bb}$$

$$+ S_{v'} v'_b) X_m^{b^o} > 0. \quad (3.5.8)$$

Since $X_m^{a^o} > X_m^{b^o}$ it is possible before the subsidy that

$$(VC_a + R_{K_a} k_{aa}) \overset{\geq}{\underset{<}{=}} (VC_b + P_{K_b} k_{bb}). \qquad (3.5.9)$$

Any of these three conditions can exist along with (3.5.2). No matter which of the three situations holds in (3.5.9) an implied switch could be reversed by a sufficient rise in P_{x_m}. The latter is so since condition (3.5.8) and $X^{a^o}_m > X^{b^o}_m$ imply that

$$(VC_b + P_{K_b} k_{bb} - S_{v'} v'_b) < (VC_a + R_{K_a} k_{aa} - S_{v'} v'_a)$$

which further implies that after the switch A does not dominate B.

After the subsidy the firm will be receiving more net revenue than prior to the subsidy no matter which technique it chooses to use. This statement is true of all models and this policy. As with previous models the possibility for firm expansion due to the subsidy also exists. Consequently, caution must be exercised in application of the subsidy if the situation is not to be degraded.

Subsidizing a Fixed Factor

A possibility for inducing a technique switch does exist. If a switch occurs it definitely will be sensitive to positive changes in the price of X_m. The justification for the previous statement is similar to that given for the previous policy. The other possibilities for influencing the firm are the same for this model as for 3.1.

Specification and Policy Applications for Model 3.5'

This model is the same as 3.5 except K_a is assumed to be the initial relevant constraint. Even though $k_a < k_b$ it is possible, disregarding other fixed factors, that the beginning level for X^a_m, $X^{a'}_m$, is less than $X^{b^o}_m$. Recall that with Model 3.5, $X^{a^o}_m > X^{b^o}_m$. Whether or not $X^{a'}_m \underset{<}{\geq} X^{b^o}_m$ depends on the level of K_a available initially.

The following are relevant initial conditions consistent with the assumption that the firm is using method A.

$$(P^o_{x_m} - VC_a)\, X^{a'}_m \underset{<}{\geq} (P^o_{x_m} - VC_b)\, X^{b^o}_m, \qquad (3.5'.1)$$

$$(P^o_{x_m} - VC_a)\, X^{a'}_m + R_K\,(K^o - K') > (P^o_{x_m} - VC_b)\, X^{b^o}_m - P_{K_b}\, k_{bb}\, X^{b^o}_m$$

$$+ R_{K_a}\, k_{aa}\, X^{a'}_m \qquad (3.5'.2)$$

$$P_{K_a}\, k_{aa} > (P^o_{x_m} - VC_a) + R_K\,(K^o - K'') \text{ where } K'' = K'$$

$$+ k_a, \text{ and} \qquad (3.5'.3)$$

$$P_{K_b}\, k_{bb} > (P^o_{x_m} - VC_b) + R_K\,(K^o - K'') \text{ where } K''$$

$$= k_a\, X^{a'}_m + k_b. \qquad (3.5'.4)$$

Which of the three conditions hold in (3. 5'. 1) is dependent on the level of $X_m^{a'}$ compared to $X_m^{b^o}$ as discussed above. The other conditions have the same explanations as discussed in Model 3. 1'.

Taxing the Market Product

Depending on how restrictive K_a is this model will take on similarities to some already discussed. If K_a restricts X_m^a such that $X_m^{a'} > X_m^{b^o}$ then 3. 5' will be similar to 3. 5. One difference is that (3. 5'. 4) is relevant initially. Other than that the analysis can follow that for 3. 5. In other words there is the possibility that a tax on X_m could induce method switching. Notice, however, that due to (3. 5'. 4) the firm must depend (if it switches) on income from salvaging K_a to obtain positive net returns.

If K_a is so restrictive that $X_m^{a'}$ is limited to some point less than $X_m^{b^o}$, then there is no chance for the tax to induce a switch. The tax will reduce the right side of (3. 5'. 2) (the smallest) more than the left which will leave the direction of the inequality unchanged.

Summary and Implications

This model has pointed out the need for being aware of what truly constitutes the effective constraint. The level which is constraining is also important if the outcome of a tax on X_m is to be predicted. If conditions exist so that the tax does imply a technique switch, increases in P_{x_m} could prompt a switch back to A.

Taxing a Variable Factor

This is a case analogous to the other "prime" models. The tax on V could alter the initial conditions so that a switch would be indicated but it will depend on salvage values of fixed resources to make the switch with ensuing positive net returns. Taxing V could prompt a methods switch with any of the three conditions in (3.5'.1).

If $X_m^{a'} < X_m^{b^o}$ it is conceivable that after the tax on V, method B could totally dominate A. Under those conditions changes in P_{x_m} will not bring about any technique changes. It may also be that a fall in price of X_m could bring about a switch back if $X_m^{a'} < X_m^{b^o}$ and if B did not dominate A. If $X_m^{a'} > X_m^{b^o}$ technique B cannot dominate A; consequently, a switch to B will not be permanent if P_{x_m} increases substantially.

Taxing the Non-market Externality

A tax on X_n could induce a technique switch. The analysis is practically identical to a tax on the variable factor, V. Whether or not the switch will be affected by changes in P_{x_m} depends on the same factors as the previous policy.

A Standard on the Quality of the Externality

The analysis for this case is similar to Model 3.5 and the other "prime" models. As with 3.5 an implied technique change can either be sensitive or insensitive to P_{x_m}. For 3.5' one must not only consider the cost levels but also the fact that there will be excess K which may or may not permit converting all X_n produced by method A to X'_n. If initially $VC_a + R_{K_a} k_{aa} \geq VC_b + P_{K_b} k_{bb}$ then the variable costs for converting X_n to X'_n will not change the initial cost relationship. Under these cost conditions a switch could be implied whether $X^{a''}_m \gtreqless X^{b'}_m$. The main difference between this model and 3.5 is that $X^{a''}_m$ might equal $X^{a'}_m$ for 3.5' whereas $X^{a''}_m < X^{a^o}_m$ for 3.5 ($X^{a'}_m$ and $X^{a^o}_m$ are the initial amounts of X_m produced by A). With the previously described initial cost relationship the switch could be insensitive to P_{x_m} provided $X^{a''}_m \leq X^{b'}_m$ which would imply dominance of method B.

If $VC_a + R_{K_a} k_{aa} < VC_b + P_{K_b} k_{bb}$ the analysis can be conducted similar to Model 3.5 with the same policy. Depending on the relative size of n_a and n_b and the size of VC_n, method B could dominate A after the standard. The possibility of dominance of B seems less for this cost assumption compared to the one in the previous paragraph.

Summary and Implications

The possibility for method B to dominate A after the standard does exist. For dominance $X^{a''}_m$ must be less than $X^{b'}_m$ plus per unit net revenue for B must exceed that from A. Whether or not $X^{a''}_m \gtrless X^{b'}_m$ depends in part on the level of the constraint K_a. It also depends on the relative size of n_a and n_b (amounts of X_n produced per unit of X_m by A and B, respectively). The major implication is that to predict *a priori* the effects of changes in P_{x_m} after the standard requires a high level of knowledge about internal process coefficients.

At the very least this policy if properly enforced will insure release of X'_n instead of X_n to the environment. A decrease in absolute volume through a technique change could be an additional spinoff.

Subsidizing a Variable Factor

The analysis for this model is quite similar to the other "prime" models and to 3.5. It differs from 3.5 in one respect which is that there exists one set of circumstances that could imply insensitivity to P_{x_m} after a switch.

Suppose that K_a restricted X^a_m to a point such that $X^{a'}_m < X^{b^o}_m$. After the subsidy it is possible that $(VC_b + P_{K_b} k_{bb} - S_{v'} v'_b) <$ $(VC_a + R_{K_a} k_{aa} - S_{v'} v'_a)$. The latter two situations together imply that B dominates A and the switch will be insensitive to P_{x_m}. If the inequality in the cost relationship in this paragraph were the other direction and a switch occurred, falling prices for X_m could imply a switch back also.

It can be shown that for Model 3.5' compared to 3.5 a smaller subsidy is required to induce a switch if all other conditions are assumed the same.[40] The general implication of the latter statement is that a firm constrained by specialized resources is likely to switch techniques with a smaller subsidy than one that is constrained initially by a common non-specialized resource.

Implications common to the other "prime" models are also common to this one.

Subsidizing a Fixed Factor

The conclusions for this model are also similar to the other "prime" models and 3.5. The analysis differs from 3.5 only with respect to sensitivity of P_{x_m}. It is possible for 3.5' that decreases in P_{x_m} could induce a switch back if $X_m^{a'} < X_m^{b^o}$. Circumstances can also be described for which the switch will be insensitive to price of X_m.[41] If $X_m^{a'} > X_m^{b^o}$ price increases could result in a switch

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[40] See Appendix A, Part 2.

[41] See discussion concerning the application of a subsidy of V' for this model. The conditions will be similar with only notation changes.

back.

An argument can be developed showing that 3.5' requires a smaller subsidy than 3.5 to imply a technique switch.[42] Once again the importance of knowing the internal structures of the firm's economic situation for a priori predictions of effects can be observed.

Summary and Conclusions for Models 3.1 through 3.5'

These models were distinguished from previous ones by the introduction of specialized fixed factors. Since changes in availability of fixed resources was permitted this chapter in a sense considered a longer run view of policy effects. Assumptions concerning relative use by the two techniques of the common fixed factor, short run variable factors, and short run variable costs were changed resulting in ten different but similar models.

All models were assumed to use technique A initially. Consequently the results summarized in Table 4.1 refer to changes from technique A to B. Only the policy effects that either resulted in no change

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[42] Utilize the framework for Appendix A, Part 2. An example of the difference in notation would be as follows: Statement (5) would be

$$A(X_m^{a^o} - X_m^{b^o}) - \lambda X_m^{b^o} < S_{K_b}^{o} (k_{bb} X_m^{b^o}).$$

Table 4. 1. Summary of Effects of Policies on Models 3. 1 through 3. 5.

Policy	Model	Model conditions				Possible effects of policies on[a]						
		$k_a \gtrless k_b$	$X_m^{a^o} \gtrless X_m^{b^o}$	$X_m^{a} \gtrless X_m^{b^o}$	$VC_a \gtrless VC_b$	Tech.	Net revenue	Level of X_m	Level of X_n	Use of V or V'	Use of K	Needs resale from K_a
Taxing the market product	3. 1	(>)	(<)	NA	(<)	0	-	0	0	0	0	NA
	3. 1'	(>)	NA	(<)	(<)	0	-	0	0	0	0	NA
	3. 2	(>)	(<)	NA	(<)	0	-	0	0	0	0	NA
	3. 2'	(>)	NA	(<)	(<)	0	-	0	0	0	0	NA
	3. 3[c]	(<)	(>)	NA	(<)	change	-	-	-	- (V)	0^d	Yes
	3. 3'	(<)	NA	either	(<)	change	-	-	-	- (V)	+	Yes
	3. 4	(>)	(<)	NA	(>)	0	-	0	0	0	0	NA
	3. 4'	(>)	NA	(<)	(>)	0	-	0	0	0	0	NA
	3. 5	(<)	(>)	NA	(>)	change	-	-	-	- (V)	0	No
	3. 5'	(<)	NA	either	(>)	change	-	-	-	- (V)	+	Yes
Taxing variable factor, V	3. 1	(>)	(<)	NA	(≶)	change	-	+	I^e	- (V)	0	No
	3. 1'	(>)	NA	(<)	(≯)	change	-	+	I^e	- (V)	+	Yes
	3. 2	(>)	(<)	NA	(≯)	change	-	+	I^e	- (V)	0	No
	3. 2'	(>)	NA	(<)	(<)	change	-	+	I^e	- (V)	+	Yes
	3. 3	(<)	(>)	NA	(<)	change	-	-	-	- (V)	0	No
	3. 3'	(<)	NA	either	(<)	change	-	I^g	I^g	- (V)	+	Yes
	3. 4	(>)	(<)	NA	(>)	change	-	+	I^e	- (V)	0	No
	3. 4'	(>)	NA	(<)	(>)	change	-	+	I^e	- (V)	+	Yes
	3. 5	(<)	(>)	NA	(>)	change	-	-	-	- (V)	0	No
	3. 5'	(<)	NA	either	(>)	change	-	I^g	I^g	- (V)	+	Yes
Taxing the non-market externality	3. 1	(>)	(<)	NA	(<)	change	-	+	$-^i$	I^j	0	No
	3. 1'	(>)	NA	(<)	(<)	change	-	+	$-^i$	I^j	+	Yes
	3. 2	(>)	(<)	NA	(<)	change	-	+	$-^i$	I^j	0	No
	3. 2'	(>)	NA	(<)	(<)	change	-	+	$-^i$	I^j	+	Yes
	3. 3	(<)	(>)	NA	(<)	change	-	-	$-^i$	- (V)	0	No
	3. 3'	(<)	NA	either	(<)	change	-	-	$-^i$	- (V)	+	Yes
	3. 4	(>)	(<)	NA	(>)	change	-	+	$-^i$	I^j	0	No
	3. 4'	(>)	NA	(<)	(>)	change	-	+	$-^i$	I^j	+	Yes
	3. 5	(<)	(>)	NA	(>)	change	-	-	$-^i$	I^j	0	No
	3. 5'	(<)	NA	either	(>)	change	-	I^g	$-^i$	I^j	+	Yes

Table 4. 1 (continued)

Policy	Model	Model conditions $k_a \gtrless k_b$	$X^{a\,o}_m \gtrless X^{b\,o}_m$	$X^{a'}_m \gtrless X^{b\,o}_m$	$VC_a \gtrless VC_b$	Tech.	Possible effects on policies on Net revenue	Level of X_m	Level of X_n	Use of V or V'	Use of K	Needs resale from K_a
A standard on the quality of the externality^k	3. 1	(>)	(<)	NA	(<)	change	-	I^l	$I^l(X'_n)$	I^l	0	No
	3. 1'	(>)	NA	(<)	(<)	change	-	I^l	I^l "	I^l	+	Yes
	3. 2	(>)	(<)	NA	(<)	change	-	I^l	I^l "	I^l	0	No
	3. 2'	(>)	NA	(<)	(<)	change	-	I^l	I^l "	I^l	+	Yes
	3. 3	(<)	(>)	NA	(<)	change	-	-	- "	- (V)	0	No
	3. 3'	(<)	NA	either	(<)	change	-	I^l	I^l "	I^l	+	Yes
	3. 4	(>)	(<)	NA	(>)	change	-	I^l	I^l "	I^l	0	No
	3. 4'	(>)	NA	(<)	(>)	change	-	I^l	I^l "	I^l	+	Yes
	3. 5	(<)	(>)	NA	(>)	change	-	-	- "	- (V)	0	No
	3. 5'	(<)	NA	either	(>)	change	-	I^g	I^g "	I^g	+	Yes
Subsidizing a variable factor $(V')^n$	3. 1	(>)	(<)	NA	(<)	change	+	+	I^e	+ (V')	0	No
	3. 1'	(>)	NA	(<)	(<)	change	+	+	I^e	+ (V')	+	No
	3. 2	(>)	(<)	NA	(<)	change	+	+	I^e	+ (V')	0	No
	3. 2'	(>)	NA	(<)	(<)	change	+	+	I^e	+ (V')	+	No
	3. 3	(<)	(>)	NA	(<)	change	+	-	-	+ (V')	0	No
	3. 3'	(<)	NA	either	(<)	change	+	I^g	I^g	+ (V')	+	No
	3. 4	(>)	(<)	NA	(>)	change	+	+	I^e	+ (V')	0	No
	3. 4'	(>)	NA	(<)	(>)	change	+	+	I^e	+ (V')	+	No
	3. 5	(<)	(>)	NA	(>)	change	+	-	-	+ (V')	0	No
	3. 5'	(<)	NA	either	(>)	change	+	I^g	I^g	+ (V')	+	No
Subsidizing a fixed factor	3. 1	(>)	(<)	NA	(<)	change	+	+	I^e	I^j (V)	0	No
	3. 1'	(>)	NA	(<)	(<)	change	+	+	I^e	I^j (V)	+	No
	3. 2	(>)	(<)	NA	(<)	change	+	+	I^e	I^j (V)	0	No
	3. 2'	(>)	NA	(<)	(<)	change	+	+	I^e	I^j (V)	+	No
	3. 3	(<)	(>)	NA	(<)	change	+	-	-	- (V)	0	No
	3. 3'	(<)	NA	either	(<)	change	+	I^g	I^g	I^g (V)	+	No
	3. 4	(>)	(<)	NA	(>)	change	+	+	I^e	I^j (V)	0	No
	3. 4'	(>)	NA	(<)	(>)	change	+	+	I^e	I^j (V)	+	No
	3. 5	(<)	(>)	NA	(>)	change	+	-	-	- (V)	0	No
	3. 5'	(<)	NA	either	(>)	change	+	I^g	I^g	I^g (V)	+	No

Table 4.1 (continued)

Policy	Model	Possible after policy effects of			
		Increasing P_{x_m} on		Decreasing P_{x_m} on	
		Technique	X_n	Technique	X_n
Taxing the market product,	3.1	0[b]	0	0[b]	0
	3.1'	0	0	0	0
	3.2	0	0	0	0
	3.2'	0	0	0	0
	3.3[c]	change back	+	0	0
	3.3'	change back	+	0	0
	3.4	0	0	0	0
	3.4'	0	0	0	0
	3.5	change back	+	0	0
	3.5'	change back	+	0	0
Taxing variable factor, V	3.1	0	0	change back[f]	I[e]
	3.1'	0	0	change back[f]	I[e]
	3.2	0	0	change back[f]	I[e]
	3.2'	0	0	change back[f]	I[e]
	3.3	change back	+	0	0
	3.3'	change back[h]	I[g]	change back[h]	I[g]
	3.4	0	0	change back[f]	I[e]
	3.4'	0	0	change back[f]	I[e]
	3.5	change back	+	0	0
	3.5'	change back[h]	I[g]	change back[h]	I[g]
Taxing the non-market externality	3.1	0	0	change back[f]	+
	3.1'	0	0	change back[f]	+
	3.2	0	0	change back[f]	+
	3.2'	0	0	change back[f]	+
	3.3	change back	+	0	0
	3.3'	change back[h]	+	change back[h]	+
	3.4	0	0	change back[f]	+
	3.4'	0	0	change back[f]	+
	3.5	change back	+	0	0
	3.5'	change back[h]	+	change back[h]	0

Table 4. 1 (continued)

Policy	Model	Possible after policy effects of			
		Increasing P_{x_m} on		Decreasing P_{x_m} on	
		Technique	X_n	Technique	X_n
A standard on the quality of the externality	3. 1	0	0	change back^f	I^e (X'_n)
	3. 1'	0	0	change back^f	I^l "
	3. 2	0	0	change back^f	I^l "
	3. 2'	0	0	change back^f	I^l "
	3. 3	change back^f	+	0	0
	3. 3'	change back^f	I^l (X'_n)	change back^m	I^l (X'_n)
	3. 4	0	0	change back^f	I^l "
	3. 4'	change back^m	I^l (X'_h)	change back^m	I^l "
	3. 5	change back^m	+	change back^m	+
	3. 5'	change back^m	I^g	change back^m	I^g
Subsidizing a variable factor (V')	3. 1	0	0	change back^f	I^e
	3. 1'	0	0	change back^f	I^e
	3. 2	0	0	change back^f	I^e
	3. 2'	0	0	change back^f	I^e
	3. 3	change back	+	0	0
	3. 3'	change back^h	I^g	change back^h	I^g
	3. 4	0	0	change back^f	I^e
	3. 4'	0	0	change back^f	I^e
	3. 5	change back	+	0	0
	3. 5'	change back^h	I^g	change back^h	I^g
Subsidizing a fixed vactor	3. 1	0	0	change back^f	I^e
	3. 1'	0	0	change back^f	I^e
	3. 2	0	0	change back^f	I^e
	3. 2'	0	0	change back^f	I^e
	3. 3	change back	+	0	0
	3. 3'	change back^h	I^g	change back^h	I^g
	3. 4	0	0	change back^f	I^e
	3. 4'	0	0	change back^f	I^e
	3. 5	change back	+	0	0
	3. 5'	change back^h	I^g	change back^h	I^g

Table 4.1 (continued)

[a] Positive changes are indicated by a (+), while the (0) and (-) indicate no change and a negative change (decrease), respectively. The letter I indicates an indeterminant situation. NA stands for not applicable, irrelevant, or not considered.

[b] Price effects are only shown for instances where the policy brought about some change in technique and/or production of X_n. If the price change could force the firm from business, the effect is still shown as no change.

[c] Where the possibility of a change exists depending only on tax or subsidy size, the effects of only the change are shown. In instances where a change was possible it was also possible that no change could take place. It is possible that a tax large enough to induce a switch might force the firm from business first.

[d] There is no change since the switch was from $X_m^{a^o}$ to $X_m^{b^o}$, both of which require K^o.

[e] The level of X_n depends on whether $n_a X_m^{a^o} \gtrless n_b X_m^{b^o}$, or $n_a X_m^{a'} \gtrless n_b X_m^{b^o}$. Since for this model $X_m^{a^o} < X_m^{b^o}$, just because $n_a > n_b$ does not necessarily mean $n_a X_m^{a^o} > n_b X_m^{b^o}$ or that $n_a X_m^{a'} > n_b X_m^{b^o}$.

[f] It is possible after the policy that B could dominate A. If such should be the case then no price changes will alter the situation.

[g] It depends on initial relationship between $X_m^{a'}$ and $X_m^{b^o}$, which could be either way depending on the restriction, K_a.

[h] Depending on the relationship of $X_m^{a'}$ and $X_m^{b^o}$ and the after policy cost situation, these price changes could induce a switch back or not affect the situation. Dominance of B over A is possible after the tax, which implies insensitivity to price changes for X_m.

[i] Before this policy will work, it must be that $n_a X_m^{a^o} > n_b X_m^{b^o}$ or $n_a X_m^{a'} > n_b X_m^{b^o}$; consequently, if a change occurs X_n will decline.

[j] Although $v_a > v_b$, $X_m^{a^o} < X_m^{b^o}$ or $X_m^{a'} < X_m^{b^o}$; therefore, no conclusion without further assumptions can be reached.

[k] Summary is for the case where conversion of X_n to X'_n requires variable costs, and some of factor K. Also note that under effects on X_n, volume of X'_n is relevant.

[l] Indeterminant since it is not possible to determine the relationship between $X_m^{a^o}$ and $X_m^{b'}$ or $X_m^{a'}$ and $X_m^{b'}$.

[m] It is possible that after the policy $X_m^{a''} \gtrless X_m^{b'}$; consequently, under proper conditions price increases or decreases could induce a switch back. If $X_m^{b'} > X_m^{a''}$, dominance of B over A is possible.

[n] Subsidy size could be such that more X_m and X_n are produced after the policy than before. Results summarized here are for instances where only a switch to method B occurs without changing X_m production beyond $X_m^{b^o}$.

or changes to technique B are shown in Table 4. 1. It should also be noted that the summarized effects are only "possible" effects. Usually when a "change" was possible it was also possible that "no change" could occur depending mainly on the level of the tax, VC_n, or subsidy.

In many instances policy effects on different models appeared the same. However, tax or subsidy size required for a change which is not reflected in Table 4. 1, was usually different. For example Model 3. 4 in which method B dominated A in the short run required a smaller subsidy or tax to induce a technique switch compared to Model 3. 3 where A dominated B.

Table 4. 1 also does not indicate the negative aspects of the subsidy policies. Subsidizing variable or fixed factors could result in increases in X_n production due to increased size of operations. The conclusion is that care needs to be used in applying subsidies so that they do not become large enough for the firm to grow. It was also found for Model 3. 1' that the subsidy could be too small. It was shown that a small subsidy could induce the firm to add production by technique A. A larger subsidy was needed to prompt a switch completely to method B. Even though in all instances subsidies could result in changes to technique B, the sensitivity of size would make it questionable as a good policy for reducing X_n. However, it appears that subsidizing fixed factors may be more practical

than variable factors. It should be easier to determine a subsidy large enough to prompt firms to buy the needed fixed factor for switching techniques without increasing operating size compared to variable factors. Then too, fixed factor subsidies should not result in outlays for subsidies each year.

It is shown in Table 4.1 that the effects of most policies for most models on externality production was indeterminant. The main implication of this indeterminancy is that the policy agency must know whether total externality production is likely to be decreased by a change to another technique.

Taxing the externality itself did not result in that problem. For the latter policy to result in a method change it was necessary that X_n produced by A be greater than that produced by B. Consequently, taxing the externality (effluent) appears to be the policy with most consistent results and least informational requirements as far as volume of X_n is concerned. However, in instances where the source of the externality is not easily identifiable taxing it could be difficult if not impossible to administer. Continual monitoring also seems necessary for this policy.

The policy of taxing a variable factor used more in total by the existing technique also produced consistent results quite similar to the externality tax. However, information requirements appear to be higher. First, it must be determined that alternative techniques that generate less total X_n are available. Then a variable factor used more in total by the existing method must be found. The only times taxing a variable factor resulted in changes were when more V was used by method A. That is the reason that use of V always is shown as declining in Table 4.1, when a change occurred. Provided that determination of the two facts above can be made relatively easy, taxing a variable factor might serve as a good substitute for taxing X_n.

Taxing X_m was the only policy that could not induce a technique change for all ten models. It was necessary for a change with this policy that more X_m be produced with the initial method. When a change did occur externality production did decline. The inconsistency in inducing changes seems to be the largest disadvantage for taxing X_m.

Placing a standard on the quality of X_n resulted in X'_n being released instead of X_n. It was also possible that technique changes could occur with this policy. The main problem with this type of policy appears to be administration. It is necessary to be able to identify externality sources. Monitoring coupled with strong enforcement also seem to be prerequisites for effective use of standards.

All policies for at least some models were subject to after the fact neutralization by price changes. The possibility for changes back to method A were most prevalent for declining prices of the market product. This phenomena indicates that there may be less need for flexible taxes or subsidies for products whose prices seem to be going up. However, that conclusion is somewhat weak without further tests of the policies. These results are in no small part due to the peculiarities of these particular models.

The "prime" models in this chapter were initially constrained by the specialized fixed factor. For policies other than subsidies these models required significant returns from resale of the specialized fixed resource if a technique change was to occur. Consequently if production is being restricted by a specialized factor the policy agency needs to consider the salvage value of such resources. If salvage value is essentially zero there may be a need in some instances to use a fixed factor subsidy in conjunction with a tax.

V. POLICY EFFECTS ON MODEL 4--TWO MARKET PRODUCTS UTILIZING TWO NON-SPECIALIZED FIXED FACTORS

All previous models considered only one market product, X_m. The present model will involve two market products which both produce the same externality, X_n, in different proportions.

As with Model 3, changes in certain assumptions and relationships will generate submodels. Variable costs of each technique, the relative use of fixed resources by technique, and the relative production of X_n are the main assumptions that are altered.

The following assumptions are relevant for all models in this chapter.

1. Production of the two market products, X_{m1} and X_{m2}, by technique A will generate relatively more of the nonmarket externality, X_n, than production by technique B. Furthermore the reduction in X_n production associated with switching from A to B is proportionately the same for both X_{m1} and X_{m2}.
2. There are two non-specialized fixed factors. These factors are assumed to be invariant over the time span considered, (except when subsidized). Both factors are used to produce each product.

3. The techniques maintain their same relative position in terms of variable costs between products. That is, if variable costs of producing one product by Technique A are lower than by B, they will also be lower for producing the other product with technique A. This will be relaxed when policies are applied.
4. The relative price of X_{m1} and X_{m2} is assumed to remain constant throughout the analysis. The variable costs will also remain constant except between methods.
5. Initially it will be assumed that whichever technique is used to produce one product will also be used to produce the other product.
6. Jointly produced with one product is relatively more of the externality X_n than with the other product.

The outcome of various policies may be different depending on the initial firm position. However, not all initial points possible will be considered. It will generally be assumed that the firm starts with technique A producing X_{m1}. A few of the other possible points will be considered to see if policy results change.

Specification and Policy Applications for Model 4.1

It is conceivable that one technique may dominate another with respect to the use of both fixed resources. Such a situation is depicted in Figure 5.1 where method A dominates method B. Notice that F_1 and F_2 refer to the resource constraints. The superscripts (A and B) refer to the technique associated with this particular constraint. In other words F_1^A and F_1^B represent the same amount of F_1 but different techniques; thus, they appear similar to two constraints. The dominance arises since technique B requires relatively more of both fixed inputs to produce X_{m1} and X_{m2}. Also notice that the fixed constraints intersect one another; if they did not, the initial conditions would be similar to Models 1 and/or 2.

A gross revenue line could be shown on Figure 5.1 by assuming something about the relative prices of X_{m1} and X_{m2}. This line would be the same for both techniques. Unfortunately a gross revenue line would not diagramatically show the net revenue maximization point; therefore, net revenue lines seem more relevant. Introduction of net revenue brings with it added complications unless the variable costs are the same between techniques. Rather than generate several more models by changing the assumptions concerning variable costs, this assumption will be altered within each model.

If $VC_a < VC_b$ for both products then in Figure 5.1 the profit maximizing firm will be utilizing only technique A. Depending on the relative positions of the per unit net revenue of each product the firm will be producing at A_1, A_2, or A_3. If $VC_a > VC_b$ for both products the situation becomes more complex. Under the latter

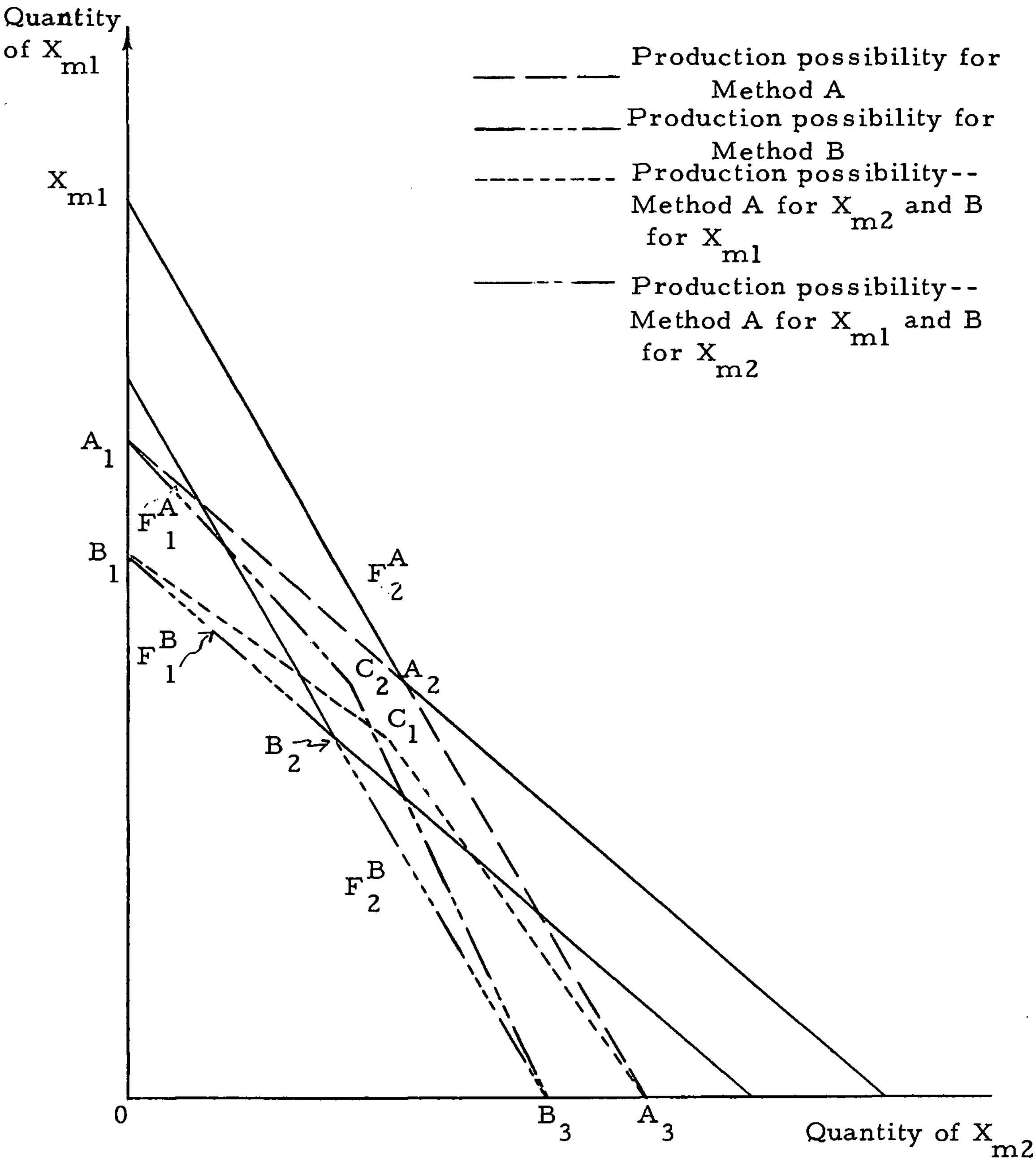

Figure 5. 1. Production possibilities, Model 4. 1--Dominance of technique A over B

assumption it is possible that the net revenue at any of the points B_1, B_2, or B_3 could exceed that at A_1, A_2, or A_3. It is impossible to tell what point is the profit maximization without additional assumptions. When the various policies are applied the relevant assumptions concerning this latter point will be made.

By assumption the firm is initially using method A which implies the following:

$$(P^o_{x_{m1}} - VC_{a1})\, X^a_{m1} + (P^o_{x_{m2}} - VC_{a2})\, X^a_{m2} > (P^o_{x_{m1}} - VC_{b1})\, X^b_{m1} + (P^o_{x_{m2}} - VC_{b2})\, X^b_{m2} \qquad (4.1.1)$$

where VC_{a1} is the variable costs for producing one unit of X_{m1} with technique A, etc. If it is assumed that $VC_{a1} < VC_{b1}$ and $VC_{a2} < VC_{b2}$ then (4.0.1) is a sufficient initial condition. However, if $VC_{a1} > VC_{b1}$ and $VC_{a2} > VC_{b2}$ then other initial conditions must be set. With the latter variable cost assumptions one must also assume

$$(P^o_{x_{m1}} - VC_{a1})\, X^a_{m1} + (P^o_{x_{m2}} - VC_{b2})\, X^b_{m2} < (P^o_{x_{m1}} - VC_{a1})\, X^a_{m1} + (P^o_{x_{m2}} - VC_{a2})\, X^a_{m2} \quad \text{and} \qquad (4.1.2)$$

$$(P^o_{x_{m1}} - VC_{b1})\, X^b_{m1} + (P^o_{x_{m2}} - VC_{a2})\, X^a_{m2} < (P^o_{x_{m1}} - VC_{a1})\, X^a_{m1} + (P^o_{x_{m2}} - VC_{a2})\, X^a_{m2}.$$

Condition (4. 1. 2) states that it is more profitable to operate exclusively with technique A than with a combination of techniques. Note that in (4. 1. 1) and (4. 1. 2) it is possible that either X^a_{m1} or X^a_{m2} could equal zero. Even if some do equal zero, conditions (4. 1. 1) and (4. 1. 2) are still relevant since they must hold for all possible values of the product variables (i. e., X^a_{m1}, X^b_{m1}, etc.).

Taxing the Market Products

Suppose that initially the firm is producing only X_{m1} with technique A (point A_1 in Figure 5. 1). Furthermore, assume that more X_n is produced per unit of X_{m1} than is produced per unit of X_{m2} and that more total X_n is produced at A_1 than if production were at any other of the production possibilities shown in Figure 5. 1.

Based on the policy criterion discussed in Chapter II, there are two extremes in possible outcomes with several "second best" type situations. The least ideal outcome would be if the policy left the situation where it is initially, i. e., the highest externality method being used to produce the highest externality generating product. The "best" outcome then would be if the firm were induced to switch to product X_{m2} using technique B. The "second best" alternatives would be producing X_{m1} with technique B, X_{m2} with method A, some X_{m1} and some X_{m2} with technique A or some X_{m1} and some X_{m2} with method B. If assumption five is relaxed then two more "second best" solutions are possible--producing some X_{m1} with method A and some X_{m2} with method B or producing some X_{m1} with method B and some X_{m2} with technique A. Any of the above positions would be preferred to the initial one.

Suppose the tax is placed on X_{m1}. The initial position is

$$(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_o > (P^o_{x_{m1}} - VC_{b1})X^b_{m1} + (P^o_{x_{m2}} - VC_{b2})X^b_{m2}, \quad (a)$$

$$(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_o > (P^o_{x_{m2}} - VC_{b2})X^b_{m2} + (P^o_{x_{m1}} - VC_{a1})X^a_{m1}, \quad (b)$$

$$(4.1.3)$$

$$(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_o > (P^o_{x_{m1}} - VC_{b1})X^b_{m1} + (P^o_{x_{m2}} - VC_{a2})X^a_{m2} \quad (c)$$

and

$$(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_o > (P^o_{x_{m1}} - VC_{a1})X^a_{m1} + (P^o_{x_{m2}} - VC_{a2})X^a_{m2} \quad (d)$$

In Figure 5.1 the beginning point is A_1. If $VC_{a1} \leq VC_{b1}$ then a tax on X_{m1} cannot induce the firm to move to point B_1 (producing X_{m1} entirely with technique B). Furthermore, if $VC_{a2} \leq VC_{b2}$ at the same time then the tax cannot induce the firm to produce anything with technique B. The best the policy can do is induce the firm to switch to one of the second best points, A_2 or A_3.[43] In other words

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[43] This is so since for any point such as B_2 or B_3 that is common to the possibility sets of both A and B, higher net revenue can be achieved using method A. For example,

$$(P^o_{x_{m1}} - VC_{a1})\left(X^a_{m1}\right)_1 + (P^o_{x_{m2}} - VC_{a2})\left(X^a_{m_2}\right)_1 > (P^o_{x_{m1}} - VC_{b2})\left(X^b_{m1}\right)_1$$

under the assumed conditions the firm will continue to operate with the same basic technique; however, it may produce at least in part a different product. A tax only on X_{m2} will not induce the firm to move from its initial point.

By utilizing the same assumptions and initial conditions similar to (4.1.3) except assuming that the firm starts at A_2 (producing only X_{m2} with method A) it can be shown that a tax only on X_{m2} could degrade rather than improve the situation. The degradation occurs since the firm may then start producing some X_{m1} which has been assumed to generate more X_n than generated by X_{m2}.

Taxing both X_{m1} and X_{m2} is less likely to cause the firm to move from its initial point than taxing either product by itself. A tax proportional to the net revenues of each product will cause a parallel shift in the net revenue lines thus not influencing the optimum production point.

Suppose that $VC_{a1} \leq VC_{b1}$ but $VC_{a2} > VC_{b2}$. Furthermore, assume that (4.1.3) holds. Under these assumptions it is not as easy to determine what the tax effects might be. The conditions (4.1.3) imply that

$$(P^o_{x_{m1}} - VC_{a1}) \left(X^a_{m1}\right)_o > (P^o_{x_{m1}} - VC_{b1}) \left(X^b_{m1}\right)_o \qquad (4.1.4)$$

$+ (P^o_{x_{m2}} - VC_{b2}) \left(X^b_{m2}\right)_1$ where $\left(X^a_{m1}\right)_1 = \left(X^b_{m1}\right)_1$ and $\left(X^a_{m1}\right)_1 = \left(X^b_{m2}\right)_1$ as represented by point B_2 in Figure 5.1.

where $\left(X^a_{m1}\right)_o$ is equivalent to point A_1 and $\left(X^b_{m1}\right)_o$ to point B_1 in Figure 5. 1. As can be observed a tax on X_{m1} only cannot alter the direction of (4. 1. 4) unless (4. 1. 4) is permitted to become negative. A negative number in (4. 1. 4) would imply that the firm would be forced out of business. Of course a tax on X_{m2} will not affect (4. 1. 4) so the firm would stay at its initial position. The result is that B_1, i. e., production of X_{m1} only by technique B, is eliminated as a possible production point after a tax on X_{m1}.

What about the other possibility points? Point B_3 was proposed as the optimum policy point. The initial conditions imply

$$(P^o_{x_{m1}} - VC_{a1}) \left(X^a_{m1}\right)_o \geq (P^o_{x_{m2}} - VC_{b2}) \left(X^b_{m2}\right)_o \qquad (4.1.5)$$

where $\left(X^a_{m1}\right)_o$ corresponds to A_1 and $\left(X^b_{m2}\right)_o$ to B_3 in Figure 5. 1. A tax only on X_{m1} does raise the possibility that (4. 1. 5) could become

$$(P^o_{x_{m1}} - VC_{a1} - T_1) \left(X^a{}_{m1}\right)_o < (P^o_{x_{m2}} - VC_{b2}) \left(X^b_{m2}\right)_o \quad (4.1.6)$$

which implies a switch from technique A to B and a change in production from X_{m1} to X_{m2}. Whether or not the switch will actually be made to the optimum policy point is still indeterminant, however. Condition (4. 1. 6) is necessary and it must also be true that

$(P^o_{x_{m2}} - VC_{b2}) \left(X^b_{m2}\right)_o > (P^o_{x_{m2}} - VC_{a2}) \left(X^a_{m2}\right)_o$ where $\left(X^a_{m2}\right)_o$

corresponds to A_3 on Figure 5.1, and $(P^o_{x_{m2}} - VC_{b2}) \left(X^b_{m2}\right) o >$ (net revenue after tax from all other combinations of products and techniques). (4.1.7)

It is possible but very unpredictable that the tax could induce the firm to switch to any number of second best points. Whichever point becomes the optimum it will not be stable if the relative prices of X_{m1} and X_{m2} vary greatly. Without knowing magnitudes of prices and input coefficients, one cannot make any stronger a priori statements.

As before, a tax on both market products appears to hinder rather than help achieve the policy goal. A tax on X_{m2} would have no effect on the firm's initial position unless that position were at some point which involved X_{m2}. In such an instance the tax on X_{m2} could induce a change that would be undesirable from a policy viewpoint.

Summary and Implications

The effects of taxing one or both market products was dependent on the initial position of the firm and on the cost assumptions. If the firm were producing X_{m1} with method A and the tax were placed on X_{m1}, no technique change was possible provided $VC_{a1} \leq VC_{b1}$ and $VC_{a2} \leq VC_{b2}$. The tax on X_{m1} could, however, have induced a switch of products with the same technique (A) being used. Depending on the level of T_1, the firm might have produced some of both X_{m1} and X_{m2} or only X_{m2}. Either of these results reduced production of X_n. Under the conditions stated above a tax on X_{m2} did not result in any changes. Furthermore, a tax on both X_{m1} and X_{m2} appeared not likely to result in any changes either. The latter is so particularly if the tax is proportional to both products.

If $VC_{a1} \leq VC_{b1}$ and $VC_{a2} \leq VC_{b2}$ but the firm started at point A_3 (producing only X_{m2} by A), the results were a little different. First of all, a tax on X_{m1} resulted in no change. Secondly, a tax on X_{m2} might have induced the firm to change products but not techniques. In this instance, a product change would have been "bad" since X_{m1} generates more X_n than X_{m2}.

Taxing X_{m1} where $VC_{a1} \leq VC_{b1}$ and $VC_{a2} > VC_{b2}$ did not induce a technique change alone. However, such a tax could have persuaded the firm to switch techniques and products which would have been to the optimum policy point. The tax could conceivably have implied a change to any number of several "second best" points. If a technique and product switch resulted from the policy, a relative price change in favor of X_{m1} could induce a switch back to the original technique (A) and product (X_{m1}). Taxing X_{m2} again did not result in any switch if the initial firm position was at A_1 in Figure 5.1. If the firm started at A_3 a tax on X_{m2} could have induced a switch of products which would have increased production of X_n.

Taxing a Variable Factor

As before, the challenge with this policy seems to be choosing the proper variable factor to tax. With the present model a variable factor could be chosen that is product and/or technique oriented. A factor that is used relatively more by X_{m1} and technique A than by X_{m2} and technique B would be the ideal one to tax if the optimum policy point is to be achieved. With very little analysis it is not difficult to envision how a tax on the wrong variable factor would not decrease the production of X_n; furthermore, if the initial position of this firm happened to be somewhere other than A_1 the situation could be degraded.

Consider a case where $VC_{a1} < VC_{b1}$ and $VC_{a2} < VC_{b2}$. Also assume that the firm is producing X_{m1} with technique A. Let the factor to be taxed be represented by V_p for variable factor-product oriented. In other words, V_p requirements are the same for a given product no matter which technique is used. Assume that V_p is used relative more by X_{m1} than by X_{m2}, i.e., $v_{p1} > v_{p2}$ where v_{p1} and v_{p2} represent the amounts of V_p required to produce one unit of X_{m1} and X_{m2}, respectively.

Initial conditions are as shown in (4.1.3). After the tax is implemented (a) in (4.1.3) becomes

$$(P^o_{x_{m1}} - VC_{a1}) \left(X^a_{m1}\right)_o - T_{vp}\, v_{p1} \left(X^a_{m1}\right)_o \overset{?}{\substack{\geq \\ <}} (P^o_{x_{m1}} - VC_{b1})\, X^b_{m1}$$

$$+ (P^o_{x_{m2}} - VC_{b2})\, x^b_{m2} - T_{vp} (v_{p1}\, X^b_{m1} + V_{p2}\, X^b_{m2}). \qquad (4.1.8)$$

The statements b, c, and d take on similar changes. To imply a switch to production of some X_{m1} and X_{m2} by technique B it is necessary that (<) hold in (4.1.8). Rewriting and assuming the (<) holds, (4.1.8) becomes

$$T_{vp} (v_{p1} \left(X^a_{m1}\right)_o - v_{p1}\, X^b_{m1} - v_{p2}\, X^b_{m2}) > (P^o_{x_{m1}} - VC_{a1}) \left(X^a_{m1}\right)_o - (P^o_{x_{m1}}$$

$$- VC_{b1})\, X^b_{m1} - (P^o_{x_{m2}} - VC_{b2})\, X^b_{m2} \qquad (4.1.9)$$

$$\rightarrow T_{vp} > \frac{(P^o_{x_{m1}} - VC_{a1}) \left(X^a_{m1}\right)_o - (P^o_{x_{m1}} - VC_{b1})\, X^b_{m1} - (P^o_{x_{m2}} - VC_{b2})\, X^b_{m2}}{v_{p1} \left(X^a_{m1}\right)_o - v_{p1}\, X^b_{m1} - v_{p2}\, X^b_{m2}}$$

(4.1.10)

In words the tax per unit of the variable factor must exceed the net revenue advantage per unit of additional v_p required at point A_1 compared to some point such as B_2 in Figure 5.1. For T_{vp} to be positive it must be that the total variable factor used at A_1 is greater than that used at B_2. Otherwise T_{vp} will be negative which would imply a subsidy to bring about a switch.

A relevant question at this point is whether the tax on V_p is likely to induce a technique or a product switch. First of all is <u>any</u>

size tax likely to cause a switch from producing X_{m1} by A to producing X_{m1} by B? The following statement describes the situation

$$(P^o_{x_{m1}} - VC_{a1} - T_{vp} v_{p1}) \left(X^a_{m1}\right)_o \overset{?}{\substack{\geq \\ <}} (P^o_{x_{m1}} - VC_{b1} - T_{vp} v_{p1} \left(X^b_{m1}\right)_o.$$

(4. 1. 11)

It is known that $(P^o_{x_{m1}} - VC_{a1} - T_{vp} v_{p1}) > P_{x_{m1}} - VC_{b1} - T_{vp} v_{p1})$ since $VC_{a1} < VC_{b1}$. Also $\left(X^a_{m1}\right)_o > \left(X^b_{m1}\right)_o$, therefore, only the (>) can hold in (4. 1. 11). For a switch to $\left(X^b_{m1}\right)_o$ the (<) must hold; therefore, a tax on V_p will <u>not</u> induce a technique change <u>only.</u>

Is the tax likely to induce a switch to X_{m2} <u>and</u> technique B? If that be the case then

$$(P^o_{x_{m1}} - VC_{a1} - T_{vp} v_{p1}) \left(X^a_{m1}\right)_o < (P^o_{x_{m2}} - VC_{b2} - T_{vp} v_{p2}) \left(X^b_{m2}\right)_o.$$

(4. 1. 12)

There is nothing in the assumed conditions that would make (4. 1. 12) impossible since $v_{p1} > v_{p2}$. However, even if (4. 1. 12) holds, what about

$$(P^o_{x_{m2}} - VC_{b2} - T_{vp} v_{p2}) \left(X^b_{m2}\right)_o \overset{?}{\substack{\geq \\ <}} (P^o_{x_{m2}} - VC_{a2} - T_{vp} v_{p2}) \left(X^a_{m2}\right)_o.$$

(4. 1. 13)

Due to the assumed conditions only (<) will hold in (4. 1. 13) so even if (4. 1. 12) holds, (4. 1. 13) implies that X_{m2} will be produced by method A. The two points between the extremes (B_2 and A_2) will be affected similarly; consequently, the tax on V_p may induce a product change but not a technique change.

Suppose the variable cost assumptions are changed to the other extreme, i.e., $VC_{a1} > VC_{b1}$ and $VC_{a2} > VC_{b2}$. Then it is true that

$(P^o_{x_{m1}} - VC_{a1} - T_{vp} v_{p1}) < (P^o_{x_{m1}} - VC_{b1} - T_{vp} v_{p1})$, so that it is

possible for the (<) to hold in (4.1.11), which would imply a switch of techniques. However, before a technique switch is implied for sure, the following question must be answered:

$$(P^o_{x_{m1}} - VC_{b1} - T_{vp} v_{p1}) \left(X^b_{m1}\right) o \overset{?}{\substack{\geq \\ <}} (P^o_{x_{m2}} - VC_{a2} - T_{vp} v_{p2}) \left(X^a_{m2}\right) o. \tag{4.1.14}$$

Again, nothing in the assumptions sheds light on (4.1.14). It appears that any of the three possibilities could occur. It is also possible that the (>) could hold in (4.1.13) implying a change of both products and techniques.

Intermediate variable cost assumptions as expected result in a combination of the first two outcomes. Assume $VC_{a1} < VC_{b1}$ but $VC_{a2} > VC_{b2}$. The tax on V_p will not induce a switch to producing X_{m1} by technique B since it is impossible for the (<) to hold in (4.1.11). However, it is possible that (4.1.12) could hold and the (>) in (4.1.13) which would imply a switch of products and techniques. It is conceivable that one of the intermediate points involving a combination of products and techniques could be the net revenue maximizing point. It is not possible to make a general statement concerning the latter point since whether or not a combination is optimal depends on price relationships and requirements for fixed resources as well as variable costs. It does seem likely that if a combination of techniques and products is the net revenue maximum it will be technique A for X_{m1} and B for X_{m2}. The last statement is made since the per unit net revenue from X_{m1} will be highest if A is used but per unit net revenue will be highest for X_{m2} if B is used.

Another possibility for taxing is a variable factor that is technique specific, V_T. Assume that V_T is used more heavily by technique A than by technique B or $v_{ta} > v_{tb}$. Assume the same initial conditions as shown in (4.1.3) and that $VC_{a1} \leq VC_{b1}$ and $VC_{a2} \leq VC_{b2}$. After implementation the (a) of (4.1.3) becomes

$$(P^{o}_{x_{m1}} - VC_{a1}) \left(X^{a}_{m1}\right) o - T_{vt}\, v_{ta} \left(X^{a}_{m1}\right) o \overset{?}{\substack{\geq \\ <}} (P^{o}_{x_{m1}} - VC_{b1})\, X^{b}_{m1}$$

$$+ (P^{o}_{x_{m2}} - VC_{b2})\, X^{b}_{m2} - T_{vt}\, v_{tb}\, (X^{b}_{m1} + X^{b}_{m2}). \qquad (4.1.15)$$

The other statements in (4.1.3) take on similar changes.

Will the tax on V_T induce a change of technique and/or products? First examine the technique change. If only a technique change is implied

$$(P^o_{x_{m1}} - VC_{a1} - T_{vt}\, v_{ta}) \left(X^a_{m1}\right)o < (P^o_{x_{m1}} - VC_{b1} - T_{vt} v_{tb}) \left(X^b_{m1}\right)o. \tag{4.1.16}$$

Since $v_{ta} > v_{tb}$ it is possible that (4.1.16) could hold. If (4.1.16) holds then it implies that

$$T_{vt} > \frac{(P^o_{x_{m1}} - VC_{a1}) \left(X^a_{m1}\right)o - (P^o_{x_{m1}} - VC_{b1}) \left(X^b_{m1}\right)o}{v_{ta} \left(X^a_{m1}\right)o - v_{tb} \left(X^b_{m1}\right)o} \tag{4.1.17}$$

In words the per unit tax (cost of not switching) must be greater than the net revenue lost per unit of reduced variable factor, V_T, by switching (return from not switching).

Is the tax likely to induce only a product change? If so, it is necessary (not sufficient) that

$$(P^o_{x_{m1}} - VC_{a1} - T_{vt} v_{ta}) \left(X^a_{m1}\right)o < (P^o_{x_{m2}} - VC_{a2} - T_{vt} v_{ta}) \left(X^a_{m2}\right)o \tag{4.1.18}$$

The assumed conditions do <u>not</u> rule out the possibility of (4.1.18) since it was assumed only that total net revenue at $\left(X^a_{m1}\right)o$ is greater than net revenue at $\left(X^a_{m2}\right)o$. It is also possible that

$$(P^o_{x_{m2}} - VC_{a2} - T_{vt} v_{ta}) \left(X^a_{m2}\right)o < (P^o_{x_{m2}} - VC_{b2} - T_{vt} v_{tb}) \left(X^b_{m_2}\right)o \tag{4.1.19}$$

since $v_{tb} < v_{ta}$. Statements (4.1.18) and (4.1.19) would be part of the necessary conditions for achieving the optimum policy point with this tax. Now assume that $VC_{a1} > VC_{b1}$ and $VC_{a2} > VC_{b2}$. With these assumptions the tax on V_T could induce product and/or technique switches. If prior to policy implementation the relationship among various points is known, then something about tax magnitude required to move the firm to various points can be stated.[44]

One would expect a variable factor that was both technique and product specific to be the most likely prospect for taxing provided

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[44] For example assume that initially $(P^o_{x_{m1}} - VC_{a1})\left(X^a_{m1}\right)_o > (P^o_{x_{m1}} - VC_{b1})\left(X^b_{m1}\right)_o > (P^o_{x_{m2}} - VC_{a2})\left(X^a_{m2}\right)_o > (P^o_{x_{m2}} - VC_{b2})\left(X^b_{m2}\right)_o$. (1)

A tax to move the firm from producing X_{m1} with A to X_{m1} with B must be such that

$$T_{vt} > \frac{(P^o_{x_{m1}} - VC_{a1})\left(X^a_{m1}\right)_o - (P^o_{x_{m1}} - VC_{b1})\left(X^b_{m1}\right)_o}{v_{ta}\left(X^a_{m1}\right)_o - v_{tb}\left(X^b_{m1}\right)_o} = T^o_{vt} \quad (2)$$

whereas a tax to induce the firm to produce X_{m2} with technique A instead of X_{m1} by method A must be such that

$$T_{vt} > \frac{(P^o_{x_{m1}} - VC_{a1})\left(X^a_{m1}\right)_o - (P^o_{x_{m2}} - VC_{a2})\left(X^a_{m2}\right)_o}{v_{ta}\left(X^a_{m1}\right)_o - v_{ta}\left(X^a_{m2}\right)_o} = T'_{vt} \quad (3)$$

From (1) it can be determined that the numerator of T'_{vt} is larger than the numerator of T^o_{vt}. One would expect the denominator of T'_{vt} to be smaller than the denominator of T^o_{vt} (although it does not <u>have</u> to be so). Consequently $T'_{vt} > T^o_{vt}$ which implies a larger tax for bringing about a product change versus a technique change.

the specificity for both factors favored the optimum policy point. Assume that V_D is a variable factor that is used relatively more by method A and by product X_{m1} versus method B and X_{m2}. That is $v_{da1} > v_{db1}$, $v_{da2} > v_{db2}$, $v_{da1} > v_{da2}$, and $v_{db1} > v_{db2}$ where v_{da1} represents the amount of V_D required to produce one unit of X_{m1} with technique A, etc. From the previous inequalities it can be determined that v_{da1} is the largest coefficient and v_{db2} the smallest.

Briefly the necessary and sufficient conditions for achieving the optimum policy point are that the net revenue, after the tax, for producing X_{m2} by technique B must exceed the net revenue from all other alternatives. Also the net revenue must be greater than zero.

Using reasoning as established with V_P and V_T it can be demonstrated that under all circumstances this policy could induce a technique and product switch. Without specific coefficients it is not possible to prove whether the tax on V_D is more or less likely to lead to the policy optimum than taxes on either V_P or V_T; however, intuitively it is logical. For one thing there are differences in coefficients for all comparisons whereas with V_P and V_T there were differences in variable requirements only when products or techniques differed. Consequently, with V_D there is a better chance of inducing disproportional changes in variable costs thus inducing technique and product changes.

Depending on the prevailing conditions the policy results may be sensitive to product price changes. The relative per unit net revenue of the products and techniques is quite crucial in making the sensitivity analysis. For example if after the tax per unit net revenue for producing X_{m2} by method B is the highest and if it is also the net revenue maximizing alternative then an equal absolute drop in all prices will not affect the situation.[45] That is $\left(X^{b}_{m2}\right)_{o}$ would still be the optimum. However, a rise in price of X_{m2} only, could cause the firm to switch to another alternative since $\left(X^{b}_{m2}\right)_{o}$ is less than $\left(X^{a}_{m2}\right)_{o}$. Certainly a decline in the price of X_{m2} while the price of X_{m1} stayed constant (or a rising of X_{m1} faster than X_{m2}) could induce switches back to other alternatives.

Summary and Implications

To summarize, the policy results could be sensitive to changes in both absolute and relative prices of the market products. Without knowledge of specific coefficients and values it is not possible to predict the amount of sensitivity to prices and/or the types of price movements which will induce switches back. A tax on a product specific variable factor was less likely to induce technique changes than was a tax on a technique specific[46] variable factor. If

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[45] Of course if prices dropped far enough the firm would be be forced to shut down.

[46] This statement is made since under certain assumptions the tax on V_P was shown not to induce a technique switch no matter how large.

technique is the critical element in controlling externality production then it will be better to search for a technique specific variable. A tax on the technique specific variable could induce a change in products also. It was not possible to compare whether or not taxing V_P rather than V_T was more or less likely to result in a product change.

Taxing the Non-market Externality

As before assume initial starting position is A_1 in Figure 5.1. Also assume that $VC_{a1} \leq VC_{b1}$ and $VC_{a2} \leq VC_{b2}$. Initial conditions are represented by (4.1.3).

Since it has been assumed that more X_n is produced at A_1 than at any of the other possibility points it is possible the tax on X_n could induce a switch from A_1. After the tax all statements in (4.1.3) are altered similarly as (a) shown here

$$(P^o_{x_{m1}} - VC_{a1} - T_n n_{a1}) \left(X^a_{m1}\right)_o \overset{?}{\underset{<}{\geq}} (P^o_{x_{m1}} - VC_{b1} - T_n n_{b1} +$$

$$(P^o_{x_{m2}} - VC_{b2} - T_n n_{b2}) X^b_{m2} \qquad (4.1.20)$$

where n_{a1} represents the amount of X_n produced by technique A per unit of X_{m1}, n_{b2} the amount of X_n produced by B per unit of X_{m2}, etc. All that is necessary for a switch from point A_1 is for the inequality of any of the altered statements in (4.1.3), a, b, c, or d, to be (<).

If after the tax all statements are (<) then one must look further to determine at what point net revenue maximization will occur. If technique B is to be used at all it is necessary that

$$(VC_{a1} + T_n n_{a1}) > (VC_{b1} + T_n n_{b1})$$

$$\text{and/or} \qquad (4.1.21)$$

$$(VC_{a2} + T_n n_{a2}) > (VC_{b2} + T_n n_{b2}).$$

Condition (4. 1. 21) is necessary since A dominates B.

It is also conceivable that this policy could induce the firm to switch market products or at least to start producing some X_{m2}. A priori it is not possible to determine which point the firm will move to. Necessary and sufficient conditions for a switch to the optimum policy point, however, can be delineated.[47]

If the firm starting position happens to be at A_3 (producing only X_{m2} by technique A) the tax on X_n will at least not degrade the situation. Under these circumstances it is possible that the optimum policy point may still be achieved.

Now assume that $VC_{a1} \leq VC_{b1}$ and that $VC_{a2} > VC_{b2}$. The only difference that this assumption is going to make is that it is more

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[47] See Appendix A, Part 3.

likely that a switch from X_{m1} production with method A will include production of X_{m2} by technique B.[48] Even if the assumption is that $VC_{a1} > VC_{b1}$ and $VC_{a2} > VC_{b2}$ and initial starting position is the same the analysis will not be altered. The necessary and sufficient conditions for the optimal policy conditions are still as shown in Appendix A, Part 3.

The results of this policy are not likely to remain stable in the face of relative price changes of the market products. If the price of X_{m2} should fall relative to the price of X_{m1}, the firm conceivably could change back to the original starting position. It is fruitless to set out the conditions necessary for a switch back to each possible point; consequently, that detail is omitted. If X_{m2} should go up relative to X_{m1} then the firm possibly would switch back to producing X_{m2} with technique A; however, there would be no inducement for producing any X_{m1} by either technique.

Summary and Implications

This policy could have induced a switch to the optimum policy point. This statement is true whether the firm started at point A_1 or A_3. If the firm happened to be at the best point (i. e., B_3), the policy would not prompt any change from it.

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[48] See Appendix A, Part 4.

A fall in the price of X_{m2} relative to X_{m1} could result in a change from the best point to the original. Conceivably a rise in the relative price of X_{m2} compared to X_{m1} could result in a technique switch back to A. A change in products would not occur with the latter price movements.

A Standard on the Quality of the Externality

Assume that a standard on the externality is placed such that X_n does not meet the standard. The policy agency has enforcement power strong enough to force the firm either to alter X_n or cease operations.

This problem could be handled in at least two ways. One way would be to assume that there exist other techniques capable of producing X_{m1}, X_{m2}, and X'_n where X'_n would meet the standard. Another way would be to consider the same basic techniques but with alterations which would permit conversion of X_n to X'_n. There is little difference between the two approaches. The difference that does exist is with respect to emphasis in the models. The first approach emphasizes different techniques for producing externalities, while the latter approach emphasizes different techniques for producing the market products. Of course, if for the first approach the basic techniques, A and B could produce only X_n, the policy being considered would immediately make A and B infeasible. The second approach in a sense involves different techniques since A and B are modified. However, the emphasis is still on these two techniques and how they will be modified by the policy. The second approach is the one that will be used.

With the introduction of more fixed factors and products treatment of this policy becomes more complex. First of all, if it is assumed that only variable factors are required to convert X_n to X'_n then the analysis can proceed similar to the preceding section. The main difference is that instead of a controllable (by policy agency) factor, T_{vp} or T_{vt}, an uncontrollable element, VC_n, will be used. By substituting VC_n for T_{vp} or T_{vt} and n_{a1} for v_{p1} and/or v_{ta} depending on technique or product, etc., in the previous section the analysis for this part can be conducted.

If the assumption is made that conversion of X_n to X'_n requires in addition to variable factors, equi-proportional amounts of each fixed factor, F_1 and F_2, for all volumes of X_n, then the analysis again will be quite similar to the preceding section. The only difference will be that the maximum amounts of market products that can be produced will be less than before given the same levels of F_1 and F_2. The resource constraints will make parallel shifts to the left in Figure 5.1.

Now suppose the conversion of X_n to X'_n requires disproportionate amounts of F_1 and F_2. One way of showing this would be for the conversion to require some of F_1 and none of F_2. Also assume that X_{m1} and X_{m2} both jointly produce X_n. Furthermore, X_n is produced relatively more by technique A than by B and relatively more by X_{m1} than X_{m2}. The amount of F_1 has not changed but the quantity that can be devoted to production of market products has decreased. Also assume that the amount of F_1 required is proportional to the volume of X_n to be processed, i. e., f_{1n} is the amount of F_1 required to convert one unit of X_n to X'_n . Notice that $(X^a_m)_{-1}$ in Figure 5.2 is less than $\left(X^a_{m1}\right)_o$ since less F_1 is available for X^a_{m1} production.[49] It will also be true that the maximum

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[49] $F_1^o = f_{1n} n_{a1} (X^a_{m1})_{-1} + f_{1a1} (X^a_{m1})_{-1}$ where f_{1a1} is the amount of F_1 required to produce one unit of X_{m1} by technique A.

$$F_1^o = f_{1n} n_{a1} \frac{F'_1}{f_{1a1}} + f_{1a1} \frac{F'_1}{f_{1a1}}, \text{ since } (X^a_{m1})_{-1} = \frac{F'_1}{f_{1a1}}$$

$$= F'_1 \; (f_{1a1} + f_{1n} n_{a1}) \quad f_{1a1}$$

$$\rightarrow F'_1 = F_1^o \left\{ \frac{f_{1a1}}{f_{1a1} + f_{1n} n_{a1}} \right\} < F_1^o \quad \text{since } f_{1a1}, f_{1n},$$

$na_1 > 0$.

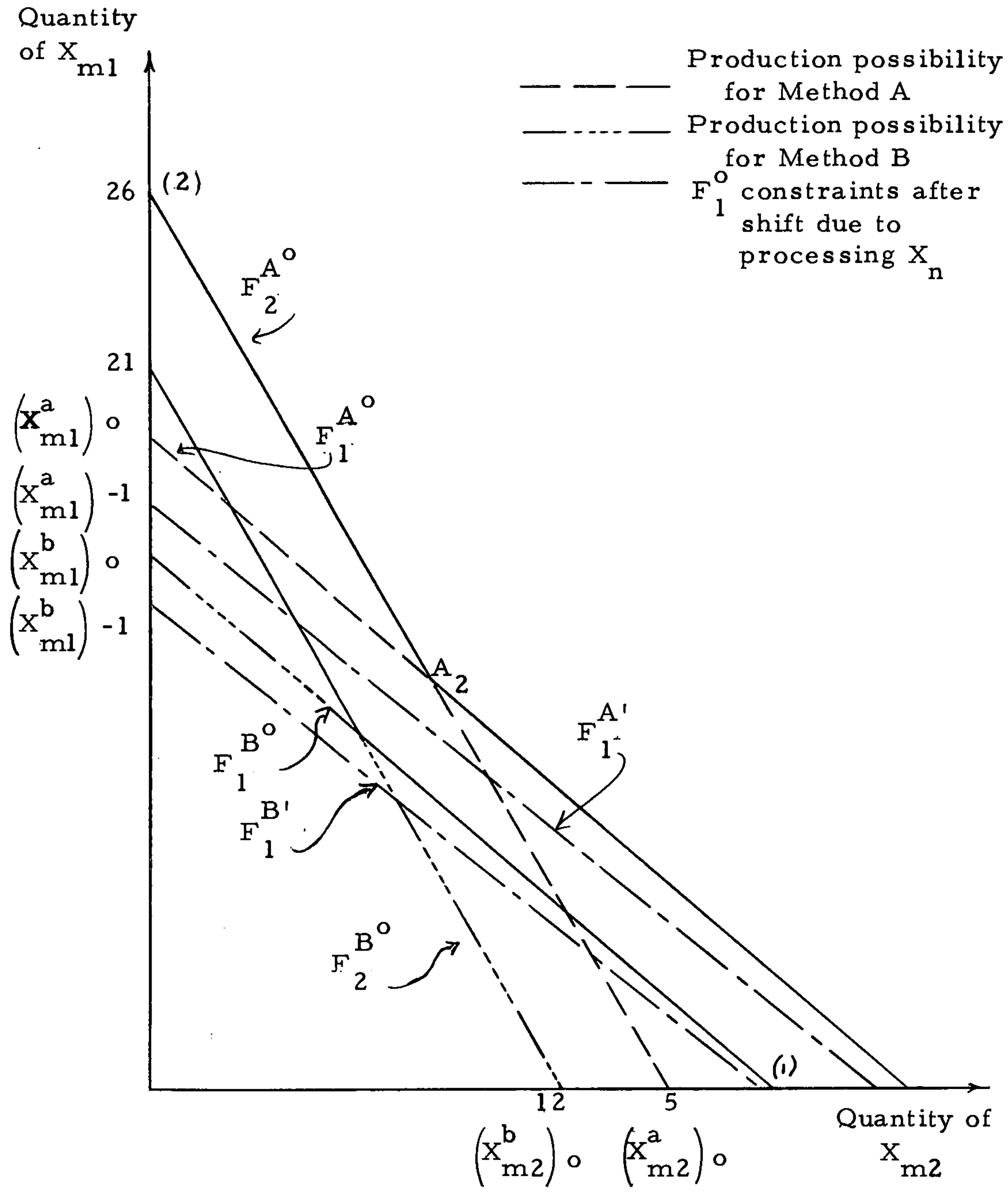

Figure 5.2. Factor shifts due to mandatory standards--Model 4.1

X_{m2} as constrained by F_1 is reduced since some F_1 will be needed to process X_n. However X^a_{m2} will not be reduced proportionately as much as X^a_{m1} since less X_n is produced with X_{m2}. As a consequence the F^A_1 constraint shift to the left is not a parallel shift.

The F_1 constraint will also shift to the left for technique B but relatively not as much as for technique A. By assumption the amount of X_n produced by technique B is less than by technique A both totally and per unit of X_{m1} and X_{m2}; consequently, less F_1 is required to convert X_n to X'_n. Again the shift will not be parallel since less X_n is produced jointly with X_{m2} than with X_{m1}. The types of shifts that could be expected are shown by the dashed lines in Figure 5. 2. Notice that after the shifts the difference in maximum amounts of X_{m1} that can be produced by the two techniques is reduced. That is $(X^a_{m1})_{-1} - (X^b_{m1})_{-1} < (X^a_{m1})_o - (X^b_{m1})_o$.
Since F_2 is the relevant constraint for producing X_{m2} the maximums for X_{m2} are not affected.

Assume that initially the firm's position is represented by (4. 1. 3). After the standard is enforced (4. 1. 3), (a), will appear as

$$(P^o_{x_{m1}} - VC_{a1} - VC_n n_{a1})(X^a_{m1})_{-1} \overset{?}{\gtrless} (P^o_{x_{m1}} - VC_{b1} - VC_n n_{b1}) X^b_{m1}$$

$$+ (P^o_{x_{m2}} - VC_{b2} - VC_n n_{b2}) X^b_{m2} . \qquad (4. 1. 22)$$

The other statements in (4. 1. 3) are altered similarly. Whether or not the firm switches techniques and/or products is fairly irrelevant as far as the standard is concerned because X'_n will be released no matter what is produced or how. Unless the standard is set at a zero effluent rate or very high quality level there still is the chance that the standard could do better than just insure release of X'_n without forcing the firm from business. If the amount of X'_n is reduced then total stream loading could be reduced thus helping even more.

Does the possibility of a technique and product switch exist for the present policy? If so, the following must hold.

$$(P^o_{x_{m1}} - VC_{a1} - VC_n n_{a1})(X^a_{m1})_{-1} < (P^o_{x_{m2}} - VC_{b2} - VC_n n_{b2})(X^b_{m2})_o. \tag{4. 1. 23}$$

Statement (4. 1. 23) can hold since compared to the initial position per unit net revenue on the left is reduced more than the per unit net revenue on the right (since $n_{a1} > n_{b2}$). Furthermore, $(X^a_{m1})_{-1}$ is less than $(X^a_{m1})_o$ whereas $(X^b_{m2})_o$ has not changed. The various assumptions concerning relative size of variable costs will not change the conclusions.

If the firm starts at any of the other possible positions it is likely to switch to a less externality producing point. The latter is so since $n_{a1} > n_{a2} > n_{b2}$ and $n_{a1} > n_{b1} > n_{b2}$. The relationship between n_{b1} and n_{a2} is not clear and the assumed conditions say nothing about it. In any case, since VC_n will decrease the revenue from the higher externality producing point the most, it is not likely a switch to a higher point will result.

Summary and Implications

If only variable factors were required to convert X_n to X'_n the results of the standard were similar to those resulting from taxing X_n. In other words, a technique and product switch could be induced. If the firm started from any of the various positions, the standard would not result in an increase in X_n. After the policy price changes were similar to the previous section.

It was also possible for the optimum policy point to be achieved if conversion of X_n to X'_n required a disproportionate amount of F_1 compared to F_2 as well as variable factors. No matter what the starting point, any switch would be to a less externality producing point. The results will be sensitive to changes in the relative prices of X_{m1} and X_{m2}. If $P_{x_{m1}}$ increases relative to $P_{x_{m2}}$ the firm may switch back to produce X_{m1} and/or technique A. A decrease in relative price of X_{m1} versus X_{m2} will not alter (4. 1. 23).

The results of this policy with respect to product and technique switching were not unlike those of other policies in many ways. However, one difference was that this policy encouraged a firm to just meet the standard and nothing more. If the firm should happen to switch techniques and/or products it is purely by accident since the policy agency has little control over VC_n and the units of fixed resource requirements. The only control is the setting of the quality for the externality. If cost and price conditions change the policy agency may not much care as long as the standard is met. The other policies, however, are more active in the sense that they can be used specifically to influence internal operating procedures.

Subsidizing a Variable Factor

The analytical framework for a subsidy is practically identical to the tax; consequently, this section will not contain additional mathematical statements. A subsidy is nothing more than a negative tax so in the tax section, the instances where a negative tax was required to induce technique change are the instances where subsidies are relevant.

Similarities in effects of the subsidy and tax can be found. For example, when it was assumed that $VC_{a1} < VC_{b1}$ and $VC_{a2} < VC_{b2}$, the tax on V_p could not induce only a technique change. With the same assumptions a subsidy on V_p cannot produce only a technique switch either. The latter is due to the fact that $(X^a_{m1})_o > (X^b_{m1})_o$ and $(X^a_{m2})_o > (X^b_{m2})_o$. If variable cost assumptions are changed, the above results also change.

As with the tax, the subsidy on variable factors can induce switches of techniques and products. Instead of searching for factors used relatively more by technique A versus B, with the subsidy the search must be for just the opposite. That is, factors used relatively more by B than A. The same reversal is necessary regarding products.

Choosing the wrong variables to subsidize could either not change or degrade the situation depending on the initial firm position. If technique switching is the desired goal subsidizing a technique specific variable factor would be better than a product specific.

The other possibility opened up by subsidies revolves around the fact that the firm will be in a better income position after the policy than before. Over a longer time period the firm may be able to obtain additional fixed factors (fixed in short run) and increase production. Under such a possibility total production of X_n may increase even if the desirable techniques and products are used. Such a possibility is very real and must not be discounted when evaluating this policy.

Subsidizing a Fixed Factor

To consider changes in fixed factors a longer time span must be considered. Most analyses so far have been directed at the short run situation where fixed factors cannot be altered.

Initial conditions as set out in (4. 1. 3) are still relevant to the longer time span. In addition it must also be assumed that the firm is in a long run equilibrium position. That is, the returns to fixed factors are equivalent to their cost.[50] If the latter were not the case the firm would either expand or contract without the proposed policy.

Subsidizing one of the fixed factors will have the same effect as lowering its price. Assuming ceterius paribus conditions on all other prices and coefficients the lowering in price of a fixed factor could induce the firm to expand operations since the return to the fixed factor then would exceed the cost.

If it is assumed that per unit net revenue from X^a_{m2} exceeds per unit net revenue from X^b_{m2} then subsidizing F_2 will not induce a technique change (providing the subsidy is less than the cost of F_2). It is possible that a change of products could occur, however. If per unit net revenue from X^b_{m2} happens to exceed that from X^a_{m2} there is a possibility that increasing F_2 could induce other changes. If F_2 is increased by ΔF_2 the following must hold initially; otherwise, the firm might switch techniques by adding ΔF_2 prior

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[50] If the firm is operating at point $(X^a_{m1})_o$ then F_1 is the relevant constraint. At $(X^a_{m1})_o$ there is excess F_2 so it seems that the cost of F_2 should be zero. The return from additional F_2 if one only considers producing X_{m1} by technique A also appears to be zero. However, if switching techniques and products is feasible then additional F_2 may have a value. This point is taken up later.

to the subsidy.

$$(P^o_{x_{m1}} - VC_{a1}) (X^a_{m1})_o > (P^o_{x_{m2}} - VC_{b2}) (X^b_{m2})_o + (P^o_{x_{m2}} - VC_{b2}) \frac{\Delta F_2}{f_{2b2}} - P_{F_2} \Delta F_2 \qquad (4.1.24)$$

where f_{2b2} is the amount of F_2 required to produce one unit of X_{m2} by method B.

Furthermore it must also be that

$$(P^o_{x_{m1}} - VC_{a1}) \frac{\Delta F_1}{f_{1a1}} < P_{F1} \Delta F_1 . \qquad (4.1.25)$$

Condition (4.1.25) implies that the cost of additional F_1 is such that the firm cannot expand production of X_{m1} by technique A.

If a subsidy on F_2 is going to induce a switch to technique B and product X_{m2} it is necessary that the inequality in (4.1.24) be reversed. In addition it must be that

$$(P^o_{x_{m2}} - VC_{b2}) (X^b_{m2})_o + (P^o_{x_{m2}} - VC_{b2}) \frac{\Delta F_2}{f_{2b2}} > (P^o_{x_{m2}} - VC_{a2}) (X^a_{m2})_o + (P^o_{x_{m2}} - VC_{a2}) \frac{\Delta F_2}{f_{2a2}} . \qquad (4.1.26)$$

The latter is necessary since otherwise a product switch without a technique switch might occur. Since $(X^b_{m2})_o < (X^a_{m2})_o$ and $f_{2b2} > f_{2a2}$ it is necessary for (4.1.26) that $(P^o_{x_{m2}} - VC_{b2}) > (P^o_{x_{m2}} - VC_{a2})$. The size of ΔF_2 that is relevant for (4.1.24) and (4.1.26) is limited by the level of the constraint $F_1^{B^o}$. In other words at $(X^b_{m2})_o$ there is enough excess F_1 available to increase production of X^b_{m2} only to a certain maximum (point (1) in Figure 5.2). Additional F_2 is required to achieve that maximum. To go beyond that point not only requires additional F_2 but also more F_1.

If (4.1.26) holds and if the subsidy is large enough to reverse (4.1.24) then a switch of products and techniques will be inferred. After a successful subsidy (4.1.26) becomes

$$(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_o < (P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_o + (P^o_{x_{m2}} - VC_{b2}) \frac{\Delta F_2}{f_{2b2}} - (P_{F_2} - S_{F_2})\,\Delta F_2. \qquad (4.1.27)$$

Notice that the firm will not end up at the optimum policy point. Rather, the final point will be (1) in Figure 5.2. Since $n_{b2} < n_{a1}$ it is likely that

$$n_{a1}(X^a_{m1})_o > n_{b2}\left[(X^b_{m})^o + \frac{F_2}{f_{2b2}}\right].$$

Subsidizing F_1 could induce the firm to produce more (X^a_{m1}) providing the subsidy lowers the effective price for F_1 below the return to F_1. In other words the subsidy would need to reverse (4.1.25). The level of factor F_2, $F_2^{A^o}$, sets an upper bound (point (2) in Figure 5.2) on expansion through subsidizing F_1.

If the starting position of the firm should happen to be A_2 in

Figure 5. 2 conditions are somewhat different. At A_2 both factors are constraining. Under that situation an increase in either fixed factors will provide added net revenue.[51] Subsidizing F_2 so that its cost is less than net revenue added would induce the firm to move down the $F_1^{A^o}$ constraint thus reducing X^a_{m1} production and increasing X^a_{m2}. That type of movement would likely be an improvement since X_{m2} generates less X_n than X_{m1}. It is conceivable that the F_2 subsidy could induce a technique as well as product switch. If so (4. 1. 26) would need to hold for this case also. Whether or not (4. 1. 26) will hold is more difficult to tell compared to the case where the initial position was A_1. Just the fact that it has been assumed the firm initially was at A_2 could imply that $(P^o_{x_{m2}} - VC_{a2})$ is closer in size to $(P^o_{x_{m2}} - VC_{b2})$ than when the firm started at A_1. In any event the subsidy could improve the situation providing it is not too large. If the firm started at A_2 subsidizing F_1 would also

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[51] For example if F_2 is increased one unit the added revenue would be the net revenue from an increase in $\frac{1}{f_{2a2}}$ units of X_{m2} minus the net revenue lost due to a decrease in production of X_{m1} by $\left(\frac{f_{1a2}}{f_{1a1}}\right) \frac{1}{f_{2a2}}$ units, i.e., net revenue added by F_2 is

$$\frac{1}{f_{2a2}} \quad (P^o_{x_{m2}} - VC_{a2}) - \left(\frac{f_{1a2}}{f_{1a1}}\right) \frac{1}{f_{2a2}} \quad (P^o_{x_{m1}} - VC_{a1}).$$

induce production of more X_{m1} and less X_{m2} which would be degrading the situation.

Summary and Implications

With this model and assuming initial firm position to be at A_1 subsidizing fixed factors could bring about a technique and product switch; consequently, a reduction in X_n could result. Subsidizing both factors simultaneously will lead to continued expansion of production; therefore, that should be avoided.

It appears that where two factors are constraining this policy might be useful. Even in such cases it will be critical to determine which factor favors X_{m2} versus X_{m1}. Since X_{m1} is the product which produces the most X_n it will be desirable to subsidize the factor which constrains X_{m2} production the most. In this case the factor is F_2.

A rise in $P_{x_{m1}}$ relative to $P_{x_{m2}}$ could result in a switch back to the original firm position. If $P_{x_{m2}}$ should rise relative to $P_{x_{m1}}$ no switch would occur providing the after subsidy firm position did not include production of any X_{m1}. However, if the after policy optimum happened to be some point such as A_2 a rise in $P_{x_{m2}}$ relative to $P_{x_{m1}}$ could result in a switch to production of only X_{m2}. The latter would likely reduce X_n.

Specification and Policy Applications for Model 4. 2

Another model can be generated if it is assumed that one technique does not dominate the other for both fixed resources. Suppose that technique A requires less of resource F_2 than method B, but more of resource F_1. This situation is depicted by Figure 5. 3.

This model is more complex diagramatically than Model 4. 1. No matter which assumptions are made concerning variable costs it is not possible to make an *a priori* statement about the net revenue maximization point. For example, assume that $VC_{a1} = VC_{b1}$ and $VC_{a2} = VC_{b2}$ or that the variable costs of producing each market product are the same regardless of technique. Under the equality assumption net revenue for both techniques can be shown by the same iso-net revenue line on Figure 5. 3. The best that one can do *a priori* is eliminate two points from the net revenue maximization set. Points A_1 and B_3 are, under the equality assumption, obviously inferior points. Depending on the relative prices and variable costs of X_{m1} and X_{m2} maximum net revenue could be at B_1, B_2, A_2, or A_3.

Suppose that the variable costs between techniques are such that $VC_{a1} < VC_{b1}$ and $VC_{a2} < VC_{b2}$. Under the latter situation there will be two net revenue lines in Figure 5. 3, one for each technique. The only points that can be compared are those that are common to the attainable sets of both techniques. Points A_1 and B_3 are such

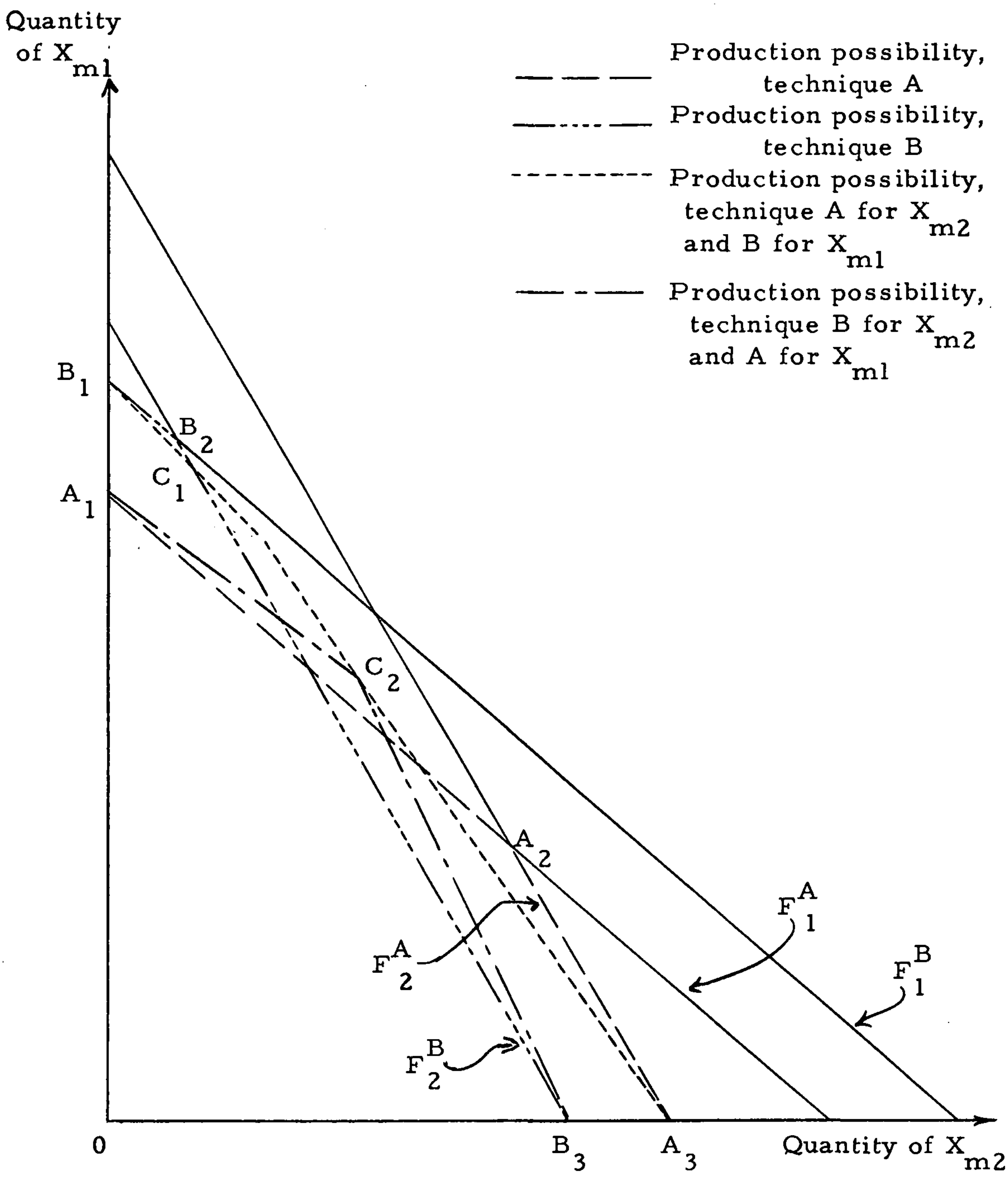

Figure 5.3. Production possibilities--Model 4.2--technique A does not dominate B

points. Due to the assumed conditions the net revenue at these common points will be greater for technique A. Only one of these points however is inferior to another point in the entire attainable set. If the net revenue line is such that it would indicate producing at B_3 then obviously if the entire set is considered it would also indicate higher net revenue at A_3. The same is not true for point A_1. Even though B_1 involves more X_{m1} than A_1, the per unit net revenue at A_1 is higher. As a result it is not possible to tell whether total net revenue is higher or lower at A_1 versus B_1. To summarize then the possible net revenue maximizing points are A_1, B_1, B_2, A_2, or A_3.

If $VC_{a1} > VC_{b1}$ and $VC_{a2} > VC_{b2}$ then by an argument similar to the previous paragraph only A_1 could be eliminated a priori from the possible set of net revenue maximizing points.

Initial conditions are similar to the previous model. The only difference is that (4.1.1) and (4.1.2) are both necessary no matter what assumption is used concerning variable costs. In other words even if $VC_{a1} < VC_{b1}$ and $VC_{a2} < VC_{b2}$ (4.1.1) is not a sufficient condition as before. There are three possible points in Figure 5.3 that could meet the initial conditions, A_1, A_2, and A_3.

The broken and dashed lines in Figure 5.3 represent production possibilities when one technique is used to produce one product and another technique is used to produce the other product. Even though one of the initial assumptions rules this possibility out it seems desirable to relax that assumption, especially when the policies are considered.

Taxing the Market Products

The assumption that the initial point is A_1 in Figure 5. 3 (producing only X_{ml} with method A) implies that $VC_{al} < VC_{bl}$. Suppose also that $VC_{a2} < VC_{b2}$. Under these assumptions it has already been demonstrated that point B_3 in Figure 5. 3 is inferior to A_3. In other words even after the policy is applied B_3 will not be an optimum point. The best the policy can do then is move the firm to one of the second best positions. The original conditions must be the same as those in (4. 1. 3). These also imply (4. 1. 4). It is impossible for the tax on X_{ml} to reverse (4. 1.4), i. e.,

$$(P^o_{x_{ml}} - VC_{al})\,(X^a_{ml})_o - T_1\,(X^a_{ml})_o > (P^o_{x_{ml}} - VC_{bl})\,(X^b_{ml})_o$$

$$- T_1\,(X^b_{ml})_o \qquad (4. 2. 1)$$

since $T_1\,(X^a_{ml})_o < T_1\,(X^b_{ml})_o$ and by (4. 1. 4). The way that Figure 5. 3 is constructed it would also be impossible for the tax on X_{ml} to induce the firm to change to point B_2 since $(X^a_{ml})_o < (X^b_{ml})_1 \equiv B_4$. The latter need not be the case, however, since different construction could meet all the assumptions designated yet result in a situation where $(X^a_{ml})_o > (X^b_{ml})_1$. So it would appear that the tax could bring about a switch to points B_2, A_2, or one of the technique-product combinations represented by the dashed and broken lines in Figure 5.3. As with Model 4.1, the new firm optimum will change with changes in the relative prices of X_{m1} and X_{m2}.

Now assume that $VC_{a1} < VC_{b1}$ but $VC_{a2} > VC_{b2}$. With these assumptions it would appear possible to achieve the optimum policy point by taxing X_{m1}. At least that point cannot be ruled out as before. If after the policy B_3 is to be the optimum, (4.1.6) and (4.1.7) must hold. The tax must first of all move the firm from point A_1. Also the first part of (4.1.7) must exist prior to policy implementation; otherwise, a second best solution will result.

The conditions that are necessary and sufficient for the tax to induce movement to the optimum policy point are shown in Appendix A, Part 5. From the necessary and sufficient conditions it is apparent that much information with respect to the internal structure of the firm is necessary if the proper magnitude of tax is to be set by the policy agency. It is entirely conceivable that for each possibility point there exists a tax level which would induce the firm to move to that point. If the tax is on X_{m1} there does not seem to be a risk of degrading the existing situation.

A tax on both X_{m1} and X_{m2} would as before reduce the possibility of achieving internal technique and/or product switches. A tax only on X_{m2} would not affect the initial situation unless the firm were producing some of X_{m2}. If the latter were the case the tax on X_{m2} would degrade the initial position.

Observation of the necessary and sufficient conditions in Appendix A, Part 5, indicates that an effective tax (causing a switch to B_3) would be sensitive to change in relative prices between X_{m1} and X_{m2}. If the price of X_{m1} were to increase, the firm might switch from the optimum policy point to one of the second best positions or even back to the original point.

Summary and Implications

If technique A dominated B with respect to variable costs of production, i.e., $VC_{a1} < VC_{b1}$ and $VC_{a2} < VC_{b2}$, a tax on X_{m1} did not induce the firm to switch to the "best" policy point. The tax could have, however, resulted in a switch to one of the second best points. If a switch does occur to a second best position an increase in the price of X_{m2} relative to X_{m1} could result in a switch back. A fall in the price of X_{m1} relative to X_{m2} could induce a switch to more X_{m2}. Of course, the latter could happen without the policy.

If $VC_{a1} < VC_{b1}$ but $VC_{a2} > VC_{b2}$ the tax on X_{m1} could prompt the firm to switch to the optimum policy point. Such a change would also be subject to reversal by a rise in $P_{x_{m1}}$ relative to $P_{x_{m2}}$. Taxing X_{m1} does not appear to present the danger of increasing X_n production as was true of other models. Of course if the firm started at some point other than A_1 and with certain cost assumptions it is possible the previous statement may not hold. For example, it is conceivable that the firm could start at B_1. A tax on X_{m1} could result in a switch to A_1. If $n_{a1}\ (X^a_{m1})_o > n_{b1}\ (X^b_{m1})_o$ then such a change would increase X_n production.

Taxing X_{m2} with the assumed conditions would not result in any change. If the initial firm position was something other than A_1 the tax on X_{m2} could increase X_n production. However, under the right circumstances taxing X_{m2} might result in a technique switch that could be beneficial. Such an event could occur if A_3 were the initial point and $VC_{a2} > VC_{b2}$. A tax on X_{m2} then might induce a switch to B_3. If the tax were too large a switch to producing X_{m1} by technique A might result, however.

Taxing a Variable Factor

Assume the initial firm position to be at A_1 in Figure 5.3 and that $VC_{a1} \leq VC_{b1}$ and $VC_{a2} \leq VC_{b2}$. Consider initially taxing a variable factor, product specific, V_p. Can such a tax induce the firm to switch techniques only? To do the latter the inequality in (4.1.4) must be reversed, i.e.,

$$(P^o_{x_{m1}} - VC_{a1} - T_{vp}v_{p1})\ (X^a_{m1})_o < (P^o_{x_{m1}} - VC_{b1} - T_{vp}v_{p1})\ (X^b_{m1})_o. \tag{4.2.2}$$

However, since $(X^a_{m1})_o < (X^b_{m1})_o$ the existence of (4. 2. 2) is <u>not</u> possible. The answer to the question then is <u>no</u>.

Can tax induce any types of change? To answer this question examine for example

$$(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_o - T_{vp}v_{p1}(X^a_{m1})_o \overset{?}{\underset{<}{\geq}} (P^o_{x_{m2}} - VC_{a2})(X^a_{m2})_o$$

$$- T_{vp}v_{p2}(X^a_{m2})_o. \qquad (4.2.3)$$

As for Model 4. 1, $v_{p1} > v_{p2}$; consequently, it is possible that the (<) could hold in (4. 2. 3) implying a product change. The changes in assumptions for Model 4. 2 do not affect the conclusions reached for Model 4. 1 concerning the possibility of this tax inducing a switch of products <u>and</u> technique, i. e., it will <u>not</u>. This is so since (4. 1. 13) is relevant to the present model also.

As noted in the previous section if the firm starts at A_1 this implies that $VC_{a1} < VC_{b1}$. It is possible, however, to change the other variable cost assumption, i. e., $VC_{a2} > VC_{b2}$. Again the results for this model are identical to the ones for Model 4. 1 which made it feasible to attain the optimum policy point. That is a switch to producing X_{m2} with method B is conceivable. With this model it is not possible to obtain a technique switch only, since $VC_{a1} < VC_{b1}$.

The differences between Models 4. 1 and 4. 2 do not affect conclusions concerning taxing a technique specific factor V_T. The same arguments and inequalities, (4. 1. 15) through (4. 1. 19), as used for

Model 4.1 can be used for this model. Between the two models there is likely to be differences in magnitudes of taxes required to bring about certain changes; however, specific numbers for costs, input coefficients, etc., would be needed to determine that. In general for this model the tax on V_T could induce both technique and product changes. The information contained in footnote 44 is not relevant to this model since it must be that $VC_{a1} < VC_{b1}$ to insure initial starting point A_1.

Conclusions reached concerning taxing a product and technique specific factor, V_D, are the same also as for Model 4.1. The discussion for Model 4.1 regarding sensitivity to price of this policy is also relevant to the present model.

If the firm started from a position other than A_1 there is a chance that one of these taxes could induce a product or technique change that would be detrimental. For example, suppose the firm started at B_1. Since $(X^a_{m1})_o < (X^b_{m1})_o$ there is a chance that a tax on a product specific variable could induce the firm to switch to A_1 which could result in more X_n. Other instances might also degrade the situation but will not be considered here.

Summary and Implications

Changes in specification between the present and previous models make little difference in the implications for this policy. First of all a tax on a product specific variable will not induce only a technique switch if $VC_{a1} \leqslant VC_{b1}$ and $VC_{a2} < VC_{b2}$. A product switch could occur, however.

It was not possible for Model 4.2 to change the assumption that $VC_{a1} < VC_{b1}$; therefore, changing to $VC_{a2} > VC_{b2}$ still did not permit the product specific tax to imply only a technique change. With these latter cost assumptions it was possible for the optimum policy point to be achieved.

A tax on a technique specific variable factor could induce product and/or technique changes. In other words this policy could under all cost assumptions prompt the firm to switch to the optimum policy point.

If a switch to the "best" point is made an increase in $P_{x_{m1}}$ relative to $P_{x_{m2}}$ could reverse the switch. A switch made to a "second best" point which did not generate any X_{m2} would not be influenced by such a change in relative prices.

Taxing the Non-market Externality

Assume that $VC_{a1} < VC_{b1}$ and $VC_{a2} \leq VC_{b2}$. The starting position for the firm is at point A_1. After the tax on the externality one of the necessary conditions for a technique change is

$$(P^o_{x_{m1}} - VC_{a1}) (X^a_{m1})_o - T_n n_{a1} (X^a_{m1})_o < (P^o_{x_{m1}} - VC_{b1}) (X^b_{m1})_o$$

$$- T_n n_{b1} (X^b_{m1})_o. \qquad (4.2.4)$$

It is possible that the externality tax could bring (4.2.4) about. One difference between Model 4.1 and the present is that it is likely to take a smaller tax to prompt a technique change for Model 4.2 than for 4.1.[52] Another difference between these two models is that (4.1.21) is not necessary for a switch to technique B for Model 4.2 since $(X^b_{m1})_o > (X^a_{m1})_o$.

Changing the variable cost assumptions to $VC_{a1} < VC_{b1}$ and $VC_{a2} > VC_{b2}$ does not result in any different conclusions than found for Model 4.1. Conclusions concerning price sensitivity are also the same as Model 4.1.

Summary and Implications

Taxing X_n could induce the firm to switch to the "best" point. If such a switch occurs an increase in $P_{x_{m1}}$ relative to $P_{x_{m2}}$ could negate the switch at a later date. A rise in $P_{x_{m2}}$ relative to $P_{x_{m1}}$ could result in a switch back to technique A but not a switch of products.

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[52] See Appendix A, Part 6.

Since the amount of X_{ml} that was produced by technique A initially was less than the amount of X_{ml} feasible with B, a technique change could occur with a smaller tax compared to Model 4. 1. The main implication of the smaller tax is that tax size is sensitive to possible production levels. In other words, a tax that might induce one firm to switch techniques may not necessarily cause another firm to change even though the firms were similar in operation.

A Standard on the Quality of the Externality

In most respects the changes in Model 4. 2 do not affect the conclusions reached when this policy was applied to 4. 1. The main difference concerns the maximum amounts of X_{ml} that can be produced by the two techniques. For Model 4. 1 the enforcement of the standard brought about a shift in the F_1 constraint in such a way that the difference between maximums of X_{ml} by the two techniques was reduced. In the present model the shifts of F_1 under the same assumptions as before will widen the difference between X^b_{ml} and X^a_{ml}. Assuming however that the initial firm position is at A_1 as in Figure 5. 3 the widening of the difference has similar effects as the narrowing did in Model 4. 1. To visualize why, examine

$$(P^o_{x_{ml}} - VC_{a1}) (X^a_{ml})_o > (P^o_{x_{ml}} - VC_{b1}) (X^b_{ml})_o. \qquad (4. 2. 5)$$

After the standard the situation will appear as

$$(P^o_{x_{ml}} - VC_{al} - VC_{n^n al}) (X^a_{ml})_{-1} \overset{?}{\gtrless} (P^o_{x_{ml}} - VC_{bl}$$

$$VC_{n^n bl}) (X^b_{ml})_{-1} . \qquad (4.2.6)$$

In both models X^a_{ml} is decreased relatively more than X^b_{ml}; therefore, the chances for the (<) to hold are improved. The only difference is that for Model 4.1, $(X^a_{ml})_o > (X^b_{ml})_o$; therefore, reducing X^a_{ml} the most brings them together. In Model 4.2, $(X^a_{ml})_o < (X^b_{ml})_o$ so that reducing X^a_{ml} the most causes the two to diverge. The conclusions are not changed by this difference in the models.

Subsidizing Fixed and Variable Factors

Changing from Model 4.1 to 4.2 does not affect the conclusions concerning these policies. Subsidy sizes will likely differ between models but such comparisons are not made. The procedures for comparing subsidy sizes would be similar to those shown in Appendix A, Part 4, for comparing tax sizes.

Specification and Policy Applications for Model 4.3

To this point it has been assumed that a change in technique will cause a parallel shift in a given resource constraint. In this model that assumption is relaxed. There are many conceivable ways these shifts could occur. For purposes of analysis only one configuration will be considered, mainly to see if policy effects change. One configuration is shown in Figure 5.4. As shown in Figure 5.4 the technique shifts favor one product or the other depending on the fixed resource. Resource F_1 is used most efficiently by product X_{m2} if technique B is used. On the other hand resource F_2 is used most efficiently by product X_{m1} with technique B. Another conceivable type shift would be when both resources are used most efficiently for producing X_{m2} with technique B and X_{m1} with technique A as shown in Figure 5.5. That is $f_{1a1} < f_{1b1}$, $f_{2a1} < f_{2b1}$, $f_{1b2} < f_{1a2}$, and $f_{2b2} < f_{2a2}$.

Staying with the assumption that the firm is initially using technique A requires conditions (4.1.1) and (4.1.2). When the policies are applied the assumption of starting with technique A will be relaxed in some instances.

Taxing the Market Products

For purposes of discussion reference will be made to Figure 5.5. The shifts due to technique changes as depicted in Figure 5.5 are a more radical departure from Models 4.1 and 4.2 than those shown in Figure 5.4.

It is interesting to note in Figure 5.5 that the production possibility for producing X_{m1} with technique A and X_{m2} with B lies entirely outside the other production possibility lines. If net

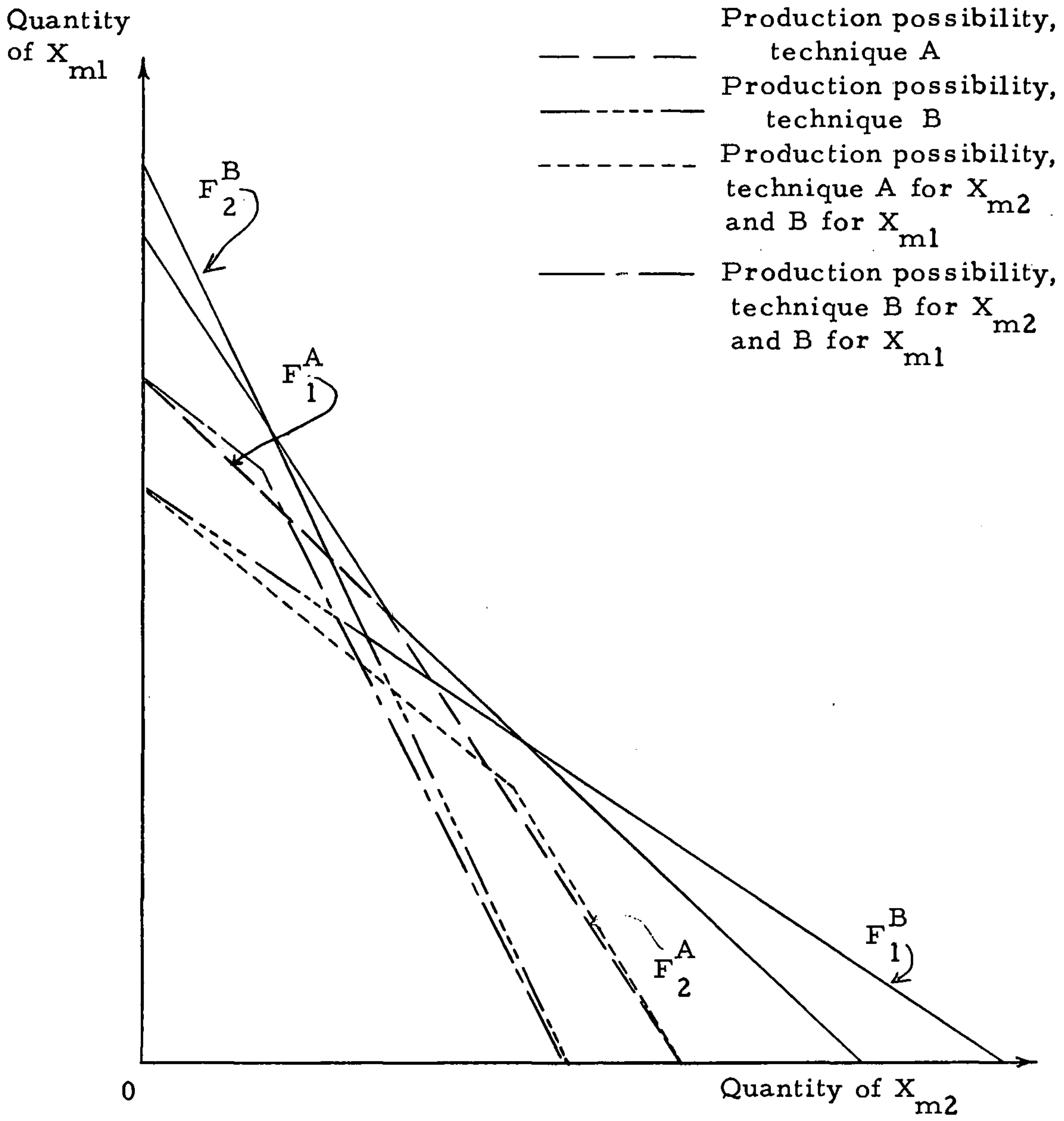

Figure 5.4. Production possibilities, Model 4.3 where technique changes favor given products for different resources

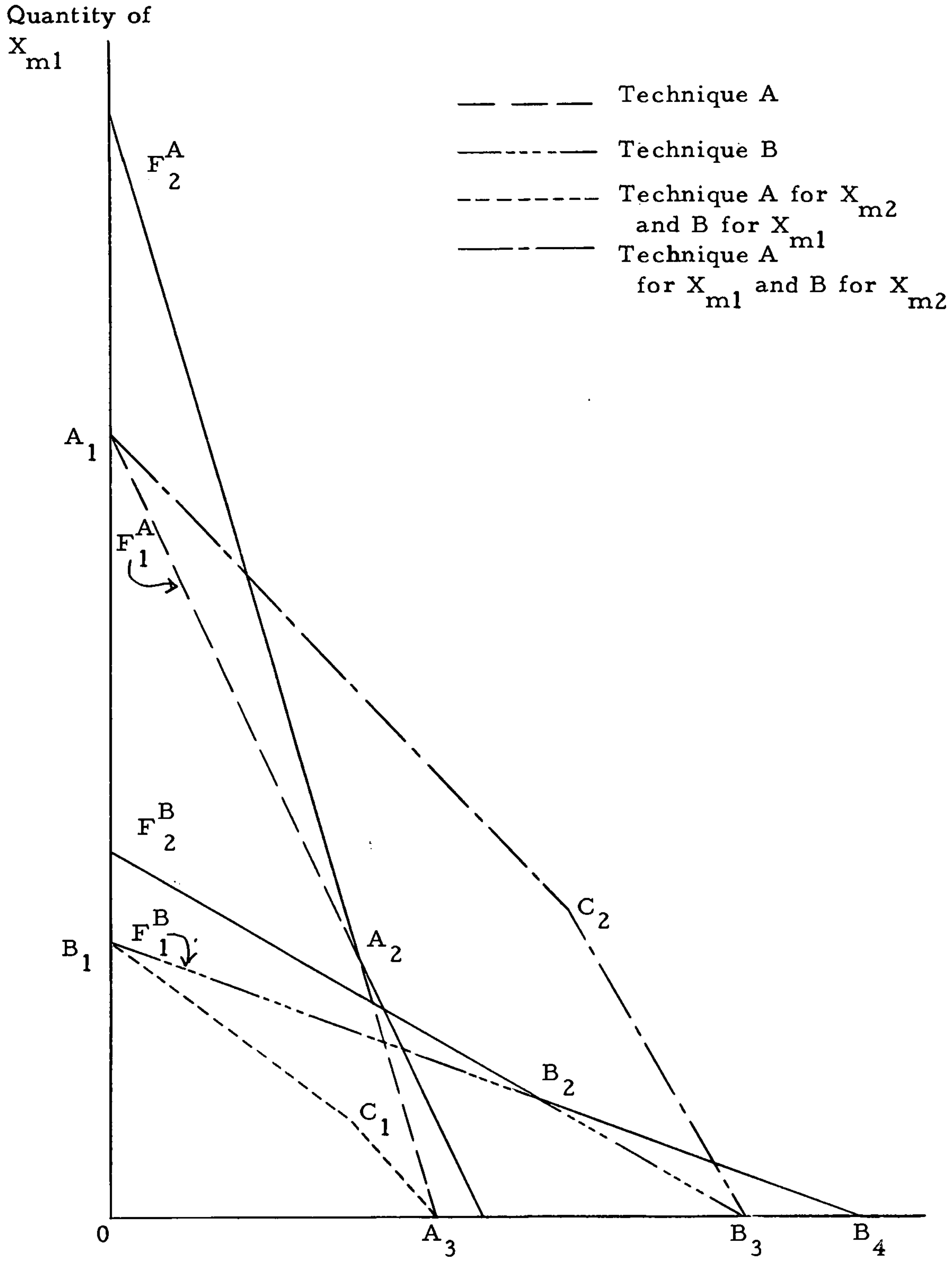

Figure 5.5. Production possibilities, Model 4.3 with technique A favoring X_{m1} and B favoring X_{m2}

revenues for each product and both techniques are equal (i. e., $VC_{a1} = VC_{b1}$ and $VC_{a2} = VC_{b2}$) then the firm will use both techniques unless only one product is produced. Such a configuration of possibility sets was not possible with the assumptions of Models 4. 1 and 4. 2.

Again assume that the initial firm position is A_1 in Figure 5. 5. Also assume that $VC_{a1} \leq VC_{b1}$ and $VC_{a2} \leq VC_{b2}$. Conditions (4. 1. 3) are also relevant. Point B_1 can be immediately ruled out as a possible net revenue maximizing position after the tax since $(P^o_{x_{m1}} - VC_{a1} - T_1) > (P^o_{x_{m1}} - VC_{b1} - T_1)$ and X^a_{m1} at A_1 is larger than X^b_{m1} at B_1. As with previous models B_3 is the optimum policy position. It is definitely possible that the tax on X_{m1} could induce switching to B_3. The necessary and sufficient conditions for a switch to B_3 are the same as in Appendix A, Part 5.

Of the "second best" positions possible C_2 in Figure 5. 5 would be the worst since more of both products would be produced at C_2 than at A_2, A_3, B_2 or C_1. So it would seem desirable not to induce the firm to move to point C_2. If

$$(P^o_{x_{m1}} - VC_{a1}) (X^a_{m1})_2 + (P^o_{x_{m2}} - VC_{b2}) (X^b_{m2})_2 \leq (P^o_{x_{m1}} - VC_{a1}) (X^a_{m1})_1 + (P^o_{x_{m2}} - VC_{a2}) (X^a_{m2})_1 \qquad (4. 3. 1)$$

then a tax on X_{m1} cannot bring about a movement to point C_2 since

$(X^a_{m1})_2 > (X^a_{m1})_1$.[53] With the presently assumed conditions, (4.3.1) is possible since $VC_{a2} < VC_{b2}$. If the inequality in (4.3.1) is "greater than" then a movement from the initial firm position, A_1, could be to point C_2. Such a movement could be prevented by a <u>larger</u> tax.[54]

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[53] One could make the same statement if $(P^o_{x_{m1}} - VC_{a1})$ $(X^a_{m1})_2 + (P^o_{x_{m2}} - VC_{b2})\,(X^b_{m2})_2 \leq (P^o_{x_{m1}} - VC_{b1})\,(X^b_{m1})_1$ $+ (P^o_{x_{m2}} - VC_{b2})\,(X^b_{m2})_1$ since $(X^a_{m1})_2 > (X^b_{m1})_1$.

[54] Suppose initially

$$(P^o_{x_{m1}} - VC_{a1})\,(X^a_{m1})_2 + (P^o_{x_{m2}} - VC_{b2})\,(X^b_{m2})_2 > (P^o_{x_{m1}} - VC_{a1})\,(X^a_{m1})_1 + (P^o_{x_{m1}} - VC_{a2})\,(X^a_{m2})_1. \qquad (1)$$

After the tax it is desirable if

$$(P^o_{x_{m1}} - VC_{a1} - T_1)\,(X^a_{m1})_2 + (P^o_{x_{m2}} - VC_{b2})\,(X^b_{m2})_2 < (P^o_{x_{m1}} - VC_{a1} - T_1)\,(X^a_{m1})_1 + (P^o_{x_{m2}} - VC_{a2})\,(X^a_{m2})_1 \qquad (2)$$

$$\Rightarrow T^o_1 > \left.\begin{array}{l} \dfrac{(P^o_{x_{m1}} - VC_{a1})\,[(X^a_{m1})_2 - X^a_{m1})_1] + (P^o_{x_{m2}} - VC_{b2})\,(X^b_{m2})_2}{(X^a_{m1})_2 - (X^a_{m1})_1} \\ \underline{(P^o_{x_{m2}} - VC_{a2})\,(X^a_{m2})_1} \qquad (3) \end{array}\right\}$$

If $VC_{a1} \leq VC_{b1}$ and $VC_{a2} > VC_{b2}$ then B_1 cannot be ruled out as a possible net revenue maximization point after the tax, but A_3 can be ruled out. It is still possible to achieve point B_3 providing all conditions in Appendix A, Part 5, are met. The possibility of the firm moving to C_2 is stronger with the present cost assumptions since (4. 3. 1) is <u>not</u> possible. The conditions as shown in footnote 54 are relevant here since $(P^o_{x_{m2}} - VC_{a2}) < (P^o_{x_{m2}} - VC_{b2})$.

If $VC_{a1} > VC_{b1}$ and $VC_{a2} > VC_{b2}$, A_3 can be eliminated as a possible net revenue maximizing point after the tax. Again (4. 3. 1) is not possible so that the tax could induce movement to C_2. However, footnote 54 could be relevant so tax size is critical. As with the other assumed cost conditions B_3 is achievable with a tax on X_{m1}.

The previous discussion strengthens the argument that predicting the reactions of firms to this type policy requires detailed knowledge of the firm. Furthermore, it is easy to see how a tax of improper size could induce movement to a less than optimal policy point.

Without even more knowledge of the firm's production process it is <u>not</u> possible to <u>a priori</u> predict the effects on the firm's demand

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therefore a tax that is less than or equal to T^o_1, will not change the first relationship in this note.

for variable services. If variable services tend to be specific to product and technique then some general statements could be made.

The present model is one instance where a tax on both market products might be desirable. Suppose that a tax is levied against X_{m1} and X_{m2} (the tax on each product need not be equal). If $VC_{b2} > VC_{a2}$ then the tax on both products could make it less likely that the firm would move to C_2 rather than say A_2.[55] If $VC_{b2} \leq VC_{a2}$ the previous statements about taxes on both products would not be true. Under the latter cost assumption taxes on both products would tend to either induce the firm to stay at A_1 or could even encourage it to move to C_2.[56]

A rise in prices of X_{m1} relative to X_{m2} could cause a firm that had switched to the optimum policy point to switch back to the original or a second best viewpoint.

If the initial firm position happens to be at some point other than A_1 there is a danger that a tax could degrade the situation. For

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[55] Suppose that (1) from footnote 54 holds initially. The tax on X_{m1}, T_1, will result in proportional reductions in the net revenue from X_{m1} for both sides of (1). A tax on X_{m2}, T_2, however will result in a proportionally higher reduction in net revenue to the left side of (1) since $(P^o_{x_{m2}} - VC_{b2}) < (P^o_{x_{m2}} - VC_{a2})$ thus making (2) more likely.

[56] Condition (1) of footnote 54 would be necessary before this statement would be true.

example, suppose B_3 were the initial position and a tax were placed on X_{m2}. One of the initial conditions would be

$$(P^o_{x_{m2}} - VC_{b2})\,(X^b_{m2})_o > (P^o_{x_{m1}} - VC_{a1})\,(X^a_{m1})_2 + (P^o_{x_{m2}} - VC_{b2})\,(X^b_{m2})_2. \tag{4.3.2}$$

A tax on X_{m2} could reverse the above thus implying a switch to C_2. Furthermore a switch to A_1 could even be implied. The point is that the initial firm position is critical in determining whether or not a desirable switch will be made.

Summary and Implications

Taxing the market product could induce the firm to move to what would be considered a policy optimum assuming A_1 is the initial firm position. Which product is taxed, however, is a crucial point. Taxing the wrong product could degrade rather than improve the situation, particularly if the firm started at some point other than A_1. Generally, taxing both products appears to be incorrect; however, under certain circumstances such a policy may be desirable.

To be able to predict *a priori* the effects on firms of a given tax level appears quite difficult. Without much information concerning the cost structure of a given firm such predictions are likely to be nothing more than educated guesses.

If the policy is being applied to firms which can produce several alternative products then the effectiveness of the policy over time will depend on relative prices of the alternative products. Such dependence tends to reduce the effectiveness of this policy.

Taxing a Variable Factor

Ideally this policy should do two things. One is to prevent the firm from switching to point C_2 and the other is to induce it to produce at B_3, both points as shown in Figure 5.5.

Suppose that the initial firm position is at A_1 in Figure 5.5 and that prior to policy implementation A_1 is the net revenue maximizing point. Assume that a product specific factor is taxed. Can such a tax induce a switch to point B_3? One of the necessary conditions for such a switch is

$$(P^o_{x_{m1}} - VC_{a1} - T_{vp} v_{p1}) (X^a_{m1})_o < (P^o_{x_{m2}} - VC_{b2} - T_{vp} v_{p2}) (X^b_{m2})_o \tag{4.3.3}$$

where v_{p1} and v_{p2} refer to the amount of V_P used to produce one unit of X_{m1} and one of X_{m2}, respectively. Also assume $v_{p1} > v_{p2}$, $VC_{a1} < VC_{b1}$ and $VC_{a2} < VC_{b2}$. There is no reason why (4.3.3) could not occur, unless $v_{p1} (X^a_{m1})_o < V_{p2}(X^b_{m2})_o$.[57]

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[57] Suppose $v_{p1} (X^a_{m1})_o < v_{p2} (X^b_{m2})_o$. Let $NA_1 \equiv (P^o_{x_{m1}} - VC_{a1})$

If $v_{p1} (X^a_{m1})_o > v_{p2} (X^b_{m2})_o$ then (4. 3. 3) is possible with a tax.

The following must also exist to prevent the firm from moving to C_2 instead of B_3:

$$(P^o_{x_{m2}} - VC_{b2} - T_{vp} v_{p2}) (X^b_{m2})_o > (P^o_{x_{m1}} - VC_{a1} - T_{vp} v_{p1}) X^a_{m1})_2$$

$$+ (P^o_{x_{m2}} - VC_{b2} - T_{vp} v_{p2}) (X^b_{m2})_2 \,. \qquad (4.3.4)$$

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and $NB_2 \equiv (P^o_{x_{m2}} - VC_{b2})$. Also assume that $(X^b_{m2})_o > (X^a_{m1})_o$ and that $NA_1 > NB_2$. Initially when the situation is $NA_1 (X^a_{m1})_o > NB_2 (X^b_{m2})_o$. (1) After the tax the necessary condition for a switch from (4. 3. 3) is $NA_1 (X^a_{m1})_o - T_{vp} v_{p1} (X^a_{m1})_o < NB_2 (X^b_{m2})_o - T_{vp} v_{p2} (X^b_{m2})_o$. (2)

$$\rightarrow T_{vp} < \frac{NB_2 (X^b_{m2})_o - NA_1 (X^a_{m1})_o}{v_{p2} (X^b_{m2})_o - v_{p1} (X^a_{m1})_o} \qquad (3)$$

Notice that the right side of (3) is negative since the denominator is positive by assumption and the numerator is negative by (1). Consequently to induce (4. 3. 3) a subsidy is implied instead of a tax.

Whether or not (4.3.4) is possible with a tax depends on two things. First of all the net revenue relationship between B_3 and C_2 prior to policy implementation and the amount of V_P used at these two points.[58] If it is assumed that more V_P is used at C_2 and if net revenue at C_2 prior to the tax is greater than at B_3 a tax will be implied by (4.3.4). Otherwise, a subsidy could be implied for part of the range by (4.3.4).[59] In other words any tax greater than the left

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[58] Using similar notation as in footnote 57 (4.3.4) can be rewritten as:

$$T_{vp} > \frac{NA_1 (X^a_{m1})_2 + NB_2 (X^b_{m2})_2 - NB_2 (X^b_{m2})_o}{v_{p1} (X^a_{m1})_2 + v_{p2} (X^b_{m2})_2 - v_{p2} (X^b_{m2})_o} . \qquad (1)$$

For the right side of (1) to be positive and for the (>) to be valid the denominator and numerator must both be positive.

[59] If $v_{p1} (X^a_{m1})_2 + v_{p2} (X^b_{m2})_2 > v_{p2} (X^b_{m2})_o$ and $NB_2 (X^b_{m2})_o > NA_1 (X^a_{m1})_2 + NB_2 (X^b_{m2})_2$ then from (4.3.4) one gets

$$NB_2 (X^b_{m2})_o - (NA_1 (X^a_{m1})_2 + NB_2 (X^b_{m2})_2) > T_{vp} \left[v_{p2} (X^b_{m2})_o - v_{p1} (X^a_{m1})_2 - v_{p2} (X^b_{m2})_2 \right] \qquad (1)$$

$$\longrightarrow - \frac{NB_2 (X^b_{m2})_o - NA_1 (X^a_{m1})_2 - NB_2 (X^b_{m2})_2}{v_{p1} (X^a_{m1})_2 + v_{p2} (X^b_{m2})_2 - v_{p2} (X^b_{m2})_o} < T_{vp} \qquad (2) \text{ since}$$

the quantity in the brackets on the right of (1) is negative.

of (2) in footnote 59 would insure (4.3.4). So under the conditions assumed in footnote 59 even a small subsidy on V_p can be used without encouraging the firm to switch to C_2. However suppose that originally net revenue at C_2 exceeds net revenue at B_3 and that more V_p is used at B_3 than at C_2. Then, only a subsidy will induce the firm to operate at B_3 instead of C_2.[60]

Size of the tax required to bring about movement to the optimum policy point is again critical. Under certain conditions tax sizes required to move the firm from A_1 to C_2 can be compared with the size required to move the firm from A_1 to B_3. Suppose that (using notation established in footnote 56) $NA_1 (X^a_{m1})_o > NB_2 (X^b_{m2})_o > NA_1 (X^a_{m1})_2 + NB_2 (X^b_{m2})_2$ and that $v_{p1} (X^a_{m1})_o > v_{p1} (X^a_{m1})_2 + v_{p2} (X^b_{m2})_2 > v_{p2} (X^b_{m2})_o$. A necessary condition for the switch from point A_1 to B_3 in Figure 5.5 is (4.3.3) which implies

$$T_{vp} > \frac{NA_1 (X^a_{m1})_o - NB_2 (X^b_{m2})_o}{v_{p1} (X^a_{m1})_o - v_{p2} (X^b_{m2})_o} \equiv T^o_{vp}. \qquad (4.3.5)$$

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[60] In footnote 59 the left of (1) will be negative and the right positive. Statement (2) will then read

$$T_{vp} < -\frac{NA_1 (X^a_{m1})_2 + NB_2 (X^b_{m2})_2 - NB_2 (X^b_{m2})_o}{v_{p2} (X^b_{m2})_o - v_{p1} (X^a_{m1})_2 - v_{p2} (X^b_{m2})_2}$$

which only makes sense if $T_{vp} < 0$ which implies a subsidy.

A similar necessary condition for a switch from A_1 to C_2 in Figure 5.5 implies that

$$T_{vp} > \frac{NA_1\,(X^a_{m1})_o - (NA_1\,(X^a_{m1})_2 + NB_2\,(X^b_{m2})_2)}{v_{p1}\,(X^a_{m1})_o - (v_{p1}\,(X^a_{m1})_2 + v_{p2}\,(x^b_{m2})_2} \equiv T'_{vp} \qquad (4.3.6)$$

One would expect that $T'_{vp} > T^o_{vp}$ which would imply a smaller tax to get the firm to switch to the desired point. Compare the following ratio

$$\frac{T^o_{vp}}{T'_{vp}} = \frac{\left[NA_1(X^a_{m1})_o - NB_2(X^b_{m2})_o\right]\left[v_{p1}(X^a_{m1})_o - v_{p1}(X^a_{m1})_2 - v_{p2}(X^b_{m2})_2\right]}{\left[NA_1(X^a_{m1})_o - NA_1(X^a_{m1})_2 - NB_2\,(X^b_{m2})_2\right]\left[v_{p1}(X^a_{m1})_o - v_{p2}(X^b_{m2})_o\right]}$$

$$< 1. \qquad (4.3.7)$$

Statement (4.3.7) is less than one since

$$\left[NA_1\,(X^a_{m1})_o - NB_2(X^b_{m2})_o\right] < \left[NA_1(X^a_{m1})_o - NA_1\,(X^a_{m1})_2 - NB_2(X^b_{m2})_2\right]$$

$$\text{and} \qquad (4.3.8)$$

$$\left[v_{p1}(X^a_{m1})_o - v_{p2}(X^b_{m2})_o\right] > \left[v_{p1}(X^a_{m1})_o - v_{p1}(X^a_{m1})_2 - v_{p2}(X^b_{m2})_2\right].$$

It is less likely that the first part of (4.3.8) will hold if $NA_1 > NB_2$; however, it is entirely conceivable that $NB_2 > NA_1$. Since the ratio in (4.3.7) is less than one, under the assumed conditions it requires a smaller tax to induce movement to B_2 than to C_2. A change in assumptions could cause (4.3.7) to be greater than one or indeterminant. The implication of this last section is that the agency could, by increasing the tax too much induce the firm to go to the worst possible "second best" point.

Changes in the variable cost assumptions will not alter the analysis appreciably. In other words it is possible for the tax to induce a switch to B_3 under any of the three cost assumptions, i.e., $VC_{a1} \lesseqgtr VC_{b1}$, and $VC_{a2} \lesseqgtr VC_{b2}$.

Now suppose a technique specific variable factor V_T ix taxed. Assume the initial firm position to again be at A_1, Figure 5.5. One necessary condition for a switch to B_3 is

$$(P^o_{x_{m1}} - VC_{a1} - T_{vt} v_{ta}) (X^a_{m1})_o < (P^o_{x_{m2}} - VC_{b2} - T_{vt} v_{tb}) (X^b_{m2})_o. \qquad (4.3.9)$$

Assuming that $v_{ta} > v_{tb}$, (4.3.9) is feasible given also that $v_{ta} (X^a_{m1})_o > v_{tb} (X^b_{m2})_o$.

Without knowing something about the relative sizes of the coefficients for V_T and V_P it is not possible to compare which type of variable factor requires the larger tax. The comparison of size of T_{vt} needed to move the firm to various points however can be made as above making the appropriate notation substitutions. In other words it is possible that T_{vt} could be too large and induce movement to C_2 instead of B_3, the same as T_{vp}.

Other conclusions are similar to those reached for Model 4.1. A tax on V_P will not induce strictly a change in techniques if $VC_{a1} < VC_{b1}$ and $VC_{a2} < VC_{b2}$; however, if

$VC_{a1} > VC_{b1}$ and $VC_{a2} > VC_{b2}$ a technique and/or product change could result. One difference the change in models makes is that T_{vp} can induce a product and technique change for the present model even with the cost assumption $VC_{a1} < VC_{b1}$ and $VC_{a2} < VC_{b2}$. The reason for the difference is that in (4. 1. 13) for Model 4. 1, $(X^{b}_{m2})_{o} < (X^{a}_{m2})_{o}$, while with the present model $(X^{b}_{m2})_{o} > (X^{a}_{m2})_{o}$.

Another difference between Models 4. 1 and 4. 3 concerns T_{vt}. If T_{vt} is to induce a product change only, then (4. 1. 18) must hold as well as

$$(P^{o}_{x_{m2}} - VC_{a2} - T_{vt}v_{ta})\,(X^{a}_{m2})_{o} > (P^{o}_{x_{m2}} - VC_{b2} - T_{vt}v_{tb})\,(X^{b}_{m2})_{o}. \tag{4. 3. 10}$$

If $VC_{a2} < VC_{b2}$ it is possible for both Models 4. 1 and 4. 3 that (4. 3. 10) could hold or that a (<) could hold in (4. 3. 10). If the latter is the case then a product and technique change is implied. Suppose $VC_{a2} > VC_{b2}$ then for Model 4. 1 either the (>) or (<) might hold in (4. 3. 10). For Model 4. 3, however, with the latter cost assumption it is not possible for (4. 3. 10) to hold as depicted thus eliminating the chance for a purely product change. Such a situation occurs since in Model 4. 3, $(X^{b}_{m2})_{o} > (X^{a}_{m2})_{o}$ and $v_{ta} > v_{tb}$. In other words, Model 4. 3 with $VC_{a2} > VC_{b2}$ makes the tax on V_{T} appear as a good policy prospect since if a switch occurs it is likely to involve product and technique.

If (4. 3. 10) holds then a fall in $P_{x_{m2}}$ could cause the (>) to become a (<) which could imply a switch to the optimum policy point. An increase in $P_{x_{m2}}$ would not affect (4. 3. 10). If the (<) held in (4. 3. 10) then a rise in $P_{x_{m2}}$ will not affect (4. 3. 10). Depending then on which situation holds prior to changes in $P_{x_{m2}}$ it does not appear that (4. 3. 10) is very sensitive to absolute movements of $P_{x_{m2}}$. Relative price changes, ceteris paribus, between $P_{x_{m1}}$ and $P_{x_{m2}}$ could cause problems, however, with other conditions such as (4. 1. 18). If $P_{x_{m1}}$ goes up relative to $P_{x_{m2}}$ it is possible (4. 1. 18) could be reversed.

The conclusions reached concerning taxing a product and technique specific variable factor do not vary for this model. As before if the factor chosen favors the optimum policy point then it seems logical that the product and technique specific variable would be the type to tax. Identification of such a variable may be more difficult, however, than either the product or technique specific types.

If the firm starts at a position other than A_1 results may be different. Whether or not the firm moves to a point that produces more X_n depends mainly on the relative total amounts of the taxed variable factor used at various points. For example, assume the initial point to be B_3. If V_P is to be taxed a condition necessary for a movement to another point is that more V_P be used at B_3 than some other position. Since $(X^b_{m2})_o > (X^a_{m2})_o$ more V_P would be used at B_3 than A_3. A tax on V_P then could induce a switch to A_3 if $VC_{a2} < VC_{b2}$. The same would not necessarily apply to a tax on V_T of V_D since the use of these two could still be higher at A_3. If the initial point were A_2 a tax on V_P would definitely not induce a switch to C_2 and a tax on V_T or V_D will not likely induce such a switch either. The latter is so since more of both products is produced at C_2 than A_2 thus increasing the probability that factor use at C_2 exceeds use at A_2. The implication is that the relative total amounts of various factors used is important in determining firm behavior.

Summary and Implications

Under assumed conditions a tax on a product specific variable factor did induce a switch to the "best" point. However, conditions could exist where too large a tax could prompt a switch to point C_2 instead of the optimum. While C_2 may be an improvement over A_1 it is worse as far as externality production is concerned than any of the other points. Again, the implication is that while this policy could be effective, it is quite sensitive to tax size.

If $VC_{a2} > VC_{b2}$ a tax on a technique specific factor did not result in a product switch only. If a switch occurred it was both a switch of products and techniques.

The relative use of the total amounts of taxed variable factors was important in determining the type of switch. The implication is that care must be exercised in determining the variable factor to be taxed. The appropriate factor should be required in larger amounts by the least desirable products and techniques.

Even if the firm were initially using only one technique of production it was conceivable that after the policy it could have used both techniques. The use of two techniques could have been associated with only one product or different techniques for each product.

If the firm were induced to switch to the "best" point a fall in the absolute price levels (i. e., $P^o_{x_{m1}}$ and $P^o_{x_{m2}}$ falling in proportion) would not reverse products. However, a fall in absolute price levels could prompt a switch back to technique A. The latter result can be reasoned by examining

$$(P^o_{x_{m2}} - VC_{a2} - T_{vt}v_{ta})\,(X^a_{m2})_o < (P^o_{x_{m2}} - VC_{b2} - T_{vt}v_{ta})\,(X^b_{m_2})_o . \qquad (4.3.11)$$

If the per unit net revenue after tax for X^a_{m2} is greater than that for X^b_{m2} then a fall in $P^o_{x_{m2}}$ could reverse (4.3.11). Changes in relative prices could negate policy induced switches also.

Taxing the Non-market Externality

The main difference created by this model is that condition (4.1.21) is no longer necessary before use of technique B will be implied. It can be observed in Figure 5.5 technique A does not dominate B. Point C_2 will be easier to avoid with this policy than with previous ones. It is highly probable that the total externality produced at C_2 will exceed that at other points (except A_1 by assumption). Certainly more X_n is produced at C_2 than A_3 and probably more than at B_3. Since the tax is on X_n directly, net revenue at C_2 will be decreased more than at most other comparison points.

If the policy has brought about a switch to B_3 then an increase in $P_{x_{m2}}$ will not cause a change back to technique A as was possible with Model 4.1. A decrease in $P_{x_{m2}}$ might however result in such a switch. Other than that the other conclusions for Model 4.1 are appropriate for this one also.

A Standard on the Quality of the Externality

Since assuming that conversion of X_n to X'_n requires variable factors only leads to a situation similar to previous policies that possibility will be ignored. Assume that conversion requires only F_1 (or at least a larger relative amount of F_1 versus F_2) plus some variable factors.

The changes in this model as shown in Figure 5.6 make little difference in the analysis as compared to Models 4.1 and 4.2. Since F_2 is the limiting factor for producing X_{m2} by either technique the shift in the production possibility line due to processing X_n might not affect the maximum amounts of X_{m2}. As with Model 4.1, X^a_{m1} is reduced more by the shift of F_1 than is X^b_{m1}. One possible effect of the shift of F_1 is more evident with this model. Notice in Figure 5.6 that it is very possible that the shift in $F^{A^o}_1$ could easily place it to the left of $(X^a_{m2})_o$ and the shift in $F^{B^o}_1$ could place it to the left of $(X^b_{m2})_o$. If such shifts take place F_2 is no longer an effective constraint for either product; consequently, the analysis is very similar to a one factor case.

Subsidizing a Variable Factor

Conclusions regarding subsidies are changed very little between this model and 4.1. The critical problem with this model involves the point C_2. Assume the firm starts at A_2 in Figure 5.5. Initially then

$$(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_1 + (P^o_{x_{m2}} - VC_{a2})(X^a_{m2})_1$$

$$> (P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_2 + (P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_2. \qquad (4.3.12)$$

For (4.3.12) to hold initially it must be that $VC_{a2} < VC_{b2}$ since

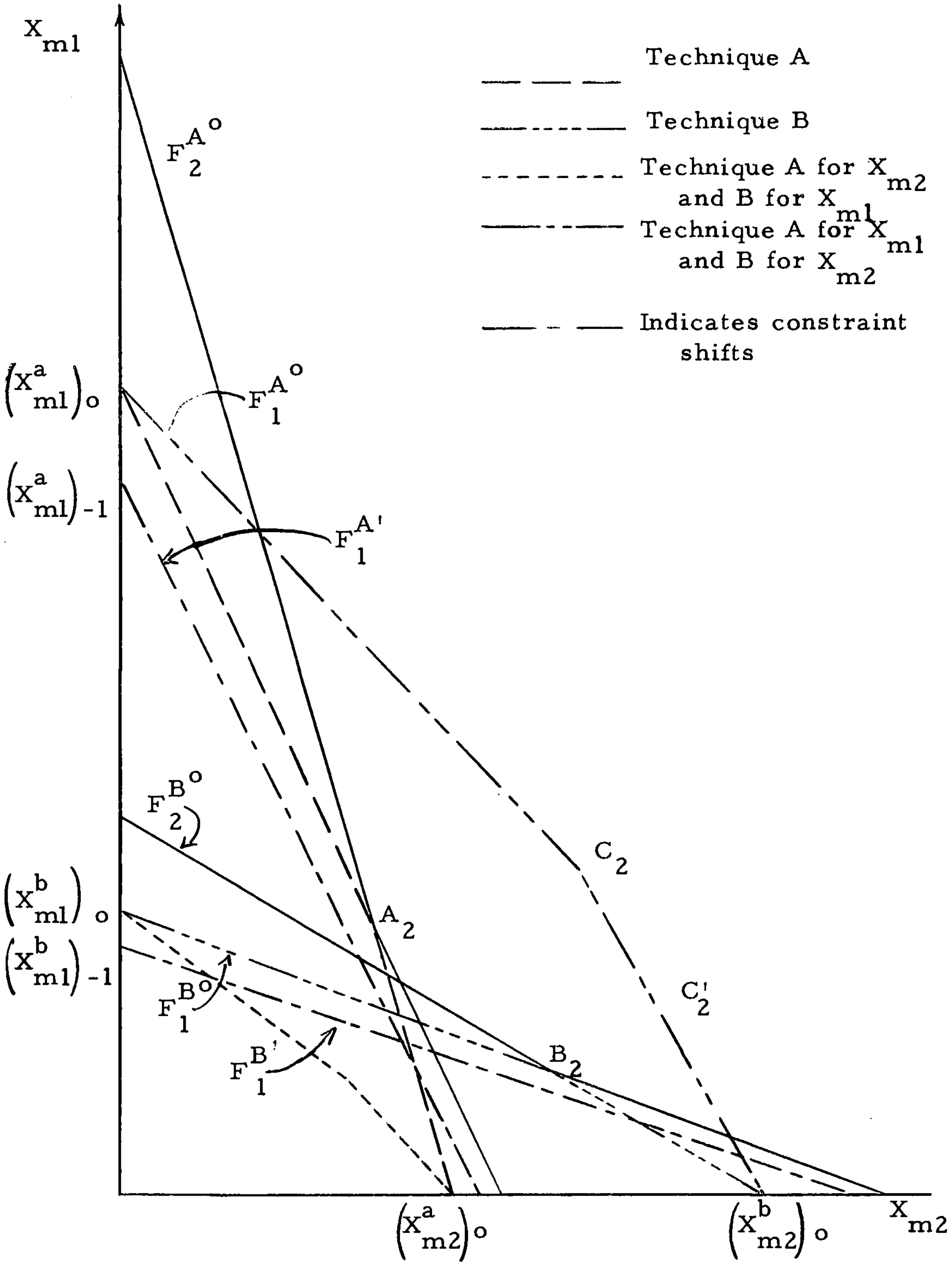

Figure 5.6. Constraint shifts due to mandatory standards, Model 4.3

$(X^a_{m1})_1 < (X^a_{m1})_2$ and $(X^a_{m2})_1 < (X^b_{m2})_2$. Assume a subsidy is placed on a product specific variable factor V_P. After the subsidy (4.3.12) becomes

$$\begin{aligned}&(P^o_{x_{m1}} - VC_{a1}((X^a_{m1})_1 + (P^o_{x_{m2}} - VC_{a2})(X^a_{m2})_1\\ &\quad + S_{vp}(v_{p1}(X^a_{m1})_1 + v_{p2}(X^a_{m2})_1) \overset{?}{\substack{\geq\\<}}\\ &\quad (P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_2 + (P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_2\\ &\quad + S_{vp}(v_{p1}(X^a_{m1})_2 + v_{p2}(X^b_{m2})_2). \qquad (4.3.13)\end{aligned}$$

Whether or not $v_{p1} \substack{\geq\\<} v_{p2}$ makes little difference for this analysis. Since more will be added to the right of (4.3.13) than to the left it is entirely possible the subsidy could induce a switch to C_2, i.e., the $(<)$ would hold in (4.3.13).

Suppose instead of a product specific variable, a technique specific variable were subsidized. Statement (4.3.12) would then become

$$\begin{aligned}&(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_1 + (P^o_{x_{m2}} - VC_{a2})(X^a_{m2})_1\\ &\quad + S_{vt}\, v_{ta}\left((X^a_{m1})_1 + (X^a_{m2})_1\right) \overset{?}{\substack{\geq\\<}}\\ &\quad (P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_2 + (P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_2\\ &\quad + S_{vt}\left(v_{ta}(X^a_{m1})_2 + v_{tb}(X^a_{m2})_2\right). \qquad (4.3.14)\end{aligned}$$

If $v_{ta} > v_{tb}$ there is a chance that the (>) will hold in (4.3.14) which implies the firm at least will not switch to C_2. However, finding a variable factor such that $v_{ta} > v_{tb}$ is contrary to the logic used previously which indicated that a variable factor which had coefficients such that $v_{ta} < v_{tb}$ would be the type to subsidize. Actually the condition necessary for a switch from A_2 to B_3 relates to total V_T instead of the relationship between v_{ta} and v_{tb}. If a switch from A_2 to B_3 is implied

$$\begin{aligned} &(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_1 + (P^o_{x_{m2}} - VC_{a2})(X^a_{m2})_1 \\ &\quad + S_{vt}\, v_{ta} \left((X^a_{m1})_1 + (X^a_{m2})_2\right) < \\ &\quad (P^o_{x_{m2}} - VC_{b2} + S_{vt}v_{tb})(X^b_{m2})_o \end{aligned} \tag{4.3.15}$$

$$\left.\begin{aligned} \Rightarrow S_{vt} > &\frac{(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_1 + (P^o_{x_{m2}} - VC_{a2})(X^a_{m2})_1}{v_{tb}(X^b_{m2})_o - v_{ta}\left((X^a_{m1})_1 + (X^a_{m2})_1\right)} \\ &\underline{- (P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_o} . \end{aligned}\right\} \tag{4.3.16}$$

For a positive subsidy to be indicated in (4.3.16) the total V_T used at B_3 must exceed that used at A_2. The latter is possible even if $v_{ta} > v_{tb}$. If $v_{ta} < v_{tb}$ the likelihood that the denominator of (4.3.16) is positive is improved; however, the possibility that the switch may be to C_2 also arises. To prevent a switch to C_2 requires just the right size of subsidy. Whether the subsidy to prevent a switch to C_2 needs to be smaller or greater than one that would induce a change to B_3 depends on the assumed initial conditions.[61]

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[61] For example, (use notation established in footnote 57) suppose that prior to the subsidy $NA_1\ (X^a_{m1})_1 + NA_2(X^a_{m2})_1 > NA_1\ (X^a_{m1})_2 + NB_2(X^b_{m2})_2 > NB_2(X^b_{m2})_o$. The subsidy must be greater than the right of (4. 3. 16) to induce a switch from A_2 to B_3. For a switch from B_3 to C_2 not to occur it must be that

$$NB_2(X^b_{m2})_o + S_{vt} v_{tb}\ (X^b_{m2})_o > NA_1\ (X^a_{m1})_2 + NB_2\ (X^b_{m2})_2 + S_{vt}\ \left(v_{ta}\ (X^a_{m1})_2 + v_{tb}\ (X^b_{m2})_2\right)$$

$$\Rightarrow S_{vt} > \frac{NA_1(X^a_{m1})_2 + NB_2(X^b_{m2})_2 - NB_2(X^b_{m2})_o}{v_{tb}\ (X^b_{m2})_o - \left(v_{ta}(X^a_{m1})_2 + v_{tb}(X^b_{m1})_2\right)} \qquad (1)$$

Notice that due to the assumed relationship between net revenues the numerator on the right of (1) is less than the numerator on the right of (4. 3. 16). The size of the denominators depends on the amount of variable factors used in total at A_2 and C_2. It is likely that the denominator in (1) could be smaller than the denominator in (4. 3. 16). If such be so (1) could be larger than (4. 3. 16) implying a larger subsidy is needed if the firm is not to switch just to C_2. Also the amount of V_T used at B_3 must exceed that at A_2 and C_2 if S_{vt} is going to be positive. If $v_{tb} > v_{ta}$ the likelihood of the latter is better than if $v_{ta} > v_{tb}$ although it is possible with either case.

Even if the denominator in (1) is smaller than that in (4. 3. 16) it is possible that (1) could be smaller than (4. 3. 16). The latter would imply that any subsidy large enough to induce a switch to B_3 would also induce a switch on to C_2. However, if total V_T use at A_2 is greater than at C_2 only a (>) can hold in (4. 3. 14) implying no change from A_2.

If in footnote 61, $NB_2(X^b_{m2})_o > NA_1(X^a_{m1})_2 + NB_2(X^b_{m2})_2$ then (1) would be negative implying that no subsidy would be necessary to exclude C_2 as a possible optimum. Of course one must at the same time assume that the denominator in (1) is positive.

If the initial firm position were at A_1 instead of A_2 some of the same types of problems could arise. However, subsidizing a product specific factor could alleviate some problems if it were used more by X_{m2} and X_{m1}.

Conclusions concerning the subsidy of a product <u>and</u> technique specific variable factor are not changed by this model. The factor subsidized should be used more by product X_{m2} and technique B to achieve the optimum policy point.

Changes in relative prices will affect the results achieved by this policy. If the optimum policy point is achieved the lowering of $P_{x_{m2}}$ relative to $P_{x_{m1}}$ could bring about a switch back to one of the less desirable points. Other conclusions are similar as for the previous models.

Summary and Implications

The application of a variable factor subsidy to this model demonstrated more clearly the possibilities of degrading the initial situation. The beginning position of the firm was quite important. Also the <u>total</u> amounts of the variable factors used at alternative points was critical. Furthermore, the relative per unit net revenues at alternative points was crucial in determining subsidy size. It was possible that too small a subsidy could move the firm to point C_2 from A_2 which would increase externality production. In some cases any subsidy large enough to induce a change from A_2 could prompt the firm to go to C_2. The main implication is that the subsidy size is quite sensitive which would hinder administering this policy.

Subsidizing a Fixed Factor

The conclusions and implications reached with Model 4.3 are similar to those for Model 4.1. There are some differences possible.

Suppose the firm is operating at A_1 in Figure 5.5. Assume a subsidy is placed on F_2. Since F_1 is the constraining factor for production of X_{m1} by technique A there can be no increase in production of X^a_{m1}. However F_2 is the constraining factor for producing X_{m2} by technique B. A lowering in the price of F_2 may make it feasible for the firm to switch to producing X_{m2} by technique B. The situation would be

$$(P^o_{x_{m1}} - VC_{a1})\,(X^a_{m1})_o \overset{?/}{\substack{\geq \\ <}} (P^o_{x_{m2}} - VC_{b2})\,(X^b_{m2})_o + (P^o_{x_{m2}} - VC_{b2})\frac{\Delta F_2}{f_{2b2}} - (P_{F_2} - S_{F_2})\,\Delta F_2. \qquad (4.3.17)$$

If the (<) holds in (4.3.17) the following is implied:

$$(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_o - (P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_o <$$

$$[(P^o_{x_{m2}} - VC_{b2})/f_{2b2} - (P_{F_2} - S_{F_2})]\Delta F_2. \qquad (4.3.18)$$

In words the net revenue added by ΔF_2 must exceed the cost of moving from A_1 to B_3 in Figure 5.5. There is a limit, however, on the size of ΔF_2 that can be used since at point B_4, F_1 becomes constraining. That is

max. $\Delta F_2 = f_{2b2}[(X^b_{m2})_4 - (X^b_{m2})_o]$ where $(X^b_{m2})_4 \equiv B_4$ in Figure 5.5.

It is also necessary that (4.1.26) hold so that just a product switch does not occur. In this case a subsidy on F_2 could bring about a technique switch regardless of whether per unit net revenue from X^a_{m2} equals, exceeds or is less than per unit net revenue from X^b_{m2}.[62] If $n_{b2}[(X^b_{m2})_o + \frac{\Delta F_2}{f_{2b2}}] < n_{a1}(X^a_{m1})_o$ the externality situation would be improved; however, there is the possibility that the

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[62] For Model 4.1 it was necessary that per unit net revenue from X^b_{m2} be greater than that from X^a_{m2} for the validity of (4.1.26). For the present model $(X^b_{m2})_o > (X^a_{m2})_o$ and $f_{2b2} < f_{2a2}$ which does away with that requirement.

switch could increase X_n production if the previous inequality went the other way. The rest of the conclusions concerning a subsidy are not different from those of Model 4.1.

Summary and Conclusions for Models 4.1 through 4.3

Two market products both generating the same externality were incorporated into these models. It was assumed that each product required some of each of two non-specialized fixed factors. The market products could be produced by two techniques. It was assumed that product X_{m2} generated less X_n per unit than did product X_{m1}. Furthermore if technique B were used with either product less X_n would be produced compared to method A. The three models differed as to the relative amounts of fixed factors required by products and techniques. Changes in variable cost assumptions were made as policies were applied to each model.

It was generally assumed that each firm would start producing X_{m1} with technique A. However in some instances that assumption was changed to examine effects on results. Not all possible starting points were analyzed for each model; consequently, results in Table 5.1 do not appear for many potential initial positions. The only initial point that was used consistently between models and policies was A_1. Therefore, viewing the policy results when A_1 was the beginning may provide more comparable information.

Table 5. 1. Summary of Effects of Policies on Models 4. 1-4. 3.

Policy			Model conditions					Possible effects of policies on[a]					
	Tax on	Model	Initial position	$VC_{a1} \gtreqless VC_{b1}$	$VC_{a2} \lesseqgtr VC_{b2}$	$(X^a_{m1})_o \gtreqless (X^b_{m1})_o$	$(X^a_{m2})_o \gtreqless (X^b_{m2})_o$	Switch		Net revenue	Output of		
								Tech.	To points		X_{ml}	X_{m2}	X_n
Taxing	X_{m1}	4. 1[b]	A_1	(≤)	(≤)	(>)	(>)	No	A_2, A_3	(-)	(-)	(+)	(-)
market	X_{m2}	4. 1	A_1	(≤)	(≤)	(>)	(>)	No	0	(-)	0	0	0
products	X_{m1}	4. 1	A_3	(≤)	any	(>)	(>)	No	0	(-)	0	0	0
	X_{m2}	4. 1	A_3	(≤)	any	(>)	(>)	No	A_1, A_2	(-)	+	(-)	+
	X_{m1}	4. 1	A_1	(≤)	(>)	(>)	(>)	Yes	B_3[c]	(-)	(-)	+	(-)
	X_{m2}	4. 1	A_1	(≤)	(>)	(>)	(>)	No	0	(-)	0	0	0
	X_{m1}	4.2	A_1	(<)	(<)	(<)	(>)	Yes	A_2, B_2[d]	(-)	(-)	+	(-)
	X_{m2}	4. 2	A_1	(<)	(<)	(<)	(>)	No	0	(-)	0	0	0
	X_{m1}	4. 2	A_3	(<)	(<)	(<)	(>)	No	0	(-)	0	0	0
	X_{m2}	4. 2	A_3	(<)	(<)	(<)	(>)	Yes	A_1, A_2, B_1, B_2	(-)	(+)	(-)	(+)
	X_{m2}	4. 2	A_3	(<)	(>)	(<)	(>)	Yes	B_3	(-)	0	(-)	(-)
	X_{m1}	4. 2	A_1	(<)	(>)	(<)	(>)	Yes	B_3[b]	(-)	(-)	(+)	(-)
	X_{m2}	4. 2	A_1	(<)	(>)	(<)	(>)	No	0	(-)	0	0	0
	X_{m1}	4. 2	B_1	any	any	(<)	(>)	Yes	A_1[f]	(-)	(-)	0	I
	X_{m1}	4. 3	A_1	(≤)	(≤)	(>)	(<)	Yes	B_3, C_2	(-)	(-)	+	(-)
	X_{m2}	4. 3	A_1	(≤)	any	(>)	(<)	No	0	(-)	0	0	0
	X_{m1}	4. 3	A_1	(≤)	(>)	(>)	(<)	Yes	B_3[c]	(-)	(-)	+	-
	X_{m2}	4. 3	B_3	any	any	(>)	(<)	Yes	A_1, C_2	(-)	+	-	+

Table 5. 1 (continued)

Policy			Model conditions					Possible effects of policy on[a]					
								Switch		Net	Output of		
	Tax on	Model	Initial position	$VC_{a1} \gtrless VC_{b1}$	$VC_{a2} \gtrless VC_{b2}$	$(X^a_{m1})_o \gtrless (X^b_{m1})_o$	$(X^a_{m2})_o \gtrless (X^b_{m2})_o$	Tech.	To points	revenue	X_{m1}	X_{m2}	X_n
Taxing	V_P	4. 1	A_1	(≤)	(≤)	(>)	(>)	No	A_2	(-)	(-)	+	(-)
variable	V_P	4. 1	A_1	(>)	(>)	(>)	(>)	Yes	B_3[c]	(-)	(-)	+	(-)
factors	V_T	4. 1	A_1	(≤)	(≤)	(>)	(>)	Yes	B_3[c]	(-)	(-)	+	(-)
	V_T	4. 1	A_1	(>)	(>)	(>)	(>)	Yes	B_3[c]	(-)	(-)	+	(-)
	V_D	4. 1	A_1	any	any	(>)	(>)	Yes	B_3[c]	(-)	(-)	+	(-)
	V_P	4. 2	A_1	(<)	(≤)	(<)	(>)	No	A_2	(-)	(-)	+	(-)
	V_P	4. 2	A_1	(<)	(>)	(<)	(>)	Yes	B_3[c]	(-)	(-)	+	(-)
	V_T	4. 2	A_1	(<)	any	(<)	(>)	Yes	B_3	(-)	(-)	+	(-)
	V_D	4. 2	A_1	(<)	any	(<)	(>)	Yes	B_3	(-)	(-)	+	(-)
	V_P	4. 2	B_1	(<)	any	(<)	(>)	Yes	A_1	(-)	(-)	0	+
	V_P	4. 3	A_1	any	any	(>)	(<)	Yes[g]	B_3	(-)	(-)	+	(-)
	V_T	4. 3	A_1	any	any	(>)	(<)	Yes[h]	B_3	(-)	(-)	+	(-)
	V_P	4. 3	A_1	any	any	(>)	(<)	Yes	B_3	(-)	(-)	+	(-)
	V_P	4. 3	B_3	any	(<)	(>)	(<)	Yes	A_3	(-)	0	(-)	+
	V_T or V_D	4. 3	B_3	any	any	(>	(<)	No[j]	0	(-)	0	0	0

Table 5.1 (continued)

Policy		Model	Model conditions					Possible effects of policies on[a]					
			Initial position	$VC_{a1} \gtrless VC_{b1}$	$VC_{a2} \gtrless VC_{b2}$	$(X^a_{m1})_o \gtrless (X^b_{m1})_o$	$(X^a_{m2})_o \gtrless (X^b_{m2})_o$	Switch Tech.	Switch To points	Net revenue	Output of X_{m1}	Output of X_{m2}	Output of X_n
Taxing the non-market externality		4.1	A_1	(≤)	(≤)	(>)	(>)	Yes	B_3	(-)	-	+	-
		4.1	A_3	(≤)	any	(>)	(>)	Yes	B_3	(-)	0	-	-
		4.1	A_1	any	(≥)	(>)	(>)	Yes	B_3	(-)	-	+	-
		4.1	B_3	any	(≥)	(>)	(≥)	No	0	(-)	0	0	0
		4.2	A_1	(<)	any	(<)	(>)	Yes	B_3	(-)	-	+	-
		4.2	A_3	any	any	(<)	(>)	Yes	B_3	(-)	0	-	-
		4.2	B_3	any	any	(<)	(≥)	No	0	(-)	0	0	0
		4.3	A_1	any	any	(>)	(<)	Yes	B_3	(-)	-	+	-
		4.3	A_3	any	(<)	(>)	(<)	Yes	B_3	(-)	0	+	-
		4.3	B_3	any	any	(>)	(<)	No	0	(-)	0	0	0
Standard on the quality of the externality[k]		4.1	A_1	any	any	(>)	(>)	Yes	B_3	(-)	-	+	-[l]
		4.1	B_3	any	(≥)	(>)	(≥)	No	0	(-)	0	0	0[l]
		4.2	A_1	(<)	any	(<)	(>)	Yes	B_3	(-)	-	+	-[l]
		4.2	B_3	any	(>)	(<)	(>)	No	0	(-)	0	0	0[l]
		4.3	A_1	any	any	(>)	(<)	Yes	B_3	(-)	-	+	-[l]
		4.3	B_3	any	any	(>)	(<)	No	0	(-)	-	+	-[l]
	Subsidy on												
Subsidizing variable factors	V_P	4.1	A_1	<	<	(>)	(>)	No	A_2	+	(-)	+	(-)
	V_P	4.1	A_1	>	>	(>)	(>)	Yes	B_3[c]	+	(-)	+	(-)
	V_P	4.1	A_3	any	any	( )	( )	No	0	+	0	0	0

Table 5. 1 (continued)

Policy			Model conditions					Possible effects of policies on[a]					
								Switch		Net	Output of		
	Subsidy on	Model	Initial position	$VC_{a1} \gtrless VC_{b1}$	$VC_{a2} \gtrless VC_{b2}$	$(X^a_{m1})_o \gtrless (X^b_{m1})_o$	$(X^a_{m2})_o \gtrless (X^b_{m2})_o$	Tech.	To points	revenue	X_{m1}	X_{m2}	X_n
Subsidizing variable factors	V_T	4. 1	A_1	any	any	$(>)$	$(>)$	Yes	B_3[c]	+	-	+	(-)
	V_D	4. 1	A_1	any	any	$(>)$	$(>)$	Yes	B_3[c]	+	-	+	(-)
	V_P	4. 2	A_1	$<$	$<$	$(<)$	$(>)$	No	A_2	+	(-)	+	-
	V_P	4. 2	A_1	$<$	$>$	$(<)$	$(>)$	Yes	B_3[c]	+	(-)	+	(-)
	V_P	4. 2	A_3	any	any	$(<)$	$(>)$	No	0	+	0	0	0
	V_T or V_D	4. 2	A_1	$<$	any	$(<)$	$(>)$	Yes	B_3[c]	+	(-)	+	(-)
	V_P	4. 3	A_1	any	any	$(>)$	$(<)$	Yes	B_3[c]	+	-	+	-
	V_P	4. 3	A_2	any	$<$	$(>)$	$(<)$	Yes	B_3, C_2[n]	+	(-), +	+	(-), +
	V_T	4. 3	A_1	any	any	$(>)$	$(<)$	Yes	B_3, C_2	+	(-)	+	(-)
	V_T	4. 3	A_2	any	$(<)$	$(>)$	$(<)$	Yes	B_3, C_2	+	+	+	-
	V_D	4. 3	A_1	any	any	$(>)$	$(<)$	Yes	B_3[c]	+	-	+	-
Subsidizing fixed factors	F_1	4. 1	A_1	any	any	$(>)$	$(>)$	No	[o]	+	+	0	+
	F_2	4. 1	A_1	any	$(<)$	$(>)$	$(>)$	No	[p]	+	-	+	I[r]
	F_2	4. 1	A_1	any	$(>)$	$(>)$	$(>)$	Yes	[q]	+	-	+	I[r]
	F_1	4. 1	A_2	any	any	$(>)$	$(>)$	No	[o]	+	+	-	+
	F_2	4. 1	A_2	any	$(>)$	$(>)$	$(>)$	Yes	[q]	+	-	+	I[r]

Table 5.1 (continued)

Policy			Model conditions					Possible effects of policies on[a]					
	Subsidy on	Model	Initial position	$VC_{a1} \gtreqless VC_{b1}$	$VC_{a2} \gtreqless VC_{b2}$	$(X^a_{m1})_o \gtreqless (X^b_{m1})_o$	$(X^a_{m2})_o \gtreqless (X^b_{m2})_o$	Switch Tech.	To points	Net revenue	Output of X_{m1}	X_{m2}	X_n
Subsidizing	F_1	4.2	A_1	≮	any	(≮)	(>)	No	o	+	+	0	+
fixed	F_2	4.2	A_1	<	(≮)	(≮)	(>)	No	p	+	+	+	I^r
factors	F_2	4.2	A_1	<	(>)	(≮)	(>)	Yes	q	+	-	+	I^r
	F_1	4.2	A_2	any	any	(≮)	(>)	No	o	+	+	-	+
	F_2	4.2	A_2	any	(>)	(≮)	(>)	Yes	q	+	-	+	I^r
	F_1	4.3	A_1	any	any	(>)	(<)	No	o	+	+	0	+
	F_2	4.3	A_1	any	any	(>)	(<)	Yes	B_4	+	-	+	I^r
	F_1	4.3	A_2	any	(≮)	(>)	(<)	No	o	+	+	-	+
	F_2	4.3	A_2	any	(>)	(>)	(<)	Yes	q	+	-	+	I^r

Table 5.1. (continued)

Policy			Possible effects of policies on[a]				
			Use of				
	Tax on	Model	V_P	V_T	V_D	F_1	F_2
Taxing market products	X_{m1}	4.1	NA	NA	NA	0, (-)	(+)
	X_{m2}	4.1	NA	NA	NA	0	0
	X_{m1}	4.1	NA	NA	NA	0	0
	X_{m2}	4.1	NA	NA	NA	+	(-), 0
	X_{m1}	4.1	NA	NA	NA	(-)	(+)
	X_{m2}	4.1	NA	NA	NA	0	0
	X_{m1}	4.2	NA	NA	NA	0, (-)	(+)
	X_{m2}	4.2	NA	NA	NA	0	0
	X_{m1}	4.2	NA	NA	NA	0	0
	X_{m2}	4.2	NA	NA	NA	+	(-), 0, (-), 0
	X_{m2}	4.2	NA	NA	NA	I[e]	(0)
	X_{m1}	4.2	NA	NA	NA	(-)	(+)
	X_{m2}	4.2	NA	NA	NA	0	0
	X_{m1}	4.2	NA	NA	NA	0	0
	X_{m1}	4.3	NA	NA	NA	(-), I	(+), I
	X_{m2}	4.3	NA	NA	NA	0	0
	X_{m1}	4.3	NA	NA	NA	(-)	+
	X_{m2}	4.3	NA	NA	NA	+, I	(-), I

Table 5.1 (continued)

Policy			Possible effects of policies on[a]				
	Tax on	Model	Use of				
			V_P	V_T	V_D	F_1	F_2
Taxing variable factors	V_P	4.1	(-)	NA	NA	(0)	+
	V_P	4.1	(-)	NA	NA	(-)	+
	V_T	4.1	NA	(-)	NA	(-)	+
	V_T	4.1	NA	(-)	NA	(-)	+
	V_D	4.1	NA	NA	(-)	(-)	+
	V_P	4.2	(-)	NA	NA	(0)	+
	V_P	4.2	(-)	NA	NA	(-)	+
	V_T	4.2	NA	(-)	NA	(-)	+
	V_D	4.2	NA	NA	(-)	(-)	+
	V_P	4.2	(-)	NA	NA	0	0
	V_P	4.3	(-)	NA	NA	(-)	+
	V_T	4.3	NA	(-)	NA	(-)	+
	V_P	4.3	NA	(-)	NA	(-)	+
	V_P	4.3	(-)	NA	NA	I[i]	0
	V_T or V_D	4.3	0	0	0	0	0

Table 5. 1 (continued)

Policy		Model	Possible effects of policies on[a]				
			Use of				
			V_P	V_T	V_D	F_1	F_2
Taxing the non-market externality		4. 1	NA	NA	NA	-	+
		4. 1	NA	NA	NA	I^e	0
		4. 1	NA	NA	NA	-	+
		4. 1	0	0	0	0	0
		4. 2	NA	NA	NA	-	+
		4. 2	NA	NA	NA	I^e	0
		4. 2	0	0	0	0	0
		4. 3	NA	NA	NA	-	+
		4. 3	NA	NA	NA	I^e	0
		4. 3	0	0	0	0	0
Standard on the quality of the externality[k]		4. 1	NA	NA	NA	-	+
		4. 1	0	0	0	0	0
		4. 2	NA	NA	NA	-	+
		4. 2	0	0	0	0	0
		4. 3	NA	NA	NA	-	+
		4. 3	0	0	0	0	0
	Subsidy on						
Subsidizing variable factors	V_P	4. 1	$+^m$	NA	NA	0	+
	V_P	4. 1	+	NA	NA	-	+
	V_P	4. 1	0	0	0	0	0

Table 5. 1 (continued)

Policy			Possible effects of policies on[a]				
			Use of				
	Subsidy on	Model	V_P	V_T	V_D	F_1	F_2
Subsidizing variable factors	V_T	4. 1	NA	+	NA	-	+
	V_D	4. 1	NA	NA	+	-	+
	V_P	4. 2	$+^m$	NA	NA	0	+
	V_P	4. 2	+	NA	NA	0	+
	V_P	4. 2	0	0	0	0	0
	V_T or V_D	4. 2	NA	+ or	+	-	+
	V_P	4. 3	$+^m$	NA	NA	-	+
	V_P	4. 3	+	NA	NA	-, I	0, I
	V_T	4. 3	NA	+	NA	-, I	+, I
	V_T	4. 3	NA	+	NA	-, I	0, I
	V_D	4. 3	NA	NA	+	-	+
Subsidizing fixed factors	F_1	4. 1	NA	NA	NA	+	+
	F_2	4. 1	NA	NA	NA	0	+
	F_2	4. 1	NA	NA	NA	0	+
	F_1	4. 1	NA	NA	NA	+	0
	F_2	4. 1	NA	NA	NA	0	+
	F_1	4. 2	NA	NA	NA	+	+
	F_2	4. 2	NA	NA	NA	0	+
	F_2	4. 2	NA	NA	NA	0	+
	F_1	4. 2	NA	NA	NA	+	0
	F_2	4. 2	NA	NA	NA	0	+

Table 5. 1 (continued)

Policy			Possible effects of policies on[a]				
			Use of				
	Subsidy on	Model	V_P	V_T	V_D	F_1	F_2
Subsidizing fixed factors	F_1	4. 3	NA	NA	NA	+	+
	F_2	4. 3	NA	NA	NA	0	+
	F_1	4. 3	NA	NA	NA	+	0
	F_2	4. 3	NA	NA	NA	0	+

[a] The (+), (-), and (0) refer to increases, decreases, and no change, respectively. If only one designation appears it implies that this type of change held for switches to all points indicated. If more than one designation appears it indicates the change associated with the particular point appearing in the same order under the tolumn, Technique Switch To. The letter I indicates an indeterminant situation under the assumed condition; a NA stands for not applicable, irrelevant to this policy or not considered in the analysis.

[b] Where the possibility of a change depended only on tax or subsidy size, the effects of only the change are shown. In instances where a change was possible, it was also possible that no change would occur with a given size of tax or subsidy. It is also possible that tax size necessary to induce a switch could force the firm from business.

[c] Other second best points such as A_2 or B_2 may be optimal after the tax or subsidy. However, it is possible for a switch to the optimum policy point to occur so the other points are ignored.

[d] It is possible to construct the model such that $(X^a_{m1})_o > (X^b_{m1})_1$. Such construction would not violate initial assumptions. The switch could conceivably be to C_1 or C_2 also.

[e] Indeterminant since without specific coefficients it is not possible to determine how F_1 use changes. Even though X_{m2} decreases, technique B requires more F_1 per unit of X_{m2} than A. Comparison of F_1 use at $(X^a_{m2})_o$ versus $(X^b_{m2})_o$ would need to be made.

[f] This merely points out one possibility which could be bad. Other points may be possible after the tax.

[g] A switch to B_3 depends on $v_{p1}(X^a_{m1})_o > v_{p2}(X^b_{m2})_o$.

[h] A switch to B_3 depends on $v_{ta}(X^a_{m1})_o > v_{tb}(X^b_{m2})_o$.

Table 5.1 (continued)

[i] Indeterminant since assumed conditions do not give enough information. That is, the relevant comparison is

$$\frac{f_{1a2}(X^{a}_{m2})_{o}}{f_{1b2}(X^{b}_{m2})_{o}} = \frac{f_{1a2}\ \dfrac{F^{o}_{2}}{f_{2a2}}}{f_{1b2}\ \dfrac{F^{o}_{2}}{f_{2b2}}} = \frac{f_{1a2}f_{2b2}}{f_{1b2} \cdot f_{2a2}}$$

Initial assumptions were $f_{1a2} > f_{1b2}$ and $f_{2a2} > f_{2b2}$ which does not permit assessment of the above ratio.

[j] No switch will occur if V_T or V_D use at all other points exceeds that at B_3.

[k] The only cases summarized are where conversion of X_n to X'_n requires variable costs and use of F_1.

[l] Actually no X_n will be released if enforcement is effective. However, a (-) indicates that the volume of X'_n released could be decreased by the switch whereas a + would indicate an increase in volume of X'_n.

[m] Notice that the V_P, V_T, and V_D for this policy are not the same as the same designated variables for taxes. That is $v_{p1} < v_{p2}$, $v_{ta} < v_{tb}$, and $v_{dal} < v_{db1}$, $v_{da2} < v_{db2}$, $v_{dal} < v_{da2}$, and $v_{db1} < v_{db2}$.

[n] Both possibilities are shown since there is a very real danger the switch could be to C_2 instead of B_3 depending on subsidy size and relative amounts of V_P or V_T used at C_2 versus B_3. A switch to C_2 would only constitute a partial technique switch.

[o] The point it could shift to does not have a letter. Without augmenting F_2 it could shift to the maximum X^{a}_{m1} as constrained by F_2.

[p] It could shift to the maximum X^{a}_{m2} as constrained by F_1.

[q] It could shift to the maximum X^{b}_{m2} as constrained by F_1.

[r] Indeterminant since no comparison is made between amount of X_n produced at these maximum points.

Notice that the summarized effects are only "possible" effects. In other words other outcomes may be possible but those shown are either considered to be the most important or the most likely. Sometimes more than one point is shown as being a possible optimum after the policy. In such cases the effects on other items such as use of F_2 are shown in multiple also. For example taxing X_{m1} for Model 4.1 (first row in Table 5.1) resulted in a possible switch to points A_2 or A_3. The use of F_1 would not change if the move were to A_2 or decline if it were to A_3. If only one designation is shown in some column associated with a multiple point switch it indicates that that designation is relevant for all points.

Table 5.1 does not show after policy effects of price changes. This item was omitted since there are several types of price changes to consider. Also in most cases there was some type of price movement that could negate the effects of the various policies. Yet, one must be aware of relative price changes particularly. However, absolute price changes (where relative prices are constant) were important in some instances.

The particular market product taxed was important to the outcome of the first policy. Also the relationships between variable costs of techniques and the initial firm position were also important. Depending on the previously mentioned items and the particular model it can be observed that taxing X_{m2} could increase production of X_n. Taxing X_{m1} did not, for the situations considered, appear to be as likely to result in increasing X_n. The implication is that when a firm is producing more than one product, selective taxing may be a better policy than taxing all products. Also improper selection of the product to be taxed could aggravate externality production.

Taxing variable factors was more complicated for these compared to previous models. However, the general implications seem not unlike earlier analyses. In other words the variable factor to tax is one that is used more in total at the least desirable production possibility point.

Product specific variable factors appeared to be, in a sense, more dangerous to tax than technique specific factors. The latter is so since in some cases taxing V_P at best could result in a product switch only. Taxes on technique specific factors could in the same cases result in technique switches as well as product. Of course it appears that taxing a factor that is both product and technique specific would be the most desirable.

Taxing the non-market externality again resulted in the most consistent results with respect to effects on X_n production. Whenever a change occurred it could have been to the least externality producing point. This policy was not as sensitive to changes in variable cost assumptions as were the previous two. This can be observed by noticing that "any" variable cost assumption appears quite frequently in the rows for the present policy. The "any" appears infrequently for the first policy and somewhat more frequently for the second.

If the firm should happen to be at the optimum policy point initially the externality tax would not prompt any changes. However, taxing variable factors and market products could possibly induce a switch away from the optimum.

Placing a standard on the externality appeared to provide consistent results also. One advantage of this policy is that X'_n will be released instead of X_n. If a switch did occur due to the standard it was possible it could be to B_3 (the policy optimum).

As discussed in previous chapters the standard must be enforceable to be effective. This implies a need for identification of externality production. If a firm should happen to switch techniques and/or products due to the policy it is just coincidental. The latter is so since the policy agency has virtually no control over VC_n once the level of the standard is set. If an externality tax were used then the responsible agency could vary tax size to prompt internal changes.

Subsidizing variable factors could result in desirable internal firm adjustments. As with a tax, the proper variable to be subsidized must be selected with care. A subsidy on a variable that is used more in total at the optimum point than any other production possibility point would be ideal. Depending on starting points, the subsidy could possibly result in an undesirable switch. However, if care is exercised in variable selection such problems could be minimized.

Care must also be exercised with respect to subsidy levels. It is possible that too large a subsidy could stimulate firm growth and eventually result in the production of more X_n.

Subsidizing factors that are fixed in the short run provided results that were difficult to interpret and inconsistent. Depending on initial position there appeared to be a good chance that externality production might be increased.

If subsidizing of fixed factors is going to work at all, the proper factor must be chosen. The more of a given factor required by the least externality generating technique and product, the more likely a desirable switch will occur. The danger also exists for this policy that fixed factor subsidies could stimulate firm growth and end up aggravating the problems.

VI. POLICY EFFECTS ON ALL MODELS

In the four previous chapters expected effects of various quality control policies on alternative models were examined. The purpose of the present chapter is to summarize the results by policy. Emphasis will be given to the particular policy aspects suggested in Objectives 2 and 3. That is, policies will be examined to determine consistency over all models and whether or not any policy may generate effects similar to effluent charges.

Finally policies that seem theoretically feasible for controlling irrigation return flow pollution will be set out. Any apparent shortcomings of such policies will then be discussed.

Taxing Market Products

All models except the last three were assumed to produce only one market product. Such an assumption is not entirely realistic although many things could be cast in the one product framework. Specifically, firms that produce several products in relatively fixed proportions could be represented by the single product cases. Nevertheless, a two product case was considered to see if policy effects might differ.

Taxing the only market product under certain conditions did result in desirable effects. However, it was more generally true

true that this policy did not result in any change internally as far as technique of production was concerned. In some instances externality (X_n) production was increased rather than decreased by this tax. In other cases X_n production declined after X_m was taxed. The conclusion is that taxing the only market product produced is not a consistent policy as far as effects on externality production are concerned.

When two products were considered results were not greatly different. Under certain model conditions taxes on either market product could induce the firm to switch to the optimum policy point. However, taxing X_{m1}, the highest externality generating product more consistently prompted switches which would reduce X_n. Taxing X_{m2} sometimes actually resulted in an increase in production of X_n.

If a switch resulted from taxing a market product the switch was always accompanied by a decline in the level of the market product taxed. This result held true for the two market product models as well as the single product ones.

Effects of the policy on variable and fixed factor use depended quite strongly on the particular model assumed. Consequently, no general statements seem appropriate.

Even when the policy was effective its results could change over time if prices of the market products shifted. Even though it was not specifically considered in the analyses it should be apparent that changes in variable factor costs could also affect policy results over time. In other words in instances where price increases could induce a switch back, falling variable costs might have the same effect.

As expected the tax on market products always decreased net revenue. Such an occurrence had to occur due to the assumed model conditions. It is possible that outside the theoretical world such an event may not occur, but this possibility will be discussed later.

The policy of taxing market products as a means of controlling jointly produced externalities does not appear to be theoretically sound. Even though the linear models considered here are only a minor subset of all possible firm models, the lack of consistency of effects should in itself make this policy questionable.

Taxing Variable Factors

When this policy was applied to the various models fixed factors were not permitted to vary from original levels. In other words this policy was viewed strictly in a short run framework.

For most models the tax was applied to only one variable factor. When more than one product was possible, taxing of three variable factor types was considered. Specifically the types were product specific, technique specific and product and technique specific.

In general, it was possible for all models, to specify variable factors such that when taxed internal changes occurred. The types of changes depended on the initial firm position, the relative variable costs between techniques and products, and the total amount of the variable factor used at various production possibility points. The only times changes of some type did not occur were when total variable factor use at the initial point was less than total use at other possibilities. The latter implies that a necessary characteristic of a prospective taxable variable factor is that it be used more in total by the least desirable techniques and products. It is important to recall that points of production compared were at certain levels due mainly to fixed factor constraints. In other words when a firm is examined for the purpose of determining likely taxable factors, the constrained conditions must be considered.

The effects of this policy on externality production were reasonably consistent over all models. There were, however, some notable exceptions. First of all it was necessary that total externality production be highest at the same points variable factor use was the largest. If the latter were the case then any changes induced by the tax had to have reduced externality production. In some models the necessary condition above was not automatically assumed; consequently, effects on X_n production were indeterminant. One relevant implication then is that inspection of a firm to be affected by this tax must consider total externality production at alternative production possibility points. Obviously such an informational requirement could make administration of the policy quite difficult. Another implication is that firm types whose production processes demonstrate a strong and positive relationship between a given variable factor and externality production are likely to respond in a favorable (i.e., reduce X_n) direction if that factor were taxed. Determination of the previous relationship may in some cases be less difficult than determining relative levels of externality production at various points.

The other exception to the consistency of this policy effects on X_n involved the two product models. Under certain conditions with some initial starting points taxing a product specific variable factor increased X_n production. With the same model conditions, taxing a technique specific or technique and product specific variable factor appeared less likely to result in a change that would increase X_n. Of course if the necessary condition discussed in the previous paragraph had held such results would not have been possible. A weak implication from the above is that technique or technique-product specific factors are better candidates for taxation than product specific factors.

The effects of the policy on production of market products was influenced by the model and initial firm positions. For example in instances where X_m or X_{m1} production at the initial starting point was less than other possibility points induced changes could have resulted in increases in X_m or X_{m1}. For the two product models it was generally assumed that initially the firm was producing only X_{m1}. Consequently, changes induced by the taxes usually resulted in reduction in X_{m1} level and an increase in X_{m2}. If the initial firm position involved production of both products this policy at least did not increase production of both products.

Size of tax was found to be quite critical. The most common result was that a tax could be too small and not result in any internal changes. However, under certain models and conditions a tax could be too large. The latter situation resulted in a switch that did reduce X_n production some but not as much as a smaller tax. The implication is that tax size can be quite critical; consequently, accurate _a priori_ prediction of effects will require very good information.

In most situations where the tax induced a switch, changes in prices of market products could have induced a reversal. The only times such an event did not occur was when, after the tax, the optimal technique and product dominated all others. The implication to be drawn is that a variable factor tax may need to be flexible. To explain further, suppose a tax level were finally determined that was effective. If prices changed the reduction of X_n prompted by the tax could be reversed. The policy agency would then need to increase or decrease the tax to maintain the reduced level of X_n.

In reality what is needed is a tax that will automatically maintain a given difference in net revenues between points. One easy solution would be for the tax to increase in absolute terms exactly the same per unit of output as price went up. However, the latter then in effect places a product tax on top of a factor tax in the face of rising prices. Or in essence returns to the firm are controlled.

Determining the size of adjustment in a factor tax needed due to changing prices appears to be quite difficult. This point can be demonstrated. Suppose that after a tax a single product firm switched to technique B. After the tax conditions are

$$(P^o_{x_m} - VC_b - T_v v_b)\, X^{b^o}_m > (P^o_{x_m} - VC_a - T_v v_a)\, X^{a^o}_m, \qquad (6.1)$$

$$\longrightarrow \frac{(P^o_{x_m} - VC_b - T_v v_b)}{(P^o_{x_m} - VC_a - T_v v_a)} > \frac{X^{a^o}_m}{X^{b^o}_m} \,. \qquad (6.2)$$

As can be observed the ratio in (6.2) must be maintained in the face of changing prices to maintain the switch. If P_{x_m} goes up the numerator and denominator of (6.2) will increase by the same absolute amounts. To determine how much T_v should be increased the values of v_a, v_b and the ratio size must be known. Also notice that if the ratio on the left of (6.2) is greater than one, price increases will decrease the ratio which could lead to a switch back to A. If the original ratio is less than one increasing P_{x_m} will increase ratio size thus not affecting the inequality direction in (6. 2). Consequently, depending on conditions a tax increase may or may not be necessary to maintain the desired production point in face of rising prices. If it is not needed the increasing tax would place an unnecessary cost burden on the firm.

The solution to the problems of tax levels due to changing prices may lie in combining control policies. Possibly the controlling agency could monitor certain industries periodically after and during periods of rising (falling) prices. If effluent quality were to change then the tax level may be increased (decreased). The problem with this solution is timing and the fact that the effluent may already have damaged the environment before the higher tax is effective. It may be that for certain types of industries determining the factors mentioned as necessary for assessing tax size may be relatively easy. For these types of firms the problem may be dealt with. Also industries for which product prices do not tend to fluctuate rapidly may better lend themselves to variable factor taxes.

One other obvious effect of the tax was that net revenue was decreased. This result occurred for all policies involving taxation.

Taxing the Non-market Externality

This policy generated the most consistent results of all policies as far as X_n production was concerned. If the firm started at some position other than the optimum policy point the externality tax was always capable of inducing a switch to that "best" point. If the beginning position were the "best" point then the tax did not induce a switch away from it. These effects resulted since a necessary condition for a switch was that net revenue be decreased more at the initial position than at some other points. For the latter to occur the externality production at the initial point had to be higher than at these other points. Consequently, this policy appears to be the best as far as reducing X_n is concerned if it can be administered.

The effects of externality tax on other variables was somewhat less predictable. Like other policies, the models and initial starting positions were important determinants of the policy effects on such things as market product production and variable factor use. It appears that the only time variable factor use may be accurately predicted is when a factor is strongly related in a positive sense to X_n production.

As with previous policies changes in the prices of market products did affect the results. The types of price changes (i.e., increases or decreases) necessary to prompt a switch back depended on the model, variable cost relationships and fixed factor constraints. Again some form of a flexible tax appears necessary if prices tend to fluctuate over time. Similar to the previous policy, however, working out the relationship between price changes and needed tax level adjustments would be difficult.

A major problem concerning the use of this policy involves administration. The problems created for administration by changing product prices was already mentioned. Possibly more important are identification problems. That is, before an externality can be taxed it must be identified. Furthermore once identified its source must also be determined. In many instances identification and source will not be a problem. Of course there will be administrative costs for monitoring and tracing but these may not be high. For example pollution from a large feedlot on the bank of a live stream should be relatively easy to identify and trace to its source. Also many industries with discharge pipes emptying directly into waterways should be evident. Air pollution from industries may, however, present a more difficult problem. How great the problem depends in part on the procedures used. For example suppose air quality in general is being monitored. Assume an increase in the level of some detrimental chemical is noted. If there are many sources of that pollutant (ignore synergistic effects) then the tracing job could become quite difficult. Of course synergistic effects could make identification of the cause even more difficult. On the other hand if the monitoring devices were on individual industrial stacks then the source of most changes would be known. However, monitoring costs would be higher. Similar problems could arise in water pollution if the stream, only, is being monitored versus individual discharge points. Research concerning the tradeoffs between monitoring costs and source tracing would be quite helpful.

There are some instances where identification of source could be almost impossible. The problem to be considered in the next chapter is a case in point. Return flows from irrigation are known to alter water quality in receiving streams. Tracing the changes to their sources appears to be almost impossible. For example a given drain may collect return flows from several farms. Also, farmer A may use water that came from farmer B's fields. By the time water reaches the stream being affected so much intermingling could have occurred that tracing back to A or B would at best be very costly and at worst, impossible.

Another example where source identification could be a problem involves underground streams. Many industries could be involved with these problems. Irrigation definitely affects ground water. Industries using settling basins may also be changing ground water quality.

One advantage of a taxing scheme is that it generates income for administration. Such income could help alleviate the costs of some of the problems mentioned.

In conclusion, taxing the externality appears to be a very good policy if identification of the externality as to type and source could be achieved. In instances where such identification is extremely difficult and/or costly other policies may be better.

A Standard on the Quality of the Externality

The standard was defined for this work in terms of proportions. In other words if a standard were set on X_n it would be in terms of parts per million of certain characteristics. Nothing in the definition was said about total load, which many standards consider. Within the framework of these models total load can be affected in at least two ways. One way is by the policy agency setting lower proportion requirements in its standard. For example the standard could be set at 100 parts per million for total dissolved solids versus 200 parts per million. Secondly, the policy could conceivably induce the firm to make internal adjustments which would decrease volume of X'_n thus reducing total stream load. For some models and conditions it was possible that a standard as defined above could increase volume of X'_n released over the level of X_n. Such an event could conceivably degrade the stream quality if the increase were large. The most common effect among all models was a decrease in volume of X'_n versus X_n.

Since a possible volume increase might occur with some models it may be better to place a standard that would also control total load. Even though such a policy was not specifically considered it seems that the analytical procedures for models in this analysis would not be much different. If a firm knew what its total load could be it could adjust the process of converting X_n to X'_n accordingly. In other words the firm may need to increase the levels of variable and fixed factors used to convert X_n to X'_n. The latter would be reflected in the models by changing the sizes of VC_n, k_n and f_{1n}.

If the standard effected a change in internal processes it usually resulted in a decrease in production of the market product. In the two product models the higher externality generating product declined and the other usually increased. In some cases the effect on market product production was indeterminant since what happened depended on initial conditions.

One distinct advantage of a standard is that the quality of the externality being produced would have to be altered. Of course that statement assumes the standard could be enforced. The other policies tend to influence level of externality production although they might inadventently alter the quality. The latter possibility will be discussed subsequently.

The standard has many of the same administrative problems as associated with the externality tax. In addition, the enforcement problem looms larger for the standard. In other words, even if the identification problems could be solved the policy must be enforced using the police power concept. Such enforcement may make continual or at least periodic monitoring a necessity. Additionally, the standard does not generate any income unless fines are levied against violators. If everyone is in compliance with the standard no income would be forthcoming. With taxing schemes there is likely to be continual income even if the externality being released is not considered to be a problem.

The fact that the standard if enforced will definitely alter the externality quality makes it attractive as a prospective policy. Changes in product prices may affect internal processes but cannot affect the externality produced. The removal of changes in X'_n due to price changes also adds to the desirability of this policy. Costs of administration need to be weighed against these positive aspects of improving quality, however. It was not within the scope of this study to make such comparisons.

Subsidizing Variable Factors

This policy is the antithesis of taxing variable factors. Instead of searching for factors used most by high externality producing products and techniques the search here must be for factors used least by these. In other words the idea here is to in a sense use bribery to induce firms to use the more desirable production methods and produce less X_n generating products.

Depending on initial starting points the policy could result in declines in externality levels. However, under some conditions X_n actually increased. The effect of the subsidy on market product levels varied between models. It was not uncommon for X_m production to be increased in the single product models while X_{m1} generally declined for the dual product models. Market product price changes could negate the subsidy effects also. Consequently, variable subsidy size should be considered.

After the subsidy was applied net revenue increased. This feature would tend to make this policy appealing to the firms being affected. On the other hand since the policy will require outlays of money in addition to administrative costs it does not seem likely to be appealing to the public.

The increase in net revenue could result in another problem alluded to in earlier chapters. Additional revenue could stimulate firm growth. This growth could then result in increases in the production of the externality as well as the market products. The growth possibility seems to be a serious drawback to the use of the subsidy.

One possibility may be to use the variable factor tax and subsidy policies together. In other words pay the firms to use certain variable inputs and tax them for using others. Hopefully it could be worked out so that the firm's net income would not be affected greatly. This could alleviate the growth problem as well as apply two stimuli to firms for making internal method changes. If the tax generated enough income to pay for the subsidy, public opposition may be less. However, the possibility that over time, tax income would go down and subsidy costs would go up is very real and should be considered. A periodic reassessment of the policies and their use could help alleviate the latter possibility. Maybe a tax on certain factors that would increase over time could be coupled with a time declining subsidy on other factors.

Subsidizing Fixed Factors

This policy was only applied to the models which used more than one fixed factor. Subsidizing fixed factors where only one is used appeared to only increase size of operation.

The effects of the policy on externality production was usually indeterminant. This situation does not improve the usefulness of the policy. Part of the indeterminancy was due to the growth concept introduced by the subsidy. Some indeterminancy arose since level of externality production depended on which factors were initially constraining. The latter problem was most noticeable in Models

3.1-3.5' where factors specialized as to technique were assumed.

Subsidizing fixed factors of production appears to be more applicable where the factors are specialized by technique and/or product. One reason for the latter is that specialized factors are likely to be easier to associate with given techniques and/or products than are non-specialized factors. Consequently, if a specialized factor associated with a desirable technique is subsidized, the direction of change due to the subsidy should be quite predictable. On the other hand, subsidizing multi-purpose factors may make the results quite difficult to predict especially since effects depend strongly on initial conditions and beginning production possibility points.

The desirability of fixed factor subsidies for specialized factor models became more apparent when such factors were the relevant initial constraint. When the latter occurred all policies other than the subsidies did not induce internal changes unless significant income was forthcoming from resale of other fixed factors. With a subsidy, changes could occur without this salvage income. In many instances specialized factors will have very low salvage value. A good example is a surface irrigation system on a farm. One way of inducing a change if the system were the effective constraint would be to highly subsidize for example automated sprinkler systems.

It also seems likely that specialized factors are common in many industries. The latter certainly is true in agriculture. Subsidizing non-specialized factors may do nothing except increase production of the market products and the externality.

The main undesirable effect from a policy viewpoint of any subsidy is the growth potential. The implication is that subsidy size is quite important and care must be exercised to avoid growth. Of course growth accompanied by technique changes may be okay. If only the factor subsidized is constraining it will make little difference whether it is specialized by technique or not. In other words the number of fixed factors likely to be constraining is still an important consideration. If other factors that could be constraining are used in production then growth could at least be limited to the upper level of the other constraint.

Subsidizing fixed factors did induce technique and/or product changes. These changes were accompanied by an increase in use of the factor subsidized. In the case of non-specialized factors, the other factor also increased in use due to the induced changes. Widespread subsidies then might eventually raise the prices of these fixed factors necessitating higher subsidies, etc. Such a spiral could increase costs of the subsidy policy. As with the previous subsidy these added costs to public may well be unpopular. However, to the firms affected subsidies are likely to be welcomed since after the subsidy revenues should be higher.

In one sense the fixed factor subsidies may be better than the variable factor type. Many fixed factors are relatively indivisible. Indivisibility implies an all or nothing situation. That is, to get any subsidy at all the firm must make a major step or large change. With variable factors and divisible fixed factors some subsidy could be acquired with small changes. Again the importance of specialized factors comes up. A firm might get some subsidy income from a non-specialized subsidy without making any changes. However, no subsidy income would be forthcoming if the specialized factor were subsidized unless the firm acquired that factor and put it to use. Thus, an internal technique change would result.

The policy effects are susceptible to reversal by changes in prices of market products. However, it appears that such price changes must be sustained over longer time periods than where variable factors are involved. The latter is particularly true where specialized, non-divisible fixed factors are involved.

In conclusion subsidizing fixed factors could be a viable policy for certain situations. The most likely situations are where production requires specialized, non-divisible factors. Of course public costs in form of subsidy payments and administrative expenditures must eventually be considered.

Comparisons of Policies to Externality Tax

None of the policies discussed generated results exactly comparable to the externality tax. Most policies fell short of the externality tax in terms of predictability of effects and consistency of results.

Of all other policies considered taxing variable factors appeared to result in effects most nearly comparable to the externality tax. If the proper type variable factors as discussed above can be recognized taxing such factors could be a good substitute for taxing externalities. As already suggested caution must be exercised in choosing the factors and in setting tax size. In instances where there is a direct positive relationship between a variable factor and externality production the proper variable choice should be easier. In some cases a tax may need to be placed on several variable factors especially if they are closely related in terms of effects. An example of this was cited in Chapter I in a quotation from Mills.[62] By taxing a group of inputs that are close substitutes the firm is likely to be forced to change techniques.

A combination of a variable factor tax and subsidy scheme appears also to be a reasonable alternative to the externality tax. The subsidy alone opens the potential for increasing firm growth

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[62] See page 6.

which diminishes its acceptability as a policy.

A standard on quality of the externality generates fairly consistent results; however, it has the same sort of shortcomings as the externality tax. So if the externality tax will not work, then the standard will not work either for the same reasons.

For longer run changes, subsidizing specialized fixed (in short run) factors appears to be a good alternative policy also. The latter is especially true where a technique change is dependent upon acquiring some factor not already owned.

Taxing market products resulted in effects the least like an externality tax. For this and reasons already discussed it was concluded that this policy is the least desirable from any point of view of those considered.

Other Possible Problems

Throughout the analysis it has been assumed that technique changes altered the volume of X_n only. That is, the possibility of X_n changing in character due to a technique and/or product change was not considered explicitly. However, in reality the likelihood of X_n changing to say Y_n is probably high. Such a possibility does not seem likely to affect the analysis of the externality standard and tax. The product X_n could be thought of as a vector of characteristics say of a water discharge. The tax, T_n, would then be a vector also, with elements of the vector being a tax on individual effluent characteristics. Any change in X_n then would be affected by the tax. Since the standard is likely to be based on several characteristics then X'_n would still need to be released no matter what happened to X_n internally.

The problem of changing X_n is more pertinent to the other policies which purposes were to indirectly improve externality quality. There is the chance that internal changes induced by the tax on variable factors and by other policies could change X_n to Y_n where Y_n is less desirable than X_n. Even in these cases such a possibility does not affect the basic analysis conducted on the alternative models. What does change, however, is the effects of policies on X_n production.

What can an agency do to get around the above problem? The answer to this question will likely vary depending on the situation but at least two generalizations can be made. One is that the agency must have good information, before it taxes for example variable factors, concerning the techniques available to the firm. Furthermore the effects of the techniques on externality production should be known in a general way. Such information, however, was discussed previously as necessary anyway if one of these other policies is to be used.

The other generalization is that some of these policies such as a tax on variable factors may need to be used in conjunction with an externality standard of some kind. In the case of water quality, stream standards coupled with variable factor taxes may alleviate that problem.

Another problem which is likely to occur concerns the reasonableness of the net revenue maximizing assumption. Of course the linear models have also implicitly assumed perfect knowledge. Many firms may not even currently be using the net revenue maximizing technique and product combination. The latter could indicate that they are not net revenue maximizers, or they have imperfect knowledge. Whether or not firms actually are net revenue maximizers is an empirical question which is likely to vary from firm to firm. However, it is almost a certainty that firms do not have perfect knowledge. The fact that uncertainty situations do exist has two main implications. One is that even after a policy changes net revenue maximizing points, any given firm may not change due to uncertainties. Secondly, observed initial positions may not in fact be net revenue maximizing points. The firms may be at these "other" points due to uncertainty and/or due to the use of decision criteria other than net revenue maximization. Application of some of the policies then is not going to automatically prompt changes. Even if they do change, such a switch may not be due to the policy. Of course if the switch is in the desired direction, the cause makes little difference.

The main point is that uncertainty and different decision criteria may affect the results of various policies. One thing that might happen due to a variable factor tax for example is that the tax may make the cost of not switching high enough that the firm ignores the uncertainties and switches. So in some ways these policies may not be strongly affected by uncertainty. It appears that more work needs to be accomplished on the effects of uncertainties. That will be left for someone else or another time.

Policies for Controlling Irrigation Return Flow Pollution

Water pollution from irrigation return flows was cited earlier as an example of a case where an externality standard or effluent tax would not be feasible. Consequently what policies at this point in the analysis seem appropriate?

The tax on a variable factor is one policy that offers potential. For one thing there appears to be a positive relationship between water diverted and return flow volumes. There seems to be a negative relationship between labor use and return flows if the present irrigation systems are maintained. In other words water and labor may offer potential variables for a tax or a subsidy, respectively.

There are also longer term adjustments that could possibly be made. For example switching from a surface irrigation system to a sprinkler method would reduce return flows. The latter change would require specialized fixed factors that few farmers in many areas have. Consequently, a subsidy on sprinkler installations appears also to be a viable possibility.

One policy that might be used that has not been considered specifically is education. Making farmers more aware of the effects of their irrigation practices may in itself offer a solution.

Some of the suggested policies may have shortcomings particularly with respect to implementation. Charging for water use may be difficult to use due to legal constraints. Many water districts presently charge for water delivery but these charges are justified on the basis of operation, maintenance, and replacement (O, M, and R) of the delivery systems. A question may arise concerning the legality of any charges above O, M and R. The legal question must be investigated before recommendations are made. Of course legislation could correct any such deficiencies. Taxing other types of inputs could be even more difficult to carry out. Again legislation might be required.

Subsidizing sprinkler equipment might also present difficult implementation problems. Frictions could be created between businesses that have traditionally serviced surface systems and those who service sprinkler systems. How much of a subsidy could also be a knotty problem particularly if some types of sprinklers are more desirable than others.

VII. ESTIMATION OF COST TO AND EFFECTS ON AN IRRIGATED SYSTEM OF SPECIFIC RETURN FLOW CONTROL POLICIES

Introduction

Theoretical analyses have indicated that taxing some variable factor could be a viable alternative to effluent taxes and/or standards. Since the case to be discussed involves water pollution due to irrigation return flows, the sources of which are not readily traceable, a variable factor tax seems logical to consider. Subsidizing specialized fixed factors (in the short run) appeared to offer another logical policy alternative. Due to factors discussed later, the factor subsidy was not utilized in the empirical analysis.

One policy not considered in the theoretical section was applied to the farm model. The policy referred to is a regulation on the amount of a variable factor (water) that can be used. This policy was applied because it seems compatible with regulatory powers already available and in use in Wyoming. Also, due to the apparent relationship between water use and return flows, the policy seems to be a roundabout way of enforcing an effluent standard based on volume only.

The subsequent discussion will be broken into six sections, three of which are subsections to the introduction. A brief description of the study area and problems due to return flows is presented. Next the farm model is delineated in general terms. The development of specific coefficients based on enterprise budgets is then presented and the relationship of the farm model to the theoretical models is discussed. The results of policy applications to the farm model are shown. Finally, conclusions are summarized.

Study Area and Problems of Irrigation Return Flows

The area of interest is located in north central Wyoming and is known locally as the Bighorn Basin. This basin is traversed by several streams which eventually join together and empty into the Yellowstone River near Bighorn, Montana. Before leaving Wyoming, the major streams join the Bighorn River, the stream of concern. The area is characterized by low precipitation levels (5.66 inches average at Worland for April through September [6, p. 9]), which makes irrigation a necessity if intensive farming is to occur. Worland has an average of 133 days between the last spring and first fall occurrences of 32 degree Fahrenheit temperatures (7, p. 10).

The specific area considered is near Worland, Wyoming (Figure 7.1). The town of Worland is located on the banks of Bighorn River, from which it draws its water supply. About 50 road miles upstream from Worland is located Boysen Reservoir, which serves as the storage source for much of the water used for irrigation along the Bighorn River. Water leaving Boysen during all months of the

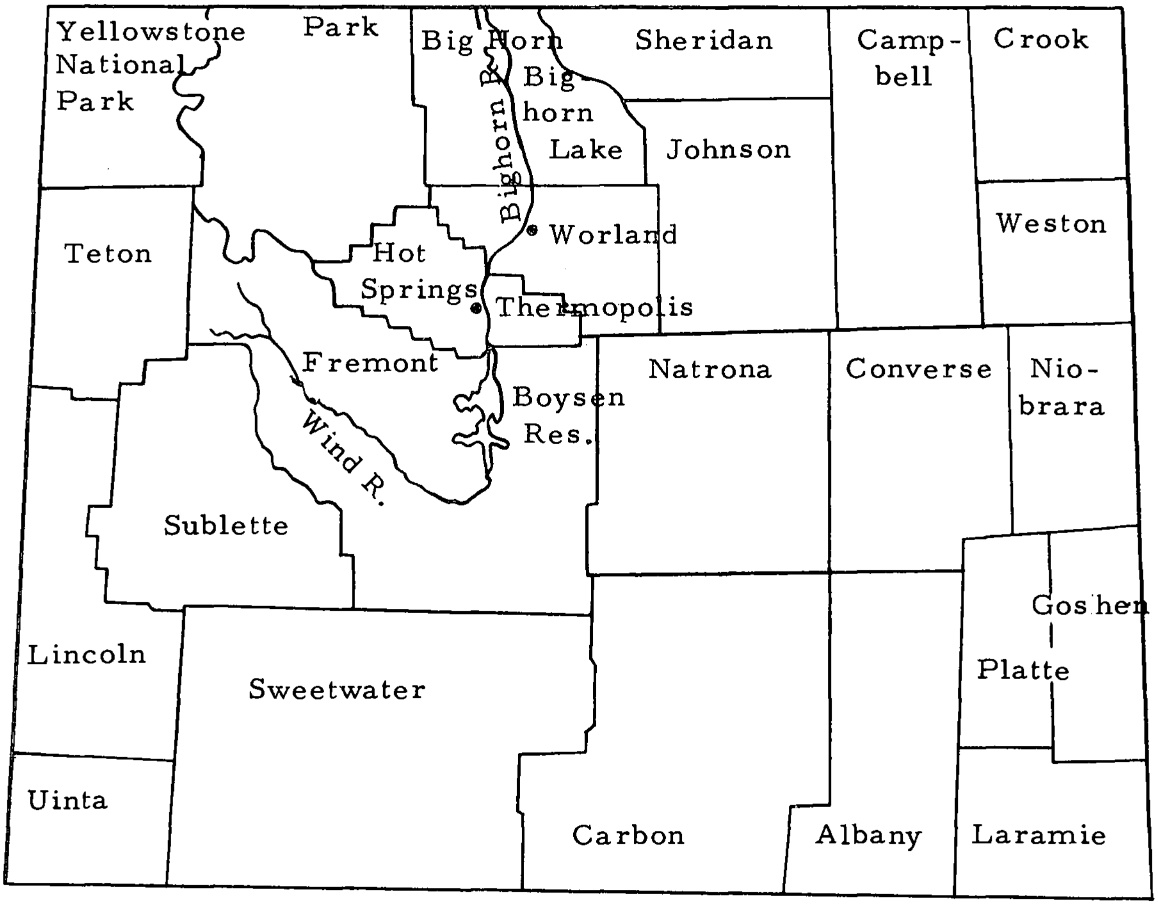

Figure 7.1. Study area in Wyoming

year is relatively clear and free of heavy sediment loads.[63] Data are not available for comparing actual sediment loads immediately below Boysen to those at Worland; however, personal conversations with officials quite familiar with the area and comparison of total dissolved solids (TDS) at the two points substantiate the statement.

For example, during 1969 water year (October 1, 1968 to September 30, 1969) TDS were noticeably higher at Worland. The average tons of TDS per acre foot of water below Boysen were .62, with extremes of .53 and .76. At Worland during the 1969 water year the average tons per acre foot were .83, with variations from .69 to 1.00 (43, p. 49 and 57). Another factor which made the TDS values noticeably different was that the months which showed lowest tons per acre foot below Boysen (October-November of 1968, June-September of 1969) were months of many of the highest loads at Worland. These months correspond closely to the irrigation season in the area. Visual appraisal of per acre foot TDS figures for other years showed similar patterns.

Irrigation in the area is characterized by surface systems of the border and furrow types. Few delivery canals are lined and

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[63] Prior to leaving Wind River Canyon the Bighorn River is known as the Wind River. The name changes at the mouth of this canyon to the Bighorn River at a point referred to as the "Wedding of the Waters" by Indians in that area.

there is little ditch lining used on farms. Most of the irrigated lands lie close to the river (one mile or less). The irrigated area tends to be long and narrow, paralleling the river. Shortage of water for irrigation has not been a problem according to irrigation district water managers and farmers in the area. Irrigation diversions into the major canals above Worland averaged 335, 850 acre feet over the seven year period 1964 through 1970.[64] The discharge of the river below Boysen has averaged 975, 200 acre feet per year since 1951 (44, p. 39). Daily discharge records also indicate there has been sufficient water available during the irrigation seasons.

Data are not available for estimating accurately water deliveries to farms. A survey of water district managers indicated that 3.5 to 7.0 acre feet per acre were not uncommon deliveries to the individual farms. These figures are considerably higher than the irrigation consumptive use requirements[65] of crops in that area.

The major implication is that return irrigation flows are a significant contributor to the sediment and turbidity increases that occur as the river traverses the irrigated area. Granted, TDS figures say nothing about suspended sediment and only indirectly

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[64] Data from Office of Wyoming State Engineer.

[65] Consumptive irrigation requirements are defined as "the depth of irrigation water exclusive of precipitation, stored soil moisture, or groundwater that is required consumptively for crop production" (38, p. 2).

give readings on turbidity. However, lack of data does not permit any more definite quantification of the problems at the present time. Circumstantial evidence alluded to above does indict agriculture as a major source of the problems. As early as 1965, Gertel and Chryst stated the following concerning a report they had received from the Wyoming State Department of Health:

> This state agency put high turbidities and heavy sediment loads in the Bighorn River during the irrigation season at the head of its list of water quality problems due to agriculture. The Bighorn is described as a stream with a good recreational potential if it were not for this problem (19, p. 73).

Recent personal conversations with the Director of the Wyoming State Department of Health (charged with the responsibility for water quality control) and Game and Fish personnel in Wyoming substantiate this attitude. It is recognized that runoff from range lands which drain into the Bighorn also contributes to the turbidity and sediment.

Quantification of problems created by high turbidity and sediment is not feasible under current data constraints. However, problem types can be delineated. During periods of high turbidity and sediment levels, the town of Worland is forced to obtain its water from another source. The Wyoming Game and Fish Commission has in effect written off the Bighorn River downstream from Worland as a trout fishery. Discussions with fishery biologists for the Game and Fish Commission indicate that the high turbidity levels are at least a contributing factor to the unsuitability of the river for trout. Other fish species such as channel catfish have been tried but results as to success are not conclusive at this time.

Yellowtail dam, which forms Bighorn Lake, is on the mainstream of the Bighorn River in Montana. The lake lies on both sides of the Wyoming-Montana border. The upper and flatter parts of the lake are in Wyoming. It is usually late in the summer before turbidity levels in the upper parts of the lake drop. Also, large sediment loads are being deposited in the upper reaches of the lake.[66] Both the latter factors appear to decrease aesthetic enjoyment of Bighorn Lake as well as decrease other recreational activities such as boating and fishing.

Judging from local residents' comments, the general aesthetic value of the river is negatively affected by the turbidity in particular. As stated earlier, quantification of these negative effects does not seem feasible. The important point is that pollution, as defined in Chapter I, does appear to be occurring.

General Description of Farm Model

During 1969 and 1971 studies of crop production costs in the

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[66] From personal interviews with U. S. Geological Survey personnel, local residents and Game and Fish personnel.

Worland, Wyoming, area were conducted by Agee (1, 2). The studies were quite comprehensive and costs of producing all major crops were determined. Agee used the group interview technique to obtain data concerning crops, labor and machinery use. From this data a crop plan was devised for a "representative farm." The representative farm was consistent with larger farms in the Worland area. Besides costs and returns for producing individual crops, the studies also show such information as per acre tractor and machinery use coefficients, per acre labor coefficients for irrigation and non-irrigation tasks, and time of irrigation. All of the previous coefficients were developed for two week time intervals from March through November. The per acre requirements were developed for each crop.

The farm model used in this study is based entirely on Agee's study. He assumed that the representative farm contained 500 acres, all in production. Farmstead and land used for roads and ditches would make the total farm somewhat larger than 500 acres. The "representative farm" assumed by Agee is shown in Table 7.1 along with per acre production levels. These same production levels were assumed for the present study.

The crops that make up the representative farm and their proportions depict closely actual operations in the Worland area. Other crop alternatives are available but their production is relatively

Table 7.1. Crop Plan and Production, 500 Acre Representative Farm, Worland Area, 1971.[a]

Crop	Acres	Percent	Production	
			Unit	Per acre
Sugarbeets	185	37.0	tons	20.8
Corn for grain	34	6.8	bushel	114.5
Corn for silage	46	9.2	tons	20.8
Feed barley, seeded with alfalfa	42	8.4	bushel	60.0
Feed barley	42	8.4	bushel	75.0
Malting barley	41	8.2	bushel	75.0
Alfalfa	110	22.0	tons	3.9

[a]Source (1, p. 9).

small at the present time. For example, dry beans are grown but their production is small and has been declining for several years.

There are no livestock alternatives shown in Agee's study. Livestock are produced, however, on these farms. Omitting livestock was done intentionally so as not to complicate the crop study. Calf wintering enterprises are fairly common and they do not compete with crop production for labor to any large degree.

The farm model assumed for the present work used the same crop alternatives as shown in Table 7.1. Total crop acreage was constrained at 500 acres. Instead of specifying rotations, upper and lower constraints were placed on certain crop acreages. The latter was done so that the initial solution would be comparable to the representative farm described by Agee. Rotations were not specified since there did not appear to be any consistent rotation among farmers or even over time for the same farmer.[67]

Livestock enterprises were not included in the farm model either so as not to complicate the model. Also, current information concerning costs and production coefficients for livestock enterprises was not available. Determination of the necessary data would be a relatively large study in itself. In other words, it was assumed that all crop production would be sold directly instead of indirectly through livestock.

Inclusion of livestock enterprises would not be expected to change the major conclusions of the study. The optimum solutions might change in that some forage and feed crops may become more profitable depending on the particular livestock enterprises used and the prices assumed. In essence including livestock enterprises provides another alternative for marketing forage and grain crops. Consequently, magnitudes of costs of policies and sizes of taxes required to induce irrigation technique switches could be changed by inclusion of livestock enterprises. However, the direction of changes and differences between the labor constrained and unconstrained situations would not likely be altered.

It was assumed that the return flow control policies would

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[67] Personal conversations with Agee and farmers in the study area.

mainly affect irrigation technique. Consequently, the enterprises in the model were broken into general and irrigation. For example, one enterprise was "sugarbeet production, general." This enterprise consisted of the costs and coefficients for all activities other than those specifically related to irrigation. Two more enterprises for sugarbeets were developed to reflect irrigation costs and coefficients. The enterprise referred to as "sugarbeet irrigation one" represents the present technique of irrigation. "Sugarbeet irrigation two" represents an alternative technique not in use presently but available. The assumed differences between irrigation techniques will be explained with respect to individual coefficients and costs below.

Production Coefficients, Costs and Prices

Since the studies by Agee are current, the production coefficients, costs and prices assumed are identical to his. Also, Agee's studies were for the specific study area.

Machine use coefficients were not specified for the linear farm model. The increase in machine use that might be necessary is reflected in enterprise costs and labor use coefficients.

Labor coefficients were developed for monthly periods instead of two week intervals reported by Agee. This was done in part to reduce the size of the linear program coefficient matrix and to allow more flexibility as to timing of various operations.

It was possible to separate irrigation labor including maintenance labor on irrigation structures from other labor requirements. The specific values for the coefficients are shown in Table 7.2 for each enterprise. As noted above, "irrigation one" reflects the requirements of the present irrigation technique. For technique two the labor coefficients were based on an assumption. That assumption was that farm irrigation efficiency[68] could be improved by using in part more labor. It was also assumed that the improved farm irrigation efficiency could occur without changing the basic structures such as diversion devices and ditches. The latter assumption was based on discussions with agricultural engineers and water district managers. The implication to the overall analysis of these two assumptions is that policy effects will be considered from a short run view. In other words, investment in the irrigation system is held constant and not permitted to change. This assumption lends more credibility to the use of constant coefficients for the "general" enterprises.

Data are not available concerning "how much" additional labor

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[68] Farm irrigation efficiency is defined as follows: "The percentage of applied irrigation water that is stored in the soil and available for consumptive use by the crop. When the water is measured at the farm headgate, it is called farm irrigation efficiency . . ." (38, p. 2-4).

Table 7.2. Coefficient Matrix for Bighorn Basin Farm Model

Resources	Constraint level	Enterprises											
		Alfalfa			Barley-alfalfa			Barley-grain			Barley-malt		
		General	Irrig. I	Irrig. II	General	Irrig. I.	Irrig. II	General	Irrig. I	Irrig. II	General	Irrig. I	Irrig. II
Objective funct. value		-45.10	-5.35	-5.46	-51.25	-5.35	-5.46	-44.43	-5.35	-5.46	-46.12	-5.35	-5.46
Land	500		1.0	1.0		1.0	1.0		1.0	1.0		1.0	1.0
Water													
April													
May			8.25	7.07		11.52	9.6		11.52	9.6		11.52	9.6
June			8.78	7.53		10.14	8.45		10.14	8.45		10.14	8.45
July			11.93	10.23		9.18	7.65		9.18	7.65		9.18	7.65
August			14.73	12.63		5.00	4.17						
Labor													
March		.18			.65			.45			.63		
April		0.00	.06	.063	.59			.67			.48		
May		0.00	.24	.252	.00	.40	.42	0.00	.40	.42		.40	.42
June		1.25	.21	.220	.00	.20	.21	0.00	.27	.284		.27	.284
July		1.10	.19	.200	.22	.54	.567	.22	.54	.567	.22	.54	.567
August			.41	.431	1.85	.27	.284	1.85			1.85		
September		.95	0.00	0.00									
October													
November													
Barley-Malt (upper)	50										1.0		
Corn-grain(upper)	50												
Corn-silage (upper)	50												
Sugarbeets (upper)	190												
Alfalfa (lower)	+80	+1											
Alfalfa prod.		-3.9											
Barley-grain					-60.0			-75.0					
Barley-malt											-75.0		
Corn-grain													
Corn silage													
Sugarbeets													
Alfalfa seeding			.333	.333	-1.0								

Table 7.2 (continued)

Resources	Constraint level	Enterprises											
		Alfalfa			Barley-alfalfa			Barley-grain			Barley-malt		
		General	Irrig. I	Irrig. II	General	Irrig. I	Irrig. II	General	Irrig. I	Irrig. II	General	Irrig. I	Irrig. II
Return flows													
April													
May			-3.30	-2.12		-5.76	-3.84		-5.76	-3.84		-5.76	-3.84
June			-3.51	-2.26		-5.07	-3.38		-5.07	-3.38		-5.07	-3.38
July			-4.77	-3.07		-4.59	-3.06		-4.59	-3.06		-4.59	-3.06
August			-5.89	-3.79		-2.50	-1.67						
September													
October													
Crop land													
Alfalfa		1.0	-1.0	-1.0									
Barley-Alfalfa					1.0	-1.0	-1.0						
Barley-grain								1.0	-1.0	-1.0			
Barley-malt											1.0	-1.0	-1.0
Corn-Grain													
Corn-Silage													
Sugarbeets													

Table 7.2 (continued)

Resources	Constraint level	Corn-grain General	Corn-grain Irrig. I	Corn-grain Irrig. II	Corn-silage General	Corn-silage Irrig. I	Corn-silage Irrig. II	Sugarbeets General	Sugarbeets Irrig. I	Sugarbeets Irrig. II
Objective funct. value		-71.82	-5.56	-5.69	-85.87	-5.56	-5.69	-202.05	-6.28	-6.48
Land	500		1.0	1.0		1.0	1.0		1.0	1.0
Water										
April									6,0	6.0
May			11.58	9.48		11.58	9.48			
June									13.11	10.72
July			14.56	11.91		14.56	11.91		13.73	11.24
August			21.04	17.22		21.04	17.22		11.64	9.53
September									5.66	4.55
Labor										
March								1.47		
April		1.66			1.66			.97	1.67	1.75
May		.58	.58	.609	.58	.58	.609	1.68	.00	.00
June		.62	.06	.063	.62	.06	.063	.72	.90	.945
July		0	.58	.609	0	.58	.609	.40	.83	.872
August		0	.54	.567	0	.59	.567	1.33	1,57	1.649
September		0	0	0	4.15	0	0	0	1.58	1.659
October		0	0	0			0	6.84	0	0
November		1.2	.05	.053						
Barley-malt (upper)	50									
Corn-grain (upper)	50	1.0								
Corn-silage (upper)	50				1.0					
Sugarbeets (upper)	190							1.0		
Alfalfa (lower)	+80									
Alfalfa prod.										
Barley-grain										
Barley-malt										
Corn-grain		-114.5								
Corn-silage					-20.8					
Sugarbeets								-20.8		
Alfalfa seeding										

Table 7.2 (continued)

Resources	Constraint level	Enterprises								
		Corn-grain			Corn-silage			Sugarbeets		
		General	Irrig. I	Irrig. II	General	Irrig. I	Irrig. II	General	Irrig. I	Irrig. II
Return flows										
April									-3.6	-3.6
May			-6.37	-4.27		-6.37	-4.27			
June									-7.21	-4.82
July			-8.01	-5.36		-8.01	-5.36		-7.55	-5.06
August			-11.57	-7.75		-11.57	-7.75		-6.04	-4.29
September									-3.06	-2.05
October										
Crop land										
Alfalfa										
Barley-Alfalfa										
Barley-grain										
Barley-malt										
Corn-grain		1.0	-1.0	-1.0						
Corn-silage					1.0	-1.0	-1.0			
Sugarbeets								1.0	-1.0	-1.0

would be needed to obtain a given increase in farm irrigation efficiency. Consequently when policies were applied, different levels of increased labor use were assumed. These levels ranged from a 5 to 30 percent increase in labor coefficients. In other words sensitivity of the solutions and water tax levels to irrigation labor use was conducted. The coefficients shown in Table 7.2 for irrigation two reflect only the five percent increase assumption.

Water use coefficients were based on a study by Trelease, et. al. (38). They utilized the Blaney-Criddle method for estimating consumptive use. Their study shows consumptive irrigation requirements by month, crop, and area of Wyoming, and was based on average climatic data.

An attempt was made to determine per acre farm diversions from canal diversion data. The Wyoming State Engineer maintains daily diversion data for major canals diverting from the Bighorn River. Private diversions are not recorded. The gross canal diversion data was adjusted by assuming various delivery efficiencies. Proportion of crops produced under various canals was determined from U.S. Bureau of Reclamation reports and Wyoming Crop Census reports by counties. Per acre and per crop diversions were then estimated using the irrigation timing information derived by Agee (1, p. 42-46). These estimates were made on a monthly basis.

When the diversion estimates per crop and per acre were compared to irrigation district estimates of yearly total per acre diversions, they appeared to be reasonable. The district estimates were not available on a crop or monthly basis. Examination of the per acre monthly diversions for each crop, however, revealed some large discrepancies that could not be reconciled with known practices. For example, diversions to sugarbeets were almost three times higher in September than in the next highest month.

Again lack of pertinent data was the major cause of the discrepancies. The gross diversion data gave no indication of amount of water diverted but _not_ delivered; i.e., some returns directly to the river. Some canals pick up return flows from lands that lie above them, which in turn are delivered to other farms. If estimates of the latter two factors were available on a monthly basis, this type of estimation might have been better. Also, proportions of various types of crops irrigated by given canals would have improved these estimates. Such estimates were only available for one canal which delivered to a small proportion of total acres in the area.

In view of the difficulties with the previous procedure, another method was used to derive water delivery coefficients. The consumptive use requirements per month and per crop as reported by Trelease _et_ _al._, served as the starting point. Time of irrigation was determined monthly and by crop from Agee's work. The irrigation consumptive use requirements were adjusted to correspond with that timing.[69] Once these irrigation consumptive use requirements were determined, a farm irrigation efficiency coefficient was estimated for each crop. There were no current studies which estimate farm irrigation efficiencies for the Worland area. However, a study in 1951 gives estimates based on soil type and topography (37, p. 11). From this study and discussions with individuals familiar with the Worland area, estimates were made. The assumed farm irrigation efficiencies were .45 for sugarbeets and both corn crops, .50 for the barley crops, and .60 for alfalfa. The monthly irrigation consumptive use requirements for each crop were then divided by these farm irrigation efficiency estimates to obtain delivery requirements. The coefficients derived are shown in Table 7.2 under "irrigation one" enterprises.

It was assumed that by using more labor and more variable costs, each of the previously given efficiencies could be increased by .10. The increased efficiencies would then be .55 for sugarbeets and

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[69] For example, a consumptive irrigation requirement is shown in Trelease's study for corn for the months of May and June. Agee's study indicated that corn was irrigated only in May. Consequently, the irrigation consumptive use requirements for May and June were combined and shown only for May. It is realized that this procedure is not 100 percent accurate, but it should provide a reasonable estimate.

corn, .60 for barley crops, and .70 for alfalfa. The higher efficiencies were then divided into the irrigation consumptive use requirements to obtain the water use coefficients for irrigation two, shown in Table 7.2.

Return flow coefficients were not separated into surface runoff and deep percolation. The technical research necessary for such a division has not been conducted in Wyoming. Estimates from other areas would not reflect irrigation practices, soils, and climatic conditions in Wyoming. Consequently, the return flow coefficients reflect all water diverted that is not consumptively used. The specific coefficients were determined by subtracting from the water coefficients the irrigation consumptive use requirements. The effect then of increased irrigation efficiency is to reduce the return flows by the same amount as the diversion is reduced.

Enterprise costs were based on the budgets by Agee (1, p. 20-27). The budgeted costs as used in the farm model are shown in Appendix Tables B-1 through B-7. These tables also give a general idea of operations for which labor and costs were required.

Labor costs are not shown in the budgets since labor buying activities were included in the initial farm model. Eventually, labor was constrained to examine effects of policies under such conditions. When the labor constraints were used, labor costs were added back into the enterprise costs.

Labor was assumed to be paid $2.31 per hour. This rate included $.15 per hour for housing depreciation. Agee used $2.16 per hour and included the $.15 for housing depreciation as a non-cash expense. Even though much of the labor could be furnished by the family, the $2.31 was charged for all labor. In a sense, this represents the opportunity cost of family labor.

The prices used for determining cost items are depicted in Appendix Table B-8. Tables for machinery inventory, investment and costs are not included but are covered in detail in the previously mentioned study (1, p. 36-39).

As stated earlier, the general cost items for each crop were assumed to be constant in the face of changing irrigation techniques. The irrigation costs shown in Appendix Tables B-1 through B-7 are for "irrigation one" enterprises and were adjusted for the "irrigation two" enterprises.

Since no information was available concerning the amount of increased costs necessary for improving farm irrigation efficiency, two different levels were assumed. These levels were 110 and 120 percent of "irrigation one" costs. The percent increases were applied only to those irrigation costs that might vary. In other words, the water and drainage cost was not increased between techniques. The latter costs are charges by the district for operation, maintenance and replacement (O, M and R) on the delivery and drainage systems in existence. These charges do vary between irrigation districts. The $4.24 per acre represents an average of payments by farmers interviewed by Agee.

The objective function value for irrigation two in Table 7.2 represents the 110 percent increase assumption. For example, irrigation one for alfalfa was -$5.35. Of this $1.11 was assumed to be variable between techniques. Taking 1.10 times $1.11 and adding to that the system O, M and R of $4.24 resulted in the -$5.46 objective function value for "alfalfa, irrigation two." Other "irrigation two" costs were determined similarly.

It must be pointed out that the enterprise costs included some fixed cost items on a yearly basis (see Appendix Tables B-1 through B-7). Fixed costs included were real estate tax, miscellaneous overhead, fixed machinery costs, and water and drainage. No charge was made, however, for management or real estate investment. Consequently, the net revenues generated by the farm model will be returns to management and land.

Prices for crops were based on information in Agee's study (1, p. 13). These are shown in Table 7.3 on a per unit of production basis.

The prices included in the linear programming activities (objective function values) are shown under the "composite" column. These composites take into account the pasture and straw values.

Table 7.3. Assumed Prices Per Production Unit for Major Crops, Worland, Wyoming, Area, 1971. [a]

Crop	Unit	Price per unit	Yield per A.	Composite[b] Unit	Composite[b] Price
Alfalfa hay				ton	$24.87
Hay (stack loose)	tons	$24.00	3.9		
Pasture	AUMs[c]	6.00	.33		
Barley (feed)--seeded with alfalfa				bu.	1.12
Barley (in bin)	bu.	1.00	60.0		
Straw	tons	10.00	.4		
Pasture	AUMs	6.00	1.0		
Barley (feed)				bu.	1.093
Barley (in bin)	bu.	1.00	75.0		
Straw	tons	10.00	.5		
Pasture	AUMs	6.00	.33		
All barley (feed)				bu.	1.10
Barley (malt)				bu.	1.28
Barley (delivered in town)	bu.	1.18	75.0		
Straw	tons	10.00	.5		
Pasture	AUMs	6.00	.33		
Corn for grain				bu.	1.266
Corn (in bin)	bu.	1.20	114.5		
Pasture	AUMs	6.00	1.33		
Corn for silage (into pit)	tons	7.00	20.8	ton	7.00
Sugarbeets				ton	17.50
Beets (at dump)	tons	16.00	20.8		
Tops	tons	1.50	20.8		

[a] Source: (1. p. 13).

[b] The composite unit and price were those used for selling activities in the linear programming model. It takes into account the straw and pasture values for the crops. These values were calculated on a per unit of the main crop basis.

[c] AUMs refers to animal unit months.

For example, the alfalfa enterprise generated $94 of hay per acre ($24 x 3. 9) and approximately $2. 00 worth of pasture per acre ($6 x . 33). Total per acre value then was $96. For each acre of alfalfa grown the model was set up such that . 333 acres of barley seeded with alfalfa must also be grown. The latter was based on the assumption of a three year life for an alfalfa stand. Barley seeded with alfalfa generated $10 of straw and pasture value per acre (. 4 x $10. 00 plus $6 x 1. 0) while barley alone only generated $7 worth of straw and pasture per acre (. 5 x $10. 00 plus $6. 00 x . 33). The difference in straw and pasture between barley seeded with alfalfa and straight barley was $3. 00, which was prorated to the alfalfa enterprise at one third, or $1. 00 per acre, to reflect the three year stand assumption. Consequently, total value for the alfalfa enterprise was set at $97. 00 per acre. The $97. 00 was divided by the per acre yield of 3. 9 tons to arrive at the composite price of $24. 87 per ton. Other composite prices were determined similarly but were not quite so complicated. To simplify calculations only one composite price, $1. 10 per bushel, was used for feed barley. This figure was used since it was between the composite barley seeded with alfalfa and straight barley composite prices. The composite price for barley seeded with alfalfa was determined by using only $7. 00 for pasture and straw since the other $3. 00 was prorated to the alfalfa.

The coefficient matrix for the farm model (Table 7.2) does not show the selling activities. Neither are the buying activities for labor and water shown. Since the coefficients other than objective function values are all -1's the previous activities were left out to condense the matrix. The prices and labor costs have been discussed. Water prices will be discussed when the policy results are presented.

Relationship Between Theoretical and Farm Models

The activities referred to in the farm model as irrigation one and two are the phases of the production process that are different. In other words, the "general" enterprise coupled with irrigation one represents the initial technique of producing the various crops. The combination of "general" and irrigation two depicts the alternative production method. Since none of the factors constrained are specialized, the farm model relates closely to theoretical Model 4.

Logically another alternative technique could be introduced which would require specialized factors. Such an alternative would be irrigation with some sprinkler system. The model here does not consider the sprinkler alternative. The main reason for omission of such an important alternative was lack of empirical data concerning sprinkler use in the area of interest. Very little technical and economic work has been done in Wyoming concerning adaptation of sprinkler systems to areas previously irrigated by surface systems. Important information such as yield comparisons, labor requirements, repairs, etc., is not available. Eventually, such data must be determined and incorporated into the present model.

The second reason for omission of sprinkler alternatives concerns adaptability to the Bighorn area. Many of the fields in existence are small and irregular in shape. It appears that the popular circular, automated systems would not be very well adapted to the area. Most of the recent work concerning sprinkler irrigation that has been accomplished in Wyoming relates to the circular automated systems. Some systems such as side rolls and hand moves may be applicable but even less information is available with respect to the latter systems.

The farm model assumes higher variable costs are required to produce with technique (irrigation) two. It also assumes more labor is required. Yields and other coefficients except water and return flows are assumed the same between techniques. Cast in the framework of the theoretical models, then, $VC_a < VC_b$. When labor is constrained, $f_{2ai} < f_{2bi}$. The 'a' in the previous inequalities refers to irrigation one, and the 'b' to irrigation two. The f_{2ai} and f_{2bi} refer to labor required to generate one unit of production of product 'i' where 'i' could refer to any of the crops. Since labor is broken into monthly time periods, each month acts as a separate constraint on production. So, in effect, the labor coefficients shown above are oversimplified, but the relationships still hold true for each month.

Land is also constrained. However, since yield levels are assumed the same between techniques, the land coefficients between techniques are equal, i. e., $f_{1ai} = f_{1bi}$ where f_{1ai} and f_{1bi} refer to land required to generate one unit of product 'i' by techniques A and B, respectively.

To summarize, the farm model specification makes it similar if not identical to theoretical Model 4. 1. Model 4. 1 was the case where two non-specialized fixed factors were used and technique A dominated B. Since specification is practically identical, one would expect effects of policy applications to the farm model also to be identical to the effects on Model 4. 1. The introduction of more than two non-specialized fixed factors and more than two products might make effects different. The farm model in a sense can be viewed as a test of expected effects of policies on theoretical Model 4. 1. That is, policy application to the farm model will test whether or not conclusions reached for Model 4. 1 hold for a linear model with more than two products and more than two non-specialized factor constraints.

Policy Applications to the Farm Model

Two policies were chosen for application to the farm model. The first policy to be discussed is a tax on gross water use. That is, various levels of taxes per acre inch of water were assessed against water delivered to the farm. The second policy to be discussed is a limit on amount of water that would be delivered per month. The two policies were not used together. In other words when water was limited no per acre inch charge was assessed. When water was taxed it was not constrained.

The farm model was maximized under various constrained conditions prior to policy applications. When the policies were applied the same constraints were used so as not to introduce changes due to changing constraints.

Taxing Water Delivered

Since a cost ranging feature for the computer program was not useable the model was maximized using various tax levels. These levels ranged from $.10 per acre inch of water to as high as $.65 per acre inch in some instances.

As already indicated additional labor required to increase farm irrigation efficiency was varied from plus 5 to 30 percent. The specific labor increases used were 5, 10, 20, and 30 percent. Variable irrigation costs were increased 10 to 20 percent to obtain the increased farm irrigation efficiencies. Consequently, eight different models were maximized using the various tax levels on water. The multi-level labor and cost increases gave some indication of sensitivity of the policy effects to irrigation labor and irrigation variable costs.

Since technique one dominates technique two the initial solutions for all labor and costs levels were identical. The initial solution was based on the constraint levels shown in Table 7. 2 and a zero tax on water.

Results with Labor Unconstrained

Necessary labor was purchased at the rate of $2. 31 per hour. The models were constrained as shown in Table 7. 2.

Crop enterprises in each "final solution" are shown in Table 7. 4 for each cost and labor level. The final solutions shown in that table are the solutions for the tax levels at which all crops in the optimum plan utilized irrigation technique two. The tax level at which the switch to irrigation two was complete is shown as the last row of the table.

Straight barley for grain was not in the optimal solution for the initial conditions. Notice that malt barley, corn-grain, corn-silage, and sugarbeets were all in the initial optimum at the level of their

Table 7.4. Initial and Final Solutions for a Water Tax When Labor was Unconstrained.[a]

	Unit	Initial Solution	Irrigation two variable costs increased by							
			+.10							
			Irrigation two labor increased by							
			.05		.10		.20		.30	
			Solution[b]	Diff.[c]	Solution	Diff.	Solution	Diff.	Solution	Diff.
Net Revenue	$	31,185	28,095	-3090	26,009	-5179	21,894	-9291	18,763	-12422
Alfalfa	acres	120	120	--	120	--	80	- 40	80	- 40
Barley-Alf.	acres	40	40	--	40	--	27	- 13	27	- 13
Barley-grain	acres	--	--	--	--	--	53	+ 53	53	+ 53
Barley-malt	acres	50	50	--	50	--	50	--	50	--
Corn-grain	acres	50	50	--	50	--	50	--	50	--
Corn-silage	acres	50	50	--	50	--	50	--	50	--
Sugarbeets	acres	190	-190	--	-190	--	190	--	190	--
Total	acres	500	500	--	500	--	500	--	500	--
Alfalfa	tons	468	468	--	468	--	312	- 146	312	- 146
Barley-feed	bu.	2398	2398	--	2398	--	5600	+3202	5600	+3202
Barley-malt	bu.	3750	3750	--	3750	--	3750	--	3750	--
Corn-grain	bu.	5725	5725	--	5725	--	5725	--	5725	--
Corn-silage	tons	1040	1040	--	1040	--	1040	--	1040	--
Sugarbeets	tons	3952	3952	--	3952	--	3952	--	3952	--
Hired labor	hours									
March		356.5	356.5	--	356.5	--	364.6	+ 8.1	364.6	+ 8.1
April		722.4	737.9	+ 15.5	755.4	+ 33.0	812.3	+ 89.9	844.5	+ 122.1
May		500.0	506.1	+ 6.1	512.3	+ 12.3	532.2	+ 32.2	545.2	+ 45.2
June		572.5	586.2	+ 13.7	599.9	+ 27.4	571.2	- 1.3	593.9	+ 21.4
July		514.9	529.4	+ 14.5	543.6	+ 28.7	553.9	+ 39.0	584.0	+ 69.1
August		833.9	852.2	+ 18.3	873.3	+ 39.4	964.0	+ 130.1	1003.2	+ 169.3
September		621.7	636.7	+ 15.0	652.1	+ 30.4	643.7	+ 22.0	673.8	+ 52.1
October		1299.6	1299.6	--	1299.6	--	1299.6	--	1299.6	--
November		62.5	62.6	+ .1	62.8	+ .3	63.0	+ .5	63.3	+ .8
Total		5484.0	5567.2	+ 83.2	5655.4	+ 171.5	5804.5	+ 320.5	5972.1	+ 488.1

Table 7.4 (continued)

			Irrigation two variable costs increased by							
			+.10							
			Irrigation two labor increased by							
	Unit	Initial Solution	.05		.10		.20		.30	
			Solution[b]	Diff.[c]	Solution	Diff.	Solution	Diff.	Solution	Diff.
Water purch.	ac. ins.									
April		1140.0	1140.0	--	1140.0	--	1140.0	--	1140.0	--
May		3184.7	2660.3	- 524.4	2660.3	- 524.4	2761.6	- 423.1	2761.6	- 423.1
June		4457.1	3700.9	- 756.2	3700.9	- 756.2	3737.7	- 719.4	3737.7	- 719.4
July		6322.6	5242.8	-1079.8	5242.8	-1079.8	5139.5	-1183.1	5139.5	-1183.1
August		6283.5	5215.4	-1068.1	5215.4	-1068.1	4654.2	-1629.3	4654.2	-1629.3
September		1056.4	864.5	- 191.9	864.5	- 191.9	864.5	- 191.9	864.5	- 191.9
Total		22,444.3	18,823.9	-3620.4	18,823.9	-3620.4	18,297.5	-4146.8	18,297.5	-4146.8
Return flow	ac. ins.									
April		684.0	684.0	--	684.0	--	684.0	--	684.0	--
May		1551.3	1026.9	- 524.4	1026.9	- 524.4	1095.8	- 455.5	1095.8	- 455.5
June		2247.4	1491.2	- 756.2	1491.2	- 756.2	1536.0	- 711.4	1536.0	- 711.4
July		3221.0	2141.2	-1079.8	2141.2	-1079.8	2140.8	-1080.2	2140.8	-1080.2
August		3179.9	2111.8	-1068.1	2111.8	-1068.1	1937.8	-1242.1	1937.8	-1242.1
September		581.4	389.5	- 191.9	389.5	- 191.9	389.5	- 191.9	389.5	- 191.1
Total		11,465.0	7844.6	-3620.4	7844.6	-3620.4	7783.9	-3681.1	7783.9	-3681.1
Changed at[d]	$	--	.15		.25		.45		.60	

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[a] Acres times yields will not always coincide with production shown since acres were rounded to nearest whole number.

[b] Solution shown is the one when irrigation two was used by all crops.

[c] Diff. refers to the difference between this solution and the initial solution.

[d] This refers to the tax on water that induced the change to irrigation two for all crops.

Table 7.4 (continued)

			Irrigation two variable costs increased by							
			+.20							
			Irrigation two labor increased by							
	Unit	Initial	.05		.10		.20		.30	
		Solution	Solution	Diff.	Solution	Diff.	Solution	Diff.	Solution	Diff.
Net Revenue	$	31,185	28,019	-3166	25,933	-5252	21,818	-9367	17,810	-13375
Alfalfa	acres	120	120	--	120	--	80	- 40	80	- 40
Barley-Alf.	acres	40	40	--	40	--	27	- 13	27	- 13
Barley-grain	acres	--	--	--	--	--	53	+ 53	53	+ 53
Barley-malt	acres	50	50	--	50	--	50	--	50	--
Corn-grain	acres	50	50		50		50	--	50	--
Corn-silage	acres	50	50		50		50		50	
Sugarbeets	acres	190	190		190		190		190	
Total	acres	500	500		500		500		500	
Alfalfa	tons	468	468	--	468	--	312	- 146	312	- 146
Barley-feed	bu.	2398	2398	--	2398	--	5600	+3202	5600	+3202
Barley-malt	bu.	3750	3750	--	3750	--	3750	--	3750	--
Corn-grain	bu.	5725	5725	--	5725	--	5725	--	5725	--
Corn-silage	tons	1040	1040	--	1040	--	1040	--	1040	--
Sugarbeets	tons	3952	3952	--	3952	--	3952	--	3952	--
Hired labor	hours									
March		356.5	356.5	--	356.5	--	364.6	+ 8.1	364.6	+ 8.1
April		722.4	737.9	+ 15.5	755.4	+ 33.0	812.3	+ 89.9	844.5	+ 122.1
May		500.0	506.1	+ 6.1	512.3	+ 12.3	532.2	+ 32.2	545.2	+ 45.2
June		572.5	586.2	+ 13.7	599.9	+ 27.4	571.2	- 1.3	593.9	+ 21.4
July		514.9	529.4	+ 14.5	543.6	+ 28.7	553.9	+ 39.0	584.0	+ 69.1
August		833.9	852.2	+ 18.3	873.3	+ 39.4	964.0	+ 130.1	1003.2	+ 169.3
September		621.7	636.7	+ 15.0	652.1	+ 30.4	643.7	+ 22.0	673.8	+ 52.1
October		1299.6	1299.6	--	1299.6	--	1299.6	--	1299.6	--
November		62.5	62.6	+ .1	62.8	+ .3	63.0	+ .5	63.3	+ .8
Total		5484.0	5567.2	+ 83.2	5655.4	+ 171.5	5804.5	+ 320.5	5972.1	+ 488.1

Table 7.4. (continued)

			Irrigation two variable costs increased by							
			+.20							
			Irrigation two labor increased by							
	Unit	Initial	.05		.10		.20		.30	
		Solution	Solution	Diff.	Solution	Diff.	Solution	Diff.	Solution	Diff.
Water purch.	ac. ins.									
April		1140.0	1140.0	--	1140.0	--	1140.0	--	1140.0	-
May		3184.7	2660.3	- 524.4	2660.3	- 524.4	2761.6	- 423.1	2761.6	- 423.1
June		4457.1	3700.9	- 756.2	3700.9	- 756.2	3737.7	- 719.4	3737.7	- 719.4
July		6322.6	5242.8	-1079.8	5242.8	-1079.8	5139.5	-1183.1	5139.5	-1183.1
August		6283.5	5215.4	-1068.1	5215.4	-1068.1	4654.2	-1629.3	4654.2	-1629.3
September		1056.4	864.5	- 191.9	864.5	- 191.9	864.5	- 191.9	864.5	- 191.9
Total		22,444.3	18,823.9	-3620.4	18,823.9	-3620.4	18,297.5	-4146.8	18,297.5	-4146.8
Return flow	ac. ins.									
April		684.0	684.0	-	684.0	-	684.0	--	684.0	--
May		1551.3	1026.9	- 524.4	1026.9	-524.4	1095.8	- 455.5	1095.8	- 455.5
June		2247.4	1491.2	- 756.2	1491.2	- 756.2	1536.0	- 711.4	1536.0	- 711.4
July		3221.0	2141.2	-1079.8	2141.2	-1079.8	2140.8	-1080.2	2140.8	-1080.2
August		3179.9	2111.8	-1068.1	2111.8	-1068.1	1937.8	-1242.1	1937.8	-1242.1
September		581.4	389.5	- 191.9	389.5	- 191.9	389.5	- 191.9	389.5	- 191.1
Total		11,465.0	7844.6	-3620.4	7844.6	-3620.4	7783.9	-3681.1	7783.9	-3681.1
Changed at[d/]	$	--	.15		.25		.45		.65	

upper constraints. Alfalfa came in higher (120 acres) than its lower constraint of 80 acres. The 120 acres of alfalfa required 40 acres of barley seeded with alfalfa to replace the stand.

As irrigation two labor was increased some changes in enterprises in the final solutions occurred. At the increased labor rates of 20 and 30 percent, alfalfa dropped to its minimum level and barley for feed entered to utilize the released acres. The latter change for these larger labor assumptions appeared to have been a result of the higher tax levels necessary to induce total switching to irrigation two. Taxes of $.45 and $.60 per acre inch of water were required to bring about a total switch for the 20 and 30 percent increased labor assumptions. The higher tax levels (above $.30) were not applied to the other two lower labor requiring models.

The crop enterprises were not affected differently by the two, increased variable cost assumptions. Note that the crops and acres for each crop are identical for the 10 and 20 percent irrigation two variable cost assumptions.

The conclusion concerning the sensitivity of crop enterprises to irrigation labor and variable cost assumptions is that labor appears to be more important. This was true at the levels considered in the farm model. In other words, to improve the model better data on irrigation labor for technique two appears important.

Labor use as expected goes up as the second irrigation technique is used. Also labor use goes up as the assumed "irrigation two" labor requirements increase, a necessary occurrence. October labor did not change as irrigation method changed for any of the assumptions since irrigation two did not require any October labor for any crop. The crops that changed did not require October labor either.

Water purchased declined from the initial solution by about 3,610 acre inches for the +.05 and +.10 labor assumptions and by approximately 4,146 acre inches for the +.20 and +.30 labor assumptions. The decline in water use was not the same for the various labor assumptions since product changes occurred for the higher labor using models which in turn resulted from the higher tax levels. If the higher tax levels had been assessed against the lower labor using models product changes may have occurred. Such changes might require tax levels higher than any used, however.

Monthly decreases in water deliveries were related to the total water diverted before any change. In other words the more water used the more the decrease was likely to be. July and August were the months with the largest drops in water use. The latter could be quite important as far as water quality is concerned since July and August appear to be months during which river turbidity is high. No decrease occurred in April due to the model. It was assumed that so much water was needed to irrigate beets, soak ditches, etc., regardless of the irrigation technique. In other words improved irrigation efficiency for irrigating beets up was not considered likely with the present structures, ditches, etc.

The decrease in water purchases alone could have positive effects on water quality of the Bighorn River. In essence more water might be left in the river for dilution purposes. Other users might, however, divert this "extra" water which could reduce dilution effects. There is definitely a need for considering such aggregate effects to help make policy recommendations more meaningful.

Return flows decreased by amounts identical to the reduction in water purchases when crops and crop acreages did not change from the initial solution. This situation arose since the model assumes that the irrigator will furnish the irrigation consumptive use requirement to the crops, no matter which technique he uses. When crop acres varied from the initial solution, return flows declined somewhat less than total deliveries.

Return flows might be decreased even more with higher taxes but costs to the farmers would also go up. When barley for grain came into the solutions for the +.20 and +.30 labor models, total return flows decreased compared to the other models. The decrease was due to the decrease in alfalfa acreage. Alfalfa requires more water in total than barley. It is possible however that even though there was less return flows in total, they might be of lower quality.

It seems likely that particularly surface return flows from barley will be more turbid than surface return flows from alfalfa. More technical work is needed concerning the last point.

Net returns to the operator's management and land were significantly reduced by this policy. The sensitivity of the "irrigation two" labor assumption also becomes more apparent when net returns are examined. If farm irrigation efficiency were increased with only 5 percent more labor and 10 percent more irrigation costs, net returns were decreased only $3, 090 (Table 7. 4). That is, the decrease was $3, 090 when a tax of $. 15 per acre inch was levied. The $. 15 tax was sufficient to induce a switch to "irrigation two" for all crops. Notice however, if the improved efficiency required 30 percent more labor and 10 percent more irrigation costs, net returns were decreased $12, 422 before all crops were irrigated by technique two. The big difference was due to the fact that the 30 percent case required a $. 60 per acre inch tax to result in a total technique switch. Again the implication is that better information concerning labor requirements of the second irrigation technique is needed. The better information would be of particular importance to the farmers. If a tax of $. 15 is sufficient to induce a technique switch the higher tax will only be a burden on the farmer. Without better information the tax level might be set at one of these higher levels.

The big cost item to the farmer of the water tax was not the variable costs or the increased labor. The tax itself accounted for the major share of the reduced net returns. The irrigation variable cost assumption appeared to make the least difference on net returns. The +.10 and +.20 columns for the same labor assumptions in Table 7.4 can be compared to demonstrate the last point. The comparison is not valid for the +.30 labor columns since when costs were +.10 a $.60 tax induced a total technique switch whereas at the +.20 cost columns a $.65 tax was needed for a total switch. The tax size necessary to induce a total switch for the other labor assumptions was not affected by the cost assumption.

Tax sizes necessary to induce technique changes are related in Table 7.5. As already indicated total technique change required increasingly higher taxes as labor requirements went up.

The irrigation method for sugarbeets was always the last to change. That is, it consistently required higher tax levels to induce changes in the method of irrigating sugarbeets. Notice that at the tax level of $.25 per acre inch the switch is complete to technique two for all crops and all assumptions except for sugarbeets. The latter suggests that some form of a discriminatory tax may be less costly to the total farm population than a uniform tax. In other words, the water tax might be higher to those who grow sugarbeets than to those who do not. Or a higher tax on water for irrigating sugarbeets might be assessed. The latter could be extremely difficult to

Table 7.5. Effects on irrigation technique of various tax levels on water when labor was unconstrained. Irrigation two variable costs increased by [a]

	10 percent Irrigation 2 labor increased by [b]				20 percent Irrigation 2 labor increased by [b]			
Tax per acre inch $	.05	.10	.20	.30	.05	.10	.20	.30
0	1[c]	1	1	1	1	1	1	1
.10	2,2,2 2,2,1	2,2,2 2,2,1	2,1,1 1,1,1	2,1,1 1,1,1	1,2,2 2,2,1	2,2,2 2,2,1	1	1
.15	2	Same	2,2,2 2,2,1	2,2,2 1,2,1	2	Same	2,2,1 1,1,2	1
.20		Same	Same	2,2,2 2,2,1		Same	2,2,2 2,2,1	2,1,1 1,2,1
.25		2	Same	Same		2	Same	2,2,2 2,2,1
.30			Same	Same			Same	Same
.35			Same	Same			Same	Same
.40			Same	Same			Same	Same
.45			2[d]	Same [d]			2[d]	Same [d]
.50				Same				Same
.55				Same				Same
.60				2				Same
.65								2

---

[a] That is, irrigation variable costs for technique two are 110 or 120 percent of technique one irrigation variable costs.

[b] That is, irrigation labor for technique two is increased from 5 to 30 percent over technique one labor requirements.

[c] The appearance of the number "1" indicates that only irrigation one appeared in the optimal solution for all crops, while "2" indicates that only irrigation two appeared. Once "2" appears alone technique two was used at all higher prices. If both techniques are used a sequence of numbers is shown. The sequence indicates which technique was used by each crop in the following order: Alfalfa, barley with alfalfa, malt barley, corn for grain, corn for silage and sugarbeets. The "same" refers to the combination above or for the previous price.

[d] Feed barley came in at this tax level. It was irrigated by technique two.

administer.

The tax level necessary to induce a total switch to method two was not affected greatly by the irrigation variable cost assumption. The only difference noticed was that at the +.30 labor assumption a $.65 tax had to be assessed when costs were +.20 versus a $.60 tax level when costs were +.10.

Product price changes are not likely to affect irrigation technique after all crops are using method two. The latter is so since after irrigation two became optimal, technique two dominated one. The domination was complete with the constraints used for these models. The situation after the tax would appear similar to Figure 7.2 for each crop. In other words the quantity of crop produced by each method was constrained to the same point. The only constraints relevant were the land and the upper and lower level crop constraints. These constraints were the same for either irrigation method. Also the use coefficients for the constraints were identical for each irrigation technique. Once the irrigation costs including taxes and labor are in the position shown in Figure 7.2, irrigation two will be used. Furthermore no price changes of the crop can induce a switch back to irrigation one. Changes in prices of variable cost items could, however, alter the relationship shown in Figure 7.2 and induce a switch back.

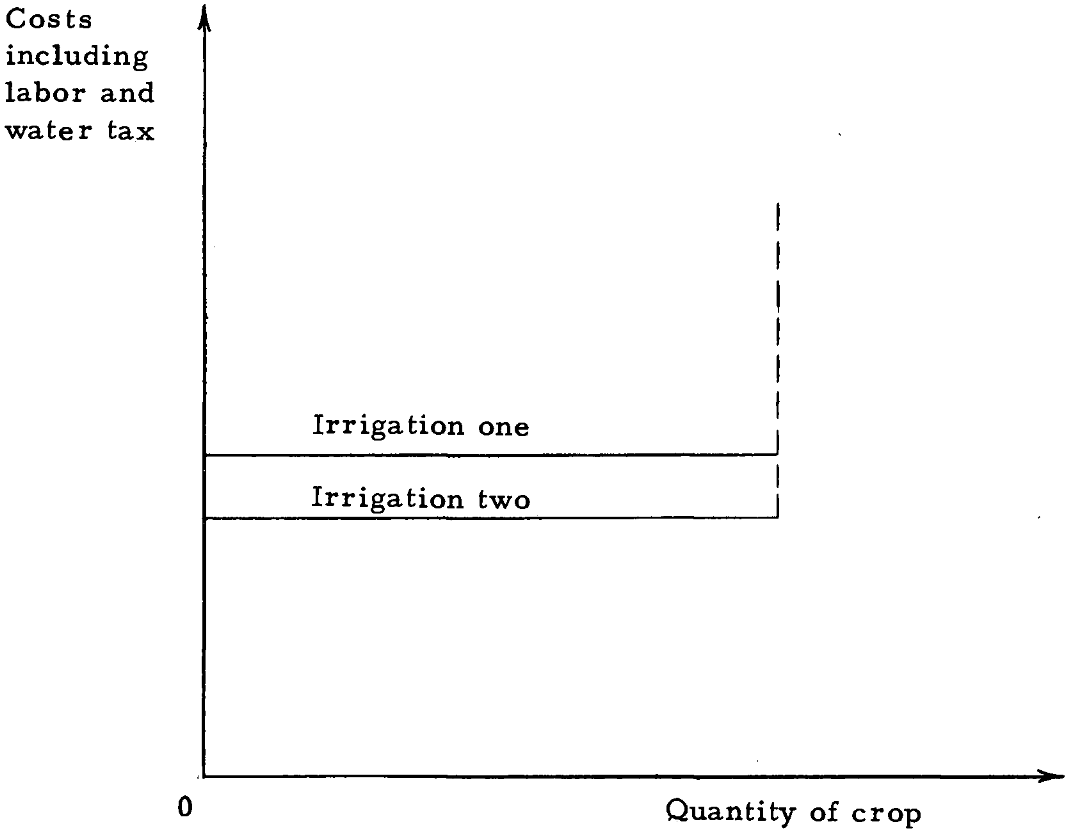

Figure 7. 2. Relationship between irrigation methods after a tax induced switch to method 2--labor unconstrained.

Relative price changes of products could certainly alter the optimal crop mix. However, a switch of crops due to relative price changes might occur whether water is taxed or not.

The conclusion is that under the conditions of this farm model the effects of the water tax will not be altered by product price changes. Consequently, once a tax level is determined that induces a switch to technique two, product price changes could be ignored. The administrative agency would only need to keep track of input price changes that might alter the cost relationship. For example a jump in the wage rates could result in a reversal of the relationship in Figure 7.1 since technique two requires more labor than one. An increased water tax may then be needed to offset the wage rate change.

Results with Labor Constrained

Labor was constrained by months at the labor use levels of the initial unconstrained solution. The initial solution as shown in Table 7.4 was similar to the "representative farm" solution described by Agee which reflects conditions on existing operations in the Worland area. It was assumed then that since existing farms were able to obtain the needed labor for operating the "representative farm" they could also obtain the labor quantities shown in the initial solution. However, what if those quantities represent the maximum amounts of obtainable labor? The levels at which labor was constrained are shown in Table 7.6 along with the other constraints used and the solutions.

Notice that the only crop constraint used was a lower constraint on alfalfa acreage. The other crops were not constrained so as to permit more flexibility in the operation when the policy was applied. Labor costs at $2.31 were added to various enterprise costs. The latter was done so that the relative returns between enterprises

Table 7.6. Initial and Final Solutions for a Water Tax When Labor was Constrained.[a]

				Irrigation two variable costs increased by							
				+.10							
				Irrigation two labor increased by							
				.05		.10		.20		.30	
	Unit	Constraint levels	Initial solution	Solution[b]	Diff.[c]	Solution	Diff.	Solution	Diff.	Solution	Diff.
Net Revenue	$		32,197	23,934	-8263	25,542	-6655	19,510	-12687	19,218	-12979
Alfalfa	acres	80 min.	120	116	- 4	107	- 13	80	- 40	80	- 40
Barley-alf.	acres		40	39	- 1	36	- 4	26	- 14	27	- 13
Barley-grain	acres		--	--	--	--	--	--	--	--	--
Barley-malt	acres		50	45	- 5	57	+ 7	59	+ 9	69	+ 19
Corn-grain	acres		100	93	- 7	100	--	92	- 8	100	--
Corn-silage	acres		--	--	--	--	--	--	--	--	--
Sugarbeets	acres		190	190	--	190	--	190	--	190	--
Total	acres	500	500	483	- 17[d]	490	- 10	447	- 53	466	- 34
Production											
Alfalfa	tons	--	468	454	- 14	419	- 49	312	- 156	312	- 156
Barley-gr.	bu.	---	2398	2324	- 74	2144	- 254	1598	- 800	1598	- 800
Barley-malt	bu.	--	3749	3348	- 401	4257	+ 508	4394	+ 645	5222	+1473
Corn-grain	bu.	--	11,449	10,619	- 830	11,405	- 44	10,492	- 957	11,408	- 41
Corn-silage	tons	--	--	--	--	--	--	--	--	--	--
Sugarbeets	tons	--	3952	3952	--	3952	--	3952	--	3952	--
Labor use	hours										
March		356.5	356.5	351.7	- 4.8	355.7	- .8	346.0	- 10.5	353.0	- 3.5
April		722.4	722.4	722.4	--	722.4	--	722.4	--	722.4	--
May		500.1	500.0	493.7	- 6.3	500.1	+ .1	500.1	+ .1	500.1	+ .1
June		572.6	572.4	571.5	- .9	557.0	- 15.4	527.0	- 4.54	522.5	- 49.9
July		515.0	514.9	515.0	+ .1	502.2	- 12.7	487.2	- 27.7	474.9	- 40.0
August		834.0	831.5	834.0	+ 2.5	834.0	+ 2.5	834.0	+ 2.5	834.0	+ 2.5
September		621.8	414.3	425.7	+ 11.4	402.2	- 12.1	394.2	- 20.1	376.2	- 38.1
October		1299.7	1299.7	1299.7	--	1299.7	--	1299.7	--	1299.7	--
November		200.0	125.0	116.2	- 8.8	124.5	- .5	113.5	- 11.5	124.5	- .5
Total			5336.7	5329.9	- 6.8	5297.8	- 3.89	5224.1	- 112.6	5207.3	- 129.4

Table 7.6 (continued)

				Irrigation two variable costs increased by						
				+.10						
				Irrigation two labor increased by						
	Unit	Constraint levels	Initial solution	.05		.10		.20		.30	
				Solution	Diff.	Solution	Diff.	Solution	Diff.	Solution	Diff.
Water purch.	Ac. Ins.										
April		--	1140.1	1140.1	--	1140.1	--	1140.1	--	1140.1	--
May		--	3184.5	2501.9	-682.6	2977.8	-206.7	2252.5	-932.0	2798.8	-385.7
June		--	4457.2	3617.3	-839.9	4237.1	-220.1	3677.4	-779.8	4043.7	-413.5
July		--	6322.6	5068.0	-1254.6	6006.2	-316.4	5028.7	-1293.9	5738.1	-584.5
August		--	6283.6	5038.5	-1245.1	5841.6	-442.0	4790.8	-1492.8	5438.9	-844.7
September		--	1056.5	864.6	-191.9	1056.5	--	998.9	-57.6	1056.5	--
Total			22444.5	18230.4	-4214.1	21259.3	-1185.2	17888.4	-4556.1	20216.1	-2228.4
Return flow	Ac. Ins.										
April		--	684.1	684.1	--	684.1	--	684.1	--	684.1	--
May		--	1551.2	962.7	-588.5	1394.8	-156.4	888.2	663.0	1329.3	-221.9
June		--	2247.4	1460.5	-786.9	2081.5	-165.9	1702.6	-544.8	2012.9	-234.5
July		--	3221.0	2070.8	-1150.2	2986.5	-234.5	2290.2	-930.8	2896.6	-324.4
August		--	3179.9	2039.5	-1140.4	2864.6	-315.3	2153.6	-1026.3	2725.9	-454.0
September		--	581.4	389.5	-191.9	581.4	--	523.8	-57.6	581.4	--
Total		--	11465.0	7607.1	-3857.9	10592.9	-872.1	8242.5	-3222.5	10230.2	-1234.8
Tax level	$	--	--	.40		.30		.60		.60	

[a] Acres times yields will not always coincide with production shown since acres were rounded to nearest whole number.

[b] Solution shown is for the highest tax level applied. See the last row for that level.

[c] Diff. refers to the difference between this solution and the initial solution.

[d] A negative figure indicates that this many acres were left idle.

Table 7.6 (continued)

				Irrigation two variable costs increased by							
				+.20							
				Irrigation two labor increased by							
	Unit	Constraint levels	Initial solution	.05		.10		.20		.30	
				Solution	Diff.	Solution	Diff.	Solution	Diff.	Solution	Diff.
Net revenue	$	--	32197	23880	-8317	25530	-6667	19468	-12729	19207	-12990
Alfalfa	acres	80 min.	120	116	-4	107	-13	80	-40	80	-40
Barley-alf.	acres		40	39	-1	36	-4	27	-13	27	-13
Barley-grn.	acres		--	--	--	--	--	--	--	--	--
Barley-malt	acres		50	48	-2	57	+7	70	+20	69	+ 19
Corn-grain	acres		100	96	-4	100	--	87	-13	100	--
Corn-sil.	acres		--	--	--	--	--	--	--	--	--
Sugarbeets	acres		190	190	--	190	--	190	--	190	--
Total	acres	500	500	489	-11	490	-10	454	-46	466	-34
Production											
Alfalfa	tons	--	468	451	-17	419	-49	312	-156	312	-156
Barley-grn.	bushels	--	2398	2312	-86	2144	-254	1598	-800	1598	-800
Barley-mlt.	bushels	--	3749	3621	-128	4257	+508	5231	+1482	5222	+1473
Corn-grain	bushels	--	11449	11099	-350	11405	-44	10012	-1437	11408	-41
Corn-sil.	tons	--	--	--	--	--	--	--	--	--	--
Sugarbeets	tons	--	3952	3952	--	3952	--	3952	--	3952	--
Labor use	hours										
March		356.5	356.5	353.7	-2.8	355.7	-.8	353.1	-3.4	353.0	-3.5
April		722.4	722.4	722.4	--	722.4	--	701.7	-20.7	722.4	--
May		500.1	500.0	500.1	+.1	500.1	+.1	500.1	+.1	500.1	+.1
June		572.6	572.4	569.7	-2.7	577.0	-15.4	517.5	-54.9	522.5	-49.9
July		515.0	514.9	515.0	+.1	502.2	-12.7	484.5	-30.4	474.9	-40.0
August		834.0	831.5	834.0	+2.5	834.0	+2.5	834.0	+2.5	834.0	+2.5
September		621.8	414.3	416.7	+2.4	402.2	-12.1	376.2	-38.1	376.2	-38.1
October		1299.7	1299.7	1299.7	--	1299.7	--	1299.7	--	1299.7	--
November		200.0	125.0	121.5	-3.5	124.5	-.5	110.2	-14.8	124.5	-.5
Total			5336.7	5332.8	-3.9	5297.8	-38.9	5177.0	-159.7	5207.3	-129.4

Table 7.6 (continued)

				Irrigation two variable costs increased by							
				+.20							
				Irrigation two labor increased by							
	Unit	Constraint levels	Initial solution	.05		.10		.20		.30	
				Solution	Diff.	Solution	Diff.	Solution	Diff.	Solution	Diff.
Water purch.	Ac. ins.										
April		--	1140.1	1140.1	--	1140.1	--	1140.1	--	1140.1	--
May		--	3184.5	2570.5	-614.0	2977.8	-206.7	2319.8	-864.7	2798.8	-385.7
June		--	4457.2	3897.2	-560.0	4237.1	-220.1	3907.9	-549.3	4043.7	-413.5
July		--	6322.6	5404.2	-918.4	6006.2	-316.4	5206.0	-1116.6	5738.1	-584.5
August		--	6283.6	5327.6	-956.0	5841.6	-442.0	4838.9	-1444.7	5438.9	-844.7
September		--	1056.5	972.5	-84.0	1056.5	--	1056.5	--	1056.5	--
Total			22444.5	19312.1	-3132.4	21259.3	-1185.2	18469.2	-3975.3	20216.1	-2228.4
Return flow	Ac. ins.										
April		--	684.1	684.1	--	684.1	--	684.1	--	684.1	--
May		--	1551.2	992.6	-558.6	1394.8	-156.4	913.1	-638.1	1329.3	-221.9
June		--	2247.4	1726.2	-521.2	2081.5	--165.9	1876.6	-370.8	2012.9	-234.5
July		--	3221.0	2368.0	-853.0	2986.5	-234.5	2443.8	-777.2	2896.6	-324.4
August		--	3179.9	2294.7	-885.2	2864.6	-315.3	2241.4	-938.5	2725.9	-454.0
September		--	581.4	497.4	-84.0	581.4	--	581.4	--	581.4	--
Total		--	11465.0	8563.0	2902.0	10592.9	-872.1	8740.4	-2724.6	10230.2	-1234.8
Tax level	$	--		.40		.30		.60		.60	

would be comparable to the unconstrained cases.

Crop enterprises in the initial and all other solutions were identical. Notice that the final solutions in Table 7.6 are for the highest tax applied to the relevant model. The only differences various model assumptions and water tax levels made were in the acres of the crops. Again the labor assumed for irrigation two seemed to affect enterprise levels more than the assumed variable irrigation cost assumptions. At the same tax levels there were some differences in crop acreages between the +.10 and +.20 cost assumptions for the +.05 and +.20 labor use coefficients (Table 7.6). For example more land (17 acres) was left idle at the +.10 cost level for the +.05 labor assumption than at the +.20 cost level for the same labor assumption (11 acres). The additional land was used by the malt barley and corn grain enterprises. At first glance such an occurrence seems incorrect. However Table 7.6 does not show the irrigation techniques used. By examining irrigation techniques used, shown in Table 7.7, the apparent error is explained. With the +.10 cost assumption at the $.40 per acre inch tax level all enterprises utilized technique two. With the +.20 cost and the $.40 tax rate on water, sugarbeets were irrigated by both techniques one and two. Since technique one requires less labor than two there was more labor available for other crops. The same explanation relates to observed differences in crop levels for the +.20 labor assumption between the two cost sizes.

Table 7.7. Effects on irrigation technique of various tax levels on water when labor was constrained. Irrigation two variable costs increased by [a]

	10 percent Irrigation 2 labor increased by [b]				20 percent Irrigation 2 labor increased by [b]			
Tax per acre inch $	.05	.10	.20	.30	.05	.10	.20	.30
0	1[c]	1	1	1	1	1	1	1
.10	1/2, 1, 1 1/2, 1	1/2, 1, 1 1/2, 1	Same	Same	1/2, 1, 1 1/2, 1, 1	1, 1, 1 1/2, 1	Same	Same
.15	Same	1/2, 1, 1 1, 1	1, 1, 1 1/2, 1	Same	Same	1/2, 1, 1 1, 1	1, 1, 1 1/2, 1	Same
.20	2, 2, 2 2, 1	Same	1/2, 1, 1 1/2, 1	1, 1, 1 1/2, 1	2, 1, 1 1/2, 1	Same	1/2, 1, 1 1/2, 1	1, 1, 1 1/2, 1
.30	Same	Same	Same	1/2, 1, 1 1, 1	Same	2, 1, 1 1, 1	Same	1/2, 1, 1 1/2, 1
.35	Same		2, 1, 1 1, 1	Same	Same		2, 1, 1 1, 1	1/2, 1, 1 1, 1
.40	2		2, 1, 2/1 1, 1	1/2, 1, 2/1 1, 1	Same		2, 1, 2/1 1, 1	1/2, 1, 2/1 1, 1
.45			Same	2, 1, 1/2 1, 1			Same	2, 1, 1/2 1, 1
.50			2, 2, 2 2, 1	Same			2, 2, 1 1/2, 1	Same
.55			Same	Same			2, 2, 2 2, 1	Same
.60			2, 2, 2 2, 1/2	2, 2/1, 1 1, 1			Same	2, 2/1, 1 1, 1

[a] That is, irrigation variable costs for technique two are 110 or 120 percent of technique one irrigation variable costs.

[b] That is, irrigation labor for technique two is increased from 5 to 30 percent over technique one labor requirements.

[c] The number "1" indicates that all crops produced used only technique one while a "2" indicates all crops used technique two only. If both techniques were used a sequence of numbers is shown. The sequence indicates the techniques used for each crop in the following order: Alfalfa, barley with alfalfa, malt barley, corn for grain, and sugarbeets. The other crops were not grown for any of the solutions. The appearance of 1/2 indicates both techniques were used for that crop and that more acres were irrigated with method one than two. The 2/1 is interpreted similarly except more acres were irrigated with method two than one.

Notice that the initial solutions for the labor constrained versus labor unconstrained models are somewhat different. Remember, there were no upper level constraints for crop acreages in the labor constrained models. Consequently corn for grain replaced corn for silage in the optimal solution. This happened since labor requirements and costs for silage corn were higher than for grain corn. The gross returns between the crops were similar. Without the upper constraints on corn acreages, the labor unconstrained model would have undoubtedly ended up at the same point.

Another noticeable difference between the labor constrained and unconstrained models concerns the acres left idle as the water tax was applied. At all of the tax levels considered all acres were utilized by the optimal solutions for the labor unconstrained models. However, as can be observed in Appendix Table B-9 land was left idle when labor was constrained.

Idle land probably does not square with reality. The latter seems particularly true when the number of idle acres is small. In reality the farmer would likely be able to find a few more hours somewhere or reduce labor for some enterprises. The important implication is that when labor is constrained the net revenue maximizing farmer will need to shift between crop enterprises as tax levels go up. Consequently he will have more decisions to make than if labor were unconstrained.

Labor use for the entire year did not vary from the initial solution by any large degree (Table 7.6). As expected the models with the most idle acres, used the least amounts of labor. Since as tax rates went up idle acreage usually went up also (Appendix Table B-9) it can be concluded that increased tax size tended to diminish total labor use. The latter result is contrary to what happened for the labor unconstrained models. There, as tax levels increased the trend was for labor use to also increase.

Some months stand out as being consistently labor constraining. For example, all available April and October labor was usually used. October labor use was mainly for sugarbeet harvest which entered all models at the upper constraint of 190 acres. No general conclusions concerning tax effects on other labor months can be drawn. There tended to be a fluctuation for these other months' labor use from total available to a few hours less than total available as tax was increased.

Water use declined for all labor use assumptions as the water tax was increased. The decline was not continuous, however, for all five cent increments. Rather the decline was in a stepwise manner, meaning that water use stayed the same for two to three increments and then would drop to a new level. Examination of Table 7.7 will give some indication of how water use changed as tax levels went up. For example, if the techniques used between tax levels did not change, the water use would not have changed either. Water use levels were consistently higher for the same tax levels for those models using more labor. The change in irrigation variable cost assumptions made little difference. For example at the 30 cent tax rate water use was 19, 312, 21, 259, 22, 426, and 22, 435 acre inches for the +. 05, +. 10, +. 20 and +. 30 labor use assumptions, respectively. These water use figures were the same for both cost assumptions. These figures are not all depicted in Table 7. 6 since only the water purchases for the highest tax rates assessed are shown. Monthly water use rates were affected similarly as the unconstrained models but not as significantly in absolute terms.

There was a noticeable difference in water purchases between the labor constrained and unconstrained models. A higher tax rate was consistently required by the various constrained models to reduce water use to levels comparable to the labor unconstrained models. In the cases of the higher assumed labor use models the largest tax rates used ($. 60 per acre inch) did not even reduce water use levels to figures as low as those generated by lower tax levels for the unconstrained models. For example a $. 60 tax rate at the 20 percent labor assumption and 20 percent cost level reduced water use in the constrained case to 18, 469 acre inches. In the unconstrained instance for the same labor and cost assumptions a $. 45 tax rate reduced water use to 7, 784 acre inches. The latter comparison can be made by looking at Tables 7.4 and 7.6.

The main conclusion to be drawn is that the existence of labor constraints made higher tax levels necessary to reduce water diversions compared to the case where unlimited labor was available at the wage rate assumed. Remember the conclusion is based on models offering as alternatives higher labor using techniques. Another conclusion is that the amount of labor needed to improve on farm irrigation efficiency appears quite important with respect to total water use.

Return flows tended to be reduced less than water deliveries for the constrained models. The difference occurs since crop acreages changed between various tax levels. If crop acreages had remained constant, return flows would have been reduced the same as farm deliveries.

The tax on water reduced return flow volumes, although, reductions were less than when labor was unconstrained. Further reductions probably could have been achieved by using higher tax rates; however, the lowest rate applied ($.30 to the +.10 labor models) diminished net returns by $6,650 or about 20 percent.

The types of changes induced by the tax were shifts from alfalfa to malt barley and reduction in total acres. Whether or not increased malt barley acreage will increase total turbidity and sediment loads even though total return flows are reduced is conjecture. The answer in part depends on what happens to the return flows after they leave the field. For example, if the return flows go to some natural channel and erode that channel, then reduction in volume could be quite helpful. More technical work is needed regarding the latter points.

For a given tax level return flows went up (still below initial levels) as the irrigation labor use coefficient increased. This increase was due to fewer acres being irrigated by method two at the higher labor use levels.

Net returns to management and land were reduced for all assumed conditions as the water tax was increased. As long as technique one alone was used the net returns were not affected by different labor use and cost assumptions for technique two. When tax levels became high enough to induce irrigation method switches net returns were then lower for the models requiring more labor and more variable costs for technique two (Appendix Table B-10).

The level of tax required to induce a total switch to irrigation two was higher for the labor constrained versus labor unconstrained models. However, in only one instance, the +.05 labor and +.10 variable irrigation costs, was the tax level applied under labor constrained conditions actually high enough to result in total use of technique two. For that case net returns were reduced by $3, 090 for the unconstrained situation versus $8, 263 for the constrained. A large part of the difference in net returns reduction can be attributed to the higher tax ($.40 per acre inch versus $.15) necessary for complete conversion to method two with the labor constrained model.

Net returns changes between the labor constrained and unconstrained situations, are not totally comparable since the initial solutions were different. However, proportions of changes due to various factors can be compared. For example, about $2,824 of the reduction in net returns for the unconstrained case discussed above were directly attributable to the $.15 tax on water. The latter represents about 91 percent of the total reduction ($3,090 in net returns). For the constrained situation about $7,292 of the lower net returns were due to the $.40 water tax or about 88 percent of the total net returns reduction ($8,263). Apparently other changes such as reduction in cropped acres accounted for relatively more of net returns reductions for the constrained case.

Other labor assumptions cannot be compared similarly since tax levels were not assessed to levels high enough to induce total changes to method two. The general pattern for both constrained and unconstrained cases when tax levels increased was for the percent of the net returns reduction attributable to water cost to go down if technique changed. However, if techniques remained constant between two tax levels the percent of decreased net revenues due to water went up. At comparable tax levels the percent of net returns reduction due to water cost was generally higher for the labor constrained cases. Such a result appears logical since the constrained cases used more water than the unconstrained models at comparable tax levels.

The constrained models consistently required higher tax levels to induce technique switches. The discussion in the previous paragraph indicated that higher tax levels accompanied by technique switches decreased the proportion of net returns attributable to water cost. The last two statements together imply that if tax levels had been increased enough for the constrained cases so that conversion to technique two were complete, then the proportion of net return reductions due to water taxes would be lower for the constrained versus unconstrained models. In other words, a result as depicted by the above discussion for the +.05 labor and +.10 cost assumption models.

There are two conclusions that can be reached from the previous discussion. First, to obtain total conversion to technique two required higher tax levels and thus was more costly to the farm for cases where labor was constraining. Second, the proportion of net return reductions (costs of the tax) due to factors other than water was higher for the farm when labor was constraining.

Another conclusion that can be reached by observing Appendix Table B-10 is that comparable tax levels consistently reduced net returns (compared to the initial) more for the labor constrained than for unconstrained models. That situation occurred since water levels were reduced less and land was left idle when labor was constrained. In summary then, various tax levels on water are likely to be more costly to farms whose production levels are constrained by labor.

Tax size and associated irrigation technique changes are shown in Table 7.7. As can be observed the tax sizes used were not generally large enough to induce complete switching to technique two. In fact when irrigation two labor was assumed to be 30 percent higher than irrigation one labor only alfalfa was irrigated totally by technique two. At lower labor assumptions more crops utilized the second technique.

A result occurred for the labor constrained models that was indicated as a possibility when the theoretical Model 4.1 was considered. That is, at certain tax levels given crops were produced with both techniques. Such a possibility was fairly common at the tax levels used (see Table 7.7).

For the models in which technique switches occurred, conversion to irrigation two for sugarbeets required higher tax levels than for other crops. Generally, crops with higher net returns per acre relative to other crops required higher tax levels to induce technique switches.

As already discussed tax levels necessary to induce technique changes were higher for the labor constrained versus labor unconstrained models. The conclusion is that the water tax needed to be higher for the farm when labor supply was constraining if return flow reducing techniques were to be adopted. It may be that some form of labor subsidy would be needed in addition to the water tax to induce farms to change to higher labor using techniques. The subsidy would at least reduce total costs to the farms of both policies.

Product price changes may affect irrigation techniques even after all crops are using method two. Unlike the labor unconstrained models technique two did not dominate one after the tax. Since labor was constrained and since method two required more labor than one, the maximum levels of production due to the labor constraint was lower for method two. The situation is depicted in Figure 7.3. At the price P^o the net revenue for irrigation two is larger than that for irrigation one. Decreases in P will not alter that relationship. However, increases in P could induce a switch back to technique one.

Input price changes could also alter the relationship shown in Figure 7.3. Since labor costs were included in the enterprise costs an increase in wage rates could cause the irrigation two costs to become higher than irrigation one. Some upward adjustment in tax level would then be necessary.

Under the conditions of the farm models considered here, flexible taxes associated with product price changes appear more necessary when labor is constraining. Increasing product prices in particular could result in reversals of irrigation techniques.

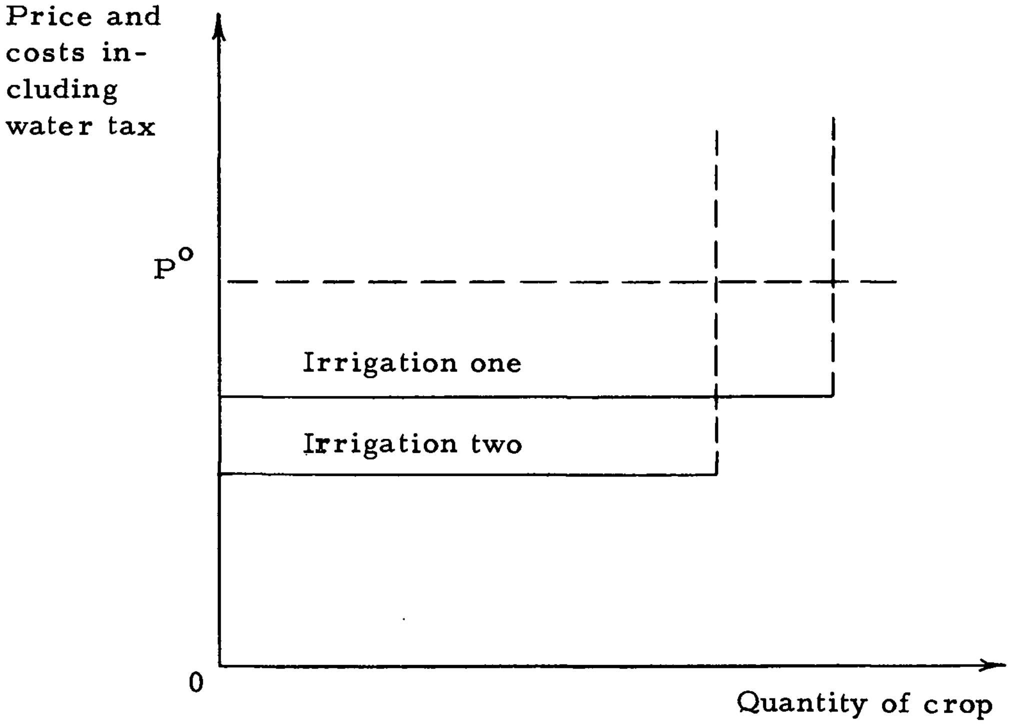

Figure 7.3. Relationship between irrigation methods after a tax induced switch to method 2--labor constrained.

Periodic review of taxes may be sufficient. Attempting to relate tax levels specifically to market prices was found to be conceptually quite difficult when discussed in the theory sections. If difficult conceptually it is likely to be difficult in practice.

Constraining Water Deliveries

The levels of the water constraints were based on the water used by the labor and water unconstrained models when irrigation two was used by all crops. In other words the same acreages of crops could be produced as initially, provided the farm used irrigation technique two. The same water constraints were applied while labor was constrained and unconstrained. The levels of the various constraints used are shown in Table 7.8. Notice that the labor constraints are only relevant for the "labor constrained" columns. Also, when labor was restricted the only crop constraint used was the 80 acre minimum on alfalfa. Total land was constrained at 500 acres in both the labor constrained and unconstrained cases.

Crop enterprises in the final solutions did not change as the labor use assumption was increased when labor was unrestricted. The acres and crops remained identical to the initial solutions. Table 7.8 shows the solutions for the irrigation two cost assumption of +.10. The +.20 cost assumption models had identical values as the +.10 except for net revenue.

Therefore, for this policy when labor was unconstrained the enterprises were insensitive to changes in cost and labor use. Data refinement does not appear as crucial for this policy as it did for the tax on water.

When labor was constrained the crop enterprises remained unchanged but acres of various crops were altered. For one thing, land was left idle for all labor use assumptions. The acres left idle increased as irrigation two labor requirements were increased. There were also shifts in acreages between crops. As labor use went up malt barley and corn for grain tended to increase while the other crop

Table 7.8. Initial and Final Solutions with Water Constrained

			Labor unconstrained							
			Irrigation two labor increased by							
			.05		.10		.20		.30	
	Units and Constraint[d]	Initial	Solution[a]	Diff.[b]	Solution	Diff.	Solution	Diff.	Solution	Diff.
Net Revenue	$	31,185	30,924	- 261	30,727	-458	30,344	-841	29,957	-1228
Cropland[c]	acres									
Alfalfa	80	120 (1)	120 (2)	--						
Barley-alf.	--	40 (1)	40 (2)	--						
Barley-malt	50	50 (1)	50 (2)	--	(same)[e]		(same)		(same)	
Corn-grain	50	50 (1)	50 (2)	--						
Corn-silage	50	50 (1)	50 (2)	--						
Sugarbeets	190	190 (1)	190 (2)	--						
Total	500	500	500	--						
Production										
Alfalfa	tons	468	468	--						
Barley-feed	bu.	2398	2398	--						
Barley-malt	bu.	3750	3750	--	(same)		(same)		(same)	
Corn-grain	bu.	5725	5725	--						
Corn-silage	tons	1040	1040	--						
Sugarbeets	tons	3952	3952	--						
Labor	hours									
March	356.5	356.5	356.5		356.5		356.5		356.5	
April	722.4	722.4	737.9	+ 15.5	755.4	+ 33.0	787.3	+ 64.9	819.7	+ 97.3
May	500.1	500.0	506.1	+ 6.1	512.3	+ 12.3	524.6	+ 24.6	536.8	+ 36.8
June	572.6	572.5	583.7	+ 11.2	594.9	+ 22.4	617.3	+ 44.8	639.7	+ 67.2
July	515.0	514.9	529.4	+ 14.5	543.6	+ 28.7	572.3	+ 57.4	601.0	+ 86.1
August	834.0	833.9	852.2	+ 18.3	873.3	+ 39.4	913.9	+ 80.0	955.1	+121.2
September	621.8	621.7	636.7	+ 15.0	652.1	+ 30.4	681.8	+ 60.1	711.8	+ 90.1
October	1299.7	1299.6	1299.6		1299.6		1299.6		1299.6	
November	200.0	62.5	62.6	+ .1	62.8	+ .3	63.0	+ .5	63.2	+ .7
Total		5484.0	5564.7	+ 80.7	5650.4	+166.5	5816.3	+332.3	5983.4	+499.4

(continued)

Table 7.8 (continued)

	Units and Constraint[d]	Initial	Labor unconstrained							
			Irrigation two labor increased by							
			.05		.10		.20		.30	
			Solution[a]	Diff.[b]	Solution	Diff.	Solution	Diff.	Solution	Diff.
Water	ac inch									
April	1140.1	1140.0	1140.0							
May	2660.4	3184.7	2660.3	- 524.4						
June	3700.9	4457.1	3700.9	- 756.2						
July	5242.8	6322.6	5242.8	-1079.8	(same)		(same)		(same)	
August	5215.4	6283.5	5215.4	-1068.1						
September	864.6	1056.4	864.5	- 191.9						
Total	18,824.2	22,444.3	18,823.9	-3620.4						
Return flow	ac inch									
April		684.0	684.0	--						
May		1551.3	1026.9	- 524.4						
June		2247.4	1491.2	- 156.2	(same)		(same)		(same)	
July		3221.0	2141.2	-1079.8						
August		3179.9	2111.8	-1068.1						
September		581.4	389.5	- 191.9						
Total		11,465.0	7844.6	-3620.4						

[a] The solution shown is for the irrigation two variable cost assumption of +.10. Solutions were identical for each labor assumption to those shown here when variable costs of irrigation two were set at +.20 except for net revenue which was some lower.

[b] Difference is between the shown solution and the initial.

[c] Numbers in parentheses e.g. (2) refer to irrigation method used. The designation (1/2) means more acres of this crop were irrigated with method 1 than 2 while (2/1) means the opposite.

[d] The alfalfa constraint is a minimum while all others are maximums. The labor constraints are only relevant for the labor constrained columns. When labor was constrained the only cropland constraints used were the minimum on alfalfa and the maximum total.

[e] The (same) indicates that solution for this block of data was the same as the previous columns for the given block.

Table 7.8 (continued)

	Units and Constraint d/	Initial	Labor constrained: Irrigation two labor increased by .05 Solution	.05 Diff.	.10 Solution	.10 Diff.	.20 Solution	.20 Diff.	.30 Solution	.30 Diff.
Net Revenue	$	32,197	31,313	- 884	30,685	-1512	29,855	-2342	29,244	-2953
Cropland c/	acres									
Alfalfa	80	120 (1)	113 (2)	- 7	108 (2)	- 12	99 (2)	- 21	93 (2)	- 27
Barley-alf.	--	40 (1)	38 (2)	- 2	36 (2)	- 4	33 (2)	- 7	31 (2)	- 9
Barley-malt	50	50 (1)	55 (2)	+ 5	58 (2)	+ 8	63 (2)	+ 13	63 (2)	+ 13
Corn-grain	50	100 (1)	102 (2)	+ 2	104 (2)	+ 4	108 (2/1)	+ 8	107 (2/1)	+ 7
Corn-silage	50	--	--	--	--	--	--	--	--	--
Sugarbeets	190	190 (1)	183 (2/1)	- 7	178 (2/1)	- 12	173 (2/1)	- 17	170 (1/2)	- 20
Total	500	500 (1)	491	- 9				- 24	464	- 36
Production										
Alfalfa	tons	468	442	- 26	422	- 46	384	- 84	362	- 106
Barley-feed	bu.	2398	2266	- 132	2160	- 238	1968	- 430	1864	- 534
Barley-malt	bu.	3749	4133	+ 384	4383	+ 634	4692	+ 943	4759	+1010
Corn-grain	bu.	11,449	11,682	+ 233	11,952	+ 503	12,338	+ 889	12,303	+ 854
Corn-silage	tons	--	--	--	--	--	--	--	--	--
Sugarbeets	tons	3952	3820	- 132	3712	- 240	3590	- 362	3534	- 418
Labor	hours									
March	356.5	356.5	347.8	- 8.7	340.3	- 16.2	330.4	- 26.1	324.8	- 31.7
April	722.4	722.4	722.4	--	722.4	--	722.4	--	722.4	--
May	500.1	500.0	497.3	- 2.7	497.0	- 3.0	500.1	+ .1	500.1	+ .1
June	572.6	572.4	564.4	- 8.0	557.5	- 14.9	547.0	- 25.4	542.5	- 29.9
July	515.0	514.9	515.0	+ .1	515.0	+ .1	515.0	+ .1	515.0	+ .1
August	834.0	831.5	834.0	+ 2.5	834.0	+ 2.5	834.0	+ 2.5	834.0	+ 2.5
September	621.8	414.3	410.1	- 4.2	404.9	- 9.4	396.0	- 18.3	394.1	- 20.2
October	1299.7	1299.7	1256.0	- 43.7	1220.8	- 78.9	1180.5	- 119.2	1162.0	- 137.7
November	200.0	125.0	127.8	+ 2.8	131.0	+ 6.0	135.6	+ 10.6	135.3	+ 10.3
Total		5336.7	5274.8	- 61.9	5222.9	- 113.8	5161.0	- 175.7	5130.2	- 206.5

Table 7.8 (continued)

	Units	Labor constrained								
	and		Irrigation two labor increased by							
			.05		.10		.20		.30	
	Constraint[d]	Initial	Solution	Diff.	Solution	Diff.	Solution	Diff.	Solution	Diff.
Water	ac inch									
April	1140.1	1140.1	1101.8	- 38.3	1070.9	- 69.2	1035.5	- 104.6	1019.3	- 120.8
May	2660.4	3184.5	2660.4	- 524.1	2660.4	- 524.1	2660.4	- 524.1	2660.4	- 524.1
June	3700.9	4457.2	3657.9	- 799.3	3649.5	- 807.7	3585.4	- 871.8	3533.9	- 923.3
July	5242.8	6322.6	5221.3	-1101.3	5207.0	-1115.6	5190.1	-1132.5	5187.1	-1135.5
August	5215.4	6283.6	5157.3	-1126.3	5123.4	-1160.2	5096.0	-1187.6	5107.1	-1176.5
September	864.6	1056.5	864.6	- 191.9	864.6	- 191.9	864.6	- 191.9	864.6	- 191.9
Total	18,824.6	22,444.5	18,663.3	-3781.2	18,575.8	-3868.7	18,432.0	-4012.5	18,372.4	-4072.1
Return flow	ac flow									
April		684.1	661.1	- 23.0	642.5	- 41.6	621.3	- 62.8	611.6	- 72.5
May		1551.2	1032.6	- 518.6	1037.5	- 513.7	1062.1	- 489.1	1097.9	- 453.3
June		2247.4	1524.1	- 723.3	1548.0	- 699.4	1564.5	- 682.9	1564.3	- 683.1
July		3221.0	2180.0	-1041.0	2212.9	-1008.1	2274.7	- 946.3	2336.1	- 884.9
August		3179.9	2132.1	-1047.8	2154.1	-1025.8	2218.4	- 961.5	2301.9	- 878.0
September		581.4	405.5	- 175.9	418.4	- 163.0	433.1	- 148.3	439.9	- 141.5
Total		11,465.0	7935.4	-3529.6	8013.4	-3451.6	8174.1	-3290.9	8351.7	-3113.3

acreages declined.

As already suggested idle land probably is not realistic. The important implication, however, is that when labor is constrained it will be more difficult for the farmer to maintain his current general production practices than when labor is not constraining. He may be able to find additional labor by reducing time spent on certain activities. The effects on yields of such reductions in time is not known. The farmer is also going to be faced with the need to make more decisions as to proper crop levels.

Labor use increased when water was constrained and labor was unrestricted. The latter happened since the model immediately used method two for irrigation. As the irrigation two labor use coefficients were increased total labor use also went up as required. March and October labor use did not change at all since none of the crops in the solutions required irrigation labor for those months.

When labor was constrained the picture was quite different. Total labor use declined as the irrigation two labor coefficients were increased. This somewhat unexpected occurrence happened since total acres cropped also declined. The pattern was not uniform by months since some crops increased while others decreased. For example labor use remained fairly constant for April, July, and August, steadily declined for March, June, September, and October, but went up for May and November.

The conclusion is that if labor is in fact constraining knowing accurately the constraint levels and labor use coefficients will help predict effects on labor use. This latter fact would likely be of little importance to the policy agency involved but it could be quite important in predicting overall effects of the policy on an area's economy.

Water diverted dropped from the initial to about the constrained level as soon as the policy was applied for the labor unconstrained case. Once at that level it did not change as labor required for irrigation two was increased.

When labor was restricted water use fell below the water constraint level for the total year. As irrigation two labor requirements went up water use continued to fall, again due to the decreasing crop acres. The drop in water use as labor requirements went up was fairly consistent by months. Water use during May and September remained constant as labor requirements increased and decreased for most other months. Water use in August did go up again at the +.30 labor assumption. The rise was apparently due to the irrigation of more sugarbeet acreage by method one at that higher labor requiring level.

If one is interested in predicting total water use for a farm then knowing the level of labor constraints and the specific coefficients for labor appears important. If individual farm models are to be aggregated to form an area model the differences in water use generated by various labor use assumptions could be quite important. The last statement is not relevant when labor is unconstrained.

Return flows for the labor unconstrained case declined by amounts identical to the decline in water diversions. As explained earlier such an occurrence is due to no changes in crops and crop acreages. The conclusion is that when labor was unconstrained changes in irrigation labor use coefficients did not affect return flow levels significantly. Of course if larger labor coefficients than those used here were assumed results might have been different.

When labor was constrained the return flow levels were affected by changes in the irrigation two, labor use coefficients. Return flows were always lower than the initial levels. However, as the irrigation labor use coefficients increased, return flows also increased. The results were somewhat surprising since total water use declined as labor requirements were increased. Total acres irrigated also fell as labor increased. A similar situation occurred when water was taxed, i. e., at a given tax level return flows increased as labor coefficients went up. However, for the tax policy water diversions also went up as labor needs increased.

The rising return flows were due to the shifting in acreages between crops and the changing of irrigation techniques. As labor use coefficients were increased, acres of malt barley and corn for grain increased while alfalfa and sugarbeet acres decreased. The return flow coefficients for malt barley and corn for grain were higher in the months these crops were irrigated than similar coefficients for alfalfa. The grain corn return flow production was higher in some months than sugarbeets. Also as labor coefficients went up more acres of sugarbeets and grain corn were irrigated by technique one. These effects combined to reduce total deliveries yet increase return flows.

The major implication is that restricting deliveries is not likely to result in equivalent reductions in return flows when labor is constraining. Furthermore, the more labor required by the alternate irrigation technique the smaller the reduction in return flows. Consequently, if the administrative agency is trying to reduce return flows by a certain volume, knowledge of labor use requirements and constraints could be quite critical in determining the proper reduction in deliveries.

Net returns were not reduced as much by this policy as by the water tax. The main reason was due to no charge for the water. As labor for irrigation two was increased net returns declined steadily. This pattern was true for both the labor constrained and unconstrained cases (see Table 7.8). Remember the net return figures in Table 7.8 are for the +.10 irrigation cost assumption. For the +.20 cost assumption each net return figure for the labor unconstrained case was $76 less than those in the table. For the labor constrained cases the net revenue figures for the higher cost assumption were from $46 to $68 less than those in the table. As labor use went up in the labor constrained models the difference in net revenue due to the cost assumptions tended to decline.

The costs of decreasing return flows for the unconstrained labor case were relatively small. Even for the +.30 labor assumption and the +.20 cost, net returns were reduced $1,304 or about 4.6 percent of the total net returns at that point.

When labor was constrained the costs of the policy were consistently higher (net returns reduced more) than for the unconstrained instances. At the +.30 labor and +.20 cost assumptions net returns were reduced by almost $3,000 or about 10 percent of net returns.

The main effect of the various labor assumptions was to increase the cost of the policy to the farm. It appeared that the costs of the policy were a long way from levels that would force the farm from business. Consequently, the main reason the policy agency would be concerned about effects on net returns would be for assessing total costs of the policy. Even if the policy agency is not interested in these costs, the agricultural sector would be. In the case of constrained labor, especially, better information concerning labor use of the second technique would be important for accurate cost estimates.

Again, the importance of whether or not labor was constraining is highlighted. When the farms operated with labor constraints and had available only higher labor and cost using alternatives the policy cost was more than when labor was unconstrained. Such a situation could bring up equity questions which will not be considered here, but may still be quite relevant.

Irrigation techniques are shown in Table 7.8 in the parentheses beside the crop acreages. It is apparent that the water constraint policy was successful in inducing a switch to technique two. Even in the labor constrained case most of the crops were utilizing technique two. Only part of the sugarbeets and corn for grain acres utilized irrigation method one.

When labor was constrained the technique used was slightly sensitive to increasing labor use coefficients. However, the water restriction results were not affected by the labor assumptions as strongly as the results of the water tax.

Product price changes could reverse the technique switches introduced by the water restraint. Before the water was constrained irrigation one dominated irrigation two for both the labor constrained and unconstrained instances. The effect of the water constraint was to destroy that dominance as shown in Figure 7.4. If the product price goes up beyond P^o the farm will continue to use irrigation two since it will generate more net returns than irrigation one. If the price of the product falls, however, there will be some point at which method one will again return the most.

It can be concluded that if a water constraint is placed on a farm similar to the one considered here, irrigation technique will then be insensitive to product price increases. Falling product prices could induce a switch back to method one. This conclusion applies whether or not labor is constrained.

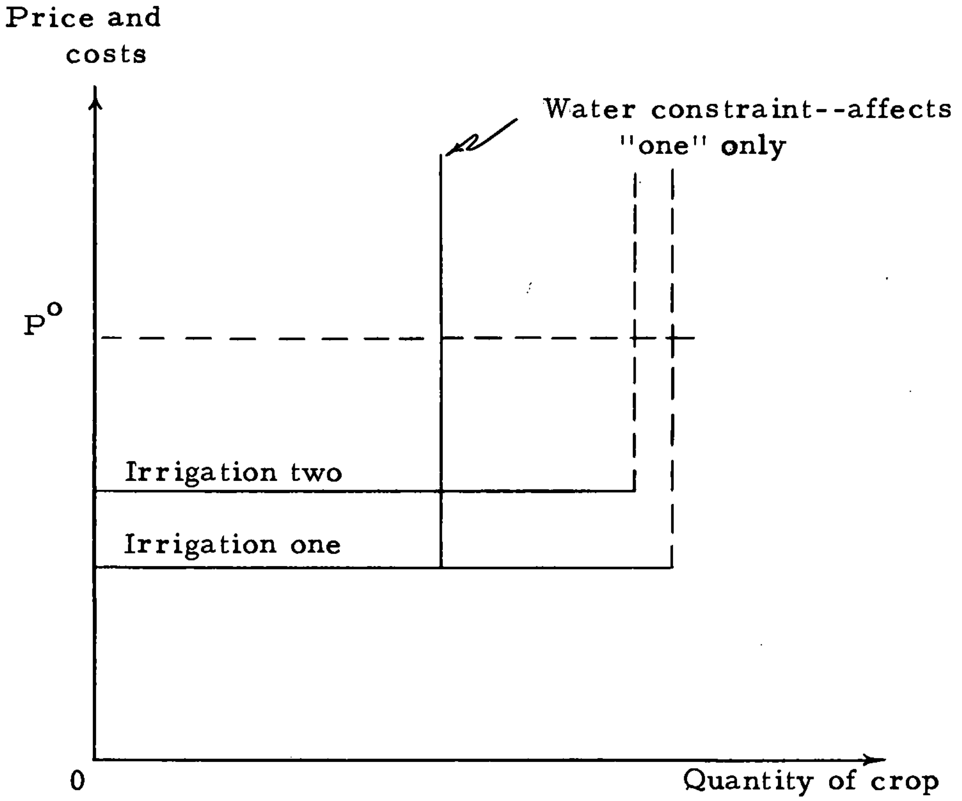

Figure 7.4. Water constraint that induced a technique switch

Summary and Conclusions

Two different policies, one pricing the other regulatory, were placed on a linear farm model. The farm model represented an irrigated farm in the Bighorn Basin area of Wyoming. The intent of the policies was to reduce return irrigation flows from the representative farm. Irrigation return flows appear to be causing pollution of the Bighorn River.

The policies were applied with the farm model being constrained and unconstrained by labor. It was assumed that there were two irrigation techniques available to the farm only one of which was used before policy implementation. It was also assumed that the two techniques both could utilize existing irrigation facilities such as ditches and turnouts. The second irrigation technique was assumed to increase farm irrigation efficiency by using more labor and additional variable costs. Since information regarding amounts of additional labor and costs required to improve farm irrigation efficiency was lacking, various labor and cost levels for irrigation two were assumed.

The first policy applied, a tax on water deliveries to the farm, did induce technique change and did reduce return flow volumes. When labor was unconstrained the tax policy was shown to be effective for all labor and cost assumptions providing the tax level was as high as $.65 per acre inch of water. Particularly the amount of labor required by irrigation two was important in determining the tax level required to induce a total switch to technique two. When labor was constrained the tax policy still appeared to be capable of inducing technique change. The level of tax necessary for the changes, however, was considerably higher than when labor was not constrained.

The regulatory policy was in the form of a monthly constraint on water deliveries. This policy also was effective in reducing return flow volumes both when labor was constrained and unconstrained. At the labor and cost levels assumed for the farm model the water restraint policy did not seem to be as sensitive to the particular labor assumptions as the tax policy. However, when water was constrained technique switches were not as complete as labor requirements increased.

There was a distinct difference in costs to the farm of the two policies. The costs to the farm were consistently and significantly lower for the regulatory policy. The main difference was due to the cost of the water since the regulatory policy did not assess any water charge.

Although product price changes were not specifically programmed, the after policy relationships between techniques provided insight into effects of possible product price change. The effects on technique of irrigation appeared to be insensitive to product price changes when irrigation two was adapted, labor was unconstrained and the switch was induced by the water tax. However, when labor was constrained it appeared that technique changes induced by the water tax could be reversed by rising product prices. Whether or not labor was constrained seemed to make less difference as to product price effects for the water restriction policy. For that policy falling product prices could induce a switch back to irrigation one. Relative price changes could result in shifts between products for both policies.

The effects of the water tax policy tended to be consistent with theoretical effects of taxing a technique specific variable factor in Model 4.1. For example, from the theoretical model it was predicted that such a policy could induce both product and technique changes. The latter occurred in the farm model. The theoretical model also indicated that one product might be produced with both techniques which also happened for the farm model. In other words, the inclusion of more than two non-specialized fixed factors and more than two products did not alter the predictive ability of the two product, two non-specialized fixed factor model.

The major conclusions reached can be summarized as follows: 1) The level of labor required to improve farm irrigation efficiency was quite important in determining tax size, costs to the farm and effects on such items as labor use and return flow reductions for the water tax policy. Labor requirements were important for the regulatory policy but not nearly as important as for the tax policy; 2) The cost to the farm of the regulatory policy was much less than the tax policy; 3) Whether or not labor was constrained made a noticeable difference on costs to the farm and return flow reductions for both policies; 4) The effects of the tax policy on techniques could be reversed by rising product prices if labor were constraining and unaffected by any product price changes if labor were not constraining. The regulatory policy effects on techniques could be reversed by falling product prices whether or not labor was constraining; and 5) Predictions based on two products, two non-specialized fixed factors, theoretical Model 4.1, were consistent with the effects of the water tax policy on the farm model where more than two products and two non-specialized factors were used.

VIII. CONCLUSIONS, HYPOTHESES, AND FURTHER RESEARCH NEEDS

This chapter is presented in three parts. The first discusses major conclusions reached in the theoretical analyses which seem to have general application to numerous types of firms. The second part discusses conclusions reached specifically for irrigation return flow pollution problems. In both of the first two sections some hypotheses are specified. The third part deals with recommendations for further work.

The main problem as delineated in Chapter I was of a theoretical nature. Review of current work in pollution control revealed a lack of knowledge concerning the effects of various policies on individual firms. The major objectives of this study were aimed at predicting policy effects on firms whose economic structures could be represented by linear economic models. To give the theory an empirical tie, two policies were applied to a linear farm model which generated irrigation return flow pollution. In the empirical example the costs of the policies to the farm were examined. The policy effects on irrigation return flows, gross water deliveries, and irrigation techniques were also considered in detail.

Attention was given to the problem of defining the terms "environmental quality" and "pollution." Many definitions from several disciplines were examined to determine definitional consistency (or lack of). Definitions were then formulated which seemed compatible with others' definitions and yet were relatable to economic theory. Pollution was defined as "a human alteration of environmental quality that presently or in the foreseeable future negatively affects someone's utility and/or cost function." As defined, pollution is definitely a value term which implies a bad or negative effect on the environment. Environmental quality was defined "as the characteristics or attributes of the environment which when taken together represent a physical, biological, and chemical description of the environment." The latter definition was intended to be value free.

There were five very basic assumptions made that were relevant to the theoretical and the empirical chapters. One assumption was that the firms were profit maximizers. Secondly, it was assumed that the firm decisions could be represented by linear models. That is the input-output coefficients were assumed to be constant. Constraints were also visualized as being linear. Third, it was assumed that all firms operated in competitive markets for both products and factors. The effect of the latter assumption was that resource and product prices were constant. Fourth, it was assumed that a non-market externality (diseconomy) was produced in fixed proportion with each product. The fifth assumption was that the firms had at least two techniques available for producing products. One of the alternative techniques was assumed to generate less of the external diseconomy than the other.

Conclusions and Hypotheses from Theoretical Models

Several theoretical models were described by making alternative assumptions about numbers of products, fixed and variable factors, variable costs and the relationship between the factors, products and costs. The following policies were then applied to most models: 1) taxing market products, 2) taxing variable factors, 3) taxing the non-market externality, 4) a standard placed on the quality of the externality, 5) subsidizing variable factors, and 6) subsidizing fixed factors.

Taxing the market products as a means of reducing production of the non-market externality appeared not to be a particularly desirable policy. The main argument against that policy was that the market product tax gave inconsistent results when applied to the various linear models. In some instances externality production was increased and in other cases it was decreased. The policy still might be useable if applied on a selective basis. Firms that produced more than one product might be candidates for such a policy. A necessary condition was that production of at least one alternative product generated noticeably less of the externality than production of product(s) initially being sold. Based on the theoretical analyses the following hypothesis can be set forth: Firms that can produce more than one market product, at least one of which generates less of the non-market externality than the others, will not reduce production of the externality when the higher externality generating product(s) is (are) taxed.

The previous hypothesis appears to be empirically testable. The test would first of all involve the identification of firms that produce or could easily produce more than one product. Also, the relationship between the products and externality production would need to be ascertained. Determination of such technical relationships is likely to be difficult particularly since there has not been a great deal of this type of technical research accomplished. Whether or not such firms had available more than one production technique would not affect the legitimacy of the test. Next, one might try to determine the firm's past behavior when relative prices of the products involved changed. If the price of the low externality product did go up relative to other products, what did the firm do? Such an analysis would depend very strongly on availability of time series data concerning the firm's production practices. An alternative test might be to determine the internal cost and decision structure of such a firm and determine hypothetically what might happen. It would seem that the previous test would be stronger than the latter.

Taxing variable factors appeared to be a possible effective policy for reducing externality production. Results of variable factor taxes were more consistent than results for taxing market products. Reductions in externality production via a factor tax depended on the characteristics of the factor. If a variable factor with the following characteristics can be identified then the variable factor tax could be effective: 1) the variable factor be strongly and positively associated with externality production, 2) total use of the variable factor be highest when the relatively high externality generating products and techniques are produced and are used, and 3) the variable factor to be taxed be product and technique specific. That is, tax a factor that is used more by the high externality generating products and techniques. The third characteristic if identified would help to insure both characteristics one and two. If factors are either product or technique specific but not both, theoretical analyses indicated that taxing the technique specific factor would likely be best.

Hypotheses can be stated for this policy. For example, taxing a variable factor which is used relatively more by a high externality generating technique will not affect externality production. This hypothesis could be tested by analyzing time series data from a given firm. First it would have to be determined whether or not alternative techniques were available to that firm. Then identification of technique specific variables would also be necessary. Finally some condition such as a significant rise in the relative cost of such a factor to the firm would have to be present in the data analyzed. One definite problem that would need to be dealt with in such an analysis is holding other things constant over time. For example changes in the prices of products and/or changes in the size of operations could camouflage the effects of changes in the relative cost of the variable factor. The hypothesis test might also be conducted by describing the internal cost and decision structure of the firm and determining how the firm would react if the cost of the relevant factor went up relative to other factors. Again, lack of technical research concerning the relationship between factors of production and externality production makes immediate testing of the hypothesis quite difficult. In other words some research by other disciplines may be necessary before the test would be achieved. Other hypotheses could be specified based on the theoretical models. However, such specification is not done here since the specific hypothesis statement will be somewhat dependent upon the researcher's interest.

Taxing the non-market externality resulted in effects that were the most consistent of any of the policies. When changes occurred due to the externality tax, they were such that the amount of the externality was reduced.

The largest problem associated with this policy concerns administration. Before the externality can be taxed it must be identified as to source and type of problems. In some instances such identification may preclude use of the externality tax.

One hypothesis that can be stated is that taxing the externality will not affect the level of the externality produced. There are already examples of empirical work that would reject the previous hypothesis.[70] These were discussed in the introductory chapter. It would seem that other similar tests could be conducted on other types of firms.

A standard on the quality of the externality was another policy applied. The standard was defined in terms of proportions. That is, a standard would be placed on the permissible concentration of various characteristics in for example water effluent. If enforceable a firm's effluent would have to meet the specified concentration if it were to be released into the environment. One advantage then would be that the administrative agency would not have to concern itself with internal firm production practices. Again the effluent will have to be identifiable if the policy is going to be enforceable. Also there will not be any revenue generated to help defray administrative costs. Consequently, the administrative costs and problems need to be weighed against the advantages this policy appears to have.

Certainly many examples of the effectiveness of this policy can be observed. What has not been ascertained is the relative costs of

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[70] See for example (17, 24).

this policy to firms versus costs of some other policies. Empirical studies should be capable of making that determination if it seems important to do so. One difference between the standard and the taxing policies relates to the permanency of the policy pressure. In other words, once the standard is met the firm is not likely to continue to search for ways of reducing externality production to lower levels. However, with a tax the firm seems more likely to continue to look for alternative ways of reducing total cost of the externality which includes costs of internal changes and the tax cost itself.

Subsidizing variable factors is a policy that worked practically opposite of the variable factor tax. Consequently, the characteristics of a variable factor for subsidizing are just the opposite of those discussed for the variable factor for taxing. That is the characteristics are opposites providing the policy intent is to reduce externality production.

The most serious fault of the variable factor subsidy concerns firm growth. It would be possible that after the subsidy the firm might become larger and in the end produce more of everything including the externality. Administrative costs would also tend to be high since no income is generated but some would need to be paid out. The firms on the other hand would be making more profit than before; consequently, they would likely prefer this policy.

Subsidizing fixed factors appeared to have potential for reducing externality production under certain conditions. If a particularly low externality generating technique not in use but available requires a specialized fixed factor (fixed in a short run sense) then subsidizing that factor could reduce externality production. The previous statement could take the form of the following hypothesis: Subsidizing a fixed (in the short run) factor that is specific to a relatively (compared to the present firm technique) low externality generating technique will not reduce production of the externality. Theoretical testing of this hypothesis has been accomplished for the linear economic model. On that basis the hypothesis would be rejected. An empirical test could again take place by analyzing time series data from a given firm or maybe cross sectional data from a group of similar firms. If analyzing time series data were the approach one might look for a situation where some new technology was introduced in an industry. This new technology, however, may not have been adopted initially due to the cost of specialized equipment. If the equipment dropped in price relative to other equipment and if it were then adopted by the firm it would seem that one might reject the above hypothesis provided externality production fell. However, the test would not be valid if the main reason for not adopting the new method initially was something other than cost. The test of this hypothesis as with the others will require research by other disciplines, particularly engineering. Without specific technical research regarding internal processes and factor requirements of such processes the test cannot be conducted.

This policy also may affect firm growth. If the subsidy level were high and/or if it were placed on factors that required large production levels to be efficient then externality production might increase. The implication is that extreme care must be exercised in applying a fixed factor subsidy.

The previous discussion has not exhausted the list of possible hypotheses derivable from the theoretical work. Depending on the researchers' interests other hypotheses could be generated. What the discussion and the related theoretical work did do was indicate that specific hypotheses can be set out. The hypotheses specified, at the very least, provide a basis for future work. However, the theoretical analyses have done more than permit hypotheses development. The types of data needed for hypothesis testing are much clearer than before. Variables that would be important to economic analyses of policy questions related to environmental problems are also considerably more apparent. Better data and model specifications provide guidelines for technical research needs. Such guidelines should give a stronger tie between the several disciplines that necessarily must be involved in environmental quality issues.

Conclusions and Hypotheses from Empirical Analysis

The theoretical sections provided a basis for ascertaining likely policies for reducing irrigation return flows. Also, specification and determination of data needs of the empirical model were greatly aided by the prior theory considerations. The farm model as specified was very similar to theoretical Model 4.1. The main difference was that the farm model contained more products and more fixed factors. In addition, the availability of predicted policy affects based on Model 4.1 provided much assistance in interpreting the results of policy applications to the irrigated farm.

Two policies were applied to a linear farm model which generated irrigation return flows. The farm model was representative of large irrigated farms along the Bighorn River of Wyoming. It was assumed that the farm could improve farm irrigation efficiency by using more labor and more variable costs and the same fixed irrigation facilities as presently in use. Improving farm irrigation efficiency would result in reducing irrigation return flow volumes. The present irrigation system was assumed to be a surface delivery system with a combination of row and border systems on individual crops. That is, a border system would be used for irrigating hay crops and a row system would be used for row crops.

The policies applied were a tax on water and a constraint on amount of water delivered to the farm. Both policies resulted in reduced return flows. The regulatory policy, however, did not appear to be as sensitive to irrigation labor requirements as the taxing policy. Whether or not labor was constrained also made a difference as to the size of tax necessary to reduce return flows and the amount of reduction in such flows.

The regulatory policy resulted in lower costs to the farm than the tax policy. The lower costs occurred with similar reductions in irrigation return flows. It can be concluded that the regulatory policy is likely, then, to be more acceptable to farmers than the tax policy.

Due to the way the linear farm model was specified return flow reductions were almost a necessity. However, the size of tax necessary to reduce return flows and the costs of both policies were items that could not be predicted _a priori_. The tax sizes necessary and costs are summarized at the end of Chapter VII.

Based on the empirical and theoretical analyses one of the major hypothesis that can be stated is: Taxing water delivered to a farm will not alter the farm's irrigation efficiency or reduce return flows. This hypothesis might be tested by observing changes in irrigation technique that may have occurred using application costs as a proxy for a water tax. For example areas might be found where irrigation is from wells and the water table has been dropping. The falling water table would tend to make pumping costs go up resulting in a higher overall cost of water delivery. If it can be determined that these increasing costs have resulted in a change to a more efficient irrigation technique then one could reject the stated hypothesis. The basis for rejection would be that a tax on water would affect decisions similar to an increased pumping cost.

Another hypothesis that could be tested is that restrictions on water deliveries will not reduce the proportion of irrigation return flows. If data were available concerning water deliveries and return flows from a given farm over a period of years one might be able to test that hypothesis. The test would need some time period represented when a farmer was for some reason forced to irrigate with less water. Such a situation sometimes occurs in certain areas when reservoir storage for the irrigation season is low. The Bighorn River area studied here, however, does not appear to have been faced with water shortages. Examination of irrigation techniques, farm irrigation efficiencies, and/or return flows from areas similar to the Bighorn but having different water supplies might provide at least a weak test of the hypothesis.

Testing of the two above hypotheses will require much technical information ,that at least for Wyoming is not presently available. For example an irrigator could conceivably reduce total water deliveries and return flows over a season by eliminating one or more irrigations. However, when he does irrigate, his farm irrigation efficiency may not be any better and the proportion of return flows for a given irrigation may not change. Better technical studies could indicate whether such behavior is likely. To help determine behavior, the effects on crop yields of fewer irrigations and/or various levels of water per irrigation would be important.

The last hypothesis to be stated explicitly is that farmers would more readily accept a given charge per acre inch of water delivered than they would a restraint on amount of water delivered. To test, this hypothesis would probably need to be made more specific, e.g., how much of a tax versus how much of a constraint on water deliveries. The test then might take the form of a behavioral study, that is, an attempt to ascertain farmers attitudes towards the alternative proposals. Rejection of this hypothesis could imply several things; consequently, it would need to be interpreted quite carefully. For example farmers' attitudes toward the two alternatives may be affected by their level of knowledge, i.e., they may or may not realize the total cost of the tax to them. Or rejection may simply imply that their decisions are not based solely on the profit maximization criterion.

Suggestions for Further Research

Empirical testing of hypotheses set forth above could provide the bases for several studies. It would seem that if sound economic recommendations concerning proper policies and costs of pollution control are to be made some of these tests are necessary. Other hypotheses should be derivable particularly from the theoretical sections of this work which could also provide the basis for research.

The theoretical and empirical work viewed the effects of the various policies in the limited linear economic model sense. Would the implications differ if the models were non-linear and were not basically static? In other words there appears to be a need for theoretically applying the policies to other types of models both static and dynamic.

Other policies exist which could be applied to similar models. Specifically combinations of the previous policies may offer the best alternatives. Some allusion was made to possible policy combinations but more specific work could prove useful.

The effects of these policies on individual firms was discussed here. However, for particular empirical studies, the aggregate effects are probably just as important. For example, there needs to be an aggregation of effects on all farms in the Bighorn Basin area of the specific policies. In such an aggregation, possibilities for adjustments in farm size, number of acres irrigated, etc., must be made. For example a given tax level may drive farmer A from business. Viewed by itself such an effect might appear to reduce total water demands and return flows. However, farmer B might acquire farmer A's land and continue to irrigate it as before thus resulting in no change in return flows and water demand.

There is a definite need for incorporating different irrigation systems into the present farm model. Specifically, activities for sprinkler irrigation techniques should be introduced. There is also a need for technical studies that would provide more refined coefficients both for the present model and any aggregate models developed. For example it has been demonstrated that the labor required for improving farm irrigation efficiency is important in assessing tax size and costs of the tax to the farmer. Studies which determined such labor needs would be quite helpful in determining the proper tax level and the total policy costs. In addition, as already discussed, research concerning the effects on crop yields of irrigation timing and of various water levels appears to be needed.

The relative administrative costs of the various policies also need to be examined. Such costs are likely to vary from industry to industry and maybe even area to area. In any event before a good decision can be made about the relative merits of some of the policies the administrative side must be considered.

As can be observed, the present study has not exhausted the topic. If anything it would seem to have opened or at least made more specific many more boxes into which the energetic researcher can jump but hopefully not suffocate.

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APPENDICES

APPENDIX A

Part 1.

The Amount of K Available for Production after Allowing for the Processing of X_n.

$K^o = k_n X_n + k_b X_m^{b'}$ where $X_m^{b'} < X_m^{b^o}$, and K^o is the fixed factor constraint

$$= k_n(n_b X_m^{b'}) + k_b X_m^{b'} \text{ since } X_n = n_b X_m^{b'}$$

$$= k_n \left(n_b \frac{K'}{k_b}\right) + k_b\left(\frac{K'}{k_b}\right) \text{ since } X_m^{b'} = \frac{K'}{k_b}$$

$$= K' \left(1 + \frac{k_n n_b}{k_b}\right)$$

$$\Rightarrow K' = \frac{K^o}{1 + \frac{k_n n_b}{k_b}} = \frac{k_b K^o}{k_b + k_n n_b} < K^o. \qquad (1)$$

To operate with technique A the following calculations are relevant

$$K^o = k_n X_n + k_a X_m^{a''}$$

$$= k_n n_a X_m^{a''} + k_a X_m^{a''} \quad \text{since } X_n = n_a X_m^{a''}$$

$$X_m^{a''} = \frac{K''}{k_a}$$

$$= K'' \left(1 + \frac{k_n n_a}{k_a}\right)$$

$$\Rightarrow K'' = \frac{K^o}{1 + \frac{k_n n_a}{k_a}} = \frac{k_a K^o}{k_a + k_n n_a} < K^o. \qquad (2)$$

But, $n_a > n_b$ and $n_a X^{a^o}_m > n_b X^{b^o}_m$ by assumption

which implies that $n_a \frac{K^o}{k_a} > n_b \frac{K^o}{k_b} \Rightarrow \frac{n_a}{k_a} > \frac{n_b}{k_b} \Rightarrow k_b n_a > k_a n_b.$ (3)

How do K' and K'' compare?

The relative size can be determined by comparing

$\frac{k_b}{k_b + k_n n_b}$ to $\frac{k_a}{k_a + k_n n_a}$ so

$$\frac{\frac{k_b}{k_b + k_n n_b}}{\frac{k_a}{k_a + k_n n_a}} = \frac{k_b(k_a + k_n n_a)}{k_a (k_b + k_n n_b)} = \frac{k_a k_b + k_b k_n n_a}{k_a k_b + k_a k_n n_a} = \frac{1 + \frac{k_n n_a}{k_a}}{1 + \frac{k_n n_b}{k_b}} \qquad (4)$$

If in (4.4) $\frac{k_n n_a}{k_a} > \frac{k_n n_b}{k_b}$ then (4.4) will be greater than one which implies that K' > K''. So the following ratio becomes relevant:

$$\frac{\frac{k_n n_a}{k_a}}{\frac{k_n n_b}{k_b}} = \frac{k_b n_a}{k_a n_b} > 1 \text{ by } (3.3) \qquad (5)$$

Therefore, it can be concluded that $K'' < K' < K^o$.

Part 2.

Comparison between Models 3.5 and 3.5' of Size of Subsidy on V' Necessary to Induce a Method Switch

The proof that the size of subsidy required to induce a switch for Model 3.5' is less than that required for 3.5 is as follows:

$$A \equiv (P^{o}_{x_m} - VC_a - R_{K_a} k_{aa}) \text{ let } VC_b + P_{K_b} k_{bb} = (VC_a + R_{K_a} k_{aa} - \lambda)$$

$$\therefore (P^{o}_{x_m} - VC_b - P_{K_b} k_{bb}) = A + \lambda, \; \lambda > 0.$$

Initially for Model 3.5, $AX^{a^o}_m > AX^{b^o}_m + \lambda X^{b^o}_m$. (1)

By assumption it is known that $A > 0$ and $X^{a^o}_m > X^{b^o}_m$.

From (1) $A(X^{a^o}_m - X^{b^o}_m) > \lambda X^{b^o}_m$. (2)

Initially for Model 3.5', $AX^{a'}_m > AX^{b^o}_m + \lambda X^{b^o}_m$. (3)

There are two cases to be proven depending on the restrictiveness of K_a. First assume $X^{a'}_m > X^{b^o}_m$ then from (3)

$$A(X^{a'}_m - X^{b^o}_m) > \lambda X^{b^o}_m. \quad (4)$$

After the tax with an implied switch (1), representing Model 3.5, becomes

$$AX^{a^o}_m + S^o_{v'} v'_a X^{a^o}_m < AX^{b^o}_m + \lambda X^{b^o}_m + S^o_{v'} v'_b X^{b^o}_m$$

$$\Rightarrow A(X^{a^o}_m - X^{b^o}_m) - \lambda X^{b^o}_m < S^o_{v'} (v'_b X^{b^o}_m - v'_a X^{a^o}_m) \qquad (5)$$

$$\Rightarrow R^o \equiv \frac{A(X^{a^o}_m - X^{b^o}_m) - \lambda X^{b^o}_m}{v'_b X^{b^o}_m - v'_a X^{a^o}_m} < S^o_{v'}. \qquad (6)$$

Since $X^{a^o}_m > X^{b^o}_m$ and from (2) the numerator of (6) is positive, and since $S^o_{v'} > 0$ (5) $\Rightarrow$ the denominator of (6) is positive. Similarly after the tax the initial condition for Model 3.5', (3), leads to

$$R' \equiv \frac{A(X^{a'}_m - X^{b^o}_m) - \lambda X^{b^o}_m}{v'_b X^{b^o}_m - v'_a X^{a'}_m} < S'_{v'} . \qquad (7)$$

Since $X^{a'}_m > X^{b^o}_m$ and by (4) the numerator of (7) is positive which as long as $S_{v'} > 0$ also $\rightarrow$ the denominator of (7) is positive. Now by observation since $X^{a'}_m < X^{a^o}_m$ the minimum value that will induce a switch for $S^o_{v'}$, $R^o + \epsilon$, is greater than the minimum for $S'_{v'}$, $R' + \epsilon$.

If $X^{a'}_m < X^{b^o}_m$, and a switch is implied then the condition that A produce the most net revenue initially implies that $(P^o_{x_m} - VC_b - P_{K_b} k_{bb}) = A - \lambda$, $\lambda > 0$. The argument goes similar as the above with a few changes. Number (2) becomes $A(X^{a^o}_m - X^{b^o}_m) > -\lambda X^{b^o}_m$ (8)
and 4 becomes $A(X^{a'}_m - X^{b^o}_m) > - \lambda X^{b^o}_m$. (9)

Carrying the change to (6)

$$R^{o} \equiv \frac{A(X^{a^{o}}_{m} - X^{b^{o}}_{m}) + \lambda X^{b^{o}}_{m}}{v'_{b}\, X^{b^{o}}_{m} - v'_{a}\, X^{a^{o}}_{m}} < S^{o}_{v'}. \qquad (10)$$

Inequality (7) will become

$$R^{o} \equiv \frac{A(X^{a'}_{m} - X^{b^{o}}_{m}) + \lambda X^{b^{o}}_{m}}{v'_{b}\, X^{b^{o}}_{m} - v'_{a}\, X^{a'}_{m}} < S'_{v'}. \qquad (11)$$

From (9) $\lambda X^{b^{o}}_{m} + A(X^{a'}_{m} - X^{b^{o}}_{m}) > 0$ but it is known that $X^{a'}_{m} - X^{b^{o}}_{m} < 0 \rightarrow X^{a'}_{m} - X^{b^{o}}_{m} < X^{a^{o}}_{m} - X^{b^{o}}_{m}$, so that by observation the numerator of R', (11), is smaller than the numerator of R^{o}, (10), and the denominator of (11) is bigger than the denominator of (10), which implies that $R^{o} > R'$, the same as before.

Part 3

Necessary and Sufficient Conditions for Achieving the Optimum Policy Solution by Taxing X_n for Model 4.1

The following notation descriptions refer to points in Figure 5.1. They are relevant for this Appendix and Part 4.

$(X^a_{m1})_o \equiv$ Point A_1, $(X^a_{m1})_1 \equiv$ Point A_2. $(X^a_{m1})_2 \equiv$ Point C_2

$(X^a_{m2})_o \equiv$ Point A_3. $(X^a_{m2})_1 \equiv$ Point A_2. $(X^a_{m2})_3 \equiv$ Point C_1

$(X^b_{m1})_o \equiv$ Point B_1, $(X^b_{m1})_1 \equiv$ Point B_2, $(X^b_{m1})_3 \equiv$ Point C_1

$(X^b_{m2})_o \equiv$ Point B_3, $(X^b_{m2})_1 \equiv$ Point B_2, $(X^b_{m2})_2 \equiv$ Point C_2

1. $(P^o_{x_{m2}} - VC_{b2} - T_n n_{b2})\,(X^b_{m2})_o > (P^o_{x_{m1}} - VC_{a1} \quad T_n n_{a1})(X^a_{m1})_o$

$$\Rightarrow T_n > \frac{(P^o_{x_{m1}} - VC_{a1})\,(X^a_{m1})_o - (P^o_{x_{m2}} - VC_{b2})(X^a_{m1})_o}{n_{a1}(X^a_{m1})_o - n_{b2}(X^b_{m2})_o}$$

2. $(P^o_{x_{m2}} - VC_{b2} - T_n n_{b2})(X^b_{m2})_o > (P^o_{x_{m2}} - VC_{a2} - T_n n_{a2})(X^a_{m2})_o$

$$\Rightarrow T_n > \frac{(P^o_{x_{m2}} - VC_{a2})(X^a_{m2})_o - (P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_o}{n_{a2}(X^a_{m2})_o - n_{b2}\,(X^b_{m2})_o}$$

3. $(P^o_{x_{m2}} - VC_{b2} - T_n n_{b2})(X^b_{m2})_o > (P^o_{x_{m1}} - VC_{b1} - T_n n_{b1}\ (X^b_{m1})_o$

$$\Rightarrow T_n > \frac{(P^o_{x_{m1}} - VC_{b1})(X^b_{m1})_o - (P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_o}{n_{b1}\ (X^b_{m1})_o - n_{b2}\ (X^b_{m2})_o}$$

4. $(P^o_{x_{m2}} - VC_{b2} - T_n n_{b2})(X^b_{m2})_o > (P^o_{x_{m1}} - VC_{a1} - T_n n_{a1})(X^a_{m1})_1$

$+ (P^o_{x_{m2}} - VC_{a2} - T_n n_{a2})(X^a_{m2})_1$

$$\Rightarrow T_n > \frac{(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_1 + (P^o_{x_{m2}} - VC_{a2})(X^a_{m2})_1 - (P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_o}{n_{a1}\ (X^a_{m1})_1 + n_{a2}\ (X^a_{m2})_1 - n_{b2}\ (X^b_{m2})_o}$$

5. $(P^o_{x_{m2}} - VC_{b2} - T_n n_{b2})(X^b_{m2})_o > (P^o_{x_{m1}} - VC_{b1} - T_n n_{b1})(X^b_{m1})_1$

$+ (P^o_{x_{m2}} - VC_{b2} - T_n n_{b2})(X^b_{m2})_1$

$$\Rightarrow T_n > \frac{(P^o_{x_{m1}} - VC_{b1})(X^b_{m1})_1 + (P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_2 - (P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_o}{n_{b1}(X^b_{m1})_1 + n_{b2}(X^b_{m2})_1 - n_{b2}\ (X^b_{m2})_o}$$

6. $(P^o_{x_{m2}} - VC_{b2} - T_n n_{b2})(X^b_{m2})_o > (P^o_{x_{m1}} - VC_{a1} - T_n n_{a1})(X^a_{m1})_2$

$+ (P^o_{x_{m2}} - VC_{b2} - T_n n_{b2})(X^b_{m2})_2$

$$\Rightarrow T_n > \frac{(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_2 + (P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_2 - (P^o_{x_{m2}} - VC_{b2})}{n_{a1}(X^a_{m1})_2 + n_{b2}(X^b_{m2})_2 - n_{b2}(X^b_{m2})_o}$$

$\underline{(X^b_{m2})_o}$

7. $(P^o_{x_{m2}} - VC_{b2} - T_n n_{b2})(X^b_{m2})_o > (P^o_{x_{m1}} - VC_{b1} - T_n n_{b1})(X^b_{m1})_3$

$+ (P^o_{x_{m2}} - VC_{a2} - T_n n_{a2})(X^a_{m2})_3$

$$\Rightarrow T_n > \frac{(P^o_{x_{m1}} - VC_{b1})(X^b_{m1})_3 + (P^o_{x_{m2}} - VC_{a2})(X^a_{m2})_3 - (P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_o}{n_{b1}(X^b_{m1})_3 + n_{a2}(X^a_{m2})_3 - n_{b2}(X^b_{m2})_o}$$

8. $(P^o_{x_{m2}} - VC_{b2} - T_n n_{b2})(X^b_{m2})_o > 0$

$$\Rightarrow T_n < \frac{(P^o_{x_{m2}} - VC_{b2})}{n_{b2}}$$

The tax must be equal to or greater than the largest of the values in (1) through (7) and less than that shown in (8).

Part 4

Comparison of Tax Size Necessary to Induce Production of X_{m2} by Method B for Alternate Cost Assumptions in Model 4.1

This statement cannot be proven to hold in all instances. However if one assumes that the variable costs of a given technique between the two assumed situations do not change relatively a great deal it will be true. For example, if $VC^o_{a2} = VC'_{a2}$ where the superscript (o) refers to the value of VC_{a2} when it is assumed that $VC_{a1} \leq VC_{b1}$, and $VC_{a2} \leq VC_{b2}$ and the (') refers to the value of VC_{a2} when $VC_{a1} \leq VC_{b1}$ and $VC_{a2} > VC_{b2}$ then it can be proven to hold. From Appendix A, Part 3, it was shown that for X_{m2} to be produced with B instead of A that

$$T_n > \frac{(P^o_{x_{m2}} - VC_{a2})(X^a_{m2})_o - (P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_o}{n_{a2}(X^a_{m2})_o - n_{b2}(X^b_{m2})_o}$$

$$\text{Let } T^o_n = \frac{(P^o_{x_{m2}} - VC^o_{a2})(X^a_{m2})_o - (P^o_{x_{m2}} - VC^o_{b2})(X^b_{m2})_o}{n_{a2}(X^a_{m2})_o - n_{b2}(X^b_{m2})_o}$$

where $(P^o_{x_{m2}} - VC^o_{a2}) > (P^o_{x_{m2}} - VC^o_{b2})$ and

$$T'_n = \frac{(P^o_{x_{m2}} - VC'_{a2})(X^a_{m2})_o - (P^o_{x_{m2}} - VC'_{b2})(X^b_{m2})_o}{n_{a2}(X^a_{m2})_o - n_{b2}(X^b_{m2})_o}$$

where $(P^o_{x_{m2}} - VC'_{a2}) < (P^o_{x_{m2}} - VC'_{b2})$.

If $T'_n < T^o_n \Longrightarrow T^o_n - T'_n > 0$ then it will indicate that it takes a smaller tax to induce production of X_{m2} with technique B under the second, ('), set of assumptions.

$$T^o_n - T'_n = \frac{[(P^o_{x_{m2}} - VC^o_{a2}) - (P^c_{x_{m2}} - VC'_{a2})]\,(X^a_{m2})_o + [(P^o_{x_{m2}} - VC'_{b2})}{n_{a2}(X^a_{m2})_o - n_{b2}\,(X^b_{m2})_o}$$

$$\underline{- (P^o_{x_{m2}} - VC^o_{b2})]\,(X^b_{m2})_o}$$

$$= \frac{[(P^o_{x_{m2}} - VC'_{b2}) - (P^o_{x_{m2}} - VC^o_{b2})]\,(X^b_{m2})_o}{n_{a2}(X^a_{m2})_o - n_{b2}\,(X^b_{m2})_o} \quad \text{since}$$

$VC^o_{a2} = VC'_{a2}$.

Note that $(P^o_{x_{m2}} - VC^o_{a2}) = (P^o_{x_{m2}} - VC'_{a2})$ and by assumption that

$(P^o_{x_{m2}} - VC^o_{a2}) > (P^o_{x_{m2}} - VC^o_{b2})$ and $(P_{x_{m2}} - VC'_{a2}) < (P_{x_{m2}} - VC'_{b2})$;

therefore, $(P^o_{x_{m2}} - VC^o_{b2}) < (P^o_{x_{m2}} - VC'_{b2})$.

Consequently $T^o_n - T'_n > 0$. Assumptions concerning amount of X_n produced make the denominator of $T^o_n - T'_n$ positive.

Part 5

Necessary and Sufficient Conditions for Achieving the Optimum Policy Solution by Using a Tax on X_{m1} for Model 4. 2.

The following notation descriptions refer to points in Figure 5. 3.

$(X^a_{m1})_o \equiv$ Point A_1, $(X^a_{m1})_1 \equiv$ Point A_2, $(X^a_{m1})_2 \equiv$ Point C_2

$(X^a_{m2})_o \equiv$ Point A_3, $(X^a_{m2})_1 \equiv$ Point A_2, $(X^a_{m2})_3 \equiv$ Point C_1

$(X^b_{m1})_o \equiv$ Point B_1, $(X^b_{m1})_1 \equiv$ Point B_2, $(X^b_{m1})_3 \equiv$ Point C_1

$(X^b_{m2})_o \equiv$ Point B_3, $(X^b_{m2})_1 \equiv$ Point B_2, $(X^b_{m2})_2 \equiv$ Point C_2

1. $(P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_o \not> (P^o_{x_{m1}} - VC_{a1} - T_1)\,(X^a_{m1})_o$

$$\Longrightarrow T_1 > [(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_o - (P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_o] \,/\, (X^a_{m1})_o$$

$$\Longrightarrow T_1 > [(P^o_{x_{m1}} - VC_{a1}) - (P^o_{x_{m2}} - VC_{b2})\,\frac{(X^b_{m2})_o}{(X^a_{m1})_o}$$

2. $(P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_o > (P^o_{x_{m1}} - VC_{b1} - T_1)(X^b_{m1})_1 + (P^o_{x_{m2}} - VC_{b2})$

$(X^b_{m2})_1$

$$\Longrightarrow T_1 > (P^o_{x_{m1}} - VC_{b1}) + (P^o_{x_{m2}} - VC_{b2}) \frac{(X^b_{m2})_1}{(X^b_{m1})_1} - (P^o_{x_{m2}} - VC_{b2}) \frac{(X^b_{m2})_o}{(X^b_{m1})_1}$$

3. $(P^o_{x_{m2}} - VC_{b2})\,(X^b_{m2})_o > (P^o_{x_{m1}} - VC_{a1} - T_1)(X^a_{m1})_2$

$+ (P^o_{x_{m2}} - VC_{b2})\,(X^b_{m2})_2$

$$\Longrightarrow T_1 > (P^o_{x_{m1}} - VC_{a1}) + (P^o_{x_{m2}} - VC_{b2}) \frac{(X^b_{m2})_2}{(X^a_{m1})_2}$$

$$- (P^o_{x_{m2}} - VC_{b2}) \frac{(X^b_{m2})_o}{(X^a_{m1})_2}$$

4. $(P^o_{x_{m2}} - VC_{b2})\,(X^b_{m2})_o > (P^o_{x_{m1}} - VC_{b1} - T_1)\,(X^b_{m1})_3$

$+ (P^o_{x_{m2}} - VC_{a2})(X^a_{m2})_3$

$$\Longrightarrow T_1 > (P^o_{x_{m1}} - VC_{b1}) + (P^o_{x_{m2}} - VC_{a2}) \frac{(X^a_{m2})_3}{(X^b_{m1})_3}$$

$$- (P^o_{x_{m2}} - VC_{b2}) \frac{(X^b_{m2})_o}{(X^b_{m1})_3}$$

5. $(P^o_{x_{m2}} - VC_{b2}) (X^b_{m2})_o > (P^o_{x_{m1}} - VC_{a1} - T_1) (X^a_{m1})_1$

$+ (P^o_{x_{m2}} - VC_{a2}) (X^a_{m2})_1$

$$\Rightarrow T_1 > (P^o_{x_{m1}} - VC_{a1}) + (P^o_{x_{m2}} - VC_{a2}) \frac{(X^a_{m2})_1}{(X^a_{m1})_1} - (P^o_{x_{m2}} - VC_{b2}) \frac{(X^b_{m2})_o}{(X^a_{m1})_1}$$

In addition the following two conditions must be included. These conditions can be established prior to the tax.

6. $(P^o_{x_{m2}} - VC_{b2})(X^b_{m2})_o > (P^o_{x_{m2}} - VC_{a2}) (X^a_{m2})_o$

7. $(P^o_{x_{m2}} - VC_{b2}) > 0$

To assume that the optimum policy point is achieved it must be that T_1 = (maximum T_1 in (1) through (5)).

Part 6

Comparison Between Model 4.1 and 4.2 of Size of Externality Tax Necessary to Induce a Technique Switch

Assume that the only difference between Models 4.1 and 4.2 is that for 4.1, $(X^a_{ml})_o > (X^b_{ml})_o$ and for 4.2 $(X^a_{ml})_o < (X^b_{ml})_o$. Then for both models (4.2.4) implies

$$T_n > \frac{(P^o_{x_{ml}} - VC_{al})(X^a_{ml})_o - (P^o_{x_{ml}} - VC_{bl})(X^b_{ml})_o}{n_{al}(X^a_{ml})_o - n_{bl}(X^b_{ml})_o} \qquad (1)$$

Assume that $(X^a_{ml})_o$ is equal for both models. The relationships between the market products then implies that $(X^b_{ml})_o$ for Model 4.2 (designated $(X^b_{ml})'_o$) is greater than $(X^b_{ml})_o$ for Model 4.1 (designated $(X^b_{ml})^o_o$. Let $T^o_n \equiv$ right side of (1) for Model 4.1 and $T^1_n \equiv$ right side of (1) for Model 4.2. If the footnoted statement is true then

$$\frac{T^o_n}{T^1_n} > 1 \text{ which implies}$$

$$\frac{\left[(P^o_{x_{ml}} - VC_{al})(X^a_{ml})_o - (P^o_{x_{ml}} - VC_{bl})(X^b_{ml})^o_o\right]}{(P^o_{x_{ml}} - VC_{al})(X^a_{ml})_o - (P^o_{x_{ml}} - VC_{bl})(X^b_{ml})'_0}$$

$$\frac{\left[n_{al}\,(X^a_{ml})_o - n_{bl}\,(X^b_{ml})'_o\right]}{\left[n_{al}\,(X^a_{ml})_o - n_{bl}\,(X^b_{ml})^o_o\right]} > 1$$

$$\begin{aligned}
\Rightarrow \Big[& n_{a1}(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})^2_o - n_{b1}(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_o \\
& (X^b_{m1})'_o - n_{a1}(P^o_{x_{m1}} - VC_{b1})(X^a_{m1})_o (X^b_{m1})^o_o + n_{b1}(P^o_{x_{m1}} - VC_{b1}) \\
& (X^b_{m1})^o_o (X^b_{m1})'_o \Big] - \Big[n_{a1}(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})^2_o \\
& - n_{b1}(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_o (X^b_{m1})^o_o - n_{a1}(P^o_{x_{m1}} - VC_{b1}) \\
& (X^b_{m1})'_o (X^a_{m1})_o + n_{b1}(P^o_{x_{m1}} - VC_{b1})(X^b_{m1})'_o (X^b_{m1})^o_o \Big] > 0
\end{aligned}$$

$$\begin{aligned}
\Rightarrow n_{b1}(P^o_{x_{m1}} - VC_{a1})(X^a_{m1})_o & \Big[(X^b_{m1})^o_o - (X^b_{m1})'_o\Big] \\
+ n_{a1}(P^o_{x_{m1}} - VC_{b1})(X^a_{m1})_o & \Big[(X^b_{m1})'_o - (X^b_{m1})^o_o\Big] > 0
\end{aligned}$$

$$\Rightarrow n_{a1}(P^o_{x_{m1}} - VC_{b1}) > n_{b1}(P^o_{x_{m1}} - VC_{a1}). \qquad (2)$$

Therefore, (2) must hold if the footnoted statement is to be true.

APPENDIX B

Appendix Table B-1. Per acre costs of producing alfalfa [a]

	Physical data per acre			Costs per acre		
Item	Materials Description	Tractor Hours	Man Hours	Materials & Custom	Fuel & Misc.[b]	Total [c]
General costs						
Spread fertilizer	100 lbs. P_2O_5	.18	.18	$ 8.90	$ .25	$ 9.15
Spray for weevil (air)	2 pts. malathion	custom		3.10		3.10
Pickup (season) 1/2T	20 miles				1.14	1.14
Swath 3 times			.75		2.58	2.58
Turn windrows 3 times		.75	.75		1.33	1.33
Stack loose 3 times	Front leader	1.80	1.80		3.34	3.34
Sub-total		2.73	3.48	12.00	8.64	20.64
Fixed machinery						16.86
Real estate tax						3.00
Overhead[d]						2.81
Interest on cash[e] costs @ 8.5% for 6 mos.						1.79
Total - General						45.10
Irrigation costs (season)						
Make ditches		.20	.20		.37	.37
Pull in ditches		.11	.11		.18	.18
First irrigation	Tubes and canvas		.20	.56		.56
4 other irrigations			.60			
Sub-total		.31	1.11	.56	.55	1.11
Water and drainage						4.24
Total - Irrigation						5.35

---

[a] Source: (1, p. 24).

[b] Fuel and miscellaneous includes fuel, engine oil, other oil, grease, filters, repairs and service labor (1, p. 14).

[c] Excludes labor costs except those in (b).

[d] Overhead was figured by Agee as 5 percent of all costs -- including labor, the general costs and irrigation costs above. The figure was held constant in this study since it was assumed it would not change significantly as irrigation technique changed.

[e] Interest costs on cash operating expenses were not increased between irrigation techniques since the difference would have been quite small.

Appendix Table B-2. Per acre costs of producing barley when seeded with alfalfa [a]

	Physical data per acre			Costs per acre		
Item	Materials Description	Tractor Hours	Man Hours	Materials & Custom	Fuel & Misc[b]	Total[c]
General costs						
Spread fertilizer	60 lbs. nitrogen	.18	.18	$ 6.12	$ .25	$ 6.37
Disc - tandem		.25	.25		.60	.60
Roller harrow		.20	.20		.41	.41
Level		.25	.25		.51	.51
Drill - corrugate	100 lbs. bar.; 14 lbs alf.	.33	.33	12.12	.84	12.96
Haul seed (2T truck)	.5 miles		.03		.08	.08
Pickup 1/2 T (season)	18 miles				1.03	1.03
Swath			.22		.76	.76
Combine			.40		1.14	1.14
Haul grain (2 trucks)	6 miles/200 bu.		.80		.28	.28
Elevate to storage					.13	.13
Rake straw		.25	.25		.45	.45
Stack straw	Front end loader	.40	.40		.67	.67
Sub-total		1.86	3.31	18.24	7.15	25.39
Fixed machinery						17.72
Real estate tax						3.00
Overhead[d]						3.12
Interest on cash [e] costs @ 8.5% for 6 mos.						2.02
Total - General						51.25
Irrigation costs (season)						
Make ditches		.20	.20		.37	.37
First irrigation	Canvas & tubes		.30	.56		.56
3 other irrigations			.60			
Pull in ditches		.11	.11		.18	.18
Irrigation after harvest			.20			
Sub-total		.31	1.41	.56	.55	1.11
Water & drainage						4.24
Total - Irrigation						5.35

---

[a] Source: (1, p. 21).

[b] Fuel and miscellaneous includes fuel, engine oil, other oil, grease filters, repairs and service labor (1, p. 14).

[c] Excludes labor costs except those in (b).

[d] Overhead was figured by Agee as 5 percent of all costs -- including labor, the general costs and irrigation costs above. The figure was held constant for this study since it was assumed it would not change significantly as irrigation technique changed.

[e] Interest costs on cash operating expenses were not increased between irrigation techniques since the difference would be quite small.

Appendix Table B-3. Per acre cost of producing barley for feed [a]

	Physical data per acre			Costs per acre		
Item	Materials Description	Tractor Hours	Man Hours	Materials & Custom	Fuel & Misc.[b]	Total[c]
General costs						
Spread fertilizer	60 lbs. nitrogen	.18	.18	$ 6.12	$ .25	$ 6.37
Disc - tandem		.25	.25		.60	.60
Roller harrow		.20	.20		.41	.41
Drill & corrugate	150 lbs. seed	.33	.33	6.00	.84	6.84
Haul seed (2T truck)	.7 miles		.04		.11	.11
Spray weeds	.75 lbs. 2, 4-D	.12	.12	.64	.24	.88
Pickup, 1/2T (season)	16 miles				.91	.91
Swath			.22		.76	.76
Combine			.40		1.14	1.14
Haul grain (2 trucks)	6 mi./200 bu.		.80		.35	.35
Elevate to storage					.16	.16
Rake straw		.25	.25		.45	.45
Stack straw	Front end loader	.40	.40		.67	.67
Sub-total		1.73	3.19	12.76	6.89	19.65
Fixed machinery						17.28
Real estate tax						3.00
Overhead [d]						2.77
Interest on cash [e] costs @ 8.5% for 6 mos.						1.73
Total - General						44.43
Irrigation costs (season)						
Make ditches		.20	.20		.37	.37
First irrigation	Canvas & tubes		.30	.56		.56
3 other irrigations			.60			
Pull in ditches		.11	.11		.18	.18
Sub-total		.31	1.21	.56	.55	1.11
Water and drainage						4.24
Total - Irrigation						5.35

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[a] Source: (1, p. 20).

[b] Fuel and miscellaneous includes fuel, engine oil, other oil, grease filters, repairs and service labor (1, p. 14).

[c] Excludes labor costs except those in (b).

[d] Overhead was figured by Agee as 5 percent of all costs -- including labor, the general costs and irrigation costs above. The figure was held constant for this study since it was assumed it would not change significantly as irrigation technique changed.

[e] Interest costs on cash operating expenses were not increased between irrigation techniques since the difference would be quite small.

Appendix Table B-4. Per acre costs for producing malting barley [a]

	Physical data per acre			Costs per acre		
Item	Materials Description	Tractor Hours	Man Hours	Materials & Custom	Fuel & misc. [b]	Total [c]
General costs						
Spread fertilizer	60 lbs. nitrogen	.18	.18	$ 6.12	$ .25	$ 6.37
Disc - tandem		.25	.25		.60	.60
Roller harrow		.20	.20		.41	.41
Drill - corrugate		.33	.33	6.50	.84	7.34
Haul seed (2T truck)	.5 miles		.03		.08	.08
Spray weeds	.75 lbs, 2, 4-D	.12	.12	.64	.24	.88
Pickup 1/2T (season)	16 miles				.91	.91
Swath			.22		.76	.76
Combine			.40		1.14	1.14
Haul grain (2 trucks)	18 mi./255 bu.		.80		.92	.92
Rake straw		.25	.25		.45	.45
Stack straw	Front end loader	.40	.40		.67	.67
Sub-total		1.73	3.18	13.26	7.27	20.53
Fixed machinery						17.97
Real estate tax						3.00
Overhead [d]						2.85
Interest on cash [e] costs @ 8.5% for 6 mos.						1.77
Total - General						46.12
Irrigation costs (season)						
Make ditches		.20	.20		.37	.37
First irrigation	Canvas & tubes		.30	.56		.56
3 other irrigations			.60			
Pull in ditches		.11	.11		.18	.18
Sub-total		.31	1.21	.56	.55	1.11
Water and drainage						4.24
Total - Irrigation						5.35

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[a] Source: (1, p. 22).

[b] Fuel and miscellaneous includes fuel, engine oil, other oil, grease filters, repairs and service labor (1, p. 14).

[c] Excludes labor costs except those in (b).

[d] Overhead was figured by Agee as 5 percent of all costs -- including labor, the general costs and irrigation costs above. The figure was held constant for this study since it was assumed it would not change significantly as irrigation technique changed.

[e] Interest costs on cash operating expenses were not increased between irrigation techniques since the difference would be quite small.

Appendix Table B-5. Per acre costs for producing grain corn[a]

	Physical data per acre			Costs per acre		
Item	Materials Description	Tractor Hours	Man Hours	Materials & Custom	Fuel & Misc.[b]	Total[c]
General costs						
Spread fertilizer	40 lbs. N + 40 lbs. P_2O_5	.18	.18	$ 7.64	$ .25	$ 7.89
Plow		.83	.83		2.08	2.08
Roller harrow (twice)		.40	.40		.82	.82
Level		.25	.25		.51	.51
Plant	18 lbs. seed	.33	.33	5.40	.86	6.26
Cultivate (3 times)		.75	.75		2.67	2.67
Spray weeds	.75 lbs. 2, 4-D	.12	.12	.64	.24	.88
Side dress (custom)	80 lbs. liquid N.			9.37		9.37
Pickup 1/2T (season)	24 miles				1.37	1.37
Combine			.40		1.94	1.94
Haul Corn (2 trucks)	6 mi./200 bu.		.80		.52	.52
Elevate to storage					.16	.16
Sub-Total		2.53	4.06	23.05	11.42	34.47
Fixed machinery						27.58
Real estate tax						3.00
Overhead[d]						4.20
Interest on cash[e] costs @ 8.5% for 6 mos.						2.57
Total - General						71.82
Irrigation costs (season)						
Make ditches		.20	.20		.37	.37
First irrigation	Tubes & canvas		.50	.75		.75
4 other irrigations			1.00			
Pull in ditches		.11	.11		.20	.20
Sub-total		.31	1.81	.75	.57	1.32
Water and drainage						4.24
Total - Irrigation						5.56

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[a] Source: (1, p. 26).

[b] Fuel and miscellaneous includes fuel, engine oil, other oil, grease filters, repairs and service labor (1, p.14).

[c] Excludes labor costs except those in (b).

[d] Overhead was figured by Agee as 5 percent of all costs -- including labor, the general costs and irrigation costs above. The figure was held constant for this study since it was assumed it would not change significantly as irrigation technique changed.

[e] Interest costs on cash operating expenses were not increased between irrigation techniques since the difference would be quite small.

Appendix Table B-6. Per acre costs for producing silage corn[a]

	Physical data per acre			Costs per acre		
Item	Materials Description	Tractor Hours	Man Hours	Materials & Custom	Fuel & Misc.[b]	Total[c]
General costs						
Spread fertilizer	40 lbs. N + 40 lbs P_2O_5	.18	.18	$ 7.64	$ .25	$ 7.89
Plow		.83	.83		2.08	2.08
Roller harrow (twice)		.40	.40		.82	.82
Level		.25	.25		.51	.51
Plant	22 lbs. seed	.33	.33	6.60	.86	7.46
Cultivate (3 times)		.75	.75		2.67	2.67
Spray weeds	.75 lbs, 2,4-D	.12	.12	.64	.24	.88
Side dress (custom)	80 lbs. liquid N.			9.37		9.37
Pickup 1/2T (season)	24 miles				1.37	1.37
Chop		.83	.83		2.20	2.20
Haul (2 trucks)	36 ton miles/load		1.66		3.23	3.23
Pack	Tractor & blade	.83	.83		1.41	1.41
Pack	Tractor only	.83	.83		1.15	1.15
Sub-total		5.35	7.01	24.25	16.79	41.04
Fixed machinery						33.49
Real estate tax						3.00
Overhead[d]						5.17
Interest on cash[e] costs @ 8.5% for 6 mos.						3.17
Total - General						85.87
Irrigation costs (season)						
Make ditches		.20	.20		.37	.37
First irrigation	Tubes & canvas		.50	.75		.75
4 other irrigations			1.00			
Pull in ditches		.11	.11		.20	.20
Sub-total		.31	1.81	.75	.57	1.32
Water and drainage						4.24
Total - Irrigation						5.56

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[a] Source: (1, p. 25).

[b] Fuel and miscellaneous includes fuel, engine oil, other oil, grease filters, repairs and service labor (1, p. 14).

[c] Excludes labor costs except those in (b).

[d] Overhead was figured by Agee as 5 percent of all costs -- including labor, the general costs and irrigation costs above. The figure was held constant for this study since it was assumed it would not change significantly as irrigation technique changed.

[e] Interest costs on cash operating expenses were not increased between irrigation techniques since the difference would be quite small.

Appendix Table B-7. Per acre costs for producing sugarbeets [a]

	Physical data per acre			Costs per acre		
Item	Materials Description	Tractor Hours	Man Hours	Materials & Custom	Fuel & Misc. [b]	Total [c]
General costs						
Pre-plant	d/	3.29	3.29	$26.51	$ 8.62	$35.13
Plant	e/	.47	.47	10.71	1.92	12.63
Cultivate (4 times)		2.00	2.00		5.24	5.24
Springtine harrow		.25	.25		.66	.66
Roll		.33	.33		.47	.47
Thinning (custom)				18.33		18.33
Weeding (custom)	Hoes, files			17.52		17.52
Labor housing	for custom work			1.83		1.83
Spray waste area	Chemicals	.22	.22	.54	.43	.97
Leaf hopper control	Custom			1.25		1.25
Webworm control	Custom			1.50		1.50
Side dress fert. (cust.)	55 lbs. liquid N.			7.53		7.53
Pickup 1/2T (season)	32 miles				1.82	1.82
Harvest	24.7 track miles	2.74	6.84		12.20	12.20
Sub-total		9.30	13.40	85.72	31.36	117.08
Fixed machinery						60.96
Real estate tax						3.00
Depreciation on housing - beet labor						1.62
Overhead [f]						11.68
Interest on cash [g] costs @ 8.5% for 6 mos.						7.71
Total - General						202.05
Irrigation costs (season)						
Make ditches		.33	.33		.61	.61
Irrigate up	Canvas, tubes		1.50	1.05		1.05
6 other irrigations			4.50			
Pull in ditches		.22	.22		.38	.38
Sub-total		.55	6.55	1.05	.99	2.04
Water and drainage						4.24
Total - Irrigation						6.28

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[a] Source: (1, p.27; 2 pp. 22 & 25).

[b] Fuel and miscellaneous includes fuel, engine oil, other oil, grease filters, repairs and service labor (1, p. 14).

[c] Excludes labor costs except those in (b).

[d] Includes 84 lbs. nitrogen, 128 lbs. phosphate, 16 lbs, potassium and 20 gallons Telon for nematodes.

[e] Includes 2 lbs. No. 1 seed and 12 lbs. RoNeet for weed control.

[f] Overhead was figured by Agee as 5 percent of all costs -- including labor, the general costs and irrigation costs above. The figure was held constant for this study since it was assumed it would not change significantly as irrigation technique changed.

[g] Interest costs on cash operating expenses were not increased between irrigation techniques since the differences would be quite small.

Appendix Table B-8. Prices paid for various input items, Worland Area, 1971 [a]

Item	Unit	Price per Unit
		dollars
Fuel and lube:		
Gasoline (after refunds)	gal	.278
Diesel	gal	.17
Engine oil	gal	1.00
Hydraulic oil	gal	1.50
Grease	lb	.25
Fertilizer:		
Nitrogen (available dry) [b]	lb	.102
Phosphate (available dry) [b]	lb	.089
Nitrogen (liquid)	lb	.089
Apply liquid nitrogen	acre	2.25
Labor:		
All labor (cash)	hr	2.16
Seed:		
Corn	cwt	30.00
Barley, for feed	cwt	4.00
for malting	cwt	6.50
Alfalfa	cwt	58.00
Pesticides:		
Malathion (weevil control-- 2 pts per A)	pint	1.05
2, 4-D (4 lbs active per gal) [c]	5 gal	17.00
Air application (no materials)	acre	1.00
Taxes:		mills
Machinery and equipment [d]	mills	61
Land and improvements [d]	mills	61

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[a] Source: (1, p. 34).

[b] Includes $3 per ton for use of spreader.

[c] Cost per pound of active material is $.85. Application rate is about 3/4 lb active per acre for corn and barley or $.64 per acre for 2, 4-D.

[d] Calculated at .25 times (X) average value of equipment, times (X) .061; Land at $30 per acre plus improvements at $19 per acre times (X) .061.

Appendix Table B-9. Effects on Crop Acreages of Water Tax Levels When Labor Was Constrained.

Crop	Assumption		Tax level per acre inch of water										
	Labor	Cost	.10[a]	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60
Alfalfa	.05	.10	120	120	117	116	Same	Same	116	NA[b]			
		.20			119	116			116				
Barley-alfalfa		.10	40	40	39	39			39				
		.20			40	39			39				
Barley-malt		.10	50	50	52	48			45				
		.20			51	48			48				
Corn-grain		.10	100	100	95	97			93				
		.20			98	97			97				
Sugarbeets		.10	190	190	190	190			190				
		.20			190	190			190				
Total		.10	500	500	493	490			483				
		.20			498	490			490				
Alfalfa	.10	.10	120	Same	Same	119	107	NA					
		.20											
Barley-alfalfa		.10	40			40	36						
		.20											
Barley-malt		.10	50			50	57						
		.20											
Corn-grain		.10	100			98	100						
		.20											
Sugarbeets		.10	190			190	190						
		.20											
Total		.10	500			497	490						
		.20											
Alfalfa	.20	.10	120					97	80		80	80	80
		.20						97	80		80	80	80
Barley-alfalfa		.10	40					32	26		26	26	26
		.20						32	27		27	27	27

Appendix Table B-9 (continued)

Crop	Assumption		Tax level per acre inch of water										
	Labor	Cost	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60
Barley-malt	.20	.10	50	Same	Same	Same	Same	61	72	Same	69	69	59
		.20						61	72		71	70	70
Corn-grain		.10	100					99	99		87	87	92
		.20						100	99		100	87	87
Sugarbeets		.10	190					190	190		190	190	190
		.20						190	190		190	190	190
Total		.10	500					479	467		452	452	477
		.20						480	468		468	454	454
Alfalfa		.10	120					113	80	80			80
	.30	.20						120					
Barley-alfalfa		.10	40					38	27	27			27
		.20		Same	Same	Same	Same	40					
Barley-malt		.10	50					54	75	70	Same	Same	70
		.20						50					
Corn-grain		.10	100					100	99	99			100
		.20						100					
Sugarbeets		.10	190					190	190	190			190
		.20						190					
Total		.10	500					495	471	466			467
		.20						500					

[a] The figures in the table refer to acres. Where only one group of numbers appears for a given set of rows in one column it indicates the acreages were the same for either cost assumption. The "same" means acreages for that tax were identical to previous tax level.

[b] NA means that this tax level and those higher were not applied to these assumed conditions.

Appendix Table B-10. Effects of Tax Levels on Net Revenues.

		Irrigation two variable costs increased by							
		10%				20%			
		Labor increased by				Labor increased by			
Labor unconst.	Tax	.05	.10	.20	.30	.05	.10	.20	.30
Initial	0	$31,185	31,185	31,185	31,185	31,185	31,185	31,185	31,185
After tax	$ .10	29,065	29,010	28,941	28,941	28,996	28,974	28,941	28,941
Cost[a]		2,120	2,175	2,244	2,244	2,819	2,211	2,244	2,244
After tax	.15	28,095	27,993	27,882	27,825	28,019	27,957	27,849	27,819
Cost		3,090	3,192	3,303	3,360	3,166	3,228	3,336	3,366
After tax	.20	27,154	26,976	26,865	26,775	27,078	26,939	26,829	26,727
Cost		4,031	4,209	4,320	3,410	4,107	4,246	4,356	4,458
After tax	.25	26,257	26,009	25,848	25,737	26,137	25,933	25,812	25,701
Cost		4,928	5,176	5,337	5,448	5,048	5,252	5,373	5,484
After tax	.30	25,272	25,068	24,831	24,720	25,196	24,992	24,794	24,684
Cost		5,913	6,117	6,354	6,465	5,989	6,193	6,391	6,501
After tax	.35	NA[b]	NA	23,836	23,737	NA	NA	23,800	23,701
Cost				7,349	7,448			7,385	7,484
After tax	.40			22,819	22,720			22,783	22,684
Cost				8,376	8,465			8,402	8,501
After tax	.45			21,894	21,724			21,818	21,688
Cost				9,291	9,461			9,367	9,497
After tax	.50			20,979	20,733			20,903	20,697
Cost				10,206	10,452			10,282	10,488
After tax	.55			NA	19,742			NA	19,706
Cost					11,443				11,479
After tax	.60				18,763				18,715
Cost					12,422				12,470
After tax	.65				NA				17,810
Cost									13,375

Appendix Table B-10 (continued)

		Irrigation two variable costs increased by							
		10%				20%			
Labor const.	Tax								
Initial	0	32,197	32,197	32,197	32,197	32,197	32,197	32,197	32,197
After tax	$ .10	29,955	29,953	29,953	29,953	29,954	29,953	29,953	29,953
Cost		2,242	2,244	2,244	2,244	2,243	2,244	2,244	2,244
After tax	.15	28,834	28,833	28,831	28,831	28,833	28,832	28,831	28,831
Cost		3,363	3,364	3,366	3,366	3,364	3,365	3,366	3,366
After tax	.20	27,796	27,712	27,709	27,709	27,766	27,710	27,709	27,709
Cost		4,401	4,485	4,488	4,488	4,431	4,486	4,488	4,488
After tax	.25	26,829	26,619	26,588	26,587	26,777	26,606	26,587	26,587
Cost		5,368	5,578	5,609	5,610	5,420	5,591	5,610	5,610
After tax	.30	25,863	25,542	25,466	25,465	25,811	25,530	25,466	25,465
Cost		6,334	6,655	6,731	6,732	6,386	6,667	6,731	6,732
After tax	.35	24,898	NA	24,374	24,345	24,845	NA	24,363	24,343
Cost		7,299		7,823	7,853	7,352		7,831	7,854
After tax	.40	23,934		23,359	23,273	23,880		23,346	23,267
Cost		8,263		8,838	8,924	8,317		8,851	8,930
After tax	.45	NA		22,351	22,254	NA		22,338	22,244
Cost				9,846	9,943			9,859	9,953
After tax	.50			21,346	21,242			21,332	21,231
Cost				10,851	10,955			10,865	10,966
After tax	.55			20,422	20,230			20,391	20,219
Cost				11,775	11,967			11,806	11,978
After tax	.60			19,510	19,218			19,468	19,207
Cost				12,687	12,979			12,729	12,990

[a] Cost is difference between the initial and after tax net revenues.

[b] NA means this tax and those higher were not applied to this model assumption.