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Title: AN EFFICIENT OPTIMIZING MODEL FOR DETERMINING THINNING REGIME AND ROTATION AGE USING THE STAND PROJECTION SYSTEM GROWTH SIMULATOR

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J. Douglas Brodie

The PATH algorithm, which is the efficient dynamic programming algorithm developed by Paredes and Brodie (1987) was interpreted from a different point of view. This modification of the PATH algorithm by the calculus of variations vastly diminished the calculation task and memory required to store optimal stands at each stage.

Using the PATH algorithm, a new dynamic programming model called Stand Optimization System (SOS) was developed. The system was incorporated into a growth simulator constructed by Arney (1985). This model optimizes timing, intensity, and type of thinning as well as rotation age based on either physical units (basal area, cubic feet, merchantable cubic feet, merchantable board feet), present net worth, or soil expectation value.

An economic analysis with Stand Optimization System

was performed so as to evaluate the impacts of interest rate, quality premium and type of thinning.

Finally, further limitation of optimality on the PATH algorithm and the relationship between the Lagrange multiplier and the decision variable were discussed.

An Efficient Optimizing Model
for Determining Thinning Regime and Rotation Age
Using the Stand Projection System Growth Simulator

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# AN EFFICIENT OPTIMIZING MODEL FOR DETERMINING THINNING REGIME AND ROTATION AGE USING THE STAND PROJECTION SYSTEM GROWTH SIMULATOR

## INTRODUCTION

Determination of the optimal thinning regime and rotation age has been a main problem in even-aged forest stand management. The widespread application of operations research techniques has been contributing to solving the stand level optimization problem. The dynamic programming approach has been developed and extensively applied in recent years.

The dynamic programming approach was applied to the forestry field by Arimizu (1959), Amidon and Akin (1968) and Schreuder (1969). Early scientists used a two-descriptor, i.e., volume and age, dynamic programming model in order to cluster each state (Kilkki and Väisänen, 1970, Brodie and others, 1978, Chen and others, 1980). Brodie and Kao (1979) proposed a three-descriptor dynamic programming model, (number of trees, basal area and age) using an existing stand growth simulator for Douglas-fir (*Pseudotsuga menziesii* [Mirb.] Franco) called DFIT. This three-descriptor dynamic programming model with forward recursive approach has been extended and adapted for other decision variables and species. Riitters and others (1982) presented a dynamic programming model with timber production and grazing control joint optimization using a ponderosa pine (*Pinus ponderosa* Dougl. ex Laws.) growth model called PPINE. Haight and others (1985) proposed a dynamic programming model with thinning and rotation age optimization using lodgepole pine (*Pinus contorta* Doug. ex Loud.). A hardwood release and thinning optimization dynamic programming model was constructed for loblolly pine (*Pinus taeda* L.) by Valsta and Brodie (1985). Torres (1987) proposed a thinning optimization dynamic programming model for a *Pinus hartwegii* growth model. In comparison with the above three or more descriptors models, a dynamic programming model optimizing both thinning and rotation age for red pine (*Pinus resinosa* Ait.) was completed by Martin and Ek (1981) with two descriptors.

Although some of the above models reduced calculation effort to select the optimal activity by using the "neighborhood storage location" technique (Brodie and Kao, 1979), dynamic programming still has computational limitations. In other words the more complicated the growth simulator, the more memory and calculations are required (Hann and Brodie, 1980).

Paredes and Brodie (1987) resolved the memory and calculation problem by utilizing both network theory and the theory of the Lagrange multipliers. Their efficient way of selecting the optimal path reduced the number of calculations and associated computer storage in comparison with the traditional dynamic programming approach. Since the number of elementary calculations in traditional dynamic programming increases exponentially with problem size, the efficiency is greater with larger problems. However, searching for the optimal value of the Lagrange multiplier remains incomplete. Two other problems called trade-off problems were not dealt with by Paredes and Brodie (1987), one of which deals with situations where no thinning is applied at a certain stage, and the other of which deals with problems associated with insufficient look-ahead period when intensive thinning is applied.

This study describes and interprets their algorithm (called the PATH algorithm) from a different point of view, in which the technique does not use the Lagrange multiplier. Then a new dynamic programming model is proposed by using the PATH algorithm and a growth simulator called Stand Projection System (SPS) (Arney, 1985). Finally the trade-off problems are solved by a new algorithm called Multi-Stage PATH (MSPATH).

# OPTIMIZING FORMULATION

In this section, the traditional dynamic programming algorithm and the PATH algorithm are briefly reviewed, then a new interpretation of the PATH algorithm is introduced.

## 1) Traditional dynamic programming algorithm

In order to simplify an example further, let's consider the problem of determining the optimal thinning regime with a management objective of maximizing present net worth, given that the thinning level is the only decision variable. The objective function $f_N$ is formulated:

$$\text{maximize } f_N = \sum_{n=1}^{N} A_n(T_n) \tag{2.1}$$

where $A_n$ represents the return generated by a decision variable $T_n$ at time n in terms of present net worth and N is a given rotation age. Using the dynamic programming expression, the above equation is transferred into the recursive equation:

$$f^*_n(Y_n) = \text{maximize } [f_n(Y_n,T_n)] \tag{2.2}$$

$$f_n(Y_n,T_n) = A_n(T_n) + f^*_{n-1}(Y_{n-1}) \tag{2.3}$$

where $Y_n$ is the state variable, which is the stand volume at stage n and $f^*_n$ is the optimal value of $f_n$. This equation is utilized in a forward recursion procedure summarized by the following steps:

Step 0 $f^*_0(Y_0) = R$
$n = 1$

where R is a given initial volume of the initial stand.

Step 1 $f_n(Y_n,T_n) = A_n(T_n) + f^*_{n-1}(Y_{n-1})$

$f^*_n(Y_n) = \max\ [f_n(Y_n,T_n)]$

Save $T_n$

Step 2 $f^*_n(Y_n=0) < f^*_{n-1}(Y_{n-1}=0) \rightarrow$ STOP
$n = n + 1$

GO TO Step 1

Each state is determined by a given interval of state node. That is, in the traditional dynamic programming algorithm, adjusting the residual stand to the corresponding state should be implemented in order to compare stands. After clustering stands with different thinning level at each state, the best stand is stored at each state of each stage, resulting in a large computational burden. After reaching the final stage, at which the current present net worth is greater than the following one, the optimal thinning regime and the optimal rotation age are obtained.

2) The PATH algorithm by the Lagrange multipliers approach

Paredes and Brodie (1987) proposed the optimization problem with the Lagrange multiplier as follows:

$$\max \quad f_N(Y_N) = \sum_{n=1}^{N} A_n(T_n) - \sum_{n=1}^{N} \lambda_n[X_n - T_n + G_{n+1}(Y_n)] \qquad (2.4)$$

where $X_n$ is the stand volume before thinning at stage n, $G_{n+1}$ is growth from stage n to stage n+1 based on $Y_n$ and $\lambda_n$ is the Lagrange multiplier at stage n.

This equation satisfies the following constrained problem:

$$\max \quad f_N(Y_N) = \sum_{n=1}^{N} A_n(T_n) \qquad (2.5)$$

subject to $X_n - T_n + G_{n+1} \geq 0, \; n = 1, \ldots . N-1.$

They modified the formulation in the dynamic programming problem as:

$$f_n(Y_n) = \max_{[T_n]}[A_n(T_n) + \lambda_n[X_n - T_n + G_{n+1}(Y_n)]] + f_{n-1}(Y_{n-1}) \qquad (2.6)$$

Searching for the optimal level of $[T_n, \lambda_n]$, this objective function can be optimized. The PATH algorithm does not utilize the state variables of the growth model for storing the optimal residual stand at each state with a fixed Lagrange multiplier, so that computational task was vastly diminished.

If the dimensions of both the first term and the second term on the right-hand side of equation (2.6) are dollars, the Lagrange multiplier, $\lambda$, can be interpreted as the average price per unit of resource. In physical objective examples, the Lagrange multiplier can be estimated as simply 1. The control decision concerns both the direct return from the thinning and the return from the future stand. Although it is possible to guess the Lagrange multiplier, it is not always guaranteed that such a value is optimal. Then it is necessary to search for the optimal Lagrange multiplier. The difficulty of searching for the optimal Lagrange multiplier $\lambda n$, can be eliminated by the following procedure.

3) The PATH algorithm by the calculus of variations

The total return at stage n is the summation of marginal return over time:

$$V(t_n) = \int_{t_0}^{t_n} M(t)\ dt \quad \text{in the continuous case,} \tag{2.7}$$

$$= \sum_{i=0}^{n} M(t_i) \quad \text{in the discrete case.} \tag{2.8}$$

where $V(t_n)$ is the total return at time $t_n$ and $M(t)$ is the marginal return at time t. Once thinning, T, is implemented at stage 1 (time $t_1$), the objective function, $V(t_n)$ is divided into two parts as:

$$V(t_n,T) = \int_{t_0}^{t_1} M(t,T)\ dt + \int_{t_1}^{t_n} M(t,T)\ dt. \tag{2.9}$$

Figure 1 depicts this integral relationship. The first integrand on the right-hand side is equal to $V_1$, which consists of the sum of thinning and residual stand after

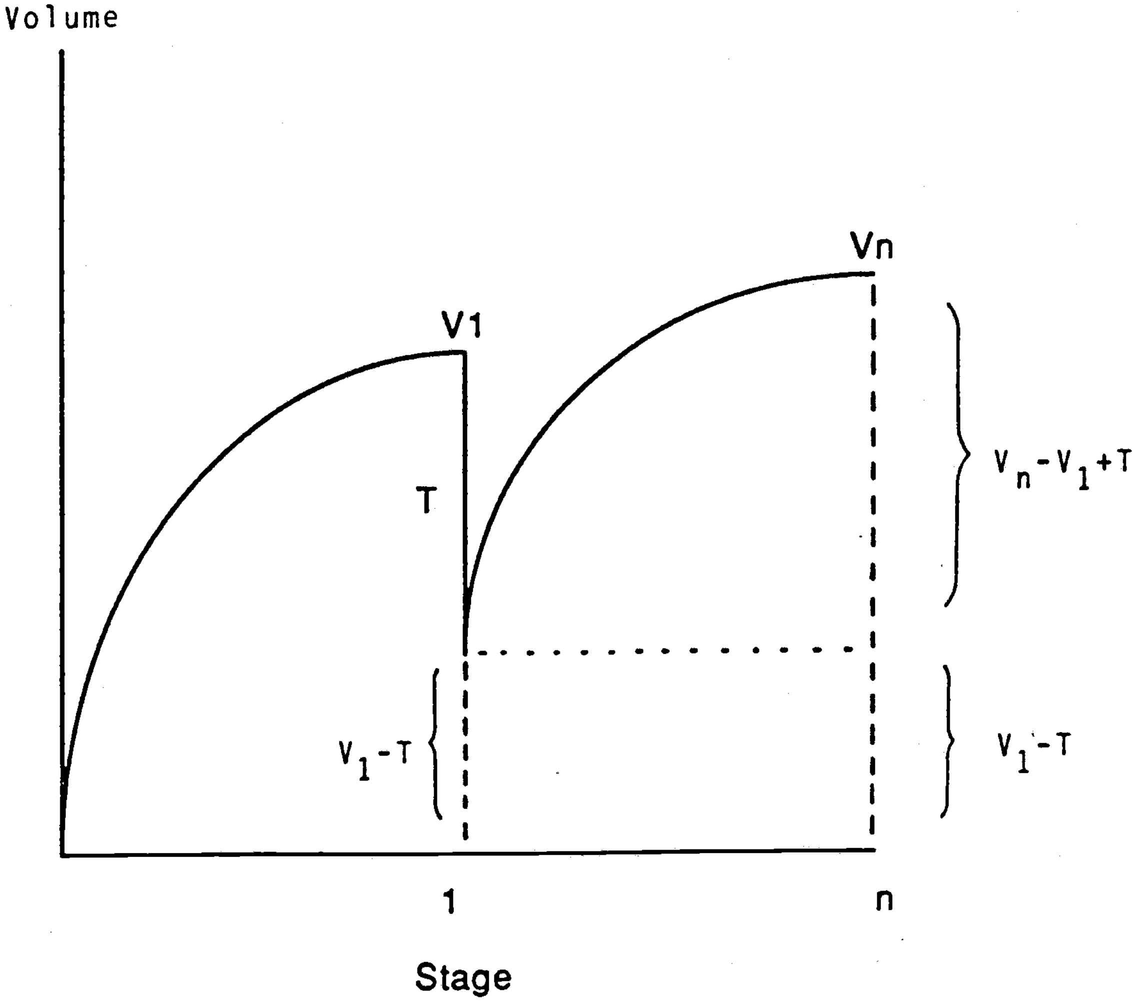



Figure 1.
The relationship between stand volume and stage when thinning T is implemented at stage 1.

thinning at stage 1. The second integrand represents the sum of growth after thinning from stage 1 to stage n (time $t_n$). Then it becomes $V_n-(V_1-T)$. Therefore, $V(t_n,T)$ represents the sum of returns from both thinning at stage 1 and the future stand at stage n, which is $V_n+T$. If the optimal thinning regime is required, the objective function becomes:

$$\underset{0<[T]\leq V(t_1)}{\text{maximize}}\ V(t_n,T) = \int_{t_0}^{t_1} M(t,T)\ dt + \int_{t_1}^{t_n} M(t,T)\ dt \qquad (2.10)$$

It is obvious that the first integrand on the right hand side is constant because the current thinning cannot affect the previous stand. Then equation (2.10) becomes:

$$\underset{[T]}{\text{maximize}}\quad V(t_n,T) = V(t_1) + \underset{[T]}{\max}\int_{t_1}^{t_n} M(t,T)\ dt \qquad (2.11)$$

This approach to interpretation of the PATH algorithm is one of the classical calculus of variations problems (Intrilligator, 1971), the canonical form of which is described as:

$$\text{maximize } J = \int_{t_0}^{t_1} I(X,X',t)\ dt \qquad (2.12)$$

$$X(t_0) = X_0$$
$$X(t_1) = X_1$$

where I is a given continuously differentiable function, $t_0$, $t_1$, $X_0$, and $X_1$ are given parameters, and X is the state variable and X' is the control variable.

The classical calculus of variations problem is to find the arc lying in a given plane and connecting two specified points in the plane by an arc of shortest length (Dreyfus, 1965). Thus, in this context, the classical calculus of variations problem can be interpreted as that of choosing the optimal thinning strategy [X(t)], which satisfies the boundary condition:

$$V(t_0) = V_0 \geq 0,$$

$$V(t_n) = V_n \geq 0,$$

where $V_0$ is the initial stand volume and $V_n$ is the final stand volume, and maximizes the integral or summation objective functional, J.

Since derivation of mathematical conditions proving that X(t) is a feasible arc is not necessary for explanation of the thinning problem, the reader is referred to Dreyfus (1965) and Intrilligator (1971) for explanation and examples. Only necessary conditions for optimization are indicated here.

The necessary conditions in the above problem are:

1. Euler equation:

$$\frac{\partial I}{\partial X} - \frac{d}{dt}\left(\frac{\partial I}{\partial X'}\right) = 0$$

(The first order necessary condition for an optimum.)

2. Boundary conditions:

$$X(t_0) = X_0,$$

$$X(t_1) = X_1$$

(Starting and ending condition.)

3. Legendre condition:

$$\frac{\delta^2 I}{\delta X'^2} \leq 0$$

(Second order condition sufficient for a maximum.)

4. Weierstrass condition:

$$E(X,X',t,Z') \leq 0, \text{ where}$$

$$E(X,X',t,Z')=I(X,Z',t)-I(X,X',t)-\frac{\delta I}{\delta X'}(X,X',t)(Z'-X')$$

for any other admissible trajectory Z(t)

(The condition for concavity.)

5. Weierstrass-Erdman corner conditions:

$$\left.\frac{\delta I}{\delta X'}\right|_{t-} = \left.\frac{\delta I}{\delta X'}\right|_{t+} ,$$

$$\left[I-\frac{\delta I}{\delta X'}X'\right]_{t-} = \left[I-\frac{\delta I}{\delta X'}X'\right]_{t+}$$

(The condition for continuity.)

Schreuder (1971) specified a problem of the optimal strategy for an even-aged forest in the calculus of variations. Although specifying a continuous problem, Schreuder (1971) turned the calculus of variations problem into a dynamic programming problem because when the necessary conditions for an optimum were obtained for the general expression, a higher than first-order nonlinear differential equation, such as the Euler equation, resulted. Moreover even if it could be solved, it would be necessary to investigate numerically all the roots to locate the global maximum because the conditions are only necessary and not sufficient. Then it was impossible for

Schreuder (1971) to obtain a closed-form expression.

However, if we know exactly what the $I(X,X',t)$ function looks like, then it is not necessary to solve the Euler equation to obtain a general solution. Furthermore, necessary conditions function as useful tests, which can eliminate candidate solutions, even if they are not sufficient to prove global optimality. In other words, those conditions can establish if the objective function has the optimal solution or not. After satisfying necessary conditions, it becomes obvious that there is the optimal trajectory. If we have such a trajectory that satisfies a given problem, that trajectory could be optimal.

Let's consider a given simple problem specification which is to maximize the total volume from thinning and final harvest as follows:

The objective is

$$\max_{[X]} \quad J = \sum_{n=0}^{N} \int_{t_n}^{t_{n+1}} I(X_n, X'_n, t)\,dt = \sum_{n=0}^{N} \int_{t_n}^{t_{n+1}} X'_n\,dt \qquad (2.13)$$

$$X = (X_0, X_1, X_2, , , X_N) \qquad X_{N+1}(t_{N+1}) = 0, \; X_0(t_0) = 0$$

where $X'$ is a vector describing a thinning regime. Control?

As mentioned by Everett (1963), if the choice of $X_i$ is decided independently in each cell, the sum is obviously maximized by simply maximizing the following objective function with respect to $X_n$:

$$J_n = \int_{t_n}^{t_{n+1}} I(X_n, X'_n, t)\,dt = \int_{t_n}^{t_{n+1}} X'_n dt \qquad (2.14)$$

In the PATH algorithm, it is assumed that the choice of $X_i$ does not affect the optimal path after the next stage, which is equivalent to the above assumption. The reason is that the stand, which provides the maximum sum of marginal return, or growth during last period, seems most likely to create the optimal stand at the next stage. However, there is a case, at which this assumption does not hold. Such a case is discussed later.

Checking necessary conditions for the calculus of variations problem,

$$\frac{\partial I}{\partial X} = 0 \quad , \qquad \frac{\partial I}{\partial X'} = 1$$

then the Euler equation is always satisfied, and so is the Legendre condition. Test the Weierstrass condition:

$$E(X_n, X'_n, t, Z'_n) = I(X_n, Z'_n, t) - I(X_n, X'_n, t) - \frac{\partial I}{\partial X'}(X_n, X'_n, t)(Z'_n - X'_n)$$

$$= Z'_n - X'_n - (Z'_n - X'_n) = 0$$

then this condition holds. In addition, the Weierstrass-Erdman corner condition is satisfied as well.

Therefore, in a given problem here, for any $X_n(t)$ satisfying boundary conditions all necessary conditions described are satisfied. Thus what should be done next is to search for the optimal trajectory among admissible ones.

When searching for the optimal trajectory in a growth model, the following conversion would reduce the numerical task. Let's define notation as follows:

$Y_n$ : vector describing the stand at stage n before a decision $T_n$

$X_n$ : vector describing the stand at stage n after a decision $T_n$

$T_n$ : vector describing the decision variable (thinning) at stage n, transferring the stand $X_n$ into $Y_n$

$X'_n$ : stand growth at range $(t_n, t_{n+1})$

Therefore, among these variables, some relationships are formulated:

$$X_n + T_n = Y_n \tag{2.15}$$

$$X_n + \int_{t_n}^{t_{n+1}} X'_n dt = Y_{n+1} \tag{2.16}$$

From these equations, the objective function(2.14) can be converted into the following function:

$$\max_{[Xn]} J_n = \int_{t_n}^{t_{n+1}} X'_n dt = \max_{[X_n]} J_n = Y_{n+1} - X_n$$

$$= \max_{[T_n]} J_n = Y_{n+1} - Y_n + T_n \tag{2.17}$$

Since $Y_n$ is constant for all admissible trajectories (the principle of optimality, Dreyfus (1965)), $Y_n$ can be eliminated from the objective function, resulting in the new objective function:

$$\underset{0<T_n\leq Y_n}{\text{maximize}}\ J_n = Y_{n+1} + T_n \tag{2.18}$$

As a result, a sequence of $T^*_i$ which optimizes the objective function (2.18) in each cell, can constitute the optimal thinning regime maximizing the original objective function (2.13). Furthermore, it is possible to determine the optimal rotation age in terms of maximizing mean annual increment of J with respect to $t_{N+1}$. In other words, setting up the maximax problem as:

$$\underset{[N]}{\max}\ \underset{[T]}{\max}\ \frac{J}{t_{N+1}} = \frac{1}{t_{N+1}} \sum_{n=0}^{N} \int_{t_n}^{t_{n+1}} I(X_n, X'_n, t)\,dt \tag{2.19}$$

the optimal rotation age can be obtained at the same time. If the optimal stand at each stage is obtained, equation (2.19) becomes:

$$\underset{[N]}{\max}\ \underset{[T]}{\max}\ \frac{1}{t_{N+1}} \sum_{n=0}^{N} (Y^*_{n+1} + T^*_n - Y^*_n)$$

$$= \underset{[N]}{\max}\ \frac{1}{t_{N+1}} (T^*_1 + T^*_2 + \cdots + T^*_N + Y^*_{N+1}) \tag{2.20}$$

where $Y^*_{N+1}$ and $T^*_n$ (n=1,2,,,N) are the optimal stand volume at age $t_{N+1}$ and optimal thinning level at age $t_n$, respectively.

Given prices of inputs, outputs and an appropriate discount rate, the production function can be converted into a net revenue equation where revenues and costs occurring at different points in time are properly discounted. Maximizing this expression yields the optimum patterns of inputs and outputs through time.

# STAND OPTIMIZATION SYSTEM (SOS)

By using the PATH algorithm and the SPS growth simulator, the Stand Optimization System model (SOS) is proposed. The acronym SOS is used to distinguish the proposed optimization framework from the SPS simulation framework. The SPS growth simulator is described in detail in APPENDIX I.

The SOS system developed here is classified as a deterministic, single descriptor, discrete-state, discrete-stage dynamic programming model. The problem solved utilizes the PATH algorithm, described in previous section, with a forward recursion. While searching for the optimal thinning level at each stage, it is possible for the user to use one of two different criteria in order to select the optimal thinning level. One of them is such that once the objective value declines, the previous thinning level becomes optimal. The other is such that after calculating all admissible solutions, the optimal thinning is selected among them. The following procedure is imbedded into the SOS model.

## 1) Optimization procedure

Employing a forward recursion to find the optimal thinning regime and rotation age, SOS searches for the optimal thinning regime at each stage in the following way:

First SOS creates the initial forest stand structure having diameter distribution with individual tree height data and crown ratio data, which result from a yield table given by the user. After thinning an amount of trees, which is decided by the number of iterations calculated by computer, and the interval of node given by the user, i.e., N x Interval, a residual stand grows until the next stage. At this stage, the sum of returns from both thinning and the future stand is compared with the previous one to store the best thinning level so far. If the user selects the first option, i.e., the unimodality assumption, once the objective function declines, SOS quits the iteration at this stage, and decides the previous thinning level is optimal. Otherwise, the iteration is continued until the number of the residual trees is less than the interval of thinning, then the best thinning level among admissible strategies is selected. SOS can also select the optimal thinning method at each stage among thinning from below, thinning from above and thinning to a cut/residual ratio fixed as 1.

If the thinning method "joint optimization" is selected by the user, after storing the best thinning level for one method at each stage, SOS does the same operation for two other methods so as to search for the best thinning level with each method at each stage. Comparing these three best objective values at each stage provides the optimal thinning method and level at each stage.

After determining the optimal trajectory at this stage, SOS sets up the initial forest structure at the next stage. Iterations continue over state and over stage until the last activity completes searching for the optimal trajectory from the initial stage to last stage. At the last activity, SOS searches for the optimal thinning level and rotation age at the same time by means of increasing rotation age by a 10 year-step. In other words, first set rotation age 10 years after last activity, store the best objective value and thinning level. Then add 10 years to the previous rotation age, search for the optimal objective value, then compare the present best value to the previous one. If the previous one is greater than the present one, the previous age is regarded as the optimal rotation age after all activities. However, it is possible for the early stage to have more objective value than the rotation age selected by the above procedure. In other words, the above procedure provides the optimal rotation age if the optimal rotation age is later than the last thinning time given by the user. Then SOS searches for the new optimal rotation age again from the initial stage to the rotation age calculated by the above procedure. As a result, optimization of both thinning regime and rotation age is completed.

This SOS optimization procedure developed can be expressed precisely in terms of a symbolic expression transformed into dynamic programming form. To begin with, let's specify the various symbols using general expressions as follows:

$r_n$ : return generated by residual stand at stage n before a decision $T_n$

$R_n$ : return generated by residual stand at stage n after a decision $T_n$

$R'_n$ : sum of marginal return generated by residual stand from stage n to stage n+1 after a decision $T_n$

$A_n$ : return generated by a decision $T_n$ at stage n

$f_n$ : total return at stage n

$N$ : last stage

Among these variables the following relationships can be indicated:

$R_n = g(X_n)$ : $R_n$ is a function of $X_n$

$r_n = h(Y_n)$ : $r_n$ is a function of $Y_n$

$A_n = k(T_n)$ : $A_n$ is a function of $T_n$

$$fn = \sum_{i=1}^{n} A_i \tag{3.1}$$

$$Y_{n+1} = X_n + \int_{t_n}^{t_{n+1}} X'_n dt \tag{2.16}$$

$$Y_n = X_n + T_n \tag{2.15}$$

$$T_{N+1} = Y_{N+1},\ X_{N+1} = 0$$

Similarly,

$$r_{n+1} = R_n + \int_{t_n}^{t_{n+1}} R'_n dt \tag{3.2}$$

$$r_n = R_n + A_n \tag{3.3}$$

These equations imply that $T_n$ affects the return from stage n through stage N. And the n-th stage return is affected by the n-th stage decision $T_n$ and its previously successive decision variable.

In terms of the PATH algorithm, the objective function can be expressed as:

$$\underset{0<T_n\leq Y_n}{\text{maximize}}\ J_n = r_{n+1} + A_n \tag{3.4}$$

$$f_n = A^*_n + f_{n-1} \tag{3.5}$$

where $A^*_n$ is the return generated by the optimal $T_n$, or $T^*_n$.

Therefore by means of recursive expression the following steps are utilized:

Step 0: Initialize conditions

$$n = 1$$

$$i = 1$$

$$t_{N+1} = t_N + 10 \text{ yr.}$$

$$f_0 = 0$$

Step 1: $J_{n,i} = r_{n+1} + A_n$

$J^*_{n,i} = \max_{[T_n]} J_{n,i}$

$n<N \rightarrow$ Save $T^*_n$

$f_n = A^*_n + f_{n-1}$

Step 2: $n<N \rightarrow n = n+1$

Go to Step 1

$n=N \rightarrow$ Go to Step 3

Step 3: $t_{N+1} = t_{N+1} + 10$

$J^*_{N,i}<J^*_{N,i-1} \rightarrow$ Save $T^*_N$

$f_N = A^*_N + f_{N-1}$

Go to Step 4

Otherwise $\rightarrow$ $i = i + 1$

Go to Step 1

Step 4: $f^* = \max_{[n]} f_n$

Print $[T^*_n]$ $n = 1,..n^*$

where i represents the number of iterations to search for the optimal rotation age, n represents the stage, and $n^*$ is the optimal stage giving the optimal rotation age.

Both Step 3 and Step 4 are added so as to search for the optimal rotation age under the positive unimodality assumption over time.

2) Features of the SOS model

Since SPS can present such data as basal area, cubic-foot volume, merchantable cubic feet, and merchantable board feet, the objective function of SOS can be based on these data. If using these data, the optimal rotation age is calculated based on mean annual increment of the corresponding unit. In addition SOS can search for the optimal thinning regime and rotation age based on present net worth (PNW) and soil expectation value (SEV). Equation (3.6) expresses PNW, and (3.7) expresses SEV.

$$PNW(tn) = \frac{r(tn)}{(1+i)^{tn}} + \frac{A(tn-1)}{(1+i)^{tn-1}} \qquad (3.6)$$

$$SEV(tn) = \frac{PNW(tn) \times (1+i)^{tn}}{[(1+i)^{tn} - 1]} \qquad (3.7)$$

where tn is the age at stage n.

The optimal rotation age is obtained based on either mean annual increment of the corresponding unit, present net worth, or soil expectation value. If optimization basis is basal area, cubic feet, merchantable board feet or merchantable cubic feet, the optimal rotation age is selected by:

$$\max_{[t_{N+1}]} J = \frac{[V_{N+1} + \sum T_i]}{t_{N+1}} \qquad (3.8)$$

where $V_{N+1}$ is the selected physical value at age $t_{N+1}$. If optimization basis is present net worth, the optimal rotation age is decided by:

$$\max_{[t_{N+1}]} J = PNW(t_{N+1}) \tag{3.9}$$

In the case of soil expectation value, it is decided by:

$$\max_{[t_{N+1}]} J = SEV(t_{N+1}) \tag{3.10}$$

Combining thinning basis, such as trees per acre or crown competition factor, and thinning method, such as thinning from below, thinning from above, or thinning to a c/r ratio of 1, can provide 2 x 3 = 6 possible thinning regimes. If thinning method joint optimization is selected, SOS indicates not only the optimal thinning level but also the optimal thinning method at each stage on either TPA or CCF basis. Modification of this approach to optimize thinning for cases involving intensive thinning will be discussed later.

3) Price equation

For the sake of making the model simple, cost and revenue from thinning and final harvest are based on entry cost, stumpage price premiums, and other constant silvicultural costs. Entry cost is fixed over time. Price equation per cubic feet is expressed as a function of DBH, which can be created by using either one linear equation or an equation with several continuous linear segments. For instance, in the former case,

$$\text{price/cuft} = .2\text{xDBH} + .08 \tag{3.11}$$

in the latter case,

$$\text{price/cuft} = 0 \qquad \text{if} \quad 0 \leq DBH \leq 7.95$$

$$= .06 \times DBH - .26 \qquad \text{if } 7.95 \leq DBH \qquad (3.12)$$

It is also possible to utilize a different price equation for different species. These financial data, with interest rate, are utilized for economic optimization and in the financial reports of physical optimization.

Since a price equation transforms a searching surface which is directly derived from physical data, into a new surface which has economic information, it is necessary to take into account what kind of price equation can be used. In other words, even if the searching surface obtained from direct physical data is nicely concave, it is possible for it to become an irregular surface when the price equation is included. In such a case, the Weierstrass condition, which shows the condition for concavity over the control variable (thinning level), would be violated, as well as the Weierstrass-Erdman corner condition. For example, if a step-wise price equation is used, these conditions are not satisfied. This violation is shown by the relationship between the objective function and the decision variable, which would not be concave. Once these conditions are violated, the solution provided by SOS becomes the better solution, and not the best one.

4) Basic required data for SOS

In essence, SOS requires the same input file as SPS does. If the user is willing to use only physical optimization, the information needed is the interval of node, or the number of trees per acre removed by unit action and the volume unit to be optimized (board feet or cubic feet). If the user is searching for the economic optimal allocation, the information needed is:

1) interest rate
2) entry cost per acre (same for both thinning and final harvest)
3) coefficient for reducing thinning value
4) price equation
5) fertilizer cost if any
6) other silvicultural costs if any.

Although it is possible to use more complicated economic data, such as logging cost identical for each thinning level, cost is limited as above.

# ANALYSIS WITH THE SOS MODEL

In the following section an example for the SOS operation is presented, and some impacts of such factors as interest rate, quality premium and type of thinning are represented. Then the trade-off problems are resolved in terms of a new algorithm, and the technique to estimate the optimal Lagrange multiplier is presented.

## 1) Application of the SOS model

### a) Illustrative example

To demonstrate SOS, an input file and financial data are required. Characteristics of the data are:

Species: Douglas-fir and western hemlock

Site index: 82 of Douglas-fir at 50-year breast-height-age basis

Region: Pacific Northwest Region

Thinning is implemented at age 20, 30, and 40 years. The basis of thinning is number of trees per acre, and the type of thinning is thinning from below with maximum DBH limit 100 inch (this value should be large so that thinning from below can be implemented at every diameter class). In addition to this data, tree height, number of trees per acre, breast height age and percent of crown ratio at each DBH class as well are given in Table 1.

Table 1. An illustrative example.
(see Arney, 1985)

| | sp | st-id | age | region |
|---|---|---|---|---|
| STAND | DF | 82 | 10 | PNW |
| MERCH | 1.0 | 16.4 | 5.0 | 9.0 |
| NAME | An illustrative example | | | |

| | | age | basis | | | method |
|---|---|---|---|---|---|---|
| THINNING | 1 | 20 | 1 | 0 | 1 | 100 |
| THINNING | 1 | 30 | 1 | 0 | 1 | 100 |
| THINNING | 1 | 40 | 1 | 0 | 1 | 100 |

TABLE

| | SP | DBH | HT | TPA | BT-AGE | CR |
|---|---|---|---|---|---|---|
| 1 | WH | 2 | 18 | 15 | 19 | 80 |
| 2 | WH | 3 | 27 | 92 | 19 | 80 |
| 3 | WH | 4 | 32 | 227 | 19 | 80 |
| 4 | WH | 5 | 35 | 121 | 19 | 80 |
| 5 | WH | 6 | 36 | 41 | 19 | 80 |
| 6 | DF | 3 | 30 | 32 | 18 | 80 |
| 7 | DF | 4 | 34 | 57 | 18 | 80 |
| 8 | DF | 5 | 38 | 138 | 18 | 80 |
| 9 | DF | 6 | 38 | 72 | 18 | 80 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| CLUMP | 0.85 | | | | | | |
| REPORTS | 10 | 20 | 30 | 40 | 50 | 60 | 70 |

Also optimization basis, interval of node, and financial data are provided as follows:

optimization basis : soil expectation value(SEV)

interval of node : 100 trees per acre

interest rate : 4 %

entry cost : \$ 50.00

coefficient for reducing thinning value : 0.80

regeneration cost : \$ 200./acre at age 0

price/1000cuft : 200xDBH + 80

Soil expectation value generated by both thinning and the future stand is calculated based on the next stage, which is assumed to be temporal rotation age. Then the optimal thinning levels at age 20, 30 and 40 are provided.

In accordance with the recursive procedure, we calculate the objective value based on present net worth, then select the optimal thinning level. After that, the return $f_{ipnw}$ produced so far is accumulated, which does not include the return from the future stand. Then the soil expectation value $f_{isev}$ based on the next stage as the rotation age is calculated, which includes the return from the future stand. The following gives the iterative calculation stage by stage:

At 0 stage

$f_{0pnw} = -200$ at age 10
$f_{0sev} = 3267.7$ at age 20

At 1 stage

$$J_1 = A_{1pnw} + r_{2pnw}$$

$$\max J_1 = \left\{\begin{array}{r} 0 + 2566 = 2566 \\ 36 + 2609 = 2645 \\ 159 + 2571 = 2730 \\ 302 + 2379 = 2680 \\ 503 + 2075 = 2578 \\ 788 + 1635 = 2423 \\ 1086 + 1165 = 2251 \end{array}\right\} = 2730$$

$A^*_{1pnw} = 159 \quad T^*_1 = 200$
$f_{1pnw} = -41$ at age 20
$f_{1sev} = 3635$ at age 30

At 2 stage

$$J_2 = A_{2pnw} + r_{3pnw}$$

$$\max J_2 = \left\{\begin{array}{r} 0 + 2672 = 2672 \\ 198 + 2581 = 2779 \\ 486 + 2265 = 2752 \\ 886 + 1790 = 2676 \\ 1341 + 1186 = 2528 \end{array}\right\} = 2779$$

$A^*_{2pnw} = 198 \quad T^*_2 = 100$
$f_{2pnw} = 157$ at age 30
$f_{2sev} = 3444$ at age 40

At 3 stage(last stage)
when rotation age is 50 year

$$J_3 = A_{3pnw} + r_{4pnw}$$

$$\max J_3 = \left\{\begin{array}{r} 0 + 2415 = 2415 \\ 311 + 2184 = 2495 \\ 757 + 1693 = 2450 \\ 1282 + 1065 = 2346 \end{array}\right\} = 2495$$

$A^*_{3pnw} = 311 \quad T^*_3 = 100$
$f_{3pnw} = 468$ at age 40
$f_{3sev} = 3078$ at age 50

when rotation age is 60 year

$$J_3 = A_{3pnw} + r_{4pnw}$$

$$\max J_3 = \left\{\begin{array}{r} 0 + 1950 = 1950 \\ 311 + 1955 = 2266 \\ 757 + 1572 = 2328 \\ 1282 + 1022 = 2303 \end{array}\right\} = 2328$$

$A^*_{3pnw} = 757 \quad T^*_3 = 200$
$f_{3pnw} = 914$ at age 40
$f_{3sev} = 2741$ at age 50

where $A_{ipnw}$ includes entry cost for thinning, $r_{ipnw}$ does not include entry cost for both thinning and final harvest.

If all activities are required, the optimal thinning regime and rotation age are:

thinning 200 trees per acre at age 20,

thinning 100 trees per acre at age 30,

thinning 100 trees per acre at age 40,

clearcut at age 50.

Figure 2 shows the network describing the possible paths at each stage and the optimal path in terms of stand volume. In comparison with the traditional dynamic programming approach, this network does not have so many paths which connect the initial stand with the final stand so that searching for the optimal rotation age may be completed much faster than traditional dynamic programming does. Then the numerical task is reduced for searching for the optimal thinning regime and rotation age.

As depicted in Figure 3 the searching surface over action with 100 trees of interval can be interpreted as being concave. In this example, even if the return generated by the future stand reached the optimal point at 100 thinning level, total return, i.e., the objective value, reached the optimal point at 200 thinning level.

Once the interval of node becomes smaller, such as 20 trees per acre, the searching surface becomes more irregular as shown in Figure 4. The reason that at 20

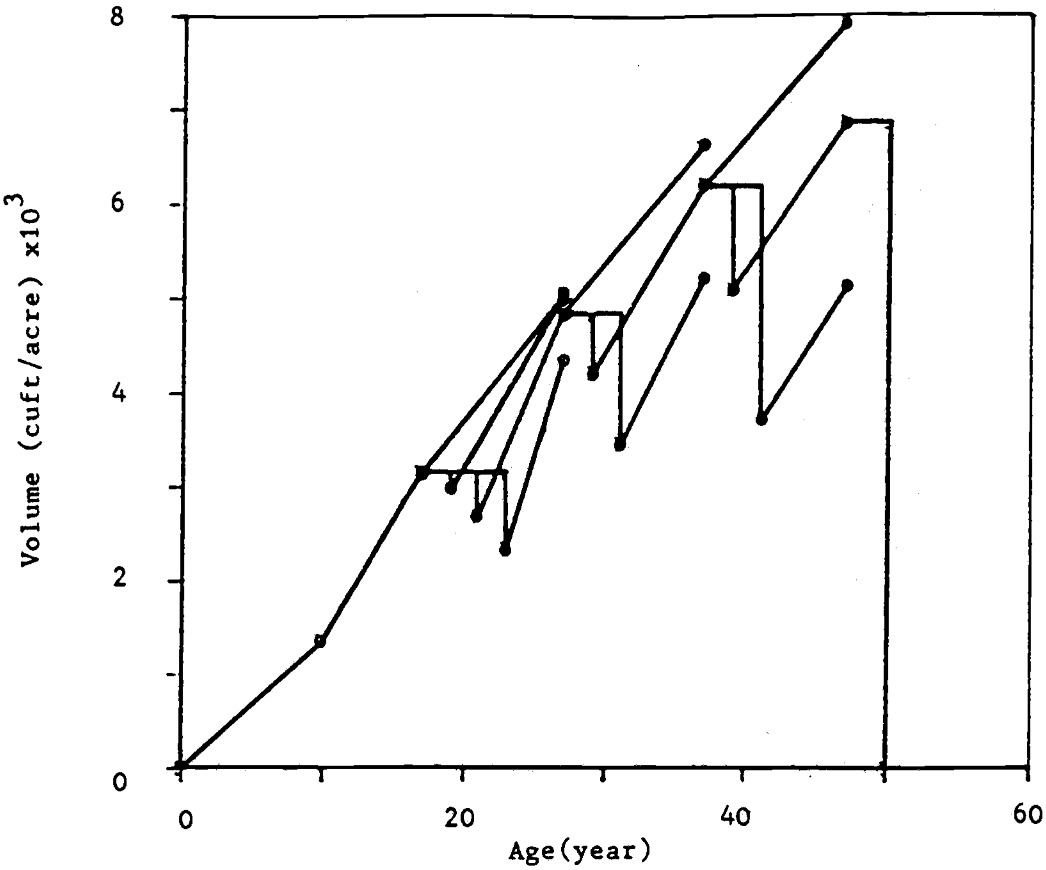



Figure 2.
Network and optimal thinning regime.

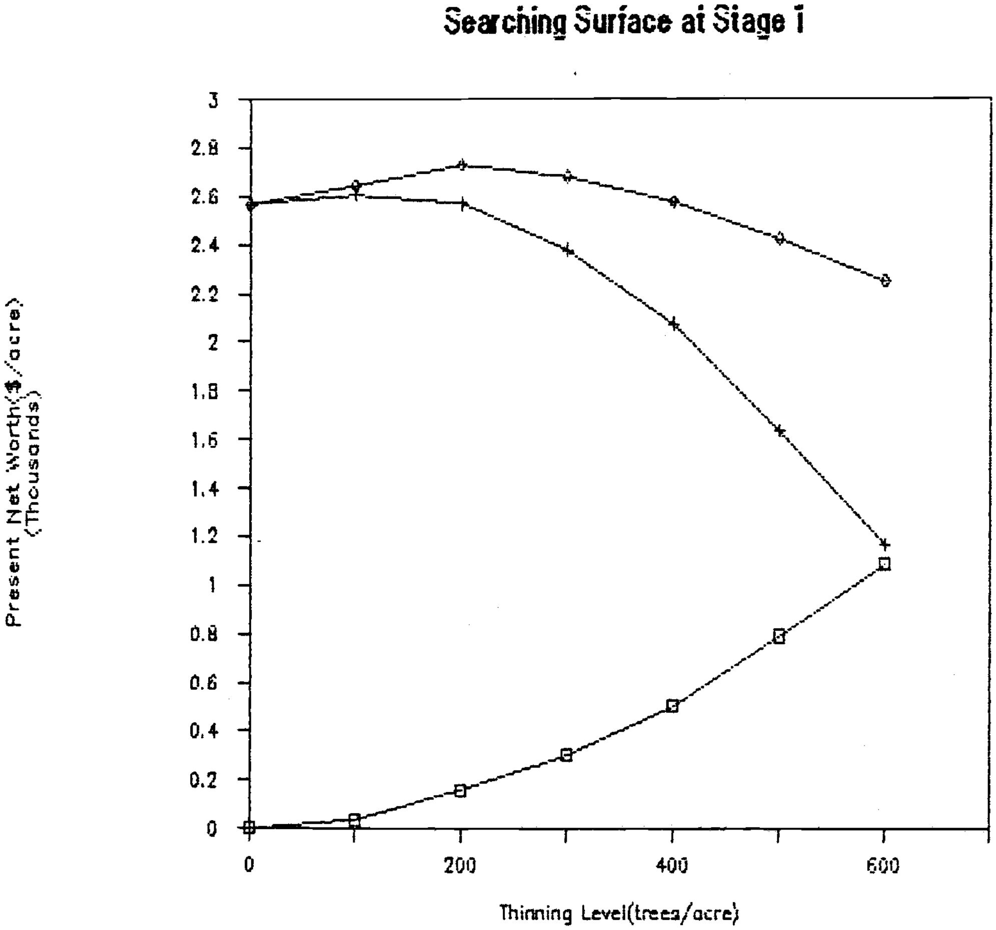



Figure 3.
Searching surface over thinning level with 100 trees per acre as interval at stage 1.

□ ----- return from thinning
+ ----- return from residual productivity
◇ ----- objective value

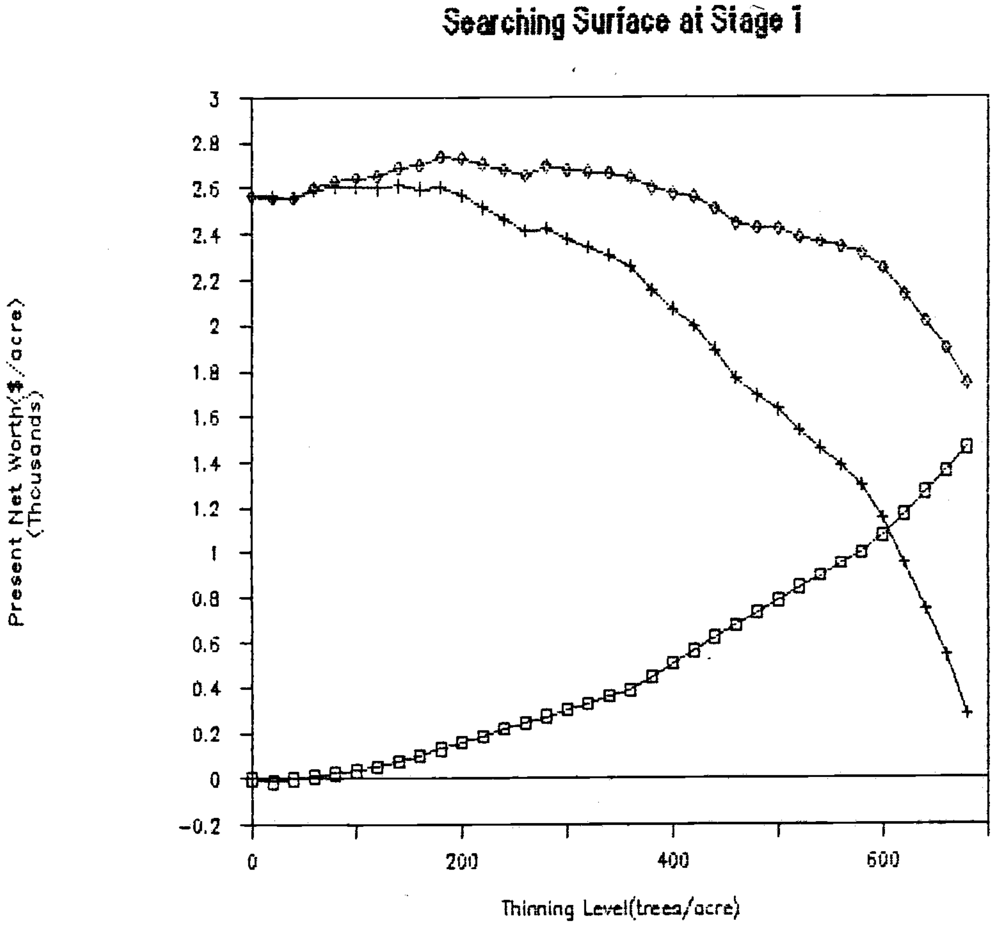



Figure 4.
Searching surface over thinning level with 20 trees per acre as interval at stage 1.

□ ----- return from thinning
+ ----- return from residual productivity
◇ ----- objective value

thinning level the objective value goes down is that when there is no thinning it does not take any entry cost so that it is possible for the second action to take more cost than the first no thinning does. Other fluctuations are due to the small difference of the decision variable. Although this irregular surface violates the Weierstrass necessary condition, this searching surface over action can be said to be concave by ignoring small fluctuations. Then it is strongly recommended to search over all feasible actions in the case of a small interval as well as sometimes in the case of a large interval, at which the first action has a larger objective value than the second action.

Figure 5 shows the searching surface over time, which can reveal the actual optimal rotation age. According to Figure 5, 30 years is the optimal rotation age and the thinning regime is:

thinning 200 trees per acre at age 20,

clearcut at age 30.

This example is based on the assumption that the future stand is correctly evaluated at each stage. Particular cases where this consideration is important will be discussed in a following section on trade-offs.

b) Effect of interest rate

In order to figure out how interest rate affects rotation age, the range of interest rate is limited to

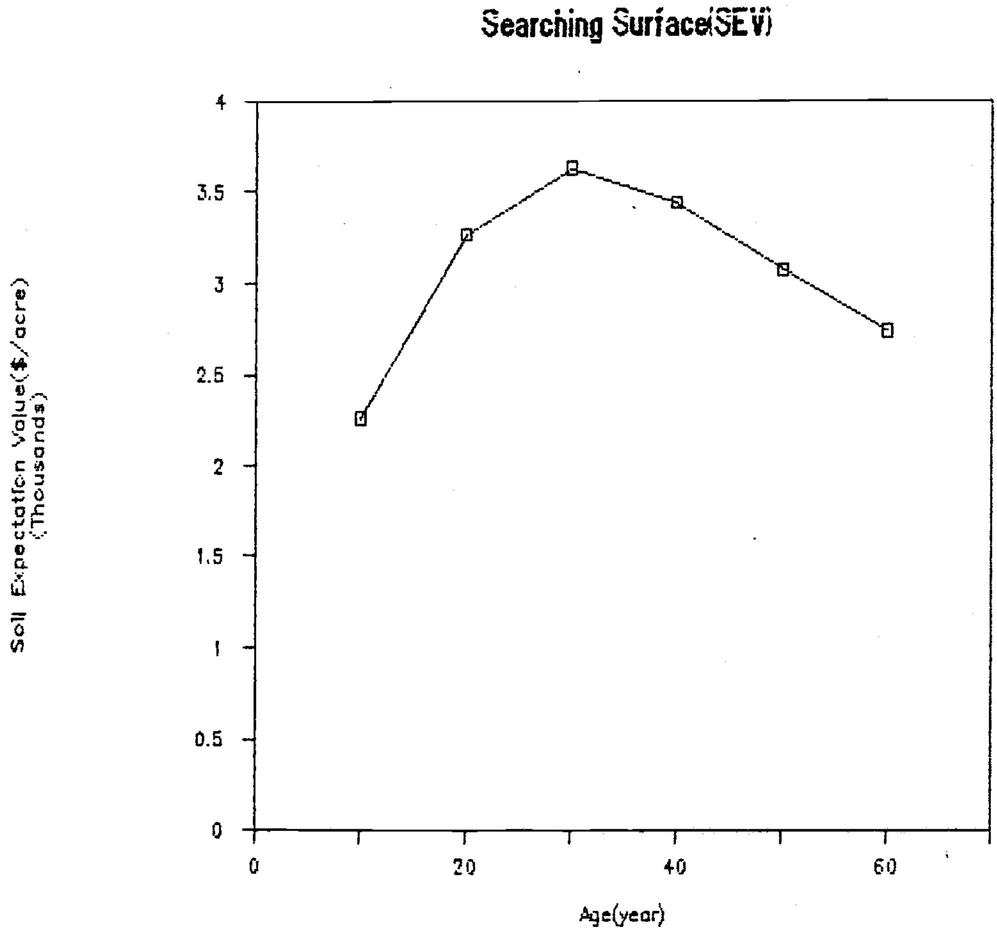



Figure 5.
Searching surface over stage based on soil expectation value.

(0,IRR], where the IRR is the maximum internal rate of return satisfying:

$$\text{Discounted Revenue} = \text{Discounted Cost} \tag{4.1}$$

Within this range PNW at the optimal rotation age can be said to be monotonically declining as interest rate increases. Mathematically expressing,

$$\frac{\partial PNW}{\partial i} < 0 \tag{4.2}$$

Under this assumption, the effect of interest rate is presented as follows:

In general, the relationship among PNW, SEV, rotation age and interest rate is:

$$SEV = \frac{PNW(1+i)^t}{(1+i)^t - 1} \tag{4.3}$$

This equation is also expressed as:

$$t = \ln\left[\frac{SEV}{SEV-PNW}\right] \frac{1}{[\ln(1+i)]} \tag{4.4}$$

Taking the first derivative with respect to i partially,

$$\frac{\partial t}{\partial i} = \ln\left[\frac{SEV}{SEV-PNW}\right] \frac{-1}{(1+i)[\ln(1+i)]^2} \tag{4.5}$$

$$PNW \geq 0$$
$$SEV \geq 0$$

Since

$$\frac{SEV}{SEV - PNW} \geq 1 \tag{4.6}$$

and

$$1 + i \geq 1 \tag{4.7}$$

then

$$\frac{\partial t}{\partial i} < 0 \tag{4.8}$$

Therefore as the interest rate increases, the optimal

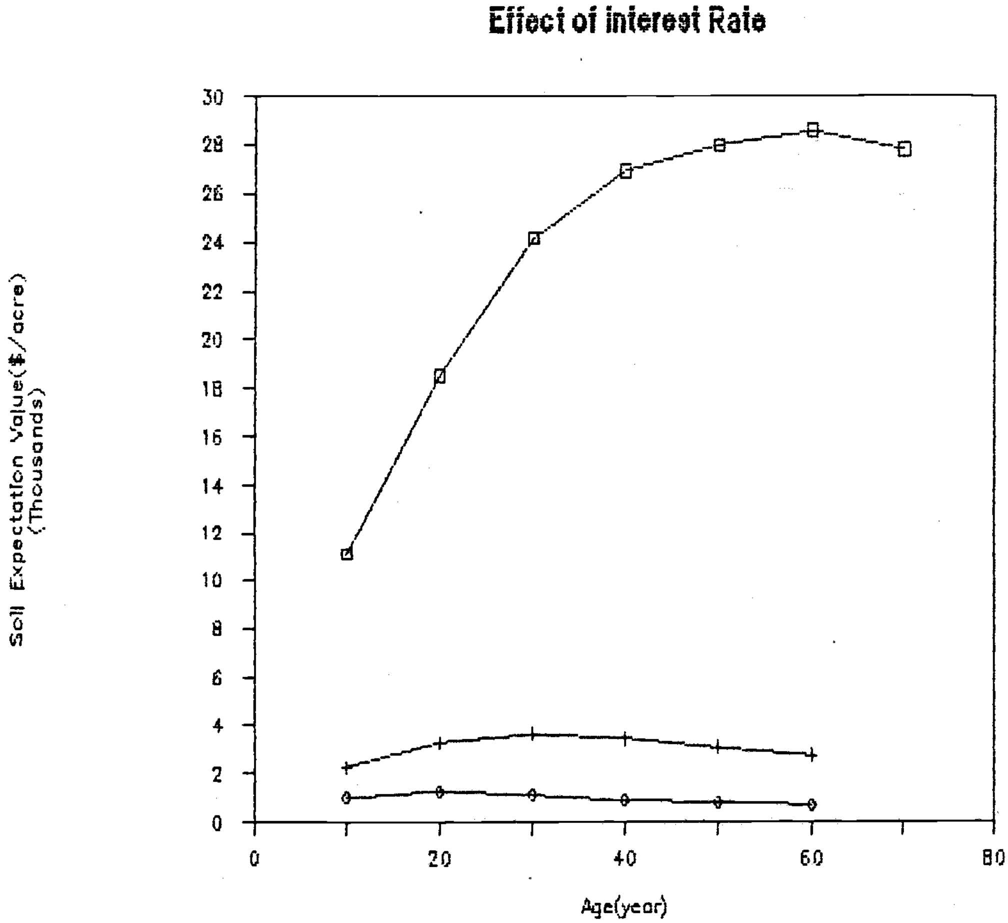



Figure 6.
Effect of interest rate on the searching surface of soil expectation value over stage.

□ ----- 1 % interest rate
+ ----- 4 % interest rate
◇ ----- 7 % interest rate

rotation age decreases within the range (0,IRR]. Figure 6 shows how the rotation age changes as interest rate increases. As was expected, the optimal rotation age declines as interest rate increases, resulting in low soil expectation value.

The interest rate also can affect the thinning regimes. Figure 7 depicted the difference of thinning regimes, at which rotation age is fixed after all activities, i.e., after 40 years. An interest rate of 1 % in Figure 7 produces more objective value at age 60 than at age 50, allowing trees to grow until age 60. On the other hand, in the case of 4 % interest rate, it is inefficient to allow trees to grow until age 60 or more, since the objective value declines after age 30 (Figure 6). In addition an interest rate of 7 % has intensive thinning at age 30, at which most profit from potential productivity is taken out. As a result, it can be said that an increase in interest rate suggests a short rotation age with low objective value, and intensive thinning at early period if rotation age is fixed after all activities as well.

c) Effect of quality premiums

To illustrate the effect of quality premiums the following four price equations are used:

Case 1: there is no quality premium

$$\text{price} = 0 \times \text{DBH} + 1000$$

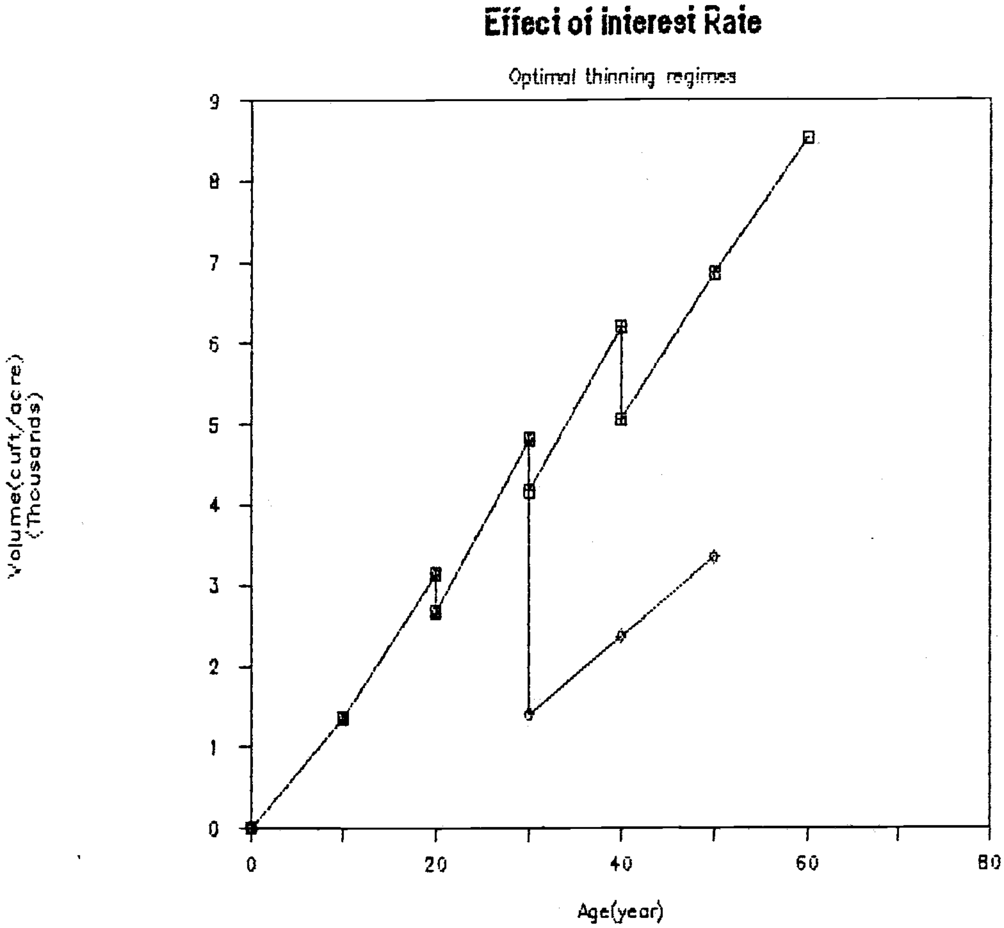



Figure 7.
Effect of interest rate on the optimal thinning regime.

□ ----- 1 % interest rate
+ ----- 4 % interest rate
◇ ----- 7 % interest rate

Case 2: there is a linear quality premium over DBH

price = 500 x DBH + 0

Case 3: after DBH = 20 inch, there is no quality premium

| | |
|---|---|
| price = 0 | if $0 \leq \text{DBH} \leq 5$ |
| = 50 x DBH - 250 | if $5 < \text{DBH} \leq 20$ |
| = 0 x DBH + 1000 | if $20 < \text{DBH}$ |

Case 4: after DBH = 11 inch there is a linear quality premium

| | |
|---|---|
| price = 0 | if $0 \leq \text{DBH} \leq 5$ |
| = 0 x DBH + 200 | if $5 < \text{DBH} \leq 8$ |
| = 0 x DBH + 500 | if $8 < \text{DBH} \leq 11$ |
| = 53 x DBH - 79 | if $11 < \text{DBH}$ |

Other input data are the same as in the illustrative example.

As depicted in Figure 8, Case 1 and Case 2 have the same optimal thinning regime. On the other hand, Case 3 and Case 4 have different optimal thinning regimes because of the different pattern of the quality premium. It can be said then that the optimal thinning regime remains the same unless the changing pattern of quality premium is differing. The reason is most likely due to the same pattern of linear increment of the quality premium as the tree diameter increases, resulting in the same impact on the objective value.

Depending upon the quality premium, the optimal

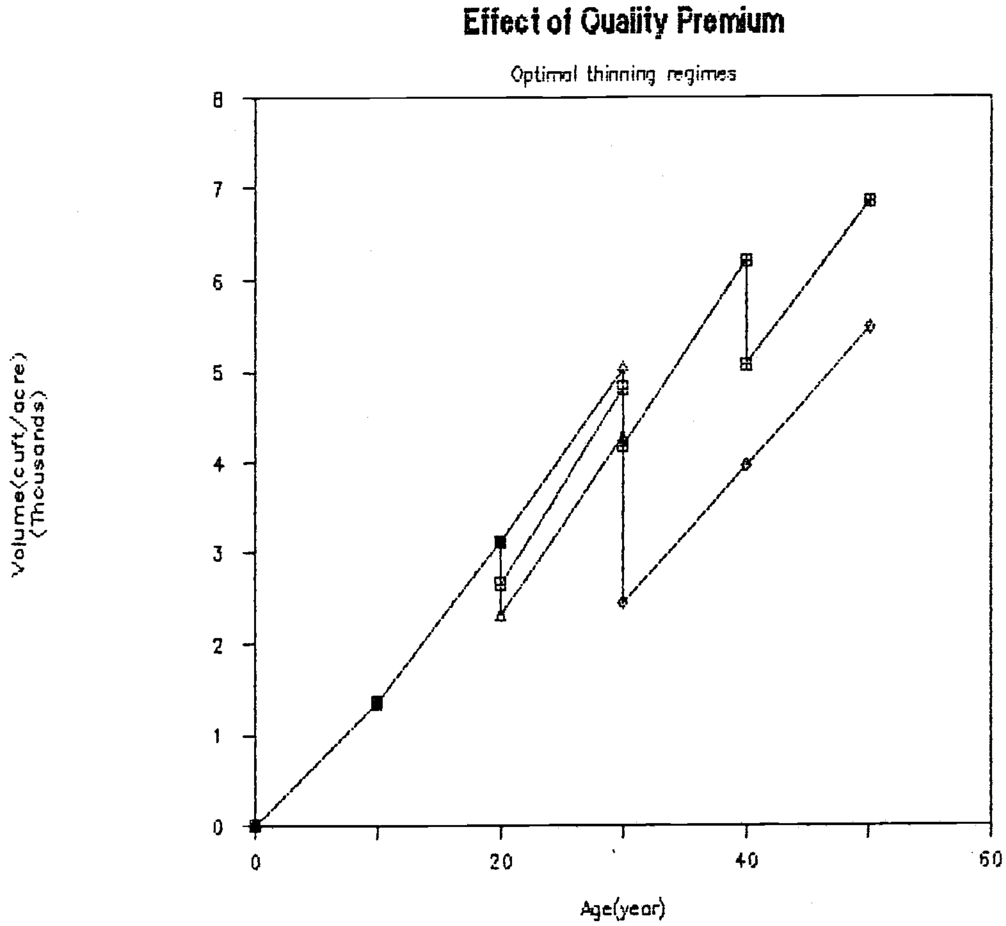



Figure 8.
Effect of quality premium on the optimal thinning regime.

□ ----- Case 1
+ ----- Case 2
◇ ----- Case 3
△ ----- Case 4

rotation age as well as soil expectation value differ. Even if the optimal thinning regimes are the same, the optimal rotation age and soil expectation value will be changed. While Case 1 has the same optimal thinning regime as Case 2 has, the optimal rotation age of Case 1 is 20 years with $2227.6/acre soil expectation value different from 30 year optimal rotation age with $9124.6/acre in Case 2 as shown in Figure 9. Case 3 has the 50 year optimal rotation age with $86.8/acre, and Case 4 has the 30 year rotation with $625.2/acre.

d) Effect of type of thinning

Thinning regime is affected by not merely interest rate and the price equation but also type of thinning. With SOS, the user can specify one of three different types of thinning. One of them is thinning from below, which allows big trees to grow more than small tree, resulting from increasing crown space by cutting small trees first. Therefore this thinning seems to weight the return from the future stand more than the return from thinning. On the other hand, thinning from above seems to weight the return from thinning more than the return from the future stand, because bigger trees are cut first. The last one is thinning to a cut/residual ratio, which is set as 1. The biggest tree is cut first, then the smallest tree until the ratio of quadratic mean of diameter of cut trees over quadratic mean of residual

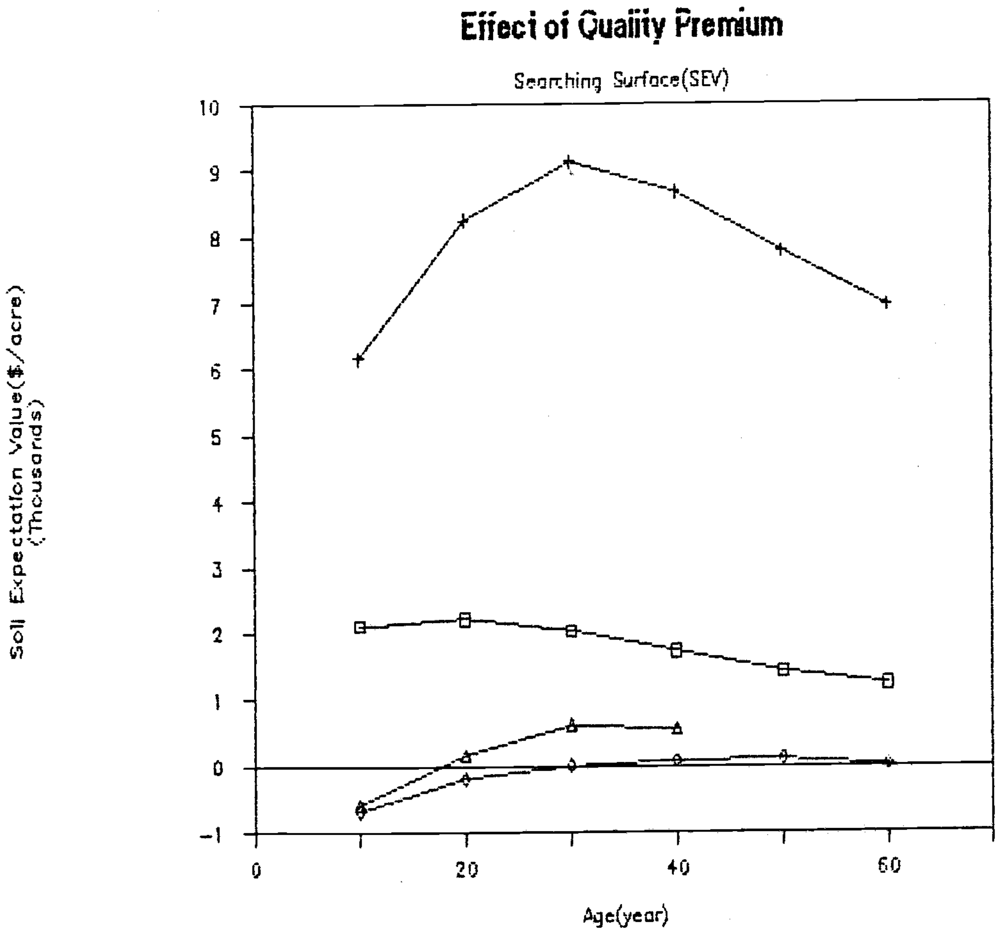



Figure 9.
Effect of quality premium on the searching surface of soil expectation value over stage.

□ ----- Case 1
+ ----- Case 2
◇ ----- Case 3
△ ----- Case 4

trees becomes equal to 1.

Input data from Table 1 are used. Financial data are the same as in the illustrative example.

As shown in Figure 10, the optimal thinning regime with the basis of thinning from above does not have any thinning at 20 and 40 years. On the other hand, both thinning from below and thinning to a c/r ratio of 1 have some thinning at every stage. Including thinning from above, which causes an inefficient thinning in this example, thinning to a c/r ratio of 1 has less soil expectation value than thinning from below does. This inefficiency results in less residual product as shown in Figure 10. Even if thinning to a c/r ratio of 1 has the same thinning level as thinning from below at stage 1, or 20 years, the future stand volume by thinning to a c/r ratio of 1 is less than the other.

In this example, the best thinning method is thinning from below, then thinning to a c/r ratio of 1 and thinning from above as shown in Figure 11. This order can be changed depending upon the financial data, especially quality premium.

2) Limitation of the optimality

Thus far, it has been assumed that the optimal path at every stage is determined based on the next stage. In

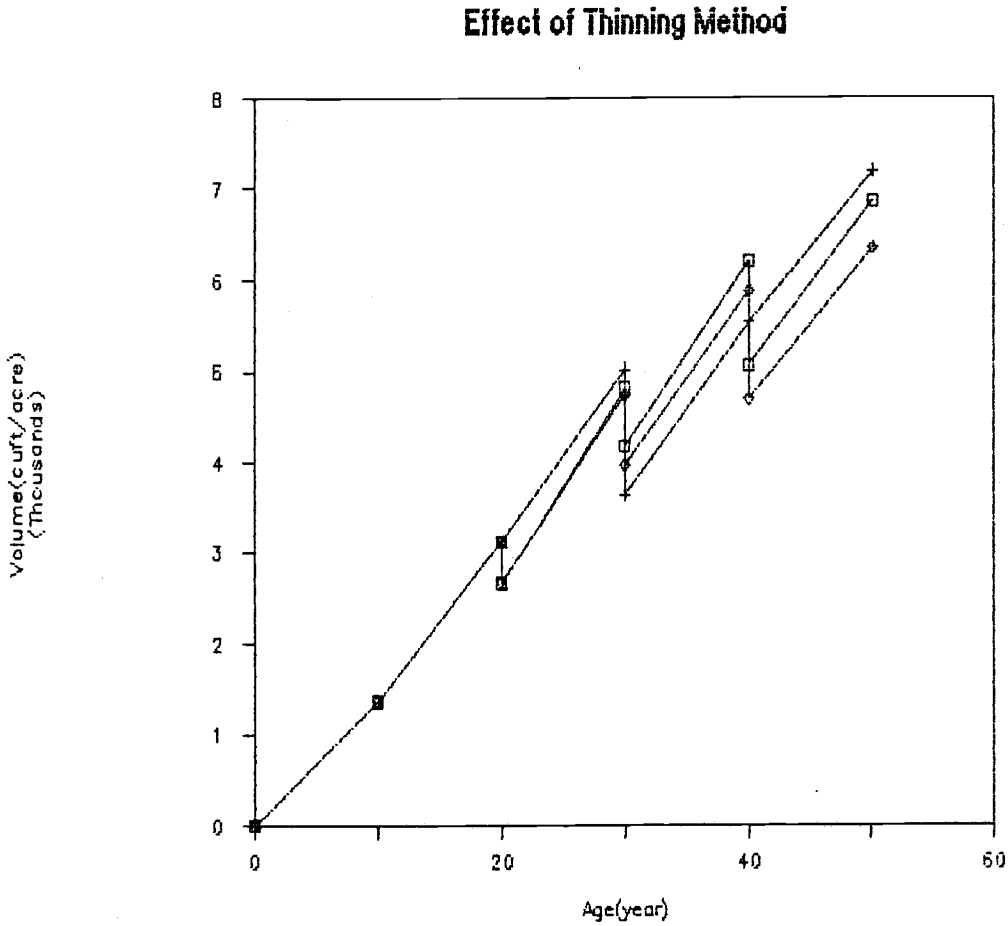



Figure 10.
Effect of thinning method on the optimal thinning regime.

□ ----- thinning from below
\+ ----- thinning from above
◇ ----- thinning to a c/r ratio of 1

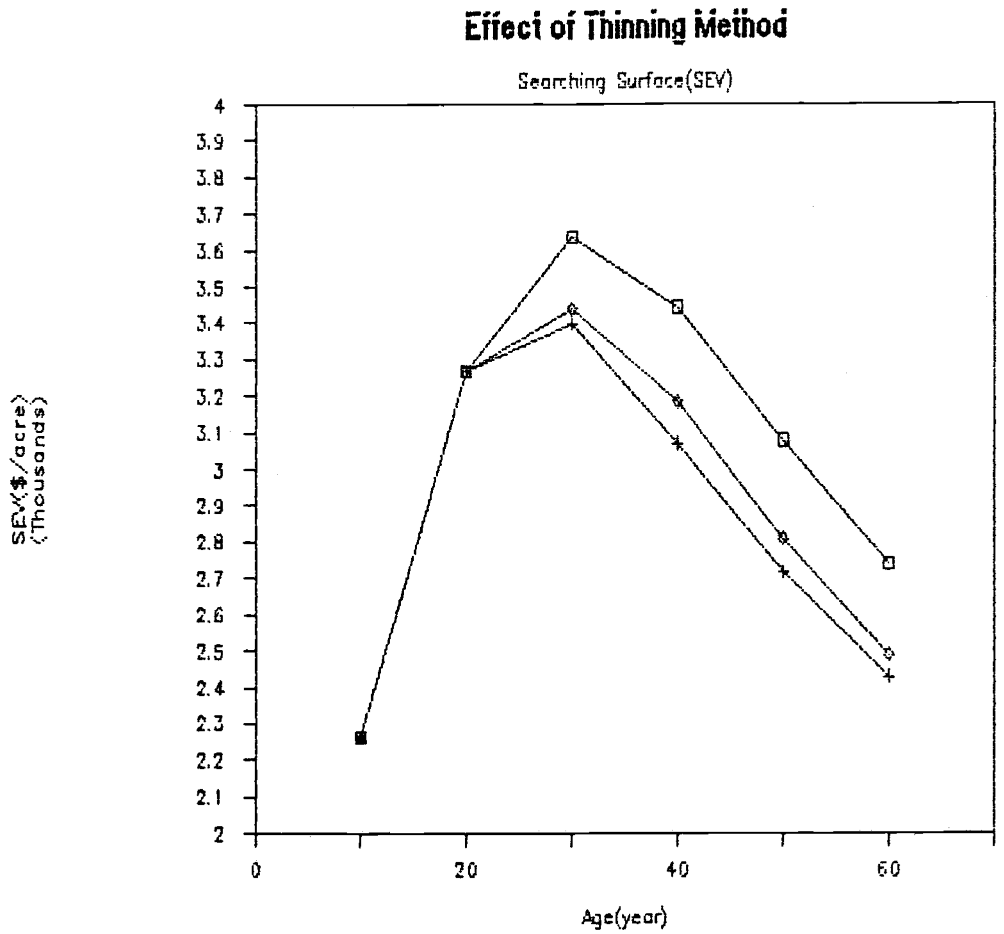



Figure 11.
Effect of thinning method on the searching surface of soil expectation value over stage.

□ ----- thinning from below
\+ ----- thinning from above
◇ ----- thinning to a c/r ratio of 1

other words, the return from the future stand is estimated on the basis of the next stage. Then the path provided is optimal so long as it is decided on this criterion as well as necessary conditions for a calculus of variations problem hold. However, two interesting situations can develop. The first situation occurs if there is no thinning at the next stage. The second situation occurs in conjunction with intensive thinning, when the look-ahead period is insufficient to evaluate impacts on the future stand. The first situation is easy to handle since the output can show if this situation happened or not. On the other hand, the second situation is not because this situation is ignored in terms of the PATH algorithm at the current stage, resulting in not showing if this situation can happen or not at the future stage. These situations are discussed below.

a) The impact on optimality with no thinning at the next stage and intensive thinning at the current stage

Let's consider the first situation in Figure 12, at which it is assumed that $T_1$ is the optimal thinning level on the basis of stage 2, and that there is no thinning at stage 2 based on stage 3.

Given that there is another optimal route $T_1'$-$Y_2'$-$Y_3'$ from stage 1 to stage 3 on the basis of stage 3, this route maximizes the objective function if stage 2 is

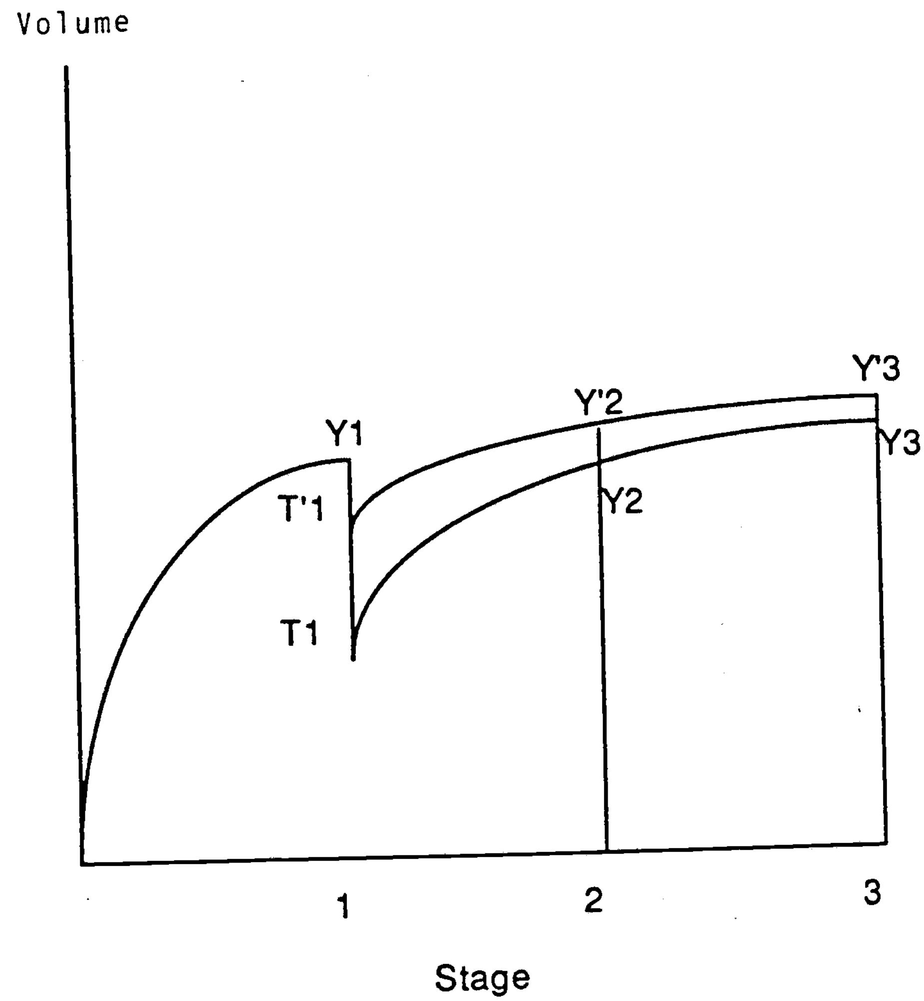



Figure 12.
The first type of trade-off.

eliminated. At stage 2, the objective value for thinning level $T_1$ is:

$$r_2(Y_2) + A_1(T_1) \tag{4.9}$$

and for thinning level $T_1'$:

$$r_2(Y_2') + A_1(T_1'). \tag{4.10}$$

According to the above assumption, suggesting that $T_1$ is optimal on the basis of stage 2, the following inequality is satisfied:

$$r_2(Y_2) + A_1(T_1) > r_2(Y_2') + A_1(T_1') \tag{4.11}$$

Therefore the difference is:

$$Dif = r_2(Y_2) - r_2(Y_2') + A_1(T_1) - A_1(T_1') \tag{4.12}$$

If there is no thinning at stage 2, route $T_1$-$Y_2$-$Y_3$ produces the objective value :

$$r_3(Y_3) + A_1(T_1) \tag{4.13}$$

and route $T_1'$-$Y_2'$-$Y_3'$ produces:

$$r_3(Y_3') + A_1(T_1') \tag{4.14}$$

According to the second assumption, under which $T_1'$ is optimal if stage 2 is eliminated, the following inequality is satisfied:

$$r_3(Y_3') + A_1(T_1') > r_3(Y_3) + A_1(T_1') \tag{4.15}$$

Then the difference is:

$$Dif' = r_3(Y_3') - r_3(Y_3) + A_1(T_1') - A_1(T_1) \tag{4.16}$$

Thus route $T_1$-$Y_2$-$Y_3$ is obtaining Dif at stage 2, and loosing Dif' at stage 3. Therefore, the trade-off from the first path to the second path should be implemented, once the indicated path starts loosing Dif'.

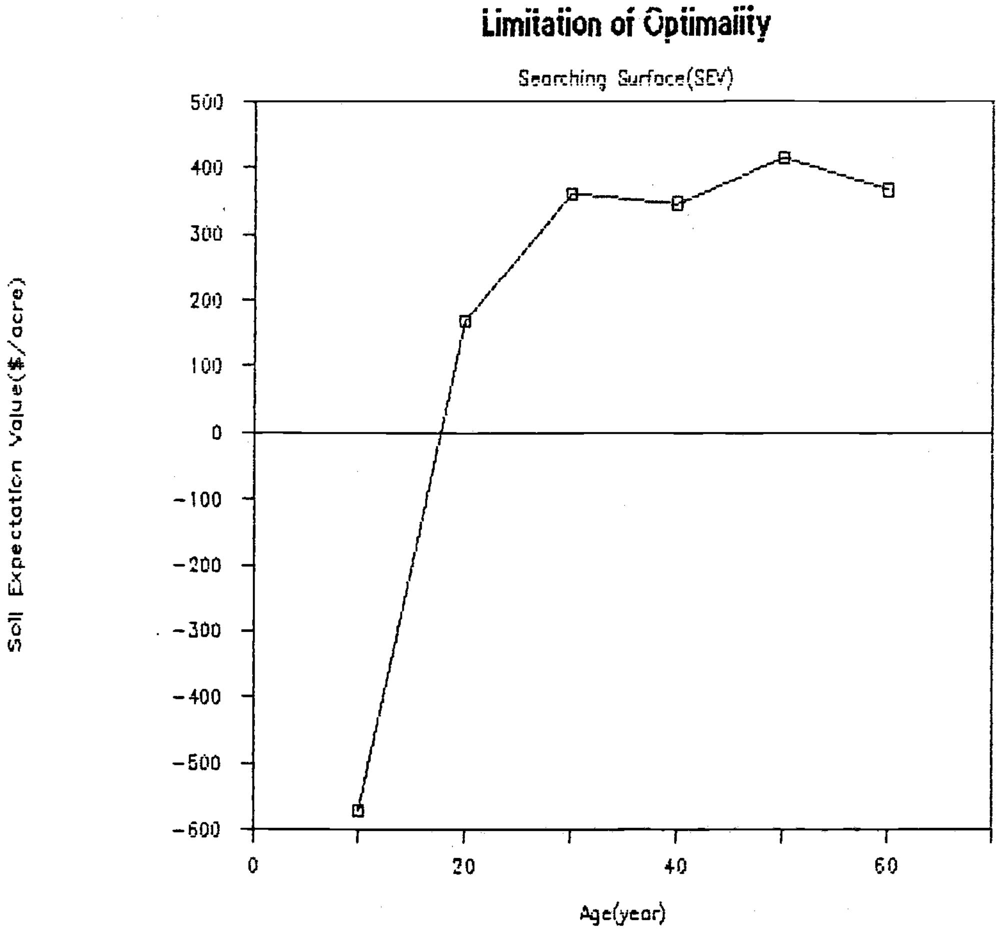



Figure 13.
Example of limitation of optimality.

Figure 13 shows the searching surface of the optimal rotation age based on soil expectation value. The reason that the searching surface is not unimodal is most likely due to the fact that the indicated thinning regime does not have thinning at age 30 and 40. Once arranging thinning requirement, i.e., thinning is implemented at age 20 and 40, the searching surface became unimodal as shown in Figure 14, allowing the optimal thinning regime to be changed. In addition the optimal rotation age is 40 years with \$423.5/acre soil expectation value more than the previous one having 50 years with \$415.3/acre. However, there is often no trade-off, even with no thinning at some stage.

The second situation can occur in the case of Figure 15, given that the path Y1-T1-Y2-T2-Y3 is the optimal path provided by the PATH algorithm. In this case, it is possible for T'1 to produce more objective value at stage 3. In other words, if sufficient evaluation of the future stand is done at stage 1, T'1 can be selected as the optimal thinning level. Then, the trade-off has to be implemented at stage 3 if T'1 produces more objective value than the path provided by the PATH algorithm.

In a complicated stand growth simulator, it is possible for the stand with few trees to create a great potential growth over the long-term. This discrepancy violates the assumption that one-stage look-ahead period

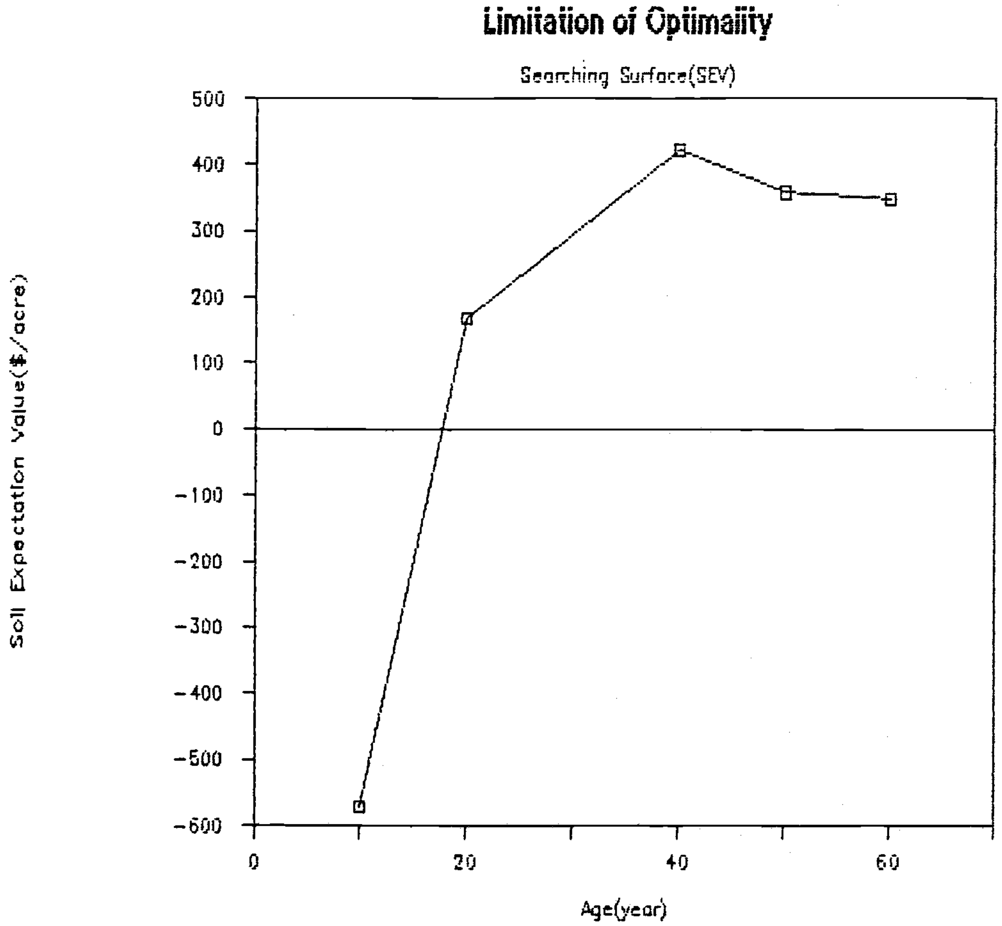



Figure 14.
Example of the first type of trade-off.

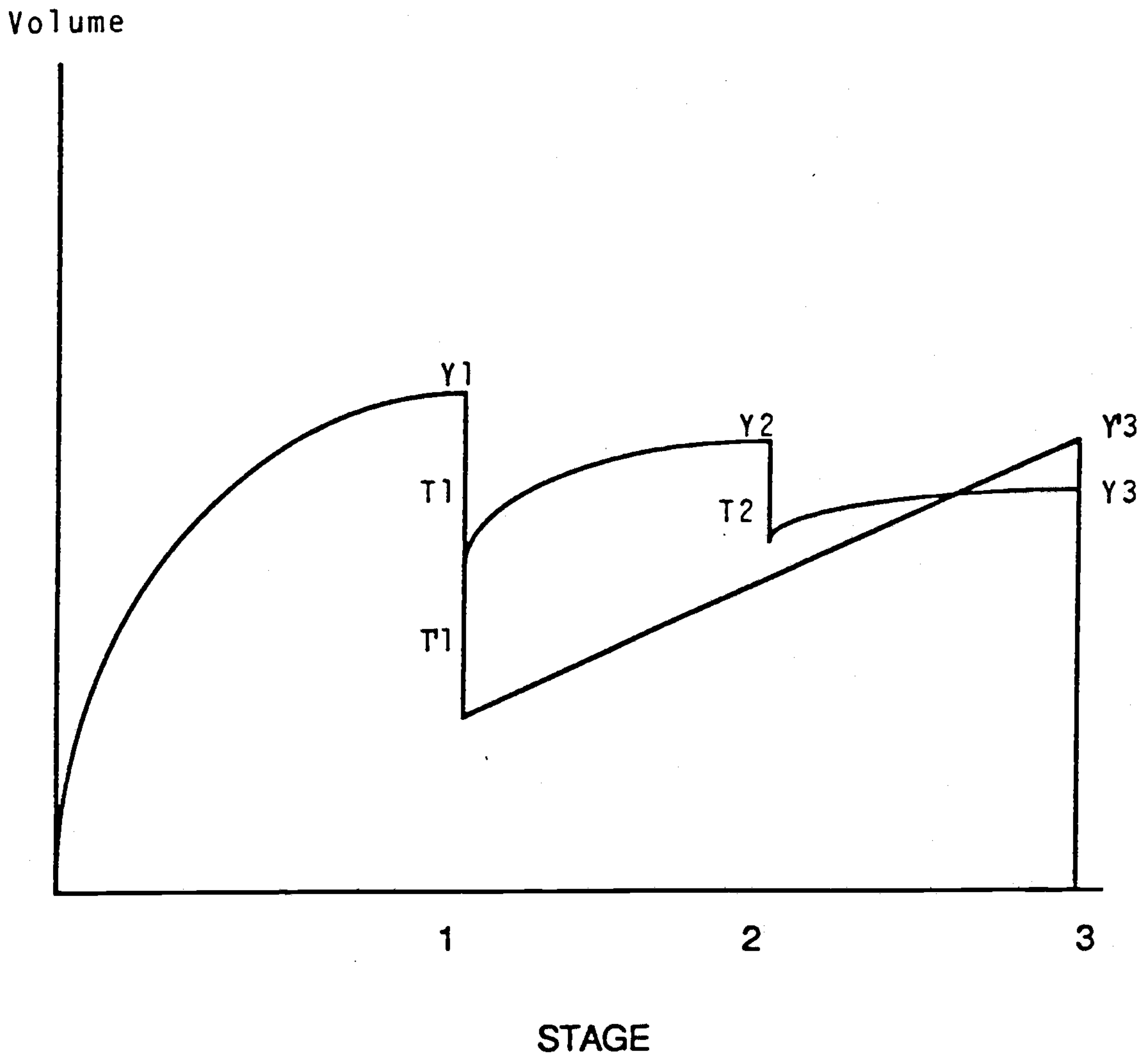



Figure 15.
The second type of trade-off.

is sufficient to evaluate impact of the future stand. Then sufficient evaluation of the future stand should be implemented especially for a complicated growth simulator. If only the path derived from the PATH algorithm is utilized, this situation is never shown.

Let's consider the illustrative example given at the previous section. In this case, the decision variable is the number of trees, then only the number of trees can control every stand structure. The path based on the one-stage look-ahead period, which is imbedded into the PATH algorithm, is:

702 trees/acre at age 20 with \$ 3267.7 SEV

Thinning 200 trees/acre at age 20

502 trees/acre at age 30 with \$ 3635.2 SEV

Thinning 100 trees/acre at age 30

402 trees/acre at age 40 with \$ 3444.4 SEV

Thinning 100 trees/acre at age 40

302 trees/acre at age 50 with \$ 3077.9 SEV

In this case, there is no stage, where there is no thinning. Then the first type of trade-off is not implemented. However, at each stage except age 30 or earlier there is another best solution. On the basis of age 40, the best path is:

702 trees/acre at age 20 with \$ 3267.7 based on SEV

Thinning 300 trees/acre at age 20

402 trees/acre at age 40 with \$ 3532.5 based on SEV

The objective value at age 40 is \$ 3532.5 which is greater than \$3444.4. In addition, on the basis of age 50, the best path is:

702 trees/acre at age 20 with \$ 3267.7 based on SEV

Thinning 400 trees/acre at age 20

302 trees/acre at age 50 with \$ 3155.9 based on SEV

The objective value at age 50, \$3155.9 is greater than \$3077.9 given by the one-stage look-ahead PATH algorithm.

In comparison with these results, an interesting observation is that in each stage, i.e., age 40 and age 50 the residual number of trees is the same as the one given by PATH even if the objective value is different. This difference of objective value is due to a difference in final size of trees, which is caused by a more intensive thinning at an early stage, e.g., 200 trees/acre at age 20, or 300 trees/acre at age 20. Since the number of residual trees is the same, the decision variable, the number of thinning trees cannot control stand any more in order to optimize the objective function.

These two situations suggested the following new PATH algorithm called <u>M</u>ulti-<u>S</u>tage PATH (MSPATH) algorithm.

b) <u>M</u>ulti-<u>S</u>tage PATH (MSPATH) algorithm

The above two situations occur when the look-ahead period is insufficient to evaluate impact on the future stand. The MSPATH algorithm uses each possible look-ahead period at each stage in order to search for the optimal objective value at each future stage based on the different combinations of look-ahead period. In other words, the MSPATH algorithm searches for the optimal combination of look-ahead period from the initial stage to the final stage at the same time when the optimal thinning level is decided. Then MSPATH can decide where one-stage look-ahead is used, two-stage look-ahead is used and so on, and how much the optimal thinning level is for the optimal combination of look-ahead period. Figure 16 shows all possible paths at each stage.

The optimal thinning regime at each stage can be obtained based on multi-stage look-ahead period. Figure 17 shows the optimal paths at each stage based on the MSPATH algorithm.

As expected, however, the difference of the objective value between the PATH algorithm and the MSPATH algorithm is quite small, around 2 %, so if the user does not care about this small difference rather than the computational time, the PATH algorithm is recommended, otherwise the MSPATH is. Figure 18 shows the optimal thinning regimes at each stage and Figure 19 shows the searching surface based on soil expectation value.

From the viewpoint of computational burden, MSPATH creates more computation than PATH. However, if the traditional dynamic programming algorithm is used in order to solve the same optimization problem as MSPATH by using

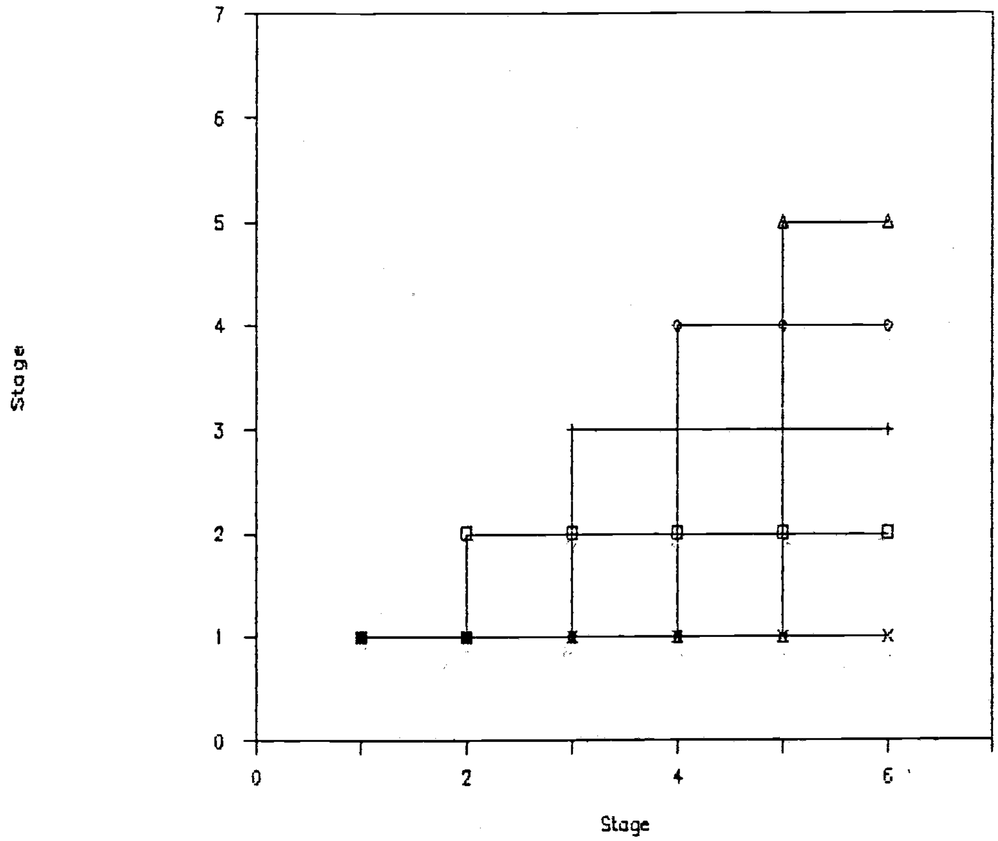



Figure 16.
The possible paths based on the MSPATH algorithm.

× ----- derived from stage 1
□ ----- derived from stage 2
\+ ----- derived from stage 3
◇ ----- derived from stage 4
△ ----- derived from stage 5

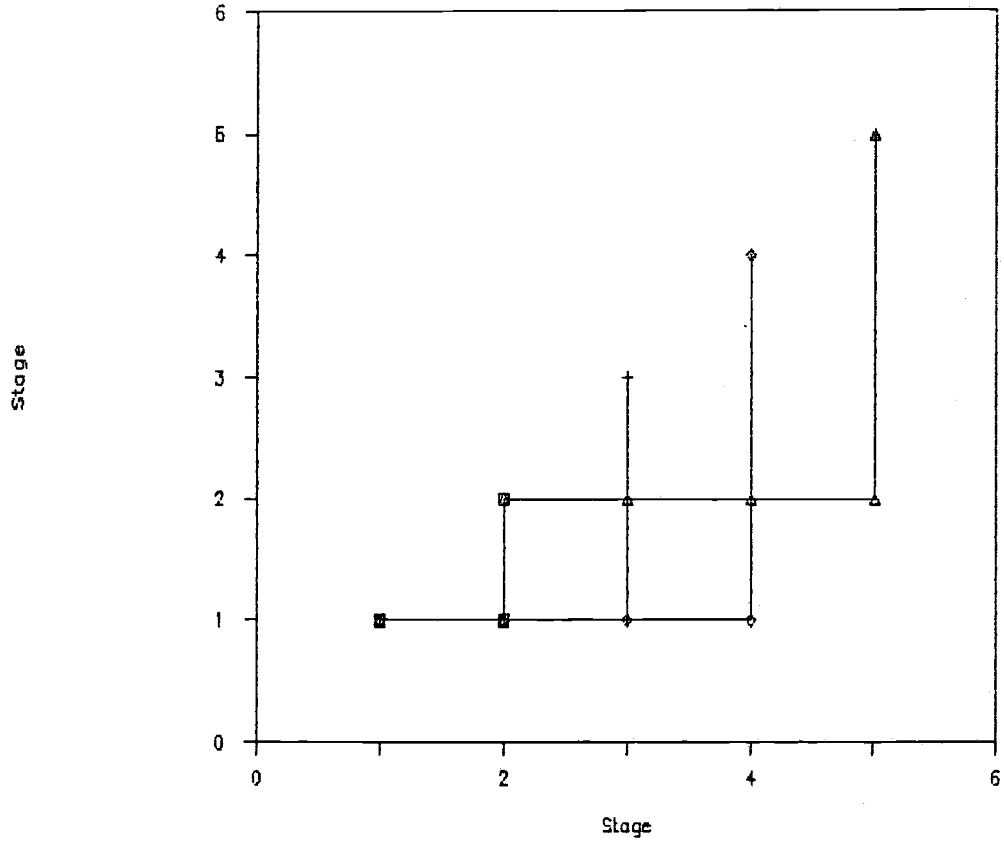



Figure 17.
The optimal path at each stage by MSPATH.

□ ----- optimal path at stage 2
\+ ----- optimal path at stage 3
◇ ----- optimal path at stage 4
△ ----- optimal path at stage 5

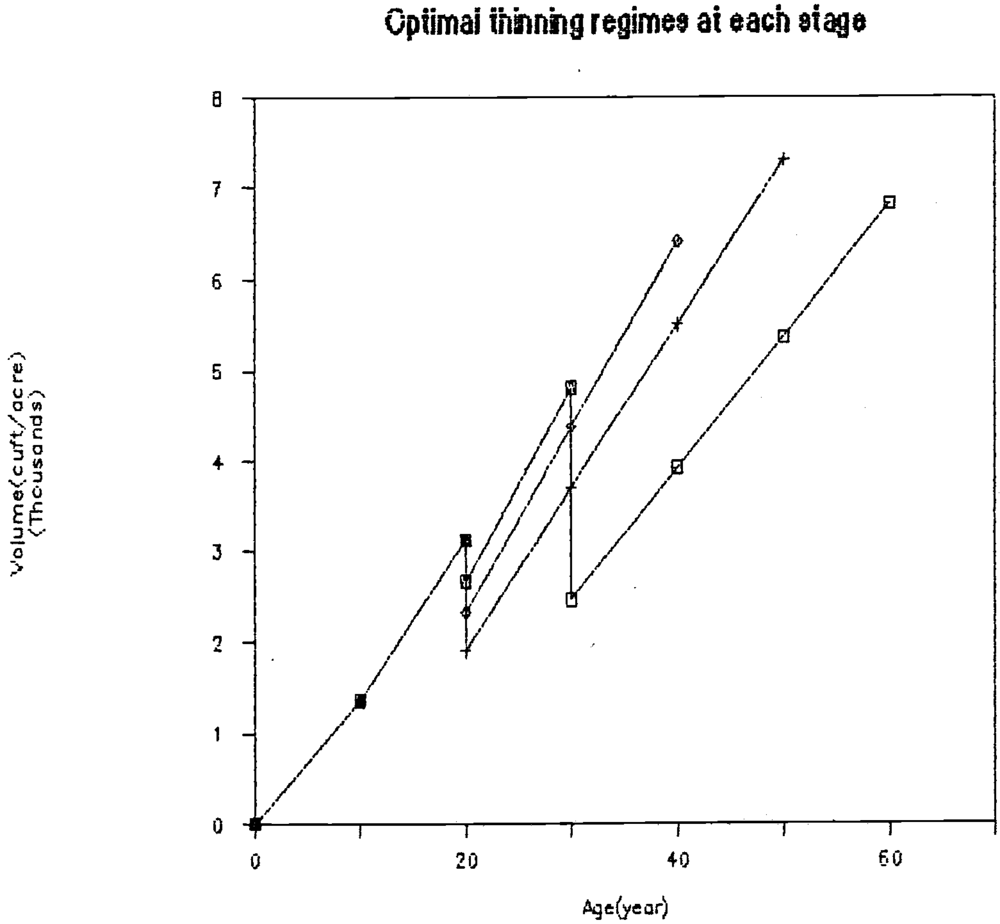



Figure 18.
The optimal thinning regime at each stage by MSPATH.

□ ----- at stage 2 age 30
◇ ----- at stage 3 age 40
+ ----- at stage 4 age 50
□ ----- at stage 5 age 60

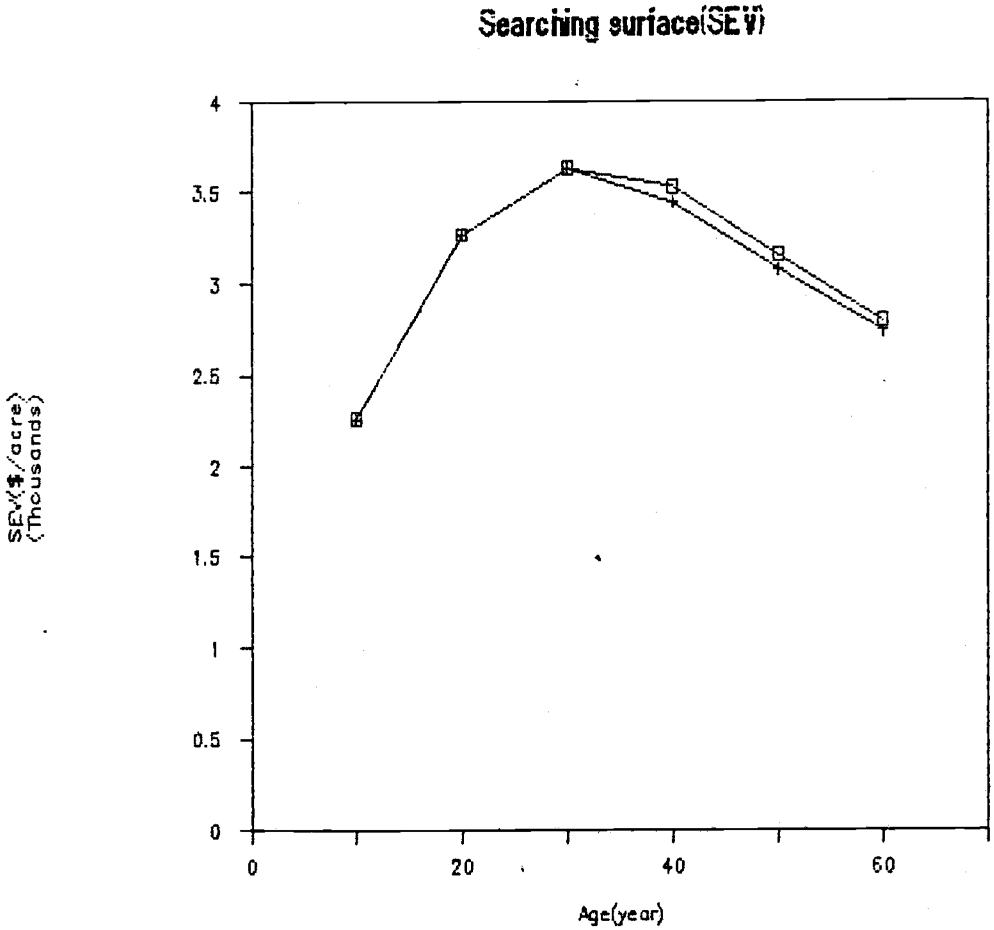



Figure 19.
Searching surfaces by PATH and MSPATH.

+ ----- PATH
□ ----- MSPATH

SPS, the problem of insufficient look-ahead period appears. That is, at the traditional dynamic programming algorithm, one-stage look-ahead period is utilized unless the principle of optimality is violated by thinning from above (Brodie and Haight, 1985). Therefore, if more accurate solution is needed for the traditional dynamic programming algorithm, the same technique as MSPATH utilizes to extend look-ahead period should be implemented. As a result, even if MSPATH creates more computational burden than PATH, it is still efficient in comparison with the traditional dynamic programming algorithm, and provides the optimal solution so long as necessary conditions for a calculus of variations problem hold.

3) Relationship between the Lagrange multiplier and optimal thinning regime

As mentioned in Section II, the PATH algorithm can be expressed in terms of the Lagrange multiplier as:

$$f_n(T_n) = \max_{[T_n]}[A_n(T_n)+\lambda_n[X_n-T_n+G_{n+1}(Y_n)]]+f_{n-1}(Y_{n-1}) \qquad (2.6)$$

at the n-th stage. The PATH algorithm is also interpreted from different point of view as:

$$J_n = \max_{[T_n]} [A_n(T_n) + r_{n+1}(Y_{n+1})] \qquad (3.4)$$

Since this objective function does not have the Lagrange multiplier, it is possible to estimate the Lagrange multiplier. Suppose that necessary conditions for a calculus of variations problem hold and sufficient look-ahead period is used. Then if the optimal solution is obtained by the above two functions respectively, the optimal thinning levels obtained by these two methods should coincide as long as the same interval of node is used. Then at the optimal point the following equation is satisfied:

$$A^*_n(T_n) + \lambda^*_n[X_n - T_n + G_{n+1}(Y_n)] = A^*_n(T_n) + r^*_{n+1}(Y_{n+1}) \qquad (4.17)$$

Solving for the Lagrange multiplier $\lambda^*_n$, we can obtain:

$$\lambda^*_n = \frac{r^*n+1(Yn+1)}{X_n - T_n + G_{n+1}(Y_n)}$$

$$= \frac{r^*n+1(Yn+1)}{Y_{n+1}} \qquad (4.18)$$

Therefore at the optimal point of the n-th stage, the Lagrange multiplier can be interpreted as the average return per unit volume at the (n+1)-th stage. If the basis of optimization is cubic-foot volume, the Lagrange multiplier becomes equal to 1 as was expected by Paredes and Brodie (1987).

Economically speaking, both the Lagrange multiplier and the decision variable correspond with each other. In other words, if the Lagrange multiplier is given, then the decision variable is determined at the optimal point. Then searching for the optimal allocation is limited to the range at which this relationship holds.

Figure 20 shows the relationship between the Lagrange multiplier derived from equation (4.18) and the decision variable, thinning level. The higher thinning level, the larger the Lagrange multiplier. Also the greater the stage, the less the Lagrange multiplier.

The Lagrange multiplier derived is interpreted economically not only as the shadow price or the opportunity cost but also as the marginal value per unit volume of resource at each stage by which the maximum attainable value of resource could be increased if an additional unit of resource were to become available (Dorfman, 1961, and Paredes and Brodie, 1987). According to equation (4.18) the Lagrange multiplier having the above interpretation at each stage should be estimated based on the future stand, not the current.

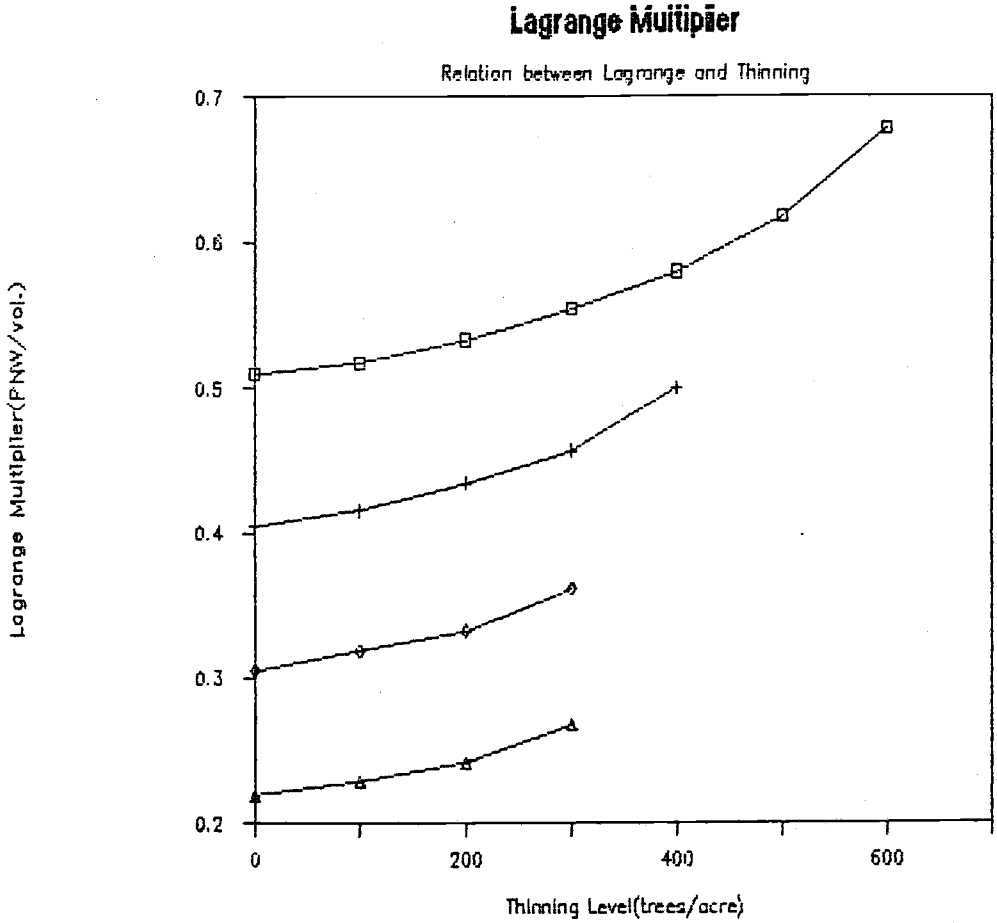



Figure 20.
The relationship between the Lagrange multiplier and thinning level.

□ ----- at age 20
\+ ----- at age 30
◇ ----- at age 40 based on rotation age 50
△ ----- at age 40 based on rotation age 60

# CONCLUSIONS

The objective of this study was to develop a new dynamic programming model using the SPS growth simulator (Arney, 1985), employing the PATH algorithm (Paredes and Brodie, 1987). This model called SOS can optimize both thinning regime and rotation age based on either mean annual increment of the given physical basis, present net worth, or soil expectation value, as long as necessary conditions for a calculus of variations problem hold and one-stage look-ahead period is sufficient to evaluate the future stand. Once either one of these conditions is violated, or one-stage look-ahead period is insufficient, the solution obtained by PATH becomes the better solution, and not the best solution. Then SOS based on the MSPATH algorithm is also proposed in order to solve the optimization problem when one-stage look-ahead period becomes insufficient. However, once one of necessary conditions is violated, the solution provided by MSPATH becomes better, and not best.

The modification of the PATH algorithm by the calculus of variations in order not to use the Lagrange multiplier allows one to recognize the efficient PATH algorithm easily. Once the optimal resource allocation is obtained, the optimal value of the Lagrange multiplier are calculated automatically as well in terms of the relationship between the objective function with the Lagrange multiplier and without.

Directly treated as a decision variable, the unit thinning level given by the user determines the number of iterations at each stage, or the residual level before thinning divided by the unit thinning level. This technique eliminated so many calculations that the joint optimization of thinning methods and the optimization of rotation age are completed with less computation in one run of SOS than the traditional dynamic programming algorithm. Thus the calculation task and memory required to store optimal stands is vastly diminished to utilize very complex forest stand level production models.

Brodie and Haight (1985) indicate that when thinning from above is incorporated in an optimization model, where growth is driven by top-height a violation of the principle of optimality can occur unless state space is expanded to include top-height. Although SOS growth is driven by top-height, the implied expansion of state space becomes unnecessary through the process of evaluating the future stand (after thinning from above) as part of the objective function. The same problem of suboptimization occurs, however, if the future stand productivity is not projected sufficiently forward. In such case, the MSPATH algorithm can resolve it.

The difference between the first type of trade-off problem and the second type of trade-off problem is such that at the first case the trade-off is implemented if the stand with more number of trees has a greater objective value at the future stage, while at the second case the trade-off is implemented if the stand with less number of trees has a greater objective value at the future stage. Also the first type of trade-off problem can be checked by the output, but the second type cannot. Therefore the new MSPATH algorithm was introduced to resolve both cases, where one-stage look-ahead period is insufficient to evaluate the return from the future stand.

According to economic analysis, the effect of interest rate on forest management is that the larger the interest rate, the earlier the rotation age. Then it is necessary to keep looking at how interest rate changes with time.

Depending upon the quality premium, the optimal type of thinning is determined. Since the optimal type of thinning for all examples used here is thinning from below, the results of the joint optimization of thinning methods were not presented. In these examples, thinning from below was the best thinning type, then thinning to a c/r ratio of 1 and thinning from above. However, thinning from above could produce the most return generated by thinning, meaning that if the forest owner needs some amount of money, thinning from above could be recommended.

# APPENDICES

# APPENDIX I

## GROWTH MODEL AND YIELD PROJECTION

Under standard classification of growth model simulators, the growth model identified by the acronym SPS (Stand Projection System) (Arney, 1985) is classified as a single-tree/distance-independent simulator. Although SPS was originally developed for coastal Douglas-fir stand, it can also simulate mixed species stands containing the following species:

| Pacific Northwest Species | Inland Northwest Species |
|---|---|
| Douglas-fir(DF) | Douglas-fir(DF) |
| Western hemlock(WH) | Grand fir(GF) |
| Western red cedar(RC) | Western larch(WL) |
| Noble fir(NF) | Ponderosa pine(PP) |
| Red alder(RA) | Lodgepole pine(LP) |

As a single-tree/distance-independent model, SPS can simulate stand in terms of different types of thinning, i.e., thinning from below, thinning from above, and thinning to a cut-residual (c/r) ratio. In the case of thinning from below, smaller trees are removed, while bigger trees are removed using thinning from above. Thinning to a c/r ratio is such that if the ratio of quadratic mean of diameter of cut trees over quadratic mean of residual trees is greater than a c/r ratio given by the user, thinning will be done from above, otherwise from below in order to make the calculated c/r ratio equal to a given value. Setting a given c/r equal to 1, is similar to mechanical thinning. In SPS, the user can also select thinning basis, i.e., trees per acre or crown competition factor (CCF) level (Krajicek and others, 1961). On the trees per acre basis, the condition to stop thinning is given by the number of residual trees after thinning. On the other hand, CCF level is based on crown competition factor after thinning, which is identified by trees per acre and residual tree diameters. Although the user can decide thinning timing using either stand age or CCF level in SPS, the SOS model proposed here is limited to stand age. Nitrogen application as a fertilizing control is also incorporated into SPS.

To run SPS it is necessary to generate an input file. This input file can be created by either word processor or the other program called EDIT (Arney, 1985). SPS can create a stand using representative data, such as the number of trees per acre, the average diameter, the average height, the standard deviation of diameter, and the breast height age, if any yield table is not given by the user. In this case, the diameter class data are created by imposing a Weibull-distribution classifying trees into each diameter class:

$$DIST = TPA[1-\exp(\frac{XW-WA}{WB})^{WC}] - CUM \qquad (A.1)$$

where WA, WB, and WC are coefficients determined by the average diameter and the standard deviation, XW is the interval of the diameter class, and CUM is the cumulative number of trees. Using the following height growth equation, the height of each diameter class can be obtained.

$$H = 4.5 + 1.4(TH - 4.5)\ \exp(\frac{-.06xTH}{D}) \qquad (A.2)$$

where TH is the given average height and D is a diameter at the corresponding diameter class.

Using the yield table obtained by the above procedure or given by the user, each component of the growth model, i.e., height growth, diameter growth, and live crown growth, is generated based on top-height growth, which is created in terms of site index (King,1966). The top-height growth, being thought of as an indicator of the potential growth in a given period of time, is estimated as a function of site index and age:

$$TOPH = B1(Site-4.5)(1-\exp(B2xAGE))^{B3} + 4.5 \qquad (A.3)$$

where TOPH is top-height, Site is site index and B1, B2 and B3 are coefficients.

Growth is calculated in steps as the number of years required for the stand to increase 12 feet in top-height or in very slow growing stands at a limit of 30 years.

Based on this growth step, an increment of height is decided by:

$$\frac{d\,HT}{dTOP} = \left(\frac{HT}{TOP}\right)^{B1}\left(1-\left(\frac{CCF}{B2}\right)^{B3}\right) \qquad (A.4)$$

where B1, B2 and B3 are coefficients and HT is height and TOP is top-height. CCF is calculated by:

$$CCF = \sum F_i x CAREA_i \qquad (A.5)$$

where $F_i$ is the frequency at the i-th diameter class and,

$$CAREAi = \frac{\pi}{4} x \frac{100}{43560}[3.91+81(1-\exp(-.0025DBH_i))] \qquad (A.6)$$

This $CAREA_i$ represents crown area of i-th tree calculated by crown width, which is estimated by the relationship between DBH and open-grown crown width of tree. An increment of diameter is also estimated by:

$$\frac{dDBH}{dTOP} = B1\left(\frac{CCF}{100}\right)^{B2}\left(1-\exp\left(B3\left(\frac{DBH}{TOP}\right)^{B4}\right)\right) \qquad (A.7)$$

Live crown ratio which is useful for estimation of vigor and mortality is estimated by:

$$CR = \left(\frac{CCF}{100}\right)^{-A} \qquad (A.8)$$

where A is a coefficient.

Mortality is also incorporated into SPS. The number of trees surviving is decided by:

$$TPA = TPA100[.76+.24(1.33-.0033CCF)^{.25}] \qquad (A.9)$$

where TPA100 is an initial number of trees per acre. After obtaining the total number of trees dying, the number of trees in each diameter class is estimated by the following way:

First calculate the average and standard deviation of the diameter (QDBH,SDEV) and the crown ratio (SCR,SDCR). Set the ratio as:

$$RATIO = \left(\frac{CR-SCR}{SDCR} + \frac{D - QDBH}{SDEV}\right) \times 100\ (\%) \qquad (A.10)$$

Then select the diameter class having the smallest ratio, take out RATIO percent of the trees from the corresponding diameter class, repeat at the next smallest diameter class until the dying trees are completely spread over the diameter classes.

If thinning is implemented, trees are removed according to thinning type given by the user, before creating any increment of growth. Since thinning affects the number of trees, then crown competition factor is changed, resulting in a growth increment change.

Fertilization with nitrogen has an effect on growth of both diameter and height. Once N pounds of nitrogen is invested, the increment of diameter is estimated as:

$$dDBH' = dDBH[1+(.248-.00495S)(1-\exp(-.0089xN))] \qquad (A.11)$$

and the increment of height is:

$$dHT' = dHT[1+(.248-.00495S)(1-\exp(-.0089xN))] \qquad (A.12)$$

where S is site index, dDBH and dHT are observed increment without any fertilizer.

# APPENDIX II

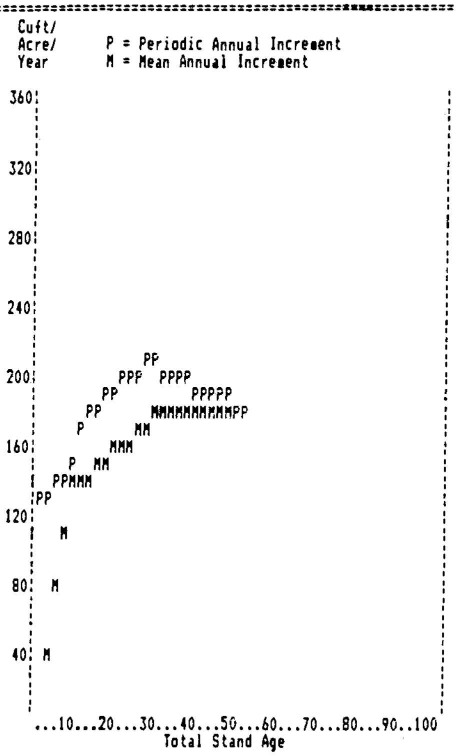
## INPUT DATA AND SUMMARY REPORTS

### a) Illustrative example

```
==================== S O S ====================
STAND       DF  82.  10.  PNW
MERCH      1.0 16.4  5.0  9.0
NAME     TEST RUN ON COMPAQ 386
THINNING     1  20.    1  10.    1 100.
THINNING     1  30.    1  10.    1 100.
THINNING     1  40.    1  10.    1 100.
REPORT     10.  20.  30.  40.  60.  70.  80.  90. 100.   0.   0.
Current stand total age =  10.
Average of  .85 acre stocked

  I SP   DBH      Ht      Tpa  BH-Age     %CR
  1 WH   2.0    18.0     15.0      19      80
  2 WH   3.0    27.0     92.0      19      80
  3 WH   4.0    32.0    227.0      19      80
  4 WH   5.0    35.0    121.0      19      80
  5 WH   6.0    36.0     41.0      19      80
  6 DF   3.0    30.0     32.0      18      80
  7 DF   4.0    34.0     57.0      18      80
  8 DF   5.0    38.0    138.0      18      80
  9 DF   6.0    38.0     72.0      18      80
Initial CCF = 248.
--- Input Complete ---

OPTIMIZATION  IS  BASED   ON : 6: SOIL EXPECTATION VALUE


THE SELECTED OPTIMIZATION IS :  1: THINNING OPTIMIZATION
INTERVAL OF NODE (THINNING LEVEL)  :100.0 TREES PER ACRE


FINANCIAL DATA

INTEREST RATE (%)  :     4.0
ENTRY COST PER ACRE ($) :     50.00
COEFFICIENT FOR REDUCING THINNING VALUE ($) : .800
 1:   REGENERATION COST           :    200.00($) at age    .00

PRICE IS PER THOUSAND CUBIC FOOT
PRICE EQUATION  : PRICE =     200.00 * DBH +      80.00
```

PACIFIC NW REGION (V85.3) <<< STAND OPTIMIZATION SYSTEM (SOS) >>> Site Index 82 DF
Stand Summary Report Base - 50 yrs BH Age
TEST RUN ON COMPAQ 386

| Age SP | DBH | Trees /Ac | Basal Area | Top Ht | Crown Ratio | Total Cuft | Merch Cuft | Merch Bdft | #Logs /Mbf | Log Dib | PNW /Ac |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | ## values on a per acre basis ## | | | | | | | | |
| 10WH | 4.2 | 496 | 49.2 | 36.0 | 80 | 740 | 00 | 00 | .0 | .0 | 502 |
| 10DF | 4.8 | 299 | 39.5 | 38.0 | 80 | 620 | 00 | 00 | .0 | .0 | 466 |
| 10Sum | 4.4 | 795 | 88.7 | 36.8 | 80 | 1360 | 00 | 00 | .0 | .0 | 968 |
| 20WH | 5.7 | 449 | 83.6 | 52.9 | 72 | 1810 | 00 | 00 | .0 | .0 | 1099 |
| 20DF | 6.6 | 253 | 61.8 | 54.2 | 57 | 1330 | 00 | 00 | .0 | .0 | 900 |
| 20Sum | 6.0 | 702 | 145.4 | 53.4 | 67 | 3140 | 00 | 00 | ,0 | .0 | 1999 |
| CutWH | 4.6 | 134 | 15.6 | 47.9 | 78 | 300 | 00 | 00 | .0 | .0 | 114 |
| CutDF | 4.9 | 66 | 8.8 | 49.4 | 66 | 170 | 00 | 00 | .0 | .0 | 68 |
| 20Sum | 4.7 | 200 | 24.4 | 48.4 | 74 | 470 | 00 | 00 | .0 | .0 | 182 |
| 30WH | 7.5 | 315 | 99.5 | 67.9 | 69 | 2810 | 560 | 2290 | 46.4 | 7.7 | 1441 |
| 30DF | 8.6 | 187 | 75.3 | 67.8 | 54 | 2020 | 820 | 3730 | 49.3 | 7.1 | 1130 |
| 30Sum | 7.9 | 502 | 174.8 | 67.8 | 64 | 4820 | 1380 | 6010 | 48.2 | 7.3 | 2571 |
| CutWH | 6.5 | 100 | 23.0 | 62.1 | 74 | 630 | 00 | 00 | .0 | .0 | 213 |
| 30Sum | 6.5 | 100 | 23.0 | 62.1 | 74 | 630 | 00 | 00 | .0 | .0 | 213 |
| 40WH | 9.0 | 215 | 97.8 | 80.8 | 67 | 3290 | 2440 | 9840 | 47.5 | 7.6 | 1348 |
| 40DF | 9.6 | 186 | 94.6 | 79.1 | 51 | 2910 | 1230 | 5220 | 44.7 | 7.7 | 1233 |
| 40Sum | 9.3 | 401 | 192.4 | 80.0 | 60 | 6210 | 3680 | 15070 | 46.5 | 7.7 | 2581 |
| CutWH | 7.3 | 59 | 17.1 | 74.4 | 71 | 550 | 00 | 00 | .0 | .0 | 141 |
| CutDF | 9.0 | 42 | 18.6 | 79.1 | 59 | 570 | 490 | 2500 | 47.7 | 6.8 | 180 |
| 40Sum | 8.0 | 101 | 35.6 | 76.4 | 66 | 1130 | 490 | 2500 | 47.7 | 6.8 | 321 |
| 50WH | 10.7 | 156 | 98.7 | 91.8 | 68 | 3790 | 3490 | 14910 | 38.1 | 8.3 | 1204 |
| 50DF | 10.7 | 144 | 90.9 | 88.5 | 52 | 3090 | 2800 | 11510 | 45.2 | 7.7 | 980 |
| 50Sum | 10.7 | 300 | 189.6 | 90.2 | 60 | 6870 | 6270 | 26420 | 41.2 | 8.0 | 2184 |




Stump Ht. 1.0 Log Length 16.4 Top Dib 5.0 Min DBH 9.0

<<SUMMARY OF SOIL EXPECTATION VALUE AT EACH ROTATION AGE>>
Optimal rotation age based on SOIL EXPECTATION VALUE basis is 30.0year

| ROTATION AGE (year) | SOIL EXPECTATION VALUE ($/acre) |
|---|---|
| 10.00 | 2262.54 |
| 20.00 | 3267.67 |
| 30.00 | 3635.15 |
| 40.00 | 3444.39 |
| 50.00 | 3077.95 |
| 60.00 | 2740.63 |

b) Effect of interest rate (1%)

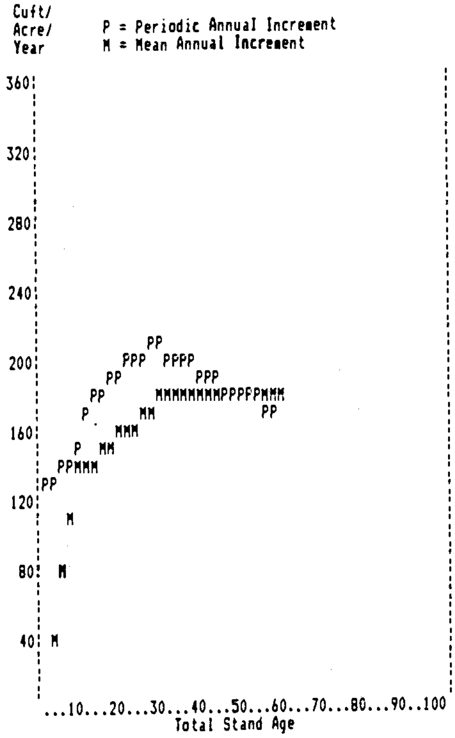
```
==================== S O S ====================
STAND         DF  82.   10.  PNW
MERCH        1.0 16.4  5.0  9.0
NAME       TEST RUN ON COMPAQ 386
THINNING      1  20.     1  10.     1 100.
THINNING      1  30.     1  10.     1 100.
THINNING      1  40.     1  10.     1 100.
REPORT     10.  20.  30.  40.  60.  70.  80.  90. 100.   0.   0.
Current stand total age =  10.
Average of  .85 acre stocked

  I SP   DBH      Ht      Tpa  BH-Age     %CR
  1 WH   2.0    18.0    15.0      19      80
  2 WH   3.0    27.0    92.0      19      80
  3 WH   4.0    32.0   227.0      19      80
  4 WH   5.0    35.0   121.0      19      80
  5 WH   6.0    36.0    41.0      19      80
  6 DF   3.0    30.0    32.0      18      80
  7 DF   4.0    34.0    57.0      18      80
  8 DF   5.0    38.0   138.0      18      80
  9 DF   6.0    38.0    72.0      18      80
Initial CCF = 248.
--- Input Complete ---

OPTIMIZATION  IS  BASED   ON : 6: SOIL EXPECTATION VALUE


THE SELECTED OPTIMIZATION IS :  1: THINNING OPTIMIZATION
INTERVAL OF NODE (THINNING LEVEL)  :100.0 TREES PER ACRE


FINANCIAL DATA

INTEREST RATE (%)  :     1.0
ENTRY COST PER ACRE ($) :      50.00
COEFFICIENT FOR REDUCING THINNING VALUE ($) : .800
 1:   REGENERATION COST             :     200.00($) at age    .00

PRICE IS PER THOUSAND CUBIC FOOT
PRICE EQUATION  : PRICE =     200.00 * DBH +      80.00
```

PACIFIC NW REGION (V85.3)
Stand Summary Report
TEST RUN ON COMPAQ 386

<<< STAND OPTIMIZATION SYSTEM (SOS) >>>

Site Index 82 DF
Base - 50 yrs BH Age

| Age SP | DBH | Trees /Ac | Basal Area | Top Ht | Crown Ratio | Total Cuft | Merch Cuft | Merch Bdft | #Logs /Mbf | Log Dib | PNW /Ac |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ** values on a per acre basis ** | | | | | | | | | | | |
| 10WH | 4.2 | 496 | 49.2 | 36.0 | 80 | 740 | 00 | 00 | .0 | .0 | 673 |
| 10DF | 4.8 | 299 | 39.5 | 38.0 | 80 | 620 | 00 | 00 | .0 | .0 | 624 |
| 10Sum | 4.4 | 795 | 88.7 | 36.8 | 80 | 1360 | 00 | 00 | .0 | .0 | 1297 |
| 20WH | 5.7 | 449 | 83.6 | 52.9 | 72 | 1810 | 00 | 00 | .0 | .0 | 1973 |
| 20DF | 6.6 | 253 | 61.8 | 54.2 | 57 | 1330 | 00 | 00 | .0 | .0 | 1617 |
| 20Sum | 6.0 | 702 | 145.4 | 53.4 | 67 | 3140 | 00 | 00 | .0 | .0 | 3590 |
| CutWH | 4.6 | 134 | 15.6 | 47.9 | 78 | 300 | 00 | 00 | .0 | .0 | 205 |
| CutDF | 4.9 | 66 | 8.8 | 49.4 | 66 | 170 | 00 | 00 | .0 | .0 | 121 |
| 20Sum | 4.7 | 200 | 24.4 | 48.4 | 74 | 470 | 00 | 00 | .0 | .0 | 326 |
| 30WH | 7.5 | 315 | 99.5 | 67.9 | 69 | 2810 | 560 | 2290 | 46.4 | 7.7 | 3468 |
| 30DF | 8.6 | 187 | 75.3 | 67.8 | 54 | 2020 | 820 | 3730 | 49.3 | 7.1 | 2719 |
| 30Sum | 7.9 | 502 | 174.8 | 67.8 | 64 | 4820 | 1380 | 6010 | 48.2 | 7.3 | 6187 |
| CutWH | 6.5 | 100 | 23.0 | 62.1 | 74 | 630 | 00 | 00 | .0 | .0 | 513 |
| 30Sum | 6.5 | 100 | 23.0 | 62.1 | 74 | 630 | 00 | 00 | .0 | .0 | 513 |
| 40WH | 9.0 | 215 | 97.8 | 80.8 | 67 | 3290 | 2440 | 9840 | 47.5 | 7.6 | 4348 |
| 40DF | 9.6 | 186 | 94.6 | 79.1 | 51 | 2910 | 1230 | 5220 | 44.7 | 7.7 | 3974 |
| 40Sum | 9.3 | 401 | 192.4 | 80.0 | 60 | 6210 | 3680 | 15070 | 46.5 | 7.7 | 8323 |
| CutWH | 7.3 | 59 | 17.1 | 74.4 | 71 | 550 | 00 | 00 | .0 | .0 | 456 |
| CutDF | 9.0 | 42 | 18.6 | 79.1 | 59 | 570 | 490 | 2500 | 47.7 | 6.8 | 581 |
| 40Sum | 8.0 | 101 | 35.6 | 76.4 | 66 | 1130 | 490 | 2500 | 47.7 | 6.8 | 1037 |
| 60WH | 11.5 | 156 | 113.5 | 101.3 | 71 | 4780 | 4450 | 20220 | 32.2 | 8.7 | 6361 |
| 60DF | 11.4 | 144 | 102.9 | 96.3 | 56 | 3760 | 3460 | 14500 | 40.6 | 8.1 | 4959 |
| 60Sum | 11.5 | 300 | 216.3 | 98.9 | 64 | 8540 | 7920 | 34720 | 35.7 | 8.5 | 11320 |




Stump Ht. 1.0   Log Length 16.4   Top Dib 5.0   Min DBH 9.0

<<SUMMARY OF SOIL EXPECTATION VALUE AT EACH ROTATION AGE>>
Optimal rotation age based on SOIL EXPECTATION VALUE basis is 60.0year

| ROTATION AGE (year) | SOIL EXPECTATION VALUE ($/acre) |
|---|---|
| 10.00 | 11103.75 |
| 20.00 | 18558.45 |
| 30.00 | 24158.45 |
| 40.00 | 26952.38 |
| 50.00 | 27993.73 |
| 60.00 | 28597.57 |
| 70.00 | 27833.82 |

c) Effect of interest rate (7%)

```
==================== S O S ====================
STAND        DF  82.  10.  PNW
MERCH       1.0 16.4  5.0  9.0
NAME      TEST RUN ON COMPAQ 386
THINNING      1  20.     1  10.    1 100.
THINNING      1  30.     1  10.    1 100.
THINNING      1  40.     1  10.    1 100.
REPORT     10.  20.  30.  40.  60.  70.   80.   90. 100.    0.    0.
Current stand total age =  10.
Average of  .85 acre stocked

  I SP   DBH      Ht      Tpa  BH-Age     %CR
  1 WH   2.0     18.0     15.0     19      80
  2 WH   3.0     27.0     92.0     19      80
  3 WH   4.0     32.0    227.0     19      80
  4 WH   5.0     35.0    121.0     19      80
  5 WH   6.0     36.0     41.0     19      80
  6 DF   3.0     30.0     32.0     18      80
  7 DF   4.0     34.0     57.0     18      80
  8 DF   5.0     38.0    138.0     18      80
  9 DF   6.0     38.0     72.0     18      80
Initial CCF = 248.
--- Input Complete ---

OPTIMIZATION  IS  BASED   ON : 6: SOIL EXPECTATION VALUE


THE SELECTED OPTIMIZATION IS :  1: THINNING OPTIMIZATION
INTERVAL OF NODE (THINNING LEVEL)  :100.0 TREES PER ACRE


FINANCIAL DATA

INTEREST RATE (%)  :     7.0
ENTRY COST PER ACRE ($) :      50.00
COEFFICIENT FOR REDUCING THINNING VALUE ($) : .800
 1:   REGENERATION COST            :      200.00($) at age     .00

PRICE IS PER THOUSAND CUBIC FOOT
PRICE EQUATION  : PRICE =      200.00 * DBH +      80.00
```

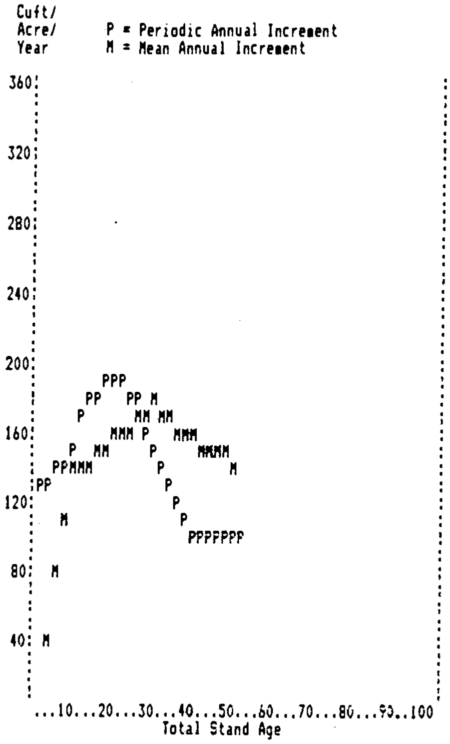
PACIFIC NW REGION (V85.3) <<< STAND OPTIMIZATION SYSTEM (SOS) >>> Site Index 82 DF

Stand Summary Report Base - 50 yrs BH Age

TEST RUN ON COMPAQ 386

** values on a per acre basis **

| Age SP | DBH | Trees /Ac | Basal Area | Top Ht | Crown Ratio | Total Cuft | Merch Cuft | Merch Bdft | #Logs /Mbf | Log Dib | PNW /Ac |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10WH | 4.2 | 496 | 49.2 | 36.0 | 80 | 740 | 00 | 00 | .0 | .0 | 378 |
| 10DF | 4.8 | 299 | 39.5 | 38.0 | 80 | 620 | 00 | 00 | .0 | .0 | 350 |
| 10Sum | 4.4 | 795 | 88.7 | 36.8 | 80 | 1360 | 00 | 00 | .0 | .0 | 728 |
| 20WH | 5.7 | 449 | 83.6 | 52.9 | 72 | 1810 | 00 | 00 | .0 | .0 | 622 |
| 20DF | 6.6 | 253 | 61.8 | 54.2 | 57 | 1330 | 00 | 00 | .0 | .0 | 510 |
| 20Sum | 6.0 | 702 | 145.4 | 53.4 | 67 | 3140 | 00 | 00 | .0 | .0 | 1132 |
| CutWH | 4.6 | 134 | 15.6 | 47.9 | 78 | 300 | 00 | 00 | .0 | .0 | 65 |
| CutDF | 4.9 | 66 | 8.8 | 49.4 | 66 | 170 | 00 | 00 | .0 | .0 | 38 |
| 20Sum | 4.7 | 200 | 24.4 | 48.4 | 74 | 470 | 00 | 00 | .0 | .0 | 103 |
| 30WH | 7.5 | 315 | 99.5 | 67.9 | 69 | 2810 | 560 | 2290 | 46.4 | 7.7 | 614 |
| 30DF | 8.6 | 187 | 75.3 | 67.8 | 54 | 2020 | 820 | 3730 | 49.3 | 7.1 | 481 |
| 30Sum | 7.9 | 502 | 174.8 | 67.8 | 64 | 4820 | 1380 | 6010 | 48.2 | 7.3 | 1095 |
| CutWH | 7.2 | 274 | 77.8 | 66.3 | 76 | 2170 | 00 | 00 | .0 | .0 | 355 |
| CutDF | 8.1 | 127 | 46.0 | 67.8 | 64 | 1240 | 140 | 620 | 49.3 | 7.0 | 223 |
| 30Sum | 7.5 | 401 | 123.8 | 66.8 | 72 | 3410 | 140 | 620 | 49.3 | 7.0 | 578 |
| 40WH | 12.0 | 41 | 32.1 | 80.8 | 80 | 1100 | 1020 | 4360 | 32.3 | 9.0 | 181 |
| 40DF | 11.3 | 60 | 41.9 | 79.2 | 69 | 1270 | 1160 | 4350 | 46.2 | 8.0 | 199 |
| 40Sum | 11.6 | 101 | 74.0 | 79.9 | 73 | 2370 | 2180 | 8710 | 39.3 | 8.5 | 380 |
| 50WH | 13.8 | 41 | 42.3 | 92.0 | 83 | 1620 | 1540 | 6980 | 24.3 | 10.1 | 156 |
| 50DF | 12.8 | 58 | 51.5 | 88.7 | 72 | 1720 | 1610 | 6080 | 38.0 | 8.8 | 154 |
| 50Sum | 13.2 | 99 | 93.8 | 90.0 | 76 | 3350 | 3150 | 13050 | 30.7 | 9.4 | 310 |



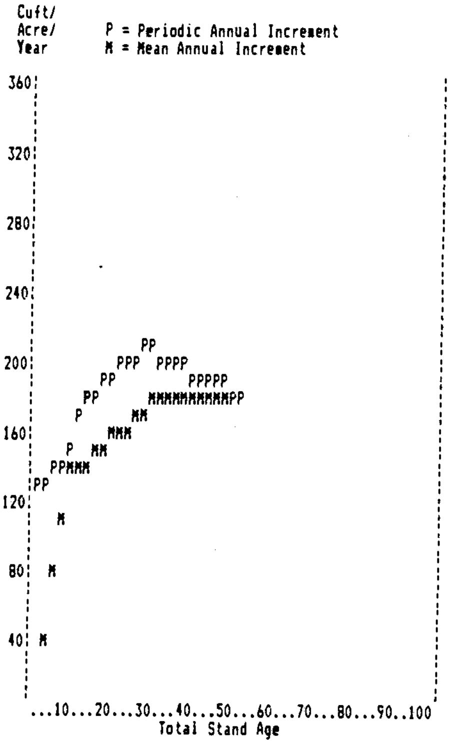

Stump Ht. 1.0 Log Length 16.4 Top Dib 5.0 Min DBH 9.0

<<SUMMARY OF SOIL EXPECTATION VALUE AT EACH ROTATION AGE>>
Optimal rotation age based on SOIL EXPECTATION VALUE basis is 20.0year

| ROTATION AGE (year) | SOIL EXPECTATION VALUE ($/acre) |
|---|---|
| 10.00 | 1022.79 |
| 20.00 | 1239.33 |
| 30.00 | 1126.81 |
| 40.00 | 898.47 |
| 50.00 | 796.65 |
| 60.00 | 691.88 |

PACIFIC NW REGION (V85.3)　　<<< STAND OPTIMIZATION SYSTEM (SOS) >>>　　Site Index 82 DF
Stand Summary Report　　Base - 50 yrs BH Age
TEST RUN ON COMPAQ 386

| Age SP | DBH | Trees /Ac | Basal Area | Top Ht | Crown Ratio | Total Cuft | Merch Cuft | Merch Bdft | #Logs /Mbf | Log Dib | PNW /Ac |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | ** values on a per acre basis ** | | | | | | | | |
| 10WH | 4.2 | 496 | 49.2 | 36.0 | 80 | 740 | 00 | 00 | .0 | .0 | 502 |
| 10DF | 4.8 | 299 | 39.5 | 38.0 | 80 | 620 | 00 | 00 | .0 | .0 | 416 |
| 10Sum | 4.4 | 795 | 88.7 | 36.8 | 80 | 1360 | 00 | 00 | .0 | .0 | 919 |
| 20WH | 5.7 | 449 | 83.6 | 52.9 | 72 | 1810 | 00 | 00 | .0 | .0 | 825 |
| 20DF | 6.6 | 253 | 61.8 | 54.2 | 57 | 1330 | 00 | 00 | .0 | .0 | 608 |
| 20Sum | 6.0 | 702 | 145.4 | 53.4 | 67 | 3140 | 00 | 00 | .0 | .0 | 1434 |
| CutWH | 4.6 | 134 | 15.6 | 47.9 | 78 | 300 | 00 | 00 | .0 | .0 | 110 |
| CutDF | 4.9 | 66 | 8.8 | 49.4 | 66 | 170 | 00 | 00 | .0 | .0 | 63 |
| 20Sum | 4.7 | 200 | 24.4 | 48.4 | 74 | 470 | 00 | 00 | .0 | .0 | 173 |
| 30WH | 7.5 | 315 | 99.5 | 67.9 | 69 | 2810 | 560 | 2290 | 46.4 | 7.7 | 865 |
| 30DF | 8.6 | 187 | 75.3 | 67.8 | 54 | 2020 | 820 | 3730 | 49.3 | 7.1 | 622 |
| 30Sum | 7.9 | 502 | 174.8 | 67.8 | 64 | 4820 | 1380 | 6010 | 48.2 | 7.3 | 1487 |
| CutWH | 6.5 | 100 | 23.0 | 62.1 | 74 | 630 | 00 | 00 | .0 | .0 | 154 |
| 30Sum | 6.5 | 100 | 23.0 | 62.1 | 74 | 630 | 00 | 00 | .0 | .0 | 154 |
| 40WH | 9.0 | 215 | 97.8 | 80.8 | 67 | 3290 | 2440 | 9840 | 47.5 | 7.6 | 686 |
| 40DF | 9.6 | 186 | 94.6 | 79.1 | 51 | 2910 | 1230 | 5220 | 44.7 | 7.7 | 606 |
| 40Sum | 9.3 | 401 | 192.4 | 80.0 | 60 | 6210 | 3680 | 15070 | 46.5 | 7.7 | 1293 |
| CutWH | 7.3 | 59 | 17.1 | 74.4 | 71 | 550 | 00 | 00 | .0 | .0 | 92 |
| CutDF | 9.0 | 42 | 18.6 | 79.1 | 59 | 570 | 490 | 2500 | 47.7 | 6.8 | 96 |
| 40Sum | 8.0 | 101 | 35.6 | 76.4 | 66 | 1130 | 490 | 2500 | 47.7 | 6.8 | 188 |
| 50WH | 10.7 | 156 | 98.7 | 91.8 | 68 | 3790 | 3480 | 14910 | 38.1 | 8.3 | 533 |
| 50DF | 10.7 | 144 | 90.9 | 88.5 | 52 | 3090 | 2800 | 11510 | 45.2 | 7.7 | 434 |
| 50Sum | 10.7 | 300 | 189.6 | 90.2 | 60 | 6870 | 6270 | 26420 | 41.2 | 8.0 | 967 |




Stump Ht. 1.0　Log Length 16.4　Top Dib 5.0　Min DBH 9.0

<<SUMMARY OF SOIL EXPECTATION VALUE AT EACH ROTATION AGE>>
Optimal rotation age based on SOIL EXPECTATION VALUE basis is 20.0year

| ROTATION AGE (year) | SOIL EXPECTATION VALUE ($/acre) |
|---|---|
| 10.00 | 2111.23 |
| 20.00 | 2227.62 |
| 30.00 | 2055.30 |
| 40.00 | 1731.87 |
| 50.00 | 1427.19 |
| 60.00 | 1246.74 |

e) Effect of quality premium (Case 2)

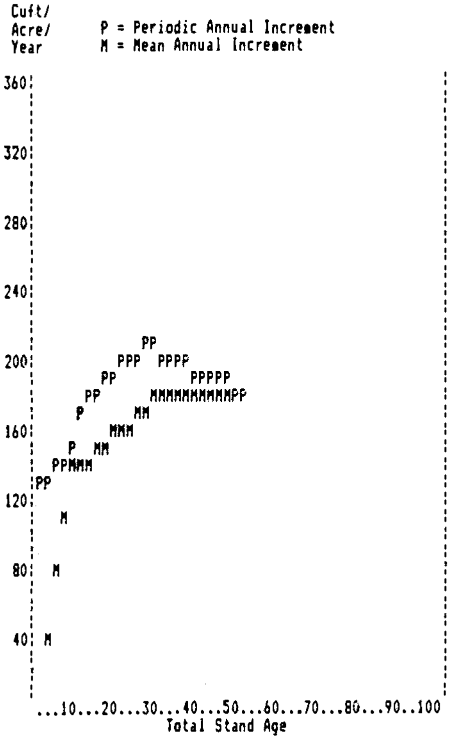
```
==================== S O S ====================
STAND        DF  82.  10.  PNW
MERCH       1.0 16.4  5.0  9.0
NAME       TEST RUN ON COMPAQ 386
THINNING     1  20.    1  10.    1 100.
THINNING     1  30.    1  10.    1 100.
THINNING     1  40.    1  10.    1 100.
REPORT     10.  20.  30.  40.  60.  70.  80.  90. 100.   0.   0.
Current stand total age =  10.
Average of  .85 acre stocked

  I SP   DBH     Ht      Tpa  BH-Age    %CR
  1 WH   2.0    18.0    15.0     19      80
  2 WH   3.0    27.0    92.0     19      80
  3 WH   4.0    32.0   227.0     19      80
  4 WH   5.0    35.0   121.0     19      80
  5 WH   6.0    36.0    41.0     19      80
  6 DF   3.0    30.0    32.0     18      80
  7 DF   4.0    34.0    57.0     18      80
  8 DF   5.0    38.0   138.0     18      80
  9 DF   6.0    38.0    72.0     18      80
Initial CCF = 248.
--- Input Complete ---

OPTIMIZATION IS BASED  ON : 6: SOIL EXPECTATION VALUE


THE SELECTED OPTIMIZATION IS :  1: THINNING OPTIMIZATION
INTERVAL OF NODE (THINNING LEVEL)  :100.0 TREES PER ACRE


FINANCIAL DATA

INTEREST RATE (%)  :    4.0
ENTRY COST PER ACRE ($) :      50.00
COEFFICIENT FOR REDUCING THINNING VALUE ($) : .800
 1:   REGENERATION COST           :     200.00($) at age     .00

PRICE IS PER THOUSAND CUBIC FOOT
PRICE EQUATION  : PRICE =     500.00 * DBH +        .00
```

PACIFIC NW REGION (V85.3)
Stand Summary Report
TEST RUN ON COMPAQ 386

<<< STAND OPTIMIZATION SYSTEM (SOS) >>>

Site Index 82 DF
Base - 50 yrs BH Age

| Age SP | DBH | Trees /Ac | Basal Area | Top Ht | Crown Ratio | Total Cuft | Merch Cuft | Merch Bdft | #Logs /Mbf | Log Dib | PNW /Ac |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | ** values on a per acre basis ** | | | | | | | | |
| 10WH | 4.2 | 496 | 49.2 | 36.0 | 80 | 740 | 00 | 00 | .0 | .0 | 1155 |
| 10DF | 4.8 | 299 | 39.5 | 38.0 | 80 | 620 | 00 | 00 | .0 | .0 | 1081 |
| 10Sum | 4.4 | 795 | 88.7 | 36.8 | 80 | 1360 | 00 | 00 | .0 | .0 | 2236 |
| 20WH | 5.7 | 449 | 83.6 | 52.9 | 72 | 1810 | 00 | 00 | .0 | .0 | 2582 |
| 20DF | 6.6 | 253 | 61.8 | 54.2 | 57 | 1330 | 00 | 00 | .0 | .0 | 2129 |
| 20Sum | 6.0 | 702 | 145.4 | 53.4 | 67 | 3140 | 00 | 00 | .0 | .0 | 4711 |
| CutWH | 4.6 | 134 | 15.6 | 47.9 | 78 | 300 | 00 | 00 | .0 | .0 | 263 |
| CutDF | 4.9 | 66 | 8.8 | 49.4 | 66 | 170 | 00 | 00 | .0 | .0 | 157 |
| 20Sum | 4.7 | 200 | 24.4 | 48.4 | 74 | 470 | 00 | 00 | .0 | .0 | 420 |
| 30WH | 7.5 | 315 | 99.5 | 67.9 | 69 | 2810 | 560 | 2290 | 46.4 | 7.7 | 3430 |
| 30DF | 8.6 | 187 | 75.3 | 67.8 | 54 | 2020 | 820 | 3730 | 49.3 | 7.1 | 2700 |
| 30Sum | 7.9 | 502 | 174.8 | 67.8 | 64 | 4920 | 1380 | 6010 | 48.2 | 7.3 | 6130 |
| CutWH | 6.5 | 100 | 23.0 | 62.1 | 74 | 630 | 00 | 00 | .0 | .0 | 502 |
| 30Sum | 6.5 | 100 | 23.0 | 62.1 | 74 | 630 | 00 | 00 | .0 | .0 | 502 |
| 40WH | 9.0 | 215 | 97.8 | 80.8 | 67 | 3290 | 2440 | 9840 | 47.5 | 7.6 | 3234 |
| 40DF | 9.6 | 186 | 94.6 | 79.1 | 51 | 2910 | 1230 | 5220 | 44.7 | 7.7 | 2960 |
| 40Sum | 9.3 | 401 | 192.4 | 80.0 | 60 | 6210 | 3680 | 15070 | 46.5 | 7.7 | 6194 |
| CutWH | 7.3 | 59 | 17.1 | 74.4 | 71 | 550 | 00 | 00 | .0 | .0 | 335 |
| CutDF | 9.0 | 42 | 18.6 | 79.1 | 59 | 570 | 490 | 2500 | 47.7 | 6.8 | 431 |
| 40Sum | 8.0 | 101 | 35.6 | 76.4 | 66 | 1130 | 490 | 2500 | 47.7 | 6.8 | 766 |
| 50WH | 10.7 | 156 | 98.7 | 91.8 | 68 | 3790 | 3480 | 14910 | 38.1 | 8.3 | 2905 |
| 50DF | 10.7 | 144 | 90.9 | 88.5 | 52 | 3090 | 2800 | 11510 | 45.2 | 7.7 | 2363 |
| 50Sum | 10.7 | 300 | 189.6 | 90.2 | 60 | 6870 | 6270 | 26420 | 41.2 | 8.0 | 5268 |




Stump Ht. 1.0 Log Length 16.4 Top Dib 5.0 Min DBH 9.0

<<SUMMARY OF SOIL EXPECTATION VALUE AT EACH ROTATION AGE>>
Optimal rotation age based on SOIL EXPECTATION VALUE basis is 30.0year

| ROTATION AGE (year) | SOIL EXPECTATION VALUE ($/acre) |
|---|---|
| 10.00 | 6170.85 |
| 20.00 | 8256.51 |
| 30.00 | 9124.58 |
| 40.00 | 8672.90 |
| 50.00 | 7796.25 |
| 60.00 | 6968.95 |

f) Effect of quality premium (Case 3)

```
==================== S O S ====================
STAND       DF  82.  10.  PNW
MERCH       1.0 16.4  5.0  9.0
NAME      TEST RUN ON COMPAQ 386
THINNING      1  20.     1  10.     1 100.
THINNING      1  30.     1  10.     1 100.
THINNING      1  40.     1  10.     1 100.
REPORT     10.  20.  30.  40.  60.  70.  80.  90. 100.   0.   0.
Current stand total age =  10.
Average of  .85 acre stocked

  I SP   DBH      Ht     Tpa  BH-Age    %CR
  1 WH   2.0    18.0    15.0      19     80
  2 WH   3.0    27.0    92.0      19     80
  3 WH   4.0    32.0   227.0      19     80
  4 WH   5.0    35.0   121.0      19     80
  5 WH   6.0    36.0    41.0      19     80
  6 DF   3.0    30.0    32.0      18     80
  7 DF   4.0    34.0    57.0      18     80
  8 DF   5.0    38.0   138.0      18     80
  9 DF   6.0    38.0    72.0      18     80
Initial CCF = 248.
--- Input Complete ---

OPTIMIZATION  IS  BASED   ON : 6: SOIL EXPECTATION VALUE


THE SELECTED OPTIMIZATION IS :  1: THINNING OPTIMIZATION
INTERVAL OF NODE (THINNING LEVEL)  :100.0 TREES PER ACRE


FINANCIAL DATA

INTEREST RATE (%)  :    4.0
ENTRY COST PER ACRE ($) :     50.00
COEFFICIENT FOR REDUCING THINNING VALUE ($) : .800
 1:   REGENERATION COST              :    200.00($) at age    .00

PRICE IS PER THOUSAND CUBIC FOOT
DISCRETE PRICE EQUATION
 DBH =   5.00 PRICE =        .00 (Y =     .00D +        .00)   DBH RANGE :   .00 TO   5.00
 DBH =  20.00 PRICE =     750.00 (Y =  50.00D +    -250.00)   DBH RANGE :  5.00 TO  20.00
 DBH =  20.00 PRICE =    1000.00 (Y = ******D + **********)   DBH RANGE : 20.00 TO  20.00
 DBH =  30.00 PRICE =    1000.00 (Y =     .00D +    1000.00)   DBH RANGE : 20.00 TO  30.00
```

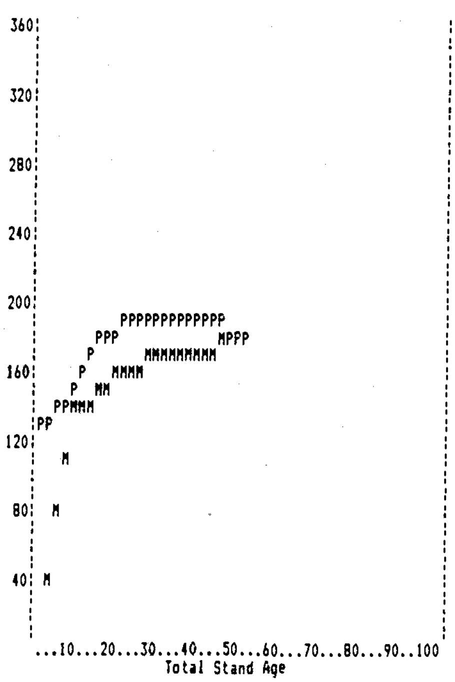
PACIFIC NW REGION (V85.3)
Stand Summary Report
TEST RUN ON COMPAQ 386

<<< STAND OPTIMIZATION SYSTEM (SOS) >>>

Site Index 82 DF
Base - 50 yrs BH Age

** values on a per acre basis **

| Age SP | DBH | Trees /Ac | Basal Area | Top Ht | Crown Ratio | Total Cuft | Merch Cuft | Merch Bdft | #Logs /Mbf | Log Dib | PNW /Ac |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10WH | 4.2 | 496 | 49.2 | 36.0 | 80 | 740 | 00 | 00 | .0 | .0 | 4 |
| 10DF | 4.8 | 299 | 39.5 | 38.0 | 80 | 620 | 00 | 00 | .0 | .0 | 8 |
| 10Sum | 4.4 | 795 | 88.7 | 36.8 | 80 | 1360 | 00 | 00 | .0 | .0 | 12 |
| 20WH | 5.7 | 449 | 83.6 | 52.9 | 72 | 1810 | 00 | 00 | .0 | .0 | 55 |
| 20DF | 6.6 | 253 | 61.8 | 54.2 | 57 | 1330 | 00 | 00 | .0 | .0 | 61 |
| 20Sum | 6.0 | 702 | 145.4 | 53.4 | 67 | 3140 | 00 | 00 | ,0 | .0 | 116 |
| 30WH | 7.0 | 407 | 112.5 | 67.7 | 67 | 3120 | 530 | 2230 | 46.0 | 7.6 | 122 |
| 30DF | 8.0 | 201 | 72.2 | 67.9 | 51 | 1920 | 760 | 3480 | 49.6 | 7.0 | 100 |
| 30Sum | 7.3 | 608 | 184.7 | 67.8 | 61 | 5030 | 1290 | 5710 | 48.2 | 7.2 | 223 |
| CutWH | 6.0 | 161 | 31.6 | 62.0 | 69 | 820 | 00 | 00 | .0 | .0 | 11 |
| CutDF | 5.9 | 39 | 7.5 | 60.6 | 53 | 190 | 00 | 00 | .0 | .0 | 2 |
| 30Sum | 6.0 | 200 | 39.1 | 61.7 | 66 | 1000 | 00 | 00 | .0 | .0 | 13 |
| 40WH | 8.7 | 246 | 104.0 | 80.6 | 67 | 3490 | 2350 | 9550 | 48.0 | 7.6 | 150 |
| 40DF | 9.6 | 162 | 81.7 | 79.2 | 51 | 2520 | 1140 | 4930 | 44.5 | 7.6 | 124 |
| 40Sum | 9.1 | 408 | 185.7 | 80.0 | 61 | 6010 | 3490 | 14480 | 46.8 | 7.6 | 274 |
| 50WH | 9.5 | 242 | 122.4 | 91.5 | 67 | 4640 | 3250 | 13480 | 40.9 | 8.1 | 163 |
| 50DF | 10.4 | 156 | 92.1 | 88.4 | 51 | 3140 | 2810 | 11450 | 48.2 | 7.6 | 122 |
| 50Sum | 9.9 | 398 | 214.5 | 90.3 | 61 | 7780 | 6060 | 24930 | 44.2 | 7.8 | 285 |




Stump Ht. 1.0 Log Length 16.4 Top Dib 5.0 Min DBH 9.0

<<SUMMARY OF SOIL EXPECTATION VALUE AT EACH ROTATION AGE>>
Optimal rotation age based on SOIL EXPECTATION VALUE basis is 50.0year

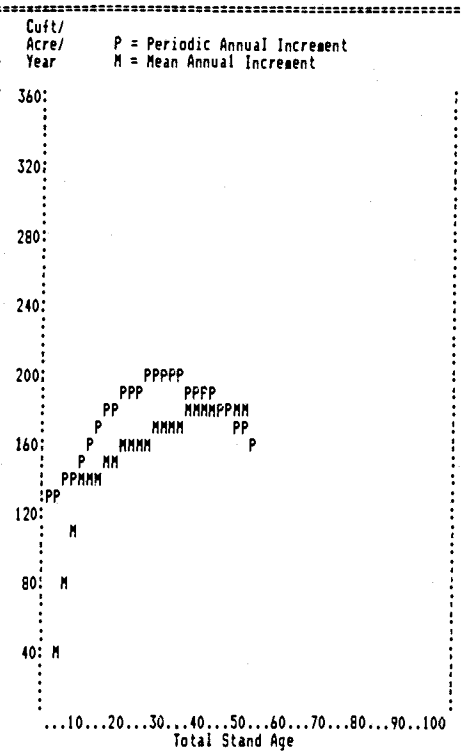
| ROTATION AGE (year) | SOIL EXPECTATION VALUE ($/acre) |
|---|---|
| 10.00 | -683.50 |
| 20.00 | -196.36 |
| 30.00 | 10.26 |
| 40.00 | 77.68 |
| 50.00 | 88.09 |
| 60.00 | 71.56 |

g) Effect of quality premium (Case 4)

```
==================== S O S ====================
STAND        DF  82.  10.  PNW
MERCH       1.0 16.4  5.0  9.0
NAME       TEST RUN ON COMPAQ 386
THINNING      1  20.    1  10.    1 100.
THINNING      1  30.    1  10.    1 100.
THINNING      1  40.    1  10.    1 100.
REPORT      10.  20.  30.  40.  60.  70.  80.  90. 100.   0.   0.
Current stand total age =  10.
Average of  .85 acre stocked

  I SP   DBH      Ht     Tpa  BH-Age    %CR
  1 WH   2.0    18.0    15.0     19      80
  2 WH   3.0    27.0    92.0     19      80
  3 WH   4.0    32.0   227.0     19      80
  4 WH   5.0    35.0   121.0     19      80
  5 WH   6.0    36.0    41.0     19      80
  6 DF   3.0    30.0    32.0     18      80
  7 DF   4.0    34.0    57.0     18      80
  8 DF   5.0    38.0   138.0     18      80
  9 DF   6.0    38.0    72.0     18      80
Initial CCF = 248.
--- Input Complete ---

OPTIMIZATION  IS  BASED   ON : 6: SOIL EXPECTATION VALUE


THE SELECTED OPTIMIZATION IS :  1: THINNING OPTIMIZATION
INTERVAL OF NODE (THINNING LEVEL)  :100.0 TREES PER ACRE


FINANCIAL DATA

INTEREST RATE (%)  :     4.0
ENTRY COST PER ACRE ($) :     50.00
COEFFICIENT FOR REDUCING THINNING VALUE ($) : .800
 1:   REGENERATION COST            :    200.00($) at age    .00

PRICE IS PER THOUSAND CUBIC FOOT
DISCRETE PRICE EQUATION
 DBH =   5.00 PRICE =        .00 (Y =     .00D +         .00)  DBH RANGE :   .00 TO   5.00
 DBH =   5.00 PRICE =     200.00 (Y = ******D + ***********)  DBH RANGE :  5.00 TO   5.00
 DBH =   8.00 PRICE =     200.00 (Y =     .00D +      200.00)  DBH RANGE :  5.00 TO   8.00
 DBH =   8.00 PRICE =     500.00 (Y = ******D + ***********)  DBH RANGE :  8.00 TO   8.00
 DBH =  11.00 PRICE =     500.00 (Y =     .00D +      500.00)  DBH RANGE :  8.00 TO  11.00
 DBH =  30.00 PRICE =    1500.00 (Y =   52.63D +      -78.95)  DBH RANGE : 11.00 TO  30.00
```

PACIFIC NW REGION (V85.3) <<< STAND OPTIMIZATION SYSTEM (SOS) >>> Site Index 82 DF
Stand Summary Reoort Base - 50 yrs BH Age
TEST RUN ON COMPAQ 386

| Age SP | DBH | Trees /Ac | Basal Area | Top Ht | Crown Ratio | Total Cuft | Merch Cuft | Merch Bdft | #Logs /Mbf | Log Dib | PNW /Ac |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | ** values on a per acre basis ** | | | | | | | | |
| 10WH | 4.2 | 496 | 49.2 | 36.0 | 80 | 740 | 00 | 00 | .0 | .0 | 17 |
| 10DF | 4.8 | 299 | 39.5 | 38.0 | 80 | 620 | 00 | 00 | .0 | .0 | 31 |
| 10Sum | 4.4 | 795 | 88.7 | 36.8 | 80 | 1360 | 00 | 00 | .0 | .0 | 48 |
| 20WH | 5.7 | 449 | 83.6 | 52.9 | 72 | 1810 | 00 | 00 | .0 | .0 | 197 |
| 20DF | 6.6 | 253 | 61.8 | 54.2 | 57 | 1330 | 00 | 00 | .0 | .0 | 118 |
| 20Sum | 6.0 | 702 | 145.4 | 53.4 | 67 | 3140 | 00 | 00 | .0 | .0 | 315 |
| CutWH | 4.9 | 234 | 31.5 | 47.9 | 78 | 640 | 00 | 00 | .0 | .0 | 35 |
| CutDF | 4.9 | 66 | 8.8 | 49.4 | 66 | 170 | 00 | 00 | .0 | .0 | 9 |
| 20Sum | 4.9 | 300 | 40.3 | 48.2 | 75 | 810 | 00 | 00 | .0 | .0 | 44 |
| 30WH | 8.1 | 215 | 78.4 | 67.9 | 72 | 2240 | 570 | 2290 | 46.7 | 7.7 | 309 |
| 30DF | 8.6 | 187 | 76.7 | 67.9 | 58 | 2060 | 830 | 3730 | 49.6 | 7.1 | 317 |
| 30Sum | 8.4 | 402 | 155.1 | 67.9 | 65 | 4290 | 1410 | 6010 | 48.5 | 7.3 | 626 |
| 40WH | 9.2 | 215 | 101.9 | 80.9 | 71 | 3440 | 2560 | 10370 | 45.9 | 7.8 | 324 |
| 40DF | 9.7 | 187 | 96.6 | 79.4 | 56 | 2980 | 2610 | 12200 | 46.2 | 7.2 | 311 |
| 40Sum | 9.4 | 402 | 198.5 | 80.2 | 64 | 6420 | 5170 | 22580 | 46.1 | 7.5 | 635 |
| 50WH | 10.2 | 208 | 119.4 | 91.9 | 70 | 4550 | 3550 | 14990 | 38.2 | 8.3 | 333 |
| 50DF | 10.5 | 166 | 101.1 | 88.7 | 54 | 3440 | 3100 | 12750 | 46.6 | 7.6 | 251 |
| 50Sum | 10.3 | 374 | 220.5 | 90.5 | 63 | 8000 | 6650 | 27740 | 42.0 | 8.0 | 584 |




Stump Ht. 1.0 Log Length 16.4 Top Dib 5.0 Min DBH 9.0

<<SUMMARY OF SOIL EXPECTATION VALUE AT EACH ROTATION AGE>>
Optimal rotation age based on SOIL EXPECTATION VALUE basis is 30.0year

| ROTATION AGE (year) | SOIL EXPECTATION VALUE ($/acre) |
|---|---|
| 10.00 | -572.29 |
| 20.00 | 169.40 |
| 30.00 | 625.23 |
| 40.00 | 563.54 |
| 50.00 | 464.21 |
| 60.00 | 416.50 |

h) Effect of type of thinning (from above)

```
==================== S O S ====================
STAND       DF  82.  10.  PNW
MERCH       1.0 16.4  5.0  9.0
NAME      TEST RUN ON COMPAQ 386
THINNING     1  20.    1   0.    2   0.
THINNING     1  30.    1   0.    2   0.
THINNING     1  40.    1   0.    2   0.
REPORT     10.  20.  30.  40.  60.  70.  80.  90. 100.   0.   0.
Current stand total age =  10.
Average of  .85 acre stocked

  I SP   DBH      Ht      Tpa  BH-Age     %CR
  1 WH   2.0    18.0     15.0      19      80
  2 WH   3.0    27.0     92.0      19      80
  3 WH   4.0    32.0    227.0      19      80
  4 WH   5.0    35.0    121.0      19      80
  5 WH   6.0    36.0     41.0      19      80
  6 DF   3.0    30.0     32.0      18      80
  7 DF   4.0    34.0     57.0      18      80
  8 DF   5.0    38.0    138.0      18      80
  9 DF   6.0    38.0     72.0      18      80
Initial CCF = 248.
--- Input Complete ---

OPTIMIZATION  IS  BASED   ON : 6: SOIL EXPECTATION VALUE


THE SELECTED OPTIMIZATION IS :  1: THINNING OPTIMIZATION
INTERVAL OF NODE (THINNING LEVEL)  :100.0 TREES PER ACRE


FINANCIAL DATA

INTEREST RATE (%)  :     4.0
ENTRY COST PER ACRE ($) :     50.00
COEFFICIENT FOR REDUCING THINNING VALUE ($) : .800
 1:   REGENERATION COST           :     200.00($) at age    .00

PRICE IS PER THOUSAND CUBIC FOOT
PRICE EQUATION  : PRICE =     200.00 * DBH +     80.00
```

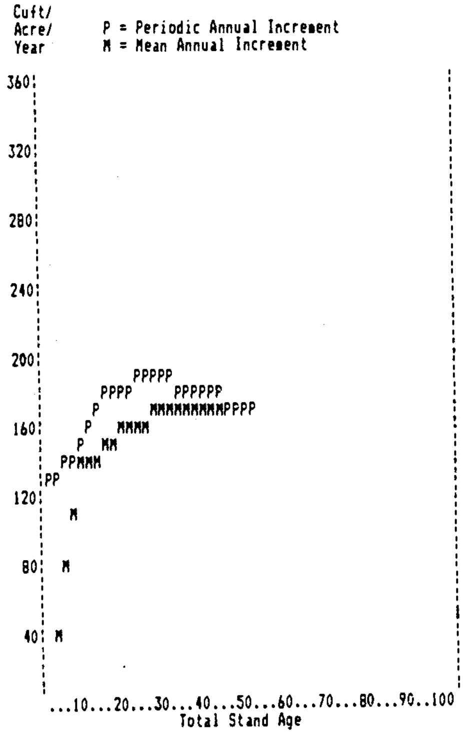
PACIFIC NW REGION (V85.3)
Stand Summary Report
TEST RUN ON COMPAQ 386

<<< STAND OPTIMIZATION SYSTEM (SOS) >>>

Site Index 82 DF
Base - 50 yrs BH Age

** values on a per acre basis **

| Age SP | DBH | Trees /Ac | Basal Area | Top Ht | Crown Ratio | Total Cuft | Merch Cuft | Merch Bdft | #Logs /Mbf | Log Dib | PNW /Ac |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10WH | 4.2 | 496 | 49.2 | 36.0 | 80 | 740 | 00 | 00 | .0 | .0 | 502 |
| 10DF | 4.8 | 299 | 39.5 | 38.0 | 80 | 620 | 00 | 00 | .0 | .0 | 466 |
| 10Sum | 4.4 | 795 | 88.7 | 36.8 | 80 | 1360 | 00 | 00 | .0 | .0 | 968 |
| 20WH | 5.7 | 449 | 83.6 | 52.9 | 72 | 1810 | 00 | 00 | .0 | .0 | 1099 |
| 20DF | 6.6 | 253 | 61.8 | 54.2 | 57 | 1330 | 00 | 00 | .0 | .0 | 900 |
| 20Sum | 6.0 | 702 | 145.4 | 53.4 | 67 | 3140 | 00 | 00 | .0 | .0 | 1999 |
| 30WH | 7.0 | 407 | 112.5 | 67.7 | 67 | 3120 | 530 | 2230 | 46.0 | 7.6 | 1526 |
| 30DF | 8.0 | 201 | 72.2 | 67.9 | 51 | 1920 | 760 | 3480 | 49.6 | 7.0 | 1040 |
| 30Sum | 7.3 | 608 | 184.7 | 67.8 | 61 | 5030 | 1290 | 5710 | 48.2 | 7.2 | 2566 |
| CutWH | 9.7 | 40 | 20.8 | 67.7 | 72 | 600 | 540 | 2260 | 45.9 | 7.6 | 301 |
| CutDF | 9.4 | 60 | 28.9 | 67.9 | 58 | 770 | 660 | 3070 | 49.6 | 7.0 | 373 |
| 30Sum | 9.5 | 100 | 49.7 | 67.8 | 63 | 1370 | 1200 | 5320 | 48.0 | 7.2 | 674 |
| 40WH | 7.7 | 367 | 119.7 | 79.1 | 66 | 3880 | 1520 | 6290 | 51.9 | 7.2 | 1354 |
| 40DF | 8.3 | 141 | 54.6 | 79.2 | 51 | 1670 | 130 | 580 | 44.5 | 7.6 | 629 |
| 40Sum | 7.9 | 508 | 174.2 | 79.1 | 62 | 5550 | 1650 | 6870 | 51.2 | 7.2 | 1984 |
| 50WH | 8.4 | 358 | 140.3 | 89.8 | 66 | 5150 | 2090 | 8190 | 47.8 | 7.7 | 1322 |
| 50DF | 9.1 | 132 | 60.3 | 88.4 | 51 | 2040 | 1510 | 6290 | 52.4 | 7.2 | 562 |
| 50Sum | 8.6 | 490 | 200.5 | 89.4 | 62 | 7190 | 3600 | 14490 | 49.8 | 7.5 | 1884 |




Stump Ht. 1.0 Log Length 16.4 Top Dib 5.0 Min DBH 9.0

<<SUMMARY OF SOIL EXPECTATION VALUE AT EACH ROTATION AGE>>
Optimal rotation age based on SOIL EXPECTATION VALUE basis is 30.0year

| ROTATION AGE (year) | SOIL EXPECTATION VALUE ($/acre) |
|---|---|
| 10.00 | 2262.54 |
| 20.00 | 3267.67 |
| 30.00 | 3398.44 |
| 40.00 | 3071.11 |
| 50.00 | 2717.99 |
| 60.00 | 2431.22 |

i) Effect of type of thinning (to a c/r ratio of 1)

```
==================== S O S ====================
STAND       DF  82.  10.  PNW
MERCH      1.0 16.4  5.0  9.0
NAME     TEST RUN ON COMPAQ 386
THINNING      1  20.     1   0.     3   1.
THINNING      1  30.     1   0.     3   1.
THINNING      1  40.     1   0.     3   1.
REPORT     10.  20.  30.  40.  60.  70.  80.  90. 100.   0.   0.
Current stand total age =  10.
Average of  .85 acre stocked

  I SP   DBH       Ht      Tpa  BH-Age      %CR
  1 WH   2.0     18.0     15.0      19       80
  2 WH   3.0     27.0     92.0      19       80
  3 WH   4.0     32.0    227.0      19       80
  4 WH   5.0     35.0    121.0      19       80
  5 WH   6.0     36.0     41.0      19       80
  6 DF   3.0     30.0     32.0      18       80
  7 DF   4.0     34.0     57.0      18       80
  8 DF   5.0     38.0    138.0      18       80
  9 DF   6.0     38.0     72.0      18       80
Initial CCF = 248.
--- Input Complete ---

OPTIMIZATION  IS  BASED   ON : 6: SOIL EXPECTATION VALUE


THE SELECTED OPTIMIZATION IS :  1: THINNING OPTIMIZATION
INTERVAL OF NODE (THINNING LEVEL)  :100.0 TREES PER ACRE


FINANCIAL DATA

INTEREST RATE (%)  :     4.0
ENTRY COST PER ACRE ($) :      50.00
COEFFICIENT FOR REDUCING THINNING VALUE ($) : .800
 1:   REGENERATION COST           :     200.00($) at age    .00

PRICE IS PER THOUSAND CUBIC FOOT
PRICE EQUATION  : PRICE =      200.00 * DBH +      80.00
```

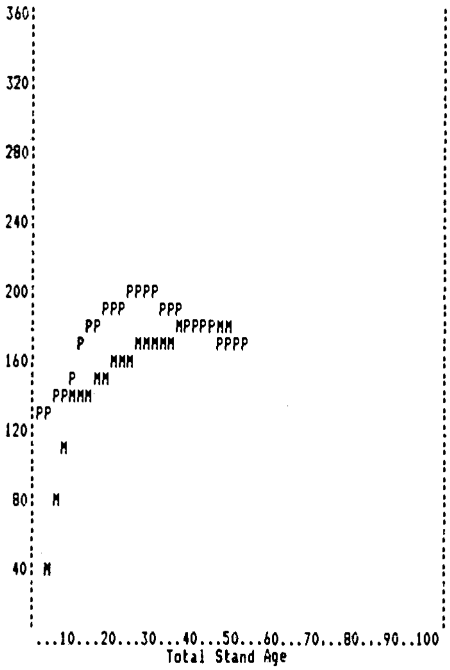
PACIFIC NW REGION (V85.3)
Stand Summary Report
TEST RUN ON COMPAQ 386

<<< STAND OPTIMIZATION SYSTEM (SOS) >>>

Site Index 82 DF
Base - 50 yrs BH Age

| Age SP | DBH | Trees /Ac | Basal Area | Top Ht | Crown Ratio | Total Cuft | Merch Cuft | Merch Bdft | #Logs /Mbf | Log Dib | PNW /Ac |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ** values on a per acre basis ** | | | | | | | | | |
| 10WH | 4.2 | 496 | 49.2 | 36.0 | 80 | 740 | 00 | 00 | .0 | .0 | 502 |
| 10DF | 4.8 | 299 | 39.5 | 38.0 | 80 | 620 | 00 | 00 | .0 | .0 | 466 |
| 10Sum | 4.4 | 795 | 88.7 | 36.8 | 80 | 1360 | 00 | 00 | .0 | .0 | 968 |
| 20WH | 5.7 | 449 | 83.6 | 52.9 | 72 | 1810 | 00 | 00 | .0 | .0 | 1099 |
| 20DF | 6.6 | 253 | 61.8 | 54.2 | 57 | 1330 | 00 | 00 | .0 | .0 | 900 |
| 20Sum | 6.0 | 702 | 145.4 | 53.4 | 67 | 3140 | 00 | 00 | .0 | .0 | 1999 |
| CutWH | 5.8 | 98 | 20.2 | 52.9 | 80 | 430 | 00 | 00 | .0 | .0 | 242 |
| CutDF | 8.0 | 2 | .7 | 54.2 | 69 | 20 | 00 | 00 | .0 | .0 | 9 |
| 20Sum | 5.9 | 100 | 20.9 | 52.9 | 80 | 450 | 00 | 00 | .0 | .0 | 251 |
| 30WH | 6.9 | 347 | 91.8 | 66.3 | 68 | 2540 | 00 | 00 | .0 | .0 | 1184 |
| 30DF | 7.8 | 243 | 83.5 | 67.8 | 53 | 2210 | 780 | 3580 | 49.6 | 7.0 | 1181 |
| 30Sum | 7.3 | 590 | 175.3 | 67.0 | 62 | 4750 | 780 | 3580 | 49.6 | 7.0 | 2365 |
| CutWH | 5.4 | 33 | 5.2 | 54.9 | 70 | 130 | 00 | 00 | .0 | .0 | 36 |
| CutDF | 8.0 | 68 | 24.7 | 67.8 | 62 | 640 | 460 | 2030 | 52.4 | 7.0 | 292 |
| 30Sum | 7.1 | 101 | 29.9 | 63.6 | 65 | 770 | 460 | 2030 | 52.4 | 7.0 | 328 |
| 40WH | 8.0 | 313 | 110.9 | 78.9 | 66 | 3660 | 1570 | 6350 | 52.2 | 7.3 | 1315 |
| 40DF | 8.7 | 173 | 72.8 | 79.1 | 51 | 2240 | 460 | 2030 | 44.2 | 7.6 | 875 |
| 40Sum | 8.3 | 486 | 183.8 | 79.0 | 61 | 5890 | 2030 | 8380 | 50.3 | 7.3 | 2191 |
| CutWH | 7.7 | 39 | 12.9 | 75.4 | 71 | 420 | 120 | 500 | 52.3 | 7.3 | 117 |
| CutDF | 8.4 | 62 | 25.2 | 77.3 | 58 | 760 | 460 | 2000 | 44.3 | 7.6 | 245 |
| 40Sum | 8.2 | 101 | 38.1 | 76.5 | 63 | 1180 | 580 | 2500 | 45.9 | 7.5 | 362 |
| 50WH | 8.8 | 274 | 117.8 | 89.8 | 67 | 4410 | 2070 | 8560 | 43.6 | 7.9 | 1179 |
| 50DF | 9.7 | 111 | 56.5 | 88.5 | 52 | 1940 | 1700 | 6970 | 53.9 | 7.1 | 550 |
| 50Sum | 9.1 | 385 | 174.3 | 89.4 | 63 | 6350 | 3780 | 15540 | 48.2 | 7.5 | 1729 |

P = Periodic Annual Increment
M = Mean Annual Increment




Stump Ht. 1.0 Log Length 16.4 Top Dib 5.0 Min DBH 9.0

<<SUMMARY OF SOIL EXPECTATION VALUE AT EACH ROTATION AGE>>
Optimal rotation age based on SOIL EXPECTATION VALUE basis is 30.0year

| ROTATION AGE (year) | SOIL EXPECTATION VALUE ($/acre) |
|---|---|
| 10.00 | 2262.54 |
| 20.00 | 3267.67 |
| 30.00 | 3438.20 |
| 40.00 | 3184.78 |
| 50.00 | 2809.77 |
| 60.00 | 2490.40 |

j) Limitation of optimality (no trade-off)

```
=================== S O S ====================
STAND        DF  82.  10.  PNW
MERCH       1.0 16.4  5.0  9.0
NAME     TEST RUN ON COMPAQ 386
THINNING      1  20.    1   0.    2   0.
THINNING      1  30.    1   0.    2   0.
THINNING      1  40.    1   0.    2   0.
REPORT     10.  20.  30.  40.  60.  70.  80.  90. 100.   0.   0.
Current stand total age =  10.
Average of  .85 acre stocked

  I SP   DBH      Ht      Tpa  BH-Age     %CR
  1 WH   2.0    18.0     15.0      19      80
  2 WH   3.0    27.0     92.0      19      80
  3 WH   4.0    32.0    227.0      19      80
  4 WH   5.0    35.0    121.0      19      80
  5 WH   6.0    36.0     41.0      19      80
  6 DF   3.0    30.0     32.0      18      80
  7 DF   4.0    34.0     57.0      18      80
  8 DF   5.0    38.0    138.0      18      80
  9 DF   6.0    38.0     72.0      18      80
Initial CCF = 248.
--- Input Complete ---

OPTIMIZATION  IS  BASED   ON : 6: SOIL EXPECTATION VALUE



THE SELECTED OPTIMIZATION IS :  1: THINNING OPTIMIZATION
INTERVAL OF NODE (THINNING LEVEL)  :100.0 TREES PER ACRE


FINANCIAL DATA

INTEREST RATE (%)  :     4.0
ENTRY COST PER ACRE ($) :     50.00
COEFFICIENT FOR REDUCING THINNING VALUE ($) : .800
 1:   REGENERATION COST            :    200.00($) at age    .00

PRICE IS PER THOUSAND CUBIC FOOT
DISCRETE PRICE EQUATION
 DBH =   5.00 PRICE =        .00 (Y =     .00D +         .00)  DBH RANGE :   .00 TO   5.00
 DBH =   5.00 PRICE =     200.00 (Y = *******D + ***********)  DBH RANGE :  5.00 TO   5.00
 DBH =   8.00 PRICE =     200.00 (Y =     .00D +      200.00)  DBH RANGE :  5.00 TO   8.00
 DBH =   8.00 PRICE =     500.00 (Y = *******D + ***********)  DBH RANGE :  8.00 TO   8.00
 DBH =  11.00 PRICE =     500.00 (Y =     .00D +      500.00)  DBH RANGE :  8.00 TO  11.00
 DBH =  30.00 PRICE =    1500.00 (Y =   52.63D +      -78.95)  DBH RANGE : 11.00 TO  30.00
```

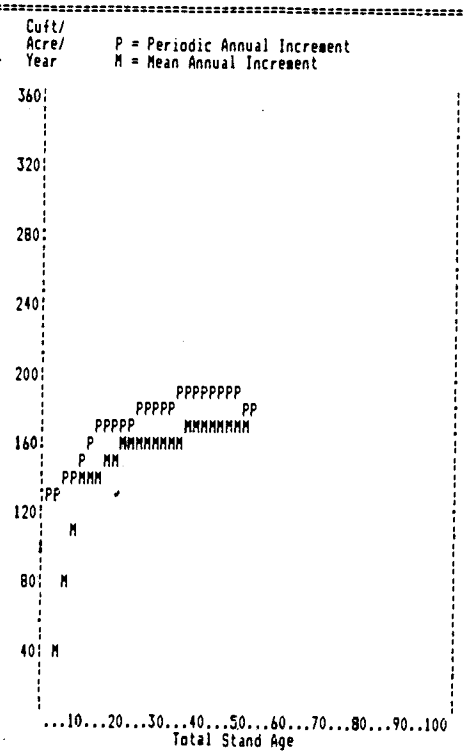
PACIFIC NW REGION (V85.3)
Stand Summary Report
TEST RUN ON COMPAQ 386

<<< STAND OPTIMIZATION SYSTEM (SOS) >>>

Site Index 82 DF
Base - 50 yrs BH Age

** values on a per acre basis **

| Age SP | DBH | Trees /Ac | Basal Area | Top Ht | Crown Ratio | Total Cuft | Merch Cuft | Merch Bdft | #Logs /Mbf | Log Dib | PNW /Ac |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10WH | 4.2 | 496 | 49.2 | 36.0 | 80 | 740 | 00 | 00 | .0 | .0 | 17 |
| 10DF | 4.8 | 299 | 39.5 | 38.0 | 80 | 620 | 00 | 00 | .0 | .0 | 31 |
| 10Sum | 4.4 | 795 | 88.7 | 36.8 | 80 | 1360 | 00 | 00 | .0 | .0 | 48 |
| 20WH | 5.7 | 449 | 83.6 | 52.9 | 72 | 1810 | 00 | 00 | .0 | .0 | 197 |
| 20DF | 6.6 | 253 | 61.8 | 54.2 | 57 | 1330 | 00 | 00 | .0 | .0 | 118 |
| 20Sum | 6.0 | 702 | 145.4 | 53.4 | 67 | 3140 | 00 | 00 | .0 | .0 | 315 |
| CutWH | 8.1 | 41 | 14.7 | 52.9 | 80 | 340 | 00 | 00 | .0 | .0 | 62 |
| CutDF | 7.3 | 159 | 46.4 | 54.2 | 69 | 1020 | 00 | 00 | .0 | .0 | 74 |
| 20Sum | 7.5 | 200 | 61.1 | 53.9 | 71 | 1350 | 00 | 00 | .0 | .0 | 136 |
| 30WH | 6.8 | 408 | 106.2 | 66.4 | 73 | 2900 | 00 | 00 | .0 | .0 | 292 |
| 30DF | 6.6 | 94 | 22.8 | 66.3 | 58 | 580 | 00 | 00 | .0 | .0 | 61 |
| 30Sum | 6.8 | 502 | 129.1 | 66.4 | 70 | 3480 | 00 | 00 | .0 | .0 | 353 |
| 40WH | 7.9 | 408 | 141.2 | 79.3 | 71 | 4570 | 1700 | 6880 | 49.6 | 7.5 | 310 |
| 40DF | 7.5 | 94 | 29.4 | 77.6 | 56 | 870 | 330 | 1700 | 47.5 | 6.8 | 61 |
| 40Sum | 7.8 | 502 | 170.6 | 79.0 | 68 | 5440 | 2030 | 8570 | 49.2 | 7.3 | 371 |
| 50WH | 8.7 | 400 | 168.3 | 90.3 | 69 | 6180 | 2370 | 9370 | 44.0 | 8.0 | 400 |
| 50DF | 8.2 | 87 | 32.7 | 86.8 | 53 | 1080 | 440 | 1850 | 50.5 | 7.2 | 51 |
| 50Sum | 8.6 | 487 | 201.0 | 89.7 | 66 | 7260 | 2810 | 11220 | 45.1 | 7.9 | 451 |




Stump Ht. 1.0 Log Length 16.4 Top Dib 5.0 Min DBH 9.0

<<SUMMARY OF SOIL EXPECTATION VALUE AT EACH ROTATION AGE>>
Optimal rotation age based on SOIL EXPECTATION VALUE basis is 50.0year

| ROTATION AGE (year) | SOIL EXPECTATION VALUE ($/acre) |
|---|---|
| 10.00 | -572.29 |
| 20.00 | 169.40 |
| 30.00 | 362.47 |
| 40.00 | 345.55 |
| 50.00 | 415.32 |
| 60.00 | 366.53 |

k) Limitation of optimality (trade-off)

```
==================== S O S ====================
STAND        DF  82.  10.  PNW
MERCH       1.0 16.4  5.0  9.0
NAME      TEST RUN ON COMPAQ 386
THINNING      1  20.     1   0.     2   0.
THINNING      1  40.     1   0.     2   0.
REPORT      10.  20.  30.  40.  60.  70.  80.  90. 100.   0.   0.
Current stand total age =  10.
Average of  .85 acre stocked

  I SP   DBH      Ht      Tpa  BH-Age     %CR
  1 WH   2.0    18.0    15.0      19      80
  2 WH   3.0    27.0    92.0      19      80
  3 WH   4.0    32.0   227.0      19      80
  4 WH   5.0    35.0   121.0      19      80
  5 WH   6.0    36.0    41.0      19      80
  6 DF   3.0    30.0    32.0      18      80
  7 DF   4.0    34.0    57.0      18      80
  8 DF   5.0    38.0   138.0      18      80
  9 DF   6.0    38.0    72.0      18      80
Initial CCF = 248.
--- Input Complete ---

OPTIMIZATION  IS  BASED   ON : 6: SOIL EXPECTATION VALUE


THE SELECTED OPTIMIZATION IS :  1: THINNING OPTIMIZATION
INTERVAL OF NODE (THINNING LEVEL)  :100.0 TREES PER ACRE


FINANCIAL DATA

INTEREST RATE (%)  :    4.0
ENTRY COST PER ACRE ($) :      50.00
COEFFICIENT FOR REDUCING THINNING VALUE ($) : .800
 1:  REGENERATION COST           :     200.00($) at age    .00

PRICE IS PER THOUSAND CUBIC FOOT
DISCRETE PRICE EQUATION
 DBH =   5.00 PRICE =        .00 (Y =    .00D +        .00)  DBH RANGE :   .00 TO   5.00
 DBH =   5.00 PRICE =     200.00 (Y = *******D + **********)  DBH RANGE :  5.00 TO   5.00
 DBH =   8.00 PRICE =     200.00 (Y =    .00D +     200.00)  DBH RANGE :  5.00 TO   8.00
 DBH =   8.00 PRICE =     500.00 (Y = *******D + **********)  DBH RANGE :  8.00 TO   8.00
 DBH =  11.00 PRICE =     500.00 (Y =    .00D +     500.00)  DBH RANGE :  8.00 TO  11.00
 DBH =  30.00 PRICE =    1500.00 (Y =  52.63D +     -78.95)  DBH RANGE : 11.00 TO  30.00
```

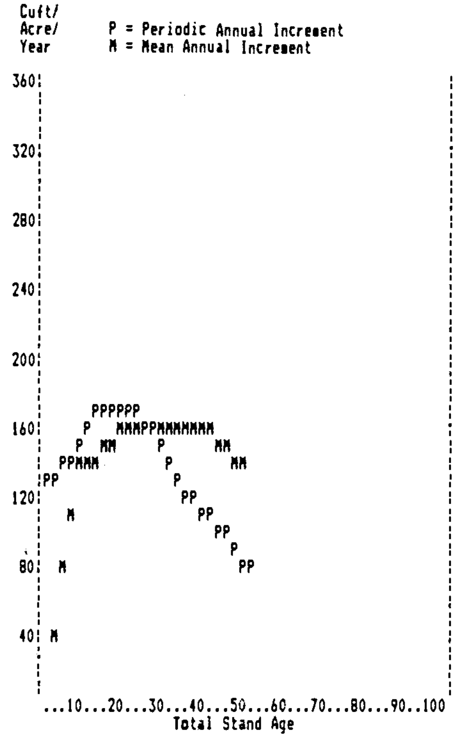
PACIFIC NW REGION (V85.3)
Stand Summary Report
TEST RUN ON COMPAQ 386

<<< STAND OPTIMIZATION SYSTEM (SOS) >>>

Site Index 82 DF
Base - 50 yrs BH Age

** values on a per acre basis **

| Age SP | DBH | Trees /Ac | Basal Area | Top Ht | Crown Ratio | Total Cuft | Merch Cuft | Merch Bdft | #Logs /Mbf | Log Dib | PNW /Ac |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10WH | 4.2 | 496 | 49.2 | 36.0 | 80 | 740 | 00 | 00 | .0 | .0 | 17 |
| 10DF | 4.8 | 299 | 39.5 | 38.0 | 80 | 620 | 00 | 00 | .0 | .0 | 31 |
| 10Sum | 4.4 | 795 | 88.7 | 36.8 | 80 | 1360 | 00 | 00 | .0 | .0 | 48 |
| 20WH | 5.7 | 449 | 83.6 | 52.9 | 72 | 1810 | 00 | 00 | .0 | .0 | 197 |
| 20DF | 6.6 | 253 | 61.8 | 54.2 | 57 | 1330 | 00 | 00 | .0 | .0 | 118 |
| 20Sum | 6.0 | 702 | 145.4 | 53.4 | 67 | 3140 | 00 | 00 | .0 | .0 | 315 |
| 40WH | 8.1 | 358 | 131.1 | 80.2 | 68 | 4300 | 820 | 3260 | 40.1 | 8.4 | 339 |
| 40DF | 9.1 | 145 | 67.5 | 78.8 | 52 | 2060 | 980 | 4350 | 44.0 | 7.6 | 207 |
| 40Sum | 8.4 | 503 | 198.6 | 79.8 | 63 | 6360 | 1800 | 7610 | 42.3 | 7.9 | 545 |
| CutWH | 9.2 | 173 | 80.6 | 80.1 | 72 | 2720 | 820 | 3300 | 40.0 | 8.3 | 217 |
| CutDF | 9.5 | 127 | 63.0 | 78.8 | 57 | 1940 | 970 | 4380 | 43.8 | 7.5 | 161 |
| 40Sum | 9.3 | 300 | 143.6 | 79.6 | 66 | 4650 | 1790 | 7680 | 42.2 | 7.9 | 378 |
| 50WH | 8.0 | 183 | 64.0 | 84.5 | 73 | 2280 | 00 | 00 | .0 | .0 | 144 |
| 50DF | 7.4 | 17 | 5.0 | 80.0 | 58 | 160 | 00 | 00 | .0 | .0 | 5 |
| 50Sum | 7.9 | 200 | 69.0 | 84.1 | 72 | 2440 | 00 | 00 | .0 | .0 | 148 |




Stump Ht. 1.0 Log Length 16.4 Top Dib 5.0 Min DBH 9.0

<<SUMMARY OF SOIL EXPECTATION VALUE AT EACH ROTATION AGE>>
Optimal rotation age based on SOIL EXPECTATION VALUE basis is 40.0year

| ROTATION AGE (year) | SOIL EXPECTATION VALUE ($/acre) |
|---|---|
| 10.00 | -572.29 |
| 20.00 | 169.40 |
| 40.00 | 423.12 |
| 50.00 | 360.05 |
| 60.00 | 349.62 |