

COMPUTER SIMULATION OF OFFTRACKING OF
TRUCK AND TRAILER COMBINATIONS
USING FOREST ROADS

By

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Abstract

Many of the analytical techniques that are currently available are not capable of analyzing the offtracking of truck and trailer combinations or the path of load overhangs through a set of curves that are commonly found on forest roads. Advances in the processing speed of microcomputers have made it possible to simulate how a truck and trailer combination negotiate a road. A microcomputer program, OFFTRACK, was developed to simulate the offtracking of six different vehicle configurations. The program will enable road designers to predict offtracking through a complex set of road geometrics, including compound and reverse curves. The program will also analyze the path of load overhangs on a vehicle such as a yarder tower on a lowboy or rubber tired undercarriage. Test results of the program compare favorably with several field studies and a scale model simulator. Outputs of the computer simulation are in the form of road widths required left and right of centerline, total width, and curve widening. A plot of the paths of the wheels and load overhang can be viewed on the screen or sent to a drum plotter.

Redacted for privacy

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1 Introduction

1.1 Purpose

Thousands of miles of forest road are constructed each year with safety, construction cost, and potential environmental impacts being major concerns. Offtracking of the design vehicle largely determines required template widths which has an effect on construction costs, potential site impacts such as sedimentation, and safety. For the forest road designer, the situation is complicated by many different types of vehicle configurations and a complex set of road geometrics. Many low volume forest roads have small radius reverse and compound curves or single curves connected by short tangents. The forest road designer must also take into account the swept paths of load overhangs on vehicle configurations. These can be in the form of a down rigged yarder tower or the overhang when hauling tree length logs.

Many of the analytical techniques that are currently available are not capable of analyzing the offtracking of truck and trailer combinations or the path of load overhangs through a set of curves. The importance of analyzing a set of curves is that the path of the rear wheels is dependent upon the road geometrics both ahead and back of the vehicles current position. A microcomputer program, OFFTRACK, was developed to simulate a truck and trailer combination negotiating a set of curves and tangents up to one mile in length. The program was written for IBM or compatible computers that are operating under DOS 2.0 or higher. The program will predict required road widths for six different vehicle configurations. It will also plot the path of a load overhang on any of the vehicle types. The numeric results are in the form of required road widths for

vehicle passage left and right of the centerline of the road. To promote further understanding of the numeric results, a plot of the wheel and load paths may be viewed on the screen or sent to a Hewlett-Packard or compatible drum plotter.

The results of the microcomputer program have been tested with the results from several field studies and a scale model simulator. The program generally agrees within one foot of the field studies and within 0.5 feet of the scale model simulator.

1.2 Organization

This report is a compendium of three papers that are either submitted or planned to be submitted in several professional engineering journals. As such, some repetition of subject matter is inevitable. A summary of the journal articles by chapter heading are as follows:

- 2 A general article that has been submitted for the Proceedings of the 1989 Council on Forest Engineering meeting. It is an overview of the methods that are currently available for analyzing offtracking, objectives and features of the OFFTRACK program, and the results of the testing that was done.
- 3 A technical article that has been submitted to the Journal of Forest Engineering. It covers the theory behind the OFFTRACK program and a summary of the results of the testing that was done.
- 4 A technical article that is planned to be submitted for the Fifth International Conference on Low-Volume Roads in 1991 sponsored by the Transportation Research Board. It is a discussion of the techniques used to develop the OFFTRACK program.

Finally, chapter 5 presents conclusions and recommendations for implementation and further development that is needed.

2 Offtracking of Truck and Trailer Combinations Using Forest Roads

By¹Thomas W. Erkert and John Sessions

Abstract

Many of the analytical techniques currently available are not capable of analyzing the offtracking of truck and trailer combinations or the path of load overhangs through sets of curves that are commonly found on forest roads. Advances in the processing speed of microcomputers have made it possible to simulate how a truck and trailer combination negotiate a road. A microcomputer program, OFFTRACK, was developed to simulate the offtracking of six different vehicle configurations. The program will enable road designers to predict offtracking through a complex set of road geometrics, including compound and reverse curves. The program will also analyze the path of load overhangs on a vehicle such as a yarder tower on a lowboy or rubber tired undercarriage. A discussion of the methods currently used to predict offtracking are presented along with the advantages and disadvantages of each. Test results of the program are presented and compare favorably with several field studies and a scale model simulator. The vehicle types that are included in the program are log truck, lowboy, lowboy with a jeep, lowboy with a jeep and a pup, short logger with a trailer, and a non-hinged vehicle such as a bus. Outputs of the computer simulation are in the form of road widths required left and right of centerline, total width, and curve widening. A plot of the paths of the wheels

¹ The authors are respectively, Civil/Logging Engineer USDA Forest Service and Professor Department of Forest Engineering, Oregon State University, Corvallis, Or 97331-5706. The mention of trade names of commercial products in this article does not constitute endorsement or recommendation by the authors, Oregon State University, or United States Department of Agriculture.

and load overhang can be viewed on the screen or sent to a Hewlett-Packard 7580B/7585B or compatible drum plotter. A discussion of further development of the program is also presented.

2.1 Introduction

The need to accurately determine the offtracking of truck and trailer combinations is one of the primary concerns of road designers. Offtracking has been defined as the difference in the paths of the inside front wheel and of the inside rear wheel as a vehicle or a combination vehicle negotiates a curve [5]. This would be the difference between R1 and R2 in Figure 2-1. Another definition commonly used is the difference in the paths of the centerline of the front and rear axles.

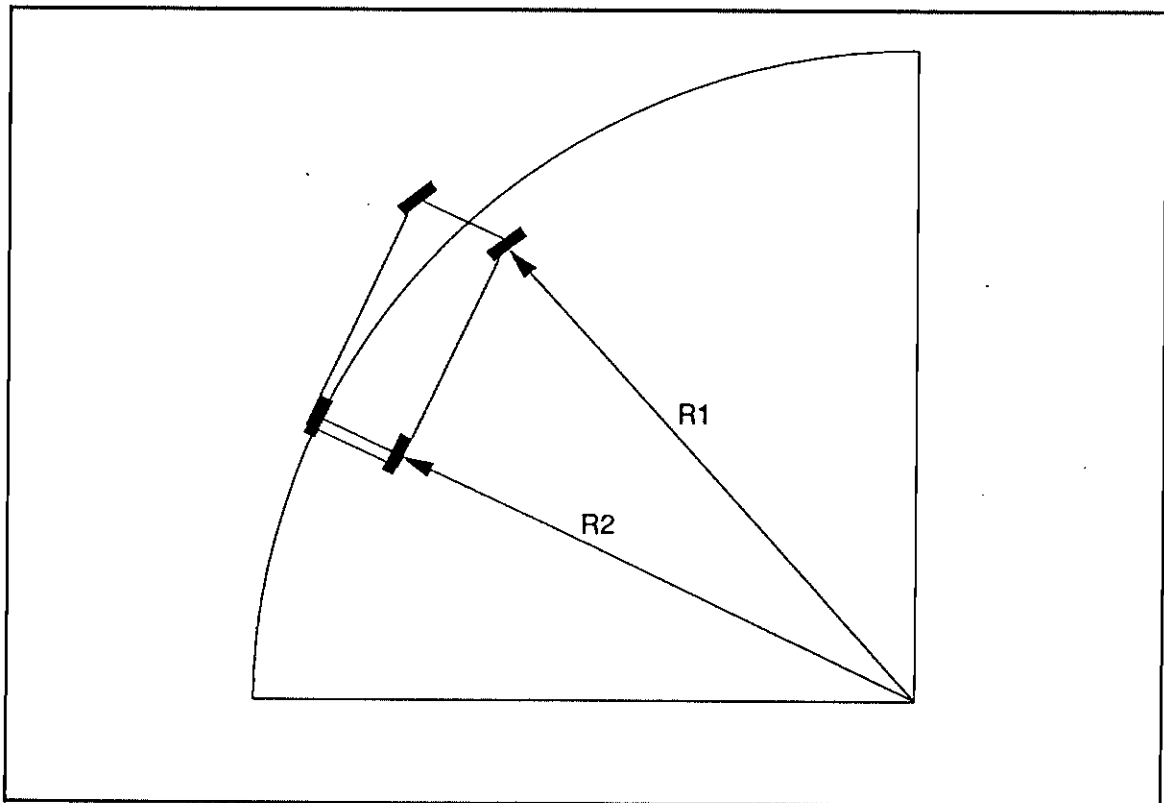


Figure 2-1. Definition of Offtracking

Thousands of miles of forest road are constructed each year with safety, construction cost, and potential environmental impacts being major concerns.

Offtracking of the design vehicle largely determines required travelway widths which has an effect on construction costs, potential site impacts such as sedimentation, and safety. For the forest road designer, the situation is complicated by many different types of vehicle configurations and a complex set of road geometrics. Many low volume forest roads have small radius reverse and compound curves or single curves connected by short tangents. The forest road designer must also take into account the swept paths of load overhangs on vehicle configurations. These can be in the form of a down rigged yarder tower or tree length logs.

Many of the available analytical techniques are not capable of analyzing the offtracking of truck and trailer combinations or the path of load overhangs through a set of curves [2, 14]. A microcomputer program, OFFTRACK, was developed to simulate a truck and trailer combination negotiating a set of curves and tangents up to one mile in length. The program will predict required road widths for six different vehicle configurations. It will also plot the path of a load overhang on any of the vehicle types. The importance of analyzing a set of curves is that the path of the rear wheels is dependent upon the road geometrics both ahead and back of the vehicles current position.

This paper presents the methods that are currently being used to analyze offtracking, describes the OFFTRACK program that was developed, and presents the results of testing the OFFTRACK program with several field studies and a scale model simulator.

2.2 Current Methods

There are five methods that are currently being utilized in varying degrees for the determination of offtracking. All of the methods discussed are for use with vehicles that use Ackerman type steering and trailing axles that are fixed perpendicular to the longitudinal axis of the vehicle. The following briefly discusses each method and the advantages and disadvantages of each. For a full discussion of any particular method, the reader should review the referenced literature.

2.2.1 Direct Solution of Maximum Offtracking

The best known method for the direct solution of maximum offtracking is that of the equations in the Society of Automotive Engineers Handbook [14]. These equations are commonly known as the "Sum of the Squares of Wheelbases". The principle behind the equation is shown in Figure 2-2. The maximum offtracking is obtained when a projected line through the centerline of a trailing axle intersects the radius point of the curve.

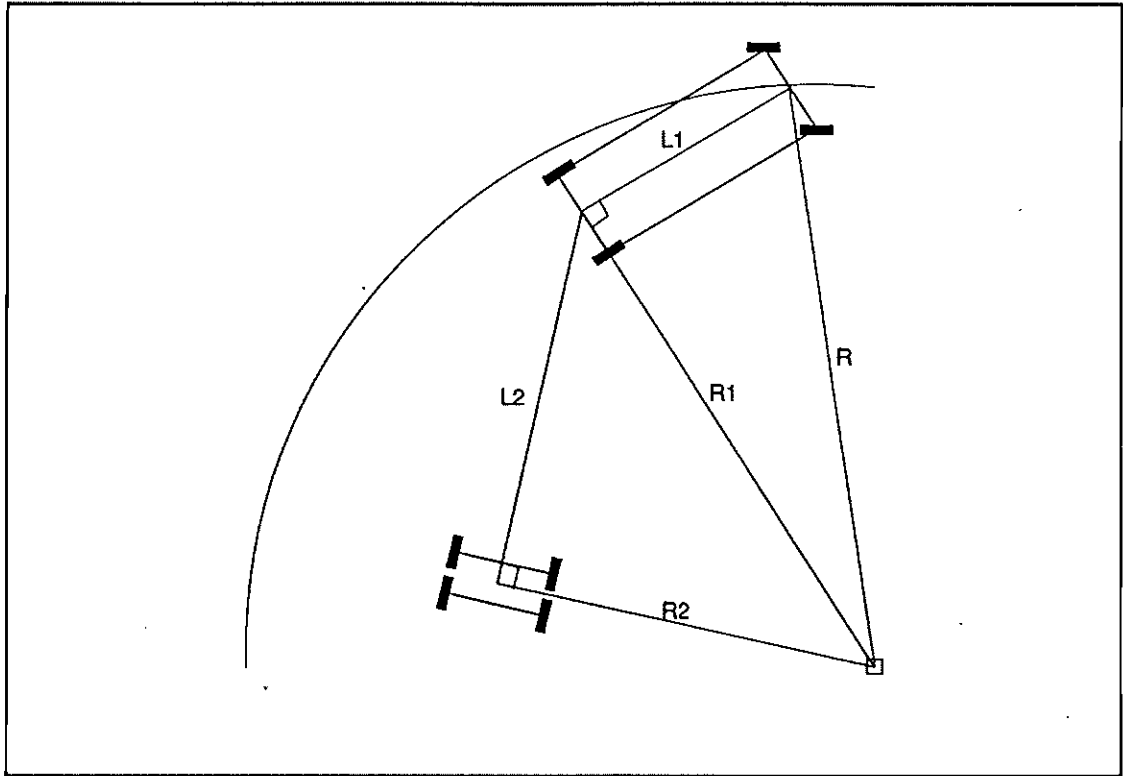


Figure 2-2. Sum of the Squares of the Wheelbases

The maximum offtracking (OT) of the vehicle combination in Figure 2-2 relative to the longitudinal centerline of the vehicle can be solved for as:

$$OT = R - \sqrt{R^2 - L_1^2 - L_2^2} \quad (1)$$

A full discussion of the use of this method can be found in a Western Highway Institute publication [11]. The main advantage of this approach is that any vehicle configuration can be analyzed fairly quickly. The disadvantages are:

1. Not valid for curves that are not long enough for the vehicle combination to develop maximum offtracking.

2. Not capable of analyzing transitional wheel paths. Many curves are not long enough for maximum offtracking to develop. The path that the rear wheels follow up to maximum offtracking is called the transitional wheel path.
3. Not capable of analyzing compound and reverse curves.

The shortcomings of this approach has lead to an empirical approach that is discussed next.

2.2.2 Empirical Methods

The USDA Forest Service developed an empirical method to predict offtracking from the sum of the squares of the wheelbases and to take into account curves that are not long enough to develop maximum offtracking [2]. The technique used a regression equation based on the sum of the squares of the wheelbases and modified it to take account of the curve central angle. The regression equation was developed from charts that were produced from a set of tractrix equations published by the San Dimas Equipment Development Center [3]. The regression equation was tested with a field test on a log truck and a scale model simulator for a lowboy vehicle combination [10]. The equation reported is:

$$C = (R - \sqrt{R^2 - L^2}) \left[1 - e^{\left(-.015\Delta \frac{R}{L} + .216\right)} \right] \quad (2)$$

Where:

C = Total vehicle offtracking

R = Radius at the centerline of the roadway

e = Base for natural logarithms

Δ = Central Angle

$$L = \sqrt{\sum_{i=0}^n L_i^2}$$

L_i = The wheelbase of a vehicle unit

This approach utilizes the fact that regardless of the vehicle configuration, as long as the sum of the squares of the wheelbases and the radius of the turn are identical, the maximum offtracking will be identical [11]. This approach is widely used by the USDA Forest Service and the USDI Bureau of Land Management. It is used in the form of design charts that give the required road width for a standard log truck and lowboy for radii of 50 to 300 feet and central angles of 10 to 180 degrees. A two foot steering correction is added to the result of the regression equation within the design charts. The advantages to using this approach are that it is quick, easy to use, and relatively accurate. However, it is only valid for one simple curve with long tangents at each end of the curve. The disadvantages are:

1. Not capable of analyzing transitional wheel paths.
2. Not capable of analyzing a load overhang path.
3. Not capable of analyzing compound and reverse curves.

2.2.3 Scale Models

Because of the deficiencies of the previous methods, scale model simulators have been used to analyze offtracking of vehicle combinations through a set of curves. There are three scale models that have been developed, U.S. Army Vee-Trans [5],

USDA Forest Service Drafting Vehicle Simulator (DVS) [7, 8], and the CalTrans Tractrix Integrator [10]. Each of the scale models are slightly different but are used with the following procedure:

1. The road traverse is drawn to a large scale. Scales used range from 1 inch = 5 feet for the Vee-Trans to 1 inch = 10 feet for the DVS.
2. The model is adjusted to the dimensions of the vehicle unit or combination.
3. The model is carefully guided over the traverse drawing and a set of pens trace the paths of the rear most wheels.

The scale models have been found to be fairly accurate when compared to field tests of actual equipment [10]. The DVS is probably the most useful of the scale models for forest road designers. It is capable of analyzing four different vehicle combinations directly. It will also plot the path of a load overhang for a down rigged yarder tower on a rubber tired undercarriage. The disadvantages of the scale models are that they are somewhat cumbersome to use for designing road sections of any length.

2.2.4 Field Simulators

Several field vehicle simulators have been designed by the USDA Forest Service for analyzing offtracking of a single unit or a two unit vehicle combination on existing roads. These models are constructed out of light weight aluminum tubing and are adjustable in length to simulate different wheelbases. No published results of the accuracy of these models exist in the literature to date. However, these simulators have proven to be quite useful in practice and are felt to be fairly accurate. The

disadvantages of these are:

1. Only a single unit vehicle and a tractor-trailer type vehicle are available.
2. The technique is of course not applicable to new construction design work.

2.2.5 Computer Simulation

Four computer simulation techniques have been developed from 1976 - 1986. Two of the techniques are not applicable to the forest environment as they were developed to look specifically at problems in urban intersection design [1, 4]. The remaining two techniques use very different approaches to the problem. The two techniques are summarized as follows:

Tractrix Algorithm [3]

There exists a general mathematical description of the path that the rear of a vehicle follows from a given input steering curve. A steering curve is defined as the set of geometrics that guide the leading vehicle in a vehicle combination. The steering curve can be defined as circular or a straight line. The path of the trailing axle is known as the general tractrix of the original steering curve [6]. To find a closed form solution, the differential equations must be integrated. This process is extremely complex and not suited for general practice [12]. In 1976, Leonard Della-Moretta of the USDA Forest Service, derived a simplified set of the original equations for a single unit vehicle that could be programmed on a computer [3]. Unfortunately, the program and the algorithm were never fully developed for combination vehicles. The differential tractrix equations are the basis of the OFFTRACK program.

Graphical Algorithm [12]

Another interesting technique is outlined by Michael Sayers [12]. In this algorithm, a series of geometric relationships were derived to describe the spiral path of the trailing vehicle unit. The centerline of the front axle of the first vehicle unit in the combination is moved along the steering curve in one foot increments. The approach can be done by hand but is of course too laborious for practical use. The general procedure is to:

1. Extend a line towards the center of the curve along the path of the rear axle of the unit.
2. Extend a line from the centerline of the front axle on the steering curve through the center of rotation of the steering curve. The point of intersection of the lines in step 1 and step 2 is the instantaneous center of rotation of the rear axle.
3. Move the centerline of the front axle along the steering curve a distance of one foot.
4. Strike an arc from the new front axle location across the path of the centerline of the rear axle. The arc has a radius equal to the wheelbase of the vehicle unit.
5. Strike an arc from the center of rotation of the rear axle across the arc from step 3. The arc has a radius equal to the distance from the center of rotation of the rear axle to the centerline of the rear axle in it's previous

location. The point of intersection of the arc's in step 3 and step 4 is the new location of the centerline of the rear axle. This completes the cycle of one movement, return to step 1 and continue.

The computer program uses a set of equations to accomplish the same set of steps above but for multi-unit vehicles. An absolute coordinate system is used to keep track of the vehicle locations. The main advantages of the program is that it is capable of analyzing any vehicle configuration and will plot the swept path of the outside front wheel and the inside rear wheel. The disadvantages of the program are:

1. It was programmed on an Apple II which is not as common in the design sector as IBM and compatible machines.
2. It is slow to execute. It takes from several minutes to 20 minutes to compute the paths for a single curve. Plotting is done on a dot matrix printer which takes another 5 to 15 minutes. The plot is done in four pieces and must be taped together [12].
3. No direct output of the required road widths is available. The dot matrix plot must be done and the widths scaled off of the plot.
4. The analysis is only accomplished in one direction. Forest roads are usually terminal facilities that the vehicle must drive in, remove the load, and drive back out.
5. The program is currently not capable of analyzing load overhang paths.

2.3 OFFTRACK Computer Simulation Program

The OFFTRACK computer simulation program was developed specifically for the forest road situation. It corrects most of the disadvantages of the currently available techniques discussed in section 2.2. It is programmed for the IBM and compatible family of computers. The tractrix algorithm is used as the method for describing how a vehicle negotiates a given road [3]. Much development work was done in linking multi-unit vehicles together as the original algorithm was for a single unit vehicle. The theory behind the program is not presented here but is discussed in other articles by the authors [16].

2.3.1 Objectives

The OFFTRACK program was designed to be a production tool to enable forest road designers to determine quickly and accurately required road widths and load overhang paths. The following objectives were established:

1. Easy to use with a simple user interface.
2. Capable of analyzing offtracking through a complex set of geometrics.
3. Capable of analyzing offtracking both ahead and back automatically.
4. Capable of analyzing load overhang paths.
5. Capable of determining required road widths left and right of centerline directly.
6. Capable of analyzing most of the vehicle combinations found on forest roads.

7. Accurate and execute quickly.
8. Provide the user with a graphical display of the wheel and load paths to promote understanding of the numeric results.

2.3.2 Program Features

The program features a pull down menu structure and on line help for ease of use. Each screen has instructions at the bottom to inform the user of valid keys and editing options. The main menu structure is shown in Figure 2-3.

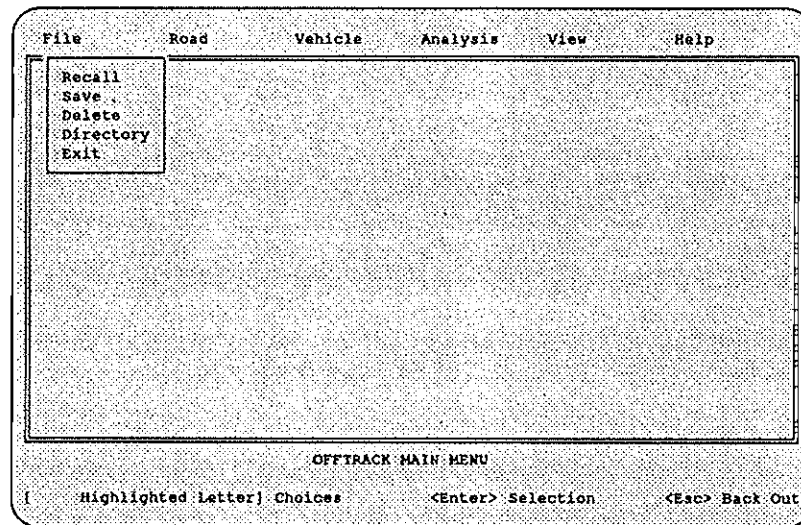


Figure 2-3. OFFTRACK Main Menu

The data inputs to the program are 1) geometry of the road segments, 2) design parameters and starting conditions, 3) definition of the vehicle configuration.

1) Geometry of Road Segments

The geometry of the road segments is input with the use of a spreadsheet type editor. Up to 100 road segments can be used. The inputs are the radius, and

the beginning and ending distance of the segment. A sign convention is used for the radius to indicate absolute direction. Curves that are clockwise in the ahead station direction are defined as positive and curves that are counter clockwise are negative. Figure 2-4 shows the road segment input screen.

The screenshot shows a terminal window titled "Road Segment Input Screen" with the following data:

File: MULTIPLE.SEG	** Road Segment Input Screen **	
Radius (ft)	Beginning Distance	Ending Distance
(+ Counter Clock)		
1	0.00	150.00
2	-50.00	307.08
3	0.00	407.08
4	-50.00	564.16
5	0.00	714.16
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		

At the bottom of the screen, the following keyboard shortcuts are listed:

- [Enter] <Tab> PgUp PgDn] Between Fields [] Within Field <Esc> Menu
- <F1> Ins Row <F2> Del Row <Ctrl End> Del Field <Ins> Toggle Insert

Figure 2-4. Road Segment Input Screen

2) Design Parameters and Starting Conditions

Design parameters consist of the minimum travelway width for design, and the minimum steering correction. A two foot minimum steering correction is customary to account for driver error. The starting conditions consist of beginning coordinates and azimuth of the first segment. Figure 2-5 shows the road design input screen.

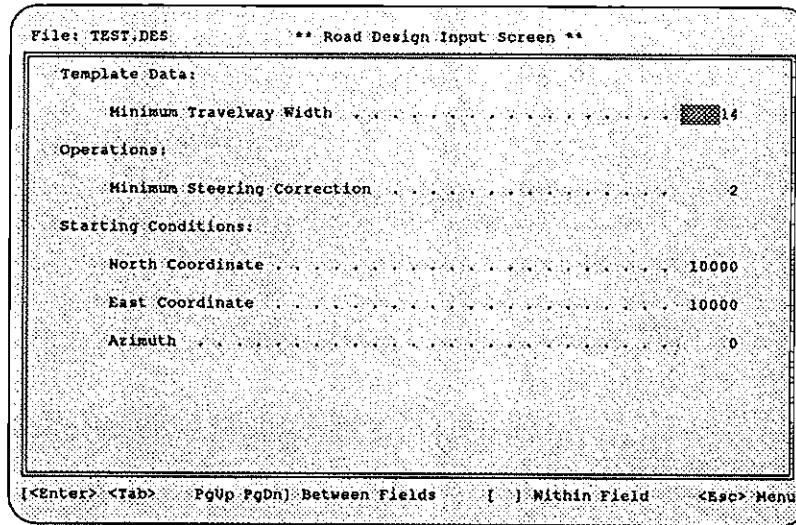


Figure 2-5. Road Design Input Screen

3) Definition of Vehicle configuration

The program is capable of analyzing six different vehicle types as shown in Figures 2-6 through 2-11 below.

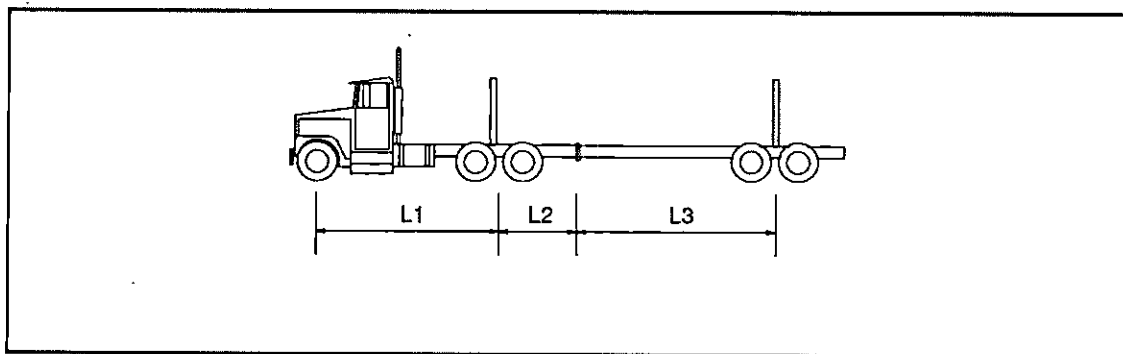


Figure 2-6. Log Truck Configuration

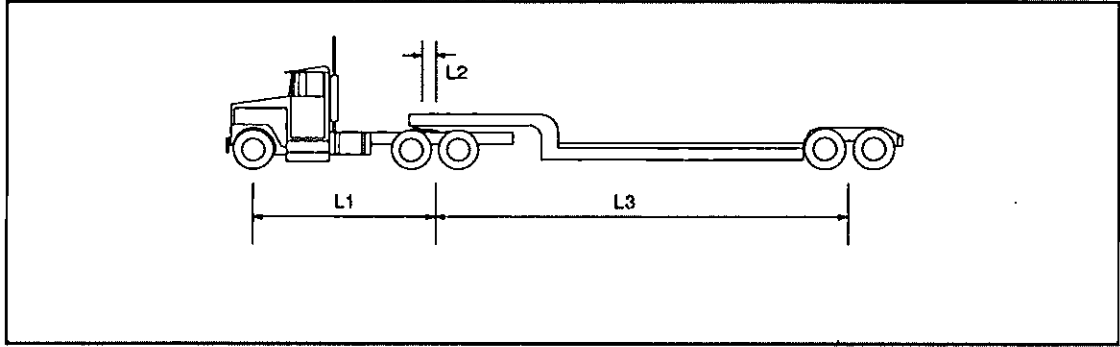


Figure 2-7. Lowboy Configuration

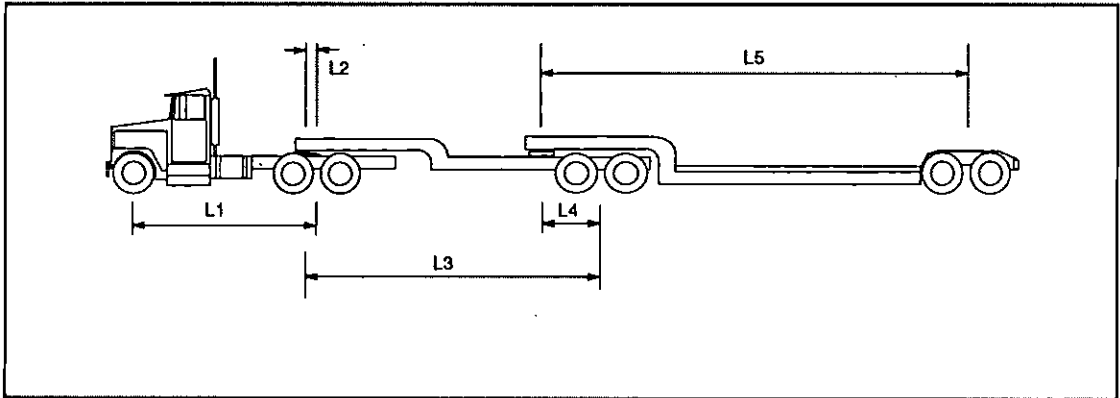


Figure 2-8. Lowboy with Jeep Configuration

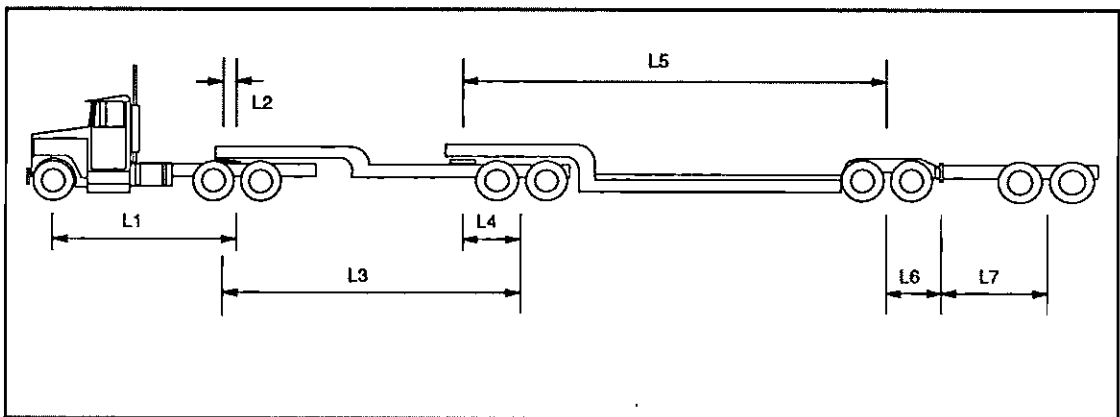


Figure 2-9. Lowboy with Jeep and Pup Configuration

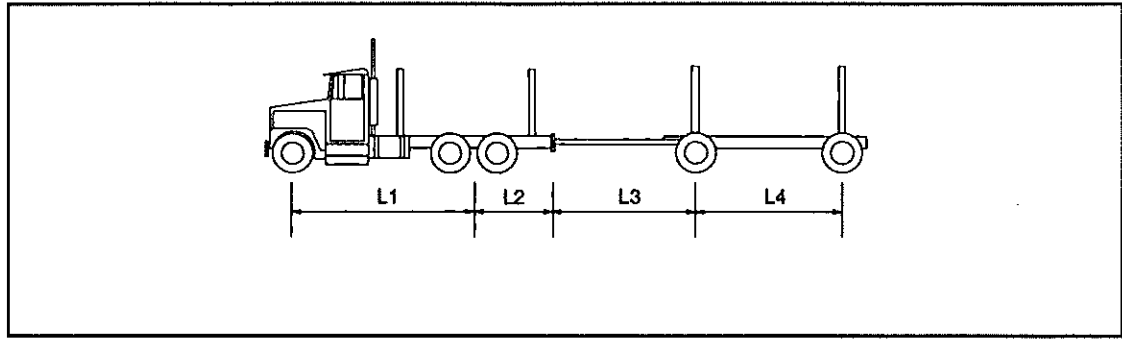


Figure 2-10. Short Logger with Trailer Configuration

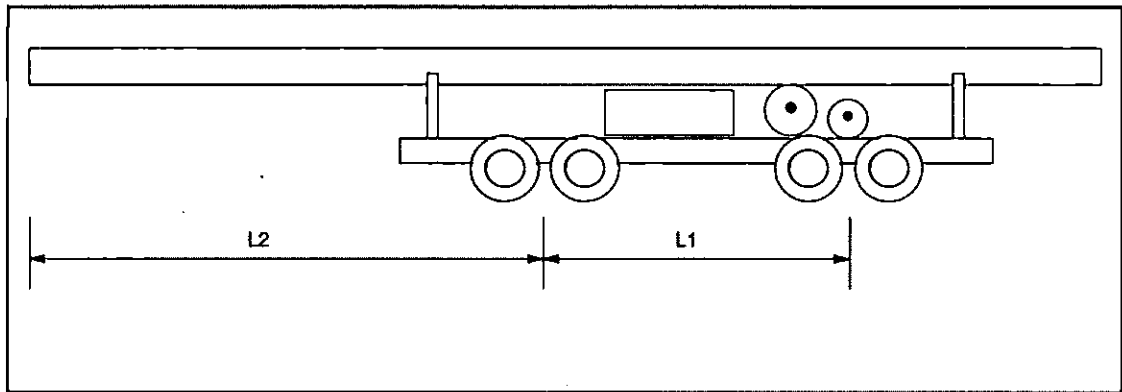


Figure 2-11. Non-Hinged Configuration (Yarder)

The inputs for any given vehicle are the wheelbases and hinge point offsets for each unit in the combination as denoted in Figures 2-6 through 2-11. Load overhangs can be specified either ahead of the hinge point or behind the rear axle for the lowboy and non-hinged type vehicles or behind the rear axle for the log truck and short logger type vehicles. Each vehicle has an input screen similar to Figure 2-12.

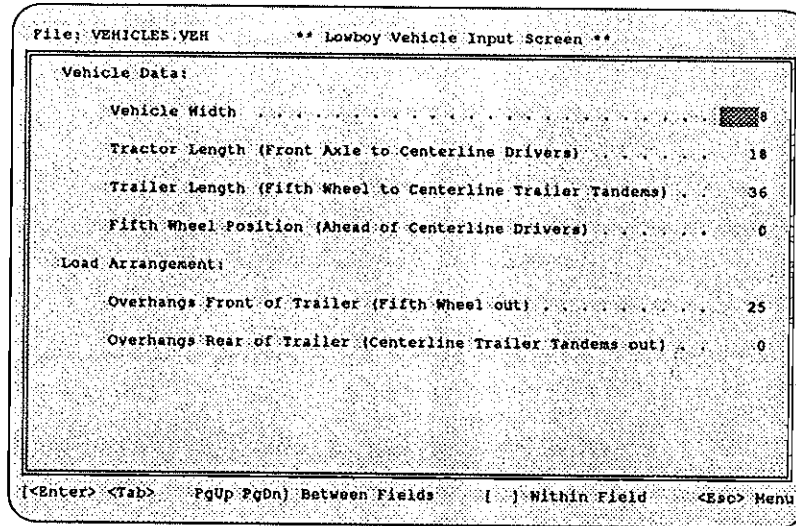


Figure 2-12. Vehicle Configuration Input Screen

The outputs of the simulation are in the form of numeric and graphical results. The numeric results can be viewed on the screen and sent to a printer. The numeric results are required road widths left and right of the centerline of the front axle, total width, and curve widening in one foot increments. It should be noted here that the simulation is done by guiding the centerline of the front axle on the centerline of the road. Therefore, the widths reported are relative to the centerline of the road. All widths reported have the steering correction added in. The curve widening is simply the total required road width less the minimum travelway width. The numeric results screen is shown below in Figure 2-13.

**** Results ****					
Section: 2	Begin Distance: 150	End Distance: 307.07			
Radius: 50	Curve Delta: 180	Vehicle: Lowboy			
Distance	Delta	Width Left	Width Right	Total Width	Widening
150.00	0	5.00	18.72	23.72	9.72
151.00	1	5.00	18.71	23.71	9.71
152.00	2	5.00	19.06	24.05	10.05
153.00	3	4.99	19.39	24.38	10.38
154.00	4	4.99	19.71	24.70	10.70
155.00	5	4.98	19.71	24.69	10.69
156.00	6	4.97	20.01	24.99	10.99
157.00	8	4.97	20.30	25.27	11.27
158.00	9	4.96	20.58	25.54	11.54
159.00	10	4.95	20.58	25.53	11.53
160.00	11	4.94	20.84	25.78	11.78
161.00	12	4.94	21.08	26.02	12.02
162.00	13	4.93	21.08	26.01	12.01
163.00	14	4.92	21.31	26.23	12.23
164.00	16	4.91	21.53	26.44	12.44

[PgUp PgDn] Scroll <Esc> Main Menu

Figure 2-13. Numeric Results Screen

To promote understanding of the numeric results, a graphical display of the wheel and load paths can be viewed on the screen or sent to a Hewlett-Packard 7580B/7585B or compatible drum plotter. The screen plot can be viewed for all road segments or for one segment at a time. The plot sent to the drum plotter can be plotted at any scale. This was done to enable the designer to plot the wheel and load paths at the same scale as the road traverse plot. The wheel and load path plot can then be overlaid under the road traverse plot to evaluate load paths relative to cut slope clearance. An example of a screen plot is shown in Figure 2-14.

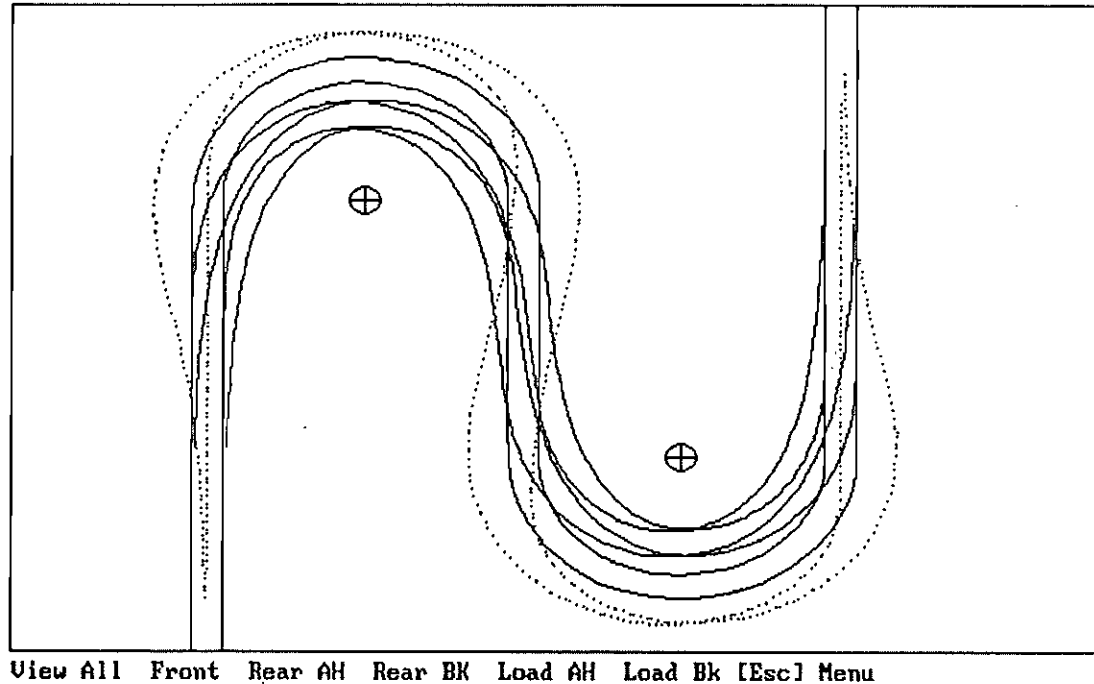


Figure 2-14. Screen Plot of Wheel and Load Paths

2.3.3 Results

The program was tested with the results of several field studies and the Drafting Vehicle Simulator (DVS). The field studies were used to test the program predicted offtracking for the log truck, lowboy, lowboy with jeep and pup for one simple curve with tangents on both ends. The DVS was used to test the program predicted wheel and load overhang path through a series of curves. A description of each test situation and the results follow.

Log Truck Field Test

A test was conducted with a loaded log truck by the USDA Forest Service in cooperation with Champion International at their yard in Bonner, Montana [2]. The test was conducted by marking curves with radii of 50 to 125 feet at centerline and

central angles of 19 to 180 degrees. The truck was driven around each curve with the driver doing his best to keep his left front tire on the mark. The path of the right rear most trailer tire was "observed" and the offtracking recorded. The authors note that measurements could be off up to 0.5 feet. The test was conducted in a unpaved yard. The truck that was used had dimensions of 19.0 foot tractor length, 9.0 foot stinger length, and 18.2 foot trailer length. A comparison of the OFFTRACK program and the field study for a 50 foot radius curve are given in Table 2-1.

Table 2-1. Log Truck Field Test vs. OFFTRACK 50 Foot Radius

		Offtracking (Ft.)							
Source	Curve Central Angle	Central Angle Ahead of P.C. (deg)							
		0	19	30	46	64	90	116	180
Field Test	19	1.8	2.1						
OFFTRACK		2.1	2.3						
Field Test	30	2.2	3.7	3.5					
OFFTRACK		2.3	3.6	3.2					
Field Test	46	2.2	4.6	5.0	3.7				
OFFTRACK		2.4	4.6	4.8	3.9				
Field Test	64	2.2	4.6	5.7	5.7	4.4			
OFFTRACK		2.2	4.5	5.2	5.5	4.4			
Field Test	90	2.2	4.6	5.7	5.8	6.3	4.8		
OFFTRACK		2.2	4.5	5.3	6.0	6.3	4.8		
Field Test	116	2.2	4.6	5.7	5.8	6.4	6.0	4.8	
OFFTRACK		2.2	4.5	5.3	5.9	6.4	6.7	5.0	
Field Test	180	2.2	4.6	5.7	5.8	6.4	6.0	6.8	4.7
OFFTRACK		2.2	4.5	5.3	5.9	6.4	6.7	6.8	5.1

The data in Table 2-1 indicate a good correlation between the field test and the OFFTRACK program. Most of the measurements agree within several tenths of a foot and the maximum deviation is 0.7 feet.

CalTrans Operational Test

As part of a broad operational test on longer combination vehicles, the California Department of Transportation conducted offtracking tests of a semi tractor trailer, Rocky Mountain Double, Turnpike Double, and Triple combinations in 1984 [9]. The tests were conducted with Viking Freight Systems, Inc. of Santa Clara California in their paved yard. The procedure of the test was that a line was painted for 60, 80, and 100 foot radius curves over a 180 degree central angle. The driver entered the curve on a tangent, placed the left front tire on the painted line through the curve and then exited on a tangent. The path of the right rear most tire was continually marked on the pavement. The OFFTRACK program was tested against the semi tractor trailer, Rocky Mountain Doubles, and the Turnpike Doubles. The semi tractor trailer was analyzed with the lowboy option and the Rocky Mountain and Turnpike Doubles were analyzed with the lowboy with a jeep and a pup option as the number of vehicle units were the same. There is currently no option in OFFTRACK that will simulate the Triples test. The dimensions of the vehicles tested can be obtained from the CalTrans publication [9]. The field test recorded swept width as the difference in radii measured to the left front wheel and the right rear wheel from the center of the curve. Measurements were recorded in 30 degree increments of central angle through the curve. A comparison of the OFFTRACK program versus the field study is given in Tables 2-2 through 2-4.

Table 2-2. CalTrans Field Test vs. OFFTRACK 60 Foot Radius

		Swept Width (Ft.)						
		Central Angle Ahead of P.C. (deg)						
Source	Vehicle	0	30	60	90	120	150	180
Field Test OFFTRACK	48' Semi	12.1	18.4	21.5	23.5	24.4	25.0	21.3
		13.0	18.9	21.9	23.5	24.5	24.3	20.2
Field Test OFFTRACK	Rocky Mtn. Double	15.1	22.5	26.5	28.7	30.1	28.9	23.6
		16.0	23.0	27.1	29.4	30.6	29.4	24.1
Field Test OFFTRACK	TurnPike Double	17.9	28.2	34.8	38.8	40.3	38.4	31.4
		20.7	30.3	36.6	40.5	41.8	40.1	33.6

Table 2-3 CalTrans Field Test vs. OFFTRACK 80 Foot Radius

		Swept Width (Ft.)						
		Central Angle Ahead of P.C. (deg)						
Source	Vehicle	0	30	60	90	120	150	180
Field Test OFFTRACK	48' Semi	10.7	16.1	18.2	19.1	19.6	19.8	16.2
		11.5	16.6	18.6	19.4	19.7	19.7	16.3
Field Test OFFTRACK	Rocky Mtn. Double	12.5	19.1	21.4	22.7	23.4	22.7	16.8
		13.4	19.6	22.2	23.3	23.8	23.4	18.6
Field Test OFFTRACK	TurnPike Double	14.4	23.0	27.1	29.1	30.4	29.4	20.4
		16.1	24.4	28.5	30.6	31.1	30.7	24.2

Table 2-4 CalTrans Field Test vs. OFFTRACK 100 Foot Radius

		Swept Width (Ft.)						
		Central Angle Ahead of P.C. (deg)						
Source	Vehicle	0	30	60	90	120	150	180
Field Test OFFTRACK	48' Semi	11.6	15.6	16.6	17.0	16.9	17.3	13.7
		10.7	15.2	16.5	16.9	17.0	17.0	14.3
Field Test OFFTRACK	Rocky Mtn. Double	12.2	18.0	19.3	19.5	19.7	19.8	16.4
		12.1	17.7	19.4	19.9	20.1	20.0	16.0
Field Test OFFTRACK	TurnPike Double	14.4	21.5	24.0	24.9	25.4	25.3	19.1
		14.1	21.5	24.4	25.5	26.0	25.6	19.9

Most of the swept width measurements in Tables 2-2 through 2-4 agree within one foot and the maximum deviation is 2.2 feet. In most cases the OFFTRACK program over predicts the swept width.

DVS Test

The DVS was used to test most of the vehicle configurations in the OFFTRACK program through a series of curves. The results of one of the tests is shown in Figures 2-15 and 2-16 for a large yarder on a rubber tired undercarriage like that shown in Figure 2-11. The vehicle used is similar to a Skagit BU-199 yarder with 110 foot tower and had the following dimensions in transport mode:

10.5 foot vehicle width plus 2 foot steering correction (total width of 12.5 feet)

21.0 foot vehicle length

36.5 foot load overhang ahead of the front axle

The test road was arbitrary and had the components as shown in Table 2-5.

Table 2-5. DVS Test Road Segments

Section	Length (Ft.)	Radius	Central Angle	Begin Distance	End Distance
1	100.0	0.0	0.0	0.0	100.0
2	120.0	50.0	137.5	100.0	220.0
3	20.0	0.0	0.0	220.0	240.0
4	120.0	-70.0	98.2	240.0	360.0
5	125.0	60.0	119.4	360.0	485.0
6	100.0	0.0	0.0	485.0	585.0

Figure 2-15 shows a comparison of wheel paths and Figure 2-16 shows a comparison of load paths. For Figure 2-16, a positive number indicates the load was on the left side of the road while a negative number indicates it was on the right side of the road of the road.

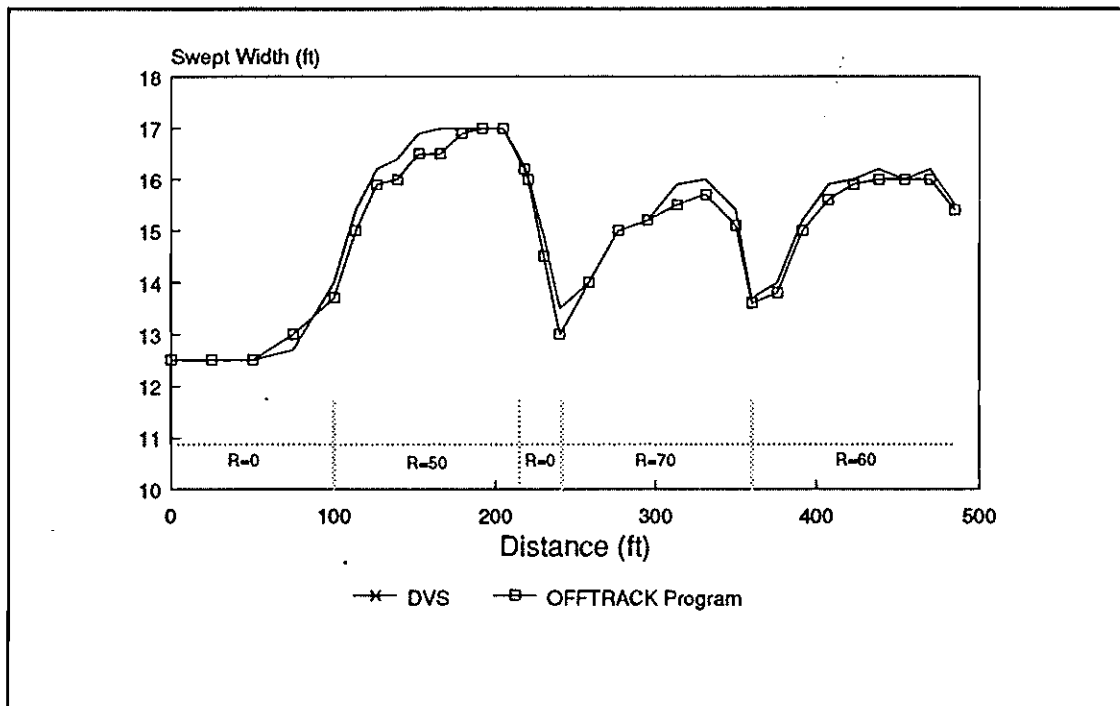


Figure 2-15. DVS vs. OFFTRACK Series of Curves

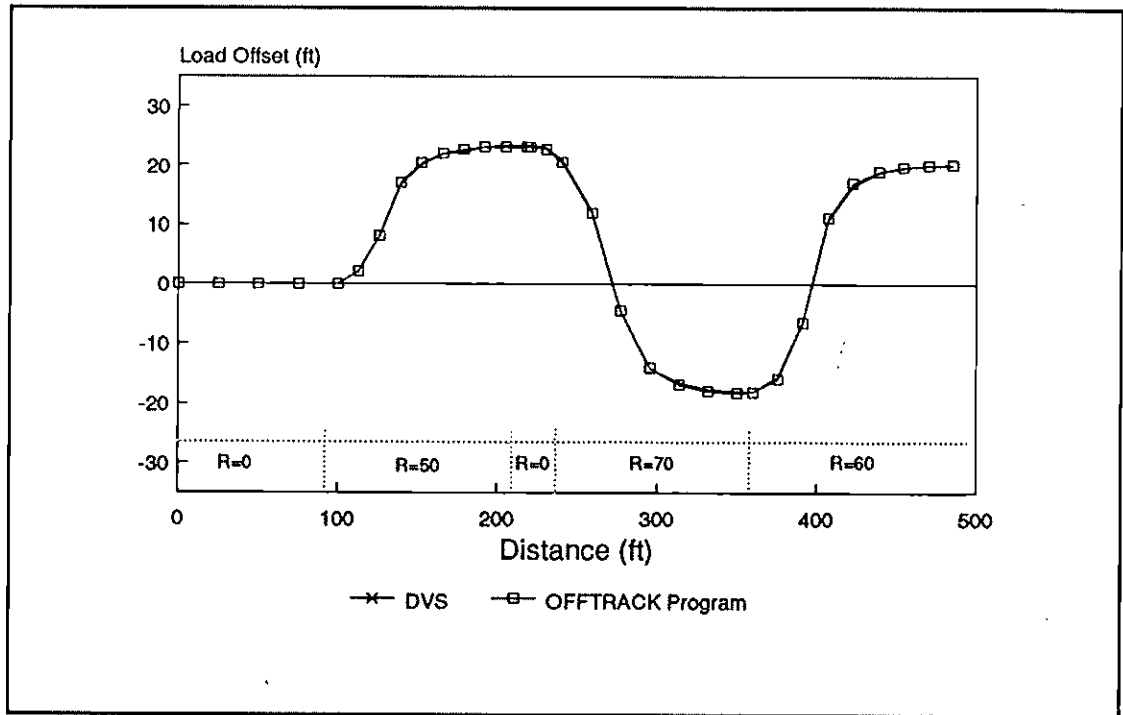


Figure 2-16. DVS vs. OFFTRACK Series of Curves Load Overhang

From the test results presented, it can be seen that the program is reasonably accurate. The test of the DVS versus the program are generally within 0.5 feet. The log truck field test is also within 0.5 feet. There is however an over estimation of swept widths for the Rocky Mountain Doubles and Turnpike Doubles in the CalTrans operational test. The program is accurate except for the shorter radius curves with the turnpike doubles combination where it over estimates the swept width by up to 2 feet. This disparity is probably due to slip and scrubbing of the tires or misalignment of axles [15]. The predicted offtracking for the field test on the log truck in a unpaved yard was very close to measured. This appears to support the influence of tire mechanics because of the lower coefficient of friction associated with the unpaved yard versus the paved yard. Tire mechanics could be taken into account by using a moment balance about the centerline of a rear axle group and adjusting the vehicle

length to achieve moment equilibrium. The inclusion of tire mechanics may not be justified for the lower frictional surfaces generally found on forest roads. The over prediction of swept width in the CalTrans test data is consistent with that reported by several other authors when comparing scale model simulators to actual equipment tests [2, 10]. In general, the program appears to predict offtracking reliably.

2.3.4 Applications

The program is well suited for determining required road widths for most vehicle configurations found operating on forest roads. It can be used for design in new construction and reconstruction. It will be especially useful on roads with many small radius curves that approximate compound and reverse curves. It will also be beneficial for the design of campground roads or roads leading to recreational sites where complex road geometrics and long vehicles are anticipated. The development of this program could be incorporated into an integrated road design package in which the designer would be relieved of the tedious task of determining required road widths. The program also has many applications in the urban environment for the design of intersections and the evaluation of new vehicle configurations.

2.3.5 Further Development

The program has several limitations for which improvements are planned in the future. The areas for further development are:

1. The ability to analyze any vehicle configuration instead of the six fixed vehicle configurations that are currently included.

2. The development of a "Driving Algorithm". This would allow the program to simulate how a driver actually drives a road. The program currently guides the lead vehicle unit by a fixed set of geometrics that are the centerline of the road. However, in practice drivers set themselves up for each curve based on the knowledge of the road segments ahead of them. This results in a input path that probably approximates a spiral instead of circular curves and tangents.
3. Account for tire slip and scrubbing for high friction surfaces.
4. Incorporate the procedure of this technique into a computerized road design system.

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3 A Method for Determining Offtracking of Multiple Unit Vehicle Combinations

By² Thomas W. Erkert, John Sessions, and Robert D. Layton

Abstract³

The required road width around curves on forest roads is largely determined by the difference in wheel paths between the inside front tractor wheel and the inside rear trailer wheel. This difference, known as offtracking, is a function of the vehicle and road geometry. This paper presents a method for determining the offtracking for common multiple unit vehicle combinations travelling over forest roads. The computational method numerically integrates the differential equations which compute the tractrix of a unit within the combination. The path that the rear of a vehicle follows from a given steering curve is called a tractrix. A unique three-point solution method is used to determine the instantaneous center of rotation for trailers in the vehicle combination. The method is shown to have good agreement with experimental data. It is suitable for use on microcomputers for single and multiple curves. A microcomputer program, OFFTRACK, was developed using this methodology.

² The authors are respectively, Civil/Logging Engineer USDA Forest Service, Professor Department of Forest Engineering, and Professor Department of Civil Engineering, Oregon State University, Corvallis, Or 97331-5706. The mention of trade names of commercial products in this article does not constitute endorsement or recommendation by the authors, Oregon State University, or United States Department of Agriculture.

³ An abridged copy of this paper was submitted to the Journal of Forest Engineering.

3.1 Introduction

When a tractor trailer combination enters a curve, the centerline of the trailer axle begins to trace a path that is different from the centerline of the tractor front axle. This difference in paths, which also coincides with the difference in paths between the inside front tractor wheel and the inside trailer wheel, is called the offtracking of the vehicle. Offtracking largely determines travelway width around curves on forest roads. This paper describes a method that can be used to determine offtracking of multiple unit vehicle combinations that use Ackerman type steering. All axles are constrained perpendicular to the longitudinal axis of the vehicle. The approach is suitable for low speed vehicle operation. Vehicle dynamics and tire mechanics are not considered. The method has been programmed for a microcomputer and will simulate offtracking of six different vehicle combinations commonly found on forest roads. A microcomputer program, OFFTRACK, has been developed to implement this methodology. The path of front and rear overhanging objects such as yarder towers or tree length logs is also computed. Notation that is used is given near the end of this paper.

3.2 Tractrix Equation

The mathematical description of the path that the rear axle of a vehicle follows from a given steering curve for low speed maneuvers is called the tractrix of the steering curve [3]. A steering curve is defined as the set of geometrics that guides a vehicle unit. For a multiple unit vehicle combination, a tractrix is also formed by the path of the hitch point for each trailer. The path of the hitch point becomes the steering curve of the trailer whose rear axle will follow a tractrix of the hitch point. The problem of determining offtracking for multiple unit vehicle combinations is solved by finding the solution of a consecutive series of tractrix curves. It is difficult to obtain a closed form solution for the tractrix curve. However, a numerical integration can be easily done for each tractrix. The steering curve for each trailer hitch point is determined through a simple curve fitting technique.

The method presented here represents the vehicle combination in what is known as a bicycle wheel model [9]. Each axle group is represented by a single wheel at the centerline of the longitudinal axis of the vehicle unit. Tandem axles are represented by a single wheel at the centerline of the bogie. The bicycle wheel model may not be appropriate for vehicle units that have axles within a group that are spaced relatively far apart [6]. For tandem axles that are spaced on four to five foot centers, the bicycle wheel model is adequate.

For a single unit vehicle combination, the formulation of the general case tractrix problem begins with the definition of the tractrix angle α . The tractrix angle is defined as the angle between the vehicle unit heading and the path heading. The path heading is the heading of a line that is tangent to the curve at a particular point (Figure 3-1).

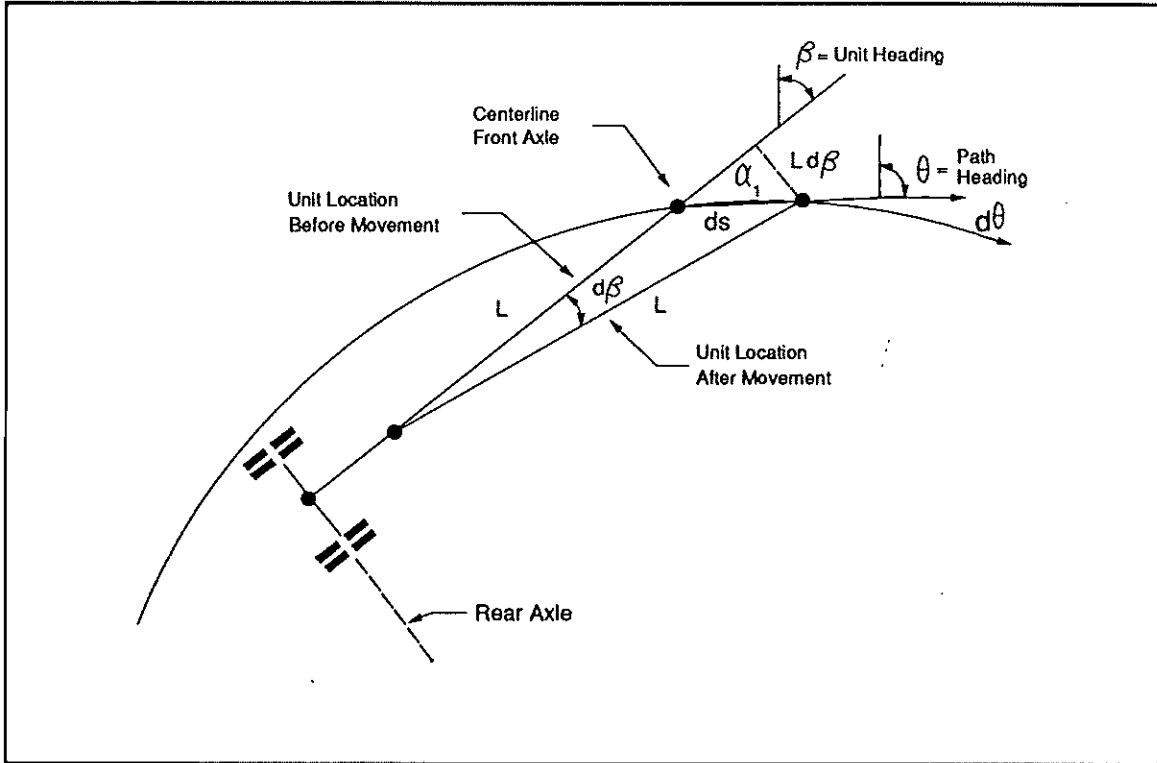


Figure 3-1. General Case Tratrix Development

The general case tratrix equation is formulated by moving the centerline of the front of the vehicle unit ahead along the steering curve a small distance ds , Figures 3-1 and 3-2. The rear of the vehicle unit will then begin to follow the direction β that the vehicle unit was headed previously [2]. The incremental distance ds should be no more than one foot or the assumptions of small angle theory will not apply [9]. For very small movements, the geometry will approach the situation as shown in Figure 3-2.

$$d\theta = \theta_2 - \theta_1$$

$$\theta_1 = \theta_2 - d\theta \quad (3)$$

$$\theta_1 = \beta_1 + \alpha_1 \quad (4)$$

$$\theta_2 = \beta_2 + \alpha_2 \quad (5)$$

Substituting equation (3) into equation (4) yields:

$$\beta_1 + \alpha_1 = \theta_2 - d\theta$$

$$\theta_2 = \beta_1 + \alpha_1 + d\theta \quad (6)$$

Setting equation (6) equal to equation (5) yields:

$$\beta_1 + \alpha_1 + d\theta = \beta_2 + \alpha_2$$

$$(\beta_1 - \beta_2) + (\alpha_1 - \alpha_2) + d\theta = 0$$

Multiplying through by -1 yields:

$$d\theta = (\beta_2 - \beta_1) + (\alpha_2 - \alpha_1)$$

$$d\theta = d\beta + d\alpha$$

$$d\beta = d\theta - d\alpha \quad (7)$$

Substituting equation (7) into equation (2) yields:

$$Ld\theta - Ld\alpha = ds \sin \alpha_1 \quad (8)$$

For a circular curve, substituting equation (1) into equation (8) yields:

$$\frac{Lds}{R} - Ld\alpha = ds \sin \alpha_1$$

$$d\alpha = ds \left[\frac{\frac{L}{R} - \sin \alpha_1}{L} \right] \quad (9)$$

Equation (9) is the differential equation to obtain the next tractrix angle once the vehicle unit moves up on the curve. Equation (7) is then used to update the heading of the vehicle unit. A standard latitude and departure coordinate system is used for recording hitch, axle, and wheel locations. Adoption of a sign convention allows equation (9) to be used for simulation of road segments with reversing curves. The sign convention used is defined as clockwise curves have a positive radius while counter-clockwise curves have a negative radius in the direction of travel.

For all vehicle units except the lead unit, equation (9) is not absolute. This is because the input radius is not of a circular curve but that of a tractrix. However, if the incremental distance ds is kept small it will approach a circular curve. The equations presented above are similar to those presented by Della-Moretta and Cisneros [2] for a single unit. The methodology developed here uses equations (7) and (9) for multiple unit vehicle combinations by linking the units through a series of steering curves.

3.3 Instantaneous Center of Rotation

When a multi-unit vehicle combination travels through a curve, it is an assemblage of rigid bodies each rotating about their own instantaneous center (Figure 3-3). If the instantaneous center of each vehicle unit can be determined, then the instantaneous radius of the steering curve can be found. This instantaneous radius is required for numerical integration of equation (9) for each vehicle unit. Equation (9) also requires the distance ds that the hitch point has moved. This can be found from the coordinates of the current and previous hitch location as shown in Figure 3-5.

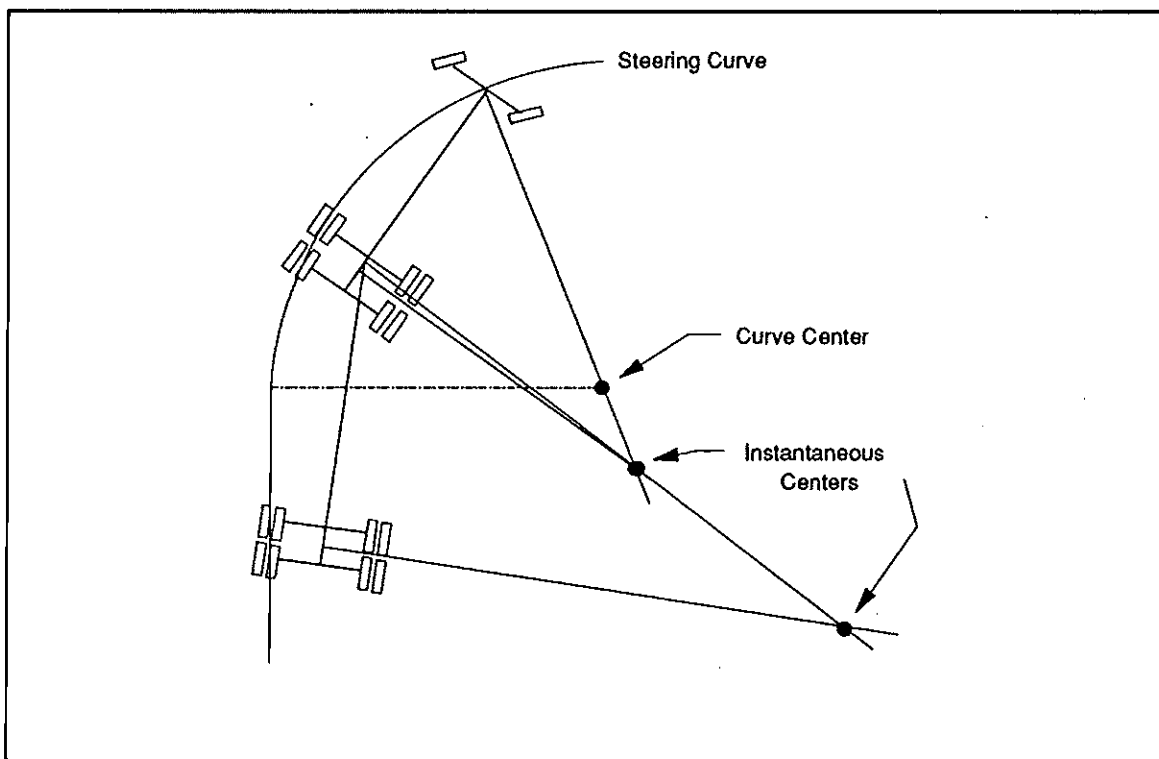


Figure 3-3. Instantaneous Centers of Rotation

3.3.1 Hinge Point Over Centerline of Rear Axle

The instantaneous radius of the hitch point for a following vehicle unit when the hitch point is directly over the centerline of the axle group, can be determined from the tractrix angle and the length of the vehicle unit. The instantaneous radius can be determined from equation (10):

$$R = L \left[\tan \left(\frac{\pi}{2} - \alpha \right) \right] \quad (10)$$

This approach was compared to the maximum offtracking that can be determined by hand computation using the "Sum of the Squares of the Wheelbases" equation [7, 10] for a 50 foot radius curve at the centerline of the front axle. A three unit vehicle combination similar to that shown in Figure 3-4 was used for two cases:

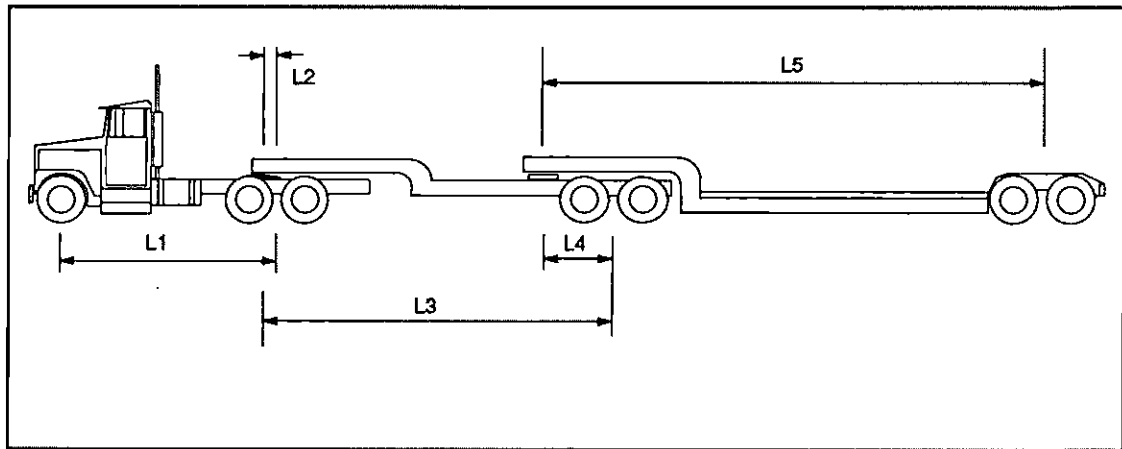


Figure 3-4. Vehicle for Instantaneous Center Test

Case 1

This case used the following vehicle dimensions with the offset (L4) of the hitch point on the second vehicle unit set to 0 feet.

L1 = 18 feet, L2 = 0 feet, L3 = 20 feet, L4 = 0 feet, L5 = 36 feet, Width = 8 feet

The maximum swept width from the Sum of the Squares of the Wheelbases equation was found to be 35.8 feet while the maximum swept width using equations (7), (9), and (10) was 32.6 feet. Swept width is the difference in radii measured to the outside front wheel and the inside rear wheel from the center of the curve.

Case 2

This case used the same vehicle dimensions as in case 1 except the offset (L4) of the hitch point on the second vehicle unit was set to 10 feet. The maximum swept width from the Sum of the Squares of the Wheelbases equation was found to be 33.7 feet while the maximum swept width using equations (7), (9), and (10) was 38.0 feet.

The swept width for case 1 was under estimated by 3.2 feet. When the hitch point was moved ahead as in case 2 the swept width was overestimated by 4.3 feet. Other vehicle configurations tested showed the same general trend. It is clear that when the hitch point is located off of the centerline of the rear axle group for the previous vehicle unit, the path of the hitch point did not rotate about the same center as the rear axle group.

3.3.2 Hinge Point Off Centerline of Rear Axle

When the hitch point is moved off of the centerline of the previous vehicle units rear axle group, the path of the hitch point did not rotate about the same center as the rear axle group of the vehicle unit. The instantaneous radius can be found directly from the coordinate locations of the hitch point. The current and the last two coordinate locations of the hitch point can be used to approximate a circular segment. For uniform movements of the hitch point the geometry will approach that of Figure 3-5.

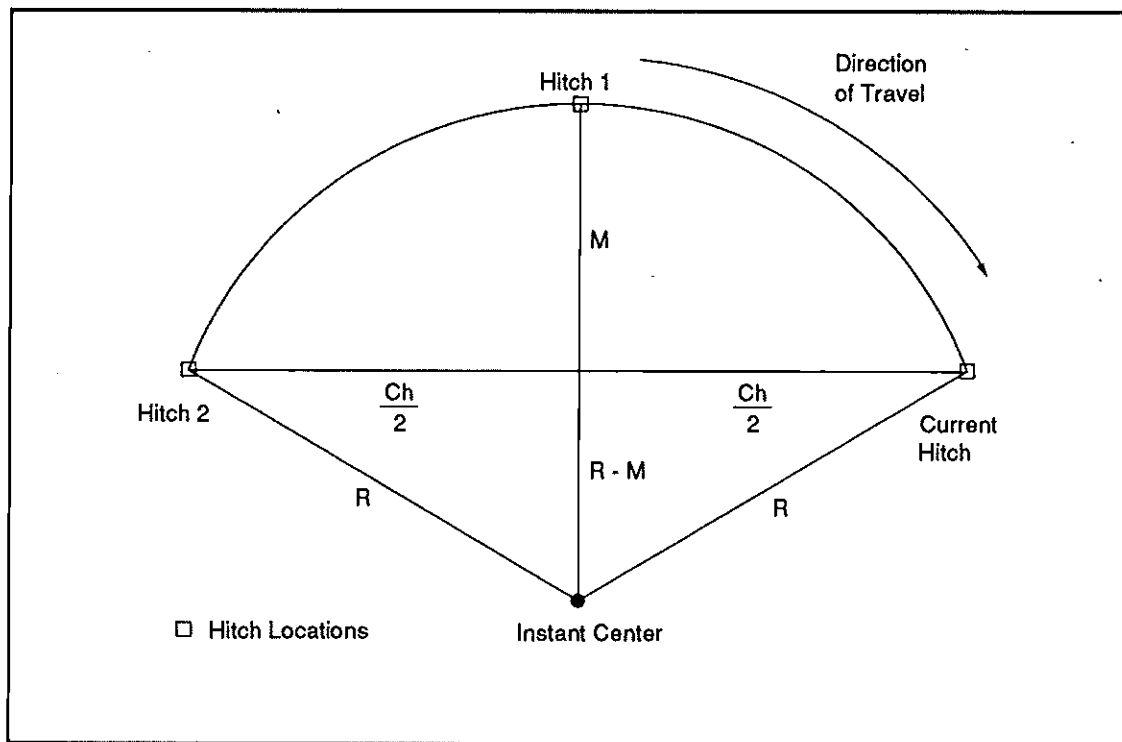


Figure 3-5. Instantaneous Radius by Three Points on Curve

The instantaneous radius is found as:

$$R^2 = \left(\frac{Ch}{2}\right)^2 + (R - M)^2$$

$$R^2 = R^2 - 2RM + M^2 + \frac{Ch^2}{4}$$

$$2RM = M^2 + \frac{Ch^2}{4}$$

$$R = \frac{M}{2} + \frac{Ch^2}{8M} \quad (11)$$

The middle ordinate M is found by finding the shortest distance from the previous hitch location (Hitch 1) to the chord by using standard analytical geometry. The chord distance Ch is found by using the coordinates of the current and second previous (Hitch 2) hitch locations.

The approach using equation (11) was much more accurate than the approach using equation (10). For comparison, the swept widths were 34.0 feet for case 1 (35.8 feet Sum of Squares - a underestimate of 1.8 feet) and 32.9 feet for case 2 (33.7 feet Sum of Squares - a underestimate of 0.8 feet).

The sign of the instantaneous radius is also needed for equation (9). The sign can be determined by noting which side of the chord the previous hitch point (Hitch 1) is located on. The approach using equation (11) has been found to have adequate accuracy for many vehicle combinations and curve radii.

3.4 Variable Length Vehicle Units

Pole type trailers that are commonly used for log trucks do not have a constant length when the vehicle combination negotiates a curve. Friction, developed between the bunks and the logs, bind the trailer to the tractor. The geometry of a variable length log truck trailer is shown in Figure 3-6. The bunk to bunk distance L_3 remains constant while the trailer length increases by means of a sliding compensator. The instantaneous length of the trailer L_4 can be determined by using the headings of the tractor β_1 and trailer β_2 at a given point in time.

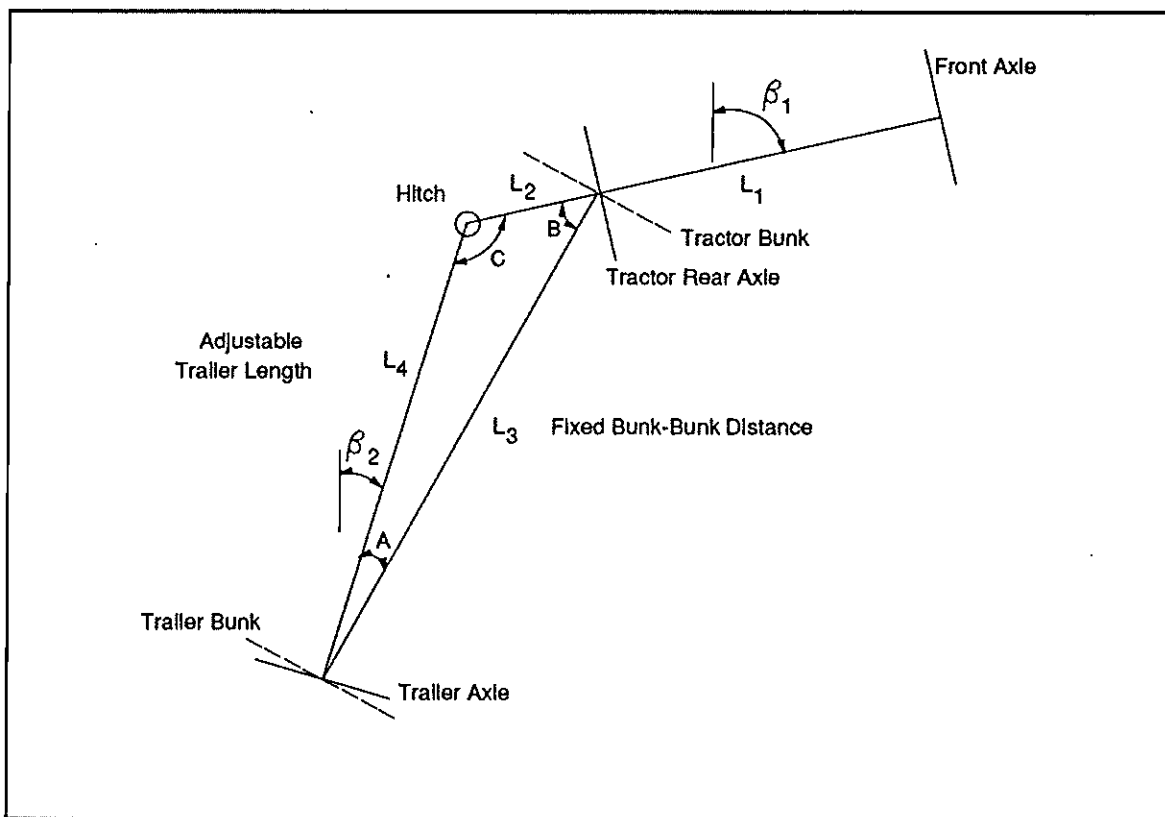


Figure 3-6. Variable Length Trailer

The adjustable trailer length L_4 is found by:

$$C = \pi - (\beta_1 - \beta_2)$$

$$A = \sin^{-1} \left[\frac{L_2 \sin C}{L_3} \right]$$

$$B = \pi - (A + C)$$

$$L_4 = \sqrt{L_2^2 + L_3^2 - 2L_2L_3 \cos B} \quad (12)$$

3.5 Front and Rear Load Overhangs

Front and rear overhang can limit vehicle passage where cut slopes or other obstacles exist at the edge of the roadway. Large front overhang such as a yarder tower on a rubber tired undercarriage is shown in Figure 3-7.

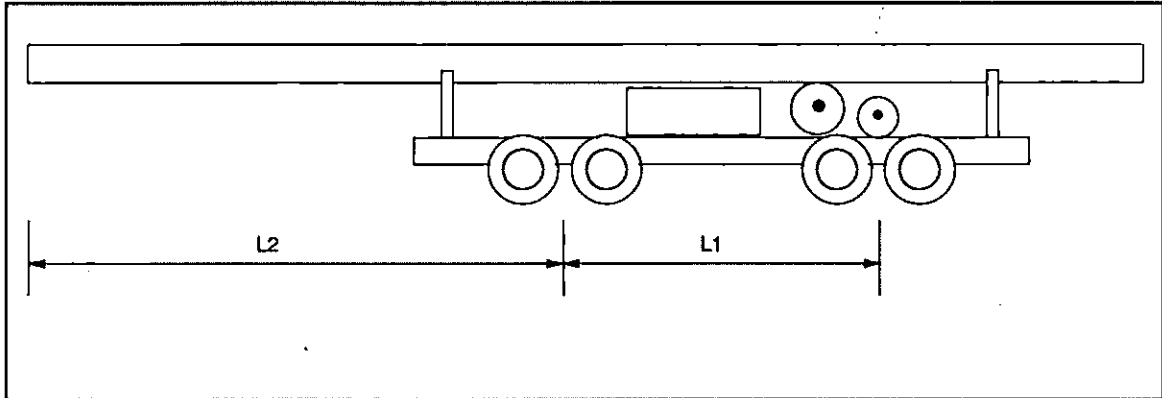


Figure 3-7. Typical Load Overhang

The coordinates of a load overhang path can be determined at any point from the heading β of the vehicle unit that it is carried on and the distance that it extends from the front or rear of the vehicle unit.

3.6 Comparison with Experimental Data

The predictions from the OFFTRACK program were compared to the experimental measurements from several field studies and scale model simulators. A description of each test situation and the results follow.

1) Log Truck Field Test

A test was conducted with a loaded log truck by the USDA Forest Service in cooperation with Champion International at an unpaved log yard in Bonner, Montana [1]. The test was conducted by marking curves with radii of 50 to 125 feet at centerline and central angles of 19 to 180 degrees. The truck was driven around each curve with the driver attempting to keep his left front tire on the mark. The path of the rear most trailer tire was noted as the wheel track left in the dirt yard and the offtracking recorded. The authors noted that measurements could be in error by as much as 0.5 feet. The truck used in the test had the dimensions as shown in Figure 3-8. A comparison of the offtracking from the OFFTRACK program and the field study for a 50 foot radius curve at centerline is given in Table 3-1.

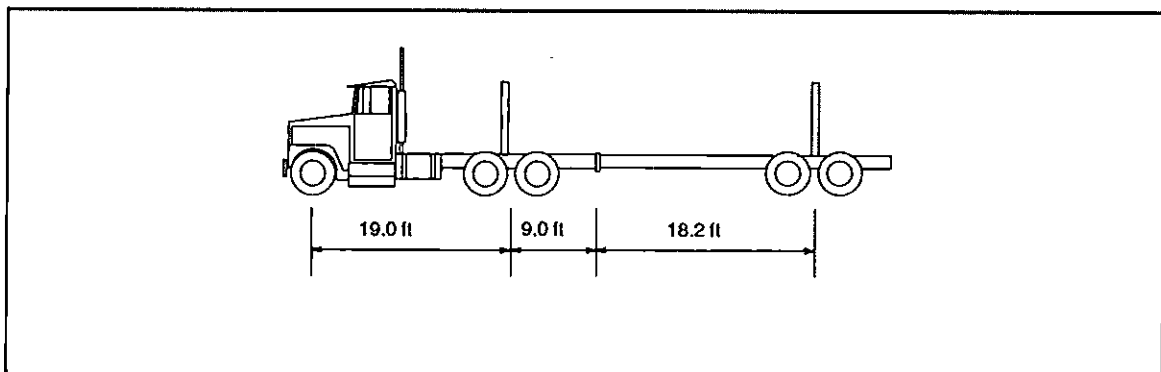


Figure 3-8. Test Log Truck Dimensions

Table 3-1. Log Truck Field Test vs. OFFTRACK 50 Foot Radius

		Offtracking (Ft.)							
		Central Angle Ahead of P.C. (deg)							
Source	Curve Central Angle	0	19	30	46	64	90	116	180
Field Test	19	1.8	2.1						
OFFTRACK		2.1	2.3						
Field Test	30	2.2	3.7	3.5					
OFFTRACK		2.3	3.6	3.2					
Field Test	46	2.2	4.6	5.0	3.7				
OFFTRACK		2.4	4.6	4.8	3.9				
Field Test	64	2.2	4.6	5.7	5.7	4.4			
OFFTRACK		2.2	4.5	5.2	5.5	4.4			
Field Test	90	2.2	4.6	5.7	5.8	6.3	4.8		
OFFTRACK		2.2	4.5	5.3	6.0	6.3	4.8		
Field Test	116	2.2	4.6	5.7	5.8	6.4	6.0	4.8	
OFFTRACK		2.2	4.5	5.3	5.9	6.4	6.7	5.0	
Field Test	180	2.2	4.6	5.7	5.8	6.4	6.0	6.8	4.7
OFFTRACK		2.2	4.5	5.3	5.9	6.4	6.7	6.8	5.1

The data in Table 3-1 indicate a good correlation between the field test and the OFFTRACK program. Most of the measurements agree within several tenths of a foot and the maximum deviation is 0.7 feet.

2) CalTrans Operational Test

The California Department of Transportation (CalTrans) conducted offtracking tests of a semi tractor trailer, Rocky Mountain Double, Turnpike Double, and Triple combinations in 1984 [5]. The tests were conducted with Viking Freight Systems, Inc. of Santa Clara, California in a paved yard. A line was painted for 60, 80, and 100 foot radius curves over a 180 degree central angle. The driver entered the curve on a tangent, placed the left front tire on the painted line through the curve and then left on a tangent.

The path of the right rear most tire was continually marked on the pavement. The OFFTRACK program was used to simulate the semi tractor trailer, Rocky Mountain Double, and the Turnpike Double. There is currently no option available in the OFFTRACK program that will analyze the offtracking of the Triple combination. The dimensions of the vehicles tested are shown in Figures 3-9 through 3-11. KP in the figures is the kingpin of a trailer. All vehicles had a trailer width of 102 inches and a tractor front axle tread width of 81 inches.

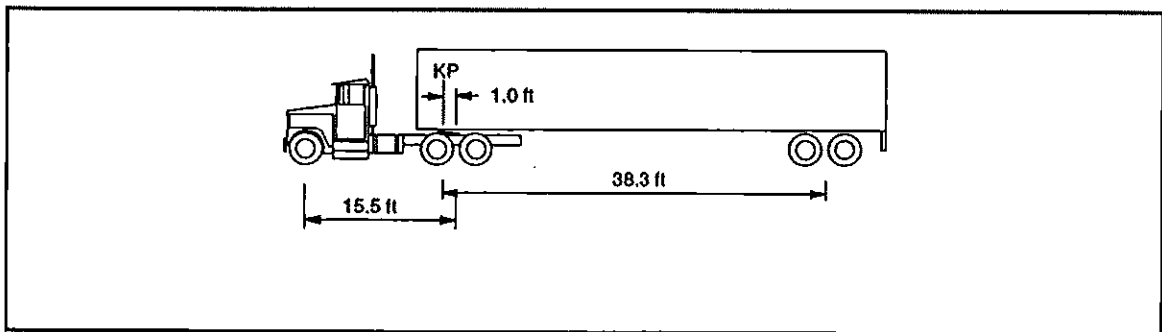


Figure 3-9. Caltrans Semi Tractor Trailer

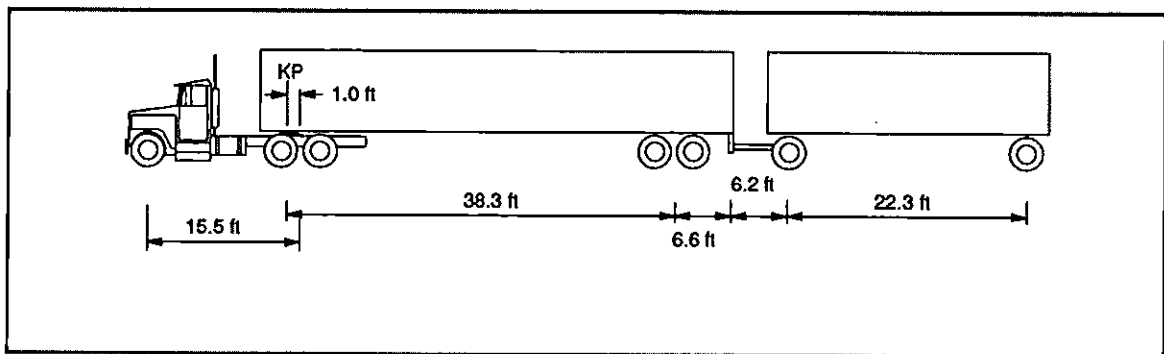


Figure 3-10. CalTrans Rocky Mountain Double

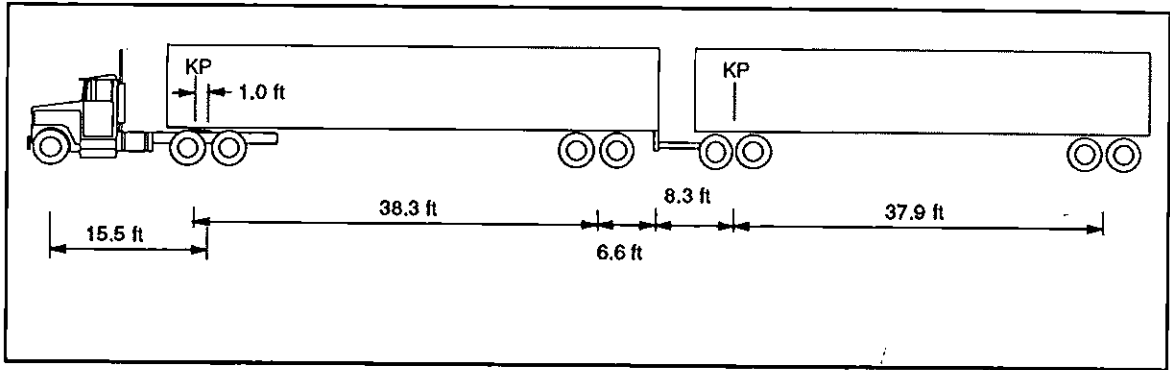


Figure 3-11. CalTrans Turnpike Double

The field test recorded swept width as the difference in radii measured to the left front wheel and the right rear wheel from the center of the curve. Measurements were recorded in 30 degree increments of central angle through the curve. A comparison of the OFFTRACK program results with the CalTrans study for singles and doubles combinations is given in Tables 3-2 through 3-4.

Table 3-2. CalTrans Field Test vs. OFFTRACK 60 Foot Radius

		Swept Width (Ft.)						
		Central Angle Ahead of P.C. (deg)						
Source	Vehicle	0	30	60	90	120	150	180
Field Test OFFTRACK	48' Semi	12.1	18.4	21.5	23.5	24.4	25.0	21.3
		13.0	18.9	21.9	23.5	24.5	24.3	20.2
Field Test OFFTRACK	Rocky Mtn. Double	15.1	22.5	26.5	28.7	30.1	28.9	23.6
		16.0	23.0	27.1	29.4	30.6	29.4	24.1
Field Test OFFTRACK	TurnPike Double	17.9	28.2	34.8	38.8	40.3	38.4	31.4
		20.7	30.3	36.6	40.5	41.8	40.1	33.6

Table 3-3. CalTrans Field Test vs. OFFTRACK 80 Foot Radius

		Swept Width (Ft.)						
		Central Angle Ahead of P.C. (deg)						
Source	Vehicle	0	30	60	90	120	150	180
Field Test OFFTRACK	48' Semi	10.7	16.1	18.2	19.1	19.6	19.8	16.2
		11.5	16.6	18.6	19.4	19.7	19.7	16.3
Field Test OFFTRACK	Rocky Mtn. Double	12.5	19.1	21.4	22.7	23.4	22.7	16.8
		13.4	19.6	22.2	23.3	23.8	23.4	18.6
Field Test OFFTRACK	TurnPike Double	14.4	23.0	27.1	29.1	30.4	29.4	20.4
		16.1	24.4	28.5	30.6	31.1	30.7	24.2

Table 3-4. CalTrans Field Test vs. OFFTRACK 100 Foot Radius

		Swept Width (Ft.)						
		Central Angle Ahead of P.C. (deg)						
Source	Vehicle	0	30	60	90	120	150	180
Field Test OFFTRACK	48' Semi	11.6	15.6	16.6	17.0	16.9	17.3	13.7
		10.7	15.2	16.5	16.9	17.0	17.0	14.3
Field Test OFFTRACK	Rocky Mtn. Double	12.2	18.0	19.3	19.5	19.7	19.8	16.4
		12.1	17.7	19.4	19.9	20.1	20.0	16.0
Field Test OFFTRACK	TurnPike Double	14.4	21.5	24.0	24.9	25.4	25.3	19.1
		14.1	21.5	24.4	25.5	26.0	25.6	19.9

Most of the swept width measurements in Tables 3-2 through 3-4 agree within one foot and the maximum deviation is 2.2 feet. In most cases the OFFTRACK program over predicts the swept width.

3) Scale Model Tests

Scale models were also used to test the methodology presented here. The predicted swept widths from the OFFTRACK program were compared to the Drafting Model Simulator (DVS) [4] and the Tractrix Integrator (TI) [8]. The test consisted of a 48 foot

radius curve on the outside front wheel through a 180 degree central angle with tangents on both ends. The vehicle used was a tractor trailer with a 18 foot tractor wheelbase, 36 foot trailer length and 0 foot fifth wheel offset from the centerline of the tractor tandems. The test results shown in Figure 3-12 show a good correlation with the maximum deviation being 0.7 feet. A comparison of swept widths using the DVS for multiple road segments of compound and reverse curves were within 0.5 feet.

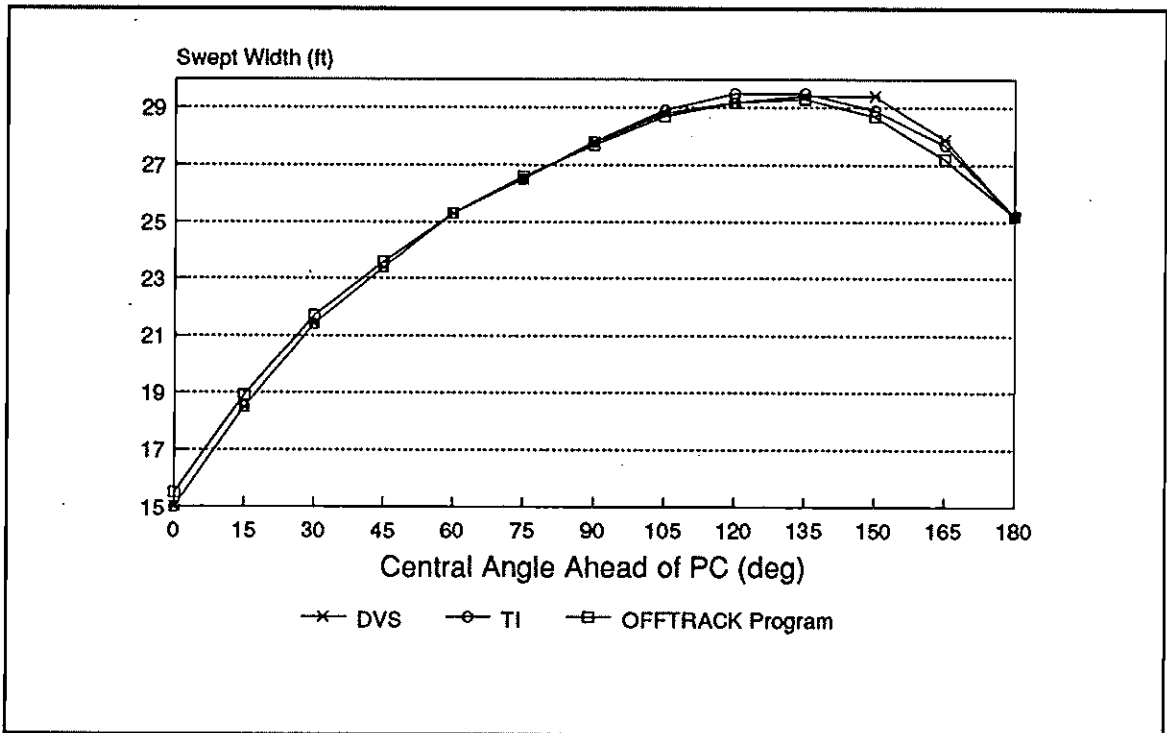


Figure 3-12. Comparison of Scale Models to OFFTRACK

3.7 Discussion and Conclusions

The OFFTRACK program simulations provided results within 0.5 feet of both scale model simulators and the log truck field measurements. There is however an over estimation of swept widths for the doubles combinations in the CalTrans test. Program results compare favorably except for the shorter radius curves with the turnpike doubles combination where overestimates of swept width of up to 2.2 feet are made. This disparity is probably due to slip and scrubbing of the tires or possibly misalignment of axles [6]. The predicted offtracking for the field test on the log truck in a unpaved yard was very close to measured. This appears to support the influence of tire mechanics because of the lower coefficient of friction associated with the unpaved yard versus the paved yard. The over prediction of swept width in the CalTrans test data is also consistent with that reported by several other authors when comparing scale model simulators to actual equipment tests [1, 8]. In general, the methodology presented appears to predict offtracking with reasonable accuracy for use as a road design tool.

3.8 Notation and Definitions

Unless otherwise noted all angles are in radians and distances are in feet.

α	Tractrix angle. Angle between the vehicle unit heading and a tangent to the steering curve.
$d\alpha$	Change in the tractrix angle after movement.
α_1, α_2	Tractrix angle before and after movement respectively.
θ	Path heading, measured as an azimuth.
$d\theta$	Change in the path heading after movement.
θ_1, θ_2	Path heading before and after movement respectively measured as azimuth's..
β	Vehicle unit heading, measured as an azimuth.
$d\beta$	Change in the vehicle unit heading after movement.
β_1, β_2	Vehicle unit heading before and after movement respectively measured as azimuth's.
R	Instantaneous radius of a steering curve.
ds	Incremental distance that the front of a vehicle unit is moved along it's steering curve.
L	Length of a vehicle unit.
M	Middle ordinate of the curve defined by three consecutive locations of a hitch point.
Ch	Chord distance between the first coordinate pair and third coordinate pair of a hitch point.
A, B, C	Interior angles formed by the centerline of the logs, trailer, and truck frame from the tractor rear axle to the hitch point for a variable length trailer.
π	3.14159
Stinger	The distance of the frame extension on a log truck from the centerline of the rear axle group to the hitch point of the trailer.

Swept Width The difference in radii measured to the outside front wheel and the inside rear wheel from the center of a curve.

3.9 References

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4 OFFTRACK - A Computer Simulation of Truck and Trailer Offtracking

By⁴Thomas W. Erkert, John Sessions, and Chris A. Bell

Abstract

Simulation programs have become an effective approach in analyzing complex systems that have been previously difficult or impossible to describe by hand computations. Offtracking of truck and trailer combinations is one such system that has been only approximated with hand computations. A microcomputer simulation program, OFFTRACK, was developed to predict the offtracking of six different vehicle combinations negotiating forest roads. Although the program was developed for application to forest roads, it may be used for other low volume roads and could be developed for high volume road applications. The program is capable of analyzing offtracking through a series of curves such as compound or reverse curves. It also predicts the path of a load overhang on a vehicle combination such as an equipment boom on a lowboy type trailer. To promote further understanding of the numeric results, plots of the actual wheel and load paths can be viewed on the screen or sent to a drum plotter. The program has been tested with several field studies and a scale model simulator, and compares favorably. This paper describes the approach taken in the development of the simulation program.

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4.1 Introduction

Offtracking of truck and trailer combinations is a very important consideration in low volume road design as it largely determines required travelway widths. In the western United States where forest roads are constructed in steep terrain, travelway widths have a direct impact on the construction cost of the road. Many forest roads are single lane terminal facilities with complex geometrics consisting of short radius curves connected by short segments of tangent or are in series such as compound and reverse curves. Many of these forest roads must also accommodate large vehicle combinations transporting logging equipment such as that shown in Figure 4-1.

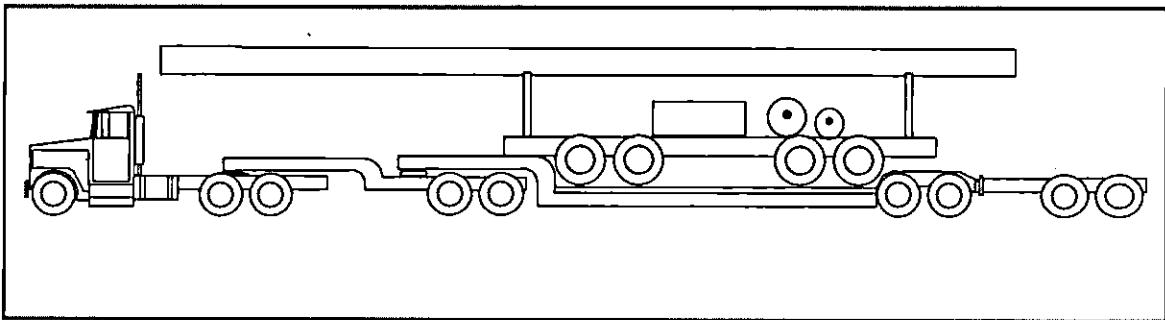


Figure 4-1. Lowboy Transporting Log Yarder

Because many forest roads are terminal facilities, loads that are brought in are off-loaded at a point where the vehicle combination can be turned around. The vehicle combination must then return over the same section of road that it came in on. Another complication in the design of forest roads is the need for adequate clearance for an equipment boom as it swings around a curve. This can be a critical design feature when a vehicle combination carrying a load such as shown in Figure 4-1 must be accommodated.

Several closed form solution techniques have been used by urban and forest road designers [1, 7, 10]. The primary limitation of these techniques is that only maximum offtracking can be solved for. Another technique that has been used is a scale model simulator [4, 8]. The scale model simulators are of several forms but are basically a scale physical model of a vehicle combination. The scale models plot the transient wheel paths as a vehicle combination negotiates a given curve or set of curves. In practice, their major disadvantage is that the road centerline must be drawn at a large scale typically from 1 inch = 5 feet to 1 inch = 10 feet and the models carefully moved along the path of the centerline. With the advent of high speed computers, several simulation programs have been developed to analyze the offtracking of a vehicle combination [2, 6, 9]. The models developed to date either use a graphical algorithm or a set of differential equations to describe the transient wheel paths. Unfortunately, the programs developed to date were written for either main frame or microcomputers that are not readily available in the design community.

The OFFTRACK program is written for IBM or compatible microcomputers operating under DOS 2.0 or higher. The program was written in a compiled BASIC and consists of six modules for the menu system, data input, file handling, analysis, printing of results, and graphics. The user interface features a pull down menu system and full field editing for data input. The program uses a set of differential equations that are integrated numerically to describe the transient wheel and load overhang paths through a series of curves. The program was written specifically for the forest road situation. It is capable of analyzing the offtracking of six different vehicle combinations that are commonly found on forest roads. It can analyze the offtracking and required road widths for up to 100 road segments that can be described by circular curves and tangents. The

results are in the form of required road widths left and right of the centerline of the road. The wheel and load paths can be viewed on the screen or sent to a drum plotter where they can be drawn at any user specified scale.

The emphasis of this paper is to describe the methodology used to develop the program. The entire development of the theory and the testing of the program are presented in other papers [11, 12] and are not discussed here. A brief discussion of the modeling of the vehicle is presented for clarity.

4.2 Modeling the Vehicle Combination

The method presented here represents the vehicle combination in what is known as a bicycle wheel model [9]. Each axle group is represented by a single wheel at the centerline of the longitudinal axis of the vehicle unit. Tandem axles are represented by a single wheel at the centerline of the bogie. The bicycle wheel model does not account for the effects of tire mechanics which for most cases have only a slight effect on offtracking [9]. The bicycle wheel model may not be appropriate for vehicle units that have axles within a group that are spaced relatively far apart [6]. For tandem axles that are spaced on 4-5 foot centers, the bicycle wheel model has proven to be adequate.

The equations used to describe how a vehicle combination moves through a given curve are known as the differential tractrix equations [2, 3]. For small movements of a single unit vehicle combination, the geometry approaches that shown in Figure 4-2.

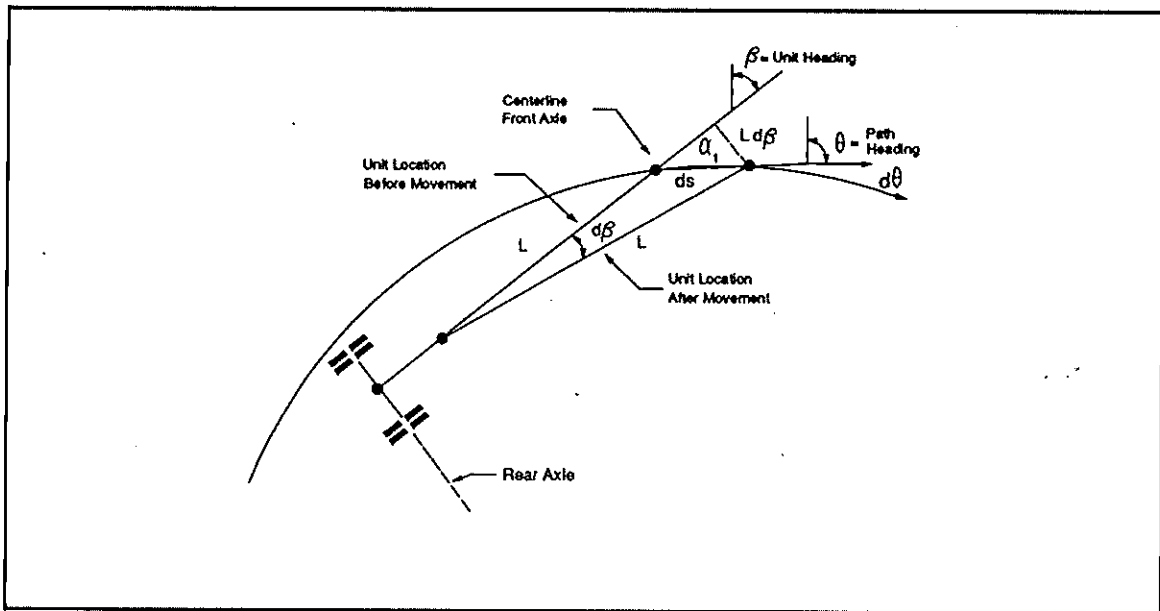


Figure 4-2. General Case Tractrix Development

The tractrix angle α is the angle between the vehicle unit heading β and the path heading θ . The path heading is the heading of a line that is tangent to the curve at a particular point. The two differential equations are as follows:

$$d\alpha = ds \left[\frac{\frac{L}{R} - \sin \alpha_1}{L} \right] \quad (1)$$

$$d\beta = d\theta - d\alpha \quad (2)$$

Equation (1) is the governing equation to obtain the next tractrix angle once the vehicle has moved up on the curve. Equation (2) is then used to update the vehicle unit heading. Adoption of a simple sign convention for the instantaneous radius allows equation (1) to be used for multiple road segments. An arbitrary sign convention of curves that are clockwise in the direction of travel are defined as positive while counter-clockwise curves are negative. For a vehicle combination, equations (1) and (2) are used for each unit in the combination in a sequential fashion. A standard latitude and departure coordinate system is used to record the wheel and load positions in arrays. For following vehicle units, the instantaneous radius R is determined from the past three coordinate positions of the hitch point on the previous vehicle unit as three points in space can define a circle. The incremental distance ds is determined from the past two coordinate hitch points as a straight line distance. It is essential to this technique that the distance ds be kept small. It has been found that a distance of one foot yields reasonably accurate results and is consistent with what has been reported by others [9].

4.3 Hardware Requirements

The program was written for IBM or compatible microcomputers operating under DOS 2.0 or higher. A graphics card is not required analysis but is needed for viewing the wheel and load paths. Graphics cards supported are:

1. Color Graphics Adapter (CGA)
2. Enhanced Graphics Adapter (EGA)
3. Video Graphics Adapter (VGA)
4. Hercules Graphics

It is not necessary but highly recommended that a hard disk and math coprocessor be installed. Because of the requirement in the theory for small distance movements, the program is numerically intensive. A math coprocessor is desirable for adequate performance of the program. For obtaining a hard copy of the numeric results, a line printer must be installed. To obtain a hard copy of the wheel and load paths, a Hewlett-Packard 7580B/7585B or compatible drum plotter must be installed.

4.4 Data Requirements

The data inputs for the program are relatively minor. The items that are needed are the definition of the road segments geometry, dimensions of the vehicle combination to be used, and initial coordinates and heading. The road segments are defined by the radius of the segment including the directional sign, and the length of the segment. The vehicle dimensions are the lengths of each unit in the combination and the location of the hitch point for a following vehicle unit if one is present. A load overhang can be specified by inputting the overhang of the load in front of the hitch point or behind the rear axle of the trailer it is being carried on. The vehicle dimensions that would be input for a standard lowboy truck and trailer combination are shown in Figure 4-3.

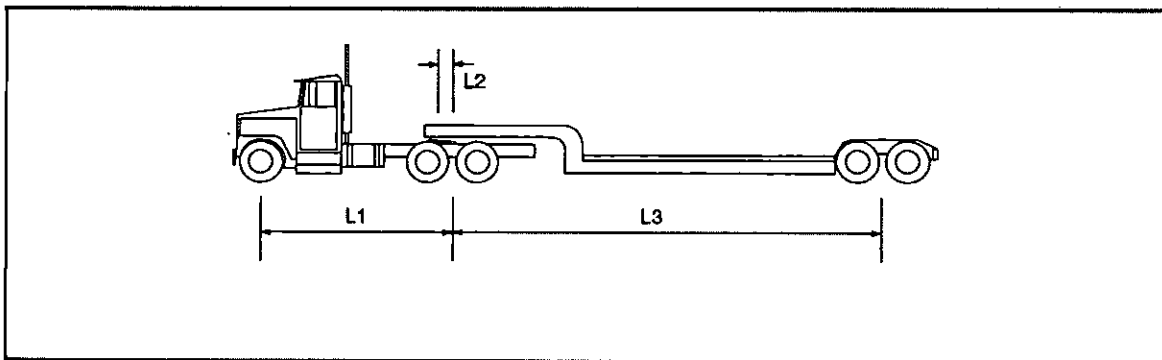


Figure 4-3. Lowboy Vehicle Dimensions

4.5 Program Procedure

At this point, it is appropriate to illustrate the main flow chart of the OFFTRACK program, Figure 4-4. The general procedure of the computational portion of the program can be summarized as follows:

1. Compute the coordinates of the front and rear most wheels and load overhang as the vehicle combination moves in the ahead direction.
2. Compute the required road widths in the ahead direction.
3. Turn the vehicle combination around and set the increments to -1 for the reverse direction.
4. Compute the coordinates of the wheel and load paths as the vehicle combination moves in the reverse direction (back).
5. Compute the required road widths in the reverse direction.
6. Compare the road widths for each distance to determine the maximum road width required for both left and right of the centerline of the road.

The innermost loop in Figure 4-4 is the mechanics of moving the vehicle combination up on the road. Equations (1) and (2) are computed and accumulated. Next, the instantaneous radius of the hitch R and the distance the hitch has moved ds are computed for the following vehicle unit if one is present. The loop is repeated using R and ds from the previous vehicle units hitch point and the current vehicle units length. When no units are left in the combination, the wheel and load overhang coordinates are updated for that iteration.

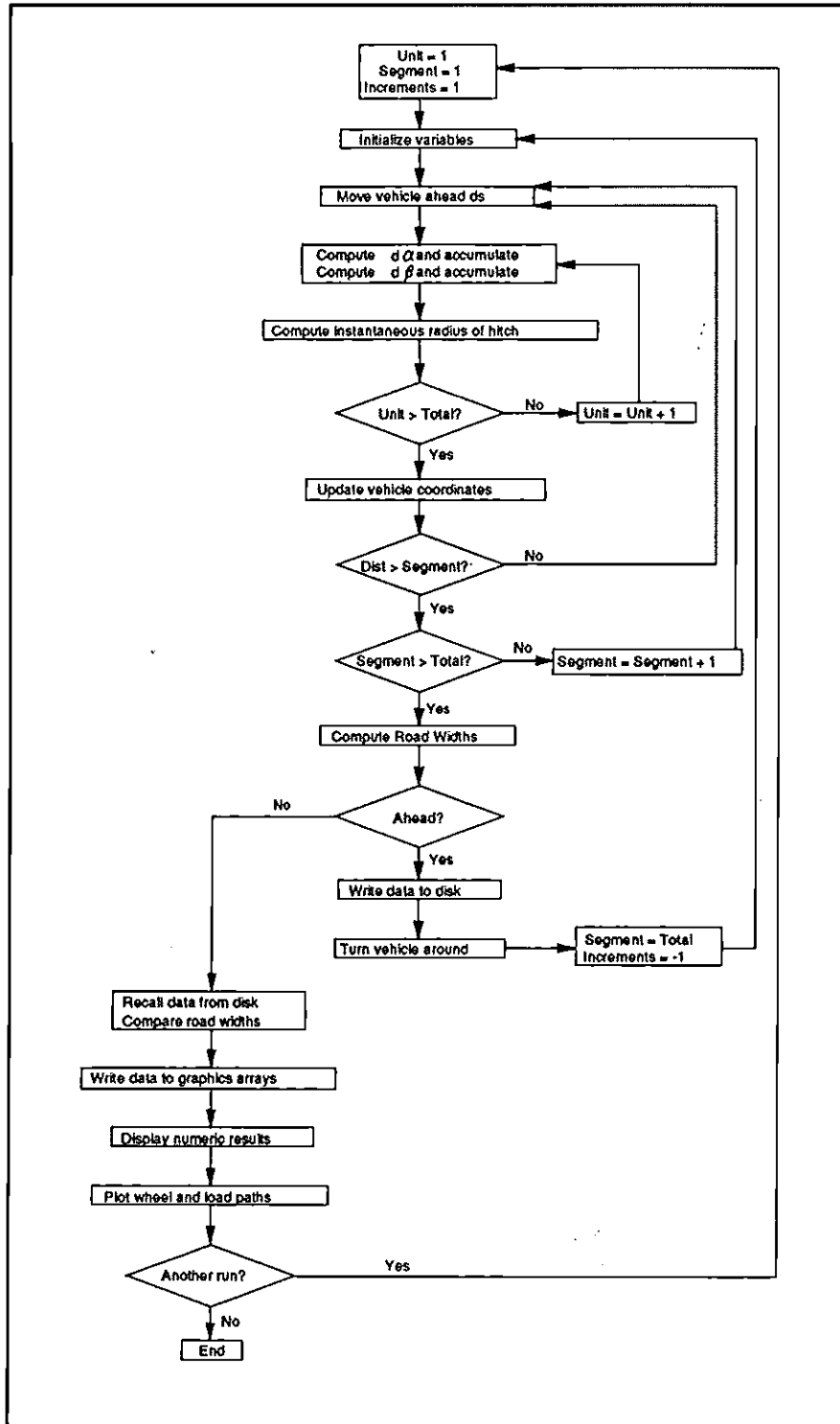


Figure 4-4. Main Program Flow Chart

With the many arrays used for storage of the wheel and load coordinate locations, it is necessary to dump the data from the simulation conducted in the ahead direction to a temporary disk file before the vehicle is turned around and the simulation is begun going backwards. With 640 Kb of memory, the program is capable of analyzing a road that is at least one mile in length. The simulation takes approximately 8-12 minutes per mile of road length on a microcomputer with a 10 Mhz 80286 processor with a 10 Mhz math coprocessor installed.

4.6 Wheel and Load Overhang Paths

The wheel and load overhang locations are updated each time the vehicle moves ahead. The four wheel locations that are tracked are the front and rear most set of wheels in the vehicle combination. The wheel locations are updated by using the vehicle units heading and centerline axle location for the appropriate wheel set and the units width. For example, the left rear wheel location at a particular point in the simulation would be computed as:

$$NorthLeftRear = NorthAxle + \frac{Width}{2} \cos\left(\beta + \frac{3\pi}{2}\right) \quad (3)$$

$$EastLeftRear = EastAxle + \frac{Width}{2} \sin\left(\beta + \frac{3\pi}{2}\right) \quad (4)$$

Load overhang locations are updated in a way similar to the wheel locations. The load overhang location is assumed to lie along the longitudinal centerline of the vehicle unit that it is being carried on. The load location can be found from the vehicle units heading, load overhang distance from the front or rear of the unit that it is being carried on, and the centerline rear axle position.

4.7 Road Widths

The computation of the required road widths is largely an exercise in analytical geometry, based on known front and rear wheel coordinates for each road station. A flow chart of the procedure used for computing road widths is shown in Figure 4-5.

From Figure 4-5 it can be seen that the approach for computation of road widths is different for road segments that are curves and tangents. Two pointers are used for indexing the wheel coordinate arrays to enable the rear wheel positions to be moved up or back relative to the front wheel positions.

The general procedure for the curve segments is to compute the central angle from the beginning of the curve to the station of interest at the centerline of the front axle. The appropriate rear wheel coordinate array is then indexed so that it has an approximately equal central angle as the centerline of the front axle. The road width is then computed as the difference in the radial distances from the center of the curve to the front and rear wheel locations. Several complications were encountered when implementing this algorithm. First, the anticipated front and rear wheel combination for the direction of the curve may not yield the maximum road width. For example, for a curve that is clockwise in the direction of travel it is anticipated that the left front and right rear wheel combination should give the maximum road width. This is because as a vehicle combination negotiates a curve, the rear wheels offtrack to the inside of the curve. In the case of reverse curves, the right front and left rear wheel combination may yield the maximum road width at the beginning or ending portion of the curve. This situation can be seen from a plot of the graphics screen generated by the program as shown in Figure 4-6. Note at the beginning of the third curve that the right front-left rear wheel combination has the maximum road width for the stations in that vicinity.

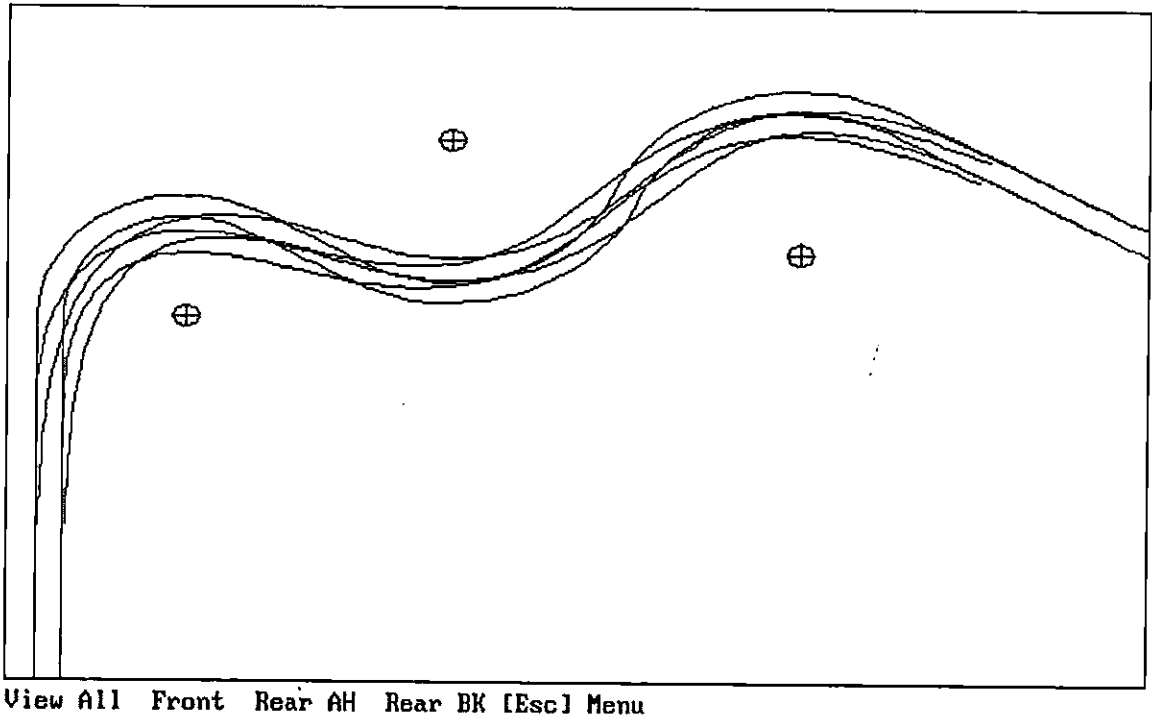


Figure 4-6. Wheel Paths for a Reverse Curve

This condition can also happen if two short radius curves are connected by a short tangent. It is also dependent on the particular vehicle configuration that is being used in the simulation. The solution to this problem is to simply check the road width from the front-rear wheel combination that is opposite of anticipated and if larger, index the rear wheel ahead to obtain the maximum road width. The road width is then recorded in the appropriate road width array by noting if the anticipated or opposite wheel combination was used to obtain the maximum road width. There is a problem with this technique if the rear wheels have tracked inside the radius point of the curve. The problem arises because the central angle computed to the rear wheels is no longer correct. This can happen when a long vehicle combination is used with a curve radius of less than 50'. A possible solution to this problem is to use a modification of the algorithm used for computation of tangent road widths which is discussed next.

Tangents are a separate case from the curve segments as there is no central angle to compare front and rear wheel locations. The solution to this problem was to use matrix algebra and redefine the coordinate locations of the front and rear wheels relative to the centerline of the front axle. The sign of the determinate of a 2x2 matrix can then be used to determine when the rear wheel is perpendicular to the road heading at the station of interest. When the sign of the determinate changes, the rear wheel is close to a perpendicular condition. The 2x2 matrix A is of the form:

$$A = \begin{pmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{pmatrix} \quad (5)$$

The determinate of the 2x2 matrix $|A|$ is given as:

$$|A| = X_1Y_2 - X_2Y_1 \quad (6)$$

Where all coordinates are redefined relative to the centerline of the front axle:

X_1 = East front wheel coordinate

Y_1 = North front wheel coordinate

X_2 = East rear wheel coordinate

Y_2 = North rear wheel coordinate

A graphical representation of the procedure is shown in Figure 4-7.

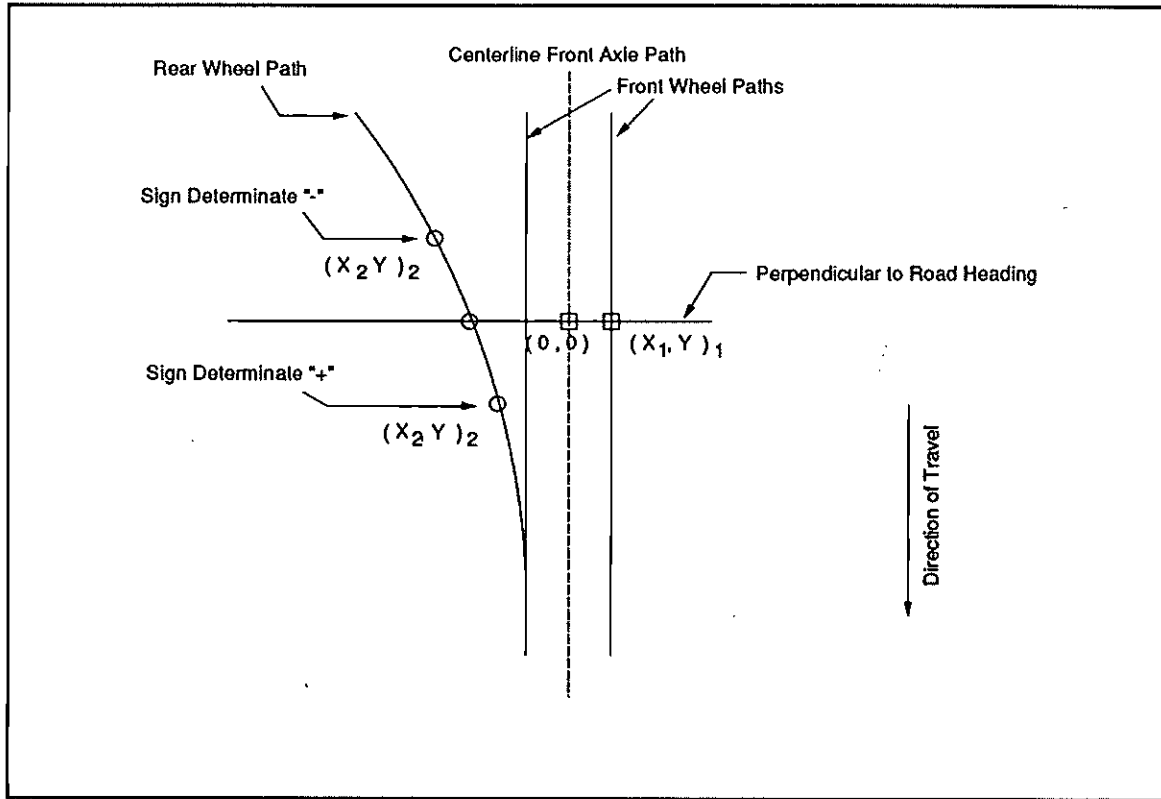


Figure 4-7. Tangent Road Width Computation

Again a problem arises in which front-rear wheel combination should be used to give the maximum road width. Because there is no way of anticipating which front-rear wheel combination yields the maximum road width, both combinations must be checked. This is done by determining the maximum width for the left front-right rear wheel combination and then setting the rear wheel index back a small amount and checking the right front-left rear wheel combination. The two road widths are compared and the maximum recorded in the appropriate road width array. It is also necessary to determine which side of the road the maximum road width is on. This was done by a translation of the coordinate axes from the absolute coordinates of the wheel locations to that relative to the road heading.

It should be noted here that the solution techniques used for the computation of road widths are not particularly elegant as indexes are incremented sequentially until a solution is found. The inclusion of a binary search routine could speed up road width computations. Future versions of the program will include a streamlined algorithm for the computation of road widths.

4.8 Conclusions and Recommendations

The program results have been tested with the results of several field equipment tests [1, 5] and a scale model simulator [4]. In general, the program is within 1 foot of the swept widths reported from the field equipment tests and within 0.5 feet of the scale model simulator. A full discussion of the testing of the program is included in a previous article by the authors [12].

The program has several limitations that are planned to be eliminated by future improvements to the program. The areas for further development are:

1. The ability to analyze any vehicle configuration instead of the six fixed vehicle configurations that are currently included.
2. The development of a faster algorithm for computation of road widths. Also, develop the tangent road width algorithm for use on curve segments to allow for the analysis of very short radius curves.
3. The development of a "Driving Algorithm". This would allow the program to simulate how a driver actually drives a road. The program currently guides the lead vehicle unit by a fixed set of geometrics that define the centerline of the road. However, in practice drivers set themselves up for each curve based on the knowledge of the road segments ahead of them. This results in an input path that probably approximates a spiral instead of circular curves and tangents.
4. Correct the theory to account for slip and scrubbing of tires.
5. Incorporate this technique into a computerized road design system.

4.9 Notation

Unless otherwise noted all angles are in radians and distances are in feet.

α	Tractrix angle
$d\alpha$	Change in the tractrix angle after movement
α_1	Tractrix angle before movement
β	Vehicle unit heading
$d\beta$	Change in the vehicle unit heading after movement
θ	Path heading
$d\theta$	Change in path heading
R	Instantaneous radius of steering curve
ds	Incremental distance that the front of a vehicle unit is moved along it's steering curve
L	Length of a vehicle unit

4.10 References

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5 Conclusions and Recommendations

5.1 Conclusions

The primary objective of this project was to design and implement a production tool that will enable road designers to predict quickly the required travelway widths on forest roads for common design vehicle combinations. The microcomputer program developed, OFFTRACK, fulfills this primary objective. The working objectives of the microcomputer program and the implemented actions are as follows:

1. Easy to use with a simple user interface. The program uses a pull down menu structure that is very user friendly. It requires minimal data input and is fully error trapped.
2. Capable of analyzing offtracking through a complex set of geometrics. The program is capable of analyzing any set of road geometrics as long as it can be described by a set of circular curves and tangents. It is capable of analyzing up to 100 road segments at once up to one mile of total road length. The program predicts the transient wheel and load overhang paths that analytical methods that are currently used are not capable of analyzing.
3. Capable of analyzing offtracking both ahead and back automatically. The simulation technique implemented accomplishes this without user intervention. It is necessary because the offtracking of a vehicle combination may not be symmetric around a given curve.
4. Capable of analyzing load overhang paths. The program developed is capable of analyzing load overhang paths for all of the six vehicle combination types

included in the program. Although no numeric results are output for load overhang paths, a graphical representation of a load overhang path can be viewed on the screen or sent to a drum plotter for further analysis.

5. Capable of determining required road widths left and right of centerline directly. The required road widths for vehicle passage are output directly. The widths reported are the total width and the widths left and right of the centerline of the road. The vehicle combination is guided by placing the centerline of the front axle on the centerline of the road. Therefore, the left and right width reported are relative to the centerline of the front axle.
6. Capable of analyzing most of the vehicle combinations found on forest roads. The program is capable of analyzing the following six vehicle types which covers most of the range of vehicle types found on forest roads:
 - Log Truck
 - Lowboy
 - Lowboy with a Jeep
 - Lowboy with a Jeep and a Pup
 - Short Logger with a Trailer
 - Non-hinged
7. Accurate with rapid execution. The program has been tested with the results of several field studies and a scale model simulator. The tests with the scale model simulator generally indicate that the road widths predicted by the program are

within 0.5 feet of the scale model simulator for one simple curve and multiple curve road segments. The programs predicted road widths are also within 0.5 feet of the log truck field tests that were conducted by the USDA Forest Service. The predicted road widths of the program also compare favorably with the field tests of a 48 foot semi, Rocky Mountain doubles, and Turnpike doubles that were conducted by CalTrans. There is an indication that as the number of units within the combination and the overall length increases, the potential error in the programs predicted road widths increases to a maximum of 2 feet. In all cases, the error is conservative in that the program over predicts the required road width. The over prediction of road width when compared to the CalTrans operational test is probably due to the effects of tire mechanics which have not been included in the modeling technique. The 2 foot maximum error reported for the Turnpike doubles in a 60 foot radius curve is within 5 percent of the field test at the midpoint of the curve.

The performance of the program is dependent on the computer that it is being run on. For a computer with a 10 Mhz 80286 processor and a 10 Mhz math coprocessor, it takes from 8 to 12 minutes per mile of road length to complete the simulation. The variation in the time required is dependent on the complexity of the vehicle combination being analyzed and the complexity of the geometrics of the road.

8. Provide the user with a graphical display of the wheel and load paths to promote understanding of the numeric results. The wheel and load paths can be viewed on the screen or sent to a Hewlett-Packard 7580B/7585B or compatible drum plotter. The plot on the drum plotter can be drawn at any user specified scale. This allows a road designer to plot the offtracking of the design vehicle at the same

scale as the road traverse. This enables the designer to check clearance of a load overhang on the vehicle combination with cut slopes or other obstacles restricting clearance.

5.2 Recommendations

The program that was developed has several limitations and opportunities for further development. The areas recommended for further development and possible solutions are as follows:

1. The vehicle combination is currently guided by placing the centerline of the front axle on the centerline of the road which is assumed to be a fixed set of geometrics of circular curves and tangents. Currently, many forest roads are not geometrically designed. That is, the alignment is fit to the topographic conditions in the field which approximate spiral curves instead of circular curves. This could be corrected by approximating the actual alignment by short circular curves defined by three stations on the road. A series of three simultaneous equations could then be solved using Gauss-Jordan elimination to define the radius of the curve in that portion of the curve. Another problem with the current assumption is that there typically is more room on the outside of the curve than half of the vehicle width. For example, if an 8 foot wide vehicle combination is guided by the centerline of the front axle with a 2 foot steering correction, the outside front wheel is approximately 5 feet from the centerline of the road. Many forest roads are designed with a 14 foot minimum travelway which means that the front wheel should be placed 7 feet from the centerline of the road. A solution to this would be to guide the vehicle combination by placing the front wheel on the edge of the road and develop a transition algorithm for guiding the vehicle between road segments.
2. A driving algorithm should be developed to enable the road designer to analyze if there is adequate clearance available on existing roads. This is necessary because

drivers do not typically drive a fixed set of geometrics on existing roads where clearance is in question. This would be a difficult algorithm to develop and would require the input of detailed road information on a section by section basis. Assuming that the survey information and the revision of the input of that information can be accomplished, a possible solution would be to derive a heuristic on when a driver thinks the rear of the vehicle combination has cleared the critical point in a curve. At that point the lead vehicle could be guided within the road template based on what the geometrics of the road are in the next section. The algorithm would need to be derived out of a full understanding of how operators currently drive forest roads. An approach to this would be to observe and possibly shoot video film of drivers making maneuvers with different vehicle combinations.

3. The program developed currently has six types of vehicle configurations included. It would be a fairly simple task to revise the program to allow for analysis of any vehicle combination. This can be accomplished by defining for each unit in the combination the length and hitch point offset for the following unit.
4. Many vehicle combinations have different widths for the front axle of the lead vehicle unit and the following axles of trailer units. Typically, the front axle of a tractor has a tread width of approximately 81 inches while trailer axles range from 96 to 102 inches. This could be accomplished in conjunction with the generic vehicle configuration described in item 3 by specifying a separate width for the first and last vehicle unit in the combination.

5. Improve the algorithm developed for the computation of road widths. Two primary improvements are needed. First, the method for computation of road widths on curves should be revised so that road widths can be computed when the rear of a vehicle combination tracks to the inside of the radius point of the curve. This could be accomplished by modifying the algorithm that was developed for tangents and using a binary search routine. Second, the algorithm for tangents should be revised to speed up computational time. Currently, the road width is computed for one wheel set combination and then the index is set back and the road width computed for the opposite wheel set combination. A solution to speed up execution could be to compute all of the road widths for each wheel set combination at one time and then compare the road widths all at one time.
6. The effects of tire mechanics could be included in the model to correct the over prediction of road width on high friction surfaces. For the forest road situation where most roads have aggregate surfaces or no surfacing at all, the inclusion of tire mechanics may not be justified. If the program were to be rewritten for use in the urban environment, tire mechanics could be an important factor. Pavements with high coefficients of friction and the presence of vehicles with axles within a axle group that are spaced farther apart than 4-5 feet would benefit from treatment of tire mechanics. The inclusion of tire mechanics could be included by using a moment balance about the centerline of a rear axle group and adjusting the vehicle length to achieve moment equilibrium.
7. The modeling technique could be expanded to address the issue of gradeability of vehicle combinations in curves. One of the primary inputs for analyzing gradeability in curves, is the location of the driver and following wheel sets as the vehicle climbs through a curve on a steep grade. The methodology presented in

this paper is capable of predicting the location of any wheel set at any given location. The additional inputs that would be required to implement this analysis would be:

- Performance characteristics of the vehicle
 - Coefficient of friction of the road surface
 - Centerline road grades for each road segment
 - Road cross slopes for each road segment
8. The inclusion of the analysis technique developed in this project in a computerized road design package.

